NONLINEAR THEORY AND EXPERIMENTAL OBSERVATIONS OF THE LOCAL COL-ETC(U)

JAN 81 M J KESKINEN, E P SZUSZCZEWICZ
Nonlinear Theory and Experimental Observations of the Local Collisional Rayleigh-Taylor Instability in a Descending Equatorial Spread-F Ionosphere

M. J. Keskinen and S. L. Ossakow
Geophysical and Plasma Dynamics Branch
Plasma Physics Division

E. P. Szuszczywicki and J. C. Holmes
E.O. Hulburt Center for Space Research
Space Science Division

January 8, 1981

This research was sponsored in part by the Defense Nuclear Agency under Subtask 99QAXX041, work unit 21, and work unit title "Plasma Structure Evolution," and under subtask 125AAAXX640, work unit 12, and work unit title "Plasma Probe Data," and the Office of Naval Research.
**Title:** Nonlinear Theory and Experimental Observations of the Local Collisional Rayleigh-Taylor Instability in a Descending Equatorial Spread-F Ionosphere

**Authors:** M. J. Keskinen, E. P. Suszczewicz, S. L. Ossakow, and J. C. Holmes

**Performing Organization:**
- **Name:** Naval Research Laboratory
- **Address:** Washington, DC 20375

**CONTROLLING OFFICE:**
- **Name:** Defense Nuclear Agency
- **Address:** Washington, DC 20305

**Report Date:** January 1981

**Number of Pages:** 38

**Security Classification:** Unclassified

**Distribution Statement:** Approved for public release; distribution unlimited.

**Supplementary Notes:**
This research was sponsored in part by the Defense Nuclear Agency under subtask S99QAXHC041, work unit 21, and work unit title “Plasma Structure Evolution;” and under subtask 125AAXHX640, work unit 12, and work unit title “Plasma Probe Data;” and the Office of Naval Research.

**Key Words:**
- Nonlinearity theory
- Kwajalein 1979 campaign
- Experimental observations
- Collisional Rayleigh-Taylor instability
- Computer simulations
- Plasma probe data
- Equatorial spread F
- Descending F layer

**Abstract:** The nonlinear evolution of the local collisional Rayleigh-Taylor instability in downward moving equatorial F layers has been studied in a coordinated theoretical and experimental program dealing with actual conditions of bottomside spread F. For ambient bottomside electron density gradient scale lengths \( L \sim 8 \) and \( 25 \) km, we find large percentage depletions and inverse power law spatial power spectra over the intermediate wavelength range \( \lambda \sim 25 \) m - 1 km. In addition we outline a nonlinear theory of the collisional Rayleigh-Taylor instability applicable to an upward or downward moving equatorial F region ionosphere. These results represent the first definitive comparison between experiment and theory with their agreement lending further support to the belief that the collisional Rayleigh-Taylor instability is responsible for large scale size irregularities that occur under conditions of equatorial spread F.
NONLINEAR THEORY AND EXPERIMENTAL OBSERVATIONS
OF THE LOCAL COLLISIONAL RAYLEIGH-TAYLOR INSTABILITY
IN A DESCENDING EQUATORIAL SPREAD-F IONOSPHERE

INTRODUCTION

Recently, coincident rocket and radar observations of equatorial spread-F were made near Kwajalein in the Marshall Islands [Szuszczewicz et al., 1980; Tsunoda, 1980a]. The in situ rocket data showed major plasma depletions distributed throughout a downward moving F-layer with regions of smaller scale in situ irregularities tending to be collocated with positive gradients of electron density. The VHF radar measurements showed that the backscatter "plumes", in addition to having large vertical extent from the bottomside to the topside [see also Kelley et al., 1976; McClure et al., 1977; Woodman and LaHoz, 1976], were identifiable with the plasma depletions [Tsunoda and Towle, 1979; Tsunoda, 1980b] and could be characterized by a growth and decay phase [Tsunoda, 1980a]. The recent rocket/radar comparison [Szuszczewicz et al., 1980] specifically showed that the most intense radar returns in the topside F-region were collocated with the upper region of a decay-phase depletion.

Several features of the small scale irregularity (λ<3m) signatures seen by the radar backscatter can be explained by various plasma kinetic instabilities [for a review, see Ossakow, 1979] which are presumably driven by steep gradients within developing plasma bubbles. On the other hand, much evidence now exists that the large scale irregularities result from a Rayleigh-Taylor instability [Balsley et al., 1972; Haerendel, 1974] driven by the ambient bottomside plasma density gradient. Global [Scannapieco and Ossakow, 1976; Ossakow et al., 1979; Zalesak and Ossakow, 1980] and local [Keskinen et al., 1980] numerical simulations of the collisional Rayleigh-Taylor instability have successfully reproduced the large plasma density depletions (bubbles) and associated spatial power spectra that have been experimentally observed.

The most recent in situ rocket observations [Szuszczewicz et al., 1980] were made in an F-layer that was moving downward, implying the existence of a large scale westward electric field. The importance of this electric field in describing the development of plasma bubbles has been discussed by several authors [Ossakow and Chaturvedi, 1978; Ossakow et al., 1979; Ott, 1978; Anderson and Haerendel, 1979]. However, all previous analytical and numerical studies of the nonlinear collisional Rayleigh-Taylor instability have not included the effects of an upward or downward moving F-layer in realistic geometries, nor have they had the opportunity for an almost ideal comparison with experimental results.

In this paper we study the nonlinear evolution of the collisional Rayleigh-Taylor instability in a vertically convecting F layer using both analytical and numerical techniques. In addition we compare the simulations with the recent in situ rocket data which provided direct measurements of gradient scale lengths and density fluctuation power spectra.

EXPERIMENTAL RESULTS

By 2100 (LT) on 17 July 1979 the bottomside of the F-region at Kwajalein had risen to an approximate altitude of 400 km. The F-layer then began a downward drift with an approximate velocity \( V_D \approx 10 \, \text{m/sec} \) [Szuszczewicz et al., 1980; R. Tsunoda, private communication, 1980] and a simultaneous onset of spread-F. With the F-layer drifting downward and spread-F conditions continuing, a rocket (designated PLUMEX I) was launched (00:31:30, July 17, 1979 LT) when the bottomside F-layer had descended to an altitude below 300 km.
A pair of pulsed probes [Szuszczyewicz and Holmes, 1977; and associated references] were diametrically extended from the forward end of the rocket payload. The sensing elements, constructed from tungsten wire, were isolated from their extension booms by coaxial guard electrodes driven at the same potential as the probes themselves. One of the probes, defined as the I-probe, operated with a baseline voltage $V_B \approx -1v$, yielding net ion baseline current $I_B^i$. The other probe, defined as the E-probe, operated with $V_B^{+} + 2v$, yielding net electron baseline current $I_B^e$. Both probes generated complete current-voltage characteristics in $\tau_s \approx 400 \text{ msec}$, yielding absolute values of $N_e$ and $T_e$ at an approximate 2.5 Hz rate. Maximum $I_B$ sampling occurred at 2048 Hz, resulting in 0.5 meter spatial resolution for relative electron density fluctuations at a vehicle velocity of 1 km/sec. (See Szuszczyewicz and Holmes, 1980 for instrument details and complete payload configuration.)

Figure 1 displays the upleg measurements of relative and absolute electron density as extracted from the pulsed-plasma-probe data. Absolute values of electron density were determined by conventional analyses of Langmuir probe characteristics [Chen, 1965] with appropriate care to eliminate perturbing effects of surface contamination [Szuszczyewicz and Holmes, 1975], density fluctuations [Holmes and Szuszczyewicz, 1975; Szuszczyewicz and Holmes, 1977] and magnetic field effects [Szuszczyewicz and Takacs, 1979]. Analysis of approximately 25 characteristics were executed over the F-layer from 340-560 km. In each case a conversion coefficient $a = N_e [\text{cm}^{-3}] / I_e (V^+)$ was determined so that the $I_e (V^+)$ profile in Figure 1 could be directly scaled to absolute electron densities. This procedure yielded $a = (5.5 \pm 0.5) \times 10^{10}$ electrons cm$^{-3}$ A$^{-1}$. The results in Figure 1 show the F-peak at 375 km with a maximum density of $1.3 \times 10^6$ cm$^{-3}$ ($\pm 10$%). The probe measurements also revealed a number of major depletions ($\delta N_e / N_e^0 \ll 90$%) distributed throughout
the F-region. Each of the depletions has been shown to have its own set of smaller scale irregularities [Szuszczewicz et al., 1980; Szuszczewicz and Holmes, 1980] with the most intense percent fluctuations occurring on the bottomside gradient near 270 km (Region "C" in Szuszczewicz et al., 1980).

The pulsed probe data provided an excellent opportunity for comparison with the numerical simulations of the collisional Rayleigh-Taylor (R-T) instability at intermediate wavelengths (e.g., Keskinen et al., 1980). Attention is focused on the depleted structure on the bottomside F-layer gradient near 270 km which is believed representative of the mid-phase development of the R-T process. (We will follow Szuszczewicz et al., 1980 and refer to this domain as region "C".) An expanded view of this domain (Fig. 2) shows that it is not a single macroscale depletion but a region of contiguous large scale structures extending over a total altitude domain of about 12 km. (Vehicle velocity in region "C" was 2.4 km/sec.)

Typically, computer simulations employ several values for the zero-order gradient scale length

\[ L = \left( \frac{1}{\frac{dN_e^0}{dy}} \right)^{-1} \]

and initialize the simulation with a two-dimensional perturbation. In the work of Keskinen et al. [1980] L was selected to be 5, 10 and 15 km and the perturbation took the form([Chaturvedi and Ossakow, 1977]).

\[ \frac{\delta N_e(x,y,t=0)}{N_e^0} = (10^{-4}) \sin (k_y y) \cos (k_x x) + 2(10^{-6}) \sin (2k_y y) \]
with $k_x$ and $k_y$ being the horizontal and vertical wavenumbers, respectively. Both $k_x$ and $k_y$ were set equal to $2\pi/960$ m in the simulation. In addition, the computation assumed that L was centered at 300 km. Under actual conditions encountered in PLUMEX I, the bottomside F-layer gradient extended from 240 to 290 km.

The question of gradient scale length can be studied in Figure 3 where it is shown that the bottomside gradient (encompassed in the 105-125 sec time frame) is not characterized by a single value of L. Experimentally, L was computed according to $<I_B^\prime> (dI_B^\prime/dy)^{-1}$ with $<I_B^\prime>$ and the altitude derivative determined from sliding linear fits to the $I_B^\prime$ data set. In region "C" (114s ≤ t ≤ 122s) L is seen to vary between 2 and 10 km, whereas adjacent domains (110s ≤ t ≤ 113s and 122s ≤ t ≤ 126s) can be characterized by L = 25 km.

Previous computer simulations [Keskinen et al., 1980], conducted with L = 5, 10 and 15 km, gave evidence that linearly unstable modes saturate by nonlinear generation of linearly damped vertical modes. The results yielded one-dimensional power laws (horizontal and vertical) that vary with a spectral index ($\equiv n$ in $P_n \propto k^{-n}$) between 2.0 and 2.5. To explore this result within the context of region "C", and to provide important comparisons with simulations in the next sections, power spectral analyses were conducted over sliding intervals of 2.4 km. The results, presented in Figure 4, show that the dominant behavior is $k^{-2.5}$ over the range $k = 2\pi/1km$ to $k = 2\pi/25$ m. The $k^{-1.85}$ behavior at t = 116.001 sec is a result of the very sharp density gradient (see region "C" Fig. 2) encompassed by the domain of the spectral analysis. (We note that the spectral analysis is executed on a time (frequency)-domain function $I_B$ and the conversion to k-space is made under the assumption $\lambda = \nu t^{-1}$, where $\nu$ is the vehicle velocity.)
In general we would conclude that our observed large depletions and spatial power spectra support the numerical simulations of Keskinen et al. (1980). Further testing of this support is achieved in the next sections which present simulations with \( L = 8 \) and \( 25 \) km with a downward drifting F-layer model that is more in keeping with the actual experimental conditions. The F-layer time-history can be important since unstable modes require times in excess of \( 4,000 \) seconds to saturate...a time during which the F-layer encountered in PLUMEX I drifted downward in excess of \( 40 \) km.

**MODEL EQUATIONS**

The two fluid equations describing the Rayleigh-Taylor instability in the presence of a zeroth-order horizontal westward electric field \( E_0 \) can be written, after invoking quasineutrality \( (n_e \approx n_i \approx n) \),

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}_\alpha) = -\nabla_t (n - n_0) \tag{1}
\]

\[
0 = -T_e \nabla n_e - e_n \left( -\nabla \varphi + \frac{V_e \times B_0}{c} + E_0 \right) \tag{2}
\]

\[
m_i n_i \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) v_i = -T_i \nabla n_i + e_n \left( -\nabla \varphi + \frac{V_i \times B_0}{c} + E_0 \right) + m_i n_i c \mathbf{v}_i \mathbf{v}_i
\]

\[
\nabla \cdot \mathbf{J} = 0 \tag{4}
\]

\[
\mathbf{J} = n_e \left( \mathbf{v}_i - \mathbf{v}_e \right) \tag{5}
\]

where \( \alpha \) denotes electrons (e) or ions (i) and \( n, \psi, \mathbf{v}, \mathbf{v}_R, \mathbf{v}_i, \mathbf{B}_0, \) and \( g \)
are the density, electrostatic potential, velocity, recombination coefficient, ion-neutral collision frequency, magnetic field, and gravity respectively. In addition, we have written the total electric field \( E = -\nabla \varphi + E_o \). All other symbols retain their conventional meaning. Since we will be interested in studying intermediate wavelengths \( \lambda \sim 100m - 1 km \) we ignore inertial and pressure effects in (1) - (5). Setting \( \nabla \varphi = \nabla \varphi_1 + \frac{m_i g}{e} [Ossakow et al., 1979] \), solving algebraically (2) and (3) for the velocities \( v_e, v_i \), using (4) and the ion continuity equation (1), we find

\[
\frac{\partial n}{\partial t} - \frac{c}{B} (\nabla \varphi \times z) \cdot \nabla n = -v_k (n - n_o) \tag{6}
\]

\[

\nabla \cdot (v \nabla \varphi) = \frac{B}{c} (g x z) \cdot \nabla n + E_o \cdot \nabla \cdot (v_i n) \tag{7}
\]

We have written eq. (6) and (7) in a downward moving frame whose velocity is \( V_D = c \frac{E_o}{E_o^2} B^2 \sim 10 m/sec \) and have adopted the following coordinate system: the magnetic field \( B \) is in the z-direction (north), the y-direction (vertical) denotes altitude with \( g = -gy \) and the x-axis points westward. By linearizing (6) and (7) and writing \( \delta n = n - n_o, \varphi_1 \propto \exp [i(k_x x + k_y y) + \gamma_k t] \), \( k \cdot B = 0 \), and \(|k| L > > 1 \) we find

\[
\gamma_k = \left( \frac{E_o}{v_{in}} + \frac{c E_o}{c B} \right) \left( \frac{k_x}{k_y} \right) \left( \frac{1}{L} - \nu_R \right) \tag{8}
\]

where \( E_o = \frac{+ E_o}{x}, L = \left( \frac{1}{n_o} \frac{\partial n_o}{\partial y} \right)^{-1} \), \( k^2 = k_x^2 + k_y^2 \).

From (8) we note that we recover the usual collisional Rayleigh-Taylor instability if \( E_o > 0 \). The growth rate \( \gamma_k \) is enhanced (reduced) by an
eastward (westward) electric field $E_0$. We will be concerned only with a
westward electric field ($E_0 = E_0$) which leads to a downward moving $F$ layer.

**NUMERICAL SIMULATIONS**

Be defining $n' (x,y) = n (x,v)/n_o (y)$, $\psi_1' (x,y) = \psi_1 (x,y)/BL$, $x' =
x/L$, $v' = y/L$, $t' = \tau/L$ where $n_o (y) = N_o (1 + y/L)$ ($N_o =$ const.) is an
equilibrium solution of (6) and (7) we can write (6) and (7) in dimension-
less form as follows (after dropping primes for clarity):

$$\frac{\partial n}{\partial t} - \frac{\partial \psi_1}{\partial v} \frac{\partial n}{\partial x} + \frac{\partial \psi_1}{\partial x} \frac{\partial n}{\partial y} = n \frac{\partial n_o}{\partial y} \frac{\partial n_o}{\partial x} - \bar{\psi}_1 (n - 1)$$

(9)

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial y^2} + \left( \frac{1}{n} \frac{\partial n}{\partial y} + \frac{1}{n_o} \frac{\partial n_o}{\partial y} \right) \frac{\partial \psi_1}{\partial y} + \frac{1}{n} \frac{\partial n}{\partial x} \frac{\partial \psi_1}{\partial x} = -\bar{\psi}_2 \frac{1}{n} \frac{\partial n}{\partial x}$$

(10)

where $\bar{\psi}_1 = \nu R/c$ and $\bar{\psi}_2 = \nu /c V_{in} - E_o /\beta$ are dimensionless constants.

Equation (9) was integrated forward in time using a flux-corrected [Boris
was solved for the self-consistent potential $\psi_1$ using a Chebychev semi-
iterative multigrid technique [McDonald, 1980]. The computational grid
described a small vertical slice oriented in the east-west direction of
the bottomside equatorial $F$ layer and is defined by 64 x 64 points with a
constant mesh spacing of $\Delta x = \Delta y = 15m$ giving an altitude and east-west extent
of 960 meters. Periodic boundary conditions were imposed on $n/n_o$ and $\psi_1$ in
both the $x$ (east-west) and $y$ (vertical) directions.

The in situ rocket data of Szuszczenicz and Holmes [1980] do not reveal
the initial conditions from which the observed irregularities developed.
Many sets of initial conditions are possible. In the context of the experimental observations of an initially upward and then downward moving F layer [Tsunoda, 1980a; Szuszczewicz et al., 1980], we make the reasonable assumption that the irregularities, whose spatial power spectra are sampled in Region "C" [Szuszczewicz and Holmes, 1980] of the bottomside, originate at a higher altitude(s) $v_0$ at earlier time(s) and are convected downward by the ambient F layer. This is in agreement with the conclusions of Narcisi and Szuszczewicz (1980) in their analysis of ion composition measurements conducted on the same rocket. From previous numerical simulations [Keskinen et al., 1980] of the intermediate wavelength collisional Rayleigh-Taylor instability for bottomside gradient scale lengths $L = 5 - 15 \text{ km}$ at an altitude of 300 km, we found that a well-developed nonlinear regime could be achieved after a time $\Delta t = 10 - 15 \gamma_m^{-1}$ where $\gamma_m$ is the maximum linear growth rate. We define the altitude of instability onset $v_0$ to be the position of the bottomside gradient at $\Delta t = 10 - 15 \gamma^{-1}$ sec prior to the time of rocket penetration. (The rocket penetrated region "C" on the bottomside at an altitude of 270 km.) This, of course, neglects the fact that $\gamma(y)$ will be decreasing since $\gamma_{in}$ and $\gamma_R$ will be increasing [see eq. (8)] for a downward moving F layer. In other words we wish to determine an altitude $v_0 = 15\gamma^{-1}(y)v_D = 270 \text{ km}$ where $v_D = 10 \text{ m/sec}$. For $L = 8 \text{ km}$ ($25 \text{ km}$) we find $v_0 \approx 335 \text{ km}$ ($365 \text{ km}$). From the Jacchia [1965] model neutral atmosphere [Ossakow et al., 1979], $\gamma_{in}$ (335 km) $\approx 0.3 \text{ sec}^{-1}$, $\gamma_{in}$ (365 km) $\approx 0.18 \text{ sec}^{-1}$, $\gamma_R$ (335 km) $\approx 1 \times 10^{-4} \text{ sec}^{-1}$, $\gamma_R$ (365 km) $\approx 4.5 \times 10^{-5} \text{ sec}^{-1}$ and $\gamma_1 = 300 \text{ sec}^{-1}$. Coupling these considerations with the probe measurements of gradient scale lengths, two different numerical simulations were made using $L = 8$ and $25 \text{ km}$. Both simulations were initialized with the following density profile: $n(x,y,t = 0)/N_0 = 1 + y/L + \epsilon(x,y)$ where $\epsilon(x,y)$ is a random function.
of position in the xy-plane with a white noise-like spatial power spectrum (no preferred wavelength) and a root-mean-square amplitude of 0.03.

Since the real space dimensions of the computational grid are 960 m by 960 m, the altitude dependent quantities $\nu_{in}(y)$ and $\nu_{R}(y)$ do not, at any fixed time, vary appreciably over the grid since their scale heights are much greater than the grid dimensions. In fact, these quantities can be fit very well with exponential variation in $y$, i.e., $\nu_{in}(y) \propto \exp(-y/L_{in})$ and $\nu_{R}(y) \propto \exp(-y/L_{R})$ with $L_{in} = 55$ km and $L_{R} = 33$ km between the altitudes of 250 and 550 km using a Jacchia [1965] model neutral atmosphere [Ossakow et al., 1979]. Ever, over several thousands of seconds, $\nu_{in}$ and $\nu_{R}$ will change (increase) in a frame moving downward with velocity $v_D = 10$ m/sec. In order to compensate for this effect, we convert the spatial (altitude) dependence of $\nu_{in}$ and $\nu_{R}$ into functions of time by making the substitution $y + v_D t$. In other words, during the course of the simulations, $\nu_{in}(t) = \nu_{in}(t_o) \exp(v_D \Delta t/L_{in})$ and $\nu_{R}(t) = \nu_{R}(t_o) \exp(v_D \Delta t/L_{R})$ where $\Delta t$ is elapsed time and $\nu_{in}(t_o), \nu_{R}(t_o)$ are the initial values depending upon initial altitude. In this simple model we ignore any variation of the bottomside density gradient scale length with altitude. We now present the important nonlinear aspects of these simulations.

Figure 5 gives an isodensity contour plot of $\delta n(x,y) = n(x,y) - n_o(y)$, at $t = 800$ sec for $L = 8$ km and shows that the random nature of the initial perturbations still prevails. The contours of the random initial perturbations in $\delta n(x,y)$ at $t = 0$ sec are very similar to Figure 5 but with more smaller scale structure. Figure 6 displays the evolution of $\delta n(x,y)$ at $t = 2000$ sec where some vertical elongation and steepening can be seen together with small scale irregularities. Figure 7 illustrates the
perturbation density contours at \( t = 4500 \) sec where further steepening and vertical elongation have occurred. This late time density configuration is similar to recent numerical simulations of the intermediate wavelength collisional Rayleigh-Taylor instability [Keskinen et al., 1980] under almost monochromatic initial conditions, i.e., with only two waves initially excited. The maximum percentage depletion \( \langle \delta n < 0 \rangle \) in Fig. 7 was 56%. Similar density contour development and late time percentage depletion were found using \( L = 25 \) km and again starting from random initial conditions. These large depletions were also noted in the in situ rocket data of Szuszczenicz and Holmes [1980].

In Figures 8a and 8b we have plotted sample one-dimensional horizontal \( P(k_x) \) and vertical \( P(k_y) \) spatial power spectra for \( 2\pi/k_x, 2\pi/k_y \) between approximately 100m and 960m in the nonlinear regime for \( L = 8 \) km at \( t = 4000 \) sec. These power spectra are obtained by integrating \( |\tilde{\delta} n(k_x,k_y)|^2 \) over \( k_x \) and \( k_y \), respectively. The spectral indices obtained from these power spectra are in agreement with those derived from bottomside irregularity power spectra as determined from rocket observations [Region "C" of Szuszczenicz and Holmes, 1980] in the wavelength domain \( \lambda: 25m - 1km \) (see Fig. 4) and recent simulations [Keskinen et al., 1980]. Similar power laws and spectral indices were found for the \( L = 25 \) km case but on a longer time scale.

**NONLINEAR THEORY**

We wish to interpret the evolution of long wavelength plasma irregularities in an upward or downward moving equatorial \( F \) region ionosphere in terms of coherent two-dimensional mode-coupling among collisional Rayleigh-Taylor modes. This type of analysis was also used to study large scale wavelengths excited by the gradient drift instability in the equatorial electrojet [Rognlien and Weinstock, 1974] and the saturation of the long wavelength local collisional [Chaturvedi and Ossakow, 1977] and collisionless
[Hudson, 1978] Rayleigh-Taylor instability in equatorial spread F. However, these latter papers did not include the effects of a convecting (rising or falling) background ionosphere which usually accompanies equatorial spread F. The linear growth rate $\gamma_k$ (eq. 8) which contains the altitude dependent parameters $\nu_R$ and $\nu_{in}$, and, by inference, the nonlinear growth rate will change in space and time in a vertically convecting ionosphere. Since with vertical drift velocities of the order of 10 m/sec the F region ionosphere can convect over a distance on the order of the scale height of $\nu_{in}$ and $\nu_R$ on time scales comparable to the nonlinear saturation times of the collisional Rayleigh-Taylor instability, this effect will be important.

We begin our nonlinear analysis with eq. (6) and (7) which can be written

$$\frac{\partial \delta n}{\partial t} - \frac{c}{B} B \cdot \frac{\delta \phi_1}{\delta z} \cdot \delta n + \nu_{in} \delta n = \frac{c}{B} B \cdot \frac{\delta \phi_1}{\delta z} \cdot \delta v \cdot \delta n \tag{11}$$

$$(c/B \omega_1) \cdot \nu_{in} B \cdot \delta \phi_1 = (c/B \cdot \frac{\delta \phi_1}{\omega_1} + \nu_{in} E/B) \cdot \delta \phi_1 \tag{12}$$

where we have made the separation $\delta n = n - n_o$. We note that the addition of a large scale electric field $E/o$ in (12) adds only a linear term. The convective nonlinear term on the righthand side of (11) is the dominant nonlinearity since the ratio of the nonlinear terms in (11) and (12) is

$$(c/B \omega_1) (\nu_{in} \cdot \nu_{in} \cdot \delta \phi_1)/(c/B) \cdot \nu_{in} \cdot \delta \phi_1 \cdot \delta \phi_1 \cdot \delta \phi_1 \cdot \delta \phi_1 \sim \frac{\nu_{in} \cdot \delta \phi_1}{\omega_1} \ll 1$$
for two dimensional perturbations $\delta_1$ and $\delta_0$. This allows one to find the potential perturbation $\delta_1$ for arbitrary density perturbation $\delta_0$ using a linearized version of (12), i.e.,

$$\frac{\epsilon \phi_1}{T} = - i \beta \frac{\delta_0}{n_0}$$  \hfill (13)

where, keeping the same notation as Chaturvedi and Ossakow [1977],

$$\beta = \frac{(g/\nu_1)(k_1/k_2^2)}{c_0}$$

with $c_0^2 = T/m_1$. Equation (11) can then be rewritten

$$\frac{\partial \delta_0}{\partial t} = \gamma_k \delta_0 + \frac{c}{B} \nabla \phi_1 \cdot \nabla \delta_0$$ \hfill (14)

with $\gamma_k$ given by (8). If we take an arbitrary two-dimensional perturbation of the form

$$\frac{\delta_0}{n_0} = A_{1,1} \sin (k_x x - \omega t) \cos k_y y$$ \hfill (15)

where the first subscript on $A$ denotes the vertical mode number and the second subscript signifies the horizontal mode number, we find using (13) the associated potential perturbation

$$\frac{\epsilon \phi_1}{T} = \beta A_{1,1} \cos (k_x x - \omega t) \cos k_y y$$ \hfill (16)

Then the nonlinear self-interaction $\frac{c}{B} \nabla \phi_1 \cdot \nabla \delta_0$ generates a term of the form $n_0 \alpha/2 A_{1,1}^2 \sin 2k_y y = A_{2,0} \sin 2k_y y$ with $\alpha = k_x^2 k_x \rho / k_y v_1$, which is linearly damped by recombination using (14). This linearly damped mode
\( A_{2,0} \) will interact through the nonlinearity in (14) with \( A_{1,1} \) to give a nonlinear damping for the linearly unstable mode \( A_{1,1} \) since

\[
\frac{c}{b} \mathbf{V} \psi_{1,1} \times \hat{z} \cdot \nabla \delta n_{2,0} = -2 \alpha_0 \ A_{1,1} \ A_{2,0} \ A_{2,0} \ A_{2,0} \sin k x \ A_{1,1} \ A_{1,1} \ A_{1,1} \ A_{1,1} \ A_{2,0} \cos k y \cos 2k y.
\]

In other words, a two dimensional linearly unstable perturbation with amplitude \( A_{1,1} \) can generate a linearly damped vertical mode \( A_{2,0} \) which, in turn, can react back to stabilize the \( A_{1,1} \) mode.

In the frame of reference of a small localized region in the bottomside of a downward convecting F layer we assume \( \mathbf{v}_n = \mathbf{v}_n(t) \) and \( \mathbf{v}_R = \mathbf{v}_R(t) \) are monotonically smooth functions of time as in the previous simulations. Then for a general perturbation of the form

\[
\frac{\delta n}{n_0} = A_{1,1} \sin (k x - \omega t) \cos k y + A_{2,0} \sin 2k y
\]

we can write

\[
\frac{\partial A_{1,1}}{\partial t} = \gamma_{1,1}(t) A_{1,1} - 2 \alpha(t) A_{1,1} A_{2,0}
\]

\[
\frac{\partial A_{2,0}}{\partial t} = -|\gamma_{2,0}(t)| A_{2,0} + \frac{\alpha(t)}{2} A_{1,1}^2
\]

where now the growth rate \( \gamma(t) \) and nonlinear coupling coefficient \( \alpha(t) \) are functions of time since they contain altitude dependent quantities.
\(\nu_{in}\) and \(\nu_{R}\). In deriving (18) and (19) we have assumed that \(\gamma(t)\) and \(\omega(t)\) are weakly time dependent. For \(E_0 \to 0\) (stationary F region), we recover the results of Chaturvedi and Ossakow [1977] who treated \(\gamma\) and \(\omega\) as space and time independent. Since \(\omega \sim 1/\nu_{in}\), one observes that the nonlinear coupling decreases (increases) for a downward (upward) drifting F layer since \(\nu_{in}\) increases (decreases). Assuming that a quasi-steady state is achieved at \(t = t_s\) with \(\partial A_{1,1} / \partial t = \partial A_{2,0} / \partial t = 0\), we obtain from (18) and (19)

\[
A_{2,0}(t_s) = \gamma_{1,1}(t_s) / 2\alpha(t_s) - 1/2 k_L
\]

\[
A_{1,1}(t_s) = (2 |\gamma_{2,0}(t_s)| A_{2,0} / \alpha(t_s))^{1/2}
\]

\[
= ((k^2/k_x^2)(\nu_{in}(t_s) \nu_{in}(t_s)/gLk_y^2))^{1/2}
\]

Since \(A_{2,0}(t_s)\) is independent of \(\nu_{in}\) and \(\nu_{R}\) we note from (20) and (21) that the two dimensional spatial power spectra of the density fluctuations in a rising or falling F region ionosphere will have the oblique linearly unstable modes, e.g., \(A_{1,1}\) being favored at low altitudes (increasing \(\nu_{R}\) and \(\nu_{in}\)) with the linearly damped waves, e.g., \(A_{2,0}\) dominating at higher altitudes (decreasing \(\nu_{R}\) and \(\nu_{in}\)) with spatial power spectra \(\propto k_y^{-2}\). Indeed for \(k_y \approx k_x\) we can write

\[
\hat{A}_{2,0}(t_s)/A_{1,1}(t_s) \approx (1/2)(\nu_{in}(t_s) \nu_{R}(t_s)L)^{1/2}
\]
so that at 300 km with $L = 25$ using a Jacchia [1965] model neutral atmosphere [Ossakow et al., 1979] we find $A_{2,0} \sim A_{1,1} \sim 12$ for $k_x L = 2\pi L / \lambda_x \approx 50$ while at 425 km $A_{2,0} / A_{1,1} \sim 10$. A more detailed examination of the static and dynamic solutions of (18) and (19) will be reserved for a future report.
SUMMARY

We have performed analytical and numerical simulations studies of the collisional Rayleigh-Taylor instability in local unstable regions of downward-moving equatorial F layers. For ambient bottomside plasma density gradient scale lengths $L = 8$ and 25 km, we have demonstrated that large percentage relative depletions can develop on time scales of several thousands of seconds from purely random initial conditions. In addition we have shown that the one-dimensional spatial power spectra of these irregularities in the vertical and east-west directions conform to power laws $\propto k^{-n}$, $n = 2-2.5$ for $2\pi/k$ between 100m and 1 km. These results are in good agreement with our recent in situ rocket observations of intermediate wavelength ($\lambda$: 25m - 1 km) irregularities equatorial spread-F, presented here in this paper, complement previous local numerical simulations [Keskinen et al., 1980], and lend further support to the belief that the collisional Rayleigh-Taylor instability is responsible for large scale size irregularities in equatorial spread-F.

Future work will include global simulations of upward and downward moving equatorial F layers together with the inclusion of neutral wind effects.
ACKNOWLEDGMENTS

We wish to thank R. Tsunoda, B. E. McDonald, and S. T. Zalesak for useful discussions. This work was supported by the Defense Nuclear Agency and the Office of Naval Research.
REFERENCES


Fig. 1 — Relative and absolute ionospheric electron density profile determined in situ during the occurrence of equatorial spread F.
Fig. 2 — Expanded view of density fluctuation observed in the bottomside irregularity (Fig. 1) centered near 270 km. This is region "C" of Szuszczewicz et al. [1980].
Fig. 3 — Gradient scale lengths on the bottomside ledge of the F-region layer shown in Figure 1.
Fig. 4 — Power spectral analyses of density fluctuations in region “C” (Figure 2).
Fig. 5 — Isodensity contours of $\delta n(x,y)/n_0(y)$ for $L = 8$ km at $t = 800$ sec. Solid contours denote $\delta n/n_0 > 0$; dashed contours denote $\delta n/n_0 < 0$. Contours are evenly spaced with y axis vertical and x axis horizontal. Tick marks and numbers denote grid point locations and size of grid in meters, respectively. Maximum enhancement ($\delta n/n_0 > 0$) and depletion ($\delta n/n_0 < 0$) are +4.8% and -3.8%.
Fig. 6 — Same as Figure 5 but at $t = 2000$ sec. Maximum enhancement and depletion are $+15\%$ and $-11\%$. 
Fig. 7 — Same as Figure 5 but at $t = 4500$ sec. Maximum enhancement and depletion are +86% and -56%.
Fig. 8 — One-dimensional (a) horizontal $P(k_x)$ and (b) vertical $P(k_y)$ spatial power spectra versus $k_x$ and $k_y$, respectively, for $L = 8$ km at $t = 4000$ sec. The solid lines are least squares fits to the numerical simulations (dots) and $k_o = 2\pi/960$ m.
DISTRIBUTION LIST

DEPARTMENT OF DEFENSE

ASSISTANT SECRETARY OF DEFENSE
COMM. CHIEF, COMM & INTELL.
WASHINGTON, D.C. 20301
01 CY ATTN J. BACO
01 CY ATTN M. EPSTEIN

ASSISTANT TO THE SECRETARY OF DEFENSE
ATOMIC ENERGY DIRECTOR
WASHINGTON, D.C. 20301
01 CY ATTN EXECUTIVE ASSISTANT

DIRECTOR
COMMAND CONTROL TECHNICAL CENTER
PENTAGON BM BE 585
WASHINGTON, D.C. 20301
01 CY ATTN C-650
01 CY ATTN C-512 R. MASON

DEFENSE COMMUNICATION ENGINEER CENTER
1500 WHEEL HOUSE AVENUE
RESTON, VA. 22090
01 CY ATTN CODE R820
01 CY ATTN CODE R410 JAMES W. MCLEAN
01 CY ATTN CODE R720 J. WORTHINGTON

DEFENSE COMMUNICATIONS AGENCY
WASHINGTON, D.C. 20305
(ADDR CMND: ATTN CODE 240 FOR)
01 CY ATTN CODE 1018

DEFENSE TECHNICAL INFORMATION CENTER
CAMERON STATION
ALEXANDRIA, VA. 22314
12 COPIES IF OPEN PUBLICATION, OTHERWISE 2 COPIES
01 CY ATTN TC

DIRECTOR
DEFENSE INTELLIGENCE AGENCY
WASHINGTON, D.C. 20301
01 CY ATTN DT-18
01 CY ATTN DB-WG E. O'FARRELL
01 CY ATTN DIARAP A. WISE
01 CY ATTN DIASA-12
01 CY ATTN DT-192 R. MORTON
01 CY ATTN M-TR J. STEWART
01 CY ATTN M. WITTIG DC-7D

DIRECTOR
DEFENSE NUCLEAR AGENCY
WASHINGTON, D.C. 20305
01 CY ATTN 5YV
01 CY ATTN TITL
01 CY ATTN DOST
01 CY ATTN KANE

COMMANDER
FIELD COMMAND
DEFENSE NUCLEAR AGENCY
KIRTLAND AFB, NM 87115
01 CY ATTN FORM

DIRECTOR
INTER-SERVICE NUCLEAR WEAPONS SCHOOL
KIRTLAND AFB, NM 87115
01 CY ATTN DOCUMENT CONTROL

JOINT CHIEFS OF STAFF
WASHINGTON, D.C. 20301
01 CY ATTN J-3 IMPECS EVALUATION OFFICE

DIRECTOR
JOINT STRATEGIC PLANNING STAFF
OFFUTT AFB
OMAHA, NE 68117
01 CY ATTN JFLW-2
01 CY ATTN JPST G. GOETZ

CHIEF
LIVERMORE DIVISION FLD COMMAND DN
DEPARTMENT OF DEFENSE
LAWRENCE LIVERMORE LABORATORY
P. O. BOX 808
LIVERMORE, CA. 94550
01 CY ATTN FCPR

DIRECTOR
NATIONAL SECURITY AGENCY
DEPARTMENT OF DEFENSE
FT. GEORGE G. MEADE, MD 20755
01 CY ATTN J-16
01 CY ATTN JOHN SKILLMAN R52
01 CY ATTN FRANK LEONARD
01 CY ATTN W. PAT CLARK
01 CY ATTN OLIVER M. BARTLETT W32
01 CY ATTN R5

COMMANDANT
NATO SCHOOL (SHAPE)
APO NEW YORK 09172
01 CY ATTN U.S. DOCUMENTS OFFICER

UNDER SECT OF DEF FOR RSC & ENGRGS
DEPARTMENT OF DEFENSE
WASHINGTON, D.C. 20301
01 CY ATTN STRATEGIC & SPACE SYSTEMS (OS)

IMPECS SYSTEM ENGINEERING DIV
WASHINGTON, D.C. 20305
01 CY ATTN R. CRAWFORD

COMMANDER/DIRECTOR
ATMOSPHERIC SCIENCES LABORATORY
U.S. ARMY ELECTRONICS COMMAND
WHITE SANDS MISSILE RANGE, NM 88002
01 CY ATTN DELAS-80 F. NILES

DIRECTOR
BMD ADVANCED TECH CTR
HUNTSVILLE OFFICE
P. O. BOX 1500
HUNTSVILLE, AL 35807
01 CY ATTN AITC-T MELVIN T. CAPPS
01 CY ATTN AITC-O W. DAVIES
01 CY ATTN AITC-R DON RUSS

PROGRAM MANAGER
BMD PROGRAM OFFICE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA. 22333
01 CY ATTN DAS-8M J. SHEA

CHIEF C-E SERVICES DIVISION
U.S. ARMY COMMUNICATIONS CMD
PENTAGON BM 8269
WASHINGTON, D.C. 20301
01 CY ATTN C-E-SERVICES DIVISION

COMMANDER
PRADC TECHNICAL SUPPORT ACTIVITY
DEPARTMENT OF THE ARMY
FORT MONMOUTH, N.J. 07703
01 CY ATTN DRAEPL-7W H. BENNET
01 CY ATTN DRAEPL-7W M. BOMKE
01 CY ATTN J. C. QUIGLEY

31
LMED