THE PRINCIPLE OF THE DISPERSEVIE FALSE RATE RECEIVER, (U)

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by
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I. INTRODUCTION

The target environment of modern radar detection is very complicated. Consequently, this calls for very complex "radar signal processing". On one hand, the "signal processing" has to strive for high signal-to-noise ratio and, on the other hand, it has to achieve a good constant false alarm rate (CFAR) characteristics.

In order to achieve good CFAR characteristics, one must consider the interference that a modern radar encounters in the environment of its targets. Besides white noise, there is ground clutter, interference from rain, snow, waves, etc., and enemy-generated active and passive jamming. The magnitude of these interferences may exceed 60 db. If one uses (digital or analog) automatic signal processing devices due to the reflected pulses of the radar in such vast regions of activity, one usually has circuit overload problems, thus causing "snow" in the PPI display.

Currently, there are two approaches to achieve the CFAR goal, one is to "unify" the effect of jamming, while the other is to use automatic adaptive gate limiting. The dispersive CFAR device setup will deal with the "unification" approach of the effect of jamming.

The dispersive CFAR device used abroad [1], [2] is two to three times better than the logarithmic-fast time constant (Log-FTC) circuit, in terms of characteristics and efficiency. It does not require modification of the transmitter, it only needs some simple addition to the receiver to achieve good results.

References [1], [2] and [3] have made definite introduction to this (CFAR) theory. Currently, there are no theoretical analysis or computational schemes. This paper discusses the theoretical problems and deals with the computations. It shows the parameter selection and provides the theoretical equations and graphs under operating conditions.
II. THE STATISTICAL CHARACTERISTICS OF AMBIGUOUS WAVES UNDERGOING THE "DISPERSE-LIMIT-COMPRESS" PROCESS

The principle of dispersive CFAR is depicted in Figure 1.

![Figure 1. Dispersive CFAR receiver block diagram](image)

Figure 1. Dispersive CFAR receiver block diagram
A) intermediate amplifier; B) amplifier; C) ideal hard limiter; D) compressor; E) detector; F) electric screening gate limiter

The amplifier and compressor shown in Figure 1 are delay lines and their dispersive characteristics are as shown in the figure. As can be seen in the figure, there are differences between CFAR receiver and pulsed radar systems. In the former, both the amplifier and compressor are in the receiver and also a limiter is inserted between them.

The two dispersive lines have equal bandwidth. The signal frequency spectrum becomes wider after limiting and a higher frequency spectrum appears. Therefore, the compression (inverse dispersion) line corresponding to the bandwidth of the limiter is a narrow band filter and the limiter and compression line together form a band-pass limiter. Also, the compression serves as a detection process of the signal which has just undergone the limiting.

The ambiguous wave of a pulse radar is composed of many single element reflected waves such as ground clutters, waves, rain, snow, etc. These waves make up a very densely packed pulse stream on the time axis and each pulse is independent.
Now, we study the statistical characteristics of ambiguous waves passing through this system.

1. The statistical characteristics of the bandwidth output.

First, we assume that the output pulse bandwidth of the ambiguous wave is $\tau_p$, and the dispersive expansion line time is $T$. Then the $i$-th pulse passing through the dispersive expansion is

$$g(t) = A_i \cos \left[ \omega (t - \tau_i) + \frac{1}{2} \mu (t - \tau_i)^2 + \phi_i \right] = \mathcal{A}_i \cos \phi_i \left( -\frac{T}{2} < \tau_i < \frac{T}{2} \right),$$

where $\mu = \frac{\Delta \omega}{T}$ is the slope, and $\phi_i$ is the initial phase amplitude. When the bandwidth $\Delta \omega << \omega_0$ the quadratic phase angles do not have much effect on the total phase amplitude, especially when $T \gg \frac{1}{\Delta \tau}$ (of course, $T$ is much greater than the period of mid-frequency), the phase position of $\phi_i$ in $[0, 2\pi]$ is nearly equal to $\omega_0 t + \phi_i$, where the second term has little effect on $\phi_i$. Therefore, we may treat the distribution characteristics of $\phi_i$ as homogeneous, its probability density distribution function of $\phi_i$ is widely known as

$$P(y_i) = \frac{1}{\pi \Delta \phi} \cos \left( \frac{y_i}{2 \Delta \phi} \right),$$

its computed average value is 0, and its variance is $\sigma_i^2 = \frac{\Delta \phi}{2}$.

Since $T \gg \tau_p$, and the ambiguous wave pulses are very densely packed, then the expansion output is composed of many overlapping frequency modulated pulse waveforms, let the pulse repetition frequency at any time be $n$, then the expansion output is

$$y(t) = \sum_{i=-\infty}^{\infty} A_i \cos \left[ \omega (t - \tau_i) + \frac{1}{2} \mu (t - \tau_i)^2 + \phi_i \right] = \sum_{i=-n}^{n} A_i \cos \phi_i.$$

In order to obtain the probability density function of $y$, we first study the characteristics function of each variable. The characteristics function of $y$ is

$$\theta_y(z) = \int_{-\infty}^{\infty} \Phi(y) e^{iny} dy = \sum_{i=-\infty}^{\infty} \frac{1}{\pi \Delta \phi} \frac{1}{\sqrt{A_i^2 - y^2}} e^{iny}.$$

Let $y_i = A_i$ when $y = A_i$, $i = 0$. When $y_i = A_i$, $i \neq 0$. Therefore,

$$\theta_{y_i}(z) = \int_{-\infty}^{\infty} \Phi(y) e^{iny} dy = \frac{1}{\pi \Delta \phi} \frac{1}{\sqrt{A_i^2 - y^2}}.$$
Note that \( \cos t = \cos(2\pi - t) \). Then
\[
e^{i2\pi e^{i\pi/4}} = e^{i\pi/4},
\]

Therefore,
\[
\theta_{\nu}^{(a)} = \frac{1}{2\pi} \int e^{ixA_{s}} \cos \omega d\omega = \Gamma_{s}(2\pi A_{s})
\]

When \( 2\pi A_{s} < 1 \)
\[
\Gamma_{s}(2\pi A_{s}) = 1 - \frac{(2\pi A_{s})^{2}}{2^{2}} + \frac{(2\pi A_{s})^{4}}{2^{4}} + \ldots
\]

From the expansion of the complex function
\[
e^{(\frac{z}{2})^{2}} = 1 + \left( \frac{z}{2} \right)^{2} + \frac{z^{4}}{2!4} + \frac{z^{6}}{3!8} + \ldots
\]

let \( z = j2\pi A_{s} \), when \( |z| < 1 \)
\[
e^{(\frac{z}{2})^{2}} \approx 1 - \frac{(2\pi A_{s})^{2}}{2^{2}} + \ldots
\]

Therefore, when \( 2\pi A_{s} < 1 \)
\[
\Gamma_{s}(2\pi A_{s}) \approx e^{z} = e^{-2\pi A_{s}^{2}/2}
\]

When \( 2\pi A_{s} \) is very large
\[
\Gamma_{s}(2\pi A_{s}) \approx \sqrt{-\frac{2\pi}{A_{s}}} \cos \left( x - \frac{x^{2}}{4} \right)
\]

\( x = 2\pi A_{s} \)

\( \Gamma_{s}(2\pi A_{s}) \) vanishes very rapidly in the coordinate system, as seen in Figure 2.
For the sake of convenience in our discussion, let us assume that the components all have the same strength (e.g., intensity). We know that the sum of the characteristic functions of each independent random variable is the product of the characteristic functions. Then the characteristic function of $y$ is

$$\theta_y(s) = \Gamma_x(2\pi A_s) = \theta_x(s)$$

We can see in Figure 2 that $\Gamma_x(2\pi A_s)$ vanishes more rapidly than $\Gamma_x(2\pi A_s)$, it is merely confined in the very small area of $2\pi A_s$ (that is, $s$). Therefore, when we consider $\theta_y(s)$, we only consider the very small area of $2\pi A_s$. Therefore

$$\theta_y(s) = \theta_x(s) e^{-isA_s}$$

Since the variance

$$\sigma^2_y = \frac{A^2}{2},$$

then

$$\theta_y(s) = e^{-isA_s}$$

The reverse transformation of the characteristic function $\theta_y(s)$ is the probability density function of $y$:
\[ p(y) = \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma_y^2}} \cdot e^{-j2\pi sy} ds = \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma_y^2}} \cdot \cos(2\pi sy) ds \]
\[ = 2\int_{0}^{\infty} e^{-\frac{y^2}{2\sigma_y^2}} \cos(2\pi sy) ds \]
\[ = \sqrt{\frac{1}{\pi \sigma_y}} e^{\frac{-y^2}{2\sigma_y^2}} \]

where \( \sigma_y = \sqrt{\mathcal{N}_o \cdot \frac{1}{2\pi}} \)

We can infer from this that when \( n \) is relatively large, that is, when the time width of the disperse line is lower, the \( n \) random amplitude sine waves constitute a noise waveform of Gauss type.

When intensity of the pulses of the ambiguous wave are not uniform, the near element intensity is not far from the average, its \( \theta_r(s) \) is

\[ \theta_r(s) = \frac{1}{\sqrt{\pi \sigma_r}} e^{\frac{-s^2}{2\sigma_r^2}} \]

where \( \sigma_r = \frac{\sum A_i^2}{2} = \sum \sigma_r \) and its \( N_{oi} \) is the power spectrum density of the \( i \)-th pulse.

The shape of the graph for \( \theta_r(s) \) is the same as that of \( \theta_r^{*}(s) \) and its \( p(y) \) is

\[ p(y) = \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma_y^2}} \cdot e^{-j2\pi sy} ds = \frac{1}{\sigma_y \sqrt{2\pi}} e^{\frac{-y^2}{2\sigma_y^2}} \]

From the above analysis, we may conclude that after the ambiguous waves have passed through the dispersive line, its amplitude distribution is Gaussian.

The physical explanation of this conclusion is as follows: Because pulses of the ambiguous wave will repeat each other.
after passing through the dispersion line, when the repetition number is large, our region of observation shows the total contribution of the n pulses of the ambiguous wave. Furthermore, the contribution by each component to the totality is very small. Also, after dispersion, \( \sigma^2 \) is \( \tau / r \) times its original strength. When \( T > \tau \), \( \sigma^2 \) is very small. Then the results of the addition will certainly satisfy the central limit theorem.

As for the situation where the output of the intermediate amplifier is Gaussian noise, since the dispersive line is a linear device and it possesses dispersive property, its output is still Gaussian noise. Many references have described this.

2. The statistical characteristics of the interference on the output of ideal band limiter.

The interference of the dispersion output is the sum of its components, it has a waveform of random modulated frequency and its phase is disorderly. Due to the non-linear effect of the limiter behind it, the amplitudes of each component suffer serious mutual adjustment (through interference) and the phase is distorted seriously. Therefore, the compression output cannot restore the interference in the component to the original shape. Thus, the compression line has only induced the effect of a narrow band filter. We shall explain this point by way of vector addition.

First analyze the addition of two vectors whose difference in intensity is not large and the time difference is \( \tau \):

\[
n_1(t) = \rho_1(t) \cos \left[ \omega_1 (t-\tau) + \frac{1}{2} \mu (t-\tau)^2 \right] \quad (t < i < t+T)
\]

\[
n_2(t) = \rho_2(t) \cos \left[ \omega_2 + \frac{1}{2} \mu t^2 \right] \quad (0 < i < T)
\]

and their difference is

\[
\varphi(t) = \frac{1}{2} \mu t^2 - \mu \tau^2 - \mu \tau t - \omega \tau
\] (1)

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The overlapped region of the combined vectors is

\[ n(t) = n_1(t) + n_2(t) = \gamma(t) \cos \left( \omega_0 t + \frac{1}{2} \mu t^2 + \theta(t) \right) \]

as shown in Figure 3.

Figure 3. Diagram of addition of two vectors and amplitude limit.

As can be seen from Figure 3, from the known condition of the two interference components, the magnitude of \( \theta(t) \) depends on \( \varphi(t) \). From equation (1), \( \varphi(t) \) is a dependent variable of time, then so is \( \theta(t) \) a dependent variable. Therefore, after the limiter, the combined vector has the phase characteristics

\[ n_v(t) = \gamma_v \cos \left[ \omega_v t + \frac{1}{2} \mu_v t^2 + \theta_v \right] \]

Obviously, it does not match the phase characteristics of the compression line.

If there are more overlapping vectors undergoing the limiting process, the effect of mutual adjustment is much more severe. If there are \( n \) vectors of approximately equal magnitude, but with different phases distributed within \( (0, 2\pi) \) in different times, then at a certain time the phase characteristic of the combined vector will certainly be close to one of the components or between some two vectors with phase characteristics near to each other (in magnitude). At another time, it is near another component. The situation is the same after a limiting. Therefore, after a compression, the interference is unaffected. At this time, the compression has a filtering effect on the interference.
Therefore, as far as the interference is concerned, limiter and compression (inverse dispersion) lines act respectively as limiter and narrow band filter. When a limiter is a hard limiter if it uses the narrow band filter to take out only part of the interference which pass through the limiter, then the output of the Gaussian noise passing through such a system has the statistical characteristics of a normal distribution [4]. When the envelope of the components that are in-phase and the normal components (both) output by the narrow band filter are correlated, its envelope is the weighted Rayleigh distribution of a Laguerre polynomial [5].

III. THE PRINCIPLE OF CONSTANT FALSE ALARMS

The foregoing paragraph indicated that the power of the i-th component is $\sigma^2_i$, and the power of the i-th component before expansion was $\sigma^2_i = \frac{T_i}{T} \sigma^2_{i'}$. Therefore, the average power of the ambiguous wave after expansion may be expressed as

$$\sigma^2 = \sum \frac{T_i}{T} \sigma^2_i$$

where $\frac{T}{T_i}$ $\geq n$.

This result explains the averaging of the n independent random elements. This kind of averaging in itself greatly lowers the fluctuation of interference near an element, thereby yielding the power $\sigma^2_{i'}$ of a smoothed interference.

Ideal bandpass limiter stabilize the power of an interference when the signal-to-noise ratio approaches zero and the limiter outputs the value of the correlation function of the interference [6], [7].

$$R_{x}(t) = \sum_{i=1}^{n} h_{ii} R_{x}^{i}(t)$$

$$h_{ii} = \frac{2^{k/2} a_{i}}{R_{x}^{1/2}(0) \Gamma(1) \Gamma\left(1 - \frac{k}{2}\right)}$$

$$= \frac{2^{k/2} a_{i}}{R_{x}^{1/2}(0) \Gamma(1) \Gamma\left(1 - \frac{k}{2}\right)}$$

$$\text{(3)}$$
where $a$ is the amplitude (value) of the limiter output, $R_\alpha(t)$ is the autocorrelation function $R_\alpha(0)=\sigma_s^2$ of an interference before limiting and $F_s(a, c, -z)$ is a hypergeometric confluent function.

Therefore, the noise power output of an ideal bandpass limiter is

$$R_s(0) = \sum_{k=1}^{L} \frac{K_k}{K_r} R_s^k(0)$$

(4)

Substituting equation (3) into equation (4), it can be seen that the noise power output of a limiter is not related to the power input of the limiter. After computations, we find

$$R_s(\omega) = \frac{2a^2}{\pi} \cdot 4 - \sigma_s^2$$

when the input signal-to-noise ratio of an ambiguous approaches zero. If we take into account only $a$, then reference [7] gives

$$\sigma_s^2 = \frac{2a^2}{\pi}$$

(5)

The above two results differ only by a factor of $\frac{4}{\pi}$. Therefore, according to the foregoing discussion, the distribution of the limiter-compressor output of an ambiguous wave is

$$p(z) = \frac{1}{\sigma_s \sqrt{2\pi}} e^{-\frac{z^2}{2\sigma_s^2}}$$

and its probability of false alarm is

$$P_{FA} = \int_{z_0}^{\infty} P(z)dz = \int_{z_0}^{\infty} \frac{1}{\sigma_s \sqrt{2\pi}} e^{-\frac{z^2}{2\sigma_s^2}} dz$$

We can see from equation (5) that $\sigma_s^2$ is only related to the limiter electric screening $a$, and the only variable in this type of integral is $z$. Thus the false alarm probability is made known after the determination of the gate limit $z_0$. When the envelope of the components that are in-phase and the normal components (both) output by the narrow band filter are correlated, the post-envelope detector false alarm probability is [5]
where $e$ is the only random variable, the other quantities are constants, and $L_n(t)$ are Laguerre polynomials:

\[
L_1(x) = 1 - x, \\
L_2(x) = 1 - 2x + \frac{x^2}{2}, \\
L_3(x) = 1 - 3x + \frac{3x^2}{2} - \frac{x^3}{6}, \\
L_4(x) = 1 - 4x + 3x^2 - \frac{2x^3}{3} + \frac{x^4}{24}
\]

Utilizing the integral

\[
\int \sqrt{2 \pi} \frac{B}{\sigma_N} \ln \left( \frac{e^t}{2} \right) \exp \left( -\frac{e^t}{2} \right) \, dt = e^{-(t/\sigma_N)} \left[ L_0 \left( \frac{Bt}{\sigma_N^2} \right) - L_{-1} \left( \frac{Bt}{\sigma_N^2} \right) \right]
\]

we may obtain the probability of false alarm. The resultant probability, by integration, differs only in coefficients from the probability obtained from Rayleigh probability density function. We can thus see from Figure 1 that we may obtain the probability of false alarm whether be it the ground clutter or random noise.

IV. ANALYSIS OF THE DETECTION CHARACTERISTICS

Since the frequencies and bandwidths of the target signal and the ground clutter are basically equal, if the receiver receives strong point targets in the reflected signal, and after expansion if the combined vector oscillates about the target (see Figure 4), then the phase characteristics of the combined vector approaches that of the target signal. Also, due to the effect of the limiter, a "cannibalism" phenomenon appears, i.e., parts of the clutter about the target signal are absorbed and the signal-to-noise ratio is increased. Therefore, the target signal is being filtered and compressed.
Figure 4. The combination of the target signal vector and the clutter vectors are combined into vector \( x(t) \).

Since the limiter can make constant the power output, then if noise exists in the system, the output intensity will remain the same regardless of the strength of the signal.

When there are multiple targets and there are overlapping of these signals after an expansion and also the differences in the signal strength are small, then there will be mutual adjustment. If there are strong and weak neighboring signals, then there will be an inhibition of the strong over the weak signal. Suppose that there are two neighboring target signals of respective different strengths

\[
S_i(t) = A_i \cos \left[ \omega_i (t - r) + \frac{1}{2} \mu_i (t - r)^2 \right] \quad (r \leq t \leq r + T)
\]

\[
S_j(t) = A_j \cos \left[ \omega_j t + \frac{1}{2} \mu_j t^2 \right] \quad (0 \leq t \leq T)
\]

After limiting, the overlapped part becomes

\[
S = A \cos \left[ \omega t + \frac{1}{2} \mu t^2 + \theta(t) \right] \quad (r \leq t \leq T)
\]

\[
\theta(t) = -\tan^{-1} \frac{A_i \sin \phi(t)}{A_i + A_j \cos \phi(t)}
\]

where the expression for \( \phi(t) \) is the same as in equation (1).

When \( A_i \gg A_j \), we have

\[
u(t) = -\tan^{-1} \frac{A_i \sin \phi(t)}{A_i \cos \phi(t)} \approx \frac{A_i}{A_i} \sin \phi(t) = 0
\]
showing that the phase characteristic of the combining signal is nearer to that of the stronger signal. We can also see that the weaker signal is seriously inhibited and that its effect on the stronger signal is insignificant. Furthermore, the frequency spectrum and phase amplitude are correspondingly affected in different ways.

Since the ambiguous wave becomes a Gaussian noise after passing through the dispersion line and its phase amplitude is disorderly and its signal is linearly, weakly adjusted, we may consider the limiter and compressor as one to detect the signal modulation in the noise. The detector characteristics of that system and the pulse compression receiver are similar.

Since the pulse compression causes a sidelobe problem, then in order to inhibit it, we usually have to utilize expanded network.

When the limiter input signal-to-noise ratio is less than 0.35, the limiter output will suffer a loss of 1 decibel in the noise [6] and the pulse compression output signal-to-noise output is [8]

$$[-\gamma + 10 \log(T \times df) - 1 - L_m] \text{db}$$

(8)

where \( \gamma \) is the limiter input signal-to-noise ratio, \( L_m \) the mismatch loss of the addition power. In order not to have significant false effect from the limiter, it only requires the limiter bandwidth to be several times that of the signal.

Consider the case of both signals and noise are in the system when the strong signal (the signal-to-noise ratio being large) and the weak signal are doubling, the inhibiting relationship of the strong signal over the weak signal of the doubling part is shown in Figure 5. The maximum effect of the inhibition is 6 db.
Figure 5. Graph for the relationship between the strong and the weak signals.

A) inhibition coefficient of weak signal (db);
B) strong signals signal/noise (db)

When a strong and a weak signal double over a small region then the weak signal-to-noise ratio of the device filter output in the concave region of an ambiguous wave may be expressed [8]

$$(-γ+10 \log (T \cdot d) \cdot (1-\rho)^2 - 1 + N_t - L_a) \, db$$

(9)

where $\rho$ is the doubling coefficient, it is the ratio of the width of the doubling portion to $T$, and $N_t$ is the quiet clutter inhibition of the doubled portion.

When the doubled portion is large, its weak signal-to-noise ratio of its signal output is [8]

$$(-γ+10 \log (T \cdot d) - 3 - L_a) \, db$$

(10)

The detection characteristic is as shown in the graph of Figure 6 [8].

Utilizing equations (8), (9), (10) and Figure 6, we can obtain through analysis the measurement of the apparent lowering of the detection probability of the doubled signals part of the time (especially for weak signals). However, if the loss in the signal-to-noise ratio is not large, we can use a model A display (scope) to observe small targets, given sufficient dynamic region of activity.
Figure 6. The probability of the gate limiting being exceeded by the voltage of the "unified" gate limiting.
In the entire system, there are two kinds of CFAR losses. One is due to the limit on the length of the dispersion line. This in turn sets a limit on the average number of single elements, thereby causing the loss. The other loss occurs when the signal-to-noise ratio input to the limiter is less than 0.35. When the limiter causes the signal-to-noise ratio to become smaller, the maximum loss is 1 db. When the ratio is greater than 0.35, after the limiter filtering, the signal-to-noise ratio will become greater, the maximum being 6 db.

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REFERENCES


Summary

In this paper we research on the principle of dispersive constant false alarm rate receiver, we analyse emphatically the statistical property of the interference to pass through the system. Based on this we discuss the principle of dispersive Constant false alarm rate Receiver, then we point out that we considered the System behind limiter as a pulse Compression receiver and discuss it's problem of signal detection.