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6 **SHAPE OPTIMIZATION OF SHEET METAL  
STRUCTURES AGAINST CRASH**

by

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Abstract

A theoretical and experimental study was undertaken into the crushing behavior of axially compressed short thin-walled open-section columns. The effect of initial geometry of panels as well as distribution and magnitude of shape imperfections on the efficiency of energy absorption was examined. Results of model tests on 0.1mm thick aluminum foil specimens have shown that the panels collapsing in the symmetric and asymmetric deformation mode provide respectively, upper and lower bound for the energy absorbed in any other buckling mode. In both of the extreme cases, the crush response of the panel was predicted theoretically with a reasonable accuracy. It is shown that an optimum design of columns against crush can be achieved by introducing a beneficial geometric imperfections of a specified magnitude so that the structure will be forced to collapse in the most energy efficient deformation mode.

### Introduction

The problem of reducing the weight of a vehicle body without sacrificing its strength, stiffness and crashworthiness properties has become a great concern to auto manufacturers. The optimization of thin-walled structures for energy absorption can be approached in many different ways. Weight reduction can be achieved by considering alternative materials such as aluminum, high strength steel or fiber reinforced composites, [1],[2]. Another possibility is to strengthen properly the shell by means of stiffeners in order to impose a desirable and energy efficient deformation mode. One could also incorporate into the design initial imperfections of a specified magnitude which again are aimed at triggering the right buckling mode and forcing the structure to collapse in that mode. Finally, great potential exists in properly shaping the sheet metal structures for maximum crash resistance under prescribed geometrical constraints.

The problem of shape optimization of thin-walled structures subjected to quasi-static crushing can be formulated as follows:

For a given weight and prescribed total crush distance, maximize the energy absorption of the structure.

Dual formulation:

For a given amount of the energy to be absorbed  
and for a prescribed total crushing distance,  
minimize the weight of the structure.

Clearly, the application of the optimal criterion requires a careful determination of the admissible class of structures and boundary conditions. Very little work on the more fundamental nature has been done in this new area. It is generally understood that closed - section members under compressive forces are far more efficient from the point of view of energy absorption than open - section structures. It has also been observed that the crushing force increases with the number of sharp corners (angles) of prismatic or channel section tubes [3]. According to various authors, the mean crushing force is proportional to the gauge thickness  $h$  raised to the power  $1.5 \div 2$ . On the other hand, only slight dependence of the force was observed in the circumference of the tube. These findings gave rise to the concept of using long, progressively collapsing tubular members for crash protection of automobile bodies. Its limitation is set by the condition of overall loss of stability which restricts the length to width ratio  $l/b$  to relatively low values. On the other hand, closed - section members with small  $l/b$  have to be excluded from most of the designs because of package requirements. In order to get the required crash resistance, one has thus to turn to open section shells.

In the present paper, the response of such shells under quasi-static compressive loading is studied to get some insight into the extremely difficult problems of the formation and motion of the

plastic folds and wrinkles. Rather than solving the optimization problem for general shells, we shall restrict the analysis to axially loaded open panels forming one quarter of prismatic thin-walled tubes. The dependence of the crushing characteristics of the panels on imperfections, boundary conditions and the transition radius was studied experimentally and theoretically and important parameters were identified which are controlling the process of energy absorption.

#### Experimental Procedure

Fourty-three model tests were run in three series on 0.1 mm thick aluminum foil specimens with dimensions shown in Figure 1. The height of all specimens was the same,  $l = 75$  mm; the ratio  $l/h$  being typical to large panels of the automobile body such as the hood, fenders, trunk lid, etc. The total circumference of the specimens was also held constant  $c = 100$  mm but the shape varied from the symmetric angle element to the cylindrical panel with the central angle equal to  $\frac{\pi}{2}$ . The considered transition radii were respectively,  $\rho = 0, 5, 20, 25, 35$  and  $65$  mm. The lateral surface of the specimens was fixed to the bottom and top plates by means of a Scotch tape whose bending stiffness was much smaller than that of the foil. Thus, the boundary conditions were of the simple supported type along the horizontal edges and free along the vertical edges. Intentionally, little care was given to eliminate the shape imperfection in order to best resemble the real world situation. On a certain number of the specimens, intial imperfections were imposed described by the amplitude  $\bar{\xi}$ , half wavelength  $\lambda$  and number

of waves  $n$ , Figure 2. All specimens were crushed up to one half of its original length in a kinematically driven testing machine. Both the bottom and top plates were fixed to the cross-head plates to prevent rotation. In each experiment, the force-shortening characteristics was recorded on the x-y plotter. From these diagrams, the energy absorbed at 30 mm crush and the average force level were calculated. Photographs of all specimens were taken before and after each test. In the first series of tests, "perfect" structures were crushed with the transition radius  $\rho$  as a design variable. In the second series, the effect of large initial imperfections was studied for specimens with selected transition radii. Finally, the effect of the deformation mode on the energy absorption of panels with  $\rho = 25$  mm was examined in the third series.

#### Experimental Results and Discussion

A typical load-deflection relationship is shown in Figure 3. One or several peaks of the load are observed, the first being usually, but not always, the maximum one. Each distinct peak is associated with the formation of a different local buckling mode. The force is generally diminishing with deflections. In most cases, the pattern of folds and wrinkles is very complicated and depends on initial random imperfections. In spite of this seemingly unpredictable behavior of panels, some interesting features of the crushing process have been identified.

#### Effect of the Panel Curvature

The total energy absorbed at 30 mm crush distance was plotted

against the transition radius  $\rho$  for all test pieces without imposed imperfections (Figure 4). While the scatter of experimental points for each value of the radius  $\rho$  is quite significant, a well defined tendency can be observed. The full line in Figure 4 represents the best fit of all experimental points\*. Thus, an optimum structure consists of a cylindrical panel supported on both sides by two flat plates. The energy absorption of the panels with the largest radius  $\rho=65\text{mm}$  was only one half of the optimum value while the efficiency of angle elements with  $\rho = 0$  falls between these two extreme cases.

#### Effect of Imperfections

The objective of the crushing tests on imperfect structures with imposed large initial imperfections was to see whether it is possible to excite deformation mode with short wavelength and get a progressive collapse of the considered open-structure members. In most cases we have introduced very large imperfections, sometimes exceeding by two orders of magnitudes, the gauge thickness of an element! Panels with large imperfections generally followed the collapse mode imposed by imperfections regardless of their wavelength. In contrast to the closed-section members, the deformation mechanism was not progressive; we have simply observed a simultaneous folding of the material along the stationary hinge line. The associated force - deflection relationship was relatively smooth but far below the corresponding curve for "perfect" shells. The dramatic decrease in the maximum force is mainly attributed to the large magnitude of initial imperfections. Examples of crushed specimens with large symmetric and asymmetric imperfections are shown

\*It has clearly a maximum for  $\rho = 25 \text{ mm}$

respectively, in Figures 5 and 6. Specimens with much smaller initial imperfections behave in a different manner, as exemplified in Figure 7. When the wavelength of imperfection is relatively short, the structure disregards its presence and eventually assumes an asymmetric rather than symmetric deformation mode with  $H = \frac{\ell}{2}$ . The associated reduction in the force level is now much less pronounced. Some aspects of the transition from the symmetric to asymmetric buckling mode will be discussed further in the paper.

The third series of tests was run on panels with an optimum radius,  $\rho = 25\text{mm}$  and no imposed initial imperfections. The random imperfections cause the structure to assume in each case a different, and in most cases, quite a complicated collapse mode leading to different load-deflection characteristics. However, two type of activated modes are of particular interest. One is a regular asymmetric mode with  $H = \frac{\ell}{2}$ , Figure 8, while the other one is a symmetric mode with  $H \approx \frac{\ell}{4}$ , Figure 9. The plastic energy dissipated in these modes were found to furnish respectively, the lower and upper bound on the energies dissipated in all other modes. This is the most important conclusion of the present experimental program. These findings bear important implications for the analysis and design of the thin panels. Now, the worse and best possible case of energy absorption can be predicted on the basis of a relatively simple collapse mechanism. Consequently, the analyst should not be worried about the effect of random imperfections and about the difficulties associated with considering in the calculations more complicated buckling pattern. On the design side, the problem of maximizing the energy absorption can be reduced to the problem of triggering the desirable symmetric

buckling mode.

In the remainder of the paper, attention will be focused on the plastic crushing analysis and elastic post-buckling analysis of panels with symmetric and asymmetric collapse mechanism.

### General Analysis of the Plastic Crushing Process

Consider a typical fold line which moves down the shell element as the deformation progresses goes on, Figure 10. Since the fold line always lies in a certain plane, (see Ref. [4]), it is reasonable to model the deformed shell element by a section of an elliptic toroidal surface, four sections of a cylindrical surface and four plane trapezoidal elements. Such a model, first suggested in [5] and [6], is fully consistent geometrically and kinematically. In previous models considered in the literature, the toroidal surface was not considered which leads to the discontinuities in the displacement field at the corner point C, [7],[8],[9]. To simplify the calculations, the elliptical toroidal surface will be approximated by a circular one defined by a central angle  $2\beta$  and a small and large radii, denoted respectively by  $r$  and  $R$ , Figure 12. The global geometry of the crushing process is described in Figure 11. The angle  $2\psi$  between two adjacent sides and the total length of segments AC+CD are kept constant while the angles  $\gamma$  ( $\alpha$ ) and  $\pi-\beta$  ( $\alpha$ ) diminish with the parameter of the process  $\alpha$ .

The angles  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\psi$  are related by (tg stands for tangent),

$$\operatorname{tg} \gamma = \frac{\operatorname{tg} \psi}{\sin \alpha}, \quad \cos^2 \beta = \frac{1 - \sin^2 \alpha}{1 + \left(\frac{\sin \alpha}{\operatorname{tg} \psi}\right)^2} \quad (1)$$

The crushing distance  $\delta$  and crushing velocity are related to the

wavelength  $H$  and  $\alpha$  through

$$\delta = 2H (1 - \cos\alpha), \quad \dot{\delta} = 2H(\sin\alpha)\dot{\alpha} \quad (2)$$

According to the assumed model, four mechanisms contribute to the energy dissipation. There are:

1. Extension in the hoop direction of the toroidal surface.
2. Change of curvature in the circumferential direction from positive  $1/R$  to negative  $-1/R$ .
3. Bending and rebending along moving plastic hinges 1 and 2.
4. Pure bending in stationary plastic hinges 3 and 4.

The strains developed in the toroidal surface may sometimes be very large thus requiring consideration of the variable thickness. The exact expressions for the dissipated energies in each of these mechanisms was calculated in [5] and [6]. In the present analysis, we shall use the following simplified formulas, valid for constant thickness:

$$\dot{E}_{\text{hoop}} = 4h\sigma_0 V_t \beta r \sin\psi \quad (3)$$

$$\dot{E}_{\text{cur}} = 4M_0 \alpha \frac{V}{\text{tg}\psi} \quad (4)$$

$$\dot{E}_{\text{br}} = M_0 \frac{VH}{r} \frac{1}{\sin\gamma \text{tg}\psi} \quad (5)$$

$$\dot{E}_b = 4M_0 c \dot{\alpha} \quad (6)$$

where  $M_0 = \frac{1}{4}\sigma_0 h^2$  and the magnitude of the normal and tangential

component of the velocity of the line AC or CD is

$$V = H(\cos\alpha)\dot{\alpha}, \quad V_t = \frac{V}{\operatorname{tg}\psi} \quad (7)$$

Equating the sum of (3) ÷ (6) to the rate of work of external forces

$$\dot{E}_{\text{ext}} = P\dot{\delta} \quad (8)$$

one gets a final expression for the magnitude of an instantaneous axial force necessary to maintain the plastic flow,

$$\frac{P}{M_0} = 8\beta \frac{r}{h} \cos\psi \operatorname{ctg}\alpha + 2\alpha \frac{\operatorname{ctg}\alpha}{\operatorname{tg}\psi} +$$

$$\frac{H}{r} \frac{\operatorname{ctg}\alpha}{\sin\gamma} \frac{1}{\operatorname{tg}\psi} + 2 \frac{C}{H} \frac{1}{\sin\alpha} \quad (9)$$

We shall consider now two special cases of Equation (9).

#### Crush Prediction in the Asymmetric Mode

Consider a panel with  $\rho = 0$ , in which an asymmetric mode is activated. The angle  $\psi$  is equal to  $\pi/4$ , so that the formula (9) is reduced to

$$\frac{P}{M} = 5.66 \frac{r}{h} \operatorname{ctg}\alpha + 2 \operatorname{ctg}\alpha + \frac{H}{r} f \operatorname{ctg}\alpha + 2 \frac{C}{H} \frac{1}{\sin\alpha} \quad (10)$$

where

$$f(\alpha) = \frac{1}{\sin\gamma}.$$

The right hand side of (10) can be formally minimized with respect to the radius  $r$  and wavelength  $H$ . The conditions  $\frac{\partial P}{\partial r} = 0$  and  $\frac{\partial P}{\partial H} = 0$  yield

$$r_{\text{opt}} = 0.42 \sqrt{Hnf\beta^{-1}} \quad (11)$$

$$H_{\text{opt}} = 0.89 \sqrt[3]{\frac{C^2 \ell}{\beta f \cos^2 \alpha}} \quad (12)$$

Thus, the optimum values of  $r$  and  $H$  turn to be a function of  $\alpha$  which is inconsistent with the assumed mechanism of deformation in which  $r$  and  $H$  were both taken as constant. While the variation of  $r$  with  $\alpha$  can easily be accommodated by the present solution without requiring much of the additional energy to be dissipated, the consequences of the condition,  $\frac{\partial P}{\partial H} = 0$  should be discussed in more detail.

The variation of dimensionless wavelength,  $\bar{H} = H/\sqrt[3]{C^2 h}$  with the parameter of the process shown in Figure 13. The wavelength of the corresponding elastic buckling mode is  $H_{e1} = \frac{\ell}{2}$ , which yields  $\bar{H}_e = 3.75$  (broken line in Figure 13). Experiments show that the panels collapse in the asymmetric mode with the elastic wavelength. A considerable variation of  $\bar{H}$ , predicted by (12) would not be possible without dissipating additional energy on the downward motion of the otherwise stationary hinge lines ACD and corner point. The necessary calculations have not been performed but it is anticipated that the new energy terms would prevent any major variation of  $H$ . It is thus, very likely that the wavelength of

the plastic collapse mode is decided early in the deformation process and a possible adjustment of the  $H$  takes place in the stage of an elastic post-buckling behaviour of the panel.

Substituting (11) and  $H = 37.5$  mm,  $c = 100$  mm,  $h = 0.1$  mm into (10), we get the parametric representation of the force-displacement relationship of the considered panel

$$\frac{P}{M_0} = \left\{ 92 \sqrt{f\beta} + 2\alpha \right\} \text{ctg}\alpha + 5.3/\text{sin}\alpha \quad (13)$$

$$\delta = 2H \{1 - \cos\alpha\}$$

The prediction of (13) has been compared with  $P - \delta$  curve measured in tests No. 32 and 35. The agreement is very good except for the initial phase when the present theory is not valid. (Figures 14 and 15). In matching the theory with experiments, we have assumed the yield stress of the aluminum foil to be  $\sigma_0 = 8 \text{ KG/mm}^2$ .

#### Crush Prediction in the Symmetric Mode

The simplified model of the symmetric collapse mode is shown in Figure 16. The existence of such a mode can be easily evidenced by making an inextensible paper model of the panel. Now, there are two hinge lines and hence, all terms in the energy balance equation, except the contribution of the stationary hinge lines, should be doubled. The angle  $\psi$  becomes  $\pi/8$  and from (9) one can get an expression for the crushing force

$$\frac{P_S}{M_0} = 14.78 \frac{r}{h} \text{ctg}\alpha + 9.65\alpha \text{ctg}\alpha + 4.82 \frac{H}{r} f \text{ctg}\alpha + 3.14 \frac{C}{H \text{sin}\alpha}$$

(14)

A minimization of the right hand side of (14) with respect to  $r$  and  $H$  gives

$$r_{\text{opt}} = 0.57 \sqrt{hHf\beta^{-1}} \quad (15)$$

$$H_{\text{opt}} = 0.52 \sqrt[3]{\frac{C^2 h}{f\beta \cos^2 \alpha}} \quad (16)$$

The plot of Eq. (16) reveals that the predicted wavelength of the symmetric mode is two times smaller than that corresponding to the asymmetric mode for all values of  $\alpha$ , Figure 13. This tendency is confirmed by test No. 30 where  $H$  was of the order of  $\frac{l}{4}$  rather than  $\frac{l}{2}$ . Substituting (15) back into (14), we obtain the sought for expression for the crush resistance of the panel

$$\frac{P}{M_0} = \left\{ 16.85 \sqrt{\frac{H}{l} f\beta} + 9.65\alpha \right\} \text{ctg}\alpha + 3.14 \frac{C}{H} \frac{1}{\sin\alpha} \quad (17)$$

Plots of Eq. (17) for several chosen values of  $H$  are shown in Figure 17. The correlation with the experimentally measured force is not as good as in the case of the asymmetric mode of deformation. The present solution correctly predicts the general character of the  $P - \delta$  relationship but overestimates the force level for a realistic value of the wavelength  $H = \frac{l}{4} = 18.75$ . It should be noted that the activation of the symmetric mode, clearly visible in Figure 6 is preceded by the development of some other modes near the bottom plate, which explains a poor agreement of the theory with test results at the initial stage of the crushing pro-

cess up to  $\delta = 10$  mm. Some discrepancies between the theory and experiments may also be attributed to the fact that the calculations were made for panels with  $\rho = 0$  while the measurements were taken for  $\rho = 25$  mm.

In the same figure shown is also the P- $\delta$  curve corresponding to the asymmetric mode (broken line). Under any circumstances, this curve is much lower than those representing a symmetric mode solutions and so is the dissipated energy.

In summing up, it can be stated that the present solution gives a good prediction of the crushing resistance of panels deformed in an asymmetric and symmetric mode, provided that the wavelength of the collapse mode is properly determined. The energy absorption calculated in these two modes furnishes a lower and upper bound on energies associated with any other deformation modes of panels with various  $\rho$  and random imperfections.

#### Post-Buckling Analysis of Elastic Panels

A further insight into the problem of an optimum design of panels against crash can be gained by analyzing the buckling and post buckling behavior of perfect and imperfect elastic structures. We have shown that the plastic energy absorption would be maximized if we succeed in activating a symmetric buckling mode.

Buckling of an angle element in the asymmetric and symmetric modes is equivalent to considering a plate element with respectively free-simply supported and free-clamped end conditions, Figure 18. The corresponding classical buckling load is given by (see for example [10])

$$P_c = \frac{k\pi^2 D}{b} \quad (18)$$

where  $D$  is the bending rigidity of a plate,  $b$  - its width and the coefficient  $k$  depends on the aspect ration  $b/a$  which in our case is  $\frac{c}{2l}$ . With the present dimensions of the plate  $c = 100$  mm,  $l = 75$  mm. This coefficient is equal to

$$k = \begin{cases} 0.9 & \text{asymmetric mode} \\ 1.34 & \text{symmetric mode} \end{cases}$$

The buckling mode corresponding to the lowest buckling force is in both cases a half of the sinusoidal wave, i.e.  $H = l/2$ . After the critical load is reached, a post buckling path is followed for both symmetric and asymmetric case, Figure 19. Thus, a perfect elastic angle element will always buckle in an asymmetric mode, which requires a much lower critical load.

The situation might be different for an imperfect structure since now the two modes may interact with each other. In the majority of cases, the most degrading effect on the load have the imperfection in the shape of the buckling mode [11]. Denote by  $\bar{\xi}_a$  and  $\bar{\xi}_s$ , the maximum amplitude of initial imperfections respectively in the asymmetric and symmetric mode. If  $\bar{\xi}_a \neq 0$  and  $\bar{\xi}_s = 0$ , the structure will follow the equilibrium paths in the asymmetric mode, Figure 19. If the compression of the plate is continued, this will eventually lead to the lowest possible energy dissipation. Suppose now that  $\bar{\xi}_a = 0$  and  $\bar{\xi}_s \neq 0$ . The structure will initially follow the symmetric mode deformation but at a certain point, such as  $F$  may bifurcate into the asymmetric mode. The question which remains to be answered is whether this will happen and if so, when and how the coordinates of the point  $F$  depend on the magnitude of the initial imperfections  $\bar{\xi}_s$ .

The important problem is being studied using Koiter's theory of post-buckling behaviour of imperfect structures with mode interactions, [12],[13] and the results will soon be published, [14]. If we succeed in controlling the deformation process so that the structure will not pop back from the symmetric mode to the preferable (giving lower force) asymmetric one, then the way for optimizing the structure for crash would be open.

#### Concluding Remarks

We have shown that the energy absorption of compressed panels can be considerably increased if a proper shape of the cross-section is assumed. Within the considered class of structures, the best shape consists of a cylindrical panel representing one quarter of a complete cylinder supported on both sides by flat plates. The presence of random initial imperfections give rise to quite a variety of collapse mechanisms. However, two specific types of imperfections are of utmost importance. It was found that the imperfections in the asymmetric and symmetric elastic buckling mode have respectively the most degrading and beneficial effect on the energy absorbed by the panel. In both of the extreme cases, the crash response of the panel can be predicted with a reasonable accuracy. Investigation is under way regarding the stability of the buckling modes imposed by symmetric imperfections. A new series of experiments is also being planned to demonstrate the effect of the beneficial imperfections on the crush response of the panels.

The required symmetric collapse mechanisms may be triggered in many different ways, for example, by attaching string stiffeners with initial curvature to the free edge of the panel. The problem

of interaction of stiffeners and a shell with the view of maximizing the energy absorption will be the subject of a future research.

It is believed that some of the factors identified in the present study would also affect the energy absorption of sheet metal structures with different geometry and boundary conditions.

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References

1. C.L. Magee, P.H. Thorton, The Interplay of Geometric and Materials Variables in Energy Absorption, J. Engineering Materials and Technology, April 1977.
2. J. Provensal, B. Cosatti, Energy Dissipation of Composite Structures (in French) Proc., 18th International FISITA Congress, May 5-8, 1980, Hamburg, pp. 70-76.
3. N. Aya, K. Takahashi, Energy Absorption Characteristics of Vehicle Body Structure, Transactions of the Society of Automotive Engineers of Japan, No. 7, May 1974.
4. W. Abramowicz, T. Wierzbicki, A Kinematic Approach to Crushing of Shell Structures, SAE paper No. 790992, Proc. 3rd International Vehicle Structural Mechanics Conference, Troy, Michigan, October 10-12, 1979, pp. 211-223.
5. W. Abramowicz, Mechanics of the Crushing of Thin-Walled Structures (in Polish), Ph.D. Thesis from the Institute of Fundamental Technological Research, Warsaw, Poland.
6. W. Abramowicz, T. Wierzbicki, Mechanics of the Crushing Process of Thin-Walled Plastic Shells, Technical Report, Department of Ocean Engineering, M.I.T., July 1981.
7. W. Johnson, P. Soden, S.T.S. Al-Hassani, Inextensional Collapse of Thin-Walled Tubes Under Axial Compression, J. Strain Analysis, Vol. 12, No. 4, 1977.
8. G.H. Tidbury, D. Kecman, Investigation into Behavior of Hinges Produced by Bending Collapse of Vehicle Structural Components, Proc. 17th International FISITA Congress, Budapest, June 4-10, 1978.
9. Y. Ohokubo, T. Akamatsu and K. Shirasawa, Mean Crushing Strength of Closed-hat section members, SAE Paper No. 740040.
10. S. Timoshenko, J.M. Gere, Theory of Elastic Stability, McGraw-Hill, 1961.
11. R.C. Tennyson, M. Booton and D. Hui, Changes in Cylindrical Shear Buckling Behaviour Resulting from System Parameter Variations, Proceedings of the 15th International Congress of Theoretical and Applied Mechanics, held at the University of Toronto, August 1980, pp. 417-430.
12. W.T. Koiter, "On the Stability of Elastic Equilibrium", Doctoral thesis, Delft, Amsterdam, 1945; English translation: NASA TT F-10, 833, 1967, and AFFDL-TR-70-25, 1970.
13. D. Hui, R.C. Tennyson and J.S. Hansen, "Mode Interaction of Axially Stiffened Cylindrical Shells: Effects of Stringer Axial Stiffness, Torsional Rigidity and Eccentricity," Journal of Applied Mechanics, in press.

14. D. Hui, T. Wierzbicki, Effect of Mode Interaction on Plastic Collapse of Short Thin-Walled Columns, to be published.

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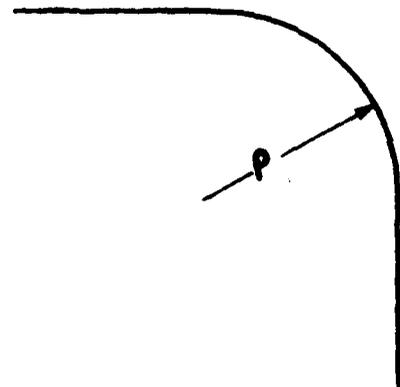
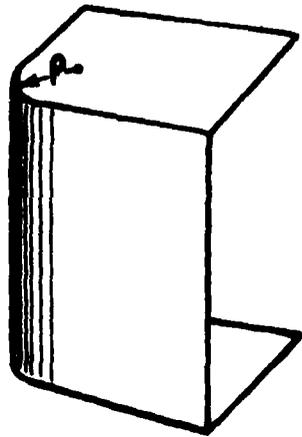
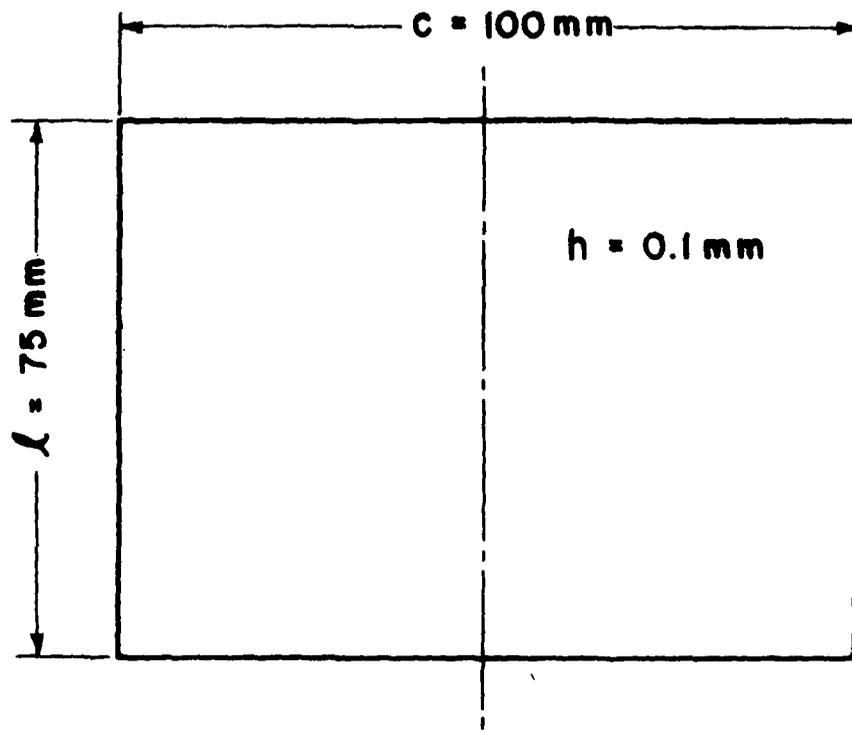
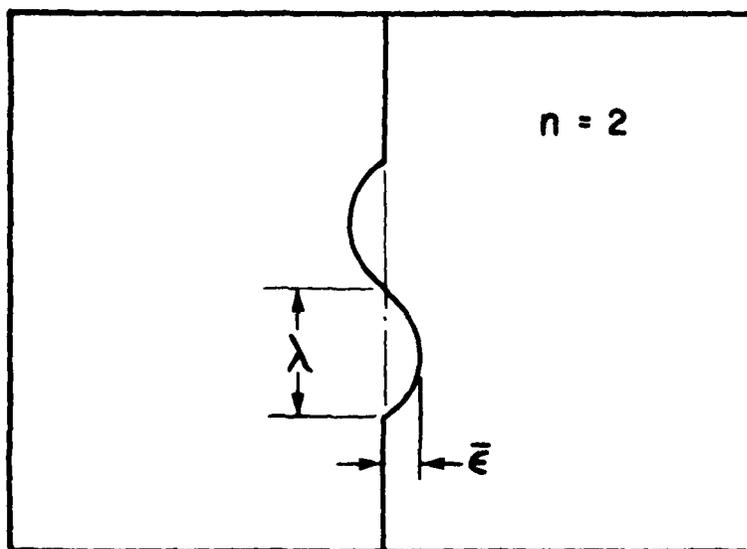
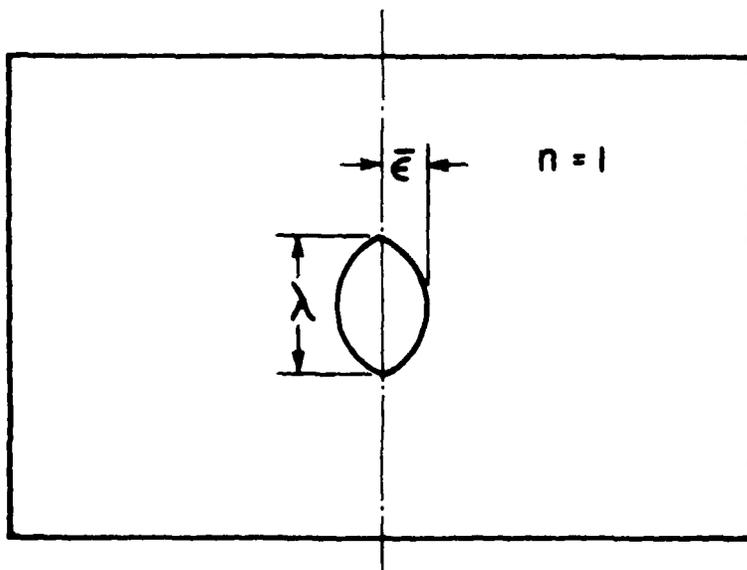


FIGURE 1



ASYMMETRIC IMPERFECTIONS



SYMMETRIC IMPERFECTIONS

FIGURE 2

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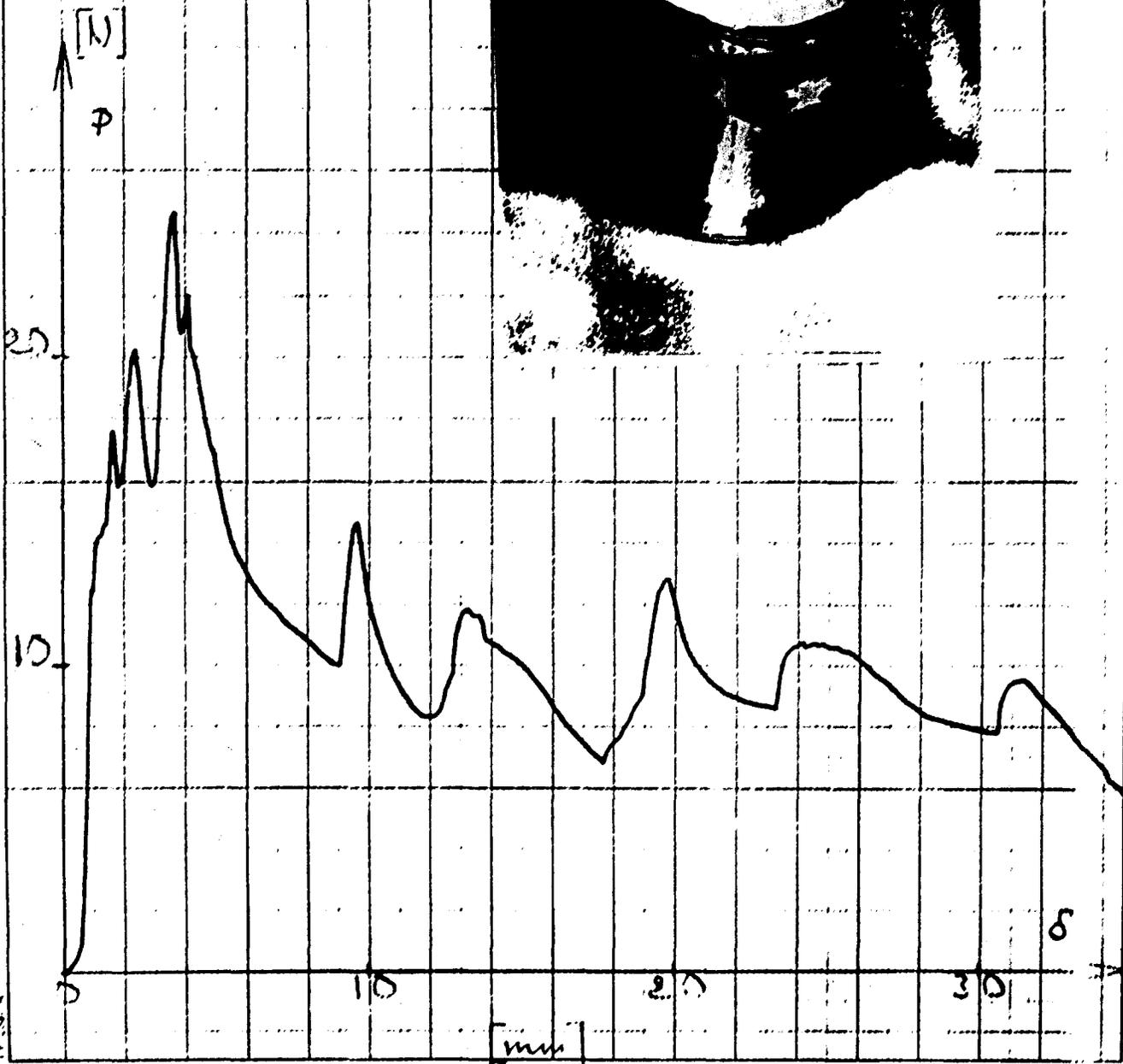


Figure 3

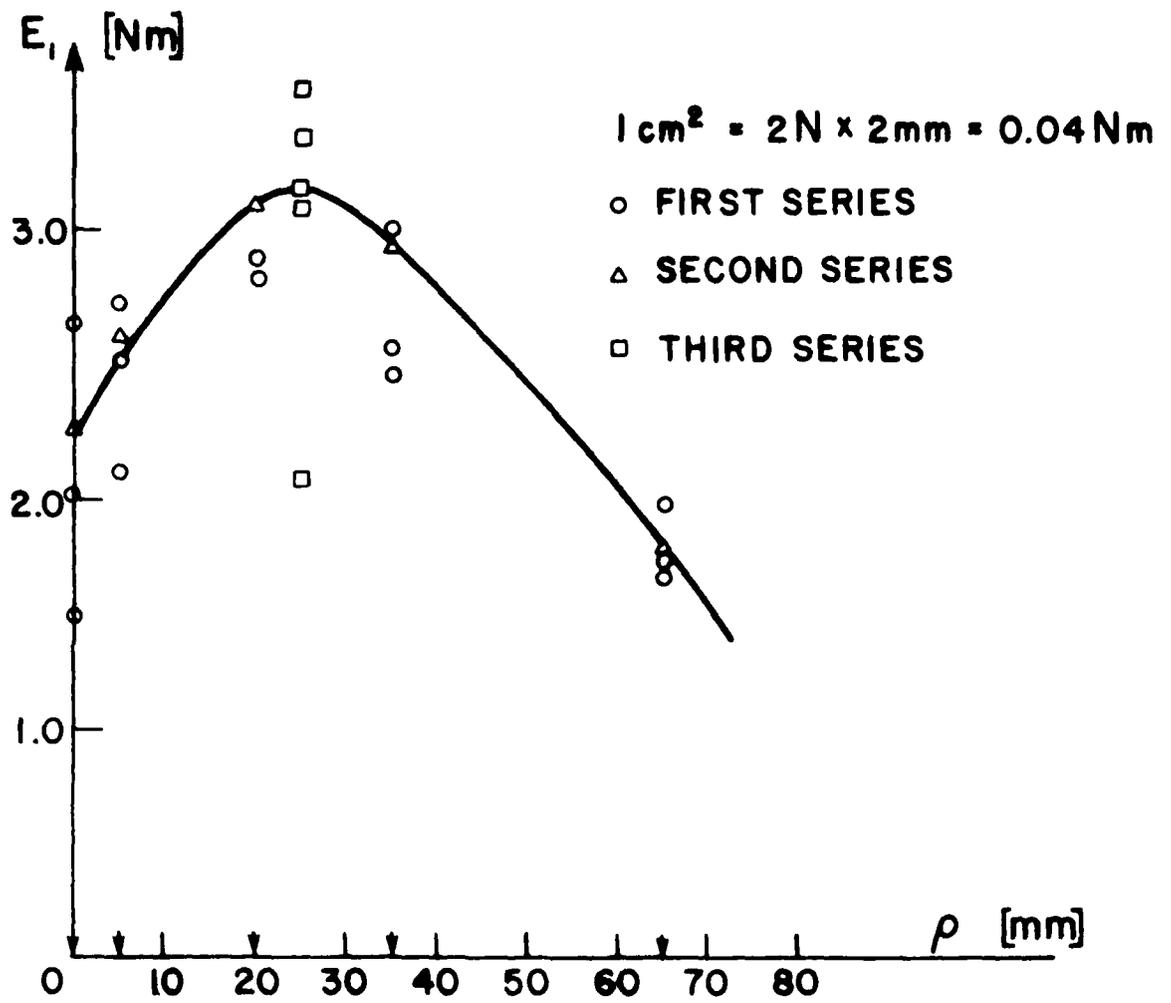


FIGURE 4

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Test no. 9,  $h=10\text{ mm}$ ,  $n=3$ ,  $r=0\text{ mm}$

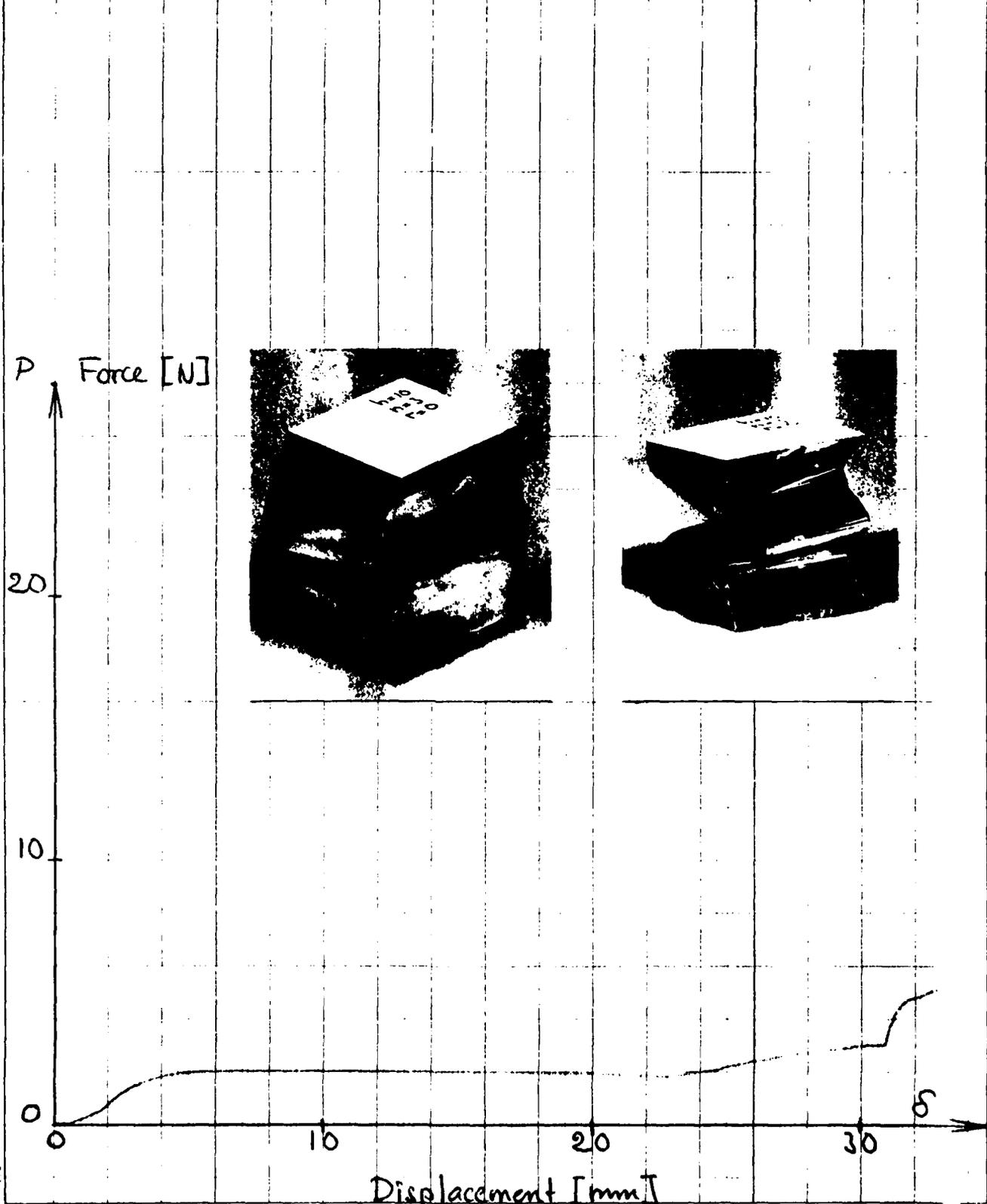


Figure 5

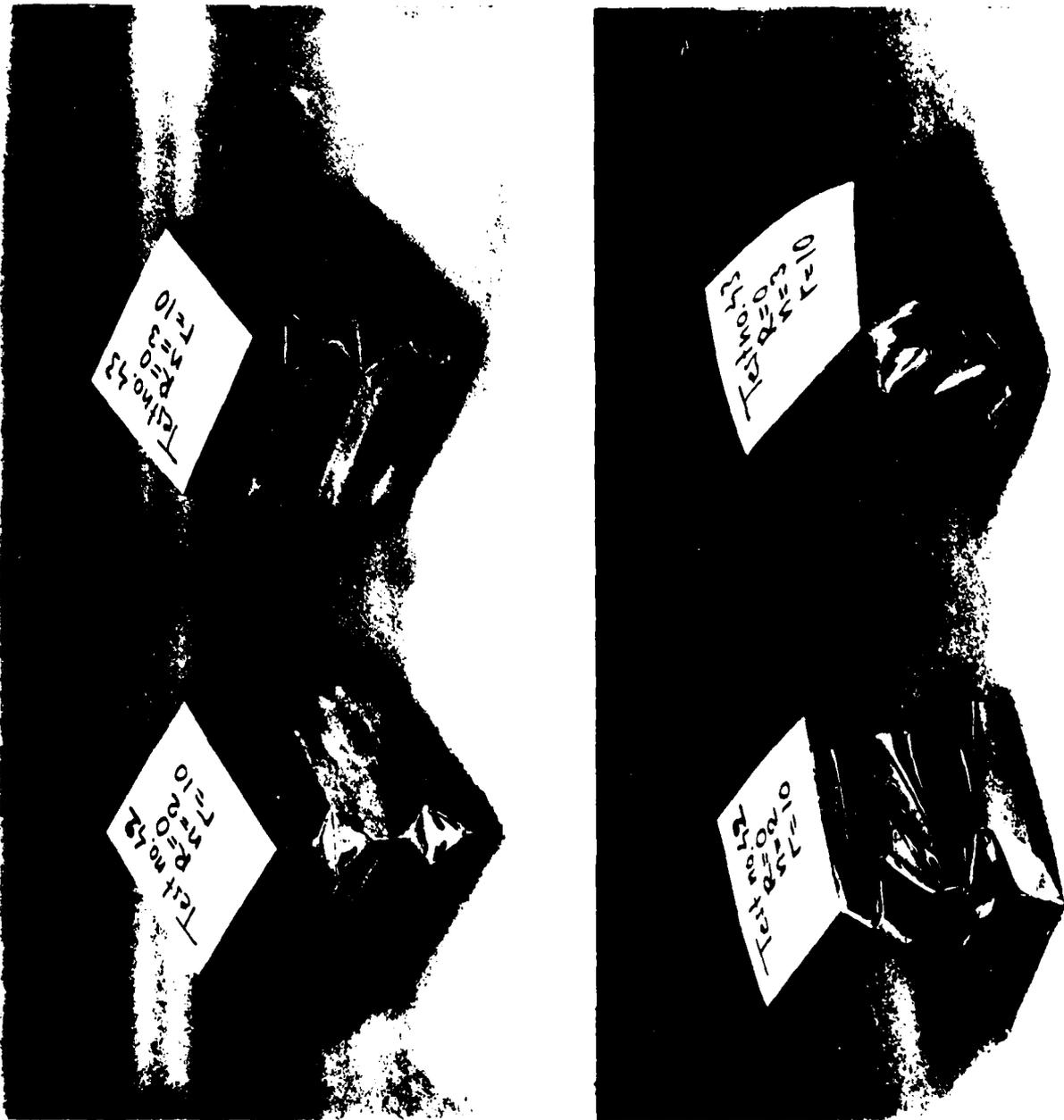


Figure 6



Figure 7



Figure 8



Figure 2

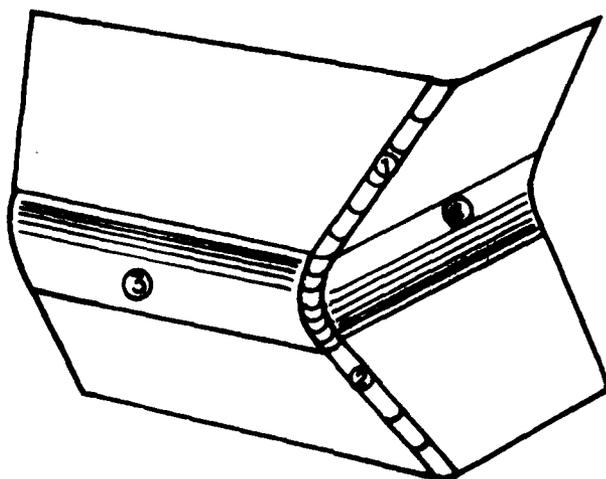


FIGURE 10

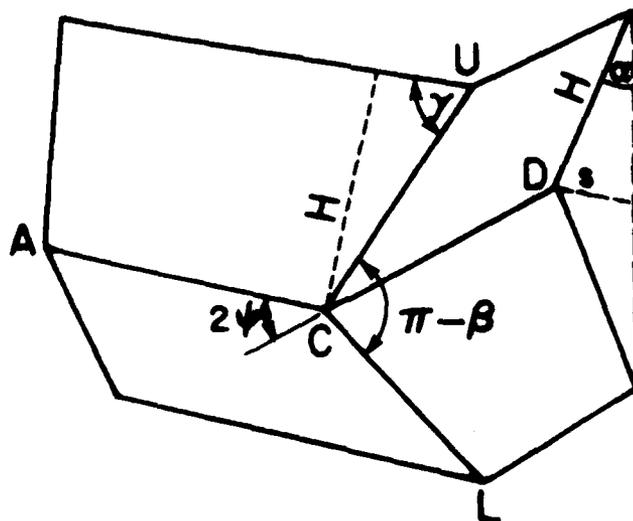


FIGURE 11

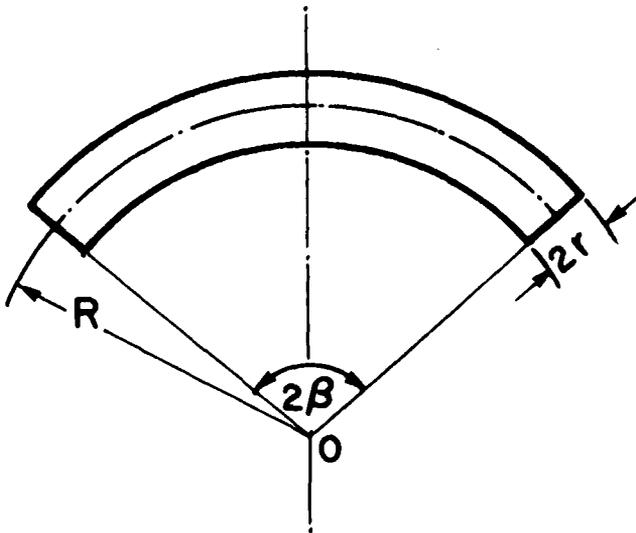
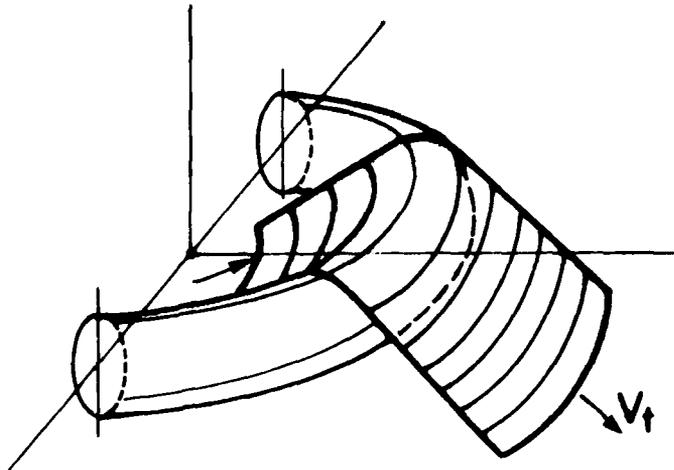


FIGURE 12

$$\bar{H} = \frac{H}{\sqrt[3]{c^2 h}}$$

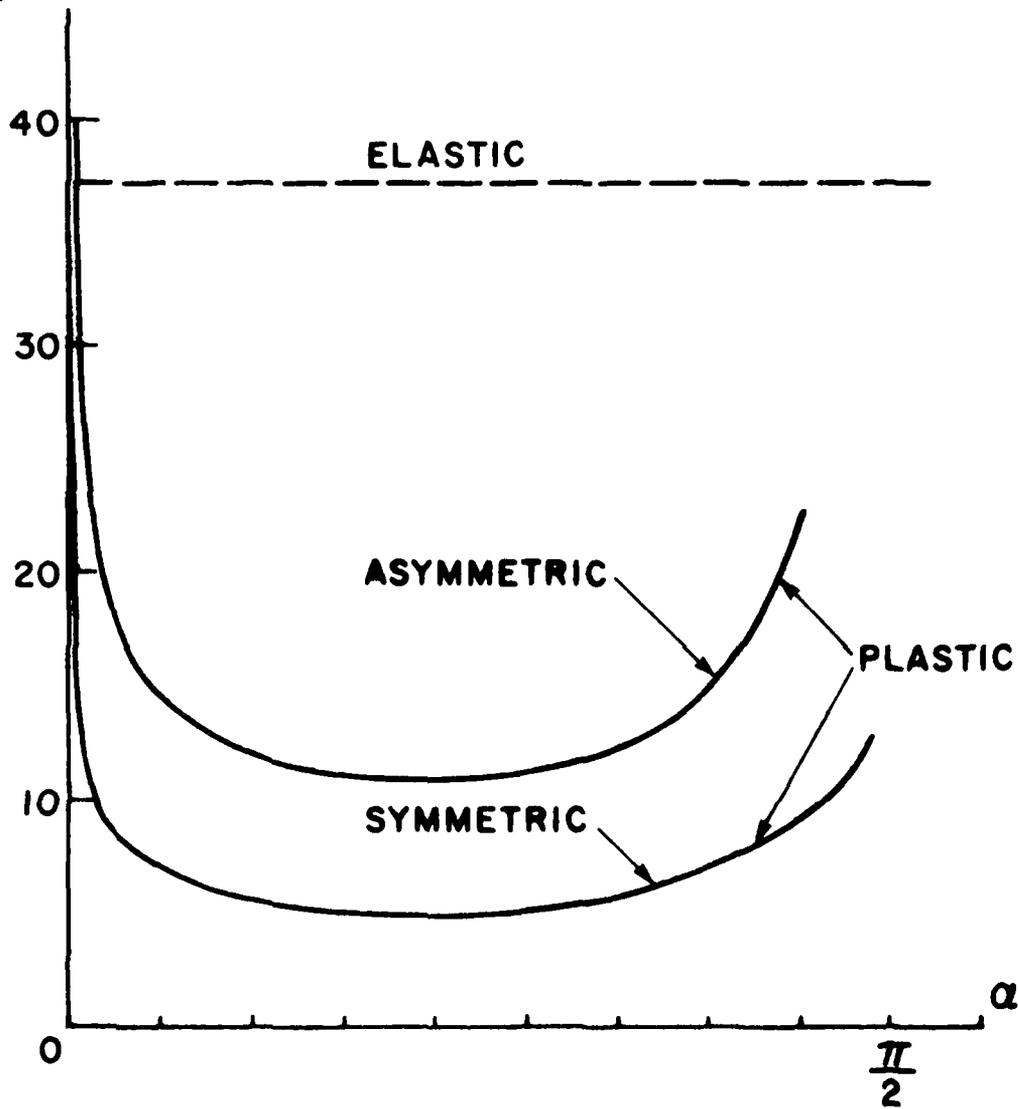


FIGURE 13

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Test no. 32.  $R=25$ ,  $n=1$ ,  $r=2$

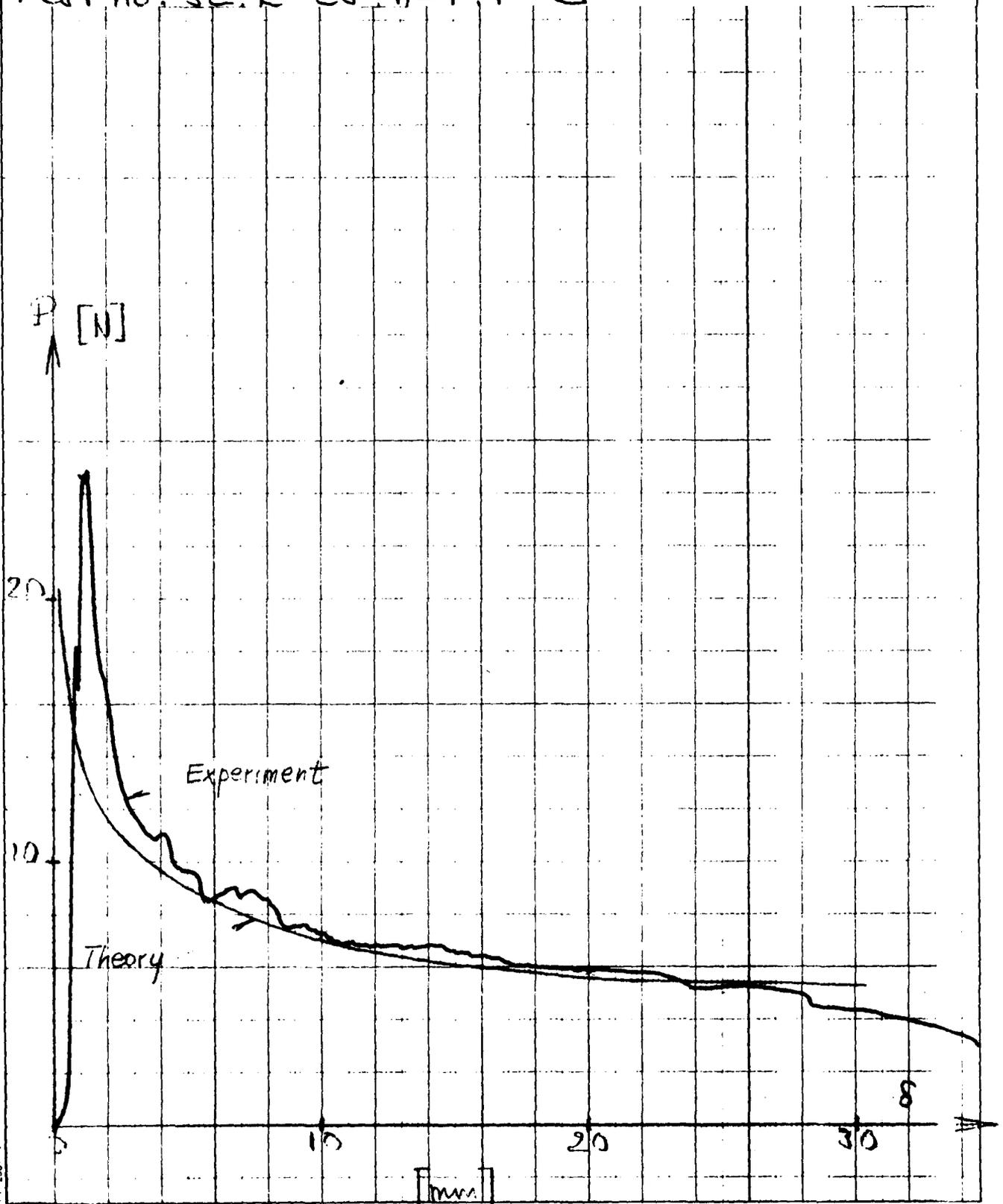


Figure 14

**VOLVO**

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test no. 35.  $R=25$ ,  $n=1$ ,  $r=10$

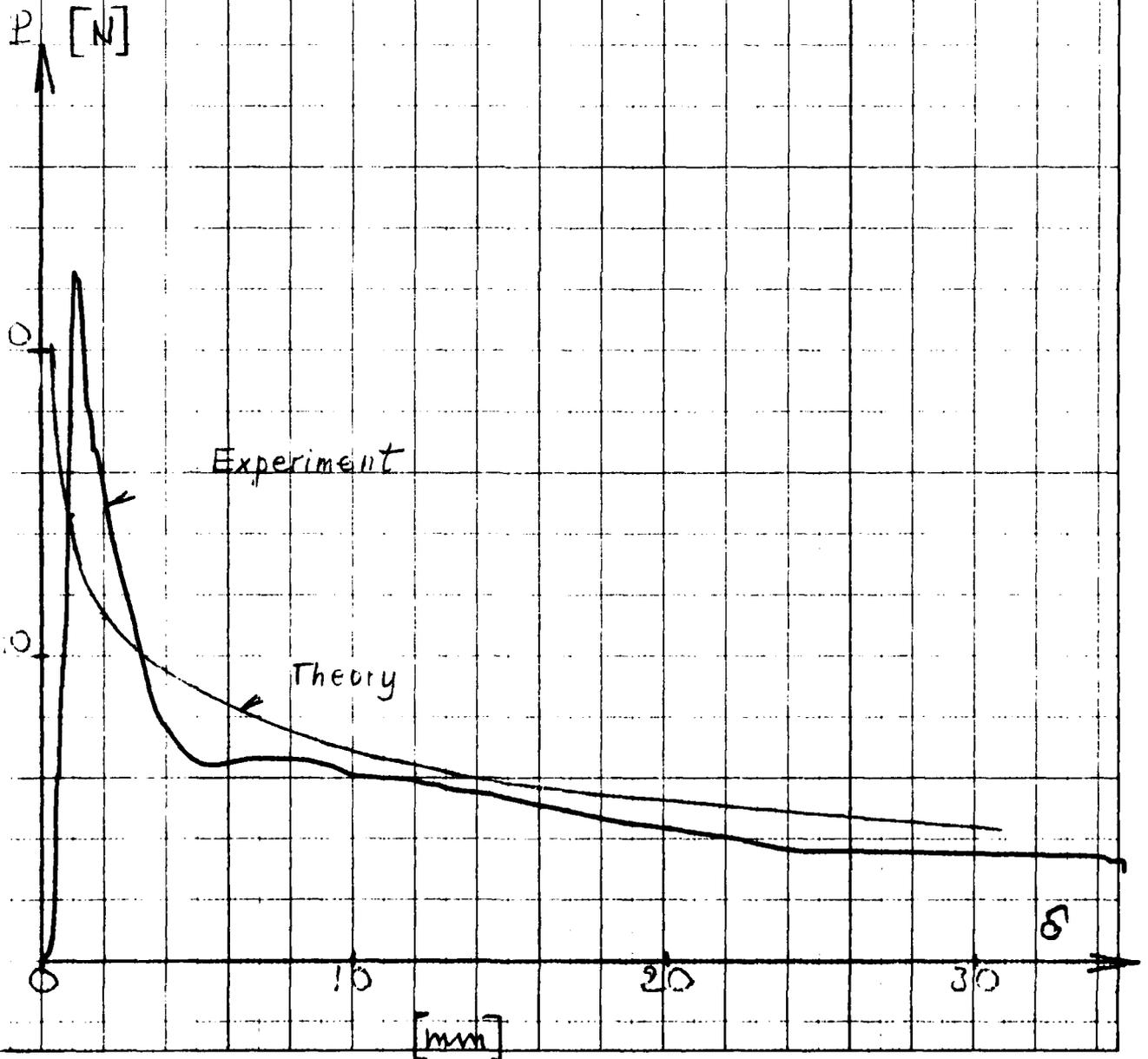
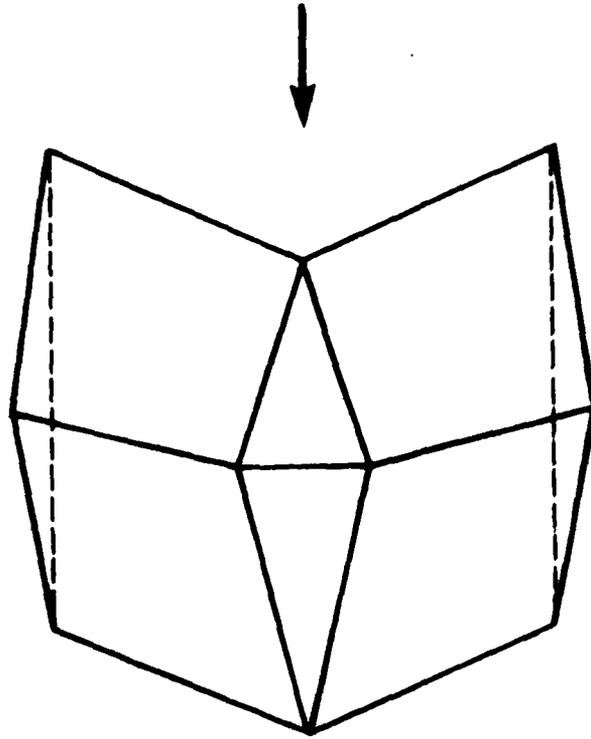


Figure 15



TOP VIEW

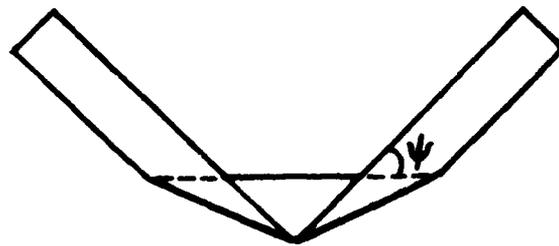


FIGURE 16

**VOLVO**

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Test no. 30, R=25

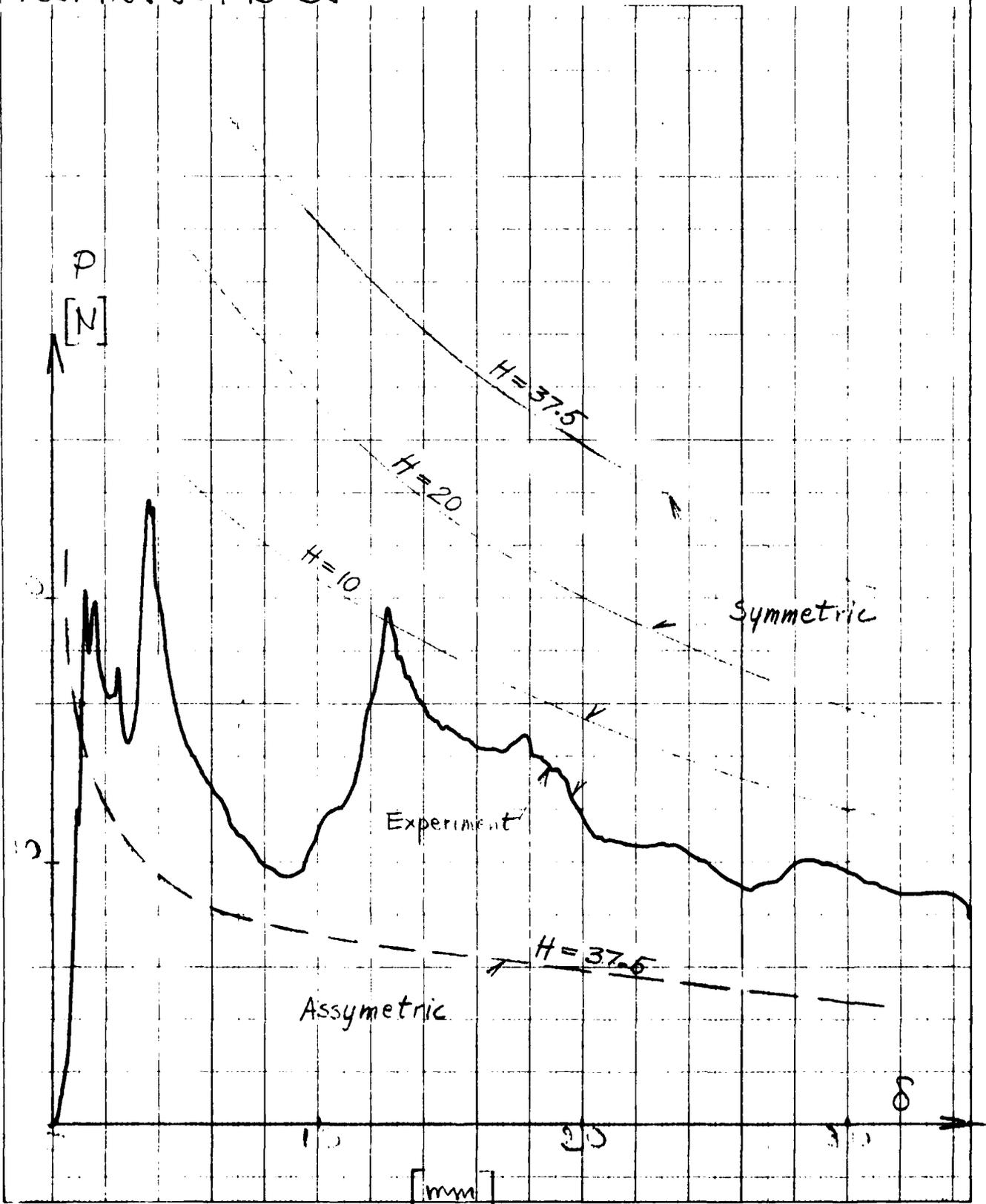
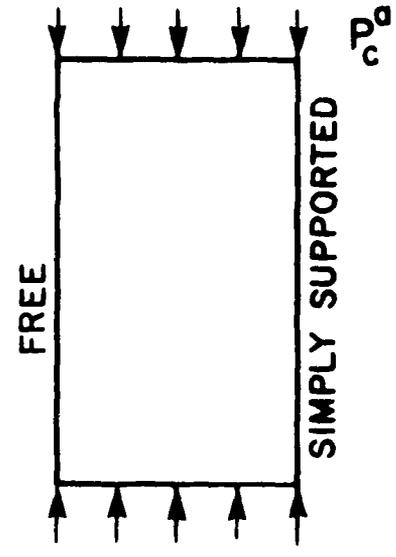
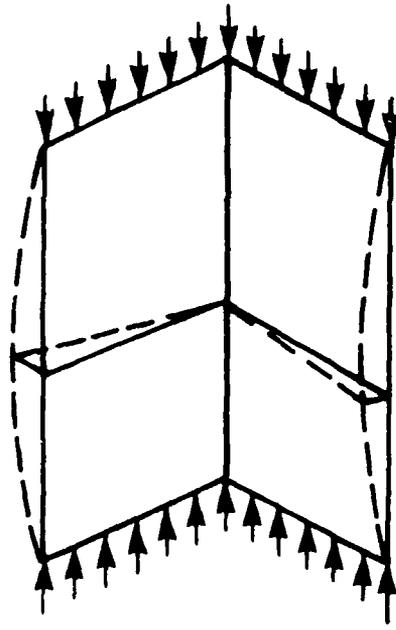
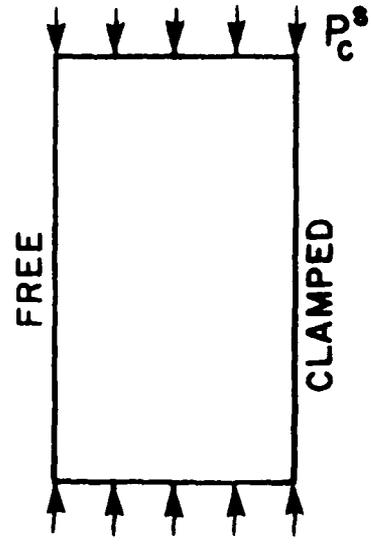
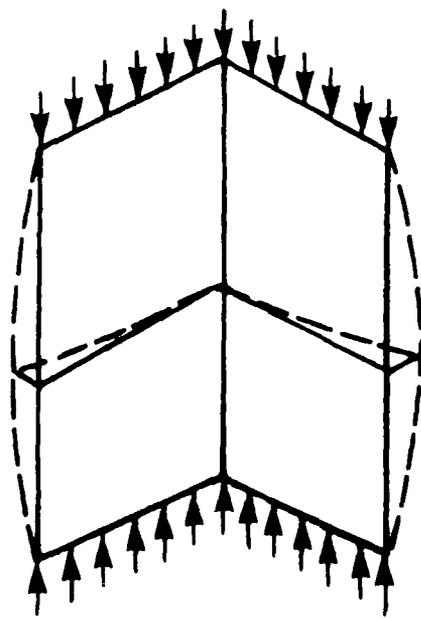


Figure 17



ASYMMETRIC MODE



SYMMETRIC MODE

FIGURE 18

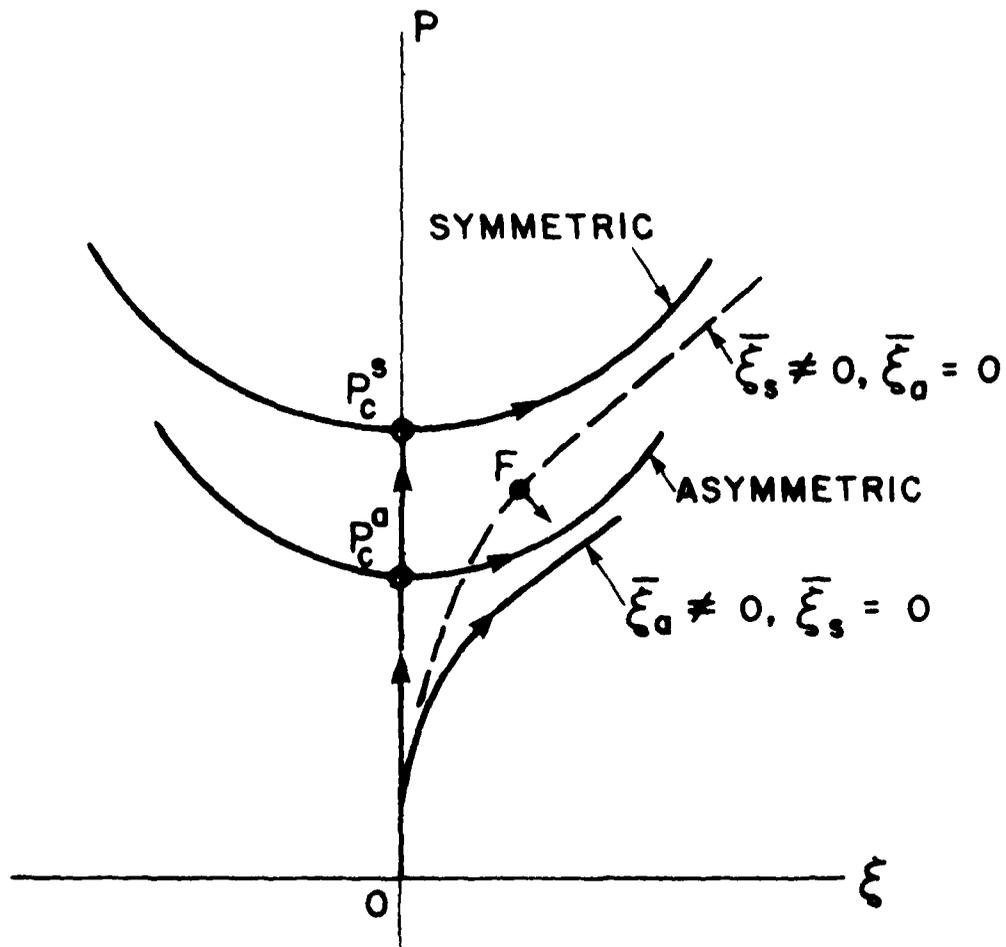


FIGURE 19

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Report Number 81-6

Shape Organization of Sheet Metal  
Structures Against Crash by Tomasz  
Wierzbicki, Ture Akerstrom and Clas  
Jersntrom, July 1981

The crushing behavior of axially compressed short thin-walled open-section columns was studied. Results of model tests on 0.1mm thick aluminum foil specimens have shown that the panels collapsing in the symmetric and asymmetric deformation mode provide respectively, upper and lower bound for the energy absorbed in any other buckling mode. In both cases, the crush response of the panel was predicted theoretically with a reasonable accuracy. Attempts were made to introduce beneficial geometric imperfections of a specified magnitude so that the structure will be forced to collapse in the most energy efficient deformation mode.

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other buckling mode. In both of the extreme cases, the crush response of the panel was predicted theoretically with a reasonable accuracy. It is shown that an optimum design of columns against crush can be achieved by introducing a beneficial geometric imperfections of a specified magnitude so that the structure will be forced to collapse in the most energy efficient deformation mode.