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DISCRIMINATION FEATURES AND SPECIFICATIONS FOR OPTICAL DECOYS.(U)

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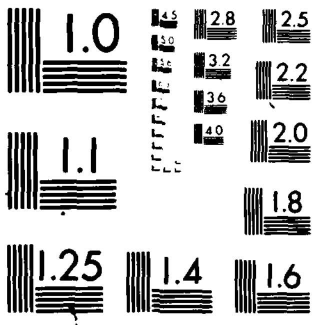

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Technical Report 547

B.J. Burdick

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Discrimination Features and Specifications for Optical Decoys

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FOR THE COMMANDER

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ABSTRACT

The formulation of optical signature features useful in discrimination studies and decoy design, and a method for evaluating a decoy's effectiveness are presented in this report. These features are based on an optical signature model which incorporates a second-order trend with a sinusoidal-type variation normally observed in optical signatures. The decoy evaluation procedure presented here provides the decoy designer with an analysis of those features and wavebands of the decoy signature which may need modification to achieve a prescribed match with an RV.

CONTENTS

I. Introduction

The purpose of this report is to introduce and document results obtained in the development and use of features derived from optical signatures. The primary motivation for and application of these features has been in the development of discrimination algorithms and in the requirements for decoys. The latter application has resulted in the development of a computer program which provides an optical decoy designer with an evaluation of a candidate decoy by means of a detailed analysis of the first- and second-moment statistics of its features.

The formulation of these features and the details of the decoy evaluation procedure are the subject of this report; a subsequent report will provide documentation of the computer program*. The use of these features in discrimination studies, as well as in decoy design, together with related topics will be presented when they become available.

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* B. J. Burdick and M. R. Colaneri, "BODE: Bhattacharyya Optical Decoy Evaluation," M.I.T. Lincoln Laboratory (to be published).

II. Signature Model and Features

As an aid to decoy design, we have developed an optical signature model which incorporates the major features observable by a multiband optical sensor. These signature features are easily calculated and are traceable to the thermal and dynamic properties of the targets so that any decoy modifications required to achieve a prescribed match with RV signature statistics can be easily ascertained.

The signature model for a given waveband, $\Delta\lambda$, is written in terms of the features as

$$H_{\Delta\lambda}(n) = A_{\Delta\lambda}\left[1 + \tilde{n}B_{\Delta\lambda} + \tilde{n}^2 C_{\Delta\lambda} + \tilde{H}_{\Delta\lambda}(n)\right] \tag{1}$$

$$n=1,2,\ldots,N$$

$$\tilde{n}=(n-n_0)/n_0 \ , \ n_0=(N+1)/2$$

where n is the observation number, N is the total number of observations, $\tilde{n}$ is a normalized observation number, and n_0 is the midpoint of the observation interval. The first three features appear explicitly in the above equation:

$A_{\Delta\lambda}$ = temporal mean* of $H_{\Delta\lambda}(n)$

$B_{\Delta\lambda}$ = linear trend of $H_{\Delta\lambda}(n)$

$C_{\Delta\lambda}$ = quadratic trend of $H_{\Delta\lambda}(n)$

The remaining two features are implicitly contained in $\tilde{H}_{\Delta\lambda}(n)$, which corresponds to the signature after trend removal, and are

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* To be precise, $A_{\Delta\lambda}$ is the temporal mean only when $C_{\Delta\lambda}=0$.

$\tilde{\sigma}_{\Delta\lambda}^{2}$ = variance of $\tilde{H}_{\Delta\lambda}(n)$

$\tilde{\rho}(\ell)$ = temporal autocovariance of $\tilde{H}_{\Delta\lambda}(n)$
$\ell=1,2,\ldots,N-1$

The assumption has been made that the temporal autocovariance function need be calculated for one band only (note that the subscript $\Delta\lambda$ is absent from $\tilde{\rho}(\ell)$). This is justified for two reasons: (1) it has been observed that, within the allowable statistical sampling errors, $\tilde{\rho}(\ell)$ is the same for all bands, and (2) it reduces the number of features and thus simplifies the task of decoy design.

These five features, then, are used to describe the major characteristics of optical signatures: the features $A_{\Delta\lambda}$, $B_{\Delta\lambda}$, and $C_{\Delta\lambda}$ describe the slowly varying trend and the features $\tilde{\sigma}_{\Delta\lambda}$ and $\tilde{\rho}(\ell)$ describe the remaining, faster varying, sinusoidal-type variation normally observed in optical signatures.

The trend features are obtained by means of a least-squares fit to the signature. That is, the residual

$$R = \sum_{n=1}^{N} \left[H_{\Delta\lambda}(n) - a_{\Delta\lambda} - \tilde{n} b_{\Delta\lambda} - \tilde{n}^2 c_{\Delta\lambda}\right]^2$$

is minimized with respect to $a_{\Delta\lambda} = A_{\Delta\lambda}$, $b_{\Delta\lambda} = A_{\Delta\lambda}\ B_{\Delta\lambda}$, and $c_{\Delta\lambda} = A_{\Delta\lambda}\ C_{\Delta\lambda}$. Defining

$$\langle N^x \rangle \equiv \sum_{n=1}^{N} \tilde{n}^x \tag{2}$$

$$\langle H^x_{\Delta\lambda} \rangle \equiv \sum_{n=1}^{N} \tilde{n}^x \; H_{\Delta\lambda}(n) \quad , \tag{3}$$

and noting that

$$\langle N^0 \rangle = N$$

$$\langle N^1 \rangle = 0$$

$$\langle N^2 \rangle = N(N-1)/3(N+1)$$

$$\langle N^3 \rangle = 0$$

$$\langle N^4 \rangle = N(N-1)(3N^2-7)/15(N+1)^3$$

We find that the minimization requirements yield the following equations

$$\frac{\partial R}{\partial a_{\Delta\lambda}} \Rightarrow a_{\Delta\lambda}\langle N^0 \rangle + b_{\Delta\lambda}\langle N^1 \rangle + c_{\Delta\lambda}\langle N^2 \rangle = \langle H^0_{\Delta\lambda} \rangle$$

$$\frac{\partial R}{\partial b_{\Delta\lambda}} \Rightarrow a_{\Delta\lambda}\langle N^1 \rangle + b_{\Delta\lambda}\langle N^2 \rangle + c_{\Delta\lambda}\langle N^3 \rangle = \langle H^1_{\Delta\lambda} \rangle$$

$$\frac{\partial R}{\partial c_{\Delta\lambda}} \Rightarrow a_{\Delta\lambda}\langle N^2 \rangle + b_{\Delta\lambda}\langle N^3 \rangle + c_{\Delta\lambda}\langle N^4 \rangle = \langle H^2_{\Delta\lambda} \rangle \quad .$$

The solution of these equations is

$$\left.\begin{aligned}
a_{\Delta\lambda} &= \frac{\langle H_{\Delta\lambda}^{0}\rangle\langle N^{4}\rangle - \langle H_{\Delta\lambda}^{2}\rangle\langle N^{2}\rangle}{\langle N^{0}\rangle\langle N^{4}\rangle - \langle N^{2}\rangle^{2}} \\
b_{\Delta\lambda} &= \frac{\langle H_{\Delta\lambda}^{1}\rangle}{\langle N^{2}\rangle} \\
c_{\Delta\lambda} &= -\frac{\langle H_{\Delta\lambda}^{0}\rangle\langle N^{2}\rangle - \langle H_{\Delta\lambda}^{2}\rangle\langle N^{0}\rangle}{\langle N^{0}\rangle\langle N^{4}\rangle - \langle N^{2}\rangle^{2}}
\end{aligned}\right\} \tag{4}$$

so that

$$\left.\begin{aligned}
A_{\Delta\lambda} &= a_{\Delta\lambda} \\
B_{\Delta\lambda} &= b_{\Delta\lambda}/a_{\Delta\lambda} \\
C_{\Delta\lambda} &= c_{\Delta\lambda}/a_{\Delta\lambda} \quad .
\end{aligned}\right\} \tag{5}$$

The features related to the sinusoidal-type variation are obtained from the signature after trend removal,

$$\tilde{H}_{\Delta\lambda}(n) = \left\{H_{\Delta\lambda}(n) - A_{\Delta\lambda}\left[1 + \tilde{n}B_{\Delta\lambda} + \tilde{n}^{2}C_{\Delta\lambda}\right]\right\}/A_{\Delta\lambda} \ ,$$

and are given by

$$\tilde{\sigma}_{\Delta\lambda}^{2} = \frac{1}{N}\sum_{n=1}^{N}\tilde{H}_{\Delta\lambda}^{2}(n) \tag{6}$$

$$\tilde{\rho}(\ell) = \frac{1}{N}\sum_{n=1}^{N-\ell}\tilde{H}_{\Delta\lambda}(n)\ \tilde{H}_{\Delta\lambda}(n+\ell)/\tilde{\sigma}_{\Delta\lambda}^{2} \tag{7}$$

$$\ell=1,2,\ldots,N-1$$

For the case when $\tilde{H}_{\Delta\lambda}(n)$ is Gaussian, these features provide a complete description.

To gain a better understanding of these features consider the following example:

$$\tilde{H}_{\Delta\lambda}(n) = \sqrt{2}\,\hat{\sigma}_{\Delta\lambda}\,\sin(n\omega\Delta t+\varphi) \quad \text{(example)}$$

where Δt is the time spacing between observations, φ is a random phase having a probability density

$$p(\varphi) = \begin{cases} 1/2\pi, & 0 \le \varphi \le 2\pi \\ 0, & \text{otherwise} \end{cases}$$

and ω is a random frequency having a probability density $p(\omega)$. (For a spin-stabilized target which is axially symmetric, ω would correspond to the precession frequency and $\hat{\sigma}_{\Delta\lambda}$ would be related to the coning angle.) The features for this signature are then given by

$$\tilde{\sigma}_{\Delta\lambda}^{2} = \frac{2\hat{\sigma}_{\Delta\lambda}^{2}}{N}\sum_{n=1}^{N}\sin^{2}(n\omega\Delta t+\varphi)$$

$$= \hat{\sigma}_{\Delta\lambda}^{2}\left\{1 + \frac{\sin(\omega\Delta t+2\varphi) - \sin\left[(2N+1)\omega\Delta t+2\varphi\right]}{2N\sin\omega\Delta t}\right\}$$

and

$$\tilde{\rho}(\ell) = \frac{2\hat{\sigma}_{\Delta\lambda}^{2}}{N}\sum_{n=1}^{N-\ell}\sin(n\omega\Delta t+\varphi)\,\sin\left[(n+\ell)\omega\Delta t+\varphi\right]/\tilde{\sigma}_{\Delta\lambda}^{2}$$

$$= \frac{\hat{\sigma}_{\Delta\lambda}^{2}}{\tilde{\sigma}_{\Delta\lambda}^{2}}\left\{\frac{N-\ell}{N}\cos\ell\omega\Delta t + \frac{\sin\left[(\ell+1)\omega\Delta t+2\varphi\right] - \sin\left[(2N-\ell+1)\omega\Delta t+2\varphi\right]}{2N\sin\omega\Delta t}\right\}$$

(Note that in the limit as $N\to\infty$: $\tilde{\sigma}_{\Delta\lambda}^{2}\to\hat{\sigma}_{\Delta\lambda}^{2}$ and $\tilde{\rho}(\ell)\to\cos\ell\omega\Delta t$.)

The ensemble averages of these features are

$$E\left\{\tilde{\sigma}_{\Delta\lambda}^{2}\right\} = \int_{\omega=-\infty}^{\infty} \int_{\varphi=0}^{2\pi} \tilde{\sigma}_{\Delta\lambda}^{2}\ p(\varphi)\ p(\omega)\, d\varphi\, d\omega$$

$$= \hat{\sigma}_{\Delta\lambda}^{2}$$

and

$$E\left\{\tilde{\rho}(\ell)\right\} = \int_{\omega=-\infty}^{\infty} \int_{\varphi=0}^{2\pi} \tilde{\rho}(\ell)\ p(\varphi)\ p(\omega)\, d\varphi\, d\omega$$

$$= \frac{N-\ell}{N} \int_{-\infty}^{\infty} p(\omega)\ \cos \ell\omega\Delta t\ d\omega$$

$$= \frac{N-\ell}{N} \times \mathrm{Re}\left[\int_{-\infty}^{\infty} p(\omega)\ e^{i\ell\omega\Delta t}\ d\omega\right]$$

That is, the ensemble average of $\tilde{\rho}(\ell)$ is the cosine transform (or the real part of the characteristic function) of the frequency distribution $p(\omega)$. This means that $E\left\{\tilde{\rho}(\ell)\right\}$ contains all the information concerning the frequency distribution of $\tilde{H}(n)$. For example, if $p(\omega)$ is Gaussian-completely characterized by a mean, m_ω, and a variance, σ_ω^2:

$$p(\omega) = \frac{1}{\sqrt{2\pi}\,\sigma_\omega} e^{-\frac{1}{2}(\omega - m_\omega)^2/\sigma_\omega^2}$$

then

$$E\left\{\tilde{\rho}(\ell)\right\} = \frac{N-\ell}{N} \times \mathrm{Re}\left[e^{i\ell m_\omega \Delta t - (\ell\sigma_\omega \Delta t)^2/2}\right]$$

$$= \frac{N-\ell}{N}\ e^{-(\ell\sigma_\omega\Delta t)^2/2} \cos \ell m_\omega \Delta t$$

which is a damped cosinusoid with frequency equal to the mean of $p(\omega)$ and a damping factor proportional to the width of $p(\omega)$.

III. Decoy Specification and Evaluation

The method used to levy decoy requirements and evaluate the effectiveness of a decoy against an RV is based on the Bhattacharyya distance.* This metric provides a measure of the separability of the RV and decoy in terms of the ensemble means and covariances of their features. It is particularly useful in as much as it is directly related to the probability of error, is additive for independent features and can be expressed in terms of differences primarily in the means and variances in each waveband and to the correlation between wavebands for each feature. That is, if $\mu(A,B,C,\tilde{\sigma},\tilde{\rho})$ represents the Bhattacharyya distance for all features in all wavebands then, if it can be assumed that the features are independent,

$$\mu(A,B,C,\tilde{\sigma},\tilde{\rho}) = \mu(A) + \mu(B) + \mu(C) + \mu(\tilde{\sigma}) + \mu(\tilde{\rho}) \qquad (8)$$

where $\mu(\cdot)$ is the Bhattacharyya distance for an individual feature. Further, as will be shown later, each $\mu(\cdot)$ can be separated into components which are primarily related tc differences in the RV and decoy means and variances in each band and the average correlation between bands

$$\mu(\cdot) = \sum_{\Delta\lambda} \mu_{MN_{\Delta\lambda}}(\cdot) + \sum_{\Delta\lambda} \mu_{VAR_{\Delta\lambda}}(\cdot) + \mu_{COR}(\cdot) \qquad (9)$$

It is in this manner that it becomes possible to pinpoint not only which decoy features may be contributing to a large

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* K. Fukunaga, Introduction to Statistical Pattern Recognition (Academic Press, New York and London, 1972).

Bhattacharyya distance but also whether this is due to the mean, variance, or band-to-band correlation and further, in which band it is occurring. The Bhattacharyya formulation also makes it possible to determine how the decoy requirements should be levied on the various features and their ensemble means, variances and correlations in the individual wavebands. This simply becomes a matter of apportioning the maximum total allowable Bhattacharyya distance among the features and then among each of the feature's first and second moment statistics.

The maximum allowable total Bhattacharyya distance, μ_{max}, is related to the total probability of error achieved by the decoy, ε, according to the equation

$$\varepsilon \approx \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2\mu_{max}}}^{\infty} e^{-t^2/2}\, dt \tag{10}$$

(which is an exact equation when the RV and decoy covariance matrices are identical, and a good approximation when they are not too dissimilar).

For example, if one wanted to design a decoy to achieve at least a 15% error rate with an RV, then a total Bhattacharyya distance of 0.5 or less would be required (from the equation above). If this were to be apportioned equally among the five features then each feature would have to have a Bhattacharyya distance of 0.1 or smaller. If this were further apportioned equally to the first-and second-order statistics of each feature (i.e. the mean and the combined variance and correlation), then they would each have to achieve a Bhattacharyya distance of 0.05 or less. If all the individual Bhattacharyya distances were less than these requirements, then the decoy would be guaranteed to achieve the desired error rate. However, it may still be possible for a decoy to achieve the desired error rate even if some of the Bhattacharyya distances are too large-all that really counts is the total Bhattacharyya distance and some components may compensate for others.

Let us now show how the Bhattacharyya distance for a single feature, $\mu(\cdot)$, can be written as a sum of components attributable to differences in the RV and decoy means and variances in each waveband and the average correlation between wavebands. In general, the mean vector and covariance matrix of a feature, X, measured in K wavebands are given by

$$M = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_K \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdot & \cdot & \cdot & \sigma_{1K}^2 \\ \sigma_{12}^2 & \sigma_{22}^2 & \cdot & & & \vdots \\ \vdots & & & \cdot & & \vdots \\ & & & & \cdot & \\ \sigma_{1K}^2 & \cdot & & \cdot & \cdot & \sigma_{KK}^2 \end{bmatrix}$$

where m_k is the ensemble mean of the kth (waveband) component of $X=(x_1,x_2,\ldots,x_k,\ldots,x_K)^T$

$$m_k = E\{x_k\}$$

and σ_{ij}^2 is the covariance of the ith and jth components of X:

$$\sigma_{ij}^2 = E\{(x_i-m_i)(x_j-m_j)\}$$

$$= \sigma_{ji}^2$$

The covariance matrix can be written in the following form to exhibit the explicit correlation between wavebands, ρ_{ij}:

$$\Sigma = \begin{bmatrix} \sigma_{11}^2 & \rho_{12}\sigma_{11}\sigma_{22} & \cdots & \rho_{1K}\sigma_{11}\sigma_{KK} \\ \rho_{12}\sigma_{11}\sigma_{22} & \sigma_{22}^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho_{1K}\sigma_{11}\sigma_{KK} & \cdot & \cdots & \sigma_{KK}^2 \end{bmatrix}$$

where

$$\rho_{ij} \equiv \sigma_{ij}^2/\sigma_{ii}\sigma_{jj} \quad .$$

(This ρ should not be confused with the feature $\tilde{\rho}(\ell)$.) To render the remaining calculations tractable, it is necessary to make the assumption that the correlation between band pairs is the same for all pairs so that the ρ_{ij}s can be replaced by their average:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \cdots & \rho\sigma_1\sigma_K \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho\sigma_1\sigma_K & \cdots & \cdots & \sigma_K^2 \end{bmatrix} \tag{11}$$

where

$$\rho = \frac{2}{K(K-1)} \sum_{j=i+1}^{K} \sum_{i=1}^{K-1} \rho_{ij}$$

We have further simplified the notation by writing σ_k^2 for σ_{kk}^2. The replacement of the individual band-to-band correlations by their average has been found to be a very good approximation and is justified on the grounds of simplifying the decoy design procedure.

The determinant and inverse of a matrix of this form can be shown to be

$$|\Sigma| = \sigma_1^2 \cdots \sigma_K^2 (1-\rho)^{K-1}\left[1+(K-1)\rho\right] \tag{12}$$

$$\Sigma^{-1} = \frac{1}{s^2} \begin{bmatrix} \left[1+(K-2)\rho\right]/\sigma_1^2 & -\rho/\sigma_1\sigma_2 & \cdots & -\rho/\sigma_1\sigma_K \\ -\rho/\sigma_1\sigma_2 & \left[1+(K-2)\rho\right]/\sigma_2^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ -\rho/\sigma_1\sigma_K & \cdots & \cdots & \left[1+(K-2)\rho\right]/\sigma_K^2 \end{bmatrix} \tag{13a}$$

where

$$s^2 = (1-\rho)\left[1+(K-1)\rho\right] \tag{13b}$$

The Bhattacharyya distance for a given feature (under the Gaussian assumption) is a function of the mean vectors and covariance matrices of the RV and decoy (DY)

$$\mu(\cdot) = \frac{1}{8}\,\Delta M^T \bar{\Sigma}^{-1} \Delta M + \tfrac{1}{2} \ln |\bar{\Sigma}| / \sqrt{|\Sigma^{RV}| \cdot |\Sigma^{DY}|} \tag{14a}$$

where ΔM is the difference in their mean vectors:

$$\Delta M = M^{RV} - M^{DY} \tag{14b}$$

and $\bar{\Sigma}$ is the average of their covariance matrices:

$$\bar{\Sigma} = \tfrac{1}{2}\Sigma^{RV} + \tfrac{1}{2}\Sigma^{DY} \quad . \tag{14c}$$

With the assumption that the correlation between band pairs is the same for all band pairs $\bar{\Sigma}$ can be written as

$$\bar{\Sigma} = \begin{bmatrix} \bar{\sigma}_1^2 & \bar{\rho}\bar{\sigma}_1\bar{\sigma}_2 & \cdots & \bar{\rho}\bar{\sigma}_1\bar{\sigma}_K \\ \bar{\rho}\bar{\sigma}_1\bar{\sigma}_2 & \bar{\sigma}_2^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \bar{\rho}\bar{\sigma}_1\bar{\sigma}_K & \cdots & \cdots & \bar{\sigma}_K^2 \end{bmatrix} \tag{15a}$$

where $\bar{\sigma}_k^2$ is the average of the RV and decoy variances in band k:

$$\bar{\sigma}_k^2 \equiv \tfrac{1}{2}(\sigma_k^{RV})^2 + \tfrac{1}{2}(\sigma_k^{DY})^2 \tag{15b}$$

and $\bar{\rho}$ is the average band-to-band correlation of the average of the RV and decoy covariance matrices:

$$\bar{\rho} \equiv \frac{2}{K(K-1)} \sum_{j=i+1}^{K} \sum_{i=1}^{K-1} \tfrac{1}{2}(\rho^{RV}\sigma_i^{RV}\sigma_j^{RV} + \rho^{DY}\sigma_i^{DY}\sigma_j^{DY})/\bar{\sigma}_i\bar{\sigma}_j \quad . \tag{15c}$$

The determinant and inverse of $\bar{\Sigma}$ are then given by the same equations derived previously so that the Bhattacharyya distance becomes

$$\mu(\cdot) = \sum_{k=1}^{K} \mu_{MN_k}(\cdot) + \sum_{k=1}^{K} \mu_{VAR_k}(\cdot) + \mu_{COR}(\cdot) \tag{16}$$

where

$$\mu_{MN_k}(\cdot) = \frac{1}{8\bar{s}^2} \left\{ \left[1+(K-2)\bar{\rho}\right]\left(\frac{\Delta m_k}{\bar{\sigma}_k}\right)^2 - \bar{\rho}\left(\frac{\Delta m_k}{\bar{\sigma}_k}\right) \sum_{\substack{j=1 \\ j\neq k}}^{K} \left(\frac{\Delta m_j}{\bar{\sigma}_j}\right) \right\} \tag{17}$$

$$\mu_{VAR_k}(\cdot) = \tfrac{1}{2} \ln\left[\bar{\sigma}_k^2 / \sigma_k^{RV} \sigma_k^{DY}\right] \tag{18}$$

$$\mu_{COR}(\cdot) = \tfrac{1}{2} \ln\left\{ \frac{(1-\bar{\rho})^{K-1}\left[1+(K-1)\bar{\rho}\right]}{\sqrt{(1-\rho^{RV})^{K-1}\left[1+(K-1)\rho^{RV}\right](1-\rho^{DY})^{K-1}\left[1+(K-1)\rho^{DY}\right]}} \right\} \tag{19}$$

where Δm_k is the kth (waveband) component of ΔM and

$$\bar{s}^2 = (1-\bar{\rho})\left[1+(K-1)\bar{\rho}\right] \quad . \tag{20}$$

The individual Bhattacharyya distances $\mu_{MN_k}(\cdot)$ and $\mu_{VAR_k}(\cdot)$ provide information on mismatches between the decoy and the RV due mainly to the mean and variance, respectively, in band k while $\mu_{COR}(\cdot)$ contains information on mismatches attributable mainly to the average band-to-band correlation. By "mainly" we mean: if the only difference between the decoy and the RV is the mean in band k, then the only nonzero Bhattacharyya distance will be $\mu_{MN_k}(\cdot)$; if the only difference is the band-to-band correlation, then only $\mu_{COR}(\cdot)$ will be nonzero; and if the only difference is the variance in band k, then not only $\mu_{VAR_k}(\cdot)$ but also $\mu_{COR}(\cdot)$ will be nonzero. That is, the variance and correlation components cannot be separated when there are variance differences between the decoy and the RV. An examination of the equation for $\bar{\rho}$ shows how this occurs. Consider the case when the decoy band-to-band correlation is the same as that of one RV ($\rho^{DY}=\rho^{RV}$) and the decoy standard deviations are the same as the RV except for one band ($\sigma_i^{DY}=\sigma_i^{RV}$ for $i\neq k$). Then $\bar{\rho}$ becomes

$$\bar{\rho} = r\cdot\rho^{RV}$$

where

$$r = \frac{1}{K}\left[(K-2) + (\sigma_k^{RV}+\sigma_k^{DY})/\bar{\sigma}_k\right] \qquad (21a)$$

so that

$$\mu'_{COR}(\cdot) = \tfrac{1}{2}\ln\left\{\frac{(1-r\rho^{RV})^{K-1}\left[1+(K-1)r\rho^{RV}\right]}{(1-\rho^{RV})^{K-1}\left[1+(K-1)\rho^{RV}\right]}\right\} \quad . \qquad (21b)$$

From this equation we see that $\mu'_{COR}(\cdot)$ will be nonzero when the decoy standard deviation differs from the RV in band k (even though they have the same band-to-band correlation). Therefore, in this case, the Bhattacharyya distance due solely to a variance difference in band k, will be

$$\mu'_{VAR_k}(\cdot) = \mu_{VAR_k}(\cdot) + \mu'_{COR}(\cdot) \quad . \qquad (22)$$

To gain a clearer understanding of the dependencies of these various Bhattacharyya distances on the statistics of the RV and decoy features, plots of their variations are shown in Figs. 1-3 for the case of three wavebands.

Figure 1 illustrates the variation of the Bhattacharyya distance due to mean differences in a single band, $\mu_{MN_k}(\cdot)$, with the mean differences normalized by the average standard deviation, $\Delta m_k/\bar{\sigma}_k$, for various values of the average band-to-band correlation, $\bar{\rho}$. Figure 2 shows the variation of Bhattacharyya distances due to variance differences in a single band, $\mu_{VAR_k}(\cdot)$ and $\mu'_{COR}(\cdot)$, with the ratio of standard deviations, $\sigma_k^{DY}/\sigma_k^{RV}$ or its inverse, for various values of the average band-to-band correlation, $\bar{\rho}$, in the case of $\mu'_{COR}(\cdot)$. Figure 3 shows the variation of the Bhattacharyya distance due to differences in the average band-to-band correlation, $\mu_{COR}(\cdot)$, with the RV and decoy average band-to-band correlations, ρ^{RV} and ρ^{DY}. This is presented as a function of a parameter η,

$$\eta = \frac{1-\rho^{DY}}{1-\rho^{RV}} \quad ,$$

so that a single curve (approximately) represents the variation of $\mu_{COR}(\cdot)$ with ρ^{DY} for any value of ρ^{RV}. Shown at the bottom of this figure are various scales of ρ^{DY} that can be used for some particular values of ρ^{RV}.

Given the RV feature statistics and the desired error rate to be achieved by the decoy, it is possible to levy requirements on the decoy feature statistics using these curves. Consider the example presented earlier where a total Bhattacharyya distance of 0.5 was required (corresponding to a total error

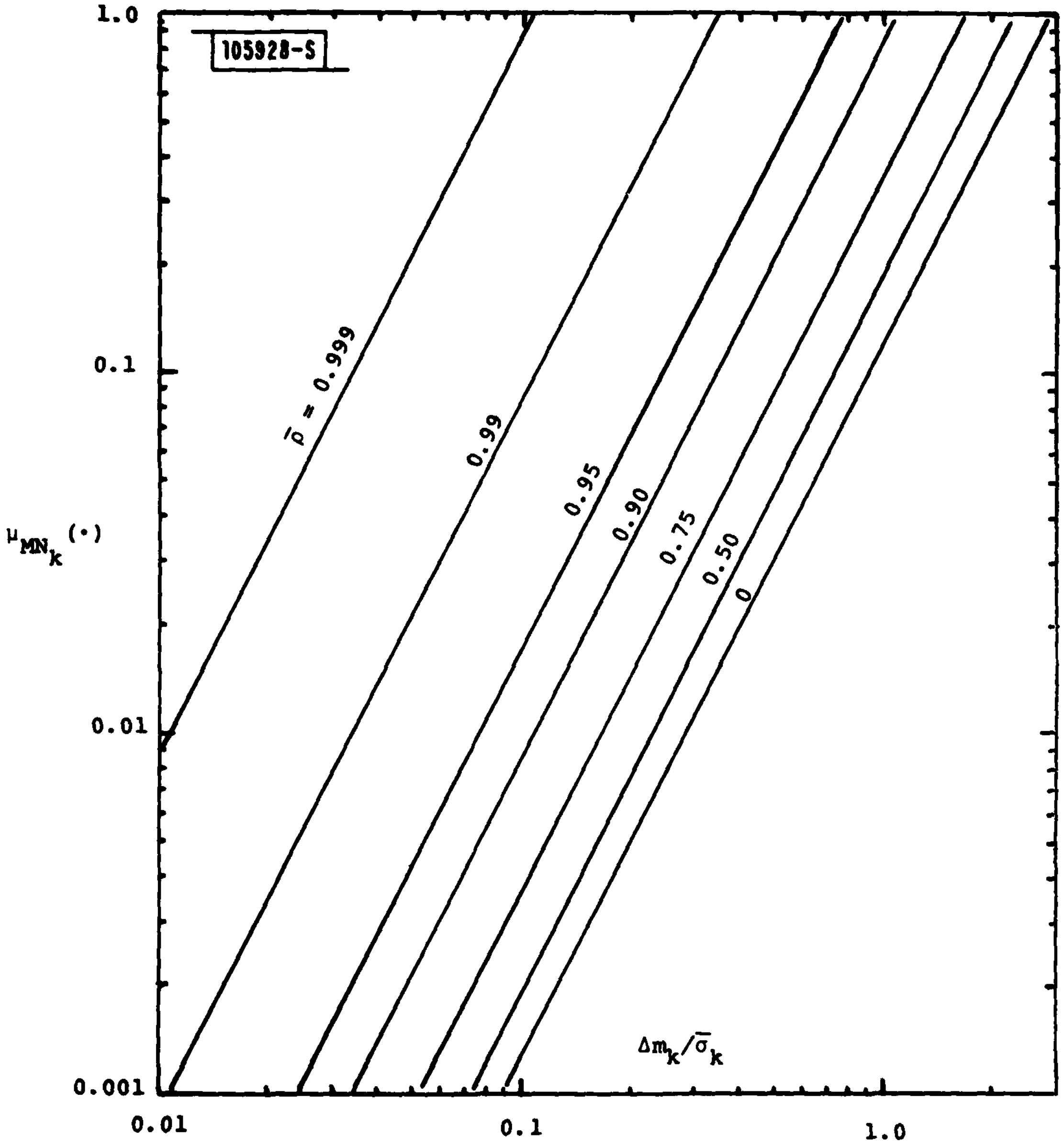

Fig. 1. Variation of the Bhattacharyya distance due to a difference in means in band k for several values of average band-to-band correlation.

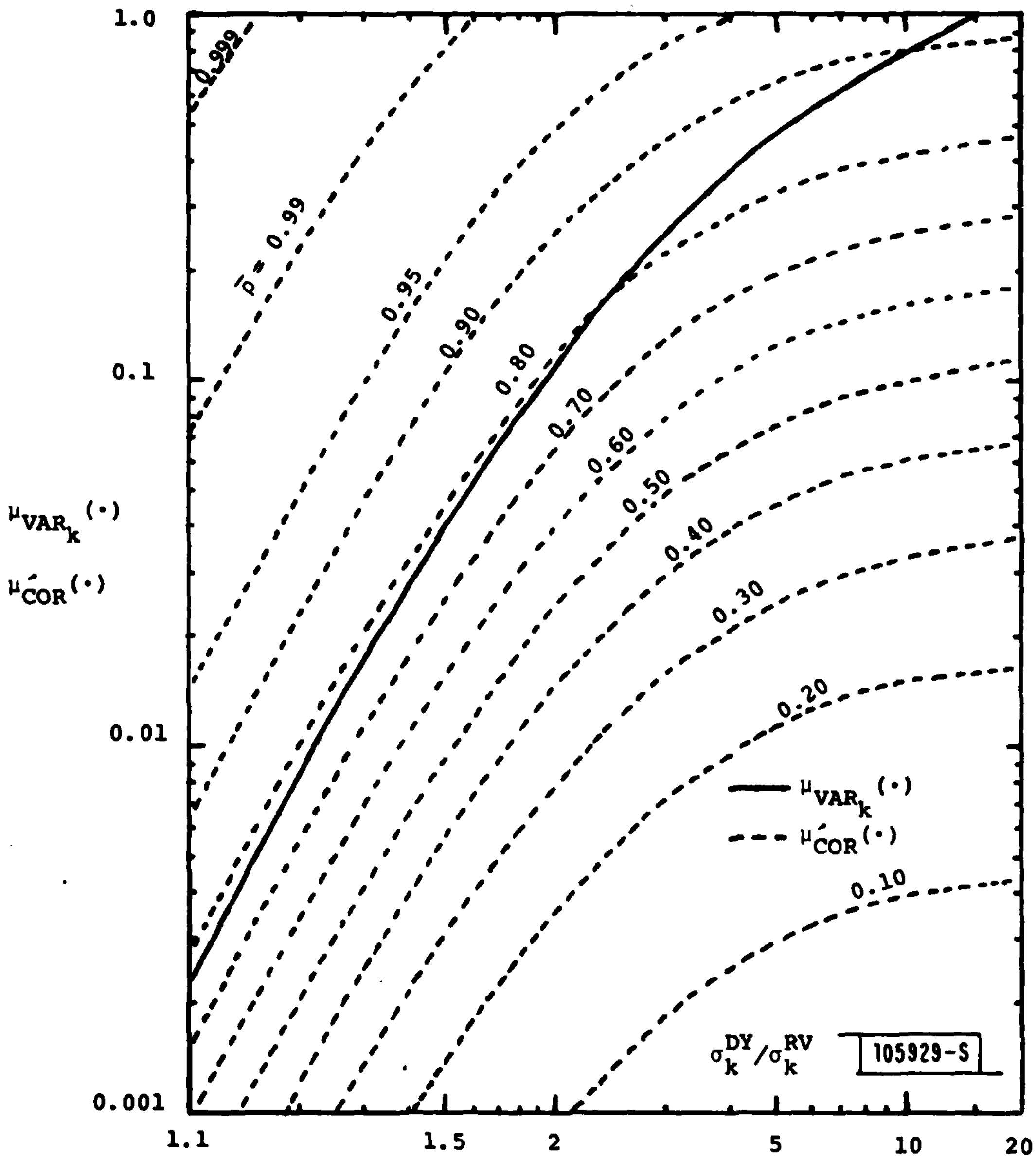

Fig. 2. Variation of Bhattacharyya distances due to variance differences in band k, with $\mu'_{COR}(\cdot)$ shown for several values of average band-to-band correlation.

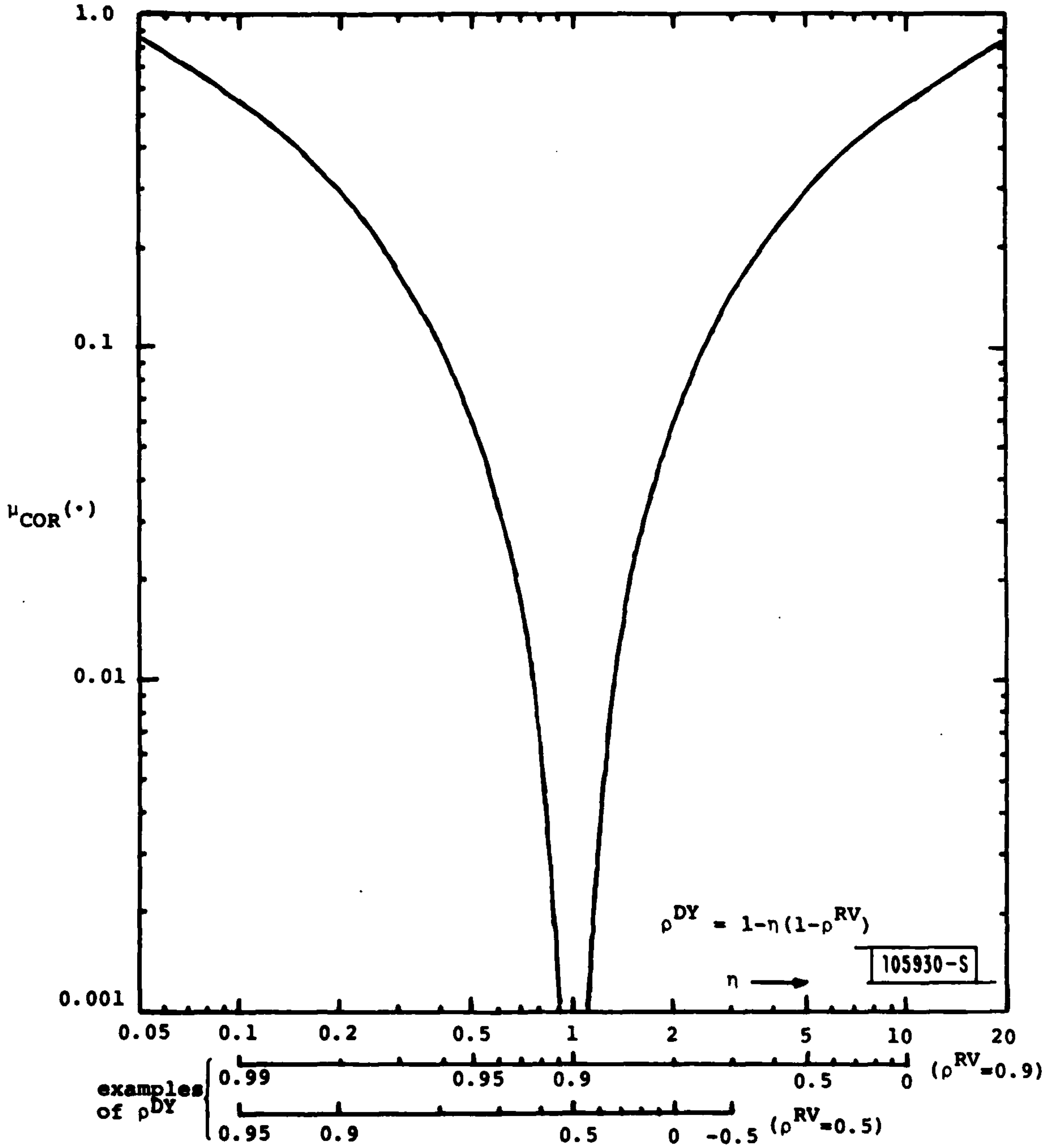

Fig. 3. Variation of Bhattacharyya distance due to band-to-band correlation differences.

probability of 15%). If this is split up equally among the five features, then the Bhattacharyya distance for each feature should have a value of 0.1 or less. Splitting this up equally between the first and second moment statistics results in a value of 0.05 for the mean and 0.05 for the variance plus correlation (which can further be split equally among the variance and the correlation to give 0.025 for each). If there are three wavebands, then the mean and variance distances should be apportioned equally among the three bands so that we arrive at:

$$\mu_{MN_k}(\cdot) \leq 0.017$$

$$\mu_{VAR_k}(\cdot) \leq 0.0083$$

$$\mu_{COR}(\cdot) \leq 0.025$$

If the average correlation between bands is 0.90 for the RV then, reading from the plots, this means that the decoy feature statistics must satisfy:

$$m_k^{RV} - 0.14\bar{\sigma}_k \leq m_k^{DY} \leq m_k^{RV} + 0.14\bar{\sigma}_k$$

$$0.91\sigma_k^{RV} \leq \sigma_k^{DY} \leq 1.10\sigma_k^{RV}$$

$$0.84 \leq \rho^{DY} \leq 0.936 \quad .$$

These would be the requirements that each of the decoy's features in each band would have to meet in order for the decoy to be assured of achieving a 15% error rate. This is an illustrative example only; the precise decoy requirements will, of course, depend on the RV's average band-to-band correlation for each feature and on the total error rate desired for the decoy.

These results apply only to the features $A_{\Delta\lambda}$, $B_{\Delta\lambda}$, $C_{\Delta\lambda}$, and $\sigma_{\Delta\lambda}$*; the Bhattacharyya distance and the decoy requirements for the temporal autocorrelation feature, $\tilde{\rho}(\ell)$, are computed in a different manner. There are basically two distinct, and complementary, ways in which the Bhattacharyya distance corresponding to the RV and decoy $\tilde{\rho}(\ell)$s can be calculated.

The first method consists of computing the spectra of the RV and decoy $\tilde{\rho}(\ell)$s, estimating the mean and standard deviation of these spectra:

m_ω^{RV}, σ_ω^{RV} = RV spectrum mean, standard deviation

m_ω^{DY}, σ_ω^{DY} = decoy spectrum mean, standard deviation

and computing the univariate Bhattacharyya distance (which is again separable into mean and a variance contribution):

$$\mu(\tilde{\rho}) = \mu_{MN}(\omega) + \mu_{VAR}(\omega) \tag{23a}$$

$$\mu_{MN}(\omega) = \frac{1}{8}\left(\frac{\Delta m_\omega}{\bar{\sigma}_\omega}\right)^2 \tag{23b}$$

$$\mu_{VAR}(\omega) = \tfrac{1}{2} \ln \frac{\bar{\sigma}_\omega^2}{\sigma_\omega^{RV}\sigma_\omega^{DY}} \tag{23c}$$

---

* For the feature $\sigma_{\Delta\lambda}$ statistics should be computed for $\ln\sigma_{\Delta\lambda}$ since it is more likely to be lognormally distributed than normally distributed (i.e., negative values are not allowed).

where

$$\Delta m_\omega = m_\omega^{RV} - m_\omega^{DY} \tag{23d}$$

$$\bar{\sigma}_\omega^2 = \tfrac{1}{2}(\sigma_\omega^{RV})^2 + \tfrac{1}{2}(\sigma_\omega^{DY})^2 \tag{23e}$$

The levying of decoy requirements on m_ω^{DY} and σ_ω^{DY} can be achieved in the same manner as described for the other features using the plots of Figs. 1 and 2 (with $\bar{\rho}=0$). This method is useful since it provides statistics on the frequency of the sinusoidal-like variation of the signature. However, it assumes that the frequency distributions are unimodal and can thus be characterized by a single mean and standard deviation.

The second method is based directly on the first-and second-moment statistics of the sinusoidal-like signature remaining after trend removal, $\tilde{H}_{\Delta\lambda}(n)$, and makes no assumptions about the nature of the frequency distribution. If, after trend removal, the signature is normalized to unit variance (by extracting the feature $\tilde{\sigma}_{\Delta\lambda}$) then it is completely characterized (for Gaussian statistics) by its ensemble mean vector and covariance matrix, which are given by:

$$\tilde{M} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

$$\tilde{\Sigma} = \begin{bmatrix} 1 & E\{\tilde{\rho}(1)\} & \cdots & E\{\tilde{\rho}(L)\} \\ E\{\tilde{\rho}(1)\} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & E\{\tilde{\rho}(1)\} \\ E\{\tilde{\rho}(L)\} & \cdots & E\{\tilde{\rho}(1)\} & 1 \end{bmatrix} \tag{24}$$

where L is the number of lags (which is less than or equal to N-1, where N is the number of observations). The mean vector is zero because the trend has been removed and the covariance matrix has the form that it does because of the definition of $\tilde{\rho}(\ell)$. That is, an element of $\tilde{\Sigma}$ is defined as

$$\tilde{\sigma}^2_{mn} = E\{\tilde{H}_{\Delta\lambda}(m)\tilde{H}_{\Delta\lambda}(n)/\tilde{\sigma}^2_{\Delta\lambda}\} \quad ,$$

and, if we can assume stationarity, $\tilde{\sigma}_{mn}$ is a function only of the difference $\ell=|m-n|$ so that

$$\tilde{\sigma}^2_{mn} = \tilde{\sigma}^2_{\ell} = \frac{1}{N}\sum_{n=1}^{N-\ell} E\{\tilde{H}_{\Delta\lambda}(\ell+n)\tilde{H}_{\Delta\lambda}(n)/\tilde{\sigma}^2_{\Delta\lambda}\} \ .$$

Interchanging the summation and expectation yields

$$\tilde{\sigma}^2_{mn} = E\left\{\frac{1}{N}\sum_{n=1}^{N-\ell} \tilde{H}_{\Delta\lambda}(\ell+n)\tilde{H}_{\Delta\lambda}(n)/\tilde{\sigma}^2_{\Delta\lambda}\right\}$$

$$= E\{\tilde{\rho}(\ell)\} \ .$$

It should be noted here that we are using a biased estimate for $\tilde{\sigma}_{mn}$ and $\tilde{\rho}(\ell)$ since the sums are divided by N and not by $N-\ell$ (which is the actual number of terms in the sum). This is done to ensure that the covariance matrix, $\tilde{\tilde{\Sigma}}$, is positive definite.

The Bhattacharyya distance corresponding to the feature $\tilde{\rho}(\ell)$ is then given by (only the variance term contributes since the means are zero):

$$\mu(\tilde{\rho}) = \tfrac{1}{2} \ln |\tilde{\tilde{\Sigma}}| / \sqrt{|\tilde{\Sigma}^{RV}| \cdot |\tilde{\Sigma}^{DY}|} \tag{25a}$$

where

$$\tilde{\tilde{\Sigma}} = \tfrac{1}{2} \tilde{\Sigma}^{RV} + \tfrac{1}{2} \tilde{\Sigma}^{DY} \ . \tag{25b}$$

NOTATION

Subscripts

i, j, k	waveband
K	total number of wavebands
$\Delta\lambda$	waveband
ℓ, m, n	observation
ω	frequency

Superscripts

DY	decoy
RV	reentry vehicle

Overscripts

$\sim$	normalized or based on normalized signature
$\overline{}$	average (over wavebands or of RV and DY)

Indices

(ℓ)	lag
L	total number of lags
(n)	observation
N	total number of observations
n_0	midpoint of observation interval
$\tilde{n}$	normalized observation number

NOTATION (cont.)

Operations, Functions

$\langle\ \rangle$	summation over observations (temporal sum)
$E\{\cdot\}$	ensemble average
$(\cdot)^T$	transpose
$\lvert\cdot\rvert$	determinant
$(\cdot)^{-1}$	inverse
$p(\cdot)$	probability density

Variables

$A_{\Delta\lambda}$	temporal mean feature (of $H_{\Delta\lambda}(n)$)
$a_{\Delta\lambda}$	$=A_{\Delta\lambda}$
$B_{\Delta\lambda}$	linear trend feature (of $H_{\Delta\lambda}(n)$)
$b_{\Delta\lambda}$	$=A_{\Delta\lambda}B_{\Delta\lambda}$
$C_{\Delta\lambda}$	quadratic trend feature (of $H_{\Delta\lambda}(n)$)
$c_{\Delta\lambda}$	$=A_{\Delta\lambda}C_{\Delta\lambda}$
ε	probability of error
$H_{\Delta\lambda}(n)$	optical signature
$\tilde{H}_{\Delta\lambda}(n)$	normalized optical signature (zero mean, unit variance)
$\langle H^x_{\Delta\lambda}\rangle$	x^{th} temporal moment of $H_{\Delta\lambda}(n)$
M	ensemble mean vector of a feature

NOTATION (cont.)

Variables (cont.)

ΔM	difference in RV and decoy ensemble means
$\tilde{M}$	ensemble mean of $\tilde{H}_{\Delta\lambda}(n)$
m_k	k^{th} waveband component of M
Δm_k	k^{th} waveband component of ΔM
m_ω	mean of spectrum of $\tilde{\rho}(\ell)$
Δm_ω	difference in RV and decoy spectral means
$\mu(\cdot)$	Bhattacharyya distance
$\mu_{MN}(\cdot)$	Bhattacharyya distance due to mean differences
$\mu_{VAR}(\cdot)$	Bhattacharyya distance due mainly to variance differences
$\mu_{COR}(\cdot)$	Bhattacharyya distance due mainly to correlation differences
$\mu'_{COR}(\cdot)$	Bhattacharyya distance due to variance differences reflected in $\mu_{COR}(\cdot)$
$\mu'_{VAR}(\cdot)$	$= \mu_{VAR}(\cdot) + \mu'_{COR}(\cdot)$
μ_{max}	maximum allowable total Bhattacharyya distance
η	$= (1-\rho^{DY})/(1-\rho^{RV})$
$\langle N^x \rangle$	sum of $\tilde{n}^x$
φ	random phase
R	residual in mean-square minimization
$\tilde{\rho}(\ell)$	temporal autocovariance feature (of $\tilde{H}_{\Delta\lambda}(n)$)

NOTATION (cont.)

Variables (cont.)

ρ_{ij}	correlation between wavebands i and j
ρ	average band-to-band correlation
$\bar{\rho}$	average of RV and decoy average band-to-band correlations
Σ	ensemble covariance matrix of a feature
$\bar{\Sigma}$	average of RV and decoy covariance matrices
$\tilde{\Sigma}$	ensemble covariance matrix of $\tilde{H}_{\Delta\lambda}(n)$
σ_{ij}^2	i,j^{th} element of Σ
σ_k^2	simplified notation for σ_{kk}^2
$\bar{\sigma}_k^2$	average of RV and decoy σ_k^2 s
$\tilde{\sigma}_{\Delta\lambda}^2$	temporal variance feature (of $\tilde{H}_{\Delta\lambda}(n)$)
$\tilde{\sigma}_{mn}^2$	m,n^{th} element of $\tilde{\Sigma}$
$\tilde{\sigma}_\ell^2$	$= \tilde{\sigma}_{mn}^2$, $\ell=\|m-n\|$
σ_ω^2	variance of spectrum of $\tilde{\rho}(\ell)$
$\bar{\sigma}_\omega^2$	average of RV and decoy spectral variances
$\hat{\sigma}_{\Delta\lambda}$	standard deviation of example
Δt	time spacing between observations
ω	frequency
X	arbitrary feature
x_k	k^{th} waveband component of X

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The formulation of optical signature features useful in discrimination studies and decoy design, and a method for evaluating a decoy's effectiveness are presented in this report. These features are based on an optical signature model which incorporates a second-order trend with a sinusoidal-type variation normally observed in optical signatures. The decoy evaluation procedure presented here provides the decoy designer with an analysis of those features and wavebands of the decoy signature which may need modification to achieve a prescribed match with an RV.

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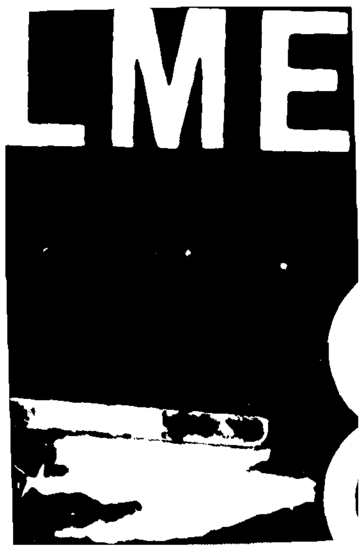
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