FATIGUE PREDICTION USING STANDARDISED LOADING SEQUENCE DATA (U)

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FATIGUE PREDICTION USING
STANDARDISED LOADING SEQUENCE DATA

by

G. S. JOST

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SUMMARY

A new proposal for predicting pre-crack fatigue lives under service loading sequences is given. It is characterised by the replacement of fatigue data obtained under constant amplitude loading by that obtained under appropriate variable amplitude loading sequences.
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16. ABSTRACT
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NOTATION

$i$
load level index, standard spectrum

$j$
load level index, related spectrum

$l$
number of fatigue cycle levels, standard spectrum

$m$
number of fatigue cycle levels, related spectrum

$n$
number of cycles

$n_i$
number of pre-crack cycles at $i$th level, standard spectrum

$n_r$
number of pre-crack cycles at $j$th level, related spectrum

$r$
subscript, related spectrum

$s$
subscript, standard spectrum

$D$
(Miner) pre-crack fatigue damage

$N$
number of cycles to cracking (notional)

$N_i$
number of cycles to cracking at $i$th level, standard spectrum

$N_r$
number of cycles to cracking at $j$th level, related spectrum

$S_r$
characteristic stress, related spectrum

$S_i$
characteristic stress, standard spectrum

$a_i$
ratio of number of cycles at $i$th level to those at level $i = 1$, standard spectrum

$a_r$
ratio of number of cycles at $j$th level to those at level $j = 1$, related spectrum

$\beta_i$
ratio of notional cycles to cracking at $i$th level to those at level $i = 1$, standard spectrum

$\beta_r$
ratio of notional cycles to cracking at $j$th level to those at level $j = 1$, related spectrum
1. BACKGROUND

Attempts at predicting, with confidence, the fatigue lives of items undergoing typical service loadings have had a long and unsatisfactory history. Even today, it is rarely possible to predict, with acceptable accuracy, the fatigue life of a given component, specimen or structure subjected to a specified loading history (trivial examples excepted). The recent competition advertised internationally by the Engineering Sciences Data Unit in which life predictions were sought for a notched specimen under given loading sequences highlights the fact that real difficulties still plague predictive fatigue.

Success in any predictive situation requires two basic ingredients: relevant and appropriate fundamental or basic data, and an acceptable (and usually mathematical) model of the process with which to use it. Given the basic data and the model, reliable predictions may be expected only if both are appropriately adequate. Although there is, at least in principle, unlimited scope for reworking the mathematical model, the same cannot be said for the basic data. If this is fundamentally deficient, then nothing is likely to remedy the deficiency.

These two elements appear in all attempts to improve predicted fatigue life. Typically, basic fatigue data are represented by those obtained under constant amplitude loading, and the model of fatigue damage accumulation is represented by the Palmgren-Miner hypothesis or a variant. Thus, when a predicted life fails to agree with that established experimentally, it is clear that either the basic data, or the damage accumulation model (including the manner in which the data are used in the model), or both, must be inadequate.

The present proposal has arisen out of difficulties associated with fatigue life prediction in aircraft structures. In this field the (highly) variable amplitude service loading sequences are typically quasi-random, quasi-deterministic flight-by-flight in nature. Several characteristically different flights may be identifiable, the numbers of fatigue cycles within a given flight ranging from some tens to several hundreds. Such sequences might be considered very far removed from constant amplitude sequences.

In this report fatigue life refers to the pre-crack stage, i.e. the number of cycles (or other appropriate units) to cause the initiation of a fatigue crack. Once this point is reached there are, of course, appropriate techniques for dealing with the crack growth phase, although they are not without their difficulties.

2. ACCUMULATION OF FATIGUE DAMAGE

Miner proposed that, for the pre-crack stage, a linear accumulation of damage be assumed. His restriction to the pre-crack regime has not, however, prevented others from extending the notion to include the crack growth regime as well. Discrepancies arising from the use of this rule in conjunction with constant amplitude fatigue data have led to much research aimed at modifying Miner’s rule and or developing alternatives. Excepting perhaps for an appreciation of the significance of residual stresses, little headway has been made in improving the precision of these predictions.

The application of fracture mechanics to fatigue crack growth has focused attention on the influence and extreme significance of load interaction or sequence effects on the rate of crack growth. Because of these effects, crack growth predictions based upon the simple accumulation of crack growth increments on a cycle-by-cycle basis, making use of constant amplitude crack growth data, fail to tally with reality for variable amplitude sequences. Numerous crack growth models have been proposed to account for interaction effects, with mixed success.

It is not unreasonable to suspect that cycle interaction and or analogous effects may operate also in the pre-crack or crack initiation stage. Should the magnitude of their influence even
distantly approach that of those in the crack growth stage, then it follows that the combination of a cumulative damage rule and data based on constant amplitude loading is inappropriate and is also unlikely to result in consistently accurate predictions of the life to crack initiation. But it is possible that such a cumulative damage rule combined with more appropriate fatigue data might lead to more reliable life estimates.

3. BASIC FATIGUE DATA

Constant amplitude fatigue data and those described by the above heading have become, for practical purposes, almost synonymous. Yet the foregoing indicates that there are shortcomings of real consequence in them and their applications to variable amplitude sequences. It is suggested that the basic fatigue data to be used predictively must already include sequence and other significant effects whether or not these effects are understood, quantified, or, for that matter, even identified. At the present time, the fulfilment of this requirement may be approached only by the adoption of realistic loading sequence fatigue data in place of constant amplitude data. This leads immediately to the concept of the generation of fatigue data under standard spectra, each appropriate to a given type or class of loading action. The recently announced TWIST and FALSTAFF sequences, which represent the type of loading history occurring in the lower wing root region of civil and military aircraft, are perhaps the first examples of such specific standard spectra.

Having obtained appropriate loading sequence fatigue data under a standard spectrum, the problem remains as to how to use such data in predicting fatigue life under other similar or "related" spectra. One proposal is outlined in Section 5.

4. FATIGUE TESTS UNDER REALISTIC LOADING SEQUENCES

Realistic fatigue tests may now be carried out to any desired degree of sophistication. Service sequence tests usually aim to establish, amongst other things, crack growth characteristics and of fatigue life, from which inspection intervals and or safe lives may be estimated. Such tests are necessary, of course, only because predicted crack growth rates and fatigue lives under specified sequences cannot be relied upon.

The problem of transposing a fatigue life established by test under one spectrum to its equivalent under some other related spectrum is often solved by the relative Miner method. This operates as follows:

(a) For the test spectrum, say using constant amplitude $S-N$ data, the Miner damage is calculated. Thus the known fatigue life $(\sum n)$, say has associated with it a calculated fatigue damage of $(\sum (nN))_1$.

(b) The Miner damage for the related spectrum $2, (\sum (nN))_2$ is also calculated.

(c) Assuming that for both spectra the total Miner damage at failure is the same

$$(\sum n) = (\sum (nN))_1 = (\sum (nN))_2$$

and

$$(\sum n) = (\sum n) (\sum (nN))_1 (\sum (nN))_2$$

Provided the two service spectra do not differ significantly, some confidence is usually placed in the new life estimate. Certainly its basis is better founded than that from a direct Miner calculation in which $\sum n N$ is equated to unity at failure. Unfortunately, however since its application does not lead invariably to satisfactory results, it cannot always be relied upon.

Another application of the relative Miner rule, which uses fatigue data obtained under the original test spectrum and omits reference to constant amplitude fatigue data altogether, is discussed in the next Section.

5. PROPOSAL

For the purpose of estimating pre-crack fatigue lives under typical service loadings or spectra it is proposed that, as basic fatigue data, constant amplitude data be replaced with those from...
appropriate standardised loading sequence fatigue tests. These data would then be used, at least initially, in Miner type calculations to predict lives to crack initiation under related spectra, i.e. under those deemed to be of the same general character or class as the source data spectrum.

The proposal provides nothing new by way of assessing the accumulation of fatigue damage; however the substitution for constant amplitude data of more realistic variable amplitude data is seen as crucial to the possibility of a significant improvement in Miner type fatigue life prediction. In principle, the proposal would work along the following lines:

(a) Establish the (set of) characteristic stress versus pre-crack cycles curve(s) under the standard spectrum, Fig. 1. It may be useful here to think in terms of a sequence in which the characteristic stress $S$, is designated as the maximum occurring in the sequence. Appropriate stress scaling about, for example, a given $lg$ level, and testing, will provide the required fatigue data for various stress levels. Corresponding data for different $lg$ levels will clearly require additional testing and may be required for full implementation of the proposal. The important point is that graphs such as Fig. 1 represent actual data obtained under the standard spectrum according to the magnitude of its characteristic stress.

(b) For the standard spectrum (subscript $s$) at any specified characteristic stress, $S$, say, a Miner damage model is set up.

$$D_s = \sum_{i=1}^{m} (n_i, N_s)$$

at the pre-crack life. The $n_i$ here are the numbers of cycles occurring at each of the $l$ stress levels in the standard sequence, and the $N_s$ are, by analogy with those in a conventional Miner sum, the pre-crack cycles at corresponding stress levels, but these cycles must be regarded as notional only. They cannot be evaluated directly by test, since that would require a change in test sequence and hence the validity of their means of estimation. They must remain (at this stage) simply the unspecified dividers in the Miner damage equation (1).

(c) For the related spectrum (subscript $r$) at characteristic stress level, $S$, say, for which the life estimate is sought, the corresponding damage equation will be given by

$$D_r = \sum_{i=1}^{m} (n_i, N_r)$$

Assuming that $D_r$ at the onset of cracking gives

$$D_r = \sum_{i=1}^{m} (n_i, N_r)$$

(d) For both standard and related spectra the relative numbers of cycles at each of the $l$ and $m$ stress levels respectively are presumed known. For the standard spectrum the total number of pre-crack cycles is

$$\sum_{i=1}^{l} n_i, n_1, n_2, \ldots, n_l.$$

By expressing all cycles in terms of one of these, $n_1$, say, the above may be expressed as

$$\sum_{i=1}^{l} n_i (1, n_1, n_2, \ldots, n_l)$$

$$\sum_{i=1}^{l} n_i (1, n_2, \ldots, n_l)$$

$$n_1, \sum_{i=1}^{l} n_i, \frac{1}{n_1}$$

* Such a suggestion, in relation to narrow-band random loading, was made by Kirkby and Edwards. 10
where \( a_n = n_i / n_{1r} \).

Similarly, for the related spectrum

\[
\sum_{i=1}^{m} \eta_i = n_{1r} \sum_{i=1}^{m} x_{i1r} \beta_{1r} = 1
\]

(5)

where

\( x_{i1r} = \eta_i / n_{1r} \).

The corresponding notional \( N_i \) and \( N_{1r} \) may be analogously expressed:

\[
\sum_{i=1}^{m} N_i = N_{1r} \sum_{i=1}^{m} \beta_{1r} = 1
\]

(6)

and

\[
\sum_{i=1}^{m} N_{1r} = N_{1r} = \sum_{i=1}^{m} \beta_{1r} = 1
\]

(7)

Substituting from (4), (5), (6) and (7) in (3) and rearranging gives

\[
n_{1r} = N_{1r} \sum_{i=1}^{m} \beta_{1r} \beta_{1r}
\]

(8)

and resubstitution for \( n_{1r} \) and \( n_i \) from (4) and (5) yields the required ratio of the fatigue life under the related spectrum to that under the standard

\[
\sum_{i=1}^{m} \eta_i = N_{1r} \sum_{i=1}^{m} \beta_{1r} \beta_{1r} \beta_{1r} \frac{S_i}{S_i}
\]

(9)

Further progress requires some tightening of relationships and definitions. First, in any practical application, the stress characterising the related spectrum, \( S_{1r} \), will be equal to that for the standard, \( S_i \); it would seem unreasonable to deliberately choose to estimate fatigue life under the related spectrum from that under the standard at a different \( S_{1r} \). Thus \( S_{1r} = S_i \), in practice. Secondly, the \( N_i \) and \( N_{1r} \) above are expressed in terms of their values \( N_{1r} \) and \( N_{1r} \) each at one particular stress level. No loss of generality ensues from assigning this level to the characteristic stress level of the spectrum. Thus, when \( S_i \) and \( S_{1r} \) are equal, the notional \( N_{1r} \) and \( N_{1r} \) will also be equal, and (8) becomes

\[
\sum_{i=1}^{m} \eta_i = \sum_{i=1}^{m} \frac{a_i}{n_{1r}} x_{i1r} \beta_{1r} \frac{S_i}{S_i}
\]

(9)

Some simple examples will serve to clarify the application of (8) and (9).

5.1 Examples

5.1.1 Related spectrum is identical to the standard, at the same characteristic stress level, \( S_i = S_{1r} \). In this (trivial) example \( l = m \), and for all \( i \neq j \), \( x_{i1r} = x_{i1r} \) and \( \beta_{1r} = \beta_{1r} \). Thus all corresponding numerator and denominator terms in (9) are identical and \( \sum \eta_i = \sum \eta_{1r} \) as required.

5.1.2 Related spectrum is identical to the standard, but at a different characteristic stress level, \( S_{1r} \neq S_i \). In this case only the \( x \) summations are identical, and (8) simplifies to

\[
\sum_{i=1}^{m} \eta_i = \frac{\sum_{i=1}^{m} x_{i1r} \beta_{1r} \frac{S_i}{S_i}} {\sum_{i=1}^{m} x_{i1r} \beta_{1r} \frac{S_i}{S_i}}
\]

This reduces to the above when \( S_{1r} = S_i \).

* Quantification of the \( N_i \) and \( N_{1r} \) is considered in Section 5.2.
5.1.3

Related spectrum is constant amplitude loading at level \( j \), \( S_j = S_j \).

Here \( m = 1 \), so that
\[
\begin{align*}
\sum a_p & = a_{1r} = 1, \\
\sum a_p/\beta_p & = a_{1r}/\beta_{1r} = 1, \\
\sum n_p & = n_{1r},
\end{align*}
\]

and (8) gives
\[
n_{1r} \sum n_n = (N_{1r} N_1)/[\sum (a_p/\beta_p)] \sum a_n.
\]

When \( S_j = S_j \), \( N_{1r} = N_1 \), and the above simplifies to
\[
n_{1r} \sum n_n = [\sum (a_p/\beta_p)] \sum a_n.
\]

The same result follows directly from (9).

Finally, for the limiting case of the multi-load level standard spectrum degenerating to the same single \( j \)th load level as the (constant amplitude) related spectrum, i.e. for the standard spectrum becoming a constant amplitude one
\[
\begin{align*}
I - 1 \\
\sum x_n & = x_{1r}, 1 \\
\sum (x_n/\beta_n) & = x_{1r}/\beta_{1r}, 1 \\
\sum n_n & = n_{1r},
\end{align*}
\]

and the above gives
\[
n_{1r} = n_{1r},
\]
as required.

5.2 Evaluation of \( \beta_i \) and \( \beta_j \)

An appropriate quantification of \( \beta_i \) and \( \beta_j \) is indicated by application 5.1.2; predicted lives at different characteristic stresses when both related and standard spectra are otherwise identical must clearly agree with the test data, Fig. 1.

Consider the equation at 5.1.2.
\[
\begin{align*}
\sum n_p & = N_{1r} \sum a_p/\beta_p \\
\sum n_n & = N_1 \sum a_p/\beta_p
\end{align*}
\]

Suppose now that \( \beta_n = \beta_n \), i.e. \( N_n N_1 = N_p N_{1r} \). This is a possibility, at least over a limited range of the variables, and follows from a linear log stress - log notional \( N \) relationship.

In that case, and since \( a_n = a_p \) here, the \( \sigma \beta \) summation terms above cancel, and there remains
\[
\sum n_p/\sum n_n = N_{1r} N_1.
\]

This equation is satisfied automatically for all \( i, j \) combinations by adopting the relationship of Fig. 1 as corresponding to the cycle stress - notional \( N \) relationship for both standard and related spectra, Fig. 2. According to this proposal
\[
\sum n_n \propto N_1,
\]
and
\[
\sum n_n \propto N_1,
\]
here, and in general
\[
\sum n_n \propto N_n.
\]
Thus

\[ \sum n_i \propto N_i. \]

\[ \beta_i = N_i, N_i, \sum n_i \sum n_i, \] (10a)

\[ \beta_i = N_i, N_i, \sum n_i \sum n_i, \] (10b)

The proposal amounts to assigning to the characteristic stress and cycles to failure scales of Fig. 1 cycle stress and corresponding notational cycles to failure, Fig. 2. The notional \( N \) and \( N_i \) then become the dummy variables in the fatigue life calculation. Because they appear in equations (8), (9) and (10) only as ratios, i.e. as \( N_i, N_i, \beta_i, \) and \( \beta_i \), their actual magnitudes are of no consequence. The ratios combine to form, in (8) and (9), the weighting factors to modify the known fatigue life under the standard spectrum to that under the related spectrum.

5.3 Numerical Examples

Fig. 3. shows a linear log stress - log life relationship plotted on conventional semi-log axes. Suppose that it had arisen from tests using a stress-scaled standard spectrum of five levels having the characteristics listed in Table 1. The spectrum is “typical” in that smaller loads occur more frequently than do larger loads. For all examples here the characteristic stress is chosen at level \( i = 1 \), i.e. the maximum occurring in the sequence. Fig. 3. shows that when the characteristic stress of the spectrum is, for example, 40 units, the fatigue life is 6 cycle units:

\[ \sum S_{i,0} = 20, \sum n_i = 30. \]

Given the fatigue data of Fig. 3 under the standard spectrum of Table 1, it is desired to calculate fatigue life under related spectra. These may differ from the standard in two basic ways: the number of cycles at each stress level may differ, or the stresses themselves may differ in level and or number. An example of each follows:

Example 1. Suppose first that the related spectrum differs from the standard only in its cycle ratios, Column 4. Table 2(a). Thus, the \( x_i \) and \( x_i \) are given, and the \( \beta_i \) (which are identical to the \( \beta_i \) since the cycle stress levels are common to both spectra) are calculated from (10a) and (10b) and Fig. 3. The full procedure is shown in Table 2(a), and the outcome is that

\[ \sum n_i \sum n_i \propto 0.97 \]

If the fatigue cycles applied are thought of in terms of groups or blocks (of 31 standard cycles and 37 related cycles in this example), then

related blocks standard blocks \( (\sum x_i \beta_i) (\sum x_i \beta_i) \) 0.81.

Example 2. In the second example, the number of stress levels and their magnitudes are changed, Table 2(b). For this case

\[ \sum n_i \sum n_i \propto 0.81 \]

and

related blocks standard blocks 2.28

These examples serve to illustrate the application of the proposal in practice. The sense of the predictions seem reasonable: in the first example, the number of intermediate load level occurrences is increased a little, compared with the standard, and relatively fewer blocks to pre-crack life are predicted. For the second case, omission of some load cycles per block results in a greater predicted number of pre-crack blocks. The efficacy or otherwise of such predictions can, of course, be judged only on the basis of appropriate test data.

* The combination of a stress scaled spectrum and a linear log stress - log life relationship provides the satisfying result that \( \sum n_i \sum n_i \) is independent of the characteristic stress level chosen as the basis for performing the calculations.
6. DISCUSSION

It has been argued above that, for life prediction purposes, constant amplitude data might with profit be replaced by appropriate (standardised) fatigue data. These could then be used, initially at least, in Miner calculations for predicting fatigue lives under similar, or related, spectra. Since the basic fatigue data result from realistic testing, any discrepancies between predicted and actual fatigue lives under related spectra would then be forcefully attributable to shortcomings in either the Miner rule or the manner in which it is applied. The proposal still leaves open, for example, the question of the specification of fatigue cycles. Clearly, the sensitivity of actual and predicted lives to cycle definition will need to be an early area for investigation.

Although the present suggestion has been made on the basis of pre-crack life, it would be tempting, when the data are available, to apply the procedure to final fatigue lives as well. Further, given fracture surfaces for which striation counting between repeating identifiable events is possible, crack growth rate data under standard and related spectra should indicate whether data obtained under the standard spectrum better predict crack growth rates, and hence crack growth lives, for related spectra than do constant amplitude data.

It is considered that the proposal is likely to given an accuracy at least equivalent to that of current procedures and it may prove significantly better.

Tests are being planned to make a first practical evaluation of the proposal using a standardized loading sequence and others which might be deemed, on physical and statistical grounds, to be related to it. The FALSTAFF sequence is to be adopted as the standard in these tests.

7. CONCLUSION

A proposal for estimating pre-crack fatigue lives has been made in which standardised fatigue loading data become the basic or fundamental data to be used in place of constant amplitude fatigue data. It is argued that the adoption of representative data may well result in better fatigue predictions; the hypothesis is to be tested by experiment.
REFERENCES

1. Engineering Sciences Data Unit. How long will this last? Leaflet on a fatigue competition in conjunction with the Society of Environmental Engineers, 1976.


4. Edwards, P. R. Cumulative Damage in Fatigue with Particular Reference to the Effects of Residual Stress. RAE TN 69237, November 1969.


**TABLE 1**

Standard Spectrum and Derived Data for $S_{char} = 40$

Characteristic Stress Level: 1

<table>
<thead>
<tr>
<th>Stress Level $i$</th>
<th>Stress Ratio $S_i/S_{char}$</th>
<th>Cycle Ratio $a_i$</th>
<th>Stress $S_i$</th>
<th>$N_{ci}$</th>
<th>$\beta_{ci}$</th>
<th>$\gamma_i$</th>
<th>$\beta_{ci}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>40</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>2</td>
<td>36</td>
<td>7.66</td>
<td>1.28</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>4</td>
<td>32</td>
<td>10.08</td>
<td>1.68</td>
<td>2.38</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>8</td>
<td>28</td>
<td>13.76</td>
<td>2.29</td>
<td>3.49</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>16</td>
<td>24</td>
<td>19.71</td>
<td>3.29</td>
<td>4.87</td>
<td></td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td></td>
<td></td>
<td><strong>31</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>13.31</strong></td>
</tr>
</tbody>
</table>

(1): given
(2): given
(3): given
(4): (2) $\cdot S_{char}$
(5): Data from Fig. 3 using (4)
(6): $\beta_{ci} = N_{ci}/N_1$ (from (5))
TABLE 2
Calculation of Predicted Lives

(a) First Example: $S_{char} = 40$

<table>
<thead>
<tr>
<th>$j$</th>
<th>$S_j/S_{char}$</th>
<th>$\sigma_{pr}$</th>
<th>$S_j$</th>
<th>$N_{pr}$</th>
<th>$\beta_{pr}$</th>
<th>$\sigma_{pr}$</th>
<th>$\beta_{pr}$</th>
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</thead>
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<td>1</td>
<td>40</td>
<td>6</td>
<td>1</td>
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<td>7.66</td>
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<td>2.34</td>
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<td>1.68</td>
<td>2.98</td>
<td></td>
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<td>13.76</td>
<td>2.29</td>
<td>5.24</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>16</td>
<td>24</td>
<td>19.71</td>
<td>3.29</td>
<td>4.86</td>
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<tr>
<td>TOTALS</td>
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<td>37</td>
<td></td>
<td></td>
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</tbody>
</table>

Eqn. (9) $\sum n_{pr}\cdot \sum n_r = (37.31)(13.31/16.42) = 0.97$

(standard data from Table 1)

or related blocks/standard blocks $= 13.31/16.43 = 0.81$

(b) Second Example: $S_{char} = 40$

<table>
<thead>
<tr>
<th>$j$</th>
<th>$S_j/S_{char}$</th>
<th>$\sigma_{pr}$</th>
<th>$S_j$</th>
<th>$N_{pr}$</th>
<th>$\beta_{pr}$</th>
<th>$\sigma_{pr}$</th>
<th>$\beta_{pr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1</td>
<td>1</td>
<td>40</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>3</td>
<td>32</td>
<td>10.08</td>
<td>1.68</td>
<td>1.79</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>7</td>
<td>28</td>
<td>13.76</td>
<td>2.29</td>
<td>3.06</td>
<td></td>
</tr>
<tr>
<td>TOTALS</td>
<td></td>
<td></td>
<td>11</td>
<td></td>
<td></td>
<td>5.85</td>
<td></td>
</tr>
</tbody>
</table>

Eqn. (9) $\sum n_{pr}\cdot \sum n_r = (11.31)(13.31/5.85) = 0.81$

(standard data from Table 1)

or related blocks/standard blocks $= 13.31/5.85 = 2.28$

(1), (2), (3): given

(4): $2 \cdot S_{char}$

(5): Data from Fig. 3 using (4)

(6): $\beta_{pr} = N_{pr}/N_r$ (from (5))
Characteristic stress

\[ S_S \]

Total cycles to crack initiation

\[ \Sigma \eta \]

FIG. 1  FATIGUE DATA FOR STANDARD SPECTRUM

Identical curve to that of Fig 1.

Notional cycles to crack initiation, \( N_p, N_y \)

FIG. 2  NOTIONAL FATIGUE DATA
FIG. 3  FATIGUE DATA FOR EXAMPLES
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