The demand for water transportation: application of discriminant analysis.
THE DEMAND FOR WATER TRANSPORTATION:
APPLICATION OF DISCRIMINANT ANALYSIS
TO COMMODITIES SHIPPED BY BARGE AND
COMPETING MODES IN OHIO RIVER AND
ARKANSAS RIVER AREAS.

LEVEL

Submitted to:
U.S. ARMY ENGINEER
INSTITUTE FOR
WATER RESOURCES
KINGMAN BUILDING
FORT BELVOIR, VIRGINIA 22060

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This report is not to be construed as necessarily representing the views of the Federal Government nor of the U.S. Army Corps of Engineers.
This dissertation develops a methodology for simulating transport demand functions for barge transportation. Discriminant analysis is utilized to calibrate a modal choice model from disaggregate observations of individual shipments. The modal choice model stems from earlier development by Moses and Lave (1971), Allen (1969) and Beuthe (1968).
Transport demand functions for barge shipments are developed for coal used for energy and metallurgical markets. Barge demand functions are developed for commodities other than coal including chemicals and refined petroleum products.

Data utilized in the analysis represents a total of 815 shipments totaling 145.9 million tons shipped annually for 14 major commodity groups utilizing 9 transport modes.

When the discriminant models were calibrated from logarithmic transformed data, 83-100 percent of the individual shipments were classified correctly. Further testing by a random holdout procedure showed no significant upward bias in classification.

Conventional rail and unit train modes comprised about 54 percent of the shipments and 33 percent of annual tonnage in the sample. Barge accounted for 18 percent of the shipments and 55 percent of the annual tonnage. Truck, pipelines and mixed mode shipments comprise the remainder.

Demand estimates, from the data, show the demand for barge transportation to be inelastic. Relative price escalation from 2 to 5 times the values reflected in the data is required to achieve substantial decreases in the quantity shipped by barge. Two significant policy implications are apparent. First, in the short run, the impact of waterway user charges is not likely to cause significant diversion of traffic to other modes. This would, however, allow competing modes to raise rates. Thus revenue deficiencies experienced by many rail carriers could be reduced by a waterway user charge. Second, the technological attributes of barge transportation remains as an advantage for many waterways users over competing modes.
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A Report Submitted to:
U.S. Army Engineer Institute for Water Resources
Kingman Building
Fort Belvoir, Virginia 22060

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AUGUST 1980
IWR Research Report 80-82
ACKNOWLEDGEMENTS

The research reported in this dissertation would not have been possible without the cooperation of many people. My dissertation committee provided comments and criticism which greatly aided the presentation of the assumptions, analysis and evaluation of results. Dr. Howard J. McBride chaired the committee and persistently supported my research. Dr. Charles A. Berry provided both the carrot and stick to bring the dissertation to conclusion. Dr. Gordon S. Skinner provided the kind of probing comments which forced the argument into a cohesive/logical structure. All of the faults of the analysis are, of course, those of the author.

Computer programming support was ably provided by R.S. Kaplan of the U.S. Army, Office of the Chief of Engineers, and by Diane French and Marilyn Fleming of the U.S. Army Engineer Coastal Engineering Research Center. Bob Swett of the U.S. Army Engineer Institute for Water Resources provided graphics support. Typing was done by Sandy Phillips, Barbara Wright, Lisa McPherson and Ann Nance of the U.S. Army Engineer Institute for Water Resources and the final draft was prepared by Joan Drinnen. To all of them, my thanks.
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CHAPTER I

INTRODUCTION

Scope and Purpose

This dissertation systematically develops estimates of demand functions for barge transportation for a number of commodities. The barge demand functions reflect competition from a full range of alternative transport modes. At issue are ways of collecting data for the full range of attributes which have a significant impact on the choice of transport mode, the analysis of the attributes of the data, the formulation of a theoretically sufficient model of transport mode selection by transport shippers and implementation of the empirical analysis to generate demand functions. The analysis is framed in the short run behavior of transport users and thus omits response which includes the relocation of economic activity.

The analysis is performed for two large regions of the nation. Because of the large amount of data representing several transport modes and commodities, determination of the proper level of disaggregation can be evaluated. The model developed stems from an extensive investigation of models using disaggregate modal choice. The analysis can replicate individual shipper mode selection decision-making, taking into account variables, identifiable in
cost accounts or not, which influence the profitability of a business firm. If these variables are not represented by economic cost estimates relevant to each shipment, the model accepts non-price variables and imputes an economic weight to those variables.

Policy Issues and Background

Demand estimates of barge transportation have become more significant to policy and project analysis as Federal policy interests have become sensitized to intermodal competition for traffic and Federal subsidies. Federal policy interest in intermodal competition among freight haulers has increased sharply in response to the decreasing profitability of the rail mode. Bankruptcy proceedings, proposals to close unprofitable rail line segments, requests for massive subsidies, and many other problems have become prominent influences on federal policy toward railroads. Important for the decreasing rail profitability explanation is the claim that federal barge transportation investment policy results in subsidies to the barge mode that allows it to attract traffic and potential profit away from the competing rail mode.

The U.S. Army Corps of Engineers' proposal to replace Lock and Dam 26 on the Mississippi River near Alton, Illinois, is a subject of a suit brought to court by Western
Railroads and environmental interests in 1972. A Federal
court ruled that authority for lock and dam replacement
of the extent proposed by the Corps, was insufficient and
thereby restrained construction until Congress explicitly
authorized the project. One prominent point in the suit
was the claim that additional barge capacity was not needed
because of excess capacity in competing rail lines and
that extra barge mode system capacity would adversely
affect rail profitability. One of the political repercussions of this claim was a proposal for a barge user
tax coupled with the appropriations action for Lock and
Dam 26.

1The Rivers & Harbors Act of 1909 authorizes the
replacement of existing projects as a major rehabilitation
action. The Corps replaced 38 low head navigational locks
and dams on the Ohio River with 18 high lift structures
and changed the lock size from 110' x 600' to twin 110' x
1200' and 110' x 600'. These changes increased the capac-
ity of the system and reduced the waiting time for tows
and maintenance costs. However, the U.S. District Court
for the District of Columbia (Civil Action 741190, Atchi-
son, Topeka and Santa Fe et al vs. Howard Calloway et al
and Civil Action 741191, Isaac Walton League of America
et al vs. Howard Calloway et al) ruled in the Lock and
Dam 26 case that the Corps had exceeded its replacement
authority and that explicit Congressional authorization
for a new project was required.

2An extensive series of newspaper articles have ac-
companied the Lock and Dam 26 case and legislation requir-
ing waterway user fees, R.T. Reid, "A Deft Maneuver by
Barge Fee Backers," The Washington Post, Tuesday, May 2,
1978, p. A4. Mr. Reid wrote 38 articles on this issue be-
If the topology of the demand function for water transportation is known, the benefits of adding new waterway capacity, the impacts of imposition of new policies for cost sharing, and the impacts of changes in operations policies can be determined. Figure 1.1 shows the amount that waterway users would be willing to pay for increased capacity that permits additional traffic to move at lower costs.

Barge Price

![Diagram](image)

Quantity Shipped by Barge

Figure 1.1 A shift in the supply function for barge transportation resulting from improvements in operating conditions.

An increase in the supply schedule $S$ to $S_1$, results in a decline in price from $P$ to $P_1$ and an increase in
quantity from Q to Q₁. This model shows a simple case in which the added capacity does not induce a shift in the demand schedule. This supply schedule is of the towing industry and does not reflect the costs of altering the waterway system to provide additional capacity. To accomplish a benefit cost analysis the area Q₁ Q AB, which reflects the amount that transport users would be willing to pay for the added capacity, is compared to the costs of adding the capacity. The difference between P₁ and P multiplied by quantity Q is cost savings to traffic using the waterway in the "without project" condition. Area A C B is a consumer surplus for users which ship additional quantities as a result of lowered barge transport costs in the "with project" condition.⁵

Current Benefit Evaluation Requirements

The rules for federal evaluation of a navigation project benefits are stated in the Procedures Manual.⁴

³Charles W. Howe, et al., Inland Waterway Transportation, Studies in Public & Private Management and Investment Decisions, Johns Hopkins Press (1969), presents a much more extensive discussion of the economic issues embodied in the analysis of public and private investment in shallow-water inland transportation. Effective public decision-making regarding investment and operating policies must take into account the response of private carriers and similarly private carrier decisions must take into account present and anticipated future public policies.

The manual defines four categories of benefits of a navigation project:

a. Cost Reduction Benefit (Same Origin-Destination; Same Mode). The same amount of traffic would use the waterway both "with" and "without project." The measure of benefit is the reduction in costs on an existing waterway where improved operating conditions occur with the project.

![Diagram showing cost reduction benefit](image)

**Figure 1.2 Cost Reduction Benefit Case**

A change in costs from $P_1$ to $P_2$ would result in benefits represented by the hatched area in Figure 1.2. The following is a list of cost reduction possibilities:

1. Reduction in total trip delays
2. Increases in efficiency due to improvements which allow longer or larger tows, and result in decreases in transportation costs per unit of goods hauled.

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Costs in Water Resources Planning (Level C); Proposed Rules and Procedures of Implementation," *Federal Register*, 44; No. 102 (Thursday, May 24, 1979), see paragraph 704.127, pp. 30220-30225.
(3) Increased efficiency due to barges being more fully loaded, as a result of deeper channel depths.

b. Shift of Mode Benefit (Same Origin-Destination, Different Mode). In this case shippers use the waterway "with" the project but another mode "without" the project. The problem is to estimate diversion from one mode to another. Impacts, if any, on barge costs would derive from the impacts discussed in the cost reduction benefit case above. Substitutability between modes is required for existence of this set of benefits.

c. Shift of Origin-Destination Benefit (Different Mode, Different Origin-Destination). In this case the location of the origin or destination and the mode of transportation is different "with" the project than "without" the project. Reductions in costs other than transportation costs may be included in the transportation benefit. The analytical requirements for this case are much more extensive than for the cases where origin and destination do not change as a result of the project. Because lowered transport costs could shift comparative advantage to an area with lower costs for factors other than transportation, benefits estimated may include the new location advantages for all factors.

d. New Movement Benefit. This case results from new traffic due to reductions in transport cost which
permit a commodity to be produced and transported in the "with" project and not in the "without" project case. Increases in production and consumption provide the basis for new movements, and the benefit is defined to include increases in producer and consumer surplus.

For each of these benefit categories, a special assumption about the nature of the transportation demand function is made:

a. Cost Reduction Benefit. It is assumed that an absolutely price inelastic demand function for a given transport mode exists.

b. Shift of Mode Benefit. It is assumed that a demand function exists which has price elastic properties. Substitution between modes is the key analytical element in estimating the demand function.

c. Shift of Origin-Destination Benefit. It is assumed that a price elastic demand function exists with location economies the key element in the benefit set.

d. New Movement Benefit. It is assumed that the transportation demand function may shift to the right due to increased goods consumption (with induced increases in transportation demand), or to increased production resulting from decreased transportation costs and subsequent use of the barge mode.
T.3: Manual defines the steps required for benefit estimates. "Without" project conditions are defined to be those most likely conditions expected to exist in the future in the absence of a new project and any associated changes in public policy towards barge transportation, including:

(1) All economically feasible moves within the discretion of the operating agency will be implemented at the appropriate time (including helper boats and lock operating policies which reduce delays).

(2) User charges provided by law at the time of estimation.

(3) Normal operation and maintenance work will be performed.

(4) Alternative modes will have infinite capacity at current costs.

(5) Only waterway investments currently in place or under construction are assumed to be in place over the period of estimation.

"With" project conditions are defined as the most likely conditions expected to exist in the future with the new project in place, including:
(1) Demand management measures, including the use of congestion or lockage fees, that are to be evaluated as nonstructural alternatives.  

(2) Actions which increase the supply capacity such as traffic management are to be evaluated as non-structural alternatives.

(3) Physical changes which increase supply capacity, such as a new waterway, larger locks, increasing channel depth, improving channel geometry in curves are to be evaluated as structural alternatives.

(4) Combinations of nonstructural and structural alternatives.

(5) Alternative timing of structural measures.

(6) Inclusion of authorized additions in the waterway system.

Procedures to be followed in navigation benefit estimation incorporate ten steps:

(1) Identify commodity types.

(2) Identify study area.

(3) Determine current commodity flows.

---

5The manual classifies alternative actions which reduce the cost of transportation as structural or non-structural. Structural measures include traditional means of increasing the physical capacity of a waterway, such as larger locks, deeper or straighter channels. Non-structural measures tend to be nontraditional means for decreasing costs, which might include the imposition of congestion fees or provision of helper boats to reduce the waiting times at congested locks.
(4) Determine current cost of moving commodity flows by competing modes.

(5) Forecast commodity demands.

(6) Project cost of competing modes.

(7) Determine current cost of waterway use.

(8) Determine future cost of waterway use.

(9) Determine waterway use with and without project.

(10) Compute annual benefits and discount.

Steps 9 and 10 require an approximation of or assumption about the transportation demand function for each commodity and each origin-destination pair. In addition, the choice of procedures used to estimate transport demand can influence the estimate of benefits substantially. Indeed, estimation procedures which omit transport demand determinants and favor supply determinants can cause substantial errors in estimating benefits in every case except the cost reduction benefit. The historic emphasis on supply-oriented benefit methodology comes from the fact that estimates of product and transportation demand schedule are very difficult to measure empirically, even with good data representing demand determinants. Coupled with the difficulties of estimating transport demand is the fact that transport demand data are difficult for public
agencies to obtain from privately owned shippers or transport companies. 6

In this dissertation the demand for waterway transportation is estimated from data obtained from transport users selecting between competing modes. The data are used to form a mode choice model which replicates transport user behavior. Profit maximizing behavior is assumed and nonpriced elements of transportation choice are allowed to affect users internal economies. Data elements include the time of transit, delivery time, handling costs

6 The U.S. Bureau of the Budget issued evaluation criteria in the early 1960s which required benefits of a waterway project to be derived from estimates of savings in the carrier costs rather than savings to shippers who would use a waterway in the "with project" but another mode in the "without project" condition. This led to research conducted by Northwestern University for the U.S. Army Corps of Engineers, Leon Moses and Lester Lave (Eds.), Cost Benefit Analysis for Inland Navigation 3 Vols., IWR Report 70-4 (U.S. Army Engineer Institute for Water Resources, Ft. Belvoir, Va., 1970). The report developed an engineering process cost function for barge and rail modes. The cost functions gave a good approximation of line haul costs. However, indirect costs are not amenable to estimation by this procedure and in the case of rail and pipeline modes, decreasing marginal costs are held to be typical at least until capacity limits are approached. These indirect costs are significant increments of total costs. The report evaluated the use of statistical cost functions to estimate indirect costs and recommended further development of an integrated Engineering-Statistical approach. Moses and Lave advanced the modal split model based on discriminant analysis and oriented to estimating the demand function for barge transportation as a superior alternative. However, by the time that the research was underway, the U.S. Congress intervened with a definition of benefits in the 1966 Transportation Act (PL 89-670). That definition holds today, namely that benefits should be measured by savings to transport users based on rates at the time of the study.
encumbered, and per mile transportation cost. Basic attributes of commodity shipments such as the amount shipped each year and the amount of each shipment are included. Using these data from actual shipments and records, rather than asking a series of "what if" questions of the shippers, an estimate of transport demand is generated which is free of the bias expectable when potential waterway users are asked whether a new (or improved) waterway would enhance their profits and they would overstate potential traffic and savings which could accrue from a potential project.

A general case is presented here, for the total transportation system. Assume two transport modes, rail and barge, and assume that the quantity demanded to be shipped by each mode is a function of the relative price and service attributes of each mode:

$$T_{q_{r,b}} = f\left(\frac{P_{b}}{P_{r}}, \frac{S_{b}}{S_{r}}, t\right)$$

(1.1)

where

- \(T_{q_{r,b}}\) = Traffic shipped by rail or barge transport
- \(P_{r,b}\) = Price of rail or barge transport
- \(S_{r,b}\) = Service attributes of rail or barge transport
- \(t\) = Technology factors (size of shipment, annual shipments)
The demand for each mode is limited by the technology which limits mode substitutability. For example, a fully loaded barge hauls 1,000 to 1,500 tons and most frequently hauls one commodity in each shipment. Shippers of 40-100 ton quantities per shipment would find rail the only feasible mode. Thus, the demand function for the rail mode would have a segment relatively price inelastic representing small shippers demand and the demand function would become more price elastic as shipment size approaches barge load size. When an expansion in waterway capacity reduces barge transport rates, goods previously shipped by rail would be diverted to barges. The size of the diversion would depend on the importance of relative service attributes of each mode, and the ability of former rail users to handle larger shipment sizes.

Organization of the Study

In Chapter Two, the theory of derived demand is presented as in Marshall.\textsuperscript{7} This is followed with the theory of derived demand for transportation developed by Moses and Lave,\textsuperscript{8} and extended to include influence of time in transit, perishability, and uncertainty in market


\textsuperscript{8}Ibid., 1, pp. 1-7.
prices developed by Allen,\textsuperscript{9} and inventory holding developed by Baumol.\textsuperscript{10}

Chapter Three reviews the competing procedures for estimating transportation demand, including the gravity models in several specifications, the abstract mode model, regression models utilizing aggregate quantity and price observations across regions and commodity groups, and a number of disaggregate mode choice models.

Chapter Four discusses the econometric assumptions and problems contained in each of the basic procedures. Three generic procedures are used in calculating the various demand models: linear regression, nonlinear regression, and linear discriminant analysis. Linear discriminant analysis of the two group case is shown to be a special case of linear regression.

Chapter Five presents the estimates of water transportation demand and analysis using linear discriminant analysis. Transformation of the data to a log normal distribution provided a means for developing the modal split model and ultimately the simulation of the demand surface. The data were disaggregated to major destination


and major commodity groups. This disaggregation improved the effectiveness of the estimating model.

Chapter Six presents the summary and conclusions drawn from the analysis. The economic interpretation of the demand model and the potential importance of shipper behavioral decision-making in transport demand estimates are presented.

Appendix A describes and criticizes the data utilized in the empirical estimates of demand for water transportation. The data utilized represent 815 individual shipments totaling 122 million tons per year, with 14 major commodity groups shipped by nine transport modes.
CHAPTER II

THE THEORY OF DEMAND FOR FREIGHT TRANSPORTATION

Introduction

This chapter reviews the theory of derived demand developed by Marshall, then the theory of derived demand for transportation developed by Moses, Lave and Allen, and finally, the demand for transportation in the context of an inventory theoretic cost function as developed by Baumol and others. Choice of transport mode is accommodated by the model. The simplest case is based on transport rates, then additional transport choice variables are added (delivery time and perishability). Next uncertainty in market prices, delivery time and damage due to perishability are added. The inventory theoretic model introduces speculative inventory holding due to uncertainty in market price, delivery time and perishability. The discussion is limited to behavior of the profit maximizing firm in the short run.

Extensive literature in transport planning presents the choice of transportation problem in the context of physical flows. Economic variables are noticeably absent.
in most gravity models.\textsuperscript{1} However, this dissertation emphasizes transport demand to reflect economic choices by managers who decide how much to produce and to ship to any destination.

\textbf{Derived Demand}

Transportation, like other inputs to production, is characterized by demand which is indirect and derived from demand for those products which it helps to produce. Following is the classic definition of derived demand by Marshall:

The price that will be offered for anything used in producing a commodity, for each separate unit of the commodity, is limited by the excess of the price at which that amount can find purchasers, over the sum of the prices at which the corresponding supplies of the other things needed for making it will be forthcoming.

To use technical terms, the demand schedule for any factor of production of a commodity can be derived from that for the commodity by subtracting from the demand price of each separate amount of the commodity the supply prices for corresponding amounts of the other factors.\textsuperscript{2}


\textsuperscript{2}Marshall, \textit{Principles}, see Chapter 6 of Book V, p. 383.
The problem is to determine the demand for knife handles, given the demand schedule for knives (D' D_k) and the supply schedules for knives (S' S_k) and handles (S'S_h) as shown in Figure 2.1. The equilibrium price for knives is OA at quantity OB. The demand for handles (D'D_h) can be derived by subtracting the differences in supply prices of knives and handles from the demand schedule for knives (D D_k). Let M P_1 cut S'S_h at M_q and S'S_k at Mq. The supply price of handles is Oq and the supply price of knives is OQ. Subtracting Qq from OP_1 gives OP which is the demand price for OM handles. D'D_h is the locus of all such points.
Derived Demand for Transportation

Following Moses and Lave, the demand of a firm for transportation is derived from the demand for the final products of the firm and the supply schedule for all production costs except transportation.\(^3\) Demand and production functions are given and the firm produces one product and operates in a perfectly competitive environment.

\[
\pi = (P - t) Q - f(Q) \tag{2.1}
\]

where

\[\pi = \text{producing firm's profit},\]
\[P = \text{market price of the firm's product},\]
\[Q = \text{quantity the firm produces and ships},\]
\[t = \text{transportation charge}.
\]

The first term of equation 2.1 represents net revenue, the second is the cost of production.

\[
\frac{\partial \pi}{\partial Q} = (P - t) - f'(Q) = 0 \tag{2.2}
\]

and

\[P - t = f'(Q) \tag{2.3}
\]

with \(f''(Q) > 0\).

\(^3\)Moses and Lave, Cost Benefit Analysis, see Part 1.
Equation 2.3 states the usual profit maximizing condition that output is determined by the intersection of marginal cost and marginal revenue (P) at a constant price overall output reflecting a competitive market. The equation also yields the firm's demand for transportation with t variable and P constant.

Figure 2.2 shows the relationship between the short run supply schedule (MC) of the firm producing one product, the market price (P_m) and derived transport demand schedule.

![Diagram showing Derived Demand for Transportation by Firm Given Average and Marginal Cost Schedules and Market Price]

Figure 2.2 Derived Demand for Transportation by Firm Given Average and Marginal Cost Schedules and Market Price

While the average variable cost curve (AVC) is U shaped, the demand function is truncated but has the usual
negative slope. The truncation occurs because the firm will shut down production of \( P-t_1 < P_1 \) since transport costs would reduce marginal revenue to less than average variable cost. The diagrams have illustrated the case that at rate \( t_1 \), \( Q_1 \) is produced and shipped, at lower rates greater quantities are shipped but at higher rates nothing is produced and shipped.

**Additional Transportation Demand Variables**

The above formulation can be extended to introduce additional transportation choice variables. Positive costs for time of transit can be added as follows:

\[
\pi = \frac{(P-t)Q}{(1+i)^a} \cdot f(Q) \tag{2.5}
\]

where \( i \) = interest rate per unit of time and, \( a \) = time of transit.

\[
\frac{d\pi}{dQ} = \frac{P-t}{(1+i)^a} - f'(Q) = 0 \tag{2.6}
\]

and,

\[
\frac{P-t}{(1+i)^a} = f'(Q), \tag{2.7}
\]

with \( f''(Q) > 0 \). \tag{2.7a}

\(^4\text{Ibid, p. 3.}\)

\(^5\text{Allen, A Model of the Demand for Transportation, Chapter 2.}\)
For equation 2.5 it is assumed that costs of production are incurred instantly, and that the total time horizon is one time period, therefore the total product is sold instantaneously once the goods arrive at the market and payment for the product is COD. There is a time separation between production of goods and the receipt of revenues from their sale. In this equation, the second term is the total cost of production and the first term is the present value of the revenue. The derivation of the demand for transportation follows exactly in the same fashion as developed on page 20. If the average cost function is U shaped, the transport demand function is truncated but has the usual negative slope. The truncation occurs because the firm will stop production if the transport cost rises to the point where

\[
\frac{(P-t)}{(1+i)^a} = \hat{p}
\]

is less than minimum average variable cost.

Allen introduced a term for perishability as follows:

\[
\pi = \frac{(1-B)(P-t)}{(1+i)^a} - f(Q)
\]  

(2.8)

Where B = damage, pilferage losses or perishability rate, and the numerator of the first term represents net revenue after deduction for perishability. Taking the
partial derivative of profit with respect to quantity, we have:

\[ \frac{\partial \pi}{\partial Q} = \frac{(1-B)(P-t)}{(1+i)^a} - f'(Q) = 0 \]  
(2.9)

and \[ \frac{(1-B)(P-t)}{(1+i)^a} = f'(Q) \]  
(2.10)

with \( f''(Q) < 0 \).  
(2.11)

Again by separating the time of production of the goods and realization of revenues, the model accounts for losses in revenue due to perishability during transit (B). Net revenues reflect both perishability and costs of time during transit. The transportation demand function derived from this specification is truncated and negatively sloped.

Allen developed the model to include uncertainty in market prices and damage rate due to perishability. Given uncertainty of transport costs (t), and interest rates (i), the firm would assume an average market price \( \bar{P} \) and average damage rate \( \bar{B} \).

Define \( \bar{P} = \sum_{k=1}^{n} P_k X_k \) where \( \{X_k\} \) is the probability distribution of prices.

\[ ^6 \text{Ibid., p. 27.} \]
Define \( B = \sum_{z=1}^{n} B_z Y_z \) where \( \{Y_z\} \) is the probability distribution of damage factors.

\[
\pi = \frac{(1-B)(\bar{P}-t)}{(1+i)^a} Q - f(Q) \tag{2.12}
\]

\[
\frac{\partial \pi}{\partial Q} = \frac{(1-B)(\bar{P}-t)}{(1+i)^a} - f'(Q) = 0 \tag{2.13}
\]

and

\[
\frac{(1-B)(\bar{P}-t)}{(1+i)^a} = f'(Q) \tag{2.14}
\]

With \( f''(Q) > 0 \). \tag{2.15}

This model gives the same general result as previously discussed. The derived demand for transportation is truncated but negatively sloped.

If delay times are varied the analysis is altered. Following Allen, instead of assuming an expected delay time prevails, the firm would maximize profits by producing the outputs determined by the expected net discounted price \( \hat{P} \):

\[
\hat{P} = \frac{(1-B)(\bar{P}-t)}{(1+i)^a m} \hat{\gamma}_m \tag{2.16}
\]

where \( \hat{\gamma}_m \) is the probability distribution of delays, and

\[
\hat{P} > \frac{(1-B)(\bar{P}-t)}{(1+i)^a} \tag{2.17}
\]

because the denominator is not linear in its parameter.
With all three variables random, the net discounted price would be

\[ \hat{P} = \sum_{m,z,k} \gamma_{m} Y_{m} Z_{k} (1-B_{k})(P_{k}-t) \]

\[ (1+i)^{a_{m}} \] (2.18)

Assuming speculative inventory holding, the analysis shifts to a desire to hold inventory because of variable market prices \( \{P_{k}\} \). Define \( \{P_{k}\} = \{P_{m}\}, \{X_{k}\} \) where \( \{X_{k}\} \) is the probability distribution of market prices \( \{P_{m}\} \) and \( q \) = inventory costs per unit per day.

Simplifying the case by constraining the inventory holding period to one day, the firm will engage in speculative inventory holding in its output decision if,

\[ \frac{\hat{P}-q}{(1+i)} > P_{1} \] (2.19)

The above (2.19) states that if the discounted value of production for inventory is greater than current market price \( P_{1} \), then firm will produce for inventory and shipment.

Where \( \hat{P} \) is the expected value of market prices \( \{P_{m}\} \) and \( q \) is discounted, inventory costs and profit maximizing output is equal to:

\[ \tau = \left[ \frac{\hat{P} - q}{(1+i)} X_{1} + \hat{P}_{2} X_{2} + \hat{P}_{3} X_{3} \right] Q - f(Q) \] (2.20)
\[ \frac{\hat{P} - q}{(1+i)} X_1 + P_2 X_2 + P_3 X_3 - f'(Q) = 0 \quad (2.21) \]

and
\[ \frac{\hat{P} - q}{(1+i)} X_1 + P_2 X_2 + P_3 X_3 = f'(Q) \quad (2.22) \]

with \( f''(q) > 0 \). (2.23)

Baumol, et al, extends the case to inventory holding in a multi-time period analysis based on an inventory theoretic cost function.\(^7\) Define costs (C) equal to direct shipping costs plus intratransit carrying costs plus inventory carrying costs.

\[ C = RT + v z T + w s T/2 + a/s \quad (2.24) \]

Where
- \( C \) = expected total variable cost,
- \( R \) = transport rate,
- \( T \) = transport flow per year,
- \( v \) = carrying cost in transit (interest (plus deterioration plus pilferage),
- \( z \) = transit time,
- \( a \) = order cost,
- \( s \) = time between shipments,
- \( w \) = warehouse carrying costs per unit per year.

The shipper controls reorder frequency and his optimal decision will follow the rule:

\(^7\) Baumol et al., Studies on the Demand for Freight Transportation, Vol. 1, Chapters 1-3.
\[
\frac{3C}{3s} = -\frac{a}{s^2} + \frac{wT}{2} = 0 \tag{2.25}
\]

therefore, \( s = \sqrt{2a/wT}, \)

and substituting for \( s \) in 2.24 and simplifying

\[
C = RT + vT + a\sqrt{2a/wT} + (wT/2) \sqrt{2a/wT} \tag{2.26}
\]

\[
= RT + vT + a\sqrt{wT/2a} + (\sqrt{wT/2})^2 \sqrt{2a/wT} \tag{2.27}
\]

\[
= RT + vT + \sqrt{a} \sqrt{wT/2} + \sqrt{wT/2} \sqrt{a} \tag{2.28}
\]

\[
= RT + vT + \sqrt{2awT} \tag{2.29}
\]

Uncertainty of delivery time and demand requirements on inventory suggests a safety stock. If the uncertainty is characterized by a Poisson distribution, safety stock level (\( S \)) is:

\[
S = k \sqrt{(s + z)T} \tag{2.30}
\]

where \( k \) = a constant,

and \( \sqrt{(s+z)T} \) = standard deviation of available inventory, which leads to an expanded cost equation,

\[
C = RT + vT + \sqrt{2awT} + k \sqrt{(s+z)T} \tag{2.31}
\]

In this case, delivery time (\( z \)) enters the inventory figure, and given uncertain timing in demands for the commodity, slower delivery would require an increase in safety stock. Differentiating (2.31) with respect to \( T \) allows the marginal shipping costs to be determined:
\[ \frac{3C}{3T} = R + vz + \sqrt{2aw} \left( \frac{1}{2\sqrt{T}} \right) + k \sqrt{(s+z)} \left( \frac{1}{2\sqrt{T}} \right) \] (2.32)

But this formulation indicates the shipper is simply trying to minimize transport costs rather than maximizing profits, so a revenue term should be added:

\[
MR = \Delta P + t \frac{d\Delta P}{dT} \] (2.33)

Where MR = marginal revenue

and \( \Delta P \) = price differences between origin and destination (normally equal to transportation costs) and substituting \( b \) for \( \frac{d\Delta P}{dT} \), set marginal costs equal to marginal revenue:

\[
R + vz + k \sqrt{s+z} \left( \frac{1}{2\sqrt{T}} \right) + \sqrt{2aw} \left( \frac{1}{2\sqrt{T}} \right) = \Delta P + bT \] (2.34)

This results in a nonlinear equation requiring estimation by simultaneous utility search techniques.\(^8\)

Because space and distance do influence price of transportation (in terms of price per unit of distance and the carrying costs associated with time in transit), the above formulation is incomplete. Distance and competition between modes are introduced in the next chapter.

\(^8\)Ibid, p. 53. The Gradx computer routine developed by R.E. Quandt is one of the developments which permits estimates of the parameters.
CHAPTER III
TRANSPORT DEMAND MODELS

Gravity Models

Numerous studies of transport flows have been organized around the physical concept of attraction and impedance.\(^1\)

\[ T_{ij} = a_0 \frac{P_i P_j}{d_{ij}} \]  

(3.1)

Where \( T_{ij} \) is the measure of traffic between \( i \) and \( j \) cities,

\( P_i, P_j \) are the populations of cities \( i \) and \( j \),

\( d_{ij} \) represents the distance between \( i \) and \( j \), and

\( a_0 \) is a parameter to be estimated from the data.

This "gravity" model is roughly similar to the gravitational force formula in Newtonian mechanics. The volume of traffic between cities \( i \) and \( j \) is directly related to

the product of the two populations, i.e., the attraction, 
and inversely related to the distance between them, i.e.,
the impedance, with the relation between attraction and 
impedance being proportional.

According to Isard:

A basic attribute of it (gravity model) used for 
projections is the lack of theory to explain the 
values or functions which we can assign to weights 
of exponents. Currently, the justification for 
the gravity model is simply that, everything else 
being equal, the interaction between any two popu-
lations can be expected to be directly related to 
their size, and since distance involves friction, 
inconvenience, and cost, such interactions can be 
expected to be inversely related to distance.²

A more sophisticated formulation is:

\[ T_{ij} = a_0 \frac{P_1 P_2}{d_{ij}} \]  

(3.2)

where \( a_0, a_1, a_2 \), are parameters to be estimated from 
the data. This equation is not a simple statement of 
proportionality of product of population and distance 
but indicates an attenuation of the attraction. The 
parameter estimates are generated by regression of cross 
section data. According to Quandt, such regressions often 
yield high correlation coefficients with regression co-
efficients significantly different from zero, but frequent-
ly yield bad predictions for projected travel for year \( x \neq n \)

²Isard, Methods of Regional Analysis, p. 515.
from the cross section of data obtained for the year $x$.\footnote{Quandt, "Some Perspectives of Gravity Models," p. 35.}

It follows that the bad predictions are a result of a

fundamentally erroneous paradigm or the result of specifica-

tion errors. Quandt proposed a greatly expanded formu-

lation to correct errors of specification as follows:

$$
T_{ij} = k a(x) p_{it} a_1(x) p_{jt} a_2(x) y_{it} a_3(x) y_{jt} a_4(x) \\
\quad m_{it} a_5(x) m_{jt} a_6(x) n_{ijt} a_7(x) t_{ijt} a_8(x) \\
s_{ijt} a_9(x)
$$

(3.3)

where $x$ = time period,

$i, j$ = transport nodes (location $i, j$),

$y$ = income,

$m$ = city type classified by portion of labor

force in mining and manufacturing,

$n$ = travel time,

$t$ = travel cost,

$s$ = transportation supply characteristics, and

$a, k =$ parameters.

The resulting regression equation is linear in

logarithms which allows for decreasing returns to the

independent variables. The expanded formulation was used

in the analysis of intercity passenger demand in the
Northeast Corridor Study for the U.S. Department of Transportation. Meyer & Strazheim argue that individual elements of travel time should be disaggregated, but data problems were deemed to be unfeasible. Cross-section data were used to estimate the functions with a result that many of the variables were highly correlated.

Limited empirical applications of the gravity models are contained in the literature. Alcaly fitted gravity models as formulated in (3.3) for air travel in seven cities in the northwest corridor of the U.S. and found that the regression coefficient of the distance variable to be insignificant. While the equations were fitted to predict demand values on given air travel routes in 1950, the results were uniformly low except for two cities. Alcaly cites Kessler for an example in which the basic gravity model formulation of the demand for transportation was modified to include various socio-economic and


5 Meyer and Strazheim, Techniques of Transport Planning, pp. 139-140.

6 Alcaly, "The Demand for Air Travel," p. 69.
demographic characteristics of the cities involved, although Kessler's analysis did not include all suggested characteristics.\(^7\)

Alcaly later explored the effect of aggregation on gravity models specifically to determine "the effects of aggregation over modes of travel on the performance of equation (3.3) in explaining the demand for travel."\(^8\) The study used 16 pairs of cities in California and 1960 data. Travel time as well as cost were used in some of the regressions. Taking account of the explanatory power and significance of the postulated log linear relationships, aggregated mode (to total transport) equations performed better as a group than did individual mode equations. An exception was the automobile mode. Air mode equations were characterized by high explanatory power and significance but distance coefficients were not significant. The study concluded that the gravity model explains travel by all modes better than travel by individual modes. It follows that the theoretical objections are minimized when the model uses aggregated data.

\(^7\)D.S. Kessler, Relationship Between Inner City Air Passenger and Demographic Factors - A Multiple Region Analysis, Princeton University, Princeton, N.J., 1965), Thesis.

Intervening Opportunities Model

The intervening opportunities model has the objective to minimize the travel time between every region, subject to the constraint that every potential destination (intervening and ultimate) is considered. Thus a trip originating in region i has less probability of ending in region j as the number of intervening opportunities increases.

\[ T_{ij} = 0 \left[ p(T_{j+1}) - p(T_j) \right] \] (3.4)

\[ p(T_j) = \text{total probability that a trip will terminate before the } j\text{th possible destination is considered}, \]

\[ (T_j) = \text{possible destinations already considered that are reached before reaching zone } j, \text{and} \]

\[ O_i = \text{the constant probability of a possible destination being accepted if considered}. \]

Thus, the number of trips taken between i and j is the number of regions multiplied by the probability of the trip terminating in j, or

\[ T_{ij} = 0 \left[ e^{-LT_j} + 1 - e^{-LT_j} \right] \] (3.5)

where L shapes the distribution of trips between i and j.

As Isard proves, the results are identical with the gravity

\footnote{Meyer & Strazheim, Techniques of Transportation Planning, p. 121.}
model when the number of intervening opportunities are a linear function of distance.10

**Fratar Method**

The Fratar method is a procedure for averaging differential growth factors for existing traffic counts and represents a logical extension from the simple growth factor model.11 Future interzonal travel forecasts are derived from the present level of interzonal trips and the different zonal growth factors. Each zone growth factor, factor $g_i$, is a ratio of future to present trip generation:

$$g_i = \frac{T'_i}{T_i} \quad (3.6)$$

Where $T'_i$ = future trip generation, and $T_i$ = present trip generation.

Total expected future zonal interchange, $T'_{ij}$, is given by

$$= T'_{ij} \cdot g_j = \sum_{j=1}^{N} \left( T'_{ij} \cdot g_i \right) \quad (3.7)$$


The model is used in a successive approximation technique iteratively to update growth factors based on trip destinations.

All of the procedures described above are dominated by empirical generalization and lack a fully articulated behavioral basis with the exception of Quandt's sophisticated specifications given in equation 3.3 above. They are not useful to derive fully and implement a model of transport demand.

**Abstract Mode Model**

The demand models discussed above define each mode by administrative ownership or type of physical equipment utilized and a single demand equation is estimated for each mode. In contrast, the abstract mode approach defines choice of mode in one equation. This approach results in significant data savings, since the number of observations available for estimating one abstract mode equation will be equal to the number of modes times the number of city pairs in the transport system. By comparison any formulation requiring an equation for each model results in a sample size equal to the number of city pairs.

Quandt and Baumol formulated the model of abstract mode to model passenger travel in the United States.

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Northeast Corridor. A single equation represents passenger choice for all modes and is linear. Each mode is characterized by several independent variables which describe its supply characteristics.

The formulation defines the characteristics for which a mode should be developed in the future. With changing technology the approach offers a definite forward thrust in analysis.

\[ T_{kij} = a_0 p_i^{a_1} p_j^{a_2} y_i^{a_3} y_j^{a_4} m_i^{a_5} m_j^{a_6} n_{ij}^{a_7} f_1(h)f_2(c)f_3(d) \]

where

- \( T_{kij} \) = transport flow from node \( i \) to node \( j \) by mode \( k \),
- \( p_i, p_j \) = population of nodes \( i \) and \( j \),
- \( y_i, y_j \) = median income of nodes \( i \) and \( j \),
- \( m_i, m_j \) = an index representing industrial characteristics of nodes \( i \) and \( j \),
- \( h_{ij} \) = best available travel time between nodes \( i \) and \( j \),
- \( h_{kij} \) = relative travel time for kth mode,
- \( c_{ij} \) = least cost to travel between mode \( i \) and \( j \),
- \( c_{kij} \) = relative costs for kth mode,
- \( d_{ij} \) = best departure figure between nodes \( i \) and \( j \),
- \( d_{kij} \) = relative departure frequency for kth mode,
- \( n_{ij} \) = number of modes serving nodes \( i \) and \( j \).
\[
\begin{align*}
    f_1(h) &= (h_{ij}^b)^{B_0} (h_{kij}^r)^{B_1}, \\
    f_2(h) &= (c_{ij}^b)^{B_2} (c_{kij}^r)^{B_3}, \\
    f_3(h) &= (d_{ij}^b)^{B_4} (d_{kij}^r)^{B_5}, \text{ and}
\end{align*}
\]

\[a, B \text{ = parameters.}\]

The derived model is linear in logarithms with attractiveness and impedance variables. Fundamentally, the specification argues that the choice of transport mode is based on a comparison of a particular mode's performance to that of the best available mode in one particular variable (cost, time, or other single service factor). This leads to a discontinuity of preferences with respect to mode of choice since the improvements of several characteristics by any mode, short of becoming the best mode with that characteristic, have no effect on demand.

Meyer and Strazheim conclude that "empirical application of the abstract mode formulation has not been notably successful." The model was applied to sixteen city pairs for air and bus and automobile traffic. Quandt and Baumol concluded that the model has promise on the

\[14\] Meyer and Strazheim, Techniques of Transport Planning, pp. 144-151.
basis of correctness and signs on regression coefficients, stability of exponents from regression and lack of problems from multicollinearity. \(^{15}\) However, Young found that the single mode demand equations, using the F test to compare residual sums of squares of combined regression and the sum of two residual sum of squares of the two separate regressions was characterized by poor results. \(^{16}\)

One application was to reduce "abstractness" by introduction of dummy variables or other modifications in the functional form. \(^{17}\) The following equation shows introduction of dummy variables for auto and bus

\[
T_{kij} = a_0(p_{ij}) a_1(c_{ij}) a_2(c_{kij}) a_3(r_{ij}) a_4(h_{ij}) a_5(n_{ij}) a_6(y_{ij}) a_7(g_{kij} + b_{kij})
\]

where \(g_{kij}\) and \(b_{kij}\) are dummy variables representing auto and bus mode.


Quandt and Young developed a number of other models to introduce nonabstractness. The hypothesis that is not the "best" cost and journey time that are relevant was developed by expressing the same figures relevant to income as measured by \( c_{ij}^{b/y_{ij}} \) and \( h_{ij}^{b/y_{ij}} \). Other variations assume income elasticities along different routes to be different (but the same for different modes on the same route). Another variation relaxes the assumption of equal income elasticities for different modes by introducing dummy variables for each mode. Further variations were added by assuming that it is the ratio of relative cost, journey time and departure frequency to income that is relevant to travel demand. Two versions introduced the concepts embodied in intervening opportunities which assume that an individual considering a trip will consider not only the absolute and relative characteristics of the modes between \( i \) and \( j \) but also may include the option to go to another mode \( l \). Equations were fitted using both the California data and data from a sample of intercity flows in the Northeast Corridor. Significant differences in income elasticity among city pairs were revealed. Values ranging from almost one to over three resulted; an outcome not easy to interpret.

It may be concluded that an abstract mode formulation, which represents travel choices among modes as a single
equation, with each mode's performance reported relative to the time or cost of the best mode serving a city pair, does not appear superior to conventional multiequation gravity formulation. The use of the more disaggregate form of modal choice model is supported by the arguments that (1) the underlying utility function and the demand functions are too complex to be represented in one equation and (2) by empirical tests.

**Freight Demand Modeling**

The above models were developed primarily to estimate derived demand for and flows of passenger transportation, although the principles used have been extended to freight traffic models. Transport flows of freight used in production processes and passengers behave quite differently since in the first case managers are maximizing profits and in the second case passengers are maximizing utility, at least in cases other than business travel. Transportation demand analysts are also concerned with the relationship between price and quantity demanded as well as in the maximization of an objective function.

The following are reviews of the literature of freight demand modeling including modal split models.
Regression Models

Perle carried out an extensive analysis of five commodity groups based on aggregate data from nine regions of the United States.\(^{18}\) Two demand models were developed. The basic model was:

\[
T_{r,t} = a_0 + a_1 c_r + a_2 c_t \tag{3.10}
\]

where \(T_{r,t}\) = quantity transported by mode \(r\) or \(t\), and \(c_r, c_t\) = price of transport by rail \((r)\) or truck \((t)\).

Analysis was conducted with all observations aggregated to the nation, then separated by commodity.

Perle's results show the expected inverse relationship between price and quantity demanded and the results permitted estimates of the price elasticity of demand. His analysis would permit the evaluation of regional impacts of changes in rail or truck rates on transportation demand but does not permit analysis of the impacts of rate changes on the level of service, change in economic activity or on shipment size.

The second model, which emphasized the elasticity of substitution was formulated as:

Changes in the elasticity of substitution can be a result of the relative changes in prices or relative changes in quantities. Thus, the first model related absolute quantities and prices and treated substitutability tangentially. The second model focused on relative quantities and prices and therefore treated substitutability directly. With nine regions, five commodity groups, and five years of data, Perle's analysis is based on a total 225 observations. The data, 1956 through 1960, showed a sharp six percent gain in truck traffic over rail. The analysis failed to consider handling costs, travel time, frequency of service or other factors in transportation demand and the price information was limited to published rates, which fail to represent a large share of truck movements.

Sloss developed a model for estimating intercity freight shipments to be carried by truck, using Canadian statistics. His formulation utilized aggregate province statistics for intercity truck traffic ($T_t$), average truck revenue per ton ($c_t$), average rail revenue per ton ($c_r$),

$$\frac{T_t}{T_r} = a_0 + a_1 \frac{c_t}{c_r} \quad (3.11)$$

---

and a variable representing general economic activity (h). The general economic activity variable was composed of the sum of farm income, value of building permits and value of shipments of manufactured goods. The following model was formulated:

\[ T_t = a_0 c_t^{a_1} + c_r^{a_2} + h^{a_3} \]  

where \( a_1, a_2 \) and \( a_3 \) are parameters to be estimated from the data.

Sloss found that the volume of truck traffic is directly related to the price of rail service and inversely related to the price of truck service. Further, the quantity of truck traffic is positively correlated with the general level of economic activity. Thus the model could be used in a macro level evaluation of alternative pricing strategies but does not include service attributes such as transit time and frequency of service or handling costs. In addition, it has limited capacity to predict the impacts of changes in prices on transport flows between particular origins and destinations.

Subsequently, A.D. Little developed an aggregate level modal split analysis in a contract study for the Maritime Administration. The data was aggregated by BEA

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economic region and the model was directed toward correlation of the fraction of plants located on water bodies to the demand for barge transportation. The model is formulated as:

\[ \% \text{ manufacturing plants on water} = \frac{T_{ij} \cdot P_c \cdot D_{rij} \cdot D_{wij}}{T_{ij} \cdot P_c \cdot D_{rij} \cdot D_{wij}} \]  

where \( T_{ij} \) = total flows between regions \( i \) and \( j \),  
\( P_c \) = value per ton of commodity \( c \),  
\( b_{ij} \) = percent commodities transported in bulk,  
\( s_{ij} \) = seasonality of shipment between regions \( i \) and \( j \),  
\( d_{ij} \) = distance between regions \( i \) and \( j \), and  
\( r, v, y \) = rail, water and truck modes.

A nonlinear model was estimated for 15 commodity groups between BEA areas. The model results shed considerable insight into the interaction of nonrate modal performance characteristics with shipper and market attributes on location of manufacturing plants. Since rate relationships are missing, the model is inadequate to analyze the impact of rate and level of service changes on traffic flows and is therefore inadequate to estimate transportation demand.
Mathematica (1967) adapted the Quandt/Baumol abstract mode model in a contract for the Northwest Corridor project.21 The general form of the adapted model is:

\[ T_{kij} = \frac{a_1 p_i a_2 y_i a_3 y_j a_4 m_i a_5 m_j a_6 n_{ij} (b_{h_i j} r_{h_{kij}}) B_1 B_2 (c_{ij} b_{c_{kij}})^{1/2} }{c_{kij}^{B_4}} \]  

(3.14)

where

- \( T_{kij} \) = quantity of freight traffic from \( i \) to \( j \) by mode \( k \),
- \( p_i, p_j \) = population of origin and destination locations,
- \( y_i, y_j \) = gross regional product of origin and destination regions,
- \( m_i, m_j \) = indicators of industrial character of regions (such as percent manufacturing),
- \( h_{ij} \) = best or least shipping time from \( i \) to \( j \),
- \( h_{kij} \) = ratio of travel time by mode \( k \) to the best time,
- \( c_{ij} \) = best or least cost of shipping from \( i \) to \( j \),
- \( c_{kij} \) = ratio of the cost of mode \( k \) to the least cost,
- \( n_{ij} \) = number of modes serving \( i \) and \( j \), and
- \( a, B \) = parameters.

21 Baumol, Studies on the Demand for Freight Transportation, 1, Chapter 1.
The data for this model are disaggregated by origin and destination pairs and the model contains a larger range of level of service and market attributes than others. However, its major disadvantage is the lack of change in mode preference due to any improvements in performance characteristics of any given mode short of becoming the best mode with that characteristic. The "best" definition may be inconsistent under some circumstances, such as truck being the least cost for very small shipments but the most expensive for very large shipments.

The Mathematica study also developed a model based on microeconomic theory of the firm similar to the general inventory theoretic cost function discussed in Chapter 2. This is a least cost model based on annual variable transportation costs of the industry in city j, receiving commodity q from city i. Variable costs are defined as the sum of direct shipper costs, total in-transit carrying costs and safety inventory costs.\(^2\)

The transportation demand function is as follows:

\[
VC = a_0 + a_1 (c_{kij} q T_{kij} q) + \frac{a_2 (h_{kij} q T_{kij} q)}{(1 + r)^8} + a_3 (b_j q T_{kij} q s_j q) \tag{3.15}
\]

\(^{22}\)Ibid, p. 55.
where VC = annual variable transport related costs, 
\( r \) = carrying costs per day (g), 
\( c_{kij}^q \) = shipping cost per unit of commodity q from city i to city j by mode k, 
\( s_j^q \) = time between shipments of commodity q to j, the reciprocal of the number of shipments per year, 
\( T_{kij}^q \) = quantity of commodity q shipped from city i to city j by mode k, 
\( h_{kij}^q \) = travel time for shipping commodity q from city i to city j by mode k, and 
\( b_j^q \) = inventory cost per unit of commodity q per year at j.

Kullman used a binary logit (logistic) model on aggregate data to predict the division of traffic between truck and rail.23 The model includes service, commodity, and market attributes as follows:

\[
T_{kij}^q = f(d_{ij}, P_k, r_1/r_2, h_1/h_2, a_1/a_2) \tag{3.21}
\]

where \( T_{kij}^q \) = quantity of commodity q moving by mode k from origin i to destination j 
\( d_{ij} \) = distance from i to j, 
\( P_k \) = value of commodity k per ton, 
\( r_{1,2} \) = transport rate of mode 1 or 2, 
\( h_{1,2} \) = transport time of mode 1 or 2, and

\[ a_{1,2} = \text{reliability of mode 1 or 2}. \]

This formulation is very similar to that used in the discriminant model discussed later in Chapters 4, 5 and 6 and utilized in the analysis of demand for transportation in this dissertation. Production economies of the firm and transport mode are included. The model has been used to estimate impacts of changes in mode performance and cost, and the impact of shifting relative market prices between origin and destination.

Kullman's model does allow the evaluation of the impact of changes in service and rate attributes and changes in reliability, although the aggregate formulation limits variability in the data and reduces its empirical usefulness for estimating non aggregated behavior.\(^{24}\) As

stated in Chapter 4, the linear logit model has been criticized by Oum as inappropriate to use for transport demand studies due to many rigid a priori restrictions on parameters of price responsiveness of demand and the underlying structure of preference is irregular and inconsistent.  

Discriminant Models

Beuthe developed a modal split model based on discriminant analysis which accounts for the spatial distribution of shipments of corn from Illinois via barge, rail or truck modes. 26 Regional boundaries are defined by the following discriminant model:

\[ T_{kj}^q = f (r_k, n_k, q_k) \]  

where \( T_{kj}^q \) = quantity of commodity q shipped by mode k from region j, where k = 1,2,3),

\( r_k \) = transport rate of mode k,

\( n_k \) = time costs by mode k, and

\( q_k \) = quality differences between modes.


26 Michael V. Beuthe, Freight Transportation Modal Choice: An Application to Corn Transportation (Northwestern University, Evanston, Ill., Dissertation, 1968), Chapter 4, pp. 4-78.
Beuthe also developed a spatial model by extension of traditional location theory. Location of shipments by each mode are relative to distance and costs, and the shipment predictions take the form of boundaries between market areas for the respective modes. The discriminant model estimated the quality differential between modes and provided substantial insight into the comparative advantage held by each transport mode whereas the locational model was limited to describing mode differences in relative transportation costs alone. The empirical study was of corn shipments from Illinois.

Allen utilized discriminant analysis to analyze the division of traffic between air and sea modes of transportation in the North Atlantic trade route.\textsuperscript{27} The linear discriminant model was defined as follows:

\[ T_{kq} = \mathbf{f}(v_q, r_k) \]  \hspace{1cm} (3.17)

where

\[ T_{kq} = \text{quantity of commodity } q \text{ shipped by mode } k, \]
\[ v_q = \text{value per pound of commodity } q, \text{ and} \]
\[ r_k = \text{rate of mode } k. \]

Allen found that the discriminant approach worked best on commodities and routes which resulted in exclusive

\textsuperscript{27}Allen, A Model of Demand for Transportation, pp. 60-72.
modal choice. Regression performed poorly on the entire sample. The analysis suffered because of lack of data on transit time and shipment size.

Antle and Haynes used linear discriminant analysis in a two mode model for shipments in the Upper Ohio River area. The model was formulated as follows:

\[ T_{kij}^q = f(d_{kij}, h_{kij}, s_{kij}, r_{kij}, c_{kij}, b_{aij}) \]  

where

- \( T_{kij}^q \) = quantity of commodity \( q \) shipped by mode \( k \) from origin \( i \) to destination \( j \), with \( k = 1 \) or \( 2 \),
- \( d_{kij} \) = distance from \( i \) to \( j \) by mode \( k \),
- \( s_{kij} \) = size of shipment from \( i \) to \( j \) by mode \( k \),
- \( h_{kij} \) = travel time from \( i \) to \( j \) by mode \( k \),
- \( r_{kij} \) = transport rate from \( i \) to \( j \) by mode \( k \),
- \( c_{kij} \) = handling cost from \( i \) to \( j \) by mode \( k \), and
- \( b_{aij} \) = transport cost of alternative mode from \( i \) to \( j \).

The model was estimated for a relatively small sample of coal, coke, petroleum, and chemical shipments by barge and rail. Costs of the alternative mode failed to appear.

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in the calibrated model due to lack of significance. The two group procedure developed by Tintner was utilized as follows: 29

\[ Z = k_1d_1 + k_2d_2 + \ldots + k_pd_n \]  (3.19)

where \( Z \) = test value,
\( k_p \) = linear weights, and
\( d_n \) = differences in means in each group of \( n \) variable.

The weights (\( k_p \)'s) are assigned in a way to maximize the value of \( Z^2 \) relative to its variance. The resulting

\[ p(x) \]

\[ \begin{align*}
\text{Group 1} & \quad 0 \\
\text{Group 2} & \quad Z \\
\text{X (weighted vector of characteristics)} & \quad X
\end{align*} \]

Figure 3.1 Probability of occurrence in groups 1 or 2 depending on weighted characteristics & Discriminant Function (Z)

computations are similar to linear regression and classification is based on a critical value of Z which minimizes classification errors.

The resulting model performed relatively well predicting the demand for coal shipments, but the sample was too small to permit good prediction for other commodity groups. The discriminant function allowed an estimate of the demand function by means of parametric shifting of barge rates to derive the quantity shipped by barge to zero. The model allows an assessment of the impacts of rate change and other attributes on the choice of mode and therefore the demand by mode.

Southwestern Division, Corps of Engineers, applied discriminant analysis to selected commodity shipments on the Arkansas River area. The Southwestern Division analysis was based on 194 shipments data by rail, truck and barge modes for six commodity groups which were moving by barge transportation for the year 1971. The analysis found, essentially, a highly price inelastic demand function for water transportation based on the data. Since the data were collected in the first year that navigation

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was fully operational on the Arkansas River, one of the possible explanations for the inelastic properties of the demand function is that the shippers were attempting to stimulate downward reductions in rail rates and that was a prominent part of the motivation for using water. This hypothesis was confirmed later in a study of rail rate adjustments which showed that, at least in iron and steel products, substantial drops in rail rates were begun in 1971. The model adopted was exactly the same as was used in the study by Antle and Haynes in 1971.

Sasaki developed a comprehensive commodity demand, supply and transport demand model to project the demand for freight transport by mode.31 The comprehensive model was composed of three submodels. Model 1 projects output of firms which utilize coal as a factor of production. Model 2 allocates the available supply of coal in a way that minimizes transportation costs to satisfy demands. Model 3 is a mode choice model which introduces nonmoneitized mode choice variables.

The discriminant mode choice model was developed for rail, unit train and barge modes as follows, with the variables defined as in 3.18:

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31 Brion R. Sasaki, A Regional Model of the Future Demand for Transportation: The Case of Barge Transportation (University of Cincinnati, Cincinnati, Ohio, Dissertation, 1974).
\[ T_{kij}^q = f (d_{kij}, h_{kij}, s_{kij}, c_{kij}) \] (3.20)

Sasaki's model was estimated from 97 observations in three Bureau of Economic Analysis (Department of Commerce) economic regions in the Ohio Valley. His work confronts the primary problems in projecting freight demands: changing technology of product users (now complicated by environmental regulations), supply constraints which affect the quantity available (also affected by environmental regulations), and shifting service and cost characteristics of transport modes, but does not deal with long-run locational changes which would affect transport demand.

The comprehensive model contains some redundancy in the allocation of aggregate shipments from origin to destination in model 2 and the mode choice calculation in model 3. The allocation and choice of mode could have been made in one step if all variables had been converted to total cost in advance of the cost minimization calculation. This suggests a possible use of modal split in an earlier step to determine the implicit prices that would be assigned to nonmonetary variables in selection of mode. Shipment size, however, would resist monetization but perhaps is reflected in the scale economy of rates.
The model is capable of evaluating the impact of changes in service and rate attributes and is uniquely able to accept constraints in both the demand and supply functions for coal.

Comparative Studies

Hartwig and Linton developed logit, probit and discriminant functions to model individual shipper choice of mode between full load truck and full load rail for intercity freight movements. The model was estimated from 1213 freight way bills for truck and rail shipments of consumer durables as follows:

$$\frac{T_{1ij}}{T_{2ij}} = f(p^q, r_{1ij} - r_{2ij}, s_1 - s_2, a_{1ij} - a_{2ij})$$

(3.22)

where $T_{1,2ij} =$ quantity transported by rail (1) or truck (2) from origin i to destination j,

$p^q =$ price per ton of commodity q,

$r_{1,2ij} =$ actual transport rate by mode 1, 2 from i to j,

$s_{1,2} =$ actual shipment size by mode 1, 2, and

$a_{1,2ij} =$ reliability of mode 1, 2 from i to j.

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They found that the probit and logit formulation performed somewhat better than discriminant functions for the commodities selected. The model allows for evaluation of impacts from shifts in transport costs and reliability measures but does not include commodity attributes or length of time in transit measures.

Summary

Analysis of the literature in freight demand modeling shows the steady progress from dependence on aggregate data models which tended to reduce applicability to particular demand and suppresses variability, to models using a much larger set of variables which influence the amount of freight to be shipped, its origin and destination, and the choice of transport mode. Models, furthermore, which concentrate on traffic flows and transport rates miss important explanatory variables. Behavioral models in which individual shipper decisions are optimizing choices are more likely to produce informed analysis of the causal variables relating to quantities shipped. Mode choice models (whether estimated by some form of regression or by discriminant analysis) offer the possibility of estimating both the quantity of traffic to be
shipped by a mode and the demand function for that mode (that relationship between transport rate and quantity shipped). A description of these models is in the next chapter.
CHAPTER IV

ECONOMETRIC CHARACTERISTICS OF
REGRESSION AND DISCRIMINANT METHODS

Introduction

The following chapter discusses, in turn, the econometric characteristic of several varieties of regression and discriminant analysis. Linear regression is used to fit the parameters of gravity, aggregate demand and abstract mode models. Nonlinear regression models are utilized in probit and logit form of modal split models. It is shown that the linear discriminant analysis of a two group case is a special case of linear regression.

Linear Regression

Following Johnston, assume a linear relationship between variable Y and k-1 explanatory variables $X_2, X_3$ through $X_k$ and a disturbance term $u$. With a sample of n observations on y and x's the following can be written:

$$ Y_i = b_1 + b_2 X_{2i} \ldots b_k X_{ki} + u_i $$  (4.1)

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1J. Johnston, Econometric Methods (McGraw Hill, 1960). See Chapters 1 and 2 for the two variable linear model and Chapter 4 for the general linear model.
The coefficients and disturbance are unknown and the problem is to obtain estimates of the unknown. In matrix notation the results are the following:

\[ Y = XB + u \] (4.2)

The simplest set of assumptions are:

\[ E(u) = 0 \] (4.3a)
\[ E(uu') = \sigma^2 I_n \] (4.3b)
\[ X \text{ is a set of fixed numbers} \] (4.3c)
\[ X \text{ has rank } k < n \] (4.3d)

It is assumed that the expected value of the \( u_i \)'s is zero, that the \( u_i \)'s have constant variance and that off diagonal elements are pairwise uncorrelated, that the sole source of variation on the \( Y \) vector is variations in the \( u \) vector and that the properties of the estimators and tests are conditional upon the \( X \)'s and that the number of observations exceeds the number of parameters to be estimated and no exact linear relationship exists between the \( X \) variables.

Least square estimates of the form:

\[ Y = XB + e \] (4.4)

can be derived by minimizing the square of the \( e_i \) term,
\[ e_i^2 = e'e = (Y - X \hat{B})' (Y - X \hat{B}) \]  \hspace{1cm} (4.4a)

\[ = Y'Y - 2 \hat{B}'X'Y - \hat{B}'X'X \hat{B} \]  \hspace{1cm} (4.4b)

then finding the value of \( \hat{B} \) which minimizes squared residuals by taking the derivative:

\[ \frac{\partial}{\partial \hat{B}} (e'e) = -2X'Y + 2X'XB \]  \hspace{1cm} (4.5)

then \( X'X \hat{B} = X'Y \) ,  \hspace{1cm} (4.6)

and \( \hat{B} = (X'X)^{-1}(X'Y) \).  \hspace{1cm} (4.7)

The least square estimates are best linear unbiased.\(^2\)

In the special case where the joint distribution of \( X \) and \( Y \) is multivariate normal, the least square estimate has the properties of consistency, efficiency, minimum variance, unbiased, and sufficiency.\(^3\)

Estimation by maximum likelihood methods can accommodate nonlinear relationships in the parameters but require assumptions of normality of the \( u_i \)'s:

\[ u_i \sim N(0, \sigma^2 I_n) \]  \hspace{1cm} (4.8)

Form the likelihood function:

\[ L = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp \left( \frac{-u'u}{2\sigma^2} \right) \]  \hspace{1cm} (4.9)

\(^2\)Ibid., p. 111.

\(^3\)Ibid., p. 133, and Franklin A. Graybill, An Introduction to Linear Statistical Models (McGraw-Hill, 1961); Vol. 1, Chapters 5-10.
\[
\frac{1}{(2\pi\sigma^2)^{n/2}} \exp \left[ - \frac{(Y-X\beta)'(Y-X\beta)}{2\sigma^2} \right]
\]  
(4.10)

\[\frac{\partial L}{\partial \beta} = (2\sigma^2)^{-1} 2X' (Y-X\beta) = 0\]  
(4.11)

\[\frac{\partial L}{\partial \sigma^2} = \frac{n}{2} (\sigma^2)^{-1} + \frac{1}{2} (\sigma^2)^{-2} (Y-X\beta)'(Y-X\beta) = 0,\]  
(4.12)

simplify and solve for the maximum likelihood estimators \(\hat{\beta}\) and \(\hat{\sigma}\):

\[\hat{\beta} = (X'X)^{-1} X'Y,\]  
(4.13)

\[\hat{\sigma}^2 = n^{-1} (Y-X\hat{\beta})'(Y-X\hat{\beta}).\]  
(4.14)

Any estimate \(\hat{\beta}_i\) is equal to \(\beta_i\) plus a linear function of \(u_i\), which has a multivariate normal distribution, therefore \(\hat{\beta}_i\) has a normal distribution. This allows the derivation of significance tests and confidence intervals for \(\hat{\beta}_i\). The critical assumption for regression analysis is that the error term is distributed normally. Procedures have been developed to cope with problems related to heteroskedasticity and the variety of cases where the independent variables are not pairwise uncorrelated.

**Logistic Regression (Logit)**

Logit analysis is based on the premise (from studies of toxicology) that members of a population subjected to a stimulus which can range over an infinite scale will
respond (in a binary choice situation) according to a simple sigmoid curve. In the context of choice or transport mode, the division of traffic between two modes is between 0 and 1 and their sum is unity. Mode choice is a monotonic function of independent variables and the transportation variables are expressed in units such that if there is an increasing or decreasing utility of transportation by a given mode, then the share of that mode decreases or increases when any of its transportation variables increases or decreases relative to the other mode. Figure 4.1 shows the assumed response function.

Figure 4.1 Cumulative Response to a Varying Stimulus.

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4Reichman and Stopher, "Disaggregate Stochastic Models of Travel-Mode Choice," p. 95.
This nonlinear response can be written:

\[ P_i = \frac{1}{1 - e^{g(x)}} \]  

(4.14)

where \( P_i \) = the conditional probability of choosing mode \( i \), and

\( g(x) \) = a utility function.

In choosing between two transport modes, it is assumed in the logit model that the reaction to stimulus is determined by the utilities of each mode and the characteristics of the user. In the two mode case the logit probability function and the resultant logit model can be written as follows:

\[ \frac{P(i)}{1-P(i)} = \frac{1}{1 + \exp \left( -a_0 - a_i x_i - \sum_{n=2}^{N} a_n x_n \right)} \]  

(4.15)

expressed as,

\[ \log \left( \frac{P(i)}{1-P(i)} \right) = a_0 + a_i x_i + \sum_{n=2}^{N} a_n x_n \]  

(4.16)

where \( P(i) \) = the conditional probability of choosing mode \( i \),

\( x_i \) = the price of mode \( i \) relative to the other modes,

\( x_n \) = other exogenous factors affecting mode choice \( n=2,3...N \), and

\( a_n \) = the parameter of the logistic function \( n = 0, 1 \ldots N \).

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The parameters are estimated by linear regression. The dependent variable is the logarithm of the market share ratio of two modes. Two forms of the utility function have been postulated. Ranson et al, and Kullman, specified the price variable as a ratio of the price of the ith mode to the base mode. McFadden along with Richards and Ben-Akiva expressed the price variable as the difference between the price of the ith mode and the base mode. If the response function shown in Figure 4.1 represents the cumulative normal distribution, the resulting probability (probit) model is estimated by maximum likelihood methods.

Discriminant Analysis

The discussion begins with the two group case presented by Morrison, then moves to the k group case presented by Tatsuoka. The function of discriminant analysis is to form a linear combination of predictor variables

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which shows large differences in group means. Figure 4.2 shows the relationship between regression using a dummy variable to indicate group membership and the discriminant function. Figure 4.3 shows the discriminant functions required for three groups.

Figure 4.2 Regression and Discriminant Functions, (a) Regression Where Group Membership is Indicated By a Dummy (1,0) Variable; (b) Discriminant Function for a Two Group Case.
In Figure 4.2 and 4.3 the coordinates represent variables which distinguish between groups. In the transport case each group represents a transport mode and one variable could be costs and the other service. With this model of mode choice, discriminant analysis derives the boundaries which minimize the group members which would be classified in error.
For notation let,

\[ x_{ji} = \text{the } i\text{th group members value of the } j\text{th independent variable}, \]

\[ b_j = \text{the discriminant coefficient for the } j\text{th variable}, \]

\[ Z_i = \text{the } i\text{th group members individual discriminant score, and} \]

\[ Z_{\text{crit}} = \text{critical values of the discriminant function.} \]

Then let each group member's discriminant score \( Z_i \), be a linear function of the independent predictor variables,

\[ Z_i = b_0 + b_1x_{1i} + b_2x_{2i} + \ldots + b_nx_{ni}. \] (4.17)

If \( Z_i > Z_{\text{crit}} \), classify \( i \) to group 1 and if \( Z_i < Z_{\text{crit}} \), classify \( i \) to group 2. The locus of points where,

\[ b_0 + b_1x_{1i} + \ldots + b_nx_{ni} = Z_{\text{crit}}, \] (4.18)

would be the classification boundary. In the two group case the classification boundary is a two dimensional plane in three dimensional space. In general the classification boundary is an \( n-1 \) dimensional hyperplane in \( n \) space.

The following discussion develops the general discriminant analysis case, continuing with the explanation by Tatsuoka. The first step is to decide on a criterion
for measuring group mean differences. The F ratio could serve as an appropriate criterion. For K groups with a total of N individuals, the F ratio is given by:

\[
F = \frac{SS_b/(K-1)}{SS_w/(N-K)} = \frac{SS_b}{SS_w} \frac{N-K}{K-1}
\]  

(4.19)

where \(SS_b\) = sum-of-squares between group means, and \(SS_w\) = sum-of-squares within group means.

The second term \(\frac{N-K}{K-1}\) is a constant for any given problem, therefore, the essential term is the relation \(SS_b/SS_w\).

With p predictor variables, \(X_1, X_2 \ldots X_p\), form the linear combination:

\[
Y = v_1 X_1 + v_2 X_2 \ldots v_p X_p
\]  

(4.20)

where the \(v_i\)'s are linear weights, and for which the within group means and between group means sum-of-squares are expressible as quadratic terms.

Denote the sum of squares of \(Y\) within the kth group by \(SS_k(Y)\) and let \(v' = (v_1, v_2 \ldots v_p)\), then,

\[
SS_w(Y) = SS_1(Y) + SS_2(Y) + \ldots SS_K(Y)
\]

(4.21)

where \(SS_k(Y) = \begin{bmatrix} S_{k,yy} & S_{k,yx} \\ S_{k,xy} & S_{k,xx} \end{bmatrix} = s_k\)  

(4.22)
Then $SS_w(Y) = v'S_1 v + v'S_2 v + \ldots + v'S_K v$

$$= v'(S_1 + S_2 + \ldots + S_K)v$$

(4.23)

or $SS_w(Y) = v'Wv$

(4.24)

since $\sum_{k=1}^{K} S_k = W$, by definition in (4.21).

(4.25)

To obtain the formulation for between-group means sum-of-squares, define the diagonal elements of $B$ as the usual between-group means sum-of-squares for the variables taken one at a time:

$$b_{ij} = \sum_{k=1}^{K} n_k (\bar{X}_{ik} - \bar{X}_i)^2$$

for all $i$, \hspace{1cm} (4.26)

where $n_k$ = size of $k$th group,

$\bar{X}_{ik}$ = $k$th group mean of $X_i$,

$\bar{X}_i$ = grand means of $X_i$.

The off diagonal elements of $B$ are then between group mean sums of products for pairs of variables over the $K$ groups:

$$b_{ij} = \sum_{k=1}^{K} n_k (\bar{X}_{ik} - \bar{X}_i) (\bar{X}_{jk} - \bar{X}_j)$$

for $i \neq j$, \hspace{1cm} (4.27)

Changing the notation to a more general form let

$\bar{X}$ = group means and,

$\bar{X}$ = grand means,
then, \( B = (\bar{x} - \bar{\bar{x}})'(\bar{x} - \bar{x}) \) \hspace{1cm} (4.28)

Pre and post multiply both sides of (4.28) by \( v' \) and \( v \) respectively,

\[
v' B v = v' (\bar{x} - \bar{\bar{x}})' (\bar{x} - \bar{x}) v
\]

\[
= (\bar{x} v - \bar{\bar{x}} v)' (\bar{x} v - \bar{\bar{x}} v)
\]

\hspace{1cm} (4.29)

(4.30)

Since \( \bar{x} v \) and \( \bar{\bar{x}} v \) are vectors which elements consist of the group means of \( Y \) and the grand means of \( Y \) respectively, the product is:

\[
(\bar{x} v - \bar{\bar{x}} v)'(\bar{x} v - \bar{\bar{x}} v) = \sum_{k=1}^{k=n} \left[ \bar{y}_k - \bar{\bar{y}} \right]^2
\]

\hspace{1cm} (4.31)

where \( \bar{y}_k = \) group means of \( Y \),

and \( \bar{\bar{y}} = \) grand mean of \( Y \), which is the between-group means sum-of-squares of the transformed variable \( Y \). Thus,

\[
SS_b(Y) = v' B v
\]

\hspace{1cm} (4.32)

and rewriting (4.24) and (4.32) gives:

\[
\frac{SS_b(Y)}{SS_w(Y)} = \frac{v' B v}{v' W v} \equiv \lambda.
\]

\hspace{1cm} (4.33)

The ratio \( \lambda \) (defined in (4.39)) provides a criterion for measuring group differences in the dimensions specified
by vector $v$. This is the discriminant criterion proposed by Fisher in 1936.10

The next step is to maximize the discriminant criterion (subject to $v'v = 1.0$) by determining a set of weights $(v_1, v_2 ... v_n)$ which,

$$
\frac{\partial \lambda}{\partial v} = \frac{2[(Bv)(v'Wv)- \lambda(v'Bv)(Wv)]}{(v'Wv)^2} = 0 \quad (4.34)
$$

Simplifying the equation reduces to:

$$
\frac{2(Bv - \lambda Wv)}{v'Wv} = 0, \quad (4.35)
$$

which implies

$$
Bv - \lambda Wv = 0, \quad (4.36)
$$

since $(v'Wv) = 0$ ,

$$
(B - \lambda W)v = 0. \quad (4.37)
$$

Assume $W$ is nonsingular and therefore possesses an inverse, premultiply both sides by $W^{-1}$

$$
(W^{-1}B - \lambda I)v = 0. \quad (4.38)
$$

The solution yields eigenvalues $\lambda_m$ and associated eigenvectors $v_m$ which maximize the discriminant criterion.


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The eigenvector, \( \mathbf{v}' = (v_{11}, v_{12}, \ldots, v_{1p}) \) corresponding to the largest eigenvalue, \( \lambda_1 \), provides a set of weights such that the transformed variable,

\[
Y_1 = v_{11}X_1 + v_{12}X_2 + \ldots + v_{1p}X_p
\]  

(4.39)

has the largest discriminant criterion \( \lambda_1 \) attainable by any linear combination of \( p \) predictor variables.

In the two group case, computation of the single discriminant function is considerably simplified. In particular \( \mathbf{B} \) assumes a particularly simple form. The expression reduces to:

\[
b_{ij} = n_1 (\bar{X}_{i1} - \bar{X}_1) (\bar{X}_{j1} - \bar{X}_1) + n_2 (\bar{X}_{i2} - \bar{X}_2) (\bar{X}_{j2} - \bar{X}_1) 
\]  

(4.40)

and the grand mean \( \bar{X} \) is equal to:

\[
\frac{n_1 \bar{X}_{i1} + n_2 \bar{X}_{i2}}{n_1 + n_2}
\]  

(4.41)

and substituting:

\[
b_{ij} = \frac{n_1 n_2}{n_1 + n_2} (\bar{X}_{i1} - \bar{X}_{i2}) (\bar{X}_{j1} - \bar{X}_{j2}) 
\]  

(4.42)

Define a \( p \) dimension row vector,

\[
\mathbf{d}' = [\bar{X}_{11} - \bar{X}_{12}, \bar{X}_{21} - \bar{X}_{22}, \ldots, \bar{X}_{p1} - \bar{X}_{p2}]
\]  

(4.43)

of the differences between the group means of the \( p \) variates, then
\[ B = \frac{n_1 n_2}{n_1 + n_2} \text{dd}' \]  

(4.44)

and define \( c = \frac{n_1 n_2}{n_1 + n_2} \), substitute into 4.38,

\[ \left[ cW^{-1}(\text{dd}') - u \lambda \right] v = 0 \]  

(4.45)

where \( u = \frac{\lambda}{\lambda + \lambda} \), since equation 4.38 can be rewritten:

\[ Bv = \lambda Wv. \]  

(4.46)

then add \( \lambda Bv \) to both sides;

\[ (1+\lambda)Bv = \lambda (W+B)v, \text{ or} \]  

(4.47)

\[ (T^{-1}B - \frac{\lambda}{\lambda + \lambda} I)v = 0. \]  

(4.48)

where \( T \) is the total sum of squares and cross products matrix.

Tatsuoka shows that if \( v_0 \) is the eigenvector of \( W^{-1}B \) associated with the eigenvalue \( \lambda_0 \), it is also the eigenvector of \( T^{-1}B \), and its associated eigenvalue is \( \lambda_0 / 1 + \lambda_0 \). Thus the eigenvalues stand in relation;

\[ u = \frac{\lambda}{\lambda + \lambda} \]  

(4.49)

\[ ^{11}\text{Tatsuoka, Multivariate Analysis, pp. 172-174.} \]
\[ uv = c T^{-1}(dd')v, \]
\[ = c T^{-1}(d)(d'v). \]  

Since \( d'v \) is a scalar, but unknown quantity (\( d' \) is 1xp and \( v = p \times 1 \)), collect all scalar quantities into a single multiplier and write:

\[ v = [c(d'v)/u]T^{-1}d, \]  
\[ v = mT^{-1}d \]  

where \( v \) is an unknown scalar, since it is px1 and \( m = [ (d'v)]/u \). This gives a solution for the eigenvector \( v \) of \( T^{-1}B \) in the two group case, because \( v \) is, in any case, proportional up to an arbitrary proportionality constant. In the two group case, the single discriminant function can be solved without solving the eigenvalue problem by postmultiplying the inverse of the sums of squares and cross products (SSCP) matrix by the column of mean differences on the \( p \) predictors to get the vector of discriminant coefficients. The single discriminant function can be obtained without solving the eigenvalue by normalizing \( v \). Rewrite (4.53),

\[ v = S^{-1}_{pp} md \]  

where \( S_{pp} \) equals the sum of squares and cross products of other predictor variables. If the criterion variable
in the two group case is a **dichotomous** variable (1,0), we can infer that the sum of the products is given by:

\[ \sum xy = \frac{n_1n_2}{n_1+n_2} (\bar{x}_1 - \bar{x}_2) \]  

(4.55)

Thus the elements of \( md' \) given in 4.54 are proportional to the predicted criterion sum of products where the criterion dichotomous variable indicates group membership. Therefore in the two group case the discriminant weights are proportional to the weights of a multiple regression equation of a dichotomous group membership variable on the \( p \) predictors. Several writers have stated that discriminant analysis in general is a special case of multiple regression,\(^{12}\) but the statement is true only for the two group case. In cases of greater than two groups, discriminant analysis reduces to canonical correlation analysis.\(^{13}\) This is possible if the criterion variables are dummy variables with one fewer dummy variables than the number of groups. Then the predictor and criterion sets are treated by the means of canonical analysis.


\(^{13}\)Ibid., p. 173.
Canonical Analysis

Canonical analysis is a technique to determine a linear combination of p predictors and a linear combination of q criterion variables such that the correlation between these linear combinations in the total sample is as large as possible. The problem is to determine one set of weights, \( \mathbf{u}' = (u_1, u_2, \ldots, u_p) \) for the predictor variables and another set of weights \( \mathbf{v}' = (v_1, v_2, \ldots, v_q) \) for the criterion variables in such a way the correlation \( r_{zw} \) between

\[
Z = u_1 X_1 + u_2 X_2 + \ldots + u_p X_p
\]

and
\[
W = v_1 Y_1 + v_2 Y_2 + \ldots + v_q Y_q
\]

is the largest attainable for the sample. Partition the sums of squares and cross products matrix \( S \)

\[
S = \begin{bmatrix}
S_{pp} & S_{pc} \\
S_{cp} & S_{cc}
\end{bmatrix}
\]

then compute the quadruple matrix product:

\[
A = S_{pp}^{-1} S_{pc} S_{cp}^{-1} S_{cc}
\]

The eigenvalue's \( u_1^2 \) and the eigenvectors \( \mathbf{v}_1 \) of the matrix \( A \) are computed. The largest eigenvalue \( u_1^2 \) is the square of the maximum \( r_{zw} \) and is the first canonical
correlation between predictor and criterion sets. The elements of the corresponding eigenvector \( \mathbf{v}_j \) are the weights to be used in combining the predictor variables to obtain the optimum linear combination.

The resulting linear correlation \( Z_1, Z_2 \ldots Z_{k-1} \) are identical (within proportionality) to the discriminant function obtained in maximizing the discriminant criterion. Tatsuoka proved that the discriminant criterion value \( \lambda \) is related to the corresponding squared canonical correlate value \( u_1^2 \) by the eigenvector: \(^{14}\)

\[
\lambda_1 = \frac{u_1^2}{(1-u_1)^2} \tag{4.59}
\]

**Summary and Preview**

Having reviewed the theory of transport demand, the various approaches to demand modeling and the econometric assumptions of regression analysis, discriminant analysis and canonical correlation, the choice was made of the technique to analyze transport demand for this dissertation. The discriminant analysis technique was chosen because discriminant analysis can provide a modal choice which can simulate the demand schedule for any given mode in the model. The schedule is estimated without any \textit{a priori} restrictions, common to linear regression. Discriminant

analysis avoids the difficulties embodied in linear logit models due to the rigid a priori restrictions on price responsiveness of demand and due to irregularity and inconsistency of the structure of technology and preferences underlying the linear logit model. On the other hand, discriminant analysis requires the assumption that each group is equal size, and is distributed normally with equal variance-covariance matrices. Linear regression requires the assumption of a normal distribution of the error term, homoskedasticity and independence of the predictor variables. Certain options are available to cope with situations in which they are not met by the data. The data utilized in this analysis proved to be skewed but transformed comfortably into lognormal form. Adjustment to correct the classification bias inherent in unequal size groups is available and was utilized.

Applications of discriminant analysis to be applied economic analysis has increased sharply in the past decade. The ability to classify credit risks, the identification of completed income tax forms to be audited and application to mode choice models are some examples.  

The next chapter shows the results of analysis of several commodities moving by water and competing transport modes in the Ohio River and Arkansas River areas.
CHAPTER V

APPLICATION OF DISCRIMINANT ANALYSIS MODEL
OF TRANSPORT DEMAND

This chapter discusses application of discriminant analysis to estimate demand functions for water transportation in the Ohio and Arkansas River area. The model is calibrated on a specially altered discriminant model which first fits the discriminant functions. Then it proceeds to simulate the transportation demand function by incrementing (adding) transport costs (or any other predictor variable such as time in transit) for any mode specified. The original program was the BMD07M version from the Bio-medical (BMD) package developed by the University of California, Berkley. The Statistical Programs for Social Sciences (SPSS), which gives similar results except for the simulation feature was also utilized. Both programs are capable of computing transformations of variables. Each develops a statistical summary of the data, fits the discriminant functions, classifies each observation and displays a summary of the classification results.

The modified BMD07M program performs the incrementing step by adding prespecified increases in rates (or other variables), and reclassifies observations based on the discriminant functions fit on the original data. This
allows the simulation of the demand function for any transport mode represented in the data. The following sections discuss the results obtained in the analysis of data on commodities which are shipped by water and competing transport modes.

**Demand Model**

The modal choice model utilized to develop estimates of the demand for water transportation is as follows:

\[ C(k) = f(T_{ij}^q, S_{ij}^q, h_{kij}^q, d_{kij}^q, r_{kij}^q, c_{ks}^q) \]

where:
- \( C(k) \) = choice of mode \( k \), given shipments per year;
- \( T_{ij}^q \) = amount shipped annually from \( i \) to \( j \) of commodity \( q \),

and modal attributes:
- \( s_{ij}^q \) = shipment size from \( i \) to \( j \) for commodity \( q \),
- \( h_{kij}^q \) = travel time from \( i \) to \( j \) for commodity \( q \) by mode \( k \),
- \( d_{kij}^q \) = distance from \( i \) to \( j \) by mode \( k \),
- \( r_{kij}^q \) = transport rate from \( i \) to \( j \) for commodity \( q \) by mode \( k \),
- \( c_{ks}^q \) = handling costs for commodity \( q \) shipped by mode \( k \) and shipment size \( s \).

The model was tried using data representing shipments in the Ohio and Arkansas River areas. The following
THE DEMAND FOR WATER TRANSPORTATION: APPLICATION OF DISCRIMINANT-ETC(U)

AUG 80 L G ANGLE
discussion begins with aggregated coal shipments disaggregate by type of user and destination, then moves to other commodities. A detailed summary of the data used in this analysis is shown in Appendix A.

Aggregate Level. Table 5.1 shows the data from a total of 395 coal shipments. Five modes transported annual tonnage in excess of 122 million tons. Barge and rail barge modes haul for 61 percent of the traffic in the sample while regular and unit trains haul for 38 percent, leaving about one percent of the traffic to be hauled by truck.

**TABLE 5.1**

**Summary of Coal Traffic by Transport Mode**

<table>
<thead>
<tr>
<th>Ohio River and Arkansas River Areas</th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
<th>Rail</th>
<th>Barge</th>
<th>Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>257</td>
<td>68</td>
<td>26</td>
<td>23</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Av. Annual Shipments (1000 Tons)</td>
<td>81</td>
<td>1,013</td>
<td>27</td>
<td>263</td>
<td>1,418</td>
<td></td>
</tr>
<tr>
<td>Avg Shipments (Tons)</td>
<td>361</td>
<td>3,831</td>
<td>50</td>
<td>1,590</td>
<td>2,670</td>
<td></td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>157</td>
<td>118</td>
<td>51</td>
<td>308</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>103</td>
<td>39</td>
<td>3</td>
<td>141</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>357</td>
<td>63</td>
<td>239</td>
<td>330</td>
<td>133</td>
<td></td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>40</td>
<td>19</td>
<td>22</td>
<td>32</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Rate (Mills Per Ton Mile)</td>
<td>228</td>
<td>53</td>
<td>464</td>
<td>107</td>
<td>198</td>
<td></td>
</tr>
</tbody>
</table>

The discriminant analysis brings each variable into the equation in a stepwise procedure based on the contribution that each variable makes to the discrimination between groups. In this case the order of entry was:


<table>
<thead>
<tr>
<th>Step</th>
<th>F value to enter or remove</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rate</td>
<td>137.1935</td>
</tr>
<tr>
<td>2. Time of transit</td>
<td>109.5812</td>
</tr>
<tr>
<td>3. Length of haul</td>
<td>46.3572</td>
</tr>
<tr>
<td>4. Shipment size</td>
<td>22.3648</td>
</tr>
<tr>
<td>5. Size of annual shipment</td>
<td>5.9386</td>
</tr>
<tr>
<td>6. Handling costs</td>
<td>1.7916</td>
</tr>
</tbody>
</table>

The discriminant functions for each mode are summarized below:

<table>
<thead>
<tr>
<th>Mode</th>
<th>No. of Shipment</th>
<th>Tons Per Year</th>
<th>Miles</th>
<th>Hours</th>
<th>Shipment Size</th>
<th>Rate</th>
<th>Handling Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>207</td>
<td>106.69268</td>
<td>3.85272</td>
<td>-7.25016</td>
<td>-.65165</td>
<td>4.31258</td>
<td>31.70828</td>
</tr>
<tr>
<td>Barge</td>
<td>68</td>
<td>-79.73727</td>
<td>3.71078</td>
<td>-1.81190</td>
<td>-.97333</td>
<td>4.93595</td>
<td>20.12777</td>
</tr>
<tr>
<td>Truck</td>
<td>28</td>
<td>-97.00450</td>
<td>3.22916</td>
<td>-5.03098</td>
<td>-5.93570</td>
<td>3.04544</td>
<td>3.04544</td>
</tr>
<tr>
<td>Rail Barge</td>
<td>23</td>
<td>-111.65172</td>
<td>4.22040</td>
<td>-5.04177</td>
<td>-.68821</td>
<td>4.81422</td>
<td>29.10273</td>
</tr>
<tr>
<td>Unit Train</td>
<td>18</td>
<td>101.79012</td>
<td>4.85906</td>
<td>-5.72404</td>
<td>-2.14519</td>
<td>4.36454</td>
<td>28.90275</td>
</tr>
</tbody>
</table>
In the classification stage of the discriminant analysis each observation has a value computed by each of the functions. The observation is assigned to the group for which the computed value is nearest to zero. For example, computed discriminant function values for two actual barge cases are shown below. The first was assigned to the rail group, the second was assigned to the barge group.

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
<th>Rail Barge</th>
<th>Unit Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1.958</td>
<td>26.817</td>
<td>20.839</td>
<td>7.96</td>
<td>15.601</td>
</tr>
<tr>
<td>Case 2</td>
<td>27.058</td>
<td>3.782</td>
<td>57.706</td>
<td>14.926</td>
<td>17.301</td>
</tr>
</tbody>
</table>

Actual data for each case were:

<table>
<thead>
<tr>
<th></th>
<th>Tons Per Year</th>
<th>Miles</th>
<th>Hours</th>
<th>Shipment Size</th>
<th>Rate</th>
<th>Handling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>2,530</td>
<td>250</td>
<td>204</td>
<td>150</td>
<td>$5.48</td>
<td>$0.550</td>
</tr>
<tr>
<td>Case 2</td>
<td>1,754,000</td>
<td>309</td>
<td>108</td>
<td>11,500</td>
<td>$ .88</td>
<td>$ .088</td>
</tr>
</tbody>
</table>

The data for Case 1 looks like a rail shipment because of the shipment size and rate.

The next step in estimating the demand function is to increment the rate variable for one mode and reclassify the cases for that mode by the discriminant functions derived above. Barge rates were incremented from an average of $ .63 per ton to $1.63 per ton. Each barge shipment was adjusted upward by $1.00 per ton. Figure 5.1 shows the demand function derived from that calculation.
At the existing barge rate average of $ .63 per ton, 69 million tons of coal were actually shipped by barge. The model misclassified 5 of the 93 barge shipments. Two were classified as rail-barge and three as rail shipments. The demand function shown in Figure 5.1, therefore, shows 62 million tons per year at $ .63 per ton. Estimated quantity to be shipped by barge at $1.63 is 1,039,950 tons per year. The modified BMD07M program summarizes annual quantity classified by mode at each stage of the incremental analysis.

Figure 5.1 Demand Function for Barge Traffic, All Coal Shipments
Table 5.2 summarizes the initial classification of 395 individual coal shipments by 5 transport modes. Off diagonal cases are misclassified.

TABLE 5.2

Number of Coal Shipments Classified by Mode

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
<th>Rail</th>
<th>Barge</th>
<th>Unit</th>
<th>Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>199</td>
<td>3</td>
<td>8</td>
<td>39</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Barge</td>
<td>1</td>
<td>64</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Truck</td>
<td>2</td>
<td>0</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Rail Barge</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Unit Train</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>13</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Percent correctly classified 95 93 73 23 54

Few barge and rail shipments were misclassified, however, many rail-barge and unit train shipments were misclassified. Several actual unit train shipments were classified as conventional rail shipments. This happened because unit train rates from Western Kentucky coal fields to Louisville Gas and Electric steam electric generating plants were based on total system shipments per year. Therefore individual shipments to any given plant resemble quantities which move otherwise by conventional rail. The data in the sample representing rail-barge suffers from some
ambiguity, since shipment size is a function of which mode completes the shipment.

In summary, the demand function estimated from all coal shipments (N=395) is price inelastic at current rates and quantities. Elasticity of price, estimated at current rates and quantities is .62, that is a one percent increase in rate results in a .62 percent decrease in quantity shipped by barge.

**Destination Level.** Observations were coded to indicate the Bureau of Economic Analysis Region (BEAR) of destination. Distinct characteristics of the demand function for barge can be noted at this level of aggregation.

**Pittsburg Region.** Table 5.3 shows the data for 35 coal shipments destined for the Pittsburg region. Total shipments in excess of 44 million tons are hauled by three modes, although barge accounts for 98 percent of the traffic.

**TABLE 5.3**

Summary of Coal Shipments to Pittsburg Region

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Rail/Barge</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>6</td>
<td>27</td>
<td>2</td>
</tr>
<tr>
<td>Annual Shipment (1000 Tons)</td>
<td>39</td>
<td>1,621</td>
<td>246</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>215</td>
<td>4,696</td>
<td>3,950</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>158</td>
<td>93</td>
<td>272</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>35</td>
<td>50</td>
<td>102</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>444</td>
<td>53</td>
<td>322</td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>27</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>Rate (Mills Per Ton Mile)</td>
<td>27</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

The demand function, generated from data summarized in Table 5.3 is essentially inelastic near the current rate, with increasing price elasticity thereafter. Estimated price elasticity of demand for barge transport is .0002 at average rates and quantities represented in the sample.

**Huntington-Ashland Region.** Data for 26 coal shipments destined for BEAR 52 are shown in Table 5.4. Total annual shipments are in excess of 9 million tons.

The demand function for water transportation for coal shipments generated from this data and shown in Figure 5.3 is essentially price inelastic.
Figure 5.2 Demand Function for Coal Shipped by Barge to Pittsburg Region
TABLE 5.4

Summary of Coal Shipments to Huntington-Ashland Region

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Rail/Barge</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>15</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Annual Shipment (1000 Tons)</td>
<td>247</td>
<td>1,314</td>
<td>171</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>864</td>
<td>4,600</td>
<td>1,675</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>131</td>
<td>170</td>
<td>145</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>91</td>
<td>38</td>
<td>92</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>318</td>
<td>90</td>
<td>244</td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>52</td>
<td>30</td>
<td>41</td>
</tr>
<tr>
<td>Rate (Mills Per Ton Mile)</td>
<td>24</td>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>


![Figure 5.3 Demand Function for Coal Shipped by Barge to Huntington-Ashland Region.](image-url)
An obvious reason for barge transport demand being inelastic in this region is that there are many coal fields which are as accessible by water as by rail, and the data summary shows barge rates to be 20 percent of rail rates and there is a time advantage by barge. Since a stepwise computational model was used, there is some evidence about the relative importance of each variable. The order of entry of the variables was:

<table>
<thead>
<tr>
<th>Step entered</th>
<th>F value to enter or remove</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rate</td>
<td>6.6814</td>
</tr>
<tr>
<td>2. Shipment Size</td>
<td>4.5075</td>
</tr>
<tr>
<td>3. Size of Annual Shipment</td>
<td>2.0051</td>
</tr>
<tr>
<td>4. Length of Haul</td>
<td>1.3795</td>
</tr>
<tr>
<td>5. Time of Transit</td>
<td>.3272</td>
</tr>
<tr>
<td>6. Handling Cost</td>
<td>.1437</td>
</tr>
</tbody>
</table>

**Cincinnati Region.** Three modes haul 69 coal shipments destined for the Cincinnati Region (BEAR 62). The barge mode hauls 77 percent of the total annual shipment (in excess of 11 million tons). The share of the total hauled by each mode is summarized in Table 5.5
TABLE 5.5

Summary of Coal Shipments to Cincinnati Region

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Water</th>
<th>Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>47</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>Annual Shipment (1000 Tons)</td>
<td>53</td>
<td>547</td>
<td>18</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>537</td>
<td>6,988</td>
<td>48</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>200</td>
<td>302</td>
<td>66</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>156</td>
<td>92</td>
<td>18</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>449</td>
<td>98</td>
<td>232</td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>42</td>
<td>52</td>
<td>43</td>
</tr>
<tr>
<td>Rate (Mills Per Ton Mile)</td>
<td>22</td>
<td>3</td>
<td>35</td>
</tr>
</tbody>
</table>


The demand function generated from these data is shown in Figure 5.4. Note that at the current rate the demand function is price inelastic, but that the upper range of rate increases would result in a price elastic demand function.
Figure 5.4 Demand Function for Coal Shipped by Barge to Cincinnati Area.
Louisville Region. Data on 68 shipments destined to Louisville Region are summarized in Table 5.6.

**TABLE 5.6**

Summary of Coal Shipments to Louisville Region

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Water</th>
<th>Truck</th>
<th>Unit Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>40</td>
<td>16</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Annual Shipment (1000 Tons)</td>
<td>26</td>
<td>290</td>
<td>3</td>
<td>984</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>744</td>
<td>5,432</td>
<td>37</td>
<td>4,167</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>457</td>
<td>703</td>
<td>45</td>
<td>55</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>438</td>
<td>168</td>
<td>5</td>
<td>55</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>715</td>
<td>183</td>
<td>3,133</td>
<td>143</td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>38</td>
<td>77</td>
<td>109</td>
<td>23</td>
</tr>
<tr>
<td>Rate (Mills Per Ton Mile)</td>
<td>16</td>
<td>3</td>
<td>85</td>
<td>14</td>
</tr>
</tbody>
</table>


The demand function for water transportation generated from these data is shown in Figure 5.5. The demand function is essentially price inelastic near current price-quantity relationships but is price elastic at higher barge rates. A demand function for rail transportation was also estimated from the model. It also shows price inelastic behavior near current prices, which seems to indicate a stable equilibrium in intermodal competition. One important factor is that rail hauls (conventional and unit train) are far shorter than water hauls. Shorter
hulls, faster speeds, and lower handling cost for the rail mode are tradeoffs against lower transport costs by barge.

**Using Industry Aggregation.** Coal-using industries were coded as electrical utilities (1), iron and steel (2), and others (3). Coal shipments were sorted on this basis and barge demand estimates made for each industry.

![Graph](image)

**Figure 5.5** Demand Function for Coal Shipped by Barge to Louisville Area.
Electrical Utility Industry. One hundred twenty-two coal shipments were destined for the electrical utility industry. The sample included over 98 million tons of which 62 million tons moved by barge.

Figure 5.6 Demand Function for Coal Shipped by Barge to Electrical Utility Users.
Figure 5.6 shows the estimated demand function for barge transportation by utilities. The calculation underestimated actual shipments at current prices by about 14 million tons.

**Metallurgical Coal.** Coal shipped to iron and steelmakers is frequently mined in captive mines and shipped on captive barges or rail cars. Captive mines and barges are owned by the coal and steelmakers. The data is summarized in Table 5.7.

**TABLE 5.7**

Summary of Metallurgical Coal Shipments

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Rail/Barge</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>31</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>Annual Shipments (1000 Tons)</td>
<td>171</td>
<td>596</td>
<td>288</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>1,183</td>
<td>4,989</td>
<td>1,976</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>201</td>
<td>157</td>
<td>775</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>107</td>
<td>113</td>
<td>233</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>419</td>
<td>74</td>
<td>398</td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>28</td>
<td>34</td>
<td>38</td>
</tr>
<tr>
<td>Rate (Mills Per Ton)</td>
<td>21</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>


The demand function, shown in Figure 5.7, for barge transportation of coal delivered to the iron and steel industry, is somewhat less price elastic than of the electrical utility industry. Figure 5.7 shows the savings.
Figure 5.7 Demand Function for Metallurgical Coal Shipments by Barge and Average Savings for Barge Users.
resulting from barge shipment over shipment by rail. In this case, benefits computed by traditional methods, i.e., savings in cost to shippers, would substantially exceed those computed by measuring the area under the demand function, since the area ABDE, representing savings computed on average savings to barge users, is greater than the area AFE, representing the area under the demand schedule for barge transportation (or "willingness to pay").

Commodities Other Than Coal. For all commodities other than coal, the demand for barge transportation indicates the demand is essentially price inelastic at the aggregate level. Figure 5.8 shows the estimated demand function based on the total of 212 shipments which include data from the Arkansas River area. Table 5.8 shows the data for all commodities other than coal.
Figure 5.8 Demand Function for Commodities Other than Coal Shipped by Barge.
TABLE 5.8
Summary of Shipments Other Than Coal

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
<th>Pipeline</th>
<th>Pipeline-Barge</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>80</td>
<td>81</td>
<td>44</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Annual Shipments (Tons)</td>
<td>23</td>
<td>155</td>
<td>5</td>
<td>225</td>
<td>1,973</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>236</td>
<td>2,930</td>
<td>26</td>
<td>2,354</td>
<td>12,000</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>562</td>
<td>952</td>
<td>63</td>
<td>162</td>
<td>1,048</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>161</td>
<td>198</td>
<td>7</td>
<td>109</td>
<td>536</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>2,415</td>
<td>283</td>
<td>901</td>
<td>120</td>
<td>202</td>
</tr>
<tr>
<td>Handling Cost (Cents Per Ton)</td>
<td>43</td>
<td>41</td>
<td>50</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>Rate (Mills Per Ton)</td>
<td>43</td>
<td>3</td>
<td>14</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Source: U.S. Army Engineer Survey, 1970 and 1971

Chemicals. For shipment of chemicals, the demand function for barge transportation (see Figure 5.9) is price inelastic through a substantial range, then highly elastic. One of the established characteristics of chemical transportation is the need for highly specialized, often dedicated, equipment because of corrosiveness and other problems. Summary data are presented in Table 5.9.
Figure 5.9  Demand Function for Chemical Shipments by Barge.
### TABLE 5.9
Summary of Chemical Shipments

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Annual Shipments (1000 Tons)</td>
<td>2</td>
<td>116</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>78</td>
<td>2,924</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>1,100</td>
<td>981</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>164</td>
<td>263</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>1,733</td>
<td>296</td>
</tr>
<tr>
<td>Handling Costs (Cents Per Ton)</td>
<td>73</td>
<td>6</td>
</tr>
<tr>
<td>Rate (Mills Per Ton)</td>
<td>16</td>
<td>3</td>
</tr>
</tbody>
</table>

Source: U.S. Army Engineer Survey, 1970 and 1971

Refined Petroleum Products. Data on 172 shipments were obtained in the survey and are shown in Table 5.10.

### TABLE 5.10
Summary of Shipments of Refined Petroleum

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Shipments</td>
<td>71</td>
<td>57</td>
<td>44</td>
</tr>
<tr>
<td>Annual Shipments (1000 Tons)</td>
<td>22</td>
<td>116</td>
<td>5</td>
</tr>
<tr>
<td>Average Shipment (Tons)</td>
<td>75</td>
<td>3,254</td>
<td>26</td>
</tr>
<tr>
<td>Length of Haul (Miles)</td>
<td>550</td>
<td>1,015</td>
<td>63</td>
</tr>
<tr>
<td>Time of Transit (Hours)</td>
<td>165</td>
<td>195</td>
<td>7</td>
</tr>
<tr>
<td>Rate (Cents Per Ton)</td>
<td>1,587</td>
<td>264</td>
<td>901</td>
</tr>
<tr>
<td>Handling Costs (Cents Per Ton)</td>
<td>42</td>
<td>51</td>
<td>50</td>
</tr>
<tr>
<td>Rate (Mills Per Ton)</td>
<td>47</td>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

The demand function for barge transportation carrying refined petroleum products is price inelastic. The explanation is that refined petroleum products are often transported by user-owned barges and towboats. Obviously, a huge rate differential exists between rail or truck modes and barge. Large shipment sizes over long distances are typical for barge.

**Efficiency of the Discriminant Function.** Besides using the discriminant analysis to estimate a demand function, a more obvious use in transportation analysis would be to classify shipments with certain attributes by the mode most likely to be chosen by those who make mode choice.

The discriminant analysis program produces a summary of the results of classification of the cases used to calibrate the discriminant functions. Obviously there should be an upward bias in the number correctly classified because the data are used to calibrate the model.¹

To estimate the bias a run of the model on coal to all destinations was made with 10 percent of the observations held out. The hold out members of the population were selected randomly. The following results were obtained:

Figure 5.10 Demand Function for Refined Petroleum Shipments by Barge.
Classification of 380 Observations

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
<th>Rail-Barge</th>
<th>Unit Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rail</td>
<td>175</td>
<td>10</td>
<td>38</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>2. Barge</td>
<td>1</td>
<td>36</td>
<td>8</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3. Truck</td>
<td>1</td>
<td>0</td>
<td>21</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4. Rail-Barge</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>5. Unit Train</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

Over 65 percent of the shipments were classified correctly. The classification functions were utilized to classify holdout observations as follows:

Classification of 40 Holdout Observations

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Barge</th>
<th>Truck</th>
<th>Rail-Barge</th>
<th>Unit Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rail</td>
<td>21</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2. Barge</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. Truck</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4. Rail-Barge</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5. Unit Train</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Of the 40 holdout observations, 27 or 68 percent were classified correctly.

Frank et al, suggest the following test of significance between the proportion of correct classifications and the proportion which could be attributed to chance: \(^2\)

\[ t = \frac{Q-P}{\sqrt{P(1-P)/n}} \]

where \( Q \) = the proportion of the sample observations correctly classified by the discriminant analysis.

\( P \) = the proportion one would expect by chance (if groups are equal size, \( P = 0.5 \))

\( t \) = Students' t distribution

The value of \( t \) computed from the sample of 380 cases is 5.69, assuming the proportion one would expect by chance to be 0.5, and the \( t \) value for the 40 case sample is 2.28. Both estimates are significant at the 0.05 level and therefore there is no evidence of significant bias due to sampling errors.

Table 5.11 presents a summary of classification results for the runs discussed in the demand analysis section.

**Summary**

Demand functions for barge transportation for coal and other commodities at various levels of disaggregation have been estimated. Classification results were in excess of 92 percent correctly classified except for shipments of
refined petroleum products. However, comparisons by tonnage showed as much as 14 percent of coal shipments by barge to electrical utility users were misclassified, an indication that large annual shipments may be more likely to be misclassified, especially where unit train technology is competing. Using a holdout sample of 40 shipments, selected at random, the expected upward bias in classification did not materialize. The following chapter develops a summary of, and conclusions reached, in this dissertation.
<table>
<thead>
<tr>
<th>Demand Model</th>
<th>% Correctly Classified</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Coal</td>
<td>95</td>
</tr>
<tr>
<td>Coal to Pittsburg Region Destination BEAR 66</td>
<td>94</td>
</tr>
<tr>
<td>Coal to Huntington-Ashland Region Destination BEAR 62</td>
<td>96</td>
</tr>
<tr>
<td>Coal to Cincinnati Region Destination BEAR 54</td>
<td>97</td>
</tr>
<tr>
<td>Coal to Louisville Region Destination BEAR 52</td>
<td>93</td>
</tr>
<tr>
<td>Coal to Electrical Utility Users</td>
<td>95</td>
</tr>
<tr>
<td>Coal to Iron and Steel Industry</td>
<td>92</td>
</tr>
<tr>
<td>Commodities Other Than Coal</td>
<td>93</td>
</tr>
<tr>
<td>Refined Petroleum Products</td>
<td>83</td>
</tr>
<tr>
<td>Chemical Products</td>
<td>100</td>
</tr>
</tbody>
</table>
CHAPTER VI

SUMMARY AND CONCLUSIONS

Characteristics of the Data

Data on nine transport modes, 14 major commodity groups, totaling 145.9 million freight tons were analyzed for this study. The number of observations for automotive equipment, transportation equipment, machinery (except electrical), and furniture industries were too few to calibrate a mode choice model. In several commodity groups, too few observations were obtained for one or more modes. Commodity groups for which adequate numbers of observations were available for calibrating the modal split model were: coal, chemicals, petroleum products, primary metals and fabricated metals.

The data for each mode and commodity were analyzed for statistical properties. In general, the raw data were found to be skewed positively and characterized by significant kurtosis. These characteristics diverge from the assumption that the groups are multivariate normal. The data were transformed to logarithmic, exponential and square root forms and evaluated. The logarithmic transform is characterized by small, negative skewness.
and small values of kurtosis. Thus the data in logarithmic transform approach properties of the normal distribution, at least in the univariate case.

**Application of Discriminant Analysis**

Discriminant analysis is based on three basic assumptions:

1. The probability density function of the linear discriminant is multivariate normal.

2. The variance-covariance matrices for the groups are equal.

3. The prior probability of being in any group is equal.

Watson reached the conclusion that since discriminant models do not estimate probability of choice of mode directly and since discriminant models performed poorly in prediction of mode choice and classification that the probit model would serve the purposes of investigating the value of time in passenger traffic.¹ He found linear regression gave an unbiased but inefficient estimate of probability.

The SPSS and BMD versions of discriminant analysis allow an adjustment for size of groups which corrects bias in classification due to unequal size groups. It has been shown that the univariate distribution of each

parameter approaches normality when transformed to logarithmic form. Box's M and its associated F test for equality of group covariance is available in SPSS, but requires a substantial increase in required core storage space. Because of limitations in core storage space in the CDC 6600 facility used for this analysis, the test could not be run. The discriminant model was run in many combinations of disaggregation of shipments to explore the robustness of the model under different assumptions. The model shows a high percent of correctly classified observations (from 83 to 100 percent) which improves as the classes of shipments are disaggregated to more homogeneous groupings. Tests of the model using holdout data showed no substantial upward bias in classification. Therefore, it is concluded that the discriminant models replicate the behavior of freight transport users in the Ohio and Arkansas River areas.

Would alternative procedures offer preferable results? Modal split models calibrated by probit and logit regression models offer the advantage of relaxing the linear assumption for parameters and the assumption of equal variance/covariance matrices.

Oum criticizes the use of linear logit models because they impose many rigid a priori restrictions on parameters of price responsiveness of demand, including
the elasticity of substitution and cross price elasticity.² He concludes that the structure of technology (or preference) underlying the linear logit model is severely irregular and inconsistent. In the price ratio versions of the linear logit model, elasticity of substitution depends upon the choice of the base mode and the multinomial case restricts cross price elasticities, with respect to any given nonbase modes, to be identical. In the price ratio case, the elasticity of substitution between any two modes at a given data point is a function of the selected mode; however, the underlying elasticities of substitution do not depend upon the selection of the base mode. The model does hold all cross price elasticities between nonbased modes to be identical. Therefore, while the linear logit model may improve the predictive ability of a mode choice model, its utility to evaluate cross mode elasticities of substitution is severely limited. The probit model contains similar problems. Computational capacity to calibrate the probit model by maximum likelihood methods have been discussed in the literature.³


However, it was not possible to utilize an operational version of the maximum likelihood methods for this classification, since SPSS and BMD program packages do not include the method.

Economic Interpretation of the Demand Model

Even though the statistical problems with the discriminant model appear to be relatively minor, some of the analysis results are difficult to justify economically. Some of the simulations suggest that the demand for barge transportation is significantly price inelastic in the short run. Escalation of the price variable from 3 to 5 times current relative prices are required to shift the slope to a relative price elastic position. This suggests less substitution between barge and other modes than one would expect. Following is a summary of the real price and service ratios between barge and other modes for a number of destination and commodity groups:
### TABLE 6.1

Comparison of Cost and Service Ratios Between Competing Modes

<table>
<thead>
<tr>
<th></th>
<th>Rail Barge Ratio</th>
<th>Truck Barge Ratio</th>
<th>Unit Train Barge Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>All coal</td>
<td>3.7</td>
<td>7.6</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>.54</td>
<td>4.0</td>
<td>.95</td>
</tr>
<tr>
<td>Coal Dest 66</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dest 52</td>
<td>4.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dest 62</td>
<td>5.2</td>
<td>9.1</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>.87</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td>3.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>58.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref. Petro</td>
<td>15.9</td>
<td>5.0</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>.64</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

*a Cost = rate and handling per ton mile; speed = miles per hour

If the data are representative, barges hold an unusual advantage in rates and a frequent advantage in speed (a surrogate for service). The following is a summary of ratios of annual shipment and average shipment sizes for various commodity destinations.
<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>Truck</th>
<th>Unit Train</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Barge</td>
<td>Barge</td>
<td>Barge</td>
</tr>
<tr>
<td>All Coal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.786</td>
<td>.27</td>
<td>1.39</td>
</tr>
<tr>
<td>Average</td>
<td>.197</td>
<td>.023</td>
<td>.58</td>
</tr>
<tr>
<td>Coal Dest 66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.053</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coal Dest 52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.188</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>.188</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coal Dest 62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.096</td>
<td>.033</td>
<td>1.154</td>
</tr>
<tr>
<td>Average</td>
<td>.077</td>
<td>.007</td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.286</td>
<td>.020</td>
<td>1.154</td>
</tr>
<tr>
<td>Average</td>
<td>.237</td>
<td>.055</td>
<td>.53</td>
</tr>
<tr>
<td>Chemicals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>.027</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref. Petro</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.191</td>
<td>.04</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>.023</td>
<td>.007</td>
<td></td>
</tr>
</tbody>
</table>

In all cases barge users are large consumers and can receive large shipment sizes. The combined impact of lower rates, good relative service characteristics, and propensity to serve large customers suggest the limited economic substitutability of other transport modes given the same shipment characteristics of barge, except when other modes can combine technology and cost effectiveness.
as is the case with pipelines and unit trains. If small users are partitioned out, only unit train technology and pipeline offers a substantial economic substitution.

What does this mean to the policy issue of waterway user charges and increasing congestion at certain points in the waterway system? This analysis shows that user charges are unlikely to displace much traffic from barge to rail in the short run. Sasaki showed little long run mode substitution in coal shipments by electrical utility users. Technological adjustment by the electrical utility industry is characterized by long lead times (10-20 years) and thermal plants have a normal useful life of 30 to 35 years. However, the location of coal mine development could shift toward areas served primarily by rail if barge rates rise and service deteriorates due to the waterway congestion. There are, however, countering trends. Rail rates for coal are increasing faster than rail rates for other commodities. Unit train operations are creating accelerated maintenance problems in track and roadbed. Utilities use their coal inventory policies as a hedge against potential strikes in the coal industry and generally prefer to keep part of their coal acquisition scattered

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4Sasaki, A Regional Model of the Future Demand for Transportation: The Case of Barge Transportation.
among small independent mines who normally use truck and rail. Therefore, utility users keep more transport technology alternatives open to them and maintain 3-4 months stockpiles of coal to deal with uncertainty in coal production and with potential service problems with transport modes.

Other commodity users have special conditions which influence their choice of mode and demand for barge transport. Chemical products are often corrosive and to some degree a safety hazard. Users frequently own specialized equipment to haul chemicals whether by rail, truck or barge. Substitution is limited by this technological specialization, especially in the short-run. Barge transport has a safety advantage since the adverse impacts of an accident can be mitigated by the dilution effects of water. This can backfire if water supply intakes are located near a spill site. However, in general, hazardous and corrosive chemicals are much less likely to cause health and safety problems if shipped by water.

The divergence of mode options between large and small users are significant in chemicals, petroleum products and to a lesser degree, users of iron and steel products. Large users can accommodate barge load quantities and use enough annually to justify barge and dock investments. Small users do not have this option unless
an intermediary can introduce a barge handling service. Most waterways induce the development of wholesaler/jobbers who can handle barge load quantities and pass along part of the economic advantage of water transportation to smaller users.

**Weakness of this Analysis.** Although there was no attempt to construct a theoretical and empirical analysis of the long-run economic demand for barge transportation, most significant investment and policy questions require an analysis of the long-run. If the commodity users of barge transportation are confronting constant cost and predictable parametric shifts in the demand function, and if the relative prices of barge and its competing modes are expected to be constant, a competent analysis of the short-run can satisfy a large share of the long-run policy issues. If these conditions are not anticipated, a substantial number of projection and analytical issues are raised. The earlier discussion of the coal using electrical utility suggest that a fair appraisal can be made from short-run behavior models since the technological state of the industry can change only very slowly. In addition, the uncertainties of determining the economic and environmental efficacy of nuclear power, the environmental policies of handling effluent requirements of coal fired plants and the risk involved from sole dependence
on softer energy producing technology suggest that an enlightened policy is to keep many options open. This entails substantial economic costs, and the uncertainties involved in the best available analytical capability cannot be reduced to provide an obvious limited set of choices with small risk.

The data acquired for this analysis would have been greatly enhanced if more data on service time and handling costs for individual shipments for each origin-destination pair had been obtained. More observations would have provided a basis for determining the variance around the expected service time and therefore would have allowed the explicit introduction of uncertainty and service time into the modal split model. Much of the rail data was obtained at a time when the New York Central-Pennsylvania system was operating at its worst. Thus relative service times obtained for rail may have been biased due to below normal performance.

Handling costs, defined as those costs entailed in excess of the transport rate to complete delivery and unloading, are difficult to estimate. With rare exceptions, handling costs are not identified in shipper or receiver cost accounting. Thus the persons interviewed would have to estimate the annual manpower and equipment costs involved in handling and divide by the tonnage involved to
approximate the handling cost estimate. The data from electrical utility users are considered to be better since they make a substantial investment in material handling to unload coal and normally have the costs identified in their capital accounts. Yet the material handling costs due to managing three to four month coal stockpiles are significant and certainly not solely attributable to the transport mode selected.

Transportation data seldom are obtained in a way to correlate it with economic trade flows. Although barge shipments are monitored in a complete annual census by the Corps of Engineers, rail shipment data are available only from a one percent way bill sample collected by the Interstate Commerce Commission and systematic national truck traffic is unregulated. There's no good way to know whether the data collected for this or any other study is consistent with actual trade flows. This problem is receiving more attention at the Department of Transportation and better overall data are becoming available. However, transportation data are greatest in number about physical rather than the economic flows.

The Need for Behavioral Models for Transportation Demand. Transport planners tend to think of the transportation as a physical network flow problem. This is because many are trained in engineering or geography.
In addition, the nation's transport system reflects a mixed public and private decision-making structure. Most transport models, even economic ones, tend to be designed under the rubric of a social optimum which places a single monopolistic decision-maker in control. As long as individual shippers control the decisions about how much to ship, by what route and mode, there is a substantial basis to develop the demand models to replicate the behavior of these independent decision-makers. Aggregate models tend to ignore or to place different weights on perceived costs of elements of transport demand than the individual decision-makers. Aggregate models tend to ignore or to place different weights on perceived costs of elements of transport demand than the individual decisions. The best example of this tendency is the propensity to use rates as a basic and frequently only element of a cost minimizing transport flow model. The consideration of nonpriced service elements, the tendency to handle technology as an independent factor, and the tendency to use aggregate cost minimizing models bias the actual behavior

5Howe et al., Inland Waterway Transportation: Studies in Public and Private Management and Investment Decisions, see Chapter 1.

of transport users. Demands, or requests from transportation users and other interested participants in public transport policy formulation, for a more realistic analysis can be facilitated by an emphasis on disaggregate behavioral models. The data needs are little more than would be required to calibrate more highly aggregated models, and there is a variety of analytical procedures available to calibrate the models.
BIBLIOGRAPHY


Wherry, G.S. "Multiple Bi-Serial and Multiple Point Bi-Serial Correlation." Psychometrica. Vol. 12 (1947).
APPENDIX A

DESCRIPTION OF DATA

This appendix displays and critiques the data used in this study. Basic data gathering was performed by a U.S. Army Engineer Division, Ohio team during the summers of 1970 and 1971 and a U.S. Army Engineer Division, Southwestern team during 1971. A total of 815 observations are available with complete data for the attributes of shipments and mode characteristics needed for the mode choice model:

- Quantities shipped per year in tons
- Miles of haul
- Time of transit (hour)
- Shipment size (ton)
- Transport rate (cents per ton)
- Handling costs (cents per ton)

The observations were coded for commodity groups by SIC codes at the 4 digit level. All analyses group the commodities at the 2 digit level as shown in Table A.1.
### TABLE A.1

**Commodity Groups by SIC and Number of Observations**

<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>SIC Code</th>
<th>No. of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>12</td>
<td>398</td>
</tr>
<tr>
<td>Crude Petroleum</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>Food (grains)</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Furniture</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>Paper</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>Chemicals</td>
<td>28</td>
<td>27</td>
</tr>
<tr>
<td>Refined Petroleum Products</td>
<td>29</td>
<td>177</td>
</tr>
<tr>
<td>Rubber and Plastic Products</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>Primary Metal Products</td>
<td>33</td>
<td>48</td>
</tr>
<tr>
<td>Fabricated Metal Products</td>
<td>34</td>
<td>64</td>
</tr>
<tr>
<td>Machinery, except electrical</td>
<td>35</td>
<td>5</td>
</tr>
<tr>
<td>Electrical Machinery</td>
<td>36</td>
<td>14</td>
</tr>
<tr>
<td>Transportation Equipment</td>
<td>37</td>
<td>1</td>
</tr>
<tr>
<td>Automotive Equipment</td>
<td>50</td>
<td>2</td>
</tr>
</tbody>
</table>

Table A.2 summarizes the data by commodity group, average tons per year and total tonnage per year by mode.

TABLE A.2

Summary of Sample Data
Tons Per Year by Commodity by Mode, 1970-1971

<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>Number of Observations</th>
<th>Average Tons Per Shipment (000)</th>
<th>Total Tons Per Year (000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>398</td>
<td>307,141</td>
<td>122,203</td>
</tr>
<tr>
<td>Coal</td>
<td>398</td>
<td>307,141</td>
<td>122,203</td>
</tr>
<tr>
<td>Rail</td>
<td>257</td>
<td>81,141</td>
<td>20,853</td>
</tr>
<tr>
<td>Barge</td>
<td>68</td>
<td>1,013,449</td>
<td>68,915</td>
</tr>
<tr>
<td>Truck</td>
<td>26</td>
<td>27,418</td>
<td>713</td>
</tr>
<tr>
<td>Rail-Barge</td>
<td>23</td>
<td>262,840</td>
<td>6,045</td>
</tr>
<tr>
<td>Truck-Barge</td>
<td>6</td>
<td>15,449</td>
<td>93</td>
</tr>
<tr>
<td>Truck-Rail-Barge</td>
<td>7</td>
<td>8,000</td>
<td>56</td>
</tr>
<tr>
<td>Unit Train</td>
<td>18</td>
<td>1,418,222</td>
<td>25,528</td>
</tr>
<tr>
<td>Crude Petroleum</td>
<td>16</td>
<td>702,712</td>
<td>11,246</td>
</tr>
<tr>
<td>Rail</td>
<td>1</td>
<td>72,800</td>
<td>73</td>
</tr>
<tr>
<td>Barge</td>
<td>10</td>
<td>466,098</td>
<td>4,661</td>
</tr>
<tr>
<td>Pipeline</td>
<td>2</td>
<td>295,887</td>
<td>592</td>
</tr>
<tr>
<td>Pipeline-Barge</td>
<td>3</td>
<td>1,972,610</td>
<td>5,918</td>
</tr>
<tr>
<td>Grain</td>
<td>22</td>
<td>6,315</td>
<td>135</td>
</tr>
<tr>
<td>Coal</td>
<td>398</td>
<td>307,141</td>
<td>122,203</td>
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<tr>
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<td>23</td>
<td>262,840</td>
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<td>6</td>
<td>15,449</td>
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<td>135</td>
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<tr>
<td>Commodity Group</td>
<td>Number of Observations</td>
<td>Average Tons Per Shipment Per Year</td>
<td>Total Tons Per Year (000)</td>
</tr>
<tr>
<td>---------------------------</td>
<td>------------------------</td>
<td>-----------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td><strong>Chemicals</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>9</td>
<td>5,742</td>
<td>52</td>
</tr>
<tr>
<td>Barge</td>
<td>13</td>
<td>92,425</td>
<td>1,202</td>
</tr>
<tr>
<td>Truck</td>
<td>5</td>
<td>2,200</td>
<td>11</td>
</tr>
<tr>
<td><strong>Petroleum Products</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>73</td>
<td>21,941</td>
<td>1,602</td>
</tr>
<tr>
<td>Barge</td>
<td>57</td>
<td>115,611</td>
<td>6,590</td>
</tr>
<tr>
<td>Truck</td>
<td>44</td>
<td>4,638</td>
<td>204</td>
</tr>
<tr>
<td>Rail-Barge</td>
<td>1</td>
<td>2,725</td>
<td>3</td>
</tr>
<tr>
<td>Pipeline</td>
<td>2</td>
<td>155,203</td>
<td>310</td>
</tr>
<tr>
<td><strong>Rubber and Plastic Products</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>3</td>
<td>863</td>
<td>3</td>
</tr>
<tr>
<td>Truck</td>
<td>9</td>
<td>1,803</td>
<td>16</td>
</tr>
<tr>
<td><strong>Primary Metals</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>27</td>
<td>59,202</td>
<td>1,598</td>
</tr>
<tr>
<td>Barge</td>
<td>6</td>
<td>30,167</td>
<td>181</td>
</tr>
<tr>
<td>Truck</td>
<td>15</td>
<td>4,496</td>
<td>67</td>
</tr>
<tr>
<td><strong>Fabricated Metals</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>25</td>
<td>1,563</td>
<td>39</td>
</tr>
<tr>
<td>Barge</td>
<td>7</td>
<td>22,421</td>
<td>137</td>
</tr>
<tr>
<td>Truck</td>
<td>20</td>
<td>1,727</td>
<td>35</td>
</tr>
<tr>
<td>Rail-Barge</td>
<td>1</td>
<td>2,000</td>
<td>2</td>
</tr>
<tr>
<td>Truck-Barge</td>
<td>11</td>
<td>647</td>
<td>7</td>
</tr>
<tr>
<td><strong>Machinery Except Electrical</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>2</td>
<td>1,742</td>
<td>3</td>
</tr>
<tr>
<td>Barge</td>
<td>1</td>
<td>3,996</td>
<td>4</td>
</tr>
<tr>
<td>Truck-Barge</td>
<td>1</td>
<td>500</td>
<td>1</td>
</tr>
</tbody>
</table>
TABLE A.2 - Continued

<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>Number of Observations</th>
<th>Average Tons Per Shipment Per Year</th>
<th>Total Tons Per Year (000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Machinery</td>
<td>14</td>
<td>8,851</td>
<td>126</td>
</tr>
<tr>
<td>Rail</td>
<td>1</td>
<td>15,000</td>
<td>15</td>
</tr>
<tr>
<td>Barge</td>
<td>6</td>
<td>11,833</td>
<td>71</td>
</tr>
<tr>
<td>Truck</td>
<td>5</td>
<td>782</td>
<td>6</td>
</tr>
<tr>
<td>Truck-Barge</td>
<td>2</td>
<td>17,000</td>
<td>34</td>
</tr>
<tr>
<td><strong>Transportation Equipment</strong></td>
<td><strong>1</strong></td>
<td><strong>5,000</strong></td>
<td><strong>5</strong></td>
</tr>
<tr>
<td>Rail</td>
<td>1</td>
<td>5,000</td>
<td>5</td>
</tr>
<tr>
<td><strong>Wholesale Trade (Automotive Equipment)</strong></td>
<td><strong>2</strong></td>
<td><strong>28,500</strong></td>
<td><strong>57</strong></td>
</tr>
<tr>
<td>Rail</td>
<td>2</td>
<td>28,500</td>
<td>57</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>815</strong></td>
<td><strong>145,963</strong></td>
<td></td>
</tr>
</tbody>
</table>


For 595 observations there is complete origin and destination coding. Table A.3 shows the number of observations by origin and destination for the data from the Ohio and Arkansas River areas. The origin and destination code is the designation of the Department of Commerce for functional economic areas (see Figure A.1). Table A.4 summarizes the total data set by mode and attributes of modal choice.
### TABLE A.3

Summary by Origin and Distribution
By Destinations With Greater Than 10 Shipments

<table>
<thead>
<tr>
<th>Destinations</th>
<th>No. of Origins</th>
<th>No. Shipments</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>12</td>
<td>48</td>
</tr>
<tr>
<td>56</td>
<td>11</td>
<td>25</td>
</tr>
<tr>
<td>60</td>
<td>14</td>
<td>50</td>
</tr>
<tr>
<td>61</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>62</td>
<td>18</td>
<td>64</td>
</tr>
<tr>
<td>64</td>
<td>16</td>
<td>34</td>
</tr>
<tr>
<td>66</td>
<td>8</td>
<td>42</td>
</tr>
<tr>
<td>115</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>117</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>118</td>
<td>30</td>
<td>54</td>
</tr>
<tr>
<td>119</td>
<td>21</td>
<td>61</td>
</tr>
</tbody>
</table>
### TABLE A.4

Summary of Observations by Mode and Modal Choice Variables

All Data, N = 815

<table>
<thead>
<tr>
<th>Mode</th>
<th>Average Tons/Year</th>
<th>Total Tons/Year (Millions)</th>
<th>Miles in Transit</th>
<th>Time of Transit (Hours)</th>
<th>Average Haul (Tons)</th>
<th>Rate Cents</th>
<th>Handling Costs (Cents)</th>
<th>No. of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>57,027</td>
<td>24.4</td>
<td>343</td>
<td>670</td>
<td>724</td>
<td>1,012</td>
<td>69</td>
<td>428</td>
</tr>
<tr>
<td>Barge</td>
<td>472,973</td>
<td>81.8</td>
<td>671</td>
<td>248</td>
<td>3,754</td>
<td>275</td>
<td>56</td>
<td>173</td>
</tr>
<tr>
<td>Truck</td>
<td>7,497</td>
<td>1.1</td>
<td>248</td>
<td>23</td>
<td>45</td>
<td>1,271</td>
<td>123</td>
<td>144</td>
</tr>
<tr>
<td>Rail-Barge</td>
<td>242,002</td>
<td>6.1</td>
<td>617</td>
<td>217</td>
<td>2,204</td>
<td>408</td>
<td>45</td>
<td>25</td>
</tr>
<tr>
<td>Truck-Barge</td>
<td>4,532</td>
<td>0.07</td>
<td>783</td>
<td>316</td>
<td>687</td>
<td>789</td>
<td>118</td>
<td>16</td>
</tr>
<tr>
<td>Rail-Truck-Barge</td>
<td>8,000</td>
<td>0.03</td>
<td>189</td>
<td>240</td>
<td>20</td>
<td>639</td>
<td>35</td>
<td>4</td>
</tr>
<tr>
<td>Unit Train</td>
<td>1,418,222</td>
<td>25.2</td>
<td>117</td>
<td>46</td>
<td>3,417</td>
<td>179</td>
<td>21</td>
<td>18</td>
</tr>
<tr>
<td>Pipeline</td>
<td>225,545</td>
<td>0.9</td>
<td>162</td>
<td>109</td>
<td>2,354</td>
<td>120</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Pipeline-Barge</td>
<td>1,972,610</td>
<td>5.9</td>
<td>1,048</td>
<td>536</td>
<td>12,000</td>
<td>202</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>
Table A.5 shows cost per ton mile and speed for the entire sample by mode. Speed was computed by dividing miles of transit by hours in transit.

Since the form of the modal split and demand model is sensitive to the distributional characteristics of the variables, these attributes were evaluated for each variable. In particular, skewness and kurtosis show how well the data fit the properties of a normal distribution. In Table A.6 and Figure A.2 the natural data shows substantial variation from desired properties especially for annual tonnage and rate. The values for data transformed into natural logarithms and into the exponential form are also shown. The exponential transform uniformly exaggerates the variations whereas the logarithmic transform provides almost perfect properties of a normal distribution. The logarithmic transform performs much better in discriminant and regression analyses.
FIGURE A.2
COMPARISON OF SKEWNESS & KURTOSIS VALUES OF TRANSFORMED VARIABLES
<table>
<thead>
<tr>
<th>Mode</th>
<th>N</th>
<th>Cost Per Ton Mile (Cents)</th>
<th>Speed (mph)</th>
<th>Length of Haul (Miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>427</td>
<td>3.07</td>
<td>2.47</td>
<td>343</td>
</tr>
<tr>
<td>Barge</td>
<td>171</td>
<td>.50</td>
<td>3.98</td>
<td>673</td>
</tr>
<tr>
<td>Truck</td>
<td>145</td>
<td>5.31</td>
<td>9.63</td>
<td>248</td>
</tr>
<tr>
<td>Rail-Barge</td>
<td>25</td>
<td>.72</td>
<td>2.84</td>
<td>617</td>
</tr>
<tr>
<td>Truck-Barge</td>
<td>16</td>
<td>1.16</td>
<td>2.49</td>
<td>783</td>
</tr>
<tr>
<td>Rail-Truck-Barge</td>
<td>4</td>
<td>3.57</td>
<td>.78</td>
<td>189</td>
</tr>
<tr>
<td>Unit Train</td>
<td>18</td>
<td>1.71</td>
<td>2.56</td>
<td>117</td>
</tr>
<tr>
<td>Pipeline</td>
<td>4</td>
<td>.79</td>
<td>1.49</td>
<td>162</td>
</tr>
<tr>
<td>Pipeline-Barge</td>
<td>3</td>
<td>.21</td>
<td>1.95</td>
<td>1,048</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tons per year</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>natural</td>
<td>4178.913</td>
<td>567.333</td>
<td>5.685</td>
<td>42.276</td>
</tr>
<tr>
<td>transformed-ln</td>
<td>2.460</td>
<td>2.610</td>
<td>-.094</td>
<td>-1.175</td>
</tr>
<tr>
<td>transformed exp*</td>
<td>1.020</td>
<td>.069</td>
<td>6.995</td>
<td>68.859</td>
</tr>
<tr>
<td><strong>Distance in miles from origin to destination</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>natural</td>
<td>409</td>
<td>456</td>
<td>1.804</td>
<td>2.911</td>
</tr>
<tr>
<td>transformed-ln</td>
<td>5.387</td>
<td>1.265</td>
<td>-.786</td>
<td>1.472</td>
</tr>
<tr>
<td>transformed exp*</td>
<td>1.725</td>
<td>1.25</td>
<td>3.612</td>
<td>16.93</td>
</tr>
<tr>
<td><strong>Transmit time in hours</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>natural</td>
<td>133</td>
<td>121</td>
<td>1.553</td>
<td>3.281</td>
</tr>
<tr>
<td>transformed-ln</td>
<td>4.224</td>
<td>1.508</td>
<td>-.255</td>
<td>.913</td>
</tr>
<tr>
<td>transformed exp*</td>
<td>1.151</td>
<td>.155</td>
<td>2.153</td>
<td>6.729</td>
</tr>
<tr>
<td><strong>Shipment size in tons</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>natural</td>
<td>1397</td>
<td>2064</td>
<td>3.439</td>
<td>13.914</td>
</tr>
<tr>
<td>transformed-ln</td>
<td>5.442</td>
<td>2.072</td>
<td>-.974</td>
<td>.184</td>
</tr>
<tr>
<td>transformed exp*</td>
<td>1.014</td>
<td>.031</td>
<td>3.625</td>
<td>15.818</td>
</tr>
<tr>
<td><strong>Rate of selected mode in cents per ton</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>natural</td>
<td>851</td>
<td>2116</td>
<td>11.613</td>
<td>170.834</td>
</tr>
<tr>
<td>transformed-ln</td>
<td>6.018</td>
<td>1.158</td>
<td>-.124</td>
<td>1.320</td>
</tr>
<tr>
<td>transformed exp*</td>
<td>1.170</td>
<td>1.058</td>
<td>21.766</td>
<td>478.317</td>
</tr>
<tr>
<td><strong>Handling costs in cents per ton</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>natural</td>
<td>74</td>
<td>103</td>
<td>3.356</td>
<td>13.276</td>
</tr>
<tr>
<td>transformed-ln</td>
<td>3.614</td>
<td>1.323</td>
<td>-.816</td>
<td>1.357</td>
</tr>
<tr>
<td>transformed exp*</td>
<td>12.62</td>
<td>06.170</td>
<td>15.005</td>
<td>292.310</td>
</tr>
</tbody>
</table>

*All values were scaled to maintain range of transform between -1.76 and 1.76*