COMPUTER SIMULATION OF SOLAR AIR HEATING SYSTEMS USING ROCK BED-EXCHANGERS
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COMPUTER SIMULATION OF SOLAR
AIR HEATING SYSTEMS USING
ROCK BED THERMAL STORAGE UNITS

Thesis

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USING ROCK BED THERMAL STORAGE UNITS

THESIS

Presented to the Faculty of the School of Engineering
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by
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Preface

Rapid interest in solar energy has come about as a result of increasing costs of energy from conventional fuels. However, due to the intermittent and diffuse nature of solar energy some unique problems have also arisen. One in particular is thermal storage, and this motivated the study of solar air heating systems utilizing rock beds as thermal storage units. A computer simulation model capable of estimating the long-term performance of an air heating system is described in this work, along with complete instructions on how to operate the program. Various systems can be simulated by simply making the appropriate change in input parameters.

I wish to thank the Air Force for accepting me into this program and providing the exceptional staff and faculty which made this educational experience most worthwhile. I also wish to thank Mr. Stan Boyd for obtaining research material that was not readily available from the AFIT library.

I would especially like to thank Dr. James E. Hitchcock, my thesis advisor, for his many hours of assistance, encouragement and suggestions which made this thesis possible. But most of all I would like to thank my parents for their reassurance and support which in so many ways kept me going and made me strong.

Daniel B. Fant
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This thesis is concerned with the analysis and design of solar air heating systems utilizing rock beds as thermal storage units. A computer simulation model capable of estimating the response of both the solar collector and the rock bed is described. Differential equations describing the rock bed were approximated in a finite-difference form and solved numerically on a digital computer. The temperature of both the solid (rock) and the fluid (air) is determined as a function of time and distance along the bed. The simulation required both charging and discharging of the rock bed for time-varying inlet fluid temperatures. The numerical method used to solve the rock bed equations proved to be stable and convergent and showed satisfactory agreement in comparison to an analytical solution for constant-inlet air temperatures. A cost analysis was also incorporated within this program, by varying the collector area one could determine the optimum collector size for maximum savings. Pressure drop relationships for flat-plate collectors, duct work and packed beds were used to determine operating costs. The particular air system tested proved to be cost effective when compared with natural gas fuel costs for an economic term of 30 years.
Background

The incident solar energy on earth is approximately $1.2 \times 10^{19}$ Btu per year, while the world's total energy consumption is about $7 \times 10^{16}$ Btu per year (Ref 17). However, solar energy is a variable source of energy. Here on earth, the sun shines only during the day; and on cloudy days it is intermittent. Therefore, proper storage of this abundant, cheap, and pollution free energy is necessary for its efficient utilization.

The idea of thermal storage goes way back to the caveman or Roman era when they brought sun heated stones or bricks to bed to supply them with warmth during the night. The Romans also dug deep cellars in the side of hills which they filled with snow during the winter and it would remain throughout the summer to be used when needed. These above happenings can best be explained by the fact that rocks act as a good storage material (Ref 17).

The direct mode of operation of air heating systems is to transfer heated air directly from the collector to the heated space. When more energy is collected than needed, it is stored sensibly in a storage bed by blowing heated air from the collector through the bed. This is referred to as the bed charging mode. When energy is not available from the collector to satisfy the space heating loads, sensible energy is removed from the rock bed. This is known as the bed discharging mode. Discharging is normally accomplished by a reversal of the air flow, thus, causing the outlet air temperature to approach the temperature at the hot end of the bed.
Purpose

The purpose of this thesis was to develop a computer program which completely simulates the three-modal operation of a solar air heating system utilizing rock beds for thermal storage. This involves a numerical method for computing the temperatures of both the air and rock as functions of time and distance for the charging and discharging modes of the bed.

With a proper theoretical analysis of rock beds, duct work and collectors, a simulation model would provide the useful purpose of estimating the performance of a solar air heater much more quickly and cheaply than would be possible by experimentation. In order to simulate various systems, a change in only certain parameters is necessary.

Approach

Before simulating an air system along with the thermal performance of a rock bed storage unit with varying inlet air temperatures, one must first develop a scheme for solar energy collection. Fortunately, this was accomplished by utilizing the previous work of Captain Prins (Ref 16) on solar water heating systems. His long term analysis of flat-plate collectors was used for determining the useful amount of solar energy collected per unit time. Here, long term is defined to be that period of time necessary to describe the performance of a heating system in any particular climate, normally a one to two year period is sufficient for long term simulations (Ref 12).

The amount of useful energy collected is the difference between the solar energy absorbed and the energy lost by the collector. It is a function of the collector area (AC), the collector heat removal efficiency (FR), the monthly average instantaneous total radiation (\( \bar{T}_{\text{IT}} \)), the utilizability (UT) and the effective transmittance-absorptance product \((\bar{t}a)\).

\[
Q = AC \times FR \times \bar{T}_{\text{IT}} \times UT \times (\bar{t}a)
\]  

(1)
FR is a function of the fluid flowrate and the absorber plate design. $T_{T_t}$ depends on the location and orientation of the collector, also on the hour of day and the time of year. The utilizability is a function of variableness of the incident solar energy and the loss coefficient of the collector, which in turn is a function of the number of cover plates, cover material and amount of insulation. Lastly, $(\bar{\alpha})$ also depends on absorber plate properties, the number of cover plates and the angle at which solar radiation strikes the collector.

The useful energy collected in equation (1) also determines the temperature rise of the collector air.

$$Q = (\dot{m}_a c_{pa}) (T_{oc} - T_{ic})$$  \hspace{1cm} (2)

where $\dot{m}_a$ is the mass flowrate of air into the collector, $c_{pa}$ is the specific heat of air at the constant pressure, $T_{ic}$ is the air temperature into the collector and $T_{oc}$ is the air temperature out of the collector. Therefore, if $Q$ is determined from equation (1), one can then solve for $T_{oc}$ from equation (2).

$$T_{oc} = \frac{Q}{c_{pa} \dot{m}_a} + T_{ic}$$  \hspace{1cm} (3)

Equation (3) is utilized in the program to determine the varying air temperatures into the rock bed during the charging mode.

Flat-plate collector results must be combined with a rock bed performance analysis to completely simulate air heating systems. This involved describing a control strategy for the various operational modes of the system; along with developing an adequate space heating model and determining auxiliary energy requirements.

A slight modification of Prins' cost analysis was also necessary to deal with air system simulations.
The heart of this thesis remains to follow, it deals with the theoretical heat transfer analysis of rock beds in Chapter II, pressure drop relationships in Chapter III and a control strategy in Chapter IV. Finally, the results of a typical computer simulation are presented and discussed in Chapter V.
In this chapter, the study of the flow rate of air through rock beds is examined. Heat transfer coefficient formulas are discussed and the differential equations which describe the response of the bed are developed. A finite-difference numerical solution to these equations is also presented.

**Rock Bed Geometry**

The rock bed is a relatively simply looking storage unit, for its geometry refer to Fig 1. The bed consists of a container to hold the rocks plus an inlet and outlet for the flow of air through the unit. For computer purposes it is divided into N-equal segments, all assumed isothermal and of length Δx; this assumption becomes better as the number of segments is increased. Also, insulation surrounds all sides of the bed to minimum energy losses.

A well designed bed is cubical in shape and utilizes rocks that may vary between .5 and 2.5 inches in diameter; these two facts result from pressure drop and heat transfer considerations (Ref 17).

**Rock Bed Assumptions**

a. Rocks are spherical with void fractions of 30 to 60 per cent.

b. Fluid must be a gas such as air.

c. Constant fluid and material properties (for air this is a very good approximation for the temperature range, 70-200 F).

d. For practical purposes, it is advisable to deal with a uniform or average volumetric heat transfer coefficient, rather than focus on local values.

e. Constant volumetric flowrates.

f. Fluid flow and heat conduction in the solid material are one-dimensional in the flow direction.
g. Thermal storage within the fluid (air) is neglected.

h. Heat conduction within the fluid (air) is neglected.

i. Fluid properties are evaluated at a film temperature, $t_f$, where

$$t_f = \frac{t_0 + t_\infty}{2}$$

and

$t_\infty = \text{free stream air temperature}$

$t_0 = \text{rock temperature}$

Heat Transfer Coefficients

There are several methods for determining the heat transfer coefficient between the air and the solid. However, in this section a relationship which takes into account both laminar and turbulent flow contributions will be considered. The following expressions and equations were cited from Ref 18. For alternate methods refer to Appendix B.

The void fraction, $\epsilon_g$, is defined as

$$\epsilon_g = \frac{\text{void volume of bed}}{\text{total volume of bed}} = \frac{V_g}{V}$$ (4)

therefore, the particle or rock fraction, $\epsilon_p$, is

$$\epsilon_p = \frac{\text{rock volume}}{\text{total volume of bed}} = 1 - \epsilon_g$$ (5)

For a bed with $N$ particles which have a volume, $V_p$, the volume occupied by the rocks is

$$V_p N = V - \epsilon_g V = V(1 - \epsilon_g)$$ (6)

Then, the number of particles per unit volume is

$$N = \frac{V}{V_p} = \frac{(1 - \epsilon_g)}{\epsilon_p}$$ (7)
The hydraulic radius, $R_h$, is defined as

$$R_h = \frac{\text{void volume of bed}}{\text{surface area of particles}} = \frac{\epsilon gV}{\lambda_p N}$$  \hspace{1cm} (8)

where $\lambda_p$ is the surface area of a particle. Substituting for $\frac{N}{V}$ from equation (7) yields

$$R_h = \frac{V_p}{\lambda_p \left(1 - \frac{\epsilon}{\lambda_p} g\right)}$$  \hspace{1cm} (9)

For heat transfer purposes, a characteristic length and velocity are needed. Ref 18 used six times the hydraulic radius for the characteristic length, $L^*$. Substituting for $R_h$ from equation (9) yields

$$L^* = 6R_h = \frac{6V_p}{\lambda_p \left(1 - \frac{\epsilon}{\lambda_p} g\right)}$$  \hspace{1cm} (10)

A spherical particle diameter, $D_p$, is defined as

$$D_p = \frac{6V_p}{\lambda_p}$$  \hspace{1cm} (11)

where

$$V_p = \frac{4}{3} \pi D_p^{3/2}$$

$$\lambda_p = \frac{4}{3} \pi D_p^2$$

So now the characteristic length becomes

$$L^* = \frac{D_p}{\lambda_p} \frac{\epsilon}{g} \left(1 - \frac{\epsilon}{\lambda_p} g\right)$$  \hspace{1cm} (12)

The characteristic velocity, $u^*$, is defined as the average velocity of the fluid flowing through the voids. From continuity
\[ \rho A_{\text{void}} u^* = \rho V_o \lambda \]  \hspace{1cm} (13)

where \( V_o \) is the superficial or free-stream velocity. The void area, \( A_{\text{void}} \), is related to \( A \), the cross-sectional area of the bed by

\[ A_{\text{void}} = \epsilon g A \]  \hspace{1cm} (14)

Combining equation (14) and equation (13) the characteristic velocity becomes

\[ u^* = V_o / \epsilon g \]  \hspace{1cm} (15)

This is also referred to as the initial velocity.

Finally, an expression for the characteristic Reynolds number can be written as

\[ Re^* = u^* L^*/\nu \]  \hspace{1cm} (16)

where \( \nu \), the momentum diffusivity, is

\[ \nu = \mu g_c / \rho \]  \hspace{1cm} (17)

Combining equations (12), (15), (17) and (16) yields

\[ Re^* = D_p G / \mu g_c \left( 1 - \epsilon g \right) \]  \hspace{1cm} (18)

where \( G \) = superficial mass velocity = \( \rho V_o \).

The characteristic Nusselt number is defined as

\[ Nu^* = h L^*/K_f \]  \hspace{1cm} (19)

Substituting \( h L^* \) from equation (17) into equation (18) gives
where \( h \) is the average coefficient of heat transfer and \( K_f \), the thermal conductivity of the fluid.

The empirical formulation of the Nusselt number is represented by the following equation (Ref 18):

\[
\text{Nu}^* = (1.5 \text{Re}^{1/2} + 0.2 \text{Re}^{2/3}) \text{Pr}^{1/3}
\]  

(21)

Reynolds number to the one-half power accounts for laminar flow effects while \( \text{Re}^{2/3} \) is its turbulent counterpart.

Combining equations (20) and (21) and solving for \( h \) gives

\[
h = \frac{(1 - \epsilon_g)}{\epsilon_g} \frac{K_f}{D_p} \left(0.5\text{Re}^{1/2} + 0.2 \text{Re}^{2/3}\right) \text{Pr}^{1/3}
\]  

(22)

where \( \text{Pr} \) is the Prandtl number of the fluid. Remember, all fluid properties are evaluated at a film temperature as discussed earlier.

Equation (22) is valid for void fractions less than .65 and Reynolds numbers greater than or equal to 100. This equation can be used for either spherical or cylindrical shaped objects, and it can be modified for staggered tube arrangements. For quick approximations of \( h \), an experimental curve of Nusselt number versus Reynolds number is provided in Ref 18.

To determine the volumetric heat transfer coefficient, \( h_V \), the following expression is used

\[
h_V = h \frac{A_x}{\lambda dx}
\]  

(23)

where

- \( A = \) cross sectional area of bed segment
- \( \lambda dx = \) total volume of bed segment
\[ dA_h = \text{total surface area of particles.} \]

Therefore,

\[ dA_h = 4\pi R_p^2 \text{ [number of particles]} \]
\[ = 4\pi R_p^2 \text{ [Adx} (1 - \epsilon_g)/\pi D_p^2/6] \]

or

\[ dA_h = \frac{6}{D_p} \text{ Adx} (1 - \epsilon_g) \quad (24) \]

After substituting \( dA_h \) from equation (24) into equation (27), \( h_v \) becomes

\[ h_v = 6h (1 - \epsilon_g)/D_p \quad (25) \]

**Internal Resistance**

To account for internal temperature gradients within the rocks a non-dimensional heat transfer parameter known as the Biot number must first be described.

The Biot number, \( Bi \), is defined as

\[ Bi = \frac{\text{internal resistance}}{\text{external resistance}} = \frac{R_i}{R_o} \quad (26) \]

**Fig 2. Spherical Rock Resistances**

where, the internal resistance is

\[ R_i = \frac{R_s}{K_s A_s} \quad (27) \]
and

\[ K_s = \text{thermal conductivity of solid} \]

\[ A_s = \text{surface area of rock}. \]

Ro, the external resistance is

\[ Ro = \frac{1}{hKs} \quad (28) \]

where \( h \) is determined by equation (22). Combining equations (26), (27) and (28) yields

\[ Bi = \frac{R_i}{R_t} = \frac{hR_p}{Ks} \quad (29) \]

It is only when the Biot number is less than 0.1 can heat conduction in the solid material be neglected. In this case, the thermal resistance of the solid particles is small in comparison to the thermal resistance of the convective film between the air and solid. Therefore, for \( Bi < 0.1 \) the effect of temperature gradients within the solid particles can be ignored.

A Biot number correction proposed by "Babcock" in an article written by Jefferson (Ref 9) is a factor which can be incorporated into the heat transfer coefficient to account for the following effects: axial (static) fluid-solid-fluid conduction, fluid phase axial dispersion, fluid-solid heat transfer resistance, and most importantly, internal particle thermal resistance. The result according to Babcock is

\[ h_{\text{eff}} = \frac{hv}{(1 + Bi/5) - \beta^2} \quad (30) \]

where

\[ \beta = \frac{HCS}{HCS + HCA} \]

and

\[ HCS = \text{heat capacity of solid} = \rho_s c_p \left( 1 - \epsilon \right) \]
HCA = heat capacity of air = $\rho_a c_{pa} \epsilon_g$

Note that HCA is much less than HCS since the density of air is much less than the rock density, therefore $\beta$ is approximately one; so for air the effect of $\beta$ on equation (30) can be neglected.

Also, the formula proposed by 'Babcock' was proven experimentally for Biot numbers up to about 4.

Development of Differential Equations

To develop the equations which describe the response of a rock bed, two energy balances on the following control volume must be made. (A similar set of equations can also be found in Ref 6). The first energy balance is for the fluid (air) equation while the other is for the solid.

\[ E_{in} = m_a c_{pa} T_a \]
\[ T_a (x) \]
\[ m_a c_{pa} \left[ T_a + \frac{3}{\delta x} T_a \right] dx = E_{out} \]

Fig 3. Differential Bed Segment

The time rate of change of energy within the control volume is

\[ E_{stored} = \epsilon_{go} c_{pa} Adx \left( \frac{\partial T_a}{\partial t} \right) \]  \hspace{1cm} (31)
where, $\varepsilon_g A dx$ is the void volume. Applying the conservation of energy law

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} + \dot{E}_{\text{stored}} = h \frac{dA}{Adx} (T_a - T_b) dA$$

(32)

where

$$\frac{h dA}{Adx} = h_v$$

as seen from equation (23).

With some rearrangement and substituting for $\dot{E}_{\text{in}}, \dot{E}_{\text{out}}$ and $\dot{E}_{\text{stored}},$

equation (32) simplifies to

$$-\rho_a C_p a \varepsilon_g \frac{\partial T_a}{\partial x} dx - \varepsilon_g \rho_a C_p a \frac{\partial T_a}{\partial t} dx = h_v (T_a - T_b) dx$$

(33)

Now, dividing through by $\rho_a C_p a$ and substituting $u^*$ for $V_o / \varepsilon_g$ yields

$$\frac{\partial T_a}{\partial t} + u^* \frac{\partial T_a}{\partial x} = \frac{-h_v}{\rho_a C_p a} (T_a - T_b)$$

(34)

But since the heat capacity of the bed ($\rho_s d vol c_{ps}$) is much greater than the heat capacity of the air ($\rho_a d vol c_{pa}$), the temperature of the air as a function of time may be neglected. Therefore, equation (34) becomes:

$$\frac{\partial T_a}{\partial x} = \frac{-h_v}{m_a C_p a} (T_a - T_b)$$

(35)

where $m_a = \rho_a A \varepsilon_g u^*$.

The following control volume will again be used to represent the bed temperature as a function of time.

Fig 4. Bed Control Volume
Energy equations for this case are

\[ E_{\text{into bed}} = \dot{m}_{a} c_{pa} [T_{a,i} - T_{a,i+1}] \]  

\[ E_{\text{out of bed}} = Q_{L} \]  

where \( Q_{L} \) is the amount of energy lost from the bed is

\[ Q_{L} = U_{I} (T_{b} - TOB) \]  

and

\[ U_{I} = \frac{K_{ins} A_{ins}}{t_{ins}} \]

where

\( K_{ins} \) = conductivity of the insulation

\( A_{ins} \) = surface area of the bed

\( t_{ins} \) = thickness of insulation surrounding bed

and, \( TOB = \) air temperature outside the bed. Also,

\[ E_{\text{stored}} = (1 - \epsilon_{g}) \rho_{s} c_{ps} \Delta x \frac{\partial T_{b}}{\partial t} \]  

where; \( (1 - \epsilon_{g}) \Delta x \) is the solid volume.

Therefore, again applying the conservation law and rearranging gives

\[ (1 - \epsilon_{g}) \rho_{s} c_{ps} \Delta x \frac{\partial T_{b}}{\partial t} = \dot{m}_{a} c_{pa} [T_{a,i} - T_{a,i+1}] - Q_{L} \]  

Solving for \( \frac{\partial T_{b}}{\partial t} \) yields

\[ \frac{\partial T_{b}}{\partial t} = \frac{\dot{m}_{a} c_{pa}}{(1 - \epsilon_{g}) \rho_{s} c_{ps} \Delta x} [T_{a,i} - T_{a,i+1}] - \frac{Q_{L}}{(1 - \epsilon_{g}) \rho_{s} c_{ps} \Delta x} \]

Solution of Equations

The two equations now derived can be approximated in a finite-difference
form. In this section an implicit solution scheme will be presented. This scheme provides stability and thus allows for larger time steps, therefore, computer costs are reduced while still maintaining reasonable results. Refer to Appendix C if an explicit or a modified-implicit scheme is desired. For the equations that follow, the subscript "i" refers to the distance increment and superscript "j" refers to the time step. As mentioned before, the bed is broken up into N-equal segments. Therefore, the equations are used to solve for N-unknown bed and N-unknown fluid temperatures.

Our partial differential equations in an implicit form, as functions of new time and position, become

\[
\frac{\partial T_a}{\partial x} (i+1) = \frac{-h_v A}{\rho_a c_p a} \left( T_{a,i+1} - T_{b,i+1} \right)
\]

and

\[
\frac{\partial T_b}{\partial t} (i+1) = \frac{\rho_a c_p a}{(1 - \varepsilon_g) \rho_s c_p s A \Delta x} \left( T_{a,i} - T_{a,i+1} \right) - \frac{Q_L (1 - \varepsilon_g) \rho_s c_p s A \Delta x}{c_p}
\]

Finite-differencing equation (41) yields

\[
\frac{T_{a,i+1} - T_{a,i}}{\Delta x} (i+1) = \frac{-h_v A}{\rho_a c_p a} \left( T_{a,i+1} - T_{b,i+1} \right)
\]

After some more rearrangement and letting \( C_1 = \frac{h_v A \Delta x}{\rho_a c_p a} \)

\[
T_{a,i+1} = \frac{1}{1 + C_1} T_{a,i} + \frac{C_1}{1 + C_1} T_{b,i+1}
\]

Since \( C_1 \) is always positive, the quantity \((1/1 + C_1)\) will never become negative; therefore, this particular form of the air equation will remain
stable for any $\Delta x$. In both the explicit and modified-implicit solution schemes, Appendix C, a $\Delta x$ stability criteria is required.

Equation (42) in a fully-implicit form becomes:

$$
\frac{T_{b,j+1} - T_{b,j}}{\Delta t} = \frac{h_c c_p a}{Q^*} T_{a,i+1} - \frac{h_c c_p a}{Q^*} T_{a,i} - \frac{Q_L}{Q^*} \tag{45}
$$

where: $Q^* = (1 - \epsilon g) \rho S c_p A \Delta x$.

Multiplying through by $\Delta t$ and letting $C_2 = \frac{h_c c_p a}{Q^*} \Delta t$

$$
T_{b,j+1} = T_{b,j} + C_2 \left( T_{a,i+1} - T_{a,i} \right) - \frac{Q_L \Delta t}{Q^*} \tag{46}
$$

This form of the bed equation will always be stable, for any $\Delta t$. Now substituting for $T_{b,j+1}$ from equation (46) into equation (44) and doing some more manipulation yields:

$$
T_{a,i+1} = \frac{1 + C_1 C_2}{1 + C_1 C_1 C_2} T_{b,j} + \frac{C_1}{1 + C_1 C_1 C_2} T_{b,j} + \frac{C_1}{1 + C_1 C_1 C_2} Q_L \Delta t \tag{47}
$$

where $Q_L = U I (T_{b,j+1} - T_{O B})$ as in equation (37). Equation (47) represents the fully-implicit finite-differenced air equation as a function of time and position in the bed.

Substituting for $T_{a,i+1}$ from equation (47) into equation (46) yields the final bed equation.

$$
T_{b,j+1} = \frac{C_1 C_2}{1 + C_1 C_1 C_2} T_{a,j+1} + \frac{1 + C_1}{1 + C_1 C_1 C_2} \left[ T_{b,j+1} - \frac{Q_L \Delta t}{Q^*} \right] \tag{48}
$$
Equations (47) and (48) will completely describe the response of the rock bed for nodes 2 through k.

However, in the discharging mode where a complete reversal of the bed temperature distribution is required, the bed temperature at node 1 must also be determined. To do this, a second-order, polynomial curve fit is used. The curve is as follows:

\[ T_{b1} + B\Delta x + C\Delta x^2 = T_{b2}^{j+1} \]
\[ T_{b1} + 2B\Delta x + 4C\Delta x^2 = T_{b3}^{j+1} \]  \hspace{1cm} (49)
\[ T_{b1} + 3B\Delta x + 9C\Delta x^2 = T_{b4}^{j+1} \]

These equations were solved using the Gauss-Siedel Reduction method; the resultant bed temperature at node 1 is

\[ T_{b1} = T_{b4} - 3T_{b3} + 3T_{b2}^{j+1} \]  \hspace{1cm} (50)

Since monotonic temperature curves were usually present in both modes, equation (50) proved to be a fairly accurate extrapolation.

Analytical Solution

An analytical result serves the useful purpose of evaluating the completeness and validity of the numerical method.

The analytical solution proposed by Clark and Arpaci in Ref 5 is as follows: For the air temperature

\[ T_a(S,\delta) - T_{a,0} = D(\delta)F_1(S,\delta) + \frac{D(S) - D(\delta)}{G_1(S,\delta)} \]  \hspace{1cm} (51)
and for the bed temperature

\[ T_b(S, \delta) - T_{b, o} = D(o)f_1(S, \delta) + \]

\[ \frac{D(\delta_1) - D(o)}{\delta_1} g_1(S, \delta) \]  

(52)

The initial and boundary conditions are:

- \( T_a(x, 0) = T_{a, o} \) = initial air temperature
- \( T_b(x, 0) = T_{b, o} \) = initial bed temperature
- \( T_a(0, t) = T_a(t) \) = inlet air temperature

For stability purposes these equations were only valid for small time steps and small distance increments. Also, the equations were derived assuming negligible heat loss from the bed. In equations (51) and (52)

\[ D(o) = T_a(o) - T_{a, o} \]

\[ D(\delta_1) = T_a(\delta_1) - T_{a, o} \]

and

\[ S = \frac{\frac{H}{P_w}}{\frac{a}{c} \frac{P_w}{A_{\text{void}}}} \left( \frac{x}{u^*} \right) \]  

(53)

\[ \delta = \frac{\frac{H}{P_w}}{\frac{c}{s} \frac{P_w}{A}} \left( t - \frac{x}{u^*} \right) \]  

(54)

where

- \( u^* \) = interstitial velocity
- \( A_{\text{void}} \) = flow area
- \( A = (1 - \varepsilon g) A = \text{rock area} \)
- \( t = \text{time in bed} \)
\( x = \) position in bed
\( \bar{h} = \) average heat transfer coefficient
\( P_w = \) wetted perimeter of rock

and \( P_w/A_{\text{void}} = \) wetted perimeter; but since \( R_h \), the hydraulic radius, can also be written as the ratio of flow area to wetted perimeter, one obtains from equation (10)

\[
P_w/A_{\text{void}} = 6/L^* \tag{55}
\]

Substituting for \( L^* \) from equation (12) into equation (55) and letting \( A^* = dA_h/Adx \) from equation (24) yields

\[
P_w/A_{\text{void}} = A^*/g \tag{56}
\]

Also note that in Figs 5 and 6
\( V = u^* \)
\( \theta = t \)
\( \rho^l = \rho_s \)
\( C_p^l = C_{ps} \)
\( \rho = \rho_a \)
\( C_p = C_{pa} \)
\( A = A_{\text{void}} \)

The functions, \( F_1, G_1 \) and \( f_1 \) and \( g \), from equations (51) and (52) can be determined graphically as functions of \( S \) and \( \delta \), or they can be obtained mathematically with the use of Bessel functions. To check the validity of the numerical program, a constant inlet air temperature was tested, and the graphical method was employed.
For a constant inlet air temperature case

\[ T_a(\delta_n) = T_a(o); \ n = 1, 2, 3... \]

and

\[ D(\delta_n) = D(o) \]

Equation (51) becomes

\[ T_a - T_{a,0} = (T_g(o) - T_{a,0})F_1 \]

Equation (57) represents the analytical air equation for constant-inlet air temperatures. Equation (52) yields the analytical bed equation.

\[ T_b - T_{b,0} = f_1 (T_g(o) - T_{b,0}) \]

For the graphs of \( F_1 \) and \( \xi_1 \) refer to Figs 5 and 6.

An analytical result (Ref 7) for the bed temperature at node 1 (x=0) can be obtained from the explicit bed equation in Appendix D, namely:

\[ \frac{\partial T_b}{\partial t} = \frac{-hA^* (T_b - T_a)}{(1 - \epsilon_g)\rho_s c_p} \]

integrating and rearranging we get:

\[ \int_{T_{b,0}}^{T_b} \frac{\partial T_b}{T_b - T_{a,0}} = \frac{-hA^*}{(1 - \epsilon_g)\rho_s c_p} \int_{0}^{t} \partial t \]

At x=0, \( T_a \) is constant for a particular \( \Delta \tau \), therefore equation (59) becomes:

\[ \ln(T_b - T_{a,0})_{T_{b,0}}^{T_b} = \frac{-hA^*}{(1 - \epsilon_g)\rho_s c_p} \Delta \tau \]

and again rearranging yields:

\[ \frac{T_b - T_{a,0}}{T_{b,0} - T_{a,0}} = \exp \left( \frac{-hA^* \Delta \tau}{(1 - \epsilon_g)\rho_s c_p} \right) \]
where:

\[ T_{d0} = \text{initial air temperature} \]
\[ T_{b0} = \text{initial bed temperature} \]

Equation (61) represents the analytical response of the bed at \( x=0 \). This result can be used for comparison purposes with the numerical curve fit solution at node 1.
Fig 5. The function $F_1$ versus $\delta$.

($\eta = 1$ signifies no heat loss)
\( f_1(S, \delta, \eta) \) for \( \eta = 1.0 \)

\[
S = \frac{\bar{h} P}{\rho C_p A} \left( \frac{\theta - X}{V} \right)
\]

\( \eta = 1 \) signifies no heat loss

Fig 6. The function \( f_1 \) versus \( \delta(\bar{h} P \rho C_p A) \)

24
III Pressure Drop Equations

This chapter deals with the discussion of pressure drop relationships for rock beds, flat plate collectors, and duct work.

Power Calculation

Once pressure drop has been determined a power calculation is necessary to obtain fan blowing costs. The theoretical work per unit mass, $w_k$, is given by the pressure drop divided by the air density:

$$ w_k = \frac{\Delta P}{\rho_a} \quad (62) $$

Power, the rate of doing work, is therefore

$$ P = \text{power} = w_k \dot{m} = w_k (\rho_a \dot{V}) \quad (63) $$

In terms of the volumetric flowrate, $Q = \dot{V}A$

$$ P = \rho_a Q w_k \quad (64) $$

Power estimates can be easily converted to operating costs by using standard utility prices and a fan efficiency factor.

Rock Bed

Ergun's equation (Ref 3) was used to determine rock bed pressure drop. His particular relationship is valid for both laminar and turbulent flows and for void fractions between .40 and .65.

$$ \frac{-\Delta P}{L} = \rho \frac{V^2}{p} \left( \frac{1-\epsilon}{\epsilon/3} \right) \left[ \frac{1-\epsilon}{\epsilon} + 1.75 \right] \quad (65) $$

where

$\rho = \text{density of fluid (air)}$
\[ V_0 = \text{free-stream velocity} \]

\[ R_e = \frac{V_0 D_p}{\nu_f} = \text{Reynolds number} \]

Ergun's equation can also be written in the following form which accents the presence of both a viscous term and a dynamic pressure term.

\[ -\Delta P = a V_0 + b \rho V_0^2 \]

where

\[ a V_0 = \text{viscous term} \]

\[ b \rho V_0^2 = \text{pressure term} \]

After some algebraic manipulation

\[ a = \left[ \frac{170}{\epsilon^3} \left( \frac{1 - \epsilon}{L} \right)^2 \right] \]

and

\[ b = \left[ \frac{1.75 (1 - \epsilon) L}{\epsilon^3 \frac{\rho}{D_p}} \right] \]

A tube bundle pressure drop expression is included in Appendix B.

**Collector**

A flat-plate collector consists of an absorber plate, made of metal for strength and good thermal conduction, insulaton to reduce energy losses from both the bottom and sides of the collector, and one or more cover plates of either glass or plastic to reduce heat losses from the top of the collector (Fig 7).
Fig 7. Collector Geometry

For this simple flat-plate air heater:

\[ A_{col} = aB = \text{air duct cross sectional area} \]
\[ P_c = 2(a+b) = \text{air duct perimeter} \]
\[ a = B/a = \text{aspect ratio of collector} \]
\[ L = \text{length of collector} \]
\[ B = \text{width of collector} \]
\[ a = \text{collector air gap} \]

The collector pressure drop can be expressed in terms of a friction factor, \( f \), (Ref 10).

\[ \Delta p = \frac{4f\rho L V^2}{2\bar{D}_h} \quad (69) \]

where:

\( f \) = Fanning-friction factor
\( \rho \) = air density
\( \bar{D}_h \) = hydraulic-diameter

For parallel flat plates:

\[ \bar{D}_h = \frac{4A_w}{\rho} = \frac{2ab}{a+b} \quad (70) \]
Substituting for $D_h$ in equation (69) gives

$$\Delta P = \frac{fL_0 V^2}{g_c a B} (a+b)$$

(71)

For fully developed turbulent flow in tubes (this is also valid for parallel plates if the hydraulic diameter is used) and a Reynolds number range of 5,000 to 30,000, the following Fanning friction factor may be used (Ref 10):

$$f = 0.079 Re^{-0.25}$$

(72)

and for $30,000 < Re < 1,000,000$:

$$f = 0.046 Re^{-0.20}$$

(73)

where $Re = \frac{\rho V D_h}{\mu g_c}$ and $V = Q/A_{col}$. Now substituting equations (72) and (73) into equation (71) yields:

$$\Delta P = \frac{0.079 L}{g_c a B} \rho V^2 \left( \frac{1}{Re} \right)^{0.25} (a+b)$$

(74)

for $5,000 < Re < 30,000$; or

$$\Delta P = \frac{0.046 L}{g_c a B} \rho V^2 \left( \frac{1}{Re} \right)^{0.20} (a+b)$$

(75)

for $30,000 < Re < 1,000,000$.

The Reynolds number for collectors is normally less than 30,000 since "a" is usually quite small; therefore equation (74) is frequently used in calculations. This equation can also be written as

$$\Delta P = \frac{0.079 L}{g_c p (a B)^3} \rho (\mu)^2 \left( \frac{1}{Re} \right)^{0.25} (a+b)$$

(76)
And since power \( P = \frac{\Delta P}{2} \)

\[
p = 0.079 L(a+b)\left(\frac{\dot{m}}{\dot{m}_b}\right)^3 \frac{1}{Re^{.25}} \tag{77}
\]

Pressure drop is extremely sensitive to variations in "a" or the collector air gap spacing. As "a" increases, \( \Delta P \) decreases; but since "a" is much less than "B", the increase in surface area for heat loss is minimal. Therefore, since the shape of the air duct is normally arbitrary, a somewhat larger gap spacing is recommended when one considers the significant reduction in pressure drop and blowing power requirements.

Duct Work

Our design analysis consists of:

a. straight, round smooth radius ducts

b. 90° - elbows, smooth and round.

For most solar energy applications, the duct design is fairly simple, involving mainly straight channels and elbows. Therefore, pressure drop calculations will be considered for these two important and essential cases only.

90° elbows: To determine pressure drop in elbows one needs to know the loss coefficient which is represented by the ratio of the total pressure loss to dynamic pressure at a referenced cross section, "o" usually at the beginning of the configuration.

\[
C_o = \text{loss coefficient} = \frac{-\Delta P}{\frac{\rho V^2}{2g_c}} \tag{78}
\]

or

\[
\Delta P = C_o \left(\frac{\rho V^2}{2g_c}\right) \tag{79}
\]
where "$C_0$" can be found in the ASHRAE Handbook of Fundamentals. The geometry for elbows as depicted in ASHRAE (Ref 1) is shown in Fig 8.

$$C_0 = KC_1^1$$  \hspace{1cm} (80)

where $C_0^1$ is dependent on $r/D$ ratios and $K$ is dependent on $\theta$.

Note that $K = 1$ when $\theta = 90^\circ$.

<table>
<thead>
<tr>
<th>$r/D$</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
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<tbody>
<tr>
<td>$C_0^1$</td>
<td>0.71</td>
<td>0.33</td>
<td>0.22</td>
<td>0.15</td>
<td>0.13</td>
<td>0.12</td>
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</table>

For angles other than $90^\circ$ multiply by the following factors:

<table>
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<tr>
<th>$\theta$</th>
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<th>20</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>110</th>
<th>130</th>
<th>150</th>
<th>180</th>
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</thead>
<tbody>
<tr>
<td>$K$</td>
<td>0</td>
<td>0.31</td>
<td>0.45</td>
<td>0.60</td>
<td>0.78</td>
<td>0.90</td>
<td>1.00</td>
<td>1.13</td>
<td>1.20</td>
<td>1.28</td>
<td>1.40</td>
</tr>
</tbody>
</table>

For design purposes each straight channel is referred to as a "length" and each $90^\circ$ elbow, a "turn". The motivation for circular ducts as opposed to rectangular ducts is:
a. simpler to manufacture
b. less leakage and easier to insulate
c. less costly
d. most important; less surface area, therefore, less pressure drop and thus less blowing power required.

Therefore, for these reasons, it should prove likely that one's final decision would be to use circular cross sections. The only real advantage with rectangular ducts is probably that they fit neatly in between beams, thus creating somewhat of a hideaway design.

Straight lengths: Pressure drop in straight channels can again be expressed in terms of the Fanning friction factor, $f$.

$$ -\Delta p = \frac{4fL\rho V^2}{2\pi D_h L} $$

where $D_h = \frac{4Av}{p} = D_T$; or simply the actual tube diameter when dealing with circular ducts.

As before,

$$ f = \frac{0.079}{Re^{0.25}} \text{ for } 5,000 < Re < 30,000 $$

and

$$ f = \frac{0.046}{Re^{0.20}} \text{ for } 30,000 < Re < 1,000,000. $$

Therefore,

$$ -\Delta p = \frac{4 \left( \frac{0.079}{Re^{0.25}} \right) L\rho V^2}{2\pi D_h L} $$

or

$$ -\Delta p = \frac{4 \left( \frac{0.046}{Re^{0.20}} \right) L\rho V^2}{2\pi D_h L} $$
where:

\[ L = \text{duct length} \]

\[ \text{Re} = \frac{\rho V D_h}{\mu g_c} \]

After substituting equation (82) into equation (63)

\[
P = \frac{\rho \pi D_l V^3 \left( \frac{0.079}{\text{Re}^{2.5}} \right)}{2g_c} L
\]  

(84)

A rule of thumb (ASHRAE, Ref 1) for duct work is that duct velocity should not exceed 700 fpm for low noise purposes;

\[
\text{Duct velocity} = V = \frac{\text{volumetric flowrate}}{\text{cross sectional area of duct}} = \frac{Q}{A_D}
\]

where: \[ A_D = \frac{\pi D^2}{4} \]

Design recommendations for pressure drop in all three configurations are (Ref 2):

a. rock beds - .1 to .4 in H₂O
b. collectors - .2 to .8 in H₂O
c. duct work - .08 in H₂O/100 ft duct length

Note that duct heat losses had to be minimal to validate the assumption that the inlet air temperature for the bed during its charging mode was nearly the same as the air temperature out of the collector. This involved the prediction of heat transfer from ducts as presented and discussed in Appendix C.
IV  Performance Simulation of an Air Heating System

This chapter is concerned with developing a model to predict the operational modes of an air heating system. Also, a space heating load model along with auxiliary energy requirements are discussed.

Control Strategy

A suitable method for long term simulations, is to assume that during any time period, the system operates in whatever modes necessary to maintain the building temperature at the desired level. During each time period, the amount of energy collected is compared with the amount of energy required to meet the load (Ref 13).

To fully complete a simulation model there are three modes that must be satisfied for a solar air heating system. For the workings of an air system refer to the schematic diagrams in Figs 9 and 10, note that only one fan is necessary for its operation.

In the first mode of operation solar energy is available but not enough is collected to meet the space heating load; therefore, the collected energy is delivered directly from the collector to the house. In mode two, solar energy is sufficient to meet the space load; therefore, the energy collected is delivered to the house until the load is satisfied; then for the remaining time period the rest of the energy is transferred to the storage bed. This is commonly referred to as the charging mode. The third mode becomes operational when solar energy is no longer available (usually at night); then hot air is drawn from the hot end of the bed until the space load is met and room temperature air is returned. This is known as the discharging mode. In modes one and three, auxiliary energy may be required if the amount of energy transported from either the collector or the storage unit is depleted before satisfying the load.
Fig 9. Schematic Diagram of a Solar Air Heating System (Ref 2)
Fig 10. Three Modal Diagram of Control Box
Space Heating Loads

A variety of factors influence space heating requirements such as the location of the building, its design, orientation and particular habits of the occupants.

Many space heating models have been proposed, ranging in detail from simple to relatively complex approaches. However, due to computer time and costs, complex models for load calculations are not justified. Therefore, a simple space heating load model, the degree-day model, expresses the heating load, QSHL, as a function of the difference between the inside temperature of the building (reduced to account for heat generation due to lights, people, etc; usually about 65°F) and the mean ambient (outside) temperature (Ref 12).

\[
Q_{\text{SHL}} = UA \left( T_{\text{inside}} - T_a \right)
\]

where:

\( UA \) = the building overall loss coefficient \times area product

The number of degree-days in a month, DD, is

\[
DD = \sum_{i=1}^{N} (65°F - T_a)^{\circ}F
\]

where \( N \) is the number of days in a month and \((65°F - T_a)^{\circ}F\) is taken to be zero for mean daily temperatures above 65°F.

Therefore, a monthly space heating load, \( Q_{\text{SHLM}} \), can be written as

\[
Q_{\text{SHLM}} = UD \times DD
\]

For simulation purposes, an hourly heating load was used, namely:

\[
Q_{\text{SHL}} = \frac{Q_{\text{SHLM}}}{24 \times N}
\]
Auxiliary Energy

Auxiliary energy is provided for space heating whenever the amount of solar energy collected is insufficient to meet the space heating load. In practice, this condition is detected by a thermostat monitoring the temperature within the building. In the present space heating model, however, the temperature within the building is assumed to be constant and auxiliary energy is supplied whenever the rate at which solar energy can be provided, $Q_{AVG}$, is less than the space heating load, $Q_{SHT}$. Therefore, the amount of auxiliary energy, $Q_{AVX}$, needed is:

$$Q_{AVX} = Q_{SHT} - Q_{AVG}$$

where it has been assumed that any heat losses ($Q_{LOS}$) actually reduce the space heating load (Ref 12).

In the discharging mode, when energy is drawn from the storage unit, the auxiliary energy needed is:

$$Q_{AVX} = Q_{SHD} - Q_{BDRH}$$

where, $Q_{SHD}$, the amount of energy drawn from the bed is:

$$Q_{SHD} = m_c p_c (T_{EB} - T_{ROOM})$$

and

$$T_{EB} = \text{Exit air temperature of bed}$$

$$T_{ROOM} = \text{Room air temperature}$$

The $Q_{AVX}$ calculations were used to determine backup energy costs in the air system simulation.

For the complete logic of an air system simulation, a flowchart of the computer program is presented in Appendix E.
V Results and Conclusions

Rock Bed Simulation Results

The major objective in using a fully-implicit finite-difference solution scheme was to be able to use time steps as large as one hour and still obtain adequate results for realistic simulations. In order to justify this criteria the rock bed was tested separately for a constant inlet air temperature and specified initial conditions as indicated in Table 1.

For this particular rock bed simulation, the air and bed temperatures are somewhat more sensitive to changes in \( \Delta x \) than to changes in time step. The corresponding temperatures for a \( \Delta x \) of six inches and a \( \Delta t \) of one hour were reasonably close for engineering purposes to the temperatures for a one inch distance increment and a .1 hour time step. Therefore, these particular increments were used in the air system simulation.

To further appreciate this selection, the air and bed temperatures were plotted as functions of position and time in the bed at various time steps. In Fig 11 one notices that for a \( \Delta t \) of one hour the air temperatures near the beginning of the bed were slightly lower than those corresponding to a \( \Delta t \) of .1 hour. However, at \( x=2 \) feet the trend began to reverse itself as it should, since energy must be conserved. But for all positions in the bed, the temperature difference between the largest and smallest time increments were again relatively small. In Fig 12, the same general trend is observed. Only for this case, the bed temperatures are a bit lower than the corresponding air temperatures of Fig 11, but as the volumetric heat transfer coefficient approaches infinity, the air and bed temperatures at a particular point in the bed become nearly equal. Note that in both these figures a \( \Delta x \) of 1 foot was used; therefore, a smaller \( \Delta x \) would obviously result in reducing the temperature difference.
Table 1. Temperature Convergence Data

Case Tested: Rock Diameter = 1 inch

Initial Bed Temperature = 70°F
Inlet Air Temperature = 140°F
Volumetric Flowrate = 1200 CSM
Average Heat Transfer Coefficient = 3.22 B/hr-ft²-F

<table>
<thead>
<tr>
<th>Δx (inches)</th>
<th>Δt (hours)</th>
<th>*TA (F)</th>
<th>*TB (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>96.1</td>
<td>91.1</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>94.94</td>
<td>91.3</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>93.37</td>
<td>90.5</td>
</tr>
<tr>
<td>4.0</td>
<td>1.0</td>
<td>96.5</td>
<td>93.3</td>
</tr>
<tr>
<td><strong>6.0</strong></td>
<td>1.0</td>
<td>98.2</td>
<td>94.7</td>
</tr>
</tbody>
</table>

* - air & bed temperatures at x=1 foot after one hour of charging

** - increments used for the simulation runs
Fig 12. Bed Temperature Profile (function of x)
between the various time steps. This again reinforces the choice of selecting a $\Delta x$ of six inches and a time step increment of one hour.

In Fig 13, the air temperatures were plotted as a function of time for a particular $x$-location in the bed ($x=1$ foot). With increasing time, the air temperatures at $x=1$ foot began approaching 140°F, the inlet air temperature, $T_{IN}$. Also, the air temperatures for the one hour time step were all slightly lower than the corresponding air temperatures at the other three time increments. These two occurrences followed both a natural and expected trend.

To test the validity of the numerical scheme an analytical solution as discussed in Chapter II was used for comparison purposes. The analytical solution was limited to relatively small time and distance increments, for stability purposes. The solution also assumed negligible heat loss from the bed; this was simulated in the numerical case by simply setting the conductivity of the insulation, $K_{INS}$, to zero. In Figs 14 and 15, the air and bed temperatures were plotted as functions of $x$ for a charging period of only .1 hour. In both graphs the numerical solution, represented by the circles, was extremely close to the analytical result. Also note that for the first few inches of the bed the air temperatures were significantly larger than the corresponding bed temperatures, and at $x=12$ inches, both the temperatures approached 70°F, the initial bed temperature.

In Figs 16 and 17, the air and bed temperatures were plotted as functions of time at $x=1$ inch, for a $\Delta t$ of .1 hour. Again, the numerical temperatures were quite similar in comparison to the analytical ones. Also for increasing time the air and bed temperatures approached 140°F, the constant-inlet air temperature; at $t=1$ hour the air temperature was approximately 139°F while the corresponding bed temperature was around 138°F.
In addition, in Fig 15 the numerical curve fit extrapolation at \( x=0 \) resulted in a bed temperature of 98F; and the dotted line joining the bed temperatures at \( x=1 \) and \( x=0 \) obviously displayed a rather nice consistency with the natural extension of the solid analytical curve.

Therefore, Figs 14 - 17 essentially prove the validity of the numerical scheme and also the second-order polynomial curve fit solution at \( x=0 \).

The next three figures, Figs 18, 19 and 20, represent the bed, collector and duct work pressure drops, respectively. The bed pressure drop is plotted as a function of rock diameter for various flowrates, while the collector and duct work pressure drops are plotted as functions of \( Q \), the volumetric flowrate.

It can be seen from Fig 18 that the bed pressure drop varies directly with \( Q \) and is inversely proportional to the rock diameter. For rock diameters less than one inch significant pressure drop may result, especially at the higher flowrates. In Figs 19 and 20, both the collector and the duct work pressure drop vary directly and almost linearly with the flowrate. Here, it is worthwhile to mention that even though heat transfer coefficients increase with \( Q \), pressure drop and thus blowing costs also increase. However, this cost increase generally outweighs any possible gain in the thermal performance of the rock bed that results due to increasing flowrates. Therefore, one is normally advised to use existing rules of thumb which provide the satisfactory range of flowrates per square foot of collector area.

The last two figures represent total monthly blowing costs as related to various rock diameters. Total cost refers to the combination of bed, collector and duct work blowing costs. Monthly costs are also directly related to \( Q \) but inversely related to rock diameter. Blowing costs remain relatively constant for rock diameters of one inch or more. However, for larger diameter rocks the rate of heat transfer is substantially reduced, since the heat transfer
coefficient is inversely proportional to rock size. Therefore, these two considerations alone appear to suggest that rock diameters of approximately one inch should prove most economical. According to Fig 21, for one inch rocks, the blowing costs at 1200 CFM correspond to approximately two dollars per month and at 1800 CFM, about six dollars per month; these results were based on an eight hour period of operation.

In the air system simulation, a volumetric flowrate of 1200 CFM with a bed of one inch diameter rocks was used. Therefore, for this particular simulation one could safely assume that the total operating costs of two dollars per month contributes rather insignificantly when determining the life cycle cost effectiveness of the air heating system. Also, with the above conditions but for 24 hours of operation, the resulting blowing cost, as indicated in Fig 22, is approximately $5.50 per month; this however is still a relatively small amount when one considers the total investment costs required per square foot of collector area.
Fig 15. Air Temperature Profile (function of time)
Fig 14. Comparison of Analytical and Numerical Air Temperatures (function of x)
BED TEMP VERSUS BED POSITION

CHARGING MODE
DP=1 INCH
T=.1 HOUR
DELX=1 INCH
TIB=70F
TIN=140F
Q=1200CFM
H=3.22 B/HR-SQFT-F
KINS=0.0 B/HR-FT-F
O=NUMERICAL
---=ANALYTICAL
---=EXTRAPOLATED

Fig 15. Comparison of Analytical and Numerical Bed Temperatures (function of x)
AIR TEMP VERSUS TIME

CHARGING MODE
DP=1 INCH
DELT=0.1 HOUR
X=1 INCH
TIB=70F
TIN=140F
Q=1200CFM
H=3.22 B/HR-SQFT-F
KINS=0.0 B/HR-FT-F
O=Numerical
———=Analytical

Fig 16. Comparison of Analytical and Numerical Air Temperatures (function of time)
Fig 17. Comparison of Analytical and Numerical Bed Temperatures (function of time)
BED PRESSURE DROP VERSUS ROCK DIAMETER

BED = 800 CU FT

\[
\begin{align*}
\Delta = 1200 \text{ CFM} \\
\square = 1400 \text{ CFM} \\
\times = 1600 \text{ CFM} \\
\ast = 1800 \text{ CFM}
\end{align*}
\]

HOURS = 8.0

Fig. 18. Bed Pressure Drop

PRESSURE DROP IN BED (INCHES OF WATER)
COLLECTOR PRESSURE DROP VERSUS FLOW RATE

BED = 800 CU FT
HOURS = 8.0
AC = 1000 SQ FT

Fig 19. Collector Pressure Drop
Fig. 20. Duct Work Pressure Drop

DUCT PRESSURE DROP VERSUS FLOW RATE

BED=800 CU FT
HOURS=6.0
DUCT DIAMETER=1.5FT
LID=200FT

VOLUMETRIC FLOW RATE (CFM)

130.00
140.00
150.00
160.00
170.00
180.00

PRESS. DROP IN DUCT (INCHES OF WATER)
Fig 21. Total Monthly Cost (8 hours of operation)
Fig 22. Total Monthly Cost (24 hours of operation)
System Simulation Results

For illustrative purposes the computer program was used to simulate a residence in Columbus, Ohio. Data for the weather, position and orientation of the collector is the same as that used in Prins' program (Ref 16). The parameter values used are as follows:

1. Latitude = 40 degrees
2. Number of covers = 2
3. Slope of collector = 55 degrees
4. Azimuth angle = 0 degrees
5. Area of collector (varied parameter) = 400, 500, 600 and 700 sq. ft.
6. Thickness of cover material = .23 cm
7. Extinction coefficient of cover material = .161/cm
8. Index of refraction of cover material = 1.526
9. Absorptivity of the absorber plate = .92
10. Heat removal efficiency = .62
11. Heat loss coefficient = .60
12. Mass flowrate of fluid through collector = 5112 lbm/hr
13. Specific heat of collector fluid = .24 B/lbm-F
14. Dirt factor = .02
15. Initial temperature of air in bed = 70.0°F
16. Efficiency of heat exchanger = 1.0
17. System in use = 4.0
18. Number of months program is to run = 8
19. Time increment for discharge = 2.0 minutes
20. Epsilon for time increment = .01 hours
21. Minimum pump energy = 275.0 B/hr
22. Structure conductance = 1000.0 B/hr-F
23. Time step = 1.0 hours
24. Volumetric flowrate = 20.1 cfs
25. Bed height = 8.0 ft
26. Bed width = 10.0 ft
27. Air density = 0.071 lbm/ft$^3$
28. Rock diameter = 0.0833 ft
29. Prandtl number for air = 0.711
30. Conductivity of air = 0.0155 B/hr-ft-F
31. Momentum diffusivity of air = 0.649 ft$^2$/hr
32. Specific heat of air = 0.24 B/lbm-F
33. Air temperature outside bed = 70.0°F
34. Rock density = 165 lbm/ft$^3$
35. Specific heat of rock = 0.21 B/lbm-F
36. Bed length = 10.0 ft
37. Number of nodes in bed = 21
38. Void fraction of bed = 0.42
39. Conductivity of rock = 1.0 B/hr-ft-F
40. Initial bed temperature = 70.0°F
41. Conductivity of insulation = 0.023 B/hr-ft-F
42. Thickness of insulation = 1.0 ft
43. Surface area of insulation = 420.0 ft$^2$
44. Dynamic viscosity of air = 0.046 lbm-hr/ft$^2$
45. Fan efficiency = 0.55
46. Collector air gap spacing = 0.333 ft
47. Width of collector = 4.0 ft
48. Length of collector = 8.0 ft
49. Length of duct work = 200.0 ft
50. Duct diameter = 1.5 ft
51. Number of elbows = 5.0
52. Hours of operation = 8.0 hours
53. Primary Federal tax credit = .40
54. Secondary Federal tax credit = .30
55. Ohio tax credit = .10
56. Initial tax credit amount = 2000.0 dollars
57. Secondary tax credit amount = 1000.0 dollars
58. Maximum tax credit amount = 4000.0 dollars
59. Annual mortgage interest rate = .12
60. Down payment = .10
62. Backup furnace cost = $3.26/million Btu (Gas)
63. Conventional furnace cost = $3.26/million Btu (Gas)
64. Efficiency of solar backup furnace = .55
65. Efficiency of conventional furnace = .55
66. Property tax rate = 0.0
67. Income tax bracket = .40
68. Extra insurance and maintenance costs = .01
69. General inflation rate = .08
70. Fuel inflation rate = .12
71. Discount rate (varied parameter) = .10 and .20
72. Term of economic analysis = 20.0 years
73. Area dependent costs (air collector) = $15/sq. ft
74. Area independent costs (fans, etc.) = 1000.0 dollars
Three parameters were varied in this simulation while the others were held fixed. The varied parameters were the collector size, the discount rate and the mortgage loan term. Varying the collector area was necessary to determine the optimum collector size for maximum savings. A cost performance curve is shown below.

Net savings refers to the total fuel savings minus investment costs.
For this simulation, the optimum collector area was approximately 600 square feet, this corresponded to a maximum savings of 8572 dollars for a discount rate of 10 per cent and a 20 year mortgage term. At a discount rate of 20 per cent a slight savings was still achieved, therefore the system provided at least a 20 per cent return on investment. In this case, it is interesting to note that the amount of savings actually increases with longer mortgage periods. This is due to the fact that the rate of return (20%) is greater than the interest rate for the mortgage loan. It is also important to note that fuel costs were based on natural gas prices, the cheapest of the fossil fuels.

For other key results of our simulation refer to Tables 2 and 3. A plot of solar energy percentage versus collector area is presented below. Note that the slope of the curve begins to drop off at higher collector areas. Because a substantial portion of the solar system cost increases linearly with collector area and thermal performance increases less than linearly, a maximum amount of savings results at some optimum collector size.
Table 2. Solar Energy Savings

<table>
<thead>
<tr>
<th>Collector area (ft²)</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage term (years)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Discount rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel savings</td>
<td>$6996</td>
<td>$10653</td>
<td>$1443</td>
<td>$1443</td>
</tr>
<tr>
<td>Expenses</td>
<td>$4554</td>
<td>$5334</td>
<td>$6531</td>
<td>$8164</td>
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<td>Net savings</td>
<td>$2462</td>
<td>$5319</td>
<td>$7905</td>
<td>$6554</td>
</tr>
<tr>
<td>.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel savings</td>
<td>$3017</td>
<td>$4594</td>
<td>$6226</td>
<td>$6548</td>
</tr>
<tr>
<td>Expenses</td>
<td>$3312</td>
<td>$4057</td>
<td>$4963</td>
<td>$6209</td>
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<tr>
<td>Net Savings</td>
<td>$-294</td>
<td>$553</td>
<td>$1258</td>
<td>$138</td>
</tr>
<tr>
<td>Mortgage term (years)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Discount rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel savings</td>
<td>$6996</td>
<td>$10653</td>
<td>$1443</td>
<td>$1443</td>
</tr>
<tr>
<td>Expenses</td>
<td>$5909</td>
<td>$4790</td>
<td>$5865</td>
<td>$5350</td>
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<tr>
<td>Net savings</td>
<td>$3086</td>
<td>$5863</td>
<td>$8572</td>
<td>$38</td>
</tr>
<tr>
<td>.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel savings</td>
<td>$3017</td>
<td>$4594</td>
<td>$6226</td>
<td>$6548</td>
</tr>
<tr>
<td>Expenses</td>
<td>$2288</td>
<td>$3002</td>
<td>$3432</td>
<td>$4290</td>
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<tr>
<td>Net Savings</td>
<td>$-29</td>
<td>$1791</td>
<td>$2793</td>
<td>$205</td>
</tr>
</tbody>
</table>

Structure Conductance = 1000 B/hr-F

Average Computer Costs = $1.00 per run
Table 3. Solar Energy Data

<table>
<thead>
<tr>
<th>Collector area (ft²)</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum temperature out of collector (°F)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oct</td>
<td>102.0</td>
<td>110.0</td>
<td>120.0</td>
<td>129.0</td>
</tr>
<tr>
<td>Nov</td>
<td>93.0</td>
<td>100.0</td>
<td>109.0</td>
<td>118.0</td>
</tr>
<tr>
<td>April</td>
<td>95.0</td>
<td>102.0</td>
<td>119.0</td>
<td>131.0</td>
</tr>
<tr>
<td>May</td>
<td>97.0</td>
<td>104.0</td>
<td>128.0</td>
<td>139.0</td>
</tr>
<tr>
<td>Total solar energy collected (BTU)</td>
<td>39016704</td>
<td>48615483</td>
<td>56853558</td>
<td>65190400</td>
</tr>
<tr>
<td>Total backup energy used (BTU)</td>
<td>79027744</td>
<td>50589481</td>
<td>21162806</td>
<td>18969020</td>
</tr>
<tr>
<td>Per cent of energy supplied by solar</td>
<td>.33</td>
<td>.49</td>
<td>.72</td>
<td>.78</td>
</tr>
</tbody>
</table>

Structure Conductance = 1000 B/hr·°F
Average Computer Costs = $1.00 per run
General Remarks on Rock Bed Air Systems

The basic advantages of a solar air heating system are that the heat transfer fluid, air, will not freeze or boil and materials used in the system generally have a long life expectancy. Maintenance of the system is expected to be low if quality collectors are used, and corrosion problems are usually quite minimal when compared to water systems. Normal maintenance is confined to cleaning or replacement of filters and proper care of the blowers or fans. Another advantage of the air system is that the house can be heated directly from the collector, the mode 1 operation.

The rock bed storage has the following advantages (Ref 17):

- a. Rocks are plentiful and cheap to purchase.
- b. They are non-toxic, non-corrosive and non-flammable.
- c. Rocks have a high heat storage capacity.
- d. Thermal energy is efficiently transmitted to the rocks by air circulating through the bed due to the large heat transfer area. Also, heat conduction through the bed of rocks is low, because the area of contact between the rocks is small; this leads to low heat losses from the bed. These advantages make the rock bed a fairly efficient storage unit because air can leave the bed at a temperature nearly equal to that of the rocks at that point, which in turn is nearly equal to the temperature of the hot air entering the bed.

The disadvantages of a rock bed air system are:

- a. Significant blowing power may result at high volumetric flowrates. For water systems, pumping power requirements are usually minimal.
- b. Rock beds generally require a larger volume for thermal storage than water storage tanks.

For design purposes a rule of thumb of 1 to 4 cfm per square foot of
collector area is recommended (Ref 2). Also, the economic optimum storage capacity is 0.50 to 2.00 cubic feet of rock storage per square foot of collector area (Ref 17). Washed stones or crushed rock one half inch to two inches in diameter should be used. The type of rock does not affect the performance of the storage unit as long as the density is approximately 160 lbm/ft$^3$ (Ref 17). There are two kinds of rocks: igneous rocks and sedimentary rocks. Table 4 shows the various densities of these two types of rocks. Also, Table 5 provides a list of some common heat storage materials (Ref 17).

Conclusions

In conclusion, a solar air heater can effectively provide space heating in residential buildings. Even when natural gas prices were used, the air system still remained cost effective. For this particular simulation, greater than 20 per cent rate of return on investment was achieved. The numerical method proved to be stable and convergent and showed satisfactory agreement in comparison to an analytical solution for constant-inlet air temperatures. Also, the three-modal simulation model demonstrated the capability of estimating the long term response of both the air system and the rock bed. In addition, blowing costs proved to be relatively insignificant in this particular simulation, resulting in a total operating cost of only about two dollars per month. Lastly, computer costs were relatively low, approximately one dollar per simulation.
Table 4. Types of Rocks and Their Densities

<table>
<thead>
<tr>
<th>IGNEOUS ROCK</th>
<th>SEDIMENTARY ROCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Density (lbm/ft³)</td>
</tr>
<tr>
<td>Granite</td>
<td>163-172</td>
</tr>
<tr>
<td>Syenite</td>
<td>163-172</td>
</tr>
<tr>
<td>Ryolite</td>
<td>163-172</td>
</tr>
<tr>
<td>Trachyte</td>
<td>160-172</td>
</tr>
<tr>
<td>Diorite</td>
<td>175-181</td>
</tr>
<tr>
<td>Quartz diorite</td>
<td>169-178</td>
</tr>
<tr>
<td>Dacite</td>
<td>169-178</td>
</tr>
<tr>
<td>Andesite</td>
<td>169-178</td>
</tr>
<tr>
<td>Basalt</td>
<td>181-194</td>
</tr>
<tr>
<td>Obsidian</td>
<td>144-169</td>
</tr>
<tr>
<td>Substance</td>
<td>Specific Heat ( c_p ) (B/lbm-F)</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>Air</td>
<td>0.240</td>
</tr>
<tr>
<td>Asbestos</td>
<td>0.250</td>
</tr>
<tr>
<td>Alcohol</td>
<td>0.55-0.65</td>
</tr>
<tr>
<td>Brine</td>
<td>0.71</td>
</tr>
<tr>
<td>Brick</td>
<td>0.200</td>
</tr>
<tr>
<td>Charcoal</td>
<td>0.242</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.200</td>
</tr>
<tr>
<td>Glass</td>
<td>0.16-0.20</td>
</tr>
<tr>
<td>Gasoline</td>
<td>0.530</td>
</tr>
<tr>
<td>Ice</td>
<td>0.500</td>
</tr>
<tr>
<td>Porcelain</td>
<td>0.255</td>
</tr>
<tr>
<td>Rock</td>
<td>0.210</td>
</tr>
<tr>
<td>Sand (dry)</td>
<td>0.191</td>
</tr>
<tr>
<td>Steel</td>
<td>0.120</td>
</tr>
<tr>
<td>Water</td>
<td>1.00</td>
</tr>
<tr>
<td>Wood (oak)</td>
<td>0.570</td>
</tr>
</tbody>
</table>
References


Appendix A

Review of Method for Determining Useful Energy Collected

The collector heat removal factor, \( F_R \), of equation (1) from Chapter II is defined as a quantity that relates the actual useful energy gain of a collector to the useful gain if the whole collector surface were at the fluid inlet temperature. \( F_R \) is represented by (Ref 2)

\[
F_R = \frac{\dot{m}_f c_p f (T_{f_o} - T_{f,i})}{[(T_{f,i} - T_a)^2] U (T_{f,i} - T_a)}
\]  

(A1)

where:

\((\overline{\alpha})_{TTt} = \) amount of solar energy absorbed by collector plate

\( U = \) energy loss coefficient of collector

The effect of \( F_R \) is to reduce the calculated useful energy gain from what it would have been had the whole collector been at \( T_{f,i} \) to what actually is using a fluid that increases in temperature as it flows through the collector (Ref 2). As the mass flow rate through the collector increases, the temperature rise through the collector decreases.

\( T_{TTt} \) in equation (A1) is given by

\[
T_{TTt} = \frac{\bar{R} r_{TTt}}{11}
\]  

(A2)

where \( r_{TTt} \) is the ratio of hourly total radiation on a horizontal surface to daily total radiation on a horizontal surface. A much better approximation of \( r_{TTt} \) than the one Prins used has now become available (Ref 14). The new equation is:

\[
r_{TTt} = \frac{n}{24} (a + b \cos \omega) \frac{\cos \omega - \cos \omega_{av}}{\sin \omega_{av} - \omega_{av} \cos \omega_{av}}
\]  

(A3)
where

\[ a = 0.409 + 0.5016 \sin (w_s - 1.047) \]
\[ b = 0.6609 - 0.4767 \sin (w_s - 1.047) \]

and \( w \) is the hour angle and \( w_s \) is sunset hour angle. In equation (A2), \( \bar{R} \) is the monthly average ratio of total radiation on a tilted surface to total radiation on a horizontal surface; therefore, \( \bar{R} = 1 \). \( \bar{R} \) is a function of both the beam and diffuse radiation incident on a horizontal surface. Since these measurements are generally unavailable they are estimated (as discussed in Prins' program) at hourly intervals from \( H \) by using the results of Liu and Jordan (Ref 13). They found that the ratio of the monthly average daily total radiation (\( \bar{T} / \bar{H} \)) was related to \( \bar{K} \), the monthly average long term clearness index. \( \bar{K} \) is the ratio of \( H \) to \( H_0 \), is the extraterrestrial radiation on a horizontal surface. To find the incident beam radiation on a tilted surface when the incident amount is known on a horizontal surface, the ratio \( R_b \) is used. \( R_b \) is the ratio of beam radiation on a tilted surface to that on a horizontal surface. If the tilted surface is oriented towards the equator, then

\[
R_b = \frac{\cos (\phi - s) \cos \delta \cos w + \sin (\phi - s) \sin \delta}{\cos \phi \cos \delta \cos w + \sin \phi \sin \delta}
\]

where \( w \) is the hour angle, \( \phi \) the latitude of the collector, \( s \) the slope of the collector from the horizontal, and \( \delta \) the declination angle.

The utilisability, \( UT \), is a function of the \( T_\alpha \) product, the average instantaneous total radiation, \( T_{\alpha T} \), and \( \eta \), the energy loss coefficient of the collector. \( UT \) is the fraction of incident radiation that can be collected, or utilized, by an idealized collector. Utilisability is less than unity because of the heat loss through the sides, back and cover plates of the col-
lector (Ref 16). The overall energy loss coefficient, $U$, is a function of the collector construction and its operating conditions.

The $(\tau a)$ product is a function of the dirt factor (DF), the shading factor (SF), and the angle of incidence for both beam and diffuse radiation.

$$(\tau a) = 0.98 (1-DF)(1-SF)(\tau a)$$  \hspace{1cm} (A5)

The factor of 0.98 was used to account for diffuse radiation, and $(\tau a)$ is evaluated for beam radiation.

For a more complete and detailed discussion of energy collection one can refer directly to Captain Prins thesis (Ref 16).
Appendix B

Alternate Methods for Determining Heat Transfer Coefficients and Pressure Drop

Rock Bed

Leff and Hawley (Ref 6) also investigated packed bed energy storage, they arrived at the following expression for the volumetric heat transfer coefficient in W/m³·°C, $h_v$:

$$h_v = 650 \left( \frac{G}{D} \right)^{0.7} \quad (B1)$$

where $G$ is the superficial mass velocity in kg/s·m², and $D$ is the equivalent spherical diameter of the particles in meters given by:

$$D = \left[ \frac{6 \times \text{net volume of particles}}{\pi \times \text{number of particles}} \right]^{1/3} \quad (B2)$$

As before

$$h = \frac{h_{\Delta x}}{A_h} = \frac{h_D}{6 \left( 1 - \epsilon \right)}$$

Another empirical correlation for packed bed heat transfer coefficients is (Ref 4):

$$j_{\text{I}} = 0.91 \text{Re}^{-0.51}; \text{ for } \text{Re} < 50 \quad (B3)$$

and

$$j_{\text{II}} = 0.61 \text{Re}^{-0.41}; \text{ for } \text{Re} > 50 \quad (B4)$$

where the Colburn factor, $j_I$, and Reynolds number, Re, are defined by:

$$j_I = \frac{h}{c_p \epsilon} \left( \frac{\text{Pr}}{\text{Pr}_f} \right)^{2/3} \quad (B5)$$

and
\[ \text{Re} = \frac{G}{(A_p/V_p) \mu g_c \phi} \quad (B6) \]

where

\[ \text{Pr} = \frac{\mu_g \text{ccp}}{k_f} = \text{Prandtl number} \]

\[ G = \rho V_o = \text{superficial mass velocity} \]

\[ \phi = \text{empirical coefficient which depends on the shape of the particle} \]

(See Table B1).

\[ \frac{A_p}{V_p} = \frac{6}{D_p} (1-c_g) \] = solid particle surface area per unit volume

The subscript of \( f \) denotes properties evaluated at the film temperatures, \( t_f \).

**Table B1: Particle Shape Factors for Packed Bed Correlations (Ref 4)**

<table>
<thead>
<tr>
<th>Shape</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spheres</td>
<td>1.00</td>
</tr>
<tr>
<td>Cylinders</td>
<td>0.91</td>
</tr>
<tr>
<td>Flakes</td>
<td>0.86</td>
</tr>
<tr>
<td>Rasching rings</td>
<td>0.79</td>
</tr>
<tr>
<td>Partition rings</td>
<td>0.67</td>
</tr>
<tr>
<td>Berl Saddles</td>
<td>0.80</td>
</tr>
</tbody>
</table>

**Tube Bundles**

A tube bundle arrangement may also be used for thermal storage. The empirical relationship for air flowing normal to a bank of staggered tubes is (Ref 15)

\[ \frac{V_m}{h_m n_o} = 1.12 h_2 (\text{Re})^{n \cdot (\text{Pr})^{1/3}} \quad (B7) \]
where

\[ Re = \text{Reynolds number} = \frac{\rho V_{\text{max}} D_0}{\mu g_c} \]

\[ V_{\text{max}} = \text{velocity based on minimum flow area} \]

\[ Pr = \text{Prandtl number of fluid} \]

\[ h_m = \text{mean heat transfer coefficient} \]

\[ D_0 = \text{outside tube diameter} \]

\[ K_f = \text{thermal conductivity of air} \]

\( b_2 \) and \( n \) are the Grimison coefficients; evaluated from tables (Ref 15) and dependent on tube spacing.

This equation normally holds true for tube banks with ten or more rows. However, for smaller number of rows a correction factor exists; these factors are listed in tables for row numbers of ten or less (Ref 15).

Fig B1. Staggered Tube Arrangement (Top View)

Fig B2. Tube Bundle (Side View)
\[ V_{\text{max}} = \frac{Q}{A_m} = \frac{Q}{D_m} L \]  
(B8)

where:

\[ A_m = D_m L = \text{minimum flow area} \]

\[ D_m = S_T - D_o = \text{minimum clearance distance} \]

\[ L = \text{tube length} \]

\[ Q = \text{volumetric flow rate} \]

To determine the Grimison coefficients, \( b_2 \) and \( n \), two more parameters, \( x_L \) and \( x_T \) must be defined.

\[ x_L = \frac{S_L}{D_o} \]  
(B9)

and

\[ x_T = \frac{S_T}{D_o} \]  
(B10)

where:

\[ S_L = \text{center to center longitudinal distance in direction of flow} \]

\[ S_T = \text{center to center transverse distance perpendicular to flow} \]

coefficients \( b_2 \) and \( n \) are presented in tabulated form (Ref 15) as functions of \( x_L \) and \( x_T \).

Equation (B7) is valid for turbulent flows and for a Reynolds number range of 2,000 to 40,000.

A pressure drop relationship for the flow of air over a bank of tubes also exist (Ref 8):

\[ p = \frac{f_{\text{max}}^2 G_{\text{max}} N}{9 \times 10^8 \left( \frac{v_m}{V_h} \right)^{0.14}} \]  
(B11)

where

\[ G_{\text{max}} = V_{\text{max}} = \text{mass velocity at minimum flow area} \]

\[ \rho = \text{density evaluated at free stream conditions} \]
N = number of transverse rows

\( \mu_w = \) wall-surface temperature dynamic viscosity of air

\( \mu_b = \) bulk-temperature dynamic viscosity of air

and the empirical friction factor, \( f^1 \), for staggered tube arrangements is:

\[
f^1 = \left[ 0.25 + \frac{0.118}{(S_T - D) / D_{O, max}^{1.2}} \right]^{-0.16} \text{Re}_{max} \tag{B12}
\]

where

\[
\text{Re}_{max} = \frac{\rho V_{max} D}{\mu g_c} = \text{Reynolds number based on minimum flow area.}
\]
Appendix C

Duct Heat Transfer Relationships

In the introduction it was mentioned that the inlet air temperature for the bed during its charging mode was approximately the same as the temperature out of the collector. However, it is only when duct heat losses are at a minimum can this assumption be made.

Therefore, to validate this statement a prediction of heat transfer from circular ducts is necessary. The following diagram is used for this purpose.

\[
\begin{align*}
R_1 &= \text{forced convective resistance} \\
R_2 &= \text{conductive resistance (insulation)} \\
R_3 &= \text{free convective resistance}
\end{align*}
\]

For the fully developed turbulent flow region the forced convective heat transfer within the circular duct can be represented by the following relationship (Ref 10).

\[
Nu = \frac{hD_T}{K_f} = CRe^{-0.08}Pr^n
\]

where:

\[
\begin{align*}
Nu &= \text{Nusselt number} \\
CRe &= \text{Courant-Reynolds number} \\
Pr &= \text{Prandtl number}
\end{align*}
\]

\[
C = \frac{0.61}{(Re)^{0.15}}
\]

\[
(1)
\]
where the value of the constant is:

\[ C = 0.021 \] for constant axial surface temperature

or \[ C = 0.022 \] for constant axial heat flux

and

\[ h_i = \text{forced convective heat transfer coefficient} \]

\[ K_f = \text{thermal conductivity of fluid (air)} \]

Therefore,

\[ \text{Nu}_T = 0.021 Re^{0.8} Pr^n \] (C2)

\[ ; \quad 0.4 < n < 0.6 \]

\[ \text{Nu}_H = 0.022 Re^{0.8} Pr^n \] (C3)

and \( n = 0.6 \) is commonly used for circular ducts exposed to outside ambient conditions. Also, these two algebraic equations are valid for \( 0.5 < Pr < 1.0 \):
or normally for gases, such as air.

The first two resistances are:

\[ R_1 = \frac{1}{h_i} \] (C4)

\[ R_2 = \frac{t_{ins}}{K_{ins}} \] (C5)

where

\[ t_{ins} = \text{thickness of insulation} \]

\[ K_{ins} = \text{conductivity of insulation} \]

and \( h_i \) is determined from either equation (C2) or (C3). The recommended thickness of insulation (Ref 1) is usually one inch of fiberglass insulation for duct work in the heated area and about 1-2 inches for ducts near the collector or storage unit.
To complete the design calculation of heat losses from a duct to an ambient fluid the outside resistance due to free convection must also be taken into account. For free convection the mean Nusselt number can be represented by the following equation (Ref 8).

\[ \bar{N}_u = \bar{C}(Gr_D Pr)^m \]  

(C6)

where the constants \( C \) and \( m \) depend on geometry and whether the flow is laminar or turbulent. Also, it is recommended that fluid properties should be evaluated at a mean-film temperature, \( t_f \).

\[ t_f = \frac{(t_w + t_\infty)}{2} \]

where:

- \( t_w \) = wall temperature
- \( t_\infty \) = ambient (free-stream) temperature

Also, \( Gr_D \) = Grashof number (based on diameter) where:

\[ Gr_D = \frac{g \beta (t_w - t_\infty) H^3}{\nu^2} \]  

(C7)

and \( \beta \) = coefficient of volumetric expansion \( = \frac{1}{T} \); for an ideal gas.

For horizontal ducts, the following results may be used:
Approximate Mean Coefficients

<table>
<thead>
<tr>
<th>Laminar</th>
<th>$10^4 \cdot 10^9$</th>
<th>.525</th>
<th>$1/4$</th>
<th>$h_0 = 0.27 \frac{\Delta t}{D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>$10^9 \cdot 10^{12}$</td>
<td>.129</td>
<td>$1/3$</td>
<td>$h_0 = 0.18 (\Delta t)^{1/3}$</td>
</tr>
</tbody>
</table>

where: $\Delta t = t_2 - t_3$

$D = D_T + 2 (t_{ins})$

and $R_3 = \frac{1}{h_0}$; "$h_0$" can be found from either equations (C3) and (C9) or equation (C6).

The total heat loss relationship can now be expressed as:

$$Q_L = U A_s (\Delta t)_{overall}$$ (C10)

where: $U = \text{overall heat transfer coefficient} = \frac{1}{\sum R}$

or

$$U = \frac{1}{R_1 + R_2 + R_3} = \frac{1}{R_{eff}}$$

and $A_s = \text{total surface area} = \pi (D_T + 2 t_{ins})L_D$. Therefore, the heat loss per unit length can be written as:

$$\frac{Q_L}{L_D} = \frac{\pi (D_T + 2 t_{ins}) (\Delta t)_{overall}}{R_{eff}}$$ (C11)

where: $L_D = \text{duct length}$

and $(\Delta t)_{overall} = t_1 - t_3$ or the overall temperature difference.

---

Fig C2. Duct Length Section
QL can also be expressed in the following form:

\[ Q_L = \dot{m} \, c_p \, (t_{id} - t_{od}) \]  \hspace{1cm} (C12)

where:

- \( t_{id} \) = air temperature into duct
- \( t_{od} \) = air temperature out of duct

Solving equation (C12) for "t\_od" we get:

\[ t_{od} = t_{id} - \frac{Q_L}{\dot{m} \, c_p} \]  \hspace{1cm} (C13)

where \( Q_L \) is determined from equation (C11).

To examine the changes in temperature through a straight duct, several test cases with variable duct lengths were carried out. In all cases a negligible temperature difference, \( t_{id} - t_{od} \), resulted. Therefore, the assumption the "t\_od" is essentially the same as the outlet collector temperature proves to be fairly valid for our particular purposes.
Appendix D

Explicit and Modified-Implicit Schemes

The governing differential air equation is the same as that derived in Chapter II, equation (35).

\[ \frac{dT_a}{dx} = -\frac{h_A}{a a c \rho a} (T_a - T_b) \]  

(D1)

Finite-differencing equation (D1) explicitly:

\[ \frac{T_{a,i+1} - T_{a,i}}{\Delta x} = -\frac{h_A}{a a c \rho a} (T_{a,i} - T_b) \]  

(D2)

and rearranging gives

\[ T_{a,i+1} = \left(1 - \frac{h_A \Delta x}{a a c \rho a}\right) T_{a,i} + \frac{h_A \Delta x}{a a c \rho a} T_b \]  

(D3)

For stability purposes

\[ \frac{h_A \Delta x}{a a c \rho a} < 1 \]  

(D4)

or

\[ \Delta x < \frac{a a c \rho a}{h_A} \]  

(D5)

Equation (D1) in a modified-implicit form becomes

\[ \frac{T_{a,i+1} - T_{a,i}}{\Delta x} = -\frac{h_A}{a a c \rho a} \left[ \frac{T_{a,i} + T_{a,i+1}}{2} - T_b \right] \]  

(D6)

Therefore

\[ T_{a,i+1} = \left[ 1 - \frac{h_A \Delta x}{2 a a c \rho a} \right] T_{a,i} + \frac{h_A \Delta x}{2 a a c \rho a} T_b \]  

\[ + \frac{1 + \frac{h_A \Delta x}{2 a a c \rho a}}{1 + \frac{h_A \Delta x}{2 a a c \rho a}} \]  

(D7)
In this case, stability is achieved when

\[ \frac{h_v A_x}{\frac{2h_c}{a} \rho a} < 1 \]  \hspace{1cm} (D8)

or

\[ \Delta x < \frac{2h_c \rho a}{h_v} = \frac{2a \rho c}{h_v} \]  \hspace{1cm} (D9)

Here, \( \Delta x \) is twice that of the explicit result.

For these two schemes, the bed equation is derived in the same manner as the air equation, the result is:

\[ \frac{dT_b}{dt} = \frac{h_v A_x}{(1-\varepsilon) \rho s \Delta a x} (T_a - T_b) - \frac{Q_L}{(1-\varepsilon) \rho s c_p s A x} \]  \hspace{1cm} (D10)

In explicit form, letting \( B = \frac{h_v \Delta t}{(1-\varepsilon) \rho s c_p s A x} \), equation (D10) becomes

\[ T_{b,i}^{j+1} - T_{b,i}^j = B(T_{a,i}^j - T_{b,i}^j) - \frac{Q_L \Delta t}{(1-\varepsilon) \rho s c_p s A x} \]  \hspace{1cm} (D11)

or

\[ T_{b,i}^{j+1} = (1 - B) T_{b,i}^j + B T_{a,i}^j - \frac{Q_L \Delta t}{(1-\varepsilon) \rho s c_p s A x} \]  \hspace{1cm} (D12)

For stability, \( B < 1 \) or

\[ \Delta t < \frac{(1-\varepsilon) \rho s c_p s A x}{h_v} \]  \hspace{1cm} (D13)

Now letting \( C = \frac{h_v A_x}{\frac{2h_c}{a} \rho a} \), equation (D13) becomes
UNCLASSIFIED

END

3-8
Therefore, equations (D12) and (D14) may be used when an explicit solution scheme is desired.

The bed equation in a modified-implicit form is

\[ T_{b,i+1}^j - T_{b,i+1}^j = B \left( T_{a,i}^* - T_{b,i+1}^j \right) \]

\[ - \frac{Q_L \Delta t}{(1-\varepsilon) \rho c_p} A \Delta x \]

where

\[ T_{a,i}^* = \frac{T_{a,i}^j + T_{a,i+1}^j + T_{a,i}^{j+1} + T_{a,i+1}^{j+1}}{4} \]

Substituting for \( T_{a,i}^* \) from equation (D16) into equation (D15) and rearranging yields

\[ T_{b,i+1}^{j+1} = T_{b,i+1}^j \left[ 1 - B \right] + \]

\[ \frac{B}{4} \left[ T_{a,i}^j + T_{a,i+1}^j + T_{a,i}^{j+1} + T_{a,i+1}^{j+1} \right] \]

\[ - \frac{Q_L \Delta t}{(1-\varepsilon) \rho c_p} A \Delta x \]

Since \( C = \frac{b_j A \Delta x}{\frac{n_c}{c_{pa}}} \), equation (D7) can also be written as:

\[ T_{a,i+1}^j = \frac{1-C/2}{1+C/2} T_{a,i}^j + \frac{C}{1+C/2} T_{b,i+1}^j \]

Therefore, the modified-implicit solution scheme is represented by equations (D17) and (D18).
Appendix E

Program Instructions

A revision of Captain Prins' computer program for solar water heaters was necessary to deal with the simulation of solar air systems. The modified program can handle both water and air systems, and is relatively easy to operate. However, when an air system simulation is desired some new data cards must be added; consisting of rock bed and pressure drop parameters.

Whenever the air system is in effect any type of water system analysis is overrided. To do this, a PROG=4.0 data card is all that is needed; the exact location of this card will be further explained in the next section.

To use this program, one must also input data about the weather, position and orientation of the collector and the Earth as explained in the Prins' program (Ref 16).

For the air system, rock bed parameters are used to determine the characteristic heat transfer coefficients between the air and the rock; the pressure drop parameters are used to determine pressure drop and blowing costs for the bed, collector and duct work. Next, Prins' part of the program is used to calculate the amount of useful energy collected from a flat-plate solar collector. Once this is done, the three modal operation of an air heating system can be analyzed.

This is accomplished by calculating a space heating load using the degree-day model; then for each hour of the day the useful energy collected is compared to the heating load requirement, and based on this criteria a particular mode of operation is chosen in order to satisfy the load.

The charging and discharging modes, modes 2 and 3 respectively, are the most important. Here, the differential equations which describe the storage
bed are needed to accurately simulate the system. As explained earlier in Chapter IV, the charging mode is used when the useful energy collected is greater than the space load; the load is satisfied first and any excess energy is stored in the bed. The time required to satisfy the load and the time spent charging the bed are both calculated. Also, if the discharging mode was previously in effect, a complete bed temperature distribution reversal is performed before charging the unit.

Mode 3 becomes operational whenever solar energy is no longer available for collection, in this case energy is taken from storage in order to meet the heating load. Again, if the charging mode was previously in effect a bed temperature distribution reversal is necessary before discharging. As in mode 2, the times needed to satisfy the load are also calculated.

In addition, energy losses from the bed are calculated for each period it is not in use; updated bed temperatures are then determined at each nodal point of the storage unit.

To accurately determine fuel savings, a yearly space heating load calculation is needed. Also, a tax credit calculation is necessary to obtain the net investment in a particular type of solar heating system, that is total investment minus the tax credit break. The additional parameters needed for this cost analysis and other modifications are fully explained in the next section.

This last section contains the extra parameters needed to operate an air system simulation. The required input data cards are explained along with the proper nomenclature and input format. All input data is real except where noted. Do not skip any data cards. Input all data in the following order.
<table>
<thead>
<tr>
<th>Card #</th>
<th>Symbol</th>
<th>Input Format</th>
<th>Units</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-15</td>
<td></td>
<td></td>
<td></td>
<td>(First 15 data cards are the same as in Prins' program, Ref 16).</td>
</tr>
<tr>
<td>16</td>
<td>EFF</td>
<td>x.xx</td>
<td>none</td>
<td>Efficiency of heat exchanger. Air systems with no heat exchanger enter 1.</td>
</tr>
<tr>
<td>17</td>
<td>PROG</td>
<td>x.</td>
<td>none</td>
<td>If PROG = 4.0, air system is in effect. For water system refer to Prins' work (Ref 16).</td>
</tr>
<tr>
<td>18</td>
<td>INT</td>
<td>xx</td>
<td>none</td>
<td>Number of intervals desired on K curve (INTEGER). Explained in Prins' program (Ref 16).</td>
</tr>
<tr>
<td>19</td>
<td>ND</td>
<td>x</td>
<td>none</td>
<td>Number of consecutive days the simulation is to run (INTEGER).</td>
</tr>
<tr>
<td>20</td>
<td>MON</td>
<td>xx</td>
<td>none</td>
<td>Number of months program is run (INTEGER).</td>
</tr>
<tr>
<td>21</td>
<td>TMIN</td>
<td>xx.xxx</td>
<td>minutes</td>
<td>Time increment used in discharging mode. Used in a Do-Loop to determine when load is satisfied or how long hot air should be drawn from the bed to meet the load.</td>
</tr>
<tr>
<td>22</td>
<td>EPSIL</td>
<td>xx.xxxxx</td>
<td>hours</td>
<td>Tolerance use to leave Do-Loop after 1 hour of discharging (1 hour plus or minus tolerance). A value of .01 is normally used.</td>
</tr>
<tr>
<td>23</td>
<td>QMIN</td>
<td>xxxx.xx</td>
<td>B/hr</td>
<td>Minimum collected energy required before fan turned on.</td>
</tr>
<tr>
<td>24</td>
<td>UA</td>
<td>xxxx.xxx</td>
<td>B/hr-F</td>
<td>Overall structure conductance.</td>
</tr>
<tr>
<td>25</td>
<td>DELTAS</td>
<td>xx.xxx</td>
<td>hours</td>
<td>Desired time step.</td>
</tr>
<tr>
<td>26</td>
<td>Q</td>
<td>xx.xxx</td>
<td>cfs</td>
<td>Volumetric flowrate.</td>
</tr>
<tr>
<td>27</td>
<td>HB</td>
<td>xx.xxx</td>
<td>feet</td>
<td>Bed height.</td>
</tr>
<tr>
<td>28</td>
<td>WB</td>
<td>xx.xxx</td>
<td>feet</td>
<td>Bed width.</td>
</tr>
<tr>
<td>29</td>
<td>RHOA</td>
<td>xxx.xxxxx</td>
<td>lbm/ft$^3$</td>
<td>Air density.</td>
</tr>
<tr>
<td>Card #</td>
<td>Symbol</td>
<td>Input Format</td>
<td>Units</td>
<td>Explanation</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>--------------</td>
<td>----------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>30</td>
<td>DP</td>
<td>xx.xxxxx</td>
<td>1bm/ft³</td>
<td>Rock diameter.</td>
</tr>
<tr>
<td>31</td>
<td>PRN</td>
<td>xx.xxxxx</td>
<td>none</td>
<td>Prandtl number of air.</td>
</tr>
<tr>
<td>32</td>
<td>CONDA</td>
<td>xxx.xxxxx</td>
<td>B/hr-ft-F</td>
<td>Conductivity of air.</td>
</tr>
<tr>
<td>33</td>
<td>NEUA</td>
<td>xxx.xxxxx</td>
<td>ft²/hr</td>
<td>Momentum diffusivity of air.</td>
</tr>
<tr>
<td>34</td>
<td>CPA</td>
<td>xx.xxxxx</td>
<td>B/1bm-F</td>
<td>Specific heat of air.</td>
</tr>
<tr>
<td>35</td>
<td>TOB</td>
<td>xx.xx</td>
<td>Degrees-F</td>
<td>Temperature of air outside the bed.</td>
</tr>
<tr>
<td>36</td>
<td>RHOS</td>
<td>xxx.xxxxx</td>
<td>1bm/ft³</td>
<td>Density of rock.</td>
</tr>
<tr>
<td>37</td>
<td>CPS</td>
<td>xxx.xxxxx</td>
<td>B/1bm-F</td>
<td>Specific heat of rock.</td>
</tr>
<tr>
<td>38</td>
<td>LB</td>
<td>xx.xx</td>
<td>feet</td>
<td>Bed length.</td>
</tr>
<tr>
<td>39</td>
<td>KJ</td>
<td>xxx</td>
<td>none</td>
<td>Number of nodes in bed (INTEGER).</td>
</tr>
<tr>
<td>40</td>
<td>EG</td>
<td>xx.xxxx</td>
<td>none</td>
<td>Void fraction of bed.</td>
</tr>
<tr>
<td>41</td>
<td>KS</td>
<td>xx.xxx</td>
<td>B/hr-ft-F</td>
<td>Thermal conductivity of rock.</td>
</tr>
<tr>
<td>42</td>
<td>PDIA</td>
<td>xx.xxxx</td>
<td>inches</td>
<td>Rock diameter.</td>
</tr>
<tr>
<td>43</td>
<td>TBI</td>
<td>xx.xx</td>
<td>Degrees-F</td>
<td>Initial bed temperature.</td>
</tr>
<tr>
<td>44</td>
<td>KINS</td>
<td>xxx.xxxx</td>
<td>B/hr-ft-F</td>
<td>Thermal conductivity of insulation.</td>
</tr>
<tr>
<td>45</td>
<td>TINS</td>
<td>xx.xxx</td>
<td>feet</td>
<td>Insulation thickness.</td>
</tr>
<tr>
<td>46</td>
<td>AINS</td>
<td>xxx.xxx.xxx</td>
<td>sq ft</td>
<td>Surface area of bed (top and sides only).</td>
</tr>
</tbody>
</table>

Cards 47 through 55 are needed for pressure drop calculations.

<table>
<thead>
<tr>
<th>Card #</th>
<th>Symbol</th>
<th>Input Format</th>
<th>Units</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>UG</td>
<td>xx.xxxxx</td>
<td>lbm-hr/ft²</td>
<td>Dynamic viscosity of air.</td>
</tr>
<tr>
<td>48</td>
<td>FFFP</td>
<td>x.xxxxx</td>
<td>feet</td>
<td>Fan efficiency.</td>
</tr>
<tr>
<td>49</td>
<td>ACC</td>
<td>xx.xxxxx</td>
<td>feet</td>
<td>Collector air gap spacing.</td>
</tr>
<tr>
<td>50</td>
<td>BC</td>
<td>xx.xxxx</td>
<td>feet</td>
<td>Width of collector.</td>
</tr>
<tr>
<td>51</td>
<td>LC</td>
<td>xx.xxx</td>
<td>feet</td>
<td>Length of collector.</td>
</tr>
<tr>
<td>52</td>
<td>LD</td>
<td>xx.xxx</td>
<td>feet</td>
<td>Length of duct work.</td>
</tr>
<tr>
<td>Card #</td>
<td>Symbol</td>
<td>Input Format</td>
<td>Units</td>
<td>Explanation</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>--------------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>53</td>
<td>DTD</td>
<td>xx.xxx</td>
<td>feet</td>
<td>Duct diameter.</td>
</tr>
<tr>
<td>54</td>
<td>TN</td>
<td>xx.xxx</td>
<td>none</td>
<td>Number of turns or elbows.</td>
</tr>
<tr>
<td>55</td>
<td>NH</td>
<td>xx.xxx</td>
<td>hours</td>
<td>Hours of operation</td>
</tr>
<tr>
<td>56</td>
<td>DDAY</td>
<td>xxxxx.xx</td>
<td>Degrees-F</td>
<td>Number of Degree-Days. (for the average number of Degree-Days corresponding to each month of the year in Vandalia, Ohio refer to Table E1).</td>
</tr>
</tbody>
</table>

Card 64 through 85 are for the cost analysis portion. If a cost analysis is not desired, do not enter any more data cards and the program will stop automatically.

<table>
<thead>
<tr>
<th>64</th>
<th>FTCI</th>
<th>xx.xxxx</th>
<th>none</th>
<th>Primary Federal tax credit percentage (ex., 40% of the first 2000 dollars invested; per cent is entered in decimal form, 40% = .40).</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>FTC2</td>
<td>xx.xxxx</td>
<td>none</td>
<td>Secondary Federal tax credit percentage (per cent is entered in decimal form).</td>
</tr>
<tr>
<td>66</td>
<td>OTC</td>
<td>xx.xxxx</td>
<td>none</td>
<td>Ohio tax credit percentage (per cent in decimal form).</td>
</tr>
<tr>
<td>67</td>
<td>STCA</td>
<td>xxxxx.xx</td>
<td>dollars</td>
<td>Initial tax credit amount (i.e., first 2000 dollars).</td>
</tr>
<tr>
<td>68</td>
<td>LTCA</td>
<td>xxxxx.xx</td>
<td>dollars</td>
<td>Secondary tax credit amount.</td>
</tr>
<tr>
<td>69</td>
<td>TMAYC</td>
<td>xxxxx.xx</td>
<td>dollars</td>
<td>Maximum return from tax credit (i.e., amount from tax credit can not exceed 4000 dollars).</td>
</tr>
</tbody>
</table>

70-85  (Same as cards 36-51 in Prins' program, Ref 16).
Table E1. Degree-Days, Vandalia, Ohio, Dayton Power and Light Co.
Appendix F

Air System Flowchart

Start

Mode Trigger
Flow 2=0.0
Flow 3=0.0
JI=1

IF

PROG EQ 4

READ IN BED
PARAMETERS

CROSS SEC AREA
Superficial & Interstitial Velocities

Mass Flowrate
Reynolds #
Heat transfer coefficient

Riot #
Effective Coeff
Constants for Fin-Dif Eqtns

Bed Initialize
for each position & time
TR(1,1) = TBI
TBI = 70F

GO TO 333
Read in Press Drop Parameters for Bed, C, D

Bed Press Drop Collector & Duct Work Pressure Drop

Prins' Program, as before, with slight modifications, for each month of the year and each hour of the day.

Prins Water System

IF PROG EQ 4

NO GO TO WATER SYSTEM

YES GO TO AIR SYSTEM

Space Heating Load Calculations

% of Solar then to Cost Anal

Bed Mass & If statements for various modes

GO TO 240

Y: QAVG < QMIN
N: QAVG > QMIN

GO TO 241

Y: QAVG > QMIN
N: QAVG < QMIN
Mode 1: Back up Heat Needed

Collector to House

$T_3 = T_{ROOM}$

QLOST CALCULATIONS FOR EACH NODE

UPDATE OLD BED TEMPERATURE

$Q_{AUX} = Q_{SHL} - Q_{AVG} - Q_{LT}$

GO TO 18

$Q_{AUX} = 0.0$

Time Mode 1
Time Mode 2

QLOST - For each node at DELT1

MODE 1 - till satisfied
MODE 2 - remainder of Energy (Heat) to storage
UPDATE OLD BED TEMPERATURES

FLAG 2= 1.0

NO

NECESSARY REVERSAL MADE FOR PROPER CHARGE

YES

GO TO 260

TIC=ROOM

IF IJ EQ 1 AND J1 EQ 1

YES

NO

BYPASS REVERSAL FOR FIRST DAY

NECESSARY REVERSAL MADE FOR PROPER CHARGE

FLAG 2=1.0

FLAG 3=0.0

FLAGS RETURNED

GO TO 270

TIC=TA(KJ,J)

TOC = QAVG

\frac{h}{c_p} + TIC

93
\[ \text{TA}(1) = \text{TOC} \]

\[ \text{GO TO 666} \]

\[ \text{IF } J1 = 1 \quad \& \quad IJ = 1 \quad \& \quad J \leq 10 \]

\[ \text{GO TO 246} \]

\[ \text{GO TO 249} \]

\[ \text{FLAG 3 EQ 1} \]

\[ \text{TB Reversal for Discharging} \]

\[ \text{Flag 3} = 1.0 \]
\[ \text{Flag 2} = 0.0 \]
\[ \text{Flags Returned} \]

\[ \text{Delti} = 0.0 \]

\[ \text{Initialize for Discharging} \]

\[ \text{TA}(1) = \text{TROOM} \]

\[ \text{Discharging YT, °} \]

\[ \text{Flags Returned} \]
DELTI = Time Step used for Discharging

DELTI + (TMIN/60)

TIC = TROOM

T3 = TIC

C2 coefficient calculated

TA(IH, J+1)

TB(I+1, IH)

TB1 = TB4 - (3TB3) + (3TB2)

95
YES

AVG > QSHL

NO

TEB = 
TA(KJ)


QBEDH = 
thc (TEB - TROOM)

QBEDH ≥ QSHL

YES

NO

DELTAS1 = 
DELTAS - EPSIL

QLOSS

CALCULATIONS

DELT3 = 
(QSHL * DELTAS) / QBEDH

DELT4 = 1 - DELT3

GO TO 680

GO TO 667

DELTI > DELTAS1

TIME STEP CHANGE
FOR DISCHARGING

QAUX = 0.0

GO TO 18
UPDATE OLD BED TEMPERATURES
GO TO 18
246
QAUX = QSHL
T3 = TROOM
WRITE OUT QAUX
RETURN TO 1 FOR NEXT HOUR OF THE DAY
TB(1) = TB(25)
For new Day or month update
Monthly QSHL
% Solar %
Continue QAUX T QSOLT
New Month
Read in Cost Analysis Parameters

Fuel Expense Calculations

Residential Expenses minus tax credit

Fuel Savings

Residential Savings

END
Appendix G

Computer Program Listing
PROGRAM COL3J INPUT OUTPUT TAPE3 INPUT TAPE4 OUTPUT
DIMENSION JI(L1), AVG(L1), DT(I1), E(L1), F(L1), FLOW(L1)
DIMENSION TI(G1, G2), TB(S1, S2), T(C1, C2)
DIMENSION TRT(S1, S2)
REAL H, N, AVG, J, I, T, T, KT, AVG, LAT, RATE, LOAD, MASS
REAL TI, JJ, KK, LL, M, L
REAL LR, KS, LSTAR, MDOA, NEUM
REAL LC, LD, HL, C, Q, NR, NR2, NR3, R, S, T, U, V, W, W1, W2,
C, 2, 2, 4, 3, 4, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25/5
JT=1
FLAG2=0.0
FLAG3=0.0
READ LATD
WRITE (6, 112E) LATD
READ SD
WRITE (6, 115E) SD
READ GN
WRITE (6, 116E) GN
READ D GAMMA
WRITE (6, 117E) D GAMMA
READ AC
WRITE (6, 118E) AC
READ THC
WRITE (6, 119E) THC
READ EC
WRITE (6, 120E) EC
READ TC
WRITE (6, 121E) TC
READ ALPHAN
WRITE (6, 122E) ALPHAN
WRITE (6, 123E) FR
READ UE
WRITE (6, 124E) UE
WRITE (6, 125E) U
```
WRITE(5, 121)
WRITE(5, 121) FLOWC
READ, OPC
WRITE(5, 122) OPC
READ, DF
WRITE(5, 1145) DF
READ, T
WRITE(5, 123) T
READ, EFF
WRITE(5, 124) EFF
READ, PROG
WRITE(5, 125) PROG
IF(PROG EQ '4') GO TO 49F
READ, TIN
WRITE(5, 213) TIN
READ, MASS
WRITE(5, 12) MISS
MASS = MISS * 8.33
READ, ACAST
WRITE(5, 126) ACAST
READ, THICKST
WRITE(5, 127) THICKST
READ, CONST
WRITE(5, 128) CONST
READ, CPS
WRITE(5, 129) CPS
READ, DAYFLOW
WRITE(5, 130) DAYFLOW
49F WRITE(5, 1235) INT
FIND, NO
WRITE(5, 1275) 1
C HOW MANY MONTHS DO YOU WANT THIS PROGRAM TO RUN?
READ, MON
WRITE(5, 1345) MON
```
WRITE(6,874) RHOS
READ(CPS)
WRITE(6,98) CPS
READ(L)
WRITE(6,985) L
READ(K)
WRITE(6,99) K
READ(ER)
WRITE(6,995) ER
READ(KS)
WRITE(6,996) KS
READ(DP)
WRITE(6,999) DP
READ(TBI)
WRITE(6,12) TBI
READ(KS)
WRITE(6,13) KS
READ(TINS)
WRITE(6,14) TINS
READ(AINS)
WRITE(6,15) AINS

108

3. DEFINE THE EFFECTIVE REYNOLDS NUMBER AND THE EFFECTIVE HEAT TRANSFER COEFFICIENT TO TAKE INTO ACCOUNT INTERNAL TEMP. GRADIENT:

\[ \Delta T = \frac{\Delta T_{\text{int}}}{L} \]

\[ V = \frac{\Delta T}{\alpha T} \]

\[ V = \frac{\Delta T}{\alpha T} \]

\[ G = \frac{\Delta T}{\alpha T} \]

\[ V_{OLP} = \left( 1.0 / \Delta T \right) \times \left( 3.0 / 16 \right) \times (DP / 3.0) \]

\[ V_{DF} = \left( \frac{\alpha T}{\Delta T} / \left( 3.0 / 16 \right) \right) \times (L / 3.0) \]

\[ LST1 = \Delta P \times \left( \Delta T / (L / 3.0) \right) \]

\[ VSTA = \Delta P \times \left( \Delta T / (L / 3.0) \right) \]

\[ R7 = (\text{UST1} \times \Delta P) / \text{NEUA} \]

\[ H = \left( 1.0 \times (\text{RE} \times \frac{1}{2}) - \frac{2}{3} \times (\text{RE} \times \frac{1}{2}) \right) \times \left( \text{COND} / \Delta P \right) \]

\[ \text{H} = \left( (\text{COND} / \Delta P) \times \left( 1.0 \times \Delta P \right) \right) / \Delta P \]

\[ G = \left( (\text{COND} / \Delta P) \times \left( 1.0 \times \Delta P \right) \right) / \Delta P \]
MOS = MOS * CPM + (1.0 - CPM)
MOS = MOS * CPM + CPM
1 = MOS / (MOS + MOS)
BIF = (1.0 + 2.0 / 7.1) + (9.0 * 2.0)
HVeff = HV / BIF
ELX = L 1 / (FLAT(MJ) - M)
MLOA = (MLOA + M) * 340
C1 = (HVeff * M) * ELX / (MLOA * CPM)
N2 = 2
N1 = (I / (J - 1))
I1 = N2 + 1
DO 1 J = 1, N1
DO 1 I = 1, I1
1 CONTINUE
READ IN PRESSURE DROP PARAMETERS FOR BED, COLLECTOR, AND DUCT WORK
READ, IG
WRITE(K, 110) IG
READ, EFP
WRITE(K, 111) EFP
READ, ACC
WRITE(K, 122) ACC
READ, GC
WRITE(K, 133) GC
READ, LC
WRITE(K, 144) LC
READ, LD
WRITE(K, 155) LD
READ, N1
WRITE(K, 166) N1
READ, N2
WRITE(K, 177) N2
337 DO 3 I = 1, N2
3 CONTINUE
105

C READ IN THE NUMBER OF DEGREE-DAYS FOR EACH MONTH TO BE USED IN
C FINDING THE SPACE-HEATING LOAD

READ DAY
WRITE (5, 993) DAY

334 READ DAY
WRITE (5, 115) DAY
READ TA46
WRITE (6, 131) T Amp
READ HAVG
WRITE (6, 1135)
WRITE (5, 14) HAVG
READ K1/4V
WRITE (3, 1021) XTA VG
READ DP
WRITE (5, 1135)
WRITE (5, 133) DP
READ TR00CD
WRITE (5, 1235) TR00CD
READ DAY0N
WRITE (5, 25) DAY0N
IF (PROG*1.E4*) GO TO 333

C PUMP PRESSURE DROP AND PUMPING COST DETERMINED

WRITE (5, 35) LIN
A = ((1.75/(6*1.35)) + ((1.5-0.05)/(DP)**2.3)) * L3
B = (1.25 * (1.5-0.05) - L3)/(60*1.35) * DP
DPLPB = (4*UG*V01)/(32.2*(3600*(1.5-0.05))) + (3*P*CA*(V01-2.0))
A = (32.2*15.35) *(2*2.3)
DPB = (DPLPB/1-E1) *(27*673)
WRITE (5, -1) DPB

490 FORMAT ("**PRESS DROP OF BEC IN INCHES OF WATER= ",&13.4)
WX = DPLPB/RHDA
P = DPLPB*0.381
P1 = P/778.
P01 = P1*93
P0B = (P01-15.35)/(1.0*1.10)
PC = PCON * DAYIN
PPCD = PCON / EFF
WRITE (E, 1, (*) PPCB

41 FORMAT("" "", "PUMPING COST OF PED PER MONTH IN DOLLARS=", 10.4)

6 COLLECT PRESSURE DROP AND PUMPING COST DETERMINED

DH = (2. * ACC / 10.) / (ACC + PC)
VD = (2. * 1.) / (3. * ACC)
WEG = (WGA + VEG) / 10
MLPC = 4. * HVAC + .3 * 
QELPC = ((.73 + LC) * (32.2 * (35.6 * .".3", )) * (MLPC + 2.* (ACC + CO)) / 

32.3 (ENC-*.45) * (4.1) * ((ACc+BC) +.3))
PC = (WELPC / 1.4) * (ACc+BC)) * 22.5

WRITE (K, 22) PPC

2 FORMAT("" "", "PRESSURE DROP IN COL. IN INCHES OF WATER=", 10.4)
PC = (3.3LPC * FLGC) / (H YA
PC = (PC / 78.) / (H4 / (ACc+LC))
PC = (PC / 23.3) / (1.033768)
PC = (PC / 30.0) + YH
PC = (PC / VEF) + EPC
WRITE (K, 22) PPC

33 FORMAT("" "", "PUMP COST OF COL. PER MONTH IN DOLLARS=", 10.4)

3 DUCT WORK PRESSURE DROP AND PUMPING COST DETERMINED

V = ((4. * ACC) / (31. + (DID + 2.*))) / 75
F = (RHOA * VD) / 4
SPLPA = (((.23 + VD)) / (17 + RHA) / (VD * 2.)) / (2.0 * (32.2)
R(8), (.2*2)) / (70)
SPLP = (TNR + 1.) + RHA * (VD * 2.)) / (2.2 * 32.2 * (REL + .2))
SPLP = SPLPA + EPLP72
PC = (PC / 78.) / (H2
WRITE (K, 22) PPC

41 FORMAT("" "", "PRESSURE DROP IN DUCT IN INCHES OF WATER=", 10.4)
PC = (3.3LPC * FLGC) / (H YA
PC = (PC / 78.) / (H4 / (ACc+BC))
PC = (PC / 23.3) / (1.033768)
PC = (PC / VEF) + EPC
PC = (PC / 30.0) + YH
PC = (PC / VEF) + EPC
WHITE (6, 40) I100
45 FORMAT("".,1X,""PUMP COST OF DUCT WK. PER MONTH IN $= ",',F1.,4)
3 TOTAL (COLLECTOR AND DUCTS) MONTHLY PUMPING COST IS CALCULATED
TOOS=PPOR +PCC +PPG
WRITE(6,60) TOOS
46 FORMAT("".,1X,""TOTAL PUMP COST PER MONTH IN DOLLARS= ",',F1.,4)
5 FIRST, DETERMINE THE AMOUNT OF RADIATION STRIKING THE COLLECTOR SURFACE.
5 FIND THE DECLINATION (ELTA), THE SUNSET HOUR ANGLE (HS), THE
5 EXTRATERRESTIAL RADIATION ON A HORIZONTAL SURFACE (HJ), THE LONG
5 TERM CLEARNESS INDEX (KAVG), AND THE MONTHLY AVERAGE DAILY DIFFUSE
5 RADIATION ON A HORIZONTAL SURFACE (HAVG)
1 IF (PROG.EQ.0) GO TO 390
375 NINT=CONST*AREA/THICKST
65 R=3.40560
700
100 K=7.2973131
200 LAT=LATO/RAD
400 DEL=7. * 1.5 * SIN(D)
500 DELTA=DELTA / PAT
600 WRITE(6,700) DELTA
700 WS=3.1415926
800 N=13.904
900 IF (KAVG.GT.4) GO TO 40
100 KAVG=KAVG/40
110 IF (KAVG.LE.3) GO TO 5
120 KAVG=KAVG*1.52
130 DAVG=KAVG* (0.73 - (0.32 * KAVG))
GO TO 10
5      DAYG=AVG*(1,2-(2,KTAVG))
10     CONTINUE
      INTV=INTV+1
      DO 5 L=1,INTV,1
      R=AVG,1,KT(L)
      N=K(L)
      WRITE(*,1234) N,KT(L)
5     CONTINUE
      WRITE(*,1025)
      WRITE(4,1030) AVG
      S=SN/PAD
      GAMMA=3GAMMA/PAD
      DO 2 N=1,N7,1
      WN=-3.*N1;7
      CNUM=0;J
      QNUM=0;J
2    CONTINUE
   100   NO LOOP 1 IS COMPLETED FOR EACH HOUR OF THE DAY
   101   HR=1.?.
   102   HR=1.
   103   DO 1 J=1,N4
   104     IF(HP.GT.12.) GO TO 26
   105     HP=HP
   106     GO TO 27
   107   26   HR=4P-12.
   108   27   IF(HP.GT.12.) GO TO 28
   109     HP=HP
   110   28   GO TO 29
   111   29   HP=HP+1.
   112   26   WRIT(E,S,1070) 4R,HP
   113   28   HP=HP+1.
   114   29   HP=HP+1.
C FIND THE ANGLE BETWEEN THE COLLECTOR SURFACE NORMAL AND THE SUN'S RAYS
C (THETA1) AND THE SOLAR ZENITH ANGLE (THETA2)
C TFG(9GAMMA)*Z,110
AS1 = COS(LAT) / (SIN(GAMMA) * TAN(S))
AS2 = SIN(LAT) / TAN(GAMMA)
AS3 = S1 + S2
BS1 = COS(LAT) / TAN(GAMMA)
BS2 = SIN(LAT) / (SIN(GAMMA) * TAN(S))
BS3 = TAN(DELTA) * (BS1 - BS2)
WS0 = SQRT(((AS1 * BS1) - (ASR * BS) + 1) * )
WS1 = COS(((ASR * BS) + ABSOT) / (ASR * BS + 1))
WS2 = COS(((ASR * BS) - ABSOT) / (ASR * BS + 1))
WS3 = WS0
IF(WD .LT. WSS) GO TO 13
IF(WD .GT. WSS) GO TO 13
GO TO 21
AS1 = COS(LAT) / (SIN(GAMMA) * TAN(S))
AS2 = SIN(LAT) / TAN(GAMMA)
AS3 = S1 + S2
BS1 = COS(LAT) / TAN(GAMMA)
BS2 = SIN(LAT) / (SIN(GAMMA) * TAN(S))
BS3 = TAN(DELTA) * (BS1 - BS2)
WS0 = SQRT(((AS1 * BS1) - (ASR * BS) + 1) * )
WS1 = COS(((ASR * BS) + ABSOT) / (ASR * BS + 1))
WS2 = COS(((ASR * BS) - ABSOT) / (ASR * BS + 1))
WS3 = WS0
WSS = WS4
IF(WD .LT. WSS) GO TO 13
IF(WD .GT. WSS) GO TO 13
GO TO 21
WA=ASIN(WS)
IF(WA=180) GO TO 13

21
A1=SIN(DELTA)*SIN(LAT)*COS(S)
A2=SIN(DELTA)*COS(LAT)*SIN(S)*COS(GAMMA)
A3=COS(DELTA)*COS(LAT)*COS(S)*COS(W)
A4=COS(DELTA)*SIN(LAT)*SIN(GAMMA)*SIN(W)
A5=A1-A2+A3+A4+A5
THETA=AATG(A)
THETA=THETA*RA
WRITE(6,138)THETA
IF(THETA,.LT.,150) GO TO 13
THETA=ACOS(COS(LAT)*COS(DELTA)*COS(W)+SIN(DELTA)*SIN(LAT))
THETA=THETA*RA
WRITE(6,138)THETA
THETA=THETA/RAD

C FIND THE MONTHLY AVERAGE RATIO OF TOTAL RADIATION ON A TILTED SURFACE TO C TOTAL RADIATION ON A HORIZONTAL SURFACE (RADS):
RADS=(PI/2)*((COS(W)-COS(WS))/(SIN(W)-(WS*COS(WS))))

C FIND THE MONTHLY AVERAGE INSTANTANEOUS TOTAL RADIATION ON A TILTED C SURFACE (ITTR):
ITTR=RADS+RADS
IF(ITY,GE,7.5) GO TO 12
ITY=0

C FIND THE MONTHLY AVERAGE INSTANTANEOUS TOTAL RADIATION ON A TILTED C SURFACE (ITTR):
ITTR=RADS+RADS
IF(ITY,GE,7.5) GO TO 12
ITY=0
GO TO 13
CONTINUE
WRITE (6,1051)
WRITE (6,1052)
C Second, find the amount of radiation utilized by the collector system
C find the transmittance (TAU) of the covers, accounting for reflection
C (TAU) and for absorptance (TAUA)

12 IF(THETAR,GT,1.0) GO TO 15
THETAC=THETAR
GO TO 13

13 THETAC=ASIN((SIN(THETAR))/ETAC)
THETAM=THETAC+RAC

15 CONTINUE
IF(THETAR,GT,1.0) GO TO 25
PHO=((ETAC-1.0)/(ETAC+1.0))**2
TAU=(1.-PHO)/(1.+(2.*PHO-1.0)*RHO)
GO TO 17

23 TCA=A(1.(THETAC-THETAR))
TCA=THETAC+THETAR
PHO1=(SIN(TC)/SIN(TCC))**2
PHO2=(TAN(TC))/TAN(TCC)**2
T1=(1.-PHO1)/(1.+(2.*PHO-1.0)*PHO1))
T2=(1.-PHO2)/(1.+(2.*PHO-1.0)*PHO2))
TAU=(TAU+T1)/2.

71 TAU=EXP(-((PHO+THC)/COS(THETAC)))
TAU=TAU**1.1TAIJ
WRITE (6,1063) TAU
C find the effective transmittance-absorptance product (TAUAA)
OTHER=(COS(THETAD))**(1/5)
ALPHA=ALPHA+OTHER
DO=CN-2.
IF(TDC,GT,47,45

45 PHD=15.
GO TO 50

47 PHO=24
GO TO 59

`45  RH0= .29
51  CONTINUE
52  TAU= (TAU4*THETA)/(1.0 - ((1.0 - ALPHA) * RH0))
55  IF(DN)55,6,65
55  B1=27
56  B2=1.7
57  B3=1.9
58  GO TO 76
61  B1=1.7
62  B2=1.2
63  B3=1.9
64  GO TO 76
65  B1=1.4
66  B2=1.5
67  B3=1.9
69  IF(THETAR=SF1,1.) GO TO 42
72  T1= ((1.0 + RH0)/(1.0 + RH01)) * EXP((-EC*THC)/COS(THETAC))
73  T2= ((1.0 + RH0)/(1.0 + (3.0*RH01))) * EXP((-2.0*EC*THC)/COS(THETAC))
78  GO TO 13
42  SD1= (SIN(T2)*SIN(T2))/SIN(T2)
43  SD2= (TAN(T2)/TAN(T22))**2
46  T1= (1.0 + RH0)/(1.0 + RH01)
47  T2= (1.0 + RH02)/(1.0 + RH02)
49  T3= (T1 + T2)**2
50  T1=T1*EXP((-EC*THC)/COS(THETAC))
51  T2= (1.0 - RH01)/(1.0 + (3.0*RH01))
52  T2= (1.0 - RH02)/(1.0 + (3.0*RH02))
54  T2= (T1 + T2)**2
55  T2=T2*EXP((-2.0*EC*THC)/COS(THETAC))
58  TAU= TAU4*(1.0 - TAU1)*(B1 + (32 + T1) + (13*T2))
61  SF= (0.0, 0.0, 0.0) * TAN(THETAR)
62  WRITE(6,177) SF
65  TAU= .99*(1.0 - SF) * TAU4
68  WRITE(6,177) TAU4
71  CONTINUE
75  GO TO 42

40  FIND THE RATIO OF CRITICAL RADIATION TO MONTHLY AVERAGE INSTANT TOTAL
6  TOTAL RADIATION (CRIT) AND THEN INTEGRATE THE AREA UNDER THE RT CURVE.
0 TO FIND THE UTILIZABILITY
PORIT = (U - (2 - TAN 2))/ (TAN 2 + U)
WRITE ( 3, 131)
WRITE ( 2, 131)
WRITE ( 3, 131) ROKIT
TNI = 1, INT
K = 0
E = ".
DO " U = 1, INT, 1
K = K + 1
IF (RKT > .00) GO TO 20
DHE = (1.32 * 2 * KT - 3.2) * KT * KT + 1.
GO TO 90
10 DHE = 1
65 CONTINUE
T = 2 / KT
C = 3. (1 - COS (S)) * PHUG
S = (1 - (D2) * D2) * M + (1 + COS (S)) * D2 * D2 + S
I DTH = (1 - AVG) * KT / KTAVG
AVG(K) = 1. INT
IF (K > 20) 30 TO 31
IF (K > 10) 30 TO 91
UT(K) = (AVG(K) - ROCIT) * TNI
GO TO 90
30 UT(K) = (AVG(K) - ROCIT) * TNI / 2.
32 CONTINUE
IF (UT(K) > 0) GO TO 90
UT(K) = 0
30 CONTINUE
K = K + 1
E = +1 VT
75 CONTINUE
UTIL =
DO 30 M = 1, INT, 1
L = 1
30 UTIL = UTIL + UTIL(L)
90 CONTINUE
WRITE(6111) UTIL
C FIND THE ENERGY TRANSFERRED TO STORAGE (OAVG)

G=FLOW/AC
FRE=YE=FR/(1.+(FR-U)/(G+CPU))**((1./EFF)-1.))
OAVG=AC*FREME*TAUAM*ITT*UTIL
WRITE(5,112) OAVG
GO TO 14

14 OAVG=1.
WRITE(5,1110)
CONTINUE

C THIRD, FIND THE AMOUNT OF ENERGY TRANSFERRED TO STORAGE.
C WHEN PROG EQUALS 0, THE SOLAR AIR HEATING PROGRAM IS IN EFFECT AND THE
C WATER-SYSTEM ANALYSIS IS SURPASSED

IF (PROG.EQ.0.) GO TO 213
HLOAD=FLOW((1.-DAYFLOW+CPS*(T3-TTN))*.335
IF (PROG.EQ.0.) GO TO 16
IF (PROG.EQ.2.) GO TO 17
IF (PROG.EQ.3.) GO TO 19
GO TO 13

C SYSTEM 1 IS A SIMPLE WATER HEATING SYSTEM WITH NO BACKUP HEAT
15 T4=T3+(OAVG*(JAST*(T3-TROOM))/MASS
T3=T4
GO TO 11

C SYSTEM 2 IS A WATER HEATING SYSTEM WITH A CONVENTIONAL WATER HEATER THAT
C PRODUCES SUPPLY FROM A SOLAR HEATED STORAGE TANK
17 T4=T3+(OAVG-HLOAD-(JAST*(T3-TROOM))/MASS
T3=(1*T*T)/2
T3=T8
OAVX=FLOW((1.-DAYFLOW+CPS*(144.-TF))*.335
TF(OAVX*GTS)--) GO TO 41
OAVX=1.
GO TO 41

C SYSTEM 3 IS LIKE SYSTEM 2 EXCEPT THAT AN ANTI-FREEZE TYPE SOLUTION IS
C USED AS THE COLLECTOR FLUID
19 T4=T3+(OAVG-HLOAD-(JAST*(T3-TROOM))/MASS
T3=(T3+T4)/2

114
T3=T4
Q1UX=FLOW(I)*DAYFLOW*CPS*(140.-T5)*8.335
IF(Q1UX.GT.4.5) GO TO 41
Q1UX=Q3
41 WRITE(6,1115) T4
C SPACE-HEATING LOAD IS CALCULATED ALONG WITH VARIOUS "IF" STATEMENTS
C NECESSARY TO OBTAIN THE AMOUNT OF AUXILIARY ENERGY REQUIRED
21 TSHL=(UA*DLY)/DAYMUN
RH11=((AX*L1)/(FLOAT(KJ-1)))*RHCS*(1.-EG)
Q42=(Q41/2.0)
IF(Q4AVG.LE.Q42) GO TO 24
IF(Q4AVG.GT.Q42) GO TO 241
2 MODE 1
T3=ROOM
QLOST=0.0
DO 226 I=1,KJ
QLOSS1=((KINS*AIN5)/(FLOAT(KJ)*TINS))*(TB(I,J)-TOP)*DELTAS
IF((I.EQ.1).OR.(I.EQ.KJ)) GO TO 211
QLOSS1=QLOSS1/(CM1+CPS)
GO TO 214
214 TBLLOSS=QLOSS1/(RA2*CPS)
219 QLOSS=QLOSS+QLOSS31
225 TRI(J,J+1)=TRI(J,J)-QLOSS1
Q1UX=0.8HL-0.8VG-0.8LOST
GO TO 18
C MODE 2 AND REJ REVERSAL
241 Q1UX=1.0
DELTAS=(Q1UX*DELTA5)/QAVG
WRITE(6,245) DELTAS
DELT2=1.-DELT1
WRITE(6,125) DELT2
DELT2=DELT2
DELT2=DELT2
247 QLOSTT=1.0
DO 274 I=1,KJ
QLOSS2=((KINS*AIN5)/(FLOAT(KJ)*TINS))*(TE(I,J)-TOP)*DELT2
GO TO 247
274 END
IF((I.EQ.1) .OR. (I.EQ.KJ)) GO TO 215
TALoss2=ALoss2/(A+1-CPS)
GO TO 217
215 TALoss2=ALoss2/(A+2-CPS)
217 ALost=TALost+Tloss2
218 TR(I,J)=TR(I,J)-TALoss2
IF(FLAG2.EQ.1) GO TO 260
TCC(I)=TR0
IF((J.EQ.1) .AND. (J.EQ.KJ)) GO TO 231
DO 2: I=1,KJ
MN=KJ-J
27 TBT=M+1,J)=TBT(I,J)
GO 218 I=1,J
218 TBT(I,J)=TBT(I,J)
251 FLAG2=1.
FLAG2=.F.
GO TO 27.
26 TCC(J)=TCC(KJ,J)
2 TEMPERATURE OUT OF THE COLLECTOR IS DETERMINED
27 TCC(J+1)=(TAVG+(4*FLOA+CPA))+TCC(J)
WRITE (5,34) TCC(J+1)
TA(J+1)=TCC(J+1)
GO TO 65.
24 CONTINUE
IF((J.EQ.1) .AND. (J.EQ.KJ) .AND. (J.LE.1)) GO TO 245
IF(FLAG3.EQ.1) GO TO 249
C MODE 3 AND FREE REVERSAL
DO 2: I=1,KJ
MN=K-J-1
24 TBT=M+1,J)=TBT(I,J)
GO 217 I=1,J
218 TBT(I,J)=TBT(I,J)
FLAG2=.
FLAG2=1.
2 DELTA=4.*J
56 TA(I,J+1)=TR0
DELT1=DELTA**2*THETA**2
TIC(1)=TROOM
6A T=TIC(1)
C2=(N*FLDA*FLPA*DELTA)/(1.+EG)*RDOS*CPS*(AX*NELX)
DO 23 I=1,4
C IMPLICIT FINITE-DIFFERENCED AIR TEMPERATURE EQUATION AS A
C FUNCTION OF TIME AND AIR POSITION IS DEVELOPED. ENERGY LOSSES FROM
C THE RED ARE ALSO TAKEN INTO ACCOUNT
TA(I+1,J+1)=(1.+C1-C2)/(1.+C1+C2)*TA(I,J+1)+
2*(C1)/(1.+C1+C2)*T8(I,J)+(C3/FLDA*FLPA*C2)*
3*(KINS*A1NS/TINS)*(T8(I,J)-T0P)*(DELTA)/(1.+C1+C2)
DO 23 I=1,4
C IMPLICIT FINITE-DIFFERENCED BED TEMPERATURE EQUATION AS A FUNCTION
C OF TIME AND POSITION IS ALSO DEVELOPED
T8(I,J+1)=(C1-C2)/(1.+C1+C2)*T8(I,J)+
5*(C1)/(1.+C1+C2)*(T8(I,J)+3*(T8(I,J)+3*(CINS*AINNS/TINS)+
6*(T8(I,J)+3*(CINS*AINNS/TINS)+
23 CONTINUE
C RED TEMP AT X=0, CURVE FIT SOLUTION
TQ(I,J+1)=TQ(I,J+1)-(3+TR(3,J))/2*TR(3,J+1)
IF (TQ.GT.TS) GO TO 10
TQ=TQ(K,J+1)
WRITE (5,63) TQ
C666=
C666=(TQ+100)*C666(TQ-TROOM)
IF (TQ.GT.TS) GO TO 67
DELTA=DELTA+C666
T8(T8+6)*DELTA GO TO 3:3
GO TO 567
57 C AUX=C8
DELTA=(C8SIL+DELTA)/3*POSH
WRITE (5,64) DELTA
DELT4=DELT4+DELTA
DO 23 I=1,KJ
QLASS=((KINS*AIN*LS)/(FLOAT(KJ)*TIME)) *(TB(I,J+1)-TB)*L
IF((I=EO.1).OR.(I.EQ.KJ)) GO TO 279
TBLOSS=QLASS/(341*CPS)
GO TO 279
278
TBLOSS=QLASS/(342*CPS)
273 T3(I,J+1)=T3(I,J+1)-TBLOSS
235 CONTINUE
GO TO 19
19 AUX=OHL-DEH
GO TO 13
256 OHLX=OHL
T3=T3OM
19 CONTINUE
WD=WD+.2525
WRITE(6,131) AUX
IF((JT,EO.1).AND.(IJ,EO.1)) GO TO 1
OUH=OUH+DEV
OAU=OAU+DEUX
1 CONTINUE
IF((JT,EO.1).AND.(IJ,EO.1)) GO TO 332
WRITE(6,131) OUH
332 T3(I,J)=TB(I,J)
IF((IJ,EO.1).AND.(IJ,EO.1)) GO TO 112
GO TO 2
112 J=J+1
GO TO 20
2 CONTINUE
MONTHLY SPACE-HEATING LOAD AND MONTHLY SOLAR ENERGY
WRITE(6,1235) TOT
IF((S=EO.1)) GO TO 271
MLOW=DITFL0W*DAYM0N
WRITE(6,1310) MLOW
FOURTH, DO A COST ANALYSIS TO DETERMINE THE ECONOMIC FEASIBILITY

THE COST ANALYSIS IS A LIFE CYCLE COST ANALYSIS PROGRAM

READ*,FC1
IF(END(*LINFUT),NE.,0.) GO TO 99
WRITE(6,730) FC1
READ*,FC2
WRITE(6,731) FC2
READ*,TOC
WRITE(6,732) TOC
READ*,STCA
WRITE(6,733) STCA
READ*,LICA
WRITE*(6,74.*) LICA
READ*,TMAXC
WRITE*(6,745) TMAXC
READ*,AMIR
WRITE*(6,210) AMIR
READ*,YRMORT
WRITE*(6,215) YRMORT
READ*,DNFMT
WRITE*(6,211) DNPMT
READ*,BUFCOST
WRITE*(6,218) BUFCOST
READ*,CONFCST
WRITE*(6,212) CONFCST
READ*,QUEFF
WRITE*(6,221) QUEFF
READ*,CONEFF
WRITE*(5,230) CONEFF
READ*,TAXRT
WRITE*(6,235) TAXRT
READ*,TAXRKT
WRITE*(5,240) TAXRKT
READ*,XTRINS
WRITE*(5,245) XTRINS
READ*,GENINF
WRITE*(6,260) GENINF
READ*,FLINF
WRITE*(5,265) FLINF
READ*,DISCRT
WRITE*(6,260) DISCRT
READ*,ANALTM
WRITE*(6,285) ANALTRM
C COST ANALYSIS MODIFIED FOR AIR HEATER
IF(PROG.EQ.4.) GO TO 505
VR1FL=(YFL/O/1.05)*CGOST/CNNEFF
GO TO 51
505 VR1FL=(OS/1.05)*CGOST/CNNEFF
515 WRITE(6,276) VR1FL
READ*,ARACST
WRITE(6,2126) ARACST
READ*,APINCST
WRITE(6,2125) ARINCST
C THE INFLATION-DISCOUNT FACTOR IS USED TO REDUCE COSTS TO PERSENT YEAR COSTS
CALL D1(ANALT,W,FLINF,DISCRT,AA)
CALL D1(ANALT,W,GENINF,DISCRT,BB)
IF(YM=t,GO TO 115
CALL D1(YMORT,AMIR,DISCRT,CC)
CALL D1(YMORT,J,J,DISCRT,FF)
GO TO 116
115 CALL D1(W,ANALT,W,AMIR,DISCRT,GG)
116 CONTINUE
CALL D1(YMORT,J,J,AMIR,EE)
WRITE(6,346) AA
WRITE(6,346) AA
WRITE(6,346) AA
WRITE(6,346) AA
WRITE(6,346) EE
WRITE(6,346) FF
G=GG/EE
WRITE(6,2.36) GG
HH=GG*(CG*(AMIR-(1.0/EE)))
WRITE(6,2.36) HH
IT=DNMT+(1.0-0.1)*(GG*(HH+TAXBKTI)))
WRITE(6,2.36) II
JJ=(XTRINS*BB)
WRITE(6,2.36) JJ
KK=TAXRT*BB*(1.0-TAXBKTI)
GG=II JJ+KK
WRITE(6,21.0) KK
WRITE (6, 211F) 00
RTC=(ARFACST*AC)+ARINCST
IF(RTC.GT.2LTCA) GO TO 600
3 INVESTMENT IN SOLAR MINUS TAX CREDIT
DFTC1=((FTC1+OTC)*(STCA)+(FTC2+OTC)*(RTCA-STCA))
IF(DFTC1.GT.TMAXC) GO TO 612
R2=RTC-DFTC1
GO TO 611
50 F DFTC2=((FTC1+OTC)*(STCA)+(FTC2+OTC)*(LTCA-STCA))
IF(DFTC2.GT.TMAXC) GO TO 612
R3=RTC-DFTC2
GO TO 611
612 R7=RTC-TMAXC
51 IF(PROG,E0,L) GO TO 630
R2=UNTOT/YFLOW
P4=YFLOW*(1.-R2)*BUFCST/(BUEFF*(10.*E6))
GO TO 641
53 R2=UNTOT/OSHT
P4=OSHT*((1.-R2)*BUFCST/(BUEFF*(10.*E6))
54 R5=(YR1F*L1)*AA
R5=00*R3
F3=R3-R6
WRITE (6, 216F) R2
WRITE (6, 211F) F3
WRITE (6, 211F) F4
WRITE (6, 211F) F5
WRITE (6, 211F) F6
99 CONTINUE
74 FORMAT("" TIME TO HOUSE, MODE 2",35X,F3,5)
742 FORMAT("" TIME TO 9ED, MODE 2",35X,F3,5)
744 FORMAT("" TEMP OUT OF COLLECTOR",35X,F9,3)
746 FORMAT("" TIME TO SATISFY LOAD",35X,F8,5)
748 FORMAT("" T A AT EXIT, MODE 3",35X,F9,3)
71. FORMAT("FEDERAL TAX CREDIT PERCENTAGE INITIAL",3X,F8.4)
72. FORMAT("FEDERAL TAX CREDIT PERCENTAGE SECONDARY",3X,F8.4)
73. FORMAT("OHIO TAX CREDIT PERCENTAGE",3X,F6.4)
74. FORMAT("INITIAL TAX CREDIT AMOUNT",3X,F10.2)
75. FORMAT("SECONDARY TAX CREDIT AMOUNT",3X,F10.2)
76. FORMAT("MAX TAX CREDIT AMOUNT",3X,F10.2)
77. FORMAT("MINUTE TIME INCREMENT FOR DISCHARGE",3X,F7.3)
78. FORMAT("EPSILON FOR TIME INCREMENT",3X,F6.5)
79. FORMAT("MINIMUM PUMP ENERGY",3X,F9.2)
80. FORMAT("STRENGTH CONDUCTANCE",3X,F9.3)
81. FORMAT("TIME STEP",4X,F6.3)
82. FORMAT("VOLUMETRIC FLOW RATE (CFE)",4X,F5.3)
83. FORMAT("RED HEIGHT",4X,F6.3)
84. FORMAT("RED WIDTH",6X,F6.3)
85. FORMAT("PARTICLE DIAMETER (FT)",5X,F10.3)
86. FORMAT("PLANT ANNUAL NUMBER",5X,F9.3)
87. FORMAT("CONDUCTIVITY OF AIR",3X,F9.3)
88. FORMAT("MOENTUM DIFFUSIVITY",4X,F9.3)
89. FORMAT("SPECIFIC HEAT (AIR)",4X,F9.3)
90. FORMAT("OUTSIDE BARE TEMPERATURE",4X,F9.3)
91. FORMAT("SOLID DENSITY",4X,F9.3)
92. FORMAT("SOLID SPECIFIC HEAT",4X,F9.3)
93. FORMAT("RED LENGTH",4X,F6.2)
94. FORMAT("NUMBER OF NODES",4X,F3.0)
95. FORMAT("VOID FRACTION OF BARE",4X,F7.3)
96. FORMAT("NUMBER OF DEGREE DAYS",4X,F9.3)
97. FORMAT("DAY OF YEAR",9X,F5.0)
98. FORMAT("DECLINATION (DEGREES)",5X,F8.4)
99. FORMAT("EXTREME RADIATION ON A HORIZON SURFACE",13X,F9.4)
100. FORMAT("SUNSET HOUR ANGLE (DEGREES)",3X,F8.4)
101. FORMAT("LONGTERM CLEANNESS INDEX",37X,F5.4)
102. FORMAT("MONTHLY AVERAGE DIFFUSE RADIATION ON A")
103. FORMAT("HORIZONTAL SURFACE",2X,F8.4)
104. FORMAT("ANGLE BETWEEN SUN AND COLLECTOR NORMAL",22X,F8.4)
105. FORMAT("SOLAR ZENITH ANGLE",12X,F8.4)
106. FORMAT("SOLAR ZENITH ANGLE",12X,F8.4)
124 FORMAT(" "EFFICIENCY OF HEAT EXCHANGER",3F10.2)
125 FORMAT(" "TYPE OF SYSTEM IN USE",3F10.2)
126 FORMAT(" "AMOUNT OF FLUID IN STORAGE TANK (GAL)",24X,F7.1)
127 FORMAT(" "SUFFIX AREA OF STORAGE TANK (SQ FT)",2Fx,F5.1)
128 FORMAT(" "INSULATION THICKNESS ON STORAGE TANK (IN)",20X,F6.2)
129 FORMAT(" "SUFFIX AREA OF STORAGE TANK INSULATION",24X,F4.2)
130 FORMAT(" "COEFFICIENT OF SPECIFIC HEAT OF STORAGE FLUID",13X,4.2)
131 FORMAT(" "NUMBER OF DAYS PROGRAM IS TO RUN",3X,12)
132 FORMAT(" "KT(",12") = ",F6.4)
133 FORMAT(" "NUMBER OF DAYS IN THE MONTH",38X,4.0)
134 FORMAT(" "TOTAL USEFUL SOLAR ENERGY FOR MONTH",24X,F10.1)
135 FORMAT(" "AVERAGE DAILY FLOW USED (GAL)",31X,F10.1)
136 FORMAT(" "MONTHLY FLOW USED (GAL)",33X,F11.1)
137 FORMAT(" "BACKUP HEAT USED THIS 10R (BTU)",27X,F11.1)
138 FORMAT(" "DAILY BACKUP HEAT USED (BTU)",31X,F11.1)
139 FORMAT(" "MONTHLY BACKUP USED (BTU)",26X,F11.1)
140 FORMAT(" "MONTHLY LOAD USED (BTU)",35X,F11.1)
141 FORMAT(" "YEARELY BACKUP ENERGY USED (BTU)",25X,F13.1)
142 FORMAT(" "PERCENT SOLAR ENERGY",33X,F9.1)
143 FORMAT(" "YEARELY SOLAR ENERGY USED (BTU)",26X,F13.1)
144 FORMAT(" "YEARLY LOAD USED (BTU)",34X,F13.1)
145 FORMAT(" "NUMBER OF MONTHS PROGRAM IS TO RUN",31X,12)
146 FORMAT(" "YEARLY SOLAR ENERGY ACCOUNTING FOR TANK LOSS",12X,F13.1)
147 FORMAT(" "ANNUAL MORTGAGE INTEREST RATE",3F5.4)
148 FORMAT(" "TERM OF MORTGAGE",7F10.0)
149 FORMAT(" "DOWN PAYMENT , AS FRACTION OF INVESTMENT",24X,F6.4)
150 FORMAT(" "BACKUP FURNACE COST",4X,F10.2)
151 FORMAT(" "CONVENTIONAL FURNACE COST",34X,F11.2)
152 FORMAT(" "EFFICIENCY OF SOLAR BACKUP FURNACE",31X,F6.2)
153 FORMAT(" "EFFICIENCY OF CONVENTIONAL SYSTEM FURNACE",24X,F4.2)
154 FORMAT(" "TAX RATE, FRACTION OF INVESTMENT",32X,F6.4)
155 FORMAT(" "EFFECTIVE INCOME TAX Bracket",35X,F9.3)
156 FORMAT(" "EXTRA INS AND MAINT COST, FRACT OF INVESTMENT",21X,F4.2)
157 FORMAT(" "GENERAL INFLATION RATE PER YEAR",33X,F5.3)
158 FORMAT(" "FUEL INFLATION RATE PER YEAR",35X,F6.3)
159 FORMAT(" "DISCOUNT RATE",51X,F5.3)
STOP; "END OF PROGRAM"

END

DELIMITER DISC, DISC

IF (RTIMF>0.0) GO TO 105

RETURN

RETURN

RETURN

RETURN
Vita

Daniel Bartholomew Fant was born on 2 July 1957 in Queens, New York. He attended Newtown High School in Connecticut and graduated in June, 1975. He then attended the University of Connecticut and graduated in May, 1979 with the degree of Bachelor of Science in Mechanical Engineering. Upon graduation, he received a commission in the USAF through the AFROTC program. He then entered the School of Engineering at the Air Force Institute of Technology in June, 1979.

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Thesis was typed by Patricia Browder.
Computer Simulation of Solar Air Heating Systems Using Rock Bed Thermal Storage Units

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Solar Air Heating System Simulation
Rock Bed Thermal Storage
Fully-Implicit Numerical Scheme

This thesis is concerned with the analysis and design of solar air heating systems utilizing rock beds as thermal storage units. A computer simulation model capable of estimating the response of both the solar collector and the rock bed is described.
Differential equations describing the rock bed were approximated in a finite-difference form and solved numerically on a digital computer. The temperature of both the solid (rock) and the fluid (air) is determined as a function of time and distance along the bed. The simulation required both charging and discharging of the rock bed for time-varying inlet fluid temperatures. The numerical method used to solve the rock bed equations proved to be stable and convergent and showed satisfactory agreement in comparison to an analytical solution for constant-inlet air temperatures. A cost-analysis was also incorporated within this program; by varying the collector area, one could determine the optimum collector size for maximum savings. Pressure drop relationships for flat-plate collectors, duct work and packed beds were used to determine operating costs. The particular air system tested proved to be cost effective when compared with natural gas fuel costs for an economic term of 20 years.