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DECISION SUPPORT WITH PARTIALLY IDENTIFIED PARAMETERS

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DECISION SUPPORT WITH PARTIALLY IDENTIFIED PARAMETERS*

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ABSTRACT:

In this paper, we investigate the problem of determining a preference structure on the set of alternatives for a general class of single-stage, choice making models with imprecisely known parameters. A variety of decision making problems under certainty and under uncertainty are modeled by the general problem formulation. The imprecisely known parameters can be, for example, attribute trade-off weights, value scores, probabilities, and utility values. Parameter imprecision is described by assuming that certain groups of parameters are members of given sets. This description forms the basis for a general and behaviorally relevant assessment model. Solution procedures for four important special cases of the general problem formulation are determined. A hypothetical automobile purchasing problem is used to illustrate the decision aiding applicability of the results.

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I. INTRODUCTION

Analytic, normative models of decisionmaking often require precise identification of various parameter values prior to the calculation of a most preferred alternative. For example, the multiattribute decision aids described by Kelly (1978) and Edwards (1977) require that the decision maker (DM) provide attribute trade-off weights which give a quantitative description of the relative importance of the various objectives under consideration. As another example, the subjective expected utility approach associated with decisionmaking problems under risk assumes knowledge of subjective outcome probabilities for each alternative (Spetzler, 1975). Sensitivity analysis is often used to develop measures of confidence in the optimality of the most preferred alternative with regards to the perceived credibility of the identified parameter values. In practice, such sensitivity analyses usually can deal only with the variation of a single parameter at a time, c. f. (Kelly, 1978), although in reality several parameters are often not known exactly and multi-parameter variations may produce results significantly different than a direct combination of a series of single-parametric variations (White, 1979).

Exact knowledge of parameter values has its advantages and disadvantages. An obvious advantage is that once all parameters are precisely specified, a real-valued criterion will always be able to select a most preferred alternative. A disadvantage is that precise parameter identification, through either objective or subjective assessment, can be stressful and time and effort consuming, c. f. (Fischhoff, et. al., 1980) for studies in cognitive psychology which support this statement in more general contexts. Such demands, and the fact that the DM may find the elicitation effort so strange and strained that perspectives are lost rather than strengthened, may render the entire decision aiding effort less than completely useful. For example, precise determination of trade-off weights in a military crisis management situation may require more time than is available for the entire decision aiding process.

Interestingly, preliminary evaluation results of a recently developed medical decision aid for diagnosing a common ambulatory complaint (White, et. al. 1980) support the following hypothesis: DM's may not require identification of the most preferred alternative by a decision aid but may only require the elimination of all but a few alternatives, with appropriate data display, in order to confidently select a most preferred alternative for implementation. This hypothesis suggests that if DM's do not necessarily find it essential to totally order the alternatives, then it may also be unnecessary to precisely identify all parameters. Relaxing the need to determine all parameter values exactly, or even to elicit them, may significantly enhance the acceptability of the decision aiding approach by reducing the likelihood that various institutional, organizational and behavioral constraints will be violated.

The above comments have stimulated the development of a decision aiding approach for multiobjective decision aiding that allows the DM to interactively determine a preferred mix of alternative discrimination specificity and parameter estimate accuracy (White and Sage, 1980, 1981). Assuming that the basic problem has already been structured, the general steps of this approach are as follows:

1. Eliminate as many alternatives as possible using currently available information about the values of the imprecisely known parameters.
2. If an alternative can be selected for implementation without further alternative elimination, then stop.
3. If a choice cannot be made, then assess further information about the values of the imprecisely known parameters and return to Step 1.

The procedure for iteratively assessing the imprecisely known parameters with increasing accuracy that is presented by White and Sage (1980) appears to have substantial behavioral acceptability. It seems likely, however, that the acceptability of the general decision aiding approach may increase significantly with a more general assessment model. The assessment model due to White and Sage (1980) requires that some but not necessarily all trade-off weight ratios be precisely specified. (In contrast, the decision aid described in (Kelly, 1978) requires that all such trade-off weight ratios be precisely specified before search for the most preferred alternative can begin.) Humans, however, may often find it easier and more natural to place bounds on these ratios. The intent of this paper is to examine the implications of allowing ratios of parameter values to be imprecisely described in terms of upper and/or lower bounds for a broad class of single-stage decisionmaking models.

The paper is outlined as follows. The general, single-stage problem formulation is presented and discussed in Section 2. We note that this problem formulation models many multiobjective decision making problems under risk. Five special and relevant cases of the general problem formulation are displayed in Section 3, four of which are treated in depth in this paper. Transitivity and set inclusion results are determined in Sections 4 and 5, respectively. In Section 6, we present three different procedures for determining a preference structure on the alternative set for one of the special cases. Procedures for determining or approximating a preference structure on the alternative set for the other three cases of interest are presented in Section 7. In Section 8, a hypothetical automobile purchasing problem, originally considered by White and Sage (1980), is reconsidered in the context of the new assessment model presented in this paper. The intent of this hypothetical example is to illustrate the behavioral relevance of our assessment model and several other decision aiding implications of our results. Conclusions are presented in the final section.

II. THE BASIC PROBLEM FORMULATION AND DISCUSSION

We now present the basic problem formulation. Let $A = \{1,2,\ldots,A\}$ be the finite set of alternatives available to the DM. The DM is allowed to select one alternative from A. The criterion on which selection is based is

$$\sum_{m=1}^{M} \sum_{n=1}^{N} \eta_m(a)\, U_{mn}(a)\, \rho_n(a) = \eta(a)\, U(a)\, \rho(a)$$

where $\eta(a) \in R_M = \{\eta \in \mathbb{R}^M: 0 \leq \eta_m, m=1,\ldots, M, \Sigma_m \eta_m = 1\}$, $\rho(a) \in R_N$, and $U(a) \in C_{M\times N} = \{U \in \mathbb{R}^{M\times N}: 0 \leq U_{mn} \leq 1, m = 1,\ldots, M, n = 1,\ldots, N\}$. (If η premultiplies (postmultiplies) a vector or a matrix, then η will be considered a row (column) vector.)

We assume that there is a set $\Lambda(a', a) \subseteq (R_M \times C_{M\times N} \times R_N)^2$ associated with every ordered pair $(a', a) \in A \times A$. Let $\Lambda = \{\Lambda(a', a): (a', a) \in A \times A\}$.

The objective of the problem is: given A and Λ, determine the subset $R(A, \Lambda) \subseteq A \times A$ such that $(a', a) \in R(A, \Lambda)$ if and only if

$$\eta' U' \rho' \geq \eta U \rho$$

for all $(\eta', U', \rho', \eta, U, \rho) \in \Lambda(a', a)$.

The motivation for examining this general problem formulation is that it models several important classes of single-stage decision making problems having partially identified parameters. The most general of these classes of problems is the multiattribute decisionmaking problem under risk, where:

- M is the number of attributes under consideration.
- N is the number of outcomes that can result from alternative selection.
- $\eta_m(a)$ is the trade-off weight assigned to attribute m if alternative a is selected ($\eta_m(a)$ is usually assumed alternative invariant).
- $\rho_n(a)$ is the probability that outcome n will occur if alternative a is selected.
- $U_{mn}(a)$ is the utility of selecting alternative a and receiving outcome n with respect to attribute m.
- $\eta(a)\ U(a)\rho(a)$ is the expected utility of selecting alternative a.
- $\Lambda(a', a)$ represents what information is available regarding the value of the 6-tuple $\{\eta(a'), U(a'), \rho(a'), \eta(a), U(a), \rho(a)\}$.
- $R(A, \Lambda)$ represents what information can be induced from A and Λ regarding preferences on A.
- *The form of the multiattribute utility function is additive.*

The set of ordered pairs $R(A, \Lambda)$ can represent a valuable aid in alternative selection. If there is an $a' \in A$ such that $(a', a) \in R(A, \Lambda)$ for all $a \in A$, then a' is an optimal alternative. Additionally, if $(a, a') \notin R(A, \Lambda)$ for all $a \neq a'$, then a' is a unique optimal alternative. More generally, the nondominated set of $R(A, \Lambda)$* is guaranteed to contain the most preferred alternative. Thus, knowledge

*Alternative $a \in A$ is said to be dominated if there is an $a' \in A$ such that $(a',a) \in R(A, \Lambda)$ and $(a,a') \notin R(A, \Lambda)$. The set of all alternatives in A that are not dominated is called the nondominated set of $R(A, \Lambda)$.

of $R(A, \Lambda)$ can enhance decisionmaking, even for the case where Λ does not provide enough information to identify an optimal alternative, by eliminating alternatives that are clearly inferior.

III. SPECIAL CASES

We now present several specializations of the basic problem formulation.

CASE 1. Let $\Lambda_1(a',a) = \{\eta(a')\} \times \{U(a')\} \times \{\rho(a')\} \times \{\eta(a)\} \times \{U(a)\} \times\{\rho(a)\}$ for all $(a'a) \varepsilon A \times A$. Then $\eta(a)$, $U(a)$, and $\rho(a)$ are known precisely for all $a \varepsilon A$. This case is a standard decision analysis problem formulation having an additive, multiattribute utility function. If $\eta(a) = (0,\ldots, 1,\ldots, 0)$ for all $a \varepsilon A$, then this case is the single attribute decision analysis problem under risk; if $\rho(a) = (0,\ldots, 1,\ldots, 0)$ for all $a \varepsilon A$, then this case is the multiattribute decision analysis problem under certainty.

CASE 2. Let $\Lambda_2(a',a)$ be the set of all 6-tuples $(\eta(a'), U(a'), \rho(a'), \eta(a), U(a), \rho(a))$ such that $U(a')$, $\rho(a')$, $U(a)$, and $\rho(a)$ are all members of sets containing a single point (and hence are known exactly) and $\eta(a') = \eta(a) \varepsilon N = \{\eta \varepsilon R_M: B\eta \leq b \} \neq \emptyset$, for given matrix B and vector b. Thus, the trade-off weights are assumed alternative invariant and only partially identified by linear inequalities.

If we interchange the interpretation of η and ρ, then this case also considers the situation where outcome probabilities are assumed alternative invariant and partially identified. In fact, it will be valuable for us to do so for comparative purposes. Therefore, define $\tilde{\Lambda}_2(a',a)$ to be the set of all 6-tuples $(\eta(a'), U(a'), \rho(a'), \eta(a), U(a), \rho(a))$ such that $\eta(a')$, $U(a')$, $\eta(a)$, $U(a)$ are known precisely and $\rho(a') = \rho(a) \varepsilon P = \{\rho \varepsilon R_N: C\rho \leq c\} = \emptyset$.

CASE 3. Let

$$\Lambda_3(a', a) = \Omega_3(a') \times \Omega_3(a)$$

where $\Omega_3(a) = \{\eta\ (a)\} \times \{U(a)\} \times P(a)$ and $P(a) = \{\rho \in R_N: \ C(a)\rho \leq c(a)\} \neq \emptyset$ for given matrix $C(a)$ and vector $c(a)$, $a \in A$. Thus, trade-off weights and utilities are known exactly but the probability mass functions are not necessarily equal and only partially identified by linear inequalities.

CASE 4. Let

$$\Lambda_4(a',a) = \Omega_4(a') \times \Omega_4(a)$$

where

$$\Omega_4(a) = N(a) \times \{U(a)\} \times P(a)$$

$$N(a) = \{\eta \in R_M: \ B(a)\eta \leq b(a)\}$$

$$P(a) = \{\eta \in R_N: \ C(a)\rho \leq c(a)\}$$

Thus, $U(a)$ is known exactly but $\eta(a)$ and $\rho(a)$ are only imprecisely known. This case considers the situation where both the single attribute utility function $\eta(a)$ and the probability mass function $\rho(a)$ are only imprecisely known, where $U(a) = I$ for all $a \in A$.

CASE 5. Let

$$\Lambda_5(a',a) = \Omega_5(a') \times \Omega_5(a)$$

where

$$\Omega_5(a') = N(a) \times U(a) \times P(a)$$

$N(a)$ is a convex polytope described by extreme points $\{\eta^{\ell}(a), \ell = 1,\ldots, L(a)\}$

$U(a)$ is a convex polytope described by extreme points $\{U^{k}(a), k = 1,\ldots, K(a)\}$

$P(a) \subseteq R_N$.

Thus, trade-off weights, outcome probabilities and utilities are all imprecisely known.

IV. TRANSITIVITY RESULTS

A clearly desirable characteristic of any relation concerned with preference is transitivity.* We now present conditions on Λ which imply that $R(A, \Lambda)$ is transitive and show that these conditions hold for all Λ_i, $i = 1,\ldots,5$.

THEOREM 1. Assume for any triple $(a, a', a") \in A^3$ such that $(a", a') \in R(A, \Lambda)$ and $(a', a) \in R(A, \Lambda)$, the sets

$$\{(\eta', U', \rho'): (\eta", U", \rho", \eta', U', \rho') \in \Lambda(a", a')\}$$

and

$$\{(\eta', U', \rho'): (\eta', U', \rho', \eta, U, \rho) \in \Lambda(a', a)\}$$

have nonnull intersection for all $(\eta", U", \rho", \eta, U, \rho) \in \Lambda(a", a)$. Then, $R(A, \Lambda)$ is transitive.

Proof: Assume $(a", a') \in R(A, \Lambda)$ and $(a', a) \in R(A, \Lambda)$; we wish to show that $(a", a) \in R(A, \Lambda)$. Consider any $(\eta", U", \rho", \eta, U, \rho) \in \Lambda(a",a)$. By assumption, there exists a triple $(\bar{\eta}, \bar{U}, \bar{\rho})$ such that

$$(\bar{\eta}, \bar{U}, \bar{\rho}) \in \{(\eta', U', \rho'): (\eta", U", \rho", \eta', U', \rho') \in \Lambda(a",a')\}$$
$$\cap \{(\eta', U', \rho'): (\eta', U", \rho', \eta, U, \rho) \in \Lambda(a',a)\}.$$

Since $(a", a')$, $(a', a) \in R(A, \Lambda)$, $\eta" U" \rho" \geq \bar{\eta}\ \bar{U}\ \bar{\rho}$ and $\bar{\eta}\ \bar{U}\ \bar{\rho} \geq \eta\ U\ \rho$ and hence $\eta" U" \rho" \geq \eta\ U\ \rho$. Since this result holds for

*The relation $R(A, \Lambda)$ is said to be transitive if for all a, a', and a" such that $(a", a') \in R(A, \Lambda)$ and $(a', a) \in R(A, \Lambda)$, it follows that $(a", a) \in R(A, \Lambda)$.

any $(\eta'', U'', \rho'', \eta, U, \rho) \in \Lambda(a'', a)$, $(a'', a) \in R(A, \Lambda)$. $\square$

COROLLARY 1. $R(A, \Lambda_i)$ is transitive for $i = 1, \ldots, 5$.

Proof: Throughout the proof, it is useful to note that $(\eta', U', \rho', \eta, U, \rho) \in \Lambda_i(a', a)$ if and only if $(\eta, U, \rho, \eta', U', \rho') \in \Lambda_i(a, a')$ for all $(a', a) \in A \times A$ and all $i = 1, \ldots, 5$.

Case 1. Trivial; in fact $R(A, \Lambda_1)$ linearly orders A.

Case 2. Note that for any $(\eta'', U'', \rho'', \eta, U, \rho) \in \Lambda_2(a'', a)$,

$$\{(\eta', U', \rho'): (\eta'', U'', \rho'', \eta', U', \rho') \in \Lambda_2(a'', a')\}$$
$$=\{(\eta', U', \rho'): (\eta', U', \rho', \eta, U, \rho) \in \Lambda_2(a', a)\}$$
$$=\{\eta(a')\} \times \{U(a')\} \times \{\rho\} \neq \emptyset.$$

The result then holds from Theorem 1.

Cases 3, 4, and 5. Note that for any $(\eta'', U'', \rho'', \eta, U, \rho) \in \Lambda_5(a'', a)$,

$$\{(\eta', U', \rho'): (\eta', U', \rho', \eta, U, \rho) \in \Lambda_5(a', a)\}$$
$$=\{(\eta', U', \rho'): (\eta'', U'', \rho'', \eta', U', \rho') \in \Lambda_5(a'', a')\}$$
$$= \mathcal{N}(a') \times \mathcal{U}(a') \times \mathcal{P}(a') \neq \emptyset.$$

Thus, the hypothesis of Theorem 1 is satisfied for cases 3, 4, and 5, since cases 3 and 4 are specializations of case 5. $\square$

V. SET INCLUSION RESULTS

In this section, we derive several useful set inclusion results. The first result, Lemma 1, suggests a general approach for decision support. The second result, Corollary 2, has potential computational significance. Proofs of both results follow directly from the appropriate definitions.

LEMMA 1. If $\Lambda' \subseteq \Lambda$, then $R(A, \Lambda) \subseteq R(A, \Lambda')$.

Lemma 1 suggests the following general approach to decision aiding:

0. Set $k = 0$ and $\Lambda^0 = R_M \times C_{M*N} \times R_N$.
1. Determine $R(A, \Lambda^k)$.
2. If $R(A, \Lambda^k)$ provides a sufficient amount of information for alternative selection, then stop. If not, then go to Step 3.
3. Perform assessment procedures to produce $\Lambda^{k+1} \subseteq \Lambda^k$, set $k = k + 1$, and go to Step 1.

We will present an application of this approach to a hypothetical automobile purchasing example in Section 8.

We now indicate the various relationships that can exist between the various $R(A, \Lambda_i)$. Proof of the following corollary follows directly from Lemma 1.

COROLLARY 2. (a) $R(A, \Lambda_5) \subseteq R(A, \Lambda_4) \subseteq R(A, \Lambda_3) \subseteq R(A, \Lambda_1)$. (b) If $P \subseteq P(a)$ for all $a \in A$, then $R(A, \Lambda_3) \subseteq R(A, \tilde{\Lambda}_2) \subseteq R(A, \Lambda_1)$.

The operational usefulness of the results presented in Corollary 2 is that if $R(A, \Lambda_i)$ is difficult to determine but that $R(A, \Lambda_j)$ and/or $R(A, \Lambda_k)$ are relatively simple to calculate and $R(A, \Lambda_j) \subseteq R(A, \Lambda_i) \subseteq R(A, \Lambda_k)$, then knowledge of $R(A, \Lambda_j)$ and/or $R(A, \Lambda_k)$ and use of the transitivity of these relations can be helpful in aiding alternative selection and/or mollifying the difficulty in computing $R(A, \Lambda_i)$.

VI. SOLUTION PROCEDURES FOR $R(A, \tilde{\Lambda}_2)$:

We now present three procedures for determining $R(A, \tilde{\Lambda}_2)$. The first two assume that the parameter set P is described in terms of linear inequalities; the third procedure assumes that P is equivalently defined as the convex hull of a finite number of extreme points.

(a) The One-Pass Procedure. The one-pass procedure is based in part on the so-called one-pass algorithm presented in (Smallwood and Sondik, 1973) for a more complex problem formulation.(See also (Potter and Anderson, 1980). Our one-pass procedure is composed of two steps:

1. Determine the set of all possible linear orders that the alternatives can have for parameters in P.
2. Generate $R(A, \tilde{\Lambda}_2)$ from the above linear orders.

The approach used to complete Step 1 will also determine the regions in P where each of the linear orders obtained is optimal. The determination of such regions has obvious use in a sensitivity analysis. We now describe each of the above two steps.

Step 1. Observe that any point in P generates a linear ordering of the alternatives. For example, assume for $\rho^0 \in P$, $\gamma(a)\, \rho^0 \geq \gamma(a+1)\rho^0$, $a = 1,\ldots, A-1$, for $\gamma(a) = \eta(a)\, U(a)$. That is, alternative 1 is preferred to alternative 2, which is preferred to alternative 3, and so forth. Thus, ρ^0 is associated with the linear order $\{1,\ldots, A\}$. In fact, all points in the region $R^0 = \{\rho \in P\colon [\gamma(a) - \gamma(a+1)]\ \rho \geq 0$, $a = 1,\ldots, A-1\}$ are associated with the linear order $\{1,\ldots, A\}$.

Note that R^o is a convex polytope; procedures for determining which constraints of the form $[\gamma(a) - \gamma(a+1)]\,\rho \geq 0$ are not redundant are surveyed in (Mattheis and Rubin, 1980). We observe that R^o is bounded by linear inequalities describing P and R and those of the form $[\gamma(a) - \gamma(a+1)]\,\rho \geq 0$. On the "other side" of the latter type of boundary, $\gamma(a+1)\rho > \gamma(a)\rho$ for some $1 \leq a \leq A-1$ and hence two alternatives which are adjacent in rank for $\rho \epsilon R^o$ have switched positions in the linear ordering, producing a new linear order, new inequalities, and a new region in P having a constant linear ordering. Clearly, this new region is also a convex polytope. The set of all necessary inequalities for R^o of the form $[\gamma(a) - \gamma(a+1)]\,\rho \geq 0$ indicates what regions in P having a constant linear ordering border R^o. Successively examining these regions, determining their associated linear orders, and discovering other convex polytopes having constant linear orders will eventually produce a set $\{R^j\}$ of convex polytopes with constant linear orders which covers P; i.e., $P = \cup_j R^j$. The objective then becomes to take the set of linear orders associated with the set $\{R^j\}$and produce a preference relation on the alternative set.

Step 2. Let S be an $A \times A$ matrix composed as follows:

(i) if for any of the linear orders determined in Step 1, alternative a' is preferred to alternative a, then set the (a', a) entry of S to 1; set all other entries equal to 0.

(ii) if the (a, a') and (a', a) entries are both 1, set them both to 0.

The matrix S, often called a subordination matrix, provides necessary information for the construction of a domination digraph. See (Sage, 1977) for details. Such a digraph is a graphical depiction of $R(A, \tilde{\Lambda}_2)$. The following example illustrates the concepts presented.

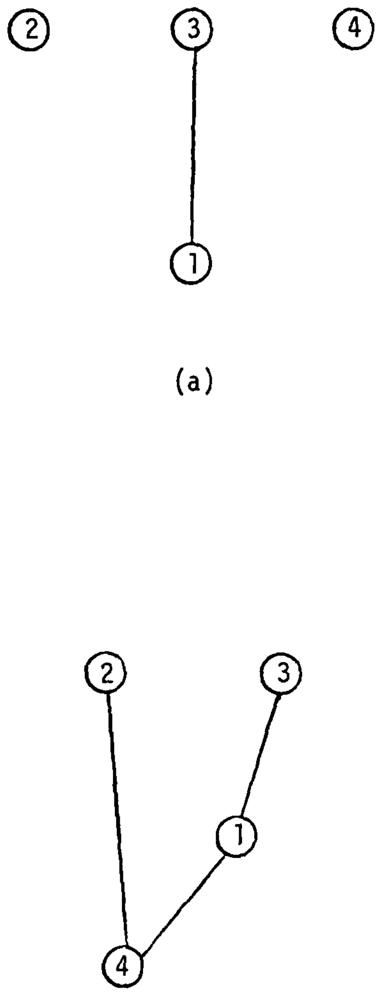
EXAMPLE 1. Assume A = 4, N = 3, and

	$\gamma_1(a)$	$\gamma_2(a)$	$\gamma_3(a)$
a=1	0.5	1	0
a=2	1	0.5	0.5
a=3	0.5	1	0.5
a=4	0.5	0	1

The associated domination digraph is given in Figure 1a, indicating that on the basis of the three objectives under consideration and the usual product order on R_3, i.e. the assumption that $\rho \in R_3$, alternative 3 dominates alternative 1. Assume the DM has revealed preferences that indicate $\rho_1 \geq \rho_3$, $\rho_2 \geq \rho_3$, and $\rho_3 \leq 0.25$. This region in R_3 is depicted graphically in Figure 2. Note that this description of P is equivalent to the existence of a 3×3 matrix C and a 3-vector c such that

$$C = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad c = \begin{bmatrix} 0 \\ 0 \\ 0.25 \end{bmatrix}$$

where $P = \{\rho \in R_3 : C\rho \leq c\}$.



(a)

(b)

Figure 1. The Domination Digraphs for (a) R_3 and (b) P for Examples 1 and 2.

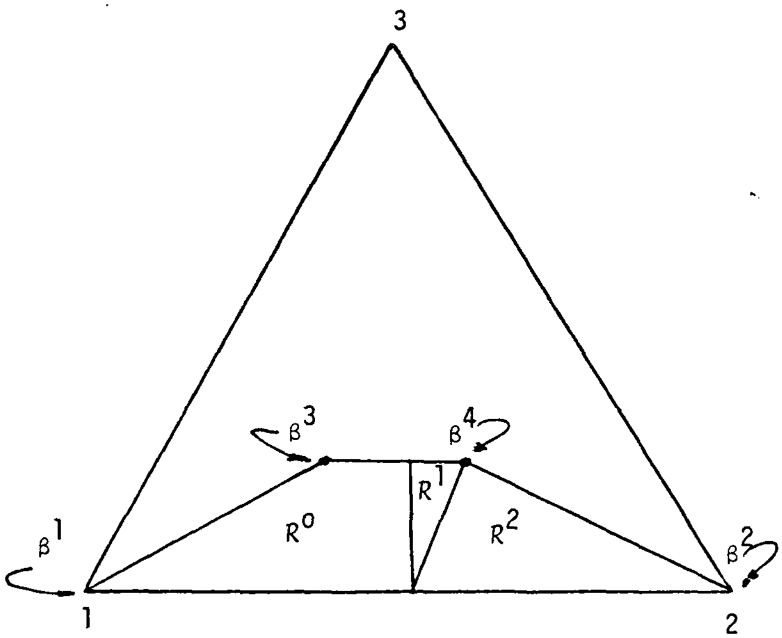


Figure 2. The Set $P = R^0 \cup R^1 \cup R^2$ and the Subregions R^0, R^1, and R^2 for Examples 1 and 2.

To start the algorithm, let ρ^0 = col (0.90, 0.07, 0.03). We note that $\gamma(2)\rho^0 \geq \gamma(3)\rho^0 \geq \gamma(1)\rho^0 \geq \gamma(4)\rho^0$. Let $R^0 = \{\rho \in P\text{: } \gamma(2)\rho \geq \gamma(3)\rho \geq \gamma(1)\rho \geq \gamma(4)\rho\}$. R^0 is bounded by the equality $\gamma(2)\rho = \gamma(3)\rho$. Thus, an adjacent subregion is $R^1 = \{\rho \in P\text{: } \gamma(3)\rho \geq \gamma(2)\rho \geq \gamma(1)\rho \geq \gamma(4)\rho\}$. R^1 is bounded by the equality $\gamma(1)\rho = \gamma(2)\rho$ and hence has an adjacent subregion $R^2 = \{\rho \in P\text{: } \gamma(3)\rho \geq \gamma(1)\rho \geq \gamma(2)\rho \geq \gamma(4)\rho\}$. Since $\cup_i R^i = P$, we have completed our "one-pass" over the region P. The various subregions of P are presented in Figure 2. The possible linear orderings of the alternatives are therefore: {2, 3, 1, 4} for R^0, {3, 2, 1, 4} for R^1, and {3, 1, 2, 4} for R^2. These linear orders produce the following subordination matrix

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The resulting domination digraph is presented in Figure 1b. We observe that restricting ρ to P has guaranteed that both 1 and 2 now dominate 4 and that 3 continues to dominate 1. □

b. <u>The Linear Programming Approach</u>. Define

$$z^*(a',a) = \min_{\rho \in P} [\gamma(a') - \gamma(a)]\rho \ .$$

Clearly,

(i) $(a', a) \in R(A, \tilde{\lambda}_2)$ if and only if $z^*(a', a) \geq 0$.

(ii) $(a', a) \notin R(A, \Lambda_2)$ if and only if there exists a $\rho \in P$ such that $[\gamma(a') - \gamma(a)]\rho < 0$.

Thus, (i) implies that the alternatives can be related by considering the sign of the optimal criterion value of A(A-1) linear programs; (ii) implies that if there is a basic feasible solution ρ such that $[\gamma(a') - \gamma(a)]\rho$ is negative, then the linear program evaluating the pair (a', a) can conclude that $(a', a) \notin R(A, \tilde{\Lambda}_2)$ without having to satisfy optimality conditions. This procedure applied to Example 1 required the consideration of 11 linear programs ($(3, 4) \in R(A, \tilde{\Lambda}_2)$ was determined by transitivity from previously obtained $(1, 4) \in R(A, \tilde{\Lambda}_2)$ and $(3, 1) \in R(A, \tilde{\Lambda}_2)$), 3 of which were terminated before completion because the criterion value went negative before optimality conditions were satisfied. The results were in agreement with the results of Example 1.

c. The Transformation Approach. Assume that P is described as the convex hull of the set of (extreme) points $\{\beta^\ell, \ell=1,\ldots,L\}$. Procedures for determining $\{\beta^\ell\}$ from C and c are contained in (Mattheis and Rubin, 1980). The transformation approach for determining the domination digraph for ρ restricted to P is based on the following fact: $\gamma'\beta^\ell \geq \gamma\beta^\ell$ for all $\ell = 1,\ldots, L$ if and only if $\gamma'\rho \geq \gamma\rho$ for all $\rho \in P$. Note that when $P = R_N$, it follows that $L = N$, $\beta^\ell = \text{col}\ (0,\ldots, 1\ldots, 0)$ where the 1 is the ℓ^{th} entry, and $\gamma'\beta^\ell \geq \gamma\beta^\ell$ for all ℓ is equivalent to $\gamma_n' \geq \gamma_n$ for all n. The above if and only if condition suggests the following procedure:

1. Determine $\gamma(a)\beta^\ell$ for all ℓ and a.

2. Construct a digraph of alternatives, based on the relation: $(a', a) \in R'$ if and only if $\gamma(a')\beta^{\ell} \geq \gamma(a)\beta^{\ell}$ $\ell = 1, \ldots, L$.

Thus, $\gamma(a)\beta^{\ell}$, $\ell=1, \ldots, L$, acts like the set of value scores for alternative a based on P.

EXAMPLE 2. Consider the problem presented in Example 1. Note that $L = 4$ and

	$\ell=1$	2	3	4
β_1^{ℓ}	1	0	.50	.25
β_2^{ℓ}	0	1	.25	.50
β_3^{ℓ}	0	0	.25	.25

The β^{ℓ} are graphically depicted in Figure 2.

It then follows that

	$\ell = 1$	2	3	4
$\gamma(1)\beta^{\ell}$	1	2	1.00	1.25
$\gamma(2)\beta^{\ell}$	2	1	1.50	1.25
$\gamma(3)\beta^{\ell}$	1	2	1.25	1.50
$\gamma(4)\beta^{\ell}$	1	0	1.00	0.75

which produces a digraph identical to the digraph in Figure 1b. $\square$

The one-pass and the linear programming procedures are preferred over the transformation procedure for two reasons. First, we feel that parameter value information is more easily and more directly described mathematically in the form $P = \{\rho \in R_N: C\rho \leq c\}$ rather than in terms of extreme points of P, a statement to which the automobile purchasing example in Section 8 provides support. Second, determining the extreme points of a set of the form $P = \{\rho \in R_N: C\rho \leq c\}$, which is required in order to use the transformation approach, appears generally to require considerably more computational effort than is saved by the relative computational simplicity of the transformation approach. A clear advantage of the linear programming procedure over the one-pass procedure is the relative availability of efficient linear programming software. The one-pass approach, however, provides more information about the ordering of the alternatives in that in Step 1, regions of P associated with total orders are determined. Such information would be necessary in order to determine how much a parameter vector would have to vary away from the nominal in order to compromise the optimality of the most preferred alternative relative to the nominal.

VII. SOLUTION PROCEDURES AND APPROXIMATIONS FOR $R(A, \Lambda_i)$, $i = 3,4,5$.

Since $\Lambda_i(a', a) = \Omega_i(a') \times \Omega_i(a)$ for $i = 3,4,5$, a necessary and sufficient condition for $(a', a) \in R(A, \Lambda_i)$ is

$$\min \ \eta' U' \rho' \ \geq \max \ \eta U \rho$$

where the minimum is taken with respect to all $(\eta', U', \rho') \in \Omega_i(a')$ and the maximum is taken with respect to all $(\eta , U, \rho) \in \Omega_i(a)$. Therefore, in order to determine $R(A, \Lambda_i)$, it is sufficient to solve 2A mathematical programming problems, half of which are minimization problems and half of which are maximization problems. We now examine cases 3, 4, and 5 on the basis of these comments.

(a) $R(A, \Lambda_3)$. The solution of

$$\max/\min \ \eta \ U \ \rho$$

for all $(\eta, U, \rho) \in \Omega_3(a)$ is a linear program of the form

$$\begin{aligned} &\max/\min \ \gamma \rho \\ &\qquad \text{s.t.} \quad C\rho \leq c \\ &\qquad\qquad\quad \rho \in R_N \end{aligned}$$

We now illustrate the determination of $R(A, \Lambda_3)$ with the following example.

EXAMPLE 3. Let γ be given as in Example 1, and assume that $P(a) = P$ for all $a \in A$ for the set P presented in Example 1. Thus, Λ_3 is identical to the $\tilde{\Lambda}_2$ given in Example 1 except that ρ and ρ' are not constrained to be equal. The solution of the requisite 8 linear programs generates the domination digraph shown in Figure 3. We note that by comparing Figure 1b and Figure 3, $R(A, \Lambda_3) \subsetneq R(A, \tilde{\Lambda}_2)$ (observe that $(3, 1) \in R(A, \tilde{\Lambda}_2)$, $(3, 1) \notin R(A, \Lambda_3)$), which is in agreement with Corollary 2b. Note also that had $R(A, \Lambda_3)$ been determined before $R(A, \tilde{\Lambda}_2)$, 3 of the (at most) 12 linear programs required to determine $R(A, \tilde{\Lambda}_2)$ could have been eliminated. □

(b) $R(A, \Lambda_4)$. The solution of

$$\max/\min \ \eta U \rho$$

for all $(\eta, U, \rho) \in \Omega_4(a)$ is a quadratic program of the form

$$\begin{aligned} \max/\min \quad & \eta U \rho \\ \text{s. t.} \quad & B\eta \le b \\ & C\rho \le c \\ & \eta \in R_M, \quad \rho \in R_N. \end{aligned}$$

Since the $2M \times 2N$ matrix

$$\begin{bmatrix} 0 & U \\ U & 0 \end{bmatrix}$$

is neither positive semidefinite nor negative semidefinite, the Kuhn-Tucker conditions for both the minimization and the maximization problems are only necessary.

The Kuhn-Tucker conditions, however, can be used to determine upper and lower bounds on the criteria associated with the minimization and maximization problems, respectively. These bounds can then be used to generate a relation on $A \times A$ that bounds $R(A, \Lambda_4)$ from above. Specifically, let $z^*(a)$ be an upper bound on the criterion associated with the minimization problem for alternative a; similarly,

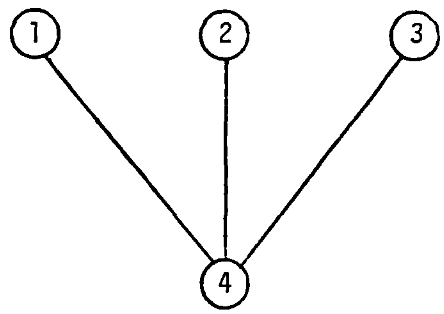


Figure 3. Domination Digraph for Λ_3 in Example 3.

let $z_*(a)$ be a lower bound on the criterion associated with the maximization problem for alternative a. Define the relation $R'(A, \Lambda_4)$ as follows: $(a', a) \in R'(A, \Lambda_4)$ if and only if $z^*(a') \geq z_*(a)$. Note that $R'(A, \Lambda_4)$ is transitive if $z_*(a) \geq z^*(a)$ for all $a \in A$. Clearly, if $(a', a) \in R(A, \Lambda_4)$, then $(a', a) \in R'(A, \Lambda_4)$, which leads to the following addition to Corollary 2.

LEMMA 2. $R(A, \Lambda_4) \subseteq R'(A, \Lambda_4)$ and hence $R(A, \Lambda_4) \subseteq R'(A, \Lambda_4) \cap R(A, \Lambda_3)$.

The following example illustrates determination of $R'(A, \Lambda_4)$.

EXAMPLE 4. Consider the problem stated in Example 3 except that

$$0.40 \leq \gamma_2(2) = 0.60$$
$$0.40 \leq \gamma_3(2) \leq 0.60$$
$$0.40 \leq \gamma_1(3) \leq 0.60$$
$$0.40 \leq \gamma_3(3) \leq 0.60$$

Thus, both utilities and probabilities are imprecisely known and can be alternative dependent. The concomitant domination digraph, determined using the solutions of the associated quadratic programming problems, is presented in Figure 4. We note that $\{(1, 4)\} = R'(A, \Lambda_4)$. It then follows from Lemma 2 that $R(A, \Lambda_4) \subseteq \{(1, 4)\}$. □

(c) $R(A, \Lambda_5)$. The solution of

$$\max/\min \quad \eta U \rho$$

for all $(\eta, U, \rho) \in \Omega_5(a)$ is in general more difficult then the mathematical programming problems associated with the determination of

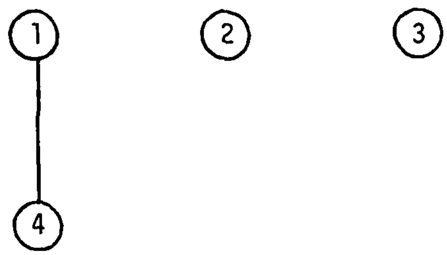


Figure 4. Domination Digraph for R'(A, Λ_4) in Example 4.

$R(A, \Lambda_i)$, $i = 1, \ldots, 4$. It is possible, however, to construct a specially structured Λ such that $R(A, \Lambda) \subsetneq R(A, \Lambda_5)$ and such that $R(A, \Lambda)$ can be determined using the quadratic programming procedure developed for determining $R(A, \Lambda_4)$. We define such a Λ after presenting the following result.

LEMMA 3. Let $N \subsetneq R_M$ and $U \subsetneq C_{M \times N}$ be convex polytopes having extreme points $\{\eta^\ell, \ell=1,\ldots, L\}$ and $\{U^k, k = 1, \ldots, K\}$, respectively. Define $\Gamma = \{\eta U: \eta \in N, U \in U\}$ and H to be the convex hull of $\{\eta^\ell U^k, \ell=1,\ldots, L, k=1,\ldots, K\}$. Then, $\Gamma \subsetneq H$. Additionally, assume that Γ is convex. Then $\Gamma = H$.

PROOF: Assume $\gamma \in \Gamma$. Then, there is a $\eta \in N$ and a $U \in U$ such that $\gamma = \eta U$. Since both N and U are both convex polytopes, there exist $\{\lambda_\ell\} \in R_L$ and $\{\sigma_k\} \in R_K$ such that $\eta = \Sigma_\ell \lambda_\ell \eta^\ell$ and $U = \Sigma_k \sigma_k U^k$. Note that $\gamma = \eta U = \Sigma_\ell \Sigma_k \lambda_\ell \sigma_k \eta^\ell U^k$ and that $\{\lambda_\ell \sigma_k\} \in R_{L \times K}$. Thus, $\gamma \in H$.

If Γ is convex and contains the extreme points of H, then $H \subsetneq \Gamma$ and hence $\Gamma = H$. $\square$

Let $\Gamma(a) = \{\eta U: \eta \in N(a), U \in U(a)\}$, and define $\Omega_4'(a) = \Gamma(a) \times \{I\} \times P(a)$ and $\Lambda_4'(a', a) = \Omega_4'(a') \times \Omega_4'(a)$. Relax the assumption that $\Gamma(a) \subsetneq R_N$ to $\Gamma(a) \subsetneq C_N$. Let $H(a)$ be the convex hull of $\{\eta^\ell(a) U^k(a)$: $\ell = 1,\ldots, L(a), k=1,\ldots, K(a)\}$, where $\{\eta^\ell(a): \ell=1,\ldots, L(a)\}$ and $\{U^k(a);\ k = 1,\ldots, K(a)\}$ are the extreme points for $N(a)$ and $U(a)$, respectively. Define $\Omega_4''(a) = H(a) \times \{I\} \times P(a)$ and $\Lambda_4''(a', a) = \Omega_4''(a') \times \Omega_4''(a)$.

COROLLARY 3. $R(A, \Lambda_4'') \subseteq R(A, \Lambda_4') = R(A, \Lambda_5)$. If $\Gamma(a)$ is convex for all $a \in A$, then $R(A, \Lambda_4'') = R(A, \Lambda_5)$.

PROOF. The proof follows from Lemmas 1 and 3. $\square$

Corollary 3 indicates that a properly constructed Λ can generate a lower bound on $R(A, \Lambda_5)$ which may determine $R(A, \Lambda_5)$ exactly and that $R(A, \Lambda)$ can be approximated using the quadratic programming procedure used for determining $R(A, \Lambda_4)$.

EXAMPLE 5. Consider the problem stated in Example 4 which we modify as follows. Let $\eta(a) = \gamma(a)$ for all $a \in A$, and assume

$$U(a) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & u_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $0.9 \le u_{22} \le 1.0$. Redefine $\gamma(a) = \eta(a)\, U(a)$. Then,

$$\begin{array}{ccc}
\gamma_1(1) = 0.5 & 0.9 \le \gamma_2(1) \le 1.0 & \gamma_3(1) = 0 \\
\gamma_1(2) = 1 & 0.36 \le \gamma_2(2) \le 0.6 & 0.4 \le \gamma_3(2) \le 0.6 \\
0.4 \le \gamma_1(3) \le 0.6 & 0.9 \le \gamma_2(3) \le 1 & 0.4 \le \gamma_3(3) \le 0.6 \\
\gamma_1(4) = 0.5 & \gamma_2(4) = 0 & \gamma_3(4) = 1
\end{array}$$

Thus, $\Gamma(a)$ is convex for all $a \in A$ and $R(A, \Lambda_4'') = R(A, \Lambda_5)$ from Corollary 3. Computations show that the parameter value imprecision is sufficient to imply that no alternative dominates any other alternative; thus, $R(A, \Lambda_5) = \emptyset$. $\square$

VIII. AN EXAMPLE APPLICATION

We now reexamine the hypothetical automobile purchasing problem presented by White and Sage (1980). This problem is modeled by Case 2 for the special case where N = 1, i.e. the decision making under certainty case. The objectives hierarchy is displayed in Figure 5. Each box corresponds to an attribute and an associated trade-off weight as described in Table 1. Each attribute corresponds to an objective; e.g. the attribute "safety" corresponds to the objective "maximize safety." When there is no confusion, we will use the attribute name as an abreviated notation for the associated objective.

The objectives hierarchy indicates what objectives can be decomposed into "lower level" objectives. For example, "cost" is composed of "initial cost", "operating cost", and "resale value". Table 2 presents value scores for each of the six (6) automobiles under consideration for each of the lowest level objectives A through H. We assume that these value scores have been assessed from a well-informed DM. Since we note that the value score associated with alternative 1 is at least as great as the value score associated with alternative 5 for each lowest level objective, alternative 1 is preferred to alternative 5 no matter what trade-off weights are applied, i.e. $(1, 5) \varepsilon R(A, \tilde{\Lambda}_2)$ for $P = R_8$. Similarly, $(2, 6) \varepsilon R(A, \tilde{\Lambda}_2)$ for $P = R_8$. A graphical depiction of this preference information is presented in Figure 6 in the form of a domination digraph. Figure 6 indicates that there are four candidates for the most preferred automobile, cars 1, 2, 3, and 4, and since the objective

TABLE 1

ATTRIBUTE NAMES AND ASSOCIATED WEIGHTS

Attribute	Name	Weight
A	safety	ρ_1
B	initial cost	ρ_2
C	fuel economy	ρ_3
D	scheduled maintenance expenses	ρ_4
E	expected unscheduled maintenance expenses	ρ_5
F	resale value	ρ_6
G	attractiveness	ρ_7
H	trunk and passenger compartment capacity	ρ_8
A-H	overall desirability	$\rho_1+\ldots+\rho_8$
C-E	operating cost	$\rho_3+\rho_4+\rho_5$
B-F	cost	$\rho_2+\ldots+\rho_6$

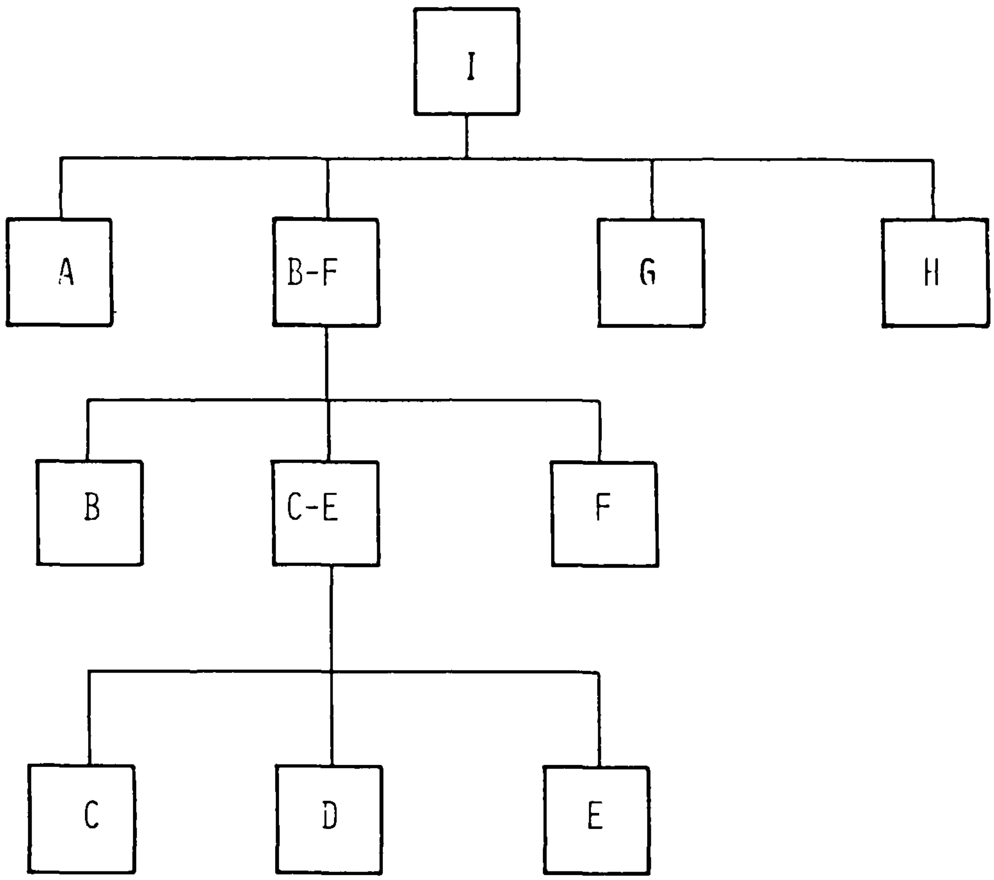


Figure 5. Objectives Hierarchy for the Automobile Purchasing Example

Table 2. Value Scores for Lowest Level Objectives

		VALUE SCORES FOR OBJECTIVES							
		A	B	C	D	E	F	G	H
ALTERNATIVES	x_1	70	100	65	40	80	10	100	60
	x_2	100	40	70	30	100	100	10	100
	x_3	60	35	70	35	10	10	40	50
	x_4	50	0	100	100	0	90	10	100
	x_5	65	40	0	40	75	0	30	55
	x_6	0	35	60	0	90	40	0	0

is to select the single most preferred automobile, cars 5 and 6 can be excluded from further consideration.

If the DM can select his or her most preferred alternative from the set {1, 2, 3, 4}, then the decision aiding process can stop. If not, additional preference information must be assessed in order to reduce the nondominated set of alternatives. Assume the DM first decides to express his or her preferences regarding the "cost" branch of the objectives hierarchy. Assume that in evaluating the relative merits of fuel economy (objective C) and scheduled maintenance expenses (objective D), the DM finds the difference in fuel economy between the car with the highest fuel economy (car 4) and the car with the lowest fuel economy (car 5) to be less important than the difference in scheduled maintenance expenses between the car with the highest scheduled maintenance expenses and the car with the lowest scheduled maintenance expenses. More succinctly, scheduled maintenance expenses are relatively at least as important as fuel economy. This preference might have a variety of explanations, e.g. all of the cars under consideration give relatively high, and relatively similar, miles per gallon. We express this preference mathematically as $\rho_3 \leq \rho_4$. Using similar arguments, assume also that the DM expresses other preferences that can be modeled by the following inequalities:

$$\rho_4 \leq \rho_5$$

$$\rho_3 + \rho_4 + \rho_5 \leq \rho_2 \leq \rho_6$$

Thus, expected unscheduled maintenance expense is considered relatively at least as important as scheduled maintenance expense, and resale

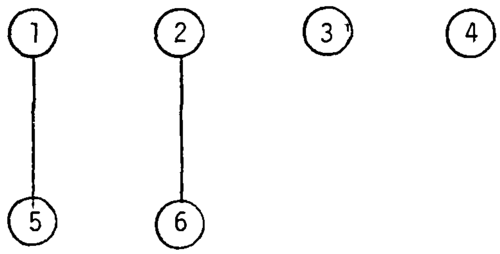


Figure 6. Domination Digraph for Table 2.

value is considered relatively at least as important as initial cost which in turn is considered relatively at least as important as operating cost. These inequalities produce the following C matrix and c vector:

$$C = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \qquad c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The associated domination digraph is given in Fig. 7, indicating that search for the most preferred automobile can be restricted to cars 1 and 2.

If the DM cannot or wishes not to decide between cars 1 and car 2, further preference information must be assessed. Assume that the DM has the following preferences:

(i) Trunk and passenger compartment capacity is relatively at least twice as important as attractiveness.

(ii) Safety is relatively at least twice as important as trunk and passenger compartment capacity and attractiveness combined.

(iii) Cost is relatively at least twice as important as trunk and passenger compartment capacity and attractiveness combined.

Thus,

Figure 7. Domination Digraph for the Automobile Purchasing Example After the First Set of Preference Inequalities.

$$\rho_8 \geq 2\,\rho_7$$

$$\rho_1 \geq 2\,(\rho_7 + \rho_8)$$

$$\rho_2 + \rho_3 + \rho_4 + \rho_5 + \rho_6 \geq 2\,(\rho_7 + \rho_8),$$

producing the following C matrix and c vector:

$$C = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & -1 & -1 & -1 & -1 & -1 & 2 & 2 \end{bmatrix} \qquad c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The resulting domination digraph is shown in Figure 8, indicating that car 2 is the most preferred. We remark that this selection was made without having to be precise about ratios of the form ρ_i / ρ_j, as may be required in (White and Sage, 1980), and without having to trade-off some relatively controversial objectives, e.g. comparing the relative worth of safety and cost.

The solution procedure used for this problem was the linear programming approach. Solution of the 30 requisite linear programs required 12.3 CPU seconds before the trade-off and 13.5 CPU seconds after the trade-off of the University of Virginia CDC 6400. We would expect these figures to at most double if the process of constructing the criteria for the linear programs was built into the software. We feel such computer times are quite adequate for interactive decision aiding.

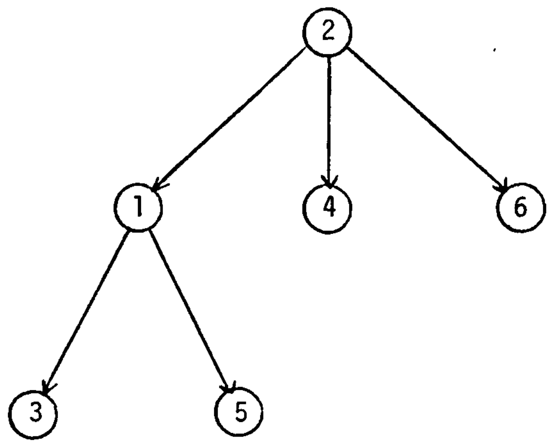


Figure 8. Domination Digraph for the Automobile Purchasing Example After the Second Set of Preference Inequalities.

IX. CONCLUSIONS

A general model of single-stage decisionmaking has been formulated and analyzed. The criterion was composed of alternative dependent parameters having values that may be only partially known. Four special cases of imprecise parameter information were considered in detail and solution or approximation techniques determined for them. These special cases model an important variety of choicemaking situations involving imprecisely known parameter values. A hypothetical automobile purchasing example was used to illustrate the potential of the decision aiding procedure implied by one of the special cases.

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UNIVERSITY OF VIRGINIA

School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,400 students with a graduate enrollment of approximately 600. There are 125 faculty members, a majority of whom conduct research in addition to teaching.

Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical and Aerospace to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering and Engineering Physics, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate; the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,530 full-time faculty with a total enrollment of about 16,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, Business Administration, and Education. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.

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