EFFECTS OF SURFACE HEAT TRANSFER ON BOUNDARY-LAYER TRANSITION

by

D. C. Wilcox and A. L. Chambers

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Principal Investigator/Program Manager:

David C. Wilcox,
(213) 990-2682

DCW Industries
13535 Ventura Boulevard, Suite 207
Sherman Oaks, California 91423
(213) 990-2682

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ABSTRACT

Effects of surface heat transfer on boundary-layer transition are analyzed in a three-part study using the Saffman-Wilcox transition model. In the first part of the study, model predictions are compared with experimental data for cooled and heated aerodynamic boundary layers on smooth flat surfaces and for cooled aerodynamic boundary layers near the stagnation point of a roughened blunt body. Consistent with measurements, the model predicts, on the one hand, that heating destabilizes a smooth-surface aerodynamic boundary layer and, on the other hand, that cooling destabilizes a rough-surface aerodynamic boundary layer. Differences between predicted and measured transition-point locations are within experimental error bounds. Then, incipient transition conditions are determined for a small, heated hydrodynamic body. Again model predictions agree with measurements which indicate that relatively small amounts of surface heating have a strong stabilizing effect on hydrodynamic boundary layers. In the final part of the study, transition location is determined for a large hydrodynamic body; results indicate that large surface heating rates are not substantially more effective than smaller rates.
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### NOTATION

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<tr>
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<tbody>
<tr>
<td>$c_p$</td>
<td>Pressure coefficient, $(p-p_\infty)/(1/2\rho U_\infty^2)$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$e$</td>
<td>Specific turbulent energy</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Parsons-Goodson-Goldschmied shape parameter</td>
</tr>
<tr>
<td>$F(\eta)$</td>
<td>Nondimensional self-similar velocity profile</td>
</tr>
<tr>
<td>$j$</td>
<td>0 for two-dimensional flow; 1 for axisymmetric flow</td>
</tr>
<tr>
<td>$k$</td>
<td>Roughness height</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Parsons-Goodson-Goldschmied shape parameter</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$Pr_L$, $Pr_T$</td>
<td>Laminar and turbulent Prandtl numbers</td>
</tr>
<tr>
<td>$q_w$, $\bar{q}_w$</td>
<td>Surface heating rate; average value of $q_w$</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial distance from symmetry axis</td>
</tr>
<tr>
<td>$r_i$, $r_n$</td>
<td>Parsons-Goodson-Goldschmied shape parameters</td>
</tr>
<tr>
<td>$R_\infty$</td>
<td>Empirical constant</td>
</tr>
<tr>
<td>$Re_s$, $Re_x$</td>
<td>Reynolds number based on arc length, plate length</td>
</tr>
<tr>
<td>$Re_\theta$</td>
<td>Reynolds number based on momentum thickness</td>
</tr>
<tr>
<td>$Re_V$</td>
<td>Incipient transition Reynolds number based on $V^{1/3}$</td>
</tr>
<tr>
<td>$s_i$</td>
<td>Parsons-Goodson-Goldschmied shape parameter</td>
</tr>
<tr>
<td>$s_t$</td>
<td>Parsons-Goodson-Goldschmied shape parameter</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_t$, $T'_t$</td>
<td>Freestream total temperature</td>
</tr>
<tr>
<td>$T'$</td>
<td>Freestream turbulence intensity</td>
</tr>
<tr>
<td>$u$, $v$</td>
<td>Velocity component in x, y direction</td>
</tr>
<tr>
<td>$U_e$</td>
<td>Boundary-layer-edge velocity</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Freestream velocity</td>
</tr>
<tr>
<td>$V$</td>
<td>Body volume</td>
</tr>
</tbody>
</table>
### SYMBOL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x, y$</td>
<td>Distance parallel to, normal to body surface</td>
</tr>
<tr>
<td>$x_m, x_i$</td>
<td>Parsons-Goodson-Goldschmied shape parameter</td>
</tr>
<tr>
<td>$\alpha, \alpha^*$</td>
<td>Empirical parameters</td>
</tr>
<tr>
<td>$\beta, \beta^*$</td>
<td>Empirical constants</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Similarity variable</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Momentum thickness</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Karman's constant; thermal conductivity</td>
</tr>
<tr>
<td>$\lambda, \lambda^*$</td>
<td>Empirical parameters</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Modified Polhausen pressure gradient parameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Molecular viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
</tr>
<tr>
<td>$\sigma, \sigma^*$</td>
<td>Empirical constants</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Specific dissipation rate</td>
</tr>
</tbody>
</table>

### Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>Boundary layer edge</td>
</tr>
<tr>
<td>$t$</td>
<td>Transition point</td>
</tr>
<tr>
<td>$w$</td>
<td>Surface</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Freestream</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

In recent years, substantial progress has been made toward maintaining laminar flow, and hence low drag, over aerodynamic/hydrodynamic bodies at practical flow speeds. In the case of aerodynamic bodies, the use of carefully designed airfoil shapes (e.g. to provide strong favorable pressure gradient) and the use of boundary-layer-control techniques (e.g. suction) has successfully delayed transition to chord-length Reynolds numbers, $Re_{St}$, of the order of 10 million. Even larger values of $Re_{St}$ have been achieved for small hydrodynamic bodies through the use of surface heating and pressure gradient. For hydrodynamic bodies, extrapolations based on linear stability theory indicate that, with practicable amounts of surface heating, values of $Re_{St}$ in excess of 200 million may be possible on relatively large hydrodynamic bodies.

The reduction in drag which can be achieved by maintaining laminar flow over any vehicle is attractive because of the reduced power requirements to move the vehicle. However, a penalty is generally paid in maintaining laminar flow on lifting bodies in that a laminar boundary layer separates much more easily than does a turbulent boundary layer, resulting in a significant reduction in lift. Hence, maintaining laminar flow is practical mainly for nonlifting bodies. Submarines and torpedos or, more generally, hydrodynamic bodies fall into the latter class.

This study focuses on the observed pronounced effects of surface heat transfer on boundary layer transition. Most importantly, this report includes transition predictions based on a relatively new transition theory for small and large heated hydrodynamic bodies.
Section 2 presents the transition equations, including a modification needed to improve transition-prediction accuracy when surface heat transfer is present. Included in Section 3 are transition computations for various aero- dynamic and hydrodynamic boundary layers. The concluding section summarizes results and conclusions.
2. FORMULATION

2.1 EQUATIONS OF MOTION

The Saffman-Wilcox transition model\(^2\) is the basic tool used in this study to analyze effects of surface heat transfer on boundary-layer transition. The model's accuracy has previously been demonstrated for a wide variety of flows ranging from incompressible boundary layers to hypersonic blunt-body flows. These applications have tested the model's ability to predict transition sensitivity to effects of free-stream turbulence, suction, surface roughness and pressure gradient. In all cases, accuracy acceptable for most engineering purposes has been obtained.

For incompressible boundary layers (i.e., for very small Mach numbers), the equations of motion which constitute the transition model are:

**Mass Conservation**

\[
\frac{\partial}{\partial x} (r^J u) + \frac{\partial}{\partial y} (r^J v) = 0 \tag{1}
\]

**Momentum Conservation**

\[
\rho \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{1}{r^J} \frac{\partial}{\partial y} \left[ r^J \left( u + \frac{e}{\Omega} \right) \frac{\partial u}{\partial y} \right] \tag{2}
\]

**Energy Conservation**

\[
\rho u \frac{\partial (C_p T)}{\partial x} + \rho v \frac{\partial (C_p T)}{\partial y} = \frac{dp}{dx} + \frac{1}{r^J} \frac{\partial}{\partial y} \left[ r^J \left( \frac{C_p}{Pr_L} + \frac{e/\Omega}{Pr_T} \right) \frac{\partial C_p T}{\partial y} \right] \tag{3}
\]
Turbulent Energy
\[
\rho u \frac{\partial e}{\partial x} + \rho v \frac{\partial e}{\partial y} = \left[ \alpha \left( \frac{\partial u}{\partial y} - \beta \rho \right) \right] \rho e + \frac{1}{r^J} \frac{\partial}{\partial y} \left[ r^J (\mu + \sigma \frac{e}{\Omega}) \frac{\partial e}{\partial y} \right]
\] (4)

Turbulent Dissipation Rate
\[
\rho u \frac{\partial \Omega^2}{\partial x} + \rho v \frac{\partial \Omega^2}{\partial y} = \left[ \alpha \left( \frac{\partial u}{\partial y} - \beta \rho \right) \right] \rho \Omega^2 + \frac{1}{r^J} \frac{\partial}{\partial y} \left[ r^J (\mu + \sigma \frac{e}{\Omega}) \frac{\partial \Omega^2}{\partial y} \right]
\] (5)

In Equations 1-5, x and y are orthogonal coordinates parallel to and normal to a body surface. The quantity r is the radial coordinate from the body's symmetry axis while j=0 for two-dimensional flow and j=1 for axisymmetric flow. The velocity components in the x and y directions are denoted by u and v; fluid density, temperature, pressure, specific heat, and viscosity are denoted by \( \rho \), T, p, C_p, and \( \mu \) respectively. The quantities e and \( \Omega \) are specific turbulent energy and specific turbulent dissipation rate; their ratio, \( e/\Omega \), is the turbulent eddy viscosity.

Both air and water are considered in this study. The various thermodynamic properties for air are related through the perfect gas law and the Sutherland viscosity law. Appendix A lists the pertinent thermodynamic properties of water.

Several empirical parameters appear in Equations 1-5. For transitional flows, past studies have established the following values:

\[
\begin{align*}
\text{Pr}_T &= 0.89 \\
\beta &= 0.15, \quad \beta^* = 0.09 \\
\sigma &= 0.50, \quad \sigma^* = 0.50
\end{align*}
\] (6)
\[ \alpha^* = 0.30 \left[ 1 - (1 - \lambda^*)(1 - \text{Re}_t/R_o) H(1 - \text{Re}_t/R_o) \right] \]  

(7)

\[ \alpha = 0.1638 \left[ 1 - (1 - \lambda)(1 - \text{Re}_t/R_o) H(1 - \text{Re}_t/R_o) \right] \]  

(8)

where \( H(x) \) is the Heaviside step function and \( \text{Re}_t \) is turbulent Reynolds number defined by

\[ \text{Re}_t = e/\Omega u \]  

(9)

Finally, the parameters \( \lambda^* \) and \( R_o \) are constants whose values are

\[ \lambda^* = 0.105 , \quad \nu_o = 0.10 \]  

(10)

The quantity \( \lambda \) depends upon the freestream turbulence level, \( T' \), and pressure gradient parameter, \( \Lambda \), defined by

\[ T' = 100 \sqrt{\frac{2}{3} \frac{e e}{U_e^2}} \]  

(11)

and

\[ \Lambda = \frac{\rho_e \partial^2}{\rho_w \nu_e} \frac{dU_e}{dx} \]  

(12)

The functional dependence of \( \lambda \) upon \( \Lambda \) and \( T' \) is

\[ \lambda = 0.105 \left( 1 + 2H(\Lambda) \left\{ 1 - \exp \left[ -40\Lambda \right] \right\} \exp \left[ -3T'^2 \right] \right) \]  

(13)

Equations 1-3 are the time-averaged conservation equations with classical eddy-viscosity and eddy-heat-diffusivity closure approximations. Equations 4 and 5 are nonlinear diffusion equations for \( \epsilon \) and \( \Omega \) which provide a description of the growth
of disturbances in a laminar flow up to, through, and beyond transition. Wilcox\(^2\) presents a thorough discussion of the way in which the model equations are used to predict boundary-layer transition. Boundary conditions suitable for flow over both smooth and rough surfaces are given by Wilcox and Chambers\(^4\).

### 2.2 NEUTRAL STABILITY CONSIDERATIONS

Qualitative features of model-predicted transition can be conveniently determined by dropping the convection and diffusion terms in the turbulent-energy equation (Equation 4). In doing this we can determine the neutral-stability point which is defined as the point in a boundary layer where turbulent energy generation, \(a^*\rho e|\partial u/\partial y|\), just balances turbulent energy dissipation, \(\beta \rho^2 \Omega e\). Hence, neutral stability is defined by the following condition:

\[
\max \frac{\alpha^*}{\beta^*} \left| \frac{\partial u}{\partial y} \right| = 1
\]  \(14\)

Then, noting for laminar boundary layers that \(\Omega\) and \(\alpha^*\) are approximately

\[
\Omega = \frac{20}{\beta} \frac{u}{\rho y^2}
\]  \(15\)

\[
\alpha^* = 0.30 \lambda^*
\]  \(16\)

Equation 14 simplifies to

\[
\max \frac{y^2}{v} \left| \frac{\partial u}{\partial y} \right| = \frac{40}{\lambda^*}
\]  \(17\)
For zero-pressure-gradient boundary layers in air, the velocity distribution can be written as

\[ u = U_e F(\eta) \]  

where

\[ \eta = \frac{1}{\sqrt[4]{\frac{U_e}{V_w}}} \]  

Using Equations 18 and 19, Equation 17 simplifies to the following:

\[ \sqrt{\frac{U_e x}{V_w}} = \frac{40/\lambda^2}{\max_n n^2 \left| \frac{\partial F}{\partial \eta} \right|} \]  

Now for incompressible flow, \( F(\eta) \) is only weakly dependent upon surface heat transfer. Hence, Equation 20 implies that the neutral stability Reynolds number, \( \frac{U_e x}{V_w} \), varies inversely as \( \lambda^2 \). Experiments\(^5\) indicate that transition Reynolds number, \( \frac{U_e x_t}{V_w} \), is inversely proportional to \( \left( \frac{T_w}{T_e} \right)^2 \). Assuming that the transition Reynolds number is proportional to the neutral stability Reynolds number (as it often is) implies that \( \lambda^* \) should be proportional to \( \frac{T_w}{T_e} \).

Computations with \( \lambda^* \) independent of \( \frac{T_w}{T_e} \) verified that such a dependence is needed. The unmodified model predicted that heating (cooling) has a stabilizing (destabilizing) effect on aerodynamic boundary layers, in contrast to the measured destabilizing (stabilizing) effect. These predictions were unsurprising as Shamroth and McDonald\(^5\) find a similar reversal in predicted transition sensitivity to heating and cooling with their turbulence-model transition method. Shamroth and McDonald resolve the problem by making a parameter similar to
\( \lambda^* \) is an increasing function of \( T_w/T_e \). While this model revision is sufficient to yield accurate transition location for aerodynamic boundary layers, it is inadequate for the hydrodynamic case. That is, according to Equation 20, transition location, \( x_t \), is proportional to \( \mu_w/\lambda^{*2} \). For a liquid, \( \mu_w \) varies as \( T_w^{-8} \) near room temperature (see Appendix a). Hence, assuming \( \lambda^* \) is proportional to \( T_w \) would imply that \( x_t \) varies as \( T_w^{-10} \). Therefore, increasing surface temperature would decrease \( x_t \), i.e., heating would destabilize a hydrodynamic boundary layer. Since heating stabilizes such a boundary layer, a different modification is clearly needed.

The parameter \( \lambda^* \) should more appropriately depend upon \( \mu_w/\mu_e \). On the one hand, \( \mu_w/\mu_e \) increases as \( T_w/T_e \) increases for air, while, on the other hand, \( \mu_w/\mu_e \) decreases as \( T_w/T_e \) increases for water. Thus, if \( \lambda^* \) were proportional to some power of \( \mu_w/\mu_e \), the model could accurately predict effects of surface heat transfer on both aerodynamic and hydrodynamic boundary layer transition. Since \( \mu_w T^{3/4} \) for air, the following revised form of Equation 20 is proposed:

\[
\lambda^* = 0.105 \left( \frac{\mu_w}{\mu_e} \right)^{4/3}
\]  

(10a)

An additional, less obvious, modification is needed. Note that the parameter \( \lambda \) partially controls the rate at which disturbances are amplified beyond the neutral stability point. Furthermore, if \( \lambda \) ever becomes sufficiently large (relative to \( \lambda^* \)) so that dissipation-rate production, \( \alpha p \Omega^2 |\partial u/\partial y| \), overtakes dissipation-rate dissipation, \( \beta p^2 \Omega^3 \), (see Equation 5) before the neutral stability point is reached, transition may never occur. Thus, if \( \lambda \) remains
unaltered, the possibility exists that a finite amount of cooling (heating) will cause an aerodynamic (hydrodynamic) boundary layer to forever remain laminar, a physically unrealistic prediction. Hence, Equation 13 must be replaced by:

$$\lambda = 0.105 \left( \frac{\mu_w}{\mu_e} \right)^{4/3} \left\{ 1 + 2H(\Lambda)[1 - \exp(-40\Lambda)] \exp(-3T^2) \right\} \quad (13a)$$

All of the computations presented in the next section have been performed using the transition model defined by Equations 1 through 9, 10a, 11, 12, and 13a; as noted earlier, appropriate surface boundary conditions are given by Wilcox and Chambers.
3. APPLICATIONS

In the first part of this section, the transition model is used to predict transition sensitivity to surface heat transfer for smooth- and rough-surface aerodynamic boundary layers; computed transition-point locations are compared with corresponding experimental data. Then, effects of surface heating on a small hydrodynamic body are computed; qualitative comparisons are made with experimental data. Finally, transition location is predicted on a large hydrodynamic body.

3.1 HEATED AND COOLED AERODYNAMIC BOUNDARY LAYERS

One of the easiest of all flows to analyze is the incompressible flat-plate boundary layer (FPBL). Furthermore, analyzing this flow provides a good test of the transition model as detailed measurements have been made to determine transition sensitivity to surface heat transfer. Using an incompressible version of DCW Industries' EDDYBL computer code, transition computations were performed for an incompressible FPBL with

\[ 0.5 \leq \frac{T_w}{T_e} \leq 3.0 \]  \hspace{2cm} (21)

As in all computations in this study, the value of \( \Omega \) at the boundary-layer edge, \( \Omega_e \), was given by

\[ \Omega_e = 0.0185U_e^2/\nu_e \]  \hspace{2cm} (22)

a value generally used in EDDYBL transition calculations. Results of the computations are shown in Figure 1; experimental
Figure 1. Computed and measured effects of surface heating and cooling on boundary layer transition in air; \((Re_{xt})_w = \) plate-length transition Reynolds number based on surface conditions; \((Re_{xt})_0 = \) the value of \(Re_{xt}\) without surface heat transfer.
data of Zysina-Molozhen and Kuznetsova are included in the figure for comparison.

Because the freestream turbulence level in the experiments was not available, computations were done for a relatively high level ($T' = 1.25\%$) and for a relatively low level ($T' = .03\%$). As shown in the figure, freestream turbulence level has only a slight effect on transition Reynolds number based on surface conditions, $(Re_{xt})_{w} = \frac{x_{t}/\nu_{w}}{U_{e}/\nu_{w}}$. Consistent with the measurements, computed $(Re_{xt})_{w}$ is approximately inversely proportional to $(T_{w}/T_{e})^{2}$; computed values of $(Re_{xt})_{w}$ generally are within about 20\% of corresponding measured values.

A second, more subtle, test of the theory is to apply the equations to rough-wall aerodynamic boundary layers. Surface cooling can actually reverse its stabilizing role to one of destabilization when the surface is rough. The explanation for this phenomenon is well understood. The destabilization occurs because cooling thins the boundary layer, thus making the roughness look larger relative to a boundary-layer thickness such as momentum thickness, $\theta$. Hence, the surface looks rougher, whereby transition occurs earlier.

To test the model's ability to predict this phenomenon, computations were performed for Mach 5 airflow past a roughened spherical body with nose radius of 2.5 inches; roughness heights, $k$, between 1.5 mils and 10 mils were used in the computations. Freestream unit Reynolds number ranged from 3 to 10 million per foot. (The compressible version of the model equations was used for these calculations). The ratio of surface temperature to freestream total temperature, $\frac{T_{t_{\infty}}}{T_{w}}$,
was varied from 0.2 to 0.8. Figure 2 compares results of the computations with a correlation of experimental data. Consistent with the measured effect, momentum thickness Reynolds number at transition, $\text{Re}_{\theta_t}$, increases with decreasing surface temperature.

Figure 2. Comparison between computed and measured effects of surface cooling on rough-wall aerodynamic boundary-layer transition.
3.2 HEATED HYDRODYNAMIC BOUNDARY LAYERS ON SMALL BODIES

The second round of applications is to a small heated hydrodynamic body. The body, shown in Figure 3, is a Parsons-Goodson-Goldschmied (PPG) minimum-drag hull shape which will be referred to as the H-2 body; the PPG body parameters are given in Appendix B. The objective of the computations was to determine incipient transition conditions for various surface temperatures. By definition, incipient transition occurs when the transition point is located at the maximum body radius.

All computations were performed with the following conditions specified:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Temperature, $T_e$</td>
<td>$71^\circ F$</td>
</tr>
<tr>
<td>Roughness Height, $k$</td>
<td>16 $\mu$m</td>
</tr>
<tr>
<td>Freestream Turbulence, $T'$</td>
<td>.01%</td>
</tr>
<tr>
<td>Freestream Dissipation Rate, $\Omega_e$</td>
<td>$0.0185 U_e^2/\nu_e$</td>
</tr>
</tbody>
</table>

The pressure distribution was obtained from the Douglas-Neumann potential flow program; Figure 4 shows the computed pressure coefficient, $c_p$, defined by

$$c_p = \frac{(p-p_\infty)/(1/2\rho U^2)}{U}$$

In the figure, $z$ is axial distance from the stagnation point. Three series of computations were performed in which surface temperature was held constant at values of $71^\circ F$, $76^\circ F$, and $81^\circ F$. Incipient transition Reynolds number, $Re_V$ (based on freestream flow conditions and $V^{1/3}$ where $V$ is body volume), was computed for each value of $T_w$. Results of the calculations are summarized in Table 1; in addition to $Re_V$, the table gives freestream velocity, $U_\infty$, transition Reynolds number based on
Figure 3. Hull shape for the small hydrodynamic body.
Figure 4. Computed pressure coefficient for the small hydrodynamic body.
arclength, $\text{Re}_{st}$, and average heating rate, $\overline{q}_w$, defined by

$$\overline{q}_w = \frac{1}{s_t} \int_{0}^{s_t} q_w(s) \, ds$$  \hspace{1cm} (24)

In Equation 24, $s_t = 4$ feet (the maximum radius point), and $q_w(s)$ is the local heat transfer. Figure 5 shows computed $\text{Re}_v$ as a function of $T_w - T_e$ and $\overline{q}_w$. As can be seen from the figure, heating stabilizes the boundary layer and hence increases the value of $\text{Re}_v$. This prediction is qualitatively consistent with the observed stabilizing effect of surface heating when the fluid is a liquid. The magnitude of the effect is also realistic.

Table 1. Computed Incipient Transition Conditions for the H-2 Body for Varying Surface Temperature.

<table>
<thead>
<tr>
<th>$T_w - T_e$ (°F)</th>
<th>$\overline{q}_w$ (kwatts/ft²)</th>
<th>$U_\infty$ (knots)</th>
<th>$10^{-6} \text{Re}_v$</th>
<th>$10^{-6} \text{Re}_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>25.3</td>
<td>9.4</td>
<td>18.2</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>35.0</td>
<td>13.0</td>
<td>24.5</td>
</tr>
<tr>
<td>10</td>
<td>1.30</td>
<td>47.4</td>
<td>17.6</td>
<td>33.8</td>
</tr>
</tbody>
</table>

An interesting feature of the predictions is revealed in Figure 5. Specifically, $\text{Re}_v$ increases approximately linearly with $T_w - T_e$ for the range of temperatures considered. However, $\text{Re}_v$ increases much less rapidly with $\overline{q}_w$. Hence, increasing the heating rate beyond about 2 kwatts/ft² may not yield substantial increases in $\text{Re}_v$.
Figure 5. Computed incipient transition Reynolds number for the small hydrodynamic body.
3.3 HEATED HYDRODYNAMIC BOUNDARY LAYERS ON LARGE BODIES

The final application of the transition model is to a large heated hydrodynamic body. Again, a PPG hull shape, referred to as the R-9a body, is used (see Appendix B). The objective of these computations was to determine transition point location for specified flow conditions.

Two computations were performed; in one, the freestream flow speed, \( U_o \), was 35 knots and in the other \( U_o \) was 30 knots. In both computations, the following conditions were imposed:

- Ambient Temperature, \( T_e \) = 55°F
- Surface Temperature, \( T_w \) = 85°F
- Roughness Height, \( k \) = 16 \( \mu \)in
- Freestream Turbulence, \( T' \) = 0.01%
- Freestream Dissipation Rate, \( \Omega_e \) = \( 0.0185 \frac{U_e^2}{\nu_e} \)

Figure 6 shows the Douglas-Neumann-computed pressure distribution.

Table 2 and Figure 7 summarize results of the computations. For both flow speeds, transition occurs well upstream of the maximum radius. Computed arclength transition Reynolds numbers, \( Res_t \), are not much larger than the value achieved on the H-2 body with \( \frac{\tau_w}{\nu} = 1.30 \) kwatts/ft\(^2\) (see Table 1). Noting the heating rates involved, the predictions are consistent with results of the preceding subsection.
Figure 6. Computed pressure coefficient for the large hydrodynamic body.
Table 2. Computed Transition Conditions for the R-9a Body.

<table>
<thead>
<tr>
<th>$U_\infty$ (knots)</th>
<th>$\overline{q}_W$ (kwatts/ft$^2$)</th>
<th>$s_t$ (ft)</th>
<th>$10^{-6} \text{Re}_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1.40</td>
<td>13.9</td>
<td>49.1</td>
</tr>
<tr>
<td>35</td>
<td>2.08</td>
<td>11.3</td>
<td>46.1</td>
</tr>
</tbody>
</table>

*To facilitate comparison with the H-2 body heating rates, the value of $\overline{q}_W$ has been computed by integrating from $s=0$ to $s=4$ feet (see Equation 24).*
Figure 7. Computed transition location on the large hydrodynamic body.
4. SUMMARY AND CONCLUSIONS

Results reported in Section 3 demonstrate the transition model's accuracy for boundary layers with surface heat transfer, provided nonuniform viscosity effects are taken into account (Subsection 2.2). With no parameter adjustment, the model accurately predicts transition Reynolds number for (a) incompressible smooth-wall aerodynamic boundary layers with surface heating and cooling, (b) cooled rough-wall boundary layers on blunt bodies in a hypersonic airstream, and (c) heated hydrodynamic boundary layers on small bodies.

Results of the hydrodynamic computations agree with the measured strong boundary-layer stabilization attending small amounts of surface heating. The model also predicts that large amounts of heating are not significantly more effective in delaying transition than the smaller rates considered in the H-2 body computations of Subsection 3.2. The larger R-9a body computations support this prediction.
APPENDIX A
THERMODYNAMIC PROPERTIES OF WATER

The following thermodynamic properties of water are pertinent to boundary layer transition: mass density ($\rho$), specific heat ($C_p$), thermal conductivity, ($\kappa$), molecular viscosity ($\mu$), and laminar Prandtl number ($Pr_L$). Values used in this study for these quantities have been obtained from Schlichting\textsuperscript{10} and are valid for temperatures ranging from about $40^\circ F$ to $110^\circ F$.

Mass density, specific heat, and thermal conductivity are approximately constant and have the following values:

\begin{align*}
\rho &= 1.936 \text{ lbf} \cdot \text{sec}^2/\text{ft}^4 \quad (A1) \\
C_p &= 1 \text{ Btu/lbf} \cdot \text{°F} \quad (A2) \\
\kappa &= 0.35 \text{ Btu/ft} \cdot \text{hr} \cdot \text{°F} \quad (A3)
\end{align*}

Viscosity and Prandtl number, by contrast, are strongly temperature dependent over this range of temperatures. An approximate polynomial fit to the Schlichting data was used to calculate $\mu$ in the computations of Section 3, i.e.,

\[ \mu = 7.943 \times 10^{-6} (T/600) \times 10^{-8} \text{ lbf} \cdot \text{sec}/\text{ft}^2 \quad (A4) \]

with temperature, $T$, in degrees Rankine. Finally, the laminar Prandtl number follows from its definition, namely, $Pr_L = \mu C_p / \kappa$, wherefore,

\[ Pr_L = 2.51(T/600)^{-8} \quad (A5) \]

Figure A1 compares Equations A4 and A5 with the measured dependences of $\mu$ and $Pr_L$ upon temperature.
Figure A1. Molecular viscosity and laminar Prandtl number for water.
APPENDIX B
HYDRODYNAMIC BODY SHAPE PARAMETERS

The two hydrodynamic bodies analyzed in this study are specified in terms of the eight dimensionless parameters used by Parsons, Goodson, and Goldschmied. The values of these parameters are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>H-2 Body</th>
<th>R-9a Body</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_r$</td>
<td>4.450</td>
<td>5.000</td>
</tr>
<tr>
<td>$x_m$</td>
<td>0.427</td>
<td>0.600</td>
</tr>
<tr>
<td>$k_1$</td>
<td>2.715</td>
<td>4.000</td>
</tr>
<tr>
<td>$r_n$</td>
<td>0.260</td>
<td>0.050</td>
</tr>
<tr>
<td>$r_i$</td>
<td>0.740</td>
<td>0.700</td>
</tr>
<tr>
<td>$s_i$</td>
<td>1.650</td>
<td>1.750</td>
</tr>
<tr>
<td>$x_i$</td>
<td>0.595</td>
<td>0.800</td>
</tr>
<tr>
<td>$t$</td>
<td>0.180</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Additionally, the H-2 body is 9.457 feet long and has a volume of 14.5 cubic feet; the R-9a body is 55 feet long and has a volume of 2500 cubic feet.
REFERENCES

1. Personal communication between D.C. Wilcox of DCW Industries and C. Gazley of the RAND Corp (May 1975).


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College of Engineering
Berkeley, California 94720
Attn: S. Berger

Stanford University
Department of Mechanical Engineering
Palo Alto, California 94305
Attn: W. Reynolds

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EFFECTS OF SURFACE HEAT TRANSFER ON BOUNDARY-LAYER TRANSITION

DAVID C. WILCOX
THOMAS L. CHAMBERS

DCW INDUSTRIES
13535 VENTURA BOULEVARD, SUITE 207
SHERMAN OAKS, CALIFORNIA 91423

UNCLASSIFIED

Effects of surface heat transfer on boundary-layer transition are analyzed in a three-part study using the Saffman-Wilcox transition model. In the first part of the study, model predictions are compared with experimental data for cooled and heated aerodynamic boundary layers on smooth flat surfaces and for cooled aerodynamic boundary layers near the stagnation point of a roughened blunt body. Consistent with measurements, the model predicts, on the one
hand, that heating destabilizes a smooth-surface aerodynamic boundary layer and, on the other hand, that cooling destabilizes a rough-surface aerodynamic boundary layer. Differences between predicted and measured transition-point locations are within experimental error bounds. Then, incipient transition conditions are determined for a small, heated hydrodynamic body. Again model predictions agree with measurements which indicate that relatively small amounts of surface heating have a strong stabilizing effect on hydrodynamic boundary layers. In the final part of the study, transition location is determined for a large hydrodynamic body; results indicate that large surface heating rates are not substantially more effective than smaller rates.