With the ceramic flat-pack design and the necessity of using four mounting supports for high-shock applications, a significant contribution to the frequency error arises from forces transmitted from the alumina package to the quartz. Frequency perturbation of quartz vibrators by external forces has received both experimental [3]-[9] and theoretical [10]-[15] attention. It is known that the frequency changes are due to both the static deformation of the crystal lattice and a nonlinear elastic constant effect [11]-[14]. In this note certain results involving the force-frequency effect are applied to the question of mounting an AT-cut circular resonator on four points in a manner adaptable to use in the ceramic flat-pack. It is found that an optimum orientation of the mounting supports exists with respect to the crystal axes and that misalignments about the optimum produce minimal frequency shifts.

The solution given here is constructed from the following features: 1) an analytic form for the force-frequency function that is new, simple, and accurate; 2) use of diametric force pairs, for which superposition holds; and 3) incorporation of the thermoelastic effect.

**FORCE-FREQUENCY EFFECT**

Fig. 1 defines the geometry under consideration. The crystallographic X axis is the datum from which angle ψ is measured; force \( F_1 \) acts along the crystal plate diameter with azimuth \( \psi \), while the diametric force \( F_2 \) is applied at azimuth \( \psi + \gamma \). The points of application of \( F_1 \) and \( F_2 \) represent the positions of the mounting supports. The problem is to find the proper values of angles \( \psi \) and \( \gamma \) to ensure a minimum sensitivity of frequency to applied force.

Although it is not necessary for the mounts to be diametrically paired as shown in Fig. 1, virtually all of the experimental data, as well as most theoretical results, exist in this form. In addition, the force-frequency effect due to opposed forces acting across the crystal diameter is found to be superposable [6]. The separate contributions to the frequency shift under loads \( F_1 \) and \( F_2 \) thus add linearly to produce the overall frequency excursion of \( F_1 \) and \( F_2 \), acting together. This fact allows a simple solution to what is otherwise a very difficult problem.

The force-frequency effect produced in circular crystal plates acted upon by diametric forces \( F \) at angle \( \psi \) is characterized by Ratka [7] by means of a force-frequency coefficient \( K_f(\psi) \). For a plate of diameter \( D \), thickness \( 2h \), nominal frequency \( f_0 \), and frequency constant \( N \), \( K_f(\psi) \) is defined as

\[
K_f(\psi) = \frac{\Delta f}{f_0} \cdot \frac{2h \cdot D}{F} \cdot \frac{1}{N}.
\]  

In (1), \( \Delta f \) is the frequency change brought about by application of compressional force \( F \). For the AT-cut of quartz,

\[
N = 2h \cdot f_0 = 1.660 \text{ MHz-mm.}
\]

\( K_f(\psi) \) in (1) can be interpreted as being a proportionality factor relating the fractional frequency change \( \Delta f/f_0 \) to the average stress acting across the crystal diameter: \( F/(2h \cdot D) \). The azimuthal dependence arises from the anisotropic nature of the crystal lattice. To determine a compensated mounting configuration, the
variation of \( K_f(\psi) \) with \( \psi \) must first be accurately determined. Although the theoretical curves have become increasingly better fits to the experimental data with time, at present the greatest accuracy in characterizing \( K_f(\psi) \) versus \( \psi \) is to be had by using the experimental results.

To this end, the data for the \( AT \)-cut given by Ratajski [7], representing a compilation from a number of sources, were subjected to a least-squares fit. From symmetry considerations the function \( K_f(\psi) \) must satisfy the relations:

\[
K_f(\psi) = K_f(\psi + \pi)
\]

(3) and

\[
K_f(\pi/2 - \psi) = K_f(\pi/2 + \psi)
\]

(4)

With due regard for these symmetries the functional form adopted for \( K_f(\psi) \) is

\[
K_f(\psi) = \sum_{n=0}^{4} A_n \cos^n \psi
\]

(5)

A two-term least-squares fit gives the coefficients listed in Table I. The resulting curve is shown in Fig. 2. A comparison between the latest theoretical results and the fit to the experimental data using (5) is given in Table II.

**FOUR-POINT MOUNTS**

The anisotropy of quartz with respect to thermal expansion will produce unequal forces on the mounting supports because the ceramic flat-pack holder is isotropic, so differential strains will be azimuth-dependent. Since the \( \Delta f \) produced by a force is proportional to both the force and to the value of \( K_f(\psi) \) at the azimuth of the force, and since it is desired that the algebraic sum of both frequency shifts equals zero, we have

\[
K_f(\psi) \cdot \rho(\psi, \gamma) \cdot K_f(\psi + \gamma) = 0
\]

(6)

In (6)

\[
\rho(\psi, \gamma) = F_2/F_1
\]

(7)

is the force ratio. Because the forces depend on the differential expansion coefficients, for a fixed temperature change, the force ratio is given by

\[
\rho(\psi, \gamma) = (\alpha_{11}(\psi + \gamma) - \alpha_0)/(\alpha_{11}(\psi) - \alpha_0)
\]

(8)

where

\[
\alpha_{11}(\psi) = \alpha_{11}(\cos^2 \psi + \sin^2 \psi \sin^2 \theta) + \alpha_{33}(\sin^2 \psi \cos^2 \theta)
\]

(9)

is the thermal expansion coefficient of the plate in the radial direction at azimuth \( \psi \), expressed in terms of the unrotated constants \( \alpha_{ij} \) and the rotation angle \( \theta = +35.25^\circ \) for \( AT \)-cut quartz. The quantity \( \alpha_0 \) in (8) is the thermal expansion coefficient of alumina.
The four-point mounting problem for circular AT-cut quartz resonator plates subjected to thermally induced mounting forces has been solved. A locus of acceptable configurations has been determined, and from this locus the position of minimum sensitivity to mounting errors has been found.

**Conclusions**

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**References**

