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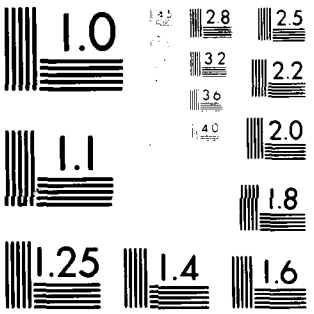

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A perturbation theory approach is taken to the problem of the diffraction of an optical plane wave by an acoustic wave propagating in an infinite slab of isotropic material. This treatment does not, like earlier treatments, neglect reflections at the interfaces, and thus can be applied to transparent solids, for which there is an abrupt wave-impedance mismatch at the interfaces.

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1. INTRODUCTION

Many treatments of the acousto-optic effect — the interaction of light with sound — have been published.[1-7] These treatments assume an infinite slab of isotropic material in which an acoustic wave is propagating parallel to the slab surfaces. A coherent, monochromatic electromagnetic plane wave is incident on the medium from one side, and emerges from the other side broken up into discrete diffracted plane waves of all integral orders (fig. 1). The acoustic disturbance modulates the refractive index of the medium, producing what may usefully be visualized as a diffraction grating.

In previous work, the researchers have used a scalar wave theory (neglecting polarization effects) and have (except for Berry[6]) assumed that reflection from the two interfaces can be neglected. Berry has considered reflections (in a scalar theory) and shows that reflected diffracted beam intensities for a slab of a certain thickness are directly proportional to the transmitted diffracted beam intensities for a slab of twice that thickness. (Actually he, like the other authors, had in mind a liquid medium.) The proportionality constant is

$$\left(\frac{n-1}{n+1}\right)^2 .$$

For liquids, this quantity tends to be small (0.01, or so), whereas for solids it may be appreciably larger.

In an attempt to model the acousto-optic effect in a transparent solid, the author has undertaken to generalize previous results to include reflected waves, and also to consider anisotropic media. The work reported here uses a scalar theory for an isotropic medium (like earlier work), but with rigorous boundary conditions — i.e., solving for reflection as well as transmission. The problem is treated by using a series expansion in the parameter

$$\delta \quad \frac{\Lambda^2}{\lambda^2}$$

(where δ is the intensity of acoustic modulation, and Λ and λ are acoustic and optical wavelengths respectively); hence, the expansion converges only when this parameter is small. A vector treatment of the interaction in an anisotropic medium will be reported in a subsequent paper.

Motivation for this research is given by the experimental work of Berg, Lee, and Udelson,[8,9] who are investigating the acousto-photorefractive effect in y-cut slabs of lithium niobate, an anisotropic material. In their work, the acoustic disturbance is given by a surface acoustic wave (SAW) excited by interdigital transducers and

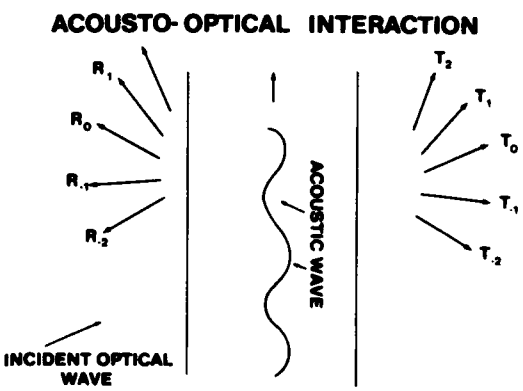

Figure 1. Incident wave is split by acoustic wave into discrete diffracted orders.

[1] *L. Brillouin, La Diffraction de la lumiere par des ultrasons, Actualites Scientifiques et Industrielles No. 59, Hermann et Cie. (Paris, 1933).*

[2] *C. V. Raman and N. S. N. Nath, The diffraction of light by high frequency sound waves: Part I, Proc. Indian Acad. Sci., 2 (1935), 406-412; Part II, 2 (1935), 413-420; Part III, 3 (1936), 75-84; Part IV, 3 (1936), 119-125; Part V, 3 (1936), 459-465; Generalized theory, 4 (1936), 222-242.*

[3] *R. Extermann and G. Wannier, Theorie de la diffraction de la lumiere par les ultrasons,* Helv. Phys. Acta. *(Switzerland), 9 (1936), 520-532.*

[4] *R. Mertens, Simon Stevin, Wis Naturk. Tijdschr., 27 (1949), 212.*

[5] *R. Phariseau, Simon Stevin, Wis Naturk. Tijdschr., 33 (1959), 72.*

[6] *M. V. Berry, The diffraction of light by ultrasound, Academic Press, London and New York (1966).*

[7] *O. Nomoto, Diffraction of light by ultrasound: Extension of the Brillouin Theory, Jap. J. Appl. Phys., 10 (1971), 611-622.*

[8] *N. J. Berg, B. J. Udelson, and J. N. Lee, A new acoustophotorefractive effect in lithium niobate, Appl. Phys. Letters, 31(1977), 555.*

[9] *J. N. Lee, N. J. Berg, and B. J. Udelson, An acousto-optic multichannel signal processor, Appl. Phys. Letters, 32 (1978), 519.*

propagating along the c-axis. The slab is subjected to a high-intensity short-duration laser pulse, which produces a "freezing" of the instantaneous refractive index, an effect which can be used in a memory correlator. Also, work is being done on real-time acousto-optic correlators and convolvers using two SAW's simultaneously. For a complete understanding of these processes, a thorough theoretical treatment of the diffraction is needed. This paper treats the isotropic time-independent case, and can be regarded as a preparation for treatment of the full time-dependent anisotropic case. (The time dependence is a trivial addition giving a frequency shift of the diffracted waves. Anisotropy, however, introduces considerable complication, because of the acoustic coupling of ordinary and extraordinary modes.) This paper does not treat the spatial dependence normal to the surface (exponential falloff).

2. THEORY

The physical system to be analyzed is an infinite, isotropic, and lossless slab of thickness L. An acoustic disturbance is propagated through the medium in the z-direction parallel to the slab surfaces, and the x-direction is chosen normal to the slab surfaces. A plane wave of monochromatic light is incident in the x-z plane, at an arbitrary angle and linearly polarized in the y-direction.

The motion of the acoustic disturbance is neglected and its effect is incorporated as a variation in the dielectric property of the material according to

$$\varepsilon \rightarrow \varepsilon + \delta \cos qz \quad ,$$

where

ε = dielectric constant of unmodulated material,

δ = modulation amplitude, and

$$q = \frac{2\pi}{\Lambda} \quad .$$

This assumption corresponds approximately to the interaction of a pulsed light wave whose duration is short compared with the acoustic wave period. The wave equation for a light beam of radian frequency ω and amplitude E is then

$$-\left(\nabla^2 + \frac{\omega^2}{c^2}\varepsilon\right) E = \left(\frac{\omega^2}{c^2}\delta \cos qz\right) E \quad , \quad (1)$$

where c is the velocity of light.

A propagating electromagnetic mode inside the medium can be resolved into plane waves as follows:

$$E = \sum_{ns} E_{ns} \exp(ikx) \exp(i\gamma_n z)\ \Delta^s \quad , \quad (2)$$

where

γ = z-component of incident light wave vector,

$$\gamma_n = \gamma + nq \quad , \quad (3)$$

$$k^2 = \sum_{l=0}^{\infty} K_l \Delta^l \quad , \quad (4)$$

$$\Delta = \delta \frac{\omega^2}{c^2 q^2} = \delta \frac{\Lambda^2}{\lambda^2} \quad . \quad (5)$$

Note that this is not a power series expansion in Δ, since $k = k(\Delta)$, and therefore it is not unique. It can more aptly be termed a plane-wave expansion. Substitution of equations (2) and (4) into the wave equation (1) yields

$$\sum_{ns}\left[\left(K_0 + \gamma_n^2 - \frac{\omega^2}{c^2}\varepsilon\right)E_{ns} + \sum_{l=1}^{s} K_l E_{n,s-l}\Delta^l\right] \exp(ikx)\exp(i\gamma_n z)\Delta^s = \sum_{ns}\frac{q^2}{2}\left(E_{n-1,s-1} + E_{n+1,s-1}\right) \exp(ikx)\exp(i\gamma_n z)\Delta^s \quad . \quad (6)$$

The zero-order component E_{00} represents a "normal" mode of propagation for the medium — i.e., a solution of the homogeneous equation

$$\left(\Delta^2 + \frac{\omega^2}{c^2}\,\varepsilon\right)E = 0 \quad .$$

From this we see that

$$K_0 = \frac{\omega^2}{c^2}\,\varepsilon - \gamma^2 \quad .$$

Equating like powers of Δ in equation (6) yields the recursion relation in E_{ns}

$$n(n+2\alpha)E_{ns} = \tfrac{1}{2}\left(E_{n-1,s-1} + E_{n+1,s-1}\right) - \frac{1}{q^2}\sum_{l=1}^{s} K_l E_{n,s-l} \quad , \qquad (7)$$

where

$$\alpha = \frac{\gamma}{q} \quad .$$

Given the angle of incidence of the light wave, θ, γ is determined by $\gamma = \omega/c \sin\theta$, and equation (7) generates the corresponding propagating optical mode of the perturbed medium.

The above solution can be written as $E = \exp(ikx)e(z)$, where $e(z)$ will be shown to be a Mathieu function. Equation (1) becomes

$$\left[\frac{d^2}{dz^2} + \frac{\omega^2}{c^2}\,\varepsilon - k^2 + \frac{\omega^2}{c^2}\,\delta\cos zq\right]e(z) = 0 \quad .$$

This is Mathieu's equation:

$$\frac{d^2 f}{d\xi^2} + (a - 2h\cos 2\xi)f = 0 \quad ,$$

where

$$2\xi = qz \quad ,$$

$$a = \frac{4}{q^2}\left[\frac{\omega^2}{c^2}\,\varepsilon - k^2\right] = (2\alpha)^2 + 0(\Delta^2) \quad ,$$

$$h = -2\,\frac{\omega^2}{c^2 q^2}\,\delta = -2\Delta \quad .$$

Thus

$$e(z) = \sum_{ns} E_{ns}\exp(i\gamma_n z)\Delta^s = f(a,\xi) \quad ,$$

where $f(a,\xi)$ is a Mathieu function. This function will not in general be one of the well-known tabulated periodic functions ce_n and se_n, but, in general, will be a Floquet solution,[10] of the form $e^{\mu z}\,\phi(z)$, where $\phi(z)$ is periodic. However, when $\alpha = N/2$ (N an integer, the Bragg condition), the solution is a linear combination of the ce_n and se_n (see appendix A).

The preceding paragraphs characterize the natural propagating modes for an infinite sinusoidally modulated medium. For a finite thickness slab, $0 < x < L$, the electromagnetic boundary conditions require that the solution be an expansion in a subset of these natural modes, requiring a new index:

$$E = \sum_{mns} E_{mns}\exp(ik_m x)\exp(i\gamma_n z)\Delta^s \quad . \qquad (8)$$

Also a backward-travelling, reflected, and a transmitted wave are required:

$$F = \sum_{mns} F_{mns}\exp(-ik_m x)\exp(i\gamma_n z)\Delta^s \quad , \qquad (9)$$

$$R = \sum_{ns} R_{ns}\exp(-ip_n x)\exp(i\gamma_n z)\Delta^s \quad , \qquad (10)$$

$$T = \sum_{ns} T_{ns}\exp(ip_n x)\exp(i\gamma_n z)\Delta^s \quad , \qquad (11)$$

where

$$p_n^2 = \frac{\omega^2}{c^2} - \gamma_n^2 \; . \qquad (12)$$

Now the wave equation (1) yields a new recursion relation:

$$(n - m)(n + m + 2\alpha)E_{mns} = \frac{1}{2}\left(E_{m,n-1,s-1} + E_{m,n+1,s-1}\right) -$$

[10] N. W. McLachlan, Theory and Application of Mathieu Functions, Oxford Press, Oxford (1947)

$$\frac{1}{q^2} \sum_{\ell=1}^{s} K_{m\ell} E_{m,n,s-\ell} \quad . \tag{13}$$

There is a similar recursion relation for F.

The boundary conditions, applied at $x = 0$ and $x = L$, guarantee continuity of tangential E and H:

$$-R_{ns} + E_{nns} + F_{nns} = -\sum_{\substack{m \\ (\neq n)}} \left(E_{mns} + F_{mns}\right) \quad , \tag{14a}$$

$$p_n R_{ns} + k_{n0} E_{nns} - k_{n0} F_{nns} = -\sum_{\substack{m \\ (\neq ns)}} \sum_{\ell=0}^{s} \left(k_{m,s-\ell} E_{mn\ell} - k_{mns-\ell} F_{mn\ell}\right) \quad , \tag{14b}$$

$$p_{n0} E_{nns} + M_{n0} F_{nns} - \exp(ip_n L) T_{ns} = -\sum_{\substack{m \\ (\neq ns)}} \sum_{\ell=0}^{s} \left(P_{m,s-\ell} E_{mn\ell} + M_{n,s-\ell} F_{mn\ell}\right) , \tag{14c}$$

$$k_{n0} P_{n0} E_{nns} - k_{n0} M_{n0} F_{nns} - p_n \exp(ip_n L) T_{ns} = -\sum_{\substack{m \\ (\neq n)}} \sum_{\ell=0}^{s} \sum_{j=0}^{s-\ell-1} \left(k_{m,s-\ell} P_{m,\ell-j} E_{mnj} - k_{m,s-\ell} M_{m,\ell-j} F_{mnj}\right) \quad , \tag{14d}$$

where

$$\exp(ik_m L) = \sum_{\ell=0}^{\infty} P_{m\ell} \Delta^{\ell} \quad ,$$

$$\exp(-ik_m L) = \sum_{\ell=0}^{\infty} M_{m\ell} \Delta^{\ell} \quad . \tag{15}$$

If the incident plane wave strikes the $x = 0$ interface with a z-direction cosine of γ_0, then the zero-order terms E_{000}, F_{000}, R_{00}, and T_{00} are just those given by the familiar Fresnel relations for an unperturbed medium. The first-order terms are then generated from the recursion relations (which yield E_{011}, E_{0-11}, F_{011}, and F_{0-11}) and the boundary conditions (which yield R_{11}, E_{111}, F_{111}, T_{11} and R_{-11}, E_{-1-11}, F_{-1-11}, T_{-11}) This process can be continued indefinitely, and is an unambiguous way of generating the solution for a given incident plane wave, provided Δ is "small," so that the series converges. The recursion relation (13) serves not only to generate the E_{mns}, but also to generate the $K_{m\ell}$. When $m = n$, the left-hand side of (13) is clearly zero. The $K_{m,s-m}$ is determined in terms of low-order values of $K_{m\ell}$ and E_{mns}. (E_{nns} and F_{nns} are determined, as previously stated, by the boundary conditions.)

The left-hand side of equation (13) is also zero whenever $n + m + 2\alpha = 0$. This is a fundamental problem and in fact invalidates the procedure, but it occurs only when $\alpha = N/2$ for some integer N — the Bragg condition. This difficulty can be dealt with in practice by simply treating an approximate problem $\alpha = N/2 + \varepsilon$. An elaboration of the theory is given in appendix A.

3. GEOMETRICAL INTERPRETATION

There is a simple geometrical construction which is a useful aid in visualizing the above treatment of the acousto-optical interaction. This construction derives from one often used to demonstrate Snell's law. Thus, in figure 2, two views of the simple unmodulated boundary-value problem are given: (a) is self-explanatory; (b) is more abstract. Here the circles (in wave-vector space) represent the naturally propagating modes in the various media. Their diameters are

$$\frac{\omega}{c} , \frac{\omega}{c} \sqrt{\varepsilon} , \text{ and } \frac{\omega}{c} \quad .$$

Figure 3 represents the acoustically modulated medium. Here are represented the higher-order (m,n) terms — plane waves generated by the recursion relation (13) and the boundary conditions (14). The (0,0) vectors are as in figure 2. Equation (13) generates (0,1) and (0,−1). Then

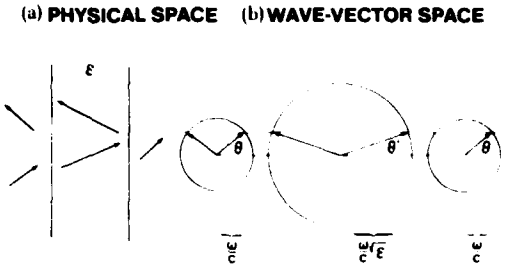

Figure 2. Representation of zero-order interactions (Snell's law) in physical and wave-vector space; boundary conditons require that $\omega_\lambda \sqrt{\varepsilon}\, c \sin\theta$ be constant for all media.

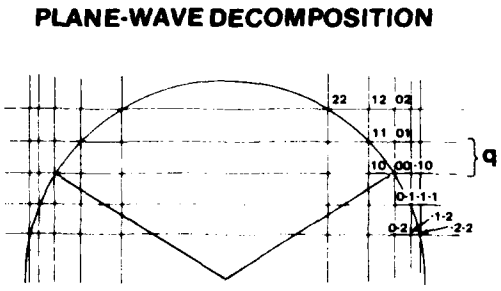

Figure 3. Internal backward and forward travel*ling waves; to each "lattice site" in wave-vector* space corresponds a complex amplitude, whose value is given by the recursion relation (eq (13)) and by the electromagnetic boundary condition.

equation (14) generates (1,1) and (−1,−1). Then equation (13) generates (1,2) (1,0), (0,2), (0,−2), (−1,0), and (−1,−2). Equation (14) generates (2,2), (−2,−2), and a second-order term for (0,0), and so on. It will be recognized that the recursion relation imposes a vertical coupling on the diagram, while the boundary conditions impose a horizontal coupling.

It is important to note that, in figure 3, the "circle" is not a true circle, but is deformed by the Δ-dependence of the k_m.

The Bragg problem ($\alpha = N/2$) can be seen from the diagram (fig. 4, where N = 1) to arise from an "identity crisis" in certain higher-order terms — i.e., "off-circle" terms will coincide with "on-circle" terms. The "off-circle" coefficient blows up as it approaches the "circle."

Also, in the more general anisotropic case, even the zero-order surface will no longer be a circle, but a more general second-, fourth-, or sixth-order surface.

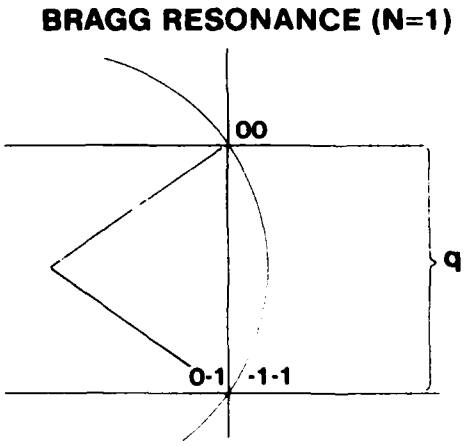

Figure 4. An example of Bragg resonance, which occurs for $\Lambda/\lambda \sin\theta = N/2$; this arises analytically from a singularity in the recursion relation, geometrically from the "double labelling" of an on-surface lattice point.

4. COMPUTATIONAL RESULTS

It is illuminating to compare results obtained from the above formalism with those obtained from earlier efforts. Nomoto[7] has done calculations for an isotropic acoustically modulated medium assuming normal incidence. In this case ($\alpha = 0$), the z-dependence of the internal fields reduces to the tabulated periodic Mathieu functions ce_n, disposing of the need for series expansions and so also of the requirement that Δ be small. (Nomoto neglects reflections — that is, he does not use the boundary conditions — but, under the assumption of a not-too-abrupt ε-transition at the interfaces, this is an acceptable approximation.)

Comparisons between the present author's and Nomoto's calculations are shown in figure 5. Here are compared the intensities of the undeflected

[7] O. Nomoto, Diffraction of light by ultrasound: Extension of the Brillouin Theory, Jap. J. Appl. Phys., 10 (1971), 611-622.

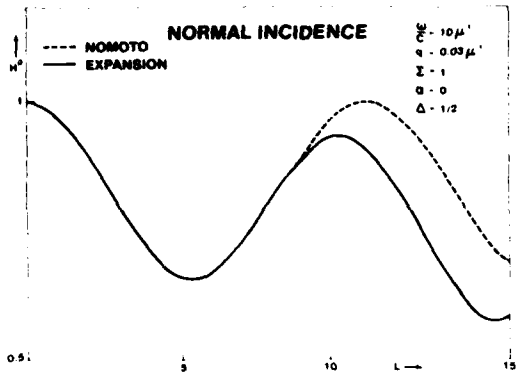

Figure 5. Comparison of Nomoto solution and seventh-order plane-wave expansion; amplitude of undeflected transmitted beam is shown as a function of slab thickness.

$$\left(T_0 = \sum_s T_{0s} \exp(ip_0x)\exp(i\gamma_0 z)\Delta^s \right)$$

transmitted wave, as functions of thickness L, using the two methods. For the system considered, $\Delta = {}^1\!/_2$. The applicability of the expansion calculation, which was performed to seventh order, is in some doubt for such a large Δ. Also, $\omega/c = 10$, $q = 0.03$, $\varepsilon = 1$, *and L varies from 0 to 20 cm.* The value $\varepsilon = 1$ was chosen to minimize the error arising from Nomoto's no-reflection assumption. Even for $\Delta = {}^1\!/_2$, the expansion technique works well up to a thickness of about 10 cm (about 500 acoustical wavelengths). It has the advantage of applying also for oblique incidence and for abrupt changes in refractive index, and of being generalizable to include anisotropic media.

Figure 6 shows, for a different set of parameter values, the amplitude of the T_{02} term as a function of incident angle. The T_{02} term can be visualized as having been doubly diffracted back to its original direction. Here $\varepsilon/c = 10$, $q = 0.5$, $\varepsilon = 5$, and $L = 1$ cm. The Bragg resonance at $\alpha = {}^1\!/_2$ $(\gamma_0 = 0.25)$ can be clearly seen.

The parameter values in figure 6 (ε, ω, q, L) quite closely resemble those of the system that this research is designed to model: modulation of an optical beam by microwave acoustical signals in a 1-cm-thick sample of lithium niobate.

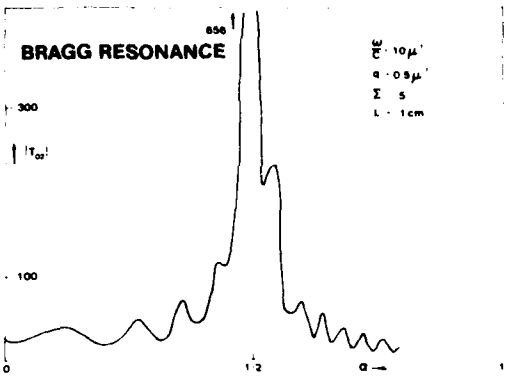

Figure 6. Second-order term of undeflected beam as function of angle, illustrating Bragg resonance.

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NOTATION

c	free-space velocity of light
ω	optical frequency
ε	dielectric constant of unmodulated material
δ	modulation amplitude
Δ	$\delta \frac{\Lambda^2}{\lambda^2}$, expansion parameter
λ	free-space optical wavelength
Λ	acoustic wavelength
q	$\frac{2\pi}{\Lambda}$
L	slab thickness
γ	z-component of incident light wave vector
α	$\frac{\gamma}{q}$
E_{mns}	term in expansion of internal forward-travelling wave
F_{mns}	term in expansion of internal backward-travelling wave
R_{ns}	term in expansion of reflected wave
T_{ns}	term in expansion of transmitted wave
k	x-component of wave vector in medium

APPENDIX A. — THE ACOUSTO-OPTIC INTERACTION FOR BRAGG ANGLES

The acousto-optic diffraction process can be described by a perturbation procedure (see main body of paper). A recursion relation, applied in conjunction with certain boundary conditions, generates plane-wave expansion terms from lower-order terms. The expansion parameter is

$$\Delta = \delta \frac{\Lambda^2}{\lambda^2} \quad ,$$

where δ is the refractive index modulation amplitude, Λ is the acoustic wavelength, and λ is the optical wavelength. The recursion relation is

$$(n - m)(n + m + 2\alpha)E_{mns} = \tfrac{1}{2}\left(E_{m,n-1,s-1} + E_{m,n+1,s-1}\right) - \frac{\Lambda^2}{4\pi^2} \sum_{\ell=1}^{s} K_{m\ell} E_{m,n,s-\ell} \quad . \qquad \text{(A-1)}$$

Here, n, m, and s label the plane-wave terms which combine to give the full field pattern. Other variables appearing are

$$\alpha = \frac{\Lambda}{\lambda} \sin\theta \quad , \qquad \text{(A-2)}$$

$$\theta = \text{angle of incidence} \quad ,$$

$$k_m^2 = \sum_{\ell=0}^{\infty} K_{m\ell} \Delta^{\ell} \quad ,$$

$$E = \sum_{mns} E_{mns} \exp\left[i(k_m x + \gamma_n z)\right] \Delta^s \quad . \qquad \text{(A-4)}$$

In the case of Bragg resonance ($\alpha = N/2$, N an integer), the method fails, since the left-hand side of (A-1) vanishes. A consideration of the solution for such cases, and how to obtain and represent them, is given here.

Physically, the Bragg condition implies perfect periodicity of the system; displacement by Λ (the acoustical wavelength) in the z-direction leaves the system unaltered. (The acoustical z-period is an integral multiple of the optical z-period.) So our solution should be periodic in z. After separation of variables, the z-part of the solution satisfies Mathieu's equation; thus, in the Bragg case, a linear combination of the ce_n and se_n is needed. Since we want a solution whose zero-order term is $\exp(ik_0x)\exp(iNqz/2)$, our solution is clearly

$$\exp(ik_0^+x)\ ce_N\left(\frac{qz}{2}, -2\Delta\right) + i\ \exp(ik_0^-x)\ se_N\left(\frac{qz}{2}, -2\Delta\right) \quad . \qquad \text{(A-5)}$$

where

$$(k_0^+)^2 = K_0^+ = \frac{\omega^2}{c^2}\varepsilon - \frac{q^2 a_N(-2\Delta)}{4} \quad .$$

$$(k_0^-)^2 = K_0^- = \frac{\omega^2}{c^2}\varepsilon - \frac{q^2 b_N(-2\Delta)}{4} \quad .$$

$$\omega = \frac{2\pi c}{\lambda} \quad .$$

$$q = \frac{2\pi}{\Lambda} \quad .$$

and a_N and b_N are the Mathieu eigenvalues in McLachlan's notation,[1] and are functions of Δ.

This solution can be obtained by a modification of the expansion method presented in the body of this paper. The modification arises from the degeneracy ($K_{-N0} = K_{00}$) which results in the Bragg case. For definiteness, we consider here the case $N = 2$ — that is, $\gamma = q$ ($\alpha = 1$). We use the usual recursion relation (A−1), but we start with nonzero E_{000} *and* E_{0-20}. Then

$$K_{00} = p_0^2\varepsilon - q^2 = K_{-20} \quad .$$

[1] N. W. McLachlan, *Theory and Application of Mathieu Functions*, Oxford Press, Oxford (1947).

APPENDIX A

For E_{011} .

$$(1)(1 + 2\alpha)E_{011} = \frac{1}{2} E_{000}$$

$$E_{011} = \frac{1}{6} E_{000}$$

$$E_{0-11}: \quad (-1)(-1 + 2\alpha)E_{0-11} = \frac{1}{2}\left(E_{000} + E_{0-20}\right)$$

$$E_{0-11} = \frac{-1}{2}\left(E_{000} + E_{0-20}\right)$$

$$E_{0-31}: \quad (-3)(-3 + 2\alpha)E_{0-31} = \frac{1}{2} E_{0-20}$$

$$E_{0-31} = \frac{1}{6} E_{0-20}$$

$$E_{022}: \quad (2)(2 + 2\alpha)E_{022} = \frac{1}{2} E_{011}$$

$$E_{022} = \frac{1}{16} E_{011} = \frac{1}{96} E_{000}$$

$$E_{002}: \quad (0)(0 + 2\alpha)E_{002} = 0 =$$

$$\frac{1}{2}\left(E_{011} + E_{0-11}\right) - \frac{1}{q^2} K_{02}E_{000}$$

$$K_{02}E_{000} = \frac{q^2}{2}\left(-\frac{1}{3} E_{000} - \frac{1}{2} E_{0-20}\right)$$

$$E_{0-22}: \quad (-2)(-2 + 2\alpha)E_{0-22} = 0 =$$

$$\frac{1}{2}\left(E_{0-11} + E_{0-31}\right) - \frac{1}{q^2} K_{02} E_{0-20}$$

$$K_{02}E_{0-20} = \frac{q^2}{2}\left(-\frac{1}{2} E_{000} - \frac{1}{3} E_{0-20}\right)$$

$$E_{0-42}: \quad (-4)(-4 + 2\alpha)E_{0-42} = \frac{1}{2}E_{0-31}$$

$$E_{0-42} = \frac{1}{16} E_{0-31} = \frac{1}{96} E_{0-20}$$

The equations for E_{002} and E_{0-20} yield

$$\begin{bmatrix} \frac{K_{02}}{q^2} + \frac{1}{6} & \frac{1}{4} \\ \frac{1}{4} & \frac{K_{02}}{q^2} + \frac{1}{6} \end{bmatrix} \begin{bmatrix} E_{000} \\ E_{0-20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} .$$

Thus,

$$\left(\frac{K_{02}}{q^2}\right)^2 + \frac{1}{3}\frac{K_{02}}{q^2} - \frac{5}{144} = 0 \quad .$$

$$K_{02} = \frac{1}{12} q^2 \quad or \quad -\frac{5}{12} q^2 \quad .$$

Thus,

$$k_0 = k_{00} + \frac{1}{24}\frac{q^2}{k_{00}} \Delta^2 + \ldots$$

or

$$k_0 = k_{00} - \frac{5}{24}\frac{q^2}{k_{00}} \Delta^2 + \ldots .$$

Call the first solution k_0^- and the second k_0^+. Substitution shows that the k_0^- solution has the property $E_{0-20} = -E_{000}$, and the k_0^+ solution has the property $E_{0-20} = +E_{000}$. We thus have, to second order, the two solutions,

$$E^- = \frac{1}{2} E_{000} \left[\exp(ik_{00}x)\exp(i\gamma_0 z) - \exp(ik_{00}x)\exp(-i\gamma_0 z)\right]\exp\left(i\frac{\Delta^2 q^2}{24k_{000}} x\right)$$

$$= i E_{000} \exp(ik_0^- x)\sin qz \quad . \qquad \text{(A-6)}$$

$$E^+ = \frac{1}{2} E_{000} \left[\exp(ik_{00}x)\exp(i\gamma_0 z) + \exp(ik_{00}x)\exp(-i\gamma_0 z)\right]\exp\left(-i\frac{5\Delta^2 q^2}{24k_{00}} x\right)$$

$$= E_{000} \exp(ik_0^+ x)\cos qz \quad . \qquad \text{(A-7)}$$

If carried out to higher order, these solutions turn out to be

$$E^- = iE_{000}\exp(ik_0^- x)se_2\left(\frac{qz}{2}, -2\Delta\right) . \quad \text{(A-8)}$$

$$E^+ = E_{000}\exp(ik_0^+ x)\, ce_2\left(\frac{qz}{2}, -2\Delta\right) . \quad \text{(A-9)}$$

As remarked earlier, the solution we want is

$$E^+ + E^- = E_{000}\left[ce_2\left(\frac{qz}{2}, -2\Delta\right)\right.$$

$$\left.\exp(ik_0^+ x) + i\, se_2\left(\frac{qz}{2}, -2\Delta\right)\exp(ik_0^- x)\right] . \quad \text{(A-10)}$$

It is of interest now to investigate the continuity properties of this solution: to consider the case $\alpha = 1 + \delta$ as $\delta \to 0$. For this case, the reader can verify the following:

$$K_{00} = p_0^2\varepsilon - \alpha^2 q^2 \simeq p_0^2\varepsilon - q^2 - 2q^2\delta ,$$

$$K_{-20} = p_0^2\varepsilon - (\alpha - 2)^2 q^2 \simeq p_0^2\varepsilon - q^2 + 2q^2\delta ,$$

$$E_{000} = 1 ,$$

$$E_{011} = \frac{1}{6} ,$$

$$E_{0-11} = -\frac{1}{2}$$

$$E_{022} = \frac{1}{96} .$$

$$K_{02} = -\frac{q^2}{6} .$$

$$E_{0-22} = \frac{1}{16\delta} ,$$

$$E_{-2-22} = -\frac{1}{16\delta} .$$

The E_{-2-22} term represents the addition of a homogeneous solution which can be considered to arise from the boundary condition. Thus, using

$$k_0^{(a)} = \frac{1}{2}\left(k_0^+ + k_0^-\right) \simeq$$

$$k_{00} - \frac{1}{12}\frac{q^2}{k_{00}}\Delta^2 ,$$

the second-order solution is given by

$$\exp\left(ik_0^{(a)}x\right)\left\{\exp(iqz) + \Delta\left[\frac{1}{6}\exp(2iqz) - \frac{1}{2}\right]\right.$$

$$+ \Delta^2\left(\frac{1}{96}\exp(3iqz) + \frac{1}{16\delta}\exp(-iqz)\right.$$

$$\left.\left.\left[\exp\left(\frac{iq^2\delta x}{k_{00}}\right) - \exp\left(\frac{iq^2\delta x}{k_{00}}\right)\right]\right)\right\}$$

$$\to \exp\left(ik_0^{(a)}x\right)\left\{\exp(iqz) + \Delta\left[\frac{1}{6}\exp(2iqz) - \frac{1}{2}\right]\right.$$

$$\left. + \Delta^2\left[\frac{1}{96}\exp(3iqz) - \frac{1}{8}\frac{iq^2x}{k_{00}}\exp(-iqz)\right]\right\} . \quad \text{(A-11)}$$

This expression also represents our Mathieu function solution to second order, where $\exp(ik_0^{(a)}x)$ has been factored out. The x-dependent coefficient arises from the series expansion of the exponentials:

$$\exp(ik_0^+ x)\, ce_2\left(\frac{qz}{2}, -2\Delta\right) +$$

$$i\exp(ik_0^- x)\, se_2\left(\frac{qz}{2}, -2\Delta\right) \cong$$

$$\exp(ik_0^{(a)}x)\left(1 - \frac{iq^2\Delta^2 x}{8k_{00}}\right)$$

$$\left(\frac{1}{2}\left[\exp(iqz) + \exp(-iqz)\right] +\right.$$

APPENDIX A

$$\Delta\left[\frac{1}{12}\exp(2iqz) - \frac{1}{2} + \frac{1}{12}\exp(-2iqz)\right] +$$

$$\Delta^2\left[\frac{1}{192}\exp(3iqz) + \frac{1}{192}\exp(-3iqz)\right]\Bigg) +$$

$$\exp\left(ik_0^{(a)}x\right)\left(1 + \frac{iq^2\Delta^2 x}{8k_{00}}\right)$$

$$\left(\frac{1}{2}\left[\exp(iqz) - \exp(-iqz)\right] +\right.$$

$$\Delta\left[\frac{1}{12}\exp(2iqz) - \frac{1}{12}\exp(-2iqz)\right] +$$

$$\left.\Delta^2\left[\frac{1}{192}\exp(3iqz) - \frac{1}{192}\exp(-3iqz)\right]\right) .$$

The reader can easily see that this matches up with the limiting form of equation (A-11), and the continuity of the solution at Bragg angles is verified. Thus, the expansion procedure described in the main body of this paper yields solutions for the Bragg case in the limit $\alpha = N/2 + \delta$, $\delta \to 0$. The fact that the coefficients themselves are not continuous arises from the inherent nonuniqueness of expansion (2).

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