A Comparison of Monopulse and Interferometric Systems

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15 March 1981

Interim Report

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Prepared for
SPACE DIVISION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
P.O. Box 92960, Worldway Postal Center
Los Angeles, Calif. 90009
This interim report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-80-C-0081 with the Space Division, Deputy for Technology, P. O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved by The Aerospace Corporation by Dr. Howard Phillips, Director, Electronics Research Laboratory. Major George A. Kuck, SD/YLXT, was the project officer for the Mission Oriented Investigation and Experimentation Program.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NITS). At NITS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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**Title**: A Comparison of Monopulse and Interferometric Systems

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**Controlling Office Name and Address**: Space Division, Air Force Systems Command, Los Angeles, Calif. 90009

**Report Date**: 15 March 1981

**Number of Pages**: 31

**Distribution Statement**: Approved for public release; distribution unlimited.

**Security Classification**: Unclassified

**Abstract**: The angular error performance of monopulse and interferometric systems is derived and used as a basis to compare the characteristics of the two techniques. The angular error performance of the interferometric system is achieved at the expense of ambiguities in its response and the use of monopulse circuitry with the interferometric elements was investigated as a means of resolving the ambiguities, a technique which appears to be previously unexplored. A statistical analysis was developed which expresses the probability of correct ambiguity resolution in terms of the monopulse performance.
and the relative size between the interferometric antenna and its baseline. The angular coverage and signal sensitivity of the two systems are dictated by their respective antenna characteristics; for the same angular error performance, the interferometric system achieves a larger coverage at the expense of signal sensitivity. The choice of a particular technique for a specified application depends on the interplay of the system requirements and the characteristics of the two techniques.
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I. INTRODUCTION

Precision angle measuring systems are required for many applications within the fields of communication, radar, direction finding, sonar, etc. Two techniques, which are commonly considered for high accuracy requirements, are (amplitude) monopulse and interferometric (phase monopulse) systems. Fundamentally, these two techniques differ because monopulse systems operate on amplitude information (although phase information is sometimes used as well), and interferometers operate on phase information. When both systems are resolution limited; i.e., the system resolution requirements dictate the antenna diameter for monopulse systems and the baseline dimensions for the interferometer, other system characteristics, such as angular coverage and hardware requirements, can significantly differ between the two techniques. The difference in these characteristics can lead to choices between the two techniques for a particular system design.

While a comparison of these techniques has been made for wideband systems operating over a ±30° field-of-view (Ref. 1), this discussion is more concerned with narrow pencil-beam systems and aperture antennas, which are capable of higher angular precision.

Interferometric systems can achieve high angular resolution by employing baselines which are multiple wavelengths in dimensions. This resolution performance is achieved at the expense of ambiguities in the angular response, which result because the system only measures the principal value of the phase difference between interferometric elements. The ambiguities must be correctly resolved in order to achieve a practical system design. One technique for ambiguity resolution is to add more antenna elements within the baseline; however, this technique can have a significant impact on system complexity and cost, particularly when the interferometric elements are large. Another technique, proposed in Ref. 2, would resolve ambiguities by varying the signal frequency; this technique is not practical for many applications. A third technique, which appears to be previously unexplored and will be discussed here, would utilize monopulse techniques in conjunction with the interfero-
metric antenna elements. This new technique is investigated in detail with a statistical analysis, which basically addresses the following question: how large must the interferometric antenna elements be relative to the baseline dimensions to correctly resolve the interferometric ambiguities within a specified probability?
II. SYSTEM ANGLE MEASUREMENT ERRORS

The basis for the angle sensing comparison for the two systems is the expression for their pointing errors. This analysis assumes the system operates on a single signal and is not degraded by other signals. These errors will be normalized to the antenna system beamwidth, and are limited by the amplitude and phase errors of the monopulse and interferometric systems, respectively. The angular error performance may be obtained most accurately from carefully measured data for the complete system; however, simple analytic derivations for these errors will be developed to provide the basis for comparison. More detailed treatments may be found elsewhere, e.g., Ref. 3. These expressions will also be used subsequently to investigate ambiguity resolution performance.

A. Monopulse Systems

The amplitude monopulse system, as shown in Fig. 1, uses two beams generated by off-axis feed elements to derive angle-sensing information. The angle-sensing performance for monopulse systems is limited by errors in \( V_1 \) and \( V_2 \). In the analysis presented here, performance in a single plane will be described; in practice, four beams would be used, two beams in each orthogonal plane. These beams would be combined as sums (\( \Sigma \)) and differences (\( \Delta \)) by a beam forming network, and angle sensing information is derived from the measured sum and difference ratio, \( \Sigma / \Delta \). The determination of the angular error as a function of the measurement errors in \( V_1 \) and \( V_2 \) basically requires a determination of the arrival angle as a function of \( V_1 \) and \( V_2 \) and a determination of the sensitivity of the arrival angle to changes in \( V_1 \) and \( V_2 \).
In this analysis the antenna beam shapes will be assumed to follow a Gaussian form. This assumption is not restrictive since the main beam response for almost any practical pencil-beam antenna is closely approximated by a Gaussian pattern, particularly since this analysis really only considers the response out to about the 3 dB width. With this assumption, the voltages $V_1$ and $V_2$ may be written

$$V_1 = A e^{-k \left( \frac{\theta - \theta_0}{\Delta} \right)^2}$$

$$V_2 = A e^{-k \left( \frac{\theta + \gamma - \theta_0}{\Delta} \right)^2}$$

where $k = 1.386$ which is obtained from evaluating the Gaussian pattern at the half-power point, $\gamma$ is the angle between the beam peak and the antenna.
axis, and $\theta_o$ is the half-power beamwidth of the antenna. The sum and difference ratio $\Sigma/\Delta$ may be written as

$$\frac{\Sigma}{\Delta} = \frac{V_1 + V_2}{V_1 - V_2}$$  \hfill (2)$$

which by using Eq. 1 may be written as

$$\frac{\Sigma}{\Delta} = \coth \frac{2k\theta_o}{\Delta}$$  \hfill (3)$$

The arrival angle $\Delta$ can be determined from this equation as

$$\Delta = \frac{\theta_o^2}{2k} \coth^{-1} \frac{V_1 + V_2}{V_1 - V_2}$$  \hfill (4)$$

The rms error in the arrival angle $\Delta$ is determined from

$$\tau_\Delta = \sqrt{\left(\frac{3\Delta^2}{\Delta V_1}\right) + \left(\frac{\theta_o}{\Delta V_2}\right)^2} \tau_V$$  \hfill (5)$$

where the errors are assumed uncorrelated with similar statistics and $\tau_V$ is the rms voltage errors. When this expression is evaluated,

$$\tau_\Delta = \frac{\theta_o^2}{4k} \tau_A$$  \hfill (6)$$

where $\tau_A$ is the relative rms voltage error, i.e., $\tau_V/V$.

The choice of the beam displacement angle $\sigma$ is a compromise; as $\sigma$ increases, $\tau_\Delta$ decreases until $\tau_A$ increases as the S/N degrades. Generally, the beams corresponding to $V_1$ and $V_2$ are overlapped at the 3 dB point so that

$$\sigma = \theta_o/2$$  \hfill (7)$$
The rms pointing error normalized to the half-power beamwidth can therefore be written as

\[
\frac{\sigma}{\lambda} = 0.510 \cdot A
\]  

(8)

The angular precision of the monopulse system is limited by the amplitude errors, \( A \), and the antenna beamwidth. An increase in the antenna diameter improves performance by reducing \( \alpha A \) (\( A \) may also be reduced because the thermal noise induced errors decrease with increased gain performance).

B. Interferometer

The angle-sensing performance of the interferometer is limited by phase errors in the output, in contrast to being limited by amplitude errors in the case of the monopulse. For this analysis, the phases of the two elements are subtracted, and nonlinear interferometric processing is not considered. The phase difference between interferometric elements for energy arriving at an angle \( \alpha \) with respect to the interferometric boresight is given by

\[
\phi = \frac{2 \pi d}{\lambda} \sin \alpha
\]  

(9)

where \( d \) is the baseline dimension between the interferometric elements as shown in Fig. 2 and \( \lambda \) is the operating wavelength.

![Functional Block Diagram of Interferometric System](image-url)
The sensitivity of the phase errors to changes in arrival angle is given by

\[ \gamma = \frac{2 \pi d}{\lambda} \cos \theta \]  

(10)

For an interferometer, the half-power beamwidth of the interferometric antenna system, \(\alpha_I\), assuming the element pattern is much broader than the interferometric array factor, is given by

\[ \alpha_I = \frac{0.881}{d \cos \beta} \]  

(11)

The rms pointing error of the interferometer normalized to its beamwidth is given by

\[ \frac{\alpha}{\alpha_I} = 0.181 \gamma \]  

(12)

The angular precision of the interferometer is limited by the phase error, \(\gamma\), and the beamwidth of the interferometric system. An increase in the interferometric baseline improves performance by reducing \(\alpha_I\).
III. MONOPULSE RESOLUTION OF INTERFEROMETRIC AMBIGUITIES

While the interferometric system has the potential of excellent angular resolution performance, such performance is achieved at the expense of angular ambiguities which result from large (multiple wavelength) baseline dimensions. Techniques must be developed to resolve these ambiguities if interferometric systems are used practically. A new technique for ambiguity resolution, described here, would utilize monopulse circuitry with the interferometric elements. In this case, the problem can be interpreted in terms of a determination of the interferometric element size relative to the baseline dimension which is required to achieve ambiguity resolution. Such a determination should be phrased in statistical terms, i.e., what should the element size be in comparison to the interferometric baseline to achieve a specified probability of correct ambiguity resolution?

The analysis will be simplified by initially investigating a single angle coordinate. The phase difference between interferometric elements previously given in Eq. 9 is

$$\Delta \phi = \frac{2\pi d}{\lambda} \sin \phi$$

$$= \alpha + 2\pi N$$

where \( \alpha \) is the principal value of the phase, which is the measured quantity and \( N \) is the ambiguity number, which requires correct determination.

The ambiguity problem can be displayed graphically. Consider Eq. 13 divided by \( 2\pi \) which yields

$$\frac{d}{\pi} \sin \phi = \frac{\alpha}{2\pi} + N$$

where the right hand side of the equation may be interpreted as the number of phase rotations as information is gathered over a range of angles. If the equation is plotted versus \( \sin \phi \), a graphic display given in Fig. 3 results for
Figure 3. Phase Rotations vs. Spatial Angle

various baseline dimensions in terms of wavelengths ($d/\lambda$). Every time the ordinate increases by one unit, another ambiguity occurs. It should be noted that $d/\lambda < 1/2$ spans less than one unit for angles between $-90^0$ and $90^0$; this baseline dimension, as is well known, has no ambiguities in its response. As another example, consider a $5\lambda$ baseline system which examines informa-
tion from $10^\circ$ to $60^\circ$. In this case the ordinate varies from $0.87 \times 10^\circ$ to $4.33 \times 60^\circ$, which is a span of 3.46 phase rotations. The system in this example would encounter 3 ambiguities.

An absolute bound on the ambiguity number $N$ is $\frac{2d}{\lambda}$ as should be clear from both the figure and equation. The number of ambiguities is reduced from the maximum value by restricting the field-of-view. Such restriction results by utilizing directive antenna elements; however, element directivity cannot completely eliminate the ambiguities since the element apertures would have to physically overlap in order to achieve a sufficiently small beamwidth to eliminate the ambiguities.

The determination of the required angular resolution from the monopulse system depends on the angular spacing between ambiguities. The angular spacing between ambiguities $\Delta \theta$ may be derived by considering

$$\frac{d}{\lambda} (\sin (\theta + \Delta \theta)) = \frac{\sigma}{2\pi} + N + 1$$

$$= \frac{d}{\lambda} \sin \theta + 1$$ (15)

which may be rewritten as

$$\frac{1}{d} = \frac{\sin (\theta + \Delta \theta) - \sin \theta}{2 \sin (\Delta \theta/2) \cos (\theta + \Delta \theta/2)}$$

$$\approx \Delta \theta \cos \theta$$ (16)

The angular spacing between ambiguities is therefore

$$\Delta \theta \approx \frac{1}{d \cos \theta}$$

$$\approx \theta_i$$ (17)

where $\theta_i$ is the interferometric half power beamwidth. This result could also be anticipated from uniform array characteristics.
The monopulse performance of the interferometric element must be better than the angular spacing between ambiguities in order to correctly identify the emitter location. The determination of this performance can be done statistically. The problem should also be viewed in two dimensions.

The ambiguities from an interferometer with orthogonal baselines to achieve angle arrival in two coordinates can be represented graphically as in Fig. 4, where $\theta$, $\phi$ are two orthogonal angular coordinates and the spacing between ambiguities $\alpha_i, \phi_i$ is $\lambda/d$ (in rad.) with $\lambda$ the operating wavelength and $d$ the interferometric baseline. This analysis will assume identical baseline dimensions in both planes and circular interferometric antenna element patterns.

Without loss of generality the correct signal location can be selected as $(\theta, \phi) = (0^0, 0^0)$, i.e., the figure is aligned with the correct location which means the system bias errors are zero and the principal phase value is subtracted out. It should be noted that the ambiguities are spaced equally about the correct location. A polar coordinate system can be defined by

$$\rho = (\rho^2 + \varphi^2)^{1/2}$$

$$\theta = \tan^{-1}(\rho/\varphi)$$

(18)

The probability of correct ambiguity resolution will be assumed equivalent to the probability $\rho < \frac{1}{2d}$, i.e., a correct measurement falls within a circle centered about the correct location with a radius equal to one-half the angular spacing between ambiguities. For the symmetric orthogonal baseline configuration, the probability of correct resolution strictly falls within a square region, however, for a regular polygon antenna array configuration, the ambiguities map into regular polygons, which approach a circle as the number of elements increase, and the radius of the circle is dictated by the overall baseline dimensions. The circular area is used here for both generality of array geometry and ease of evaluation and provides a representative assessment of system performance.
Figure 4 Angular Location of Interferometric Ambiguities for a Four Element Interferometer with Orthogonal Baselines
The monopulse system on the interferometric array elements is assumed to provide independent measurements in two orthogonal directions, \( \alpha \) and \( \sigma \), and the statistics in each direction have equal variances and zero means (no bias values). The probability density for these measurements is given by

\[
P_{\alpha} = \frac{e^{-\frac{\alpha^2}{2\sigma^2}}}{(2\pi \sigma^2)^{1/2}}
\]

\[
P_{\sigma} = \frac{e^{-\frac{\sigma^2}{2\alpha^2}}}{(2\pi \alpha^2)^{1/2}}
\]  

(19)

\[
\sigma^2 = \alpha^2 = \hat{\sigma}^2
\]

Note that

\[
P_{\alpha}P_{\sigma} = \frac{e^{-\frac{\alpha^2}{2\sigma^2}}}{(2\pi \sigma^2)^{1/2}} \cdot \frac{e^{-\frac{\sigma^2}{2\alpha^2}}}{(2\pi \alpha^2)^{1/2}}
\]

\[
= \frac{e^{-\frac{\sigma^2 + \alpha^2}{2\alpha^2}}}{2\pi \sigma^2}
\]

\[
= P_{\sigma, \alpha}
\]

and

\[
\int_0^{2\pi} \int_0^{2\pi} P_{\alpha, \sigma} d\alpha d\sigma = \int_0^{2\pi} P_{\sigma, \alpha} \sigma d\sigma d\alpha
\]

(21)

The probability of correct ambiguity resolution is given by

\[
P_{cr} = P(\sigma < \frac{\lambda}{2d})
\]

\[
= \int_0^{\frac{\lambda}{2d}} \int_0^{2\pi} P_{\sigma, \alpha} \sigma d\sigma d\alpha
\]

\[
= 1 - e^{-\frac{1}{2} \left(\frac{\lambda}{2d}\right)^2}
\]

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The monopulse accuracy previously derived in Eq. 8 is given by

$$\frac{\gamma_0}{\gamma_o} = 0.510 \frac{\gamma_A}{A}$$

(23)

where $\gamma_A$ is the amplitude error in the monopulse measurement. The half power beamwidth of the element is assumed to be

$$\beta_o = \frac{1.22\lambda}{D}$$

(24)

where $D$ is the element diameter so that

$$\gamma_o = \gamma = \frac{0.510 \gamma_A}{D} \frac{1.22\lambda}{\pi} = 0.622 \frac{\gamma_A}{D}$$

(25)

Combination of the above equations yields

$$P_{cr} = 1 - e^{-0.323 \left( \frac{D}{d\gamma_A} \right)^2}$$

(26)

which is the desired probability of correct ambiguity resolution.

This analysis applies to a single monopulse measurement. If $K$ independent measurements are performed, the random errors in the monopulse variance $\gamma^2$ are reduced by $1/K$. Independent measurements can arise from monopulse systems on more than one element or from multiple independent looks at the signal. In this case

$$P_{cr} = 1 - e^{-0.323K \left( \frac{D}{d\gamma_A} \right)^2}$$

(27)

This equation is the desired result giving the probability of correct ambiguity resolution in terms of the ratio of the element size to the baseline dimensions and the errors associated with the monopulse measurement.
The derived result is independent of the operating frequency which can be anticipated physically. While the angular separation between ambiguities decreases with increasing frequency, the field-of-view narrows and the monopulse accuracy increases with increasing frequency. As a result, the required element aperture size relative to the baseline dimensions to achieve ambiguity resolution is frequency-independent.
IV. DISCUSSION OF RESULTS

The preceding analysis has developed expressions for the angular measurement errors for the monopulse and interferometric systems and examined the probability of resolving interferometric ambiguities with monopulse circuitry. The analysis will now be used as the basis for comparison of the two angle sensing techniques. One problem in making such a comparison is the determination of the appropriate output measurement errors, $\Delta A$ for the monopulse and $\Delta \Phi$ for the interferometer.

The output measurement errors for the two systems require a careful assessment of many factors, some of which are hardware and system specific in nature and also depend on the care and precision used in calibration to eliminate systematic error sources. Generally, error budget estimates are used to determine an r.s.s. estimate of the error sources to achieve an evaluation of system performance. The final proof of performance is generally an evaluation and calibration of the entire system including the receiving electronics.

One output error source which can be readily estimated is the effect of thermal noise. Reference 4 provides a discussion of the measurement accuracy as limited by thermal noise; basically, both relative amplitude errors and phase errors are inversely proportional to the square root of the signal-to-noise ratio. The electronics and beam forming networks also contribute errors to the measurements. The antenna systems contribute errors which are dependent on polarization, frequency, and spatial responses. It should be noted that the amplitude errors associated with antenna pattern responses and monopulse circuitry are most important for monopulse systems, and the location of the phase center of the antennas, insertion phase of RF cables, and phase detector performance are important for interferometric systems. Propagation errors, such as refraction and multipath, are another important class of errors. Finally, mechanical errors associated with the system mounting and positional readout enter into the overall error budget.
The output measurement errors of monopulse systems are sometimes expressed in terms of amplitude imbalance (dB values). Such an expression naturally results from measurements of beam forming network performance, axial ratio characteristics of antennas, etc. Figure 5 presents the conversion between amplitude imbalance and the relative voltage errors used in this analysis. A "rule-of-thumb" estimate of monopulse performance, often used in first order assessments, is that the system angular error is one-tenth of the antenna half-power beamwidth; this error corresponds to $\gamma_A = 0.196$ from Eq. 8 or a 1.7 dB amplitude imbalance. Similarly, phase imbalance is sometimes used for interferometers as a means of expressing the output measurement error and again such an expression naturally evolved from phase detector performance, RF cable insertion phase differences, etc. Typical output error values, $\gamma_A$ and $\gamma_\phi$, are used parametrically in Figs. 6 and 7 to provide estimates on the system size requirements for monopulse and interferometric systems respectively.

The requirements for the interferometric aperture size relative to the baseline dimensions will now be considered using the "rule-of-thumb" performance ($\gamma_A/\theta_0 = 0.1$) for monopulse systems. The probability of correct ambiguity resolution is plotted in Fig. 6 as a function of the element size relative to the baseline dimensions with the number of independent measurements as a parameter. The element size relative to the baseline is a function of the system requirements for correct ambiguity resolution. For example, if the probability of correct ambiguity resolution is 0.995, the element diameter must be approximately 80% of the baseline dimension if one measurement is made, 56% if two measurements are made, 40% if four measurements are made, etc. The values can be conveniently computed by solving Eq. 27 for $D/d$ which yields

$$D/d = \gamma_A \left(-3.096 \frac{\ln(1-P_{cr})}{K}\right)^{1/2} \quad (28)$$

If the monopulse output error $\gamma_A$ is smaller, the element diameter relative to the baseline dimensions is reduced for a fixed value of $P_{cr}$. 

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Figure 5  Relative Voltage Error Vs. Amplitude Imbalance
Angular Error vs. Monopulse Aperture Dimensions

Figure 6
Figure 7  Angular Error Vs. Interferometric Baseline
Figure 8. Probability of Correct Ambiguity Resolution Vs. Baseline Dimension Relative to Element Diameter

\[ \sigma_A = 1.7 \text{ dB} \]
The field-of-view performance of monopulse and interferometric systems differ. In both cases, the field-of-view is dictated by the antenna aperture diameter. For comparable resolution, monopulse systems are required to have larger antennas and hence a narrower beamwidth than the antenna elements in an interferometer. As a consequence, the field-of-view of the interferometer is larger than that of the monopulse system. The larger aperture of the monopulse system results in a higher gain value and hence more system sensitivity. For example, if the interferometer element is 40\% smaller than the monopulse antenna, its gain value is 8 dB lower than the interferometric system assuming identical antenna efficiencies for both systems. For resolution-limited cases, as was assumed initially, such sensitivity losses may not be important. A fundamental tradeoff in a system design is the compromise between angular coverage and system sensitivity. For many applications, extending the field-of-view is very important; such is the case in direction finding, signal acquisition, etc.

The choice between a monopulse system and an interferometric system depends on the interplay between the system requirements and the characteristics of the two techniques. For some systems, the overall size may be a major constraint; for others, angular resolution, and field-of-view may be the overriding consideration; for still other systems, signal sensitivity may be the controlling factor; etc. Existing hardware and economics are other factors which dictate system choices.

An example system tradeoff will be used to illustrate this interplay between parameter choices. The constraints for this system design will be an overall size limitation of 30\% (wavelength), $\tau_A = 1.7$ dB for monopulse amplitude errors, 20° phase imbalance for interferometric phase errors, and a 95\% probability of correct ambiguity resolution if an interferometric system is used. The interferometric system will utilize four elements and each of these elements will have a monopulse capability. The ratio of the interferometric element size to the baseline dimension, as determined from Eq. 28, is 30\%. The baseline dimension required to keep the interferometric system
within a 30λ overall dimension is 23.1λ and each antenna element is 6.9 wavelengths in diameter. The system comparison is summarized in Table I.

The antenna technology would typically use a dish for the monopulse system and horns for the interferometric system; the costs are perhaps a subjective area, but the cost of a dish is probably comparable or more than the cost of four horns with monopulse circuitry required for the interferometric system.

The gain of the monopulse antenna is 36.9 dB (55% efficiency) while the gain of the horn is 25.8 dB (80% efficiency), which results in the 11.1 dB signal sensitivity penalty for the interferometric system. The interferometric system provides 70% better angle-sensing capability and 4.3 times larger field-of-view performance than the monopulse systems. For systems which operate in a high S/N environment, the interferometric system may be a better choice than a monopulse system.

### Table I

<table>
<thead>
<tr>
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<th>Monopulse</th>
<th>Interferometer/Monopulse</th>
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<tbody>
<tr>
<td>Antenna diameter, λ</td>
<td>30</td>
<td>6.9</td>
</tr>
<tr>
<td>Field of View, ° (3 dB beamwidth)</td>
<td>2.3</td>
<td>10.1</td>
</tr>
<tr>
<td>Antenna Gain Relative to Monopulse, dB</td>
<td>-11.1</td>
<td>-11.1</td>
</tr>
<tr>
<td>Angular error, °</td>
<td>0.233</td>
<td>0.137</td>
</tr>
</tbody>
</table>

The parameters for the interferometer in this example can be varied to achieve different values. The example was based on the requirement for ambiguity resolution. If the baseline is reduced with a corresponding increase in the element antenna size, both the probability of correct ambiguity resolution and the antenna gain are increased with a decrease in the field-of-view and the angular accuracy. The basic conclusions from these analyses are the following:

1) When the antenna size is dictated by S/N requirements for the system and the corresponding angular error performance and coverage is adequate, the monopulse system is appropriate.
2) When the angular error requirements dictate the antenna size rather than the S/N requirements, interferometric systems offer better angular coverage performance and potentially lower cost.

3) When the baseline is a significant fraction of the element diameter, the interferometric ambiguities can be resolved with acceptable probability by the use of monopulse circuitry with the interferometric antennas.
V. CONCLUSIONS

A system comparison of monopulse and interferometric angle-sensing techniques has been made. The angular errors normalized to the antenna system beamwidth have been derived by simple analytic means to estimate the performance of each system. A problem which occurs with interferometric systems is ambiguities in its angular response, which arise because only the principal value of the phase difference can be measured. A new technique for ambiguity resolution was investigated to resolve ambiguities with monopulse circuitry used in conjunction with the interferometric elements. A statistical analysis has derived the probability of correct ambiguity resolution as a function of the monopulse performance and the relative size between the interferometric antenna element and its baseline dimension. Relatively simple analytic expressions have been used to compare the two techniques on a system-level estimate basis; these estimates can be refined by using measured values to achieve a more precise performance estimate. An important task in such a refinement is the determination of appropriate entries in the r.s.s. error budget for the output errors, \( \sigma_A \) and \( \sigma_\phi \).

The choice between the two angle sensing techniques for a particular application lies with tradeoffs in angle sensing performance, angular coverage, signal sensitivity, overall size, and system complexity and costs. The interferometric system required multiple antennas which are smaller in size than the single monopulse antenna. The angle-sensing performance tradeoffs depend on the amplitude and phase errors, and a general statement contrasting the performance of the two systems is difficult to make without specific system error budgets. For high S/N situations the thermal noise errors may not be the dominant error source, i.e., the resolution-limited case initially assumed, and better angular resolution may be achieved with an interferometric system than a monopulse system, assuming both systems are constrained to the same overall dimensions. The angular coverage of the interferometric system is greater than that of the monopulse system for a specified angular value; however, this angular coverage advantage is achieved at the expense of reduced signal sensitivity performance. System complexity and costs are subjective issues which should be evaluated for each instance.
REFERENCES


LABORATORY OPERATIONS

The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military concepts and systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the Nation's rapidly developing space systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

Aerophysics Laboratory: Aerodynamics; fluid dynamics; plasmasdynamics; chemical kinetics; engineering mechanics; flight dynamics; heat transfer; high-power gas lasers, continuous and pulsed, IR, visible, UV; laser physics; laser resonator optics; laser effects and countermeasures.

Chemistry and Physics Laboratory: Atmospheric reactions and optical backgrounds; radiative transfer and atmospheric transmission; thermal and state-specific reaction rates in rocket plumes; chemical thermodynamics and propulsion chemistry; laser isotope separation; chemistry and physics of particles; space environmental and contamination effects on spacecraft materials; lubrication; surface chemistry of insulators and conductors; cathode materials; sensor materials and sensor optics; applied laser spectroscopy; atomic frequency standards; pollution and toxic materials monitoring.

Electronics Research Laboratory: Electromagnetic theory and propagation phenomena; microwave and semiconductor devices and integrated circuits; quantum electronics, lasers, and electro-optics; communication sciences, applied electronics, superconducting and electronic device physics; millimeter-wave and far-infrared technology.

Materials Sciences Laboratory: Development of new materials; composite materials; graphite and ceramics; polymeric materials; weapon effects and hardened materials; materials for electronic devices; dimensionally stable materials; chemical and structural analyses; stress corrosion; fatigue of metals.

Space Sciences Laboratory: Atmospheric and ionospheric physics, radiation from the atmosphere, density and composition of the atmosphere, aurorae and airglow; magnetospheric physics, cosmic rays, generation and propagation of plasma waves in the magnetosphere; solar physics, x-ray astronomy; the effects of nuclear explosions, magnetic storms, and solar activity on the earth's atmosphere, ionosphere, and magnetosphere; the effects of optical, electromagnetic, and particulate radiation in space on space systems.