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THE GROWTH OF HOT REDUCED DENSITY CHANNELS IN GASES DUE TO TURB--ETC(U)

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# THE GROWTH OF HOT REDUCED DENSITY CHANNELS IN GASES DUE TO TURBULENCE AND HEAT CONDUCTION

## I. INTRODUCTION

A hot channel, formed by either laser or ohmic heating of a gas,<sup>1,2</sup> expands initially due to overpressure. If spatial asymmetries exist in the initial heating, vorticity is created and degenerates into turbulence throughout the channel<sup>3</sup>. A second phase of growth then follows the attainment of pressure equilibrium as outside gas mixes into the hot channel. Viscous effects cause the turbulence to decay until heat conduction becomes significant. As the gas responds to this heat flow a final growth and filling-in of the channel occurs.

We predict the scaling of the channel radius with time during both later phases of growth. To do this we use the governing equations, the assumption of pseudo-self-similar expansion and in the first case a simple "mixing length" model of the turbulence.

## II. GROWTH DUE TO TURBULENCE

In turbulent flow a pattern of chaotic eddies with a spectrum of sizes is superimposed on the mean fluid motion. If the fluid velocity is represented as a mean value,  $\bar{v}_x$ , plus a relative fluctuation,  $v'_x$ , such that

$$v_x = \bar{v}_x + v'_x,$$

then the rms value of the fluctuation velocity is a measure of the intensity of the turbulence

$$\bar{v}_x = (\overline{v_x'^2})^{1/2}.$$

Turbulence embedded in a uniformly moving fluid will rapidly become isotropic

$$\bar{v}_x = \bar{v}_y = \bar{v}_z = \bar{v}.$$

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We shall model the channel as entirely filled with isotropic turbulence and undergoing a cylindrically symmetric average expansion.

Turbulence transports fluid properties by small scale convection. However, this effect may be modeled as a large scale version of transport at the molecular level and treated as an anomalous diffusion. We suppose small volumes of fluid move a distance  $l$  (Prandtl eddy length) before breaking up and losing their identities. The product of this "mean free path" and the relative velocity is known as the eddy diffusivity and becomes the effective mass diffusivity, kinematic viscosity, or thermal diffusivity

$$D = \nu = \alpha = l\bar{v}.$$

(Slightly different values of  $l$  actually apply to each situation.)

Experiments show isotropic turbulence has particularly simple properties.<sup>4</sup> If a uniform flow passes through a grid, the resulting turbulence becomes isotropic a short distance downstream from the grid. The intensity of the turbulence decays beyond this virtual origin according to

$$\bar{v} \sim \frac{1}{\sqrt{x}} \sim \frac{1}{\sqrt{t}}$$

where the second step follows from the constant mean velocity. The eddy length however grows

$$l \sim \sqrt{x} \sim \sqrt{t}.$$

We may therefore ascribe constant transport properties to the medium as long as the turbulence persists. The value of this constant depends however on the initial conditions under which the turbulence was created, in this case on the grid wire size.

We may take spatial averages of the variables

$$\bar{\rho} = \frac{1}{V} \int_V \rho dV \quad (1)$$

$$\bar{v} = \frac{1}{V} \int_V v dV \quad (2)$$

$$\bar{T} = \frac{1}{\bar{\rho}V} \int_V \rho T dV \quad (3)$$

where we select the averaging volume,  $V$ , as larger than the scale of the turbulence but smaller than the channel scale. The form of the governing equations,<sup>5</sup> when expressed in terms of the average properties, is found by application of Reynolds procedure<sup>6</sup>. For example we substitute in the continuity equation as follows

$$\frac{\partial}{\partial t}(\bar{\rho} + \rho') + \nabla \cdot [(\bar{\mathbf{v}} + \mathbf{v}')(\bar{\rho} + \rho')] = 0 \quad (4)$$

and then average the entire equation resulting in

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{\mathbf{v}}) + \nabla \cdot (\overline{\rho' \mathbf{v}'}) = 0. \quad (5)$$

In recognition of the transport effect of the turbulence (and following Boussinesq's momentum transport treatment<sup>7</sup>) we make the following replacement for the correlation term

$$\overline{\rho' \mathbf{v}'} = -\alpha \nabla \bar{\rho} \quad (6)$$

with the result that

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \bar{\rho} \bar{\mathbf{v}} - \alpha \nabla^2 \bar{\rho} = 0. \quad (7)$$

The conservation of energy equation may be treated similarly and the correlation terms replaced by a diffusive one. The governing equations thus become

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \bar{\rho} \bar{\mathbf{v}} - \alpha \nabla^2 \bar{\rho} = 0 \quad \text{conservation of mass} \quad (8)$$

$$\bar{\rho} c_v \left( \frac{\partial \bar{T}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \bar{T} \right) + \rho_o R T_o \nabla \cdot \bar{\mathbf{v}} - \nabla \cdot \left( \bar{\rho} c_v \alpha \nabla \bar{T} \right) = 0 \quad \text{conservation of energy} \quad (9)$$

$$\bar{\rho} \bar{T} = \rho_o T_o \quad \text{equation of state} \quad (10)$$

This set of equations incorporates a number of simplifications. The Navier-Stokes equation (conservation of momentum) has been eliminated by assuming constant pressure. A symmetric expansion creates no new turbulence and the energy contained in the existing turbulence is small. Therefore an additional equation to describe the time variation of  $\bar{\mathbf{v}}$  is not needed nor do we find a term in the energy equation to show the heat generated by the decay of turbulence. No terms proportional to the dynamic viscosity appear because the expansion is symmetric. The heat conduction,  $k$ , is neglected

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during the turbulent phase in comparison with the anomalous conduction,  $\bar{\rho}c_v\alpha$ . We may rewrite these equations in forms appropriate to a cylindrically symmetric expansion (hereafter dropping the averaging bars).

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r\rho v) - \alpha \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \rho}{\partial r} \right) = 0 \quad (11)$$

$$\rho c_v \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r} \right) + \rho_o R T_o \frac{1}{r} \frac{\partial}{\partial r} (rv) - \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho c_v \alpha \frac{\partial T}{\partial r} \right) = 0 \quad (12)$$

$$\rho T = \rho_o T_o. \quad (13)$$

The approximate scaling of the channel expansion with time may be inferred from these equations by assuming the radial profiles of  $T - T_o$ ,  $\rho - \rho_o$ , and  $v$  each expand in a pseudo-self-similar fashion. (These profiles share a common characteristic radius,  $\delta$ , at any time  $t$  under the presumption of constant pressure.) This assumption allows us to replace the derivatives by simple proportionalities when we apply these equations to a point coinciding with the "edge" of the channel. (See the Appendix) The various constants of proportionality ( $a$  through  $e$ ) are retained when we make these substitutions in order that additive cancellations among terms may be seen to be exact. The equations now reduce to a set of scaling relationships

$$a \frac{(\rho - \rho_o)}{t} + b \frac{v\rho}{\delta} - a\alpha \frac{(\rho - \rho_o)}{\delta^2} = 0 \quad (14)$$

$$\rho c_v \left[ c \frac{(T - T_o)}{t} + d v \frac{(T - T_o)}{\delta} \right] + e \rho_o R T_o \frac{v}{\delta} - c \rho c_v \alpha \frac{(T - T_o)}{\delta^2} = 0 \quad (15)$$

$$\rho T = \rho_o T_o. \quad (16)$$

The three scaling relationships would appear to be insufficient to solve for the unknowns  $T$ ,  $\rho$ ,  $v$ , and  $\delta$ . However after eliminating  $v$  and  $\rho$  as we solve for  $\delta(t)$ ,  $(T - T_o)$  divides out so that the scaling is independent of temperature as expected for a self similar expansion. The solutions that may be found,

$$\delta^2 = \alpha t \quad (17)$$

$$v = 0, \quad (18)$$

where  $\delta$  and  $v$  are the mean radius and radial velocity, show that the channel "diffuses" away at a rate determined by the anomalous coefficient. This behavior persists until the turbulence decays. Such

behavior may be seen in Fig. 1 where the variation of the square of the radius with time for an ohmically heated channel in air is shown.

### III. GROWTH DUE TO HEAT CONDUCTION

We now consider the expansion of the channel due to heat conduction after the turbulence has decayed. Dropping the anomalous diffusion terms and retaining the thermal conductivity the scaling relationships become

$$\frac{a(\rho - \rho_o)}{t} + b \frac{v\rho}{\delta} = 0 \quad (19)$$

$$\rho c_p \left[ c \frac{(T - T_o)}{t} + d v \frac{(T - T_o)}{\delta} \right] + e \rho_o R T_o \frac{v}{\delta} - c k \frac{(T - T_o)}{\delta^2} = 0 \quad (20)$$

$$\rho T = \rho_o T_o. \quad (21)$$

For the moment we treat  $k$  as constant. Then a solution may be found for  $\delta(t)$

$$\delta^2 = \frac{k}{\rho_o (c_v + \frac{ea}{bc} R)} t \approx \frac{k}{\rho_o c_p} t. \quad (22)$$

Thus the combined heat flow plus fluid response scales in the same manner as ordinary heat flow.

We must now account for the variation of the heat conductivity,  $k$ , with temperature. For a gas  $k$  is independent of density but proportional<sup>8</sup> to  $\sqrt{T}$ . The channel behavior now depends on initial conditions as evidenced by the failure of  $T$  to cancel out of the scaling relationships when we assume

$$k = \kappa \sqrt{T}. \quad (23)$$

An approximate integral of the energy equation exists which is independent of the previous results and which relates  $T$  to  $\delta$  through the initial conditions. We take a global view and treat  $T$  and  $\rho$  as representative values for the entire channel. The initial heat input,  $Q$ , must always equal the work done by expansion,  $W$ , plus the extra internal energy  $E$ . The work done is the pressure times the volume, at density  $\rho_o$ , needed to accommodate the gas expelled to beyond radius  $\delta$ .

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$$\begin{aligned}
 W &= \rho_o \frac{(\pi \delta^2 \rho_o - \pi \delta^2 \rho)}{\rho_o} \\
 &= \rho_o R T_o \pi \delta^2 \left( 1 - \frac{T_o}{T} \right).
 \end{aligned}
 \tag{24}$$

The extra internal energy is

$$E = \rho c_v \pi \delta^2 (T - T_o) = \rho_o T_o c_v \pi \delta^2 \left( 1 - \frac{T_o}{T} \right).
 \tag{25}$$

Equating the sum of  $E$  and  $W$  to  $Q$  we find

$$\frac{\delta_{\min}^2}{\delta^2} = \left( 1 - \frac{T_o}{T} \right)
 \tag{26}$$

where

$$\delta_{\min}^2 = \frac{Q}{\pi \rho_o T_o c_p}.
 \tag{27}$$

(As  $T \rightarrow \infty$ ,  $\delta \rightarrow \delta_{\min}$  which is the radius at which a channel of zero density and infinite temperature would contain  $Q J/m$  and still be at pressure  $\rho_o$ .) The previous scaling relations, including the behavior of  $k$  with  $T$  imply

$$\delta^2 = \frac{\kappa \sqrt{T}}{\rho_o c_p} t.
 \tag{28}$$

Eliminating  $T$  between Eqs. (26) and (28) we find the scaling is now

$$\delta \sqrt{\delta^2 - \delta_{\min}^2} = \frac{k_o}{\rho_o c_p} t
 \tag{29}$$

$$\text{where } k_o = \kappa \sqrt{T_o}.$$

Under normal circumstances  $\delta > \delta_{\min}$  and the scaling reduces to

$$\delta^2 = \frac{k_o}{\rho_o c_p} t
 \tag{30}$$

This limiting slope would appear as horizontal on Fig. 1\*.

\*A geometric factor which depends on the particular self-similar shape and choice of an "edge" has to this point been assimilated into  $\alpha$ . For a Gaussian profile with the  $1/e$  point as the edge, equations (17) and (30) would be of the form  $\delta^2 = 4\alpha t$ .

#### IV. APPLICATION TO OHMICALLY HEATED CHANNELS IN THE ATMOSPHERE

Figure 1 displays data  $(\delta, t)$  for an ohmically heated channel that has been used at NRL to study the propagation of relativistic electron beams in pre-formed channels<sup>9</sup>. These channels are formed by guiding an electric discharge from a Marx generator ( $V \sim 250\text{kv}$ ,  $I \sim 10\text{kA}$ ) with a laser-induced, air breakdown and have been produced with lengths up to  $\sim 2\text{m}$ . The ohmic deposition is  $\sim 3\text{ J/cm}$  and the channels expand to a  $\sim 1\text{ cm}$  radius in  $\sim 30\mu\text{s}$  before they reach pressure equilibrium and stabilize in size. After  $\sim 150\mu\text{s}$  turbulence becomes evident in Schlieren photographs of the channels and they simultaneously begin the growth evidenced in Fig. 1. The kinetic energy in the turbulence,  $\sim 10^{-3}\text{ J/cm}$ , is a small fraction of the internal energy of the channel. It nevertheless strongly influences the channel behavior. Rapid channel growth continues for  $\sim 2\text{ms}$  with an effective diffusivity of  $\sim 500\text{ cm}^2/\text{sec}$  which is  $\sim 2000$  times the thermal diffusivity.\*

The temperature of the channel strongly affects its electrical conductivity and thereby the propagation of an electron beam. In these experiments the electron beam was expelled from the channel at  $t \sim 100\mu\text{s}$  but showed slightly enhanced propagation in a channel at  $t \sim 500\mu\text{s}$  which demonstrates the importance of this anomalous cooling mechanism.

#### V. CONCLUSIONS

The treatment of turbulence as a diffusive phenomena is often used in engineering analyses to yield useful semi-empirical results. Reasonable experimental evidence exists for applying a similar approach to the cooling of a turbulent, reduced density channel. The gross behavior of the channel is thus "explained". It is implied for example that the radial density profile of a Gaussian channel with initial central density,  $\rho_i$ , and initial characteristic radius,  $\delta_i$ , evolves in time according to

$$(\rho - \rho_0) = (\rho_i - \rho_0) \left( \frac{\delta_i}{\delta} \right)^2 \exp \left[ - \left( \frac{r}{\delta} \right)^2 \right] \quad (31)$$

\*In comparing the turbulent expansion to true heat flow a Gaussian channel profile is assumed and allowance is made for the accompanying factor of 4 in Equation (17).

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where

$$\delta^2 = \delta_1^2 + 4\alpha t \quad (32)$$

The value of the eddy diffusivity,  $\alpha$ , remains however an empirical number. The treatment presented here assumes a constant specific heat,  $c_v$ . Such an assumption is valid for channels in air provided  $T < 3000^\circ\text{K}$ .

## VI. ACKNOWLEDGMENTS

Many people were very generous in their responses to my first cryptic versions of this memorandum. Substantial comments were offered by A. Cooper, R.F. Fernsler, J.R. Greig, R.E. Pechacek, J.M. Picone, and by E.P. Lee of the Lawrence Livermore National Laboratory.

## VII. APPENDIX: THE RADIAL AND TEMPORAL GRADIENTS OF SELF SIMILAR SOLUTIONS

The radial profile of any variable,  $V$ , (eg.  $T - T_0$ ,  $\rho - \rho_0$ ,  $v$ ,  $\rho v$ ) is here assumed to expand self-similarly

$$V = f(t) g\left(\frac{r}{\delta(t)}\right) \quad (33)$$

where simple scaling with time is assumed for both the overall amplitude  $f$  and the scale length  $\delta$  such that

$$f(t) = \beta t^n, \quad (34)$$

$$\delta(t) = \gamma t^m, \quad (35)$$

and the shape is always that of  $g(\chi)$ . The "edge",  $\chi_e$ , is a fixed value defined by

$$g(\chi_e) = \epsilon g(0) \quad (36)$$

where  $\epsilon$  is a fixed fraction.

Consider first the radial derivative of  $V$ , at a point  $r$ , at the moment,  $t$ , when the "edge" passes by

$$\frac{\partial V}{\partial r} \Big|_{\frac{r}{\delta} = \chi_e} = f \frac{\partial g}{\partial \chi} \frac{\partial \chi}{\partial r} \Big|_{\frac{r}{\delta} = \chi_e} = f(t) \frac{\partial g}{\partial \chi} \Big|_{\chi_e} \frac{1}{\delta(t)}. \quad (37)$$

However the slope of  $g(\chi)$  at the edge is proportional to  $g(\chi_e)/\chi_e$ .

$$\left. \frac{\partial g}{\partial \chi} \right|_{\chi_e} = \zeta \frac{g(\chi_e)}{\chi_e}. \quad (38)$$

Thus

$$\left. \frac{\partial V}{\partial r} \right|_{\frac{r}{\delta} - \chi_e} = \frac{\zeta}{\chi_e} \frac{f(t) g(\chi_e)}{\delta(t)} = \frac{\zeta}{\chi_e} \frac{V(r,t)}{\delta(t)} \quad (39)$$

i.e., the spatial derivative is proportional to the edge value of  $V$  divided by the radius of the edge.

Consider now the temporal derivative of  $V$

$$\left. \frac{\partial V}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} = f \left. \frac{\partial g}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} + g \left. \frac{\partial f}{\partial t} \right|_{\frac{r}{\delta} - \chi_e}. \quad (40)$$

Starting with the first term

$$f \left. \frac{\partial g}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} = f \frac{\partial g}{\partial \chi} \left. \frac{\partial \chi}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} = f(t) \zeta \frac{g(\chi_e)}{\chi_e} \left. \frac{\partial \chi}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} \quad (41)$$

where we have used Eq. (38). However from Eq. (35)

$$\chi = \frac{r}{\gamma t^m} \quad (42)$$

Thus

$$\left. \frac{\partial \chi}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} = \left. \frac{-mr}{\gamma t^{m+1}} \right|_{\frac{r}{\delta} - \chi_e} = \left. \frac{-m\chi}{t} \right|_{\frac{r}{\delta} - \chi_e} = \frac{-m\chi_e}{t}. \quad (43)$$

The first term thus becomes

$$f \left. \frac{\partial g}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} = -m\zeta \frac{V(r,t)}{t}. \quad (44)$$

We now consider the second term. In light of Eq. (34)

$$g \left. \frac{\partial f}{\partial t} \right|_{\frac{r}{\delta} - \chi_e} = g\beta n t^{n-1} \left. \right|_{\frac{r}{\delta} - \chi_e} = ng(\chi_e) \frac{f(t)}{t} = n \frac{V(r,t)}{t}. \quad (45)$$

Combining the results for the first and second terms we find

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$$\frac{\partial V}{\partial t} \bigg|_{\delta - \lambda_c} = -m\zeta \frac{V(r,t)}{t} + n \frac{V(r,t)}{t} = \eta \frac{V(r,t)}{t} \quad (46)$$

so that the temporal derivative is proportional to the edge value of  $V$  divided by the elapsed time  $t$ .

The following proportionalities are used in the main text

$$\frac{\partial \rho}{\partial t} = \frac{\partial(\rho - \rho_0)}{\partial t} = a \frac{(\rho - \rho_0)}{t} \quad (47)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r\rho v) = b \frac{\rho v}{\delta} \quad (48)$$

$$\frac{\partial T}{\partial t} = c \frac{(T - T_0)}{t} \quad (49)$$

$$\frac{\partial T}{\partial r} = d \frac{(T - T_0)}{\delta} \quad (50)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (rv) = e \frac{v}{\delta} \quad (51)$$

The coefficients for the second derivatives are determined by the requirement that the equations reduce to the proper mass and thermal diffusion results in the absence of the terms reflecting fluid motion

$$\alpha \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \rho}{\partial r} \right) = a\alpha \frac{(\rho - \rho_0)}{\delta^2} \quad (52)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r\rho c_v \alpha \frac{\partial T}{\partial r} \right) = c\rho c_v \alpha \frac{(T - T_0)}{\delta^2} \quad (53)$$

The various coefficients are interrelated through the equation of state. Taking derivatives of this equation we find

$$\frac{\partial \rho}{\partial t} = \frac{-\rho_0 T_0}{T^2} \frac{\partial T}{\partial t} \quad (54)$$

$$\frac{\partial \rho}{\partial r} = \frac{-\rho_0 T_0}{T^2} \frac{\partial T}{\partial r} \quad (55)$$

Dividing Eq. (54) by Eq. (55)

$$\frac{\frac{\partial \rho}{\partial t}}{\frac{\partial \rho}{\partial r}} = \frac{\frac{\partial T}{\partial t}}{\frac{\partial T}{\partial r}} \quad (56)$$

but in terms of our proportionalities

$$\frac{a \frac{(\rho - \rho_0)}{t}}{b \frac{(\rho - \rho_0)}{\delta}} = \frac{c \frac{(T - T_0)}{t}}{d \frac{(T - T_0)}{\delta}} \quad (57)$$

so we find

$$ad = bc \quad (58)$$

which we employ in reducing the scaling relationships.

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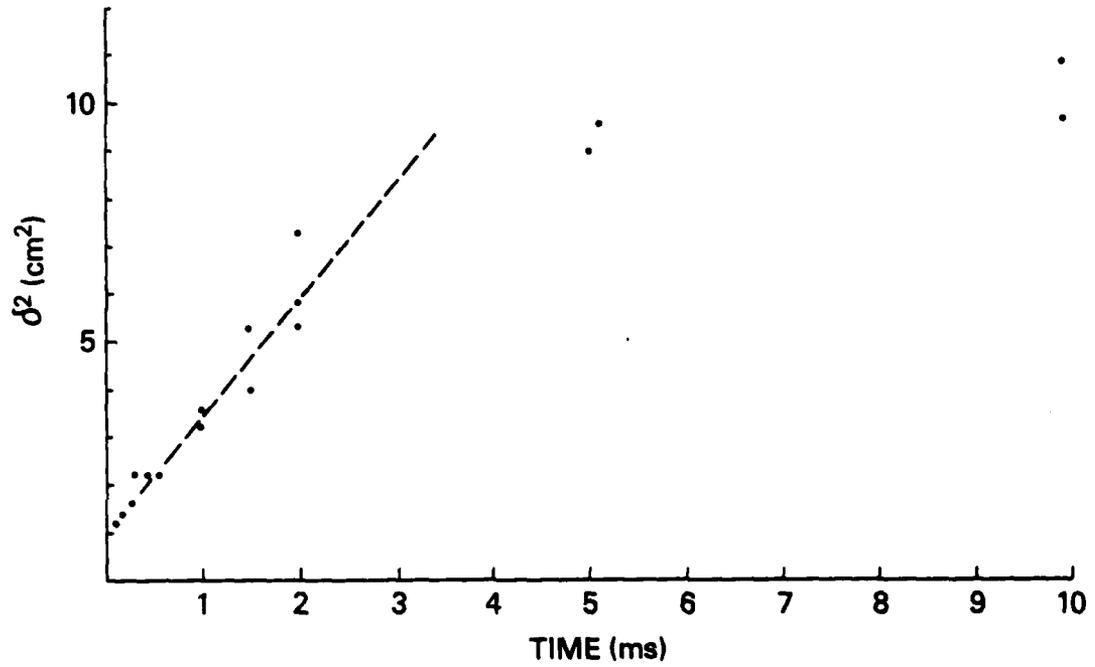


Fig. 1 - The square of the radius versus time for an ohmically heated channel in air.

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