

AD-A099 568

WISCONSIN UNIV-MADISON DEPT OF STATISTICS  
LIFE TESTING AND RELIABILITY WITH APPLICATION IN ENGINEERING SY--ETC(U)  
APR 81 R A JOHNSON, G K BHATTACHARYYA

F/8 12/1

N00014-78-C-0722

NL

UNCLASSIFIED

ARO-15660.4-M

AD-A  
00014-78-C-0722



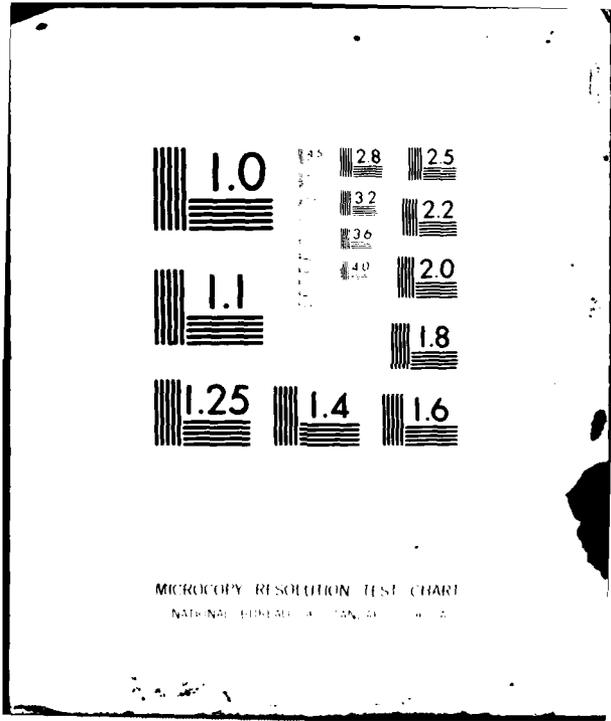
END

DATE

FILED

6-81

DTIC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

**LEVEL II**

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER <b>19</b> 15660.4-M	2. GOVT ACCESSION NO. <b>18</b> ARO AD-A099 568	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Life Testing and Reliability with Applications in Engineering Systems.	<b>12</b>	5. TYPE OF REPORT & PERIOD COVERED Final Report Oct 1, 1978 - Sept 30, 1980
7. AUTHOR(s) Richard A. Johnson Gouri K. Bhattacharyya	<b>15</b>	6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics University of Wisconsin, Madison 1210 W. Dayton Street., Madison, WI		8. CONTRACT OR GRANT NUMBER(s) N00014-78-C-0722 X ARO MIPR 20-79; 12-80
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS <b>11</b> 28 Apr 81
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) <b>9</b> Final Rept. Oct 78-30 Sep 80		12. REPORT DATE April 28, 1981
		13. NUMBER OF PAGES 4 <b>12</b> 61
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

Approved for public release; distribution unlimited.

**DTIC ELECTED**  
**S JUN 1 1981 D**  
**C**

16. DISTRIBUTION STATEMENT (of this Report)

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Stress-strength; Covariates; Accelerated life tests; Censored data; Hazard rate; Fisher information; Transformations; Model fitting

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a summary of our research under the project entitled "Life testing and reliability with applications in engineering systems." Directions of our major advances include: an extension of the classical stress-strength reliability analyses so as to include informations on the covariates; efficiency studies of the modified least squares estimators in accelerated life tests; inferences to compare two exponential distributions with jointly censored samples; study of moment properties of the hazard-rate functions and

AD A 099 568

DTIC FILE COPY

400273 xlt

Block 20 (continuation)

evaluation of the exact Fisher information in censored data; study of the properties of Box-Cox transformations in light of Kullback-Liebler information, and providing guidance for the choice of a transformation of life data towards improved model fitting and inferences.



Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	<input type="checkbox"/>
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

**Title:** Life Testing and Reliability with Application  
in Engineering Systems

**Type of Report:** Final

**Authors:** Richard A. Johnson and Gouri K. Bhattacharyya

**Date:** April 28, 1981

**Grant No.:** N00014-78-C-0722  
Office of Naval Research  
Partial funding from U. S. Army Research Office  
ARO MIPR 20-79; 12-80

**Institution:** University of Wisconsin, Madison

Approved for public release; distribution unlimited.

## A) Problems studied and important results

### 1. Accelerated life tests.

Experimental programs designed to ascertain the life expectancy of structures are often expeditiously performed by testing the structure at stress conditions that are more severe than normal operating conditions. Our work (1) focuses on a class of models which assumes a parametric family of life distributions and a linear relation of the logarithm of the scale parameter on the stress variables. The asymptotic normality of the modified least squares estimators (MLSE) and the maximum likelihood estimators (MLE) are established under easily verifiable conditions. Expressions for the asymptotic efficiency of the MLSE are derived, and numerical comparisons are investigated for the Weibull, exponential, and gamma families. The work (2) extends these results to the situations where the failure time data are type II censored on the right. The loss of efficiency of the MLSE is investigated in relation to the severity of censoring and the nature of spread of the design points.

### 2. Statistical analyses with censored life test data.

In most life testing experiments, one is confronted with situations where observations are terminated before all experimental units have failed. Savings in the cost of experimentation as well as the expediency of making early decisions are compelling reasons for observing incomplete samples. Parametric inferences with censored data confront many situations where the minimal sufficient statistics are not complete, and the standard theories of optimal estimation and hypothesis testing do not apply. Even with negative exponential failure times, this situation arises in two-sample comparative experiments where the data are censored at a specified total number of failures in the combined sample.

The problem of testing the equality of two exponential distributions is investigated in (3) under combined type II censoring. This work includes the exact distributions and moment properties of the failure counts and the total times on test, asymptotic distributions under local alternatives, and the development of an invariant test that maximizes the Pitman efficiency.

The work (4) deals with an evaluation of the exact Fisher information for censored data, and it improves over earlier results which were based on large sample approximations. The concept of hazard rate function is extended, some fundamental moment relations are established, and interesting applications are studied for the normal and gamma distributions.

### 3. Stress-strength models in reliability.

Some progress is reported in (5) concerning the formulation of statistical models that incorporate covariates into the stress-strength reliability framework. For instance, in the context of firing of rockets, the operating pressure (stress) may depend on the ambient temperature ( $t$ ),

and consequently, can be viewed as a random variable whose mean is a function of  $t$ . Identifiable concomitants of strength may also be incorporated as covariates. Thus, we are led naturally to inferences on the survival probability conditional on the specified value(s) of the covariate(s). Specific emphases of our research include Bayesian analysis with some parametric models, especially, normal and exponential, simultaneous confidence statements on the reliability for a range of values of the covariate, nonparametric estimation techniques, and methods of setting the target value of the covariate so as to ensure high reliability.

#### 4. Selecting transformations with life distributions.

Transformations have greatly expanded the domain of application of many important statistical techniques. Our work (6) concerning the large sample behavior of the Box-Cox transformations, has substantially clarified the role of key assumptions. Moreover, it relates the choice of a transformation to one that minimizes the Kullback-Liebler information number between the transformed variable and a normal.

These ideas can be extended to treat the selection of a transformation that will improve the fit of data to gamma, Weibull or other typical life distributions. Some preliminary work in this direction (7) describes how the selection of an extreme value distribution, from among all three types, reduces to the problem of choosing a transformation to the negative exponential distribution.

Oftentimes failure data are available only in grouped form. Yet, for descriptive or predictive purposes, it may be desirable to transform to either a nearly normal or exponential distribution. In (8) we introduce procedures for selecting good transformations for grouped or censored data.

#### B. List of Publications and Technical Reports

1. "Asymptotic normality and efficiency of modified least squares estimators in some accelerated life test models." Revision (1979) to appear in Sankhya, Vol. 42.
2. "On the performance of least squares estimators from censored samples in accelerated life tests." Tech. Report No. 576, Dept. of Statistics, University of Wisconsin, (1979).
3. "On testing equality of two exponential distributions under combined type II censoring." Tech. Report No. 601, Dept. of Statistics, University of Wisconsin, (1980). To appear in Journal of American Statistical Association.
4. "Exact Fisher information for censored samples and the extended hazard rate functions." Communications in Statistics — Theory & Methods, Vol. A8 (15), 1493-1510, (1979).

5. "Stress strength models for reliability: overview and recent advances." To appear in the Proceedings of the 26th Annual Design of Experiments Conference, (1981).
6. "The large sample behavior of transformations to normality." Journal of American Statistical Association, Vol. 75, 855-861, (1980).
7. "The selection of an extreme value distribution and the problem of transforming to a specified distribution." Tech. Report No. 544, Dept. of Statistics, University of Wisconsin, (1979).
8. "Transformation of grouped or censored data to near normality." Tech. Report No. 542, Dept. of Statistics, University of Wisconsin, (1979).
9. "Transformations of a discrete distribution to near normality." Tech. Report No. 546, Dept. of Statistics, University of Wisconsin, (1979).
10. "Use of the Box-Cox transformations with binary response models." Tech. Report No. 575, Dept. of Statistics, University of Wisconsin, (1979).
11. "Bayesian results for the inverse Gaussian distribution with an application." Technometrics, Vol. 21, 247-251, (1979).

C. Degrees awarded:

James R. Taylor — Ph.D., 1978  
Victor Guerrero — Ph.D., 1979  
Thore Langeland — Ph.D., 1980

D. Scientific Personnel supported (Sept. 1, 1978 to Sept. 30, 1980)

Principal Investigators

Richard A. Johnson      2 months 1979, 3 months 1980  
Gouri K. Bhattacharyya   3 months 1979, 2 months 1980

Research Assistants

Victor Guerrero	7/1/79 - 8/31/79	16.67%
Arthur Fries	7/1/79 - 8/31/79	33.33%
	9/1/79 - 1/31/80	50%
	2/1/80 - 6/30/80	25%
	9/1/80 - 9/30/80	25%
	12/1/80 - 12/31/80	25%
	1/1/81 - 6/30/81	16.67%
Steve Verrill	9/1/80 - 9/30/80	25%
	12/1/80 - 12/31/80	25%
	1/1/81 - 6/30/81	16.67%

ATE  
LMED  
-8