TRANSIENT ANALYSIS OF FLUID-COUPLED ELASTIC SHELL SYSTEMS USING ETC(U)

JUN 81 K C KIDDY

UNCLASSIFIED NSWC/TR-81-153
TRANSIENT ANALYSIS OF FLUID-COUPLED ELASTIC SHELL SYSTEMS USING FLUID FINITE ELEMENTS

BY KENNETH C. KIDDY
RESEARCH AND TECHNOLOGY DEPARTMENT

8 MAY 1981

Approved for public release, distribution unlimited.
Transient Analysis of Fluid-Coupled Elastic Shell Systems Using Fluid Finite Elements

Kenneth C. Kiddy

Naval Surface Weapons Center
White Oak, Silver Spring, Maryland 20910

Approved for public release, distribution unlimited

Fluid Finite Element
Transient Response
Fluid-Structure Interaction
An eight node, first order hexahedron finite element has been modified to simulate an ideal compressible fluid. The transient response of two submerged fluid-coupled cylindrical elastic shells impinged upon by a transversely incident plane pressure pulse is computed using the fluid elements to model the fluid between the two shells. The numerical computations are accomplished by utilizing the USA-STAGS computer code. This code employs the finite element method for the structure and the Doubly Asymptotic Approximation for the fluid-structure interaction. The results of the USA-STAGS calculations are compared to an analytical solution.

This work was completed as part of the Double Hull Lethality Program and was funded by the Defense Nuclear Agency.

The author acknowledges the help, comments, and suggestions provided by Michael Giltrud, Frank Brogan, and Dr. Hanson Huang.

J. F. PROCTOR
By direction
CONTENTS

INTRODUCTION ......................................................... 5
ANALYTICAL FORMULATION ........................................... 6
USA-STAGS ANALYSIS ................................................. 10
RESULTS ................................................................. 11
CONCLUSION ............................................................. 12
BIBLIOGRAPHY .......................................................... 17

ILLUSTRATIONS

Figure Page
1 SUBMERGED FLUID-COUPLED CYLINDRICAL SHELL SYSTEM ........ 13
2 FINITE ELEMENT MODEL FOR FLUID-COUPLED SHELL SYSTEM ...... 14
3 TIME HISTORIES OF RADIAL VELOCITIES OF THE INNER SHELL ... 15
4 TIME HISTORIES OF THE HOOP STRESSES AT THE MIDDLE SURFACE OF THE INNER SHELL ................................. 16
INTRODUCTION

The class of problems involving fluid structure interaction is indeed a complex one. They simultaneously bring together the subjects of both fluid mechanics and solid mechanics. The structure must first be modeled adequately to determine the elastic response. This is certainly within the capability of many structural analysis computer codes. However, for a structure submerged in a fluid, the surface loading depends on the state of motion of the structure in the fluid. Thus, the problem of determining the loading is one in which the state of motion of the fluid and structure are coupled.

Analytical investigations have resulted in the adoption of methods for uncoupling the structure from the fluid. Several such procedures, known as surface approximation techniques, have been compared by Geers. It is apparent from his analysis that the Doubly Asymptotic Approximation (DAA) is the most accurate for early and late times in predicting the behavior of submerged shells. The DAA also affords a smooth transition between early and late time response.

The STAGS finite element code (Structural Analysis of General Shells) has been combined with a DAA code, USA (Underwater Shock Analysis) with the


resulting code called USA-STAGS\textsuperscript{6}. This code has been used to predict the response of a linear elastic cylindrical shell immersed in a fluid through which a plane acoustic wave is propagating\textsuperscript{7}. The results of the USA-STAGS calculations have been compared to an exact analysis by Huang\textsuperscript{8}. Good agreement has been obtained for step and exponential pressure wave loading. In addition, the USA-STAGS predictions compare favorably with the late time asymptote.

This paper investigates the effectiveness of using a fluid finite element to model the entrained fluid between two submerged concentric cylindrical elastic shells impinged upon by a transversely incident pressure pulse. The results of the USA-STAGS calculations are compared to an analytical solution by Huang\textsuperscript{9}.

\textbf{ANALYTICAL FORMULATION}

The USA-STAGS code is a combination of the Underwater Shock Analysis (USA) code and the Structural Analysis of General Shells code (STAGS). In USA, the fluid is assumed to be an infinite acoustic medium whose response to the motions of the structure is described by the DAA. STAGS is a general purpose finite element code intended for analysis of a shell type structure.

At this time it is useful to highlight the essentials of the problem formulation.

\textbf{STRUCTURAL RESPONSE EQUATION.} As STAGS is based upon the finite element method, the discretized differential equation of motion for the non-linear structure is expressed as

\[ M_s \ddot{x} + C_s \dot{x} + K_s x = f \]

where \( x \) is the structural displacement vector. \( M_s \) and \( C_s \) are the structural mass and damping matrices. \( K_s \) is the non-linear stiffness matrix and \( f \) is the external force vector. Generally \( M_s \), \( C_s \) and \( K_s \) are highly banded symmetric matrices of large order. In particular, STAGS considers \( M_s \) to be diagonal and \( C_s \) to be a linear combination of \( M_s \) and \( K_s \).


For the excitation of a submerged structure by a transient acoustic wave, \( f \) is given by
\[
f = -G A_f (p_I + p_s) + f_D
\]  
(2)
where \( p_I \) is the modal incident pressure vector (a known) and \( p_s \) is the modal scattered pressure vector (unknown). The dry structure dynamic load vector is given by \( f_D \); additionally, \( A_f \) is an area matrix and \( G \) is a transformation matrix.

**FLUID RESPONSE EQUATION.** USA makes use of the DAA to describe the response of the scattered pressure at the fluid structure interface\(^{10,11}\). The DAA exhibits both excellent high frequency accuracy and excellent low frequency accuracy as well as offering a smooth transition between the two asymptotes.

The differential equation governing the fluid response is
\[
M_f \ddot{p}_s + \rho c A_f p_s = \rho c \dot{M}_f \dot{U}_s
\]  
(3)
where \( p_s \) is the scattered pressure vector; \( U_s \) is the vector of the scattered wave particle velocities; \( \rho \) and \( c \) are the fluid density and sound speed. The added mass matrix, \( M_f \), is produced by a boundary - element treatment of Laplace's equation for the irrotational flow generated in an inviscid, incompressible fluid by motions of the structure's wet surface\(^{12}\).

The above equation (3) is subject to the following kinematic compatibility equation
\[
G^T \dot{x} = U_I + U_S
\]  
(4)
where the superscript \( T \) represents the matrix transposition. The compatibility equation (4) constrains the normal fluid particle velocity \( (U_I + U_S) \) to the normal structural velocity at the wet interface. The transformation matrix, \( G \), relates the structural degrees of freedom to the fluid degrees of freedom and it follows from the invariance of virtual work with respect to coordinate systems.

---


FLUID STRUCTURE INTERACTION EQUATION. Substituting equation (2) into equation (1) and equation (4) into equation (3) yields the coupled fluid structure interaction equations.

\[
\begin{align*}
M_S \ddot{x} + C_S \dot{x} + K_S x &= f_D - G A_f (P_I + P_s) \\
M_f \dot{P}_s + \rho c A_f P_s &= \rho c M_f (GT S - \dot{U}_I)
\end{align*}
\]

The above equation (5) may be solved simultaneously at each time step by the transfer of \(-G A_f P_s\) and \(\rho c M_f G^T S\) to the left side of their respective equation. Such a procedure is exceedingly difficult for large systems because of the large coupling of the coefficient matrices. Therefore, a staggered solution procedure has been developed that is unconditionally stable with respect to the time step for the linear problem.\(^{13}\)

The computational strategy for the staggered solution procedure is embodied in the following steps assuming the solution is known at time \(t\).

1. Estimate the unknown structural restoring force vector at \(t + \Delta t\) from the extrapolation of current and past values.
2. Transform this extrapolated value into fluid node values and form the right-hand side of the fluid equation, which also involves the unknown incident pressure at \(t + \Delta t\).
3. Transform fluid pressures into structural nodal forces.
4. Solve the structural equation for the displacement and velocity vectors at \(t + \Delta t\).
5. Transform the computed structural restoring force vector at \(t + \Delta t\) into fluid node values and reform the right-hand side of the fluid equation.
6. Resolve the fluid equation and obtain refined values for the total pressures at \(t + \Delta t\).
7. Save system response.

Steps (1), (3) and (5) constitute the basic staggered solution technique, while

Steps (2) and (4) are required because of the difference between the fluid and structural surface meshes. The iteration on the fluid solution reflected in Steps (6) and (7) has been added to enhance accuracy. Inasmuch as the computation time is overwhelmed by the structural solution requirements, this requires only a small increase in total run time. The use of a three-step extrapolation method in Step (1) also improves accuracy, as discussed by Park, et al.\textsuperscript{13}

**FLUID FINITE ELEMENT FORMULATION.** By appropriately modifying their material property matrices, many solid elements, based on the displacement formulation, can be "mocked" as fluid elements. The details of implementing this modification to the material matrices are given by Kalinowski\textsuperscript{14}. A summary of the procedure used is now given.

The general relationship between stress and strain in matrix form is given by

\[
\{\sigma\} = [E] \{\varepsilon\}
\]  

(6)

where \{\sigma\} and \{\varepsilon\} are the stress and strain vectors respectively. The elasticity matrix, \([E]\), is a 6 x 6 isotropic matrix which has the general form

\[
\begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(7)

where \(\lambda\) and \(\mu\) are the Lame constants.

The development of a three degrees of freedom per node (3 displacement components) solid fluid element is straightforward. By letting the shear modulus (or equivalently the Lame constant \(\mu\)) go to zero, the dynamic elasticity field equations\textsuperscript{15} will reduce to the displacement form of the wave equation

\[
c^2\nabla^2 u_i = \ddot{u}_i; \quad c^2 = \frac{\lambda}{\rho}
\]  

(8)

The pressure is related to the displacement vector by

\[
p = -\lambda \nabla \cdot u_i
\]  

(9)


where $\lambda$ is the Lamé constant, $u_i$ is the displacement vector, $c$ is the fluid wave speed, and $\rho$ is the fluid density. Note that operating on equation (8) with the divergence $\nabla \cdot (\cdot)$ operator and employing equation (9) reduces equation (8) to the pressure wave equation

$$c^2 \nabla^2 p = p \quad (10)$$

This is the acoustical representation for the fluid field employed by practically all of the computer codes used in fluid structure interaction problems. The wave equation (8) will be enforced by letting $\mu = 0$ and $\lambda = \rho c^2$ in the general stress-strain law, equation (7).

Therefore, the local element stiffness and mass matrices for each fluid element can be created from ordinary solid linear elastic elements by appropriately choosing the values of $\mu$ and $\lambda$. Elements created in this manner have been referred to as "mock" fluid finite elements. The stiffness matrices of the remaining structural elements are then created in the usual way. Finally, the total stiffness and mass matrices of the entire system of elements are assembled in the usual manner dictated by the finite element method.

A boundary condition is also required with fluid structure interaction methods. That is, at the fluid structure interface the normal motion of the structure equals the normal motion of the fluid. This effect is achieved with the mock elements by using a double set of nodes at the fluid structure interface. One set belongs to the fluid and the other to the solid. The motion of the two sets of nodes is then assumed to be independent of each other in the tangential direction but are forced to be equal in the normal direction.

Since STAGS does not have a solid element, the eight node, first order hexahedron element in NONSAP has been modified according to the procedure described above. The mass and stiffness matrices for the mock fluid elements, generated using NONSAP, are then introduced into STAGS. The USA-STAGS code may then be used in the normal manner to compute the response of the fluid-structure system.

USA-STAGS ANALYSIS

The plane strain problem of the transient response of two fluid coupled cylindrical elastic shells to a transversely incident pressure pulse will serve as a benchmark problem for the fluid finite element. The finite element solution will be compared with the analytical solution which has been obtained by Huang for this problem.

---

NSWC TR 81-153

The submerged fluid coupled shell system and the incident plane pressure wave are depicted in Figure 1. The fluid surrounding the outer shell and that between the two shells is considered to be an ideal compressible fluid and is characterized by its mass density, \( \rho \), and sound speed, \( c \). The Doubly Asymptotic Approximation (DAA) is used for analyzing the transient interaction of the infinite fluid exterior to the outer shell, the incident pressure wave, and the outer shell itself. This leaves only the inner and outer shells and the entrained fluid to be modeled using finite elements. Due to symmetry, modeling only the region from \( 0 \leq \theta \leq \pi \) is necessary. The finite element model used is shown in Figure 2. It contains 36 plate bending elements for the inner and outer shell and 72 hexahedral fluid elements for the entrained fluid.

The middle surface radii, thickness, mass densities, Young's moduli and Poisson's ratios of the outer and inner shells are \((a^e, h^e, \rho_s^e, E^e, \nu^e)\) and \((a, h, \rho_s, E, \nu)\), respectively. The material properties and dimensions used in the checkout problem are:

\[
\begin{align*}
\rho &= \rho^e = 0.362 \text{ lb/in}^3 \\
c &= c^e = 60000 \text{ in/sec} \\
\rho_s &= \rho_s^e = 0.282 \text{ lb/in}^3 \\
E &= E^e = 30 \times 10^6 \text{ psi} \\
\nu &= \nu^e = 0.3 \\
a^e &= 6.25 \text{ in} \\
a &= 5.0 \text{ in} \\
h^e &= 0.0363125 \text{ in} \\
h &= 0.14525 \text{ in}
\end{align*}
\]

RESULTS

An analysis of the submerged fluid-coupled shell was done for the case of a step wave of infinite duration. The USA-STAGS calculations were carried out for 600 steps using a time step of about 1/200 of the transit time (the time for the incident wave front to traverse the distance of one outer shell diameter).

Figure 3 shows the time history of the radial velocity \((\hat{w})\) at various locations of the inner shell. (Note that the velocity has been non-dimensionalized by \( \bar{w} = \frac{\rho_0}{\rho c} \), where \( \rho_0 \) is the magnitude of the incident pressure wave. The non-dimensional time, \( T' \), is given by \( T' = \frac{T-\tau}{\tau} \), where \( T = c^e t/a^e \), \( \tau = (1-c) \), \( \tau = a/a^e \), and \( t \) is the actual problem time. Physically, the motion of the outer shell starts at \( T=0 \), while that of the inner shell starts at \( T = \tau = (1-a/a^e) \). In general, the agreement between the USA-STAGS solution and Huang's analytical solution is quite good though the calculated results contain some higher frequency oscillations. The late time translational velocities of the shell compare very well.

A comparison of the analytical and USA-STAGS solutions for the time histories
of the hoop stress resultants at various locations of the inner shell is shown in Figure 4. (Note that the stress resultants, $N_o$, have been non-dimensionalized by $\alpha = \rho_0/c^2$. The time, $T'$, is non-dimensionalized as before.) Again, the agreement between the two solutions is quite good. Also shown on this figure are results from Huang's analytical solution done without the presence of an outer shell.

CONCLUSION

Comparison of the analytical and USA-STAGS solutions to the problem of two submerged fluid-coupled shells impinged upon by a transversely incident plane pressure wave is quite favorable. It is apparent from this analysis that the use of the "mock" fluid finite element is an effective technique for solving this class of fluid-structure interaction problems.
FIGURE 1  SUBMERGED FLUID-COUPLED CYLINDRICAL SHELL SYSTEM
FIGURE 2  FINITE ELEMENT MODEL FOR FLUID-COUPLED SHELL SYSTEM
FIGURE 3  TIME HISTORIES OF RADIAL VELOCITIES OF THE INNER SHELL
FIGURE 4  TIME HISTORIES OF THE HOOP STRESSES AT THE MIDDLE SURFACE OF THE INNER SHELL
NSWC TR 81-153

BIBLIOGRAPHY


DISTRIBUTION

Commander
David Taylor Naval Ship Research & Development Center
Attn: Code 17
Code 1740
Code 1740.1
Code 1720
Code 1720.3
Code 042
Bethesda, Maryland 20084

Underwater Explosions Research Division
David W. Taylor Naval Ship Research & Development Center
Attn: Technical Reference Center
Portsmouth, Virginia 23709

Director
Naval Research Laboratory
Attn: Code 2027
Code 8400
Code 8406
Washington, D.C. 20374

Commander
Naval Weapons Center
Attn: Code 533 (Technical Library)
China Lake, California 93555

Commander
Naval Ocean Systems Center
Attn: Technical Library
San Diego, California 92152

Commanding Officer
Naval Underwater Systems Center
Attn: Technical Library
Newport, Rhode Island 02840

Office of Naval Research
Attn: Code 474
Washington, D.C. 20360

Strategic Systems Project Office
Navy Department
Attn: SP-270P (J. Pitsenberger)
Washington, D.C. 20376