SCATTERING BY NONSPHERICAL PARTICULATES

FINAL REPORT
April 1981

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Sponsored by the
U.S. ARMY RESEARCH OFFICE
under
Contract DAAG29-78-G-0024
during
1 November 1978 to 31 October 1980

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Light Scattering, Extinction, Resonant light scattering, Particulates

The theory, which treats only spheres, is usually employed to predict the scattering of light by particles whose size is of the order of the wavelength. The effects due to particle shape-- a cylinder (4:1), prolate spheroids (4:1 and 2:1), a sphere, oblate spheroids (2:1 and 4:1), and a disk (4:1) are investigated here for 4 sizes spanning the resonant region. All particles have the same index of refraction, $m = 1.61 - i 0.0044$, representative of silicates.
Microwave analog and theoretical methods are used to derive the scattered intensity and degree of polarization as a function of scattering angle along with the extinction. All results refer to an ensemble or a cloud of identical particles because averages have been taken over random particle-orientations. The degree of polarization, backscatter, and the radiation pressure cross-section are most sensitive to particle shape, implying the use of Mie theory may be inappropriate for many applications.
STATEMENT OF THE PROBLEM

This investigation examines the effect of particle shape in light scattering by clouds of identical particles. The scattering and absorption of light by particles whose size is of the order of the wavelength can be predicted theoretically in only a few special cases: by arbitrary spheres (Mie, 1908), by infinite cylinders (Waite, 1955), and by arbitrary spheroids (Asano and Yamamoto, 1975). How well such idealizations correspond to scattering by ensembles of real-world particles can be determined only by a systematic investigation of the role of particle shape. For nonspherical particles, the question assumes an additional complexity because the final results require averaging over a distribution (assumed random) of all particle orientations.

We have either measured or computed the light scattered and absorbed by an ensemble of randomly oriented, identical particles for each of the 28 particles depicted in Figure 1. All particles have the same index of refraction, \( m = 1.61 - i \cdot 0.004 \), which is representative of silicates. Their shapes and sizes have been selected and arranged so that:

1. each particle has an axis of rotation that is parallel to the heavy arrow shown in the upper right of the figure;
2. from left to right along the rows, the particles change from cylinders (4:1), to prolate spheroids (4:1, 2:1), to spheres, to oblate spheroids (2:1, 4:1), to disks (4:1) where the aspect ratios are in parentheses;
3. along any one row of the matrix, the surface area of the particles is approximately conserved;
4. each row differs from the preceding one by a single scale factor — the linear dimensions change by about 25%.

The dimensions, surface areas, volumes, and radii of equivalent (surface area) spheres of the 28 particles are provided in Table 1.

The tools required to accomplish the tasks of calculation and measurement were also developed under this contract. The theory of Asano and Yamamoto was programmed to allow for a complex index of refraction and for averaging over particle orientations. More importantly, the Microwave Analog Facility at the State University of New York at Albany (now at the University of Florida) was up-
Figure 1. — The 28 particle shapes whose scattering properties are investigated in this study. All particles (targets) have an axis of symmetry parallel with the heavy arrow shown in the upper right. The wavelength is also indicated.
<table>
<thead>
<tr>
<th>Target ID No.</th>
<th>Description</th>
<th>a(cm)</th>
<th>b(cm)</th>
<th>Surface Area (cm²)</th>
<th>( x S = 2π a_S / \lambda )</th>
<th>Volume (cm³)</th>
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<td>11.34</td>
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<td>6.026</td>
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<tr>
<td>25</td>
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<td>-----</td>
<td>120.04</td>
<td>6.100</td>
<td>123.70</td>
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<td>0.892</td>
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</table>
graded and automated so that the scattering properties of any arbitrarily shaped particle could be measured and averaged over a uniform distribution of the particle's orientation. This is the first time that true "random averaging" has been accomplished experimentally.

The microwave analog method exploits the fact that in the theory of light scattering all particle dimensions enter only in units of the wavelength of the radiation. Thus, the microwave analog of visible light scattering by submicron particles is microwave scattering by targets whose sizes are of the order of centimeters. Under this contract a target suspension system was designed and constructed where, along with the use of computer-controlled stepping motors, a harness of invisible (to the microwaves) nylon threads moves the target through a predetermined sequence of orientations. This arrangement permitted the first general experimental verification of the spheroid theory, the first measurements of the scattering due to randomly oriented disks and cylinders, and makes possible the investigation of light scattering due to a cloud of particles of arbitrary shape.

IMPORTANT RESULTS

The angular intensity of scattered radiation as a function of scattering angle is described by an "F" matrix which operates on the incident (zero subscripted) Stokes vector,

\[ (I_i, I_r, U, V) = F (I_{i0}, I_{r0}, U_0, V_0). \]  \hspace{1cm} (1)

For an ensemble of randomly-oriented, axially-symmetric particles, F has the form

\[
F = \frac{i^2}{4\pi^2} \begin{bmatrix}
    i_{22}(\theta) & i_{21}(\theta) & 0 & 0 \\
    i_{12}(\theta) & i_{11}(\theta) & 0 & 0 \\
    0 & 0 & a_3(\theta) & b_2(\theta) \\
    0 & 0 & -b_2(\theta) & a_4(\theta)
\end{bmatrix}
\]  \hspace{1cm} (2)

The quantities \(i_{22}, i_{11},\) and \(i_{12} (= i_{21})\) are sufficient to describe the scattering of unpolarized or linearly polarized radiation, and these functions can be determined using the microwave method. The extinction cross-section \((C_{ext})\) can also be measured in the micro-
wave lab as described in Publication 4. For each of the 28 particles, the following quantities can be derived from \( i_{11}, i_{22}, i_{12}, \) and \( C_{\text{ext}}:\)

Degree of polarization:

\[
P(\theta) = \frac{i_{11}(\theta) - i_{22}(\theta)}{i_{11}(\theta) + 2i_{12}(\theta) + i_{22}(\theta)}
\] (3)

Total scattering cross-section:

\[
C_{\text{sca}} = \frac{\pi}{k^2} \int_0^\pi \{i_{11}(\theta) + 2i_{12}(\theta) + i_{22}(\theta)\} \sin \theta d\theta
\] (4)

Absorption cross-section:

\[
C_{\text{abs}} = C_{\text{ext}} - C_{\text{sca}}
\] (5)

The asymmetry factor:

\[
\cos \theta = \frac{\pi}{k^2 C_{\text{sca}}} \int_0^\pi \{i_{11}(\theta) + 2i_{12}(\theta) + i_{22}(\theta)\} \cos \theta \sin \theta d\theta
\] (6)

Radiation pressure cross-section:

\[
C_{\text{pr}} = C_{\text{ext}} - \cos \theta C_{\text{sca}}
\] (7)

Each cross-section, when divided by the averaged geometrical cross-section (= \( k^2 \) surface area for all particles dealt with here), yields the corresponding efficiency factor denoted by \( Q \).

Figure 2 has a dual purpose. First, it shows the format that is used to present the results for each particle. The two top plots, total brightness and degree of polarization both as a function of scattering angle, are derived from the graphs of \( i_{22}(\theta), i_{11}(\theta), i_{12}(\theta) \) at the bottom. The efficiency factors are shown in the upper right. The second purpose of Figure 2 is to display a comparison between calculations (solid line and the "Computed" column of numbers) and measurements (dots and "Analog" column) for a large, 4:1, prolate spheroid (target number 24 of Figure 1).

In Publication 1, an error analysis of the measurements is given, and the results for each of the 28 particles are presented
A comparison between theory (--) and experiment (o) for the scattering due to the 2:1 prolate spheroid listed as target number 24 in Figure 1. The various efficiency factors, designated by Q, are shown in the upper right.
in the format shown in Figure 2. The scattering patterns are then compared. Below are listed the most important findings of the role of particle shape, the sphere being used as a standard:

a) Angular Distribution of the Scattered Light

Scattering angles between $30^\circ$ and $45^\circ$ contain the least information on particle shape. The angular position of the absolute minimum intensities is always between $110^\circ < \theta < 160^\circ$ with only a slight rise toward $\theta = 180^\circ$ — the spectacular rise in backscatter for spheres (the rainbow) is missing. The perpendicular component, $i_{11}(\theta)$, always has more angular variation than the parallel, $i_{22}(\theta)$, and the cross polarization, $i_{12}(\theta)$, is usually much smaller than the other components. In many cases, the maximum of $i_{12}$ is nearly equal to the maximum of $i_{11}$ or $i_{22}$.

b) Degree of Polarization

The degree of polarization as a function of scattering angle, $P(\theta)$, depends strongly on particle shape. It never exceeds $\pm 50\%$, whereas for spheres it may come close to $-100\%$ in this size range. For $\theta < 90^\circ$, the 4:1 prolate spheroids and 4:1 cylinders of the same surface area have nearly the same signatures in $\theta$ dependence, while the 2:1 prolate spheroids produces the reverse polarization. No such statement can be made for the oblate spheroids and disks. The degree of polarization, when averaged over $90^\circ < \theta < 180^\circ$, is more negative for the prolate particles than for the oblate. And for all shapes, including spheres, the oscillations in $P(\theta)$ increase with increasing size. Thus, $P(\theta)$ is an indicator of both the size and shape of the particle, and future studies should concentrate on delineating these relationships.

c) Extinction

The 4:1 prolate spheroids and 4:1 cylinders of the same surface area produce equivalent extinctions as do the three smallest 4:1 oblate spheroids and 4:1 disks. The extinction depends on the aspect ratio; the greater the aspect ratio, the larger the particle size at which the maximum extinction occurs. Most of the extinction results agree with the recent calculations of Asano and Sato (Appl. Opt., 19, 962-974, 1980) provided that one compares phase shift parameters, $(m - 1)x$, rather than particle sizes, $x$.

d) Radiation Pressure Cross-Section

The quantity of interest here is $\beta$, the ratio of radiation to gravitational forces, which is proportional to $Q$ for $A/V$ where $A/V$ is the ratio of surface area to volume. This is an
important astrophysical quantity because particles with $\beta > 1$ are purged from the interplanetary dust complex. This process is the ultimate sink of matter in the solar system, and the manner in which a dust grain evolves to $\beta = 1$ determines its solar lifetime. For the smallest size range, the sphere has the largest $\beta$ for a given surface area; for the larger sizes, $\beta$ is greatest for the cylinders and disks and, surprisingly, the least for 4:1 spheroids. The particle shape, which can change $\beta$ by 75% for a given surface area, may therefore play a fundamental role in interplanetary dust dynamics.

As an adjunct to this investigation, an "International Workshop on Scattering by Irregularly Shaped Particles" was held at Albany, New York, on 5-9 June 1979, whose proceedings are described in Publication 2.
LIST OF PUBLICATIONS


LIST OF SCIENTIFIC PERSONNEL

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