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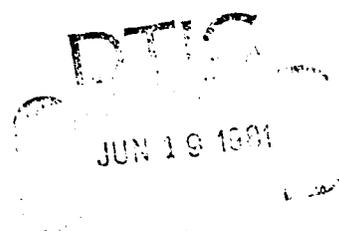
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**THE INFLUENCE OF AN INITIAL STRESS ON THE  
NON-LINEAR RESPONSE OF SUBMERGED  
CYLINDRICAL SHELLS**

BY MICHAEL E. GILTRUD    KENNETH C. KIDDY  
RESEARCH AND TECHNOLOGY DEPARTMENT

26 MARCH 1981

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of submerged general structures to transient acoustic loadings. However, these methods have just recently been applied to the non-linear case.

The approach taken by this work is numerical and in particular applies the USA-STAGS code which combines the *finite element method* for the structure with the DAA for the fluid. We first validate the approach for the linear case, then we evaluate the effect of both geometric and material non-linearities on the response of submerged cylindrical shells. We then assess the influence of an initial stress field on the non-linear response. Particular consideration is given to the amplification of individual modes by the initial stress field.

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FOREWORD

This work assesses the influence of an initial stress field on the non-linear, elastic-plastic response of a submerged isotropic cylindrical shell to transient acoustic loading. Whereas analytical solutions have been obtained for the linear response of such shells to simple temporal and spatial loadings, numerical solutions must be used for the more general case. The approach taken by this work is numerical and in particular applies the USA-STAGS code which combines the finite element method for the structure with the DAA for the fluid. We first validate the approach for the linear case. We then assess the influence of an initial stress field on the non-linear response. Particular consideration is given to the amplification of individual modes by the initial stress field. One important conclusion is that the modal responses are a function of the magnitude of the initial stress field and the intensity of the loading. This situation is not observed for the linear case.

  
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## INTRODUCTION

Fluid structure interaction problems have recently been receiving considerable attention in conjunction with efforts to gain a more thorough understanding of the response of submerged structures to underwater shock loading. The difficulty in developing predictive analytical models is twofold. First, the structure must be suitably modeled in order to determine the response. This certainly is within the capability of many structural analysis computer codes. Secondly, the loading effects are altered according to the state of motion of the structure in the fluid. Thus, the problem of determining the loading is one in which the state of motion of the fluid and structure are coupled.

Analytical solutions have been obtained for simple structural geometries with simple temporal and spatial waveforms. However, in some cases even the analytical solutions are conveniently evaluated numerically. In addition, analytical solutions have not addressed the problem of structure non-linearities. In any event, analytical investigations have resulted in the adoption of methods for uncoupling the structure from the fluid.

These procedures, known as surface approximation techniques, have been compared by Geers<sup>1</sup>. It is apparent from his analysis that the Doubly Asymptotic Approximation (DAA)<sup>2,3</sup> is accurate for early and late times in predicting the behavior of submerged shells. The DAA also affords a smooth transition between early and late time response. More recently, the DAA has been used in conjunction with several linear elastic finite element computer

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<sup>1</sup>Geers, T. L., "Transient Response Analysis of Submerged Structures," in Finite Element Analysis of Transient Non-Linear Behavior, AMD Vol. 14, ASME, New York, 1975.

<sup>2</sup>Geers, T. L., "Response of an Elastic Cylindrical Shell to a Transverse Acoustic Shock Wave in a Light Fluid Medium," J. Acoust. Soc. Am., Vol. 48, No. 3, Sep 1970, pp 692-701.

<sup>3</sup>Geers, T. L., "Excitation of an Elastic Cylindrical Shell by a Transient Acoustic Wave," J. Appl. Mech., Vol. 36, No. 3, Sep 1969, pp 459-469.

codes to predict the response of submerged targets to underwater shock loading<sup>4</sup>. It is just recently that these methods have been applied to the non-linear problem.

This work assesses the influence of an initial stress field on the non-linear, elastic-plastic response of a submerged isotropic cylindrical shell to transient acoustic loading. The approach taken is numerical and in particular applies the USA-STAGS code which combines the finite element method for the structure with the DAA for the fluid. We first validate the approach for the linear case, we then assess the influence of an initial stress field on the non-linear response by comparing the results to the case without prestress. Particular consideration is given to the amplification of individual modes by the initial stress field. One important conclusion is that the modal responses are a function of the magnitude of the initial stress field and the intensity of the loading. This situation is not observed for the linear case.

#### ANALYTICAL FORMULATION

The USA-STAGS code is a combination of the Underwater Shock Analysis code (USA)<sup>5</sup> and the Structural Analysis of General Shells code (STAGS)<sup>6</sup>. In USA, the fluid is assumed to be an infinite acoustic medium whose response to the motions of the structure is described by the DAA. STAGS is a general purpose non-linear finite element structural analysis code that is efficient for the analysis of the inelastic collapse of stiffened shell structures. At this time it is useful to highlight the essentials of the problem formulation.

#### STRUCTURAL RESPONSE EQUATION

As STAGS is based upon the finite element method, the discretized differential equation of motion for the non-linear structure is expressed as

$$\underline{M}_S \ddot{\underline{x}} + \underline{C}_S \dot{\underline{x}} + \underline{K}_S \underline{x} = \underline{f} \quad (1)$$

where  $\underline{x}$  is the structural displacement vector.  $\underline{M}_S$  and  $\underline{C}_S$  are the structural mass and damping matrices.  $\underline{K}_S$  is the non-linear stiffness matrix and  $\underline{f}$  is the

<sup>4</sup>Everstine, G. C., Wang, Yung-Fa, "The Response of a Cylindrical Hull with Internal Structure (Configuration 1) to Underwater Nuclear Shock Waves," Report No. 77-0037, DTNSRDC, April 1977.

<sup>5</sup>DeRuntz, J. A., Geers, T. L., and Felippa, C. A., "The Underwater Shock Analysis Code (USA), A Reference Manual," LMSC-D633861, Lockheed Palo Alto Research Laboratory, Palo Alto, California, January 1979.

<sup>6</sup>Almroth, B. O., Brogan, F. A., and Stanley, G. M., "Structural Analysis of General Shells, Volume II, User Instructions for STAGSC," LMSC-D633873, Lockheed Palo Alto Research Laboratory, Palo Alto, California, January 1979.

external force vector. Generally  $\underline{M}_S$ ,  $\underline{C}_S$  and  $\underline{K}_S$  are highly banded symmetric matrices of large order. In particular, STAGS considers  $\underline{M}_S$  to be diagonal and  $\underline{C}_S$  to be a linear combination of  $\underline{M}_S$  and  $\underline{K}_S$ .

For the excitation of a submerged structure by a transient acoustic wave,  $\underline{f}$  is given by

$$\underline{f} = -\underline{G} \underline{A}_f (\underline{P}_I + \underline{P}_S) + \underline{f}_D \quad (2)$$

where  $\underline{P}_I$  is the modal incident pressure vector (a known) and  $\underline{P}_S$  is the modal scattered pressure vector (unknown). The dry structure dynamic load vector is given by  $\underline{f}_D$ ; additionally,  $\underline{A}_f$  is an area matrix and  $\underline{G}$  is a transformation matrix.

#### FLUID RESPONSE EQUATION

USA makes use of the DAA to describe the response of the scattered pressure at the fluid structure interface<sup>7,8</sup>. The DAA exhibits both excellent high frequency accuracy and excellent low frequency accuracy as well as offering a smooth transition between the two asymptotes.

The differential equation governing the fluid response is

$$\underline{M}_f \dot{\underline{P}}_S + \rho c \underline{A}_f \underline{P}_S = \rho c \underline{M}_f \dot{\underline{U}}_S \quad (3)$$

where  $\underline{P}_S$  is the scattered pressure vector;  $\underline{U}_S$  is the vector of the scattered wave particle velocities;  $\rho$  and  $c$  are the fluid density and sound speed. The added mass matrix,  $\underline{M}_f$ , is produced by a boundary element treatment of the irrotational fluid by the motions of the structure's wetted surface<sup>9</sup>.

The above equation (3) is subject to the following kinematic compatibility equation

$$\underline{G}^T \dot{\underline{x}} = \underline{U}_I + \underline{U}_S \quad (4)$$

where the superscript T represents the matrix transposition. The compatibility equation (4) constrains the normal fluid particle velocity ( $\underline{U}_I + \underline{U}_S$ ) to the

<sup>7</sup>Geers, T. L., "Residual Potential and Approximate Methods for Three-Dimensional Fluid-Structure Interaction Problems," J. Acoust. Soc. Am., Vol. 49, No. 5, (Part 2), May 1971, pp 1505-1510.

<sup>8</sup>Geers, T. L., "Doubly Asymptotic Approximations for Transient Motions of Submerged Structures," J. Acoust. Soc. Am., Vol. 64, No. 5, November 1978, pp 1500-1508.

<sup>9</sup>DeRuntz, J. A., and Geers, T. L., "Added Mass Computation by the Boundary Integral Method," Int. J. Num. Meth. Eng., Vol. 12, 1978, pp 531-550.

normal structural velocity at the wet interface. The transformation matrix,  $G$ , relates the structural freedoms to the fluid freedoms and it follows from the invariance of virtual work with respect to either coordinate system.

#### FLUID STRUCTURE INTERACTION EQUATION

Substitution of equation (2) into equation (1) and equation (4) into equation (3) yields the coupled fluid structure interaction equations.

$$\begin{aligned} \underline{M}_S \ddot{\underline{x}} + \underline{C}_S \dot{\underline{x}} + \underline{K}_S \underline{x} &= \underline{f}_D - \underline{G} \underline{A}_f (\underline{P}_I + \underline{P}_S) \\ \underline{M}_f \dot{\underline{P}}_S + \rho c \underline{A}_f \underline{P}_S &= \rho c \underline{M}_f (\underline{G}^T \ddot{\underline{x}} - \dot{\underline{U}}_I) \end{aligned} \quad (5)$$

The above equation (5) may be solved simultaneously at each time step by the transfer of  $-\underline{G} \underline{A}_f \underline{P}_S$  and  $\rho c \underline{M}_f \underline{G}^T \ddot{\underline{x}}$  to the left side of their respective equation. Such a procedure is exceedingly difficult for larger systems because of the large connectivity of the coefficient matrices. Therefore, a staggered solution procedure has been developed that is unconditionally stable with respect to the time step for the linear problem<sup>10</sup>.

The computational strategy for the staggered solution procedure is embodied in the following steps assuming the solution is known at time  $t$ .

- (1) Estimate the unknown structural restoring force vector at  $t + \Delta t$  from the extrapolation of current and past values.
- (2) Transform this extrapolation into fluid node values and form the right-hand side of the fluid equation, which also involves the unknown incident pressure at  $t + \Delta t$ .
- (3) Transform fluid pressures into structural nodal forces.
- (4) Solve the structural equation for the displacement and velocity vectors at  $t + \Delta t$ .
- (5) Transform the computed structural restoring force vector at  $t + \Delta t$  into fluid node values and reform the right-hand side of the fluid equation.
- (6) Resolve the fluid equation and obtain refined values for the total pressures at  $t + \Delta t$ .
- (7) Save system response.

<sup>10</sup>Park, K. C., Felippa, C. A., and DeRuntz, J. A., "Stabilization of Staggered Solution Procedures for Fluid-Structure Interaction Analysis," pp 95-124 of Computational Methods for Fluid-Structure Interaction Problems, AMD-Vo. 26, ASME, New York, 1977.

Steps (1), (3) and (5) constitute the basic staggered solution technique, while Steps (2) and (4) are required because of the difference between the fluid and structural surface meshes. The iteration of the fluid solution reflected in Steps (6) and (7) has been added to enhance accuracy. Inasmuch as the computation time is overwhelmed by the structural solution requirements, this requires only a small increase in total run time. The use of a three-point extrapolation method in Step (1) also improves accuracy, as discussed in<sup>10</sup>.

#### LINEAR ELASTIC RESPONSE

Consider an isotropic cylindrical shell with a modulus of elasticity,  $E$ , Poisson's ratio,  $\nu$ , and density,  $\rho_s$ , immersed in an infinite acoustic medium having density,  $\rho$ , and sound speed,  $c$ . The geometry of the problem is given in Figure 1 for which the thickness to radius ratio is 0.065. Before examining the influence of an initial stress field, we review the accuracy of the procedure by comparing the analysis to an exact solution by Huang for the loading case of a step wave of infinite duration<sup>11,12</sup>.

Figure 2 gives the non-dimensional radial velocity of the leading edge of the shell. (Note that the velocity has been non-dimensionalized by  $\dot{u}_0 = P_0/\rho c$  and the time has been non-dimensionalized by  $c/a$ .) In general, we see that the agreement is quite good though the calculated results contain some higher frequency oscillations.

#### NON-LINEAR RESPONSE

Structural non-linearities arise from two sources, first geometric considerations and secondly, material considerations. Geometric non-linearities arise from retaining the non-linear terms in the strain-displacement relationship as:

$$\epsilon_{ij} = 1/2 \{u_{i,j} + u_{j,i} + u_{k,i} u_{k,j}\} \quad (6)$$

The non-linearity in the above equation (6) is the product term,  $u_{k,i} u_{k,j}$ , and physically represents the square of the rotations. For the linear case previously discussed, we have assumed that the squares of the rotations are small and may be neglected. However, for shell type structures, the rotations may not be small and neglecting their effect may not be prudent.

<sup>11</sup>Huang, H., "An Exact Analysis of the Transient Interaction of Acoustic Plane Waves with a Cylindrical Elastic Shell," J. Appl. Mech., Vol. 37, No. 4, December 1970, pp 1091-1106.

<sup>12</sup>Giltrud, M. E., and Lucas, D. S., "A Numerical Comparison with an Exact Solution for the Transient Response of a Cylinder Immersed in a Fluid," in the 49th Shock and Vibration Bulletin, September 1979.

The effect of the non-linearities gives rise to a phenomenon known as structural instability or buckling. The essence of buckling is summarized as a very large increase in displacement resulting from an infinitesimal increase in load. The pressure or load at which such instability occurs is known as the critical buckling pressure or load. For the cylindrical shell under consideration herein, the critical buckling pressure is<sup>13</sup>

$$P_c = \frac{Eh^3}{4(1-\nu^2) a^3} \quad (7)$$

Throughout the subsequent discussions we utilize the critical buckling pressure as a reference pressure to assess the influence of the non-linearities. However, even though the structure undergoes a non-linear deformation, it remains elastic and thereby satisfies the generalized Hooke's relation as:

$$\sigma = \zeta \epsilon \quad (8)$$

We therefore define the stress associated with the critical pressure  $P_c$ , as the critical stress  $\sigma_c$ . In the discussion of the inelastic behavior,  $\sigma_c$  will be used as a reference stress to characterize the material.

On the other hand, non-linearities may arise from material considerations. Such non-linearities in the stress-strain relationships is commonly called plasticity. Once a material is loaded beyond its elastic limit, it no longer satisfies the generalized Hooke's law. The effective stress now becomes a function of the integral of the plastic strain increment as:

$$\bar{\sigma} = H [\int d \epsilon^P] \quad (9)$$

The functional form of the above equation is quite complicated. It requires a yield criterion and an associated flow rule. For all the analyses described subsequently, the Mechanical Sub-Layer or White-Besseling method has been used. No attempt will be made here to describe its implementation. The reader is referred to References 14 and 15 for a further discussion.

<sup>13</sup>Brush, D. O., and Almroth, B. O., Buckling of Bars, Plates and Shells, McGraw-Hill Book Co., New York, New York, 1975.

<sup>14</sup>Besseling, J. F., "A Theory of Elastic, Plastic, and Creep Deformation of an Initially Isotropic Material Showing Strain Hardening, Creep Recovery and Secondary Creep," J. Appl. Mech., Vol. 25, No. 4, December 1958, pp 529-536.

<sup>15</sup>Hunsaker, B., Vaughan, D. K., and Sticklin, J. A., "A Comparison of the Capability of Four Hardening Rules to Predict a Material's Plastic Behavior," Texas Engineering Experiment Station, Proc. of the Office of Naval Research Plasticity Workshop, June 1975, pp 27-65.

GEOMETRIC NON-LINEARITIES

Having retained the non-linear terms in the strain displacement equation (6), we first examine the effects of an initial stress on the response of the cylinder. To do this we will expand the physical response in a Fourier series as:

$$W(\theta, t) = \sum_{n=0}^{\infty} W_n(t) \cos(n\theta) \quad (10)$$

where  $W_n$  is the response of the  $n^{\text{th}}$  Fourier mode. Now the response can be analyzed mode by mode.

For all cases the loading is a plane step wave of infinite duration whose magnitude is the elastic buckling pressure. Figure 3 shows the response of the  $n=0$  breathing mode. (Note that in all cases the displacement response has been non-dimensionalized by  $\Omega = P_0 a / \rho c^2$ . As before, the time has been non-dimensionalized by  $c/a$ .) Note that the influence of the geometric non-linearities is only slight for this mode, even when a 50% and 75% prestress has been applied prior to the shock wave. We recognize that the  $n=0$  mode is controlled by the axisymmetric pressure and as a result, it is only slightly affected by geometric considerations. Also, the response approaches the late time asymptote of the hydrostatic response.

The response of the  $n=2$  flexural mode is given in Figure 4. Recall that the  $n=2$  or ovalization mode is the collapse mode for the cylindrical shell under hydrostatic pressure loading. Therefore, the influence of the non-linearities is expected to be quite pronounced.

Without prestress the effect of the geometric non-linearities is quite pronounced, in fact. The response reaches a maximum and appears to remain there. Actually, for very late times ( $T > 50$ ) the response diverges as the shell collapses into the  $n=2$  hydrostatic collapse mode. However, when an initial prestress of 50% of the static collapse pressure has been applied, the collapse begins immediately after the shell is engulfed by the wave ( $T \geq 2.0$ ). The situation is similar for the case with the prestress equal to 75% of the collapse pressure, except the collapse is much more rapid. We observe then that the tendency of the prestress is to reduce the flexural stiffness of the shell. Therefore, the effect is much more pronounced on the  $n=2$  flexural mode than the  $n=0$  breathing mode.

GEOMETRIC AND MATERIAL NON-LINEARITIES

The influence of the combined effect of both material and geometric non-linearities is now assessed. The material stress strain curve is a physically reasonable model which is typical of many low carbon steels. A series of analyses has been performed with the incident pressure equal to the collapse pressure which are summarized in Figures 5 and 6.

Figure 5 gives the response of the  $n=0$  breathing mode. We observe that without prestress the response is unaltered by the combined effect of material and geometric non-linearities. The reason for this behavior is that the incident pressure is equal to the elastic limit pressure. Consequently, the response is still elastic. However, the influence of an initial stress is dramatic. For the case where the initial stress is equal to half the inelastic collapse pressure, the response of the  $n=0$  mode grows very rapidly without bounds. Likewise, a similar behavior is found for the case where the initial stress equals 75% of the inelastic collapse pressure. We ascribe this behavior to the total stress, i.e., the initial stress plus the applied stress, exceeding the elastic limit, whereafter hoop strength of the shell is greatly reduced, and the shell is free to respond more rapidly.

Figure 6 gives the response of the  $n=2$  flexural mode. Since the magnitude of the incident wave equals the inelastic collapse pressure, the effect of the combined material and geometric non-linearities causes the shell to buckle into a  $n=2$  inelastic static collapse mode which is similar to the situation observed for the geometric non-linear case above (Figure 4). The difference herein lies in the larger non-symmetric extensional response.

For the case of an initial stress equal to 50% of the inelastic collapse pressure, the response grows rapidly without bound. A similar behavior is also observed for the case where the initial stress equals 75% of the collapse pressure. Again, the behavior is similar to that observed for the elastic case (Figure 4). Upon closer scrutiny, we observe that the inelastic response is larger than the elastic response. Once again, the increase is due to the increase in the non-symmetric extensional response. This phenomenon has been reported by Geers<sup>16</sup> where he observed a dramatic increase in the non-symmetric extensional response for the inelastic response.

#### SUMMARY

This work has assessed the influence of an initial stress field on the non-linear elastic and inelastic response of a cylindrical shell in an acoustic medium subjected to transient loading. The approach is numerical and in particular applies the USA-STAGS code. We first reviewed the accuracy of the method for the linear elastic problem. Next, we determined the influence of an initial stress field on the non-linear elastic response. The initial stress reduces the flexural stiffness of the shell, thereby precipitating collapse. Finally, the effect of the initial stress on non-linear inelastic response was established. For the axisymmetric response, the response grows without bound once the total stress, which is the combination of the initial stress and the applied stress, exceeds the elastic limit stress. Additionally, the extensional response is a major contributor to the non-axisymmetric response unlike the case of the non-linear elastic response which is primarily flexural response. Recapitulating, we find that for the situation studied herein, the initial stress field alters significantly the non-linear response of the shell and must be addressed to thoroughly describe the response.

<sup>16</sup>Geers, T. L., and Yen, C. L., "Inelastic Response of an Infinite Cylindrical Shell to a Transient Acoustic Wave," LMSC-D676214, Lockheed Palo Alto Research Laboratory, Palo Alto, California, March 1979.

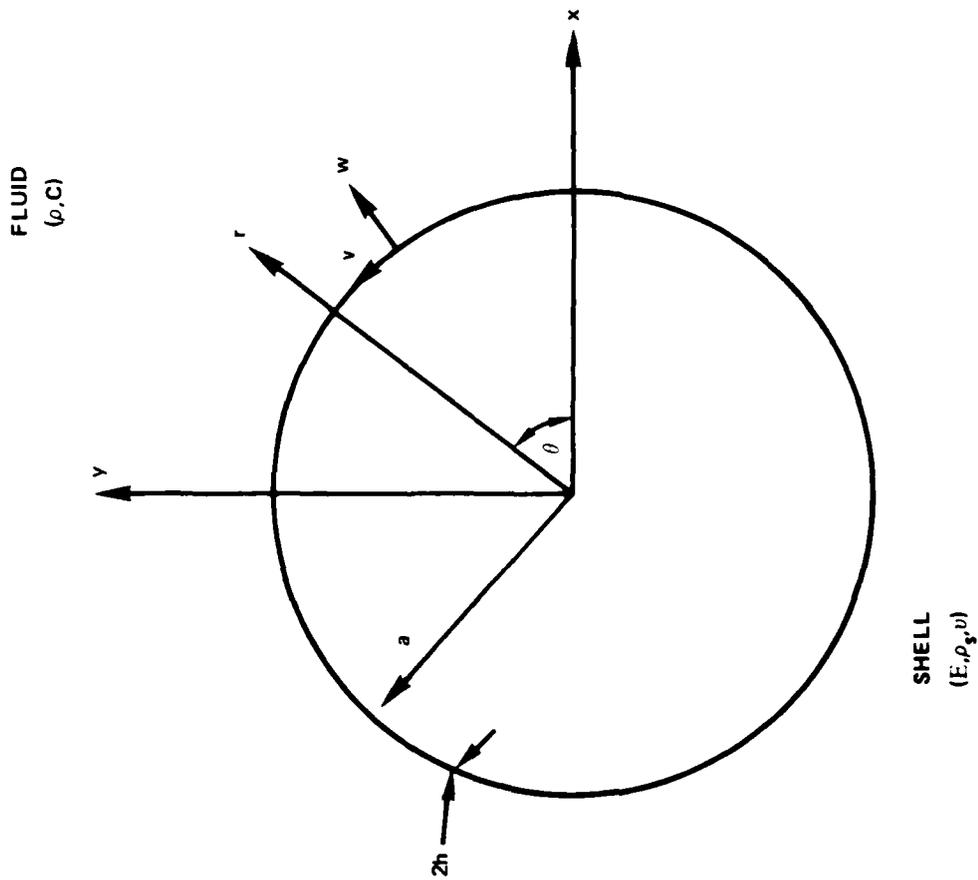
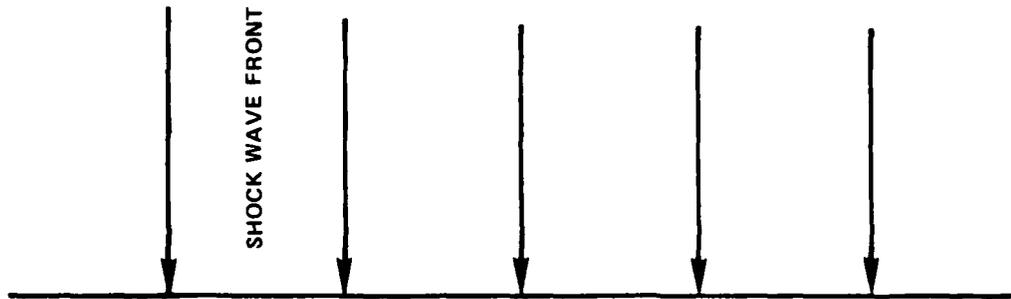


FIGURE 1 PROBLEM GEOMETRY

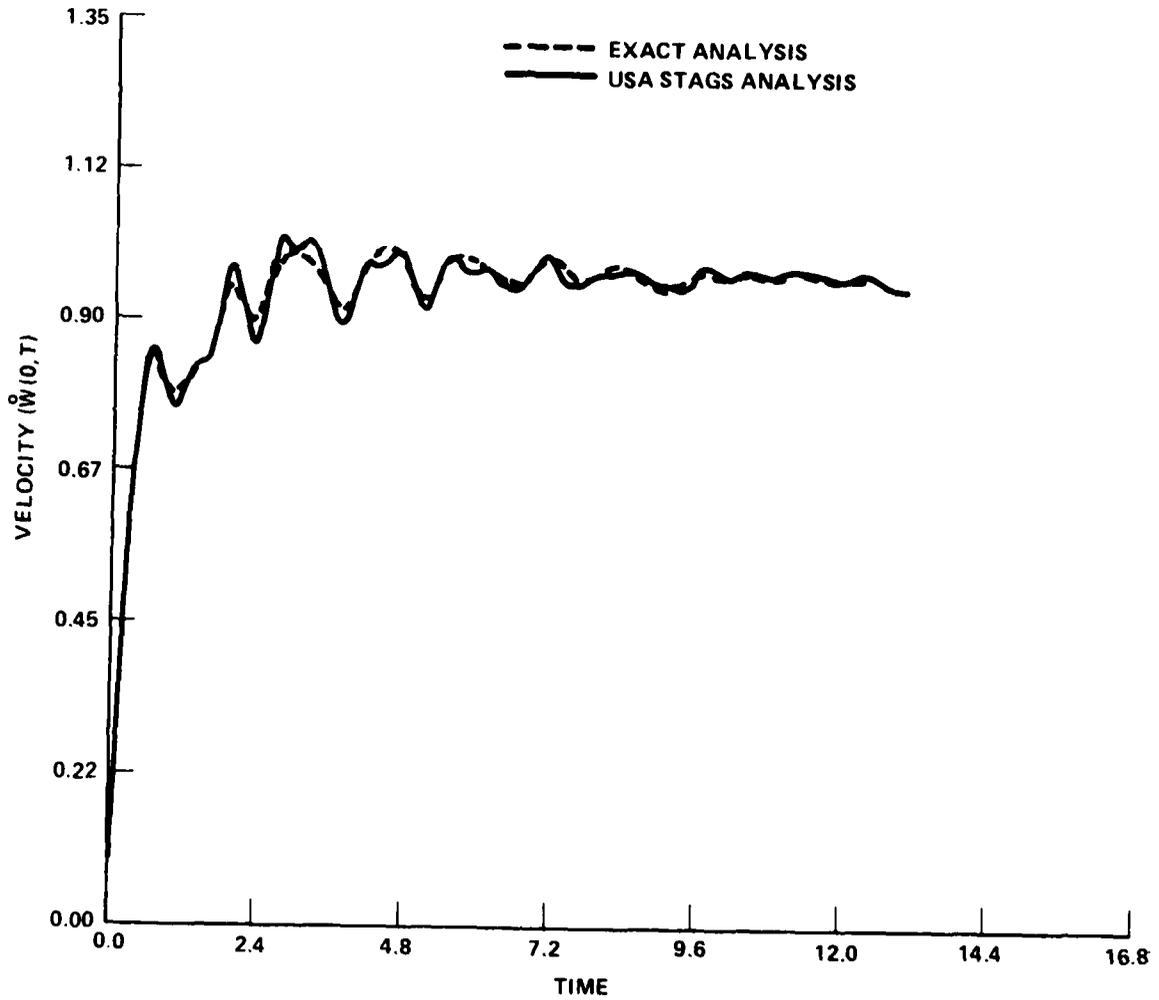


FIGURE 2 RADIAL VELOCITY OF THE POINT OF FIRST CONTACT BY THE WAVE

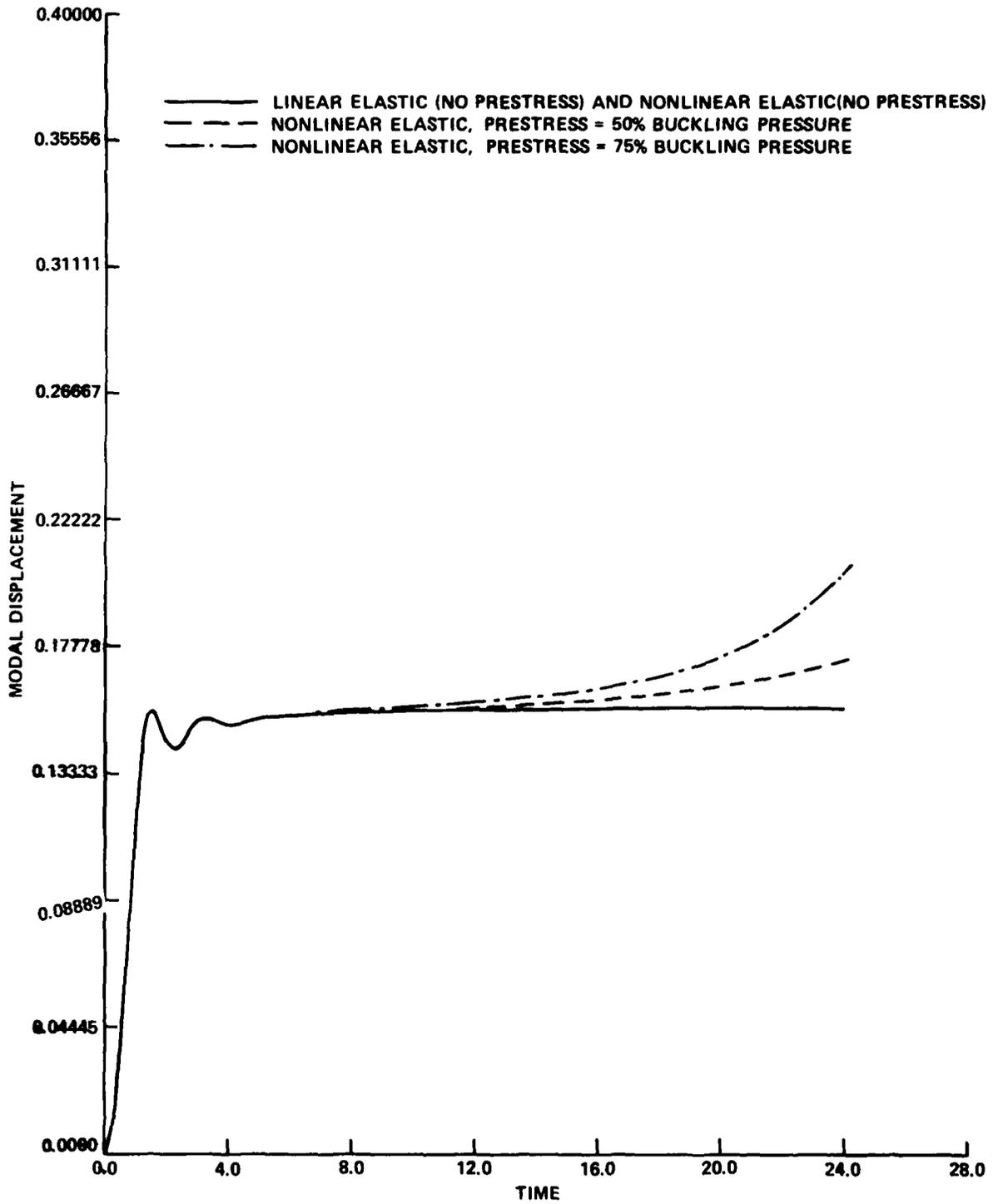


FIGURE 3 EFFECT OF PRESTRESS ON THE NONLINEAR ELASTIC RESPONSE FOR MODE 0

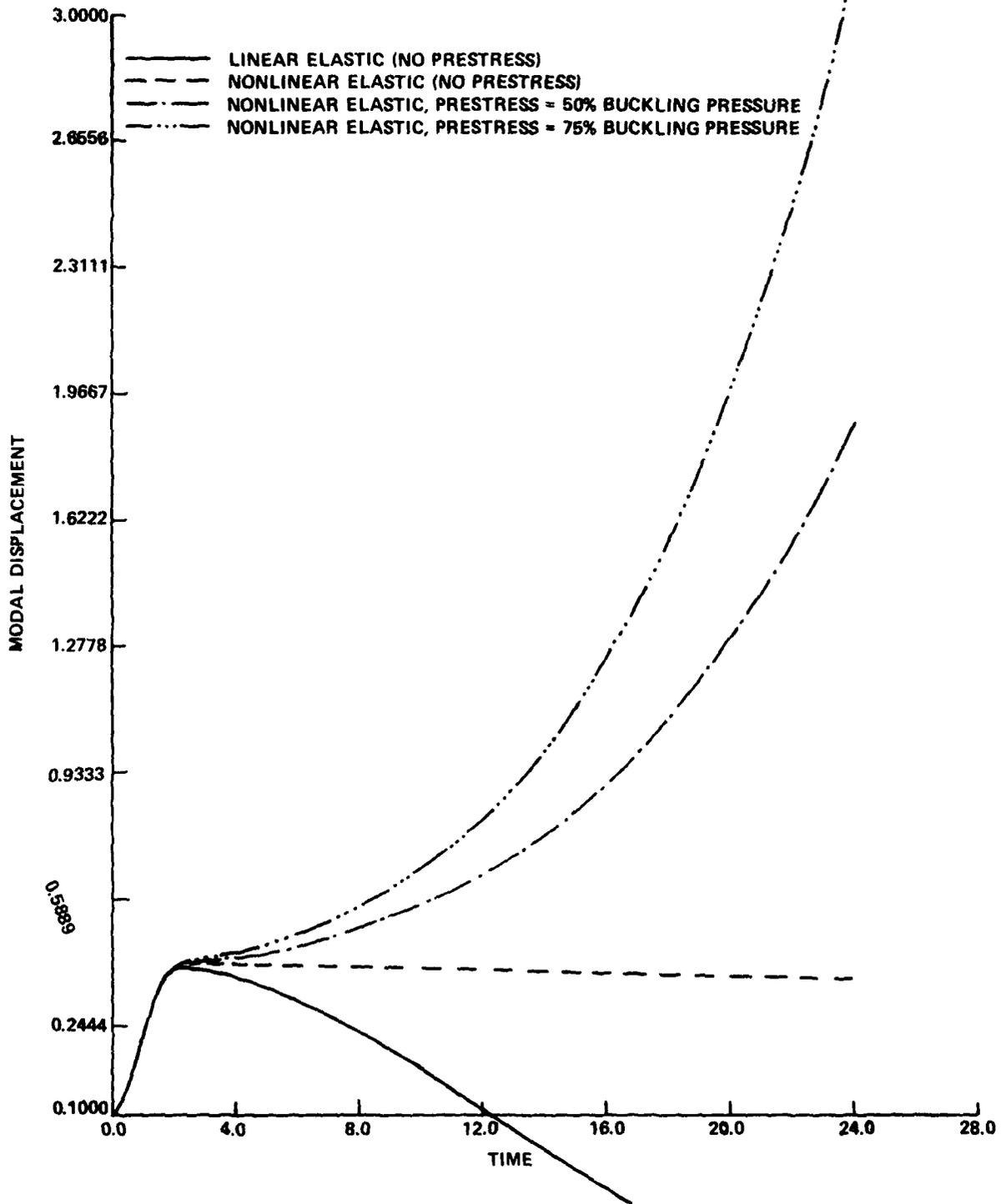


FIGURE 4 EFFECT OF PRESTRESS ON THE NONLINEAR ELASTIC RESPONSE FOR MODE 2

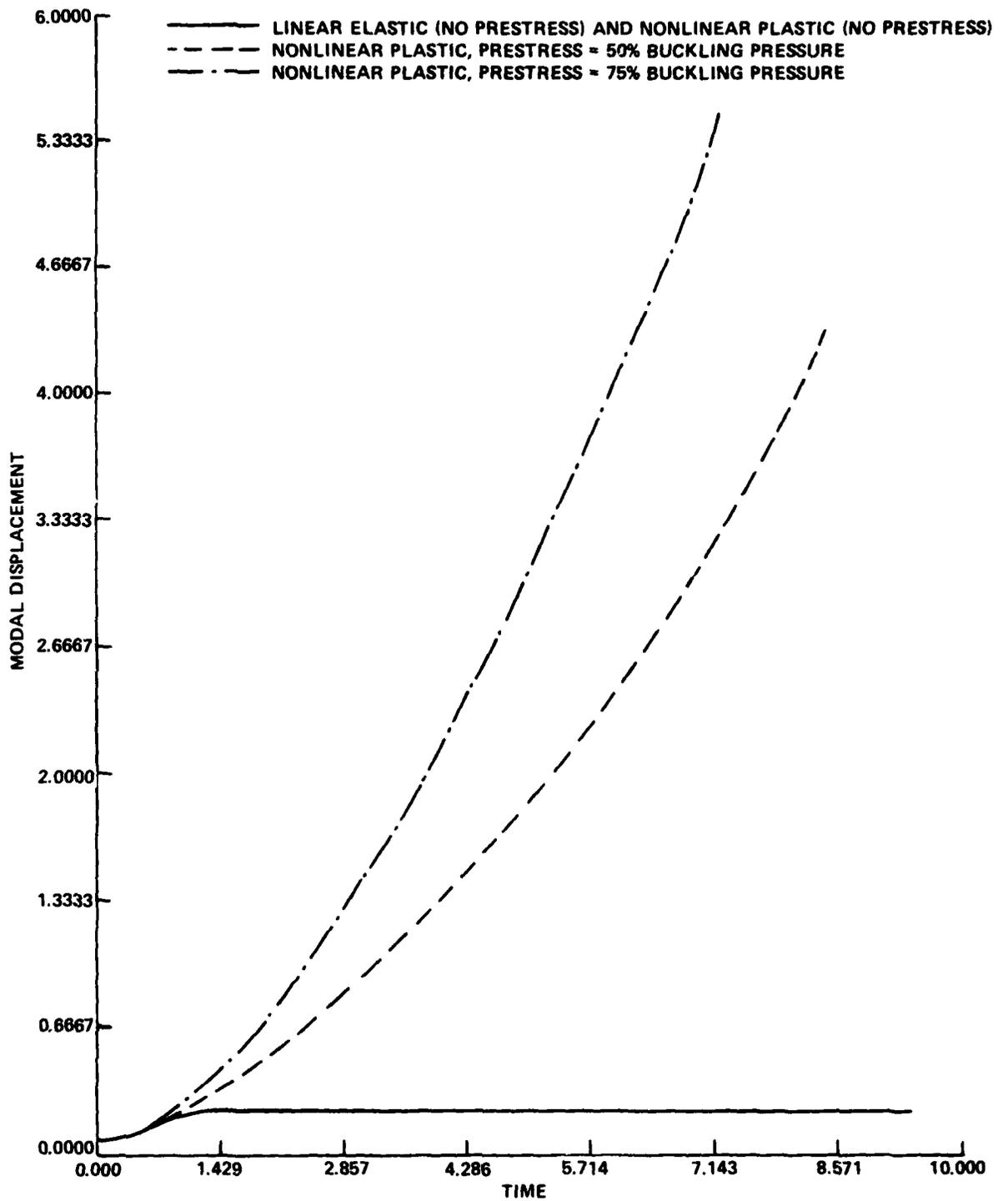


FIGURE 5 EFFECT OF PRESTRESS ON THE NONLINEAR PLASTIC RESPONSE FOR MODE 0

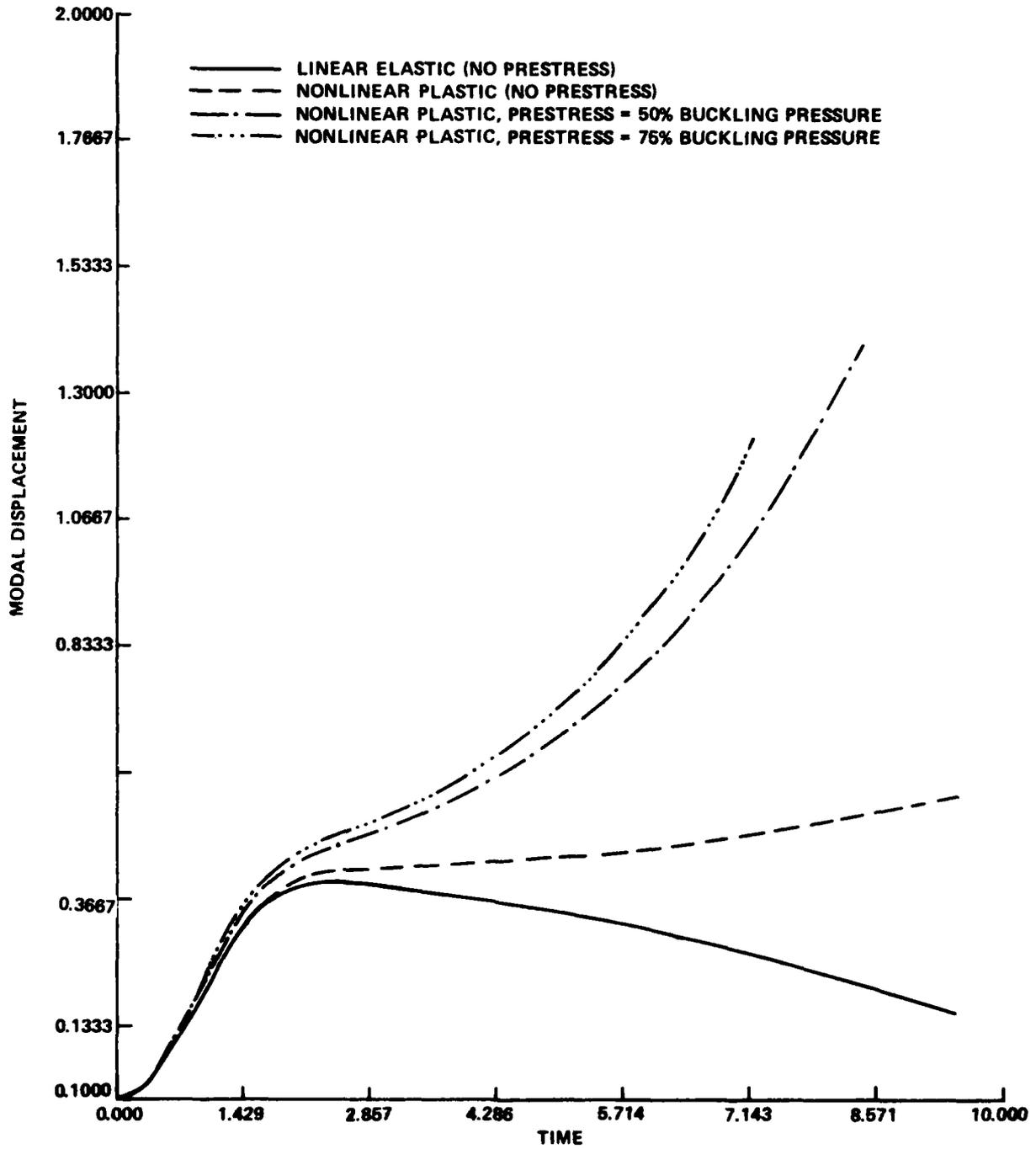


FIGURE 6 EFFECT OF PRESTRESS ON THE NONLINEAR PLASTIC RESPONSE FOR MODE 2

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4. Everstine, G. C., Wang, Yung-Fa, "The Response of a Cylindrical Hull with Internal Structure (Configuration 1) to Underwater Nuclear Shock Waves," Report No. 77-0037, DTNSRDC, April 1977.
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