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A STUDY OF TERRAIN SCATTERING PHYSICS

Applied Science Associates, Inc.

Gary S. Brown

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This report presents results of a study to improve existing models for scattering from rough terrain. The two-scale composite surface model is first corrected to properly account for shadowing. This correction shows that both the optical and Bragg scattering terms in the expression for σ° go to zero as grazing incidence is approached. The shadowing function for non-Gaussian surfaces is shown to be easily obtained from existing theories and numerical results for exponentially distributed surfaces are presented. → next page

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cont.

The two-scale composite surface based upon the Burrows perturbation theory is extended to the following cases: bistatic scattering from a dielectric surface with small scale roughness, and bistatic scattering from a dielectric surface with large and small scales of roughness. Comparisons are made with the Rice theory for both cases. It is shown how an existing theoretical relationship between the height and slope probability density functions can be used to practically obtain one from the other. Finally, a new integral equation is developed for the coherent field scattered from a perfectly conducting rough surface. Asymptotic evaluation of this equation in the limit of large Rayleigh parameter indicates that surface slopes cannot be ignored.

Evaluation

This contract was concerned with developing improved analytical models to describe the scattering of electromagnetic waves from rough surfaces.

A perturbation technique was used to develop a two-scale composite model for describing electromagnetic wave scattering from rough surfaces. The bistatic scattering cross section of lossy dielectric rough surfaces was derived. Corrections to the shadowing functions for non-Gaussian surfaces were derived in general. Explicit expressions were derived for exponentially distributed surface heights in the case of backscattering. An exact solution for the coherent wave scattered from a rough surface was obtained in the form of an integral equation. The exact solution for the coherent wave includes the effects of diffraction, which are important at low grazing angles. The objective of this contract has been successfully met.

Robert J. Papa

ROBERT J. PAPA
Project Engineer

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TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd.)

1.0 INTRODUCTION

As radar systems designs become more complex and versatile, their performance becomes increasingly sensitive to the operational environment. This places an increased burden on the designer to incorporate the effects of the environment in system design studies. However, before this can be done, it is necessary to develop an accurate model of the environment. In the case of ground clutter or multipath, this means that there is a need for rough surface scattering models. Such models must not only be based upon sound physical principles but also exhibit agreement with measurements.

The purpose of this study is to provide improved models for surface scattering from terrain in the microwave frequency range and near grazing incidence. The basic approach entails applying the composite surface scattering theory to a lossy, rough dielectric surface. As long as the incident angle is not too near grazing and the surface is reasonably free of sharp edges or cusps, the composite model should be a reasonable description of the scattering process. Since the composite model is based upon the combining of two asymptotic scattering theories, it is approximate. Thus, an additional goal of this study is to investigate new techniques for improving the composite model. As a first step toward obtaining improvements to the composite model, a rigorous new formulation of the problem of coherent scattering from a rough surface is developed. The intent of this work is an attempt to gain a better understanding of the interplay between the statistical surface parameters and the scattered field. Such understanding is an absolutely essential prerequisite to modeling more complex factors such as vegetation and snow cover.

1.1 Summary of Results

Section 2 corrects an error in the composite surface scattering model. In particular, it is shown that shadowing is improperly accounted for and this error is corrected. In the corrected version of the composite model, it is shown that the conventional shadowing function multiplies both the zeroth and first order incoherent scattered power perturbation terms. Thus, even for a perfectly conducting surface the first order term will go to zero for backscattering at grazing incidence due to the shadowing function. The first order perturbation power suffers an additional attenuation due to the shadowing of unfavorably oriented large scale surface slopes; however, this effect is relatively small compared to the impact of the conventional shadowing function.

Section 3 demonstrates how the shadowing function for non-Gaussian surfaces may be easily obtained from existing shadowing theories. The important surface characteristic in the general case is the probability density function of the large scale slopes in the plane of incidence. Explicit results are obtained for a surface characterized by a roughness whose probability density function is exponential. The results of this study are particularly important for terrain scattering because terrain height cannot always be described by a Gaussian probability density function.

Section 4 extends the composite model to bistatic scattering from a lossy, dielectric, rough surface. The details are presented for three cases of increasing complexity; backscattering from a surface with only small scale roughness, bistatic scattering from a surface with only small scale roughness, and, finally, bistatic scattering from a composite (large and small scales of roughness) surface. This approach is a logical progression from the simple to the complex and is therefore beneficial to the reader. Furthermore, this approach facilitates checking the results of the perturbation theory against existing solutions.

Section 5 discusses one technique for relating the joint probability density function of the surface heights to the joint density function for the slopes. The technique was originally obtained from an analysis of optical scattering from a rough surface but its relevance to this problem has apparently been overlooked. Although the technique is not always applicable to measured data, there are cases where it can provide the desired transformation.

Section 6 develops a new approach to the problem of coherent scattering from a perfectly conducting rough surface based upon the magnetic field integral equation. In contrast with the classical multiple scattering formalism which leads to an infinite number of integral equations, this approach results in a single integral equation of infinite dimension. The infinite dimensionality is a consequence of retaining all orders of surface height derivatives in the averaging process. The major benefit of this approach is that it is possible to put the mathematical operation of truncating the dimensionality of the integral equation into one-to-one correspondence with the neglect of higher order surface height derivatives. Comparisons with the multiple scattering approach results show very good agreement in domains where both theories are valid. This approach also shows that in order to neglect surface slopes the product of the Rayleigh roughness parameter and the rms surface slope must be much less than unity; that is it is not sufficient to simply require the mean square slope to be small.

2.0 A CORRECTION TO THE COMPOSITE SURFACE SCATTERING MODEL

2.1 Background

In [1], a solution to the problem of backscattering from a randomly rough, perfectly conducting surface comprising both large and small scales of roughness was presented. As a direct consequence of the stipulation that the surface roughness was a zero mean jointly Gaussian process, the scattering cross section per unit area was determined to be the sum of two terms, i.e. $\sigma_{pp'}^{o}(\theta,\phi) = [\sigma_{pp'}^{o}(\theta,\phi)]_0 + [\sigma_{pp'}^{o}(\theta,\phi)]_1$. The $[\sigma_{pp'}^{o}(\theta,\phi)]_0$ contribution is dominant near normal incidence and results from the shadow corrected optical like reflection from properly oriented facets or specular points on the surface. The $[\sigma_{pp'}^{o}(\theta,\phi)]_1$ term is due to Bragg resonance scattering from the small scale surface features with appropriate accounting for the resonance broadening effects of the large scale surface undulations. Although shadowing is formally accounted for in a correct manner in the $[\sigma_{pp'}^{o}(\theta,\phi)]_1$ term, there is an error in the exact representation of the shadowing function which leads to an incorrect estimate of the effects of shadowing on large angle of incidence scattering. The goal of this section is to correct the above error and to properly account for the effect of large scale shadowing on the small scale scattering term.

2.2 Discussion of the Error

The analysis presented in [1] relating to the determination of $[\sigma_{pp'}(\theta,\phi)]_1$ is correct up to and including equation (23). The problem with the analysis following (23) is a result of inadequate attention to the definition of the shadowing function $R(\theta,\phi)$. That is, $R(\theta,\phi)$, as it appears in (23) of [1], can and should be expressed as follows;

$$R(\theta,\phi) = \overline{P^{(o)}}\left(\hat{k}_i \mid \zeta_{\ell x} = \frac{k_{ox}}{B} , \zeta_{\ell y} = \frac{k_{oy}}{B}\right) \tag{2.1}$$

where

$$\overline{P^{(o)}}\left(\hat{k}_i \,\middle|\, \zeta_{\ell x}, \zeta_{\ell y}\right) = \left\langle P^{(o)}\left(\hat{k}_i \,\middle|\, \zeta_\ell, \zeta_{\ell x}, \zeta_{\ell y}\right)\right\rangle_{\zeta_\ell} \tag{2.2}$$

is the probability that an incident ray having direction $\hat{k}_i$ will intersect a point on the large scale surface with orthogonal slopes $\zeta_{\ell x}$ and $\zeta_{\ell y}$ and will not be shadowed by any other part of the surface regardless of the height ζ_ℓ of the point in question. The symbol $\langle\cdot\rangle_{\zeta_\ell}$ in (2.2) denotes the ensemble average over all values of large scale height ζ_ℓ and the notation $P^{(o)}(\cdot)$ is the same as that employed by Sancer [2] in his excellent analysis of the effect of shadowing on $[\sigma^o_{pp'}(\theta,\phi)]_0$. It should be noted that as a result of the integrations in (23) of [1], the shadowing function $R(\theta,\phi)$ is equal to $\overline{P^{(o)}}(\cdot)$ evaluated at the specific large scale slope values given by $\zeta_{\ell x} = k_{ox}/B$ and $\zeta_{\ell y} = k_{oy}/B$ where $k_{ox} = -2k_o \sin\theta\cos\phi$, $k_{oy} = -2k_o\sin\theta\sin\phi$, $B = 2k_o\cos\theta$, θ is the angle of incidence relative to the normal to the mean $(\zeta = 0)$ plane, and ϕ is the azimuth direction of incidence.

In order to more clearly understand and therefore rectify the error in [1], it is beneficial to repeat equation (23) of [1], i.e.

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle I(x_1,y_1)I(x_2,y_2)f_{pp'}\left(\zeta_{\ell x_1},\zeta_{\ell x_2},\zeta_{\ell y_1},\zeta_{\ell y_2}\right)\exp\left[jB\left(\zeta_{\ell_2}-\zeta_{\ell_1}\right)\right]\right\rangle$$

$$\cdot \exp(-jk_{ox}\Delta x - jk_{oy}\Delta y)\, d\Delta x\, d\Delta y \approx R(\theta,\phi) f_{pp'}\left(\frac{k_{ox}}{B}, \frac{k_{ox}}{B}, \frac{k_{oy}}{B}, \frac{k_{oy}}{B}\right)$$

$$\cdot \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{-j(k_{ox}\Delta x + k_{oy}\Delta y) - 4k_o^2\cos^2\theta\,\overline{\zeta_\ell^2}\left[1-\rho_\ell(\Delta x,\Delta y)\right]\right\} d\Delta x d\Delta y \tag{2.3}$$

where $\Delta x = x_2 - x_1$ and $\Delta_y = y_2 - y_1$. In (2.3) $I\left(x_{\frac{1}{2}}, y_{\frac{1}{2}}\right)$ is one if the point $\left(x_{\frac{1}{2}}, y_{\frac{1}{2}}, \zeta_{\ell_{\frac{1}{2}}}\right)$ on the large scale surface having slopes $\zeta_{\ell x_{\frac{1}{2}}}$ and $\zeta_{\ell y_{\frac{1}{2}}}$ is illuminated and zero if it is shadowed. The function $f_{pp'}$ is defined in [1], $\overline{\zeta_\ell^2}$ is the mean square height of the large scale surface, and $\rho_\ell(\cdot)$ is the normalized autocorrelation function of the large scale surface height. It should be noted that the left hand side of (2.3) represents the Fourier transform of the $<\cdot>$ term (from $\Delta x \Delta y$ - space to $k_{ox}k_{oy}$ - space) and (2.3) is supposed to show where the transform variables appear in the result. Such knowledge is essential to accomplishing the convolution of (2.3) with the small scale surface height spectrum because this convolution determines $[\sigma^o_{pp'}]_1$. That is, in the convolution k_{ox} and k_{oy} must be replaced by $k_{ox} - k_x$ and $k_{oy} - k_y$ where k_x and k_y are the new variables of integration (see (32) of [1]).

If $R(\theta,\phi)$ in (2.3) had been replaced by its precise definition, as given by (1), the problem of determining where k_{ox} and k_{oy} appear in the right hand side of (2.3) would have been correctly solved. Unfortunately, this was not done. Instead, $R(\theta,\phi)$ was incorrectly written as $(1 + C_o)^{-1}$ and, through the use of trigonometric manipulations, C_o was expressed in terms of k_{ox} and k_{oy} (see equations (24) through (28) in [1]). This development failed to recognize that C_o does not, in general, depend upon the transform variables in (2.3). That is, if one changes the transform variables from k_{ox} to k_x and from k_{oy} to k_y, C_o would still only depend on $k_{ox} = -2k_o \sin\theta\cos\phi$ and $k_{oy} = -2k_o \sin\theta\sin\phi$. The primary consequence of this error is to provide an incorrect formula for $R(\cdot)$ for use in all equations following (28) in [1]. As will be shown, this error fortunately has negligible consequence on the numerical results presented in [1].

2.3 Correct Analysis

The error identified and explained above can be rectified by determining the functional dependence of the right hand side of (2.1) upon the large scale slopes $\zeta_{\ell x}$ and $\zeta_{\ell y}$ since they are replaced by the transform variables k_{ox} and k_{oy}. This can be done in a relatively straightforward manner by generalizing Smith's [3] results to the case where $\hat{k}_i$ is at an angle $\pi/2 - \phi$ with respect to the y-axis rather than directly along the y-axis. Such a generalization leads to the following;

$$\overline{p^{(o)}}(\hat{k}_i \mid \zeta_{\ell x}, \zeta_{\ell y}) = \frac{U(\operatorname{ctn}\theta - \zeta_{\ell x}\cos\phi - \zeta_{\ell y}\sin\phi)}{1 + C_o} \tag{2.4}$$

where C_o is given by (24) of [1] and $U(\cdot)$ is the unit step function which is one if the argument is positive and zero if the argument is negative. Of particular note in (2.4) is the fact that $\zeta_{\ell x}$ and $\zeta_{\ell y}$ appear only in the argument of the unit step function. Substituting (2.4) into (2.1) yields the correct expression for $R(\theta,\phi)$, i.e.

$$R(\theta,\phi) = \frac{U\left(\operatorname{ctn}\theta - \frac{k_{ox}}{B}\cos\phi - \frac{k_{oy}}{B}\sin\phi\right)}{1 + C_o} \tag{2.5}$$

If one substitutes $k_{ox} = -2k_o \sin\theta\cos\phi$ and $k_{oy} = -2k_o\sin\theta\sin\phi$ in (2.5) such as required in the determination of $[\sigma^o_{pp'}]_0$, then $R(\theta,\phi) = (1 + C_o)^{-1}$ and it will depend only on the angles θ and ϕ and the mean square slopes $\overline{\zeta^2_{\ell x}}$ and $\overline{\zeta^2_{\ell y}}$ of the large scale surface (see equations (24) and (25) of [1]). However, in the convolution expression for $[\sigma^o_{pp'}]_1$, the unit step function must be retained. That is, one must use the following relationship;

$$R\left(\frac{k_{ox}-k_x}{2k_o\cos\theta}, \frac{k_{oy}-k_y}{2k_o\cos\theta}\right) = \frac{U\left(\operatorname{ctn}\theta - \left[\frac{k_{ox}-k_x}{2k_o\cos\theta}\right]\cos\phi - \left[\frac{k_{oy}-k_y}{2k_o\cos\theta}\right]\sin\phi\right)}{1 + C_o} \tag{2.6}$$

Equation (2.6) leads to a completely different interpretation of the effects of large scale shadowing on $[\sigma^o_{pp'}]_1$ from that erroneously presented in [1]. According to (2.6), shadowing now gives rise to an attenuation factor $(1+C_o)^{-1}$ which is common to both $[\sigma^o_{pp'}]_0$ and $[\sigma^o_{pp'}]_1$. This result is merely a consequence of the fact that small areas on the surface capable of producing strong Bragg scatter, i.e. $\zeta_{\ell x} - \zeta_{\ell y} = 0$, are as equally shadowed as the areas properly oriented for specular reflection, i.e. $\zeta_{\ell x} = k_{ox}/B$ and $\zeta_{\ell y} = k_{oy}/B$. This statement can be easily verified by noting that for both sets of the above values of $\zeta_{\ell x}$ and $\zeta_{\ell y}$, $U(\mathrm{ctn}\,\theta - \zeta_{\ell x}\cos\phi - \zeta_{\ell y}\sin\phi) = 1$ in equation (2.4) and so $R(\theta,\phi) = (1+C_o)^{-1}$. Since both the $[\sigma^o_{pp'}]_0$ and $[\sigma^o_{pp'}]_1$ terms suffer the same attenuation due to shadowing, the transition region in θ (where $[\sigma^o_{pp'}]_0$ decreases and $[\sigma^o_{pp'}]_1$ becomes predominant) is independent of shadowing effects. Since shadowing results from the slopes of the large scale surface structure and since the large scale slopes are not the important surface characteristic in determining the transition region, this result demonstrates that the theory is self consistent.

The unit step function in (2.6) serves the very important purpose of establishing the limits on the integrals in the convolutional expression for $[\sigma^o_{pp'}]_1$ (see (32) of [1]). However, before this aspect of the problem is considered, it is worthwhile reviewing the physics behind the reason for the unit step function in (2.6). The unit step function appears in (2.6) because there is a certain range of surface slope values for which the probability of having an incident ray shadowed is identically one [3]. This result may be readily understood by referring to Figure 2.1. In (a), the slope of the surface is negative at the point of intersection and the incident ray is not shadowed in a small neighborhood of the point. In (b), the surface slope is equal to the slope of the incident ray (ctnθ) and the ray is therefore tangent to the surface

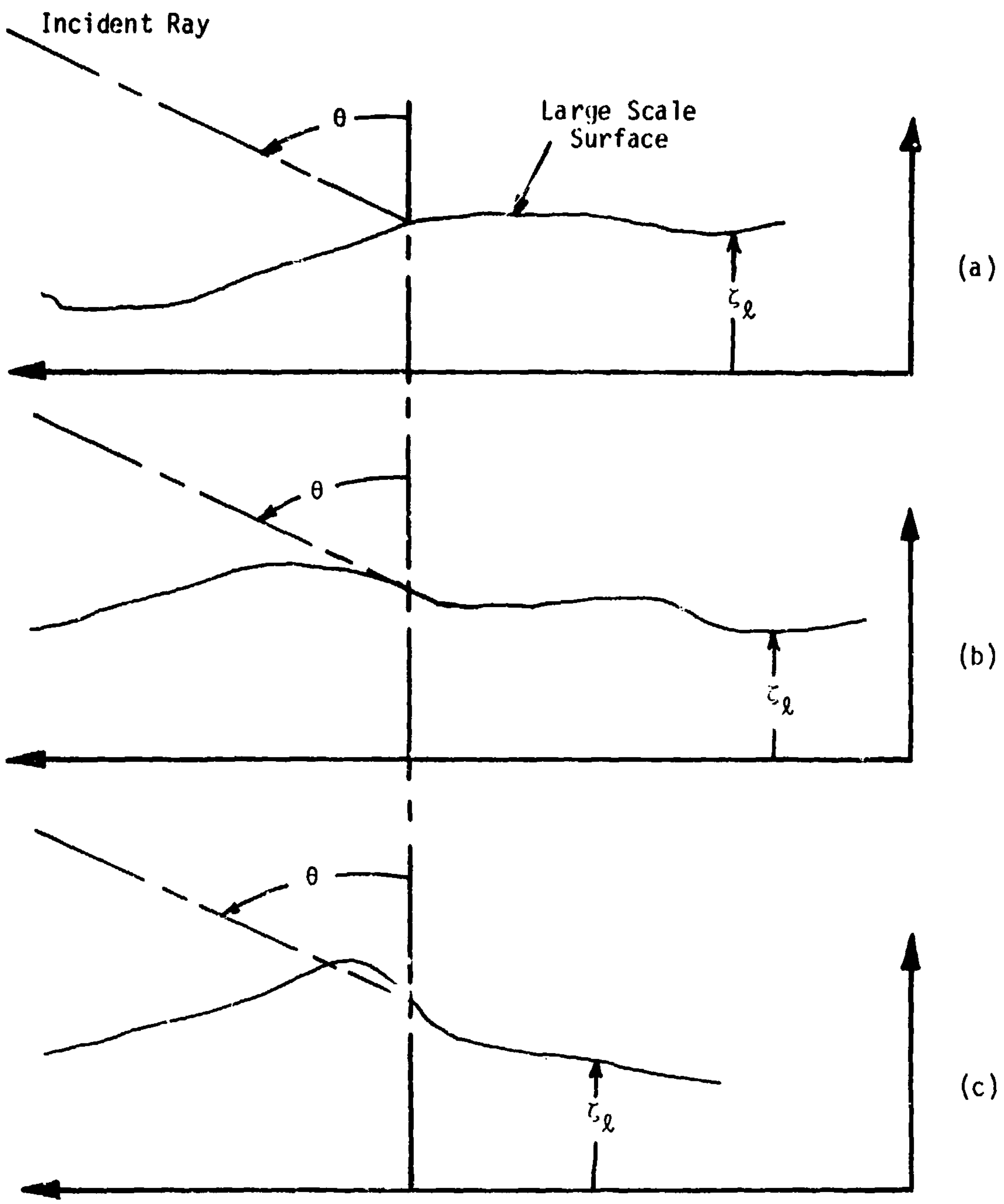

Figure 2-1. Diagrams explaining the reason for the unit step function in the shadowing function. Only portions of the large scale surface are shown since the small scale structure does not impact the shadowing.

at the point of intersection. In this case the point may or may not be shadowed in a small neighborhood of the point, depending upon the surface curvature. In (c), the surface slope exceeds the slope of the incident ray and the ray is necessarily shadowed by some portion of the surface in a small neighborhood of the point. Thus all points on the surface will be shadowed if their slope, in the direction of the incident ray projected onto mean plane, exceeds the slope of the incident ray. Stated another way, the probability of such an event is one. This is the physical reason for the unit step function in (2.4) and subsequent equations involving the shadowing function.

In order to retain the physical significance of the unit step function, it is desirable to deal with a particular form of the equation for $[\sigma^o_{pp'}(\theta,\phi)]_1$, i.e. equation (40) of [1],

$$\left[\sigma^o_{pp'}(\theta,\phi)\right]_1 = \frac{4k_o^4(1+C_o)^{-1}}{\pi\sqrt{\overline{\zeta^2_{\ell x}}\;\overline{\zeta^2_{\ell y}}}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(2k_o\cos\theta\,\xi_x + k_{ox}\,,\,2k_o\cos\theta\,\xi_y + k_{oy})$$

$$\cdot\; U(\mathrm{ctn}\theta + \xi_x\cos\phi + \xi_y\sin\phi)\;\Gamma^2_{pp'}(-\xi_x, -\xi_y)$$

$$\cdot\;\exp\left[-\frac{\xi_x^2}{2\overline{\zeta^2_{\ell x}}} - \frac{\xi_y^2}{2\overline{\zeta^2_{\ell y}}}\right] d\xi_x\, d\xi_y - I_{k_d} \qquad (2.7)$$

where I_{k_d} is defined in [1]. Equation (2.7) expresses the convolutional broadening of the spectrum about the Bragg wavenumbers k_{ox} and k_{oy} as a direct consequence of the distribution of large scale slopes; that is, ξ_x and ξ_y are equivalent to $\zeta_{\ell x}$ and $\zeta_{\ell y}$. It should be pointed out that the correct expression for the shadowing function has been used in (2.7). The curve in the $\xi_x\xi_y$-plane separating the regions where the step function is zero and one is a straight line, i.e. $\xi_x \cos\phi + \xi_y \sin\phi = -\operatorname{ctn}\theta$. Except for special values of ϕ given in Table I, the step function in (2.7) will consequently give rise to a coupling between the ξ_x and ξ_y integrals. More specifically, the lower (upper) limit on the ξ_x integral in (2.7) is given by

$$\xi_x = \frac{-(\operatorname{ctn}\theta + \xi_y \sin\phi)}{\cos\phi} \tag{2.8}$$

for $\cos\theta > 0$ (<0) . From an analytical point of view (2.8) represents an irritating consequence of shadowing. From a practical standpoint, the restrictions imposed by (2.8) may not be numerically relevant for a large range of incidence angles as demonstrated by the following argument. The dominant factor in the integrand in (2.7) is the slope dependent Gaussian term which is equivalent to the probability density function for the large scale slopes. For most practical purposes, the effect of this term is to truncate the range of integration in (2.7) to about a ± 3-sigma excursion from $\xi_x = \xi_y = 0$. The $\pm$ 3-sigma excursions for ξ_x and ξ_y are $\pm 3\sqrt{\overline{\zeta_{\ell x}^2}}$ and $\pm 3\sqrt{\overline{\zeta_{\ell y}^2}}$, respectively, and substituting these values in the unit step function argument yields the following requirement

$$\pm 3\sqrt{\overline{\zeta_{\ell x}^2}} \cos\phi \pm 3\sqrt{\overline{\zeta_{\ell y}^2}} \sin\phi > -\operatorname{ctn}\theta \tag{2.9}$$

for the unit step function to be unity. If $0 < \phi < \pi/2$, the "worst case" situation occurs when $\xi_x = -3\sqrt{\overline{\zeta_{\ell x}^2}}$ and $\xi_y = -3\sqrt{\overline{\zeta_{\ell y}^2}}$ or

$$3\sqrt{\overline{\zeta_{\ell x}^2}} \cos\phi + 3\sqrt{\overline{\zeta_{\ell y}^2}} \sin\phi < \operatorname{ctn}\theta \tag{2.10}$$

The worst case situation occurs when the left hand side of (2.9) is most negative. Thus for numerical purposes and all values of θ satisfying (2.10), one can replace the infinite limits in (2.7) by $\pm 3\sqrt{\overline{\zeta_{\ell x}^2}}$ and $\pm 3\sqrt{\overline{\zeta_{\ell y}^2}}$.

TABLE I

Integration Limits for Eqn. (2.7) and
ϕ a Multiple of $\pi/2$

ϕ	ξ_x integration limits		ξ_y integration limits	
	lower	upper	lower	upper
0	$-\text{ctn}\,\theta$	∞	$-\infty$	∞
$\pi/2$	$-\infty$	∞	$-\text{ctn}\,\theta$	∞
π	$-\infty$	$\text{ctn}\,\theta$	$-\infty$	∞
$3\pi/2$	$-\infty$	∞	$-\infty$	$\text{ctn}\,\theta$

Conditions similar to (2.10) can be obtained for other ranges of ϕ in a straightforward manner.

It is implicitly assumed that the upper limit on θ resulting from (2.10) also satisfies the optical criterion required of the large scale surface*, namely that $4k_o^2 \overline{\zeta_\ell^2} \cos^2\theta >> 1$. For moderate slopes there will generally be a gap between the maximum value of θ resulting from (2.10) and the upper bound on θ resulting from the large scale surface optical criterion. In this case, one must necessarily revert to the more exact limits such as given by (2.8). What is happening in this situation is that the $\pm$ 3-sigma support of the slope density function in (2.7) is overlapping the region of the $\xi_x\xi_y$-plane where the unit step function is zero. In fact, if one could go to the grazing incidence limit $(\theta = \pi/2)$, it is readily observed from Figure 2.2 that only half the $\xi_x\xi_y$-plane is encompassed by the integrals in (2.7). Thus, in addition to the attenuation of $[\sigma_{pp}^o]_1$ near grazing incidence due to the $(1 + C_o)^1$ factor, there is another reduction factor resulting from the shadowing of points on the large scale surface having positive slopes (see Figure 2.1). It should be remembered that although the impact of the unit step function upon the limits of the integrals in (2.7) is somewhat involved, it is a direct consequence of the rather simple slope-shadowing limitations explained in Figure 2.1.

The analysis presented above does not alter the general composite surface scattering theory set forth in [1]. It does, however, correct and expand the theory in [1] as it relates to the effects of shadowing upon large angle of incidence backscattering from a randomly rough surface. This additional analysis was necessitated by the use of an incorrect functional form for the shadowing function in [1].

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*This criterion has also been called the stationary phase approximation [4] and the deep phase modulation condition [5].

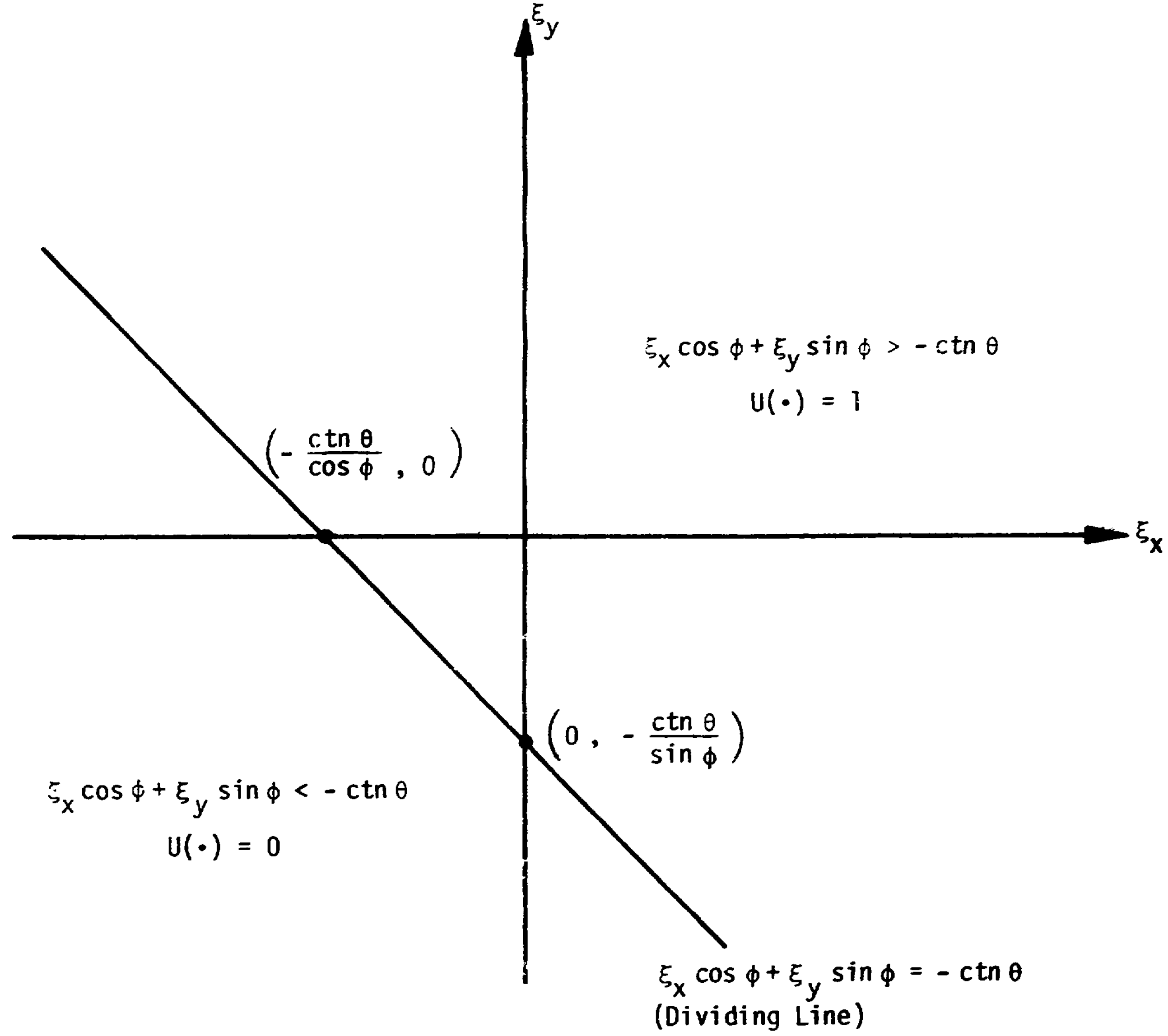

Figure 2-2. A plot of the regions in which the unit step function in the shadowing function is zero and unity.

2.4 Discussion of Numerical Example

The use of an incorrect shadowing function in Sections I through IV of [1] was primarily a sin of omission. That is, since nearly all the results in Section I through IV of [1] were formal in nature, it was not necessary to use the erroneous shadowing function given by (28) of [1]. However, the numerical example comprising Section V of [1] does require attention in order to properly account for the effects of large scale shadowing. In the example presented in [1], R was set equal to unity because the large scale slopes were relatively small. According to the analysis presented above, this step is not justified for all angles of incidence. The correct effect of shadowing is addressed below.

As in Section V of [1], it is assumed that $\Gamma^2_{pp'} \approx \Gamma^2_{pp'}(0,0)$ and the surface height spectrum is isotropic, i.e. $\overline{\zeta^2_{\ell x}} = \overline{\zeta^2_{\ell y}} = \overline{\zeta^2_{\ell t}}/2$ and $S(k_x,k_y) = S\left(\sqrt{k_x^2+k_y^2}\right)$. Substituting $k\cos\alpha = k_x$, $k\sin\alpha = k_y$ and $dk_x dk_y = k dk d\alpha$ in (32) of [1] and using (2.6) above for the shadowing function yields;

$$[\sigma^o_{pp'}]_1 = \frac{2k_o^2\, \Gamma^2_{pp'}(0,0)}{\pi\, \overline{\zeta^2_{\ell t}}\, \cos^2\theta\,(1+C_o)} \int_0^{2\pi}\int_{k_d}^{\infty} S(k)\,U\!\left(\text{ctn}\,\theta + \tan\theta + \frac{k\cos(\alpha-\phi)}{2k_o\cos\theta}\right)$$

$$\cdot \exp\left\{-\left[4k_o^2\sin^2\theta + k^2 + 4kk_o\sin\theta\cos(\alpha-\phi)\right] \Big/ 4k_o^2\, \overline{\zeta^2_{\ell t}}\cos^2\theta\right\} k\,dk\,d\alpha \qquad (2.11)$$

The unit step function is unity whenever $k\cos(\alpha-\phi) > -2k_o/\sin\theta$. Since the minimum value of $\cos(\alpha-\phi)$ is -1, this inequality will be satisfied for all α if k is restricted to less than $2k_o/\sin\theta$. Since the range of k from $2k_o/\sin\theta$ to ∞ for $\cos(\alpha-\phi) > 0$ contributes little to the integral in (2.11) (because $\overline{\zeta^2_{\ell t}}$ is small), the k-integral limits can be approximated by

$[k_d, 2k_o/\sin\theta]$. In this range of k, the step function is unity and the α-integration can be accomplished with the following result;

$$[\sigma^o_{pp'}]_1 \approx \frac{4k_o^2\,\Gamma^2_{pp'}(0,0)}{\overline{\zeta^2_{\ell t}}\cos^2\theta(1+C_o)} \int_{k_d}^{2k_o/\sin\theta} S(k)\, I_o\left(\frac{k\sin\theta}{k_o\,\overline{\zeta^2_{\ell t}}\cos^2\theta}\right)\exp\left\{-\frac{(k-2k_o\sin\theta)^2}{4k_o^2\,\overline{\zeta^2_{\ell t}}\cos^2\theta} - \frac{k\sin\theta}{k_o\,\overline{\zeta^2_{\ell t}}\cos^2\theta}\right\} k\,dk \tag{2.12}$$

where $I_o(\cdot)$ is a Bessel function of the second kind. The right hand side of (2.12) may now be compared with the $[\sigma^o_{pp'}]_1$ part in (44) of [1], i.e.

$$\frac{4k_o^2\,\Gamma^2_{pp'}(0,0)}{\overline{\zeta^2_{\ell t}}\cos^2\theta} \int_{k_d}^{\infty} S(k)\, I_o\left(\frac{k\sin\theta}{k_o\,\overline{\zeta^2_{\ell t}}\cos^2\theta}\right)\exp\left\{-\frac{(k-2k_o\sin\theta)^2}{4k_o^2\,\overline{\zeta^2_{\ell t}}\cos^2\theta} - \frac{k\sin\theta}{k_o\overline{\zeta^2_{\ell t}}\cos^2\theta}\right\} k\,dk \tag{2.13}$$

The obvious differences are the factor $(1+C_o)^{-1}$ and the finite upper limit on the integral in (2.12). Figure 2.3 illustrates how $(1+C_o)^{-1}$ varies with θ for $\overline{\zeta^2_{\ell t}} = 0.0224$ which was the value of mean square slope used to construct Figures 3 and 4 of [1]. Of particular note in the plot of $(1+C_o)^{-1}$ is the fact it does not start to decrease until θ exceeds 85°; at 87.5° $(1+C_o)^{-1} = -2.5$ dB. The value of $\theta = 87.5°$ is the point at which $4k_o^2\overline{\zeta^2_\ell}\cos^2\theta \simeq 10$ and, consequently, represents the approximate limit of the large scale theory. That is, for $\theta > 87.5°$ the analysis of the scattering from the large scale structure on the surface can no longer be accomplished using optical techniques. The effect of the finite upper limit on the integral in (2.12) is much less dramatic. In fact if the $4k_o^2\zeta^2_\ell\cos^2\theta \gtrsim 10$ criterion is ignored and the limit

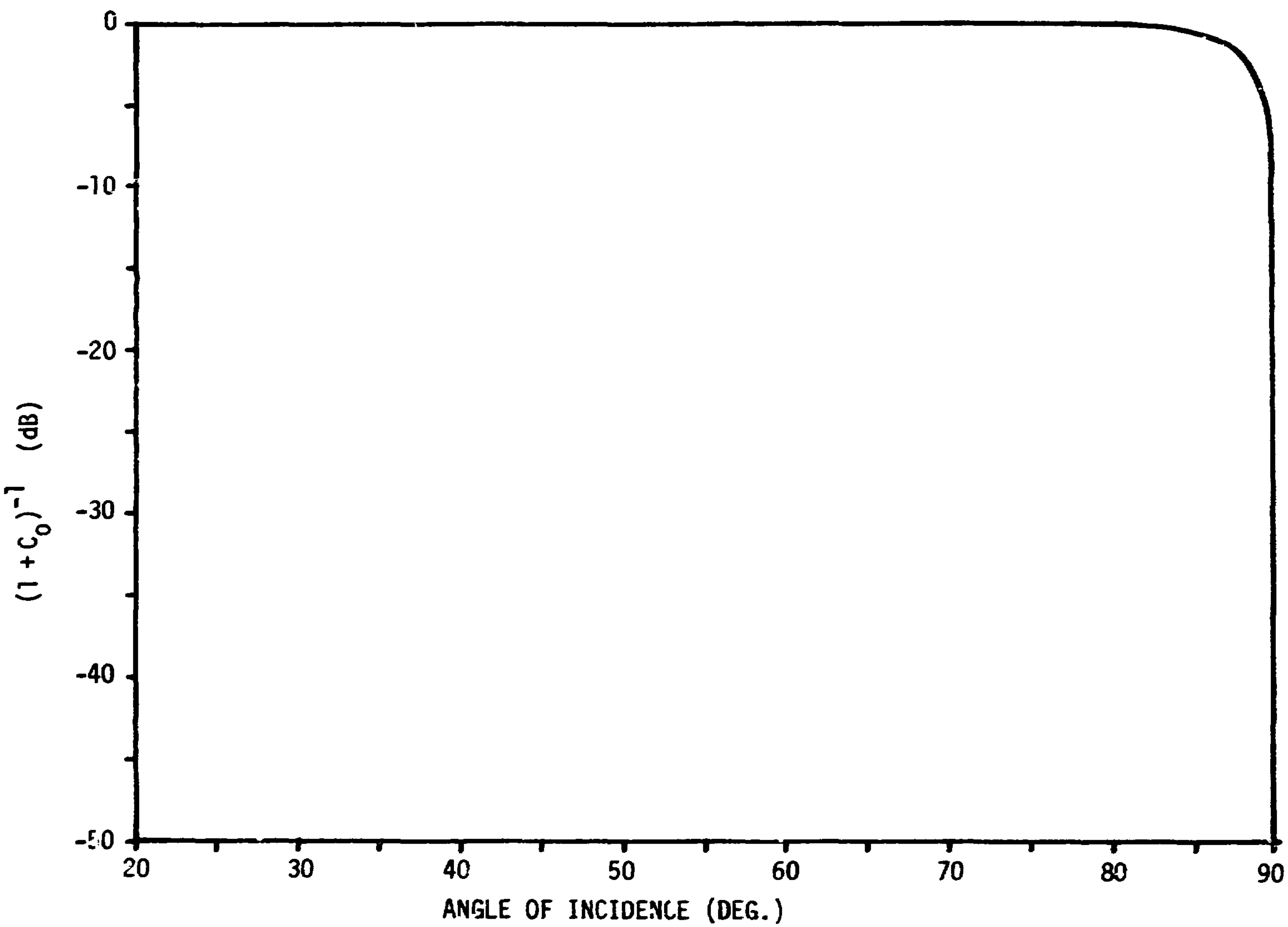

Figure 2-3. Behavior of the factor $(1+C_o)^{-1}$ resulting from the corrected shadowing function for $\overline{\zeta_{\ell t}^{\,2}} = 0.0224$.

of $\theta = 90°$ is taken in (2.12), it can be shown* that the finite upper limit gives rise to a 3 dB reduction from the value of (2.13) at $\theta = 90°$. This same conclusion can also be obtained from (2.7) and it is a direct consequence of the fact that all positive slopes are excluded from consideration at $\theta = 90°$ (see Figure 2.1). Within the range of validity of the optical criterion, the finite upper limit on the integral in (2.12) gives rise to much less attenuation than the $(1 + C_o)^{-1}$ factor.

In view of the above analysis, it is concluded that the numerical results presented in Section V of [1] are correct as they were presented. Determination of the spectral division wavenumber k_d is in no way altered by the inclusion of the correct shadowing function. The curves shown in Figures 3 through 6 of [1] are correct because they do not encompass the range of θ where shadowing is important ($\theta \gtrsim 85°$). As noted above, the onset of shadowing effects occurs approximately where the optical criterion ($4k_o^2 \zeta_\ell^2 \cos^2\theta \gtrsim 10$) is violated, i.e. $\theta \approx 87.5°$, for the numerical example presented in [1]. However, for larger slopes shadowing must be considered since it will cause a significant reduction in σ_{pp}^o, near grazing incidence.

2.5 Summary

In the analysis presented in [1], it was correctly demonstrated how one includes the effects of large scale surface feature shadowing on the backscattering cross section of a composite surface for large angles of incidence. Unfortunately, an incorrect form for the shadowing function was used in [1] which led to the erroneous evaluation of the impact of shadowing upon large angle of incidence scattering. In this section, the correct form of the shadowing

*To show this one can apply Laplace's method to asymptotically evaluate (2.12) as $\cos\theta \to 0$. However, it must be remembered that the maximum of the integrand occurs at the upper limit of the integrand as $\cos\theta \to 0$ and this impacts the evaluation of the integral [6].

function has been presented and included in the formulas obtained in [1]. Particular emphasis has been placed upon the physical significance of shadowing as it effects large angle of incidence scattering. It has been shown that shadowing leads to multiplication of both $[\sigma^o_{pp'}]_0$ and $[\sigma^o_{pp'}]_1$ by the factor $(1+C_o)^{-1}$ which causes $[\sigma^o_{pp'}]_1$ and, thus, $\sigma^o_{pp'}$, to go to zero near grazing incidence for a perfectly conducting, randomly rough, composite surface. Furthermore, there is another effect which leads to an additional 3 dB attenuation at grazing incidence $(\theta \simeq 90°)$*. This effect results from those slopes which, with probability one, will cause the point on the surface having these slopes to be shadowed. At grazing incidence, all positive slopes are in this class.

A reevaluation of the numerical results presented in [1] revealed that use of the correct shadowing function did not alter any of the results relative to the choice of the spectral dividing wavenumber k_d. Furthermore, none of the curves presented in [1] were affected because they only encompass the range of θ from 0 to 70° and the effects of shadowing were present for $\theta \gtrsim 85°$. Also, techniques were presented relative to overcoming some of the analytical difficulties resulting from the use of the correct shadowing function.

*It is reemphasized that exact grazing incidence cannot be addressed by this theory because the optical criterion assumed of the large scale surface features is violated. The -3 dB figure is significant only in its magnitude relative to the effect of the $(1+C_o)^{-1}$ factor.

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3.0 SHADOWING BY NON-GAUSSIAN RANDOM SURFACES

3.1 Background

Shadowing of random surfaces was originally introduced [1] as an *ad hoc* correction to the results provided by physical or geometrical optics approximate theories of rough surface scattering. Sancer [2] subsequently demonstrated how shadowing could be rigorously accounted for in the optical limit for random surfaces. Furthermore, he showed that previously derived expressions for the effects of shadowing based upon purely geometrical considerations [3, 4] were directly applicable. Using Sancer's results, Brown [5,6] showed how shadowing could be rigorously included in a formulation for scattering from random surfaces characterized by many scales of roughness, i.e. composite rough surfaces.

While shadowing theory is reasonably mature, it has only been applied to jointly Gaussian random surfaces. The Gaussian results are probably adequate for the ocean but they are questionable for terrain and completely inadequate for sea ice fields. For sea ice, water first fills all surface depressions below mean sea level and then freezes. This eliminates all surface height excursions below mean sea level and the probability density function of the surface roughness is clearly non-Gaussian. For these reasons, it is important to extend shadowing theory to the point where it can easily accommodate non-Gaussian surface statistics; such is the purpose of this section.

3.2 Analysis

The special case of backscattering is chosen to illustrate the approach; this minimizes some of the conceptual details associated with the more general bistatic case. It turns out that the extension of the results to the bistatic geometry can be accomplished almost by inspection. The analysis presented by Smith [4] is general to a point in the development; however, there are a number of integrations which must be accomplished in order to arrive at the final expression for the shadowing function. In the case of a jointly Gaussian surface these integrals can be performed and a closed-form result is obtained for the shadowing function. For non-Gaussian surfaces, the required integrations appear to be, at best, formidable. The purpose of this section is to show that if the height and slopes of the surface are independent random variables then the final expression for the shadowing function is drastically simplified.

For the reader's convenience Smith's notation will be employed in this section and his Figure 1 is essentially repeated here as our Figure 3.1. There are three critical relationships from Smith's paper [4] which are required. If $S(F,\theta)$ is the probability that no part of the surface will intersect the incident ray (at an angle θ with respect to the normal to the mean flat surface) on its way to point F on the surface then $S(F,\theta)$ is given by

$$S(F,\theta) = h(\mu - q_o) \exp\left\{ - \int_0^\infty g(\tau)d\tau \right\} \tag{3.1}$$

where $h(\cdot)$ is the unit step function, $\mu = \text{ctn}\,\theta$, q_o is the slope of the surface in the y-direction at F, and $g(\tau)\Delta\tau$ is the conditional probability that the surface will intersect the incident ray in the interval $(\tau, \tau + \Delta\tau)$ given that it does not intersect the ray in $(0,\tau)$. The function $g(\tau)\Delta\tau$ is determined by the behavior of $P_3(\xi, q|F,\tau)$ which is the joint probability of the height ξ and y-slope at the point $(x=0, y=\tau)$ conditioned upon the height (ξ_o) and y-slope (q_o) at point F; in particular,

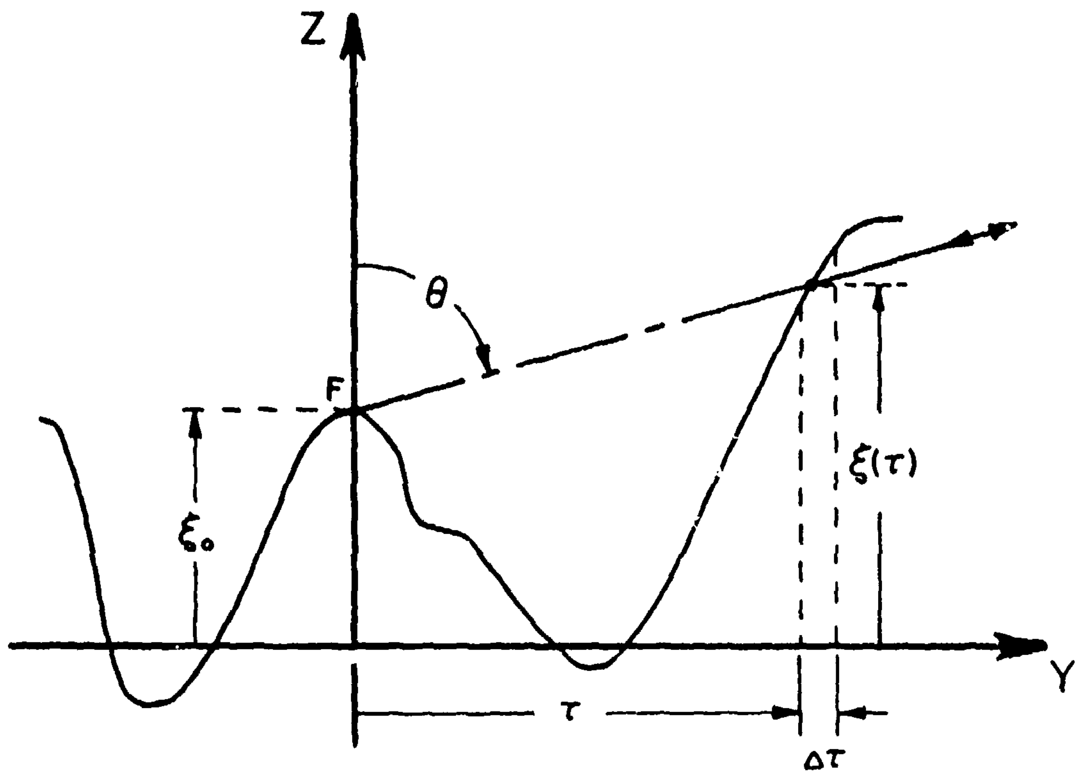

Figure 3-1. Shadowing geometry. The incident ray lies in the $x = 0$ plane and the slopes of the surface at the point F are $\partial\xi/\partial x = p_o$ and $\partial\xi/\partial y = q_o$.

$$g(\tau)\Delta\tau = \frac{\displaystyle\int_{\mu}^{\infty} (q-\mu)P_3(\xi,q|F,\tau)\Big|_{\xi=\xi_o+\mu\tau} dq}{\displaystyle\int_{-\infty}^{\infty} dq \int_{-\infty}^{\xi_o+\mu\tau} P_3(\xi,q|F,\tau)d\xi}\,\Delta\tau \tag{3.2}$$

The average of $S(F,0)$ over all surface heights with $p_o = 0$ and $q_o = -1/\mu$ is the desired shadowing function $R(\theta)$, i.e. the probability that a back-scattering specular point on the surface will not be shadowed.

Smith proceeded to evaluate (3.2) in the Gaussian case by assuming that the heights and slopes at F were uncorrelated with those at $y=\tau$. Here it will be assumed that decorrelation implies statistically independence[*] and that the height and slopes are independent; thus,

$$P_3(\xi,q|F,\tau) \approx P_1(\xi)P_2(q) \tag{3.3}$$

where $P_1(\xi)$ is the height probability density function and

$$P_2(q) = \int_{-\infty}^{\infty} P_{22}(p,q)dp \tag{3.4}$$

where $P_{22}(p,q)$ is the joint probability density function of the x and y slopes. Substituting (3.3) into (3.2) yields the following

$$g(\tau) = \frac{\Gamma\, P_1(\xi_o+\mu\tau)}{\displaystyle\int_{-\infty}^{\xi_o+\mu\tau} P_1(\xi)d\xi} \tag{3.5}$$

where

[*]This is not true in general so the following analysis applies to a restricted class of non-Gaussian surfaces.

$$\Gamma = \int_{\mu}^{\infty} (q-\mu)P_2(q)dq \tag{3.6}$$

The denominator of (3.5) is recognized as the distribution function for ξ evaluated at $\xi_o + \mu\tau$, i.e. $F_1(\xi_o + \mu\tau)$. Also, $P_1(\xi_o + \mu\tau)$ in (3.5) is equal to the derivative of the distribution function evaluated at $\xi = \xi_o + \mu\tau$. Consequently, (3.5) becomes $g(\tau) = [\Gamma/F_1(\xi_o + \mu\tau)]dF_1(\xi_o + \mu\tau)/d(\xi_o + \mu\tau)$. Substituting this result in (3.1), making the change of variable $\eta = \xi_o + \mu\tau$ and noting that $dF_1/F_1 = d(\ell n\, F_1)$ yields

$$S(\theta,F) = h(\mu - q_o)\ \exp\left\{-\Gamma/\mu \int_{\eta=\xi_o}^{\infty} d[\ell n F_1(\eta)]\right\} \tag{3.7}$$

where ℓn denotes the natural logarithm. Since $F_1(\infty) = 1$, (3.7) reduces to the following;

$$S(\theta,F) = h(\mu - q_o)\ F_1\,(\xi_o)^{\Gamma/\mu} \tag{3.8}$$

Remembering that $P_1(\xi_o) = dF_1(\xi_o)/d\xi_o$, the average of (3.8) over all values of ξ_o simplifies to

$$S(p_o,q_o,\theta) = h(\mu - q_o) \int_{\xi_o=-\infty}^{\infty} \left\{F_1(\xi_o)\right\}^{\Gamma/\mu} dF_1(\xi_o) \tag{3.9}$$

or

$$S(p_o,q_o,\theta) = \frac{h(\mu - q_o)}{\Gamma/\mu + 1} \tag{3.10}$$

since $F_1(-\infty) = 0$ and $\Gamma/\mu + 1 \geq 0$. With $q_o = -1/\mu$, the shadowing function appropriate for <u>backscatter</u> reduces to the following simple expression;

$$R(\theta) = \left\{ 1 + \int_{\mu}^{\infty} (q/\mu - 1) P_2(q) dq \right\}^{-1} \tag{3.11}$$

where, in summary, $\mu = \text{ctn}\,\theta$, θ is the incidence angle, and $P_2(q)$ is the probability density function of the slopes of the surface in the plane of incidence defined by the incident ray and the normal to the mean surface. It is interesting to note from (3.11) that for normal incidence $(\theta = 0)$ $R(0) = 1$ whereas at grazing incidence $(\theta = \pi/2)$ $R(\pi/2) = 0$ since $\mu = 0$ and $\int_0^{\infty} q P_2(q) dq > 0$. Thus, these basic properties of the shadowing function are independent of the detailed properties of the slope density function. One can easily verify that (3.11) is identical to the results obtained by Smith for the special case of a jointly Gaussian surface.

The form of (3.10) compared to Smith's results, i.e. (23) of [4], suggests that the above result can be directly translated to the bistatic case and, indeed, this is the case. For the bistatic case, a generalization of Sancer's [2] results will be given. It should be noted that the inequalities involving the angles of incidence (θ_o) and scattering (θ) , just prior to Sancer's equations (49), (50), (54) and (55) should be reversed. With $\mu = \text{ctn}\,\theta$ and $\mu_o = \text{ctn}\,\theta_o$, Sancer's results are easily generalized by replacing his C_o by $\Gamma(\mu_o)/\mu_o$ and C_2 by $\Gamma(\mu)/\mu$.

3.3 Example

To illustrate the above results, the backscattering shadowing function $R(\theta)$ will be determined for the exponential joint slope density function introduced by Barrick [7], i.e.

$$P_{22}(p,q) = \frac{3}{\pi w^2} \exp\left[- \sqrt{6(p^2 + q^2)/w^2} \right] \tag{3.12}$$

where w^2 is the mean square slope of the surface roughness. $P_{22}(p,q)$ in (3.12) represents a surface whose roughness is isotropic, $w^2/2 = \langle p^2 \rangle = \langle q^2 \rangle$, with statistically dependent slopes. That is, the joint density function cannot be expressed as a product of the marginal or individual densities. The calculation of the marginal density $P_2(q)$, using (3.4), is reasonably straightforward and the result is as follows;

$$P_2(q) = \frac{6}{\pi w^2} |q| K_1 (\sqrt{6} |q|/w) \tag{3.13}$$

where $K_1(\cdot)$ is one of the modified Bessel functions of order one [8]. It is interesting to compare this density with a Gaussian, i.e.

$$P_2(q) = \frac{1}{w\sqrt{2\pi}} \exp(-q^2/2w^2)$$

and this is done in Figure 3.2 where the normalized densities $P_2(q)w$ are plotted as a function of the normalized slope q/w. It should be noted from the plots in Figure 3.2 that the "exponential" density shows a much greater probability of occurrence of small slopes than the Gaussian. This result is in agreement with one intuitive approach for generating a surface characterized by (3.12), e.g. one strongly filters all surface height excursions below a certain level to eliminate the possibility of large negative height excursions. This process increases the probability of small slopes at the expense of the large slopes.

Substituting (3.13) in (3.11) and using tabulated integrals of Bessel functions given in [9], the following closed-form result is obtained for the backscattering shadow function $R(\theta)$;

$$R(\theta) = \left\{ \frac{x}{\pi} K_2(x) + \frac{1}{2} + \frac{x}{2} \left[K_1(x)L_o(x) + L_1(x)K_o(x) \right] \right\}^{-1} \tag{3.14}$$

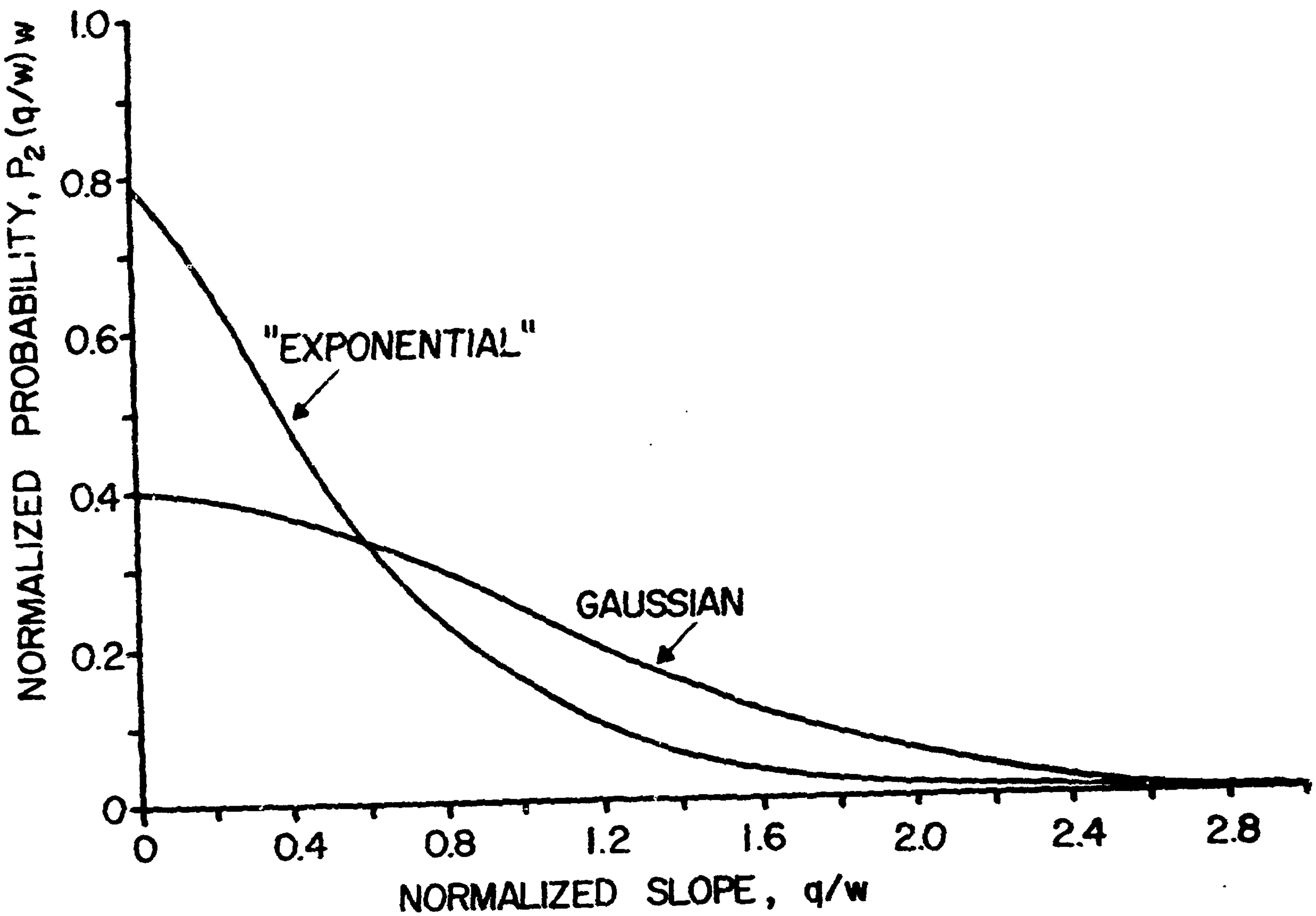

Figure 3-2. A comparison of the "exponential" marginal slope density given by (3.13) and the Gaussian.

where $x = (\sqrt{6}/w)\operatorname{ctn}\theta$ and the $L_n(\cdot)$, $n = 0$ or 1, symbol denotes the modified Struve functions [8]. Using asymptotic forms for the special functions in (3.14), it may be readily verified that $R(0) = 1 (x \to \infty)$ and $R(\pi/2) = 0 (x = 0)$. The modified Struve functions may be computed from tables given in [8] for $x \gtrsim 5$ and by a power series for smaller arguments.

Figure 3.3 compares (3.14) and the shadowing function for a Gaussian function obtained by Smith [4] for a range of rms slopes. The shadowing function for the exponential joint slope density is larger because the marginal slope density given by (3.13) exhibits less likelihood for large slopes than the corresponding Gaussian density. That is, the larger slopes are the source of more significant shadowing.

3.4 Summary

The shadowing theory developed by Smith [4], while sufficiently general to deal with any joint slope density function, involves what appears to be a number of rather complicated integrals. Under the assumption that the surface height is statistically independent of the surface slopes, it is shown that Smith's theory can be reduced to a single integration involving the marginal density function for the slopes in the plane of incidence. Using this result but without regard to the specific form of the marginal density function, it can be shown that the backscattering shadowing function is unity at normal incidence and zero at grazing incidence. Because the final result involves an integration or smoothing process, it is amenable to the use of histogram data for the marginal slope density.

This theory is applied to an exponential joint slope density representing an isotropic surface for which the slopes are not statistically independent. The backscattering shadowing function for the exponential and Gaussian joint slope densities are compared and it is found that the Gaussian surface

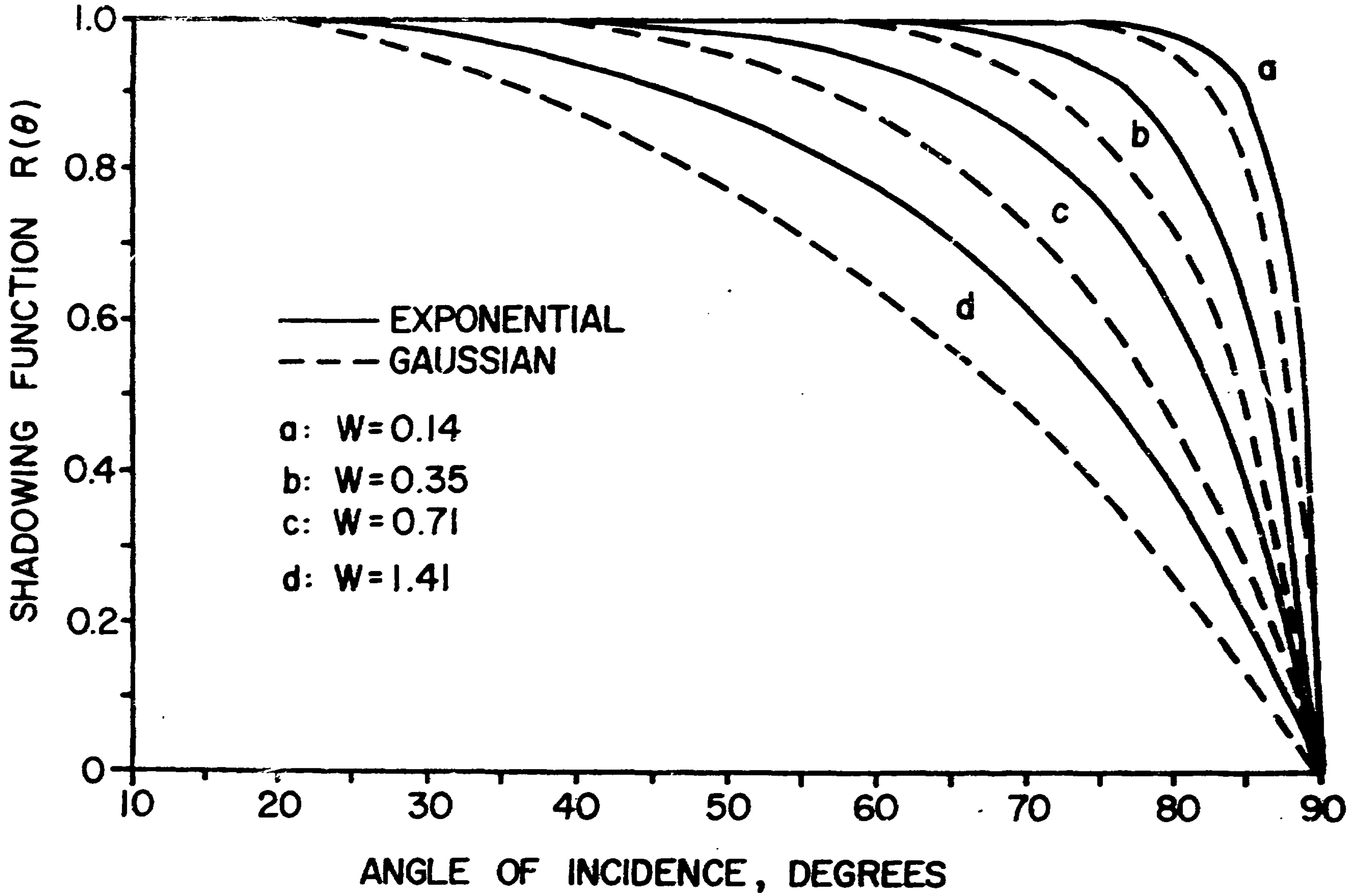

Figure 3-3. A comparison of the backscattering shadowing function for the exponential and Gaussian joint slope densities for a range of rms slopes (w).

produces stronger shadowing. This result is found to be a consequence of the greater likelihood of large slopes with the Gaussian density.

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4.0 BISTATIC SCATTERING FROM LOSSY RANDOM SURFACES

4.1 Background

Prior to the mid-1960's, electromagnetic scattering from randomly rough surfaces was modeled using either perturbation theory or physical optics [1]. First order perturbation theory appeared to do a reasonable job of analytically describing the scattering process when the surface roughness was small in terms of the electromagnetic wavelength and multiple scattering was negligible. Physical optics produced meaningful results in and about the specular scattering direction when the surface exhibited very large but smoothly undulating height variations. Unfortunately, there were numerous attempts to apply these theories to situations where the implicit assumptions in the models were violated. These attempts usually assumed some surface parameter such that the scattering measurements and the "model" were brought into agreement. However, it was very quickly recognized that these attempts were highly suspect because of their failure to satisfy certain fundamental principles.

As more and more rough surface microwave scattering measurements were acquired, it became obvious that neither first order perturbation theory nor physical optics were individually adequate for all angles of incidence and scattering. Conversely, it appeared that physical optics seemed to do a good modeling job near the specular scattering direction while first order perturbation theory was reasonably accurate for all other scattering angles. Almost simultaneously, researchers in the U.S. [2] and the U.S.S.R.[3,4] began to advocate the combining of these two diverse theories in what was later to be called the composite surface scattering model. In this model, the surface was considered to be made up of both large and small scale surface features (height and spatial wavelength) relative to the electromagnetic wavelength, λ_o . The large scale surface features were considered to be responsible for the physical optics-like scattering near the specular direction. The small scale surface structure gave rise to a perturbation field (to the optical field) which was the dominant scattering mechanism away from the specular scattering direction. The interaction between the optical and first order perturbation fields was assumed to be totally dependent upon the tilting of the small surface structure by the larger gently undulating features [5].

More recently, rigorous first order boundary perturbation theory has been applied to the problem of backscattering from a perfectly conducting, Gaussian distributed rough surface [6]. The results of this analysis indicated that much of the original work on this problem could be rigorously justified. Furthermore, additional insight was gained in regard to such aspects of the problem as shadowing (see Section 3 of this report), spectral dichotomy, and the tilting interpretation. A logical extension of this latter theory encompasses bistatic scattering from a lossy dielectric surface. The purpose of this section is to present the details associated with such an extension.

Before the details of scattering from a composite dielectric surface are presented, it is illuminating to consider two much simpler cases. The first is backscattering from a dielectric surface having only a small scale roughness while the second case addresses bistatic scattering. The advantages of this approach are that it leads to familiarity with the perturbation technique and it sets forth the principles that will be used for the composite surface.

4.2 Backscattering From A Dielectric Surface With Small Scale Roughness

The geometry for this problem is shown in Figure 4-1. The mean or average surface is the $z = 0$ plane; the random roughness ζ_s superposed upon this plane is positive for $\zeta_s > 0$ and negative for $\zeta_s < 0$. Below the rough surface $(z < \zeta_s)$, the relative dielectric constant of the medium is ε_r and the relative magnetic permeability is taken to be the same as for free space,

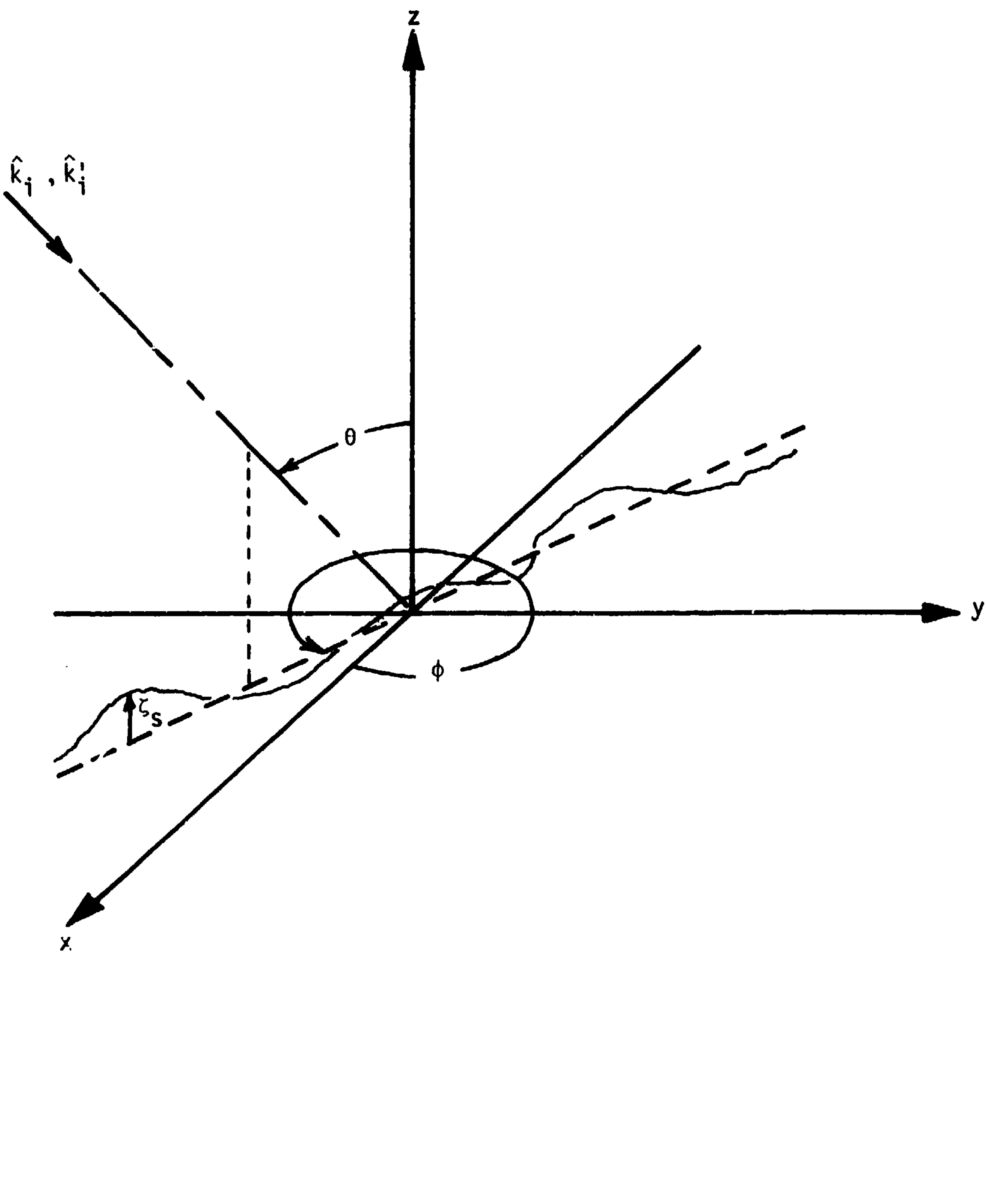

Figure 4-1. Geometry for backscattering from a randomly rough surface having only small scale roughness ζ_s .

i.e. $\mu_r = 1$. Above the rough surface $(z > \zeta_s)$, the medium is free space, i.e. $\varepsilon_r = 1$ and $\mu_r = 1$.

Provided that the roughness is small with respect to the electromagnetic wavelength λ_o , e.g. $4k_o^2 \overline{\zeta_s^2} << 1$ where $k_o = 2\pi/\lambda_o$ and $\overline{\zeta_s^2}$ is the mean square height of the roughness, the scattered field $\vec{E}^s$ can be expressed as follows;

$$\vec{E}^s \approx \delta^o \vec{E} + \delta^1 \vec{E} \tag{4.1}$$

where $\delta^o \vec{E}$ is the field scattered by a surface having no roughness (the zeroth order perturbation) and $\delta^1 \vec{E}$ is the scattered field which depends on the roughness to first order only (the first order perturbation). The primary assumption in (4.1) is that higher order terms such as $O(\zeta_s^2)$, $O(\zeta_s^3)$, etc., are negligible. The zeroth order perturbation field $\delta^o \vec{E}$ is trivially determined since it is just the field reflected by an infinite, flat dielectric interface. Both Mitzner [7] and Burrows [8] have obtained particularly useful expressions for $\delta^1 \vec{E}$. The Mitzner result is more straightforward but it is restricted to small roughness perturbations superposed on a flat plane.* Burrows' solution for $\delta^1 \vec{E}$ is somewhat more complicated but it is more general in that the unperturbed surface need not be planar or even deterministic. For small scale roughness on a plane, the Burrows formulation requires a bit more effort in computing $\delta^1 \vec{E}$ than Mitzner's result. For a composite surface, only the Burrows result is sufficiently general to address this problem.

At first glance, the Burrows expression for $\delta^1 \vec{E}$ appears to be somewhat cumbersome and confusing. However, if it is realized that the result is obtained

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*Mitzner's result can actually be applied to any unperturbed surface for which the wave equation is separable. For the problem considered here, the surface is restricted to a plane.

from an application of reciprocity then the notation becomes more meaningful. Basically, one deals with two incident electric fields of the form

$$\vec{E}_i = E_i \hat{e} \qquad E_i = E_o \exp(-j \vec{k}_i \cdot \vec{r}) \qquad (4.2a)$$

and

$$\vec{E}_i{}' = E_i{}' \hat{e}' \qquad E_i{}' = E_o \exp(-j \vec{k}_i{}' \cdot \vec{r}) \qquad (4.2b)$$

where the primed field may have a different polarization and direction of incidence than the unprimed field. Burrows' expression for the first order perturbation (electric) field scattered in the direction $-\hat{k}_i{}'$ and polarized in the $\hat{e}'$ direction is as follows [8];

$$\delta^1\vec{E}\cdot\hat{e}' = \frac{k_o^2 \exp(-jk_o R)}{4\pi R\, E_o \varepsilon_o} \int_{S_o} \left[\Delta\vec{E}\cdot\vec{D}' + \Delta\vec{B}\cdot\vec{H}' - \Delta\vec{H}\cdot\vec{B}' - \Delta\vec{D}\cdot\vec{E}'\right] \zeta_s \, dS_o \qquad (4.3)$$

where it is assumed that $\delta^1\vec{E}\cdot\hat{e}'$ is measured in the far-field of the rough surface. The distance R is measured from the origin of the reference coordinate system on the mean surface to the point of observation or measurement of $\delta^1\vec{E}\cdot\hat{e}'$ and ε_o is the permittivity of free space. The fields $\vec{E}'$, $\vec{H}'$, $\vec{D}'$ and $\vec{B}'$ are the fields on the unperturbed surface (S_o) due to the <u>primed</u> incident field while $\Delta\vec{E}$, $\Delta\vec{H}$, $\Delta\vec{D}$ and $\Delta\vec{B}$ are the discontinuities in the fields on the unperturbed surface (S_o) due to the <u>unprimed</u> incident field. Thus, to determine the $\hat{e}_p$-polarized component of the scattered first order perturbation field in the general direction $\hat{k}_s$ one merely sets $\hat{e}' = \hat{e}_p$ and $\hat{k}_i' = -\hat{k}_s$ in the expression for the primed incident field $\vec{E}_i'$, computes the resulting fields $\vec{E}'$, $\vec{H}'$, $\vec{D}'$, $\vec{B}'$ on the unperturbed surface S_o, and substitutes these results in (4.3).

For backscattering $\hat{k}_s = -\hat{k}_i$ so according to the above receipe, $\hat{k}_i{}' = \hat{k}_i$

which in the coordinates of the geometry shown in Figure 4-1 is as follows;

$$\hat{k}_i{}' = \hat{k}_i = k_o(-\sin\theta\cos\phi\,\hat{x} - \sin\theta\sin\phi\,\hat{y} - \cos\theta\,\hat{z}) \tag{4.4}$$

Contrary to previous analyses [1], the direction of incidence specified by the angle ϕ should not, at this point in the development, be arbitrarily set to some convenient value such as 0 or $\pi/2$. The reason for this is that the surface may have anisotropic roughness and the orientation of the x and y axes of the reference coordinate system should be fixed relative to this surface characteristic and not the direction of incidence.

Since the fields inside the surface integral in (4.3) are the fields induced on the infinite planar dielectric surface S_o, it is convenient to further categorize the problem according to the polarization of the incident fields. For both $\vec{E}_i$ and $\vec{E}_i{}'$ horizontally polarized, $\hat{e}$ and $\hat{e}'$ are orthogonal to the plane formed by the unit vectors $\hat{k}_i$ and $\hat{n} = \hat{z}$ where $\hat{n}$ is the normal to the mean or unperturbed surface. In this case both $\hat{e}$ and e' are totally tangential to the mean plane. For both $\vec{E}_i$ and $\vec{E}_i{}'$ vertically polarized, $\hat{e}$ and $\hat{e}'$ are parallel to the plane formed by $\hat{k}_i$ and $\hat{n} = \hat{z}$.

4.2.1 Horizontal Polarization

When $\hat{e}$ and $\hat{e}'$ are tangential to the mean or unperturbed surface, the fields $\vec{E}$, $\vec{E}'$, D and $\vec{D}'$ (when evaluated on S_o) are entirely tangential to S_o. Since the tangential component of the electric field is continuous across an interface, $\Delta\vec{E} = 0$. Furthermore, since there is no change in μ_r across the boundary and the lower medium is assumed not to be perfectly conducting, $\Delta\vec{B}$ and $\Delta\vec{H}$ are both zero on the interface. Consequently, (4.3) reduces to the following;

$$\delta^1\vec{E}\cdot\hat{e}' = -\frac{k_o^2\exp(-jk_oR)}{4\pi R\,E_o\,\varepsilon_o}\int_{S_o}(\Delta\vec{D}\cdot\vec{E}')\zeta_s\,dS_o \tag{4.5}$$

The total $\vec{E}'$-field on the surface S_o due to $\vec{E}_i'$ is given by

$$(1+R_h)\,\vec{E}_i'(S_o) = E_o(1+R_h)\exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)\,\hat{e}' \tag{4.6}$$

where R_h is the Fresnel (field) reflection coefficient for horizontal polarization and $\vec{r}_\perp = x\hat{x} + y\hat{y}$ or just $\vec{r}$ evaluated on S_o. The discontinuity in $\vec{D}$ is given by

$$\Delta\vec{D} = \varepsilon_o\left\{\vec{E}_i(z=0^+) - \varepsilon_r\,\vec{E}_i(z=0^-)\right\} \tag{4.7}$$

and

$$\vec{E}(z=0^+) = (1+R_h)\,\vec{E}_i(S_o) \tag{4.8a}$$

$$\vec{E}(z=0^-) = T_h\,\vec{E}_i(S_o) \tag{4.8b}$$

where T_h is the Fresnel (field) transmission coefficient for horizontal polarization. Combining (4.8a) and (4.8b) in (4.7) yields

$$\Delta\vec{D} = \varepsilon_o\left[1+R_h-\varepsilon_r T_h\right]\vec{E}_i(S_o)$$

or

$$\Delta\vec{D} = \varepsilon_o\left[1+R_h-\varepsilon_r T_h\right]E_o\exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)\,\hat{e} \tag{4.9}$$

Multiplying (4.7) by (4.9) and realizing that $T_h = 1 + R_h$ yields

$$\Delta\vec{D}\cdot\vec{E}' = -\varepsilon_o E_o^2(1+R_h)^2(\varepsilon_r-1)\exp(-j\,2\vec{k}_i\cdot\vec{r}_\perp)(\hat{e}\cdot\hat{e}') \tag{4.10}$$

Substituting this result in (4.5) produces the desired result for the first order perturbation field polarized in the $\hat{e}'$ direction

$$\delta^1\vec{E}\cdot\hat{e}' = \frac{k_o^2 E_o}{4\pi R}\exp(-j\,k_o R)(1+R_h)^2(\varepsilon_r - 1)\iint \exp(-j\,2\vec{k}_i\cdot\vec{r}_\perp)\,\zeta_s\,dx\,dy \tag{4.11}$$

where $\hat{e}\cdot\hat{e}' = 1$,

$$R_h = \frac{\cos\theta - \sqrt{\varepsilon_r - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon_r - \sin^2\theta}}$$

and so

$$(1+R_h)^2(\varepsilon_r - 1) = \frac{4(\varepsilon_r - 1)\cos^2\theta}{\left[\cos\theta + \sqrt{\varepsilon_r - \sin^2\theta}\right]^2} \tag{4.12}$$

There are several points to note about (4.11). The derivation of (4.11) was considerably simpler than the Rayleigh-Rice approach [1]; this is because all of the difficult work was done in obtaining (4.3). Equation (4.11) is an expression for the scattered first order perturbation field, a more meaningful quantity than the average scattered power when dealing with phase sensitive systems. The average of (4.11) is zero because $\langle\zeta_s\rangle = 0$; however, the average of $\langle|\delta^1\vec{E}\cdot\hat{e}'|^2\rangle$ or the incoherent power is not zero. At the beginning of this section $\hat{e}'$ was specified to be in the same direction as $\hat{e}$, thus $\delta^1\vec{E}\cdot\hat{e}'$ represents $\delta^1 E_{hh}$ where the double-h subscript denotes horizontal polarization on transmission and reception. It is now possible to examine the consequences of cross-polarized sampling of the scattered field. In this case $\hat{e}'$ is orthogonal to $\hat{e}$; thus, $\hat{e}$ is horizontally polarized and $\hat{e}'$ is vertically polarized, i.e.

$$\hat{e} = \hat{e}_h = \sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

$$\hat{e}' = \hat{e}_v = -\cos\theta\cos\phi\,\hat{x} - \cos\theta\sin\phi\,\hat{y} + \sin\theta\,\hat{z}$$

Returning to (4.3) for this case, it is noted that $\Delta\vec{E}$ is still zero because $\vec{E}_i$ is tangential to S_o , $\Delta\vec{B}$ and $\Delta\vec{H}$ are still zero because there is no change in μ_r across S_o and the conductivity is assumed finite, and the problem reduces to evaluating $\Delta\vec{D}\cdot\vec{E}'$ on S_o . However, $\Delta\vec{D}$ has the direction $\hat{e}_h$ while $\vec{E}'$ is polarized in the $\hat{e}_v$ direction, consequently, $\hat{e}\cdot\hat{e}' = \hat{e}_h\cdot\hat{e}_v = 0$ and there is no depolarization by the surface. This is just a confirmation of the fact that first order perturbation theory does not lead to a depolarized scattered field when the roughness is small scale.

4.2.2 Vertical Polarization

The case of vertical polarization is a bit more algebraically involved because $\Delta\vec{E}$ is no longer zero across S_o . Both magnetic field discontinuities, $\Delta\vec{B}$ and $\Delta\vec{H}$, are still zero for the same reason as given above. Thus, (4.3) reduces to

$$\delta^1\vec{E}\cdot\hat{e}' = \frac{k_o^2 \exp(-j k_o R)}{4\pi R E_o \varepsilon_o} \int_{S_o} [\Delta\vec{E}\cdot\vec{D}' - \Delta\vec{D}\cdot\vec{E}']\zeta_s \, d S_o \qquad (4.13)$$

For vertical polarization, it is customary to use the incident magnetic field $\vec{H}_i$ as the source. Thus, the incident, reflected and transmitted magnetic fields on S_o are given by

$$\vec{H}_i = H_o \hat{e}_h \exp(-j \vec{k}_i\cdot\vec{r}_\perp)$$
$$\vec{H}_r = H_o R_v \hat{e}_h \exp(-j \vec{k}_r\cdot\vec{r}_\perp)$$
$$\vec{H}_t = H_o T_v \hat{e}_h \exp(-j \vec{k}_t\cdot\vec{r}_\perp)$$

where R_v and T_v are the Fresnel (field) reflection and transmission coefficients for vertical polarization. On S_o the corresponding electric fields are as follows;

$$\vec{E}_q = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \hat{k}_q \times \vec{H}_q \quad ; \quad E_t = -\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_r}}\ \hat{k}_t \times \vec{H}_t$$

where $q = i, r$ and $\hat{k}_i$, $\hat{k}_r$ and $\hat{k}_t$ specify the direction of propagation of the incident, reflected, and transmitted fields. Note that since $\vec{r}_\perp$ is on S_o ,

$$\vec{k}_i \cdot \vec{r}_\perp = \vec{k}_r \cdot \vec{r}_\perp = \vec{k}_t \cdot \vec{r}_\perp$$

which is merely a restatement of the fact that the angle of incidence equals the angle of reflection and Snell's law [9] is obeyed. For backscattering and similar polarization sampling of the scattered field, the primed fields are the same as those above.

Although somewhat cumbersome at this stage of the development, it is desirable to split the fields into components which are tangential to and normal to S_o . The reason for introducing this transformation is that it will be very useful in the composite surface development and it is therefore beneficial to obtain some facility with the technique on this easier problem. The normal to S_o is $\hat{n} = \hat{z}$ while the tangent will be taken as $\hat{\tau} = \hat{n} \times \hat{e}_h$. This particular choice of $\hat{\tau}$ is convenient because $\hat{\tau} \cdot \hat{e}_v$ selects the component of $\hat{e}_v$ that is tangent to the surface. Since $\Delta\vec{E} \cdot \tau = 0$ and $\Delta\vec{D} \cdot \hat{n} = 0$ on S_o , (4.13) simplifies to

$$\delta^1 \vec{E} \cdot \hat{e}_v = \frac{k_o^2 \exp(-j k_o R)}{4\pi R\ \varepsilon_o} \int_{S_o} \left[(\Delta\vec{E} \cdot \hat{n})(\vec{D}' \cdot \hat{n}) - (\Delta\vec{D} \cdot \hat{\tau})(\vec{E}' \cdot \hat{\tau}) \right] \zeta_s \, d S_o \tag{4.14}$$

where $\hat{e}'$ has been replaced by its equivalent $\hat{e}_v$ since like polarization sampling of the scattered field has been specified. The boundary conditions $\Delta\vec{E} \cdot \hat{\tau} = 0$ and $\Delta\vec{D} \cdot \hat{n} = 0$ should not be discarded because they will provide some useful relationships. From $\Delta\vec{E} \cdot \hat{\tau} = 0$ there results

$$\left[(\hat{k}_i \times \hat{e}_h) + R_v(\hat{k}_r \times \hat{e}_h)\right] \cdot \hat{\tau} = \frac{T_v}{\sqrt{\varepsilon_r}} (\hat{k}_t \times \hat{e}_h) \cdot \hat{\tau} \tag{4.15}$$

while $\Delta\vec{D}\cdot\hat{n} = 0$ yields

$$\left[(\hat{k}_i \times \hat{e}_h) + R_v(\hat{k}_r \times \hat{e}_h)\right] \cdot \hat{n} = \sqrt{\varepsilon_r}\; T_v(\hat{k}_t \times \hat{e}_h) \cdot \hat{n} \tag{4.16}$$

where $\Delta\vec{E} = \vec{E}_i + \vec{E}_r - \vec{E}_t$ and $\Delta\vec{D} = \vec{D}_i + \vec{D}_r - \vec{D}_t$ have been used along with (4.15) and $\vec{D} = \varepsilon \vec{E}$.

It is now necessary to determine the field quantities inside the integration in (4.14). The quantity $\Delta\vec{E}\cdot\hat{n}$ can be reduced to the following form through the use of (4.15);

$$\Delta\vec{E}\cdot\hat{n} = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\; H_o \frac{T_v}{\sqrt{\varepsilon_r}} (\varepsilon_r - 1) \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)(\hat{k}_t \times \hat{e}_h)\cdot\hat{n} \tag{4.17}$$

while (4.16) simplifies $\vec{D}'\cdot\hat{n}$ to

$$\vec{D}\cdot\hat{n} = -\sqrt{\mu_o\,\varepsilon_o}\; H_o T_v \sqrt{\varepsilon_r}\; \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)(\hat{k}_t \times \hat{e}_h)\cdot\hat{n} \tag{4.18}$$

so

$$(\Delta\vec{E}\cdot\hat{n})(\vec{D}'\cdot\hat{n}) = \mu_o H_o^2 T_v (\varepsilon_r - 1) \exp(-j\,2\vec{k}_i\cdot\vec{r}_\perp) \left[(\hat{k}_t \times \hat{e}_h)\cdot\hat{n}\right]^2 \tag{4.19}$$

Through similar manipulations,

$$\Delta\vec{D}\cdot\hat{\tau} = -\sqrt{\mu_o\varepsilon_o}\; H_o \frac{T_v}{\sqrt{\varepsilon_r}} (1 - \varepsilon_r) \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)\,(\hat{k}_t \times \hat{e}_h)\cdot\hat{\tau} \tag{4.20}$$

and

$$\vec{E}'\cdot\hat{\tau} = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\; H_o \frac{T_v}{\sqrt{\varepsilon_r}} \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)(\hat{k}_t \times \hat{e}_h)\cdot\hat{\tau} \tag{4.21}$$

so

$$(\Delta\vec{D}\cdot\hat{\tau})(\vec{E}'\cdot\hat{\tau}) = \mu_o H_o^2 \frac{T_v^2}{\epsilon_r} (1-\epsilon_r) \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)\left[(\hat{k}_t\times\hat{e}_h)\cdot\hat{\tau}\right]^2 \tag{4.22}$$

Combining (4.19) and (4.22) and completing the unit vector operations yields

$$(\Delta\vec{E}\cdot\hat{n})(\vec{D}'\cdot\hat{n}) - (\Delta\vec{D}\cdot\hat{\tau})(\vec{E}'\cdot\hat{\tau}) = \mu_o H_o^2\, T_v^2\, \frac{(\epsilon_r-1)}{\epsilon_r^2}\left[\epsilon_r + (\epsilon_r-1)\sin^2\theta\right]\exp(-j\,2\vec{k}_i\cdot\vec{r}_\perp) \tag{4.23}$$

where

$$T_v = \frac{2\epsilon_r\cos\theta}{\epsilon_r\cos\theta + \sqrt{\epsilon_r - \sin^2\theta}}$$

Substituting this result in (4.14) and recognizing that $E_o = \sqrt{\frac{\mu_o}{\epsilon_o}}\,H_o$ and $\hat{e}_v = -\hat{k}_i\times\hat{e}_h$, the final result is obtained

$$\delta\,\vec{E}\cdot\hat{e}_v = \frac{k_o^2\exp(-j\,k_oR)}{4\pi R}\sqrt{\frac{\mu_o}{\epsilon_o}}\,H_o\,T_v^2\,\frac{(\epsilon_r-1)}{\epsilon_r^2}\left[\epsilon_r + (\epsilon_r-1)\sin^2\theta\right]\iint \exp(-j2\vec{k}_i\cdot\vec{r}_\perp)\,\zeta_s\,dx\,dy \tag{4.24}$$

Essentially the same remarks apply to the vertically polarized scattered field as for the horizontal case. In addition, it should be noted that if (4.11) and (4.24) are converted to $\sigma°$ or the scattering cross section per unit area according to

$$\sigma°(\theta,\phi) = \lim_{R\to\infty}\lim_{A\to\infty}\left\{\frac{4\pi R^2}{A}\,\frac{\langle|\delta^1\vec{E}|^2\rangle}{E_o^2}\right\}$$

where A is the illuminated area, the result is identical to the result obtained by Peake using the Rayleigh-Rice approach [1].

4.3 Bistatic Scattering From A Dielectric Surface With Small Roughness

For bistatic scattering the unit vectors specifying the directions of incidence of the unprimed and primed fields are given by (see Figure 4-2)

$$\hat{k}_i = -\sin\theta_i \cos\phi_i\ \hat{x} - \sin\theta_i \sin\phi_i\ \hat{y} - \cos\theta_i\ \hat{z}$$
$$\hat{k}_i' = -\sin\theta_s \cos\phi_s\ \hat{x} - \sin\theta_s \sin\phi_s\ \hat{y} - \cos\theta_s\ \hat{z} \qquad (4.25)$$

and $\vec{k}_i = k_o \hat{k}_i$, $\vec{k}_i' = k_o \hat{k}_i'$. The unit vectors specifying the directions of horizontal and vertical polarizations for the primed and unprimed fields are as follows;

$$\hat{e}_h = -\sin\phi_i\ \hat{x} + \cos\phi_i\ \hat{y}$$
$$\hat{e}_h' = -\sin\phi_s\ \hat{x} + \cos\phi_s\ \hat{y}$$
$$\hat{e}_v = -\cos\theta_i\ \cos\phi_i\ \hat{x} - \cos\theta_i\ \sin\phi_i\ \hat{y} + \sin\theta_i\ \hat{z} \qquad (4.26)$$
$$\hat{e}_v' = -\cos\theta_s\ \cos\phi_s\ \hat{x} - \cos\theta_s\ \sin\phi_s\ \hat{y} + \sin\theta_s\ \hat{z}$$

The normal to the unperturbed surface is $\hat{n} = \hat{z}$ while the tangents to the surface for vertical polarizations are given by

$$\hat{\tau} = \hat{n} \times \hat{e}_h \qquad \hat{\tau}' = \hat{n} \times \hat{e}_h' \qquad (4.27)$$

Equations (4.25)-(4.27) describe the basic quantities that will be required in this section.

4.3.1 Horizontal Polarization

Since $\Delta\vec{B}$ and $\Delta\vec{H}$ are zero, because there is no change in μ_r across S_o and the conductivity is finite, (4.3) becomes

$$\delta^1\vec{E} \cdot \hat{e}_h' = \frac{k_o^2 \exp(-j\,k_o R)}{4\pi R E_o\ \varepsilon_o} \int_S [\Delta\vec{E}\cdot\vec{D}' - \Delta\vec{D}\cdot\vec{E}']\ \zeta_s\, d\,S_o \qquad (4.28)$$

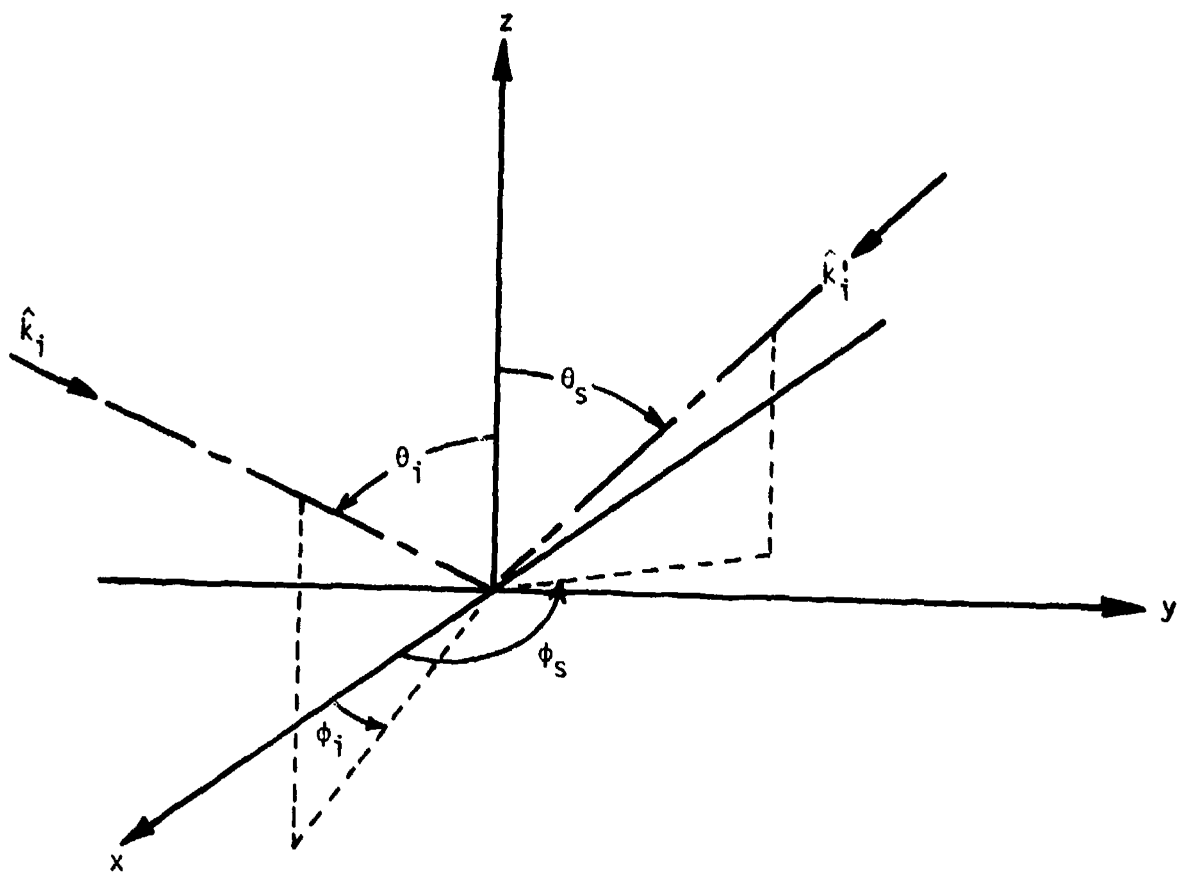

Figure 4-2. Geometry for bistatic scattering from a randomly rough surface having only small scale roughness ζ_s .

Furthermore, $\Delta\vec{E}$ is also zero across S_o because $\vec{E}$ is tangential to S_o. Thus, (4.28) becomes

$$\delta^1\vec{E}\cdot\hat{e}_h' = -\frac{k_o^2 \exp(-j k_o R)}{4\pi R E_o \varepsilon_o} \int_S [\Delta\vec{D}\cdot\vec{E}'] \zeta_s \, dS_o \tag{4.29}$$

On the surface S_o, the incident, reflected, and transmitted electric fields (unprimed and primed) are as follows;

$$\begin{aligned}
\vec{E}_i &= E_o \exp(-j \vec{k}_i\cdot\vec{r}_\perp)\hat{e}_h & \vec{E}_i' &= E_o \exp(-j \vec{k}_i'\cdot\vec{r}_\perp)\hat{e}_h' \\
\vec{E}_r &= E_o R_h \exp(-j \vec{k}_r\cdot\vec{r}_\perp)\hat{e}_h & \vec{E}_r' &= E_o R_h' \exp(-j \vec{k}_r'\cdot\vec{r}_\perp)\hat{e}_h' \\
\vec{E}_t &= E_o T_h \exp(-j \vec{k}_t\cdot\vec{r}_\perp)\hat{e}_h & \vec{E}_t' &= E_o T_h' \exp(-j \vec{k}_t'\cdot\vec{r}_\perp)\hat{e}_h'
\end{aligned} \tag{4.30}$$

where also on the surface S_o,

$$\vec{k}_i\cdot\vec{r}_\perp = \vec{k}_r\cdot\vec{r}_\perp = \vec{k}_t\cdot\vec{r}_\perp \qquad \vec{k}_i'\cdot\vec{r}_\perp = \vec{k}_r'\cdot\vec{r}_\perp = \vec{k}_t'\cdot\vec{r}_\perp \tag{4.31}$$

and the same notation as introduced earlier has been continued. The fields $\Delta\vec{D}$ and $\vec{E}'$ on S_o are as follows;

$$\Delta\vec{D} = \varepsilon_o E_o (1 + R_h - \varepsilon_r T_h) \exp(-j \vec{k}_i\cdot\vec{r}_\perp)\hat{e}_h$$

$$\vec{E}' = E_o (1 + R_h') \exp(-j \vec{k}_i'\cdot\vec{r}_\perp)\hat{e}_h'$$

Using the fact that $1 + R_h = T_h$, the product $\Delta\vec{D}\cdot\vec{E}'$ becomes

$$\Delta\vec{D}\cdot\vec{E}' = \varepsilon_o E_o^2 (1 + R_h)(1 + R_h')(1 - \varepsilon_r)(\hat{e}_h\cdot\hat{e}_h') \exp\left[-j(\vec{k}_i + \vec{k}_i')\cdot\vec{r}_\perp\right]$$

and substituting this result in (4.29) yields

$$\delta^1\vec{E}\cdot\hat{e}_h' = \frac{k_o^2 \exp(-jk_oR)}{4\pi R} E_o (1+R_h)(1+R_h')(\varepsilon_r - 1)(\hat{e}_h\cdot\hat{e}_h') \iint \exp\left[-j(\vec{k}_i+\vec{k}_i')\cdot\vec{r}_\perp\right]\zeta_s \, dxdy \tag{4.32}$$

This result can be easily translated into the angles (θ_i, ϕ_i) and (θ_s, ϕ_s) by the use of (4.25) and

$$\hat{e}_h \cdot \hat{e}_h' = \cos(\phi_i - \phi_s)$$

along with

$$1 + R_h = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\epsilon_r - \sin^2\theta_i}} \qquad 1 + R_h' = \frac{2\cos\theta_s}{\cos\theta_s + \sqrt{\epsilon_r - \sin^2\theta_s}}$$

For ease of comparison, it should be noted that most results similar to (4.32) express the direction of scattering as $\hat{k}_s$ which, in the above notation, is $-\hat{k}_i'$. Equation (4.32) yields $\delta^1 E_{hh'}$. For cross polarized sampling of the scattered field, the problem becomes somewhat more involved and it will be discussed in Section 4.3.3. It should be noted that when $\phi_s = \phi_i$ (backscattering), (4.32) reduces to the result obtained in Section 4.2.1.

4.3.2 Vertical Polarization

Since $\hat{n} \cdot \vec{E}$ and $\hat{\tau} \cdot \vec{D}$ are discontinuous across the unperturbed surface, (4.3) reduces to

$$\delta^1 \vec{E} \cdot \hat{e}_v' = \frac{k_o^2 \exp(-jk_o R)}{4\pi R E_o \epsilon_o} \int_{S_o} \left[(\Delta\vec{E} \cdot \hat{n})(\vec{D}' \cdot \hat{n}) - (\Delta\vec{D} \cdot \hat{\tau})(\vec{E}' \cdot \hat{\tau}')(\hat{\tau} \cdot \hat{\tau}') \right] \zeta_s \, dS_o \qquad (4.33)$$

where the scalar product $\hat{\tau} \cdot \hat{\tau}'$ must be included because the unit vectors $\hat{\tau}$ and $\hat{\tau}'$ are not necessarily parallel, e.g. see (4.27). The incident, reflected, and transmitted unprimed and primed magnetic field quantities are given by

$$\vec{H}_i = H_o \exp(-j\vec{k}_i \cdot \vec{r}_\perp)\hat{e}_h \qquad \vec{H}_i' = H_o \exp(-j\vec{k}_i \cdot \vec{r}_\perp)\hat{e}_h'$$

$$\vec{H}_r = H_o R_v \exp(-j\vec{k}_r \cdot \vec{r}_\perp)\hat{e}_h \qquad \vec{H}_r' = H_o R_v' \exp(-j\vec{k}_r' \cdot \vec{r}_\perp)\hat{e}_h'$$

$$\vec{H}_t = H_o T_v \exp(-j\vec{k}_t \cdot \vec{r}_\perp)\hat{e}_h \qquad \vec{H}_t' = H_o T_v' \exp(-j\vec{k}_t' \cdot \vec{r}_\perp)\hat{e}_h' \qquad (4.34)$$

while the corresponding electric fields are

$$\vec{E}_i = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \hat{k}_i \times \vec{H}_i \qquad \vec{E}_i' = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \hat{k}_i' \times \vec{H}_i'$$

$$\vec{E}_r = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \hat{k}_r \times \vec{H}_r \qquad \vec{E}_r' = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \hat{k}_r' \times \vec{H}_r' \tag{4.35}$$

$$\vec{E}_t = -\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_r}}\ \hat{k}_t \times \vec{H}_t \qquad \vec{E}_t' = -\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_r}}\ \hat{k}_t' \times \vec{H}_t'$$

and $\vec{D} = \varepsilon \vec{E}$, $\vec{D}' = \varepsilon \vec{E}'$. The same notation introduced in Section 4.3.2 is continued here. From the boundary condition that the tangential component of the electric field be continuous across S_o , i.e. $\Delta\vec{E}\cdot\hat{\tau} = 0$ and $\delta\vec{E}'\cdot\hat{\tau}' = 0$, the following relationships result;

$$(\hat{k}_i \times \hat{e}_h)\cdot\hat{\tau} + R_h(\hat{k}_r \times \hat{e}_h)\cdot\hat{\tau} = \frac{T_v}{\sqrt{\varepsilon_r}}\ (\hat{k}_t \times \hat{e}_h)\cdot\hat{\tau}$$
$$(\hat{k}_i \times \hat{e}_h')\cdot\hat{\tau}' + R_h'(\hat{k}_r' \times \hat{e}_h')\cdot\hat{\tau}' = \frac{T_v'}{\sqrt{\varepsilon_r}}(\hat{k}_t' \times \hat{e}_h')\cdot\hat{\tau}' \tag{4.36}$$

Similarly from the continuity of the normal component of the $\vec{D}$-field across S_o , i.e. $\Delta\vec{D}\cdot\hat{n} = 0$ and $\Delta\vec{D}'\cdot\hat{n} = 0$, there results

$$(\hat{k}_i \times \hat{e}_h)\cdot\hat{n} + R_v(\hat{k}_r \times \hat{e}_h)\cdot\hat{n} = \sqrt{\varepsilon_r}\ T_v(\hat{k}_t \times \hat{e}_h)\cdot\hat{n}$$
$$(\hat{k}_i' \times \hat{e}_h')\cdot\hat{n} + R_v'(\hat{k}_r' \times \hat{e}_h')\cdot\hat{n} = \sqrt{\varepsilon_r}\ T_v'(\hat{k}_t' \times \hat{e}_h')\cdot\hat{n} \tag{4.37}$$

Using (4.37) to simplify the expressions for $\Delta\vec{E}\cdot\hat{n}$ and $\vec{D}'\cdot\hat{n}$ yields

$$\Delta\vec{E}\cdot\hat{n} = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\; H_o \frac{T_v}{\sqrt{\varepsilon_r}} (\varepsilon_r - 1)\exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)(\hat{k}_t\times\hat{e}_h)\cdot\hat{n} \tag{4.38}$$

and

$$\vec{D}'\cdot\hat{n} = -\sqrt{\mu_o\,\varepsilon_o}\; H_o \sqrt{\varepsilon_r}\; T_v' \exp(-j\,\vec{k}_i'\cdot\vec{r}_\perp)(\hat{k}_t'\times\hat{e}_h')\cdot\hat{n} \tag{4.39}$$

In a similar fashion, (4.36) is used to simplify the expressions for $\Delta\vec{D}\cdot\hat{\tau}$ and $\vec{E}'\cdot\hat{\tau}'$ with the following result:

$$\Delta\vec{D}\cdot\hat{\tau} = \sqrt{\mu_o\,\varepsilon_o}\; H_o \frac{T_v}{\sqrt{\varepsilon_r}} \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)(\hat{k}_t\times\hat{e}_h)\cdot\hat{\tau} \tag{4.40}$$

and

$$\vec{E}'\cdot\hat{\tau}' = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\; H_o \frac{T_v'}{\sqrt{\varepsilon_r}} \exp(-j\,\vec{k}_i'\cdot\vec{r}_\perp)(\hat{k}_t'\times\hat{e}_h')\cdot\hat{\tau}' \tag{4.41}$$

Substituting (4.38) - (4.41) in (4.33), noting that $\hat{\tau}\cdot\hat{\tau}' = \cos(\phi_i - \phi_s)$, and simplifying the unit vector operations yields the following result for $\delta^1\vec{E}\cdot\hat{e}_v'$;

$$\delta^1\vec{E}\cdot\hat{e}_v' = \sqrt{\frac{\mu_o}{\varepsilon_o}}\; H_o \frac{\exp(-jk_oR)}{4\pi R}\, T_v T_v' \frac{(\varepsilon_r - 1)}{\varepsilon_r^2}\Big\{\varepsilon_r \sin\theta_i \sin\theta_s + \sqrt{(\varepsilon_r - \sin^2\theta_i)(\varepsilon_r - \sin^2\theta_s)}$$

$$\cdot \cos(\phi_i - \phi_s)\Big\} \iint \exp\left[-j(\vec{k}_i + \vec{k}_i')\cdot\vec{r}_\perp\right]\zeta_s \, dx\,dy \tag{4.42}$$

where $E_o = H_o\sqrt{\mu_o/\varepsilon_o}$ and

$$T_v = 1 + R_v = \frac{2\,\varepsilon_r \cos\theta_i}{\varepsilon_r \cos\theta_i + \sqrt{\varepsilon_r - \sin^2\theta_i}}$$

and $T_v' = T_v(\theta_i \to \theta_s)$. Equation (4.42) is $\delta^1 E_{vv'}$ and it is easily shown to reduce to the result for backscattering in Section 4.2.2. when $\phi_i = \phi_s$ and $\theta_i = \theta_s$. Once again it should be emphasized that the most difficult part of obtaining (4.42) is evaluating the terms $(\hat{k}_t \times \hat{e}_h)\cdot\hat{\tau}$ and $(\hat{k}_t' \times \hat{e}_h')\cdot\hat{\tau}'$ in (4.40) and (4.41). As noted previously this is a consequence of the fact that the difficult analysis was finished once (4.3) was derived and the actual evaluation of (4.3) is very straightforward. Finally, comparing the σ° values resulting from (4.32) and (4.42) with the corresponding results obtained from the Rayleigh-Rice theory [10] shows complete agreement.

4.3.3 Cross Polarization

As shown in Sections 4.2.1 and 4.2.2, depolarization for scattering by small scale roughness is a second order effect in the plane of incidence. If, however, the scattered field outside of the plane of incidence is computed, it will be found to have a nonzero cross polarized component. This result is simply a consequence of the fact that the unit vectors $\hat{e}_h'$ and $\hat{e}_v'$ are not fixed with respect to the surface-centered coordinate system and they change their directions as the observation point moves out of the plane of incidence. This, of course, is a purely geometrical effect and it has nothing to do with any change in the basic scattering mechanism.

The derivation of the results follows essentially the same pattern as set forth in the previous sections. There is one point that should be noted because it simplifies the algebra somewhat. For the case of the incident field horizontally polarized $(\hat{e}_h)$ and the scattered field vertically polarized $(\hat{e}_v')$, the unprimed field quantities should be obtained from $\vec{E}_i = E_o \exp(-j\,\vec{k}_i\cdot\vec{r}_\perp)\hat{e}_h$ while the primed fields should be obtained from $\vec{H}_i' = H_o \exp(-j\,\vec{k}_i'\cdot\vec{r}_\perp)\hat{e}_h'$. For the incident field vertically polarized $(\hat{e}_v)$

and the scattered field horizontally polarized $(\hat{e}_h')$, the unprimed fields are obtained from $\vec{H}_i = H_o \exp(-j\vec{k}_i \cdot \vec{r}_\perp)\hat{e}_h$ while the primed fields are to be derived from $\vec{E}_i' = E_o \exp(-j\vec{k}_i' \cdot \vec{r}_\perp)\hat{e}_h'$. This approach is consistent with the technique of obtaining all field quantities from the horizontal $(\hat{e}_h$ or $\hat{e}_h')$ field for planar surface reflection.

With $E_o = H_o\sqrt{\mu_o/\varepsilon_o}$, the following expressions for $\delta^1 E_{hv'}$ and $\delta^1 E_{vh'}$ result;

$$\delta^1 E_{hv'} = \delta^1\vec{E}\cdot\hat{e}_v' = \frac{k_o^2 \exp(-jk_oR)}{\pi R} E_o \left\{ \frac{\cos\theta_i \cos\theta_s \sin(\phi_s-\phi_i)(\varepsilon_r-1)\sqrt{\varepsilon_r-\sin^2\theta_s}}{(\cos\theta_i+\sqrt{\varepsilon_r+\sin^2\theta_i})(\varepsilon_r\cos\theta_s+\sqrt{\varepsilon_r+\sin^2\theta_s})} \right\}$$

$$\cdot\iint \exp\left[-j(\vec{k}_i+\vec{k}_i')\cdot\vec{r}_\perp\right]\zeta_s \, dx\, dy \qquad (4.43)$$

$$\delta^1 E_{vh'} = \delta^1\vec{E}\cdot\hat{e}_h' = \frac{k_o^2 \exp(-jk_oR)}{\pi R}\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_o \left\{ \frac{\cos\theta_i \cos\theta_s \sin(\phi_s-\phi_i)(\varepsilon_r-1)\sqrt{\varepsilon_r-\sin^2\theta_i}}{(\cos\theta_s+\sqrt{\varepsilon_r-\sin^2\theta_s})(\varepsilon_r\cos\theta_i+\sqrt{\varepsilon_r-\sin^2\theta_i})} \right\}$$

$$\cdot\iint \exp\left[-j(\vec{k}_i+\vec{k}_i')\cdot\vec{r}_\perp\right]\zeta_s \, dx\, dy \qquad (4.44)$$

where, in summary, the angles are defined in Figure 4.2, $\vec{r}_\perp = x\hat{x} + y\hat{y}$, and $\vec{k}_i$ and $\vec{k}_i'$ are defined as $k_o\hat{k}_i$ and $k_o\hat{k}_i'$, respectively, where $\hat{k}_i$ and $\hat{k}_i'$ are given in (4.25).

When comparing (4.43) and (4.44) with the cross polarized scattered fields resulting from the Rayleigh-Rice approach [10], ϕ_i should be set equal to π. The expression for $\delta^1 E_{vh'}$ agrees with the results in [10, pg. 706]. The expression for $\delta^1 E_{hv'}$ is, however, the negative of the α_{hv} coefficient in [10, pg. 706, eqn. 9.1-69]. Normally this difference is not important because

$\delta^1 E_{hv'}$ is squared and then averaged to find the incoherent power. However, if one is dealing with circular polarization the sign does become critical. To resolve this issue, a special case can be constructed whereby α_{hv} should agree with α_{vv} or $\delta^1 E_{hv'} = \delta^1 E_{vv}$. This special case involves taking $\phi_i = \pi$ and $\phi_s = 0$ and $\theta_s = 0$ in the expression for α_{vv} or $\delta^1 E_{vv'}$ and comparing this result with α_{hv} and $\delta^1 E_{hv'}$ for $\phi_i = \pi$, $\phi_s = \pi/2$, and $\theta_s = 0$. In this special case both $\delta^1 E_{vv'}$ and $\delta^1 E_{hv'}$ should be polarized in the $-\hat{x}$-direction. Comparing (4.42) and (4.43) for this special case shows that indeed $\delta^1 E_{vv'} = \delta^1 E_{hv'}$. However, evaluating α_{vv} and α_{hv} from [10] results in $\alpha_{vv} = -\alpha_{hv}$; consequently, there does appear to be a sign error in the expression for α_{hv} and (4.43) is correct.

This section completes the development for scattering from a dielectric surface having only a small scale roughness. Once again it should be emphasized that the purposes of Sections 4.2 and 4.3 are (1) to check the Burrows perturbation approach against the conventional Rayleigh-Rice results and (2) to illustrate the actual mechanics of evaluating the Burrows expression for the first order perturbation field. Hopefully, this latter purpose, if achieved, should considerably simplify the transition to the composite surface case.

4.4 Bistatic Scattering From A Dielectric Surface With Composite Roughness

For small scale roughness superposed on a planar surface, the Burrows perturbation formula (4.3) is particularly easy to evaluate. This results from the fact that one deals with the fields on an infinite planar surface and, for such a surface, the fields are easily described and related through the Fresnel coefficients, Snell's law, and the equality of the angles of incidence and reflection. For a composite surface, the unperturbed surface is not planar but it is assumed to be very gently undulating. More specifically, the unperturbed surface is actually defined such that it contains no spatial frequency components which are smaller than γk_o , where γ is a constant which is greater than unity. Of course, it is desirable to have γ as large as possible but this is not always practical since the small scale height must satisfy $4k_o^2 \overline{\zeta_s^2} \ll 1$ [6]. However, if γ can be made sufficiently large then the scattering from the unperturbed surface can be treated using physical optics. Physical optics assumes that the surface may be considered to be locally planar and the fields on the surface can be accurately approximated using Fresnel theory. This approach is recognized to be essentially the same as the small scale roughness on a planar surface problem. The one important difference is that for the gently undulating unperturbed surface, the local normal is no longer entirely z-directed and, in fact, depends upon the slopes of the large scale surface. This means that one must construct a local coordinate system on the undulating unperturbed surface and compute the surface fields required in (4.3) in terms of this system. This must be done for both the unprimed and primed fields because they have different angles and directions of incidence for the general bistatic case.

For the unprimed fields, the important unit vectors are $\hat{k}_i$ and $\hat{n}_\ell$ which is the normal to the <u>large</u> scale or unperturbed surface. These two quantities are important because they form the local plane of incidence. One next constructs unit vectors $\hat{e}_{\ell h}$ and $\hat{e}_{\ell v}$ which are orthogonal and parallel, respectively, to $\hat{k}_i$ <u>and</u> $\hat{n}_\ell$. These unit vectors are also horizontally and vertically polarized, respectively, with respect to the <u>local</u> plane of incidence. Any arbitrarily polarized unprimed incident field can now be decomposed into components parallel to $\hat{e}_{\ell h}$ and $\hat{e}_{\ell v}$ since the incident field must be transverse to $\hat{k}_i$. The unprimed field quantities required in (4.3) can then be computed as in the previous sections. The exact same construction of $\hat{e}'_{\ell h}$ and $\hat{e}'_{\ell v}$ and the decomposition of the primed incident field must be performed in order to compute the primed fields required in (4.3). Fortunately, this is easily accomplished by simply changing θ_i to θ_s and ϕ_i to ϕ_s in the unprimed quantities.

Before getting into the actual details, there are a few other points that should be noted. All of the above noted manipulations are going to lead to the following changes in $\delta^1\vec{E}$ obtained in equation (16) of [6]. First, the factor $\Gamma_{pp'}$ is going to depend on ε_r, the angles and directions of incidence and scattering $(\theta_i, \phi_i, \theta_s, \phi_s)$, and the slopes of the large scale or unperturbed surface $(\zeta_{\ell x}, \zeta_{\ell y})$. The only other change is that the exponential inside the surface integral will become $\exp[-j(\vec{k}_i + \vec{k}_i') \cdot \vec{r}_\ell]$ where $\vec{r}_\ell = x\hat{x} + y\hat{y} + \zeta_\ell \hat{z}$ because of the generalization to bistatic scattering. Except for correcting [6] to properly include shadowing, as detailed in Section 2, all other aspects of the solution presented in [6] remain the same. Combining $\delta^1\vec{E}$ from this analysis with Sancer's result [11] for essentially $\delta^0\vec{E}$ yields the total scattered field. Furthermore, it should be expected that for backscattering the dielectric nature of the surface should have an almost negligible effect upon the wavenumber at which the surface height spectrum is partitioned into large and small scale sub-spectra. Finally, because there is no discontinuity in magnetic properties across the unperturbed surface and the conductivity is assumed to be finite, $\Delta\vec{B}$ and $\Delta\vec{H}$ in (4.3) will be zero. This fact holds true regardless of any tilting of the locally planar surface.

The first task at hand is to construct $\hat{e}_{\ell h}$ and $\hat{e}_{\ell v}$. Since $\hat{e}_{\ell h}$ is orthogonal to both $\hat{k}_i$ and $\hat{n}_\ell$, it is given by

$$\hat{e}_{\ell h} = \frac{\hat{k}_i \times \hat{n}_\ell}{|\hat{k}_i \times \hat{n}_\ell|} \tag{4.45}$$

where $\hat{k}_i = k_{ix}\hat{x} + k_{iy}\hat{y} + k_{iz}\hat{z}$ and the k_{iq}, $q = x,y$ and z, are obvious from (4.25). The normal to the large scale surface is given by

$$\hat{n}_\ell = \frac{-\zeta_{\ell x}\hat{x} - \zeta_{\ell y}\hat{y} + \hat{z}}{\sqrt{1 + \zeta_{\ell x}^2 + \zeta_{\ell y}^2}} = n_{\ell x}\hat{x} + n_{\ell y}\hat{y} + n_{\ell z}\hat{z} \tag{4.46}$$

Expanding the cross product in (4.45) yields

$$\hat{e}_{\ell h} = h_{\ell x}\hat{x} + h_{\ell y}\hat{y} + h_{\ell z}\hat{z} \tag{4.47}$$

where

$$h_{\ell x} = \frac{(k_{iy} n_{\ell z} - k_{iz} n_{\ell y})}{|\hat{k}_i \times \hat{n}_\ell|} \qquad h_{\ell y} = \frac{(k_{iz} n_{\ell x} - k_{ix} n_{\ell z})}{|\hat{k}_i \times \hat{n}_\ell|}$$

$$h_{\ell z} = \frac{(k_{ix} n_{\ell y} - k_{iy} n_{\ell x})}{|\hat{k}_i \times \hat{n}_\ell|}$$

and $\hat{e}_{\ell h}$ is completely determined. For the unit vector $\hat{e}_{\ell v}$, the expressions are more involved because $\hat{e}_{\ell v}$ must be in the plane form by $\hat{k}_i$ and $\hat{n}_\ell$ and also orthogonal to $\hat{e}_{\ell h}$. For the reflected and transmitted fields $\hat{k}_i$ goes to $\hat{k}_r$ and $\hat{k}_t$. This will not change the direction of $\hat{e}_{\ell h}$ because $\hat{k}_i$, $\hat{k}_r$ and $\hat{k}_t$ are all coplanar. This will, however, alter the direction of $\hat{e}_{\ell v}$; this is easily understood by noting that $\hat{k}_q$, $\hat{e}_{\ell h}$, and $\hat{e}_{\ell v}$ form a mutually orthogonal triad of unit vectors. Thus, if $\hat{e}_{\ell h}$ does not change direction but $\hat{k}_q$ does, then $\hat{e}_{\ell v}$ must necessarily change direction. What this means is that we must find a new $\hat{e}_{\ell v}$ for each value of $\hat{k}_q$. This is easily done by the following equality;

$$\hat{e}_{\ell v}^{q} = \hat{e}_{\ell h} \times \hat{k}_q \tag{4.48}$$

where $q = i$, r, and t. Note that it is not necessary to divide the rhs of (4.48) by the magnitude because it is unity, i.e. $\hat{e}_{\ell h}$ and $\hat{k}_q$ are mutually orthogonal by (4.45). The unit vector $\hat{e}_{\ell v}^{q}$ may also be written as

$$\hat{e}_{\ell v}^{q} = v_{\ell x}^{q}\,\hat{x} + v_{\ell y}^{q}\,\hat{y} + v_{\ell z}^{q}\,\hat{z} \tag{4.49}$$

where

$$v_{\ell x}^{q} = -(k_{qy}\,h_{\ell z} - k_{qz}\,h_{\ell y}) \qquad\qquad v_{\ell y}^{q} = -(k_{qz}\,h_{\ell x} - k_{qx}\,h_{\ell z})$$

$$v_{\ell z}^{q} = -(k_{qx}\,h_{\ell y} - k_{qy}\,h_{\ell x})$$

and $\hat{e}_{\ell v}^{q}$ is completely determined for $q = i$, r, and t. For the primed fields, $\hat{e}_{\ell h}'$ and $\hat{e}_{\ell v}^{q'}$ are obtained by merely replacing $\vec{k}_q$ by $\vec{k}_q'$, $q = i$, r, and t, in the expressions for $\hat{e}_{\ell h}$ and $\hat{e}_{\ell v}$.

The incident, reflected, and transmitted unprimed fields on the unperturbed surface will now be decomposed into locally horizontal and vertical components. If the incident field is of the form $\vec{E}_i = E_o \exp(-j\,\vec{k}_i \cdot \vec{r}_\perp)\,\hat{e}_a$ on the unperturbed surface where $\hat{e}_a$ is its polarization direction then $\vec{E}_i$, $\vec{E}_r$ and $\vec{E}_t$ can be written as follows;

$$\begin{aligned}
\vec{E}_i &= E_h^i\,\hat{e}_{\ell h} + E_v^i\,\hat{e}_{\ell v}^i \\
\vec{E}_r &= E_h^r\,\hat{e}_{\ell h} + E_v^r\,\hat{e}_{\ell v}^r \\
\vec{E}_t &= E_h^t\,\hat{e}_{\ell h} + E_v^t\,\hat{e}_{\ell v}^t
\end{aligned} \tag{4.50}$$

where

$$\begin{aligned}
E_h^q &= E_o \exp(-j\,\vec{k}_q \cdot \vec{r}_\perp)(\hat{e}_a \cdot \hat{e}_{\ell h}) \\
E_v^q &= E_o \exp(-j\,\vec{k}_q \cdot \vec{r}_\perp)(\hat{e}_a \cdot \hat{e}_{\ell v}^q)
\end{aligned} \tag{4.51}$$

and $q = i, r, t$. The corresponding magnetic fields are given by

$$\vec{H}_q = \sqrt{\frac{\varepsilon_o}{\mu_o}}\;\hat{k}_q \times \vec{E}_q\;\frac{\sqrt{\varepsilon_r}}{\delta_{qt}} \tag{4.52}$$

where $\delta_{qt} = \sqrt{\varepsilon_r}$ for $q = i$ and r, and $\delta_{tt} = 1$. Expanding (4.52) yields

$$\vec{H}_i = \sqrt{\frac{\varepsilon_o}{\mu_o}} \left(- E_h^i \, \hat{e}^i_{\ell v} + E_v^i \, \hat{e}_{\ell h} \right)$$

$$\vec{H}_r = \sqrt{\frac{\varepsilon_o}{\mu_o}} \left(- E_h^r \, \hat{e}^r_{\ell v} + E_v^r \, \hat{e}_{\ell h} \right)$$

$$\vec{H}_t = \sqrt{\frac{\varepsilon_o \varepsilon_r}{\mu_o}} \left(- E_h^t \, \hat{e}^t_{\ell v} + E_v^t \, \hat{e}_{\ell h} \right)$$

From Fresnel theory, the $\hat{e}_{\ell h}$-component of $\vec{H}_r$ is equal to R_{v_ℓ} * times the $\hat{e}_{\ell h}$-component of $\vec{H}_i$, so

$$E_v^r = R_{v_\ell} E_v^i \tag{4.53}$$

Similarly, the $\hat{e}_{\ell h}$-component of $\vec{H}_t$ is equal to T_{v_ℓ} times the $\hat{e}_{\ell h}$-component of $\vec{H}_i$, so

$$E_v^t = \frac{T_{v_\ell}}{\sqrt{\varepsilon_r}} E_v^i \tag{4.54}$$

Equations (4.53) and (4.54) can now be used in (4.50) to express all of the fields in terms of E_v^i and E_h^i, i.e.

$$\begin{aligned} \vec{E}_i &= E_h^i \, \hat{e}_{\ell h} + E_v^i \, \hat{e}^i_{\ell v} \\ \vec{E}_r &= R_{h_\ell} E_h^i \, \hat{e}_{\ell h} + R_{v_\ell} E_v^i \, \hat{e}^r_{\ell v} \\ \vec{E}_t &= T_{h_\ell} E_h^i \, \hat{e}_{\ell h} + \frac{T_{v_\ell}}{\sqrt{\varepsilon_r}} E_v^i \, \hat{e}^t_{\ell v} \end{aligned} \tag{4.55}$$

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*The subscripts "ℓ" on R_v, R_h, T_v, T_h means that the angles in the appropriate Fresnel formulas must be defined with respect to the normal to the large scale surface, $\hat{n}_\lambda$.

where E_h^i and E_v^i are given by (4.52) with $q = i$. The corresponding $\vec{D}$-fields are; $\vec{D}_i = \varepsilon_o \vec{E}_i$, $\vec{D}_r = \varepsilon_o \vec{E}_r$, $\vec{D}_t = \varepsilon_o \varepsilon_r \vec{E}_t$. The primed fields may be obtained from (4 55) and (4.51) by replacing $\hat{k}_q$ by $\hat{k}_q'$, $\hat{e}_{\ell h}$ by $\hat{e}_{\ell h}'$, $\hat{e}_{\ell v}^q$ by $\hat{e}_{\ell v}^{q'}$ and by changing $\hat{e}_a$ to whatever scattered field polarization is to be sampled, say, $\hat{e}_b$.

The appropriate form of (4.3) is

$$\delta' \vec{E} \cdot \hat{e}_b = \frac{k_o^2 \exp(-j k_o R)}{4\pi R E_o \varepsilon_o} \int_{S_o} \left[(\Delta\vec{E} \cdot \hat{n}_\ell)(\vec{D}' \cdot \hat{n}_\ell) - \Delta\vec{D} \cdot \vec{E}' \right] \zeta_s \, d S_o \tag{4.56}$$

where the shadowing factor has been temporarily omitted from the integrand since it can be added at the end of the development. The $(\Delta\vec{E} \cdot \hat{n}_\ell)$ is equal to $(\vec{E}_i + \vec{E}_r - \vec{E}_t) \cdot \hat{n}_\ell$ or using (4.55) and noting that $\hat{n}_\ell \cdot \hat{e}_{\ell h} = 0$,

$$\Delta\vec{E} \cdot \hat{n}_\ell = E_v^i \left(\hat{e}_{\ell v}^i \cdot \hat{n}_\ell + R_{v_\ell} \, \hat{e}_{\ell v}^r \cdot \hat{n}_\ell - \frac{T_{v_\ell}}{\sqrt{\varepsilon_r}} \, \hat{e}_{\ell v}^t \cdot \hat{n}_\ell \right) . \tag{4.57}$$

Equation (4.57) can be simplified somewhat by using the relationship that results from $\Delta\vec{D} \cdot \hat{n} = 0$. The final result is

$$\Delta\vec{E} \cdot \hat{n}_\ell = E_v^i \, \frac{(\varepsilon_r - 1)}{\sqrt{\varepsilon_r}} \, T_{v_\ell} \, (\hat{e}_{\ell v}^t \cdot \hat{n}_\ell) \tag{4.58}$$

For $\vec{D}' \cdot \hat{n}_\ell$,

$$\vec{D}' \cdot \hat{n}_\ell = E_v^{i'} \, \varepsilon_o \sqrt{\varepsilon_r} \, T_{v_\ell}' \, (\hat{e}_{\ell v}^{t'} \cdot \hat{n}_\ell) \tag{4.59}$$

so the product of (4.58) and (4.59) can be written as follows

$$(\Delta\vec{E} \cdot \hat{n}_\ell)(\vec{D}' \cdot \hat{n}_\ell) = E_o^2 \exp\left[-j(\vec{k}_i + \vec{k}_i') \cdot \vec{r}_\perp \right] \varepsilon_o (\varepsilon_r - 1) \, T_{v_\ell}' \, T_{v_\ell} (\hat{e}_{\ell v}^t \cdot \hat{n}_\ell)(\hat{e}_{\ell v}^{t'} \cdot \hat{n}_\ell) \tag{4.60}$$

It can be shown that

$$\hat{e}^{t}_{\ell v} \cdot \hat{n}_{\ell} = \frac{\sin\theta_{\ell i}}{\sqrt{\varepsilon_r}}$$

and

$$\hat{e}^{t'}_{\ell v} \cdot \hat{n}_{\ell} = \frac{\sin\theta'_{\ell i}}{\sqrt{\varepsilon_r}}$$

where

$$\sin\theta_{\ell i} = \sqrt{1-(-\hat{k}_i \cdot \hat{n}_{\ell})^2}$$

$$\sin\theta'_{\ell i} = \sqrt{1-(-\hat{k}'_i \cdot \hat{n}_{\ell})^2}$$

so (4.60) becomes

$$(\Delta\vec{E} \cdot \hat{n}_{\ell})(\vec{D}' \cdot \hat{n}_{\ell}) = E_o^2 \varepsilon_o (\varepsilon_r - 1) T'_{v_\ell} T_{v_\ell} \frac{\sqrt{1-(-\hat{k}_i \cdot \hat{n}_{\ell})^2}\ \sqrt{1-(-\hat{k}'_i \cdot \hat{n}_{\ell})^2}}{\varepsilon_r} \exp\left[-j(\hat{k}_i + \vec{k}'_i) \cdot \vec{r}_{\ell}\right] \tag{4.61}$$

For the remaining term in (4.56), the important parts are $(\Delta\vec{D})_p$ and $(\vec{E}')_p$ where the p-subscript denotes tangential to the large scale surface, i.e. the normal component of $\vec{D}$ is continuous across the boundary so $\Delta\vec{D} \cdot \hat{n}_{\ell} = 0$. The tangential components of the $\Delta\vec{D}$-field can be found by decomposing $\Delta\vec{D}$ into components directed along $\hat{e}_{\ell h}$ and τ_{ℓ}, where $\tau_{\ell} = \hat{n}_{\ell} \times \hat{e}_{\ell h}$, i.e.

$$(\Delta\vec{D})_p = (\Delta\vec{D} \cdot \hat{e}_{\ell h})\, \hat{e}_{\ell h} + (\Delta\vec{D} \cdot \hat{\tau}_{\ell})\, \hat{\tau}_{\ell} \tag{4.62}$$

Using

$$\Delta\vec{D} = \varepsilon_o E^i_h \left(1 + R_{h_\ell} - \varepsilon_r T_{h_\ell}\right) \hat{e}_{\ell h} + \varepsilon_o E^i_v \left(\hat{e}^i_{\ell v} + R_{v_\ell} \hat{e}^r_{\ell v} - T_v \sqrt{\varepsilon_r}\, \hat{e}^t_{\ell v}\right)$$

and simplifying this expression with the aid of

$$(\hat{e}^{\,i}_{\ell v}\cdot\hat{\tau}_\ell) + R_{v_\ell}(\hat{e}^{\,r}_{\ell v}\cdot\hat{\tau}_\ell) = \frac{T_{v_\ell}}{\sqrt{\varepsilon_r}}(\hat{e}^{\,t}_{\ell v}\cdot\hat{\tau})$$

which results from $(\Delta\vec{E})_p = 0$ yields

$$(\Delta\vec{D})_p = -\varepsilon_o E_h^i (1+R_{h_\ell})(\varepsilon_r - 1)\,\hat{e}_{\ell h} - \frac{\varepsilon_o}{\sqrt{\varepsilon_r}} E_v^i T_{v_\ell}(\varepsilon_r - 1)(\hat{e}^{\,t}_{\ell v}\cdot\hat{\tau}) \tag{4.63}$$

The appropriate expression for $(\vec{E}')_p$ is

$$(\vec{E}')_p = E_h^{i'} T'_{h_\ell}\,\hat{e}'_{\ell h} + E_v^{i'}\frac{T'_{v_\ell}}{\sqrt{\varepsilon_r}}(\hat{e}^{\,t'}_{\ell v}\cdot\hat{\tau}'_\ell)\hat{\tau}'_\ell \tag{4.64}$$

since on the unperturbed surface $\vec{E}' = \vec{E}'_i + \vec{E}'_r = \vec{E}'_t$. The relationship for $(\Delta\vec{D})_p(\vec{E}')_p$ is obtained by taking the dot product of (4.63) with (4.64). Combining this result with (4.61) and substituting into (4.56) yields the following result;

$$\delta^1 E_{ab} = \frac{E_o k_o^2 \exp(-jk_oR)}{\pi R}\iint \Gamma_{ab}(\zeta_{\ell x},\zeta_{\ell y})I(x,y)\exp\left[-j(\hat{k}_i+\hat{k}_i')\cdot\vec{r}_\ell\right]\zeta_s\,dx\,dy \tag{4.65}$$

where $\delta^1 E_{ab}$ is the scattered first order perturbation field for an incident polarization $\hat{e}_a$ and scattered polarization $\hat{e}_b$, $I(x,y)$ is unity on the illuminated parts of the surface and zero for the shadowed parts, and

$$\Gamma_{ab} = \frac{(\epsilon_r-1)}{4}\sqrt{1+\zeta_{\ell_x}^2+\zeta_{\ell_y}^2}\left\{(\hat{e}_a\cdot\hat{e}^{\,i}_{\ell v})(\hat{e}_b\cdot\hat{e}^{\,i'}_{\ell v})\left(\frac{T_{v_\ell}T'_{v_\ell}}{\epsilon_r^2}\right)\left[\epsilon_r \sin\theta_{\ell i}\sin\theta'_{\ell i}\right.\right.$$

$$\left. + \sqrt{\epsilon_r-\sin^2\theta_{\ell i}}\ \sqrt{\epsilon_r-\sin^2\theta'_{\ell i}}\left(\frac{\hat{k}_i\cdot\hat{k}'_i-(\hat{n}_\ell\cdot\hat{k}_i)(\hat{n}_\ell\cdot\hat{k}'_i)}{\sin\theta_{\ell i}\sin\theta'_{\ell i}}\right)\right] + (\hat{e}_a\cdot\hat{e}_{\ell h})(\hat{e}_b\cdot\hat{e}'_{\ell h})T_{h_\ell}T'_{h_\ell}$$

$$\cdot\left[\frac{\hat{k}_i\cdot\hat{k}'_i-(\hat{n}_\ell\cdot\hat{k}_i)(\hat{n}_\ell\cdot\hat{k}'_i)}{\sin\theta_{\ell i}\ \sin\theta'_{\ell i}}\right] + \left[(\hat{e}_a\cdot\hat{e}^{\,i}_{\ell v})(\hat{e}_b\cdot\hat{e}'_{\ell h})T'_{h_\ell}\left(\frac{T_{v_\ell}}{\epsilon_r}\right)\sqrt{\epsilon_r-\sin^2\theta_{\ell i}}\right.$$

$$\left.\left. + (\hat{e}_b\cdot\hat{e}^{\,i'}_{\ell v})(\hat{e}_a\cdot\hat{e}_{\ell h})T_{h_\ell}\left(\frac{T'_{v_\ell}}{\epsilon_r}\right)\sqrt{\epsilon_r-\sin^2\theta'_{\ell i}}\right]\left[\frac{\hat{n}_\ell\cdot(\hat{k}_i\times\hat{k}'_i)}{\sin\theta_{\ell i}\sin\theta'_{\ell i}}\right]\right\} \quad (4.66)$$

For convenience, the above terms are summarized below

$\hat{e}_a$ = Polarization of the incident electric field $(\vec{E}_i = E_o^i\,\hat{e}_a)$

$\hat{e}_b$ = Polarization of the scattered electric field $(\vec{E}_s = E_o^s\,\hat{e}_b)$

$$\hat{n}_\ell = \frac{-\zeta_{\ell x}\hat{x} - \zeta_{\ell y}\hat{y} + \hat{z}}{\sqrt{1+\zeta_{\ell x}^2+\zeta_{\ell y}^2}}$$

$$\vec{k}_i = k_o\hat{k}_i \quad ; \quad \hat{k}_i = -\sin\theta_i\cos\phi_i\,\hat{x} - \sin\theta_i\sin\phi_i\,\hat{y} - \cos\theta_i\,\hat{z}$$

$$\vec{k}'_i = k_o\hat{k}'_i \quad ; \quad \hat{k}'_i = -\sin\theta_s\cos\phi_s\,\hat{x} - \sin\theta_s\sin\phi_s\,\hat{y} - \cos\theta_s\,\hat{z}$$

$$\hat{e}_{\ell h} = \frac{\hat{k}_i\times\hat{n}_\ell}{|\hat{k}_i\times\hat{n}_\ell|} \qquad \hat{e}'_{\ell h} = \frac{\hat{k}'_i\times\hat{n}_\ell}{|\hat{k}'_i\times\hat{n}_\ell|} \qquad \hat{e}^{\,i}_{\ell v} = \hat{e}_{\ell h}\times\hat{k}_i \qquad \hat{e}^{\,i'}_{\ell v} = \hat{e}'_{\ell h}\times\hat{k}'_i$$

$$\sin\theta_{\ell i} = |-\hat{k}_i\times\hat{n}_\ell| \qquad \sin\theta'_{\ell i} = |-\hat{k}'_i\times\hat{n}_\ell|$$

$$\vec{r}_\ell = x\,\hat{x} + y\,\hat{y} + \zeta_\ell\,\hat{z} \qquad \cos\theta_{\ell i} = (-\hat{k}_i)\cdot\hat{n}_\ell \qquad \cos\theta'_{\ell i} = (-\hat{k}'_i)\cdot\hat{n}_\ell$$

$$T_{v_\ell} = \frac{2\epsilon_r \cos\theta_{\ell i}}{\epsilon_r \cos\theta_{\ell i} + \sqrt{\epsilon_r - \sin^2\theta_{\ell i}}} \qquad T'_{v_\ell} = \frac{2\epsilon_r \cos\theta'_{\ell i}}{\epsilon_r \cos\theta'_{\ell i} + \sqrt{\epsilon_r - \sin^2\theta'_{\ell i}}}$$

$$T_{h_\ell} = \frac{2\cos\theta_{\ell i}}{\cos\theta_{\ell i} + \sqrt{\epsilon_r - \sin^2\theta_{\ell i}}} \qquad T'_{h_\ell} = \frac{2\cos\theta'_{\ell i}}{\cos\theta'_{\ell i} + \sqrt{\epsilon_r - \sin^2\theta'_{\ell i}}}$$

Attempts have been made to compare (4.66) with the equivalent factor resulting from the "tilted-plane" approach [5] but, unfortunately, the correspondence is not easy to establish. It appears that such a comparison might best be accomplished by comparing numerical values of (4.66) with the corresponding factor from the "tilted-plane" approach [5].

Equation (4.65) is the desired result. From this expression, one can easily obtain σ°_{ab} from

$$\sigma^\circ_{ab} = \lim_{R\to\infty} \lim_{A\to\infty} \left\{ \frac{4\pi R^2}{A} \frac{1}{E_o^2} \left[\langle|\delta^o E_{ab}|^2\rangle + \langle|\delta^1 E_{ab}|^2\rangle \right] \right\} \tag{4.67}$$

along with the development given in [6] and as corrected in Section 2. The contribution of the zeroth order incoherent power $\langle|\delta^o E_{ab}|^2\rangle$ has been previously obtained by Sancer [11] and his results can be used directly in (4.67).

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5.0 A USEFUL RELATIONSHIP FOR THE JOINT SLOPE PROBABILITY DENSITY FUNCTION

5.1 Background

The incoherent power scattered in and about the specular direction depends upon the joint probability density function (jpdf) of the large scale slopes. The jpdf for the large scale slopes is also important in determining the degree of "tilt" or k-space broadening imparted to the small scale Bragg scatterers. Ideally, one would like to measure the jpdf for the large scale slopes, the roughness spectrum of the small scale heights, and the complex dielectric constant of the surface in order to predict the average scattering properties of a specified section of terrain. That is, these surface measurements would be substituted in the rough surface scattering model which, in turn, would provide an estimate of the average coherent and incoherent scattered power. From a practical point of view, measurements of the jpdf for the slopes and the small scale roughness spectrum are very difficult to obtain and the difficulty increases as the radar or electromagnetic wavelength decreases. For example, in the case of an L-band system with λ_o = 30 cm, the small scale part of the scattering model will require surface height spectral measurements of surface undulations having wavelengths of less than about 90 cm because $\lambda_{BRAGG} = (\lambda_o/2)\csc\theta_i$ for backscatter. For the large scale features of the surface, the jpdf for the slopes representing surface features having spatial wavelengths greater than about 90 cm is required. Obviously, spectral information on the small scale features is going to be the most difficult to obtain. However, even the jpdf for the large scale slopes is going to be difficult to estimate. It is not unreasonable to expect that we can obtain measurements of the jpdf for the large scale <u>heights</u> and even the correlation function for the <u>heights</u> at least somewhat close to the 90 cm spatial resolution. However, this information must somehow be translated into the jpdf for the <u>surface slopes</u> and this is where the difficulty comes in. We ignore the possibility of a <u>direct</u> measurement of the jpdf of the large scale slopes for arbitrary terrain because such a task appears to be too difficult to even contemplate.

The question basically boils down to the feasibility of translating or converting measurements of the jpdf for the surface <u>heights</u> into the jpdf for the surface <u>slopes</u>. The purpose of this section is to point out an analytical means for accomplishing this transformation and suggest that the scheme be attempted on an experimental basis. The relationship is not new and, in fact, results from some earlier rough surface scattering analysis. However, it has apparently gone unnoticed at least insofar as it applies to this very real world problem of translating the height jpdf into the slope jpdf.

5.2 The Transformation

Perhaps the oldest approach to estimating the quasi-specular incoherent power scattered by a rough surface is now called the autocorrelation approach. Basically, one assumes the validity of physical optics, interchanges the order of spatial integration and ensemble averaging in the expression for the scattered power, and assumes Gaussian surface statistics with the final result that the average scattered power is dependent upon the behavior of the surface height correlation function near $|\Delta\vec{r}| = 0$[1] . In the mid-60's, Kodis [2] showed that the average scattered power could alternatively be interpreted in terms of the number of specular points on the surface and the absolute radii of curvature at the specular points. Barrick [3] subsequently linked these two approaches in the high frequency limit where both are valid.

In the process of establishing the similarity between the autocorrelation and specular point approaches, Barrick obtained a relationship between the jpdf's of the surface heights and the surface slopes. In particular, if $P_{\zeta_x\zeta_y}(\zeta_x,\zeta_y)$ is the jpdf of the x and y surface slope components and $\Phi_{\zeta_1\zeta_2}(k_x,k_y)$ is the joint characteristic function for the surface heights $\zeta_1(x_1,y_1)$ and $\zeta_2(x_2,y_2)$ then [3]

$$P_{\zeta_x\zeta_y}\left(\zeta_x = -\frac{q_x}{q_z}, \zeta_y = -\frac{q_y}{q_z}\right) = \frac{q_z^2}{4\pi^2} \lim_{k_o\to\infty}\left\{\iint_{-\infty}^{\infty} \Phi_{\zeta_1\zeta_2}(k_x = k_o q_z, k_y = -k_o q_z; \Delta_x, \Delta_y)\right.$$

$$\left. \cdot \exp(j\, k_o q_x \Delta x + j\, k_o q_y \Delta_y) d\Delta x\, d\Delta y \right\} \qquad (5.1)$$

where $\Delta x = x_1 - x_2$ and $\Delta y = y_1 - y_2$. In (5.1) the quantities q_x, q_y and q_z are limited as follows; $|q_i| \le 1$, $i = x,y,z$. It should be noted that $\Phi_{\zeta_1\zeta_2}(\cdot)$ is an implicit function of the surface height correlation function; this is how the $(\Delta x, \Delta y)$ variation comes about in (5.1).

If an analytical form for $\Phi_{\zeta_1\zeta_2}$ is available then (5.1) can be used directly to obtain the jpdf for the slopes. In cases where $\Phi_{\zeta_1\zeta_2}(\cdot)$ is obtained from measured data, it is not immediately obvious that (5.1) is of any practical use since the behavior of the joint height characteristic function will not be known in the limit of $k_x\to\infty$ and $k_y\to\infty$. However, consider the following reasoning as a means for obtaining estimates of $P_{\zeta_x\zeta_y}(\cdot,\cdot)$.

Since $\Phi_{\zeta_1\zeta_2}$ is the two-dimensional Fourier transform of the jpdf for the height, it can be obtained numerically by using a Fast Fourier Transform (FFT) on the measured jpdf height data. The result of this operation will be denoted by $\hat{\Phi}_{\zeta_1\zeta_2}$. Because of measurement noise and particularly quantization noise in the measured height jpdf data, $\hat{\Phi}_{\zeta_1\zeta_2}$ will be limited to values less than, say, $k_x \le K_x$ and $k_y \le K_y$. The maximum value of k_o that can be achieved in (5.1) is therefore $\max(K_x/q_z, K_y/q_z)$. If q_z is small then the resulting maximum value of k_o can be very large. The transform variables in (5.1) will be given by $K_x q_x/q_z$ and $K_y q_y/q_z$ which may also be large, depending upon q_x and q_y. Thus, if q_z is near zero and $\hat{P}_{\zeta_x\zeta_y}$ is computed using the following;

$$\hat{P}_{\zeta_x\zeta_y}\left(\zeta_x = -\frac{q_x}{q_z}, \zeta_y = -\frac{q_y}{q_z}\right) = \frac{q_z^2}{4\pi^2} \int\int_{-L}^{L} \hat{\Phi}_{\zeta_1\zeta_2}(k_x = K_x, k_y = K_y; \Delta x, \Delta y)$$

$$\cdot \exp\left(j \frac{K_x q_x}{q_z} \Delta x + j \frac{K_y q_x}{q_z} \Delta y\right) d\Delta x \, d\Delta y \qquad (5.2)$$

it may turn out that $\hat{P}_{\zeta_x\zeta_y}$ is a sufficiently good estimate of $P_{\zeta_x\zeta_y}$ as to be useful in the scattering model. Unfortunately, this approach breaks down when q_x or $q_y = 0$; however, it may be possible to get close enough to $q_x = 0$ or $q_y = 0$ to infer the behavior of $P_{\zeta_x\zeta_y}$ along these lines in the q_x, q_y-plane. The limits on the integrations in (5.2) symbolically denoted as $\pm L$, will be determined by the correlation length of the surface, i.e. the separation distance for which the surface height correlation function is essentially zero.

An alternate approach to estimating $P_{\zeta_x\zeta_y}$ is to examine the asymptotic behavior of $\hat{\Phi}_{\zeta_1\zeta_2}$ as $k_x \to K_x$ and $k_y \to K_y$. From this behavior, it may be possible to generate an asymptotic <u>functional</u> dependence of $\hat{\Phi}_{\zeta_1\zeta_2}$ on k_x and k_y. By repeating this procedure for different values of Δx and Δy, it might be possible to also generate or build-in the functional dependence of $\hat{\Phi}_{\zeta_1\zeta_2}$ on Δx and Δy. In this manner, the dependence of $\hat{\Phi}_{\zeta_1\zeta_2}$ upon k_x, k_y, Δ_x, and Δ_y is obtained at least in the limit of moderately large k_x and k_y. This functional form could then be transformed according to (5.2). The major problem here is that the accuracy of the result will depend directly upon how precisely the surface height correlation function is known near $\Delta x = 0$ and $\Delta y = 0$. This statement results from the fact that the behavior of the transform of a function as $k \to \infty$ is directly determined by the behavior of the function as $\Delta r \to 0$ [4].

The problem of converting height jpdf data into slope jpdf results is definitely not easy. Even with the use of (5.1) the problem still poses a number of numerical complexities, primarily because of the required limit as $k_o \to \infty$. However, as discussed above, (5.1) does provide some hope in solving what is otherwise a totally untractable problem. It is felt there is sufficient hope as to warrant further investigation of the utility of (5.1) in the solution of this problem.

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6.0 A NEW APPROACH TO COHERENT SCATTERING FROM A PERFECTLY CONDUCTING RANDOMLY ROUGH SURFACE

6.1 Background

Among those involved with the applications of rough surface scattering theory, the statement is frequently made that coherent scattering is reasonably well understood and adequate models exist for the phenomenon. Because of the paucity of electromagnetic scattering data [1], one must go to the acoustic field to appreciate just how truly erroneous this statement is! The acoustic data [2,3] show that for scattering from an agitated water surface all models are accurate for small Rayleigh roughness parameter. However, as either the frequency or surface roughness is increased or the angle of incidence is decreased, the data show a significantly stronger scattered field than is predicted by physical optics and the inclusion of shadowing in the model only makes the situation worse [4,5]. The acoustic experiments are important because they were designed in such a manner as to eliminate one postulated reason for why early electromagnetic data did not agree with the physical optics model [6]. A model based upon pure geometric optics has been developed [7] but it tends to overestimate the mean scattered field. Furthermore, this analysis appears to be based upon a questionable transition from a single sinusoid surface to a random surface and it provides no justification for the use of geometric optics for a situation which is clearly fraught with diffraction and multiple scattering effects. DeSanto [8] has formally solved the problem through the use of a diagram expansion method for calculating the stochastic Green's function for the rough surface. DeSanto's results became even more significant when he recently showed [9] that the first correction term to the physical optics result did indeed increase the level of the average scattered field. Unfortunately, it is difficult to interpret the physical basis of the higher order correction terms, each of which involve the solution of an integral equation whose complexity increases with order. The need for further analytical and experimental research on this problem is therefore still great.

The purpose of this section is to present a new approach to the coherent scattering problem based upon averaging the magnetic field integral equation describing the current induced on a perfectly conducting surface by an incident field. The motivation for this return to fundamentals is as follows. First, it is desirable to investigate solutions to stochastic scattering problems which do not require an arbitrary closure assumption. Second, it is absolutely essential to have a solution wherein mathematical simplifications can be put into one to one correspondence with physical approximations. Finally, acquiring a better understanding of the coherent scattering problem is vital to the accurate modeling of rough surface multipath effects.

6.2 Analysis

The analysis will be presented in two phases. In the first phase the surface roughness will be assumed to be arbitrarily distributed. In the second phase, the surface roughness will be assumed to comprise a Gaussian process. Restriction of the problem to a Gaussian surface permits the detailed examination of certain simplifying assumptions and also the comparison with DeSanto's [10] results.

6.2.1 Preliminaries

The rough surface is assumed to be perfectly conducting and infinite in extent. The surface roughness $\zeta(x,y)$ is stipulated to comprise a zero mean statistically homogeneous process with the mean surface equal to the $z = 0$ plane. In the following development, position vectors will be denoted by

$\vec{r} = \vec{r}_t + z\hat{z}$ with $\vec{r}_t = x\hat{x} + y\hat{y}$ and for a point on the surface $z = \zeta(x,y)$. Using an $\exp(j\omega t)$ time convention, the incident magnetic field is given by

$$\vec{H}^i(\vec{r}) = H_o \hat{h} \exp(-j \vec{k}_i \cdot \vec{r})$$

where for vertical polarization $\hat{h} = \hat{y}$, while for horizontal polarization $\hat{h} = \sin\theta_i \hat{z} - \cos\theta_i \hat{x}$ and the karat symbol denotes a unit vector. The field is assumed to be incident along the positive x-axis so the incident azimuth angle is also zero, $\phi_i = 0$. The incident wavevector is given by

$$\vec{k}_i = -k_o(\sin\theta_i \hat{x} + \cos\theta_i \hat{z})$$

where θ_i is the angle measured from the z-axis or the normal to the mean surface and $k_o = 2\pi/\lambda_o$ is the free space wavenumber.

The current $\vec{J}_s$ induced on the surface S_o by the incident magnetic field must satisfy the magnetic field integral equation (MFIE), i.e.

$$\vec{J}_s(\vec{r}) = 2\hat{n}(\vec{r}) \times \vec{H}^i(\vec{r}) + \frac{1}{2\pi} \hat{n}(\vec{r}) \times \int_{S_o} \vec{J}_s(\vec{r}_o) \times \nabla_o g(|\vec{r} - \vec{r}_o|) dS_o \tag{6.1}$$

for $\vec{r} \in S$. In (6.1) $\hat{n}(\vec{r})$ is the upward directed unit normal to the surface and $g(|\vec{r} - \vec{r}_o|)$ is the free space scalar Green's function where, in expanded form,

$$\hat{n}(\vec{r}) = \frac{-\zeta_x \hat{x} - \zeta_y \hat{y} + \hat{z}}{\sqrt{1 + \zeta_x^2 + \zeta_y^2}}$$

$$g(|\vec{r} - \vec{r}_o|) = \frac{\exp(-j k_o |\vec{r} - \vec{r}_o|)}{|\vec{r} - \vec{r}_o|}$$

It should be noted that the gradient operating on g in (6.1), when evaluated on the surface $z_o = \zeta_o(x_o, y_o)$, treats the random height as if it were independent of the coordinates x_o and y_o. Expanding the double cross product in (6.1), converting the surface integration to an integration over the $z_o = 0$ plane through $dS_o = \sqrt{1 + \zeta_{x_o}^2 + \zeta_{y_o}^2}\, d\vec{r}_{t_o}$, and multiplying both sides of (6.1) by $\sqrt{1 + \zeta_x^2 + \zeta_y^2}$ yields the following,*

$$\vec{J}(\vec{r}) = 2\underset{\sim}{\hat{n}}(\vec{r}) \times \vec{H}^i(\vec{r}) + \frac{1}{2\pi} \int \left\{ [\underset{\sim}{\hat{n}}(\vec{r}) \cdot \nabla_o g]\, \vec{J}(\vec{r}_o) - [\underset{\sim}{\hat{n}}(\vec{r}) \cdot \vec{J}(\vec{r}_o)] \nabla_o g \right\} d\vec{r}_{t_o} \qquad (6.2)$$

where

$$\vec{J}(\vec{r}) = \sqrt{1 + \zeta_x^2 + \zeta_y^2}\; \vec{J}_s(\vec{r}) \qquad (6.3a)$$

$$\underset{\sim}{\hat{n}}(\vec{r}) = -\zeta_x \hat{x} - \zeta_y \hat{y} + \hat{z} \qquad (6.3b)$$

and the integration is over the entire $z_o = 0$ plane. For future reference, the quantity $\vec{J}(\vec{r})$ will be called the equivalent flat plane current because it is referenced to the $z = 0$ plane. Using the fact that $\vec{J}_s(\vec{r}_o)$ must be tangential to the surface and $\sqrt{1 + \zeta_x^2 + \zeta_y^2} > 0$, there results

$$J_z(\vec{r}_o) = \zeta_{x_o}\, J_x(\vec{r}_o) + \zeta_{y_o}\, J_y(\vec{r}_o) \qquad (6.4)$$

Equation (6.4) can be substituted in the right side of (6.2) to yield coupled integral equations for $J_x(\vec{r})$ and $J_y(\vec{r})$. The coupling is a consequence of the term $[\underset{\sim}{\hat{n}}(\vec{r}) \cdot \vec{J}(\vec{r}_o)] \nabla_o g$ which, with the substitution of (6.4) in (6.2) yields the following x and y-components;

---

*All limits on the integrals in this section are $(-\infty, \infty)$ so they will not be explicitly shown.

$$[\underset{\sim}{\hat{n}}(\vec{r})\cdot\vec{J}(\vec{r}_o)]\,\frac{\partial g}{\partial \eta_o} = -\left\{(\zeta_x - \zeta_{x_o})J_x(\vec{r}_o) + (\zeta_y - \zeta_{y_o})J_y(\vec{r}_o)\right\}\frac{\partial g}{\partial \eta_o}$$

where η_o is x_o or y_o. It should be noted that these terms are proportional to the difference in slopes at $\vec{r}_t$ and $\vec{r}_{t_o}$. Thus, if the surface is very gently undulating, these terms should be very small. In the analysis to follow, these terms will be ignored; thus, the problem reduces to the assumption of no depolarization or the case of a surface having slopes which are very slowly varying with $\vec{r}_t$. Actually, the analysis can be carried through for the vector or coupled equation problem in essentially the same manner as to be presented here. However, because it does tend to symbolically complicate the equations it is better to introduce the approach with the scalar problem.

6.2.2 Arbitrarily Distributed Roughness

Ignoring the $\underset{\sim}{\hat{n}}\cdot\vec{J}$ term in (6.2) yields

$$J_q(\vec{r}) = 2\hat{q}\cdot[\underset{\sim}{\hat{n}}(\vec{r})\times\vec{H}^i(\vec{r})] + \frac{1}{2\pi}\int\left\{-\zeta_x\frac{\partial g}{\partial x_o} - \zeta_y\frac{\partial g}{\partial y_o} + \frac{\partial g}{\partial \zeta_o}\right\}J_q(\vec{r}_o)\,d\vec{r}_{t_o} \tag{6.5}$$

where $\hat{q} = \hat{x}$ or $\hat{y}$. Computation of the equivalent flat plane current is not truly the desired end result; what is really sought is the average scattered field $\langle E_s\rangle$ which in the Fraunhofer zone is proportional to

$$\int\left\langle J(\vec{r})\exp(jk_{sz}\zeta)\right\rangle\exp\left[j(k_{sx}x + k_{sy}y)\right]\,d\vec{r}_t \tag{6.6}$$

(where it has been assumed that the averaging operation denoted by $\langle\cdot\rangle$ and the surface integration can be interchanged). The averaging operation in (6.6) implies an average over ζ and <u>all other</u> random variables upon which $J(\vec{r})$ depends. Clearly, from (6.5), $J(\vec{r})$ depends on the slopes ζ_x and ζ_y; furthermore, experience indicates that $J(\vec{r})$ should also depend upon the curvature components of the surface $(\zeta_{xx},\zeta_{xy},\zeta_{yy})$. In point of fact, $J(\vec{r})$ evaluated at $\vec{r}_t$ depends upon all higher order derivatives of the surface height which are correlated with the surface height and slopes evaluated at $\vec{r}_{t_o}$. This means that, for the general case, J depends upon all higher order derivatives of the surface height. Thus, in order to accomplish the average in (6.6) the quantity $J(\vec{r})\exp(jk_{sz}\,\zeta)$ must be multiplied by the single point joint probability density function

$$p_1(\zeta,\nabla\zeta,\nabla^2\zeta,\nabla^3\zeta,\cdots)$$

and averaged over $\zeta,\nabla\zeta,\nabla^2\zeta,\cdots$, where $\nabla^n\zeta$ is a symbolic notation for all n^{th} order derivatives of the surface height evaluated at the point $\vec{r}_t$. That is,

$$\langle J(\vec{r})\exp(jk_{sz}\,\zeta)\rangle = \iint\cdots\int J(\vec{r})\exp(jk_{sz}\,\zeta)\,p_1(\zeta,\nabla\zeta,\nabla^2\zeta,\cdots)\cdot d\zeta d\nabla\zeta\cdots \tag{6.7}$$

The right side of (6.7) can also be written as the convolutions of the infinite dimensional Fourier transforms of $J(\vec{r})$ and $p_1(\cdot)$ as follows;

$$\langle\cdot\rangle = \lim_{n\to\infty}\frac{1}{(2\pi)^{n!}}\iint\cdots\int \tilde{J}(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots\vec{\beta}_n)\Phi_1(k_{sz}-\beta_1,-\vec{\beta}_2,-\vec{\beta}_3,\cdots,-\vec{\beta}_n)\cdot d\beta_1\,d\vec{\beta}_2\,d\vec{\beta}_3\cdots d\vec{\beta}_n \tag{6.8}$$

where $\vec{\beta}_n$ is an n-dimensional "vector" and

$$\tilde{J}(\beta_1,\vec{\beta}_2,\cdots,\vec{\beta}_n) = \iint\cdots\int J(\vec{r})\exp\left[j(\beta_1\zeta+\vec{\beta}_2\cdot\nabla\zeta+\cdots+\vec{\beta}_n\cdot\nabla^{n-1}\zeta)\right]\cdot d\zeta\,d\nabla\zeta\cdots d\nabla^{n-1}\zeta \tag{6.9}$$

$$\Phi_1(k_{sz}-\beta_1,-\vec{\beta}_2,\cdots,-\vec{\beta}_n) = \int\int\cdots\int p_1(\zeta,\nabla\zeta,\cdots,\nabla^{n-1}\zeta)\exp\Big[j(k_{sz}-\beta_1)\zeta$$

$$-j(\vec{\beta}_2\cdot\nabla\zeta+\cdots+\vec{\beta}_n\cdot\nabla^{n-1}\zeta)\Big]\ d\zeta\, d\nabla\zeta\cdots d\nabla^{n-1}\zeta \qquad (6.10)$$

The notation is a bit cumbersome here, so explicitly writing a few terms may be helpful, e.g.

$$\vec{\beta}_2\cdot\nabla\zeta = \beta_x\zeta_x + \beta_y\zeta_y$$

$$\vec{\beta}_3\cdot\nabla^2\zeta = \beta_{xx}\zeta_{xx} + \beta_{xy}\zeta_{xy} + \beta_{yy}\zeta_{yy}$$

$$\vec{\beta}_4\cdot\nabla^3\zeta = \beta_{xxx}\zeta_{xxx} + \beta_{xxy}\zeta_{xxy} + \beta_{xyy}\zeta_{xyy} + \beta_{yyy}\zeta_{yyy}$$

Since Φ_1 is the Fourier transform of p_1, it is the <u>single point joint characteristic function</u> for the random surface. Note also that $\tilde{J}$ is the Fourier transform of J with respect to <u>all</u> the <u>random variables</u> upon which it depends (an infinite number in general).

According to (6.8), $\tilde{J}$ is required in order to compute the average scattered field. This suggests (6.5) should be multiplied by

$$\exp(j\, k_1\,\zeta + j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta) \qquad (6.11)$$

and then averaged. By expressing the averaging integrations as convolutions of Fourier transforms, an integral equation for $\tilde{J}$ in $(k_1,\vec{k}_2,\vec{k}_3,\cdots)$-space can be obtained. The average of the term on the left side of (6.5) is given by

$$\left\langle J_q(\vec{r})\exp\left(jk_1\zeta + j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta\right)\right\rangle = \int\cdots\int J_q(\vec{r})\exp(jk_1\zeta + j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta)$$

$$\cdot\ p_1(\zeta,\nabla\zeta,\cdots)d\zeta\, d\nabla\zeta\cdots$$

$$= \lim_{n\to\infty}\frac{1}{(2\pi)^{n!}}\int\cdots\int \tilde{\underset{\sim}{J}}_q(\vec{r}_t,\beta_1,\vec{\beta}_2,\cdots)\,\Phi_1(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\cdots)$$

$$\cdot\ d\beta_1\, d\vec{\beta}_2\cdots \tag{6.12}$$

The source term on the right side of (6.5) can be written as follows;

$$2\hat{q}\cdot[\underset{\sim}{\hat{n}}\times\vec{H}^i(\vec{r})] = 2H_o[C_o + C_x\zeta_x + C_y\zeta_y]\exp(-jk_{iz}\zeta - j\vec{k}_i\cdot\vec{r}_t) \tag{6.13}$$

where C_o, C_x and C_y are determined by the polarization of $\vec{H}^i$ but are independent of the random surface variables. Thus, the average of the product of this term and (6.11) is given by

$$\left\langle 2\hat{q}\cdot[\underset{\sim}{\hat{n}}\times\vec{H}^i(\vec{r})]\right\rangle = 2H_o\exp(-j\vec{k}_i\cdot\vec{r}_t)\left[C_o + jC_x\partial_{k_{2x}} + jC_y\partial_{k_{2y}}\right]$$

$$\cdot\ \Phi_1(k_1-k_{iz},\vec{k}_2,\vec{k}_3,\cdots) \tag{6.14}$$

where $\vec{k}_2$ is symbolic for the variables k_{2x} and k_{2y}. The $(2\pi)^{-(n!)}$ term does not appear in (6.14) because no convolutions are required - only straightforward Fourier transforms. The symbols $\partial_{k_{2x}}$ and $\partial_{k_{2y}}$ denote the partial derivative operators $\partial/\partial k_{2x}$ and $\partial/\partial k_{2y}$, respectively.

The average of the product of the integral term on the right side of (6.5) and the exponential factor in (6.11) is somewhat more involved than the averages of the other terms in (6.5). First, the <u>two point joint probability density function</u>

$$p_2(\zeta,\zeta_o,\nabla\zeta,\nabla_o\zeta_o,\nabla^2\zeta,\nabla_o^2\zeta_o,\cdots ;\ \Delta\vec{r}_t = \vec{r}_t - \vec{r}_{t_o})$$

must be used because of the additional set of random variables $\zeta_o, \nabla_o\zeta_o, \nabla_o^2\zeta_o, \cdots$ in this term. Thus, the average can be written as follows;

$$\left\langle \int \left\{ -\zeta_x \frac{\partial g}{\partial x_o} - \zeta_y \frac{\partial g}{\partial y_o} + \frac{\partial g}{\partial \zeta_o} \right\} J_q(\vec{r}_o) d\vec{r}_{t_o} \exp\left(jk_1\zeta + j \sum_{n=1}^{\infty} \vec{k}_{n+1}\cdot\nabla^n\zeta \right) \right\rangle$$

$$= \int\cdots\int \left\{ -\zeta_x \frac{\partial g}{\partial x_o} - \zeta_y \frac{\partial g}{\partial y_o} + \frac{\partial g}{\partial \zeta_o} \right\} J_q(\vec{r}_o) \exp\left(jk_1\zeta + j \sum_{n=1}^{\infty} \vec{k}_{n+1}\cdot\nabla^n\zeta \right)$$

$$\cdot\ p_2(\zeta,\zeta_o,\nabla\zeta,\nabla_o\zeta_o,\cdots)\ d\vec{r}_{t_o}\, d\zeta\, d\zeta_o\, d\nabla\zeta\, d\nabla_o\zeta_o \cdots \qquad (6.15)$$

Assuming that the order of the integrations can be arbitrarily interchanged, the ζ-integration can be written as a convolution of the ζ-Fourier transforms of the Green's function derivatives and $p_2(\cdot)$. Noting that

$$F_\zeta \left\{ \frac{\partial g(\vec{r}_t - \vec{r}_{t_o}, \zeta - \zeta_o)}{\partial \zeta_o} \right\} = -\exp(j\beta_o\zeta_o)\, F_\zeta \left\{ \frac{\partial g(\vec{r}_t - \vec{r}_{t_o}, \zeta)}{\partial \zeta} \right\}$$

where F_ζ denotes the Fourier transform with respect ζ and β_1 is the transform variable, and substituting

$$\tilde{g}(\Delta\vec{r}_t,\beta_o) = F_\zeta\left\{ g(\Delta\vec{r}_t,\zeta) \right\} = \int g(\Delta\vec{r}_t,\zeta)\exp(j\beta_o\zeta)d\zeta$$

$$\tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o) = F_\zeta\left\{ \frac{\partial g(\Delta\vec{r}_t,\zeta)}{\partial\zeta} \right\} = \int \frac{\partial g(\Delta\vec{r}_t,\zeta)}{\partial\zeta} \exp(j\beta_o\zeta)d\zeta$$

in the convolution integration with $\Delta\vec{r}_t = \vec{r}_t - \vec{r}_{t_o}$ yields the following form for (6.15);

$$\langle\cdot\rangle = \frac{1}{2\pi}\int\cdots\int\left\{-\zeta_x\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial x_o} - \zeta_y\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial y_o} - \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\} J_q(\vec{r}_o)$$

$$\cdot \exp\left(j\beta_o\zeta_o + j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta\right)\tilde{p}_2(k_1-\beta_o,\zeta_o,\nabla\zeta,\nabla_o\zeta_o,\cdots)d\vec{r}_{t_o}d\beta_o d\zeta_o d\nabla\zeta d\nabla_o\zeta_o\cdots \quad (6.16)$$

where the tilde symbol denotes the Fourier transform of $p_2(\cdot)$ with respect to ζ .

The ζ_o-integration in (6.16) can also be written as a convolution as follows;

$$\langle\cdot\rangle = \frac{1}{(2\pi)^2}\int\cdots\int\left\{-\zeta_x\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial x_o} - \zeta_y\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial y_o} - \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}\tilde{J}_q(\vec{r}_o,\beta_1)$$

$$\cdot \exp\left\{j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta\right\}\tilde{\tilde{p}}_2(k_1-\beta_o,\beta_o-\beta_1,\nabla\zeta,\nabla_o\zeta_o,\cdots)d\vec{r}_{t_o}d\beta_o d\beta_1 d\nabla\zeta d\nabla_o\zeta_o\cdots \quad (6.17)$$

where the tilde over J_q and the second tilde over p_2 denote the Fourier transform with respect to ζ_o . The remaining integrations over $\nabla\zeta,\nabla^2\zeta,\cdots$ are simply Fourier transforms, so

$$\langle\cdot\rangle = \frac{1}{(2\pi)^2}\int\cdots\int\left\{j\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial x_o}\partial_{k_{2x}} + j\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial y_o}\partial_{k_{2y}} - \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}$$

$$\cdot \tilde{J}_q(\vec{r}_o,\beta_1)\ \ \underset{\sim}{\tilde{\tilde{p}}}_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,\nabla_o\zeta_o,\vec{k}_3,\nabla_o^2\zeta_o,\cdots)\ d\vec{r}_{t_o}d\beta_o d\beta_1 d\nabla_o\zeta_o d\nabla_o^2\zeta_o\cdots \quad (6.18)$$

where

$$\underset{\sim}{\tilde{\tilde{p}}}_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,\nabla_o\zeta_o,\vec{k}_3,\nabla_o^2\zeta_o,\cdots)$$

$$= \int\cdots\int \tilde{\tilde{p}}_2(k_1-\beta_o,\beta_o-\beta_1,\nabla\zeta,\nabla_o\zeta_o,\nabla^2\zeta,\nabla_o^2\zeta_o,\cdots)\exp\left[j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta\right]$$

$$\cdot\, d\nabla\zeta\, d\nabla^2\zeta\cdots$$

is the Fourier transform of p_2 over all variables <u>except</u> $\nabla_o\zeta_o,\nabla_o^2\zeta_o,\nabla_o^3\zeta_o,\cdots$. It should be noted that the differential operators $\partial_{k_{2x}}$ and $\partial_{k_{2y}}$ in (6.18) operate only on $\underset{\sim}{\tilde{\tilde{p}}}_2$. The integrations in (6.18) over $\nabla_o\zeta_o,\nabla_o^2\zeta_o,\nabla_o^3\zeta_o,\cdots$ can be written as convolutions of the Fourier transforms of $\tilde{J}_q$ and $\underset{\sim}{\tilde{\tilde{p}}}_2$, i.e.

$$\langle\cdot\rangle = \lim_{n\to\infty}\frac{1}{(2\pi)^2(2\pi)^n n!}\int\cdots\int\left\{j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial x_o}\,\partial_{k_{2x}} + j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial y_o}\,\partial_{k_{2y}} - \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}$$

$$\cdot\,\underset{\sim}{\tilde{J}}_q(\vec{r}_{t_o},\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots)\,\underset{\approx}{\tilde{\tilde{p}}}_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3,\cdots)\,d\vec{r}_{t_o}\,d\beta_o\,d\beta_1\cdots \qquad (6.19)$$

where

$$\underset{\sim}{\tilde{J}}_q(\vec{r}_{t_o},\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots) = \int\cdots\int\tilde{J}_q(\vec{r}_{t_o},\beta_1,\nabla_o\zeta_o,\nabla_o^2\zeta_o,\cdots)\exp\left\{j\sum_{n=1}^{\infty}\vec{\beta}_{n+1}\cdot\nabla_o^n\zeta_o\right\}$$

$$\cdot\, d\nabla_o\zeta_o\; d\nabla_o^2\zeta_o\cdots$$

and

$$\underset{\approx}{\tilde{\tilde{p}}}_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,\vec{\beta}_2,\vec{k}_3,\vec{\beta}_3,\cdots) = \int\cdots\int\underset{\sim}{\tilde{\tilde{p}}}_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,\nabla_o\zeta_o,\vec{k}_3,\nabla_o^2\zeta_o,\cdots$$

$$\cdot\,\exp\left\{j\sum_{n=1}^{\infty}\vec{\beta}_{n+1}\cdot\nabla_o^n\zeta_o\right\}d\nabla_o\zeta_o\; d\nabla_o^2\zeta_o\cdots$$

The function $\underset{\approx}{\tilde{\tilde{p}}}_2$ is the transform of the two point joint probability density function; thus, it is equal to the two point joint characteristic function Φ_2. Substituting (6.19), (6.14), and (6.12) back into the average of the weighted [by the term in (6.11)] equation (6.5) yields

$$\lim_{n\to\infty} \frac{1}{(2\pi)^{n!}} \int\cdots\int \underset{\sim}{\tilde{J}}_q(\vec{r}_t,\beta_1,\vec{\beta}_2,\cdots)\,\Phi_1\,(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\cdots)d\beta_1\, d\vec{\beta}_2\cdots$$

$$= 2H_o \exp(-j\vec{k}_i\cdot\vec{r}_t)\Big[C_o + j\,C_x\,\partial_{k_{2x}} + j\,C_y\,\partial_{k_{2y}}\Big]\Phi_1\,(k_1-k_{iz},\vec{k}_2,\vec{k}_3,\cdots)$$

$$+ \lim_{n\to\infty} \frac{1}{(2\pi)^2(2\pi)^{n!}} \int\cdots\int \left\{ j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial x_o}\,\partial_{k_{2x}} + j\,\frac{\partial\tilde{g}}{\partial y_o}\,\partial_{k_{2y}} - \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}$$

$$\cdot\,\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3,\cdots)\,\underset{\sim}{\tilde{J}}_q\,(r_{t_o},\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots)d\vec{r}_{t_o}\,d\beta_o d\beta_1 d\vec{\beta}_2\cdots \tag{6.20}$$

Substituting $\Delta x = x - x_o$, $\Delta y = y - y_o$, and $\Delta\vec{r}_t = \vec{r}_t - \vec{r}_{t_o}$ in the right most term in (6.20) yields the following;

$$\lim_{n\to\infty} \frac{1}{(2\pi)^{n!}} \int\cdots\int \underset{\sim}{\tilde{J}}_q(\vec{r}_t,\beta_1,\vec{\beta}_2,\cdots)\,\Phi_1\,(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\cdots)d\beta_1\, d\vec{\beta}_2$$

$$= 2H_o \exp(-j\vec{k}_i\cdot\vec{r}_t)\Big[C_o + j\,C_x\,\partial_{k_{2x}} + j\,C_y\,\partial_{k_{2y}}\Big]\Phi_1\,(k_1-k_{iz},\vec{k}_2,\vec{k}_3,\cdots)$$

$$+ \lim_{n\to\infty} \frac{(2\pi)^{-2}}{(2\pi)^{n!}} \int\cdots\int \left\{ j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta x}\,\partial_{k_{2x}} + j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta y}\,\partial_{k_{2y}} + \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}$$

$$\cdot\,\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3,\cdots)\,\underset{\sim}{\tilde{J}}_q\,(\vec{r}_t-\Delta\vec{r}_t,\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots)d\Delta\vec{r}_t\,d\beta_o d\beta_1 d\vec{\beta}_2\cdots \tag{6.21}$$

The single point joint characteristic function Φ_1 is independent of $\Delta\vec{r}_t$ while the two point joint characteristic function depends on $\Delta\vec{r}_t$ through the correlation function. Since all terms in (6.21) must exhibit the same dependence upon $\vec{r}_t$ and only $\tilde{J}_q$ and $\exp(-j\vec{k}_i\cdot\vec{r}_t)$ are functions of $\vec{r}_t$, (6.21) implies that $\tilde{J}_q$ can be written as follows;

$$\tilde{J}_q(\vec{r}_t,\beta_1,\vec{\beta}_2,\cdots) = j_q(\beta_1,\vec{\beta}_2,\cdots)\exp(-j\vec{k}_i\cdot\vec{r}_t) \tag{6.22}$$

Substituting this result in (6.21) and rearranging terms produces the following integral equation for $j_q(\beta_1,\vec{\beta}_2,\cdots)$;

$$\lim_{n\to\infty}\frac{1}{(2\pi)^n n!}\Bigg\{\int\cdots\int j_q(\beta_1,\vec{\beta}_2,\cdots)\Bigg[\Phi_1(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\cdots)$$

$$-\frac{1}{(2\pi)^2}\iint\left\{j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta x}\,\partial_{k_{2x}} + j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta y}\,\partial_{k_{2y}} + \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}$$

$$\cdot\,\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3,\cdots)\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)\,d\Delta\vec{r}_t\,d\beta_o\Bigg]d\beta_1\,d\vec{\beta}_2\,d\vec{\beta}_3\cdots\Bigg\}$$

$$= 2H_o\left[C_o + j\,C_x\,\partial_{k_{2x}} + j\,C_y\,\partial_{k_{2y}}\right]\Phi_1(k_1-k_{iz},\vec{k}_2,\vec{k}_3,\cdots) \tag{6.23}$$

Before a detailed discussion of (6.23) is presented, it is advisable to review the mathematical meaning of the various terms, e.g.

$$j_q(\beta_1,\vec{\beta}_2,\cdots) = \exp(j\vec{k}_i\cdot\vec{r}_t)\int\cdots\int J_q(\vec{r}_t,\zeta,\nabla\zeta,\nabla^2\zeta,\cdots)\exp\left[j\beta_1\zeta + j\sum_{n=1}^{\infty}\vec{\beta}_{n+1}\nabla^n\zeta\right]$$

$$\cdot\;d\zeta\,d\nabla\zeta\;\;d\nabla^2\zeta\;\;\cdots \tag{6.23a}$$

$$\Phi_1(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\cdots) = \int\cdots\int p_1(\zeta,\nabla\zeta,\nabla^2\zeta,\cdots)\exp\left[j(k_1-\beta_1)\zeta + j\sum_{n=1}^{\infty}(\vec{k}_{n+1}-\vec{\beta}_{n+1})\cdot\nabla^n\zeta\right]$$

$$d\zeta\, d\nabla\zeta\ \ d\nabla^2\zeta\cdots \tag{6.23b}$$

$$\tilde{g}(\Delta\vec{r}_t,\beta_o) = \int g(\Delta\vec{r}_t,\zeta)\exp(j\beta_o\zeta)d\zeta \tag{6.23c}$$

$$\tilde{g}_\zeta(\Delta\vec{r}_t,\vec{\beta}_o) = \int \frac{\partial g(\Delta\vec{r}_t,\zeta)}{\partial\zeta}\exp(j\beta_o\zeta)d\zeta \tag{6.23d}$$

and

$$\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3,\cdots) = \int\cdots\int p_2(\zeta,\zeta_o,\nabla\zeta,\nabla_o\zeta_o,\nabla^2\zeta,\nabla_o^2\zeta_o,\cdots)$$

$$\cdot\exp\left[j(k_1-\beta_o)\zeta + j(\beta_o-\beta_1)\zeta_o + j\sum_{n=1}^{\infty}\vec{k}_{n+1}\cdot\nabla^n\zeta - j\sum_{n=1}^{\infty}\vec{\beta}_{n+1}\cdot\nabla_o^{\,n}\zeta_o\right]$$

$$\cdot\ d\zeta\, d\zeta_o\, d\nabla\zeta\, d\nabla_o\zeta_o\cdots \tag{6.23e}$$

Also, equation (6.22) confirms DeSanto's earlier analysis [8] in that it shows that the coherent scattered field is specular in nature. That is, if the average in (6.6) is written as a convolution such as in (6.8) and (6.22) is substituted for $\tilde{J}_q$, the $\vec{r}_t$-spatial integration will yield a product of δ-functions, e.g.

$$\int\exp(-j\vec{k}_i\cdot\vec{r}_t)\exp(j\vec{k}_s\cdot\vec{r}_t)d\vec{r}_t = \delta(k_{sx}-k_{ix})\delta(k_{sy}-k_{iy}) \quad , \tag{6.24}$$

which shows that the scattered field is nonzero only for $k_{sx} = k_{ix}$ and $k_{sy} = k_{iy}$.

6.2.2.1 Discussion of Results

Equation (6.23) is an integral equation for $j_q(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots)$; if this function can be determined, the exact amplitude of the scalar scattered field may be computed as follows;

$$\lim_{n\to\infty} \frac{1}{(2\pi)^{n!}} \int\cdots\int j_q(\beta_1,\vec{\beta}_2,\cdots)\,\Phi_1\,(k_{sz}-\beta_1,-\vec{\beta}_2,-\vec{\beta}_3,\cdots)\,d\beta_1\,d\vec{\beta}_2\,d\vec{\beta}_3\cdots \tag{6.25}$$

The quantity j_q, from (6.22), is proportional to the Fourier transform of the equivalent flat plane current with respect to all the random variables of which this current is a function. Caution should be exercised in any attempt to attach a physical meaning to j_q because in transforming from $\zeta,\nabla\zeta,\nabla^2\zeta,\cdots$ to $\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots$ the stochastic character of the random variables is lost. In fact this is the fundamental reason behind working in $\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots$ space. That is, if one were to average (6.5), weighted by the exponential factor in (6.11), in the conventional manner of multiplying by the appropriate joint density functions and integrating over all random variables directly without going to the transform space, the stochastic nature of the random variables would prohibit one from obtaining a single equation such as (6.23). It is well known that a conventional average of integral equations such as (6.5) leads to an infinite set of integral equations [11] because one does not know the average of the product of J_q and the kernel inside the integral in (6.5). What has been shown here is that if the averages are expressed as convolutions in transform space rather than direct integrations over the random variables, it is possible to obtain a single integral equation because the transformed product of J_q and the kernel term can be factored. The price that one pays for the single integral equation is that it has infinite dimensions because all order derivatives of ζ which have a nonzero two-point correlation with ζ

must be included. In fact, the primary difference between this approach and the conventional method [11] which gives rise to an infinite set of equations is the following. In the conventional approach, one attempts to solve for the average of the desired unknown quantity without trying to explicitly determine its dependence upon the random parameters in the problem. In the approach presented here the exact opposite is done only in the transform domain where the stochastic character of the random parameters enters only through the kernel and the correlations between parameters. Furthermore, in the conventional approach it is frequently difficult to attach physical significance to any truncation (closure) or partial summation of the infinite set of equations. In the method presented here, truncation of the infinite dimensionality of (6.23) is determined entirely by the relative magnitude of the correlations between the random variables.

There is one potential problem with (6.23) which may make it less attractive than the conventional approach. In the infinite equation solution, all the integral equations are of the second kind and, thus, normally amenable [10] to numerical solution. On the other hand, (6.23) is of the first kind which is usually rift with problems [12]. This is certainly a point to be considered in the future; however, there are two reasons why it may not be a problem. First, the desired quantity is (6.25) and not simply j_q ; thus problems associated with the accurate recovery of j_q from (6.23) may vanish when computing (6.25). Second, since (6.23) is obtained from an integral equation of the second kind, this may also minimize some of the problems normally associated with equations of the first kind.

There are a number of interesting results that can be obtained from (6.23) without specifying the forms of the one and two-point joint characteristic functions. In the physical optics approximation, the term in (6.23) involving the Green's function is ignored and, according to (6.25), the amplitude of the average scattered field is thus given by

$$2H_o\left\{\lim_{\substack{k_{2x}\to 0\\ k_{2y}\to 0}}\left[C_o + j\,C_x\,\partial_{k_{2x}} + j\,C_y\,\partial_{k_{2y}}\right]\Phi_1\,(k_{sz}-k_{iz},k_{2x},k_{2y},0,0\cdots)\right\} \quad (6.26a)$$

The terms involving the partial derivatives with respect to k_{2x} and k_{2y} are equivalent to the following average;

$$\left\langle \frac{\partial\zeta}{\partial q}\ \exp\,[j(k_{sz}-k_{iz})\zeta]\right\rangle \quad (6.26b)$$

where $q = x$ or y . For a statistically homogeneous process, ζ is uncorrelated with $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$ [13]. Consequently, the average in (6.26b) is the product of averages of the slope and height factors and if the slopes are a zero mean process then (6.26b) is identically zero. Thus, (6.26a) reduces to $2H_o\,C_o\,\Phi(k_{sz}-k_{iz})$ where $\Phi(\cdot)$ is the marginal characteristic function for the surface height. In the physical optics limit the specularly scattered field is independent of the slopes <u>provided</u> the surface is statistically homogeneous and the slopes are zero mean.

The function j_q corresponding to the physical optics approximation (denoted by j_q^o) may be obtained from (6.23) by inspection, e.g.

$$j_q^o(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots) = 2H_o\delta(\beta_1-k_{iz})\prod_{i=3}^{\infty}\delta(\vec{\beta}_i)\Big\{C_o\delta(\beta_{2x})\,\delta(\beta_{2y}) - j\,C_x\,\delta'(\beta_{2x})\delta(\beta_{2y})$$

$$- j\,C_y\,\delta(\beta_{2x})\,\delta'(\beta_{2y})\Big\} \quad (6.27)$$

here $\delta(\cdot)$ is the Dirac delta and $\delta'(\cdot)$ is its derivative. The next level of approximation is the so-called Born approximation. In this approach, one solves for $\int j_q\Phi_1 = \langle E_s\rangle$ by moving the term containing the Green's function transform to the right side of (6.23) and substituting j_q^o [from (6.27)] in this term. This leads to an approximation for the complex amplitude of the scattered field which is valid when the term containing the Green's function transform is small compared to the source term; in particular, the result is as follows

$$\langle E_s\rangle^B = 2H_o \lim_{\substack{k_{2x}\to 0\\ k_{2y}\to 0}} \left[C_o + j\,C_x\,\partial_{k_{2x}} + j\,C_y\,\partial_{k_{2y}}\right]\Phi_1(k_{sz}-k_{iz},\vec{k}_2,0,0,\cdots)$$

$$+\frac{2H_o}{(2\pi)^2} \lim_{\substack{\beta_{2x},k_{2x}\to 0\\ \beta_{2y},k_{2y}\to 0}} \iint\left\{j\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta x}\,\partial_{k_{2x}} + j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta y} + \tilde{g}_\zeta(\Delta\vec{r}_t,\zeta_o)\right\}\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)$$

$$\cdot\left\{C_o\Phi_2(k_{sz}-\beta_o,\beta_o-k_{iz},0,0,\cdots) - j\,C_x\,\frac{\partial\Phi_2}{\partial\beta_{2x}}(k_{sz}-\beta_o,\beta_o-k_{iz},k_{2x},0,-\beta_{2x},0,0,\cdots)\right.$$

$$\left.-\,j\,C_y\,\frac{\partial\Phi_2}{\partial\beta_{2y}}(k_{sz}-\beta_o,\beta_o-k_{iz},0,k_{2y},0,-\beta_{2y},0,0,\cdots)\right\}d\Delta\vec{r}_t\,d\beta_o \qquad (6.28)$$

where, as a reminder, the arguments of Φ_2 are the transform variables corresponding to the following order of random variables $(\zeta,\zeta_o,\partial\zeta/\partial x,\partial\zeta/\partial y,$ $\partial\zeta_o/\partial x_o,\partial\zeta_o/\partial y_o,\nabla^2\zeta,\nabla_o^2\zeta_o,\cdots)$. The B superscript on $\langle E_s\rangle$ in (6.28) denotes the Born approximation.

One final result that should be demonstrated is the limit of a perfectly flat plane. In this limit the correlation function for the heights goes to the mean square height, $\langle\zeta\,\zeta_o\rangle = \sigma^2 = \langle\zeta^2\rangle$, while the mean square slope, curvature, rate of change of curvature, etc., go to zero. Consequently,

$$\Phi_1(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\vec{k}_3-\vec{\beta}_3,\cdots) = \Phi_1(k_1-\beta_1)$$

$$\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3,\cdots) = \Phi_1(k_1-\beta_o+\beta_o-\beta_1) = \Phi_1(k_1-\beta_1)$$

The Green's function transform $\tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)$ is an odd function of β_o because $\tilde{g}_\zeta = j\,\beta_o\,\tilde{g}$ and $\tilde{g}$ is an even function of β_o. Thus, the term in (6.23) involving the Green's function transforms is zero and (6.23) reduces to

$$\lim_{n\to\infty} \frac{1}{(2\pi)^n n!}\int\cdots\int j_q(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots)\,\Phi_1(k_1-\beta_1)\,d\beta_1\,d\vec{\beta}_2\cdots$$

$$= 2H_o\,C_o\,\Phi_1(k_1-k_{iz}) \tag{6.29}$$

which yields the proper value for $\langle E_s\rangle$ as $k_1\to k_{sz}$ and also

$$j_q(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots) = 2H_o\,C_o\,\delta(\beta_1-k_{iz})\prod_{i=2}^{\infty}\delta(\vec{\beta}_i) \tag{6.30}$$

Furthermore, the inverse transform of (6.22) is given, in this case, by

$$J_q(\vec{r}_t,\zeta) = 2H_o\,C_o\exp(-j\,k_{iz}\,\zeta - j\,\vec{k}_i\cdot\vec{r}_t) \tag{6.31}$$

and this is a valid transformation since ζ does not depend upon $\vec{r}_t$. It should be noted that this is the case of a randomly elevated plane and yet it produces the same results as the physical optics approximation. This correspondence leads to increased suspicion of the physical optics result. That is, physical optics treats the problem as a randomly elevated plane.

6.2.3 Jointly Gaussian Distributed Roughness

The results of the previous section are important because they provide a rigorous mathematical foundation for the multivariate approach. They are also general in that they are valid for any zero mean statistically homogeneous roughness. While it is possible to obtain certain asymptotic solutions such as with the physical optics and Born approximations or in the case of a randomly elevated plane, it is difficult to appreciate the power of this approach without seeing it applied to a specific surface height distribution. This is particularly true in regard to reducing the dimensionality of the integral equation. That is, given the form of the single and two point joint characteristic functions and (6.23), how does one go about determining which surface parameters, i.e. $\zeta, \nabla\zeta, \nabla^2\zeta, \cdots$, are important and over what range of values? To accomplish this goal, the jointly Gaussian surface has been selected. There are two reasons for this choice. First, the Gaussian surface has been extensively studied by others [2-10]. Second, the jointly Gaussian density and characteristic function have known closed mathematical forms; something which is difficult, at best, to obtain for other distributions. The purposes of this section are to demonstrate how one goes about solving (6.23) for a Gaussian surface, obtain asymptotic solutions, and compare these results with others.

The surface is assumed to be zero mean, jointly Gaussian, and statistically homogeneous. With $\bar{u}_2$ the column matrix

$$\bar{u}_2 = \begin{bmatrix} \zeta \\ \zeta_o \\ \nabla\zeta \\ \nabla_o\zeta_o \\ \vdots \end{bmatrix} \tag{6.32a}$$

and $\bar{\bar{C}}_2$ the square covariance matrix

$$\bar{\bar{C}}_2 = \begin{bmatrix} \langle\zeta^2\rangle & \langle\zeta\zeta_o\rangle & \langle\zeta\cdot\nabla\zeta\rangle \cdots \\ \langle\zeta_o\zeta\rangle & \langle\zeta_o{}^2\rangle & \langle\zeta_o\cdot\nabla\zeta\rangle \cdots \\ \langle\nabla\zeta\cdot\zeta\rangle & \langle\nabla\zeta\cdot\zeta_o\rangle & \langle(\nabla\zeta)^2\rangle \cdots \\ \langle\nabla_o\zeta_o\cdot\zeta\rangle & \langle\nabla_o\zeta_o\cdot\zeta_o\rangle & \langle\nabla_o\zeta_o\cdot\nabla\zeta\rangle \cdots \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{6.32b}$$

the two point joint probability density function is given by

$$p_2(\zeta,\zeta_o,\nabla\zeta,\nabla\zeta_o,\cdots) = \lim_{n\to\infty} \left[(2\pi)^n |\bar{\bar{C}}_2|\right]^{1/2} \exp\left[-\bar{u}_2^T (\bar{\bar{C}}_2)^{-1} \bar{u}_2/2\right] \tag{6.33}$$

where the T superscript denotes the transpose of $\bar{u}_2$, i.e. $\bar{u}_2^T$ is a row matrix, and $|\bar{\bar{C}}_2|$ is the determinant of the covariance matrix. With $\bar{V}_2$ the column matrix of transform variables

$$\bar{V}_2 = \begin{bmatrix} k_1 \\ \beta_1 \\ \vec{k}_2 \\ \vec{\beta}_2 \\ \vdots \end{bmatrix} \tag{6.34a}$$

the two point joint characteristic function is given by

$$\Phi_2(k_1,\beta_1,\vec{k}_2,\vec{\beta}_2,\cdots) = \exp\left\{-\bar{V}_2^T \bar{\bar{C}}_2 \bar{V}_2/2\right\} \tag{6.34b}$$

The terms in the covariance matrix are typically as follows;

$$\langle\zeta\,\zeta_o\rangle = R(\Delta\vec{r}_t) \qquad \langle\zeta_x\,\zeta_{x_o}\rangle = -R_{xx}(\Delta\vec{r}_t)$$
$$\langle\zeta\,\zeta_x\rangle = 0 \qquad \langle\zeta_y\,\zeta_{y_o}\rangle = -R_{yy}(\Delta\vec{r}_t)$$
$$\langle\zeta\,\zeta_{x_o}\rangle = -R_x(\Delta\vec{r}_t) \qquad \langle\zeta_x\,\zeta_{xx_o}\rangle = R_{xxx}(\Delta\vec{r}_t)$$
$$\langle\zeta\,\zeta_y\rangle = 0 \qquad \langle\zeta_x\,\zeta_{yy_o}\rangle = R_{xyy}(\Delta\vec{r}_t)$$
$$\langle\zeta\,\zeta_{y_o}\rangle = -R_y(\Delta\vec{r}_t) \qquad \langle\zeta_y\,\zeta_{xx_o}\rangle = R_{yxx}(\Delta\vec{r}_t)$$
$$\langle\zeta\,\zeta_{xx_o}\rangle = R_{xx}(\Delta\vec{r}_t) \qquad \langle\zeta_y\,\zeta_{yy_o}\rangle = R_{yyy}(\Delta\vec{r}_t)$$
$$\langle\zeta\,\zeta_{yy_o}\rangle = R_{yy}(\Delta\vec{r}_t)$$

where $\Delta\vec{r}_t = \vec{r}_t - \vec{r}_{t_o}$, the x and y-subscripts denote differentiation with respect to Δx and Δy, respectively, and $R(\Delta\vec{r}_t)$ is the surface height correlation function. The above results follow from the general relationship [13]

$$\left\langle \frac{\partial^{m+n}\zeta}{\partial x^m \partial y^n} \cdot \frac{\partial^{p+q}\zeta_o}{\partial x_o^p \partial y_o^q} \right\rangle = (-1)^{p+q} \frac{\partial^{m+n+p+q} R(\Delta\vec{r}_t)}{\partial \Delta x^{m+p} \partial \Delta y^{n+q}} \tag{6.35}$$

where m, n, p, and q are nonnegative integers.

If $\overline{u}_1$ is the column matrix

$$\overline{u}_1 = \begin{bmatrix} \zeta \\ \nabla\zeta \\ \nabla^2\zeta \\ \vdots \end{bmatrix} \tag{6.36}$$

and $\overline{\overline{C}}_1$ is the covariance matrix

$$\overline{\overline{C}}_1 = \begin{bmatrix} <\zeta^2> & <\zeta\cdot\nabla\zeta> & <\zeta\cdot\nabla^2\zeta> & \cdots \\ <\nabla\zeta\cdot\zeta> & <(\nabla\zeta)^2> & <\nabla\zeta\cdot\nabla^2\zeta> & \cdots \\ <\nabla^2\zeta\cdot\zeta> & <\nabla^2\zeta\cdot\nabla\zeta> & <(\nabla^2\zeta)^2> & \cdots \\ \vdots & & & \end{bmatrix} \tag{6.37}$$

the single point joint probability density function is given by

$$p_2(\zeta,\nabla\zeta,\nabla^2\zeta,\cdots) = \lim_{n\to\infty} \left[(2\pi)^n \left|\overline{\overline{C}}_1\right|\right]^{1/2} \exp\left[-\overline{u}_1^T (\overline{\overline{C}}_1)^{-1} \overline{u}_1 / 2\right] \tag{6.38}$$

and the corresponding joint characteristic function is

$$\Phi_1(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots) = \exp\left\{-\overline{V}_1^T \overline{\overline{C}}_1 \overline{V}_1 / 2\right\} \tag{6.39}$$

where V_1 is the column matrix of transform variables

$$\bar{V}_1 = \begin{bmatrix} \beta_1 \\ \vec{\beta}_2 \\ \vec{\beta}_3 \\ \vdots \end{bmatrix} \tag{6.40}$$

The diagonal elements in (6.37) are the mean square (or variance) parameters of the surface. The mean square height is $\sigma^2 = \langle\zeta^2\rangle$, the x and y-components of the mean square slope are, respectively, $S_x^2 = \langle\zeta_x^2\rangle$ and $S_y^2 = \langle\zeta_y^2\rangle$, and one can similarly define the mean square curvature components, rate of change of curvature, etc. It should be noted that these variance parameters can be obtained from (6.35) evaluated at $\Delta\vec{r}_t = 0$. Furthermore, in order for these variances to be finite, there are certain analytic properties required of $R(\Delta\vec{r}_t)$ in the neighborhood of $\Delta\vec{r}_t = 0$.

Equations (6.34) and (6.39) provide the joint characteristic functions required in (6.23). However, these functions still depend upon an infinite number of transform variables and this will make the manipulations more difficult to follow. The crucial points to be made can be accomplished just as completely if only two transform variables are considered. Since the two most important surface parameters are usually the height and slopes, these will be selected for the demonstration. In order to eliminate the other surface parameters, it will be necessary to assume a very gently undulating surface which has a vanishingly small mean square curvature, rate of change of curvature, etc.; that is, $\langle(\nabla^n\zeta)^2\rangle \approx 0$ for $n > 1$. Under these conditions,

$$\Phi_1(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2,\vec{k}_3-\vec{\beta}_3,\cdots) \to \Phi_1(k_1-\beta_1,\vec{k}_2-\vec{\beta}_2) \tag{6.41a}$$

$$\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2,\vec{k}_3,-\vec{\beta}_3) \to \Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2) \tag{6.41b}$$

and in (6.23)

$$J_q(\beta_1,\vec{\beta}_2,\vec{\beta}_3,\cdots) \to J_q(\beta_1,\vec{\beta}_2) \prod_{i=3}^{\infty} \delta(\vec{\beta}_i) \tag{6.41c}$$

because the joint probability density functions go to Dirac deltas, $\delta(\nabla^n\zeta)$, while J_q is independent of the $\nabla^n\zeta$. Under these circumstances, (6.23) reduces to the following form;

$$\frac{1}{(2\pi)^3}\left\{\iint J_q(\beta_1,\vec{\beta}_2)\left[\Phi(k_1-\beta_1)\Phi(\vec{k}_2-\vec{\beta}_2) - \frac{1}{(2\pi)^2}\iint\left\{j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta x}\,\partial_{k_{2x}}\right.\right.\right.$$

$$\left.\left.\left.+\,j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta y}\,\partial_{k_{2y}} + \tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o)\right\}\Phi_2(k_1-\beta_o,\beta_o-\beta_1,\vec{k}_2,-\vec{\beta}_2)\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)d\Delta\vec{r}_t\,d\,\beta_o\right]d\beta_1 d\vec{\beta}_2\right\}$$

$$= 2H_o\left[C_o + j\,C_x\,\partial_{k_{2x}} + j\,C_y\,\partial_{k_{2y}}\right]\Phi(k_1-k_{iz})\,\Phi(\vec{k}_2) \tag{6.42}$$

where, because the height and slopes are independent for a Gaussian surface, the single point joint characteristic function has been factored into a product of marginal characteristic functions for the height and slopes. It should be noted that (6.42) is exactly the equation that would result from assuming that the current depends only upon the random surface height and slopes. However, as noted above, (6.42) is valid <u>only</u> when the variances or mean square values of all the higher order surface derivatives are vanishingly small.

The remaining portion of this section shall be devoted to two goals. Using the height and slopes as examples, an attempt will be made to obtain some quantitative measure of when, say, the slopes can be neglected relative to the height. Hopefully, it will then be possible to generalize this result to the point of also estimating the required smallness of the curvature variance, the rate of change of curvature variance, etc. in order that they may also be ignored. This is an extremely important issue since the solution of (6.23) in any more than three dimensions (height and two components of slope) is highly impractical. The second goal is to study the possibility of solving (6.42). That is, since (6.42) represents surfaces which are not unreasonable, its solution would represent a considerable breakthrough in the understanding of scattering from rough surfaces.

After some straightforward manipulation of the characteristic functions, (6.42) may be written as follows;

$$\frac{1}{(2\pi)^3}\iiint j_q(\beta_1,\beta_{2x},\beta_{2y})\Phi(k_1-\beta_1)\Phi(k_{2x}-\beta_{2x})\Phi(k_{2y}-\beta_{2y})\Bigg\{1$$

$$-\frac{1}{(2\pi)^2}\iint G(\Delta\vec{r}_t,\beta_o,\vec{k}_2,\vec{\beta}_2)\exp\Big[-(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1)-(S_x^{\,2}+R_{xx})\beta_{2x}k_{2x}$$

$$-(S_y^{\,2}+R_{yy})\beta_{2y}k_{2y}-(k_1-\beta_o)(R_x\beta_{2x}+R_y\beta_{2y})+(\beta_o-\beta_1)(R_xk_{2x}+R_yk_{2y})$$

$$-R_{xy}(\beta_{2y}k_{2x}+\beta_{2x}k_{2y})\Big]\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)d\Delta\vec{r}_t\,d\beta_o\Bigg\}d\beta_1\,d\beta_{2x}\,d\beta_{2y}$$

$$=2H_o\Big[C_o+j\,C_x\,\partial_{k_{2x}}+j\,C_y\,\partial_{k_{2y}}\Big]\Phi(k_1-k_{iz})\Phi(\vec{k}_2) \tag{6.43}$$

where $S_x^{\,2}=\langle\zeta_x^{\,2}\rangle$, $S_y^{\,2}=\langle\zeta_y^{\,2}\rangle$, and G is the term involving the Green's function transforms which is as follows;

$$G(\Delta\vec{r}_t,\beta_o,\vec{k}_2,\vec{\beta}_2)=j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta x}\Big[-S_x^2(k_{2x}-\beta_{2x})+\partial_{k_{2x}}\Big]+j\,\frac{\partial\tilde{g}(\Delta\vec{r}_t,\beta_o)}{\partial\Delta y}\Big[-S_y^2(k_{2y}-\beta_{2y})$$

$$+\partial_{k_{2y}}\Big]+\tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o) \tag{6.44}$$

The term in (6.43) involving G does not converge very rapidly as $\Delta\vec{r}_t\to\infty$, so to avoid potential problems this asymptotic behavior will be subtracted out. That is, since the correlation function and its derivatives go to zero as $\Delta\vec{r}_t \to \infty$, this term asymptotically approaches, for $\Delta\vec{r}_t$ sufficiently large,

$$\iint G(\Delta\vec{r}_t,\vec{\beta}_o,\vec{k}_2,\vec{\beta}_2)\exp\left[-\sigma^2(\beta_o-\beta_1)(\beta_o-k_1)-S_x^2\beta_{2x}k_{2x}-S_y^2\beta_{2y}k_{2y}\right]$$

$$\cdot\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)\,d\Delta\vec{r}_t\,d\beta_o = \Gamma(\beta_1,k_1,\vec{k}_2,\vec{\beta}_2) \qquad (6.45)$$

The $\Delta\vec{r}_t$-integration in (6.45) can be easily accomplished with the following result;

$$\Gamma(\cdot) = \int\left\{j\,\tilde{\tilde{\tilde{g}}}_{\Delta x}(\vec{k}_{it},\beta_o)(-S_x^2k_{2x}) + j\,\tilde{\tilde{\tilde{g}}}_{\Delta y}(\vec{k}_{it},\beta_o)(-S_y^2k_{2y}) + \tilde{\tilde{\tilde{g}}}_{\zeta}(\vec{k}_{it},\beta_o)\right\}$$

$$\cdot\exp\left[-\sigma^2(\beta_o-\beta_1)(\beta_o-k_1)-S_x^2\beta_{2x}k_{2x}-S_y^2\beta_{2y}k_{2y}\right]d\beta_o \qquad (6.46)$$

where the triple tildes denote the three dimensional $(\Delta x,\Delta y,\zeta)$-Fourier transform, the subscripts on the $\tilde{\tilde{\tilde{g}}}$ denote the derivatives of $g(\Delta\vec{r}_t,\zeta)$ with respect to the indicated variable, and $\vec{k}_{it} = k_{ix}\hat{x} + k_{iy}\hat{y}$ is the transverse part of the incident wave vector. The β_o-integration is difficult to perform directly because of the Gaussian factor in (6.46); however, it can be accomplished by using Parseval's theorem to rewrite (6.46) as an integration over the product of inverse Fourier transforms (with respect to β_o). That is, with*

*The 4π factor arises because g is 4π times the conventional free space Green's function $\exp(-jk_or)/4\pi r$.

$$\tilde{\tilde{g}}_{\Delta x}(\vec{k}_{it},z) = -\frac{jk_{ix}}{2\pi}\int \tilde{\tilde{\tilde{g}}}(\vec{k}_{it},\beta_o)\exp(-j\beta_o z)d\beta_o = \frac{4\pi k_{ix}}{2}\frac{\exp(-jk_{iz}|z|)}{k_{iz}} \tag{6.47a}$$

$$\tilde{\tilde{g}}_{\Delta y}(\vec{k}_{it},z) = -\frac{jk_{iy}}{2\pi}\int \tilde{\tilde{\tilde{g}}}(\vec{k}_{it},\beta_o)\exp(-j\beta_o z)d\beta_o = \frac{4\pi k_{iy}}{2}\frac{\exp(-jk_{iz}|z|)}{k_{iz}} \tag{6.47b}$$

$$\tilde{\tilde{g}}_{\zeta}(\vec{k}_{it},z) = \frac{1}{2\pi}\int \tilde{\tilde{\tilde{g}}}_{\zeta}(\vec{k}_{it},\beta_o)\exp(-j\beta_o z)d\beta_o = \frac{4\pi}{2}\exp(-jk_{iz}|z|)\mathrm{sgn}(z) \tag{6.47c}$$

where k_{iz} is the z-component of the incident wave vector, and [14, pg. 63]

$$\mathrm{sgn}(z) = \begin{matrix} 1 & z>0 \\ 0 & z=0 \\ -1 & z<0 \end{matrix} \tag{6.47d}$$

and

$$\frac{1}{2\sqrt{\pi}\,\sigma}\exp\left\{\frac{1}{4\sigma^2}\left[\sigma^4(k_1+\beta_1)^2 - z^2 - j\,2\sigma^2 z(k_1+\beta_1) - 4\sigma^4\beta_1 k_1\right]\right\}$$

$$= \frac{1}{2\pi}\int \exp\left[-\sigma^2(\beta_o-\beta_1)(\beta_o-k_1) - j\,\beta_o z\right]d\beta_o = Q(z) \tag{6.47e}$$

(6.46) may be written as follows;

$$\Gamma(\cdot) = 2\pi\exp\left[-S_x^2 k_{2x}\beta_{2x} - S_y^2 k_{2y}\beta_{2y}\right]\int\left\{-j2\pi\left(\frac{S_x^2 k_{ix}k_{2x}}{k_{iz}}\right) - j2\pi\left(\frac{S_y^2 k_{iy}k_{2y}}{k_{iz}}\right)\right.$$

$$\left. + 2\pi\,\mathrm{sgn}(z)\right\} Q(-z)\exp(-jk_{iz}|z|)dz \tag{6.48}$$

Completing the z-integration yields

$$\Gamma(\cdot) = \gamma(\beta_1,k_1,\vec{k}_2)\exp\left[-S_x^2 k_{2x}\beta_{2x} - S_y^2 k_{2y}\beta_{2y}\right] \tag{6.49}$$

where

$$\gamma(\beta_1,k_1,\vec{k}_2) = j2\pi^2\left(\frac{S_x^2 k_{ix}k_{2x} + S_y^2 k_{iy}k_{2y}}{k_{iz}}\right)\exp(-\sigma^2 k_1\beta_1)\left\{\exp\left[-\sigma^2 k_{iz}^2\left\{1-\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\right.$$

$$\cdot\,\mathrm{erfc}\left[\frac{j\sigma k_{iz}}{2}\left\{2-\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right] + \exp\left[-\sigma^2 k_{iz}^2\left\{1+\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\cdot\mathrm{erfc}\left.\left[\frac{j\sigma k_{iz}}{2}\left\{2+\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\right\}$$

$$+ 2\pi^2\exp(-\sigma^2 k_1\beta_1)\left\{\exp\left[-\sigma^2 k_{iz}^2\left\{1-\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\cdot\mathrm{erfc}\left[\frac{j\sigma k_{iz}}{2}\left\{2-\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\right.$$

$$\left. - \exp\left[-\sigma^2 k_{iz}^2\left\{1+\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\cdot\mathrm{erfc}\left[\frac{j\sigma k_{iz}}{2}\left\{2+\left(\frac{k_1+\beta_1}{k_{iz}}\right)\right\}\right]\right\} \tag{6.50}$$

where erfc(•) is the complementary error function. Substituting (6.49) in (6.43) in such a manner as to regularize the $d\Delta\vec{r}_t$ integral results in the following;

$$\frac{1}{(2\pi)^3}\iiint j_q(\beta_1,\beta_{2x},\beta_{2y})\,\Phi(k_1-\beta_1)\,\Phi(k_{2x}-\beta_{2x})\,\Phi(k_{2y}-\beta_{2y})\left\{1\right.$$

$$-\frac{1}{(2\pi)^2}\iint\left[K(\Delta\vec{r}_t,\beta_o,\beta_1,k_1,\vec{k}_2,\vec{\beta}_2) - K(\Delta\vec{r}_t,\beta_o,\beta_1,k_1,\vec{k}_2,\vec{\beta}_2;\partial^n R=0,n=0,1,2)\right]$$

$$\left.\cdot\,\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)\,d\Delta\vec{r}_t\,d\beta_o - \Gamma(\beta_1,k_1,\vec{k}_2,\vec{\beta}_2)/(2\pi)^2\right\} d\beta_1\,d\beta_{2x}\,d\beta_{2y}$$

$$= 2H_o\left[C_o - jC_x S_x^2 k_{2x}/2 - jC_y S_y^2 k_{2y}/2\right]\Phi(k_1-k_{iz})\,\Phi(\vec{k}_2) \tag{6.51}$$

where K(•) is given below and the addendum of "$\partial^n R=0,n=0,1,2$" means that the correlation function and all of its derivatives are to be set to zero;

$$K(\Delta\vec{r}_t,\beta_o,\beta_1,k_1,\vec{k}_2,\vec{\beta}_2) = G(\Delta\vec{r}_t,\beta_o,\vec{k}_2,\vec{\beta}_2)\exp\Big[-(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1)$$

$$-(S_x^2+R_{xx})\beta_{2x}k_{2x}-(S_y^2+R_{yy})\beta_{2y}k_{2y}-(k_1-\beta_o)(R_x\beta_{2x}+R_y\beta_{2y})$$

$$+(\beta_o-\beta_1)(R_xk_{2x}+R_yk_{2y})-R_{xy}(\beta_{2y}k_{2x}+\beta_{2x}k_{2y})\Big] \tag{6.52}$$

Equation (6.51) is an exact integral equation for the function j_q under the conditions that all surface height derivatives of order two or larger are negligible in a mean square sense and depolarization is negligible. Having obtained j_q, the amplitude of the mean scattered field may be computed as follows;

$$\langle E_s\rangle = \frac{1}{(2\pi)^3}\iiint j_q(\beta_1,\beta_{2x},\beta_{2y})\,\Phi(k_{sz}-\beta_1)\,\Phi(-\beta_{2x})\,\Phi(-\beta_{2y})\,d\beta_1\,d\vec{\beta}_2 \tag{6.53}$$

6.2.3.1 Discussion of Results

Attention will now be directed toward studying the surface conditions which permit a simplification of (6.42) or (6.51). Of particular concern are the range of surface parameters which result in a one dimensional simplification of either (6.42) or (6.51). In expanded form (6.42) takes the following form;

$$\frac{1}{(2\pi)^3}\iiint j_q(\beta_1,\beta_{2x},\beta_{2y})\Phi(k_1-\beta_1)\Phi(k_{2x}-\beta_{2x})\Phi(k_{2y}-\beta_{2y})\Bigg\{1-\frac{1}{(2\pi)^2}\iint\Big[j\,\frac{\partial\tilde{g}}{\partial\Delta x}$$

$$\cdot\Big\{-S_x^2k_{2x}-R_{xx}\beta_{2x}+(\beta_o-\beta_1)R_x-R_{xy}\beta_{2y}\Big\}+j\,\frac{\partial\tilde{g}}{\partial\Delta y}\Big\{-S_y^2k_{2y}-R_{yy}\beta_{2y}+(\beta_o-\beta_1)R_y$$

$$-R_{xy}\beta_{2x}\Big\}+\tilde{g}_\zeta\Big]\exp\Big\{z_1+j\,\vec{k}_i\cdot\Delta\vec{r}_t\Big\}\,d\Delta\vec{r}_t\,d\beta_o\Bigg\}\,d\beta_1\,d\beta_{2x}\,d\beta_{2y}$$

$$= 2H_o\Big[C_o-j\,C_xS_x^2k_{2x}/2-j\,C_yS_y^2k_{2y}/2\Big]\Phi(k_1-k_{iz})\Phi(k_{2x})\Phi(k_{2y}) \tag{6.54}$$

where

$$z_1 = -(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1)-(S_x^2+R_{xx})\beta_{2x}k_{2x} - (S_y^2+R_{yy})\beta_{2y}k_{2y} - (k_1-\beta_o)(R_x\beta_{2x}+R_y\beta_{2y})$$

$$+ (\beta_o-\beta_1)(R_xk_{2x}+R_yk_{2y}) - R_{xy}(\beta_{2y}k_{2x}+\beta_{2x}k_{2y})$$

When (6.54) can be essentially reduced to an integration over the height transform variable (β_1) , it is reasonable to expect that the equivalent planar current is approximately independent of the surface slopes. If this is true then $j_q(\beta_1,\beta_{2x},\beta_{2y})$ assumes the following approximate form;

$$j_q(\beta_1,\beta_{2x},\beta_{2y}) \approx j_q(\beta_1)\delta(\beta_{2x})\delta(\beta_{2y}) \tag{6.55}$$

and substituting this simplification in (6.54) yields

$$\frac{1}{2\pi}\int j_q(\beta_1)\Phi(k_1-\beta_1)\Phi(k_{2x})\Phi(k_{2y})\left\{1 - \frac{1}{(2\pi)^2}\iint\left[j\frac{\partial\tilde{g}}{\partial\Delta x}\left\{-S_x^2k_{2x}+R_x(\beta_o-\beta_1)\right\}\right.\right.$$

$$\left.+ j\frac{\partial\tilde{g}}{\partial\Delta y}\left\{-S_y^2k_{2y}+R_y(\beta_o-\beta_1)\right\} + \tilde{g}_\zeta\right]\exp\left\{-(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1) +\right.$$

$$\left.\left.(\beta_o-\beta_1)(R_xk_{2x}+R_yk_{2y})\right\}\exp(j\,\vec{k}_i\cdot\Delta\vec{r}_t)\,d\Delta\vec{r}_t\,d\beta_o\right\}d\beta_1$$

$$= 2H_o\left[C_o - j\,C_x S_x^2 k_{2x}/2 - j\,C_y S_y^2 k_{2y}/2\right]\Phi(k_1-k_{iz})\Phi(k_{2x})\Phi(k_{2y}) \tag{6.56}$$

One way of satisfying (6.56) is to take the exponent which depends upon R_x and R_y to be very small. The maximum amplitudes of R_x and R_y are of order $S_x\sigma$ and $S_y\sigma$, respectively. Introducing the normalized height transform variables $\eta_o = \beta_o/k_{iz}$, $\eta_1 = \beta_1/k_{iz}$ and $\kappa_1 = k_1/k_{iz}$, the exponent in question can be ignored provided

$$k_{iz}\,\sigma\,S_x \ll 1 \qquad\qquad k_{iz}\,\sigma\,S_y \ll 1 \tag{6.57}$$

Assuming that (6.57) is satisfied, (6.56) reduces to the following;

$$\frac{1}{2\pi}\int j_q(\beta_1)\Phi(k_1-\beta_1)\left\{1-\frac{1}{(2\pi)^2}\iint\left[-j\,S_x^2 k_{2x}\frac{\partial\tilde{g}}{\partial\Delta x}-j\,S_y^2 k_{2y}\frac{\partial\tilde{g}}{\partial\Delta y}+\tilde{g}_\zeta\right]\right.$$

$$\left.\cdot\exp\left\{-(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1)\right\}\exp(j\,\vec{k}_i\cdot\Delta\vec{r}_t)\,d\Delta\vec{r}_t\,d\beta_o\right\}d\beta_1$$

$$=2H_o\left[C_o-j\,C_x S_x^2 k_{2x}/2-j\,C_y S_y^2 k_{2y}/2\right]\Phi(k_1-k_{iz}) \tag{6.58}$$

Equating like powers of k_{2x} and k_{2y}, (6.58) yields three equations for $j_q(\beta_1)$. Since it is not at all clear that these three equations will yield the same $j_q(\beta_1)$, it is further necessary to assume that $S_x^2 << 1$ and $S_y^2 << 1$. Thus, (6.58) finally becomes

$$\frac{1}{2\pi}\int j_q(\beta_1)\Phi(k_1-\beta_1)\left\{1-\frac{1}{(2\pi)^2}\iint\tilde{g}_\zeta\exp\left\{-(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1)+j\vec{k}_i\cdot\Delta\vec{r}_t\right\}d\Delta\vec{r}_t\,d\beta_o\right\}$$

$$\cdot\,d\beta_1\approx 2H_o\,C_o\,\Phi(k_1-k_{iz}) \tag{6.59}$$

which is the desired result. It should be noted that although the solution of (6.59) will not satisfy (6.58) exactly, the stipulation of small mean square slopes insures that the difference will be small.

The approach presented above for reducing the dimensionality of (6.42) or (6.51) is by no means unique, i.e. there may well be many other equally valid techniques. However, the end result and the conditions are expected to be the same. The condition of small mean square slopes permits ignoring the terms in (6.58) which exhibit a linear dependence upon k_{2x} and k_{2y}. Condition (6.57) or the assumption that the product of the rms slopes and the Rayleigh parameter ($k_{iz}\sigma$) is very small is necessary in order to eliminate the coupling between the height and slope resulting from the integral term in (6.5).

Of these two sets of conditions, it is obvious that (6.57) is the more difficult to satisfy because as $k_{1z}\sigma$ increases the rms slopes (S_x and S_y) will have to necessarily decrease. This is a very important condition because it shows that (6.59) is valid only when an increase in roughness height is accompanied by an increase in the horizontal scale of the roughness. Thus, the validity of (6.59) is intimately tied to the interplay between the rms surface slope and the electrical height of the surface roughness. To the author's knowledge, this is a new result and it further illustrates the power of the multivariate approach. That is, reducing the dimensionality of the multivariate integral equation can always be directly linked to ignoring, due to smallness, some physical property of the surface. This is one clear advantage that the multivariate approach enjoys over the conventional multiple scattering approach because it is most difficult to translate closure of the multiple scattering equations into an equivalent surface assumption.

Having demonstrated how one goes about determining when the slope effects in (6.42) or (6.51) can be ignored, it is now possible to back up and estimate more definitively when the curvature, rate of change of curvature, etc., effects can be ignored. This is a somewhat tedious task and will be saved for future studies. However, it is clearly obvious that the dominant smallness assumption is going to result from the statistical coupling or correlation between the surface height and the order of surface height derivative in question.

Regularizing the $\Delta\vec{r}_t$-integrand of (6.59) at infinity leads to the slope independent analog of (6.51), i.e.

$$\frac{1}{2\pi}\int j_q(\beta_1)\Phi(k_1-\beta_1)\Bigg\{1-\frac{1}{(2\pi)^2}\iint \tilde{g}_\zeta\Big[\exp\{-(\sigma^2-R)(\beta_o-\beta_1)(\beta_o-k_1)\}$$

$$-\exp\{-\sigma^2(\beta_o-\beta_1)(\beta_o-k_1)\}\Big]\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)d\Delta\vec{r}_t d\beta_o-\Gamma(\beta_1,k_1,\vec{k}_2=0,\vec{\beta}_2=0)/(2\pi)^2\Bigg\}d\beta_1$$

$$=2H_o C_o \Phi(k_1-k_{iz}) \tag{6.60}$$

where $\Gamma(\cdot)$ is given by (6.49) and (6.50). In the limit of a randomly elevated plane $R\to\sigma^2$ and (6.60) reduces, as it should, to the physical optics result for the mean scattered field. Introducing the normalized variables

$$\eta_o=\beta_o/k_{iz} \qquad \eta_1=\beta_1/k_{iz} \qquad \kappa_1=k_1/k_{iz} \quad ,$$

(6.60) becomes

$$\frac{k_{iz}}{2\pi}\int j_q(k_{iz}\eta_1)\Phi(k_{iz}[\kappa_1-\eta_1])\Bigg\{1-\frac{k_{iz}}{(2\pi)^2}\iint\tilde{g}_\zeta\Big[\exp\{-k_{iz}^2\sigma^2(1-\rho)(\eta_o-\eta_1)(\eta_o-\kappa_1)\}$$

$$-\exp\{-k_{iz}^2\sigma^2(\eta_o-\eta_1)(\eta_o-\kappa_1)\}\Big]\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)d\Delta\vec{r}_t d\eta_o-\Gamma(\eta_1,\kappa_1,\vec{k}_2=0,\vec{\beta}_2=0)/(2\pi)^2\Bigg\}$$

$$\cdot\, d\eta_1=2H_o C_o \Phi(k_{iz}[\kappa_1-1]) \tag{6.61}$$

where $\rho(\Delta x,\Delta y)=R(\Delta x,\Delta y)/\sigma^2$ is the normalized surface height correlation function. If $k_{iz}\sigma \ll 1$ then (6.61) may be approximated as follows;

$$\frac{k_{iz}}{2\pi}\int j_q(k_{iz}\eta_1)\Phi(k_{iz}[\kappa_1-\eta_1])\Bigg\{1-\frac{k_{iz}}{(2\pi)^2}(k_{iz}\sigma)^2\iint\tilde{g}_\zeta\exp\{-(k_{iz}\sigma)^2(\eta_o-\eta_1)(\eta_o-\kappa_1)\}$$

$$\cdot\,\rho(\Delta x,\Delta y)(\eta_o-\eta_1)(\eta_o-\kappa_1)\exp(j\vec{k}_i\cdot\Delta\vec{r}_t)d\Delta\vec{r}_t d\eta_o-\Gamma(\eta_1,\kappa_1,\vec{k}_2=0,\vec{\beta}_2=0)/(2\pi)^2\Bigg\}d\eta_1$$

$$\simeq 2H_o C_o \Phi(k_{iz}[\kappa_1-1]) \tag{6.62}$$

where the exponential term in (6.61) containing $\rho(\Delta x,\Delta y)$ has been approximated by a two term power series because $k_{iz}\sigma << 1$. Consistent with the previous assumptions of small mean square slope, curvature, rate of change of curvature, etc. $\rho(\Delta x,\Delta y)$ may be considered to vary much more slowly with Δx and Δy than $\tilde{g}_\zeta(\Delta\vec{r}_t,k_{iz}\eta_o)$ and so (6.62) becomes

$$\frac{k_{iz}}{2\pi}\int j_q(k_{iz}\eta_1)\Phi(k_{iz}[\kappa_1-\eta_1])\left\{1-\frac{k_{iz}}{(2\pi)^2}(k_{iz}\sigma)^2\int(\eta_o-\eta_1)(\eta_o-\kappa_1)\,\tilde{\tilde{g}}_\zeta(\vec{k}_{i_t},k_{iz}\eta_o)\right.$$

$$\left.\cdot\exp\left\{-(k_{iz}\sigma)^2(\eta_o-\eta_1)(\eta_o-\kappa_1)\right\}d\eta_o - \Gamma(\eta_1,\kappa_1,\vec{k}_2=0,\vec{\beta}_2=0)/(2\pi)^2\right\}d\eta_1$$

$$\approx 2H_o\,C_o\,\Phi(k_{iz}[\kappa_1-1]) \qquad (6.63)$$

Since the integral term is multiplied by $(k_{iz}\sigma)^2$, it is much smaller than $\Gamma(\cdot)$ because $k_{iz}\sigma << 1$; thus, for small Rayleigh parameter the integral equation for $j_q(k_{iz}\eta_1)$ becomes

$$\frac{k_{iz}}{2\pi}\int j_q(k_{iz}\eta_1)\Phi(k_{iz}[\kappa_1-\eta_1])\left\{1-\Gamma(\eta_1,\kappa_1,\vec{k}_2=0,\vec{\beta}_2=0)/(2\pi)^2\right\}d\eta_1$$

$$= 2H_o\,C_o\,\Phi(k_{iz}[\kappa_1-1]) \qquad (6.64a)$$

or in terms of β_1 and k_1

$$\frac{1}{2\pi}\int j_q(\beta_1)\,\Phi(k_1-\beta_1)\left\{1-\Gamma(\beta_1,k_1,\vec{k}_2=0\,,\vec{\beta}_2=0)/(2\pi)^2\right\}d\beta_1$$

$$= 2H_o\,C_o\,\Phi(k_1-k_{iz}) \qquad (6.64b)$$

There are some interesting consequences of (6.64) which will now be considered. First, one way of achieving a small Rayleigh parameter is to let θ_i

be very near grazing incidence $(\theta_i = \pi/2)$. Near grazing incidence is where approximate optical theories predict a strong dependence on the mean square slope due to self-shadowing [4,5]. However, (6.64) exhibits no dependence upon the surface slopes and, in fact, is determined entirely by the Rayleigh parameter $k_{iz}\sigma$. This discrepancy between (6.64) and approximate optical theories does not appear to be a consequence of any of the simplifications leading to (6.64) because these same simplifications are inherent in the approximate theory (except for $k_{iz}\sigma S << 1$ which is trivially satisfied near grazing incidence). The lack of dependence of (6.64) upon the surface slope variances is also in agreement with rigorous boundary perturbation theory which, in turn, is accurate for $k_{iz}\sigma << 1$. Thus, (6.64) clearly establishes the inaccuracy of shadow corrected optical approximations for near grazing incidence or, more correctly, for small Rayleigh parameters, at least for the case of coherent scattering. One possible reason for the failure of the shadow corrected optical approximation is that it assumes that the current induced on the surface is zero on the shadowed parts of the surface. However, this is at complete variance with (6.64) which, for $k_{iz}\sigma$ sufficiently small, predicts no shadowing of the incident field. In fact (6.64) shows that for a sufficiently small Rayleigh parameter, the average scattered field appears to result from a randomly elevated plane.

The average scattered field for the case of $k_{iz}\sigma << 1$ can be obtained by first solving (6.64) for $j_q(\beta_1)$ and then convolving this result with $\Phi(k_{sz}-\beta_1)$, $k_{sz} = -k_{iz}$, to find the average scattered field. There is, however, a more direct approach to computing $<E_s>$ which results from the special form of (6.64). Writing $\Gamma(\cdot)$ in its integral form as given by (6.46) yields

$$\frac{1}{2\pi}\int j_q(\beta_1)\Phi(k_1-\beta_1) = 2H_o C_o\Phi(k_1-k_{iz}) + \frac{1}{(2\pi)}\iint j_q(\beta_1)\Phi(k_1-\beta_1)$$

$$\cdot \exp\left[-\sigma^2(\beta_o-\beta_1)(\beta_o-k_1)\right] \tilde{\tilde{g}}_\zeta(\vec{k}_{i_t},\beta_o)d\beta_o\, d\beta_1 \Big/ (2\pi)^2 \tag{6.65}$$

Since

$$\Phi(k_1-\beta_1)\exp\left[-\sigma^2(\beta_o-\beta_1)(\beta_o-k_1)\right] = \exp\left\{-\frac{\sigma^2}{2}(k_1-\beta_o)^2 - \frac{\sigma^2}{2}(\beta_1-\beta_o)^2\right\}$$

$$= \Phi(k_1-\beta_o)\Phi(\beta_o-\beta_1)$$

and

$$\langle E_s(\beta_o)\rangle = \frac{1}{2\pi}\int j_q(\beta_1)\Phi(\beta_o-\beta_1)\,d\beta_1$$

(6.65) may be rewritten as follows

$$\langle E_s(k_1)\rangle = 2H_o C_o\Phi(k_1-k_{iz}) + \int \langle E_s(\beta_o)\rangle\Phi(k_1-\beta_o)\,\tilde{\tilde{g}}_\zeta(\vec{k}_{i_t},\beta_o)d\beta_o \Big/ (2\pi)^2 \tag{6.66}$$

Furthermore, since

$$\tilde{\tilde{g}}_\zeta(k_{i_t},\beta_o) = 4\pi\,\frac{j\,\beta_o}{\beta_o^2 - k_{iz}^2 - j\varepsilon}$$

where ε is a small positive quantity, (6.66) reduces to the following singular integral equation of the <u>second</u> kind for $\langle E_s\rangle$;

$$\langle E_s(k_1)\rangle = 2H_o C_o\Phi(k_1-k_{iz}) + \frac{j}{\pi}\int \frac{\beta_o\Phi(k_1-\beta_o)}{\beta_o^2-k_{iz}^2-j\varepsilon}\langle E_s(\beta_o)\rangle\,d\beta_o \tag{6.67}$$

This equation bears a remarkable resemblance to a result obtained by DeSanto [8,10], which he stated was valid in the limit of $R \to \sigma^2$. In comparing (6.67) with DeSanto's result, there are two factors that should be noted. First, DeSanto's equation has a β_o in the denominator of the integral rather than in the numerator. This difference appears to result from the manner in which he treats the singularity in $\partial g/\partial\zeta$. In (6.67) the β_o in the numerator results very simply from the following transform relationship;

$$\tilde{g}_\zeta(\Delta\vec{r}_t,\beta_o) = j\beta_o\ \tilde{g}\ (\Delta\vec{r}_t,\beta_o)$$

For $k_{iz}\sigma \ll 1$, this difference will have no essential effect upon $\langle E_s(k_{sz})\rangle$. A second point to be noted is the fact that DeSanto's result corresponding to (6.67) is actually based upon the assumption that

$$R(\Delta\vec{r}_t) = \begin{cases} \sigma^2 & \Delta\vec{r}_t = 0 \\ 0 & \Delta\vec{r}_t \neq 0 \end{cases} \tag{6.68}$$

rather than $R \to \sigma^2$ as stated in [9] and [10]. Since the correlation function in (6.68) differs from zero only over a domain of zero measure, substitution of (6.68) in (6.60) will thus yield (6.67). In fact (6.68) was discovered by searching for the form of $R(\Delta\vec{r}_t)$ which would reduce (6.60) to (6.67). However, it must be remembered that (6.60) is based upon the assumption that the variances of $\nabla^n\zeta, n=1,2,\ldots,$ are all very small and it is not immediately obvious that (6.68) satisfies these conditions. Thus, although (6.67) can be obtained in the same manner as DeSanto derived his corresponding result, there remains some question as to the meaning of the expressions for $\langle E_s\rangle$. Aside from these minor differences, it is most encouraging that these two significantly different mathematical approaches give rise to very similar equations for the average scattered field.

When the Rayleigh parameter becomes moderate to large, (6.67) can no longer be rigorously justified and (6.60) must be solved. Since the only major difference between (6.67) and (6.60) is the appearance in (6.60) of the term involving integrations over $\Delta\vec{r}_t$ and β_o, it would appear that (6.60) should be readily soluable also. Unfortunately, no analytical approximations have been found for this term. Since this term is roughly equivalent to the average of the ζ-derivative of the Green's function, it is extremely important to the determination of j_q or $<E_s>$. Quite obviously, future efforts on this problem should concentrate on the approximate analytical evaluation of this term. If analytical approaches prove fruitless, then numerical integration techniques should also be considered.

6.3 Summary

The purpose of this section is to develop an alternate approach to the problem of coherent scattering from a perfectly conducting rough surface. Since the conventional multiple scattering approach is difficult to interpret in terms of the statistical properties of the surface, it is desirable to have an approach which clearly shows the dependence of the mean scattered field upon the various surface parameters. The approach developed in this section leads to a single integral equation of infinite dimensions for a function which when convolved with the joint characteristic function for the height, slope, curvature, etc. leads to the coherent field. The integral equation has infinite dimension because the mean scattered field depends upon all order derivatives of the surface height. Particular attention is given to a Gaussian surface which has negligible curvature, rate of change of curvature, etc. It is shown how one links reducing the dimensionality of the integral equation to conditions on the surface parameter. In particular it is demonstrated that the equivalent planar current induced on the surface can be taken to be independent of the surface slopes whenever the slope variances are small and the product of the Rayleigh parameter and the rms slope is small. For near grazing incidence it is found that there is no justification for the inclusion of a shadowing function. Comparing these results with those obtained from multiple scattering theory shows a great deal of agreement and, in addition, some further insight into certain limiting approximations.

In summary the multivariate approach introduced here has the following positive qualities;

- it is very straightforward,
- it is sufficiently general to cover any surface height distribution,
- it clearly shows the dependence of the scattered field on measurable surface parameters,
- one can, in a straightforward manner, deduce when the various higher order surface height derivatives are important,
- the technique can be extended to the general vector case, and
- the basic approach can also be used to determine the incoherent field.

The only negative aspect of the approach is that it will require the evaluation of some very complicated and difficult integrations which, in essence, stem from averaging spatial derivatives of the Green's function.

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