FOREIGN TECHNOLOGY DIVISION

CALCULATION AND STUDY OF THE CORE AND INDUCTOR CHARACTERISTICS OF A LINEAR INDUCTION ACCELERATOR

by

P. V. Bukayev, A. I. Anatskiy, and Ye. P. Khal'chitskiy

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EDITED TRANSLATION

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Country of origin: USSR

Translated by Victor Mesenzeff

Requester: FTD/QID

Approved for public release; distribution unlimited.

Prepared by:
TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

Date 23 Jan 1981
**U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM**

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*ye initially, after vowels, and after Ь, Ъ; ё elsewhere.
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**RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS**

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Linear induction accelerators of electrons (LIU) that have been constructed in recent years [1, 2] are still rather primitive and have not been studied enough from the standpoint of their technical and economical indices. One of the ways that can be used to improve the latter — shortening the diameter of the inductor and reducing the longitudinal accelerating field — is given in work [3]. This paper describes a method for calculating the inductors, which is based on a principle of calculating peak transformers [4–6], taking into account the special features of the design, inductor's operation, and experimental data on the properties of magnetic materials [7–9]. The formulas that were obtained couple the basic parameters of the inductor (volt-ampere characteristic, efficiency, and weight) with the axial dimension of its core.

The operating principle of a linear induction accelerator is based on the use of a rotational electrical field excited in a system consisting of several toroidal single-coil transformers—inductors (Fig. 1) for accelerating the electrons. The accelerating field $E_z$ is excited in the internal cavity of a magnetic solenoid with variation of the magnetic flux in the inductor cores, which develops, in turn, under the effect of the voltage pulse $U$ applied to the exciting coils.
The LIU inductor, which transmits the energy from the power sources to a beam of accelerated particles by means of the rotational electrical field is a pulse transformed possessing a number of features.

1. The secondary winding of the transformer is a beam of accelerated particles, therefore, in principle, it is impossible to increase the voltage in the load as compared to the primary voltage and, at the same time, obtain a high secondary voltage, which accelerates the particles.

2. High rate of induction variation in the core - which is necessary for obtaining significant values of secondary voltage. This, in turn, requires the use of a material that could remagnetize completely in the course of a very short pulse.

These features have led to the fact that, in order to calculate the parameters of the inductor by means of the method devised for pulse transformers, it was necessary to conduct an experimental determination of the characteristics of the magnetic materials in a pulsed mode that is close to nominal, i.e., with high rates of induction change in the core [4] and maximum possible remagnetization.

Fig. 1. Diagram of a magnetic solenoid.
Principal parameters of an inductor, which have a significant effect on the intensity of the rotational accelerating field $E_z$, average value of the beam current $I_{\text{n.c.}}$, efficiency of an accelerator $\eta$, and the cost of construction and operation of the modulators and induction system, are: 1) properties of the magnetic material of the core and its cost; 2) voltage in the inductor's winding; 3) duration of the voltage pulse; and 4) axial dimension of the inductor's core.

Magnetic material of the core must have high values of saturation induction $B_s$, magnetic permeability $\mu$, specific resistance $\rho$, and low value of coercive force $H_c$. Furthermore, it must be inexpensive.

These requirements are best satisfied by the 50NP permalloy, which was chosen for the LIU-3000 cores [2], possessing $B_s=1.45$ T [8], initial permeability $\mu_n=900$, $u_{\text{max}}\geq 35000$, $\rho=0.45 \cdot 10^{-6}$ ohm·m, and $H_c=20$ A/m.

The voltage $U$, pulse duration $\tau$, active cross section of the core $S_\text{a}$, and induction increase value in the course of a pulse $B_C$ are connected to one another by an equation, which, for an inductor with a ratio of the number of turns in windings equalling unity, has the form [4–6]

$$\frac{U\tau}{S_\text{a}B_C} = 1,$$

where $B_C$ is an average value of induction increase through the cross section of the core, which is a function of the thickness of the strip, characteristics of the material, and the shape of the core's cross section.

The intensity of the rotational field inside the magnetic solenoid is determined by the formula

$$E_z \approx \frac{U}{l_\text{a}} = \frac{U}{a_\text{z} + 2d_\text{d} + 2d_\text{b}},$$

where $a_\text{z}$ is the axial dimension of the inductor core; $d_\text{d}$ - size of clearance between the core and winding; $d_\text{b}$ - size of clearance between inductors; $l_\text{a}$ - axial dimension of an inductor.

If $k_\text{a}=a_\text{z}/l_\text{a}$ is the active-material filling coefficient of a magnetic solenoid along the axis of the accelerator, then, with a known final energy $E_k$ (without taking into account the gaps between the sections), the length $L$ of the accelerator can be determined by the
Pulse voltage $U$ is determined by the type of the switching device used (thyatron, discharger, pulse modulation lamp) and the nature of the circuit used to shape of a rectangular voltage pulse (single or double shaping line, partial discharge of capacity). However, in all cases, as can be seen from formula (2a), the length of the accelerator diminishes with a rise of voltage $U$ in the inductor, i.e., the intensity of the accelerating field $E_z$ increases.

In LIU-3000 the voltage $U$ is equal to 15 kV, and a circuit with a single shaping line and the TGI-1-2500/35 thyatron, whose anode voltage equalled to 35 kV, was used in the modulator.

The pulse length was selected taking into account the value of $E_z$ which, as can be seen from formulas (1) and (2), increases with a decrease in $\tau$. In this case the magnitude of the mean current of the beam of accelerated electrons $I_{n\xi}$ also decreases. It is also necessary to consider the fact that the decrease in $\tau$ leads to the necessity of using a thinner strip for the core whose cost, with $d < 20 \, \mu m$, increases disproportionately with a simultaneous rise in the manufacturing cost of the core. For an efficient use of the magnetic material of the core, the thickness $d$ of the strip should be selected such that the time constant of eddy currents, determined by the expression [6]

$$\Theta = \frac{0.016 \mu \omega d^2}{p} \approx 2 \cdot 10^{-8} \frac{\mu d^2}{p},$$

satisfied the condition $\Theta = \tau / 3$; in this case the thickness of the strip and pulse length are connected by the following equality

$$d = \sqrt{\frac{2\pi}{0.048 \mu_0 \mu_{\Delta}}} \approx 4000 \sqrt{\frac{2\pi}{\mu_{\Delta}}}.$$

Here $p$ is the specific resistance of the magnetic material, ohm·m; $\tau$ - pulse duration, s; $\mu_0 = 4\pi \cdot 10^{-7}$ H/m - magnetic permeability of vacuum; and $\mu_{\Delta}$ - relative magnetic permeability in a particular cycle.

Pulse duration for the LIU-3000 was chosen to be 0.5 µs with the thickness of the strip at 20 µm, which satisfies these equations when $\mu_{\Delta} = 10000$, determined from a hysteresis loop for 50 Hz.
Axial dimension of the core $a_z$ was selected with the consideration of the technical and economic efficiency. A decrease in this dimension leads to an increase in $E_z$ and, consequently, at a given final energy, to a decrease in the length $L$ of the accelerator and the expenditures, associated with it, for the construction of a building. However, in this case, there is an increase in the weight of the core of the inductor $G$, magnitude of the remagnetizing $I$ and eddy $I_B$ currents and, consequently, in the capacity of the pulsed-feed system for the inductors. To determine these relationships, we will use formula (1) and the equality:

$$S_n = S_{kr},$$

(5)

$$l_c = \pi \frac{D_n + d_n}{2},$$

(6)

$$D_n = \alpha_s S_n + d_n,$$

(7)

$$G = g_l S_n,$$

(8)

in which $S_n$ is total cross section of the core, $m^2$; $k_r$ - radial core-filling coefficient; $l_c$ - mean geometric length of the core's cross section, $m$; $D_n$ and $d_n$ - external and internal diameters of the core, $m$; $g$ - specific weight of the magnetic material, $g/cm^3$.

Using relations (1) and (5), from equality (7), we obtain

$$D_n = \frac{2U_z}{a_s \Delta B_c} + d_n.$$ 

(9)

From expressions (6) and (9) we find

$$l_c = \pi \left( \frac{U_z}{a_s \Delta B_c} + d_n \right).$$

(10)

Taking into account relations (1), (8), and (10), we determine the entire core

$$G = \pi g \left( \frac{U_z}{a_s \Delta B_c} + d_n \right) \frac{U_z}{\Delta B_c}.$$ 

(11)

In deriving these formulas, the value of the internal diameter of the core $d_B$ was assumed to be known. It was determined in the calculation of conductivity of the system for a given current strength of a beam.
of accelerated particles in a pulse with a selected focusing system.

Average harmonic length of the magnetic circuit was used to determine the dependence of the eddy and magnetization currents and their corresponding energies on \( \alpha_z \) [10]

\[
I_r = \pi \frac{D_n - d_p}{\ln \frac{D_n}{d_p}}.
\] (12)

The value of the eddy current in the core is determined by the equality [6]

\[
I_s = \frac{U I_d \alpha}{12 S_{m_t}}.
\] (13)

which, taking into account (12) and (9), can be transformed to

\[
I_s = \frac{n U \mu_s}{6 \mu_0 \mu_s ^2 \alpha_\tau \ln \left(1 + \frac{2 U \tau}{\alpha_\tau \alpha_\delta B_c}ight)}.
\] (14)

The value of the magnetizing current at the end of the pulse is equal to

\[
I_s = \Delta H_p I_r.
\] (15)

where \( \Delta H_p \) is a change in the magnetic field in the course of the pulse, \( A/m \).

Having transformed (15) with the consideration of (12), (9), and equality

\[
\Delta H_p = \frac{\Delta B_c}{\mu_0 \mu_s}.
\] (16)

we obtain

\[
I_s = \frac{2 n U \tau}{\mu_0 \mu_s \alpha_\tau \ln \left(1 + \frac{2 U \tau}{\alpha_\tau \alpha_\delta B_c}\right)}.
\] (17)

where \( \mu_0 \) is magnetic permeability in a pulse mode with total remagnetization.

The value of total current, which compensates for the losses which are due to eddy currents and remagnetization of the core, i.e., apparent magnetization current \( I_k \) at the end of the pulse, equalling the sum of currents determined by equalities (14) and (17)

\[
I_s = \frac{n U \alpha}{\alpha_\tau \ln \left(1 + \frac{2 U \tau}{\alpha_\tau \alpha_\delta B_c}\right)} \left(\frac{\alpha}{6 \mu_0 \mu_s \alpha_\tau \tau} + \frac{2 \tau}{\mu_0 \mu_s \alpha_\tau \tau}\right).
\] (18)
We note that the instantaneous value of the remagnetizing current, which increases linearly with time in the course of the pulse, can be obtained from equation (17) by substituting the current time \( t \) for \( \tau \).

We will assume that the instantaneous value of the eddy current \( i_\lambda \) in the first approximation remains constant in the course of a pulse and equals \( I_\lambda \). Let us substitute consecutively the \( i_\mu \), \( I_\mu \), and \( I_\lambda \) in the following equality:

\[
di = U_{\text{idt}}
\]  

and, having integrated these equalities within the limits of the pulse from 0 to \( \tau \), we will determine correspondingly the remagnetization energy of the core

\[
W_\mu = \frac{nU_\mu^2 z}{\mu_0 k r \ln \left( 1 + \frac{2U_\mu}{\mu_0 k r d B_c} \right)}
\]  

and energy of losses due to eddy currents

\[
W_\lambda = \frac{nU_\lambda^2 \Delta t}{6 \mu_0 k r \ln \left( 1 + \frac{2U_\lambda}{\mu_0 k r d B_c} \right)}
\]  

Total energy used in the core in the course of a pulse is equal to

\[
W_c = \frac{nU_c^2 z}{a_2 k r \ln \left( 1 + \frac{2U_c}{a_2 k r d B_c} \right)} \left( \frac{\tau}{\mu_0 \kappa_4} + \frac{\tau}{6 \mu_0} \right)
\]  

Equalities (11), (14), (17), (18), and (20)–(22) determine the quantitative relationships between the basic parameters of the accelerator \((E_z; U; \tau; a_z)\), magnetic properties of the material, geometric dimensions and weight of the core, and also the power of the pulse-feed system.

The accelerator's efficiency \( \eta_y \) can be defined by the equality

\[
\eta_y = \frac{W_n}{W_{\text{loss}}}
\]  

Here \( W_n = U/I_\mu \) is energy transferred to the beam of accelerated particles, while the total energy supplied to the inductor from the torus modulus

\[
W_{\text{loss}} = W_n + W_c.
\]  

(23a)
Fig. 2. Increase in induction in the course of a pulse and apparent magnetic permeability as a function of change in intensity of the magnetic field.

Fig. 3. Value of magnetizing current as a function of the axial dimension of the inductor's core at various values of pulsed voltage and internal diameter of the core.

Fig. 4. Total remagnetization energy (curves 1-111) and efficiency of the accelerator (curves 1-3) as functions of the axial dimension of the core at the various values of pulsed voltage and internal diameter of the core.

Fig. 5. Weight of the core as a function of its axial dimension at the various values of pulsed voltage and internal diameter of the core.

After the transformation of equality (23), with the consideration of relations (22) and (23a), and simplification we obtain
The value of apparent magnetic permeability \( \mu_k \), which takes into account the demagnetizing effect of the eddy currents in the core, is used frequently in calculations and is easily determinable experimentally. The formula for \( \mu_k \) can be obtained from the equality

\[
\Delta B = \mu_k u_k \Delta H_e.
\]  

(25)

To determine the values of \( \mu_k \) and \( B_c \), we fabricated and tested models of four cores made of a strip of the 50NP permalloy: dimension - 0.02×20 mm; diameter - \( d_8 = 50 \) mm and \( D_H = 200 \) mm. A three-stage pulse amplifier with two powerful pulse modulation lamps in the output stage, which are connected in parallel, was used as the pulse generator for testing the models. At the output the pulse had the following parameters: \( U = 2-25 \) kV, \( \tau = 0.5 \) \( \mu s \), and \( I_k = 200 \) A.

Constant current \( I_m \), whose magnitude was regulated for obtaining optimum values of \( \mu_k \) and \( \Delta B_c \), was used for the remagnetization of cores in the intervals between the pulses. The curves of dependence of the increase in induction in the period of a pulse \( \Delta B \) and of the
The apparent magnetic permeability \( \phi' \) on the maximum value of intensity of the magnetic field at the end of the pulse are shown in Fig. 2. An optimum direct current of remagnetization during testing was found to be within pulse duration \( t = 0.5 \) μs, with pulse duration \( \tau = 2 \mu s \).

The rest of the values of the quantities entering into the equations, which were obtained as a result of the tests carried out on the models of the cores and inductors of the LLL-3000 and used for calculation, are:

- \( k_1 = 0.7 \), \( \Delta B_c = 2 \), and \( \mu = 2000 \).

The graphs in Figs. 3-5 show the calculation results derived from these formulas for some parameters of the inductors, depending on the axial dimension of the core \( a \). With pulse duration \( \tau = 0.5 \) μs, calculation was performed for the internal diameters of the inductor's core \( d_0 = 220 \) and \( 100 \) mm pulse voltages in the inductors \( U = 15 \) and \( 23 \) kV.
In order to calculate a circuit for correcting the shape of a pulse, it is necessary to determine the parameters of an equivalent circuit of the inductor, shown in Fig. 6.

The magnetization inductance $L_p$, resistance, which is equivalent to the losses due to eddy currents, $R_e$, and load resistance $R$ were calculated by the following formulas:

$$L_p = \frac{S_{m_p}^2 \omega_{m_p}}{I_r},$$  \hspace{1cm} (27)

$$R_e = \frac{12S_{m_p}}{N^2 I_r},$$  \hspace{1cm} (28)

$$R = \frac{U}{I_n}.$$  \hspace{1cm} (29)

Fig. 7 shows a photograph of a current pulse $i$ and voltage $u$ in the winding of a loaded inductor core. As can be seen, the shape of the current, in the first approximation, corresponds to a sum of currents: current which is constant in magnitude in the course of a pulse compensating for the eddy losses and magnetization current, which increases linearly in time.

Fig. 8 shows the curves $U=f(I)$, which limit the region containing the characteristics of all inductors of the LIU-3000 (upper curve corresponds to inductors, which are best in quality).

Conclusion

The dependence curves and calculation formulas that were presented confirm the significant influence exerted by the magnitude of the axial dimension of the core $a_z$ on the inductor's parameters and, consequently, on the technical and economic indices of the accelerator as a whole. It can be seen from the curves that an increase in $E_z$ can be achieved only with a simultaneous decrease in the pulse duration and thickness of the strip used for the core winding. Otherwise, this leads to an increase in cost of construction and operation of the accelerator. These curves also make it possible to make a proper selection of the basic structural dimensions and parameters of the inductor core and unit, and to determine its accelerating efficiency and consumption of power during operation. Formulas (18), (22), (27), and (28) are necessary for calculating the modulator's shaping line.
The characteristics of the 50NP permalloy 0.02 mm thick (see Fig. 2) can be used for calculating the inductor cores with different pulse durations from 0.2 to 1.0 μs and relatively large values of the diameter ratios D1/d4. In this case the values of ΔBc and μk must be recalculated by the existing method.

In conclusion the authors express their gratitude to F. G. Zheleznikov and V. A. Suslov under whose supervision this work was accomplished, and also to the comrades who took part in calculations, construction, adjustment of the modulators used for tests, devising a method for measuring currents and voltages, A. V. Belyayeva, Yu. A. Kazankin, A. F. Mikhaylov, and N. A. Ratnikov.

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