NONLINEAR STABILIZATION OF THE FARLEY-BUNEMAN INSTABILITY BY ST--ETC(U)

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NONLINEAR STABILIZATION OF THE FARLEY-BUNEMAN INSTABILITY BY STRONG E X B TURBULENCE*

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It is shown that through nonlinear mode coupling processes long wavelength low frequency strong E X B turbulence can stabilize short wavelength high frequency Farley-Buneman modes in a weakly ionized low pressure convecting plasma. Favorable comparisons are made with experimental observations.

Strong turbulence
Equatorial electrojet
Nonlinear saturation
Farley-Buneman instability
NONLINEAR STABILIZATION OF THE FARLEY-BUNEMAN
INSTABILITY BY STRONG E X B TURBULENCE

It is well known that in the absence of a magnetic field the two stream
instability can occur in a homogeneous plasma when the electron drift velo-
city with respect to the ions exceeds the electron thermal velocity.\(^1\)

Farley\(^2\) and Buneman\(^2\) have shown that, in the presence of a magnetic field,
the electron drift velocity with respect to the ions has only to exceed the
ion acoustic velocity \(C_s\) to generate unstable waves traveling perpendicular
to the magnetic field. We will consider the nonlinear evolution of the
Farley-Buneman instability in a low \(\beta\), weakly ionized, convecting plasma
which is subjected to a magnetic field \(\mathbf{B}\), an electric field \(\mathbf{E}\) and a
density gradient \((\partial n_0/\partial z)\). Differences in the collision frequencies
\(\nu_i/\Omega_i \gg 1\), \(\nu_e/\Omega_e \ll 1\) of the ions and electrons with the background neu-
tral gas results in the formation of a cross field current \(-J_0\) from the
\[\mathbf{V}_d = \mathbf{E} \times \mathbf{B}/B^2\] electron drift. For weak currents \(J_0\) long wavelength field
aligned fluctuations in density \(\delta n = \exp[i(ky-\omega t)]\) have been found by
Simon\(^3\) and Hoh\(^3\) to be linearly unstable when \(\nabla \cdot \mathbf{E} n_0 > 0\). In the nonlinear
regime this \(\mathbf{E} \times \mathbf{B}\) gradient drift instability evolves into an isotropic two-
dimensional strongly turbulent state in the plane perpendicular to the mag-
netic field as shown previously.\(^4\) For stronger currents such that \(V_d > C_s\)
the Farley-Buneman instability will develop at shorter wavelengths. These
long and short wavelength modes can coexist simultaneously (see Fig. 1) with
the former usually occurring before the latter. Previous studies of the
nonlinear evolution and saturation of these short wavelength Farley-
Buneman modes have invoked quasilinear effects,\(^5\) resonance broadening,\(^6\) and
mode coupling.\(^7\) These works have neglected the effects of the strong large
scale background \(\mathbf{E} \times \mathbf{B}\) turbulence. In this Letter we show that the long
wavelength \(\mathbf{E} \times \mathbf{B}\) turbulence can stabilize the short wavelength high fre-
quency Farley-Buneman instability. Although the following discussion is

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applicable to any weakly ionized low $\beta$ current carrying plasma convecting in regions of $\mathbf{E} \times \mathbf{B}$ turbulence it has direct bearing on density irregularities in the equatorial electrojet ionospheric plasma.

The basic equations for the electron ($N_e$) and ion ($N_i$) fluids in a low $\beta$, weakly ionized, collisional plasma can be written

\[
\frac{\partial N}{\partial t} + \nabla \cdot N \mathbf{V} = 0 \tag{1}
\]

\[
eN(\mathbf{E} + \mathbf{V} \times \mathbf{B}) + TN + N_e \mathbf{V} = 0 \tag{2}
\]

\[
\frac{\partial N}{\partial t} + \nabla \cdot N \mathbf{V} = 0 \tag{3}
\]

\[
(\partial/\partial t + \mathbf{V}_i \cdot \nabla) N_i \mathbf{V}_i = eN(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) - TN - N_i \mathbf{V} \mathbf{V}_i \tag{4}
\]

\[
\nabla \cdot (\mathbf{J}_i + \mathbf{J}_e) = 0 \tag{5}
\]

where we have assumed quasineutrality ($N_e = N_i = N$), isothermality ($T_e = T_i$), electrostatic fluctuations ($\mathbf{E} = -\nabla \Phi$), and neglected electron inertia.

Linearizing equations (1)-(5) with $N = n_o + n$, etc. and assuming fluctuations of the form $n, \phi, \mathbf{v}_e, \mathbf{v}_i \propto \exp \{i(k_y y + k_z z - (\omega_{kr} + i\gamma(k)t))\}$ with $k \cdot B = 0$ and $k_y L >> 1$ we find\(^9\) for the frequency and growth rate in the ion frame

\[
\omega_{kr} = k^2 \mathbf{v}_d / (1 + \psi) \tag{6}
\]

\[
\gamma(k) = \left[\psi / (1 + \psi)\right] \left\{ (\omega_e/\mathbf{v}_e)(\mathbf{v}_d/L)\cos^2 \theta + \omega_{kr}^2/\mathbf{v}_i - k^2 c_s^2/\mathbf{v}_i \right\} \tag{7}
\]

where $\psi = \mathbf{v}_e \mathbf{v}_i / \mathbf{v}_e \mathbf{v}_i$, $L^{-1} = (1/n_0)(\partial n_0 / \partial z)$, $c_s^2 = 2kT/m_i$, and $\theta$ is the angle defined by $k$ and $\mathbf{v}_d$. From the expression for the growth rate $\gamma(k)$ in
eq. (7), we note that at low frequencies (long wavelengths and weak currents) such that \( \omega_k / \nu_i < (\Omega_e / \nu_e) (1 / kL) \) the \( E \times B \) gradient drift term will dominate with all modes with \( k < k_c = [(\Omega_e / \nu_e) (V_d / L) (\nu_i / c_s^2)]^\frac{1}{2} \) \( \cos \theta \) unstable. At higher frequencies (short wavelengths and strong currents) Farley-Buneman modes will become unstable if \( V_d > c_s \) but with no critical wavelength. In the equatorial electrojet plasma \( V_d(t) \) is time dependent varying from \( V_d < c_s \) to \( V_d > c_s \) over a time interval \( \Delta t \gg \gamma(k)^{-1} \). As a result the Farley-Buneman instability will be excited in strong \( E \times B \) turbulence. Since these waves are nondispersive they will interact strongly. However as their amplitude increases, they do not steepen appreciably but are unstable to perturbations perpendicular to their propagation. It must be noted that this fluid approximation is valid for \( \omega \leq \nu_i \). At higher frequencies kinetic effects, e.g., ion Landau damping, will become important and introduce a high frequency cutoff. For lower frequencies both fluid and kinetic treatments are identical.  

By writing \( N(x,t) = n_o + \sum_{k \omega} n(k,\omega) \exp[-i(k \cdot x - \omega t)] \), \( \phi(x,t) = \phi_o + \sum_{k \omega} \phi(k,\omega) \exp[-i(k \cdot x - \omega t)] \), etc., expanding eqs. (1)-(5) in the small parameter \( \nu_e / \Omega_i \ll \nu_i \sim \epsilon \ll 1 \), and considering high frequencies \( \omega_k \) such that \( \omega_k / \nu_i = (\Omega_e / \nu_e) (1 / kL) \approx 10^{-2} \) (for \( \lambda \sim 3 \text{m, } L = 6 \text{ km, } \Omega_e / \nu_e = 10^2 \)) we find to second order in \( n_{k \omega} \) \( (n/n_o \ll 1) \)

\[
D(k,\omega)n(k,\omega) = \int d^2 k' d\omega' V(k,k',\omega,\omega') n(k',\omega') n(k-k',\omega-\omega')
\]

where

\[
D(k,\omega) = \omega - k \cdot V_d (1 + \psi)^{-1} - i\nu \omega^2 (1 + \psi)^{-1} \nu_i^{-1} + i\psi k^2 c_s^2 / \nu_i
\]

is the Farley-Buneman dielectric and

\[
V(k,k',\omega,\omega') = -(\dot{x} \times \dot{k} \cdot \dot{k}' / k'^2) \left[(1 + \psi)^{-1} (\nu_i / \Omega_i) k' \cdot V_d + k' \cdot \dot{x} \times V_d + \dot{k}' c_s^2 / \nu_i\right]
\]

3
By neglecting the nonlinear term on the right hand side of eq. (8) we recover the linear result from \( D(k, \omega, k_\perp + i \gamma(\omega)) = 0 \) giving \( \omega_{kr} = k^2 V_d/(1 + \psi) \) and \( \gamma(\omega) = (\psi/1 + \psi)(1/v_i) [(k \cdot V_d)^2 - k^2 c_s^2] \).

We solve eq. (8) for the high frequency short wavelength component of \( n(k, \omega) \) by considering its mode coupling to the low frequency long wavelength well developed strong \( E \times B \) turbulence. Let a Farley-Buneman wave be denoted by \((k_I, \omega_I)\) and a turbulent \( E \times B \) mode by \((k_{II}, \omega_{II})\). Physically, when a Farley-Buneman mode \((k_I, \omega_I)\) grows to such a level that it can couple with the \( E \times B \) turbulence \((k_{II}, \omega_{II})\), a beat wave component \((k_I \pm k_{II}, \omega_I = \omega_{II})\) will appear which in turn can couple with \((k_{II}, \omega_{II})\) to affect \((k_I, \omega_I)\). The evolution of \( n(k_I, \omega_I) \) can then be written

\[
D^I(k_I, \omega_I) n^I(k_I, \omega_I) = \int d^2k' \, dw' \, V(k_I, k', \omega_I, \omega') n^{II}(k', \omega') n^{I-II}(k_I - k', \omega_I - \omega')
\]

(9)

The beat wave \((k_I - k_{II}, \omega_I - \omega_{II})\) evolves according to:

\[
D^{I-II}(k_I - k, \omega_I - \omega) n^{I-II}(k_I - k, \omega_I - \omega) = \int d^2k' \, dw' \, V(k_I - k, k', \omega_I, \omega') n^{II}(k', \omega') n^{I}(k_I, \omega_I)
\]

(10)

where we have used \( V(k, k', \omega, \omega') = V(k, -k', \omega, -\omega') \). Substituting eq. (10) into eq. (9) we find the nonlinear dispersion relation to lowest order

\[
\mathcal{D}^I(k_I, \omega_I) n^I(k_I, \omega_I) = 0
\]

(11)

where \( \mathcal{D}^I(k_I, \omega_I) = D^I(k_I, \omega_I) + \delta D^I(k_I, \omega_I) \). The nonlinear part of eq. (11) can be written

\[
\delta D^I(k_I, \omega_I) = -\int d^2k' \, dw' \, \frac{V(k_I, k', \omega_I, \omega) V(k_I - k, k', \omega_I - \omega', \omega')}{D^{I-II}(k_I - k', \omega_I - \omega')} n^{II}(k', \omega')
\]

(11a)
where $I_{\text{II}}(k,\omega) = \langle n_{\text{II}}(k,\omega)n_{\text{II}}(-k,\omega)/n_0^2 \rangle$ is the power spectrum of the $E \times B$ turbulence. It has been previously shown\textsuperscript{10} that $I_{\text{II}}(k,\omega)$ can be calculated using the direct-interaction approximation of Kraichnan\textsuperscript{11}

\[ |\omega-\omega(k)|^2 I_{\text{II}}(k,\omega) = \frac{1}{2} \int d^2 k' d\omega' |\omega(k,k')|^2 I_{\text{II}}(k',\omega') I_{\text{II}}(k-k',\omega-\omega') \]

(12)

\[ \Gamma_{\text{II}}(k,\omega) = -\int d^2 k' d\omega' \frac{\omega(k,k-k',k')I_{\text{II}}(k',\omega')}{\omega-\omega'(k-k',\omega-\omega')} \]

(13)

with $\omega(k,k') = \omega(k,k-k')$ and $\omega(k) = \omega_{k-1} + i\gamma(k)$ where

\[ \gamma(k) = (\psi/1 + \psi)[(\alpha/\nu_e)(V_d/L)\cos^2 \theta - \frac{k^2 c_s^2}{\nu_1}] \]

In eq. (12) $I_{\text{II}}(k,\omega)$ is the self-damping of the long wavelength $E \times B$ fluctuations $(k_{\text{II}},\omega_{\text{II}})$. We now proceed to solve equations (11)-(13).

First, we note that since the interacting waves considered here are non-dispersive (see eq. (6)) the beat wave dielectric $D_{\text{I-II}}(k-I-w_{\text{I}}-w_{\text{II}}) = 0$ and the right hand side of eq. (11a) diverges. As a result we replace

\[ D_{\text{I-II}}(k-I-w_{\text{I}}-w_{\text{II}}) \]

by its renormalized value $D_{\text{I-II}}(k-I-w_{\text{I}}-w_{\text{II}}) + \delta D_{\text{I-II}}(k-I-w_{\text{I}}-w_{\text{II}}) \sim \delta D_{\text{I-II}}(k-I-w_{\text{I}}-w_{\text{II}}) \) in eq. (11a). In order to study the nonlinear saturation we solve eq. (11a) for $\text{Im} \delta D_{\text{I}} = -i\Gamma_{\text{II}}(k_{\text{I}},w_{\text{I}}(k_{\text{I}}))$. From previous studies\textsuperscript{10} the steady state solution of eqs. (12)-(13) can be written

\[ I_{\text{II}}(k,\omega) = I_{\text{II}}(k)(2\pi)^{-1/2}(\Gamma_{\text{II}}(k))^{-1}\exp[-(\omega-\omega(k))^2/2(\Gamma_{\text{II}}(k))^2] \]

(14)

where $\omega(k) = kV_d/(1+\psi)$, $\Gamma_{\text{II}}(k) = 3.4 n^{-2}(\nu_1/\nu_e)k^2 V_d(I_{\text{II}}(k))^{-1/2}$, and $I_{\text{II}}(k) = Ik^{-n}$ is isotropic\textsuperscript{11,12} with $n = 3-4$ while\textsuperscript{13} $I \approx V_d^m, m = 2$. Substituting eq. (14) into (11a) and assuming that $k' \sim k_{\text{II}} < k_{\text{I}} = k$, $\omega' \sim \omega_{\text{II}} < \omega_{\text{I}} = \omega$, $D_{\text{I-II}}(k_{\text{I}}-k',w_{\text{I}}-\omega') \sim D_{\text{I}}(k_{\text{I}},w_{\text{I}}) + O(k'/k_{\text{I}}) \sim \delta D_{\text{I}}(k_{\text{I}},w_{\text{I}})$ we find

5
\[ \Gamma(k)^2 = \int d^2k' V(k',k') V(k-k',k') \Gamma(k') \]
\[ = (\nu_i/\Omega_i)^2 \int d^2k' (\hat{x} \times k' \cdot k)^2 (k' \cdot V_d)^2 |k'|^{-\nu} \Gamma(k') \]
\[ = (\nu_i/\Omega_i)^2 k^2 V_d^2 \int d\theta' k' dk' \sin^2(\theta-\theta') \cos^2 \theta' \Gamma(k') \]

where \( \Gamma(k) \gg \gamma(k) \) has been assumed and \( \theta' \) and \( \theta \) are the angles made by \( k' \) and \( k \), respectively, with \( V_d \). In evaluating the quantity \( V(k,k') V(k-k',k) \) in eq. (15) we have kept only terms proportional to \( (\nu_i/\Omega_i)^2 \)

\( \nu_i/\Omega_i \gg 1 \). Experimental studies\(^{14} \) as shown in Figure 2(a) indicate that \( \Gamma(k) \approx k \) is approximately independent of angle \( \theta \). This allows the replacement of \( \Gamma(k) \) in eq. (15) by its angle averaged result \( (2\pi)^{-1} \int d\theta \Gamma(k) = \Gamma(k) \) giving

\[ \Gamma(k) = \langle \nu_i/\Omega_i \rangle \left( \frac{1}{2} \right) k V_d < |n/n_0|^2 \]

where \( < |n/n_0|^2 \rangle = \int d\theta k dk \Gamma(k) = 2\pi \int dk k \Gamma(k) \). Using \( \nu_i/\Omega_i \approx 22 \)

\[ 2\pi/k \approx 5 \text{ m}, \ V_d = 4 \times 10^2 \text{ m/sec}, \ c_s \approx 3.6 \times 10^2 \text{ m/sec}, \text{ and } < |n/n_0|^2 \rangle \approx 0.01 \]

\( \text{eq. (16) gives for the spectral width } \Gamma(k)/2\pi \approx 10 \text{ Hz (cf. Fig. 1). Figure 2(b) shows that the scaling of } \Gamma(k) \approx k \text{ from eq. (16) is in reasonable agreement with experimental results}^{12,14} \text{ which indicate that the spectral broadening } \omega_k \approx \Gamma(k) \approx k^{\frac{1}{7}} \).\]

The nonlinear dispersion relation in eq. (11) for the short wavelength Farley-Buneman instability in the long wavelength \( E \times B \) turbulent background can then be written explicitly

\[ w - k \nu_d (1 + \nu) - i \nu \omega^2 (1 + \nu)^{-1} \nu_i^{-1} + i \nu k^2 c_s^2 / \nu_i + i \frac{1}{2} (\nu_i / \Omega_i) k V_d < |n/n_0|^2 \rho_2 \]

\( = 0 \).

\[ \text{(17)} \]

Separating \( w = \omega_k + i \gamma(k) \) we find

\[ \gamma(k) = \left[ \psi/(1 + \psi) \right] \left\{ (k \nu_d)^2 / \nu_i - k^2 c_s^2 / \nu_i \right\} - \frac{1}{2} (\nu_i / \Omega_i) k V_d < |n/n_0|^2 \rho_2 \]

\( \text{(17a)} \)
For complete stabilization of the fastest growing linear mode ($\theta = 0$) the long wavelength background fluctuation level

$$< |n/n_o|^2 > \gtrsim 2 \nu_e \Omega_s^{-1}(1 + \psi)^{-1}(kV_d/\nu_i - k^2 c_s^2/\nu_i kV_d).$$

with $kV_d/\nu_i = 0.1$, $k^2 c_s^2/\nu_i kV_d = 0.05$ we find $n/n_o \gtrsim 0.002$ which is consistent with available experimental estimates.

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Fig. 1 — Simultaneous equatorial electrojet ionospheric plasma density fluctuation power spectra (vertical axis $I(k,\omega)$ vs. frequency $\omega$ in Hz (horizontal axis) from Ref. 14 for several radar backscatter observation frequencies; the radar frequencies 29.0 MHz, 9.0 MHz, 6.6 MHz, 5.6 MHz correspond to wavelengths 5.1m, 10.5m, 16.6m, 22.7m, and 26.7m, respectively.
Fig. 2 — (a) Power spectrum width vs. wave vector angle made by $k$ and $V_d$, the electrojet electron drift velocity, at several wavelengths from Ref. 14; note approximate isotropy in angle. (b) Power spectrum width vs. wave number $k$ at $\theta = 15^\circ$ from Ref. 14; solid line is best fit $\propto k^{0.7}$. 

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