LEVEL II

AD-
FINAL REPORT
IRO REPORT NO. 263

(10)

AD A097024

TECHNIQUES FOR SECONDARY ITEM CLASSES INTEGRATED FORECASTING
PART I - ACTIVE ITEMS

U.S. ARMY
INVENTORY
RESEARCH
OFFICE

DTIC
ELECTE
MAR 30 1981
S D
F

SEPTEMBER 1980

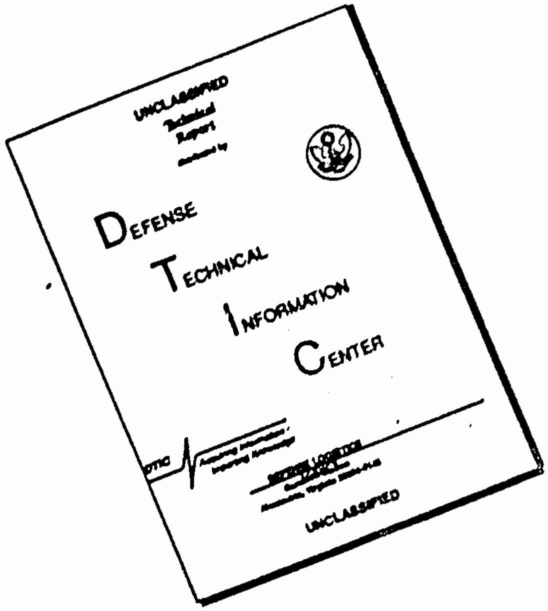
ROOM 800
U.S. CUSTOM HOUSE
2nd and Chestnut Streets
Philadelphia Pa. 19106

DTIC FILE COPY

Approved for Public Release; Distribution Unlimited

81 3 30 002

DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

Information and data contained in this document are based on input available at the time of preparation. Because the results may be subject to change, this document should not be construed to represent the official position of the U.S. Army Materiel Command unless so stated.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

14 IRO-263-PT-1

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
IRO Report No. 263

2. GOVT ACCESSION NO.
AD-A097024

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 INTEGRATED FORECASTING TECHNIQUES FOR SECONDARY ITEM CLASSES - PART I - ACTIVE ITEMS.

5. TYPE OF REPORT & PERIOD COVERED
9 Final Report,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10 EDWIN/GOTWALS, III
DONALD/ORR
P

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
US Army Inventory Research Office
Room 800, US Custom House
2nd & Chestnut Sts., Phila., PA 19106

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
US Army Materiel Development & Readiness Cmd
5001 Eisenhower Avenue
Alexandria, VA 22333

12. REPORT DATE
11 Sep[redacted]980

13. NUMBER OF PAGES
86

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
12 90

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Information and data contained in this document are based on input available at the time of preparation. Because the results may be subject to change, this document should not be construed to represent the official position of the US Army Materiel Development & Readiness Command unless so stated.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Forecast Methods
Secondary Parts Demand
Evaluation Procedures
Inventory Management

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is one of the final in a series on forecasting methods and forecast performance for Army item demands in the wholesale supply system. An extended data base of 48 quarters is used and both old and new algorithms are evaluated. A more intensive analysis is made than in previous studies. Forecast algorithms are used on various item activity classes with the intent of detecting patterns which could indicate where certain algorithms work best. It was hoped that ultimately a synthesis of procedures for forecasting by item
(cont)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

403572

classes and for interfacing across classes could be developed. However, the evaluation results as described in this report present no clear cut improvement to the current method for active items. In Part II, a new forecast procedure is recommended for inactive items.

SUMMARY

This report is one of the final in a series on forecasting methods and forecast performance for Army demands in the wholesale supply system. An extended data base of 48 quarters is used and a summary of results from both old and new forecast methods is presented. The analysis is more intense than in [8]; forecast algorithms are used on various item activity classes with the intent of detecting patterns which could indicate where certain algorithms work best. It was hoped that ultimately a synthesis of procedures for forecasting by item classes would be developed. However, the evaluation results as found in the report present no clear cut improvement to the current method for active items. In [4], a new forecast procedure is recommended for inactive items.

Accession For
NTIS GRA&I
DTIC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Dist | Avail and/or Special
A

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

1.1 Overview

This report is one of the final in a series on forecasting methods and forecast performance for Army item demands in the wholesale supply system. An extended data base of 48 quarters is used and both old and new algorithms are evaluated. A more intensive analysis is made herein than in [8]; forecast algorithms are used on various item activity classes with the intent of detecting patterns which could indicate where certain algorithms work best. It was hoped that ultimately a synthesis of procedures for forecasting by item classes and for interfacing across classes could be developed. However, the evaluation results as described in this report present no clear cut improvement to the current method for active items. In [4], a new forecast procedure is recommended for inactive items.

1.2 Scope

The specific reasons and scope for this report are listed:

a. IRO continues to maintain and expand a data base of aviation parts' demand. The data base of report [3], quarterly data for 9700 items from 1967-1973, has been extended to a file of 13900 items for years 1967-1977. The analysis herein concentrated upon the years 1971-1977, the Vietnam war having less impact than in the earlier file.

b. Several of the algorithms in [8] utilized forecast parameters (e.g. Kalman k-factors) which were obtained from statistical properties of aggregate time series of demand and of flying hours over that earlier 7-year time period. In this report we update these parameters (as well as compute parameters for new types of algorithms) based on a later 7-year period 1971-1977.

c. Reports,[7],of superior performance of moving average forecasts incorporating a Trigg tracking signal by the TARCOM Systems Analysis Group led IRO to modify such a Trigg algorithm and to test it with the expanded data base.

d. Preliminary success of the Trigg tracking signal led IRO to develop other refined algorithms which utilized other switching signals for fixing the length of past history used in a current forecast.

e. Changes have been made in the supply simulator since it was used as one of the performance measuring tools in [8]. Additional improvements developed during the course of the work have been included in the final simulation analyses.

1.3 Findings

For the cursory reader, the table on page 6 is probably the best culmination of the many results and tables in the body of the report. At a glance one can see the basic breakout of the analysis per stratification class. (See bottom of page). A 95% confidence interval was plotted for the mean difference in performance (as measured by the simulator) per item between the alternative algorithms and standard (1794) for the indicated stratification classes. In each case, none of the alternative algorithms performed significantly better than the standard (each confidence interval contains zero).

The final candidate forecast algorithms considered for this analysis were:

1794: the current 8 quarter moving average

KAL: a modified Kalman filter (see [9]) which is similar to exponential smoothing.

IROTRIGG: a switching scheme between an 8 quarter and 4 quarter moving average, with some extra weight given to the current quarter's demand.

MED4: a 4 quarter moving median.

MOVD: a 8 quarter moving average on demands only.

It should be noted that these findings are limited to the more active classes of items. Report [4] has been written which deals specifically with items exhibiting low demands.

STRATIFICATION CLASSES

YEARLY DOLLAR DEMAND

YEARLY REQUISITIONS	0 - $5000	$5000 - $50000	>$50000
0-3	ST1 N=335	ST2 N=100	ST3 N=4
3-12	ST4 N=124	ST5 N=230	ST6 N=17
>12	ST7 N=98	ST8 N=64	ST9 N=115

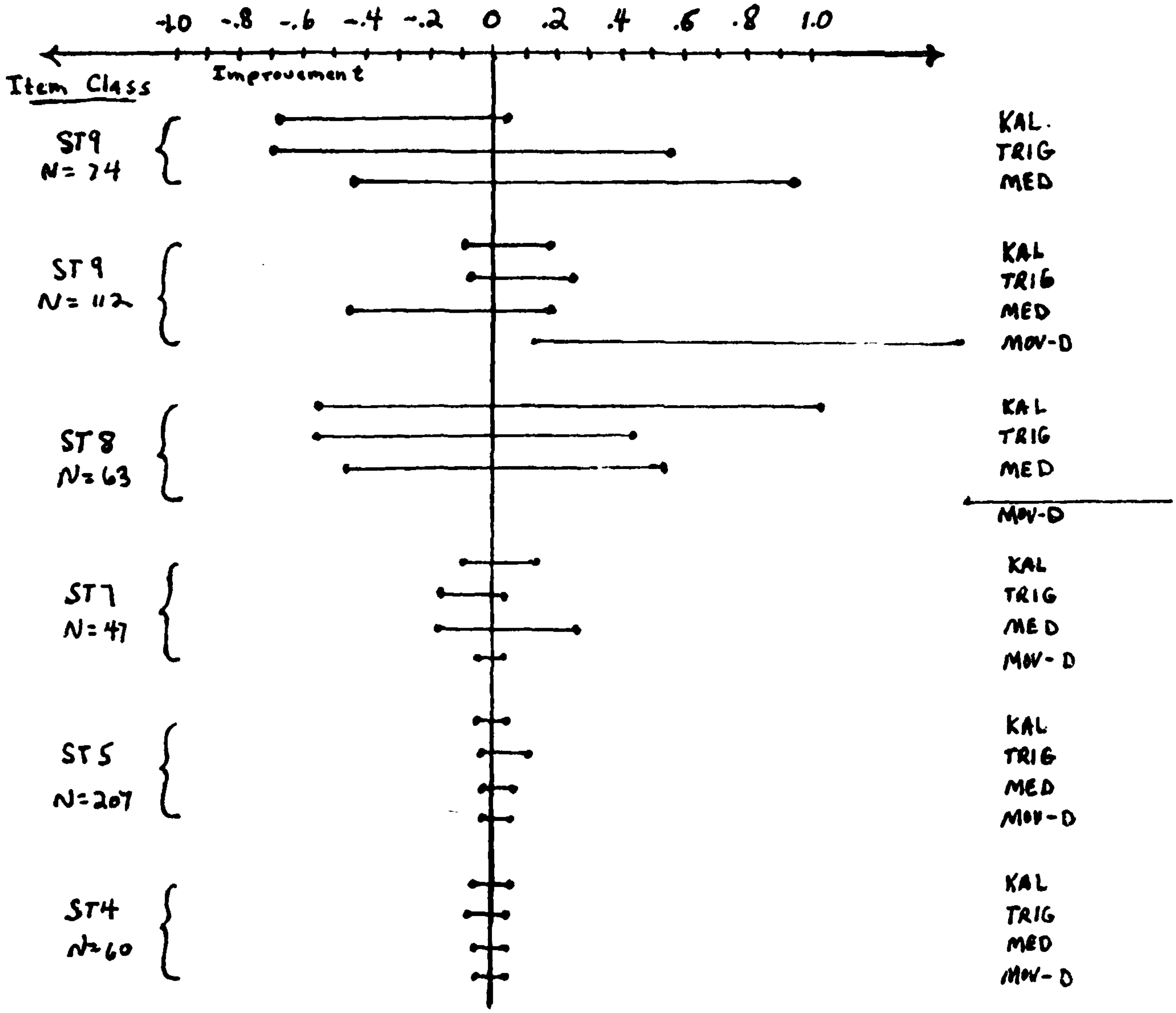
95% Confidence Intervals For

The Mean of Individual Differences Between

Time Weighted Backorders of

(test policy − std Policy)

1794 = Std Policy



CHAPTER II

DATA

2.1 Description

The IRO demand history file includes 11 years of requisitions and demand by quarter accumulated from the AVSCOM Demand Return & Disposal (DRD) files from 1967 thru 1977. Flying hours covering the same period as the demand data were obtained from DCSLOG. The file contains a sample of 20,865 items from all those in the system in 1966 and subsequently entered.

The data base is limited to recurring demands for which program data was available. SSA and Grant Aid demands were eliminated as were items not purchased thru central procurement, based on the last recorded IMPC code. Every attempt was made to drop items subject to logistical transfer.

Previous IRO forecasting projects used an older 7-year data base [3], compiled in much the same way from the DRD files. The only significant difference in the new data base is the inclusion of items with trivial demand, essential for forecasting demand for inactive items.

2.2 Classification

Each item was classified as low dollar value (LDV) or high dollar value (HDV) according to whether the demand rate averaged over the 11 years was less than or greater than or equal to $50,000; and the requisition rate was less than or greater than or equal to 100 per year. Items with over a million dollars of demand per year were dropped.

The items were further divided into dynamic (DYN) and non-dynamic (NON) based on the Federal Stock Class (FSC). The dynamic components were considered to be those that experience high rotation rates; i.e. rotor blades, transmissions, and turbine engines. For more detail see Cohen [3].

The data breaks out into the following four groups:

HDVDYN	86
HDVNON	262
LDVDYN	1169
LDVNON	19348
	20865

Some of the simulation and other forecasting work was done using the last 7 years of the 11 year data base, thus eliminating the 1967-1970 period

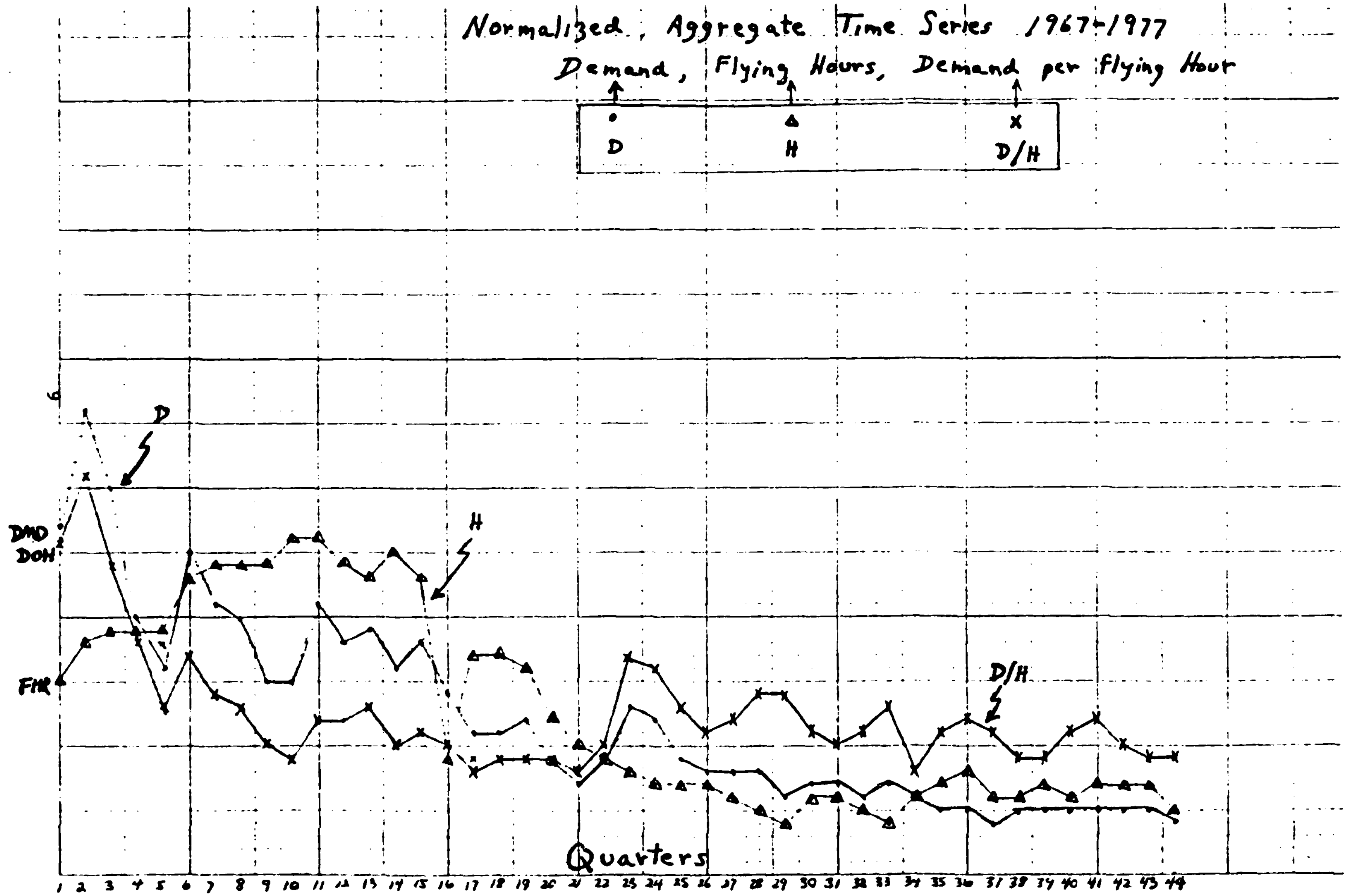
subject to the Vietnam war. LDV and HDV divisions were recreated based on the demand in the last 7 years. The new breakdown of items for this period 1971-1977 is:

HDVDYN	54
HDVNON	224
LDVDYN	1199
LDVNON	19384
	20861

Four items were eliminated which had over one million dollar demand, based on the last 7 years.

2.3 Aggregate Series

The graph on page 9 illustrates the aggregate series of all the items for demand (D), flying hours (H), and demand divided by flying hours (D/H).



Normalized, Aggregate Time Series 1967-1977
Demand, Flying Hours, Demand per flying Hour
D
H
D/H
DMD
DOH
FHR
Quarters
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

CHAPTER III

KALMAN FILTERS

3.1 Basic Forecast Methods

As noted in references [8], [9], "Kalman Filters" are, for a general class of statistical processes, optimal forecast procedures, in the sense of minimizing mean square forecast error. These algorithms have a general exponential smoothing structure where the smoothing weights themselves are variable and updated.

In this section the most important Kalman algorithms that were tested are briefly summarized and designated with code names. Complete mathematical descriptions are found in the Appendix to this chapter. All algorithms operate on the demand per flying hour time series D/H in order to predict a rate value. Forecasts for demand in a future period are then made by multiplying the rate estimate D/H by the program (flying hours) for that future period.

- KALMAN - Original Kalman filter algorithm investigated in reference [8]. Updated k factors for D/H for the period 1971-1977 are used.
- KALNEW1 - Kalman filter with a modification to the formula for updating the weights. New $k_{D/H}$ factors are used. (See Section 3.2)
- KALMANS - A switching signal is used to choose either the current observation of D/H or the KALMAN estimate.
- KALMANSMA2 - As in KALMANS but replace current observation by current 2 quarter average.
- KALNEW1S - As in KALMANS but uses the KALNEW1 estimate.
- KALNEW1SMA2 - As in KALNEW1S but replace current observation by current 2 quarter average.
- KALNEW1SMA3 - As in KALNEW1S but replace current observation by current 3 quarter average.
- KALREL - Current weighting value in original KALMAN procedure is adjusted by a "relevance" function which tends to put more weight on current observations.
- KAL2SPK - An a priori two spike distribution on zero value and a non-zero value for $k_{D/H}$ is assumed. Bayesian updating is applied to the spike probabilities and the estimates from 2 Kalman filters (for $k = 0$ and $k \neq 0$) are appropriately weighted.

3.2 K-factors for Kalman Algorithms

A parameter k_y is needed to obtain the changing values of weights applied to past and current observations of a time series variable y in a Kalman forecasting procedure. The details on the mathematical theory of Kalman k factors and on the statistic formed from empirical forecast errors can be found in [9]. Suffice it here to say that the mean square error on y over L periods when using a moving average of B periods is a function of k and of a variance q^2 of the process mean. The functional relations can be solved to find estimates of k.

The tables present the average k values for D and D/H processes for items falling in various requisition classes. The 1967-1973 table is a refinement of the table for that same period in [8], a result of correcting a not completely innocuous bias in data processing; k_D did not change much, but $k_{D/H}$ now does not increase continuously as requisition activity goes up. The period 1971-1977 indicates generally higher values of k_D, $k_{D/H}$; hence relatively more stability in the processes D, D/H than in the Vietnam era. There has also been a shift in the column trends in the later era, an indication of a change in demand patterns by requisition class. The 1971-1977 k values are probably more representative of "normality" but should be updated about every three years or with changes in war or economic environment.

3.3 Appendix A - Mathematical Addendum

Basic Notation

y_n = observed value of process in period n.
For all algorithms listed, Demand/Flying Hours, D/H was the utilized observation variable y in the empirical analysis.

x_n = mean of process in period n.

$\hat{x}_n$ = estimate of mean of process at end of period n.

$\hat{y}_n(\ell)$ = forecast at end of period n of the process value ℓ periods later.
Except for a few cases with assumed deterministic trend components in the process model y (which did not perform well), our processes assume stochastic fluctuations in the future around the current mean; therefore a best MSE forecast $\hat{y}_n(\ell) = \hat{x}_n$ is used in the cases below.

Parameters for D, D/H Processes - Years 1967-1973
(by requisition class)

Upper Bound	# Items	Avg Req.	DEMAND K_D	q_D^2	smoothed q^2	q	DEMAND per Flying Hour $K_{D/H}$
1	1637	.61	0	-			1.561
2	1554	1.52	3.368	9.65	10	3.2	5.446
3	1314	2.50	4.654	28.47	15	3.9	8.250
4	973	3.49	4.728	76.05	20	4.5	9.527
5	703	4.50	5.749	75.9.	40	6.3	11.987
6	549	5.51	4.724	33.27	80	8.9	10.016
8	866	6.96	4.730	87.90	110	10.5	10.246
12	1053	9.74	4.635	190.19	190	13.8	10.523
18	844	14.69	4.058	290.33	290	17.0	11.029
∞	1429	41.39	2.642	1869.97	1200	34.6	10.145

Parameters for D, D/H Processes — Years 1971-1977
(by requisition class)

Upper Bound	# Items	Avg Req.	DEMAND K_D	DEMAND q_D^2	DEMAND smoothed q^2	DEMAND smoothed q	DEMAND per Flying Hour $K_{D/H}$
1	7176	.59	0	-			0
2	1723	1.45	11.503	2.79	5	2.2	18.358
3	916	2.46	12.409	17.91	15	3.9	17.381
4	595	3.50	11.321	15.51	20	4.5	16.076
5	430	4.49	10.550	42.52	35	5.9	17.232
6	359	5.48	10.164	32.51	50	7.1	13.287
8	499	6.94	9.691	114.07	93	9.6	13.265
12	596	9.83	9.550	115.79	140	11.8	13.760
18	525	14.68	8.168	214.39	205	14.3	11.425
∞	1089	52.03	7.128	576.77	575	24.0	10.356

$H_{n+\ell}$ = flying hours in future period $n+\ell$.

$H(n,L)$ = total flying hours, period n+1 to n+L.

$\hat{D}(n,L)$ = forecast of demand over L periods based on information thru end of period n

$$= \sum_{n+1}^{n+L} \hat{y}_n(\ell)\, H_{n+\ell} = \hat{x}_n H(n,L)$$

KALMAN

$$\hat{x}_n = \hat{x}_{n-1} + G_n \quad (y_n - \hat{x}_{n-1}) \tag{A1}$$

$$G_{n+1} = \frac{1 + k\ G_n}{1 + k\ G_n + k\ (H_n^2/H_{n+1}^2)} \tag{A2}$$

$$\hat{y}_n(\ell) = \hat{x}_n$$

k is updated every 4 quarters based on the current 8 quarter moving average estimate of yearly requisitions. (Table lookup)

KALNEW1

As above with

$$G_{n+1} = \frac{1 + k\ G_n(H_{n+1}^2/H_n^2)}{1 + k\ G_n(H_{n+1}^2/H_n^2) + k} \tag{A3}$$

KALMANS

In reference [ ] the formula for the MSE of a one period (quarter) forecast for moving average of M periods applied to the Dynamic Mean process is

$$MSE_M = q^2[k + k/M + \frac{(M+1)(2M+1)}{6M}] \tag{A4}$$

So in terms of two moving average mean square errors, solving (A4) for k,

$$K = \frac{(MSE_N \cdot C_M) - (MSE_M \cdot C_N)}{(\frac{N+1}{N} \cdot C_M) - (\frac{M+1}{M}\ C_N)} \tag{A5}$$

where $C_I = 1 + \frac{(I-1)(2I-1)}{6I}$

For N = 4, M = 8 and R = MSE/MSE_4

Then (A5) leads to

$$k \leq .5 \text{ if and only if } R \geq 1.5 \tag{A6}$$

We approximate the effect of k inferred to be quite small by putting all weight (G_n=1) in a Kalman algorithm on the current observation y_n.

The above relations lead to the following heuristic, hybrid algorithm employing a switching signal.

Let

$\hat{Z}_n(j)$ = j quarter moving average after period n.

$$MSE_n(j) = (1-\alpha)\ MSE_{n-1}(j) + \alpha\ (\hat{Z}_{n-1}(j) - y_n)^2 \qquad (A7)$$

Then

if $[MSE_N(8)/MSE_n(4) > 1.5], \hat{y}_n(\ell) = y_n$

otherwise $\hat{y}_n(\ell) = \hat{x}_n = \hat{x}_{n-1} + G_n(y_n - \hat{x}_{n-1})$

where G_n is given by (A2).

α was chosen to be .8 based on empirical testing.

KALMANSMA2

The reasoning applied here to obtain a heuristical switching signal algorithm is similar to that of KALMANS. We wish to know when it is appropriate to use an MA2(2 quarter moving average) on y (i.e. D/H).

Note in (A2), that when flying hours are stable ($H_n \approx H_{n+1}$), G approaches .67 for k = .7. An exponential smoothing weight of .67 is equivalent (in the sense of average weighting of all past history) to a moving average base B = 2.

For N = 2, M = 8 and $R \equiv MSE_8/MSE_2$, then (A5) leads to:

$$k \leq .7 \text{ if and only if } R \geq 1.728 \qquad (A8)$$

Then using the notation of (A7), define KALMANSMA2 as

If $[MSE_n(8)/MSE_n(2) > 1.728], \hat{y}_n(\ell) = \frac{y_n + y_{n-1}}{2}$

Otherwise $\hat{y}_n(\ell) = \hat{x}_n = \hat{x}_{n-1} + G_n(y_n - \hat{x}_{n-1})$

where G_n is given by (A2)

KALNEW1S

Use G_n computed via (A3) in KALMANS procedure.

KALNEW1SMA2

Use G_n computed via (A3) in KALMANSMA2 procedure.

KALNEW1SMA3

Since k = 2 implies steady state C = .5, which in turn indicates a moving average base B = 3 (see reasoning in KALMANSMA2), equation (A5) is used with N = 3, M = 8 to find

$$k \leq 2 \text{ if and only if } MSE_8/MSE_3 \geq 1.288$$

Hence the following heuristic for KALNEW1SMA3:

$$\text{if } [MSE_n(8)/MSE_n(3) > 1.288], y_n(\ell) = \frac{y_n + y_{n-1} + y_{n-2}}{3}$$

$$\text{Otherwise } \hat{y}_n(\ell) = \hat{x}_n = \hat{x}_{n-1} + G_n(y_n - \hat{x}_{n-1})$$

where G_n is given by (A3)

KALREL

$$G_{n+1} = 1 + \frac{r}{1 + A_{n+1}} \quad , r = 1 \qquad \text{(A9)}$$

$$\text{where} \quad A_{n+1} = \frac{1 + k\, G_n}{1 + k\, H_n^2/H_{n+1}^2} \qquad \text{(A10)}$$

If $0 \leq r < 1$, then more weight is applied to the observation y_{n+1}. The factor r can be defined as the value of a relevance function, which measures in some manner the relevance of the current observation. Extending the theory of [3a] to our models, a relevant relevance function may be formulated, viz.,

$$r = \min \left\{ 1, \left(\frac{2.5\, H_{n+1} \hat{q}_D}{k\, H_n^2} \right) \cdot \frac{(1 + k\, G_n + k\, H_n^2/H_{n+1}^2)}{|y_{n+1} - \hat{x}_n|} \right\} \qquad \text{(A11)}$$

where $\hat{q}_D$ = standard deviation of demand mean based on items' requisition class. See Table

2.5 is an adjustable value which here bounds the difference between the new estimate $\hat{x}$ and the observation to be no more than 2.5 standard deviations of the process.

This procedure hedges towards y_{n+1} as a new estimate if the error magnitude $|y_{n+1} - \hat{x}_n|$ is large.

KAL2SPK

A two-spike distribution for the item's k-factor is assumed - around values $k = 0$ and $k = k_{D/H}$, where the latter is tabulated by the item's requisition class. The probabilities z_n of $k = k_{D/H}$ are updated as below and used in combining the Kalman estimate $\hat{x}_n$ from (A1) for $k \neq 0$ with y_n, the Kalman estimate for $k = 0$, i.e.

$$\hat{y}_n(\ell) = z_n\hat{x}_n + (1-z_n)y_n \qquad \text{(A12)}$$

If w_n, v_n are proportional to the probabilities p (error in period n/k) for $k = k_{D/H}$ and $k = 0$ respectively and assuming normality of errors, then it can be shown for the Dynamic Mean model,

$$w_n = \frac{1}{\sqrt{B}} \exp [- (y_n - x_{n-1})^2/2\ BV_0] \qquad \text{(A13)}$$

$$v_n = \frac{1}{\sqrt{B}} \exp [- (y_n - y_{n-1})^2/2\ V_0] \qquad \text{(A14)}$$

where

$$V_0 = \hat{q}_D^2/H_{n-1}^2,\ \hat{q}_D^2 = \text{tabulated variance of demand mean.}$$

$$B = (1 + k \frac{H_{n-1}^2}{H_n^2} + k\ G_{n-1})$$

$$BV_0 = \text{theoretical variance of error.}$$

Straightforward application of Bayesian updating yields new z_n

$$z_n = \frac{z_{n-1}w_n}{z_{n-1}w_n + (1-z_{n-1})v_n} \qquad \text{(A15)}$$

REGKBNEWSMA2

This algorithm, see [9], is the closest weighted average analogue to a Kalman filter's weighting of past history; in addition a switch is incorporated ala' KALMANSMA2. Hence referring to the section of KALMANSMA2

$$\text{if } [MSE_n(8)/MSE_n(2) > 1.728], \hat{y}_n(\ell) = \frac{y_n + y_{n-1}}{2}$$

otherwise

$$\hat{y}_n(\ell) = \sum_{j=1}^{B} w_{n-j+1} y_{n-1+1} \qquad \text{(A16)}$$

where $\quad w_{n-j+1} = H^2_{n-j+1} / \sum_{i=1}^{B} H^2_{n-i+1}$

Base period B is determined from the nearest integer $[\cdot]$

$$B = \left[\sqrt{\frac{1 + 6k}{2}} \right] \qquad \text{(A17)}$$

and k is updated every 4 quarters based on the current 8 quarter moving average of yearly requisitions (Table lookup on $k_{D/H}$).

3.4 Appendix B - Structural Changes in KALMAN to obtain KALNEW1

With a slight change in the assumed process model, the Dynamic Mean model (see [9] Chapter II), the updating formula for the weight in period n+1 applied to the current observed value y_{n+1} of the process becomes

$$G_{n+1} = \frac{q^2_{n+1} + r^2_n G_n}{q^2_{n+1} + r^2_n G_n + r^2_{n+1}} \qquad \text{(B1)}$$

instead of equation 2.5 in [9]. If as in [9] Chapter IV, it is assumed that r^2_n, the variance of $y_n \doteq D_n/H_n$, varies with $1/H^2_n$, and that $r^2_n/q^2_n = k_{D/H}$, then

$$G_{n+1} = \frac{1 + (H^2_{n+1}/H^2_n) \quad k_{D/H} G_n}{1 + (H^2_{n+1}/H^2_n) \quad k_{D/H} G_n + k_{D/H}} \qquad \text{(B2)}$$

as opposed to

$$G_{n+1} = \frac{1 + k G_n}{1 + k G_n + k H^2_n/H^2_{n+1}} \qquad \text{(B3)}$$

as given in [ ].

Equation (B2) is the basis for the KALNEW algorithms. Note as k becomes large (B2) and (B3) behave similarly.

CHAPTER IV

IRO TRIGG TRACKING SIGNAL

4.1 Background

The IRO-TRIGG is a modified version of the TACOM New Parameter method described in [7]. It utilizes a TRIGG tracking signal to determine whether the series is stationary (constant mean) or not and uses an adjusted eight or four quarter moving average respectively as the forecast.

The tracking signal is the ratio between a weighted average of the algebraic (signed forecast) errors and absolute (non signed) error, while using a standard eight quarter moving average (MA8) as the forecast. If the series is stationary, the MA8 will do well in forecasting the constant mean of the system, and the over and under forecast errors will be on average algebraically sum to zero. On the other hand, if the series is following a trend, then the MA8 will always underforecast (trend positive) or over forecast (trend negative) and thus the algebraic sum of the errors will differ by the absolute sum only in sign.

Thus if the signal is close to ±1, then the series is following a trend whereas if the signal is close to 0, then the series is stationary with a constant mean.

4.2 Computation

Let:

$$MA8(n) = 1/8 \sum_{i=n-7}^{n} Y_i, \text{ where } Y_i = \frac{\text{Demand}}{\text{Flying Hours}} \text{ in quarter } i$$

$$MA4(n) = 1/4 \sum_{i=n-3}^{n} Y_i$$

$$AMA8(n) = \alpha_1 Y_n + (1-\alpha_1)MA8(n) \qquad (0 \leq \alpha_1 \leq 1)$$

$$AMA4(n) = \alpha_2 Y_n + (1-\alpha_2)MA4(n) \qquad (0 \leq \alpha_2 \leq 1)$$

$$ERROR(n) = Y_n - MA8(n-1)$$

$$MALE(n) = \alpha ERROR(n) + (1-\alpha)MALE(n-1) \qquad (0 \leq \alpha < 1)$$

(Mean Algebraic Error)

$$MABE(n) = \alpha /ERROR(n)/ + (1-\alpha)MABE(n-1)$$

(Mean Absolute Error)

$TS(n) = MALE(n)/MABE(n)$
(Tracking Signal)

Forecast

$$\hat{Y}_{n+1} = \begin{cases} AMA8(n) & \text{if } |TS(n)| < \beta \\ AMA4(n) & \text{if } |TS(n)| \geq \beta \end{cases} \qquad (0 < \beta < 1)$$

Initialization for Tracking Signal

Ref [ ].

Let:

$$MA7(7) = 1/7 \sum_{1}^{7} Y_i$$

$$ERROR(8) = Y_8 - MA7(7)$$

$$MALE(7) = 0$$

$$MABE(7) = (1.25/7) \sum_{1}^{7} /Y_i - MA7(7)/$$

Then

$$TS(8) = \frac{\alpha \; ERROR(8)}{\alpha \; |ERROR(8)| + (1-\alpha)MABE(7)}$$

4.3 IRO-TRIGG Parameters

The IRO-TRIGG forecast procedure depends on certain parameters which have to be determined prior to the forecast. These parameters may be selected subjectively or empirically by experimentation. Remarks and observations concerning these parameters are as follows:

Remarks

a. The α_1 and α_2 parameters define the weights given to the past observation when computing AMA8(n) and AMA4(n), i.e.

$$AMA8(n) = W_1 \sum_{n-7}^{n-1} Y_i + W_2 Y_n$$

where $W_1 = 1/8\ (1-\alpha_1)$

and $W_2 = 1/8\ (1+7\ \alpha_1)$

Hence as $\alpha_1 \rightarrow 0$, $AMA8(n) \rightarrow MA8$

and as $\alpha_1 \rightarrow 1$, $AMA8(n) \rightarrow Y_n$

Similarly

$$AMA4(n) = W_1^* \sum_{n-3}^{n-1} Y_i + W_2^* Y_n$$

where $W_1^* = 1/4\ (1-\alpha_2)$

and $W_2^* = 1/4\ (1-3\ \alpha_2)$

also as $\alpha_2 \to 0$, $AMA4(n) \to MA4$

$\alpha_2 \to 1$, $AMA4(n) \to Y_n$

So if α_2 is small and α_1 is large

then the AMA8(n) will do better then AMA4(n) in forecasting a non-stationary (trend) series,

also AMA4(n) would do better then AMA8(n) in forecasting a stationary series.

Since this is counter to the logic of the switching process, careful judgement should be made when selecting the combination of values for α_1 and α_2.

b. *The α parameter determines the amount of weight* given to the latest error when using MA8 for computing the tracking signal.

$$TS(n) = \frac{\alpha\ ERROR + (1-\alpha)\ MALE(n-1)}{\alpha\,|ERROR| + (1-\alpha)\ MABE(n-1)}$$

(Tracking Signal)

Now as $\alpha \to 1$, $TS(n) \to \frac{ERROR}{|ERROR|} \to \pm 1$

independent to the type of series being forecasted. Therefore for α large,

the TRIGG signal may incorrectly identify a trend condition.

c. The β or threshold parameter identifies the region in which the tracking signal indicates a stationary or non-stationary series. The larger the β the more confidence is given to correctly identifying a non-stationary series, but less confidence is given to correctly identifying a stationary series. From the previous paragraph, it is obvious that α and β are related and that as α gets large β should likewise get large in order to maintain the same level of confidence.

Observations: (Results from experimentation with data base)

a. We found that $\alpha_1 = \alpha_2 = .15$ worked as well or better then any other values selected for experimentation and these values were somewhat robust.

b. The best α level was found to be .66, contrary to the small α implied in the remarks. The switching process was very sensitive to changes in this parameter. For $\alpha = .66$, the tracking signal indicated a non-stationary process 74% of the time.

c. It was surprising that changes in the β level made little difference in the statistical results. β of .5 was chosen.

d. The IRO-TRIGG with parameters $\alpha_1 = \alpha_2 = .15$, $\alpha = .66$, and $\beta = .5$ was the final model chosen to represent the tracking signal technique. This method performed better then the other alternatives not listed in the report.

4.4 Structural Change to the TACOM New Parameter Algorithm

As mentioned earlier the IRO-TRIGG forecast algorithm is a modified version of the TACOM New Parameter algorithm cited in [7]. The changes as described below were made in an effort to make the forecast technique more consistent with the underlying theory of the TRIGG tracking signal.

a. Computing the Tracking Signal. Instead of using the errors from the actual forecast as done by TACOM, the IRO version uses the errors from an eight quarter moving average forecast in computing the tracking signal. By so doing the tracking signal becomes a monitor for trends. The eight quarter moving average will lag behind any trend in the data which will result in a tracking signal close to $\pm$ one. Since the tracking signal is computed independently of the actual forecast, it is not effected by any switch in the forecast technique and will continue to indicate a trend as long as there is one.

b. α_1 and α_2 Constraints. The TACOM New Parameter algorithm utilizes two empirically found parameters α_1 and α_2 which adjust the amount of weight given to the current observation in both the eight and four quarter moving average computations $(0 \leq \alpha_1 \leq 1)$, $(0 \leq \alpha_2 \leq 1)$. The IRO-TRIGG version also uses these parameters but constrains their values so that the adjusted four quarter average still responds faster to changes (trends) in the data than the adjusted eight quarter average. (This will not be the case if α_1 is sufficiently large and α_2 is sufficiently small as noted under remarks, section a.).

c. <u>An Additional Empirical Parameter α.</u> When computing the TRIGG tracking signal, an exponentially smoothed algebraic error is compared with an exponentially smoothed absolute error. The TACOM New Parameter algorithm uses a smoothing constant of .66 for these calculations. The IRO-TRIGG version considers this α smoothing constant as an empirical parameter which needs to be estimated.

CHAPTER V

CURRENT AND OTHER FORECAST METHODS

5.1 Current - 1794

The current Army method of forecasting estimates the demand per program (flying hours) at end of period* n using

$$\hat{x}_n = \sum_{j=1}^{8} D_{n-j+1} / \sum_{j=1}^{8} H_{n-j+1}$$

This may be written in terms of a weighted moving average on y = D/H:

$$x_n = \sum_{j=1}^{8} W_{n-j+1} Y_{n-j+1}$$

$$W_{n-j+1} = \frac{H_{n-j+1}}{\sum_{i=1}^{8} H_{n-i+1}}$$

5.2 Moving Median (MED4)

In an effort to eliminate the effects of spikes in the data, a simple four quarter moving median on D/H as described below was used.

Let $s(n) = \{y_{n-3}, y_{n-2}, y_{n-1}, y_n\}$ be the last four observations

and let

$(y_{1,n}, y_{2,n}, y_{3,n}, y_{4,n})$ be the ordered array of this set

where $y_{i,n} \leq y_{j,n}$ for $i < j$

Then

$$MED4(n) = \frac{y_{2,n} + y_{3,n}}{2}$$

and

$$\hat{y}_{n+1} = MED4(n)$$

Several data experiments were performed on the length of the base period and the four quarter base appeared to perform best.

---

*A period = one quarter of a year, so 8 quarters or 2 years of history is used.

5.3 Moving Average on Demand

To support Cohen's [3] findings that the use of program data (flying hours) improves forecasts a simple eight quarter moving average was applied to the demand series D.

Let $\left\{x_{n-7}, x_{n-6} \ldots x_n\right\}$ be the latest eight observation

Then $\hat{x}_{n+1} = \sum_{i=0}^{7} x_{n-i}/8$

CHAPTER VI

EXPERIMENTAL DESIGN

6.1 Overview

From our previous experiences we have found that there is no clear cut way to evaluate forecast algorithms in an inventory management system. With this in mind, an ad hoc sequential step wise experiment was designed where both forecast methods and evaluation procedures were eliminated and/or refined after each step. As a result both a best forecast algorithm and a best evaluation method may be determined. The details of each step are as follows:

6.2 Step 1: (13 forecast algorithms, 9 statistical error measures)

For the first step, the four major data sets described on page were used. After appropriate initialization, each forecast algorithm was used to make a one quarter and a four quarter forecast for every quarter of each series. These forecasts were compared to the actual demand of the series and the errors were rolled up within classes of series. Various error measures were computed for 13 forecast algorithms, 6 series classes, and 2 forecast horizons. The experimental layout is as follows:

Data Sets:

Low Dollar Value Non-Dynamic (LDVNON) consisting of 54 items
Low Dollar Value Dynamic (LDVDYN) consisting of 224 items
High Dollar Value Dynamic (HDVDYN) consisting of 1199 items
High Dollar Value Non-Dynamic (HDVNON) consisting of 19384 items

Forecast Horizons

1 quarter
4 quarters

ERROR Measures:

Let x_{ij} be the demand (for the ith item) in the jth quarter

F_i = index set of forecasts for ith item

η_i = cardinal size of F_i (number of times a forecast was made)

E_{ij} = error $(x_{ij} - \hat{x}_{ij})$, $j \in F_i$

AVG_i = average demand for item i
(double 12 month moving average starting after the first non-zero demand)

Simple Averages

The first error measures considered were simple averages of traditional measures.

$$\text{MAD} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} |E_{ij}|$$

(Mean Absolute Error)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} (E_{ij})^2$$

(Mean Square Error)

$$\text{BIAS} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} E_{ij}$$

Percent Error Measures

The simple averages give more weight to items with high demand frequency. Since the items were stratified into homogeneous classes it was desirable to give equal weights to each item in the class, hence the following percent error measures were considered.

$$\text{AVG \% of Forecast} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} \frac{|E_{ij}|}{\hat{x}_{ij}}$$

$$\text{AVG \% of Actual} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} \frac{|E_{ij}|}{x_{ij}}$$

$$\text{AVG \% of Both} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} \frac{|E_{ij}|}{1/2\ (x_{ij} + \hat{x}_{ij})}$$

Relative Error Measures

Now since many of the series were quite variable, the denominator of the percent error measures did not reflect the steady state demand of the item hence the following relative measures were considered.

$$\text{RELATIVE BIAS} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} \frac{E_{ij}}{AVG_i}$$

$$\text{RELATIVE MAD} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} \frac{|E_{ij}|}{AVG_i}$$

$$\text{RELATIVE MSE} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_i} \sum_{j \in F_i} \left(\frac{E_{ij}}{AVG_i}\right)^2$$

Forecast Algorithms: (refer to Chapters III, IV, V for definition)

KALMAN	IROTRIGG15-75
KALNEW1	IROTRIGG25-25
KALMANS	REGKBNEWSMA2
KALNEWIS	CURRENT 1794
KALNEW1SMAZ	KALREL
KALNEW1SMA3	KAL2SPK
KALMANSMA2	

Series Classifications:

Class 1 Average Annual Dollar demand between \$0 and \$5000

Class 2 Average Annual Dollar demand between \$5000 and \$50000

Class 3 Average Annual Dollar demand greater than \$50000

Class 4 Average Annual Number of Requisitions between 0-3

Class 5 Average Annual Number of Requisitions between 3-12

Class 6 Average Annual Number of Requisitions greater than 12

6.3 Step 2: (5 forecast algorithms, 6 error measures)

This step consisted of evaluating five forecast algorithms over a four quarter forecast horizon. The data sets and series classifications were the same as Step 1. Four of the five algorithms were the best ranked ones from the previous step and the error measures were those which appeared most consistent and/or easiest to understand. The fifth algorithm, MED4, was suggested after Step 1 was completed. The algorithm and error measures are as follows:

ERROR MEASURES: (refer to pages 27, 28, 29 for definition)

- REL MAD
- REL MSE
- % of Actual
- % of Forecast
- % of Both
- Bias

Algorithms:

- 1794
- KALNEW1
- KALNEW13MA3
- IROTRIGG
- MED4

6.4 Step 3: (Statistical vs Simulation Evaluation)

This step is the most complex of the three. Samples from five of nine stratification classes were taken and both statistical and simulation analysis were used to evaluate the remaining four algorithms. The following table gives a count of items sampled in the 3 x 3 dollar demand versus requisition stratification.

Yearly Dollar Demand

Yearly Requisitions	0 - $5000	$5000 - $50000	> $50000
0 - 3	Strat Class 1 N = 335	2 N = 100	3 N = 4
3 - 12	Analyzed 4 N = 124	Analyzed 5 N = 230	6 N = 17
> 12	Analyzed 7 N = 98	Analyzed 8 N = 64	Analyzed 9 N = 115

Forecast Algorithms:

The five remaining forecast algorithms considered for the analysis were:

KALMAN

1794

IROTRIGG

MED4

MOVD

Moving D was added to the list of those previous tested to determine if the results from Cohen's report [3] still appear valid. That is, do forecast algorithms utilizing program data perform better than those forecasting on demand only.

Statistics:

In an effort to better relate error measures to inventory performance, the following procedural changes were made to the way the statistics were collected.

(1) Forecast only after a demand; this is the only time a reorder point may be triggered and where the forecast is actually used (alternative would be periodic review which wasn't considered).

(2) Use only the item's PLT as a forecast horizon; again this is what would be used in an inventory system.

(3) Use a simple eight quarter average for the forecast if the item had been inactive for a year prior to the demand triggering the forecast; this would handle the migration of an item from an active strat class to an inactive one without unduly penalizing the algorithm which would normally work well in an active class and does poorly in the less active class.

Along with these procedural changes, additional inventory measures were considered. An overforecast error in predicting demand impacts the inventory control system differently then does an underforecast. Overforecasts result in carrying too much stock and increase the possibility of being stuck with obsolete items, whereas underforecasts increase the possibility of not satisfying a customer's orders and in the case of the Army may reduce the readiness of a weapon system. Since there is not a natural tradeoff between these two types of errors, the following separate measures were developed.

Notation

For the given i^{th} demand series $\{x_{ij}\}$ and its corresponding $\{r_{ij}\}$ requisition series (the number of requisitions at time j)

let $D_{i,t}(\ell)=\sum_{j=1}^{\ell} x_{i,t+j}$ be the demands over ℓ periods from time t

$R_{i,t}(\ell)=\sum_{j=1}^{\ell} r_{i,t+j}$ be the number of requisitions over ℓ periods from time t

$EL_{i,t}(\ell) = (\hat{D}_{i,t}(\ell) - D_{i,t}(\ell)) = \sum_{j=1}^{\ell} (\hat{x}_{i,t+j} - x_{i,t+j})$ be the errors over lead time ℓ

UP_i = unit price of the i^{th} item

Overforecast Measure

$$OF(i) = \sum_{j \in F_i} \max\left[\frac{EL_{i,j}(8)}{D_{i,j}(8)}, 0\right] D_{i,j}(8) \quad UP_i$$

$$= \sum_{j \in F_i} \max\left[EL_{i,j}(8), 0\right] UP_i$$

is the cost of the extra stock purchased for periods of eight quarters for the i^{th} item.

$$OF = \frac{\sum_{i=1}^{N} OF(i)}{\sum_{i=1}^{N} UP_i \sum_{j \in F_i} D_{ij}(8)}$$

is a percent of the total dollar demand spent on extra stock.

The base period of 8 is used to represent the long term effect of an overforecast.

Underforecast Measure

$$UF(i) = \sum_{j \in F_i} \max \left[- \frac{EL_{ij}(PLT)}{D_{ij}(PLT)}, 0 \right] R_{ij}(PLT)$$

is an estimate of the number of requisitions not satisfied for the i^{th} item, i.e. if demand is underforecasted, say by 20%, then it is implied that 20% of the requisitions will not be satisfied.

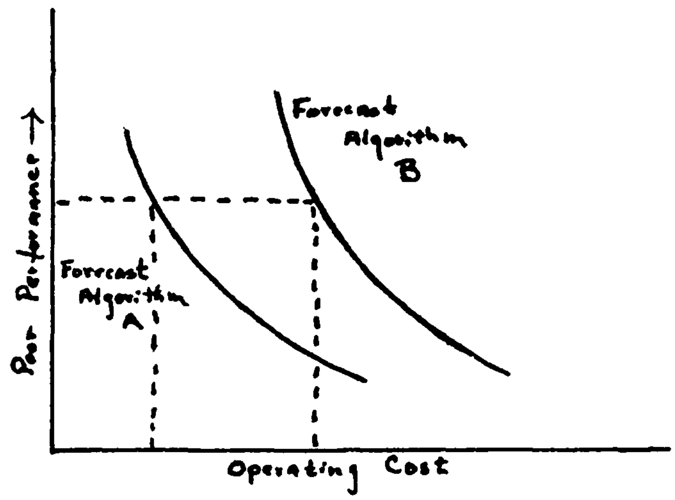
$$UF = \frac{\sum_{i=1}^{N} UF(i)}{\sum_{i=1}^{N} \sum_{j \in F_i} (R_{ij}(PLT))}$$

is an estimate of the percent of the total requisition not satisfied over all the items.

The base period is the procurement lead time of the item which is the quickest time stock could be replenished after a new order is placed. In an underforecast situation the reorder point will probably be crossed within a PLT.

Simulator

The final analysis was done with the IRO Simulator of the Army wholesale supply system, a description of which is found in Cohen [2]. Algorithms are compared in the form of cost-performance curves; the curves are traced thru several "λ" points for each forecast procedure, the lambda (λ) values reflecting an operating policy which relates to the cost of a backorder. Actual demand and flying hour time series for items in any of the various data groupings are used in particular simulation runs. All algorithms have the same starting conditions prior to accumulating performance statistics. To do this during the warmup period (2 year) all algorithms utilize an 8 quarter moving average on demand. Of course, also during warmup, the algorithms obtain their various starting values where needed.



To determine if the curves are statistically different, a fixed cost analysis is done on the difference between the performance of the current policy and the test policies for each item. Histograms of these differences are displayed and statistical tests are used for comparison. Details of the test procedure are found in Section 7.3.

There have been many changes made to the simulator since used in reference [8]. The most noteworthy are:

(1) Excess costs (projected from assets above RO at end of simulator run) are accumulated and averaged into operating costs only on items coded terminal or obsolete or on items with trivial (nearly zero AYD) demand. However, end of simulation stock is also stratified into 1 to 15 years of supply over all items and presented as a simulator output for each forecast policy.

(2) Previously, forecasts could be updated in between quarters (at time of buys) using moving, interpolated, quarters formed from actual quarterly data. Now forecasts are updated only on the actual quarters.

(3) The current PCER tables utilized in the VSL module were previously adjusted for lead time by a theoretical factor of 1/SL from base values. Now, the PCER base values do not change, in order to reflect conservatively the empirical observation that percentage error increased with lead time L.

(4) The constraint that safety level be no more than the expected lead time demand quantity is lifted.

(5) The simulator now incorporates the effects of phased deliveries.

(6) The point estimate of an item's order size in a current interpolated time interval (obtained previously from the interpolated requisition and demand history) is now smoothed (averaged) with previous order size computations.

CHAPTER VII

ANALYSIS AND RESULTS

The results from the data experiments described in the previous chapters are analyzed in the next few sections.

In each step algorithms were eliminated based on their performance when compared to the alternative methods while using several error measures. The final analysis was performed via the IRO simulator on the resulting five techniques.

7.1 Step 1 (Rankings based on 9 statistical error measures)

The tables in this section summarize the comparative performance (rankings) of the algorithms from which initial screening decisions were made. For each of the four data base groups, and within stratification classes (dollar demand and average yearly requisitions) for each group, the algorithms are ranked ("1" being best) based on values across the many error measures described in Chapter VI. Rankings are done for both 1 quarter and 4 quarter forecast error measures.

After a study of the ranking patterns in these tables, KALNEW1, CURRENT, KALNEW1SMA3, IROTRIGG 25-25* were chosen for further statistical and simulation investigation. KALNEW1SMA3 was chosen over KALNEW1SMA2 because of our conservative tendency to use the last 3 quarters (MA3) of data rather than only the last 2 quarters. Current 1794, of course, is chosen as the base for improvement. MED4 was developed later in our study.

---

*The "25-25" refers to the α_1, α_2 values in hendredths.

Item Group: LDVDYN28

Rankings, 1 Qtr Error
4 Qtr Error

Stratification Class

	$D 0-5000	$D -50000	$D - ∞	Req 0 - 3	Req - 12	Req - ∞
# in Class	547	77	2	341	131	154
Algorithm						
KALNEWI	8 3	7 7		9 9	4 1	1 1
KALMANS	5 7	8 8		5 5	7 7	6 7
KALNEWIS	6 6	9 9		4 4	6 8	7 8
KALNEWISMA2	3 4	3 2		2 2	2 4	4 4
KALNEWISMA3	4 5	1 1		6 6	3 2	2 2
KALMANSMA2	2 2	2 4		3 3	1 3	5 3
IROTRIGG15-75	7 8	10 10		7 7	10 10	10 10
IROTRIGG25-25	1 1	4 3		1 1	5 6	3 5
REGKBNEWSMA2	9 9	6 6		8 8	8 5	8 6
CURRENT 1794	10 10	5 5		10 10	9 9	9 9

Item Group: HDVDYN28

Rankings, 1 Qtr Error
4 Qtr Error

Stratification Class

	#D 0 - 5000	#D -50000	#D - ∞	Req 0 - 3	Req - 12	Req - ∞
# in Class	2	13	34	3	12	34
Algorithm						
KALMAN		11 10	11 7		8 2	11 9
KALNEW1		9 9	3 1		1 3	6 1
KALMANS		6 1	5 3		2 4	4 3
KALNEWIS		5 4	9 10		10 10	8 10
KALNEWISMA2		3 3	1 6		3 7	2 5
KALNEWISMA3		7 5	4 4		5 6	3 2
KALMANSMA2		4 2	2 5		4 8	1 4
IROTRIGG15-75		1 7	10 11		11 11	9 11
IROTRIGG25-25		2 6	6 8		6 5	5 6
REGKBNEWSMA2		8 8	7 9		9 9	7 8
CURRENT 1794		10 11	8 2		7 1	10 7
KALREL *						
KAL2SPK *						

* Ranking poor in general

Item Group: HDVNON28

Rankings, 1 Qtr Error
4 Qtr Error

Stratification Class

	$D 0 - 5000		$D -50000		$D - ∞		Req 0 - 3		Req - 12		Req - ∞	
# in Class	11		73		128		4		27		181	
Algorithm												
KALMAN	5	3	8	8	1	4			9	9	2	2
KALNEWI	2	1	7	7	3	1			8	8	1	1
KALNEWIS	7	7	6	6	9	9			6	6	9	9
KALNEWISMA2	3	5	2	2	5	6			5	3	5	5
KALNEWISMA3	1	4	1	1	4	5			1	2	4	3
KALMANSMA2	4	8	3	3	6	7			4	5	6	7
IROTRIGG15-75	10	9	9	9	10	10			7	7	10	10
IROTRIGG25-25	6	2	4	5	2	2			3	4	3	4
REGKBNEWSMA2	8	10	5	4	8	8			2	1	7	8
CURRENT 1794	9	6	10	10	7	3			10	10	8	6
KALREL *												
KAL2SPK *												

* Ranking poor in general

7.2 Step 2 (Ranks from four quarter error measures)

Four tables are presented in this section, one for each of the four item groupings - LDV dynamic and non-dynamic items, HDV dynamic and non-dynamic items. Each table presents rankings of the five candidate items for two stratifications, by annual dollar demand and by annual demand frequency (requisitions). The relative performance rankings in terms of 6 error measures are tabulated; it should be noted that all but the last are relative or "percent" error measures.

The algorithm MED4 was statistically evaluated and ranked only for the measures REL MAD and REL MSE. These two measures, incidently, are the most consistent, in the sense that their rank orderings most frequently agree with a consensus rank ordering across all the measures in a stratification class.

A pattern of some note in the tables: 1794 and KALNEW1 are often ranked closely ("paired") compared to the KALNEW1SMA3 - IROTRIGG pair. The latter pair tend to perform well in less active classes (I, II, V, VI), their tracking signals reacting to fluctuations, while the 1794-KALNEW1 algorithms weight more past history and hence perform well on items with more stable D/H values, i.e. the active classes (II, III, V, VI).

There is no dominant algorithm across all strat classes and tables. In Step 3 we focus upon the statistical and simulator performances of the algorithms in a three by three stratification.

Item Group: LDVNON28

Rankings within class

4 Quarter Error Measures

Algorithm	Strat Class, upper bound	REL MAD	REL MSE	% Actual	% Forecast	% Both	/Bias/
CURRENT 1794	I	4	4	2	2	4	4
KALNEWI	5000	3	3	1	1	2	2
KALNEWISMA3		2	1	3	4	1	3
IROTRIGG		1	2	4	3	3	1
MED4	BD	1-	1-				
CURRENT 1794	II	2	2	3	2	3	4
KALNEWI	50000	1	3	2	1	1	2
KALNEWISMA3		3	4	1	4	2	3
IROTRIGG		4	1	4	3	4	1
MED4	BD	1-	1-				
CURRENT 1794	III						
KALNEWI							
KALNEWISMA3	8						
IROTRIGG							
MED4	BD						
CURRENT 1794	IV	4	4	3	4	4	2
KALNEWI		3	3	4	2	2	1
KALNEWISMA3	3	2	1	2	1	1	3
IROTRIGG		1	2	1	3	3	4
MED4	Reg	1-	1-				
CURRENT 1794	V	2	3	4	1	2	2
KALNEWI		1	1	3	2	1	4
KALNEWISMA3	12	3	2	1	4	3	1
IROTRIGG		4	4	2	3	4	3
MED4	Reg	3.5	3.5				
CURRENT 1794	VI	3	3	4	4	4	3
KALNEWI		1	1	3	1	1	4
KALNEWISMA3	8	2	2	1	2	2	2
IROTRIGG		4	4	2	3	3	1
MED4	Reg	2.5	2.5				

...IS PAGE IS BEST QUALITY PRAC...
...M COPY FURNISHED TO DDC

Item Group: HDVDYN28

Rankings within class

4 Quarter Error Measures

Algorithm	Strat Class, upper bound	REL MAD	REL MSE	% Actual	% Forecast	% Both	/BIAS/
CURRENT 1794	I						
KALNEWI							
KALNEWISMA3	5000						
IROTRIGG							
MED4	$D						
CURRENT 1794	II	4	4	4	3	3	3
KALNEWI		3	3	3	3⁺	4	1
KALNEWISMA3	50000	1	1⁺	1	1	1	2
IROTRIGG		1⁺	1	1⁺	1⁺	2	4
MED4	$D	1⁻	1⁻				
CURRENT 1794	III	1⁺	2	4	1	1	4
KALNEWI		1	1	3	2	2	3
KALNEWISMA3	∞	3	3	1	4	3	2
IROTRIGG		4	4	2	3	4	1
MED4	$D	5	5				
CURRENT 1794	IV						
KALNEWI							
KALNEWISMA3	3						
IROTRIGG							
MED4	Req						
CURRENT 1794	V	1	1	1	1	1	2
KALNEWI		2	2	2	2	2	1
KALNEWISMA3	12	3	3	4	4	4	3
IROTRIGG		4	4	3	3	3	3⁺
MED4	Req	5	5				
CURRENT 1794	VI	3⁺	4	4	1⁺	4	3
KALNEWI		1	1	3	1	2	1
KALNEWISMA3	∞	1⁺	1⁺	1	4	1	2
IROTRIGG		3	3	2	2	3	4
MED4	Req	2	2				

Rankings within class

Item Group: LDVDYN28

4 Quarter Error Measures

Algorithm	Strat Class, upper bound	REL MAD	REL MSE	% Actual	% Forecast	% Both	/BIAS/
CURRENT 1794	I 5000 BD	4	2+	4	2	4	4
KALNEWI		3	3	3	1	1	2
KALNEWISMA3		1+	2	1	3	1+	1
IROTRIGG		1	1	1+	3+	3	3
MED4		1-	1-				
CURRENT 1794	II 50000 BD	3	2	4	2	4	4
KALNEWI		4	4	3	1	1	3
KALNEWISMA3		1	2+	1	2+	2	1
IROTRIGG		2	1	2	3	3	2
MED4		5	5				
CURRENT 1794	III ∞ BD						
KALNEWI							
KALNEWISMA3							
IROTRIGG							
MED4							
CURRENT 1794	IV 3 Reg	4	3	4	3	4	4
KALNEWI		3	3+	3	1	2	3
KALNEWISMA3		2	2	2	2	1	2
IROTRIGG		1	1	1	3+	3	1
MED4		1-	1-				
CURRENT 1794	V 12 Reg	4	4	4	2	4	4
KALNEWI		1	1	2	1	2	2
KALNEWISMA3		2	2	1	4	1	1
IROTRIGG		3	3	2+	3	3	3
MED4		2.5	2.5				
CURRENT 1794	VI ∞ Reg	4	4	4	4	4	1
KALNEWI		1	1	3	1	1	2
KALNEWISMA3		2	2	1	3	2	3
IROTRIGG		3	3	2	2	3	4
MED4		2.5	2.5				

Item Group: HDVNON28

Rankings within class

4 Quarter Error Measures

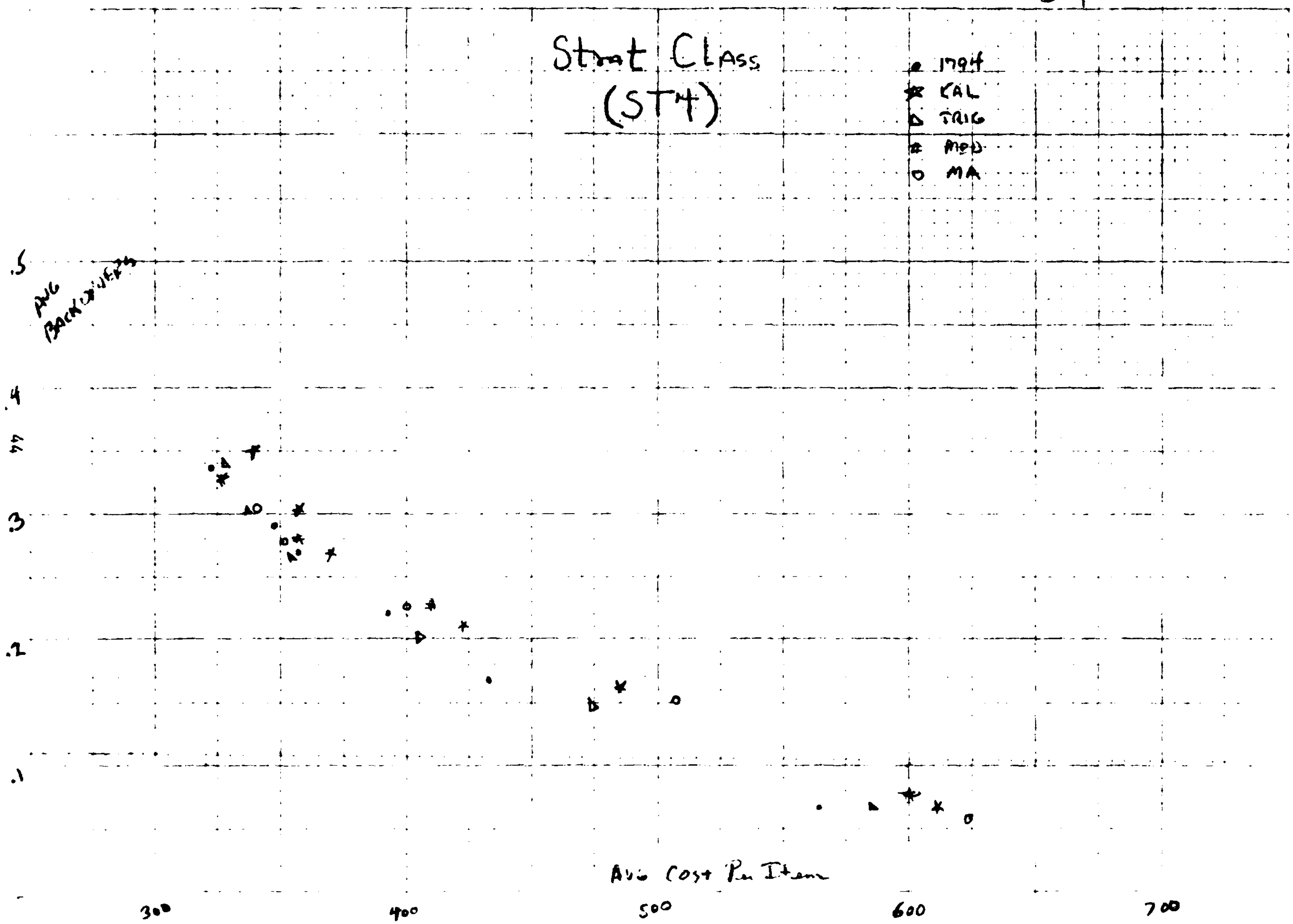
Algorithm	Strat Class, upper bound	REL MAD	REL MSE	% Actual	% Forecast	% Both	/BIAS/
CURRENT 1794	I	2	4	1^+	4	4	4
KALNEWI		1	1	1	2	1^+	3
KALNEWISMA3	5000	4	2	4	3	3	2
IROTRIGG		1^+	3	3	1	1	1
MED4	A&D	5	2.5				
CURRENT 1794	II	4	4	4	4	4	4
KALNEWI		3	3	3	1	1^+	3
KALNEWISMA3	50000	1	1	1	3	3	2
IROTRIGG		1^+	2	2	2	1	1
MED4	A&D	1^-	1^-				
CURRENT 1794	III	1	2^+	4	4	1	4
KALNEWI		1	2	3	1	2	3
KALNEWISMA3	8	2^+	4	2	3	3^+	1
IROTRIGG		2	1	1	2	3	2
MED4	A&D	1.5	1^-				
CURRENT 1794	II						
KALNEWI							
KALNEWISMA3	3						
IROTRIGG	Reg						
MED4							
CURRENT 1794	I	4	4	4	1	4	4
KALNEWI		3	3	3	3	3	3
KALNEWISMA3	12	1	1	2	4	1^+	2
IROTRIGG		2	2	1	2	1	1
MED4	Reg	1^-	1^-				
CURRENT 1794	II	2	3	4	4	3	4
KALNEWI		1	1	3	1	1	3
KALNEWISMA3	8	2^+	2	1	3	2^+	2
IROTRIGG		3	2^+	2	2	2	1
MED4	Reg	5	5				

7.3 Step 3 (Simulation and final statistical results)

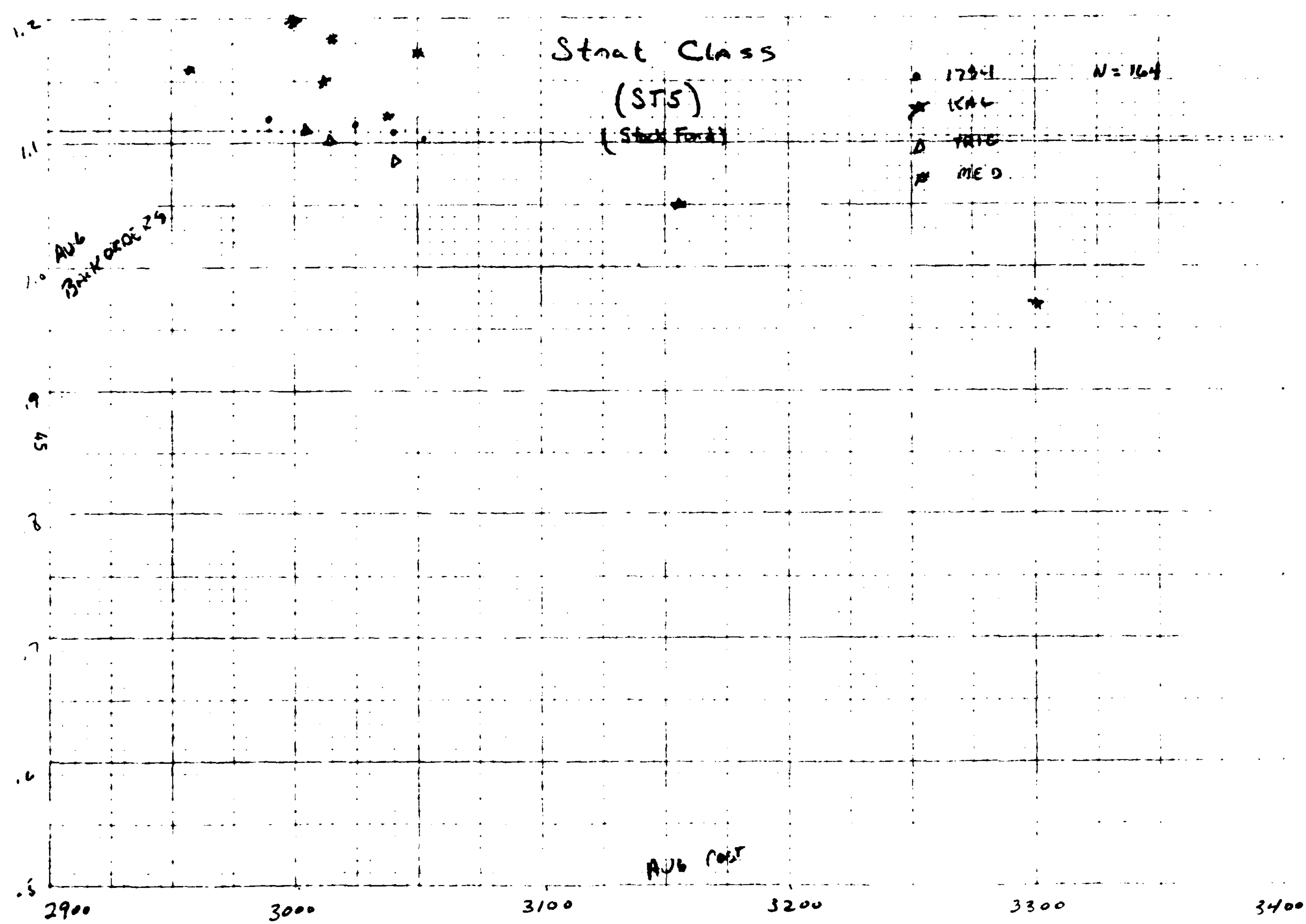
In this section the performance results via simulated cost effectiveness and statistical error measures are analyzed. The final four algorithm candidates, MED4, 1794, KAL1 (formerly coded KALNEW1), Trigg (IROTRIGG), along with MOVD (moving average on demands) were used to compute forecasts in various simulation runs over five classes (active items) of the 3 x 3 data stratification described on page 29. The simulated cost - performance curves are captured in pages 44 - 50 where performance is measured in terms of the average of the time weighted backorders as opposed to average days wait which was reported in the previous studies. (This transformation of performance measures has no impact on the simulation results but does make the comparative analysis easier, ref [6]).

The table on page 51 contains the statistics from the various error measures for each of the five data classes and for both a one quarter and PLT forecast horizon. The second part of the table compares the within class ranks of the statistical measures and the simulation results which were ranked at a fixed cost as described in the next section on Final Analysis.

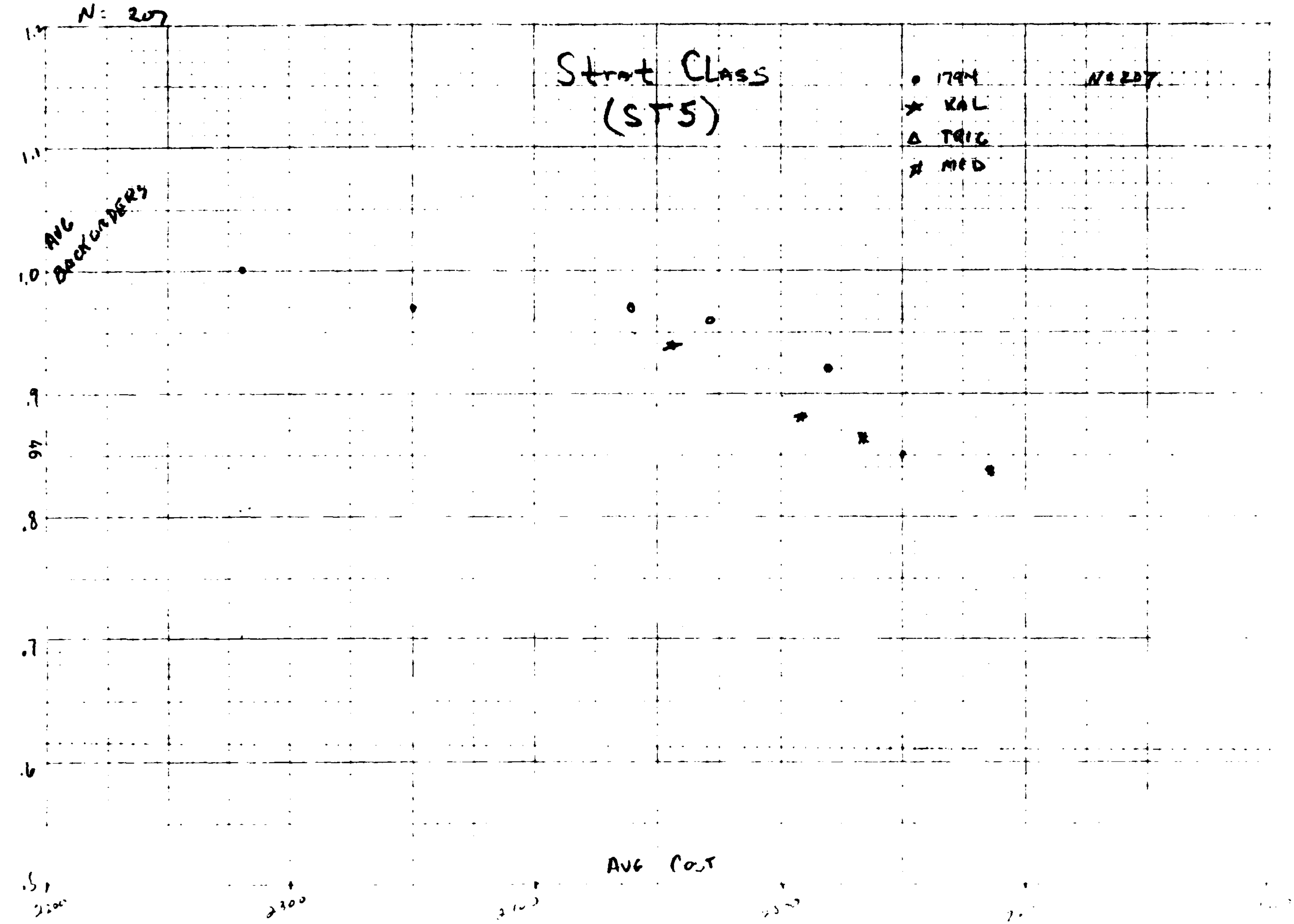
ST 4

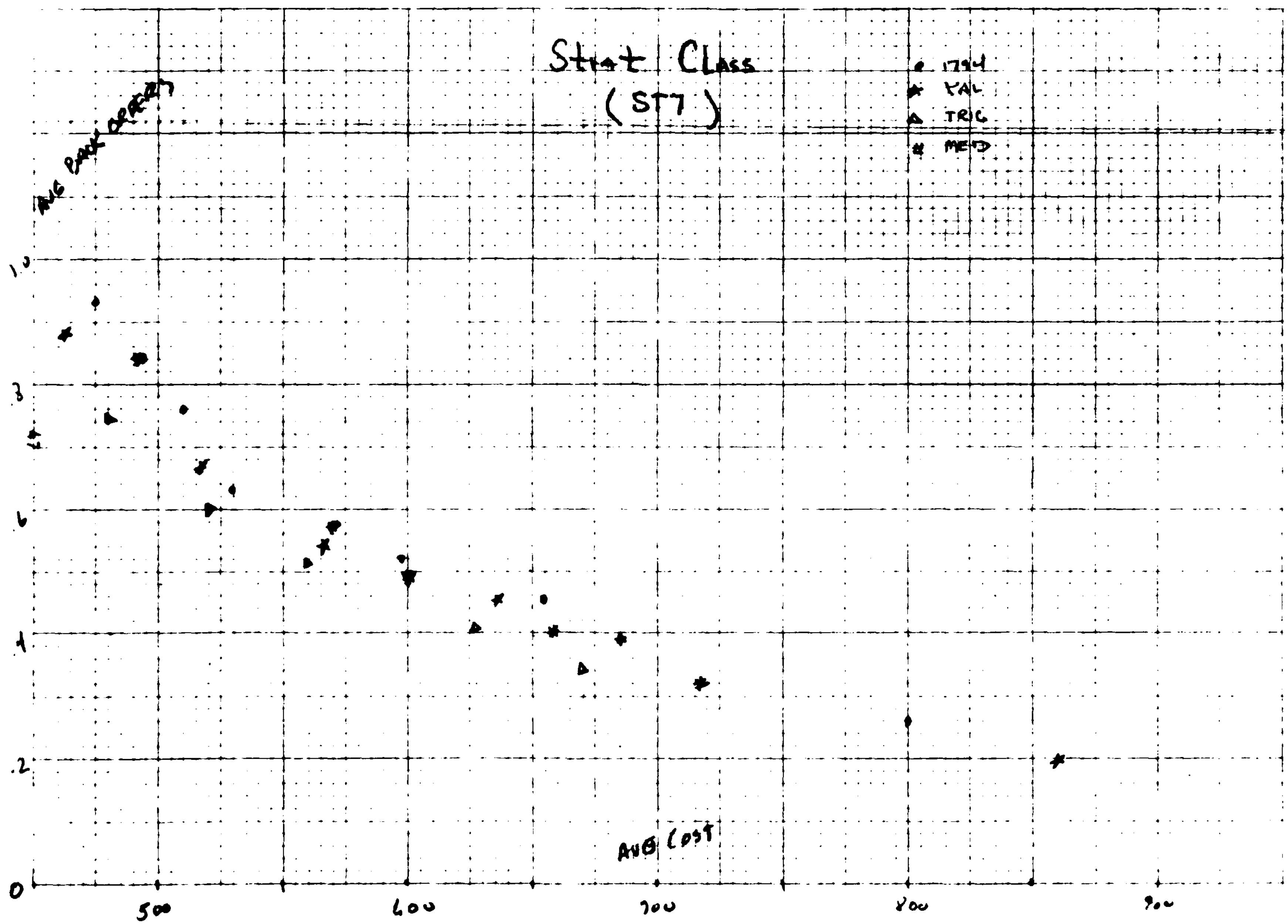


Strat Class
(ST4)
1794
CAL
TRIG
MED
MA
.5
.4
44
.3
.2
.1
Avg Backorders
300
400
500
600
700
Avg Cost Per Item

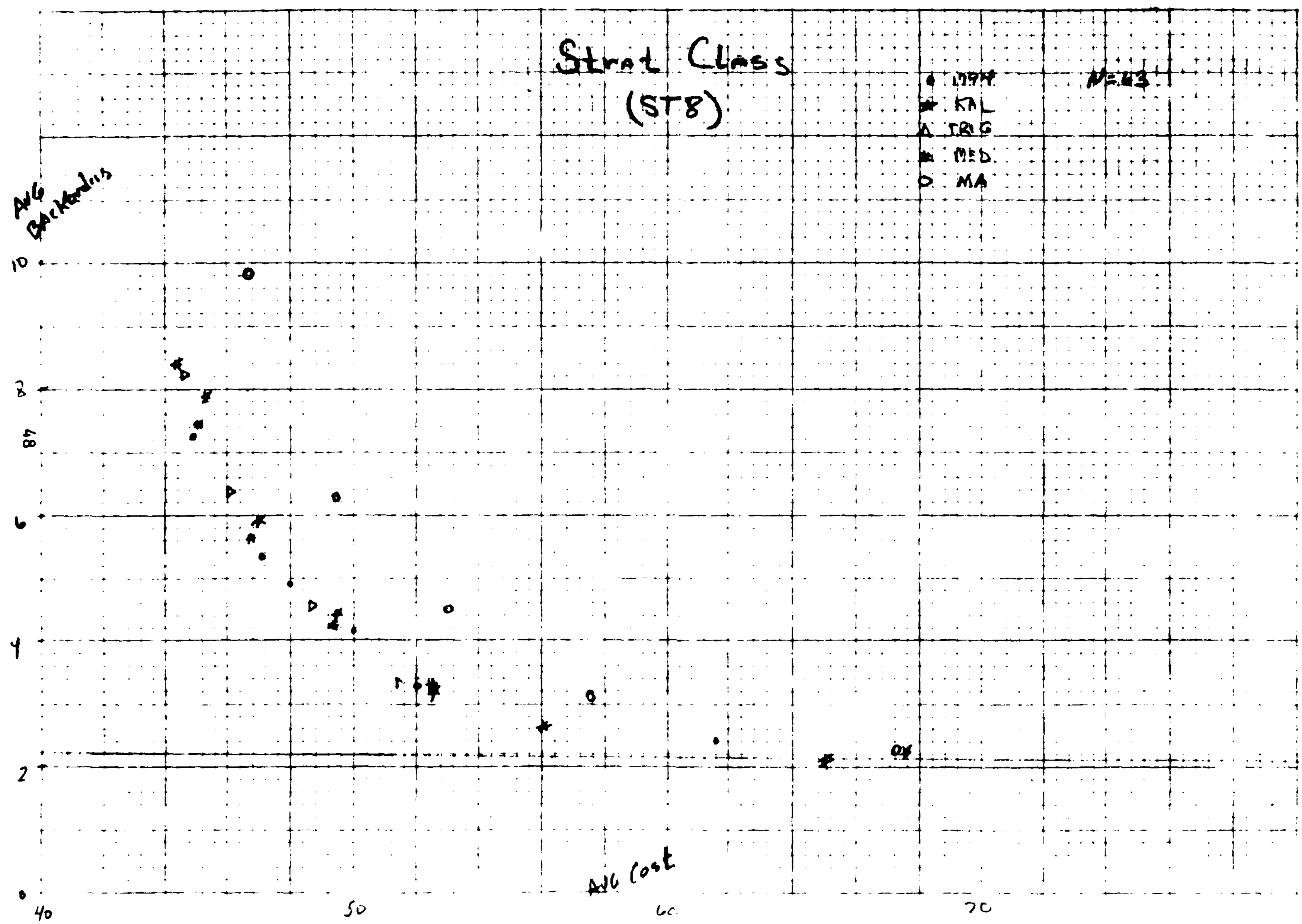


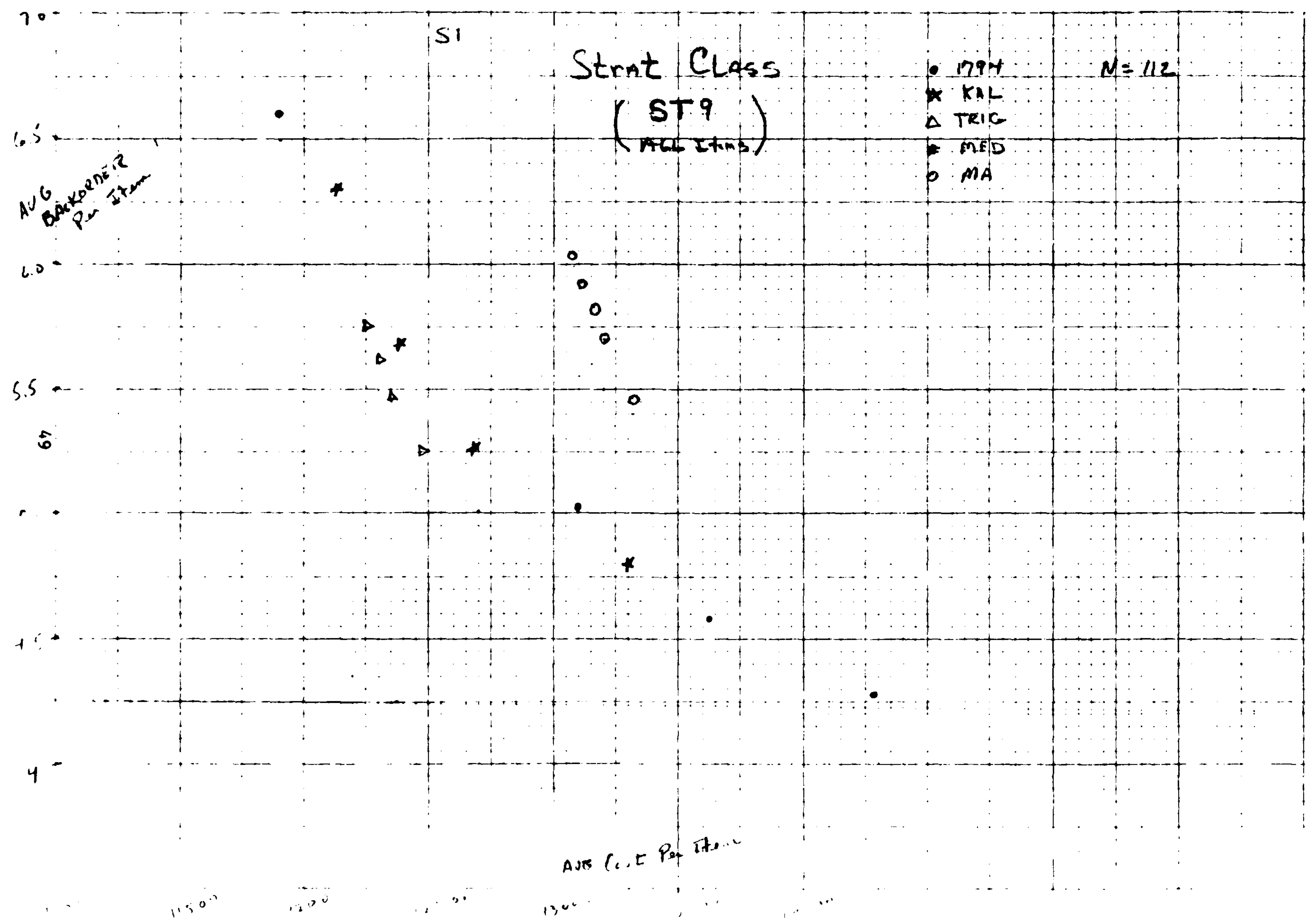
Strat Class
(STS)
Stock Fork
1794
KAL
MED
N = 164
AVG BACKORDERS
AVG COST
1.2
1.1
1.0
.9
.8
.7
.6
.5
2900
3000
3100
3200
3300
3400
45

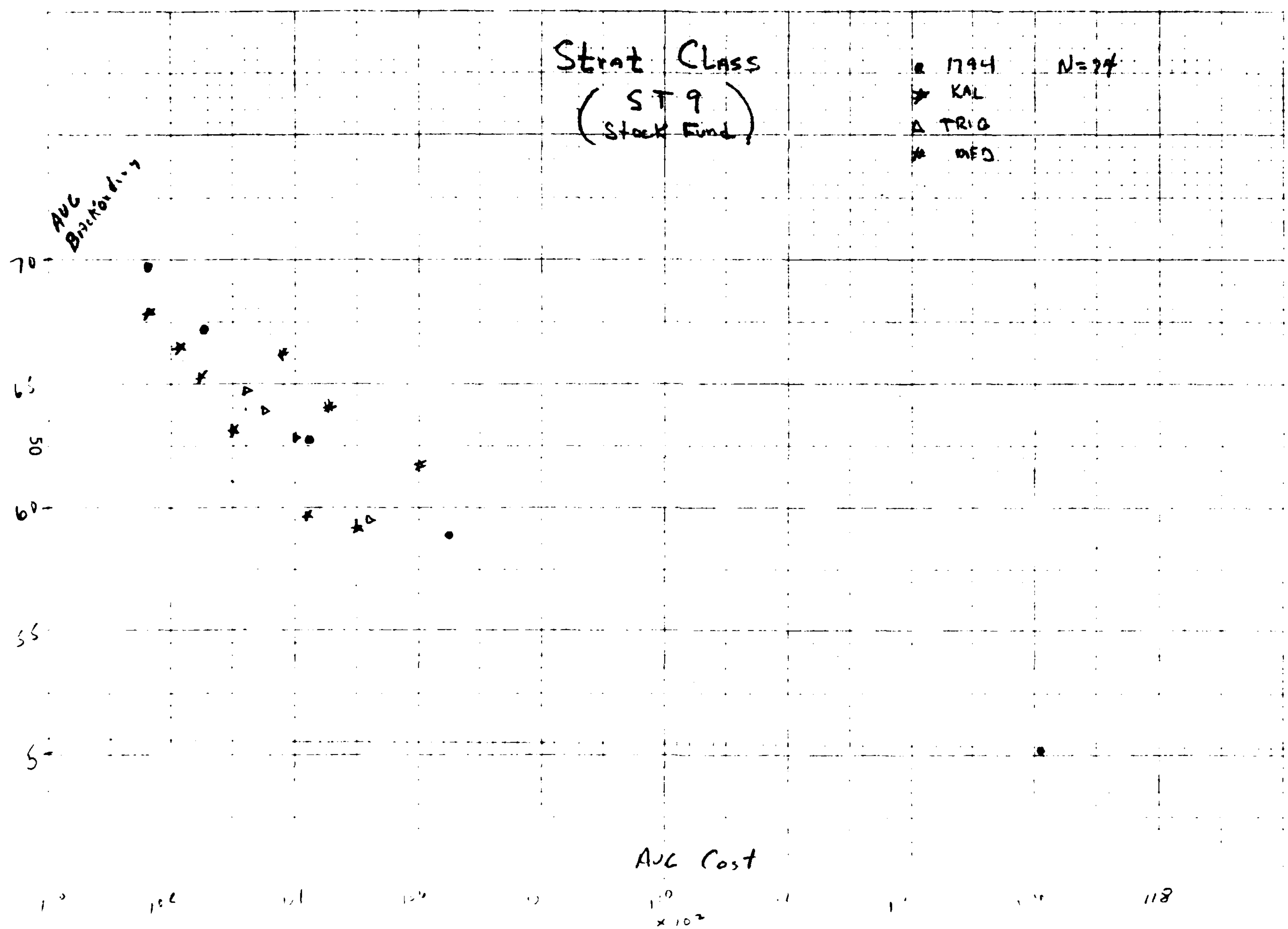

N: 207
Strat Class
(ST5)
• 1794
★ KAL
△ TRIG
# MED
N=207
AVG BACKORDERS
1.1
1.0
.9
.8
.7
.6
AVG COST
2300



Strat Class
(ST7)
1794
KAL
TRIG
MED
AVG PACK ORDER
AVG COST
1.0
.8
.6
.4
.2
0
500
600
700
800
900


Strat Class
(ST8)
N=43
KAL
TRIG
MED
MA
AVG Backorders
10
8
6
4
2
0
AVG Cost
40
50
60
70


S1
Strat Class
(ST9)
All Items
• 1794
✱ KAL
Δ TRIG
MED
o MA
N = 112
7.0
6.5
6.0
5.5
4
AVG Backorder Per Item
AVG Cost Per Item



Strat Class
(ST 9
Stock Fund)
1794
KAL
TRIO
MED
N=24
AVG Backorders
70
65
50
60
55
5
AVG Cost
x 10²
118

Low

Statistical Error Measures

Strat Class	Forecast Method	ERROR 1st gtr	ERROR PLt	RELATIVE ERROR 1st gtr	RELATIVE ERROR PLt	MAD 1st gtr	MAD PLt	REL MAD 1st gtr	REL MAD PLt	REL MSE 1st gtr	REL MSE PLt
ST 4	1791	-.117	.717	-.047	.028	4.625	9.910	1.032	.823	2.301	1.384
	KAL	.021	1.178		.077	4.506	9.414	1.00	.790	2.086	1.247
	TRIG	.342	1.968	.096	.171	4.722	9.984	1.050	.858	2.202	1.424
	MED	-.555	-.369	-.111	-.031	4.514	9.471	1.024	.804	2.281	1.313
	MOND	.453	2.435	.045	.137	4.98	10.95	1.08	.89	2.54	1.642
ST 5	1794	.615	5.323	.053	.137	2.446	20.494	1.060	.810	2.624	1.278
	KAL	1.249	8.765	.093	.155	7.882	22.950	1.056	.792	2.607	1.150
	TRIG	.957	6.54	.187	.242	2.486	21.751	1.102	.865	2.725	1.346
	MED	-1.128	-.019	-.032	.025	7.246	20.022	1.022	.778	2.500	1.104
	MOND	.743	6.204	.103	.200	7.75	21.031	1.084	.843	2.473	1.317
ST 7	1791	-2.880	-10.314	-.072	-.069	15.593	38.124	.731	.622	1.025	.630
	KAL	-2.667	-10.335	-.057	-.056	14.483	34.651	.692	.544	.912	.531
	TRIG	-1.778	-7.477	-.011	-.009	14.291	34.223	.690	.583	.926	.601
	MED	-3.838	-13.345	-.114	-.116	14.639	34.971	.703	.599	1.017	.629
	MOND	-.824	-4.090	-.008	.006	15.460	37.243	.735	.616	1.038	.630
ST 8	1714	-6.319	-22.399	-.020	-.032	44.61	107.121	.443	.377	.373	.262
	KAL	-4.657	-17.798	-.010	-.031	41.57	100.45	.409	.344	.312	.213
	TRIG	1.594	3.303	.013	.005	42.61	103.96	.409	.349	.310	.214
	MED	-1.626	-6.227	-.037	-.051	42.649	104.571	.410	.348	.309	.210
	MOND	-.201	.033	.049	.057	43.98	110.12	.474	.424	.471	.400
ST 9	1794	1.108	-4.619	-.009	-.055	23.49	58.25	.487	.372	.4.7	.274
	KAL	.358	-5.438	-.019	-.053	22.069	54.05	.473	.371	.440	.261
	TRIG	2.064	2.361	.027	.004	21.57	53.53	.454	.389	.453	.279
	MED	-.778	-4.94	-.028	-.059	22.198	57.02	.490	.407	.482	.310
	MOND	2.521	1.902	.023	-.009	23.478	61.698	.489	.402	.476	.310

COMPARATIVE RANKINGS

Error Measures Continued

	Inventory Types: Over	Inventory Types: Under	Error: 1gtn	Error: PLT	Rel Error: 1st	Rel Error: PLT	MAD: 1gtr	MAD: PLT	Rel MAD: 1stv	Rel MAD: PLT	Rel MSE: 1gtr	Rel MSE: PLT	Over	Under	Simulator: ASF	Simulator: ALL
1794	.450	.271	2	2	1	1	3	3	3	3	4	3	3	4		2
KAL	.370	.243	1	3		3	1	1	1	1	1	1	2	2		4
TRIG	.454	.241	3	4	3	5	4	4	4	4	2	4	4	1		1
MED	.295	.329	5	1	4	2	2	2	2	2	3	2	1	5		5
MOVD	.591	.245	4	5	2	4	5	5	5	5	5	5	5	3		3
(ST4)																
1794		.278	1	2	2	2	1	2	3	3	3	3		4		3
KAL	.460	.257	5	5	3	3	5	5	2	2	2	2		1		2
TRIG		.272	3	4	5	5	2	3	5	5	5	5		3		4
MED	.366	.367	4	1	1	1	3	1	1	1	1	1		5		1
MOVD	.542	.263	2	3	4	4	4	4	4	4	4	4		2		5
(ST5)																
1794	.280	.318	4	3	4	4	5	5	5	5	4	4	3	4		5
KAL	.256	.296	3	4	3	3	2	2	2	1	1	1	2	3		3
TRIG	.285	.273	2	2	2	2	1	1	1	2	2	2	4	2		1
MED	.233	.327	5	5	5	5	3	3	3	3	3	3	1	5		4
MOVD	.354	.268	1	1	1	1	4	4	4	4	5	5	5	1		2
(ST7)																
1794	.202	.198	5	5	3	3	5	4	4	4	4	4	4	5		3
KAL	.172	.182	4	4	2	2	1	1	1	1	3	2	2	4		4
TRIG	.182	.153	2	2	1	1	2	2	2	2	2	3	3	1		1
MED	.154	.176	3	3	4	4	3	3	3	3	1	1	1	3		2
MOVD	.319	.172	1	1	5	5	4	5	5	5	5	5	5	2		5
(ST8)																
1794	.114	.183	3	3	1	4	5	4	3	2	3	2	5	4	3	3
KAL	.112	.181	1	5	2	3	2	2	1	1	1	1	4	3	1	2
TRIG	.130	.157	4	2	4	1	1	1	2	3	2	3	1	1	2	1
MED	.103	.187	2	4	5	5	3	3	5	5	5	5	3	5	4	
MOVD	.132	.170	5	1	3	2	4	5	4	4	4	4	2	2		4
(ST7)																

Since there are obvious discrepancies between the simulation ranks and statistical ranks a decision had to be made as to what results should be used for the final analysis. Much experimentation was done with the simulator but inconsistencies with the statistics continued to plague the results. A discussion with Prof Mueller of the University of Ghent (Belgium) revealed that work being done on an unpublished PhD thesis using Monte Carlo methods indicate that minimum mean squared error forecast techniques are not necessarily optimal methods when applied to inventory management models. This fact along with our belief that the IRO simulator best represents the Army management system compelled us to use the simulated results for our final decisions.

7.4 Final Results (Evaluation of Simulation Results)

In an effort to determine if there is a statistical difference between the cost-performance curves generated by the simulator the following fixed cost analysis was employed, details of which are in [6].

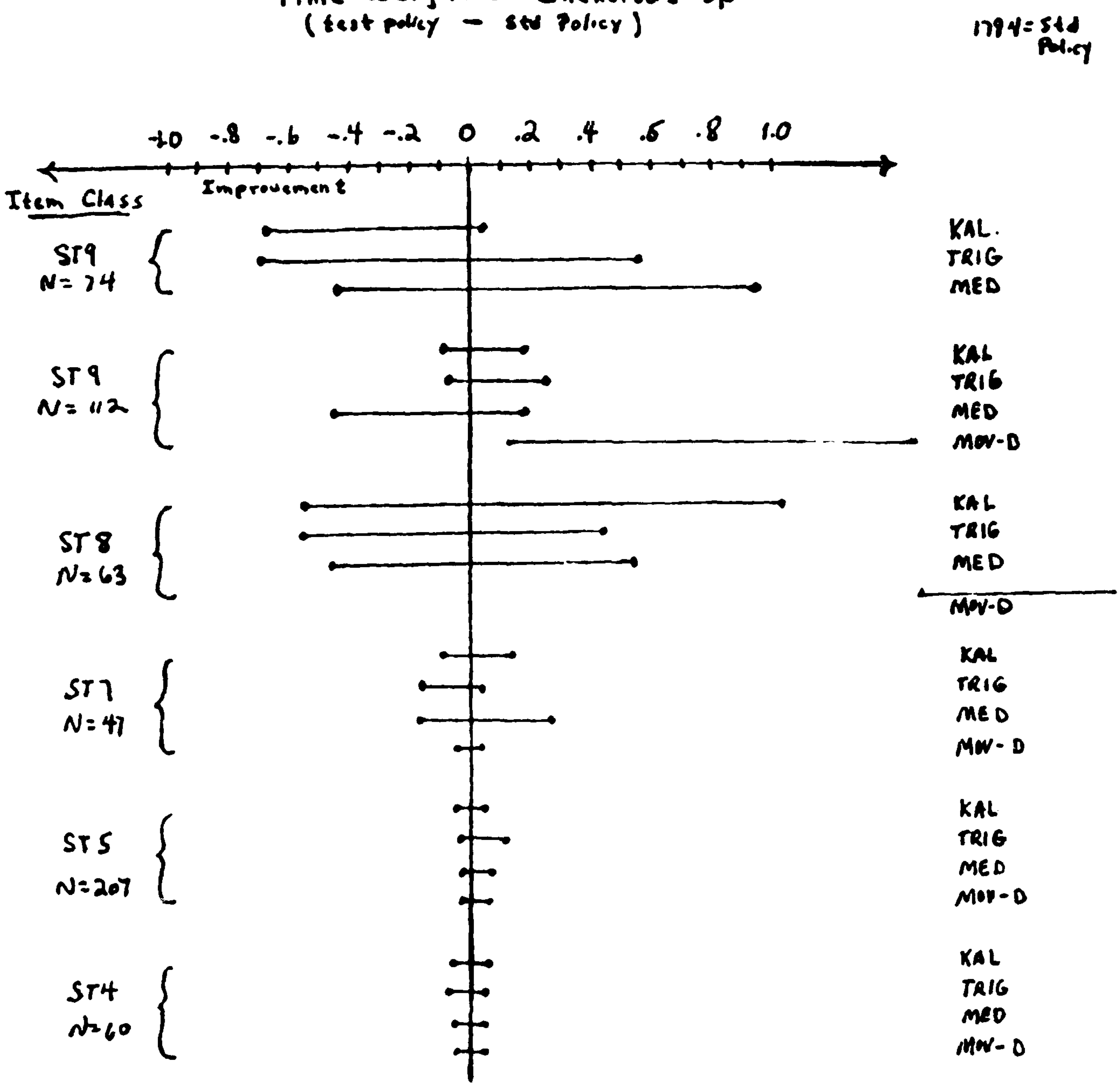
For each class of items, a fixed (current) cost is computed by running the simulator using the standard forecast algorithm (1794) and the λ value the Commodity Command presently uses. The resulting cost represents the inventory cost presently incurred to manage the class of items. Also during this run, the performance of each item is arrayed in a data file for future analysis.

For the alternative policies (forecast algorithms) several λs are used to generate the cost-performance curve for each policy. Using the shape of this curve, a spline technique is used to determine the performance of each item using the alternative policy at the fixed cost. Distributions (histograms) of item performance for each policy at the current cost are found in Appendix A.

To better measure the difference between the alternative methods and the standard, the difference between the individual item performance for the alternative and the standard were computed. (It is shown in the basic statistical literature that these differences will be less variable due to the elimination of extraneous effects and will measure only the difference in methods.) The distributions of these differences are found in Appendix B.

Statistically testing for zero means for each of these difference distributions is equivalent to testing for a difference between the standard (1794) curve and the alternatives as plotted on pages 44 to 50. The results of these tests are captured on the next page where 95% confidence intervals are

95% Confidence Intervals For
The Mean of Individual Differences Between
Time Weighted Backorders of
(test policy − std Policy)



displayed for each distribution of differences for each class of items tested. (Note if these intervals do not contain zero, then the null hypothesis of zero mean is rejected at an α level of 5%) The rankings found on page 52 taken from this table by looking at the mid-point of each interval and ranking them from left to right.

Findings

(1) Using the IRO simulator as described in this report, there is no statistical difference in the simulated performance for each of the five stratification classes for the following comparison.

KALMAN vs 1794

IROTRIGG vs 1794

MED4 vs 1794

(2) MOVD performed worse than 1794 and the other alternative algorithms for the more active items - Class 8 and 9.

Note: MOVD was the only algorithm not using program data.

(3) There was no difference between MOVD and 1794 for less active classes 7, 6, 5, 4.

CHAPTER VIII

CONCLUSION

8.1 Findings

In an earlier report by Orr [8], the candidate of final choice for forecasting was a Kalman filter algorithm. This choice was based upon its dominance in statistical forecast accuracy and its savings over the current method as projected from cost-performance curves produced from simulator runs. There was little difference amongst the algorithms' curves for LDV items, and so most of the savings was driven by HDV items.

From the current viewpoint it appears that the savings might not have been statistically significant. At the least, the algorithm's simulated performance has not been robust against the changes that have occurred (see below) since that report (although its relative forecast accuracy has held up). Presently we find "Kalman" working well on some of our groupings of items and the current (1794) method (among other algorithms) working well on other items. The improved statistical tools [6] we use now show, in any case, that no algorithm's cost-performance curve is significantly better than another's - for active items. In several cases a few items can influence the performance rankings.

It is not possible to isolate the impact of individual changes made since the earlier report when comparing the differing results. However, such changes were:

a. The data base was extended from 7 to 11 years and more items were captured. The last 7 years of the 11 were used in the current analysis, so much of the Vietnam era from the earlier time series was not influential.

b. It is possible that a different small group of items might now be driving the HDV savings.

c. The statistical analysis program was overhauled to be more flexible and capture various error measures.

d. Changes in the simulator were made. Several of these could narrow the potential difference in performance amongst the algorithms, e.g., excess cost savings were accumulated only for some items, a standard moving average forecast superseded all algorithms in periods of very little activity, forecast updates were made only after quarters with demand.

e. Some changes to the algorithms themselves were made. Theoretical adjustments were made to the "Kalman" to produce several versions; also, the "k" parameters were updated to reflect the later 7 years of history that were used.

8.2 Postscript

This table consolidates statistical results of one of the more meaningful error measures, MAD/AYD, for the four algorithms - by data group and by requisition class. This relative error when multiplied by an algorithm's current forecast of average yearly demand, AYD, yields an estimate of the mean absolute deviation (error) in a year's demand. The theory and formulas in [9] can be used to convert this estimate to a variance of lead time demands, the latter a necessary variable in computing safety levels in the VSL EOQ module of CCSS.

It is apparent that the choice of a final algorithm dictates the percent errors to be used in VSL EOQ. The present PCER tables in that module should be superseded by an expanded, refined version of these tables by requisition class (and perhaps by dollar demand). The MAD/AYD for the inactive class (0-3 requisitions) would be based upon current work on algorithms for inactive items, and not upon those values in this current table, which for some entries are suspect.

8.3 Recommendations

For active items (greater than 3 requisitions a year) the current (1794) forecast procedure should not be replaced.

The percent error (PCER) tables in the VSL EOQ module should be overhauled using the statistical byproducts of this current research and of the inactive item research.

A plan for consolidating the current algorithm (for active items), the pending algorithm (for inactive items, including NSO & insurance) and the consequent lead time demand variance procedure should be instituted.

Relative Error, 1 Year Forecasts

MAD/AYD by Requisition Class

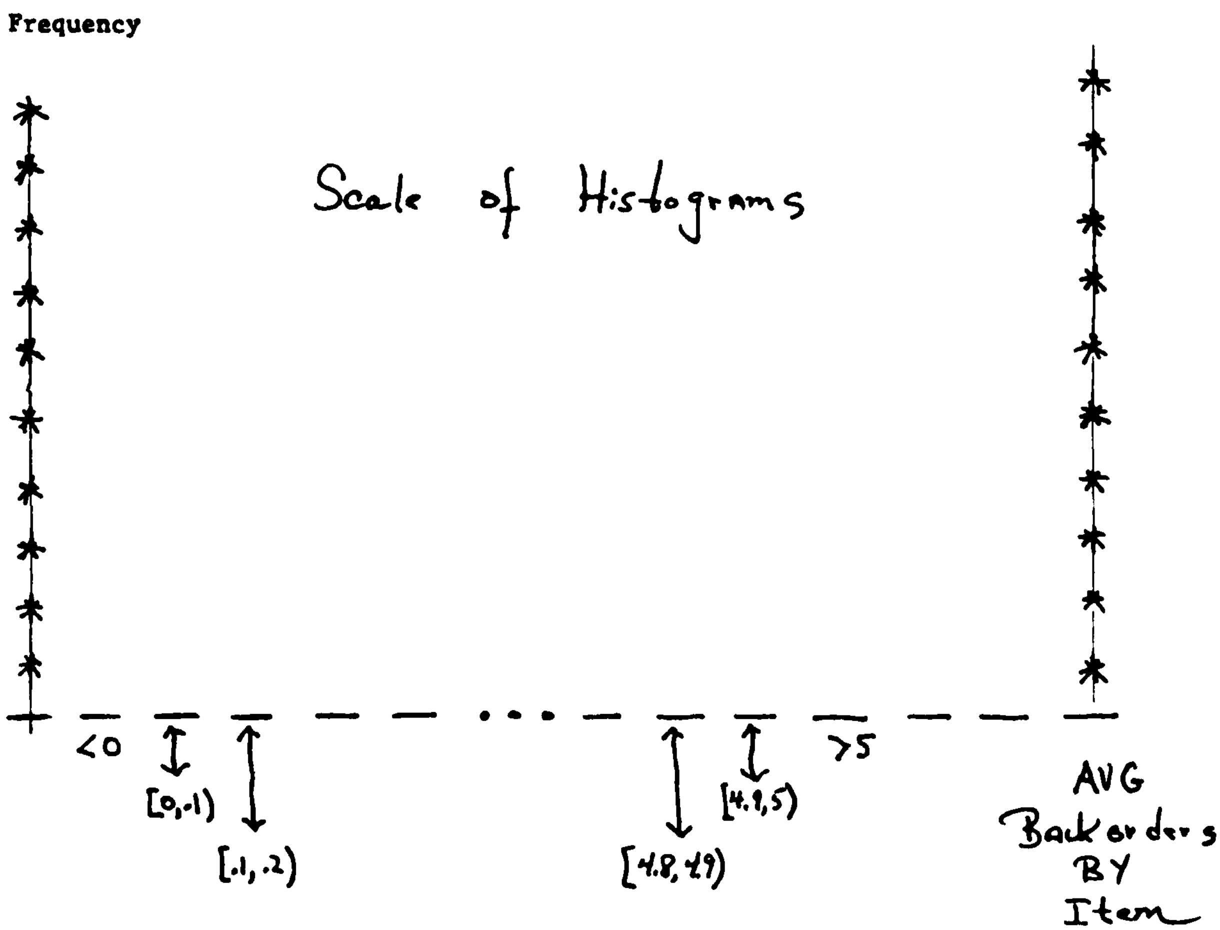
(0-3) - (3-12) - (>12)

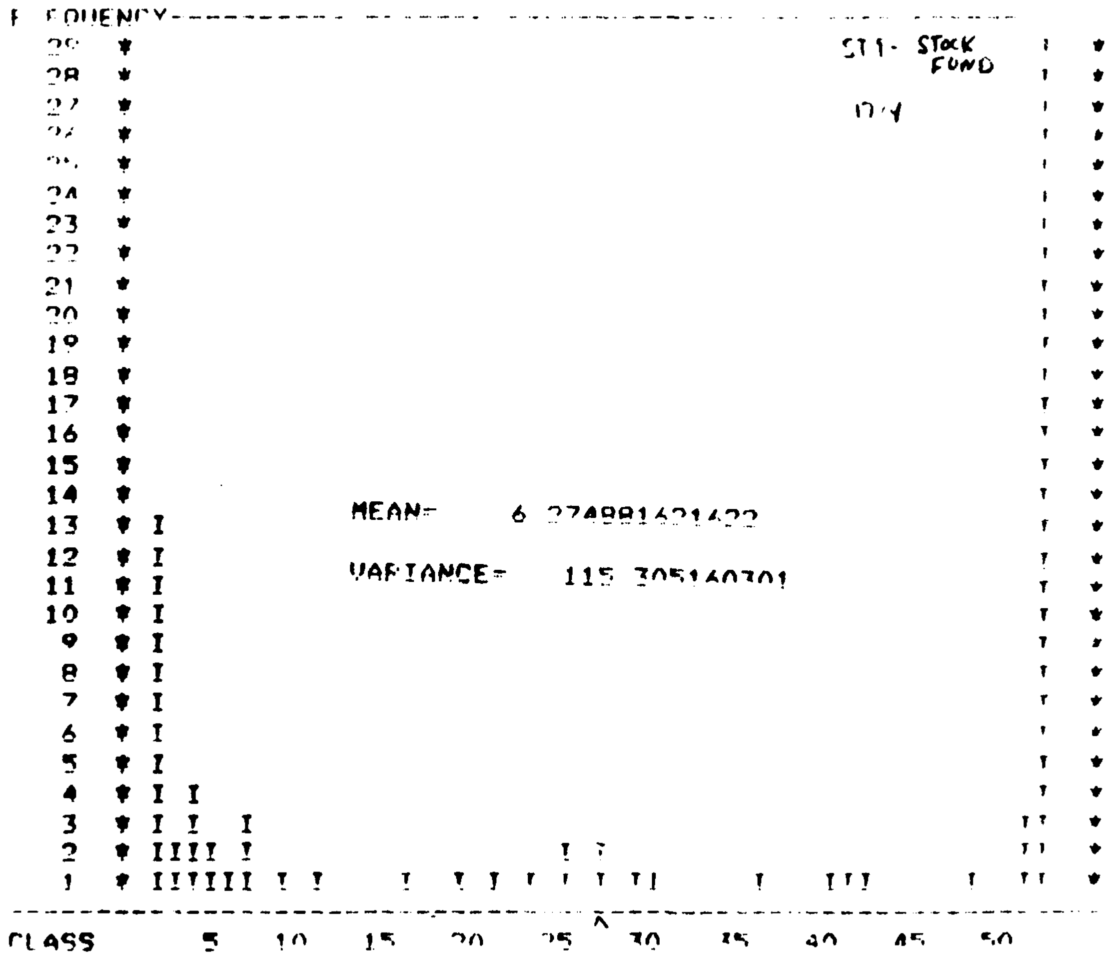
Data Base	# Items	1794	KALI	TRIG	MED4
LDVNON28	9467	1.637	1.396	1.174	.781
	2301	.797	.779	.837	.805
	1242	.512	.476	.514	.510
LDVDYN28	341	1.525	1.337	1.194	.750
	131	.802	.718	.772	.745
	154	.530	.465	.502	.495
HDVNON28	4	1.042	1.331	1.019	1.096
	27	1.043	.839	.779	.687
	181	.369	.358	.372	.381
HDVDYN28	3	10.173	6.273	1.115	.568
	12	.482	.522	.576	.643
	34	.388	.357	.380	.376

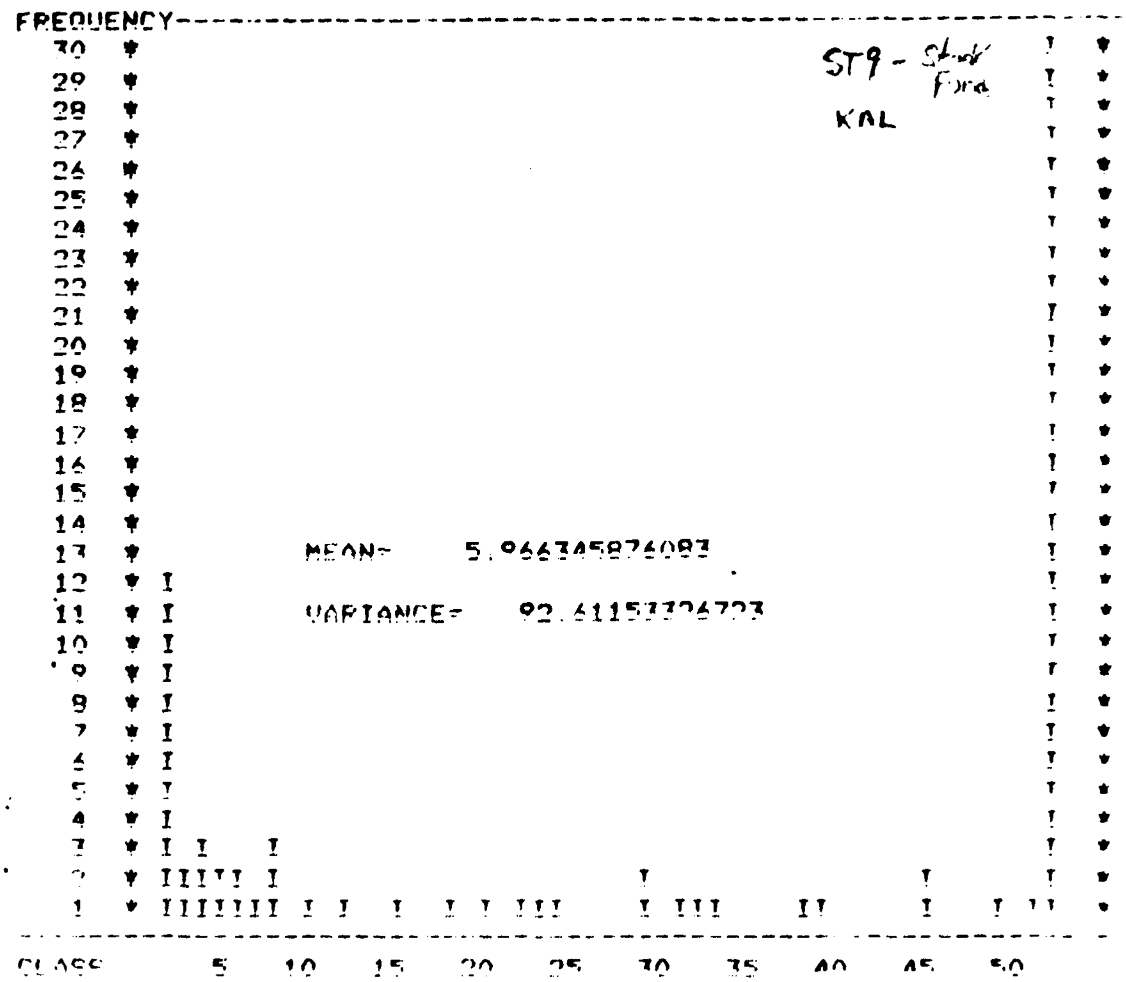
APPENDIX A

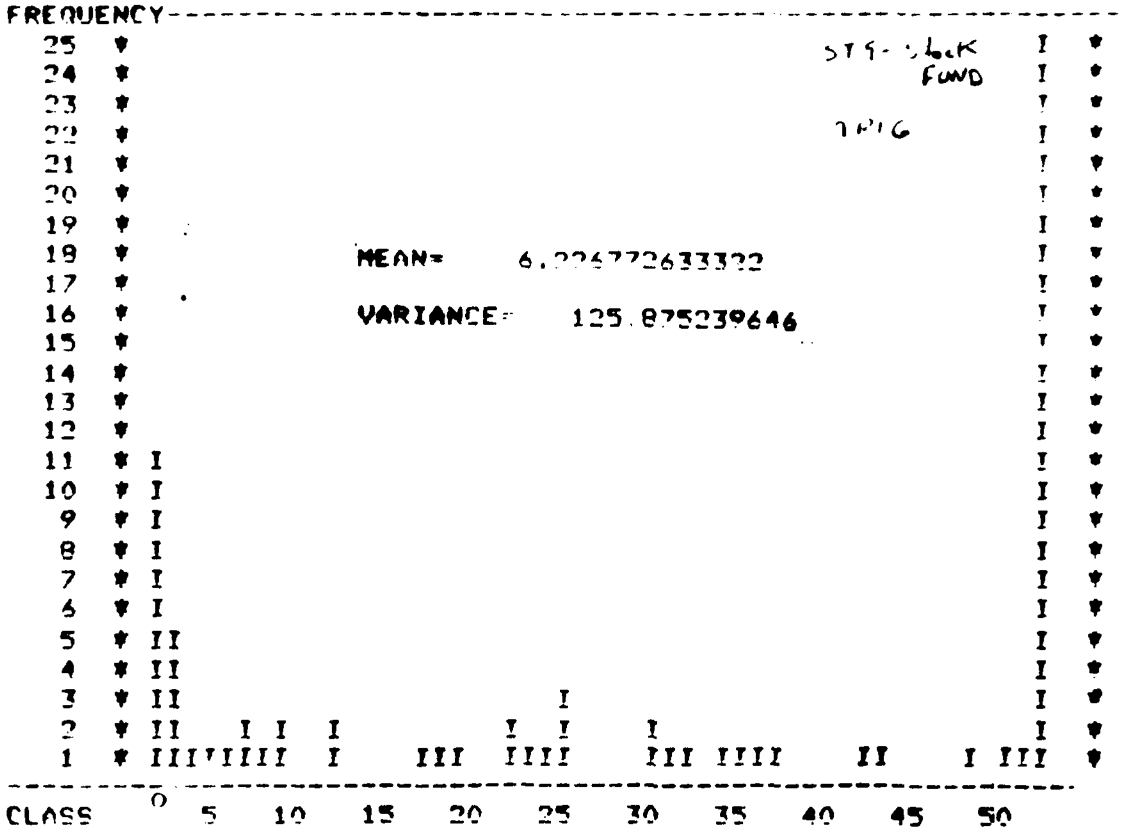
HISTOGRAMS OF INDIVIDUAL ITEM PERFORMANCE AS DETERMINED BY SIMULATOR

FORECAST METHOD	DATA STRATS
1794	ST9 (stock fund)
KAL	ST8
IROTRIGG	ST7
MED4	ST5

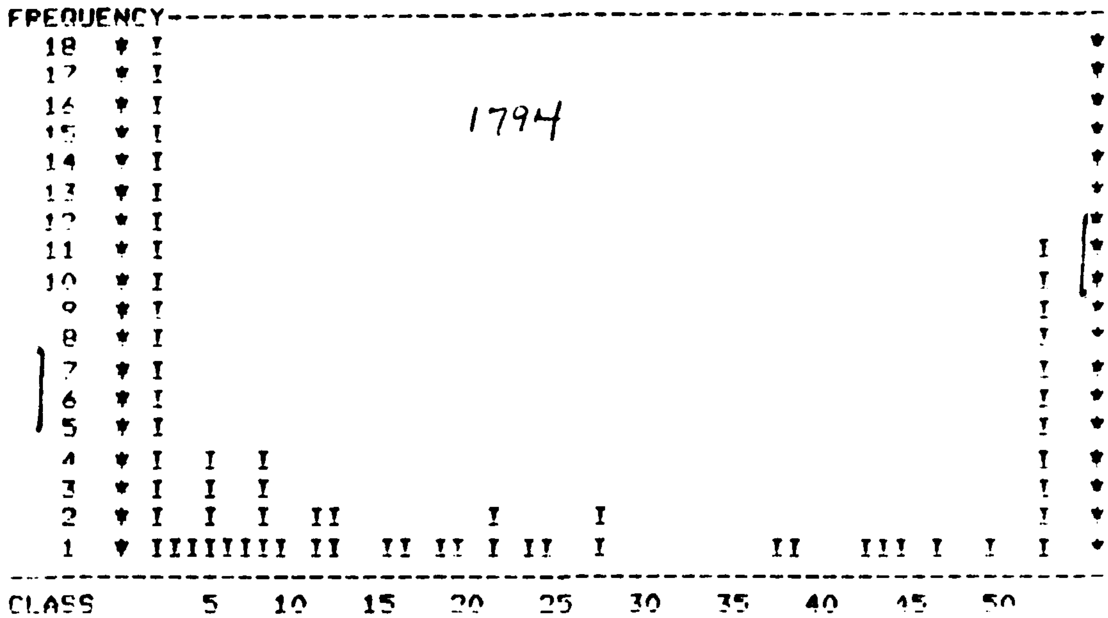



FREQUENCY
ST1- STOCK FUND
17/4
MEAN= 6
VARIANCE= 115
CLASS
5 10 15 20 25 30 35 40 45 50


FREQUENCY
ST9 - Stud Fora
KAL
MEAN= 5.966345874083
VARIANCE= 92.61153324723
CLASS
5 10 15 20 25 30 35 40 45 50

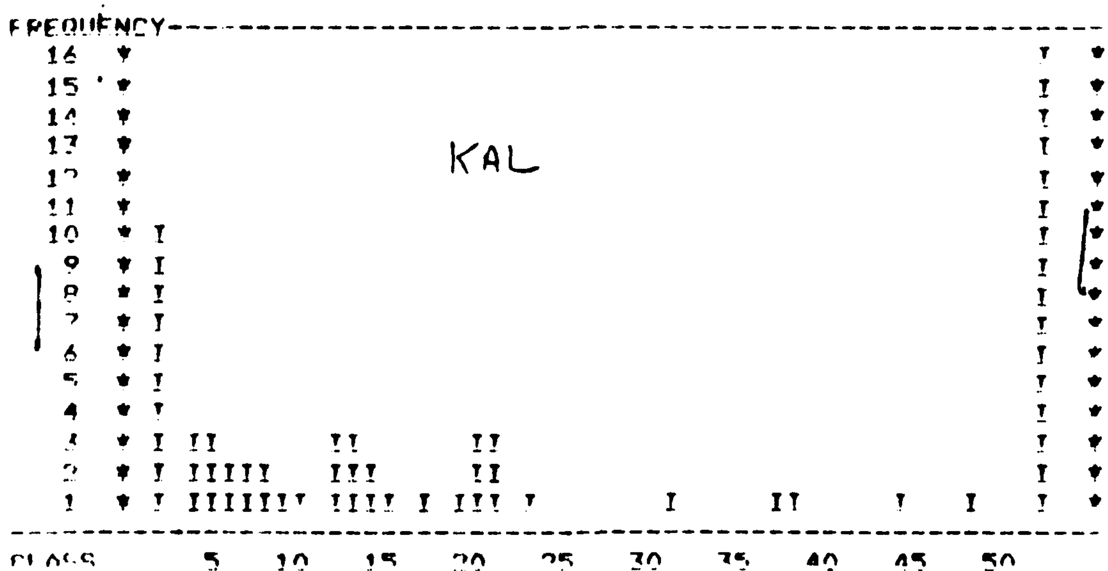

FREQUENCY
MEAN= 6.226772633322
VARIANCE= 125.875239646
CLASS
519 - Stock FUND

STRAT CLASS 8



MEAN= 4.927899047619

VARIANCE= 104.4875355832

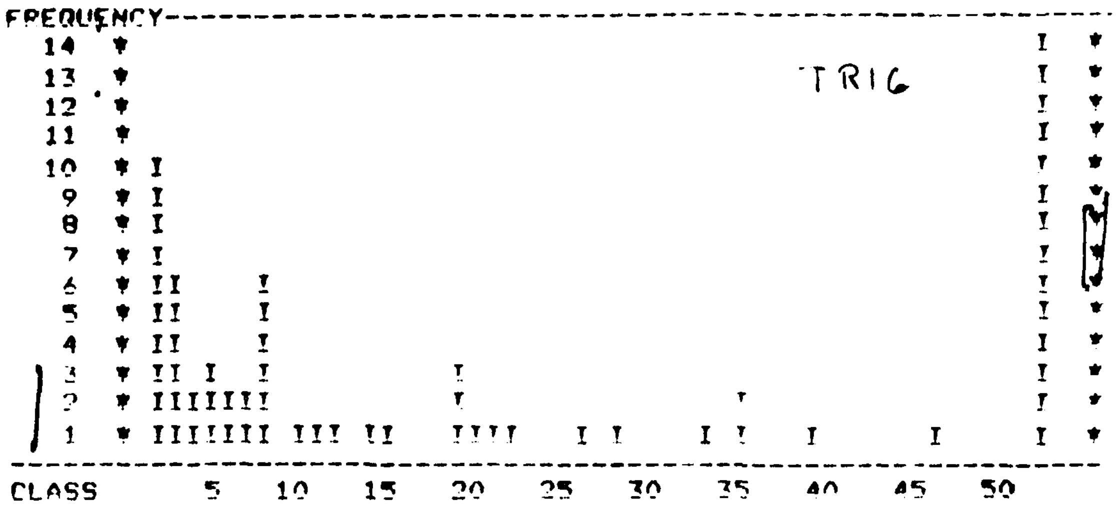


MEAN= 5.154220381447

VARIANCE= 91.70849226575

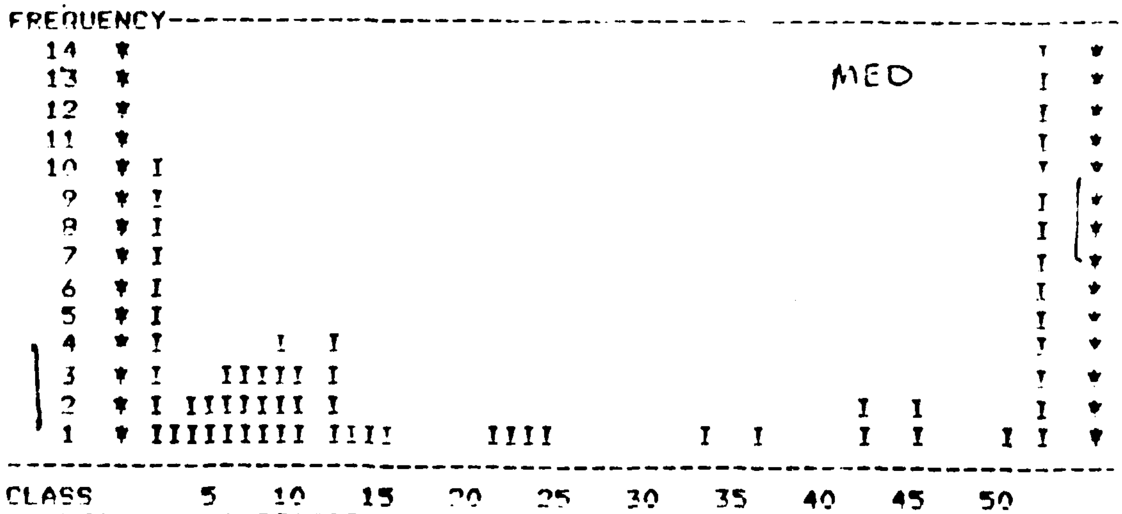
STRAT CLASS 8

N=62



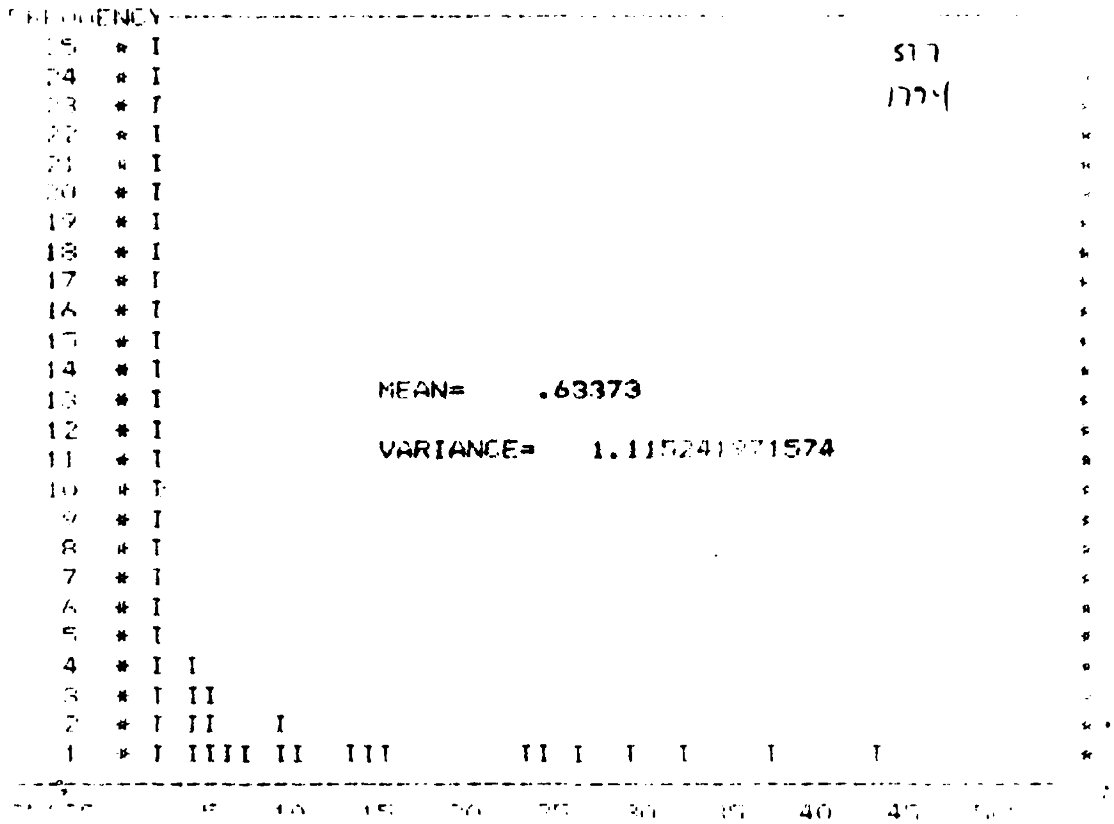
MEAN= 4.85372414878

VARIANCE= 84.96466955536



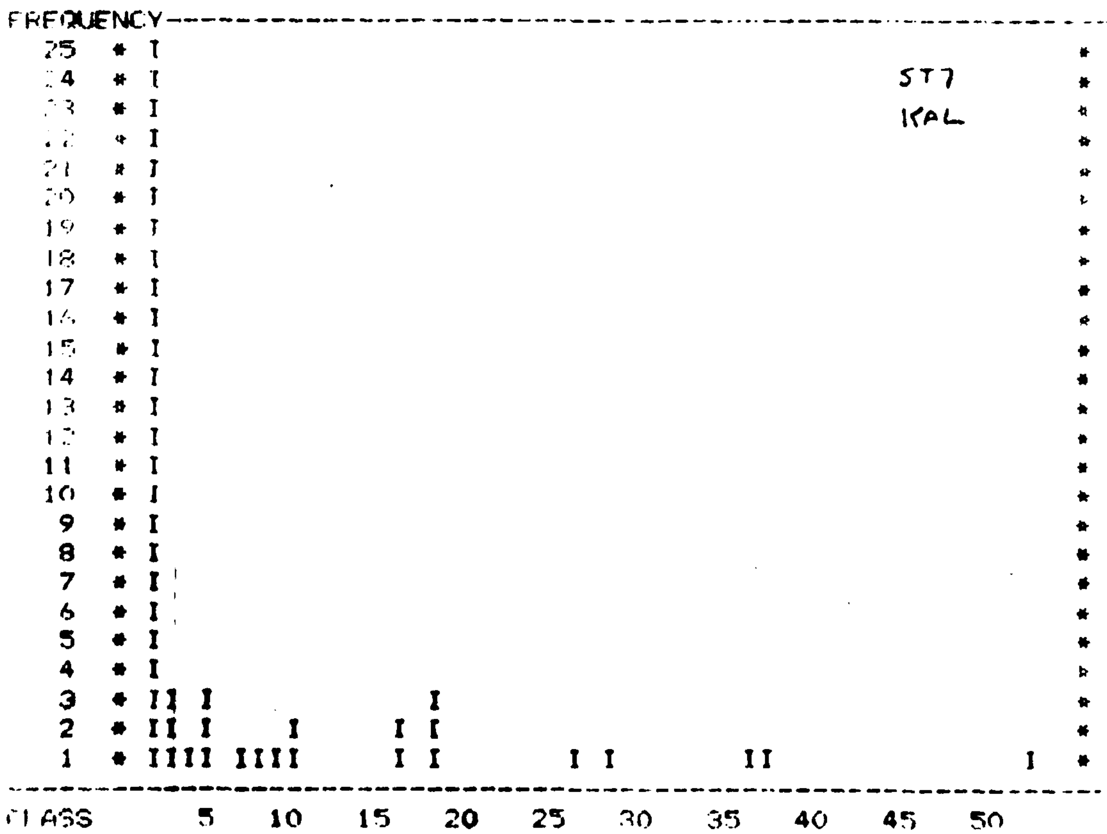
MEAN= 4.951909449766

VARIANCE= 92.52434274389



MEAN= .63373

VARIANCE= 1.115241971574



MEAN= .6705007185431

VARIANCE= 1.43568143392

577

TRIC

```
FREQUENCY----------------------------------------------------------------
  24  * I
  23  * I
  22  * I
  21  * I
  20  * I
  19  * I
  18  * I
  17  * I
  16  * I
  15  * I
  14  * I
  13  * I
  12  * I
  11  * I
  10  * I
   9  * I
   8  * I
   7  * I
   6  * I
   5  * I
   4  * I
   3  * IIII
   2  * IIII    I
   1  * IIIII III  I  II  I     II   I   I    I
------------------------------------------------------------------------
CLASS      5   10   15   20   25   30   35   40   45   50
   MEAN=      .5787237[illegible]67

   VARIANCE=    .963587094219[illegible]
```

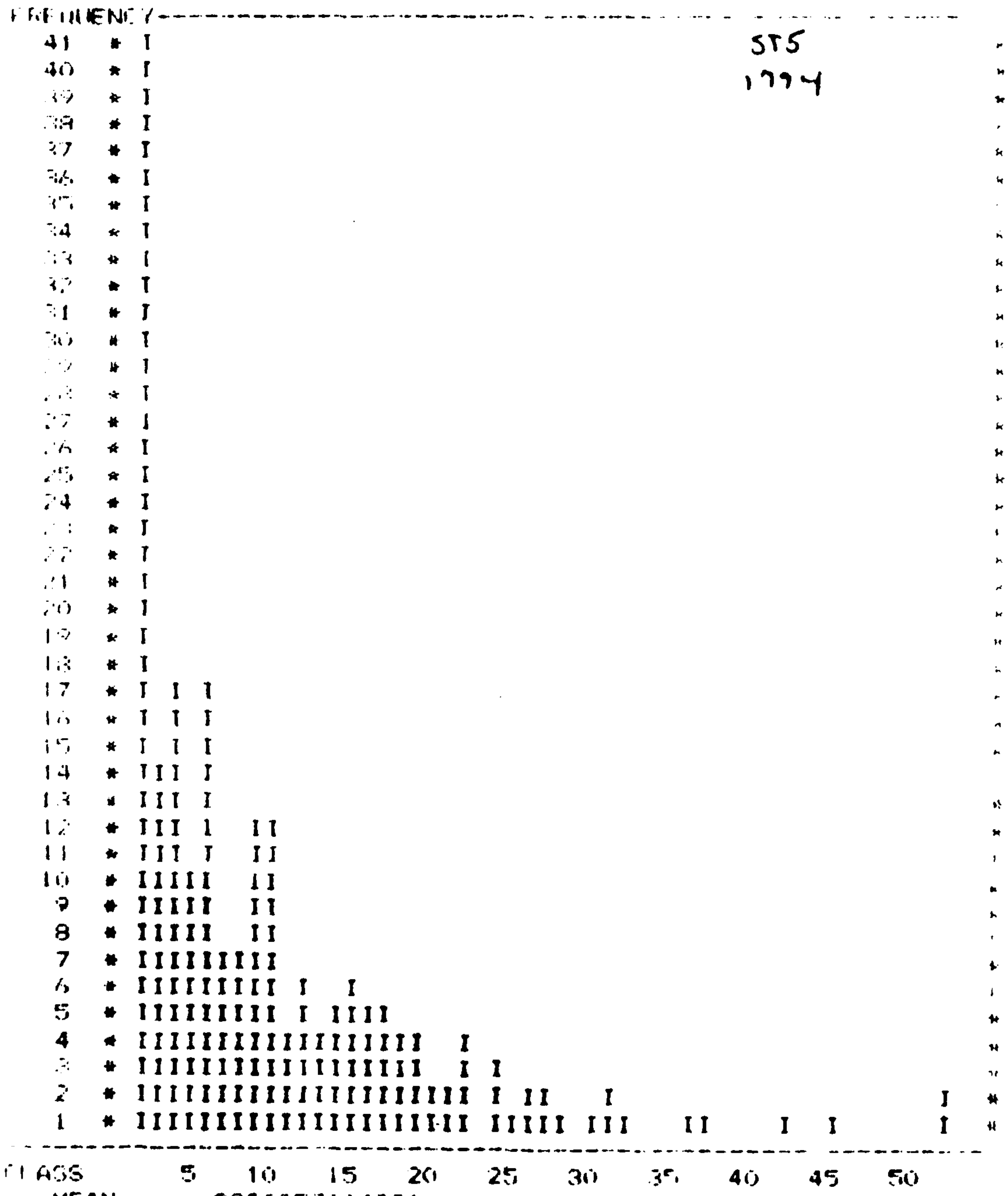
571

MF

```
FREQUENCY-------------------------------------------------------
  24  * I
  23  * I
  22  * I
  21  * I
  20  * I
  19  * I
  18  * I                        MEAN=      .7083022127[illegible]
  17  * I
  16  * I                        VARIANCE=    1.50[illegible]0146004
  15  * I
  14  * I
  13  * I
  12  * I
  11  * I
  10  * I
   9  * I
   8  * I
   7  * I
   6  * I
   5  * I
   4  * I   I
   3  * I   I
   2  * I I I  I  II
   1  * IIIII  IIIII        II      II  I     I            I
------------------------------------------------------------------------
CLASS      5   10   15   20   25   30   35   40   45   50
```


[..]TS PAGE IS BEST QUALITY [..]
FROM COPY FURNISHED TO DDC

STRAT CLASS 5



MEAN= .8839957004831

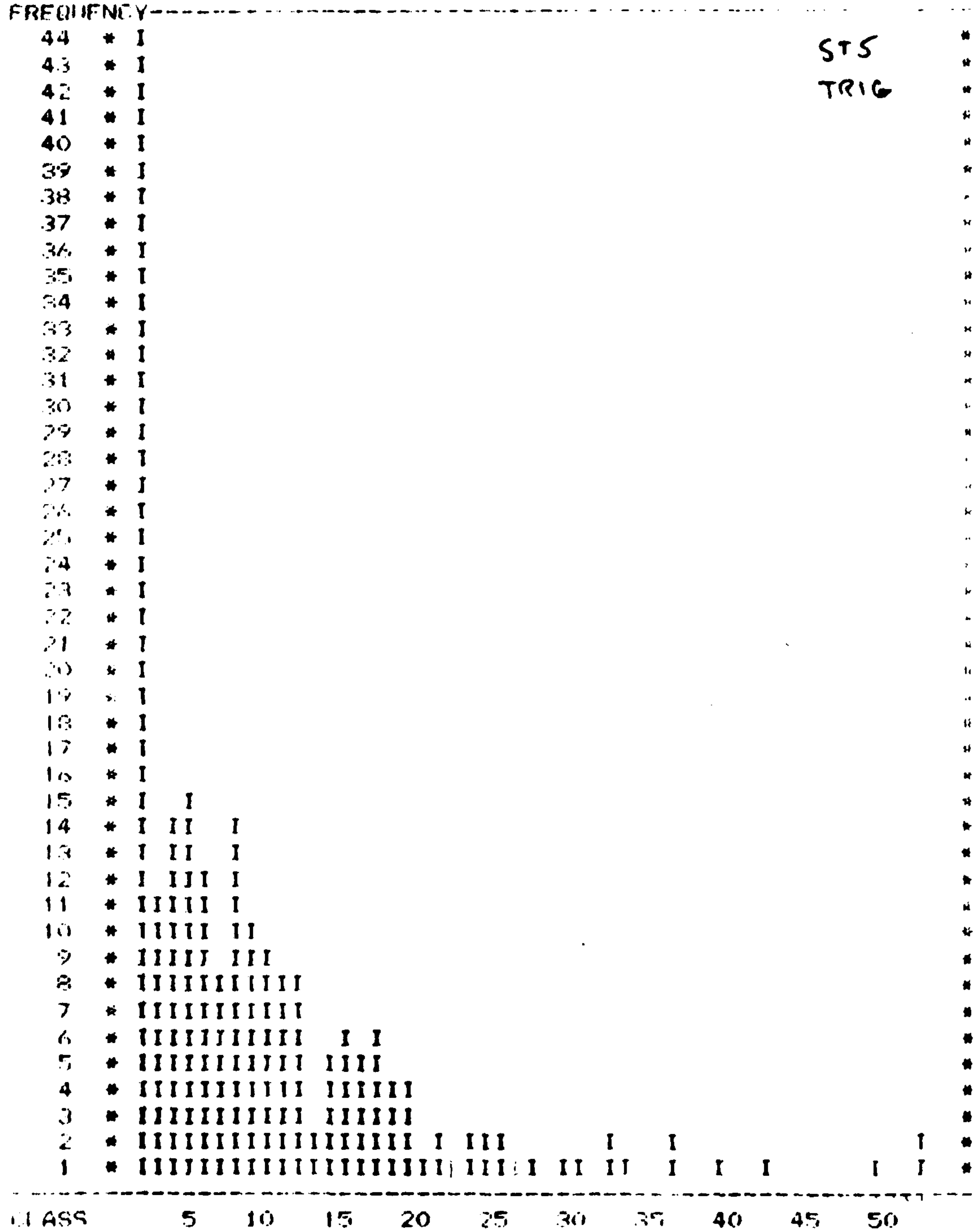
VARIANCE= 1.15650812353

STRAT CLASS 5

```
FREQUENCY---------------------------------------------------------------
  43  * I                                                    ST5      *
  42  * I                                                    KAL      *
  41  * I                                                             *
  40  * I                                                             *
  39  * I                                                             *
  38  * I                                                             *
  37  * I                                                             *
  36  * I                                                             *
  35  * I                                                             *
  34  * I                                                             *
  33  * I                                                             *
  32  * I                                                             *
  31  * I                                                             *
  30  * I                                                             *
  29  * I                                                             *
  28  * I                                                             *
  27  * I                                                             *
  26  * I                                                             *
  25  * I                                                             *
  24  * I                                                             *
  23  * I                                                             *
  22  * I                                                             *
  21  * I                                                             *
  20  * I                                                             *
  19  * I                                                             *
  18  * I                                                             *
  17  * I                                                             *
  16  * I                                                             *
  15  * I   I                                                         *
  14  * I I I                                                         *
  13  * I I I  I                                                      *
  12  * I III  I                                                      *
  11  * IIIII  I                                                      *
  10  * IIIII II     I                                                *
   9  * IIIII II     I                                                *
   8  * IIIIIIIIII   I                                                *
   7  * IIIIIIIIII   I                                                *
   6  * IIIIIIIIII   III                                              *
   5  * IIIIIIIIIIII III      I                                        *
   4  * IIIIIIIIIIII III I  I I                                        *
   3  * IIIIIIIIIIII III II I I                                        *
   2  * IIIIIIIIIIII III II I I       I   I               I           *
   1  * IIIIIIIIIIIIIIIIIIIII I II IIII   II    I    I    I           *
------------------------------------------------------------------------
CLASS     5    10   15   20   25   30   35   40   45   50
  MEAN=     .8997160002407

  VARIANCE=    1.241705621852
```

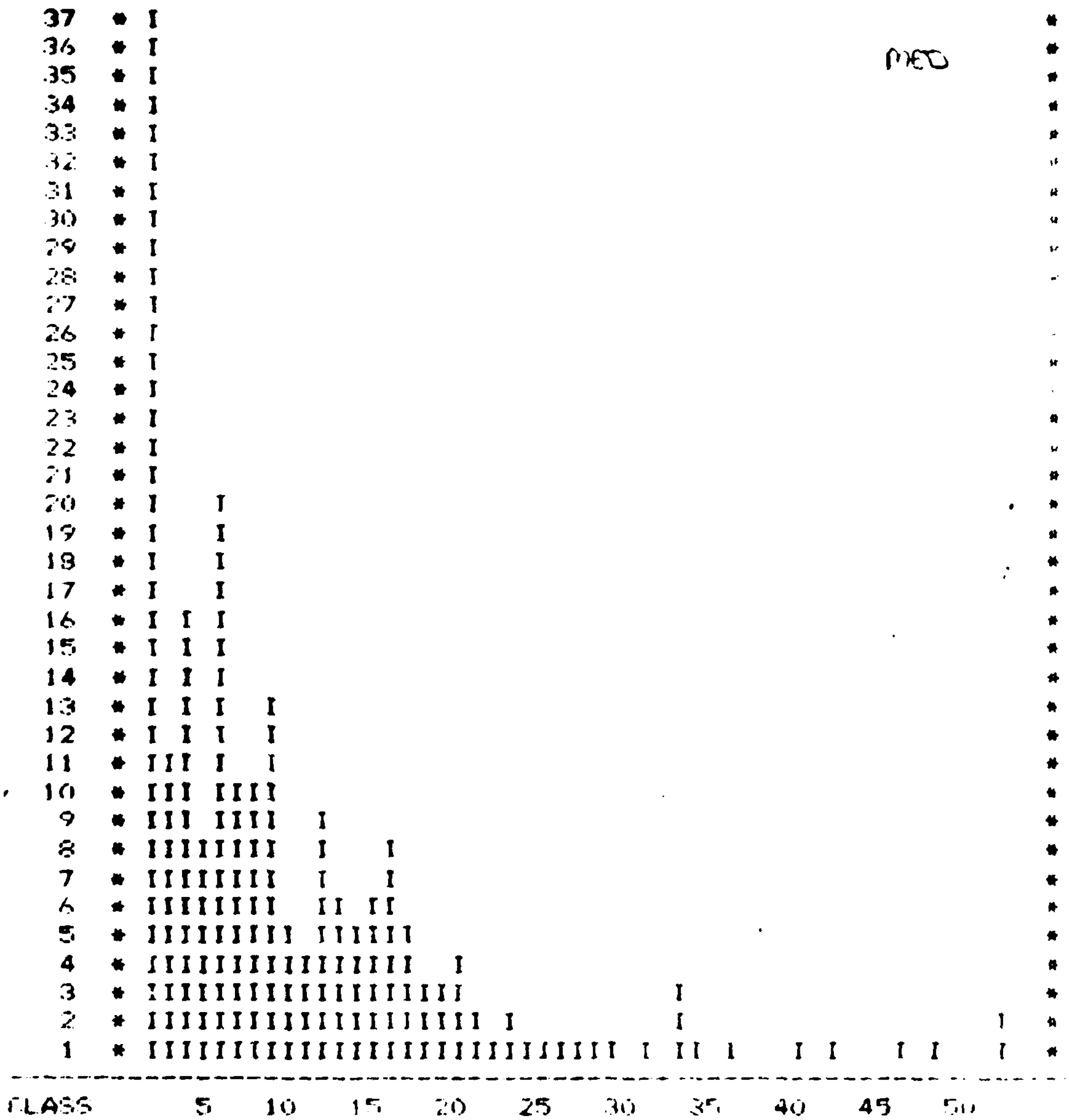
STRAT CLASS 5



MEAN= .8694979853693

VARIANCE= 1.252062547115

STRAT CLASS 5



MEAN= .9331464133597

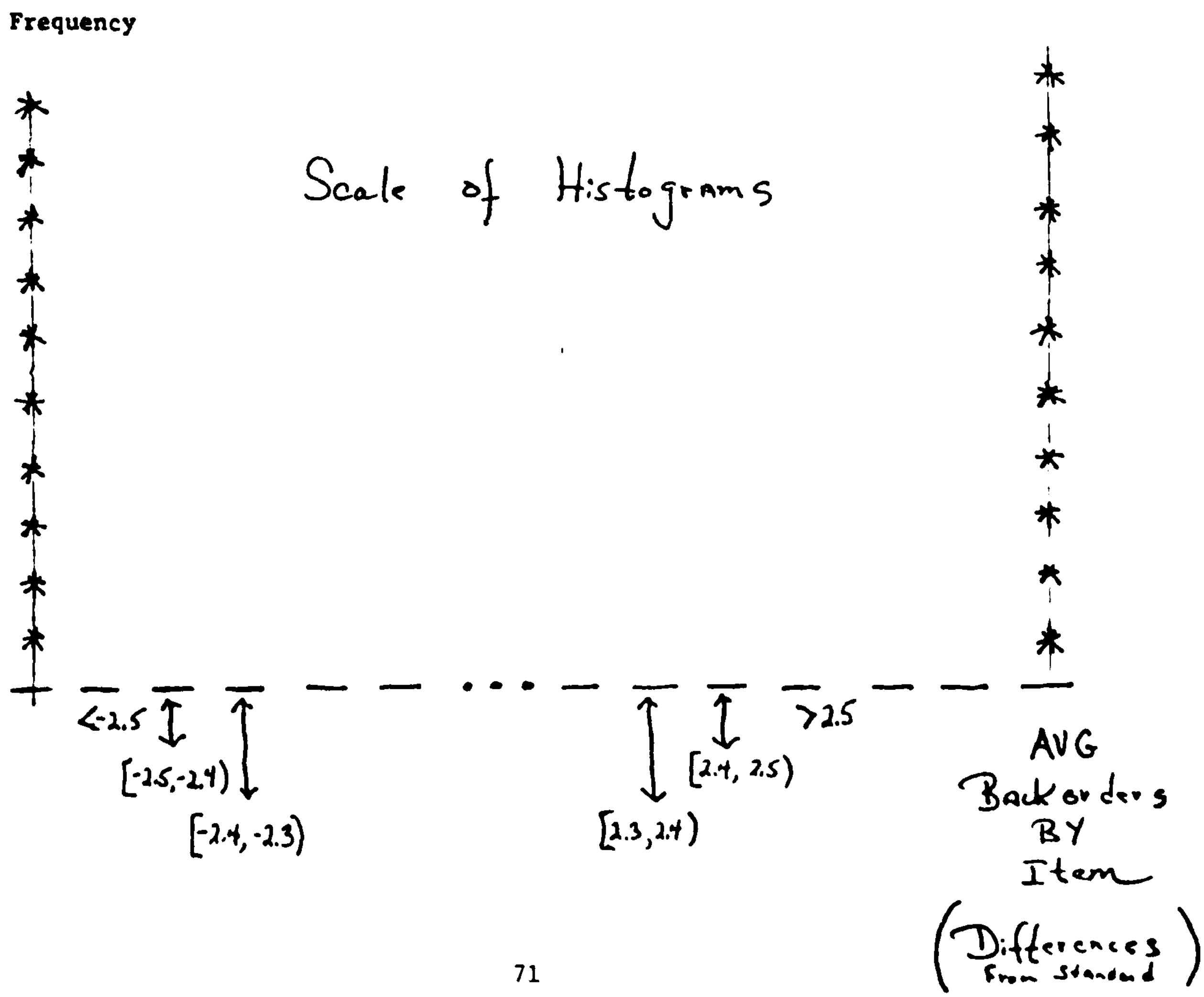
VARIANCE= 1.327564530822

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

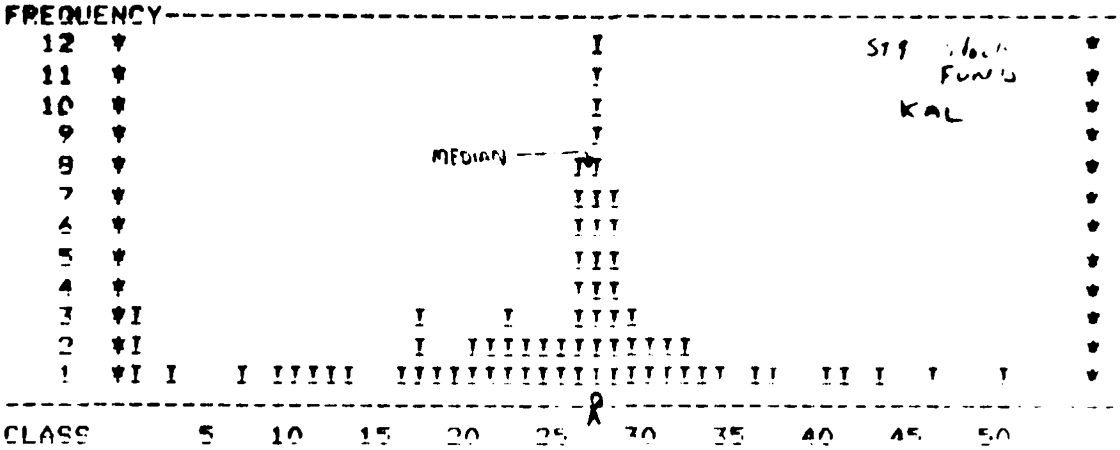
APPENDIX B

HISTOGRAM OF DIFFERENCES BETWEEN INDIVIDUAL PERFORMANCE OF ALTERNATIVE ALGORITHMS AND STANDARD (1794)

ALTERNATIVE METHOD	DATA STRATS
KAL	ST9
IROTRIGG	ST8
MED4	ST7
MOVD	ST5

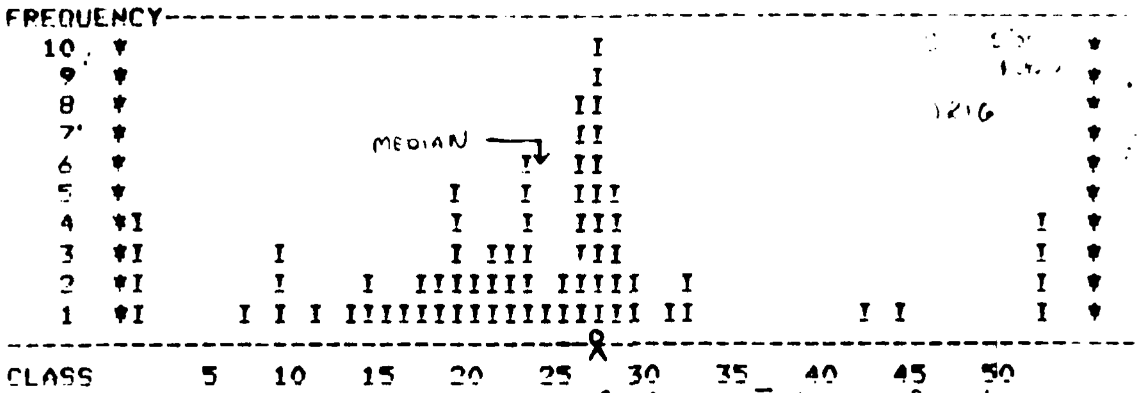


N= 14



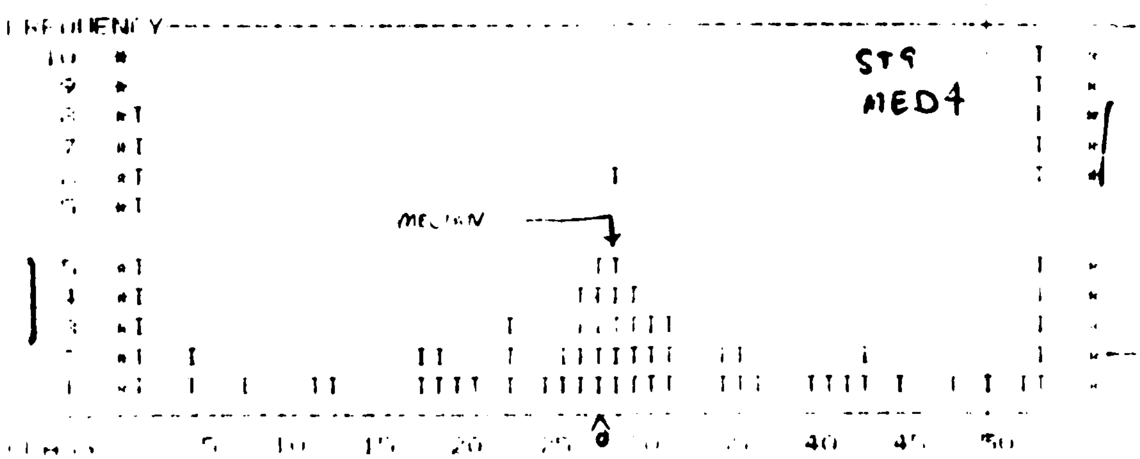
MEAN = -.3085357455397 95% Confidence interval for μ_d (-.641, .025)

VARIANCE = 2.072632392748



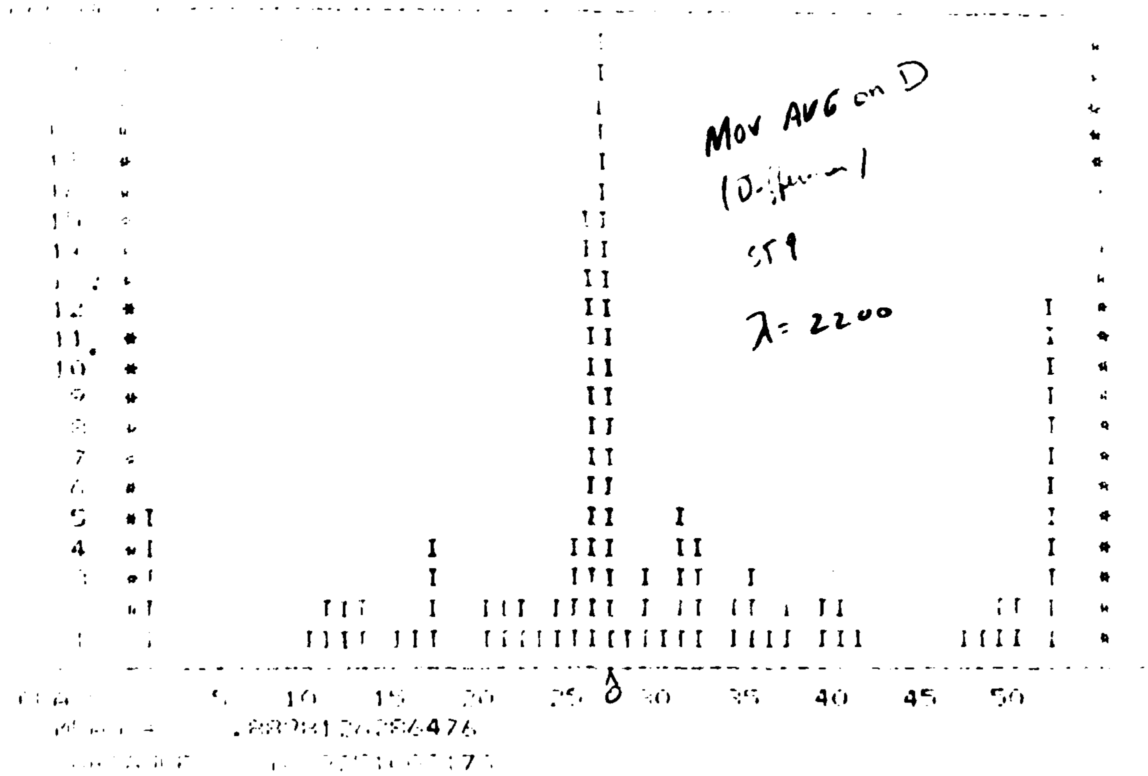
MEAN = -.04810898829984 Confidence Interval for μ_d (-.665, .569)

VARIANCE = 7.094066197033



MEAN = [illegible] 95% Confidence Interval for μ_d (-.428, .928)

VARIANCE = [illegible]


Mov AVG on D
λ= 2200
5
10
15
20
25
30
35
40
45
50

STRAT CLASS 8

N: 63

KAL

```
FREQUENCY-------------------------------------------------------
 10    *                         I                           *
  9    *                        II                           *
  8    *                        II                           *
  7    *                        II                        I  *
  6    *                        II  MEDIAN                I  *
  5    *                        II                        I  *
  4    *                        II         I              I  *
  3    *I                     I II   I     I              I  *
  2    *I                  II IIII   II    I      I       I  *
  1    *I  I            IIIII IIIIIIIII I II   IIIII    I I  *
----------------------------------------------------------------
                                 0
CLASS      5    10   15   20   25   30   35   40   45   50
  MEAN =      .2247217338284
  VARIANCE =  10.37755381773
```

95 % Confidence Interval for u_d
(-.585, 1.037)

TRIG

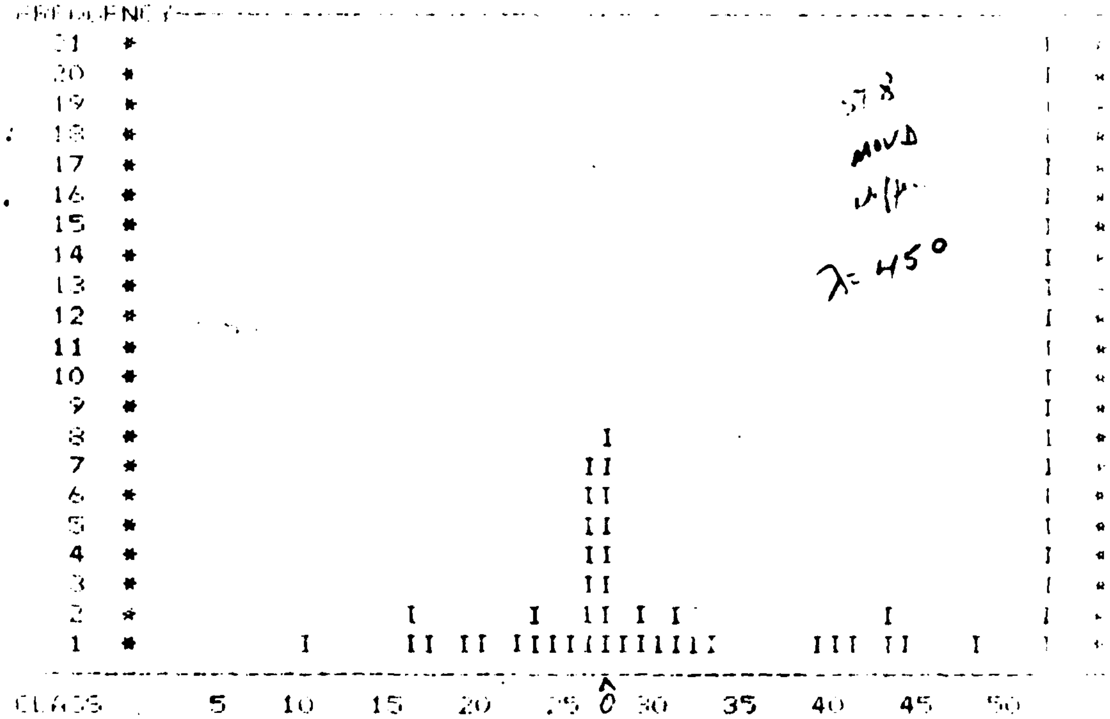
```
FREQUENCY-------------------------------------------------------
 11    *                        II                           *
 10    *                        II                           *
  9    *                        II                           *
  8    *                        II                           *
  7    *                        II  MEDIAN                   *
  6    *                        II                           *
  5    *                        II                           *
  4    *                     I  II                           *
  3    *I                  I I  II  II I                     *
  2    *I                  I I IIIIIIIII        I         I  *
  1    *I       I    I  II I IIIIIIIIIII II      II  I    II *
----------------------------------------------------------------
                                 0
CLASS      5    10   15   20   25   30   35   40   45   50
  MEAN =      -.07417487923941
  VARIANCE =  3.999041686517
```

95% Confidence Interval for u_d
(-.574, .426)

MED4

```
FREQUENCY-------------------------------------------------------
  9    *                         I                           *
  8    *                         I  MEDIAN                   *
  7    *                        II                           *
  6    *                        III                          *
  5    *                       IIII                       I  *
  4    *I                      IIII II                    I  *
  3    *I                      IIII II                    I  *
  2    *I                      IIIIIII                    I  *
  1    *I       II  I  I   IIIIIIIIIIIII   II    I  III  II  *
----------------------------------------------------------------
                                 0
CLASS      5    10   15   20   25   30   35   40   45   50
  MEAN =      .02401040214657
  VARIANCE =  4.237199390636
```

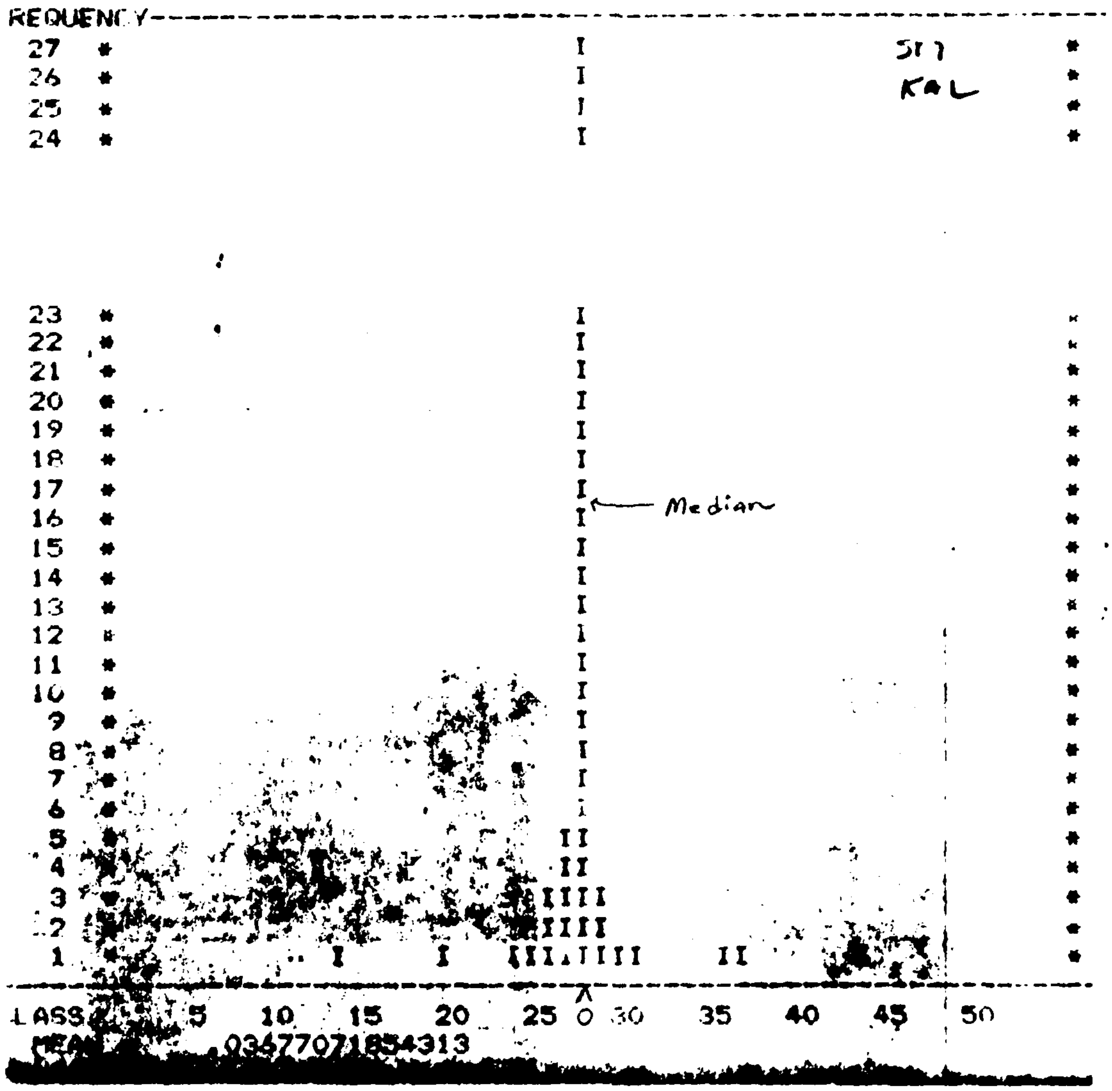
95 % Confidence Interval for u_d
(-.486, .534)



MEAN = [illegible].723175935044

VARIANCE = 20.33742227457

N=47



σ^2 = .1603

95% Confidence Interval for μ_d

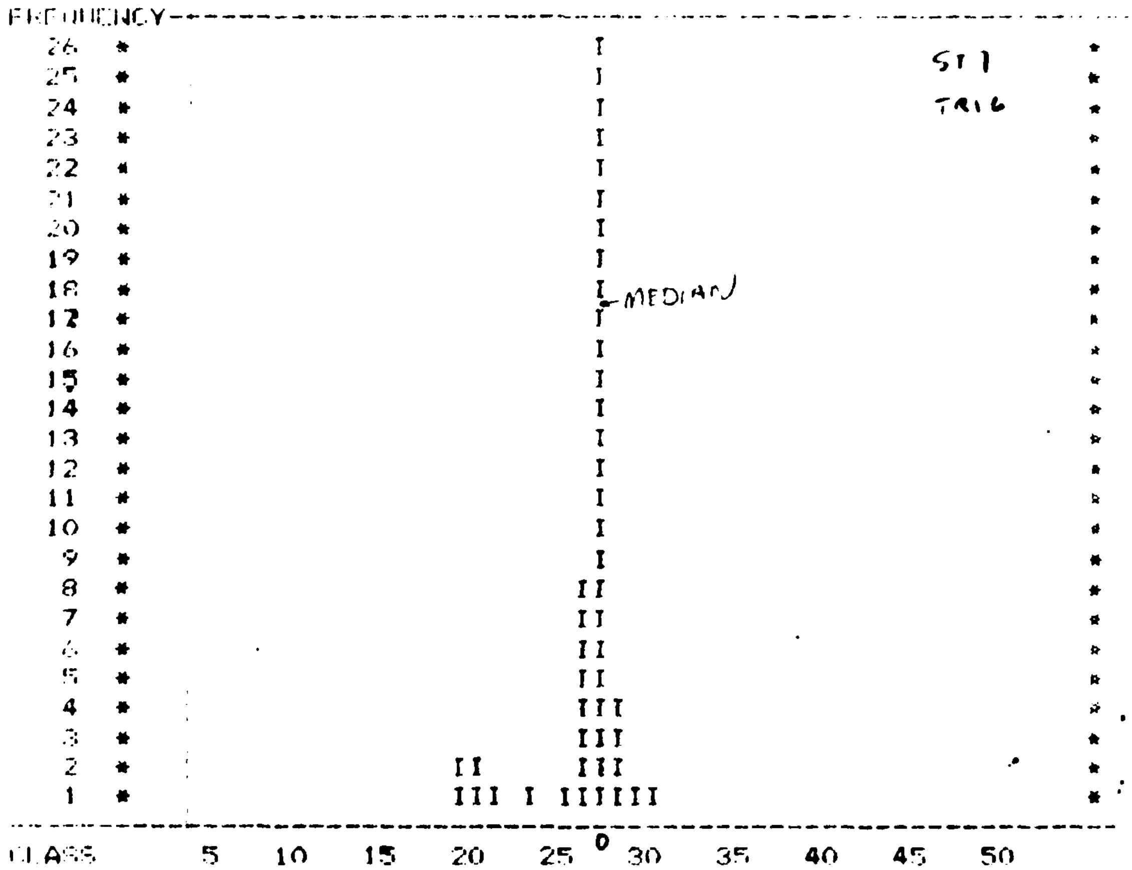
(-.080, .153)

ZEROS DELETED 19

$\bar{X}$ = .0617

σ^2 = .262

THIS PAGE IS BEST QUALITY PRAC
FROM COPY FURNISHED TO DDC

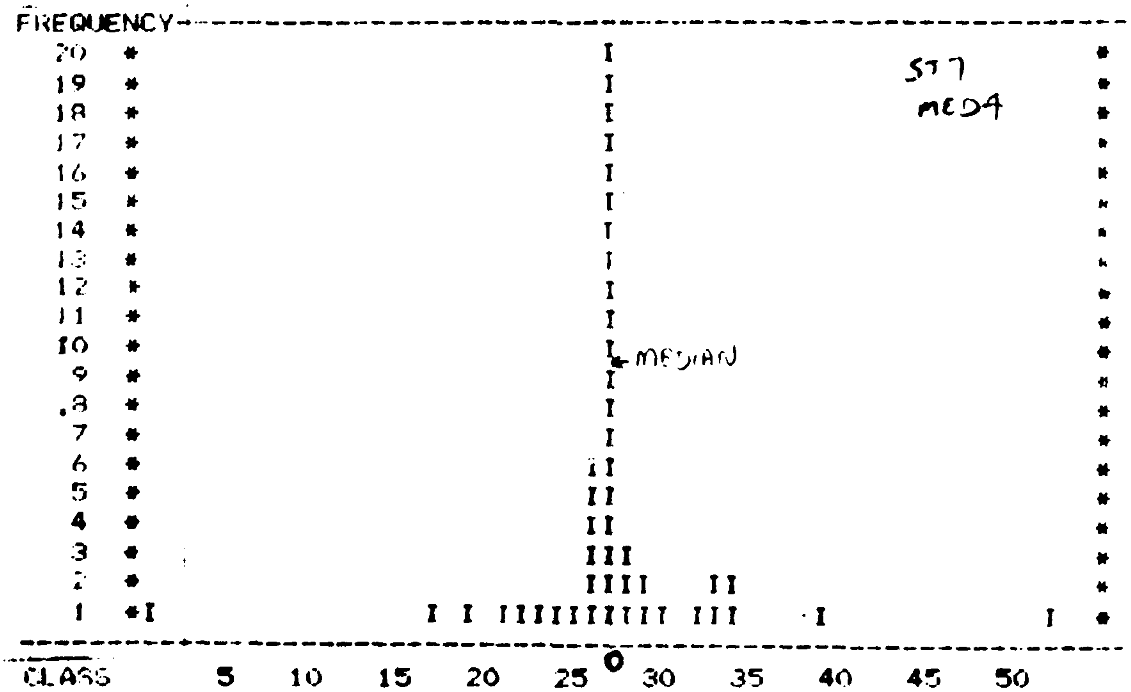


MEAN = -.0550012046613

95 % Confidence Interval for μ_d

(-.124, .014)

VARIANCE = .05571826937446

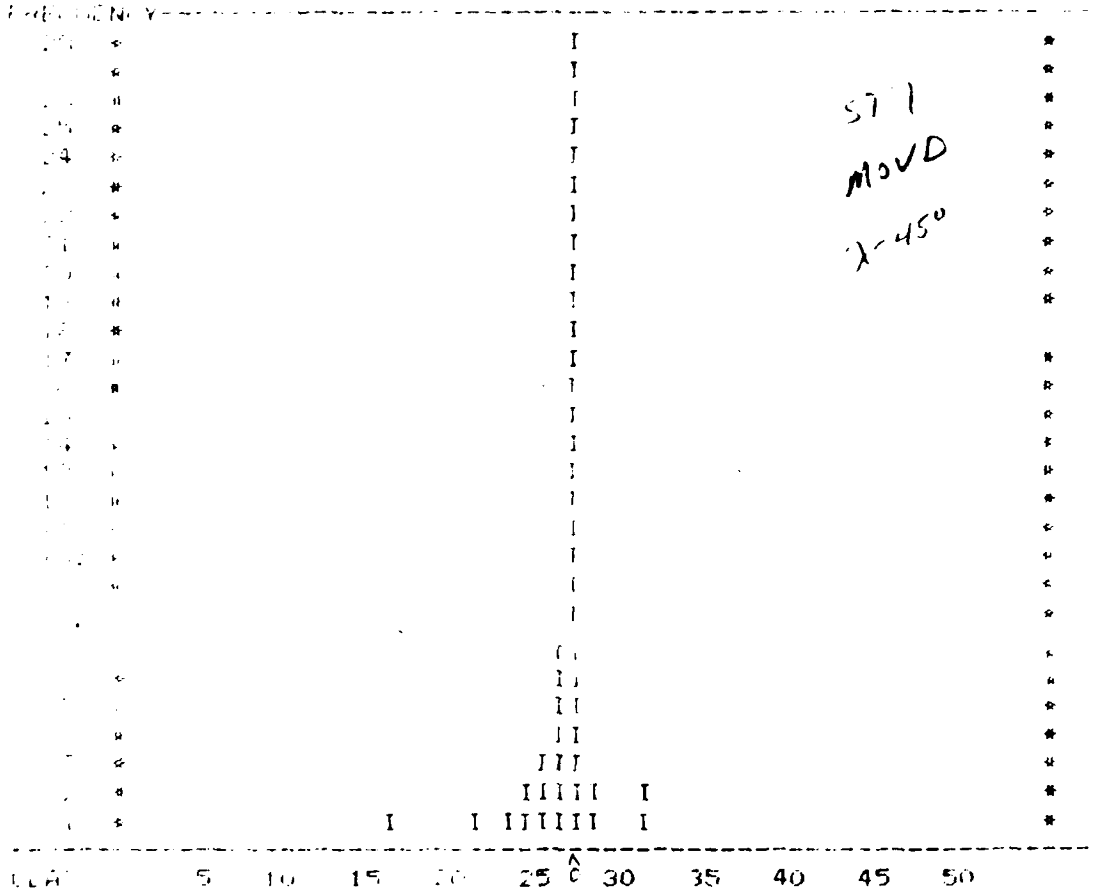


MEAN = .07457221271823

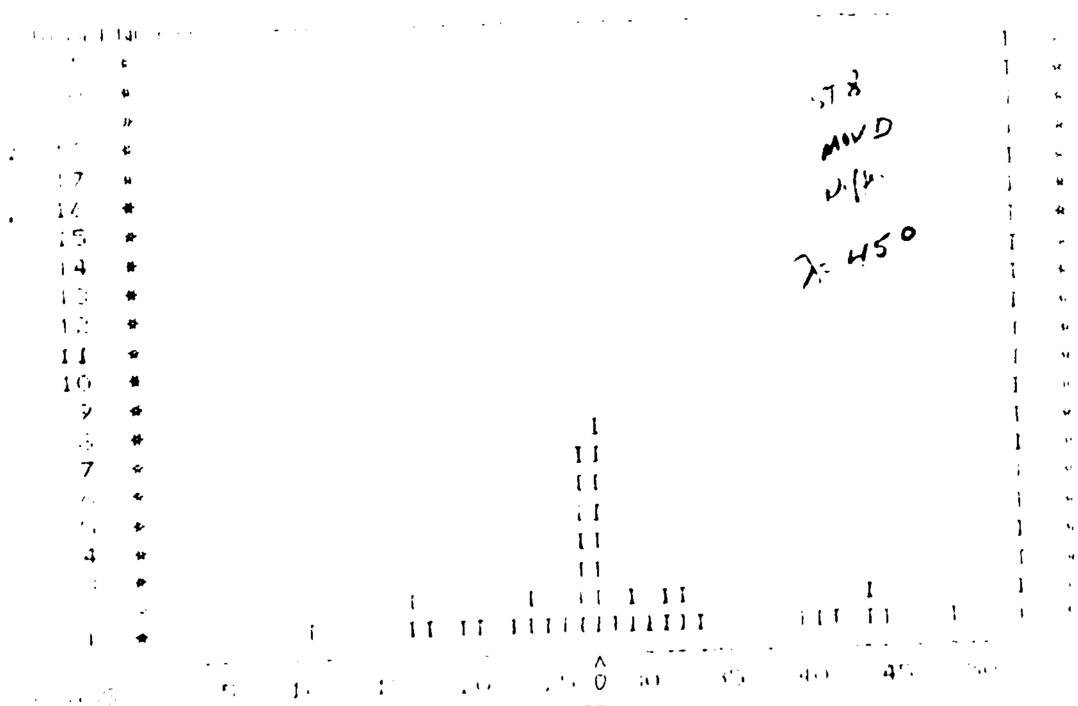
95 % Confidence Interval for μ_d

(-.1443, .2933)

VARIANCE = .5582554976329

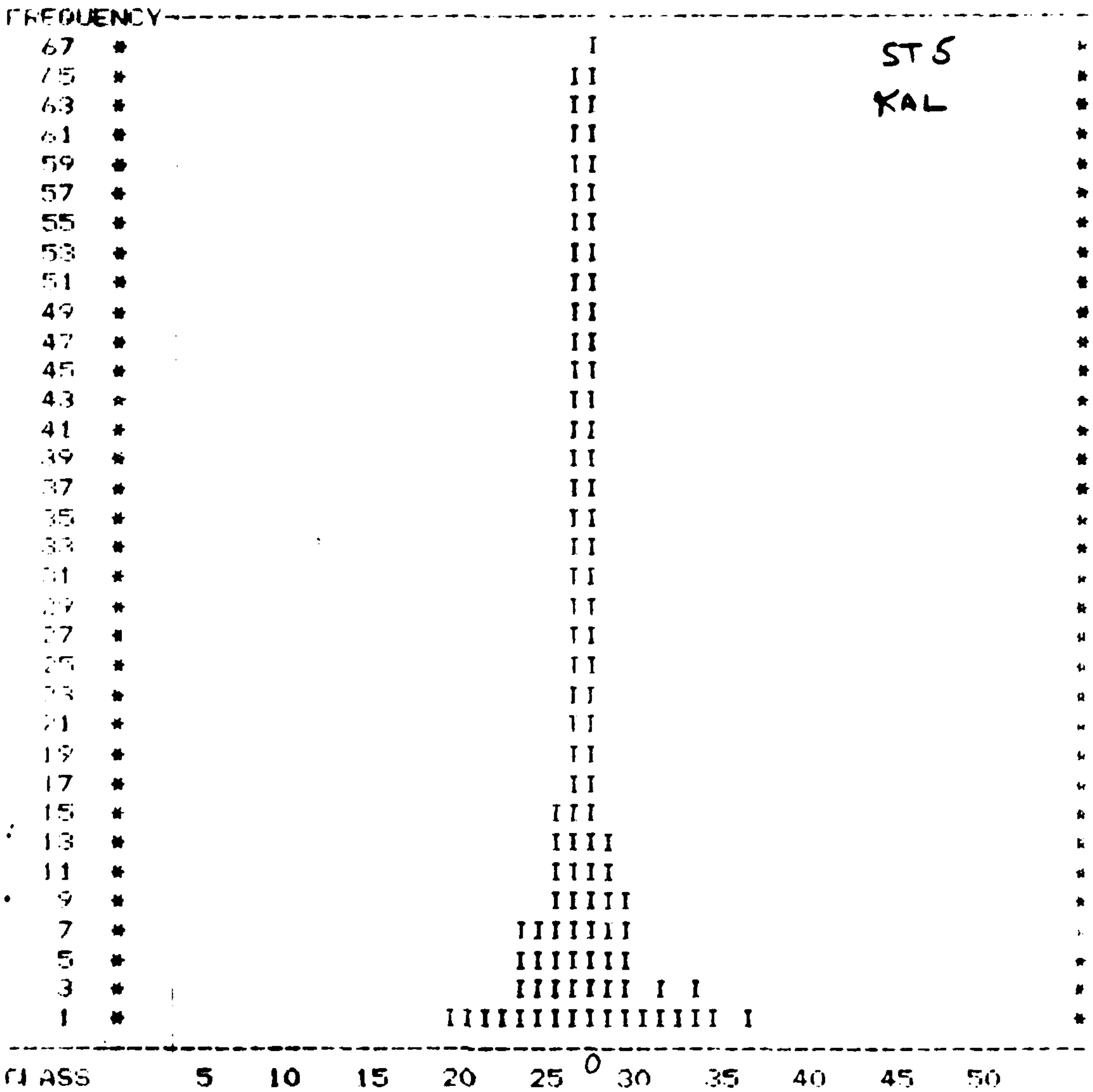


MEAN = -.02508341044056
VARIANCE = .04456227941227



THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

N: 109



MEAN = .0157202997576

N: 164

VARIANCE = .05188707045175

$\bar{X} = .0198$ } 43

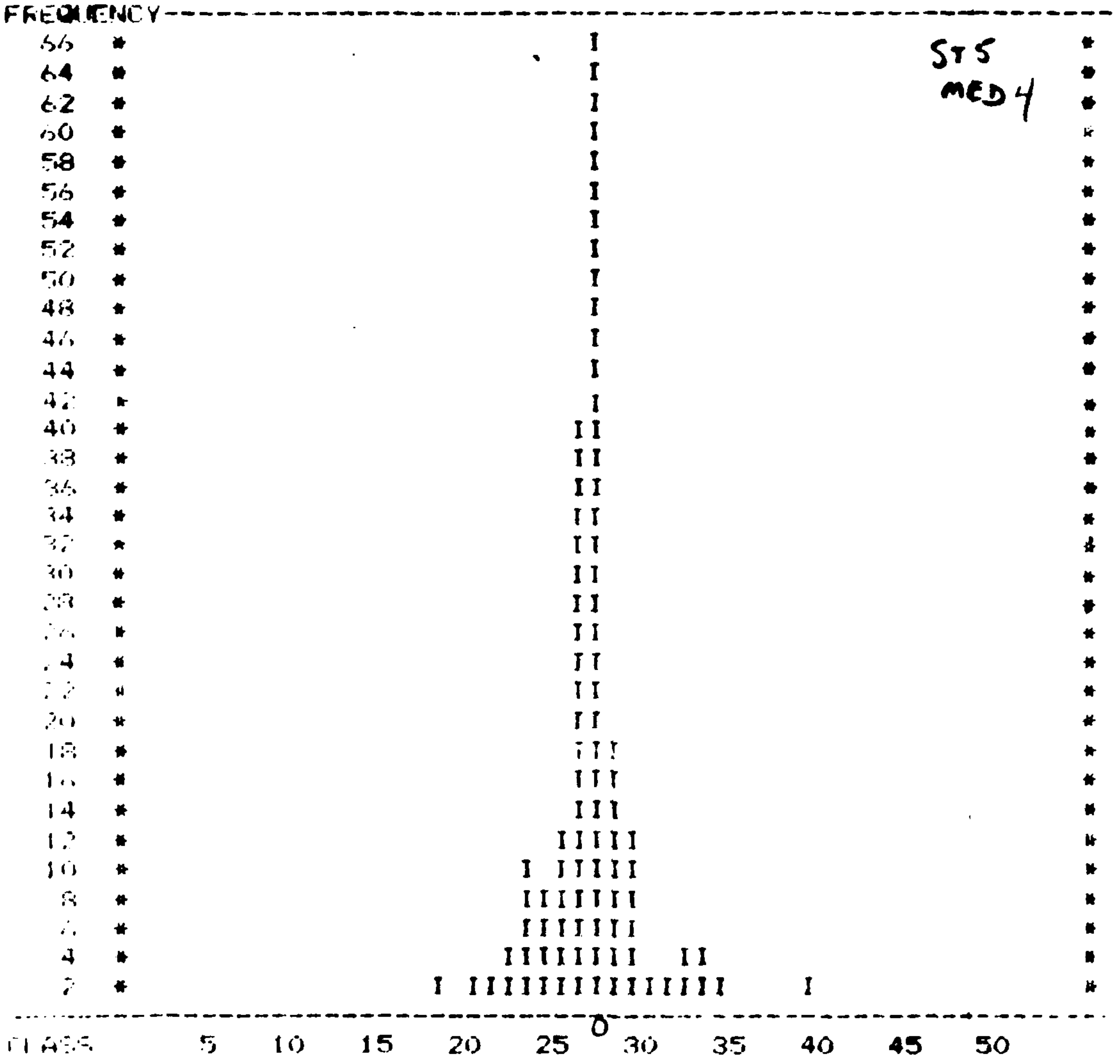
$\sigma^2 = .065$ } ZEROES DELETED

95% Confidence Interval for μ_a

(-.016, .046)

N: 207

STRAT CLASS 5



MEAN = .04915071287663 ; N: 164 $\bar{x}$: -.0183

VARIANCE = .176623729334 σ^2: .0774

95% Confidence Interval for μ_x

(-.0032, .1063)

N· 207

STRAT CLASS 5

```
FREQUENCY---------------------------------------------------------------
   69  *                         I                                  *   STS
   67  *                         I                                  *   TRIG
   65  *                         I                                  *
   63  *                         II                                 *
   61  *                         II                                 *
   59  *                         II                                 *
   57  *                         II                                 *
   55  *                         II                                 *
   53  *                         II                                 *
   51  *                         II                                 *
   49  *                         II                                 *
   47  *                         II                                 *
   45  *                         II                                 *
   43  *                         II                                 *
   41  *                         II                                 *
   39  *                         II                                 *
   37  *                         II                                 *
   35  *                         II                                 *
   33  *                         II                                 *
   31  *                         II                                 *
   29  *                         II                                 *
   27  *                         II                                 *
   25  *                        III                                 *
   23  *                        III                                 *
   21  *                        III                                 *
   19  *                        III                                 *
   17  *                        III                                 *
   15  *                        III                                 *
   13  *                        III                                 *
   11  *                        IIII                                *
    9  *                        IIII                                *
    7  *                      I IIII                                *
    5  *                      IIIIII                                *
    3  *                    I IIIIII I II                           *
    1  *                II IIIIIIIIIIIIIII  II                      *
------------------------------------------------------------------------
                                    ^
CLASS        5    10   15   20   25   30   35   40   45   50
   MEAN =        -.01449771511[illegible]
   VARIANCE =     .0[illegible]1[illegible]3208020638
```

: N=147

$\bar{Y}$: .062

σ^2 : .221

} 47 ZEROES DELETED

95% Confidence Interval for μ_a

(-.0482, .0194)

```
FREQUENCY-------------------------------------------------------
  72 *                          I
  70 *                          I
  68 *                          I
  66 *                          I
  64 *                          I
  62 *                          I
  60 *                          I
  58 *                         II
  56 *                         II
  54 *                         II
  52 *                         II
  50 *                         II
  48 *                         II
  46 *                         II
  44 *                         II
  42 *                         II
  40 *                         II
  38 *                         II
  36 *                         II
  34 *                         II
  32 *                         II
  30 *                         II
  28 *                         II
  26 *                         II
  24 *                         II
  22 *                        III
  20 *                        III
  18 *                        IIII
  16 *                        IIII
  14 *                        IIII
  12 *                        IIII
  10 *                        IIII
   8 *                        IIII
   6 *                     IIIIIII
   4 *                     IIIIIIIII I
   2 *                 I IIIIIIIIIIIIII
----------------------------------------------------------------
                                ^
CLASS     5   10   15   20   25 0 30   35   40   45   50
  MEAN =     .007430825590905
  VARIANCE =    .1128929523846
```

MOV D
D
STS

REFERENCES

1 Box & Jenkins, Time Series Analysis: Forecasting & Control, Holden-Day, 1970.

2 Cohen, Martin, "Alpha 4140.39 Simulator," AMC Inventory Research Office, May 1973, AD762348.

3 Cohen, Martin, "Demand Forecasting With Program Factors," AMC Inventory Research Office, September 1975, ADA017858.

3a Efron, B. & Morris, C., "Limiting the Risk of Bayes & Empirical Bayes Estimators, Part II," Project Rand Report R-647-PR, June 1971.

4 Gotwals, E. P., "Integrated Forecasting Techniques for Secondary Items Classes - Part II, Inactive Items," US Army Inventory Research Office, to be published Sep 80.

5 Gotwals, E. P., "Evaluation of Forecast Methods in an Inventory Management System (An Empirical Study)," US Army Inventory Research Office, to be published Nov 80.

6 Gotwals, E. P., "A Statistical Approach to Evaluating Results From the ALPHA 4140.39 Simulator," US Army Inventory Research Office, to be published.

7 Meng, Daniel, "Application of Dynamic Mean to Demand Forecast," System Analysis Division, Tank Automotive Materiel Readiness Command, September 1977, Report No. SA-77-11.

8 Orr, Donald, "Demand Forecasts Using Process Models and Item Class Parameters: Application of Ancillary Variables," DRC Inventory Research Office, April 1976, ADA026081.

9 Orr, Donald, "Kalman and Moving Average Filters for Forecasting: Systematics of Demand Processes and Extension," DRC Inventory Research Office, October 1976, ADA032476.

DISTRIBUTION

COPIES	
1	Deputy Under Sec'y of the Army, ATTN: Office of Op Resch
	Headquarters, US Army Materiel Development & Readiness Command
1	DRCPA-S
1	DRCDMR
1	DRCPS-P, ATTN: Ms. Lamb
1	DRCMM
1	DRCMM-R
1	DRCMM-RS
1	DRCMM-M
1	Dep Chf of Staff for Logistics, ATTN: DALO-SMS, Pentagon, Wash., DC 20310
1	Dep Chf of Staff for Logistics, ATTN: DALO-SML, Pentagon, Wash., DC 20310
1	Commandant, US Army Logistics Mgt Center, Ft. Lee, VA 23801
1	Office, Asst Sec'y of Defense, ATTN: MRA&L-SR, Pentagon, Wash., DC 20310
2	Defense Logistics Studies Info Exchange, DRXMC-D
10	Defense Technical Information Center, Cameron Station, Alexandria, VA 22314
	Commander, USA Armament Materiel Readiness Cmd, Rock Island, IL 61201
1	ATTN: DRSAR-MM
1	ATTN: DRSAR-SA
	Commander, USA Communications & Electronics Materiel Readiness Cmd, Ft. Monmouth, NJ 07703
1	ATTN: DRSEL-MM
1	ATTN: DRSEL-SA
	Commander, USA Missile Command, Redstone Arsenal, AL 35898
1	ATTN: DRSMI-S
1	ATTN: DRSMI-D
	Commander, USA Troop Support & Aviation Materiel Readiness Command, St. Louis, MO 63120
1	ATTN: DRSTS-SP
1	ATTN: DRSTS-SPSS
1	ATTN: DRSTS-BA(1)
	Commander, US Army Tank-Automotive Command, Warren, MI 48090
1	ATTN: DRSTA-F
1	ATTN: DRSTA-S
1	Commander, US Army Armament Research & Development Cmd, ATTN: DRDAR-SE, Dover, NJ 07801
1	Commander, US Army Aviation Research & Development Cmd, 4300 Goodfellow Blvd, St. Louis, MO 63120
1	Commander, US Army Aviation Research & Development Cmd, ATTN: DRDAV-BD (Marl Glenn), 4300 Goodfellow Blvd, St. Louis, MO 63120
1	Commander, US Army Communications Research & Development Command, ATTN: DRSEL-SA, Ft. Monmouth, NJ 07703
1	Commander, US Army Electronics Research & Development Command, ATTN: DRDEL-AP, Adelphi, MD 20783
1	Commander, US Army Mobility Equipment Research & Development Cmd, ATTN: DRDME-O, Ft. Belvoir, VA 22060

COPIES	
1	Commander, US Army Natick Research & Development Command, ATTN: DRXNM-O, Natick, MA 01760
1	Commander, US Army Logistics Center, Ft. Lee, VA 23801
1	Commander, US Army Logistics Evaluation Agency, New Cumberland Army Depot, New Cumberland, PA 17070
1	Commander, US Air Force Logistics Cmd, WPAFB, ATTN: AFLC/XRS, Dayton, Ohio 45433
1	US Navy Fleet Materiel Support Office, Naval Support Depot, Mechanicsburg, PA 17055
1	Mr. James Prichard, Navy SEA Systems Cmd, ATTN: PMS 3061, Dept of US Navy, Wash., DC 20362
1	George Washington University, Inst of Management Science & Engr., 707 22nd St., N.W., Wash., DC 20006
1	Naval Postgraduate School, ATTN: Dept of Opns Anal, Monterey, CA 93940
1	Air Force Institute of Technology, ATTN: SLGQ, Head Quantitative Studies Dept., Dayton, OH 43433
1	US Army Military Academy, West Point, NY 10996
1	Librarian, Logistics Mgt Inst., 4701 Sangamore Rd., Wash.,DC 20016
1	University of Florida, ATTN: Dept of Industrial Systems Engr., Gainesville, FL 32601
1	RAND Corp., ATTN: S. M. Drezner, 1700 Main St., Santa Monica, CA 90406
1	US Army Materiel Systems Analysis Activity, ATTN: DRXSY-CL, Aberdeen Proving Ground, MD 21005
1	Commander, US Army Logistics Center, ATTN: Concepts & Doctrine Directorate, Ft. Lee, VA 23801
1	ALOG Magazine, ATTN: Tom Johnson, USALMC, Ft. Lee, VA 2380
1	Commander, USDRC Automated Logistics Mgt Systems Activity, P.O. Box 1578, St. Louis, MO 63188
1	Director, DARCOM Logistics Systems Support Agency, Letterkenny Army Depot, Chambersburg, PA 17201
1	Commander, Materiel Readiness Support Activity, Lexington, KY 40507
1	Director, Army Management Engineering Training Agency, Rock Island Arsenal, Rock Island, IL 61202
1	Defense Logistics Agcy, ATTN: DLA-LO, Cameron Sta, Alexandria, VA 22314
1	Dep Chf of Staff (I&L), HQ USMC-LMP-2, ATTN: MAJ Sonneborn, Jr., Wash., DC 20380
1	Logistics Control Activity, Presidio of San Francisco, CA 94120
1	HQ, Dept of the Army, (DASG-HCL-P), Wash., DC 20314
1	Operations Research Center, 3115 Etcheverry Hall, University of California, Berkeley, CA 94720
1	Dr. Jack Muckstadt, Dept of Industrial Engineering & Operations Research, Upson Hall, Cornell University, Ithaca, NY 14890
1	Prof Herbert P. Galliher, Dept of Industrial Engineering, University of Michigan, Ann Arbor, MI 48104
1	Scientific Advisor, ATCL-SCA, Army Logistics Center, Ft. Lee, VA 23801
1	Commandant, USA Armor School, ATTN: MAJ Harold E. Burch, Leadership Dept, Ft. Knox, KY 40121

COPIES

1	Prof Robert M. Stark, Dept of Stat & Computer Sciences, University of Delaware, Newark, DE 19711
1	Prof E. Gerald Hurst, Jr., Dept of Decision Science, The Wharton School, University of Penna., Phila., PA 19174
1	Logistics Studies Office, DRXMC-LSO, ALMC, Ft. Lee, VA 23801
1	Procurement Research Office, DRXMC-PRO, ALMC, Ft. Lee, VA 23801
1	Dept of Industrial Engr. & Engr. Management, Stanford University, Stanford, CA 94305
1	Commander, US Army Communications Command, ATTN: Dr. Forry, CC-LOG-LEO, Ft. Huachuca, AZ 85613
1	Commander, US Army Test & Evaluation Cmd, ATTN: DRSTE-SY, Aberdeen Proving Ground, MD 21005
1	Prof Harvey M. Wagner, Dean, School of Business Adm, University of North Carolina, Chapel Hill, NC 27514
1	DARCOM Intern Training Center, Red River Army Depot, Texarkana, Tx 75501
1	Prof Leroy B. Schwarz, Dept of Management, Purdue University, Krannert Bldg, West Lafayette, Indiana 47907
1	US Army Training & Doctrine Command, Ft. Monroe, VA 23651
1	Operations & Inventory Analysis Office, NAVSUP (Code 04A) Dept of Navy, Wash., DC 20376
1	US Army Research Office, ATTN: Robert Launer, Math. Div., P.O. Box 12211, Research Triangle Park, NC 27709
1	Prof William P. Pierskalla, 3641 Locust Walk CE, Philadelphia, PA 19104
1	US Army Materiel Systems Analysis Activity, ATTN: DRXSY-MP, Aberdeen Proving Ground, MD 21005
	Air Force Logistics Management Center, Gunter Air Force Station, AL 36144
1	ATTN: AFLMC/LGY
1	ATTN: AFLMC/XRP, Bldg. 205
1	Engineer Studies Center, 6500 Brooks Lane, Wash., DC 20315
1	US Army Materiel Systems Analysis Activity, ATTN: Mr. Herbert Cohen, DRXSY-MP, Aberdeen Proving Ground, MD 21105
1	Commander, US Army Missile Cmd, ATTN: Ray Dotson, DRSMI-DS, Redstone Arsenal, AL 35898
1	Prof Peter Kubat, Graduate School of Mgmt, University of Rochester, Rochester, NY 14627