RESEARCH ON THE INVERSE SCATTERING PROBLEM FOR THE THREE-DIMENS--ETC(U)

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RESEARCH ON THE INVERSE SCATTERING PROBLEM
FOR THE
THREE-DIMENSIONAL SCHROEDINGER EQUATION.

Final Report.

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Harry E. Moses

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**Title:** Research on the Inverse Scattering Problem for the Three-Dimensional Schrödinger Equation

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**Key Words:**
Inverse problem of scattering
Inverse spectral theory for the three-dimensional problem
Gelfand-Levitan equation

**Abstract:**
The work performed under the grant is summarized.
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1. Personnel Supported by the Grant

1. H. E. Moses, Principal Investigator.

2. C. Kaiser, Assistant Professor of Mathematics, University of Lowell, a nominal amount of support (less than $4,000).

3. J. M. Cohen, Associate Professor of Physics, University of Pennsylvania, a nominal amount of support (less than $5,000).


The Principal Investigator's principal research direction has been to find appropriate triangularity conditions for the Gelfand-Levitan kernel in three-dimensions. The Gelfand-Levitan kernel for the one-dimensional problem has served as a model.

In this program we have proposed candidate for the Gelfand-Levitan kernel which we hope will give rise to local scattering potentials. A Jost wave function with appropriate completeness properties was given. Green's functions of a Jost type were also given.

We then broadened our attention to the consideration of non-local potentials in three dimensions. By introducing appropriate triangularity properties, we were able to obtain explicit non-local potentials for which the Schroedinger equation could be solved for exactly. A rich spectral theory results from the use of such potentials. Some have very unusual scattering properties, namely they do not scatter at all and thus could not be detected by a scattering experiment. These non-local potentials are the first potentials, non-local or local, which led to simple closed form solutions of the Schroedinger equations (energy-dependent potentials used by earlier workers are not really potentials, strictly speaking).
To understand the nature of triangularity better we considered the one-dimensional analogue of the non-local three-dimensional potentials. The potentials so obtained are the most general parity-dependent potentials and have very interesting scattering properties. These potentials may have technical applications. We are continuing to look into this matter. It was possible to show that a local and a non-local potential could give rise to the same scattering operator and point spectrum.

3. Papers Published or Accepted for Publication


4. "An Explicit Example of a Local and a Non-Local Potential whose Hamiltonians are Unitarily Equivalent and whose Scattering Operators are Identical," accepted for publication by Studies in Applied Mathematics (with P. B. Abraham and B. DeFacio).

4. Papers which are Being Prepared for Submission

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