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MICROPROCESSOR CONTROL OF LOW SPEED VSTOL FLIGHT

A TRIDENT SCHOLAR PROJECT REPORT

by

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ABSTRACT

The objective of this project has been to design an improved three-axis stability augmentation system (SAS) for the AV-8B Advanced Harrier VSTOL aircraft using microprocessor-based digital control. The research focuses on improving the handling qualities of the airplane through SAS redesign in the low speed flight regime. Particular attention is paid to the so-called weather-cocking instability encountered in transition (hover to Conventional and Vice Versa) ~~(hover to conventional)~~ flight. Until quite recently, there has been a dearth of information about the flight characteristics of the Harrier. A major breakthrough in this field was achieved by the development of MCAIR's X22A AV-8B mathematical model, which yielded a set of linearized stability derivatives for the aircraft. The first step toward the improvement of the AV-8B SAS requires the utilization of these coefficients in the development of an analog/hybrid model of both the aircraft itself and of the unmodified SAS. The controller design employs digital state feedback control to relocate the system closed loop poles. The conclusion reached is that this method of control represents a valid approach to the final solution of the Harrier's stability and control problems. A long range goal of this research is that this controller design concept be applied to future aerospace vehicles.

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PREFACE

This investigation was conducted at the United States Naval Academy, Annapolis, Maryland under the Trident Scholars Program. This publication is the final technical report, and it includes results through 30 May 1979. An oral presentation was made on 24 April 1979 to RADM William P. Lawrence, Superintendent, U. S. Naval Academy and other distinguished guests at the annual Trident Scholar's Dinner. The research was conducted in the Department of Weapons and Systems Engineering, and in cooperation with the Department of Aerospace Engineering.

The principal investigator for the study was myself, Midshipman Robert V. Walters. The faculty advisors for the project were Associate Professor E. E. Mitchell (principal advisor) and Assistant Professor K. A. Knowles of the Systems Engineering Department, and CDR M. D. Hewett, Chairman of the Aerospace Engineering Department. The assistance, insistence, and perseverance that Professor Mitchell, Professor Knowles, and Commander Hewett have shown has been a major catalyst in the research effort. Considering the tremendous amount of engineering expertise and experience that these men brought to bear on the problem, it is not surprising that such satisfying results were achieved.

Robert V. Walters

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I worry that the existence of a practical VSTOL aircraft today (the Harrier, in its operational A and forthcoming B versions) is being ignored in favor of the long-range development of a "high-performance" VSTOL culminating in the 1990s. Even this development has been slowed by the Defense Department, stating that before further funds are devoted to VSTOL it must be proven superior to conventional aircraft. In my judgment, if it were supported at this time, an effective VSTOL aircraft could be developed as early as 1986. The Soviets are doing it....

- ADM Elmo R. Zumwalt, USN (Ret)
  May 1979, following Department of Defense cancellation of the AV-8B program.
  [17, p. 104]

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LIST OF SYMBOLS

Variables	Description
e	Naperian base (2.71828. . . . .)
A	System dynamics matrix (continuous-time system)
f	Frequency, Hz
B	Control effect matrix (continuous-time system)
G	Total external moment
g	Gravitational acceleration
h	Angular momentum
I_n	Identity matrix of dimension nxn
K	State feedback matrix
L	Rolling moment
M	Pitch moment
m	Mass
N	Yawing moment
p	Roll rate, rad/sec
q	Pitch rate, rad/sec
r	Yaw rate, rad/sec
s	Laplace transform variable
T	Sampling interval, sec
t	Time, sec
u	x-axis velocity, ft/sec
$\underline{u}$	Control vector
V	Total velocity, ft/sec
v	y-axis velocity, ft/sec

LIST OF SYMBOLS (Cont.)

Variables	Description
w	z-axis velocity, ft/sec
X	Aerodynamic force along the x-axis
x	Axial position, ft
$\underline{x}$	State vector
Y	Aerodynamic force along the y-axis
y	Lateral position, ft
Z	Aerodynamic force along the z-axis
z	Vertical position, ft

Variables (Greek)

α	Angle of attack, deg
β	Sideslip angle, deg
Δ	Disturbance quantity
γ	Vertical flight path angle, deg
δ_A	Aileron deflection, deg
δ_E	Elevator deflection, deg
δF	Flap deflection, deg
δ_R	Rudder deflection, deg
$\delta\theta_j$	Nozzle angle deflection, deg
δ_T	Throttle deflection percent
ζ	Damping ratio
θ	Pitch attitude angle, rad
λ	Eigenvalue

LIST OF SYMBOLS (Cont.)

Variables	Description
Σ	Summation
Φ	State transition matrix
ϕ	Roll attitude angle, deg
ψ	Yaw attitude angle, deg
ω_n	Natural frequency, rad/sec

Subscripts	
c.g. or c	Center of gravity
CL	Closed-loop
d	Desired value
i	Element index for vectors and matrices
o	Nominal value

Punctuation	
$(\dot{\ })$	Derivative of quantity with respect to time
$(\underline{\ })$ or $(\rightarrow)$	Vector quantity
$\delta(\ )/\delta(\ )$	Partial derivative of one variable with respect to another
$\Delta(\ )$	Perturbation variable
$(\ )^T$	Transpose of a vector or matrix
$(\ )^{-1}$	Inverse of a matrix
$(\tilde{\ })$ or $(\bar{\ })$	Matrix quantity
$\|\ \|$	Determinant
$\Delta'(\lambda)$	Characteristic equation
$\equiv$	Definition

LIST OF SYMBOLS (Cont).

Acronyms

A/D	Analog-to-Digital
APL	A Programming Language
ASCII	American Standard Code for Information Interchange
CPU	Central Processing Unit
D/A	Digital-to-Analog
IAS	Indicated Air Speed
I/O	Input/Output
KIAS	Knots, Indicated Air Speed
NATOPS	Naval Air Training and Operating Procedures Standardization
SAS	Stability Augmentation System
sps	Samples per second
VSTOL	Vertical and Short Take-Off and Landing

CHAPTER 1

INTRODUCTION: JUSTIFICATION AND ORGANIZATION OF THE REPORT

One of the major controversies facing naval policy makers is the issue of the suitability and effectiveness of VSTOL aircraft in a sea-going combatant environment. Major policy decisions on the future of VSTOL aircraft are being formulated based on the Marine Corps experience with the Hawker Siddeley AV-8A Harrier. The success or failure of the AV-8B now in development will further influence the Navy's future VSTOL plans.

The AV-8A has experienced a very high accident rate in the Marine Corps. In the first six and one-half years of its use in Marine Corps operation, the Harrier logged twenty-eight crashes -- nearly one fifth of the fleet -- over twenty of which resulted in the total destruction of the aircraft. The aircraft's problems mirror the present state of the art in VSTOL technology. Present technology is capable of producing engines of sufficient thrust at low weight to provide the thrust-to-weight ratios demanded in VSTOL airplane design. Special VSTOL airframe structural and aerodynamic considerations are also well within present technological capabilities. Stability, control, and handling qualities

problems, however, persist. They are the major threat to the success of the VSTOL concept. The poor transition-regime handling qualities of the AV-8A due to inadequate control authority and inadequately designed stability augmentation are well documented and have contributed to the poor reputation of the airplane as a difficult, unforgiving airplane to fly.

A strong research effort in stability augmentation design is required to insure the success of the AV-8B and future VSTOL efforts. In addition to the motivation resulting from the political exigencies surrounding the Harrier program, the strong current interest in the study of digital flight control provides further justification for the research.

With these thoughts in mind, it was decided to conduct an investigation of the flight characteristics of the AV-8B Advanced Harrier, with the intent of improving the effectiveness of its stability augmentation system (SAS) in the low speed flight regime. The goal of the study was the realization of a microprocessor-compatible compensator that would stabilize the Harrier. It was planned that the new system would be capable of functioning at two levels of control authority. The "normal" mode would be designed to improve the overall handling qualities of the aircraft, but still allow the pilot sufficient control to meet the demanding maneuverability

requirements of a modern fighter/attack aircraft. The "recovery" mode would essentially take control of the aircraft, seeking, in spite of any external disturbances, to return the aircraft to some reference "trim" condition. The digital state variable controller that since has become the fruit of the study lives up to many of these initial expectations.

This report documents the year-long research effort leading up to the final controller design. The organization of the report is as follows.

The body of the report is divided into ten chapters. The first three chapters are composed primarily of background information. Chapter 2 describes the subject of the investigation, the Harrier. Particular attention is paid to the existing flight control and stability augmentation systems. The reasons for the Harrier's unstable transition flight characteristics are discussed in Chapter 3. The next two chapters concern the formulation of a mathematical model for the Harrier. Chapter 4 presents a derivation of the VSTOL equations of motion, while Chapter 5 discusses the methodology for the experimental determination of the coefficients. Chapters 6 and 7 contrast the controller design of the present conventional SAS and that of the digital state feedback device.

A description of the digital and the analog/hybrid computer simulations is contained in Chapter 8. Finally, Chapter 9 treats the results of the investigation, and conclusions are presented in Chapter 10. Thirteen Appendices document the computer programming necessary in the research, and contain, in the form of computer-generated time histories, the results of the project.

CHAPTER 2

THE HARRIER

The genesis of the design of the Harrier's basic airframe is unique in that the entire aircraft was quite literally built around its propulsion plant - the Rolls Royce Pegasus engine. The Pegasus is an axial flow twin spool turbo fan engine, and in this respect is similar in principle to the engines powering many of our modern airliners. The requirements of VSTOL flight necessitate several departures from the norm, however. The most important of these changes concerns the ducting of the jet exhaust. Instead of the conventional configuration of a single nozzle located in line with the turbine shafts, the Pegasus uses four rotatable nozzles located symmetrically on the sides of the engine (and protruding from the fuselage directly under the shoulder-mounted wings). This engine-airframe configuration gives the Hawker Siddeley aircraft unique capabilities. The AV-8 Harrier has the ability to "vector" its thrust, i.e., to change both the direction and length (magnitude) of the thrust vector. As will be explained in a moment, thrust vectoring is the Harrier's means of achieving VSTOL capability. Another unique feature is that the two compressor/turbine sections of the Pegasus engine are contrarotating. By spinning the

blading in opposite directions, the gyro-coupling effects that had the plagued many previous VSTOL designs have been all but eliminated. An additional performance criteria met by the Pegasus-Harrier package results from the location of the exhaust nozzles such that the net thrust vector always acts through the aircraft's center of gravity. This placement means that no pitching moment is caused by the rotation of the nozzles.

While seeing the Harrier in a hover gives one the impression that it is defying gravity, it must, of course, obey Newton's laws of motion. This implies that in all phases of steady flight some lifting force must balance the weight of the aircraft. In the VTOL/hover regime, the lift is produced entirely by the thrust of the Pegasus engine, the nozzles being rotated nearly perpendicular to the aircraft centerline and the thrust being directed vertically downward. For transition to conventional flight, the nozzles, which share a common mechanical linkage to a control lever in the cockpit, are slowly rotated aft. The net effect of this rotation is to reduce the magnitude of the upward (lift) component of the thrust vector while simultaneously producing a forward thrust component. The resulting forward velocity

of the aircraft causes airflow over the wings and therefore produces aerodynamic lift. As the transition progresses, wing-generated lift gradually replaces the upward component of the thrust vector until, at a speed of about 100 knots, the nozzles are rotated fully aft and the aircraft is in the conventional regime. The preceding sequence of events is reversed for a decelerating transition from fully wing-borne to fully jet-borne flight. It is worthwhile to note here that the Harrier's most acute stability and control problems occur in the transition regime, from about 30 to 80 knots. The dynamics of this phenomena will be discussed fully in the next chapter.

Knowledge of the design and performance of the Harrier's flight control system is vitally important to solution of its stability and control problems. The aircraft's primary controls for wing-borne flight consist of conventional ailerons, rudder, and stabilator, with a reaction control system (RCS) providing additional control authority in the transition and hover regimes.

This reaction control system bleeds high pressure air from the engine through lightweight ducting to special nozzles (called "puffer" valves) located at the nose tail, and each

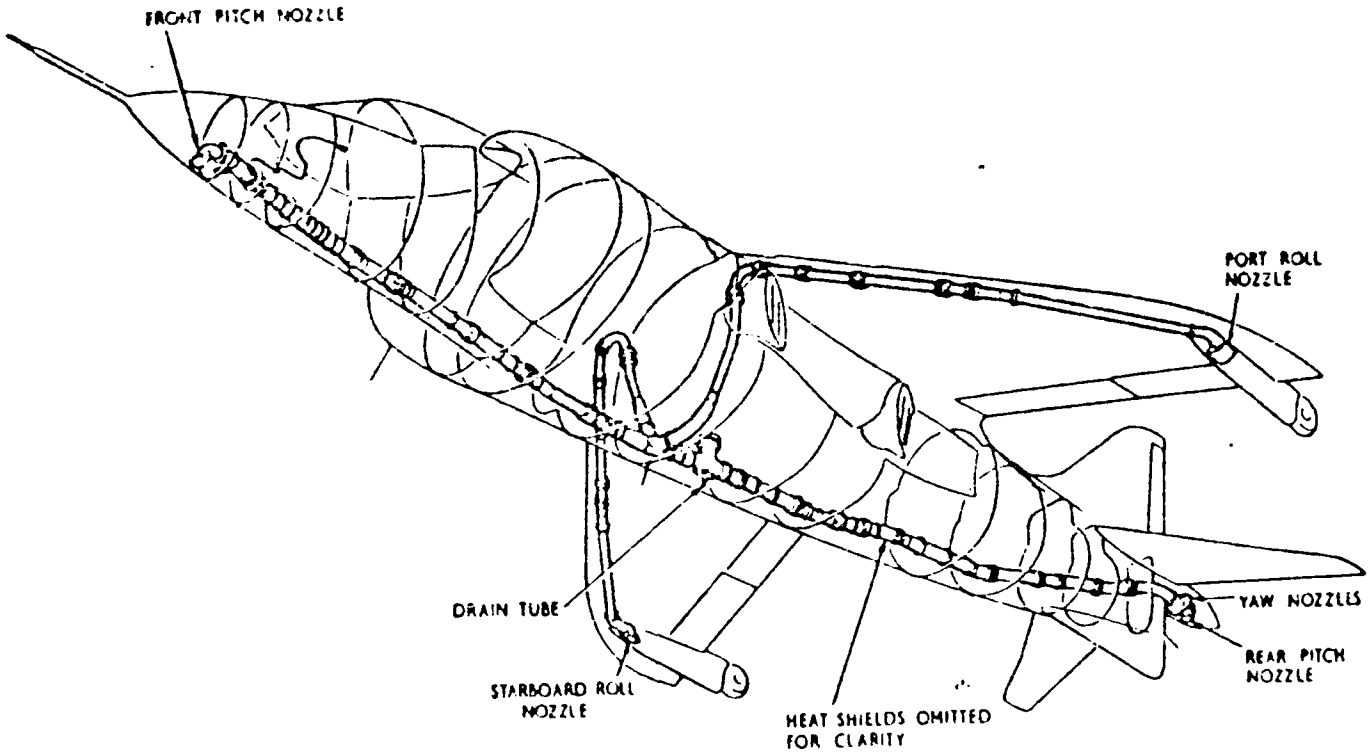


Figure 2.1 Reaction Control System

Nozzles and Ducts [4, p. 25.1]

wingtip of the aircraft. These shutter valves are mechanically linked to the conventional controls; a master control valve is geared to fully activate the system when the engine nozzles are rotated 20° below the aircraft's centerline. Since the nozzle angle is proportional to the aircraft's airspeed, the net effect of the master control valve gearing is that the RCS is activated only in the VSTOL-hover regime (below about 100 KIAS). The RCS extends the Harrier's controllability envelope all the way down to zero airspeed, long after the aerodynamic controls have lost their effectiveness.

A single channel limited authority stability augmentation system (SAS) is provided to dampen rolling, pitching, and yawing motions while in a hover or transition. The device, which contains no "autopilot" function, was added to reduce pilot workload under turbulent and low visibility conditions. This is accomplished by sensing the aircraft's angular velocities about the roll, pitch, and yaw axes and immediately initiating a correcting signal to the control actuators, thus augmenting the pilot's demands on the aerodynamic control surfaces and/or reaction control nozzles.

As it is primarily a low speed system, the SAS functions almost exclusively through the reaction control system. It should be mentioned that the control authority of the reaction control system decreases dramatically in high thrust demand

situations. The SAS is quite economical in space and weight: both the pitch and roll channels are housed in a single 5.5 pound unit that contains gyros, computing, power supply, and self-test devices. The yaw SAS is slightly more elaborate, as it utilizes feedback of both the aircraft's yaw rate and of its lateral acceleration. In general, while the addition of stability augmentation in its present form may definitely be considered as a positive and flying-quality-enhancing improvement, there is quite a lot of motivation toward optimization of the system.

Of additional interest from the controls standpoint is a lateral acceleration indication system consisting of a readout projected on the pilot's heads-up display (HUD) and a novel rudder pedal shaker system that actually vigorously vibrates the rudder pedal that the pilot should depress to eliminate the lateral acceleration. Sideslip is proportional to lateral acceleration, and the use of an accelerometer as the sensor of a sideslip-regulation circuit for turn coordination of conventional aircraft is quite common. As will be described in Chapter 3, the elimination of sideslip is of key importance to VSTOL stability studies. In fact, control of this quantity is so important to the directional stability of the Harrier

in transition that it is fed back not only to the pilot through the indication system, but also to the lateral autostabilizer through an electronic filtering/signal processing network.

The aircraft that is the subject of the research effort documented in this report is an updated version of the AV-8A now in service in the Marine Corps. It is called the AV-8B Advanced Harrier, and a brief survey of its development and capabilities is now presented.

Following the acquisition by McDonnell-Douglas (1969) of manufacturing rights for the AV-8A Harrier, a tremendous amount of research time and money was put into the full development of the Harrier's potential. A parallel effort was made in the refinement of the Pegasus engine by Pratt and Whitney (who had similarly obtained production rights from Rolls Royce). Many independent research projects flourished on the fringes of this central two-pronged advance, spurred on by publication of large quantities of previously unavailable technical data. The AV-8B Advanced Harrier is the improvement program's crowning achievement. The linearized mathematical models developed by Calspan Corporation of Buffalo, N.Y., which form the basis for the simulation described later in this report, were an offshoot of this engineering

renaissance for the Harrier.

Effective lift is the AV-8's most critical design criteria. There are two methods of increasing effective lift: increase lift or reduce weight. The AV-8B makes use of both approaches. To increase lift in jet-borne flight, 600 pounds of additional lift were obtained by Pratt and Whitney through a redesign of the Pegasus' engine inlets. A special "cross dam" was then installed under the fuselage in order to take better advantage of interference effects during VTOL. An estimated 1200 pounds of additional lift is generated by this device in the VTO mode. Lift in the conventional flight regime has been augmented by the use of a thicker, "supercritical" airfoil. This new wing also cuts down drag at high speed, thereby reducing fuel consumption - always a concern in a VSTOL aircraft. The structure of the new airfoil is the major weight reducing component of the Advanced Harrier. The entire wing is made of graphite epoxy reinforced at high stress areas with titanium and aluminum. Most of the AV-8B's other subsystems were carried over from the AV-8A with minor modifications. The net result of these increases in effective lift is a performance increase of about 100% (a useful load of about 15,000 lbs).

Like the AV-8A, the Advanced Harrier's operational effectiveness is best measured by its versatility. It can

take over 3000 pounds of ordinance straight up and to a target fifty miles distant; from a short take-off, its performance characteristics are comparable to those of the touted A-4 Skyhawk. The Harrier has proven itself capable of operating from small sea-going platforms, or from dispersed austere sites ashore. The Harrier is an excellent aircraft; it is, of course, the only VSTOL platform sufficiently advanced in development to be used operationally. But, if any of the Harrier's shortcomings can be labeled "serious", one of those must surely be its transition-regime stability characteristics. The next chapter examines this area in depth.

CHAPTER 3

STABILITY CONSIDERATIONS

A quick yet effective means of forming a preliminary evaluation of an aircraft's flight characteristics is to note its "Prohibited Maneuvers" as tabulated in its flight manual. The AV-8A NATOPS Flight Manual [3 , R-12] lists 26 prohibited maneuvers, among them:

1) Out-of-the-wind vertical take-off or landing with a wind speed greater than 20 kts.
2) In the transition regime:
   a) Over 15 units angle of attack above 50 KIAS
   b) Sideslip between 30-150 KIAS
   c) Other than gentle turns between 90-150 KIAS
   d) Any turns between 30 and 90 KIAS
3) Lateral wind component in excess of 30 kts.
4) Take-off or landing on paved surfaces with crosswind greater than approximately 5-15 knots (depending on type of landing and visibility) or on any other surface with crosswind greater than 5 kts.

Thus, it is seen that stability and control problems substantially limit the Harrier's usable flight envelope, especially in low speed situations.

The Harrier's effectiveness below about 80 knots is quite

limited, even when operating within the boundaries prescribed by NATOPS. This limitation stems from the unusually high pilot workload required during the transition, especially in a low visibility or turbulent environment. The present SAS, by damping the unstable, at times erratic motions of the aircraft, very noticeably improves the Harrier's handling characteristics in semi-wing-borne and jet-borne flight. But, as many pilots quickly attest to, an "...attitude command SAS would be desirable since this would make a positive orientation reference available to the pilot at all times." [4, 332] The control system designed herein seeks to answer this plea.

It is desirable at this point to back-off from a discussion of the aircraft at the limits of its stability/control envelope and discuss its handling characteristics in general terms. First of all, jet VSTOL aircraft operating in wing-borne flight perform very much like conventional aircraft In general, the Harrier's flight characteristics at airspeeds above 150 knots are quite admirable. However, VSTOL craft in the jet-borne regime may exhibit characteristics that appear quite peculiar to those familiar only with conventional aircraft. The Harrier in VSTOL flight, for example, exhibits neutral to negative stability in pitch and roll, and a definite, pronounced instability in yaw due to the intake momentum drag phenomena. The research effort described in this report began

as a quest for a solution to this particular control anomaly (along with the related roll-coupling and pitch sensitivity characteristics). With this in mind, our attention will now focus on the aerodynamic principles that underlie the intake momentum drag effect.

Air entering a jet engine experiences a rapid decrease in velocity at the compressor section inlet. This reduction of velocity produces a net change in momentum, which results in a "momentum drag" force. At the high engine power settings required for VSTOL flight, some 400 lb/sec of air are brought to rest in the Pegasus' intakes. At extremely low airspeeds (below 30 kts), this phenomena has little effect, as the retarding force (drag) caused by the change in momentum is quite small. At higher forward velocities, however, this drag force is substantial (equaling, according to Newton, the product of the mass of the air and the velocity).

The air intakes of the Pegasus engine are located forward of the aircraft's center of gravity. The geometry of the situation implies that this intake momentum drag, in the presence of any angular deflection about the yaw axis (so-called "sideslip") produces a destabilizing moment. Further, the magnitude of this moment increases linearly with velocity. Combating this torque is the inherently stabilizing yawing moment produced by the

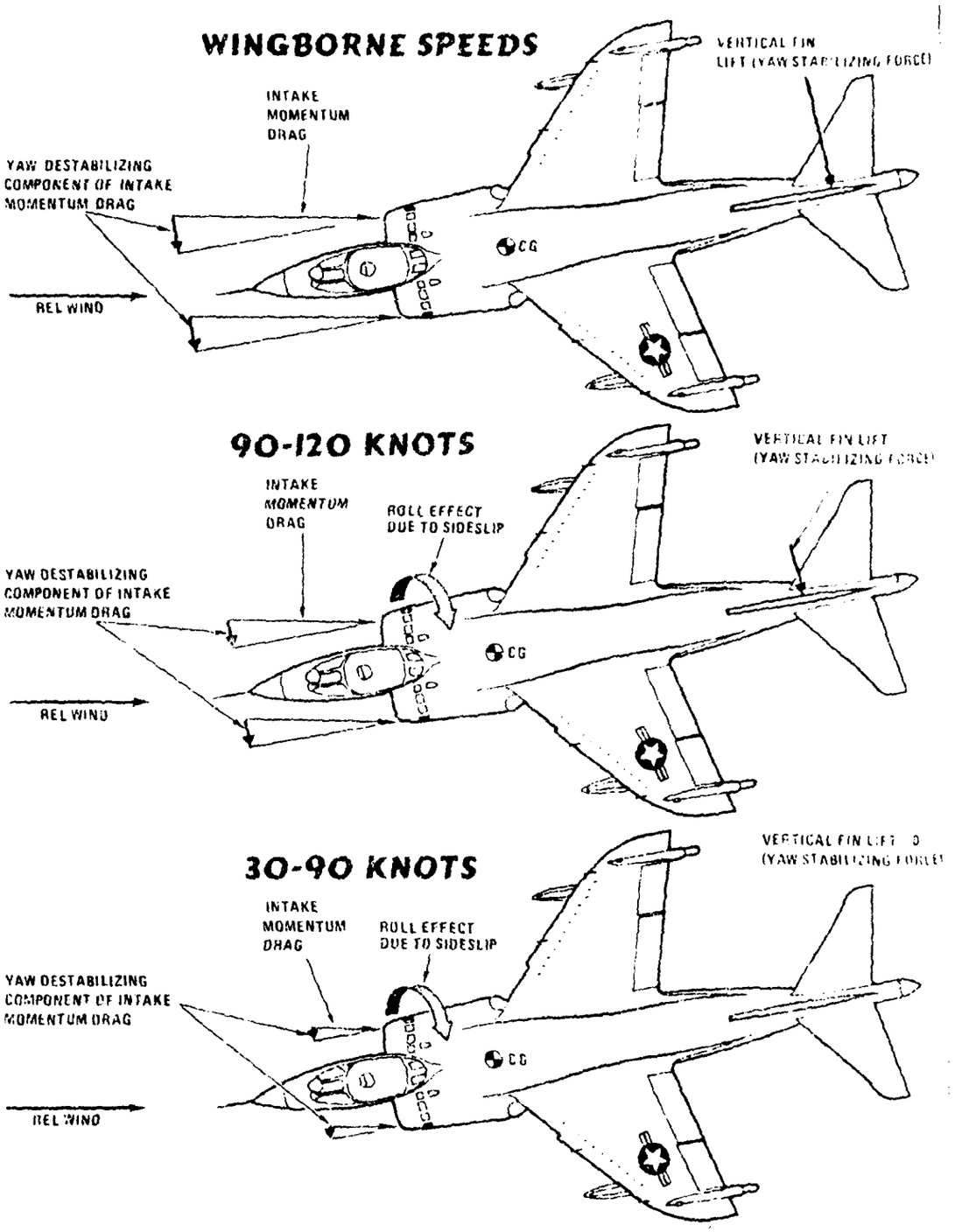


Figure 3.1 Intake Momentum
Drag Effect [3, p. 4-6]

aerodynamic lift of the vertical tail, which increases as a function of the velocity squared. The net result of these t o opposing moment mechanisms is that the Harrier is directionally very unstable between 30 and 90 knots, but essentially stable outside this velocity envelope.

In the Harrier, unless corrective control is instituted, sideslip disturbances result in coupled motions about all axes. This is the reason for the high degree of emphasis placed on its regulation. The mechanism that underlies the aerodynamic coupling associated with sideslip merits discussion.

Sideslip produces rolling moments in most aircraft, especially when swept wings are utilized. The rolling moment is generated by a lift differential between the two wings. This change in lift distribution is caused, in turn, by two mechanisms: (1) the sideslip angle causes the airflow to pass nearly normal to the leading edge of the windward wing, while the leeward wing is not only turned away from the wind, but is also shielded by the fuselage; and (2) the yaw rate produced by the destabilizing moment causes the windward wing's velocity relative to the airflow to increase, and, conversely, that of the leeward wing to decrease. Thus, both mechanisms augment the lift of the wind-ward wing and reduce the leeward wing's lift, thereby creating an unstable rolling moment. The righting characteristic about the roll axis

("dihedral effect") that is normally designed into an aircraft is not, in most cases, of sufficient strength in the Harrier in VSTOL flight to combat the roll coupling instability. Furthermore, since large deflections in yaw and roll substantially reduce the control authority of all flight controls, angle of attack (which is analogous to pitch angle, a longitudinal variable) is also a coupled variable in this intake momentum drag situation. This fact is particularly relevant, considering the acute pitch sensitivity of the Harrier in transition. So, what began as a simple sideslip disturbance has, through aerodynamic coupling, adversely affected handling qualities about all three axes. The AV-8A NATOPS Flight Manual takes this analysis one step further.

Three variables are of paramount importance to the intake momentum drag/roll coupling/pitch sensitivity phenomena:

Indicated air speed, IAS	(q)
Angle of attack, AOA	(α)
Sideslip angle	(β)

The magnitude of the rolling moment is roughly proportional to the product of the three variables (IAS, AOA, β). [3, 4-7] Furthermore, the mathematics are such that if any two of the terms are large, the rolling moment will be substantial even for a relatively small value of the third term. Typical "large" values are:

q = 100 KIAS

α = 15 units $\doteq$ 15°

β = 30°

Note that these numbers approximately coincide with several of the limits mentioned in the "Prohibited Maneuvers".

With respect to this serious VSTOL regime instability, the flight manual warns:

> Most dangerous of all, the AOA will increase instantly with roll if there is a sideslip angle present. This can result in an almost instantaneous loss of control with very little or no warning. A typical loss of control sequence at low IAS involves allowing a sideslip to develop which introduces a rolling moment which, if not counteracted, instantly increases AOA which increases the rolling moment so that the situation becomes progressive. [3, 4-7].

The three axis SAS that is presently installed in the Harrier seeks, through damping, to reduce pilot workload and lessen the effects of disturbances; the pilot would then theoretically be able to devote more attention to such concerns as sideslip control in transition. However, the lack of an autopilot function in the SAS puts a heavy burden on the pilot in itself, particularly with respect to the regulation of this yaw/bank/pitch coupled instability in turbulent and/or low visibility conditions. Also the directional (sideslip) controller, because of noise problems associated with the measurement of lateral acceleration, can be harsh and jerking in its operation. In addition, it is extremely important to note that simultaneous application of control to more than one reaction control system axis results (in high thrust demand situations) in an overall reduction

in the available control authority about all axes. Recall that the reaction control system is the Harrier's major controlling agent in hover/transition flight. This last limitation underlines the fact that a controller designed to combat this aircraft's characteristics must monitor the aircraft's motion about all axes and provide instantaneous control actuation in response to disturbances. The state variable feedback controller described and demonstrated later in this report does just that.

Having qualitatively described the Harrier's configuration and performance, the discussion now turns to a fairly rigorous mathematical formulation of these dynamic characteristics.

CHAPTER 4

THE EQUATIONS OF MOTION

It doesn't take a great deal of thought to come to the conclusion that the airplane in flight is a very complex dynamic system. Its operation requires a study of six degrees of freedom for gross motions, while the modes of motion of its many sub-systems add tremendously to the number of system states required for a model of engineering precision. But a realistic model must, of course, be defined, for practical considerations often render it impossible for the control systems engineer to have any actual contact with the system he is studying. When operating under this handicap, the best he can do is to insure that he has a thorough understanding of both the mathematical model he is using and of its relation to the physical world. With the aim of providing a greater degree of comprehension of the system presently under study, this chapter presents a derivation of the Harrier airframe equations of motion. The procedure used is based largely on that described in Bernard Etkins', The Dynamics of Flight, Editions I and II [5 and 6], an aircraft stability and control text that has become a standard in the field.

In order to provide a perspective on the ensuing derivation, a summary of the key steps in the development of the equations is in order. The aircraft is first treated as a rigid body and the

equations of motion are derived assuming the reference axes are fixed to its center of gravity. This analysis yields the classical Euler equations. A discussion is then made of the orientation of the plane in space and of the choice of axes. The treatment of spinning rotors and the modeling of the aircraft controls is then considered. Finally, small disturbance theory is used to linearize and partially decouple the equations. It should be emphasized that this discussion is meant to be more illustrative than definitive. A complete derivation is found in The Dynamics of Flight.

A rigorous derivation of the general equations of motion for aircraft begins with particle mechanics, where elementary masses and forces are related via Newton's second law and summed:

$$\Sigma\delta\vec{F} = \Sigma\delta m\dot{\vec{V}} \qquad 4.1$$

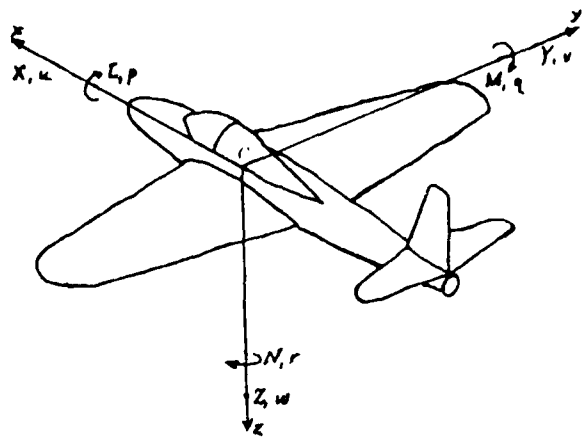
By applying the definition of the center of mass and carrying out the summation, we arrive at the familiar:

$$\vec{F} = m\,\dot{\vec{v}}_c \qquad 4.2$$

where $\vec{F}$ is the resultant external force applied, m is the total mass, and $\dot{\vec{v}}_c$ is the acceleration of the center of mass. A similar development for rotations equates the total external moment, $\vec{G}$, to the time derivative of angular momentum, $\dot{\vec{h}}$, by:

$$\vec{G} = \dot{\vec{h}} \qquad 4.3$$

These two vector equations (4.2 and 4.3) form the foundation for the expanded equations of motion.

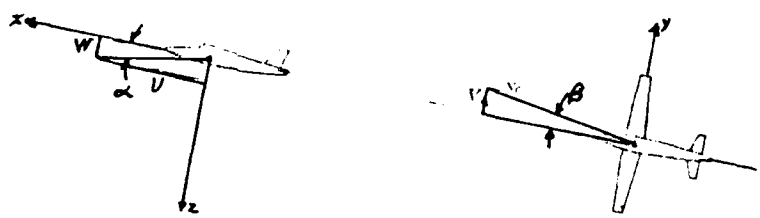


L = Rolling Moment	P = Rolling Velocity	ϕ = Bank angle
M = Pitching Moment	Q = Pitching Velocity	θ = Pitch angle
N = Yawing Moment	R = Yawing Velocity	ψ = Azimuth angle

X, Y, Z = Aerodynamic Force Components

U. V, W = Velocity Components (w.r.t. the airmass)

Figure 4.1 Standard Body-Axis Nomenclature



Angle of Attack: $\alpha = \tan^{-1} \frac{W}{U}$

Angle of Sideslip: $\beta = \sin^{-1} \frac{V}{V_{c.g.}}$

Figure 4.2 Inclination of Body Axes to $v_{c.g.}$

Before proceeding with the discussion leading to the Euler equations, a brief comment on reference axes is appropriate. First of all, the development presented here is valid for any "body axis" system -- orthogonal axes fixed in the airplane, with the center of gravity as the origin. Because of the simplifications they allow later in the derivation, the ultimate choice of axes is the "stability axis" system - with the x-axis pointing in the direction of motion, the y-axis extending out the right wing, and the z-axis pointing "downward" such that it fulfills the requirement of orthogonality. Stability axes find application primarily in the study of linear disturbance theory. Figures 4.1 and 4.2 define the necessary nomenclature.

The evaluation of the angular momentum, $\vec{h}$, is vitally important.

$$\vec{h} = \Sigma\delta\vec{h} = \Sigma(\vec{F} \times \vec{V})\delta m \qquad 4.4$$

Let the aircrafts angular velocity vector be:

$$\omega = iP + jQ + kR \qquad 4.5$$

The total velocity of a point in a rotating rigid body is

$$\vec{V} = \vec{v}_c + \vec{\omega}x\vec{r} \qquad 4.6$$

with $\vec{r} = x\vec{i} + y\vec{j} + z\vec{h}$, from which

$$\vec{h} = \Sigma r + (\vec{v}_c + \vec{w}x\vec{r})\zeta m \qquad 4.7$$

The application of vector algebra yields:

$$\vec{h} = \vec{\omega}\Sigma(x^2+y^2+z^2)\delta m - \Sigma\vec{r}(Px+Qy+Rz)\delta m \qquad 4.8$$

The summations that occur in the equations of the scalar components of equation 4.8 are the moments and products of inertia for the aircraft, so we have, after substitution:

$$\begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \qquad 4.9$$

where

[A B C] = moments of inertia about x, y, and z axes, respectively. and $D = \int yzdm$, $E = \int xzdm$, $F = \int xydm$ are the so-called products of inertia.

The derivative of a vector $\vec{Q}$ in a rotating reference frame is:

$$\dot{\vec{Q}} = \frac{\delta\vec{Q}}{\delta t} + \vec{\omega}x\vec{Q} \qquad 4.10$$

where

$$\frac{\delta\vec{Q}}{\delta t} = \vec{i}\,\dot{Q}_x + \vec{j}\,\dot{Q}_y + \vec{k}\,\dot{Q}_z$$

Applying this definition to our vector equations of motion, in a reference frame fixed to the aircraft, yields:

$$\vec{F} = m\frac{\delta v_c}{\delta t} + m\vec{\omega} \times \vec{v}_c \qquad 4.11$$

$$\vec{G} = \frac{\delta\vec{h}}{\delta t} + \vec{\omega} \times \vec{h} \qquad 4.12$$

with the following scalar components:

$$\begin{aligned} F_x &= m(\dot{U} + QW - RV) \\ F_y &= m(\dot{V} + RU - PW) \\ F_z &= m(\dot{W} + PV - QU) \end{aligned} \qquad 4.13$$

and

$$\begin{aligned} L &= \dot{h}_x + Qh_z - Rh_y \\ M &= \dot{h}_y + Rh_x - Ph_z \\ N &= \dot{h}_z + Ph_y - Qh_x \end{aligned} \qquad 4.14$$

axis system may be uniquely defined by an ordered series of three rotations - first in yaw to angle ψ, then in pitch to angle θ, and finally in roll to angle ϕ. The specific ordered set of rotations just described defines the so called Euler angles.

The next step in the development of the equations is to account for the "rotary derivatives" - the angular momentum added to the total by any spinning subsystems. This quantity would normally appear as an additive term ($\vec{h}'$) to the r.h.s. of equation 4.8, but, because of the Harrier's contrarotating turbine sections, it is not important to this discussion.

The next subsystem group to be modeled is that of the flight controls. A detailed discussion of this area is beyond the scope of this report. In general, each of the three control systems (rudder, aileron and elevator) is assumed to be a rigid, one-degree-of-freedom linkage. LaGrange's equation of motion in a moving reference frame is then applied:

$$\frac{d}{dt}\frac{\partial T}{\partial q_k} - \frac{\partial T}{\partial q_k} = F \qquad 4.18$$

where

T = kinetic energy of the system

W = work done on the system by external forces

F = generalized force

q_k = generalized coordinate

where h is defined by equation 4.9. Equations 4.13 and 4.14, together with 4.9, are the Euler equations of motion for the airplane. The physical interpretation of the variables of the Euler equations will become evident after the following discussion concerning orientation relative to fixed axes.

Because one of the basic assumptions made in the foregoing analysis was that the reference axes used were fixed to the aircraft, the linear and angular position of the aircraft cannot be described relative to them. Two sets of differential equations produce the desired transformation to a fixed frame (x', y', z'):

$$\begin{aligned}
\dot{x}' &= U\cos\theta\cos\psi+V(\sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi)+W(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi) \\
\dot{y}' &= U\cos\theta\sin\psi+V(\sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi)+W(\cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi) \\
\dot{z}' &= -U\sin\theta+V\sin\phi\cos\theta+W\cos\phi\cos\theta
\end{aligned} \tag{4.15}$$

and, to describe the angular orientation:

$$\begin{aligned}
P &= \dot{\phi} - \dot{\psi}\sin\theta \\
Q &= \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\
R &= \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi
\end{aligned} \tag{4.16}$$

therefore,

$$\begin{aligned}
\dot{\theta} &= Q\cos\phi - R\sin\phi \\
\dot{\phi} &= P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta \\
\dot{\psi} &= (Q\sin\phi + R\cos\phi)\sec\theta
\end{aligned} \tag{4.17}$$

It is obvious that the integration of equations 4.15 and 4.17 represents a formidable problem except in simplified special cases. Fortunately, linearization by small disturbance theory simplifies them. It should be mentioned that the orientation of an airplane relative to another

Application of this equation yields the three control equations:

$$H_e + F_e = I_e\ddot{\delta}_e + m_e e_e a_{c_z} + P_{ex}\,(PR-\dot{Q}) \quad \text{(elevator)}$$

$$H_r + F_r = I_r\ddot{\delta}_r - m_r e_r a_{cy} - P_{rx}\,(PQ+\dot{R}) - P_{rz}\,(RQ-\dot{P}) \quad \text{(rudder)} \qquad 4.19$$

$$2H_a + F_a = I_a\ddot{\delta}_a + 2P_{ay}\,(RQ + \dot{P}) \quad \text{(aileron)}$$

where

a_c = accelerated of center of mass

e = mass eccentricity of control surface

F = generalized control force

H = control hinge moment

I = control effective moment of inertia

M = mass of control surface

P = control product of inertia

δ = control angle

The development of the control equations represents the last step in the modeling of the aircraft from a stability and control point of view. However, the complexity of the equations renders them nearly unusable in their present form.

One minor addition needs to be added to the present equations. The total external force applied to the aircraft is the sum of aerodynamic (including propulsive) and gravitational forces. The quantities appearing on the l.h.s. of equations 4.13 are, therefore,

$$F_x = X - mg \sin\theta$$

$$F_y = Y + mg \cos\theta \sin\phi \qquad 4.20$$

$$F_z = Z + mg \cos\theta \cos\phi$$

The moment equations, being purely aerodynamic, need not be altered.

Small disturbance theory will now be used to linearize the equations. The validity of the approach rests on two facts: (1) it is often the case that the aerodynamic response is a nearly linear function of the disturbance, and (2) flight involving large external disturbances may be sustained with relatively small variations in linear and angular velocity of the aircraft. Small disturbance theory necessitates the following assumptions:

(a) all disturbances and derivatives all small, and their products and squares can therefore, be considered negligible;

(b) stability axes are used;

(c) reference flight condition symmetric and without angular velocity;

(d) small angle approximations may be used;

(e) rotary derivatives are negligible; and

(f) all controls are dynamically balanced.

The application of small disturbance theory to linearize the equations involves several steps. Initial application to equations 4.13, 4.14, 4.16, 4.17, and 4.19 yields:

$$
\begin{aligned}
X_o + \Delta X - mg(\sin\theta_o + \theta\cos\theta_o) &= m\dot{u} \\
Y_o + \Delta Y + mg\phi\cos\theta_o &= m(\dot{v} + u_o r) \qquad 4.21 \\
Z_o + \Delta Z + mg(\cos\theta_o - \theta\sin\theta_o) &= m(\dot{\omega} - u_o q)
\end{aligned}
$$

$$L_o + \Delta L = A\dot{p} - E\dot{r}$$
$$M_o + \Delta M = B\dot{q} \qquad 4.22$$
$$N_o + \Delta N = -E\dot{p} + C\dot{r}$$

$$H_{e_o} + \Delta H_e + F_{e_o} + \Delta F_e = I_e\ddot{\eta}$$
$$H_{r_o} + \Delta H_r + F_{r_o} + \Delta F_r = I_r\ddot{\zeta} \qquad 4.23$$
$$2H_{a_o} + 2\Delta H_a + F_{a_o} + \Delta F_a = I_a\ddot{\xi}$$

$$\dot{\theta} = q$$
$$\dot{\phi} = p + r \tan \theta_o \qquad p = \dot{\phi} - \dot{\psi} \sin \theta_o \qquad 4.24$$
$$\dot{\psi} = r \sec \theta_o$$

If the disturbance variables in equations 4.21 - 4.24 are set equal to zero, the equations then apply to the reference condition. By subtracting these reference values from the equations, we can eliminate all initial forces and moments:

$$\Delta X - mg\theta\cos\theta_o = m\dot{u}$$
$$\Delta Y + mg\phi\cos\theta_o = mv + mu_o r \qquad 4.25$$
$$\Delta Z - mg\theta\sin\theta_o = m\dot{\omega} - mu_o q$$

$$\Delta L = A\dot{p} - E\dot{r}$$
$$\Delta M = B\dot{q} \qquad 4.26$$
$$\Delta N = -E\dot{p} + Cr$$

$$\Delta H_e + \Delta F_e = I_e \ddot{N}$$

$$\Delta H_r + \Delta F_r = I_r \ddot{\zeta} \qquad 4.27$$

$$2\Delta H_a + \Delta F_a = I_a \ddot{\xi}$$

$$\dot{\theta} = q$$

$$\dot{\phi} = p + r \tan \theta_o \qquad p = \dot{\phi} - \dot{\psi} \sin \theta_o \qquad 4.24 \text{ (unchanged)}$$

$$\dot{\psi} = r \sec \theta_o$$

The derivation up to this point has been general in scope, i.e., is applicable to both conventional and VSTOL aircraft. It is in the treatment of the aerodynamic force and moment perturbations (the terms with a "Δ" prefix) that the differences show. This is also the point of major problems from the VSTOL modeling point of view, as the literature essentially omits this process from their discussion. The actual method used was introduced by Bryan in 1911. It is based on the assumption that the aerodynamic perturbations can be constructed from the instantaneous values of the disturbance velocities, the control deflections, and their derivatives via a Taylor series expansion. The resulting series is linearized by neglecting all higher order terms. One such "linear air reaction" takes the form

$$\Delta A = A_u u + A_{\dot{u}} \dot{u} + \ldots A_\delta \delta + A_{\dot{\delta}} \dot{\delta} \qquad 4.28$$

where $A_u = \left(\frac{\partial A}{\partial u}\right)_o$ reference condition

The derivatives "A" are called the stability derivatives of the aircraft. Engineering experience and judgement are used to determine which stability derivatives are important enough to be included in the air reaction. This decision, in turn, affects the final form of the equations. Further discussion of this decision process is beyond the scope of this text.

After careful thought and consideration of the literature, it was concluded that the VSTOL linear air reactions take the following form for the Harrier:

$$\Delta X = X_u u + X_w w + X_q q + X_{\theta_j} \theta_j + X_{\delta_T} \delta_T + X_{\delta_E} \delta_E$$

$$\Delta Y = Y_v v + Y_p p + Y_r r + Y_{\delta_A} \delta_A + Y_{\delta_R} \delta_R \qquad 4.29$$

$$\Delta Z = Z_u u + Z_w w + Z_q q + Z_{\theta_j} \theta_j + Z_{\delta_T} \delta_T + Z_{\delta_E} \delta_E$$

$$\Delta L = X_v v + L_p p + L_r r + L_{\delta_A} \delta_A + L_{\delta_R} \delta_R$$

$$\Delta M = M_u u + M_w w + M_q q + M_{\theta_j} \theta_j + M_{\delta_T} \delta_T + M_{\delta_E} \delta_E \qquad 4.30$$

$$\Delta N = N_v v + N_p p + N_r r + N_{\delta_A} \delta_A + N_{\delta_r} \delta_r$$

and $\quad I_a = I_r = I_e = 0$

Substitution of the air reactions into equations 4.24-4.27 yield equations similar in form (but not content) to those in the literature:

$$[m\frac{d}{dt} - X_u]\, u - X_w w + (mg\cos\theta_o - X_q\frac{d}{dt})\,\theta = X_{\theta_j}\theta_j - X_{\delta_T}\delta_T - X_{\delta_e}\delta_e = 0$$

$$-Z_u ut(m\frac{d}{dt} - Z_w)w - [(mu_o + Z_q)\frac{d}{dt} - mg\sin\theta_o]\,\theta - Z_{\theta_j}\theta_j - Z_{\delta_T}\delta_T - Z_{\delta_e}\delta_e = 0$$

4.31

$$-M_u u - M_w w + (B\frac{d^2}{dt^2} - M_q\frac{d}{dt})\theta - M_{\theta_j}\theta_j - M_{\delta_T}\delta_T - M_{\delta_e}\delta_e = 0$$

$$q - \frac{d}{dt}\theta = 0$$

$$(m\frac{d}{dt} - Y_v)\, v - Y_p p + (mu_o - Y_r)r - (mg\cos\theta_o)\phi - Y_{\delta_a}\delta_a - Y_{\delta_r}\delta r = 0$$

$$-L_v v + (A\frac{d}{dt} - L_p)p - (E\frac{d}{dt} + L_r)r - L_{\delta_a}\delta_a - L_{\delta_r}\delta_r = 0$$

4.32

$$-N_v v - (E\frac{d}{dt} + N_p)p + (C\frac{d}{dt} - N_r)r - N_{\delta_a}\delta_a - N_{\delta_r}\delta_r = 0$$

$$\dot{\phi} = p + \tan\theta_o r$$

OUTPUT EQUATIONS

$$\alpha = \tan^{-1} \frac{W}{U} = \tan^{-1} \left[\frac{w}{u_o+w}\right]$$

$$\beta = \tan^{-1} \frac{V}{U} = \tan^{-1} \left[\frac{v}{w_o+u}\right] \qquad 4.33$$

$$\dot{\psi} = r \sec \theta_o$$

Since $\theta_o = 5°$ for the model, the second term of the r.h.s. of the last equation of equation set 4.32 may be neglected. Algebraic manipulation is all that is needed to transform equations 4.31 and 4.32 into their final, state variable matrix form. The results are shown in Figures 4.3 and 4.4. But equations 4.31 and 4.32 are of little use without experimental knowledge of the values of their coefficients. Hence, the next chapter discusses the source from which the AV-8B's stability derivatives were obtained, and describes the methodology used to determine them experimentally.

THE LONGITUDINAL EQUATIONS

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} X_u' & X_w' & -g\cos\theta_o & -w_o - X_q' \\ Z_u' & Z_w' & -g\sin\theta_o & u_o + Z_q' \\ 0 & 0 & 0 & 1 \\ M_u' & M_w' & 0 & M_q' \end{bmatrix} \begin{bmatrix} u \\ w \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} X_{\delta_{ES}}' & X_{\delta_T}' & X_{\theta_j}' \\ Z_{\delta_{ES}}' & Z_{\delta_T}' & Z_{\theta_j}' \\ 0 & 0 & 0 \\ M_{\delta_{ES}}' & M_{\delta_T}' & M_{\theta_j}' \end{bmatrix} \begin{bmatrix} \delta_{ES} \\ \delta_T \\ \theta_j \end{bmatrix}$$

for the stability derivatives: $X' = \frac{X}{m}$ $Z' = \frac{Z}{m}$ $M' = \frac{M}{B}$ where m = mass of aircraft

B = pitch inertia

Figure 4.3 The Longitudinal State Equations

THE LATERAL EQUATIONS

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v' & w_o+Y_p' & -u_o+Y_r' & g\cos\theta_o \\ L_v' & L_p' & L_r' & 0 \\ N_v' & N_p' & N_r' & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} Y'_{\delta_{AS}} & Y'_{\delta_{RP}} \\ L'_{\delta_{AS}} & L'_{\delta_{RP}} \\ N'_{\delta_{AS}} & N'_{\delta_{RP}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{AS} \\ \delta_{RP} \end{bmatrix}$$

where $Y' = Y/m$

and

$$L'_v = \frac{CL_v+EN_v}{CA-E^2} \quad L_p' = \frac{CL_p+EN_p}{CA-E^2} \quad L_r' = \frac{CL_r+EN_r}{CA-E^2} \quad L'_{\delta_{AS}} = \frac{CL_{\delta_{AS}}+EN_{\delta_{AS}}}{CA-E^2} \quad L'_{\delta_{RP}} = \frac{CL_{\delta_{RP}}+EN_{\delta_{RP}}}{CA-E^2}$$

$$N'_v = \frac{EL_v+AN_v}{CA-E^2} \quad N_p' = \frac{CL_p+AN_B}{CA-E^2} \quad N_r' = \frac{EL_r+AN_r}{CA-E^2} \quad N'_{\delta_{AS}} = \frac{CL_{\delta_{AS}}+AN_{\delta_{AS}}}{CA-E^2} \quad N'_{\delta_{RP}} = \frac{EL_{\delta_{RP}}+AN_{\delta_{RP}}}{CA-E^2}$$

$A = I_{xx}$
$B = I_{yy}$
$C = I_{xz}$

Figure 4.4 The Lateral State Equations

CHAPTER 5

ORIGIN OF THE MODEL

The simulation described later in this report is based upon a linear, non-constant coefficient mathematical model developed by Calspan Corporation of Buffalo, New York in the summer of 1977. [7] Calspan actually laid the groundwork for the model much earlier, publishing a feasibility study which contained a generic VSTOL model in July of 1976. [8] The genesis of the linear AV-8B model as published in the final report, Calspan TM No. 98, is interesting in two major respects.

The first is that the original motivation for developing a linear math model was to determine the feedback gains for the variable stability mechanism of Calspan's X-22A in-flight simulation aircraft. In fact, hardware limitations of the X-22A avionics were the primary reason for linearization of the AV-8B equations of motion. The other particularly notable fact concerning the origin of the model is that it is, essentially, a simulation of a simulation. Simulation of the actual Advanced Harrier was not possible because at the time of the TM No. 98 report, the AV-8B was not in existence. A study of the methodology used in the TM No. 98 model development provides needed insight as to the fidelity and application of the linearized equations of motion for the Harrier used in this work.

The only model of the AV-8B Harrier in existence at the time of the Calspan report was a nonlinear table-look-up digital

computer simulation at McDonnell-Douglas' (MCAIR) St. Louis facility. Consequently, Calspan, in cooperation with NADC, Warminster, requested that two representative approach (transition→ hover) trajectories be "flown" on the MCAIR simulation. The aircraft was to be kept at trim throughout the approach. MCAIR complied, defining a 3° (flight path angle or "glide slope") approach from 105 Kt, and a 5° approach from 65 Kt. It was decided that the angle of attack would be held constant at 8° and that a "one-step" deceleration would be used, that is, nozzles rotated from full back to full down instantaneously. In all, these two trajectories may be considered representative of actual flight procedures based on the NATOPS Flight Manual. [3]

The mathematical model was to be developed in terms of these task-related reference trajectories, with linearized, but non-constant stability and control derivatives being calculated about the "trim" conditions. Small perturbation theory and the use of stability axes as the reference frame are necessary to employ this procedure. The actual calculation of the derivatives was conducted as follows.

A computer subroutine was written to interact with the MCAIR simulation. At various points along each reference trajectory, the subprogram first perturbed one of the state variables (u, v, w, etc.), and then measured the corresponding changes in the other aerodynamic and thrust forces and moments (the other state variables). These force and moment deflections were then plotted

versus the perturbation, and the appropriate linearized stability derivative determined by statistical data fit in the least squares sense. Since this entire analysis is based on perturbation studies at a single velocity along the reference trajectory, the process yields a discrete series of stability derivatives, one for each velocity tested. Calspan choose 0, 30, 50, 65, 80, and 105 knots, and TM No. 98 contains tabulated values of every stability and control derivative at each of these six velocities. [7, pp 9-10] A summary of these results is contained in Appendix A.

In terms of the equations of motion, the stability derivatives represent the coefficients of the characteristic equation. As shown in the last chapter, a linear model can be represented as a set of simultaneous first order differential equations of the form

$$\dot{\bar{X}} = \underset{\sim}{A}\,\bar{X} + \underset{\sim}{B}\bar{u} \qquad 5.1$$

where

$\underset{\sim}{A}$ = n x n Coefficient Matrix

$\underset{\sim}{B}$ = n x r Input Matrix

$\bar{X}$ = n x 1 State Vector

$\bar{u}$ = r x 1 Input Vector

The stability and control derivatives are then the elements of the A and B matrices respectively.

Thus, the airframe model of the AV-8B Harrier has been completely developed and defined. It is seen to consist of two fourth order systems of the form of equation 5.1. One equation describes the lateral/directional modes of motion, while the other treats

the longitudinal characteristics. The coefficients of the A and B matrices have been calculated by Calspan for six distinct points in the transition/hover regime, and have been shown to be functions of aircraft velocity. Having derived and defined a Harrier airframe model, the discussion can now focus on the theory of stability augmentation controllers.

CHAPTER 6

PRESENT STABILITY AUGMENTATION

It is appropriate to preface an analysis of the way the present Harrier's Stability Augmentation System (SAS) attempts to solve the instability problem in transition flight with a few comments on the development of the SAS block diagram used in this research. In the first place, no definitive model of the AV-8B SAS was found in the literature. The most thorough treatment of the AV-8A SAS is contained in Reference 8 as an appendix. [8, pp 92-5] NADC, Warminster responded to the writer's request for more information by providing a sketch of their best estimate of the SAS structure. The block diagrams of the AV-8B SAS shown in Figures 6.1 and 6.2 were constructed using information from various sources, judgements being made based on engineering experience in cases of conflicting data. This updating of the model was a dynamic and on-going process throughout the research. The final models arrived at and shown here are believed to accurately represent the AV-8B SAS.

Inspection of Figures 6.1 and 6.2 reveals that all channels of the SAS exhibit a saturation nonlinearity. These nonlinearities arise from the control authority limits imposed on the SAS.

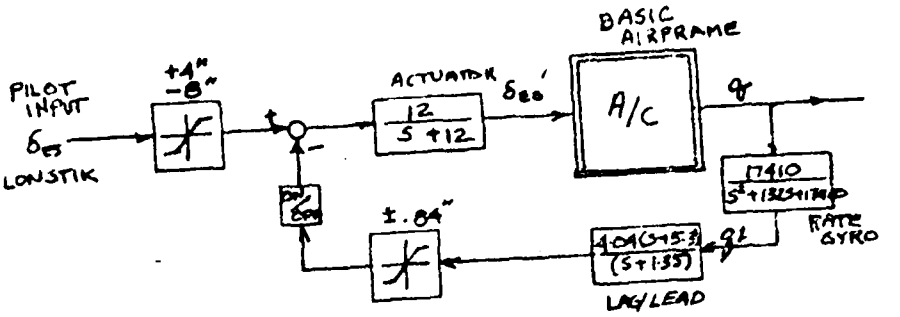


Figure 6.1 Longitudinal SAS

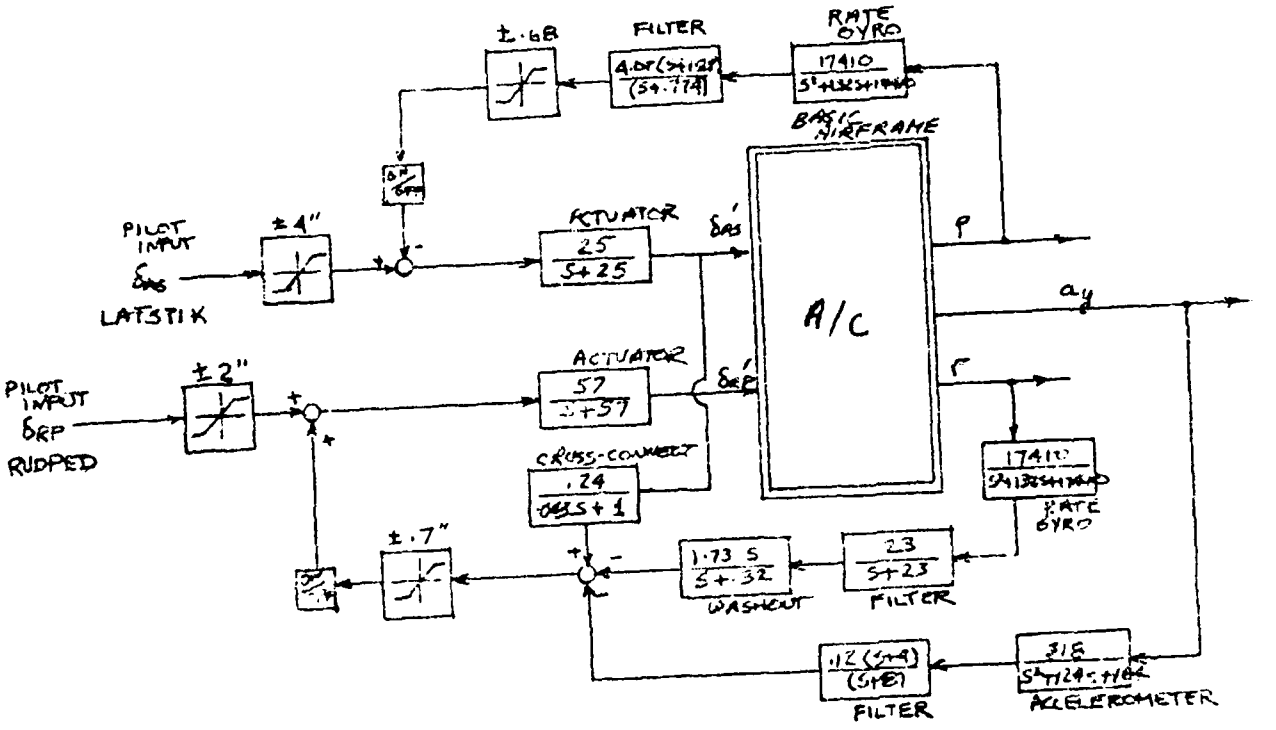


Figure 6.2 Lateral/Directonal SAS

Nonlinear system behavior cannot be compensated by classical controls techniques. In addition, it can be seen that the SAS model, by requiring complex signal filtering and processing in the feedback loops, significantly raises the order of the system. Also, except for the lateral acceleration feedback loop (which will be discussed below), angular rates are the only system variables fed back. Feedback of the first time derivative of the controlled quantity usually effects relative stability, serving to increase the damping of the motion. Thus, Harrier pilots commonly refer to the SAS as a "control damper". In the literature, angular rate feedback in aircraft controllers without accompanying feedback of angular position is virtually nonexistent.

The feedback of single states to single inputs with accompanying series compensation networks in a multi-input, multi-output system is an attempt to apply something familiar, classical transfer function compensation, to something unfamiliar, the multi-input, multi-output plants. The present Harrier SAS represents just such an attempt. It seems safe to conclude that this approach can provide neither a systematic design of, nor an optimal configuration of total system compensation.

Because the Harrier lateral and longitudinal airframe models

consist of coupled differential equations, and because both systems of equations are of the multi-input, multi-output type, conventional transfer function control design is largely ineffective. This is not to say that it is impossible to determine the input/output relation (transfer function) between any given input and output. With a knowledge of the composition of the $\underset{\sim}{A}$ and $\underset{\sim}{B}$ matrices, it is relatively easy, with the use of modern control theory, to perform the necessary matrix manipulations for transfer function determination. The problem that arises is that compensation of the determined input-output network usually has a detrimental effect on the performance of the other input-output relationships. In the interest of completeness, this method of SAS design was attempted with the Harrier airframe equations using the powerful Thayer method of classical transfer function compensation. As might be expected, the attempt was no more successful than the present SAS.

From a performance standpoint, the present SAS design does improve the Harrier's handling qualities to a limited extent. From a state-of-the-art control systems point of view, however, the system is quite rudimentary, certainly yields suboptimal results (the aircraft is still unstable), and is therefore inadequate.

CHAPTER 7

STATE FEEDBACK CONTROL

It seems intuitively correct that the more knowledge the controller has as to the state of the system being controlled, the more accurate the controlling signal produced. Similarly, if the controller has the ability to affect a greater number of control inputs, it will be better able to bring the controlled plant to the desired output state. By measuring every system output state, and then calculating an actuating signal for each control input of the system based on a specified weighted sum of the output states, the state feedback controller generates a superior solution to the situation discussed in the previous chapter. This chapter documents the development of the digitally-based, state feedback controller designed as a stability augmentation device for the AV-8B Harrier. A representative drawing of the system is presented in Figure 7.1.

The use of state feedback control eliminates, or greatly reduces all the problems inherent to the design of the present SAS. To review, the deficiencies of the present SAS are:

(1) failure to account for nonlinear effects;

(2) requirement of complex analog signal processing in the feedback loop;

(3) greatly increased system order;

(4) has no significant effect on root locations; and

(5) does not account for all the states, nor for coupling effects.

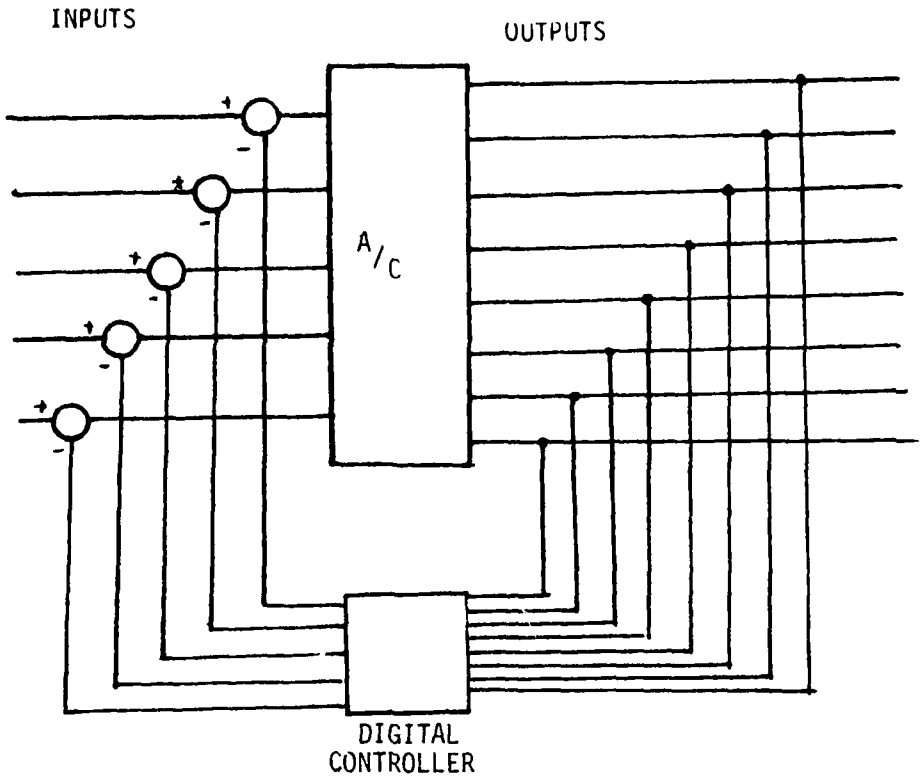


Figure 7.1 Representative Model: Digital State Feedback

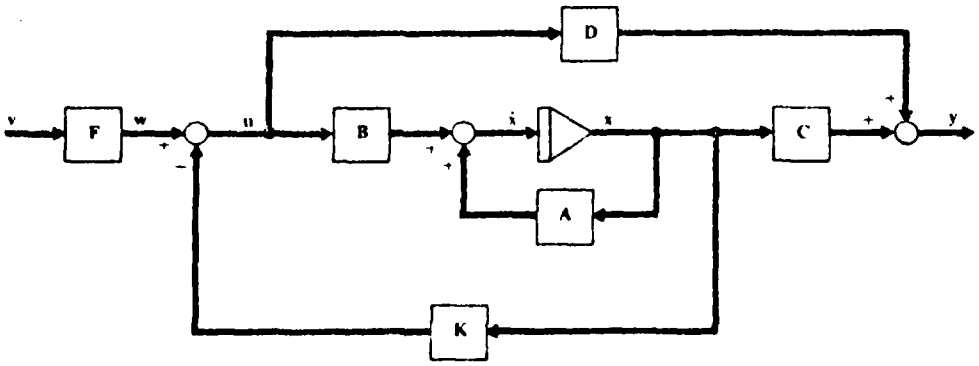


Figure 7.2 State Feedback Block Diagram
($\underset{\sim}{D} = \underset{\sim}{0}$, $\underset{\sim}{F} = \underset{\sim}{I}$, $\underset{\sim}{C} = \underset{\sim}{I}$)

These problems are not encountered in the state feedback controller. In addition, the following benefits are realized using state feedback:

(1) Because the controller reacts so efficiently to counteract disturbances of normal magnitude, saturation is seldom encountered, and the system is thus effectively linear.

(2) Output variables are multiplied by a pure gain and summed, thus no extensive signal processing is required.

(3) System order is unaffected by state feedback control.

(4) Provided no limitations are put on feedback gains, it is theoretically possible to relocate closed loop poles anywhere in the s-plane.

(5) The calculation of the state feedback gain matrix, k^T, accounts for coupling effects between all system states and inputs.

Basic modern control theory states that the characteristic differential equation for a system can be determined by setting the determinant of the $[s\underset{\sim}{I}-\underset{\sim}{A}]$ matrix equal to zero. The roots of the resulting algebraic equation in the complex variable "s" are then equal to the roots of the system itself, and therefore define the system's natural modes of motion. When this analysis is applied to the Harrier airframe equations (see Appendix B), it is found that at least one root of the lateral equation and one root of the longitudinal equation has a positive real part. This is to be expected as the flight manual contains numerous warnings of transition instabilities. The function of a stability augmentation system (SAS) then, in terms of the roots of the

characteristic equation, is to "pull" the loci of root positions (or "root loci") further to the left and, if possible, into the stable left half plane. The design criteria used to determine the SAS state feedback gains for this study was the placement of the system closed loop poles. The following discussion is a brief summary of the technique as presented in Reference 9. [9, pp. 308-314]

It can be shown that for a controllable open loop system of the form

$$\dot{x} = \underset{\sim}{A}x + \underset{\sim}{B}u \qquad 7.1$$

any desired closed loop poles may be achieved by a constant state feedback matrix $\underset{\sim}{k}^T$. It can also be shown that the numerator dynamics of the closed loop system are unaffected by state feedback. Furthermore, the closed loop eigenvalues must satisfy the equation (refer to Figure 7.2):

$$\Delta'(\lambda) = |\lambda_i \underset{\sim}{I} - \underset{\sim}{A} + \underset{\sim}{B}\underset{\sim}{k}^T| = 0 \qquad 7.2$$

for each desired root, λ_i. It is possible to transform this equation into an equivalent form (see "Nomenclature" section for explanation of symbology):

$$\Delta'(\lambda_i) = 0 = |(\lambda_i \bar{I}_n - \bar{A})[\bar{I}_n + (\lambda_i \bar{I}_n - \bar{A})^{-1}\bar{B}\bar{k}^T]|$$

$$= |\lambda_i \bar{I}_n - \bar{A}|\,|\bar{I}_n + (\lambda_i \bar{I}_n - \bar{A})^{-1}\bar{B}\bar{k}^T|$$

$$= \Delta(\lambda_i)|\bar{I}_n + \bar{\phi}(\lambda)\bar{B}\bar{k}^T| \qquad 7.3$$

since $\Delta(\lambda_i) \equiv |\lambda_i \bar{I}_n - \bar{A}|$

$$= \Delta(\lambda_i)|\bar{I}_r + \bar{k}^T\bar{\phi}(\lambda_i)\,\bar{B}^T| \qquad 7.4$$

Here $\phi(\lambda_i) \equiv (\lambda_i I_n - A)^{-1}$ and $\Delta(\lambda_i)$ is the open loop characteristic equation. The $\bar{k}^T$ that drives the determinant specified by either equation 7.2 or 7.3 to zero for each λ_i is the solution to the pole placement problem.

Elementary matrix theory insures a zero determinant if any row or column is zero. Associate Professor E. E. Mitchell, the principal faculty advisor for this project, programmed an algorithm to force the columns of equation 7.4 to zero. A listing of this program is included as an appendix.

After suitable manipulation of the equations, it is possible to determine that there are nr unique solutions to the feedback gains problem. This is fortunate as the algorithm proved susceptible to computer round-off error noise in this application. (In general, the round-off error is more prominant when the plant is a large, unstable system). To combat the round-off problem, it is good practice to use double precision variables and to test for a zero determinant for each solution.

The final design procedure was as follows. First, the gain-calculating program (see Appendix D) was employed to study the magnitudes of the gains required for various pole placement areas. While infinite gain is theoretically possible in a digital controller, extremely high gains are still undesirable. Time responses for the various gain matrices were studied on the digital computer as a part of this effort. As a result, the following system pole locations were somewhat subjectively chosen:

Longitudinal: -2, -2.2, -2.4, -3

Lateral: -3, -3.2, -3.5, -4

Not only did these roots give satisfactory system time responses, but, for V_{cg} = 30 kt, they were small enough to be magnitude scaled to run on the analog section of a hybrid computer. There are two major reasons for the desirability of selecting roots on the real axis: (1) It was thought that roundoff errors would make the pole placement a "rough" process, and so a high damping ratio was desired; and (2) the gain-calculating algorithm proved especially unstable for complex pole placement. Like the stability derivatives, the state feedback gains are also velocity dependent. A table-look-up routine was thus utilized to implement variable gains as functions of velocity.

On paper, state feedback control provides a plethora of advantages including: (1) positive placement of poles in the s-plane; (2) "total system" compensation accounting for coupling

effects; and (3) no real increase in model complexity. The summary of simulation results leaves little doubt as to the validity of the technique.

CHAPTER 8

COMPUTER SIMULATION

The final model of the Harrier airframe consists of two fourth order, linear differential equations of the form:

$$\dot{\bar{x}} = \underset{\sim}{A}\bar{x} + \underset{\sim}{B}\bar{u} \qquad 8.1$$

with the elements of the $\underset{\sim}{A}$ and $\underset{\sim}{B}$ matrices being the velocity-dependent stability and control derivatives, respectively. The first step toward a computer simulation of this set of equations was the development of a subroutine to calculate and read the stability derivatives at any velocity (see Appendix E). Since the values of the derivatives were known only for six distinct velocities, some sort of interpolation scheme was required. The method chosen was a polynomial fit in the least squares sense. The coefficient-calculating subprogram, MATCOF, reads the coefficients of the least-squares polynomials, and then uses them to calculate the stability and control derivatives. The derivatives are returned in the arrays ALON, BLON, ALAT, and BLAT. This subprogram was used, without modification, on both the large scale digital computer simulation and the hybrid simulation.

The initial testbed for the mathematical model was the Naval Academy's Time-Sharing System (NATS). A FORTRAN digital

simulation utility program, DIGISIM, was used as the basis for the simulation. DIGISIM, which was also programmed by Associate Professor Mitchell, uses a predictor-corrector algorithm to handle the integrations. The many user-defined options available in DIGISIM make it a particularly powerful simulation tool. These features are described in Appendix G of this report. Using an existing utility program has the additional advantage of some assurance of the stability of the integration routine itself.

The DIGISIM-based digital simulation, named AHEAD - Advanced Harrier Electronic Augmentation Device (see Appendix H), was a tremendous aid to the development of both the model of the present SAS and the analog/hybrid simulation. It proved particularly helpful in the determination of scaling constants for the analog implementation of the equations of motion. It was also used to check the accuracy of the analog model. There is every reason to believe that AHEAD is an excellent computer model of the AV-8B, its present SAS, and the newly-designed digital state feedback controller. The major drawback to its use as a controller-design tool is that the simulation consumes approximately ninety seconds of NATS C.P.U. time for ten seconds of run time.

Because it possesses (in its analog section) the ability to integrate in real time coupled with no loss of the convenience of digital program control and input/output, the hybrid implementation of the Harrier's equations of motion represents the ultimate tool in controller design. Because of the complexity of the system, this implementation was a long and painstaking process. Noise in the analog section is by far the most formidable problem to be overcome when modeling a large, unstable system. Persistance paid off, however, and the final version of the hybrid simulation yielded responses that compared closely to those of AHEAD for simulation times up to four seconds for the lateral and longitudinal equations (maximum error recorded was approximately 29%).

Noise is the limiting factor as far as accuracy is concerned, especially in the longitudinal simulation. Two items should be noted regarding simulation accuracy: (1) the noise had little effect on the position variables, probably due to the filtering effect of the pure integration; and (2) a maximum error of 29% on one state of an unstable fourth order system is actually quite small. Thus, the hybrid simulation is also judged to yield an excellent representation of the Harrier's flight in the VSTOL regime.

The one really major problem encountered in the analog/hybrid simulation involved the lateral acceleration feedback loop of the present SAS model. In order to generate the lateral acceleration on the analog model, the derivatives of three of the states were required. The derivative is available by summing the inputs to the particular state's integrator. This was tried, but the resulting lateral acceleration signal was so noisy that its feedback actually had a destabilizing effect on system response. The circuit was thereafter removed from the hybrid model. It is interesting to note that the actual lateral accelerometer used in the Harrier is very sensitive, having a threshold acceleration of 0.06 G. As such, this signal is highly sensitive to spurious system motions. In addition, the literature suggests that the feedback of lateral acceleration is usually accompanied by noise problems [1, p. 145 and 2, p. 17]. This report supports that belief.

The final hybrid model represents a truly versatile tool for the evaluation of control system performance. The initial analog parameters (including potentiometer settings) are set by a digital program that shares its name with its NATS brother, AHEAD (see Appendix J). This program automatically loads the digital state feedback control subroutine SFBCON (see Appendix K). Rather than use an actual microprocessor and associated interface equipment, it was decided that a small portion of the PDP-15's memory would be used to "simulate" a microprocessor. Considering

the present state of microprocessor technology, this substitution can be made without loss of applicability. Following the initial setup, program control is transferred to the logic and switching section of the analog computer. SFBCON runs constantly in an endless loop, sampling the state variables at a rate of about 71 samples per second and calculating feedback control signals based on each set of sampled data. Periphery devices include a functional miniature joystick and an analog generated, pictorial output, both used for pilot-in-the-loop studies. Of course, in addition to the real-time mode, the system response may be observed using the hybrid's high speed repetitive operation (rep-op) mode. Push-button logic allows instantaneous switching between the four feedback options: (1) no feedback, no SAS; (2) present SAS; (3) continuous (analog) state feedback; and (4) digital state feedback. User documentation and system diagrams are contained in Appendix I.

CHAPTER 9
SUMMARY OF RESULTS

The results of this research effort can be described in terms of the project's two major accomplishments: The successful implementation of a proven mathematical model of the AV-8B on both a large scale digital system and an analog/hybrid system, and the design and implementation of an effective SAS for that model. If it can be conclusively determined that the present simulation accurately describes the dynamic behavior of the Harrier, and if it can be shown that the state feedback control system stabilizes that model, then there is a strong logical argument that this control system satisfies the goals of the research. This summary of results will focus on showing that the newly designed digital state variable controller fulfills this project's stated objective.

The analysis of results must begin with the mathematical model, for if it is invalid, the possibility exists that any results obtained from it could be fruit of the proverbial poisoned tree. Of the validity of the final AV-8B model, Lebacqx [7, p.9] comments: "...it was judged that the linear model represented the nonlinear time histories reasonably well at these flight conditions." It therefore seems reasonable to conclude that, providing conditions remain such that small perturbation theory holds (as described in Chapter 4), the Calspan TM No. 98 linearized stability and control

derivatives define a valid mathematical model of the AV-8B.

The next logical concern is the accuracy with which the digital and hybrid simulations described earlier in this report represent the Calspan model. The answer to this question is illustrated by comparison of the graphical outputs shown in Part I of Appendix L with the Calspan-generated curves presented in Appendix M. The plots are nearly identical in every respect. Also, since the hybrid model has been shown to be in close accord with the NATS digital simulation, it can be concluded that both computer simulations describe the actual behavior of the Harrier in flight.

Having reached this favorable conclusion concerning the fidelity of the current computer representations of the unmodified Harrier, attention can be focused on the performance of the digital state feedback SAS. Reference to Parts II through IV of Appendix L indicates that application of the newly-designed digital controller results in stabilization of both the lateral and the longitudinal modes of motion of the model. The stabilization is evidenced by the fact that the state variables are driven to zero. Recall that, due to the nature of the linearization process, the state variables of the model represent disturbance quantities. Thus, the regulation of these variables to zero has the physical significance of returning

the aircraft to its "trim" condition, and therefore of stabilizing it. A closer examination of the time responses in Appendix L is in order.

The original objectives of this research called for an analysis of the intake momentum drag instability in the transition region. By setting the initial conditions of the model to reflect actual values the state variables might equal at the outset of the "weathercocking" phenomena, Parts II through IV of Appendix L provide an interesting comparative study of the performance of the present SAS versus that of the state feedback controller in this situation. Parts II and III demonstrate the system's effectiveness at various velocities in the transition region. Part II treats the lateral modes, while Part III contains longitudinal responses. The curves of Part IV present a progression of response characteristics, from that of the unstable bare airframe to the completely stable, near first order response of the state feedback equipped system.

As a further demonstration of the ability of the system to recover the aircraft, Parts V and VI of Appendix L illustrate the expansion of the flight envelope that implementation of the digital SAS allows. As discussed earlier, the flight manual prohibits angle of attack exceeding 15 units and side velocities exceeding 30 kts. The state feedback controller permits these limits to be safely expanded, at least as far as the linearized model is concerned.

A thorough study of the time trajectories of Appendix L allows one to formulate several generalizations regarding the relative performance of the two stability augmentation packages. The lateral motion of the aircraft (presented primarily in Part II of Appendix L) appears to be close to neutrally stable. In addition, the relative stability of the aircraft seems to increase with velocity, a finding not in keeping with the predictions of earlier chapters that the aircraft's motion was most unstable between 30 and 90 KIAS. Despite this discrepancy, it was judged that the curves were a fairly accurate description of the Harrier with existing stability augmentation. The trajectories of the state feedback equipped aircraft, in contrast to those just studied, show a stable, nearly first order response in all cases. Furthermore, the gain scheduling algorithm is seen to do an adequate job in maintaining a damping ratio greater than 0.5. Recall that since the design locations for the system poles lie on the real axis, the desired damping ratio is unity. Also, note how intolerable the system is to large disturbances -- it exhibits a "willingness" to reach comparatively large instantaneous values of angular velocities in order to eliminate a large side velocity, for example. (See Parts II and VI of Appendix L) This tenacity to remain at or near trim is a convenient characteristic as far as the model is concerned, for it also implies that the system seeks to remain in the linear region (where the linearized derivatives are valid).

The longitudinal response characteristics (highlighted in Part III of Appendix L) of the model in many respects resemble those of the lateral case. Under conventional stability augmentation, the aircraft experiences a very slight divergence in pitch at all velocities in the VSTOL regime. Also, a pitch distrubance causes a rapid reduction in forward velocity (U) in all cases. As in the lateral case, the motion of the state feedback equipped system is characterized by stable responses of high damping ratio.

Preliminary pilot-in-the-loop handling qualities studies conducted on the hybrid computer demonstrate the superiority of the state feedback system. The pilot evaluation task was defined as maintaining steady transition-regime (usually 30 KIAS) flight down a 100' x 100' corridor. The pilot controlled the aileron and elevator via a miniature joystick. His performance was observed on an oscilloscope using an analog-generated pictoral output. The experimenter had the ability to introduce wind gusts of varying magnitudes from any direction. The pilot was seldom able to maintain control of the aircraft under conventional stability augmentation for more than several seconds. On the other hand, the state feedback system allowed the pilot to retain control for an extended period of time, even in the presence of turbulence. The pictoral responses of both the handling qualities studies and of the open loop studies were video-taped to facilitate further study.

As stated earlier, the objective of this research has been to design an improved, microprocessor-compatible three-axis SAS for the AV-8B Harrier. Further, the SAS was to be capable of functioning at two levels of control authority - a "normal" mode and a "recovery" mode. Closed loop pole placement by digital state feedback is a suitable method to meet these criteria. The performance of the "recovery" mode is demonstrated in this paper. Note again the tenacity with which the system seeks to return the aircraft to a trimmed condition. Further, this level of performance is achieved without exceeding the control authority limit imposed by military specifications. [10] There are two possible avenues that may be followed in implementing a "normal" control authority mode: either to define new design pole locations and recalculate the feedback gain matrices, or to simply reduce the control authority of the present SAS. Preliminary studies indicate that, from a handling qualities point of view, the latter approach is preferable.

CHAPTER 10

CONCLUSIONS

The general conclusion that may be drawn from this study is that it is feasible to stabilize the motion of the AV-8B Harrier in low speed flight by using a microprocessor-based state feedback controller to "directly" relocate the system poles. This controller design has been shown to be capable of fully stabilizing a proven computer simulation of the Harrier. The achievement of these objectives has required the accomplishment of the following tasks:

- The computer implementation and verification of the Calspan linearized mathematical model for the AV-8B Harrier airframe. The computer model includes a least-squares interpolation routine to accommodate the assignment of the model's variable stability gains. The research required the development of both a digital and a hybrid computer simulation. The models were verified by comparison of time trajectories to those in the literature.

- The construction and computer implementation of a valid model for the existing Harrier SAS. The model required the use of information from various sources in the literature.

- The development of a computer algorithm for the direct relocation of the closed loop system poles. The program was then used to determine the lateral and longitudinal state feedback matrices required to meet the desired pole location criteria over the studied velocity range (0-100 KIAS).

- The design of a microprocessor-compatible controller to implement and test the calculated state feedback gains.

- A comparison of the performance of the existing conventional SAS design versus that of the digital device.

A number of conclusions concerning the simulation of VSTOL aircraft, digital flight control, and the relative performance of the two Harrier stability augmentation packages may be formulated based on this research. The statements made regarding flying qualities are the result of preliminary, informally conducted studies. The conclusions are summarized below, and additional details are contained in the text.

- The present computer simulations represent a fairly accurate and versatile tool for the design and evaluation of VSTOL flight control systems. The Naval Academy's digital and hybrid computer facilities are clearly well adapted to the study of such systems.

- Direct relocation of closed loop poles using state feedback is an effective method of flight control design. Consisting of a blend of modern (state feedback) and classical (root locus) control theories, this procedure benefits from the user's ability to rapidly visualize expected results -- a feature lacking in many control design methodologies.

- The sampling rate allowed by this controller design (a rapid 71 sps.) is more than adequate to insure performance quite comparable to that of an analogous continuous system.

- In all cases, the state variable controller was found to stabilize the Harrier, providing clean, nearly first order response. It is judged, based on the response characteristics, that the digital controller facilitated movement of the system poles to locations in close proximity to the design values.

- In contrast, the existing conventional SAS, while providing definite improvement in the aircraft's handling qualities is unable to fully stabilize its motion. In general, the responses of the conventional SAS model exhibit a near neutral stability.

- Pilot-in-the-loop handling qualities studies demonstrate clearly the superiority of the digital state feedback system. While the pilot is, in general, unable to maintain control of the conventionally-augmented system, the system equipped with the state feedback device is quite controllable, even in turbulent conditions.

- The lateral acceleration feedback loop of the hybrid simulation's conventional SAS model proved so susceptible to the effects of spurious noise in the system that it had to be removed from the circuit. This result, coupled with similar findings found in the literature, casts some degree of doubt on the use of the actual system on the Harrier, and particularly on its very light detection threshold (.06G).

The determination of whether the controller designed in this project is capable of stabilizing the Harrier rests on a number of as yet unanswered questions. The following are among the key considerations:

- The effect of many of the higher order terms (such as instrumentation delays, more complete actuator dynamics models, etc.) that have been omitted from this model.

- The limitations associated with the use of a linearized model. As shown in Chapter 4, linearization of the stability derivatives decouples the lateral and longitudinal equations of motion. However, our discussion of the

- intake momentum drag phenomena alluded to probable coupling between the lateral and longitudinal modes. The strength of this coupling and its effect on the controller have yet to be determined.

- The effects of sampling rate, pure time delays, and quantization errors. A discussion of ongoing research in regard to these and other important parameters in the study of digital flight control is contained in reference 16.

- The ability to instrument the aircraft to measure all necessary system states. The use of an observer may be required to approximate unavailable system states.

- The controller's ability to cope with model inaccuracies and to adapt to inevitable changes in plant dynamics. Complete sensitivity and robustness studies are clearly in order.

As the study of the state feedback control of aircraft is in its infancy, the literature provides little help with the many questions that this research has prompted.

In summary, an accurate representation of the dynamic behavior of the AV-8B Harrier has been programmed on both the Naval Academy PDP-15/EAI 681 Hybrid Computer System and on the Naval Academy Time-Sharing System (NATS). In addition, a digitally-based variable gain state feedback controller has been added to the simulations, and proved to be capable of fully stabilizing them. While it is doubtful that the controller in its present form is the final answer, it has been demonstrated beyond a reasonable doubt that microprocessor-based variable gain state feedback control is a valid approach to the final solution of the Harrier's stability and control problems.

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APPENDIX A

THE AV-8B STABILITY DERIVATIVES

This appendix presents the stability and control derivatives for the AV-8B Harrier linear mathematical model formulated by Calspan Corporation. This data has been extracted from TM No. 98, and is included here in the interest of completeness. The $\underset{\sim}{A}$ and $\underset{\sim}{B}$ matrices shown in Figures 4.3 and 4.4 may be derived from this data by simple substitution.

AV-8B LONGITUDINAL MODEL

STABILITY AND CONTROL DERIVATIVES

V_o ft/sec	0	50.668	84.447	109.781	135.116	177.339
V_o (Kt)	0	30	50	65	80	105
X_u'	-.044	-.044	-.044	-.044	-.044	-.044
Z_u'	0.0	-.023	-.054	-.092	-.101	-.105
M_u'	0.0	-.0009	-.0020	-.0026	-.0003	0.0
X_w'	0.0	0.0	+.0035	+.010	+.0175	+.035
Z_w'	-.018	-.125	-.195	-.240	-.300	-.390
M_w'	+.0042	+.0047	+.0040	+.0021	-.0015	-.0070
X_q'	0	+.03	+.05	+.062	+.085	.11
Z_q'	-.05	-.29	-.40	-.47	-.61	-.79
M_q'	-.056	-.118	-.160	-.191	-.224	-.250
$X_{\delta es}'$	-.161	-.151	-.145	-.139	-.134	-.126
$Z_{\delta es}'$	-.350	-.340	-.335	-.330	-.295	-.205
$M_{\delta es}'$	.230	.235	.237	.240	.241	.243
$X_{\delta T}'$	.34	.30	.262	.225	.180	.140
$Z_{\delta T}'$	-2.55	-2.46	-2.37	-2.26	-2.10	-1.75
$M_{\delta T}'$	-0.036	-0.036	-0.036	-0.036	-0.036	-0.036
X_{θ_i}'	-.555	-.546	-.530	-.510	-.475	-.400
Z_{θ_i}'	-.066	-.061	-.050	-.028	-.023	-.040
M_{θ_i}'	0	0	0	0	0	0

AV-8B LATERAL MODEL

STABILITY AND CONTROL DERIVATIVES

V_0(ft/sec)	0	50.668	84.447	109.781	135.116	177.339
V_0(Kt)	0	30	50	65	80	105
Y_v'	-.034	-.063	-.088	-.104	-.120	-.138
L_v'	-.0020	-.0144	-.0197	-.0204	-.0184	-.0104
N_v'	-.0030	-.0021	-.0010	0.0	+.0014	.0047
Y_p'	0	0	0	0	0	0
L_p'	-.13	-.42	-.62	-.79	-1.0	-1.38
N_p'	-.005	-.032	-.053	-.072	-.091	-.126
Y_r'	-.225	-.24	-.24	-.235	-.235	-.215
L_r'	.015	.15	.24	.31	.375	.49
N_r'	-.042	-.088	-.12	-.142	-.164	-.203
$Y_{\delta_{AS}}'$	0	-.006	-.015	-.026	-.039	-.065
$L_{\delta_{AS}}'$	0.5	0.5	0.5	0.5	0.5	0.5
$N_{\delta_{AS}}'$	.030	.030	.031	.033	.035	.040
$Y_{\delta_{RP}}'$	-.68	-.67	-.70	-.75	-.80	-.90
$L_{\delta_{RP}}'$	-.060	-.065	-.075	-.080	-.095	-.13
$N_{\delta_{RP}}'$	.225	.235	.243	.248	.255	.265
ω_0	0	50.175	83.625	108.713	133.801	175.613
μ_0	0	7.052	11.753	15.279	18.804	24.681

APPENDIX B

ROOTS OF THE AV-8B CHARACTERISTIC EQUATION

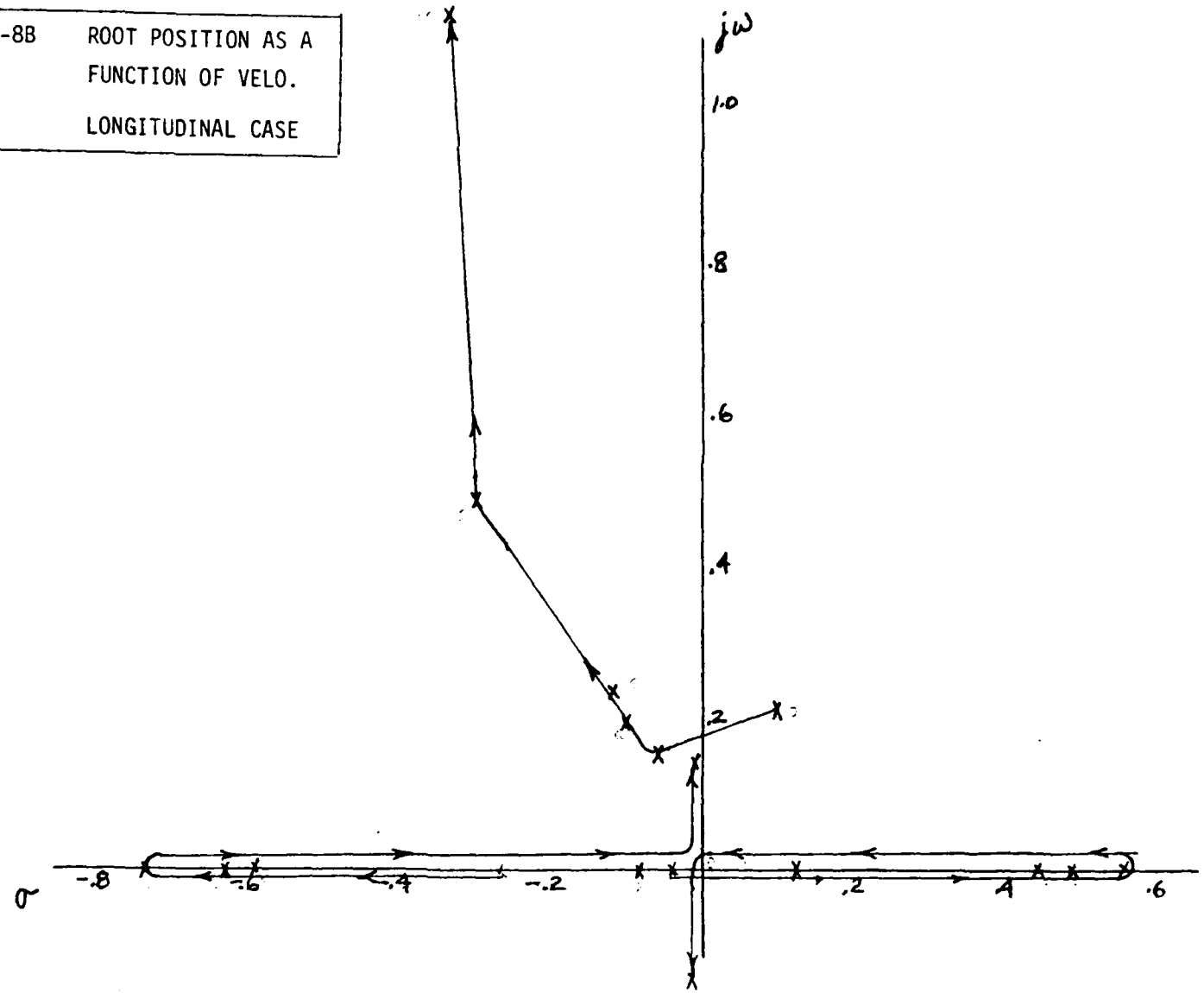
The roots of the system characteristic equation are tabulated and plotted as a function of velocity.

ROOTS OF THE AV-8B LINEAR AIRFRAME MODEL

V (KTS)	LATERAL	LONGITUDINAL
0.0	-.451	-.044
	.157 ± j .344	.097 ± j .209
	-.069	-.267
30.0	-.947	.444
	.244 ± .648 j	-.594
	-.112	-.069 ± j .144
50.0	-1.086	.548
	.190 ± j .770	-.731
	-.123	-.108 ± j .191
65.0	-1.158	.496
	.117 ± j .837	-.627
	-.113	-.172 ± j .233
80.0	-1.238	.116
	.016 ± j .907	-.083
	-.079	-.300 ± j .466
105.0	-1.429	-.021 ± j .130
	-.148 ± j 1.103	-.336 ± j 1.105
	.0035	

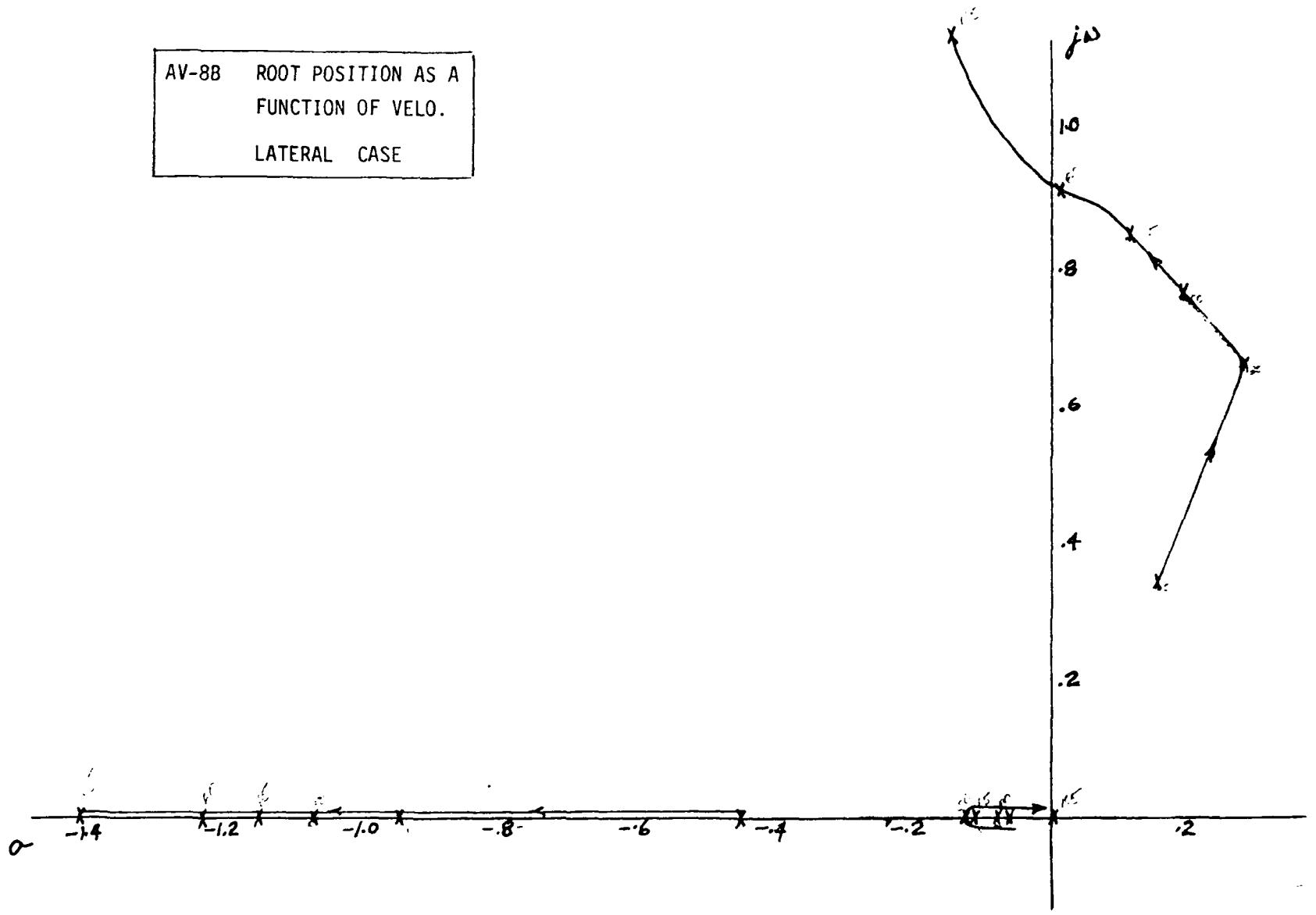
AV-8B ROOT POSITION AS A FUNCTION OF VELO.

LONGITUDINAL CASE



AV-8B ROOT POSITION AS A FUNCTION OF VELO.

LATERAL CASE



APPENDIX C

MATRIX MANIPULATION SUBROUTINES

("MATPAK")

Nearly all of the programs utilized in conjunction with this research rely heavily on matrix techniques. The file listed in this appendix contains many of the most commonly used algorithms.

```
***  SUBROUTINE TO TAKE THE NEGATIVE TRANSPOSE
***  OF A MATRIX
SUBROUTINE NTRAN(X,K,L,Y,M,N)
DIMENSION X(20,20),Y(20,20)
DO 10 I=1,K
DO 10 J=1,L
10 Y(J,I) = -X(I,J)
M = L
N = K
RETURN
END
***  SUBROUTINE TO TAKE THE TRANSPOSE
***  OF A MATRIX
SUBROUTINE TRAN(X,K,L,Y,M,N)
 DIMENSION X(20,20),Y(20,20)
DO 10 I=1,K
DO 10 J=1,L
10 Y(J,I) = X(I,J)
M = L
N = K
RETURN
END
***  SUBROUTINE TO ADD TWO MATRICES
 SUBROUTINE MADD(R,A,N,M,B)
DIMENSION R(20,20),A(20,20),B(20,20)
*
DO 10 I=1,N
DO 10 J=1,M
10 R(I,J) = A(I,J) + B(I,J)
RETURN
END
***  SUBROUTINE TO SUBTRACT TWO MATRICES
SUBROUTINE MSUB(R,A,N,M,B)
DIMENSION R(20,20),A(20,20),B(20,20)
*
DO 10 I=1,N
DO 10 J=1,M
10 R(I,J) = A(I,J) - B(I,J)
RETURN
END
***  SUBROUTINE TO MULTIPLY TWO MATRICES
SUBROUTINEMMULT(R,A,N,M,L,B)
DIMENSION R(N,L),A(N,M),B(M,L)
DIMENSION V(20,20)
*
DO 10 I=1,N
DO 10 J=1,L
V(I,J) = 0.
DO 10 K=1,M
 V(I,J) = A(I,K)*B(K,J) + V(I,J)
10 CONTINUE
```

AD-A094 696 NAVAL ACADEMY ANNAPOLIS MD DIV OF ENGINEERING AND WEAPONS F/G 1/3
MICROPROCESSOR CONTROL OF LOW SPEED VSTOL FLIGHT.(U)
JUN 79 R V WALTERS

unclassified NL

1 OF 3
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A094696

```
DO 20 I=1,N
DO 20 J=1,L
20 R(I,J) = V(I,J)
RETURN
END
 *
 *      ..................................................................
 *
 *         SUBROUTINE MINV
 *
 *         PURPOSE
 *            INVERT A MATRIX
 *
 *         USAGE
 *            CALL MINV(A,N,D,L,M)
 *
 *         DESCRIPTION OF PARAMETERS
 *            A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
 *                RESULTANT INVERSE.
 *            N - ORDER OF MATRIX A
 *            D - RESULTANT DETERMINANT
 *            L - WORK VECTOR OF LENGTH N
 *            M - WORK VECTOR OF LENGTH N
 *
 *         REMARKS
 *            MATRIX A MUST BE A GENERAL MATRIX
 *
 *         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
 *            NONE
 *
 *         METHOD
 *            THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
 *            IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
 *            THE MATRIX IS SINGULAR.
 *
 *      ..................................................................
      SUBROUTINE MINV(R,X,N,D1)
DOUBLE PRECISION A,D,BIGA,HOLD
      DIMENSION A(100),R(10,10),L(100),M(100)
DIMENSION X(10,10)
K=0
DO 9999 I=1,N
DO 9999 J=1,N
K=K+1
9999 A(K)=X(I,J)
 *
 *         ..............................................................
 *
 *         IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
 *         C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
 *         STATEMENT WHICH FOLLOWS.
```

```
 *
*        DOUBLE PRECISION A,D,BIGA,HOLD
* SEE ABOVE
 *
 *         THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
 *         APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
 *         ROUTINE.
 *
 *         THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
 *         CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT
 *         10 MUST BE CHANGED TO DABS.
 *
 *         ..............................................................
 *
 *         SEARCH FOR LARGEST ELEMENT
 *
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( DABS(BIGA)- DABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
 *
 *         INTERCHANGE ROWS
 *
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
 *
 *         INTERCHANGE COLUMNS
 *
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
```

```
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
*
*        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
*        CONTAINED IN BIGA)
*
   45 IF(BIGA) 48,46,48
   46 D=0.0
      GO TO 160
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
*
*        REDUCE MATRIX
*
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
*
*        DIVIDE ROW BY PIVOT
*
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
*
*        PRODUCT OF PIVOTS
*
      D=D*BIGA
*
*        REPLACE PIVOT BY RECIPROCAL
*
      A(KK)=1.0/BIGA
   80 CONTINUE
*
```

```
          FINAL ROW AND COLUMN INTERCHANGE
 *
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 GO TO 160
1   K=0
DO 170 I=1,N
DO 170 J=1,N
K=K+1
170 R(I,J) = A(K)
D1=D
RETURN
END
```

```
* PROGRAMMER: E.E. MITCHELL
* PROGRAM NAME: FADDM1
* SYSTEM: DTSS FORTRAN
* FUNCTION: COMPUTES STATE FEEDBACK GAINS FOR
*           THE SOLUTION OF THE ARBITRARY POLE
*           PLACEMENT PROBLEM FOR MULTI-INPUT
*           MULTI-OUTPUT SYSTEMS
*
***  MAIN PROGRAM FOR FADD
*
**********************************************
*
DIMENSION A(10,10),H(10,10),S(10,10)
DIMENSION P(10),AX(10,10),B(10,10,10)
DIMENSION C(10,10)
DIMENSION Q(10),RR(10),RI(10),D(10,10,10)
CHARACTER YES,FILE1,FILE2
LIBRARY"*E05006:FADD"
LIBRARY"*E05006:MATPAK"
LIBRARY"*E05006:PRNT"
LIBRARY"*E05006:EVAL"
M = 10
IN=0
IOUT=0
* DATA FROM FILE OR TTY
PRINT,"INPUT FROM A FILE?"
INPUT,YES
IF(YES .NE. "YES") GO TO 5
PRINT,"FILE NAME IS?"
INPUT,FILE1
OPENFILE 2,FILE1
IN=2
*
*
* OUTPUT TO FILE OR TTY
*
5 CONTINUE
PRINT,"OUTPUT TO A FILE?"
INPUT,YES
IF(YES .NE. "YES") GO TO 10
PRINT,"FILE NAME IS ?"
INPUT,FILE2
OPENFILE 3,FILE2
REWIND 3
ENDFILE 3
IO1=3
10 CONTINUE
*
* READ DATA FROM FILE OR TTY DEPENDING UPON IN=0 OR 2
* IN=0 FOR TTY, IN=2 FOR FILE1
*
```

```
IF(IN .EQ. 0)PRINT
IF(IN .EQ. 0)PRINT,"A = N BY N MATRIX"
IF(IN .EQ. 0)PRINT
IF(IN .EQ. 0)PRINT,"N ="
READ(IN,500)N
IF(IN .EQ. 0)PRINT,"ENTER A BY ROWS, 1 ROW/LINE"
PRINT
DO 8 I=1,N
8 READ(IN,500)(A(I,J),J=1,N)
IF(IOUT .NE. 0)WRITE(IOUT,510)
WRITE(IO,510)
DO 30 I=1,N
IF(IOUT .NE. 0)WRITE(IOUT,520)(A(I,J),J=1,N)
30 WRITE(IO,520)(A(I,J),J=1,N)
CALL FADD(A,B,AX,P,N,M,IO1,IO2)
*
*
500 FORMAT(V)
510 FORMAT(////"  A MATRIX",
520 FORMAT("0",5(G12.5,2X),5(/"    ",4(G12.5,2X)))
IF(IN .EQ. 0)PRINT,"N,L, B BY ROWS"
READ(IN,500)N,L
DO 40 I=1,N
40 READ(IN,500)(C(I,J),J=1,L)
DO 45 I=1,N
DO 45 J=1,L
45 H(I,J)=C(I,J)
CALL PRM(H,"INPUT B",10,10,N,L,0)
DO 70 K=1,N
DO 50 I=1,N
DO 50 J=1,N
50 AX(I,J)=B(K,I,J)
CALL MMULT(AX,AX,N,N,L,C)
DO 60 I=1,N
DO 60 J=1,L
60 B(K,I,J)=AX(I,J)
70 CONTINUE
IF(IN .EQ. 0)PRINT,"ENTER N ROOT LOCATIONS, RR,RI, 1/LINE"
DO 80 I=1,N
80 READ(IN,500)RR(I),RI(I)
PRINT,"DESIRED ROOT LOCATIONS"
DO 85 I=1,N
85 PRINT 520,RR(I),RI(I)
DO 100 L1=1,N
* CALL PRRV(P,"P",N+1,0)
PR1=RR(L1)
PI1=RI(L1)
CALL EVAL(P,N+1,PR1,PI1)
DO 100 I=1,N
DO 100 J=1,L
DO 90 K=1,N
```

```
90 Q(K)=B(K,I,J)
* CALL PRRV(Q,"Q",N,0)
PR=RR(L1)
PI=RI(L1)
CALL EVAL(Q,N,PR,PI)
D(L1,I,J)=PR/PR1
* PRINT,D(L1,I,J),PR,PR1
100 CONTINUE
DO 110 I=1,L
DO 110 J=1,N
C(I,J)=0.
110 C(1,J)=1.
DO 180 KK=1,L*N
DO 130 I=1,N
DO 130 J=1,L
S(J,I)=-C(J,I)
IF(C(J,I) .EQ. 0.)GO TO 130
DO 120 K=1,N
120 AX(K,I) = D(I,K,J)
130 CONTINUE
I=1
150 DO 160 J=1,L
IF(C(J,I) .EQ. 0)GO TO 160
C(J,I)=0.
IF(J .EQ. L)GO TO 170
C(J+1,I)=1.
GO TO 175
160 CONTINUE
170 C(1,I)=1.
I=I+1
IF(I .GT. N)GO TO 175
GO TO 150
175 CONTINUE
* CALL PRM(C,"C",10,10,N,N,0)
* CALL PRM(AX,"AX",10,10,N,N,0)
CALL MINV(AX,AX,N,DET)
IF(DET .NE. 0) GO TO 190
PRINT,"DET = 0"
CALL PRM(S,"C",10,10,N,L,0)
GO TO 180
190 CALL MMULT(AX,S,L,N,N,AX)
CALL PRM(AX,"K GAINS",10,10,L,N,0)
CALL MMULT(AX,H,N,L,N,AX)
CALL PRM(AX,"B*K",10,10,N,N,0)
CALL MSUB(AX,A,N,N,AX)
CALL PRM(AX,"A-BK",10,10,N,N,0)
IF(IO3 .NE. 3) GO TO 180
CALL PRM(AX,"A-BK",10,10,N,N,IO3)
180 CONTINUE
END
```

```
   SUBROUTINE FADD(A,B,AX,P,N,M,IO,IO2)
*
*
* SUBROUTINE TO FIND THE INVERSE OF THE (SI-A)
*          MATRIX - A MATRIX IN GENERAL FORM
*
*     E.E. MITCHELL      6/78
*
* CALL VARIABLES
*
*  A = A MATRIX IN (SI-A)
*  B = COEFFICIENTS OF NUMERATOR POLYNOMINALS
*      IN THE (SI-A) INVERSE
*      B(1,I,J)*S**(N-1) + B(2,I,J)*S**(N-2) +...+
*         B(N,I,J)
*  P = VECTOR OF CHARACTERISTIC EQN COEFFICIENTS
**       P(1)*S**N + P(2)*S**(N-1) +...+ P(N)
*  AX = INVERSE OF A MATRIX UPON RETURN
*  B(N+1,1,1) = DETERMINANT OF A MATRIX UPON RETURN
*  M = OR > N+1, EQUALS THE DIMENSION OF A,B,AX & P
*      MATRICES IN THE CALLING PROGRAM
*  N = ORDER OF A MATRIX
*  IO1 = OUTPUT FILE # FOR INVERSE OF (SI-A)
*  IO2 = OUTPUT FILE # FOR INVERSE OF A MATRIX
*
*
*********************************************
DIMENSION P(M),A(M,M),AX(M,M),B(M,M,M)
LIBRARY"*E05006:MATPAK"
15  DO 20 I=1,N
DO 20 J=1,N
20 B(1,I,J)=0.0
DO 25 I=1,N
25 B(1,I,I) = 1.0
*
* MAKE A1 = A
*
DO 35 I=1,N
DO 35 J=1,N
35 AX(I,J) = A(I,J)
P(1) = -1.0
*
*   NEED A COUNTER, K=2,N+1 FOR THE N MATRICES
*
K=1
*
*   THE LOOP AROUND THE ALGORITHM
*
50 K=K+1
XK=K-1
*
```

```
* FIRST P(K)
* PRINT,"G1"
*
P(K) = 0.
DO 60 I=1,N
60 P(K) = P(K) + AX(I,I)
TOL=1.E-6
P(K) = P(K)/XK
* PRINT,"P(K)",K,P(K)
*  THE B MATRICES
DO 80 I=1,N
DO 80 J=1,N
 B(K,I,J) = AX(I,J) - P(K)*B(1,I,J)
80 CONTINUE
* WRITE(IO,620)K
* DO 82 I=1,N
* 82 WRITE(IO,560)(B(K,I,J),J=1,N)
DO 86 I=1,N
DO 86 J=1,N
86 IF(ABS(B(K,I,J)) .LE. TOL) B(K,I,J) = 0.0
*  THEN THE AI MATRICES
IF(K .EQ. N+1) GO TO 120
DO 100 I=1,N
DO 100 J=1,N
AX(I,J) = 0.0
DO 100 L=1,N
AX(I,J) = AX(I,J) + A(I,L)*B(K,L,J)
100 CONTINUE
* WRITE(IO,622)
* DO 103 I=1,N
* 103 WRITE(IO,560)(AX(I,J),J=1,N)
DO 105 I=1,N
DO 105 J=1,N
105 IF(ABS(AX(I,J)) .LE. TOL) AX(I,J) = 0.0
GO TO 50
* END OF LOOP
120 WRITE(IO,505)
ISW=0
DO 140 I=1,N
WRITE(IO,560)(B(N+1,I,J),J=1,N)
140 IF(B(N+1,I,J) .GT. 1.E-05)ISW = 1
IF(ISW .EQ. 0) GO TO 160
PRINT,"STOP=1, CONTINUE=0"
IF(IN .EQ. 0)GO TO 160
INPUT,ISW
IF(ISW .EQ. 1) STOP
160 CONTINUE
*
*  HERE TO OUTPUT RESOLVANT MATRIX
*
WRITE(IO,510)N
```

```
DO 180 I=1,N+1
J = N+1-I
P(I)=-P(I)
180 WRITE(IO,520)P(I),J
WRITE(IO,530)
DO 200 K=1,N
WRITE(IO,550)K
DO 200 I=1,N
200 WRITE(IO,560)(B(K,I,J),J=1,N)
GO TO 285
210 CONTINUE
WRITE(IO,570)
WRITE(IO,580)
WRITE(IO,585)N,N
DO 220 I=1,N
DO 220 J=1,N
WRITE(IO,580)
WRITE(IO,590)I,J
DO 220 K=2,N+1
L = N+1-K
220 WRITE(IO,595)B(K,I,J),L
250 CONTINUE
* OUTPUT THE INVERSE OF A
260 XK =-((-1)**(N-1))*P(N+1)
B(N+1,1,1) = XK
WRITE(IO,600)XK
WRITE(IO,610)
XK=-P(N+1)
DO 270 I=1,N
WRITE(IO,560)(B(N,I,J)/XK,J=1,N)
DO 280 J=1,N
280 AX(I,J) = -B(N,I,J)/P(N+1)
270 CONTINUE
285 RETURN
**
**
**
500 FORMAT(" ",6(G8.2,2X)/" ",3X,5(G8.2,2X))
505 FORMAT("1",////,"0 SOLUTION CHECK, IS B=0 ?")
510 FORMAT(///" CHARACTERISTIC EQN, ORDER=",I4)
515 FORMAT(///" INPUT 1 FOR RESOLVENT MATRIX"
 &  /" INPUT 2 FOR INVERSE OF A MATRIX"
 &  /" INPUT 0 FOR BOTH")
520 FORMAT(" ",G14.5,"*S**",I2)
530 FORMAT(///" ADJOINT MATRICES, B(I), WHERE"
 & /"B(0)*S**N + B(1)*S**(N-1) +   + B(N-1)*S"
 & " + B(N)")
550 FORMAT(///" B(",I2,")",)
560 FORMAT("0",5(G12.5,2X),5(/"    ",4(G12.5,2X)))
570 FORMAT(///" RESOLVENT MARTIX POLYNOMINALS")
580 FORMAT("    0")
```

```
585 FORMAT("+    1(S) IS A ",I2," BY",I2," MATRIX")
590 FORMAT("+    /(",I2,",",I2,")",)
595 FORMAT("0",4(G11.4,"*S**",I2,2X),
 & 5(/"    ",3(G11.4,"*S**",I2,2X)))
600 FORMAT(///" DETERMINANT OF A = ",G14.5)
610 FORMAT(" A INVERSE")
620 FORMAT(" B,K",I2)
622 FORMAT(" A,K",I2)
*
END
```

```
   SUBROUTINE EVAL(B,N,Q,S)
*
* PROGRAMMER: E.E. MITCHELL
* PROGRAM NAME: EVAL
* SYSTEM: DTSS FORTRAN
* FUNCTION: POLYNOMIAL EVALUATION AT
*           A COMPLEX POINT:
*           A(1)+A(2)*X+A(3)*X**2+...A(N+1)*X**N
*
*************************************************
*
DIMENSION A(10)
DIMENSION B(10)
COMPLEX X,R
* PRINT,"ENTER POINT,  REAL PART, IMAG PART"
* INPUT,Q,S
DO 5 I=1,N
5 A(I)=B(N+1-I)
X=CMPLX(Q,S)
R=CMPLX(0.,0.)
J=N
1 IF(J)3,3,2
2 R=R*X+A(J)
J=J-1
GO TO 1
3 CONTINUE
* PRINT 500,Q,S
* PRINT 510,R
Q=REAL(R)
S=AIMAG(R)
RETURN
12 CONTINUE
Q=SQRT(Q*Q+S*S)
IF(Q.EQ. 0.0.AND. S.EQ. 0.0)GO TO 10
S=ATAN2(S,REAL(R))*180./3.1415926
PRINT 520,Q,S
10 CONTINUE
RETURN
*
500 FORMAT("0VALUE OF POLYNOMINAL AT ",2G14.5)
510 FORMAT(" RR,RI",2G14.5)
520 FORMAT("0 OR MAGNITUDE =",G14.5,/4X,"ANGLE =",G14.5)
END
```

Sample Run

```
RUN

INPUT FROM A FILE?? YES
FILE NAME IS?? LCN5
OUTPUT TO A FILE?? YES
FILE NAME IS ?? F



A MATRIX

-.44000E-01    -.57910E-03    -32.160        1.3329

 .72470E-03    -.38160E-01    -1.6900       -9.6633

 .00000E-38     .00000E-38     .00000E-38    1.0000

-.13680E-       .45910E-02     .00000E-38   -.66380E-01

            INPUT B  MATRIX
-0.15940       0.33711
-0.34610      -2.5380
 .00000E-38    .00000E-38
 0.23080      -.36000E-01
DESIRED ROOT LOCATIONS

-2.0000        .00000E-38

-2.2000        .00000E-38

-2.4000        .00000E-38

-3.0000        .00000E-38

            K GAINS  MATRIX
-239.15       5396.0       52881.       7967.4
 .00000E-38   .00000E-38   .00000E-38   .00000E-38

              B*K  MATRIX
 38.120      -860.13      -8429.3      -1279.0
 82.769      -1867.6      -18302.      -2757.5
 .00000E-38   .00000E-38   .00000E-38   .00000E-38
-55.195       1245.4       12205.       1839.9

              A-BK  MATRIX
-38.164       860.12       8397.1       1271.3
-82.768       1867.5       18301.       2747.9
 .00000E-38   .00000E-38   .00000E-38   1.0000
 55.194      -1245.4      -12205.      -1839.0
```

K GAINS MATRIX

-2.0055	-0.34484	80.280	30.519
.33202E-01	-0.75553	-7.3926	-1.1112

B*K MATRIX

0.33086	-0.19973	-15.289	-5.2393
0.60983	2.0369	-9.0225	-7.7422
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.46406	-.52391E-01	18.795	7.0837

A-BK MATRIX

-0.37486	0.19915	-16.871	6.5713
-0.60910	-2.0751	7.3325	-1.9211
.00000E-38	.00000E-38	.00000E-38	1.0000
0.46269	.56982E-01	-18.795	-7.1501

K GAINS MATRIX

-1.8190	-0.33227	75.516	29.800
.36604E-01	-0.83229	-8.1443	-1.2242

B*K MATRIX

0.30228	-0.22761	-14.783	-5.1628
0.53664	2.2273	-5.4660	-7.2066
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.42113	-.46727E-01	17.722	6.9218

A-BK MATRIX

-0.34628	0.22703	-17.377	6.4948
-0.53592	-2.2655	3.7760	-2.4567
.00000E-38	.00000E-38	.00000E-38	1.0000
0.41977	.51318E-01	-17.722	-6.9882

K GAINS MATRIX

-4.7658	-0.69854	149.33	40.941
-20.043	-3.3280	494.84	74.695

B*K MATRIX

-5.9970	-1.0106	143.01	18.654
52.518	8.6882	-1307.6	-203.75
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.37840	-.41415E-01	16.652	6.7603

A-BK MATRIX

5.9530	1.0100	-175.17	-17.322
-52.518	-8.7264	1305.9	194.08
.00000E-38	.00000E-38	.00000E-38	1.0000
0.37703	.46006E-01	-16.652	-6.8266

K GAINS MATRIX

-1.6640	-0.32584	71.081	29.059
.40005E-01	-0.90904	-8.8961	-1.3373

B*K MATRIX

0.27872	-0.25451	-14.329	-5.0813
0.47436	2.4199	-2.0229	-6.6602
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.38548	-.42478E-01	16.726	6.7528

A-BK MATRIX

-0.32272	0.25393	-17.831	6.4133
-0.47364	-2.4581	0.33289	-3.0031
.00000E-38	.00000E-38	.00000E-38	1.0000
0.38411	.47069E-01	-16.726	-6.8192

K GAINS MATRIX

-6.7910	-0.94612	206.31	50.673
-33.072	-4.9150	864.41	138.31

B*K MATRIX

-10.066	-1.5061	258.52	38.548
86.286	12.802	-2265.3	-368.57
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.37678	-.41425E-01	16.496	6.7161

A-BK MATRIX

10.022	1.5055	-290.68	-37.216
-86.285	-12.840	2263.6	358.90
.00000E-38	.00000E-38	.00000E-38	1.0000
0.37541	.46016E-01	-16.496	-6.7825

K GAINS MATRIX

-17.114	-2.1710	502.91	102.28
-97.707	-12.583	2723.1	461.99

B*K MATRIX

-30.210	-3.8956	837.83	139.44
253.90	32.686	-7085.3	-1207.9
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.43246	-.48088E-01	18.039	6.9755

A-BK MATRIX

30.166	3.8951	-869.99	-138.10
-253.90	-32.724	7083.6	1198.3
.00000E-38	.00000E-38	.00000E-38	1.0000
0.43109	.52679E-01	-18.039	-7.0419

K GAINS MATRIX

-0.11082	-0.17151	-3.8712	11.223
7.7953	-0.17620	-421.37	-103.03

B*K MATRIX

2.6455	-.32061E-01	-141.43	-36.521
-19.746	0.50657	1070.8	257.61
.00000E-38	.00000E-38	.00000E-38	.00000E-38
-0.30621	-.33242E-01	14.276	6.2994

A-BK MATRIX

-2.6895	.31482E-01	109.27	37.853
19.747	-0.54473	-1072.5	-267.27
.00000E-38	.00000E-38	.00000E-38	1.0000
0.30484	.37833E-01	-14.276	-6.3657

*

APPENDIX E

CALCULATION OF AV-8B STABILITY DERIVATIVES

("MATCOF")

The subroutine contained in this appendix calculates the AV-8B Harrier stability and control derivatives at any velocity, V. The derivatives are returned in the arrays ALON, BLON, ALAT, and BLAT. The subroutine first reads in the coefficients for the predetermined least squares polynomials. Each stability derivative is the solution of a polynomial in V, the input velocity.

```
SUBROUTINE MATCOF(V,ALON,BLON,ALAT,BLAT)
*
*PROGRAMMER: R.V. WALTERS
*FILE NAME: MATCOF
*SYSTEM: DTSS FORTRAN
*FUNCTION: SUBROUTINE FOR CALCULATION OF
*          COEFF@S OF A AND B MATRICES FOR
*          AV-8 HARRIER SIMULATION AT ANY V.
**************************
*
DIMENSION AALON(16,6),BBLON(9,4),AALAT(11,4),BBLAT(6,4)
DIMENSION V5(6,1),V3(4,1)
DIMENSION ALON(16,1),ALAT(16,1),BLON(12,1),BLAT(12,1)
*
LIBRARY "MATPAK"
*
V5(6,1)=1.
DO 1 I=5,1,-1
V5(I,1)=V5(I+1,1)*V
1 CONTINUE
*
V3(4,1)=1.
DO 2 I=3,1,-1
V3(I,1)=V3(I+1,1)*V
2 CONTINUE
*
OPENFILE 6,"ALON"
REWIND 6
*
DO 4 J=1,6
DO 5 I=1,16
READ (6,)AALON(I,J)
5 CONTINUE
4 CONTINUE
*
OPENFILE 7,"BLON"
REWIND 7
*
DO 7 J=1,4
DO 8 I=1,9
READ (7,)BBLON(I,J)
8 CONTINUE
7 CONTINUE
*
OPENFILE 8,"ALAT"
REWIND 8
*
DO 9 J=1,4
DO 10 I=1,11
READ (8,)AALAT(I,J)
10 CONTINUE
```

```
9 CONTINUE
*
OPENFILE 9,"BLAT"
REWIND 9
*
DO 11 J=1,4
DO 12 I=1,6
READ (9,)BBLAT(I,J)
12 CONTINUE
11 CONTINUE
*
CALL MMULT(ALON,AALON,16,6,1,V5)
IF (V.LT.5.) ALON(3,1)=-32.02
IF (V.LT.5.) ALON(7,1)=-3.37
*
CALL MMULT(BLON,BBLON,9,4,1,V3)
BLON(12,1)=BLON(9,1)
BLON(11,1)=BLON(8,1)
BLON(10,1)=BLON(7,1)
BLON(7,1)=0.
BLON(8,1)=0.
BLON(9,1)=0.
*
CALL MMULT(ALAT,AALAT,11,4,1,V3)
DO 3 I=12,16
  ALAT(I,1)=0.
3 CONTINUE
ALAT(14,1)=1.
*
CALL MMULT(BLAT,BBLAT,6,4,1,V3)
BLAT(8,1)=BLAT(6,1)
BLAT(7,1)=BLAT(5,1)
BLAT(5,1)=BLAT(4,1)
BLAT(4,1)=BLAT(3,1)
BLAT(3,1)=0.
BLAT(6,1)=0
DO 6 I=9,12
  BLAT(I,1)=0
6 CONTINUE
*
RETURN
END
```

APPENDIX F

DETERMINATION OF STATE FEEDBACK GAINS

("SFBCOF")

This subroutine employs a table look-up algorithm to determine the feedback gains. Feedback gains were calculated for three design points: V = 5, 30, and 60 kt. The SFBCOF program employs the following logic:

$0 \leq V \leq 15$	USE	V = 5 DESIGN
$15 \leq V \leq 45$	USE	V = 30 DESIGN
$45 \leq V$	USE	V = 60 DESIGN

```
SUBROUTINE SFBCOF(V,XKLON,XKLAT)
*  PROGRAMMER:  R V  WALTERS
*  PROGRAM NAME:  SFBCOF
*  SYSTEM:  DTSS FORTRAN
*  FUNCTION:  DETERMINES AV-8B HARRIER SFB
*             GAINS AT ANY VELOCITY
*
*****************************************
*
*
*LIBRARY "KLON5","KLON30","KLON60"
*
DIMENSION XKLON(8),XKLAT(8)
*
IF(V GT 15.) GO TO 2
*
OPENFILE 1,"KLON5"
REWIND 1
OPENFILE 2,"KLAT5"
REWIND 2
*
DO 1 I=1,8
READ(1,)XKLON(I)
READ(2,)XKLAT(I)
1 CONTINUE
RETURN
*
2 CONTINUE
*
IF(V.GT.45.) GO TO 4
*
OPENFILE 3,"KLON30"
REWIND 3
OPENFILE 4,"KLAT30"
REWIND 4
*
DO 3 I=1,8
READ(3,)XKLON(I)
READ(4,)XKLAT(I)
3 CONTINUE
RETURN
*
4 CONTINUE
*
OPENFILE 5,"KLON60"
REWIND 5
OPENFILE 6,"KLAT60"
REWIND 6
*
DO 5 I=1,8
READ(5,)XKLON(I)
```

```
READ(6,)XELAT(I)
5 CONTINUE
*
RETURN
END
```

APPENDIX G

"DIGISIM" DOCUMENTATION

DIGISIM, programmed by Associate Professor E. E. Mitchell is the digital simulation utility program on which my DTSS model was based. This appendix describes its features and user-defined options.

```
******   DIGISIM  ---  DIGITAL SIMULATOR  --  E.E. MITCHELL 2/77
*
***  REMEMBER  --  RUN FORTRAN
*
*  LINES 400-650:     INITIAL TIME DATA BLOCK
*  LINES 750-1000:    SYSTEM DYNAMIC EQUATIONS
*  LINES 1200-1500:   OUTPUT STATEMENTS
*  LINES 1700-1850:   TERMINAL CALCULATION REGION
*
* BASIC DATA:  TMAX,DELT,NSAMPL
* BASIC STATEMENTS:  X=ENTGRL(XD,X(0)),CALL OUTPUT(X,"X")
* ADDITIONAL FUNCTIONS:  STEP,FRSTOR,SCNDOR,PULSE,SQUARE,DELAY,DIGTAL,LIM
*                         PTRAIN
* FOR MORE DETAILS CONTINUE LISTING
*
****  INITIAL TIME REGION
*
*  THIS PROGRAM HAS AN INITIAL TIME DATA REGION EXTENDING FROM LINES 400-
* THIS SPACE IS USED TO DEFINE CONSTANTS, INITIAL VALUES AND OTHER NON
* CHANGING PARAMETERS REQUIRED BY YOUR PROGRAM.  ALSO INITIAL CALCULATION
* MAY BE MADE HERE (IF YOU DEFINE THE REQUIRED PARAMETERS).
*  IN ADDITION, SPECIAL CONSTANTS FOR BUILT IN DIGISIM FUNCTIONS ARE DEF
* HERE.  FOR INSTANCE, COMMONLY DEFINED TERMS IN THIS AREA ARE:
*    HEAD="    "    ONE LINE OF 80 OR LESS CHARACTERS BETWEEN THE QUOTES
*                    THIS IS TYPED OUT AS A HEADING FOR THE RUN
*    DELT           INTEGRATION STEP SIZE (DELT=0.05 DEFAULT)
*    TMAX           TOTAL SIMULATED RUN TIME (TMAX=3 DEFAULT)
*    NSAMPL         OUTPUT SPACING- ANSWERS PRINTED EVERY DELT*NSAMPL
*                    TIME INTERVAL (NSAMPL=2 DEFAULT)
*    ZK             DIGITAL CONTROLLER GAIN
*    ZA(1-6),ZB(1-6)  DIGITAL CONTROLLER COEFFICIENTS (ZA()=ZB()=0 DEFAU
*    TSAM          DIGITAL CONTROLLER SAMP TIME (TSAM =0 DEFAULT)
*    TLAG           DELAY PERIOD FOR TIME DELAY (TLAG=0 DEFAULT)
*    TO             INITIAL TIME IF OTHER THAN 0 (TO=0 DEFAULT)
*    TKPLOT         IF TKPLOT=1 THE OUTPUT RESPONSE IS WRITTEN INTO A FILE
*                       SAVED BY YOU.  IT IS IN THE PROPER FORM FOR PLOTTIN
*                       ON THE TEKTRONIC GRAPHIC TERMINALS (TKPLOT=0 DEFAU
*    PPLANE         IF PPLANE=1 THE TKPLOT FILE IS WRITTEN TO MAKE A PHASE
*                       PLANE PLOT OF THE FIRST TWO CALL OUTPUT VARIABLES
*
****  DYNAMIC REGION
*
*  THE PROGRAM HAS A DYNAMIC REGION FROM STATEMENT NUMBERS 750-1000  IN
* AREA THE SYSTEM DYNAMICS, CONSISTING OF DIFFERENTIAL AND ALGEBRATIC
* EQUATIONS, ARE DEFINED
*  IN THE DYNAMIC REGION, THE FULL REGULAR FORTRAN LIBRARY IS AVAILABLE
* FOR THAT MATTER, IT IS AVALIABLE ANYWHERE IN THE PROGRAM  IN ADDITION
* THE FOLLOWING FUNCTIONS ARE FURNISHED:  (X=INPUT,Y=OUTPUT)
*
*    T = TIME
*    Y = ENTGRL(X,X0) -- X0=INITIAL VALUE - DECIMAL POINT REQUIRED IF NU
```

```
*     Y = STEP(T1) -- Y=0 FOR T<T1, Y=1 FOR T>=T1
*     Y = PULSE(T1,T2) -- Y=0 EXCEPT Y=1 WHEN T1<=T<=T2
*     Y = SQUARE(T1,T2,T3,P) -- SQUARE WAVE WITH AMPLITUDE OF Y=1  LEADIN
*                               OF FIRST PULSE AT T3, PULSE WIDTH IS T1 T
*                               REPEATS EVERY P SECONDS
*     Y = FRSTOR(X,A,B,C,D) --  FIRST ORDER TRANSFER FUNCTION DEFINED AS
*                                (A*S+B)/(C*S+D) C IS NOT ZERO
*     Y = SCNDOR(X,A,B,C,D,E,F) -- SECOND ORDER TRANSFER FUNCTION DEFINED
*                                  (A*S*S+B*S+C)/(D*S*S+E*S+F) D IS NOT Z
*     Y = DELAY(X,TLAG) -- TIME DELAY, Y = X AFTER X IS DELAYED TLAG SECO
*     Y = DIGTAL(X) -- DIGTAL FILTER OR CONTROLLER SIMULATOR. DELT SHOULD
*                      <=TSAM/10. TSAM=SAMPLE INPUT-OUTPUT TIMES OF THE D
*                      CONTROLLER. TRANS FUNCTION HAS 6 ZA,ZB TERMS AND I
*                      (ZK*(ZA(1)*X+ZA(2)*X-1+ZA(3)*X-3) - ZB(2)*Y-1
*                              -ZB(3)*Y-2 )/ZB(1)
*                      X=CURRENT INPUT,X-1 = LAST INPUT,ETC. DITTO FOR Y
*     Y = LIMIT(U1,X,U2) -- Y=U1 IF X<U1, Y=U2 IF X>U2, Y=X OTHERWISE
*     Y = PTRAIN(T1) -- Y=1 IF T = N*T1; THIS IS A PULSE TRAIN, UNIT PULS
*                        EVERY N*T1 SECONDS
*
*
****  OUTPUT REGION
*
*  THE PROGRAM HAS AN OUTPUT REGION FROM STATEMENT NUMBER 1200-1500. IN
* REGION THE VARIABLES TO BE PRINTED AND PLOTTED ARE DEFINED. TO OUTPUT
* X, XDOT AND X2DOT, THE STATEMENTS ARE:
*
*     CALL OUTPUT(X,"X")
*     CALL OUTPUT(XDOT,"XDOT")
*     CALL OUTPUT(X2DOT,"ACCEL")
*
*  ANY 8 CHARACTERS MAY BE USED BETWEEN THE " ", THESE ARE THE COLUMN
* HEADINGS OVER THE PRINTED VALUES.
** NOTE **  TIME IS OUTPUT AUTOMATICALLY
*
** NOTE ** THE FIRST 6 CALL OUTPUT VARIABLES ARE PLOTTED
*
****  TERMINAL CALCULATION REGION
*
*  THE PROGRAM HAS A TERMINAL REGION FROM STATEMENT NUMBERS 1700-1850  T
* AREA IS USED TO MODIFY PARAMETERS, ECT. AT THE END OF A RUN. ONE CAN
* THEN GO BACK AND INTEGRATE AGAIN AND AGAIN. A TYPICAL PROBLEM THAT
* WOULD USE THIS AREA IS A BOUNDARY VALUE PROBLEM.
*
* IF YOUR PROGRAM CONTAINS THE STATEMENT
*  GO TO 9000
* CONTROL IS TRANSFERED BACK TO THE TOP OF THE INITIAL AREA
* IN A LIKE MANNER, THE STATEMENT
*  GO TO 9100
* TAKES CONTROL TO THE BOTTOM OF THE INITIAL AREA,(TOP OF DYNAMIC AREA)
*
```

APPENDIX H

"AHEAD" - ADVANCED HARRIER ELECTRONIC AUGMENTATION DEVICE

AHEAD is the DIGISIM-based digital Harrier simulation. The output traces of Appendix L were made using AHEAD in conjunction with a special APL graphing routine. AHEAD is an extremely versatile design tool. The user must program control inputs and initial conditions prior to run time. At run time, user inputs TMAX, his desired SAS option, desired output format, and output file. System response data is then output to the user-defined output file.

```
* PROGRAMMER: R.V. WALTERS
* PROGRAM NAME: AHEAD
* SYSTEM: DTSS FORTRAN
* FUNCTION: SIMULATES THE LAT AND LONG
*           DYNAMICS OF THE AV-8B HARRIER,
*           INCLUDING BOTH EXISTING STABILITY
*           AUG AND A STATE FB CONTROLLER
*
*
*******************************************
LIBRARY "MATCOF"
LIBRARY "SFBCOF"
IMPLICIT REAL(I-N)
INTEGER ICZ,NICZ,ITZ,NITZ,ITMAX,KERR,IAZ,NZZ,KPZ,NSAMPL,KTZ,IIZ,JJZ,IZ
INTEGER ICNT,NVAR,IDIGL,ICN,IADCNT,ISZZ
DIMENSION ALON(16,1),ALAT(16,1),BLON(12,1),BLAT(12,1)
COMMON ZOUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT,ICZ,NICZ,ITZ,NITZ,
& ITMAX,KERR,IAZ(100),Y1PZ(100),Y4PZ(100),X1PZ(100),X4PZ(100),NZZ,KPZ,
& NSAMPL,KTZ
DIMENSION XKLON(8),XKLAT(8)
COMMON /HOLD/IADCNT,ITQA,JCNT,IQCNT,HLD(500)
COMMON /OUTPZ/ IIZ,JJZ,AZZ,ISZZ,AMXZ(25),AMNZ(25),HEAD,NPLOT,TKPLOT,PPLA
COMMON/DIGITA/IDIGL,ZA(6),ZB(6),TSAM,ZK,NCNT
*
CHARACTER YES
CHARACTER PDQ,PDQ2,IPQ,AZZ(25),HEAD*80
REAL LIMIT
INTEGER TKPLOT
DATA ICN,IIZ,JJZ/0,0,0/
ZOUT=ICNT=IDIGL=0
TKPLOT=0
NPLOT=6
DO 109 IZ=1,4
ZA(IZ)=0.
109 ZB(IZ)=0.
ZK=0.
IADCNT=0
DO 108 IZ=1,500
108 HLD(IZ)=0.
1 DO 2 IZ=1,100
2 IAZ(IZ)=0
9000 CONTINUE
TO=0
DELT= 05
NICZ=10
NITZ=4
****   INITIAL REGION ***
*
PRINT,"IC'S & INPUTS CORRECT?"
PRINT,
PRINT,"IF NOT,STOP!"
```

```
PRINT,
PRINT,
*
PRINT,"INPUT TMAX"
INPUT,TMAX
PRINT,
DELT=.002
NSAMPL=50
HEAD=" AV-8B TIME RESPONSE"
TKPLOT=1
VMUL=1.
*
XLON10=0.
XLON20=0.
XLON30=0.
XLON40=0
*
XLAT10=0.
XLAT20=0.
XLAT30=0.
XLAT40=0.
*
QSAS=0.
RSAS=0.
PSAS=0.
SFBLON=0.
SFBLAT=0.
*
PRINT,"CONVENTIONAL SAS ON?"
*
INPUT,YES
IF (YES.NE."YES") GO TO 506
QSAS=1.
RSAS=1.
PSAS=1.
*
506 PRINT,"SFB SAS ON?"
INPUT,YES
IF (YES.NE."YES") GO TO 516
SFBLON=1.
SFBLAT=1.
*
516 CONTINUE
*
IFLAG1=1
IFLAG2=1
PRINT,"SUPPRESS LONG DYNAMICS?"
INPUT,YES
IF(YES.EQ."YES")IFLAG1=0
PRINT,"SUPPRESS LAT DYNAMICS?"
INPUT,YES
```

```
IF(YES EQ "YES")IFLAG2=0
PRINT,
*
PRINT,"INPUT VELOCITY"
INPUT,V
VINIT=V
PRINT,
*
CALL MATCOF(V,ALON,BLON,ALAT,BLAT)
CALL SFBCOF(V,XKLON,XKLAT)
*
PRINT,"INITIAL COEFFICIENTS CALCULATED"
PRINT,
*
PRINT,"BEGIN SIMULATION"
PRINT,
*
*   LET T=0?
*
*
*
*
****
IF(TKPLOT .EQ. 0)GO TO 9100
WRITE(ZOUT,87)
87 FORMAT(" WHAT IS YOUR SAVED FILE NAME")
READ(ZOUT,14)IPQ
OPENFILE 3,IPQ
REWIND 3
ENDFILE 3
9100 KTZ=0
T=TQ
HDELT=0.5*DELT
KERR=0
ICZ=1
ITMAX=NICZ
4 ITZ=1
5 NZZ=0
****  DYNAMIC REGION ***
IF(IFLAG1.EQ.0) GO TO 833
*
****************************
*   LONGITUDINAL SIMULATION
****************************
*
```

```
*
BKLON1=XKLON(1)*XLON1+XKLON(2)*XLON2+XKLON(3)*XLON3+XKLON(4)*XLON4
BKLON2=XKLON(5)*XLON1+XKLON(6)*XLON2+XKLON(7)*XLON3+XKLON(8)*XLON4
*
762 THROT=0.     !THROTTLE
ULON3=0.       !JET ANGLE
*
ULON2=THROT-SFBLON*BKLON2
*
LONSTK=0.
LONSTK=LIMIT(-8.,LONSTK,4.)    ! LONSTK
*
Q1DD=17410.*XLON4-132.*Q1D-17410.*Q1
Q1D=ENTGRL(Q1DD,0.)
Q1=ENTGRL(Q1D,0.)
*
BXLOND=4.04*(Q1D+5.3*Q1)-1.35*BXLON
BXLON=ENTGRL(BXLOND,0.)
BXLON=QSAS*LIMIT(-.84,BXLON,.84)
*
ELON=LONSTK-BXLON-SFBLON*BKLON1
*
ULON1D=12.*(ELON-ULON1)
ULON1=ENTGRL(ULON1D,0.)    ! ELEV/STK INPUT
ULON1=LIMIT(-8.84,ULON1,4.84)
*
XLON1D=ALON(1,1)*XLON1+ALON(2,1)*XLON2+ALON(3,1)*XLON3+ALON(4,1)*XLON4
XLON1D=XLON1D+BLON(1,1)*ULON1+BLON(2,1)*ULON2+BLON(3,1)*ULON3
XLON2D=ALON(5,1)*XLON1+ALON(6,1)*XLON2+ALON(7,1)*XLON3+ALON(8,1)*XLON4
XLON2D=XLON2D+BLON(4,1)*ULON1+BLON(5,1)*ULON2+BLON(6,1)*ULON3
XLON3D=ALON(9,1)*XLON1+ALON(10,1)*XLON2+ALON(11,1)*XLON3+ALON(12,1)*XLON
XLON3D=XLON3D+BLON(7,1)*ULON1+BLON(8,1)*ULON2+BLON(9,1)*ULON3
XLON4D=ALON(13,1)*XLON1+ALON(14,1)*XLON2+ALON(15,1)*XLON3+ALON(16,1)*XLO
XLON4D=XLON4D+BLON(10,1)*ULON1+BLON(11,1)*ULON2+BLON(12,1)*ULON3
*
XLON1=ENTGRL(XLON1D,XLON10)
XLON2=ENTGRL(XLON2D,XLON20)
XLON3=ENTGRL(XLON3D,XLON30)
XLON4=ENTGRL(XLON4D,XLON40)
*
838 CONTINUE
IF(IFLAG2.EQ.0) GO TO 978
*
**********************
*   LATERAL SIMULATION
**********************
*
BKLAT1=XKLAT(1)*XLAT1+XKLAT(2)*XLAT2+XKLAT(3)*XLAT3+XKLAT(4)*XLAT4
BKLAT2=XKLAT(5)*XLAT1+XKLAT(6)*XLAT2+XKLAT(7)*XLAT3+XKLAT(8)*XLAT4
*
```

```
ULAT3=0.                ! NO INPUT
*
P1DD=17410. *XLAT2-132. *P1D-17410. *P1
P1D=ENTGRL(P1DD,0. )
P1=ENTGRL(P1D,0. )
*
P2D=4.07*(P1D+1.28*P1)-.774*P2
P2=ENTGRL(P2D,0. )
P2=PSAS*LIMIT(-.68,P2,.68)
*
LATSTK=0.               ! LATSTK
LATSTK=LIMIT(-4. ,LATSTK,4. )
*
EPLAT=LATSTK-P2-SFBLAT*BKLAT1
*
ULAT1D=25. *(EPLAT-ULAT1)
ULAT1=ENTGRL(ULAT1D,0. )    ! ALRN/STK INPUT
ULAT1=LIMIT(-4.68,ULAT1,4.68)
*
R1DD=17410. *XLAT3-132. *R1D-17410. *R1
R1D=ENTGRL(R1DD,0. )
R1=ENTGRL(R1D,0. )
*
R2D=23. *(R1-R2)
R2=ENTGRL(R2D,0. )
*
R3D=1.73*R2D-.32*R3
R3=ENTGRL(R3D,0. )
*
R4D=23.26*(.24*ULAT1-R4)
R4=ENTGRL(R4D,0. )
*
AY=-1. *XLAT1D+2.32*XLAT2D-11. *XLAT3D+32.2*XLAT4
*
AY1DD=318. *AY-19.45*AY1D-164. *AY1
AY1D=ENTGRL(AY1DD,0. )
AY1=ENTGRL(AY1D,0. )
*
AY2D=.12*(AY1D+4. *AY1)-8. *AY2
AY2=ENTGRL(AY2D,0. )
*
BRLAT=R4-R3-AY2
BRLAT=RSAS*LIMIT(-.7,BRLAT,.7)
*
RUDPED=0                ! RUDPED
RUDPED=LIMIT(-2. ,RUDPED,2. )
*
ERLAT=RUDPED+BRLAT-SFBLAT*BRLAT2
*
ULAT2D=57. *(ERLAT-ULAT2)
ULAT2=ENTGRL(ULAT2D,0. )    ! RUDR/STK INPUT
```

```
ULAT2=LIMIT(-2.7,ULAT2,2.7)
*
XLAT1D=ALAT(1,1)*XLAT1+ALAT(2,1)*XLAT2+ALAT(3,1)*XLAT3+ALAT(4,1)*XLAT4
XLAT1D=XLAT1D+BLAT(1,1)*ULAT1+BLAT(2,1)*ULAT2+BLAT(3,1)*ULAT3
XLAT2D=ALAT(5,1)*XLAT1+ALAT(6,1)*XLAT2+ALAT(7,1)*XLAT3+ALAT(8,1)*XLAT4
XLAT2D=XLAT2D+BLAT(4,1)*ULAT1+BLAT(5,1)*ULAT2+BLAT(6,1)*ULAT3
XLAT3D=ALAT(9,1)*XLAT1+ALAT(10,1)*XLAT2+ALAT(11,1)*XLAT3+ALAT(12,1)*XLAT
XLAT3D=XLAT3D+BLAT(7,1)*ULAT1+BLAT(8,1)*ULAT2+BLAT(9,1)*ULAT3
XLAT4D=ALAT(13,1)*XLAT1+ALAT(14,1)*XLAT2+ALAT(15,1)*XLAT3+ALAT(16,1)*XLA
XLAT4D=XLAT4D+BLAT(10,1)*ULAT1+BLAT(11,1)*ULAT2+BLAT(12,1)*ULAT3
*
XLAT1=ENTGRL(XLAT1D,XLAT10)
XLAT2=ENTGRL(XLAT2D,XLAT20)
XLAT3=ENTGRL(XLAT3D,XLAT30)
XLAT4=ENTGRL(XLAT4D,XLAT40)
*
978 CONTINUE
*
IF (T.NE.1.) GO TO 1003
****
IF(NF1.NE.1)PRINT,"T=1"
NF1=1
1003 IF (T.NE.2.) GO TO 1006
IF(NF2.NE.1)PRINT,"T=2"
NF2=1
1006 IF (T.NE.3.) GO TO 1009
IF(NF3.NE.1)PRINT,"T=3"
NF3=1
1009 IF (T.NE.4.) GO TO 1012
IF(NF4.NE.1)PRINT,"T=4"
NF4=1
1012 IF (T.NE.TMAX) GO TO 1015
IF(NF5.NE.1)PRINT,"T=TMAX"
NF5=1
1015 CONTINUE
IF(KERR .NE. 0) GO TO 9
ITZ=ITZ+1
IF(ITZ .LE. ITMAX) GO TO 5
KPZ=0
***  OUTPUT REGION ***  LINES 1200-1500
*
IF(IFLAG1 NE.1)GO TO 1245
*
CALL OUTPUT(XLON1,"U(X-VEL)")
CALL OUTPUT(XLON2,"W(Z-VEL)")
CALL OUTPUT(XLON3,"THETA")
*
IF(IFLAG2.EQ.1)GO TO 1245
*
CALL OUTPUT(XLON4,"Q(DPIT)")
*
```

```
1245 CONTINUE
*
IF(IFLAG2 NE. 1) GO TO 1267
*
CALL OUTPUT(XLAT1,"V(Y-VEL)")
*
IF(IFLAG1 EQ. 1) GO TO 1263
*
CALL OUTPUT(XLAT2,"P(DROLL)")
*
1263 CONTINUE
CALL OUTPUT(XLAT3,"R(DYAW)")
CALL OUTPUT(XLAT4,"PHI")
*
1267 CONTINUE
*
*
*
*
*
*
*
*
*
*
****
JJZ=JJZ+2
IF(ICZ .EQ. 0)GO TO 6
ICZ=0
ITMAX=NITZ
6 IF(T .LT. (TMAX+.00001))GO TO 68
IF(TKPLOT .NE. 0)GO TO 7
WRITE(ZOUT,12)
12 FORMAT("  WANT TO INCREASE TMAX")
READ(ZOUT,14)IPQ
14 FORMAT(V)
IF(IPQ .NE  "YES") GO TO 7
WRITE(ZOUT,13)
13 FORMAT(" TMAX=")
READ(ZOUT,14)TMAX
GO TO 6
68 KTZ=KTZ+1
T=KTZ
T=T*DELT+TO
GO TO 4
9 WRITE(ZOUT,10)
10 FORMAT(5X," INTEGRATION ASSIGNMENT ERROR")
STOP
7 ISZZ=2
****   TERMINAL REGION  ***
*
```

```
*
*
*
*
****
IF(TKPLOT .EQ. 0)CALL RANGE
IF(TKPLOT .EQ. 0) CALL PLOTZ
IF(TKPLOT .NE. 0) CALL TEKPLT
STOP
END
FUNCTION ENTGRL(X,Y0)
COMMON OUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT,IC,NIC,IT,NIT,ITMA
&IA(100),Y1(100),Y2(100),X1(100),X2(100),N,KP,NSAMPL,KT
N=N+1
IF(IC.EQ.0) GO TO 20
IF(IT.NE.1) GO TO 10
IF(IA(N).NE.0) GO TO 50
IA(N)=1
GO TO 15
10 IF(IA(N).EQ.0) GO TO 50
15 Y2N=Y0
GO TO 40
20 IF(IA(N).EQ.0) GO TO 50
IF(IT.NE.1) GO TO 30
*      PREDICTOR
Y1N=Y2(N)
X1N=X2(N)
Y1(N)=Y1N
X1(N)=X1N
Y2N=Y1N+X1N*DELT
*      CORRECTOR
30 Y2N=Y1(N)+(X1(N)+X)*HDELT
40 Y2(N)=Y2N
X2(N)=X
ENTGRL=Y2N
RETURN
50 KERR=1
RETURN
END
FUNCTION STEP(T1)
COMMON OUT,XVZ(6,402),ICNT,NVAR,T
Y=1.
IF(T.LT.T1) Y=0
STEP=Y
RETURN
END
FUNCTION PULSE(T1,T2)
COMMON OUT,XVZ(6,402),ICNT,NVAR,T
Y=0
IF(T.GE.T1 AND.T.LE.T2) Y=1.
PULSE=Y
```

```
RETURN
END
FUNCTION SQUARE(T1,T2,T3)
COMMON OUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT
Y=0
TM=AMOD(T-T1,T3)
IF(TM .GE. 0. .AND. TM .LE. T2) Y = 1.
SQUARE=Y
RETURN
END
FUNCTION FRSTOR(X,A,B,C,D)
COMMON /FRST/ SZ,Z
IF(C.EQ.ZERO) GO TO 1
SZ=X-D*Z/C
Z=ENTGRL(SZ,0.)
FRSTOR=(A*SZ+B*Z)/C
RETURN
1 PRINT 2
2 FORMAT(5X,"FIRST ORDER TRANSFER FUNCTION OUT OF ORDER")
STOP
END
FUNCTION SCNDOR(X,A,B,C,D,E,F)
COMMON /SCND/ SZZ,SZ,Z
IF(D EQ ZERO) GO TO 1
SZZ=X-(E*SZ+F*Z)/D
SZ=ENTGRL(SZZ,0.)
Z=ENTGRL(SZ,0.)
SCNDOR=(A*SZZ+B*SZ+C*Z)/D
RETURN
1 PRINT 2
2 FORMAT(5X,"SECOND ORDER TRANSFER FUNCTION OUT OF ORDER")
STOP
END
SUBROUTINE OUTPUT(X,A1)
CHARACTER A1,A(25),HEAD*80
COMMON OUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT,IC,NIC,IT,NIT,ITMA
&IA(100),Y1(100),Y2(100),X1(100),X2(100),N,KP,NSAMPL,KT
COMMON /OUTP2/II,JJ,A,ISZZ,AMXZ(25),AMNZ(25),HEAD,NPLOT,TKPLOT,PPLANE
COMMON /OUTP1/ V(25),IP,KASE,KS,TSAV,ICN
*
IF(T .NE. TO)GO TO 10
KS = 0
IF(JJ .GT. 0) GO TO 40
II=II+1
A(II)=A1
ISZZ=1
NVAR = MIN(II,6,NPLOT)
GO TO 40
10 IF(KP .NE. 0) GO TO 40
IF(KS .EQ. 0) GO TO 60
20 KS=KS+1
```

```
IF(KS .GE. NSAMPL) KS = 0
40 KP=KP+1
IF(KS .NE. 0) RETURN
TSAV = T
V(KP) = X
RETURN
*  PRINT LAST TIME PERIOD
60 CONTINUE
ICNT=ICNT+1
DO 120 I=1,NVAR
120 XVZ(I,ICNT) = V(I)
IF(TKPLOT .NE. 0)GO TO 20
IF(ICN .EQ. 0) WRITE(OUT,900)
IF(ICN .EQ. 0) PRINT,HEAD
IF(ICN .LT. 51) GO TO 70
ICN = 0
WRITE(OUT,915)
WRITE(OUT,900)
70 CONTINUE
KT1=MIN(5,II)
IF(JJ .EQ. 2)WRITE(OUT,910)(A(I),I=1,KT1)
IF(II .GT. 5)WRITE(OUT,905)
WRITE(OUT,920)ICNT,TSAV,(V(I),I=1,KT1)
ICN=ICN+1
IF(II .LE. 5) GO TO 100
*
ICN = ICN + 1
80KT1=KT1 + 1
KT2=MIN(KT1+4,II)
IF(JJ .EQ. 2)WRITE(OUT,930)(A(I),I=KT1,KT2)
WRITE(OUT,940)(V(I),I=KT1,KT2)
ICN=ICN+1
KT1 = KT2
IF(II .GT. KT2) GO TO 80
100 JJ=4
CALL RANGE
GO TO 20
900 FORMAT(1H1,//////)
905 FORMAT(1H )
910 FORMAT(/11X,"TIME",3X,5(2X,A8,2X))
915 FORMAT(////)
920 FORMAT(1H ,I3,1X,6G12.5)
930 FORMAT(/,18X,5(2X,A8,2X))
940 FORMAT(1H ,15X,5G12.5)
END
SUBROUTINE PLOT2
*
*
*              N= NUMBER OF POINTS
*              NVAR= NUMBER OF PLOTS
*
```

```
*
*
*               V= NVAR X N MATRIX; EACH COLUMN CORRESPONDS TO A VECTOR
*                    COMPOSED OF POINTS TO BE PLOTTED
*
*
*               MAX NVAR = 6
*               MAX N = 400  WITHOUT CHANGING DIMENSIONS
*
*
*
COMMON OUT,V(6,402),N,NVAR
COMMON /OUTP2/II,JJ,A2,ISZZ,AMXZ(25),AMNZ(25),HEAD,NPLOT,TKPLOT,PPLANE
DIMENSION SF(7),IS(7),H(7),IC(7)
CHARACTER BLANK,DOT,STR,MINS,HEAD*80
CHARACTER X(8),LINE(120),A2(25)
BLANK=" "
DOT="I"
STR="+"
MINS="-"
X(1)="+"
X(2)="*"
DO 9999 I=1,7,2
9999 X(I)=X(1)
DO 9998 I=2,8,2
9998 X(I)=X(2)
*
*
*
    DO 1 I=1,NVAR
    H(I)=0
    SF(I)=0.0
    IS(I)=I
1   IC(I)=1
*
    DO 6 I=1,NVAR
       V(I,1)=V(I,1)+1.E-6
    DO 6 J=1,N
    IF(V(I,J)    )3,3,4
 3  IC(I)=2
 4  IF(ABS(V(I,J))-H(I))6,6,5
 5     H(I)=ABS(V(I,J))
 6  CONTINUE
*
*
*                    GENERATE SCALE FACTORS
*
*
    DO 7 I=1,NVAR
    A=60/(NVAR*IC(I))
    SF(I)=A/(H(I)+.1*H(I))
```

```
      A=(30 *(IC(I)-1)+60 *(I-1))/NVAR
         DO 7 J=1,N
 7    V(I,J)=SF(I)*V(I,J)+A
*
*
*                    WRITE HEADING
*
*
      WRITE(OUT,500)NVAR
      IF(NVAR-4)8,8,9
 8    NS=1
      NQ=1
      NP=NVAR
      GO TO 10
 9    NS=2
      NQ=1
      NP=4
 10      DO 11 K=1,NS
       WRITE(OUT,501)(A2(I),I=NQ,NP)
      WRITE(OUT,502)(SF(I),I=NQ,NP)
      NQ=5
 11   NP=NVAR
      WRITE(OUT,505)
PRINT,HEAD
      WRITE(OUT,503)
*
*
*                    MAKE AXIS
*
*
       LLNTH = 61
      K=0
       DO 12 I=1,LLNTH
 12      LINE(I)=MINS
         WRITE(OUT,504)K,(LINE(I),I=1,LLNTH)
*
*
*                    BLANK THE LINE
*
*
       DO 13 I=1,LLNTH
 13 LINE(I)=BLANK
*
*
*                    PLOT THE VARIABLES
*
*
         DO 15 J=1,N
         LINE(1)=DOT
      DO 14 I=1,NVAR
      JK=(60 *I)/NVAR +1
```

```
      JP=JK-(30 *(IC(I)-1))/NVAR
      JL=V(I,J)+2.5
      LINE(JK)=DOT
      LINE(JP)=DOT
 14      LINE(JL)=X(I)
        WRITE(OUT,504)J,(LINE(K),K=1,LLNTH)
                DO 15 K=1,LLNTH
 15     LINE(K)=BLANK
*
*
      RETURN
*
*
*
*
500 FORMAT( ////5X,"SYSTEM PLOT IN"/5X,I2,5X,"VARIABLES"/)
501 FORMAT(5X,"VARIABLES;",5X,4(A8,6X))
502 FORMAT(3X,"SCALE FACTORS",4(G14.5))
503 FORMAT(///1X,6(10X,"1"))
504 FORMAT( 1X,I4,6X,121A1)
505 FORMAT(//)
            END
FUNCTION DELAY(C,TLAG)
CHARACTER A(25),HEAD*80
COMMON OUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT,IC,NIC,
&IT,NIT,ITMAX,KERR,IA(100),Y1(100),Y4(100),X1(100),X4(100),N,KP,
%NSAMPL,KT
COMMON /OUTP2/ II,JJ,A,ISZZ,AMX(25),AMN(25),HEAD,NPLOT,TKPLOT,PPLANE
COMMON /OUTP1/ V(25),IP,KASE,KS,TSAV,ICN
COMMON /HOLD/ IADCNT,ITQA,JCNT,IQCNT,HLD(500)
IF(IADCNT.GE.1) GO TO 102
IADCNT=1
ITQA=1
JCNT=0
IQCNT=0
102 IF(T.GT.TLAG) GO TO 103
DELAY=HLD(ITQA)
HLD(ITQA)=C
IQCNT=IQCNT+1
GO TO 104
103 IF(JCNT.GE.1) GO TO 106
ITQA=1
JCNT=1
GO TO 107
106 IF(ITQA LT.IQCNT) GO TO 107
JCNT=0
107 DELAY=HLD(ITQA)
HLD(ITQA)=C
104 ITQA=ITQA+1
RETURN
END
```

```
   FUNCTION DIGTAL(U1)
COMMON OUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT,IC,NIC,IT,NIT,ITMA
&IA(100),Y1PZ(100),Y4PZ(100),X1PZ(100),X4PZ(100),N,KP,NSAMPL,KT
COMMON/DIGITA/IDIGL,ZA(6),ZB(6),TSAM,ZK,NCNT
COMMON/ EXTRA/ NTERM,U(6),Y(6)
*
*
IF(IDIGL  GT  0) GO TO 100
IDIGL=1
NCNT=0
NTERM=6
DO 10 I=1,NTERM
U(I) = 0
10 Y(I) = 0
*
*
*  ADJUST DELT TO BE INTEGRAL MULTIPLE OF TSAM
*
IF(DELT .GT. TSAM/10.) GO TO 50
PP=1.
 20  RR=TSAM/PP/DELT
JJ=RR
IF((RR-FLOAT(JJ)) .LT. .001) GO TO 70
PP=PP+1.
IF(PP .LE. 3.) GO TO 20
PP=1.
  30  XDEL=TSAM/20./PP
IF(XDEL .LT. DELT) GO TO 60
PP=PP+1.
GO TO 30
  50   XDEL=TSAM/10.
  60   NSAMPL=NSAMPL*DELT/XDEL
DELT=XDEL
GO TO 100
  70   DELT=DELT/PP
NSAMPL=NSAMPL*PP
  100  CONTINUE
*
* HERE FOR NORMAL OPERATION
*
IF(ABS(KT*DELT - NCNT*TSAM) .GT. 1.E-6 ) GO TO 300
 IF(IT  EQ  ITMAX) NCNT=NCNT+1
U(1) = U1
YX = 0.
DO 110 I=1,NTERM
110 YX=YX + ZK*ZA(I)*U(I)
DO 120 I=2,NTERM
120 YX=YX - ZB(I)*Y(I)
Y(1) = YX/ZB(1)
IF(IT .LT. ITMAX) GO TO 300
DO 130 I=NTERM,2,-1
```

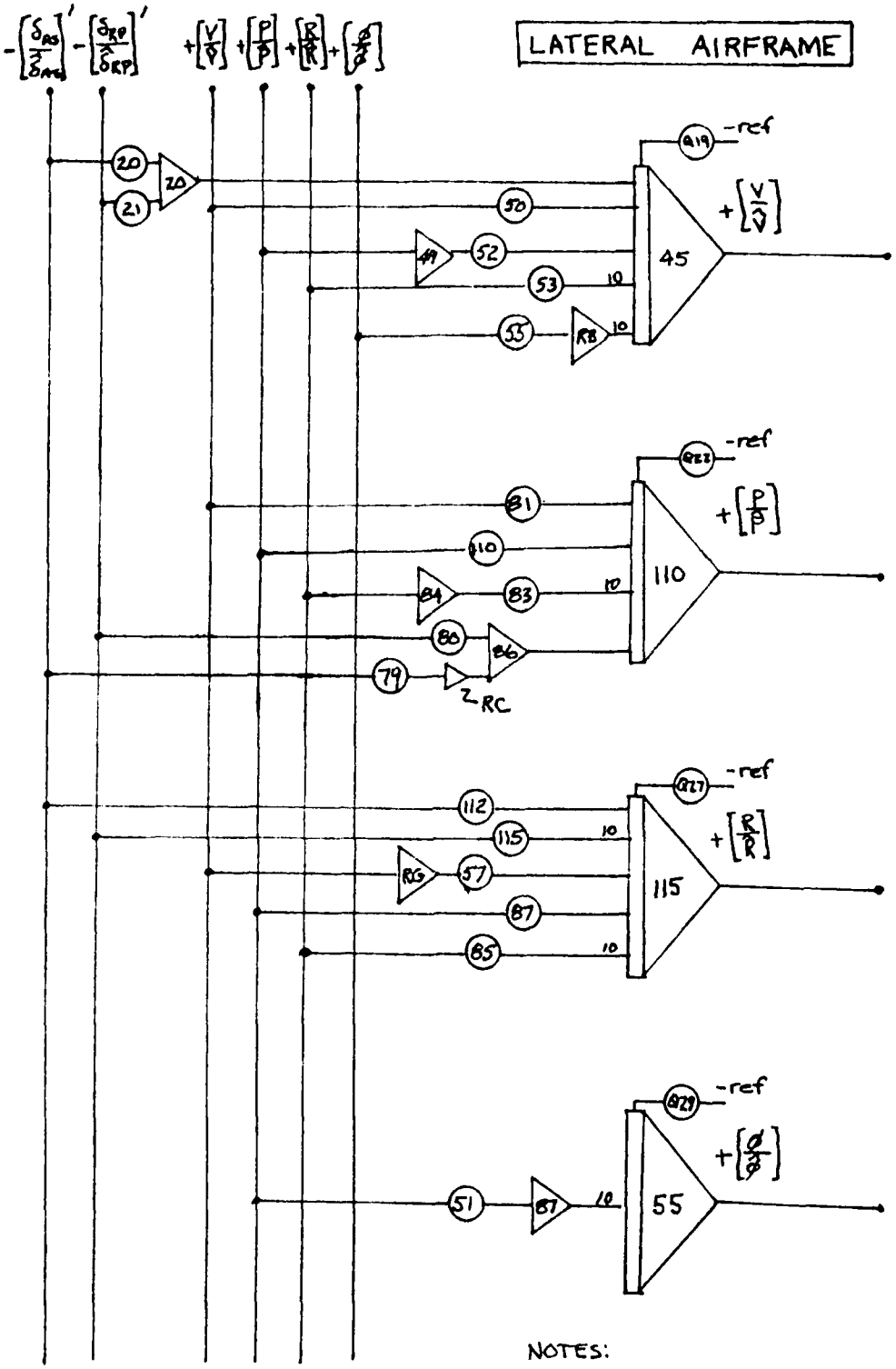
```
U(1) = U(I-1)
130 Y(I) = Y(I-1)
*
 300  DIGTAL = Y(1)
RETURN
END
    SUBROUTINE RANGE
CHARACTER A(25),HEAD*80
COMMON OUT,XVZ(6,402),ICNT
COMMON /OUTP2/ II,JJ,A,IS,AMX(25),AMN(25),HEAD,NPLOT,TKPLOT,PPLANE
COMMON /OUTP1/ V(25),IP,KASE,KS,TSAV,ICN
*
IF(ICNT .GT. 1) GO TO 20
DO 10 I=1,25
AMX(I) = -1.E+6
10 AMN(I) = 1.E6
20 GO TO (30,80),IS
30 DO 50 I=1,II
AMX(I) = AMAX1(AMX(I),V(I))
50 AMN(I) = AMIN1(AMN(I),V(I))
70 RETURN
*
80 WRITE(OUT,900)
DO 100 I=1,II
100 WRITE(OUT,910)I,A(I),AMX(I),AMN(I)
GO TO 70
900 FORMAT(1H0,"   MAX AND MIN VALUES")
910 FORMAT(1H ,I2,3X,A8," MAX =",G12.5,"  MIN =",G12.5)
END
REAL FUNCTION LIMIT(U1,X,U2)
LIMIT = X
IF(X .GT. U2) LIMIT=U2
IF(X .LT. U1) LIMIT=U1
RETURN
END
SUBROUTINE TEKPLT
*
* THIS SUBROUTINE PREPARS A FILE FOR PLOTTING
* ON THE TEKTRONICS GRAPHIC TERMINALS.
* YOU MUST OPEN A FILE IN YOUR CATALOG
*
*
CHARACTER A(25),HEAD*80
COMMON OUT,X(6,402),ICNT,NVAR,T,TO,TMAX,DELT,HDELT,IC,NIC,IT,NI[illegible]AX,[illegible]
&IA(100),Y1PZ(100),Y4PZ(100),X1PZ(100),X4PZ(100),N,KP,NSAMPL,KT
COMMON /OUTP2/ II,JJ,A,IS,AX(25),AZ(25),HEAD,NPLOT,TKPLOT,PPLANE
*
*
IF(PPLANE .NE. 0)GO TO 70
DO 50 I=1,NVAR
K=0
```

```
T=TO
DO 20 J=1,ICNT
WRITE(3,500)T,X(I,J)
K=K + 1
T = K*NSAMPL
T = T*DELT + TO
20 CONTINUE
WRITE(3,510)
50 CONTINUE
RETURN
70 DO 80 J=1,ICNT
80 WRITE(3,500)X(1,J),X(2,J)
WRITE(3,510)
RETURN
500 FORMAT(E12.4,E12.4)
510 FORMAT(" 1.E37 , 1.E37 ")
END
REAL FUNCTION PTRAIN(T1)
COMMON ZOUT,XVZ(6,402),ICNT,NVAR,T,TO,TMAX,DELT
SAVE P,J
DATA J/0/
PTRAIN=0.
P=J
P=P*T1
IF(T .GT. P)J=J+1
IF(ABS(T-P) .LT. 0.5*DELT)PTRAIN=1/DELT
RETURN
END
```

APPENDIX I

ANALOG CIRCUIT DIAGRAMS

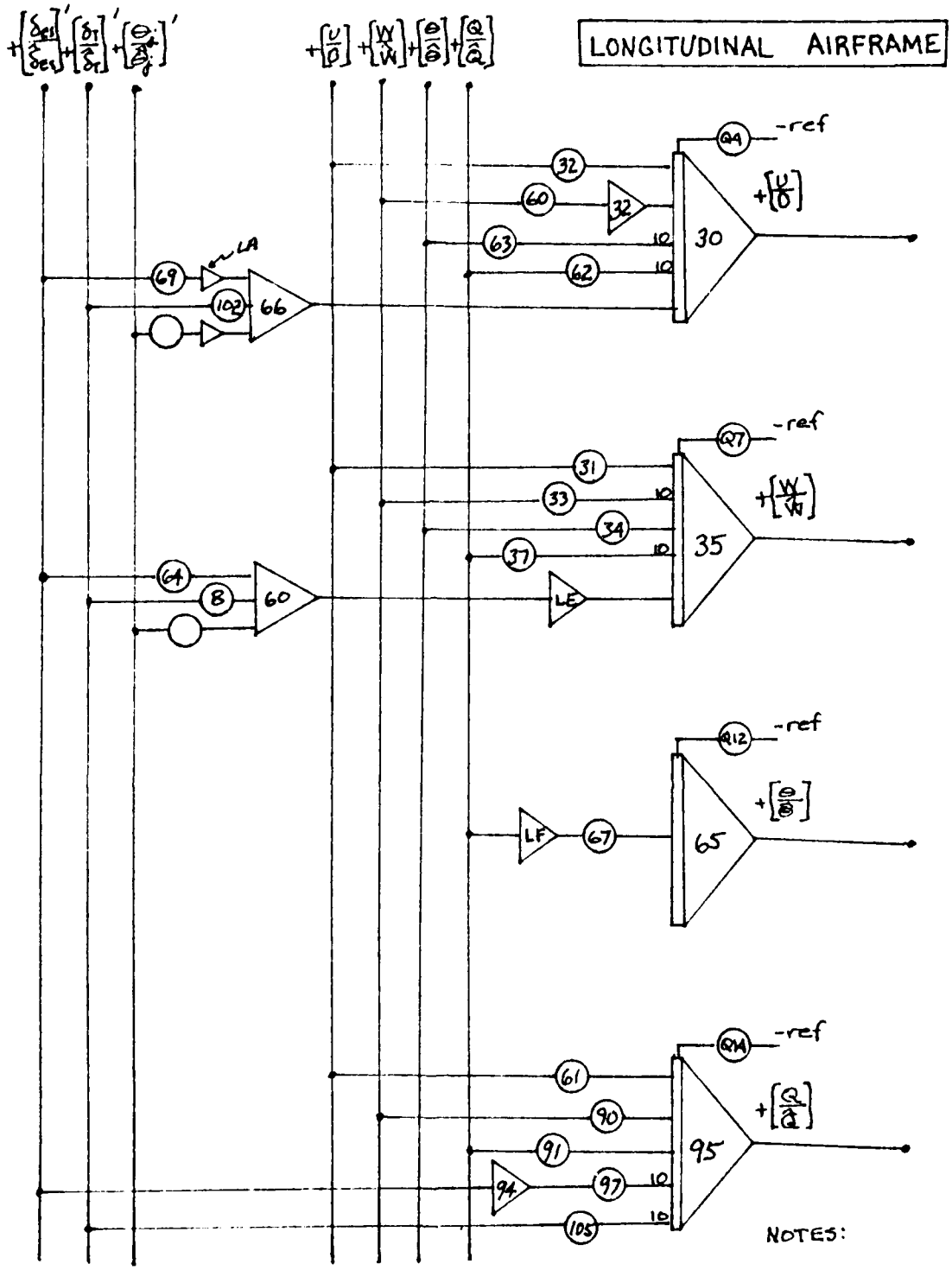
This appendix contains the necessary analog flow diagrams to implement the Harrier airframe equations, present SAS, continuous state feedback, and digital state feedback controller. Component numbers reference the present implementation of the simulation on the EAI-681 large scale analog computer in the hybrid technology lab of Nimitz Hall, U. S. Naval Academy.



NOTES:

1) SEE APPENDIX J FOR POT SETTINGS

2) COEFFICIENT AT POT 57 CHANGES SIGN (SET RELAY B4 "+" FOR NEGATIVE a_9)

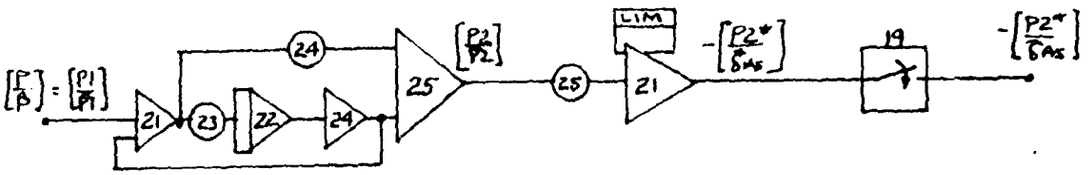


NOTES:

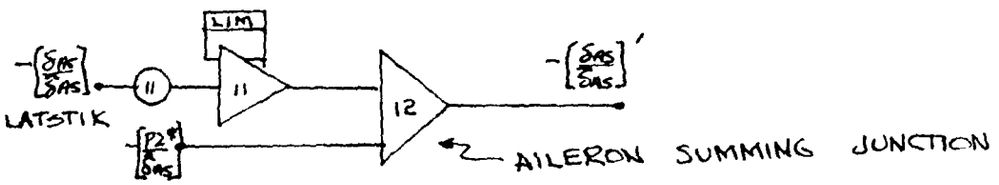
1) SEE APPENDIX J FOR POT SETTINGS

2) θ_j CONNECTIONS WERE NOT WIRED BECAUSE THEY WERE NOT NEEDED.

ROLL RATE FEEDBACK



CALCULATION OF δ_{AS}'

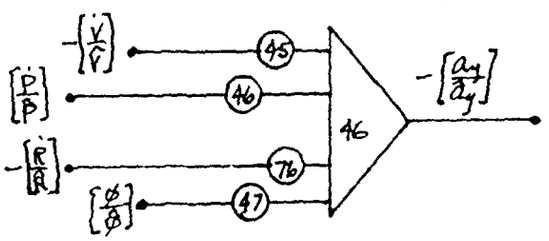


LATERAL STABILITY AUGMENTATION

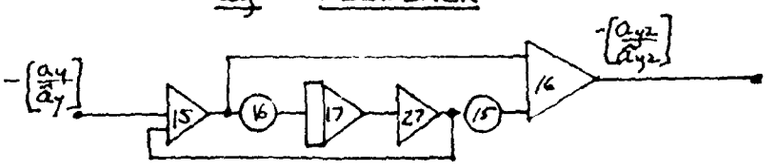
CALCULATION OF a_y

NOTE:

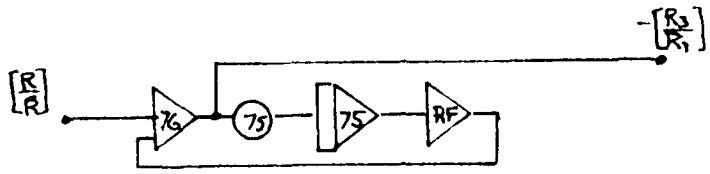
. BECAUSE OF ACUTE NOISE SENSITIVITY, LAT. ACCEL. FEEDBACK WAS REMOVED FROM THE CIRCUIT.



a_y FEEDBACK



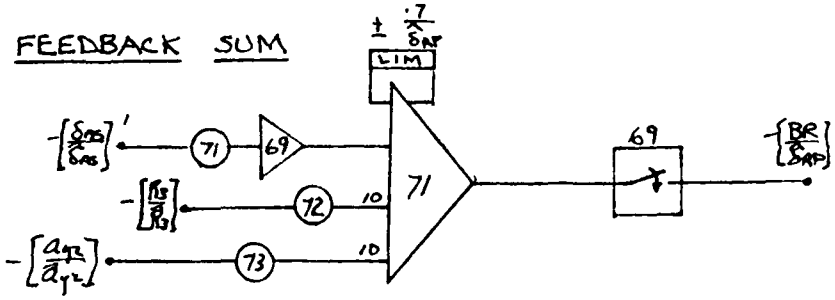
YAW RATE FEEDBACK



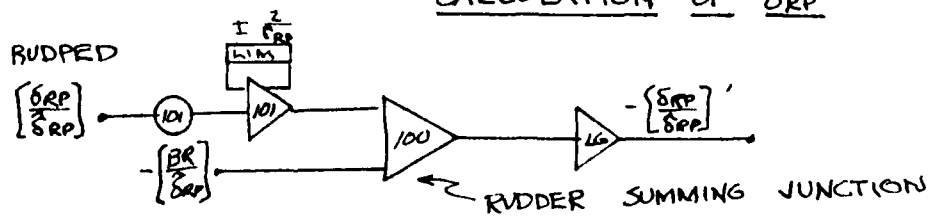
LATERAL STABILITY AUGMENTATION

(CONTINUED)

RUDDER FEEDBACK SUM

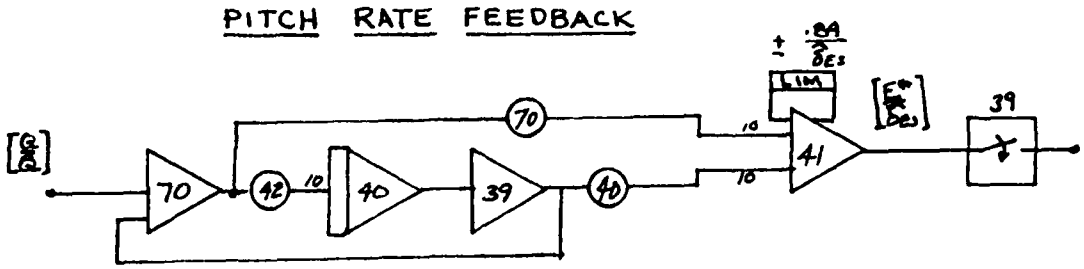


CALCULATION OF δ_{RP}'

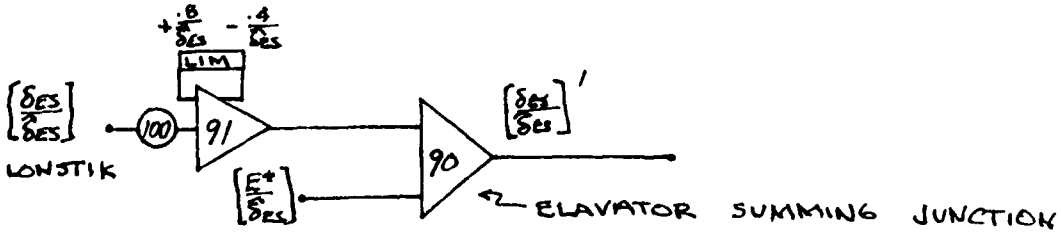


LONGITUDINAL STABILITY AUGMENTATION

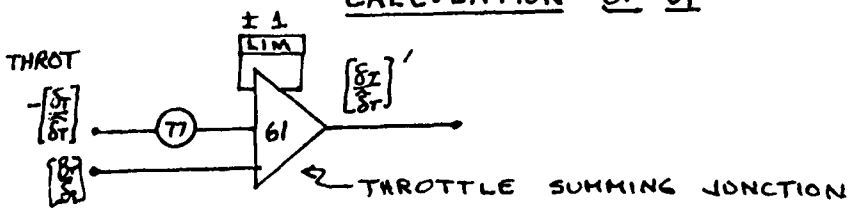
PITCH RATE FEEDBACK



CALCULATION OF δ_{ES}'

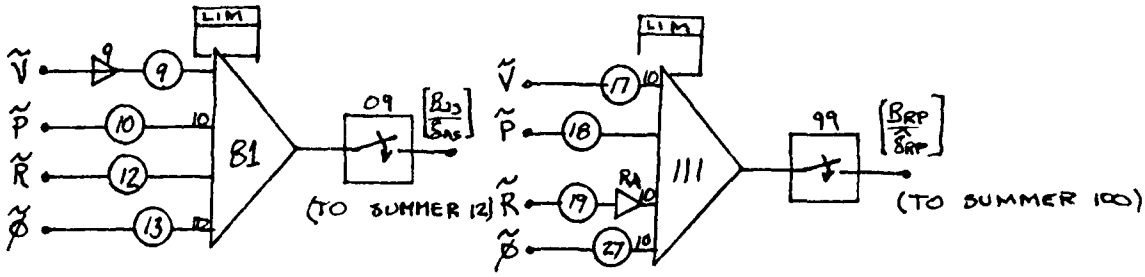


CALCULATION OF δ_T

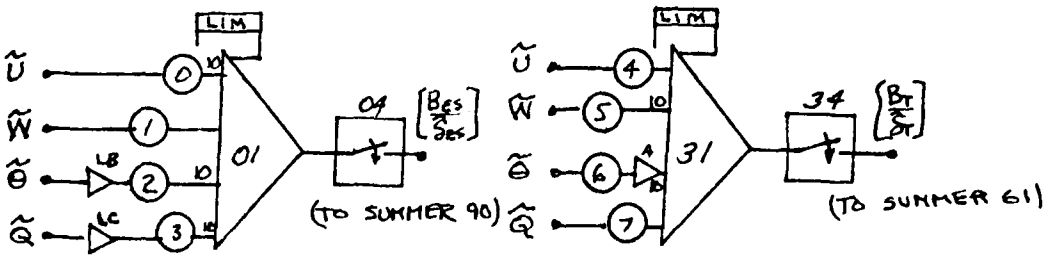


CONTINUOUS STATE FEEDBACK IMPLEMENTATION

LET $\tilde{X} \equiv \begin{bmatrix} X \\ \dot{X} \end{bmatrix}$



LATERAL CASE



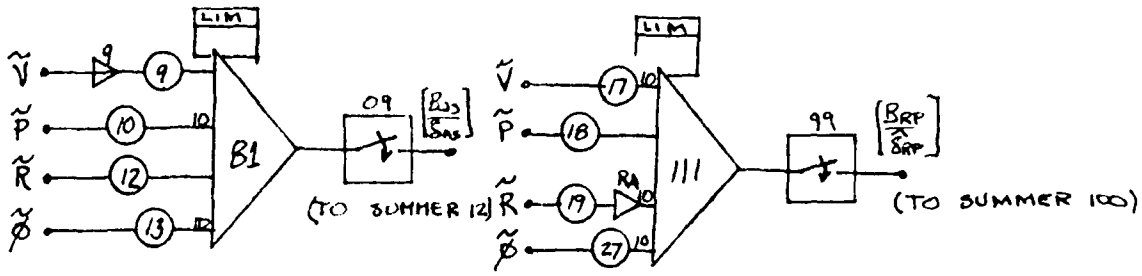
LONGITUDINAL CASE

NOTES:

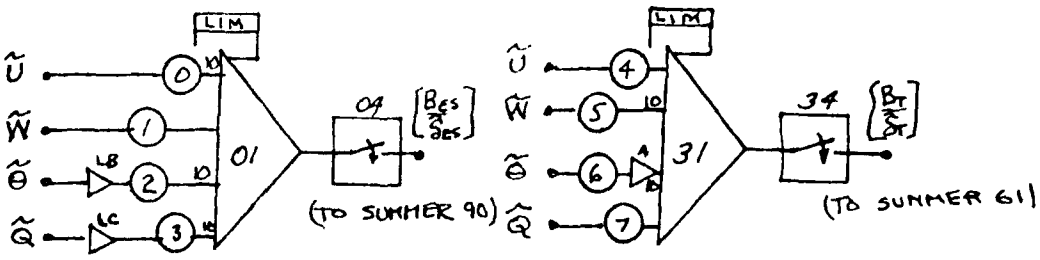
1) SEE APPENDIX J FOR POT SETTINGS
2) SEE APPENDIX K FOR DIGITAL STATE FEEDBACK CONTROLLER

CONTINUOUS STATE FEEDBACK IMPLEMENTATION

LET $\tilde{x} \equiv \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$



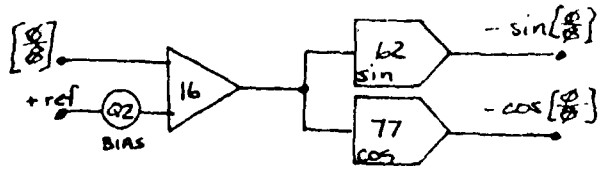
LATERAL CASE



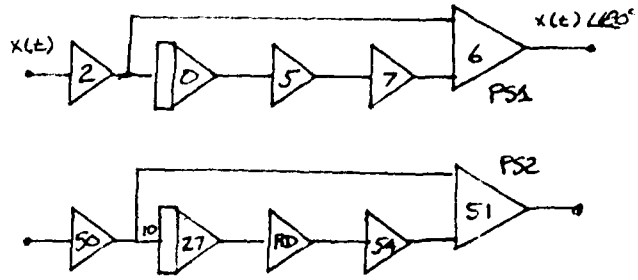
LONGITUDINAL CASE

NOTES:

1) SEE APPENDIX J FOR POT SETTINGS
2) SEE APPENDIX K FOR DIGITAL STATE FEEDBACK CONTROLLER



SIN/COS GENERATION

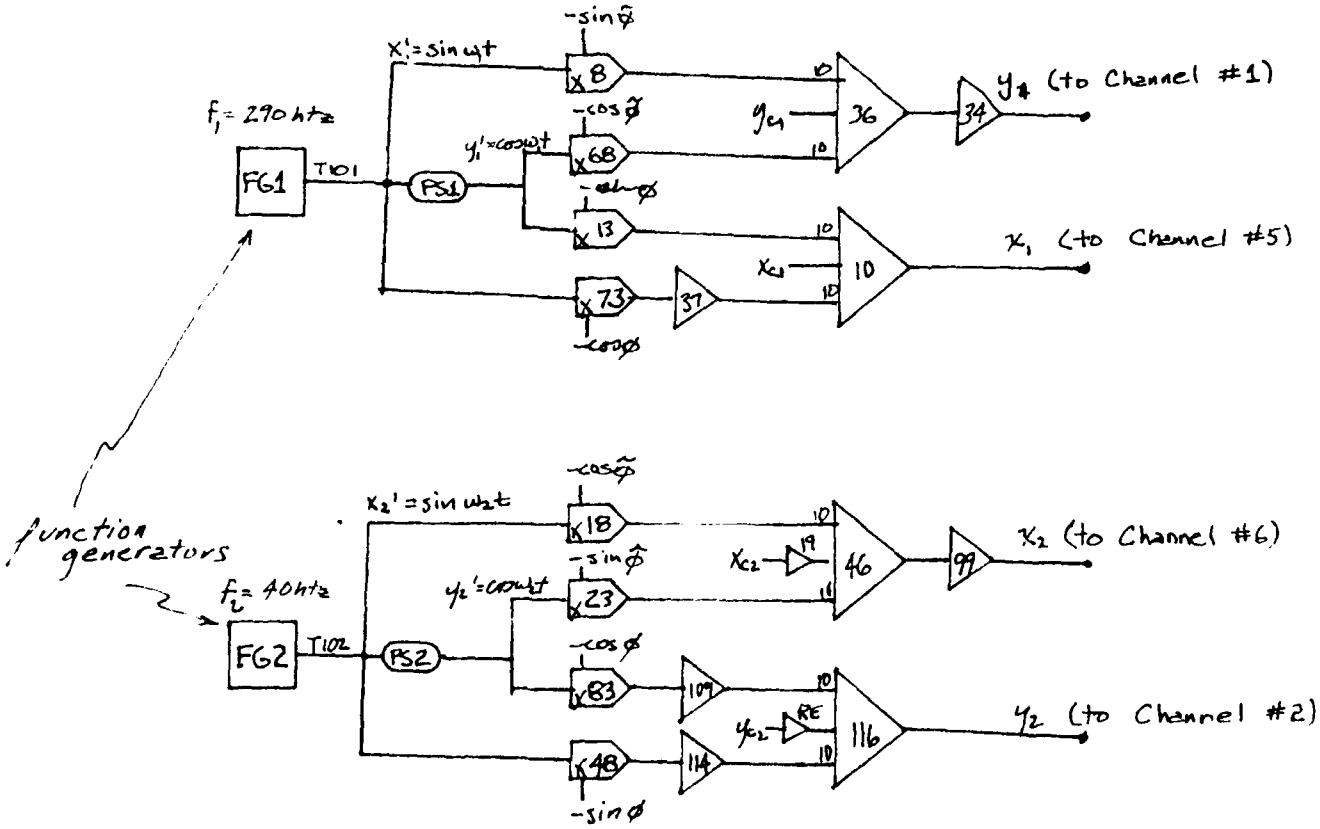


180° PHASE SHIFTERS

ANALOG GRAPHICS IMPLEMENTATION

GOVERNING EQUATIONS:

$$\begin{cases} x = x' \cos\phi - y' \sin\phi + h \\ y = x' \sin\phi + y' \cos\phi + k \end{cases}$$



APPENDIX J

DIGITAL HYBRID INITIATION ROUTINE

("AHEAD")

Also named AHEAD, this program sets the pots and initializes the analog computer simulation parameters. Upon completion of this task, it automatically loads the digital state feedback controller-SFBCON (see Appendix K).

```
C   PROGRAMMER: R.V. WALTERS
C   TITLE: AHEAD
C   FUNCTION: INITIALIZATION OF AV-8B
C             HYBRID SIMULATION PARAMETERS
C
          REAL LONSTK
          REAL LATSTK
C
C
          DIMENSION AALON(16,6),BBLON(9,4),AALAT(11,4),BBLAT(6,4)
          DIMENSION ALON(16,1),BLON(12,1),ALAT(16,1),BLAT(12,1)
          DIMENSION XKLON(8),XKLAT(8)
          DIMENSION V5(6,1),V3(4,1)
          DIMENSION FILE1(2),FILE2(2),FILE3(2),FILE4(2)
          DIMENSION FILE5(2),FILE6(2),FILE7(2)
          DIMENSION POTS(118)
C
          COMMON XKLON,XKLAT
          COMMON /BLK1/VCAP,PCAP,RCAP,PHICAP,AYCAP
          COMMON /BLK2/UCAP,WCAP,THACAP,QCAP
          COMMON /BLK3/DASCAP,DRPCAP,DESCAP,DTCAP
C
          DATA FILE1(1),FILE1(2)/'ALON ','  SRC'/
          DATA FILE2(1),FILE2(2)/'BLON ','  SRC'/
          DATA FILE3(1),FILE3(2)/'ALAT ','  SRC'/
          DATA FILE4(1),FILE4(2)/'BLAT ','  SRC'/
          DATA FILE5(1),FILE5(2)/'KLAT1','  SRC'/
          DATA FILE6(1),FILE6(2)/'KLON1','  SRC'/
          DATA FILE7(1),FILE7(2)/'KLON2','  SRC'/
C
C   SET CONSTANTS
C
          IT=2
          CALL HINIT(IE)
          CALL SAMO(1,IE1)
          CALL SLMO(1,IE2)
          CALL STCO(0,IE3)
C
          WRITE(4,81)
          WRITE(4,18)
C
81        FORMAT(1X,'ANALOG SET UP')
82        FORMAT(1X,'V5 VECTOR CALCULATED')
83        FORMAT(1X,'V3 VECTOR CALCULATED')
88        FORMAT(1X,'FILES OPENED')
```

```
C   SET SCALE FACTORS
C
          VCAP=50
          PCAP=2.5
          RCAP=2.5
          PHICAP=2.5
          DUCAP=10
          DRCAP=10
          AYCAP=100.
C
          UCAP=50
          WCAP=50
          THACAP=2.5
          QCAP=2.5
          DESCAP=20
          DTCAP=20.
C
C   INPUT FLIGHT VELOCITY
C
C
          WRITE(4,21)
21        FORMAT(1X,'INPUT U (VEL OF A/C C.G.)')
          READ(4,)V
          WRITE(4,18)
C
C   INPUT PRINTING INFO
C
          WRITE(4,22)
22        FORMAT(1X,'PRINT MATRICES?  (YES=1)')
          READ(4,)MATOUT
          WRITE(4,18)
C
          WRITE(4,23)
23        FORMAT(1X,'PRINT OUT SETTINGS?  (YES=1)')
          READ(4,)POTOUT
          WRITE(4,18)
C
C   CHOOSE LONGITUDINAL SPB MATRIX
C
          WRITE(4,24)
24        FORMAT(1X,'USE IMPROVED KLONG?  (YES=1)')
          READ(4,)KIMP
          WRITE(4,18)
```

```
C  SET INITIAL CONDITIONS (UNSCALED)
C
        VZER=0.
        PZER=0.
        RZER=0.
        PHIZER=0
C
        UZER=0.
        NZER=0.
        THAZER=0.
        RZER=0.
C
C  SET INPUTS
C
        LONFOR=0.
        LATETA=0
        RUDRED=0.
        THROT=0.
C
C CALCULATE THE VS VECTOR
C
        VS(0,1)=1.
        DO 4 I=0,1,-1
        VS(I,1)=VS(I+1,1)+V
C       CONTINUE
C
        WRITE(4,82)
C
C
C CALCULATE THE VS VECTOR
C
        VS(0,1)=1.
        DO 4 I=0,1,-1
        VS(I,1)=VS(I+1,1)+V
C       CONTINUE
C
        WRITE(4,82)
```

```
C READ AALON
C
      CALL SEEK(1,FILE1)
C
      WRITE(4,E2)
C
      DO 6 J=1,6
      DO 7 I=1,16
      READ(1,)AALON(I,J)
7     CONTINUE
6     CONTINUE
      ENDFILE 1
C
C
C
C  READ BBLON
C
      CALL SEEK(1,FILE2)
      DO 8 J=1,4
      DO 9 I=1,9
      READ(1,) BBLON(I,J)
9     CONTINUE
8     CONTINUE
      ENDFILE 1
C
C
C  READ AALAT
C
      CALL SEEK(1,FILE3)
      DO 10 J=1,4
      DO 11 I=1,11
      READ(1,) AALAT(I,J)
11    CONTINUE
10    CONTINUE
      ENDFILE 1
C
C
C  READ BBLAT
C
      CALL SEEK(1,FILE4)
      DO 12 J=1,4
      DO 13 I=1,5
      READ(1,) BBLAT(I,J)
13    CONTINUE
12    CONTINUE
      ENDFILE 1
```

```
C  READ WKLAT
C
      CALL SEEK(1,FILE5)
      DO 27 I=1,8
      READ(1,)WKLAT(I)
27    CONTINUE
      ENDFILE 1
C
C  READ WKLON
C
      IF (KIMP.NE.1) GO TO 25
      CALL SEEK(1,FILE7)
      DO 27 I=1,8
      READ(1,)WKLON(I)
27    CONTINUE
      ENDFILE 1
      GO TO 26
C
25    CONTINUE
      CALL SEEK(1,FILE6)
      DO 28 I=1,8
      READ(1,)WKLON(I)
28    CONTINUE
      ENDFILE 1
C
26    CONTINUE
C
C
C
C
C CALCULATE ALON
C
      WRITE(4,14)
14    FORMAT(1X,'DATA READ')
C
      CALL AMULT(ALON,AALON,16,8,1,V5)
      IF (V.LT.5.) ALON(2,1)=-22.02
      IF (V.LT.5.) ALON(7,1)=-2.37
C
C CALCULATE BLON
C
      CALL AMULT(BLON,BBLON,9,4,1,V2)
      BLON(12,1)=BLON(2,1)
      BLON(11,1)=BLON(2,1)
      BLON(10,1)=BLON(7,1)
      BLON(7,1)=0
      BLON(8,1)=0
      BLON(9,1)=0
```

```
C  CALCULATE ALAT
C
      CALL MMULT(ALAT,AALAT,11,4,1,V1)
      DO 15 I=12,16
      ALAT(I,1)=0
15    CONTINUE
      ALAT(14,1)=1.
C
C  CALCULATE BLAT
C
      CALL MMULT(BLAT,BBLAT,6,4,1,V1)
      BLAT(8,1)=BLAT(6,1)
      BLAT(7,1)=BLAT(5,1)
      BLAT(5,1)=BLAT(4,1)
      BLAT(4,1)=BLAT(3,1)
      BLAT(3,1)=0.
      BLAT(6,1)=0.
      DO 16 I=9,12
      BLAT(I,1)=0.
16    CONTINUE
C
      WRITE(4,17)
17    FORMAT(1X,'COEFFICIENTS CALCULATED')
      WRITE(4,18)
      WRITE(4,18)
18    FORMAT(1H )
C
      IF(IATOUT NE.1) GO TO 30
C
      WRITE(4,19)
19    FORMAT(1H1,7X,'ALON',7X,'ALAT')
      WRITE(4,18)
      DO 20 I=1,16
      WRITE(4,21) ALON(I,1),ALAT(I,1)
20    CONTINUE
21    FORMAT(1X,4X,E11.4,4X,E11.4)
      WRITE(4,18)
      WRITE(4,18)
      WRITE(4,22)
22    FORMAT(6X,'BLON',7X,'BLAT')
      WRITE(4,18)
      DO 23 I=1,12
      WRITE(4,21) BLON(I,1),BLAT(I,1)
23    CONTINUE
      WRITE(4,18)
      WRITE(4,18)
      WRITE(4,24)
24    FORMAT(6X,'XLON',6X,'XLAT')
      WRITE(4,18)
      DO 25 I=1,8
      WRITE(4,21) XLON(I),XLAT(I)
25    CONTINUE
      WRITE(4,18)
      WRITE(4,18)
```

```
C   ZERO THE POTS
C
          DO 24 I=1,118
          POTS(I)=5.
24        CONTINUE
C
C   CALCULATE THE POT SETTINGS
C
          POTS(1)=XKLON(2)*NCAP/DESCAP
          POTS(2)=XKLON(3)*THACAP/DESCAP/10.
          POTS(3)=XKLON(4)*QCAP/DESCAP/10.
          POTS(4)=XKLON(5)*UCAP/DTCAP
          POTS(5)=XKLON(6)*NCAP/DTCAP/10.
          POTS(6)=XKLON(7)*THACAP/DTCAP/10.
          POTS(7)=XKLON(8)*QCAP/DTCAP
          POTS(8)=BLON(5,1)*DTCAP/WCAP
          POTS(9)=XKLAT(1)*VCAP/DASCAP
          POTS(10)=XKLAT(2)*PCAP/DASCAP/10.
          POTS(11)=LATSTR/DASCAP
          POTS(12)=XKLAT(3)*RCAP/DASCAP
          POTS(13)=XKLAT(4)*PHICAP/DASCAP/10.
          POTS(17)=XKLAT(5)*VCAP/DRPCAP/10.
          POTS(18)=XKLAT(6)*PCAP/DRPCAP
          POTS(19)=XKLAT(7)*RCAP/DRPCAP/10.
          POTS(20)=BLAT(1,1)*DASCAP/VCAP
          POTS(21)=BLAT(2,1)*DRPCAP/VCAP
          POTS(22)=.774
          POTS(24)=.805
          POTS(25)=6.72*PCAP/DASCAP/10.
          POTS(27)=XKLAT(8)*PHICAP/DRPCAP/10.
          POTS(30)=XKLON(1)*UCAP/DESCAP/10.
          POTS(31)=ALON(5,1)*UCAP/WCAP
          POTS(32)=ALON(1,1)
          POTS(33)=ALON(6,1)/10.
          POTS(34)=ALON(7,1)*THACAP/WCAP
          POTS(35)=ALON(8,1)*QCAP/WCAP/10.
          POTS(40)=15.95*QCAP/DESCAP/10.
          POTS(42)=-135
          POTS(50)=ALAT(1,1)
          POTS(51)=ALAT(14,1)*PCAP/PHICAP/10.
          POTS(52)=ALAT(2,1)*PCAP/VCAP
          POTS(53)=ALAT(3,1)*PCAP/VCAP/10.
          POTS(54)=ALAT(4,1)*PHICAP/VCAP/10
```

```
      POTS(57)=ALAT(9,1)+VCAP/RCAP
      POTS(60)=ALON(2,1)+WCAP/UCAP
      POTS(61)=ALON(3,1)+UCAP/QCAP
      POTS(62)=ALON(4,1)+QCAP/UCAP/10
      POTS(63)=ALON(3,1)+THACAP/QCAP/10.
      POTS(64)=BLON(4,1)+DESCAP/WCAP
      POTS(67)=ALON(12,1)+QCAP/THACAP/10.
      POTS(69)=BLON(1,1)+DESCAP/UCAP
      POTS(70)=15.86+QCAP/DESCAP/40.
      POTS(71)=.24+DASCAP/DRPCAP
      POTS(72)=1.73+RCAP/DRPCAP/10.
      POTS(75)=.32
      POTS(77)=THROT/DTCAP
      POTS(79)=BLAT(4,1)+DASCAP/PCAP/10.
      POTS(80)=BLAT(5,1)+DRPCAP/PCAP
      POTS(81)=ALAT(5,1)+VCAP/PCAP
      POTS(82)=ALAT(7,1)+RCAP/PCAP/10.
      POTS(85)=ALAT(11,1)/10.
      POTS(87)=ALAT(10,1)+PCAP/RCAP
      POTS(90)=ALON(14,1)+WCAP/QCAP
      POTS(91)=ALON(15,1)
      POTS(97)=BLON(10,1)+DESCAP/QCAP/10.
      POTS(100)=LONSTK/DESCAP
      POTS(101)=RUDPED/DRPCAP
      POTS(102)=BLON(2,1)+DTCAP/UCAP
      POTS(105)=BLON(11,1)+DTCAP/QCAP/10.
      POTS(110)=ALAT(6,1)
      POTS(112)=BLAT(7,1)+DASCAP/RCAP
      POTS(115)=BLAT(6,1)+DRPCAP/RCAP
C
C
      WRITE(4,50)
50    FORMAT(1X, POT SETTINGS CALCULATED')
      WRITE(4,18)
      WRITE(4,18)
```

```
C  SET THE POTS
C
        CALL SAMO(7,IE6)
C
        DO 51 I=1,118
        ICOF=ABS(10000.*POTS(I))
        IF (ICOF.EQ.50000) GO TO 51
        IF (ICOF.LE.10000) GO TO 25
        WRITE(4,26) I
26      FORMAT(1X,'POT SETTING TOO LARGE:',I3)
        ICOF=9999
25      CONTINUE
        CALL SPOT(I,ICOF,IT,IE7)
C
        IF(POTOUT.NE.1) GO TO 51
C
        WRITE(4,) POTS(I),IE7
51      CONTINUE
C
C
C  ALAT(9,1) SIGN TEST
C
        IF (ALAT(9,1).GT.0.) GO TO 52
        CALL SSRP(11,IE4)
        GO TO 110
52      CALL SSRM(11,IE5)
110     CONTINUE
C
C
C
        CALL SAMO(1,IE8)
C
        WRITE(4,18)
        WRITE(4,54)
        WRITE(4,18)
54      FORMAT(1X,'POT SETTING COMPLETE')
C
C  BEGIN SIMULATION
C
C
        CALL IFBCON(W)
C
        STOP
        END
```

APPENDIX K

DIGITAL STATE FEEDBACK CONTROLLER SUBROUTINE

("SFBCON")

This program implements the digital state feedback controller. Note that it operates continuously in an endless loop. It is activated from the logic and switching controls of the analog computer. It has been determined that its sampling rate is approximately 71 samples per second (T = 14 ms.).

```
      SUBROUTINE SFBCON(V)
C
      DIMENSION XKLON(8),XKLAT(8)
C
      COMMON XKLON,XKLAT
      COMMON /BLK1/VCAP,PCAP,RCAP,PHICAP,AYCAP
      COMMON /BLK2/UCAP,WCAP,THACAP,QCAP
      COMMON /BLK3/DASCAP,DRPCAP,DESCAP,DTCAP
C
      WRITE(4,2)
2     FORMAT(1X,'DIGITAL CONTROL ACTIVATED')
C
1     CONTINUE
C
C  READ AND CONVERT ANALOG STATES
C
      CALL CRACS(6,IV,IE1)
      CALL CRACS(7,IP,IE2)
      CALL CRACS(8,IR,IE3)
      CALL CRACS(9,IPHI,IE4)
      CALL CRACS(2,IU,IE5)
      CALL CRACS(3,IW,IE6)
      CALL CRACS(4,ITHA,IE7)
      CALL CRACS(5,IQ,IE8)
C
C
C  CALC LATERAL FEEDBACK SIGNAL (SCALED)
C
      IDAS=.1*(XKLAT(1)*IV*VCAP+XKLAT(2)*IP*PCAP)/DASCAP
      IDAS=IDAS+.1*(XKLAT(3)*IR*RCAP+XKLAT(4)*IPHI*PHICAP)/DASCA
      IDAS=-IDAS
      IDRP=.1*(XKLAT(5)*IV*VCAP+XKLAT(6)*IP*PCAP)/DRPCAP
      IDRP=IDRP+.1*(XKLAT(7)*RCAP*IR+XKLAT(8)*IPHI*PHICAP)/DRPCA
C
C  CALC LONGITUDINAL CONTROL INPUT (SCALED)
C
      IDES=.1*(XKLON(1)*IU*UCAP+XKLON(2)*IW*WCAP)/DESCAP
      IDES=IDES+.1*(XKLON(3)*ITHA*THACAP+XKLON(4)*IQ*QCAP)/DESCA
      IDT=(XKLON(5)*IU*UCAP+XKLON(6)*IW*WCAP)/DTCAP
      IDT=IDT+(XKLON(7)*ITHA*THACAP+XKLON(8)*IQ*QCAP)/DTCAP
C
C  TEST FOR SATURATION
C
      IF(IDAS.GT.10000) IDAS=10000
      IF(IDRP.GT.10000) IDRP=10000
      IF(IDES.GT.10000) IDES=10000
      IF(IDT.GT.10000) IDT=10000
```

```
C   PERFORM DIGITAL TO ANALOG CONVERSION
C
      CALL LTDAS(0,IDAS,IE9)
      CALL TLDA
      CALL LTDAS(1,IDRP,IE10)
      CALL TLDA
      CALL LTDAS(2,IDES,IE11)
      CALL TLDA
      CALL LTDAS(3,IDT,IE12)
      CALL TLDA
C
C
      GO TO 1
C
      RETURN
      END


      SUBROUTINE MMULT(R,A,N,M,L,B)
      DIMENSION R(N,L),A(N,M),B(M,L)
      DIMENSION V(20,20)
C
      DO 101 I=1,N
      DO 101 J=1,L
        V(I,J)=0.
          DO 101 K=1,M
            V(I,J)=A(I,K)*B(K,J)+V(I,J)
101   CONTINUE
      DO 102 I=1,N
      DO 102 J=1,L
        R(I,J)=V(I,J)
102   CONTINUE
      RETURN
      END
```

APPENDIX L

AV-8B OUTPUT TIME TRACES

This appendix contains time traces of the AHEAD AV-8B, illustrating the results of the research. In all cases, the x-axis variable is time in seconds, and the ordinate represents the system states in:

Radians - θ, ϕ
Radians per sec - Q, P, R
Feet per sec - U, W, V

The results are divided into six sections:

I. V = 30; No stability augmentation; no I.C.'s.

Shows open loop system response to specified control inputs. Simulation fidelity can be assessed by comparison with Calspan results, Appendix M.

II-IV

$\phi_o = 15^o = .26R$

v_o = 10 kt - 16.9 fps

$r_o = 5^o$/sec - .09 R/s

$\theta_o = 15^o - .26R$

Initial condition response for sections II through IV is meant to be representative of the onset of the coupled intake momentum drag pehnomena.

II. LATERAL RESPONSE; V = 0, 30, 50, 80

Compares performance of present SAS and state feedback controller at various velocities.

APPENDIX I

(Continued)

III. LONGITUDINAL RESPONSE; V = 0, 30, 50, 90

Same as II, except longitudinal case.

IV. LATERAL AND LONGITUDINAL; V = 30

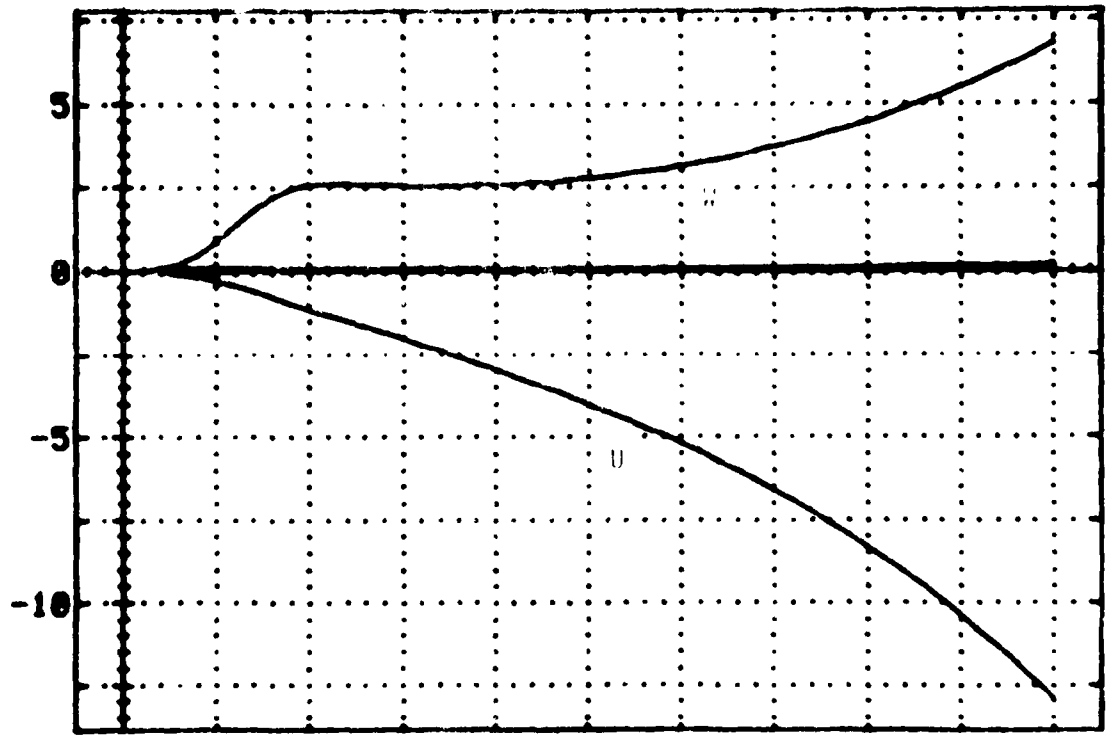
Compares performance of the various stability augmentation options.

V. LONGITUDINAL; V = 30; INPUT PITCH ANGLE

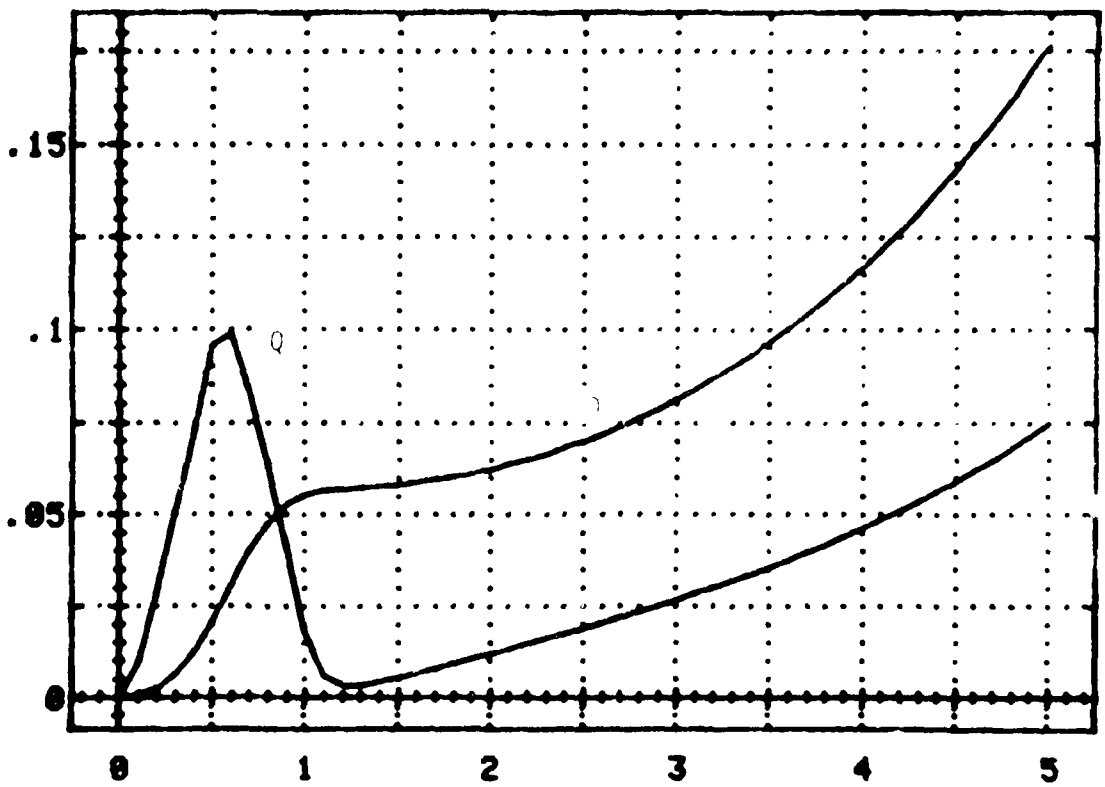
Shows response of SFB-controlled system to θ_0 = 15^0, 20^0, 25^0, and 30^0.

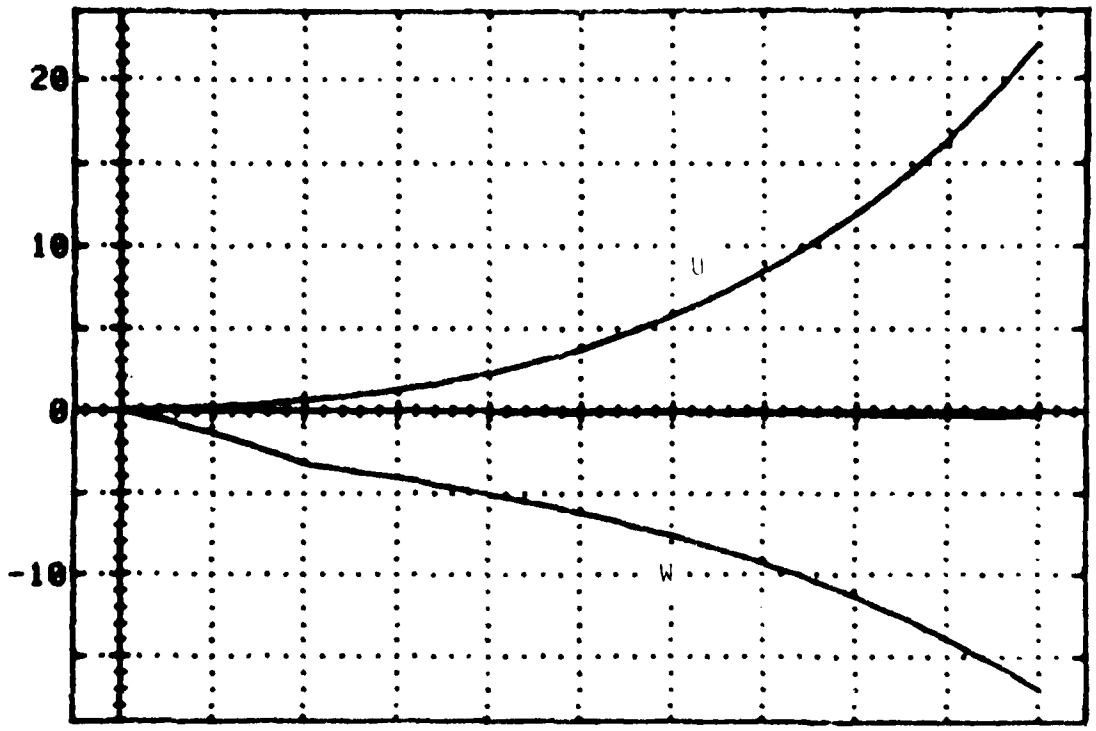
VI. LATERAL; r_0 = 5° s; ϕ_0 = 15°; INPUT SIDE VELOCITY

Shows response of SFB-controlled system to v_0 = 10, 20, 30 and 40 kts. in presence of an initial bank angle and yaw rate (V = 30)

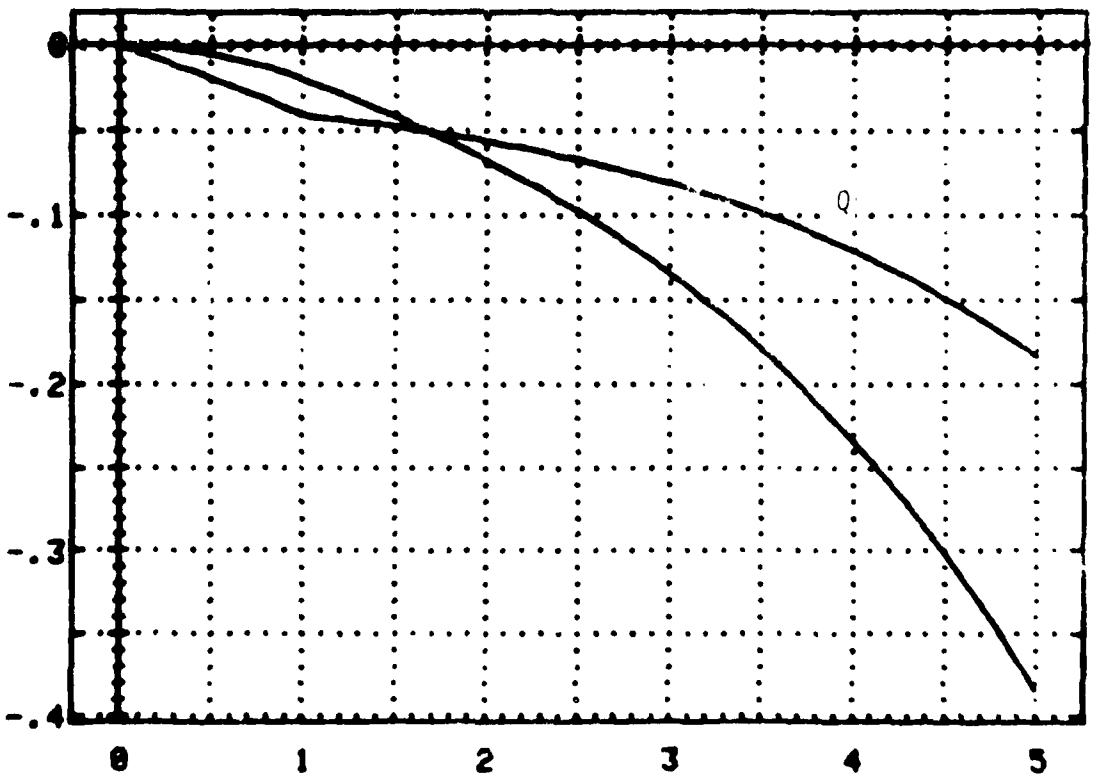


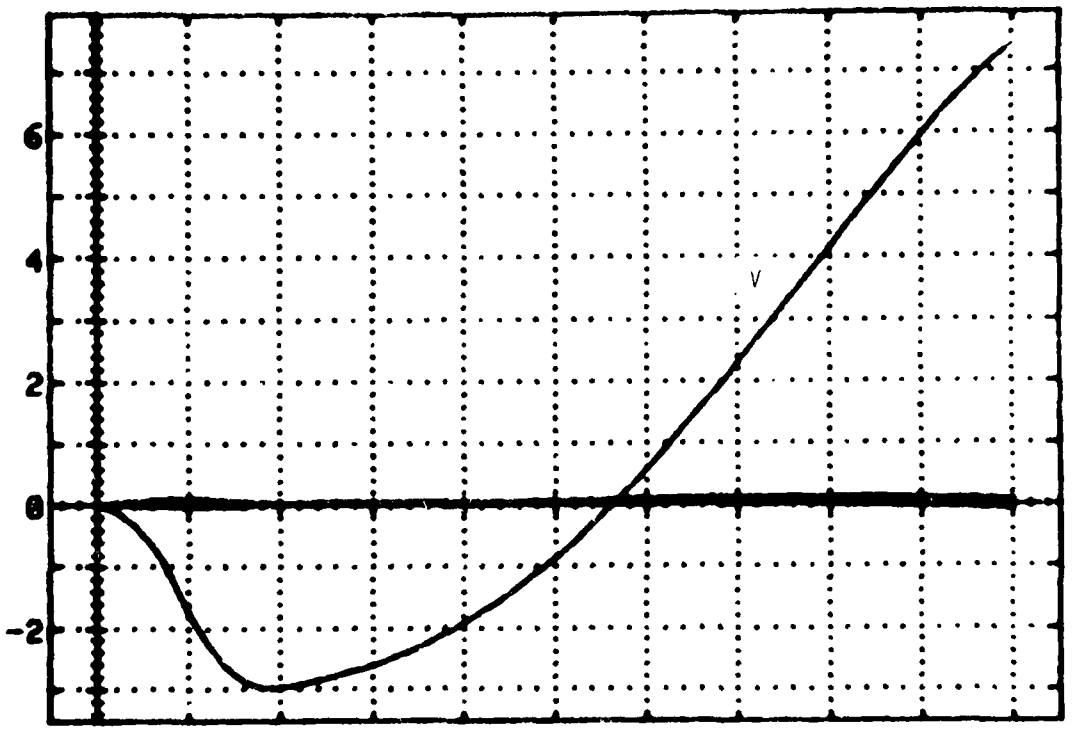
I.A) LONSTK = 1 - 2*STEP(.5) + STEP(1)



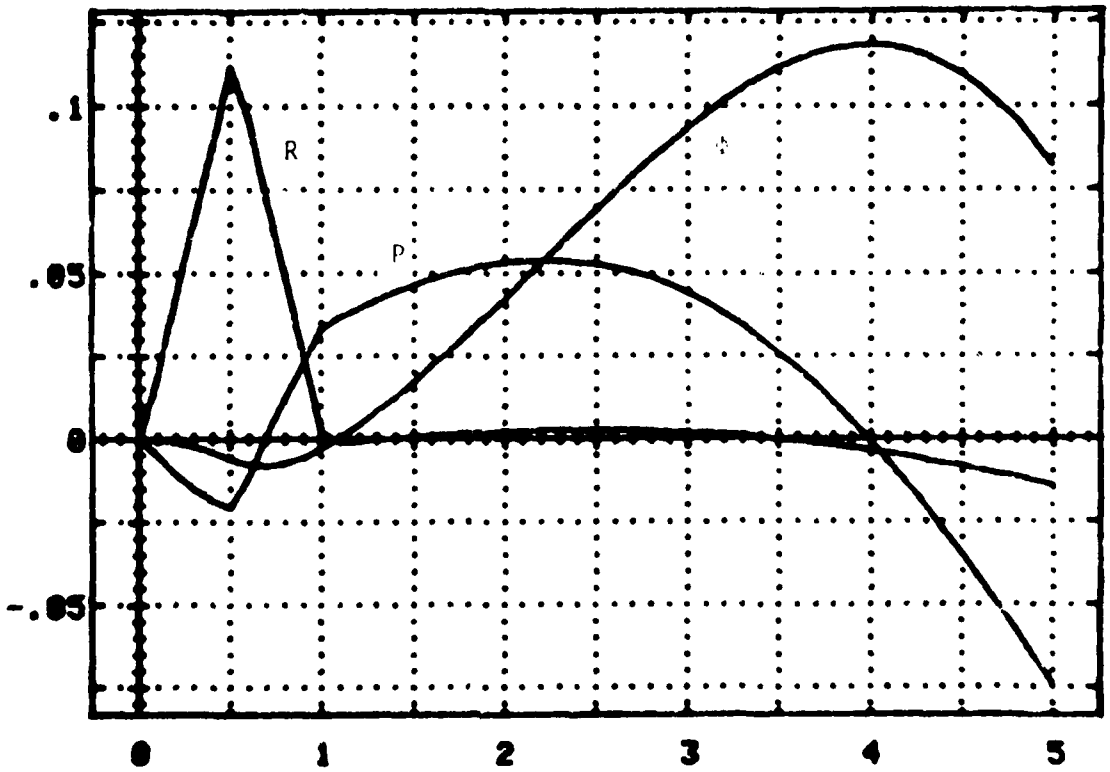


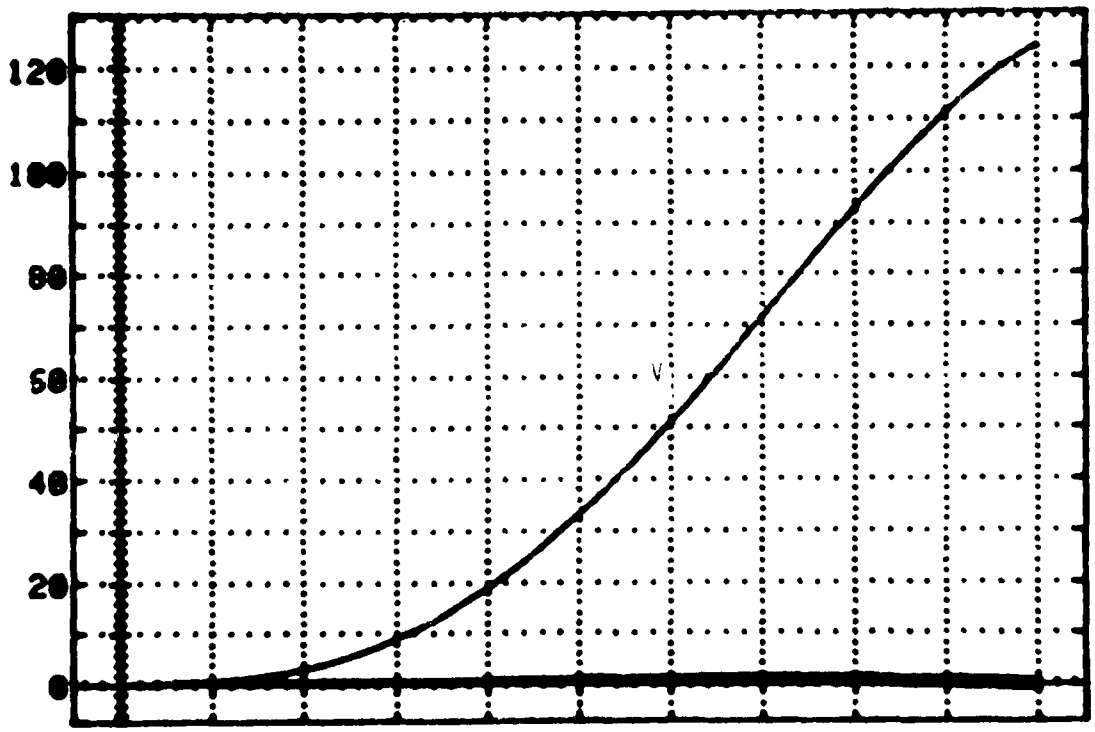
I.B) THROT = 1 - STEP(1)



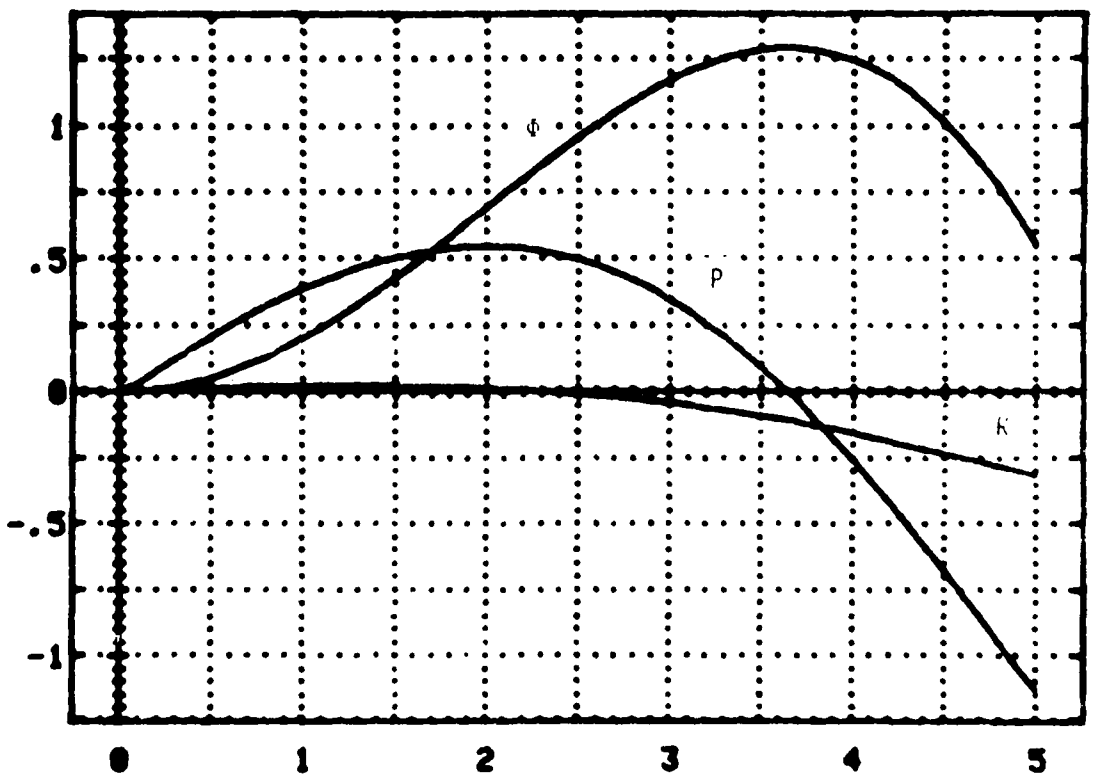


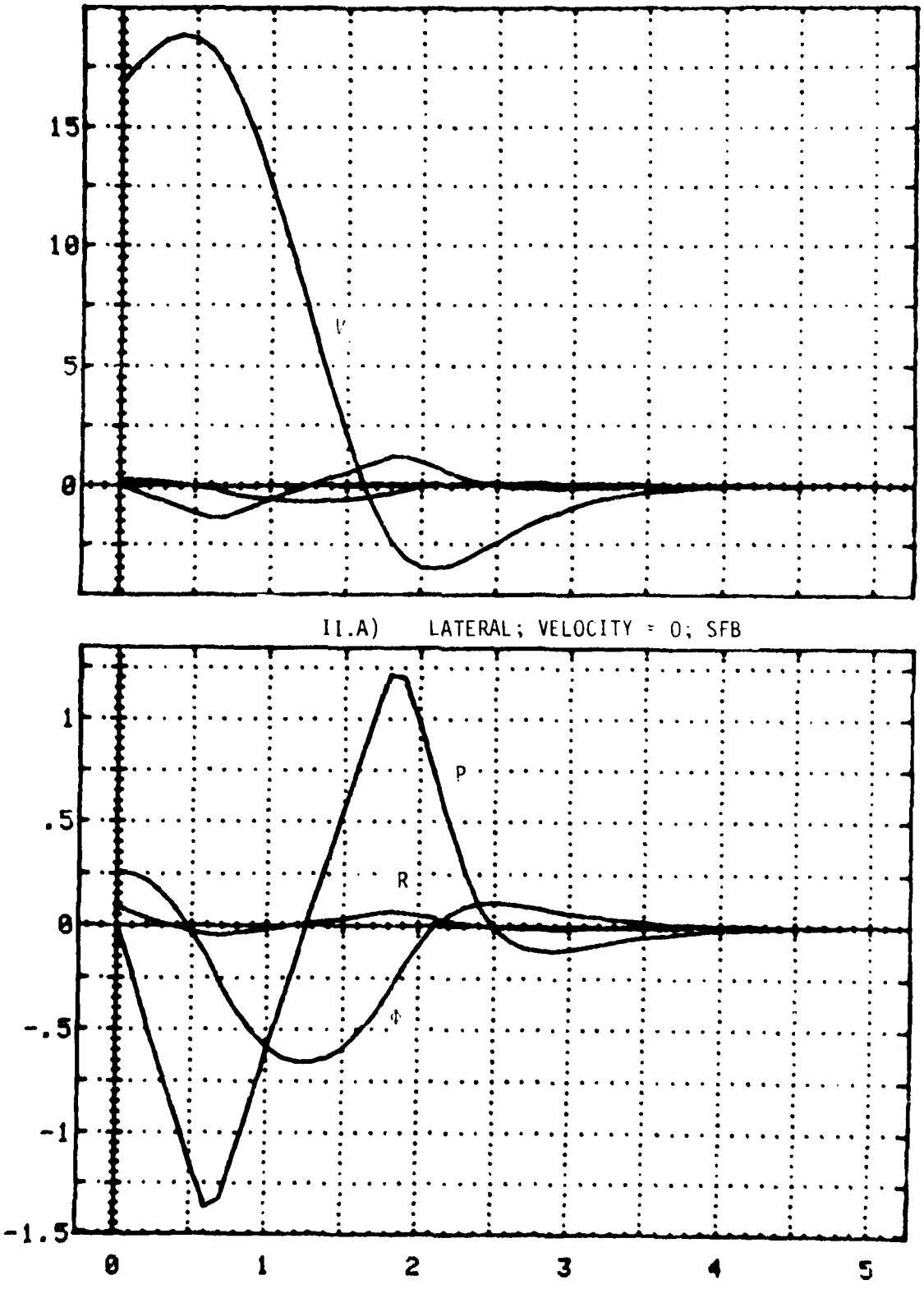
I.C) RUDPED = 1 - 2*STEP(.5) + STEP(1)



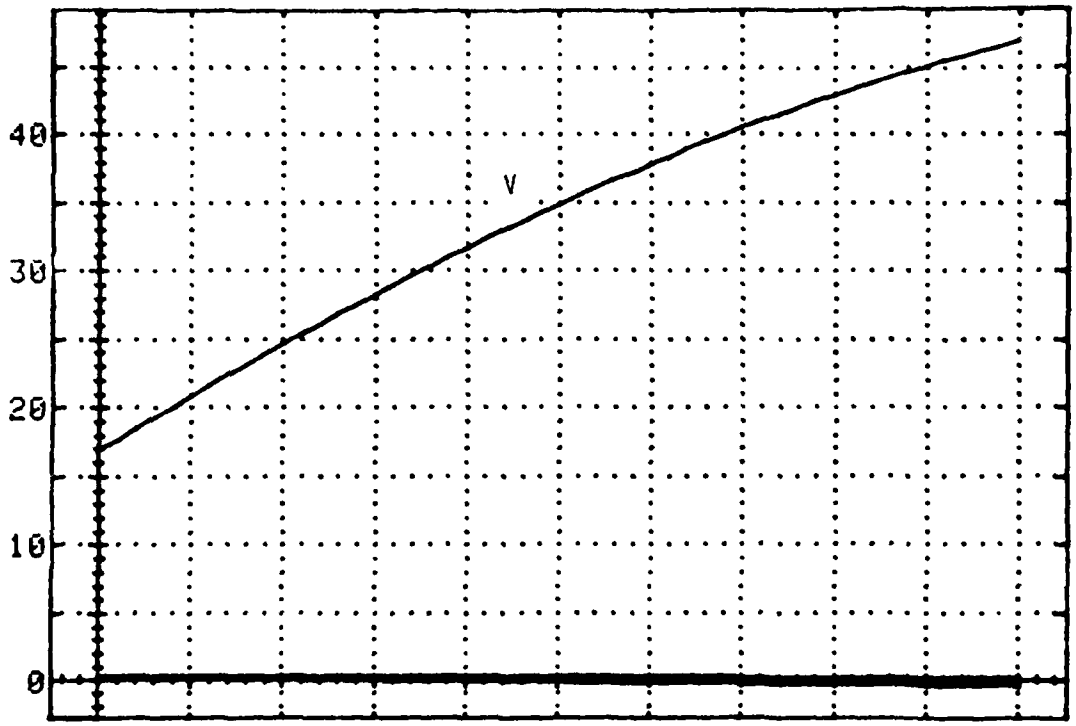


I.D) LATSTK = 1

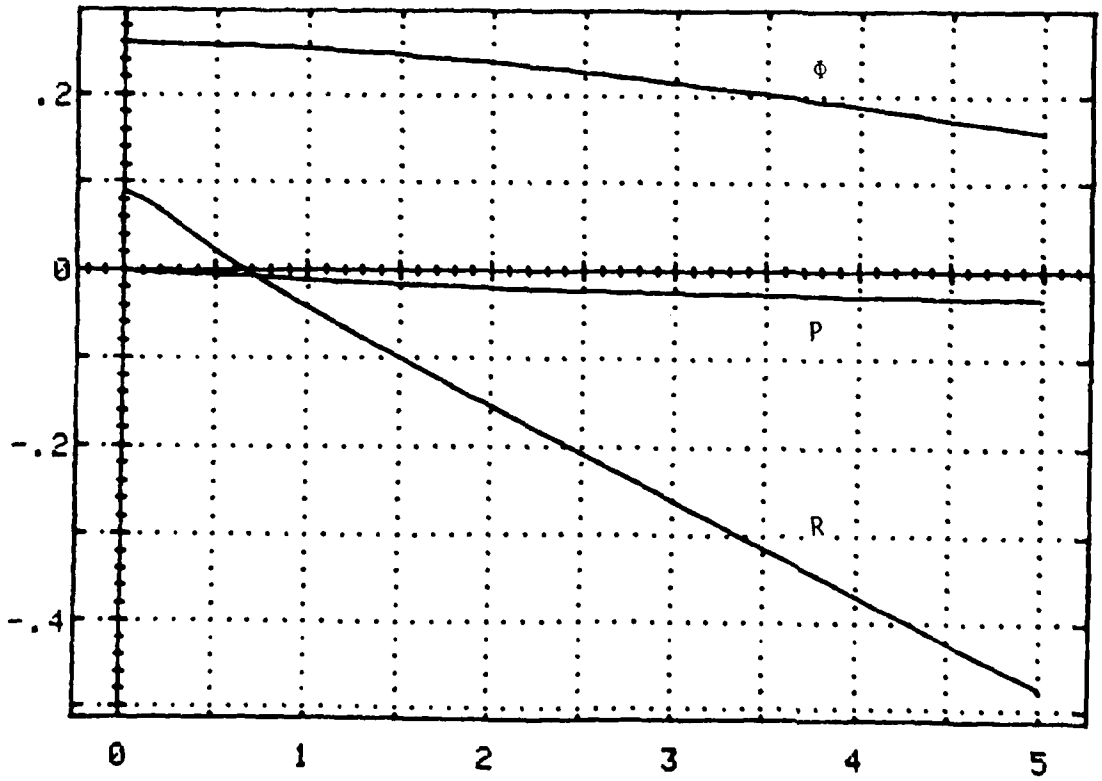


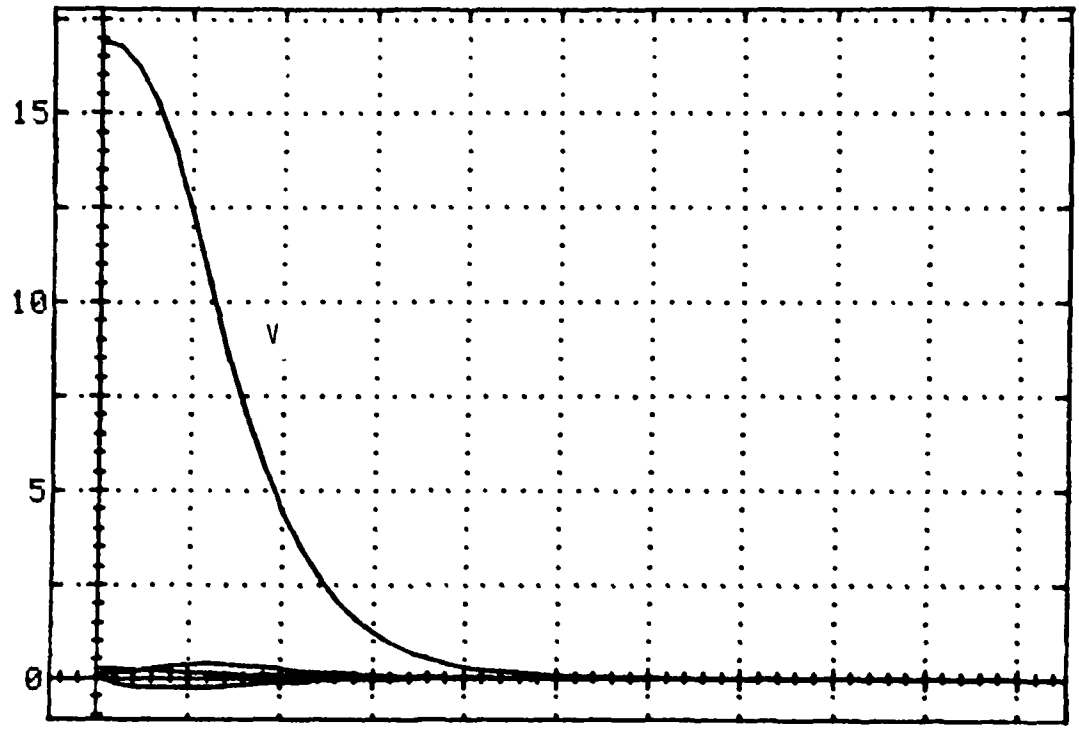


II.A) LATERAL; VELOCITY = 0; SFB

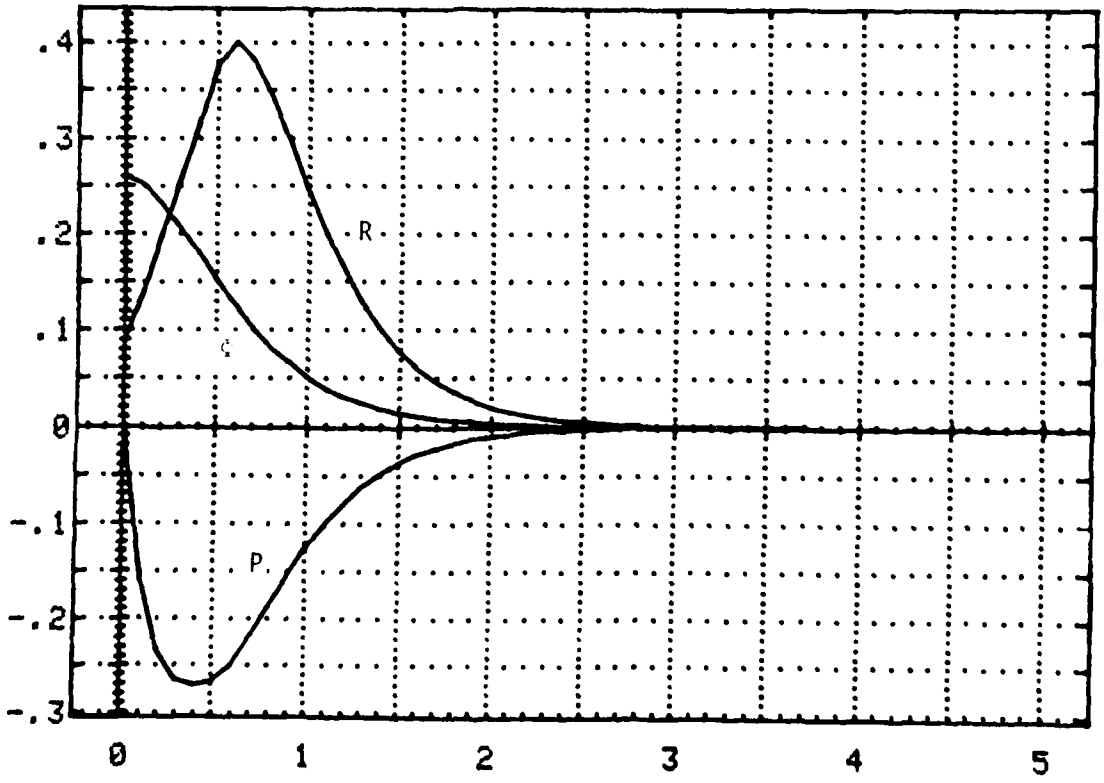


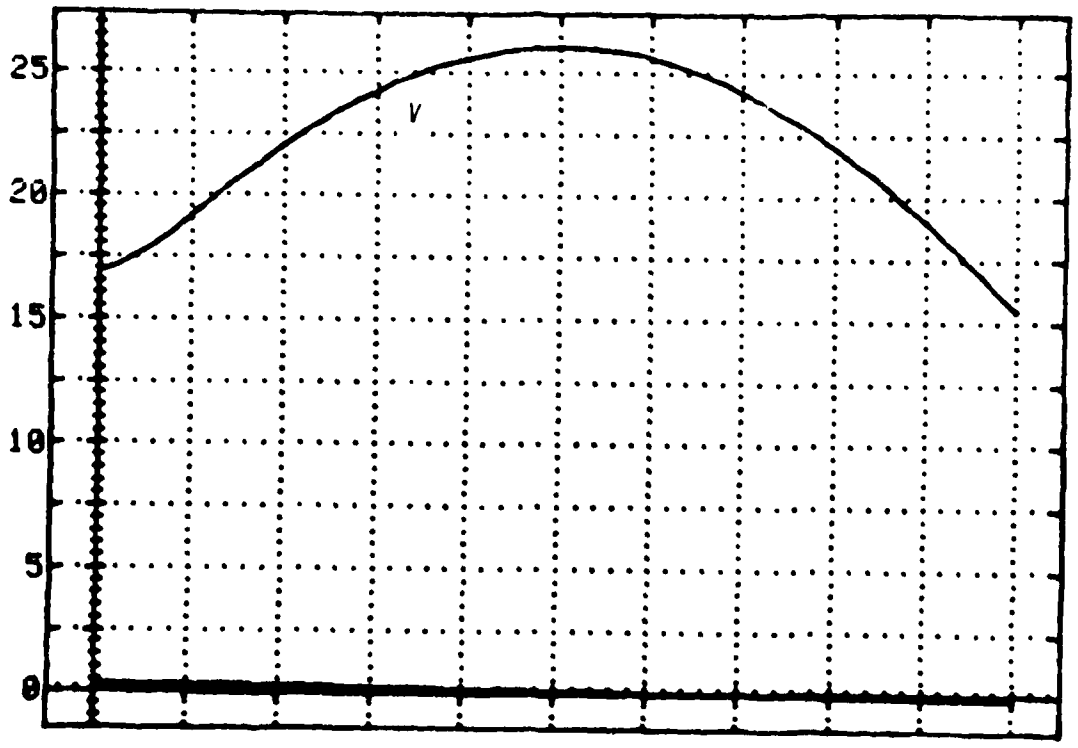
II.A) LATERAL; VELOCITY = 0; CONV



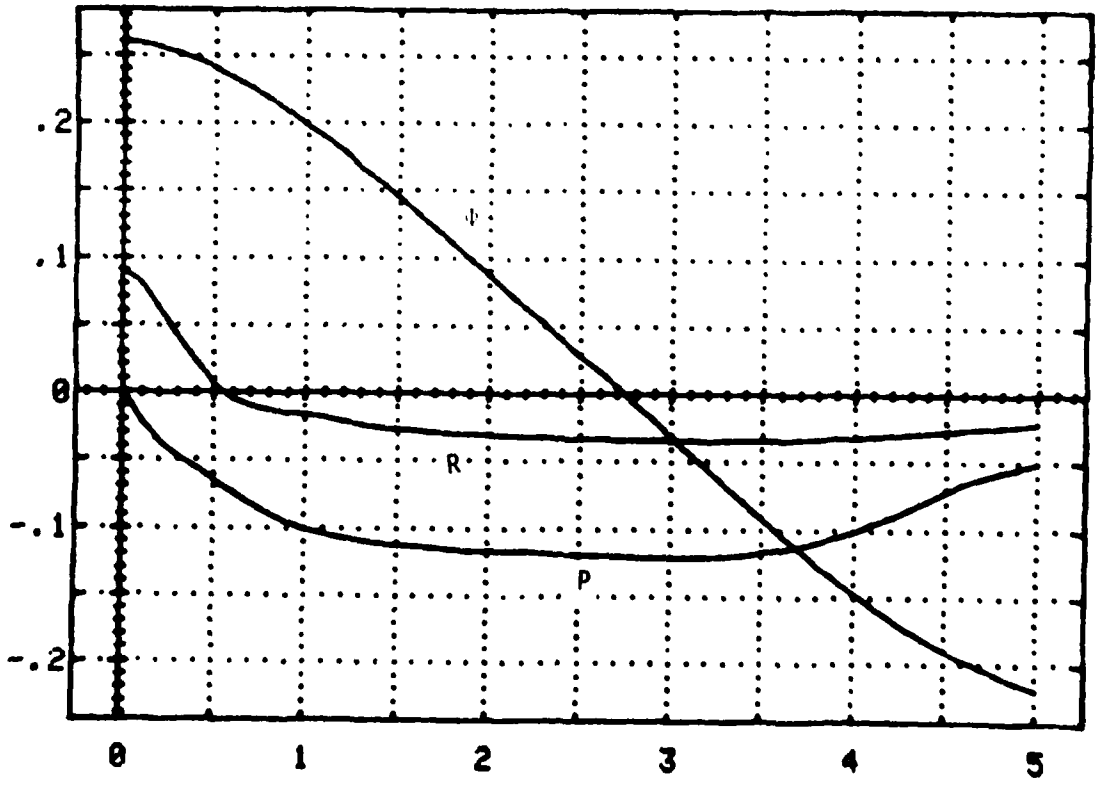


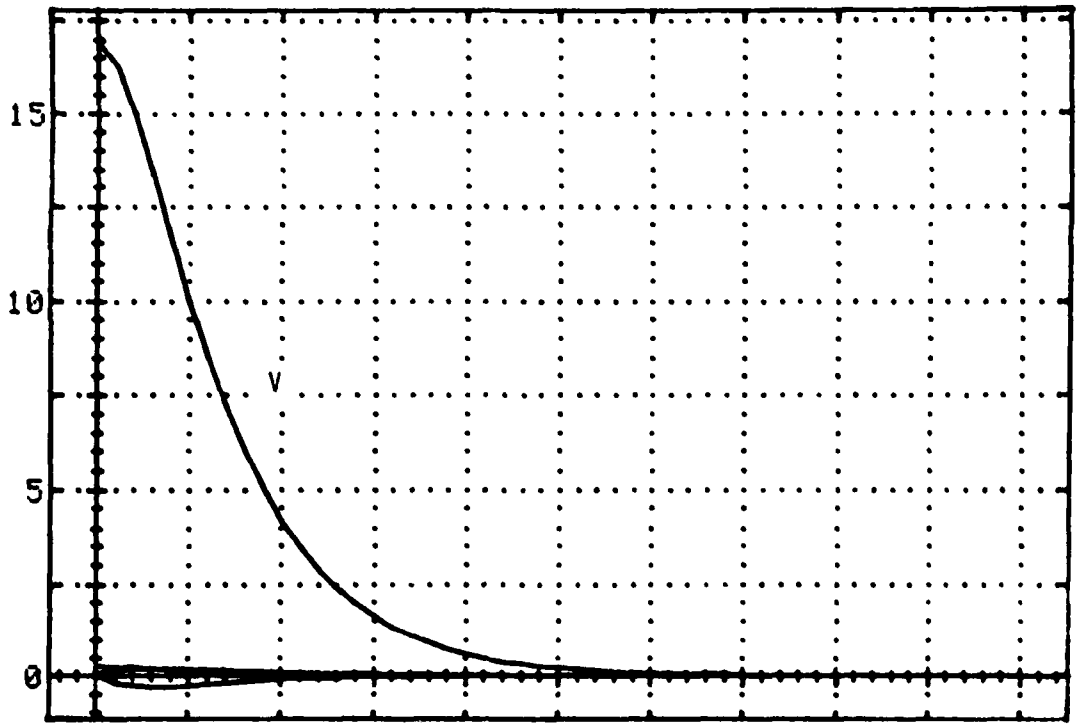
II.B) LATERAL; VELOCITY = 30; SFB



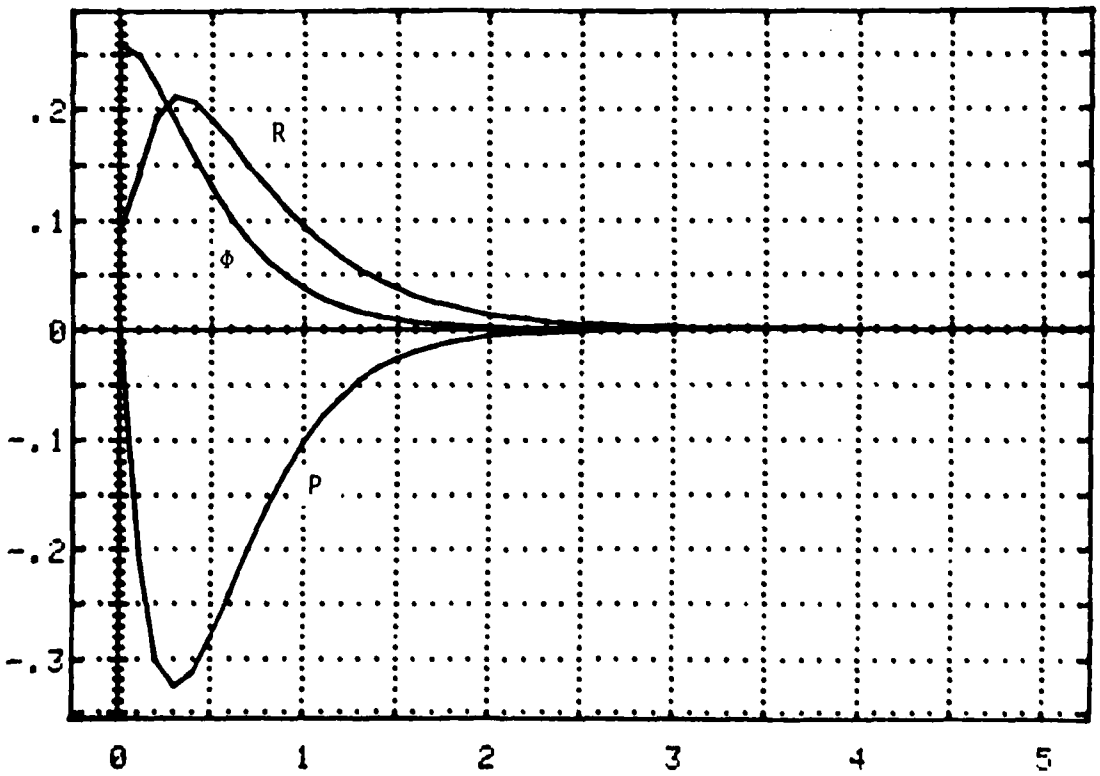


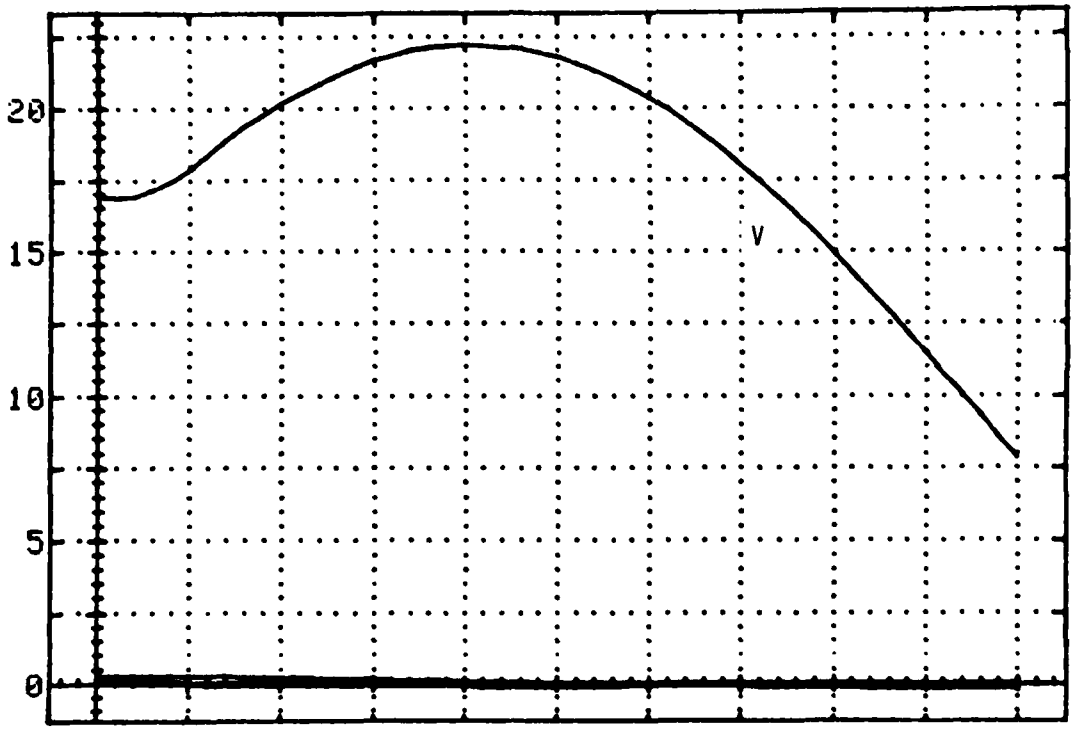
II.B) LATERAL; VELOCITY = 30; CONV



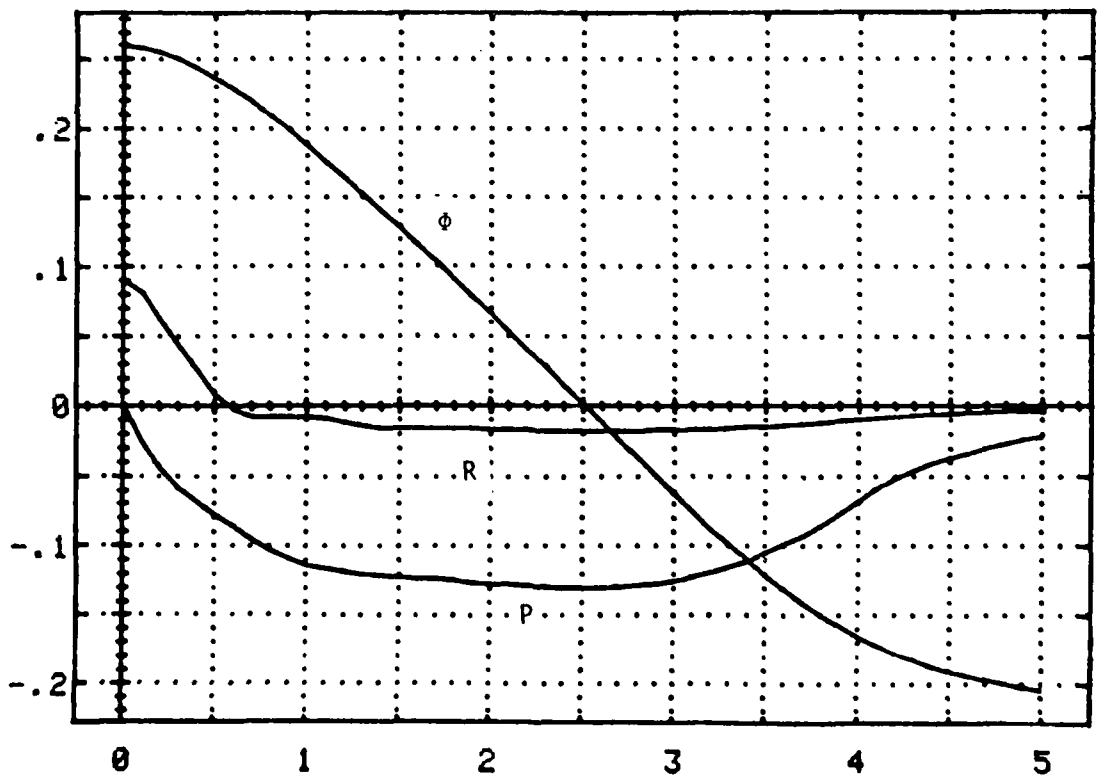


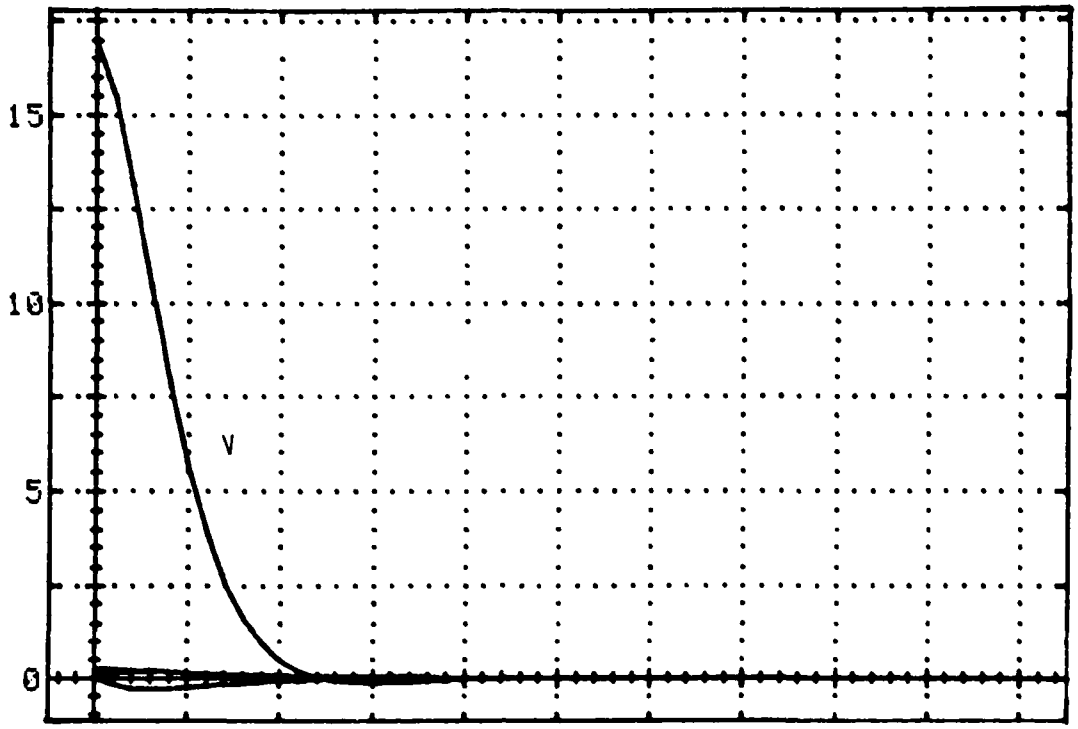
II.C) LATERAL; VELOCITY = 50; SFB



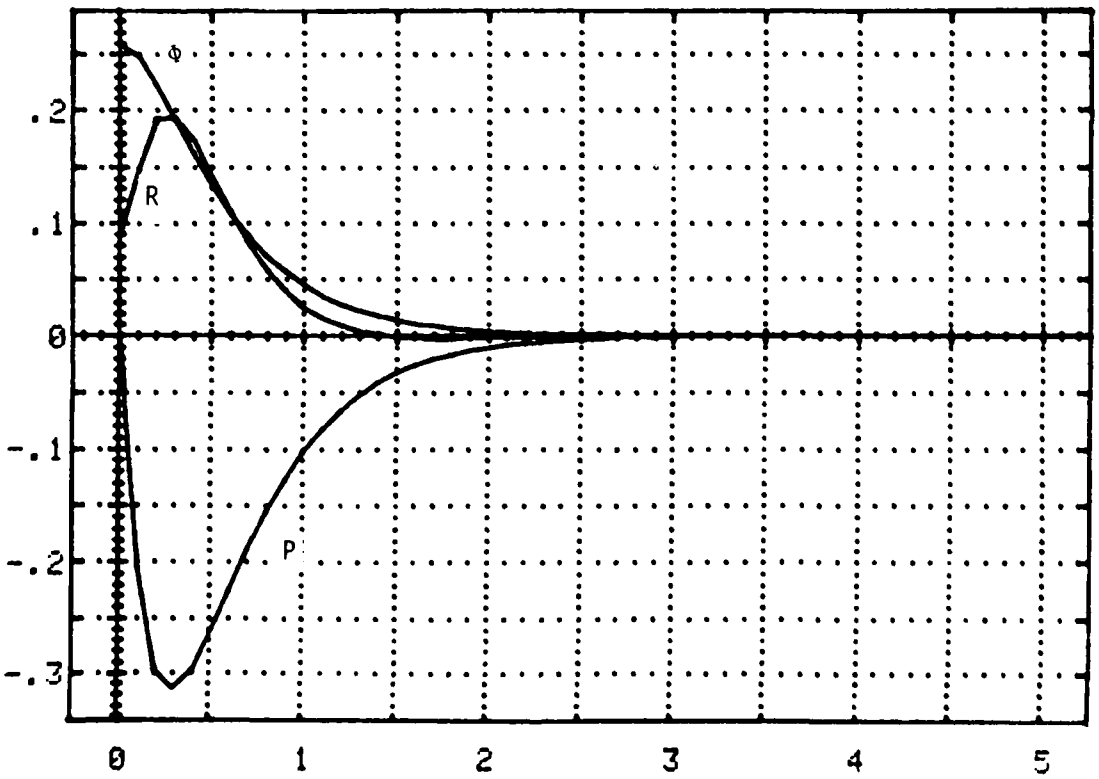


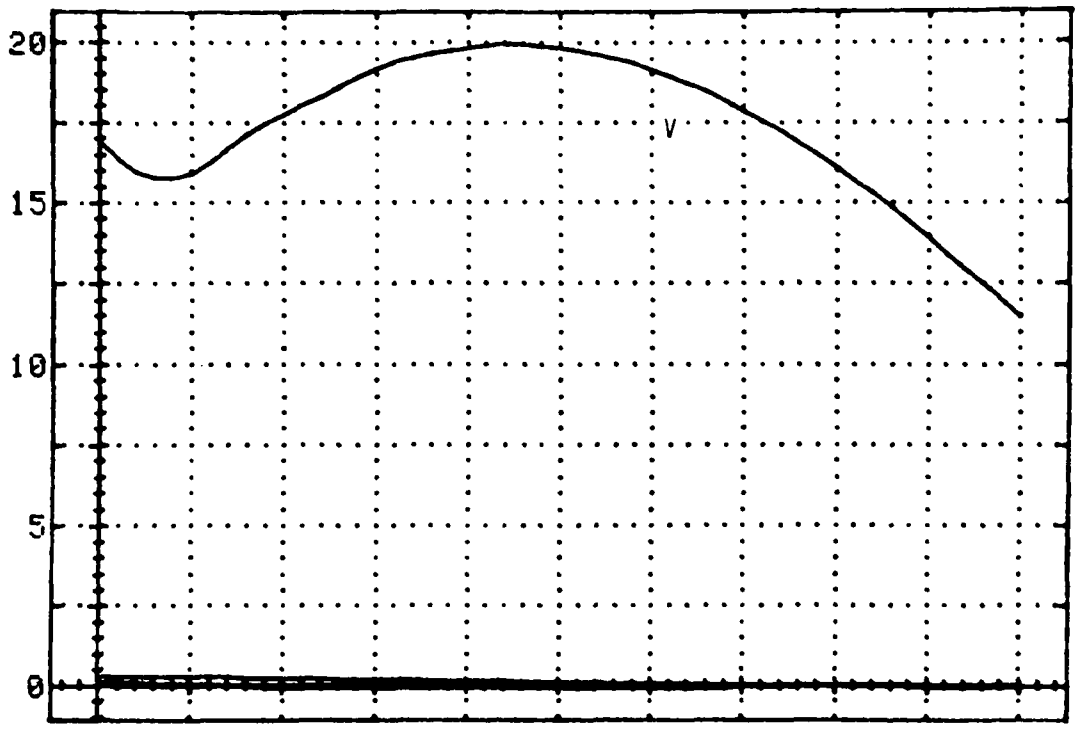
II.C) LATERAL; VELOCITY = 50; CONV



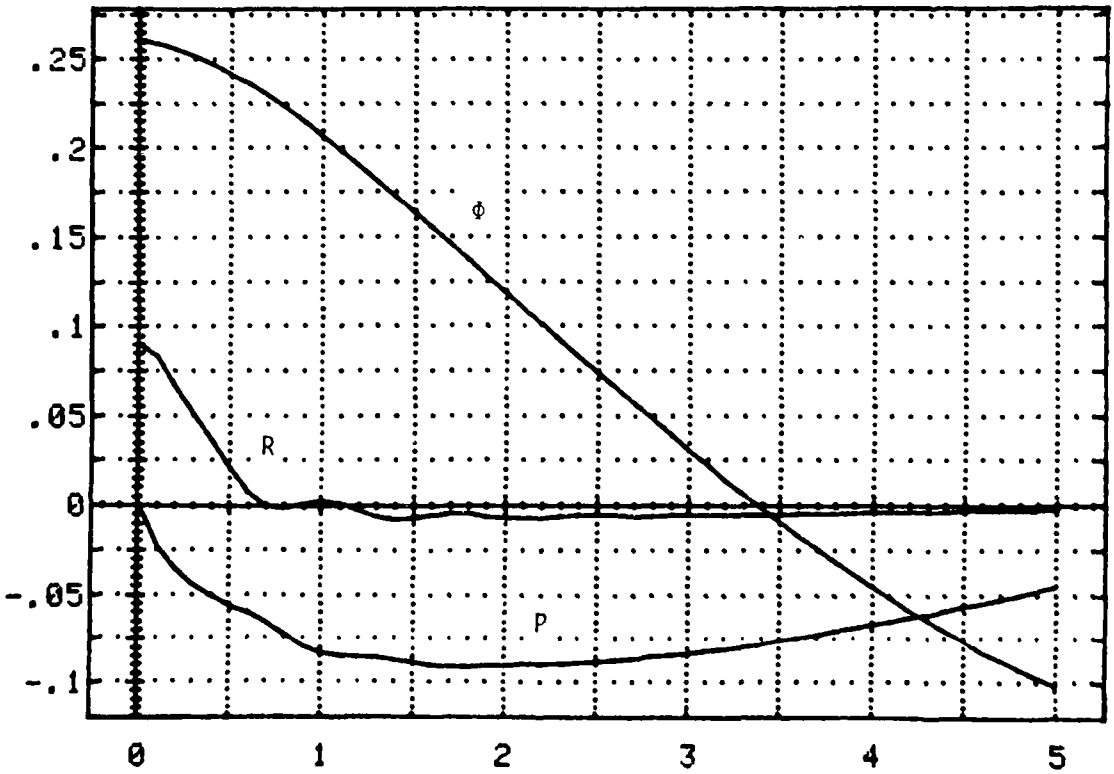


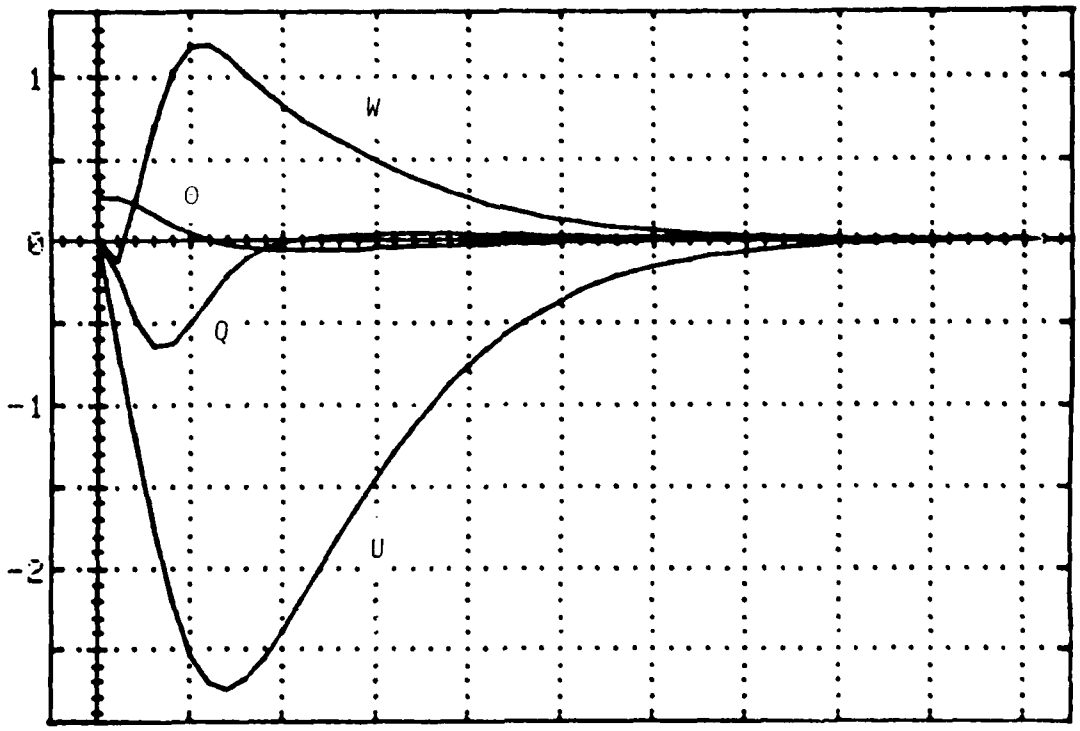
II.D) LATERAL; VELOCITY = 80; SFB



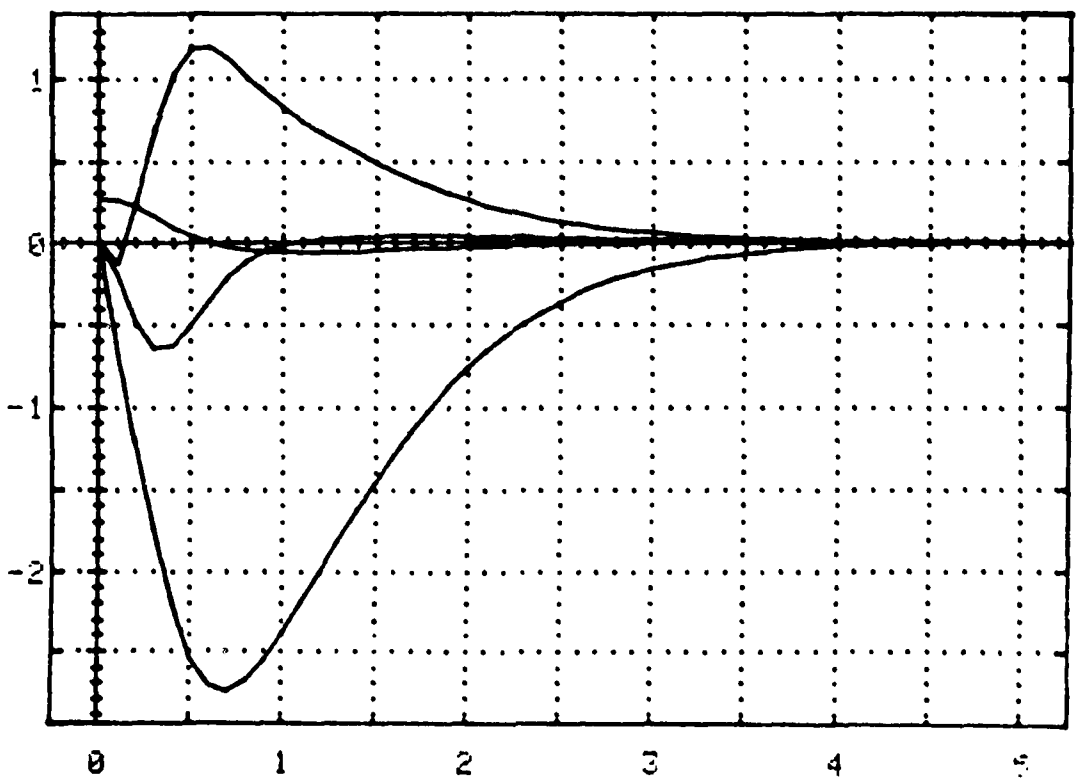


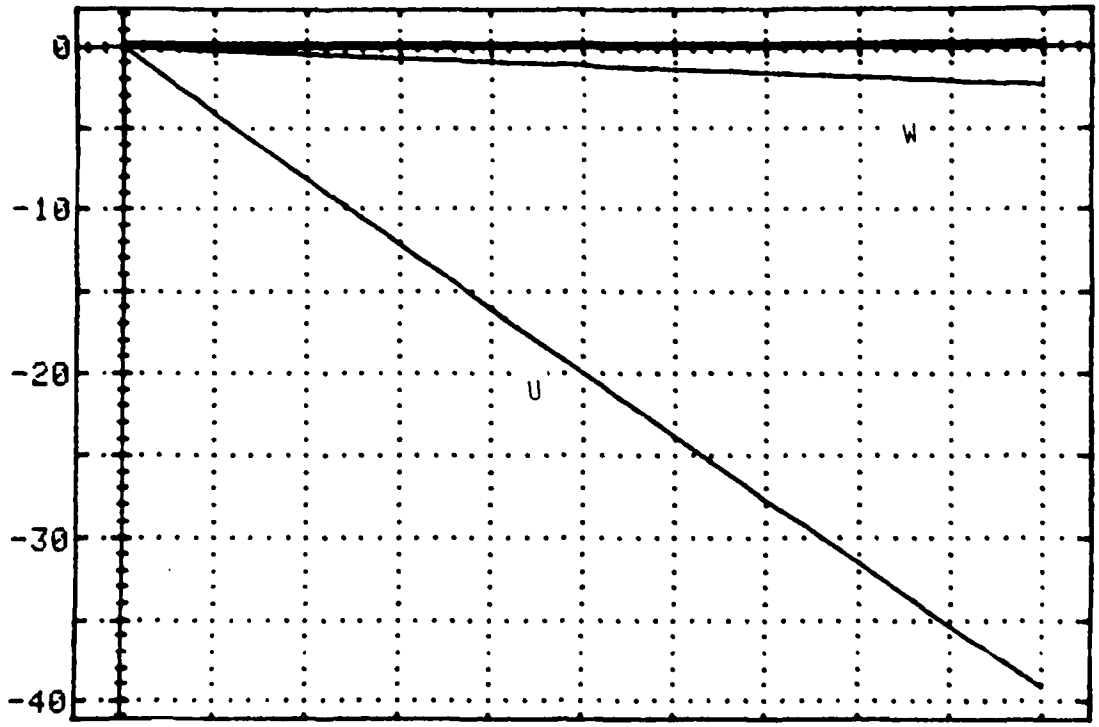
II.D) LATERAL; VELOCITY = 80; CONV



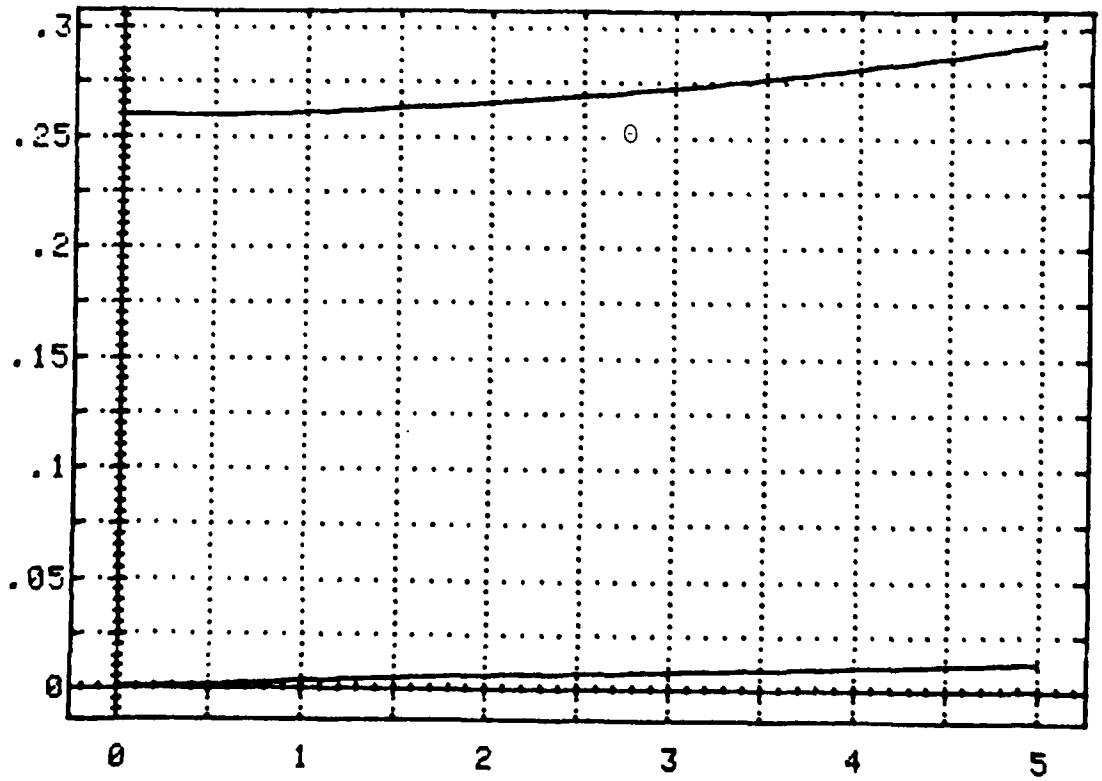


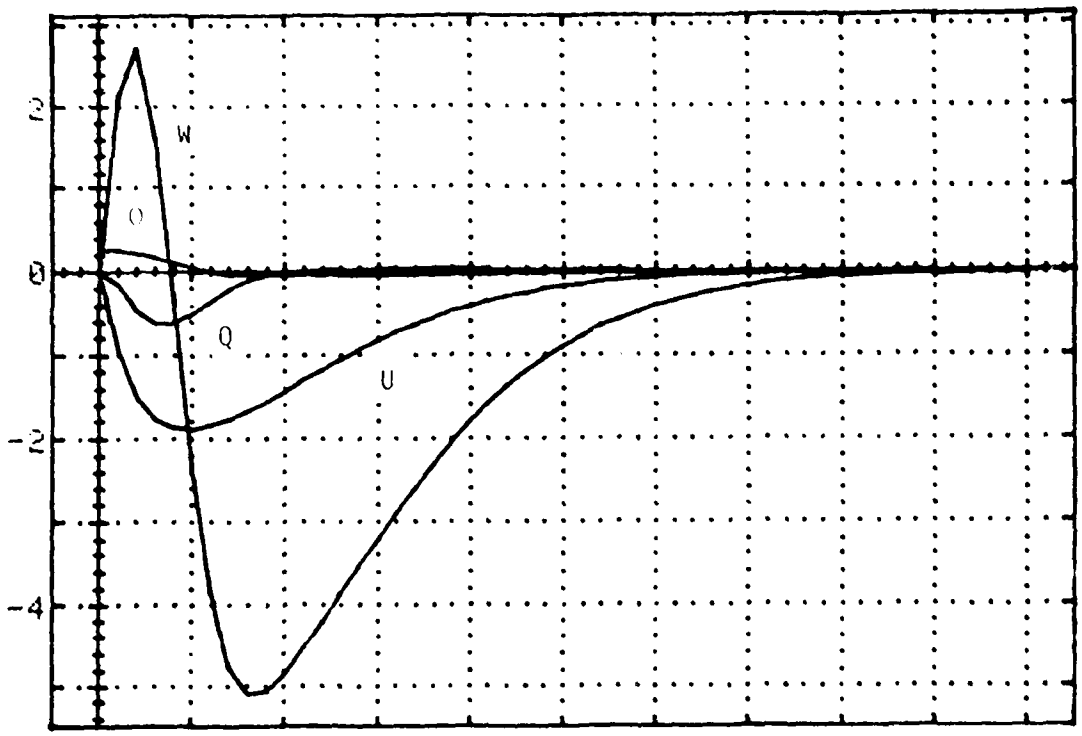
III.A) LONGITUDINAL; VELOCITY = 0; SFB



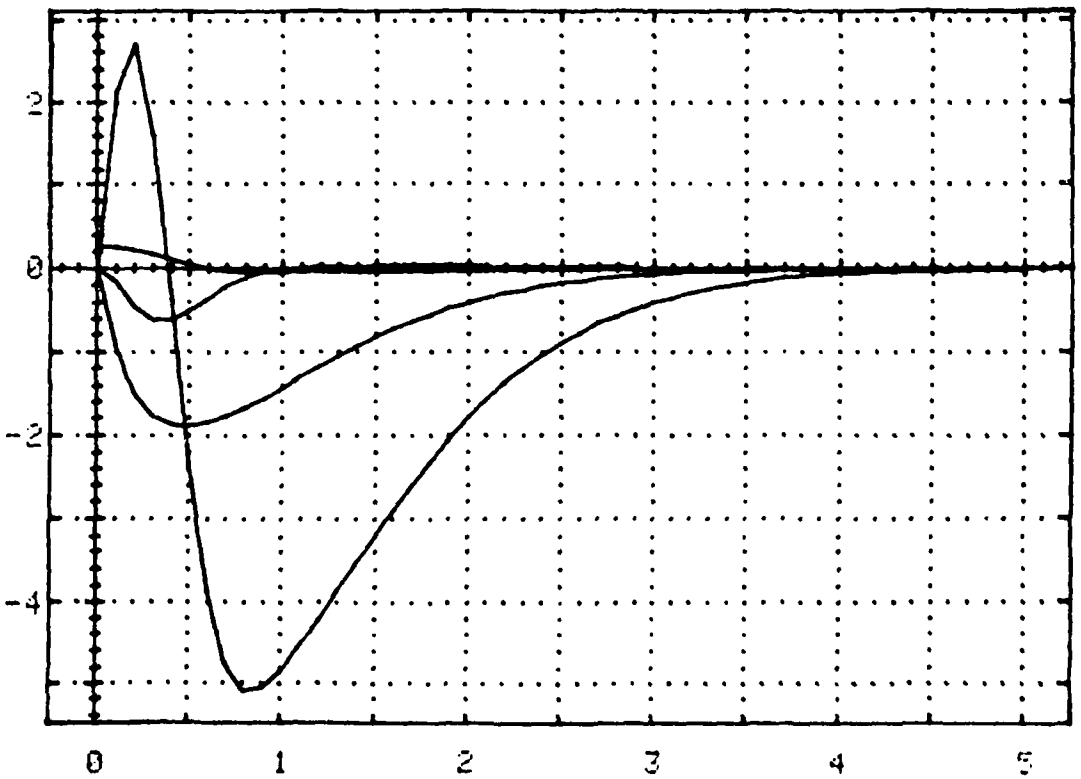


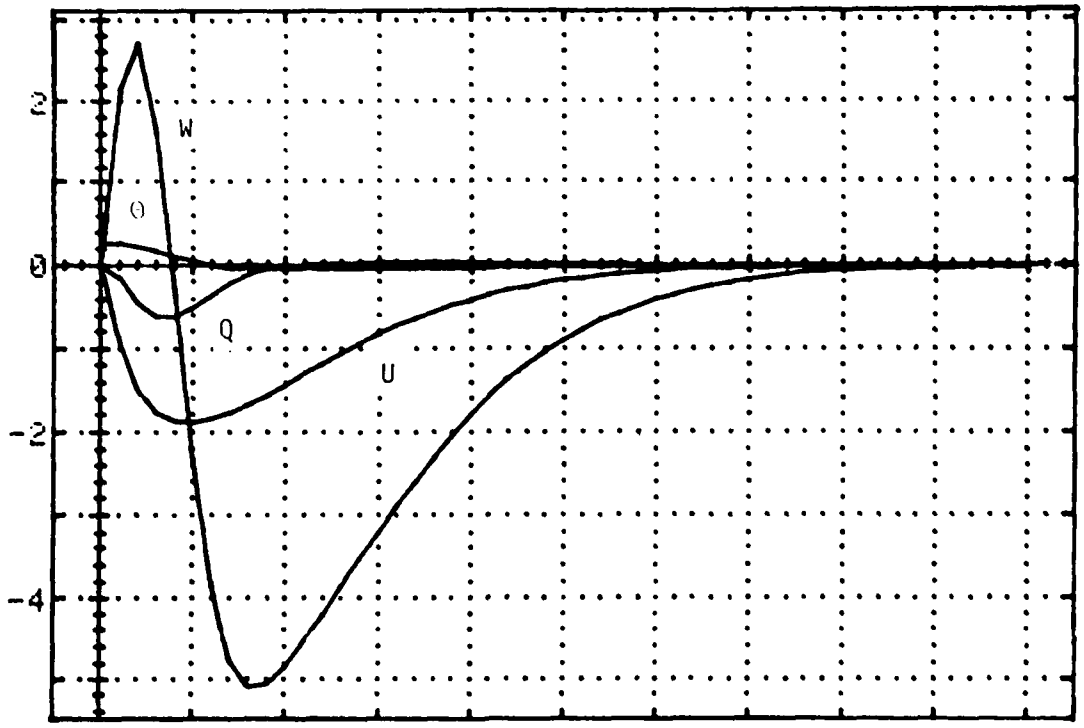
III.A) LONGITUDINAL; VELOCITY = 0; CONV



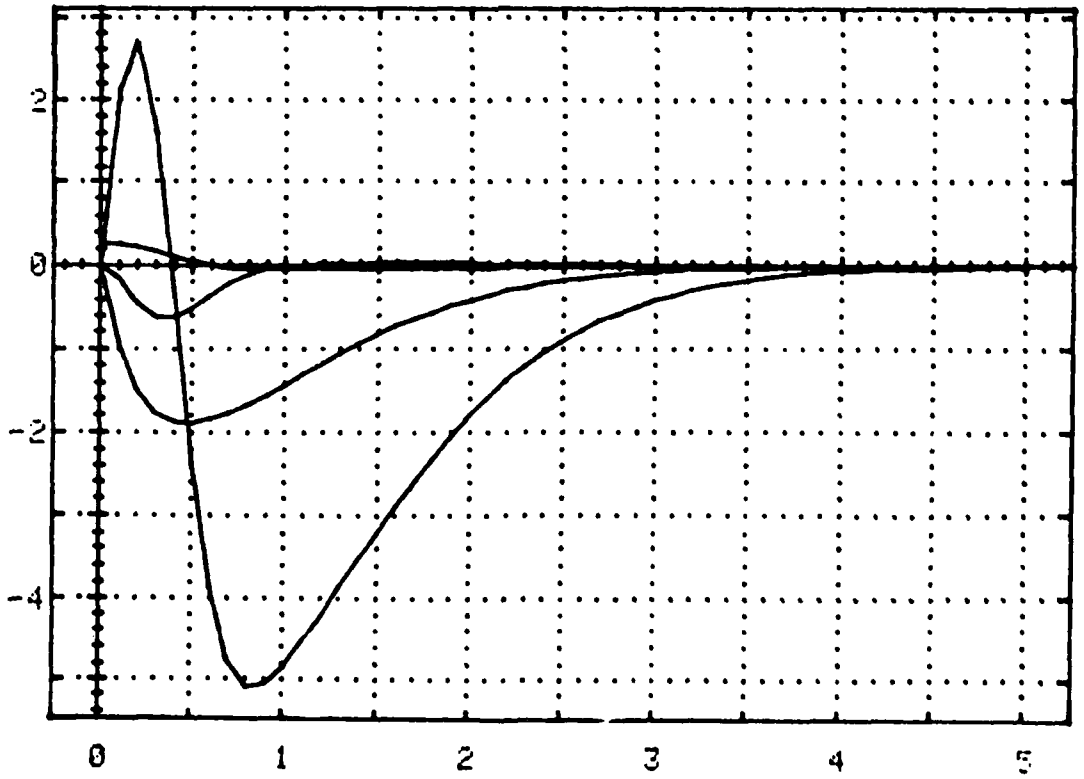


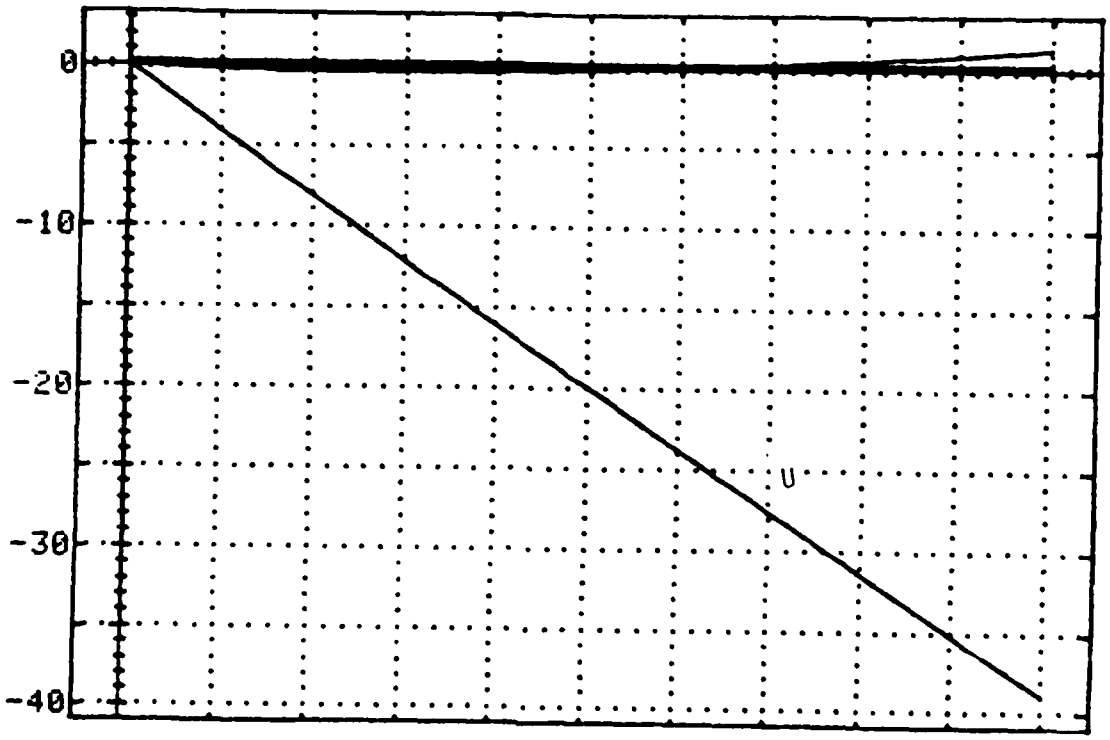
III.B) LONGITUDINAL; VELOCITY = 30; SFB



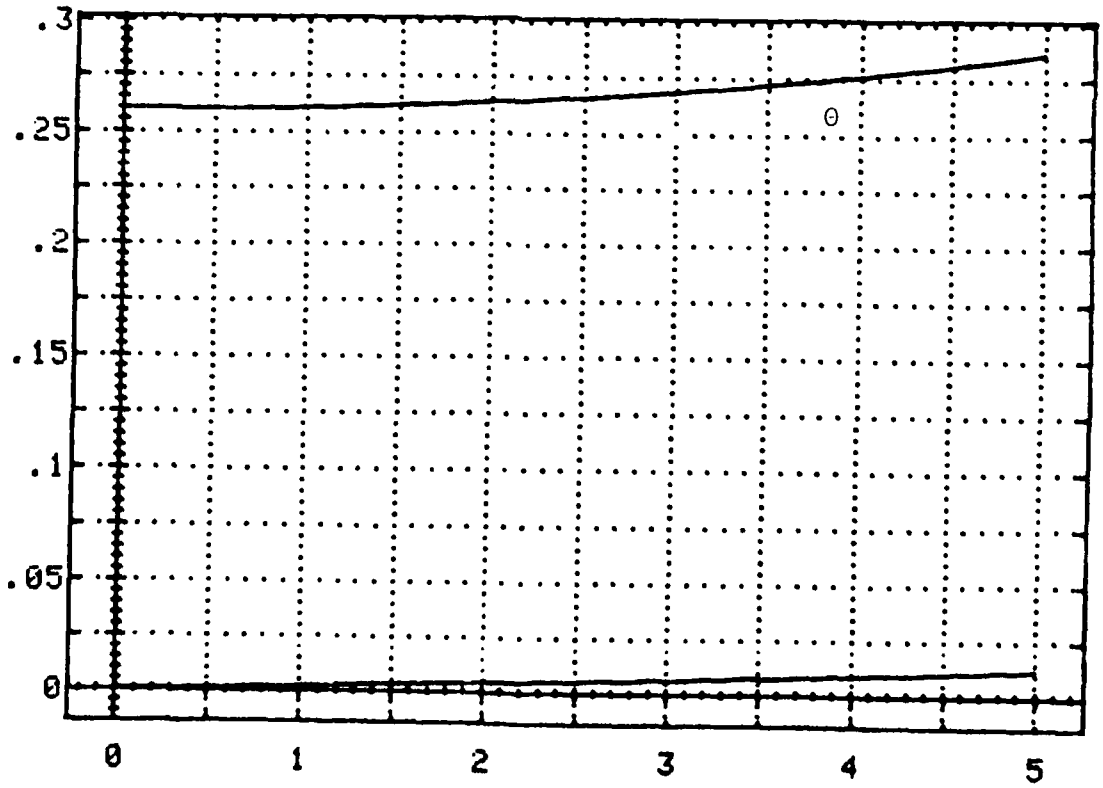


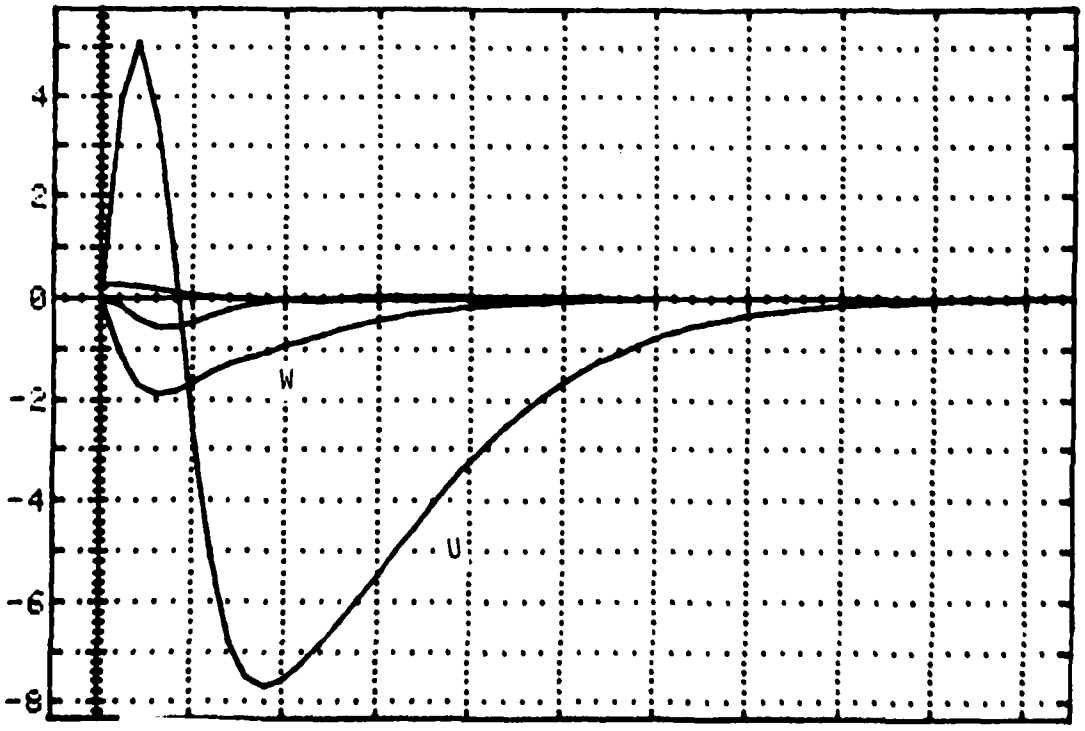
III.B) LONGITUDINAL; VELOCITY = 30; SFB



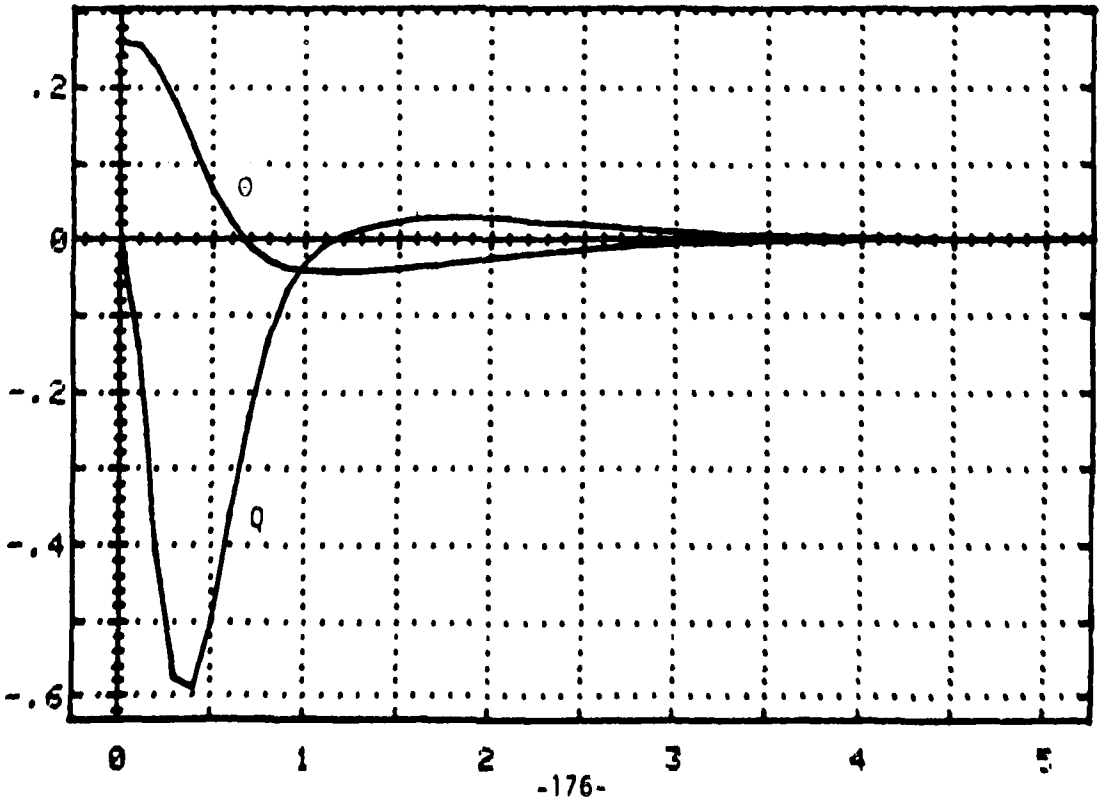


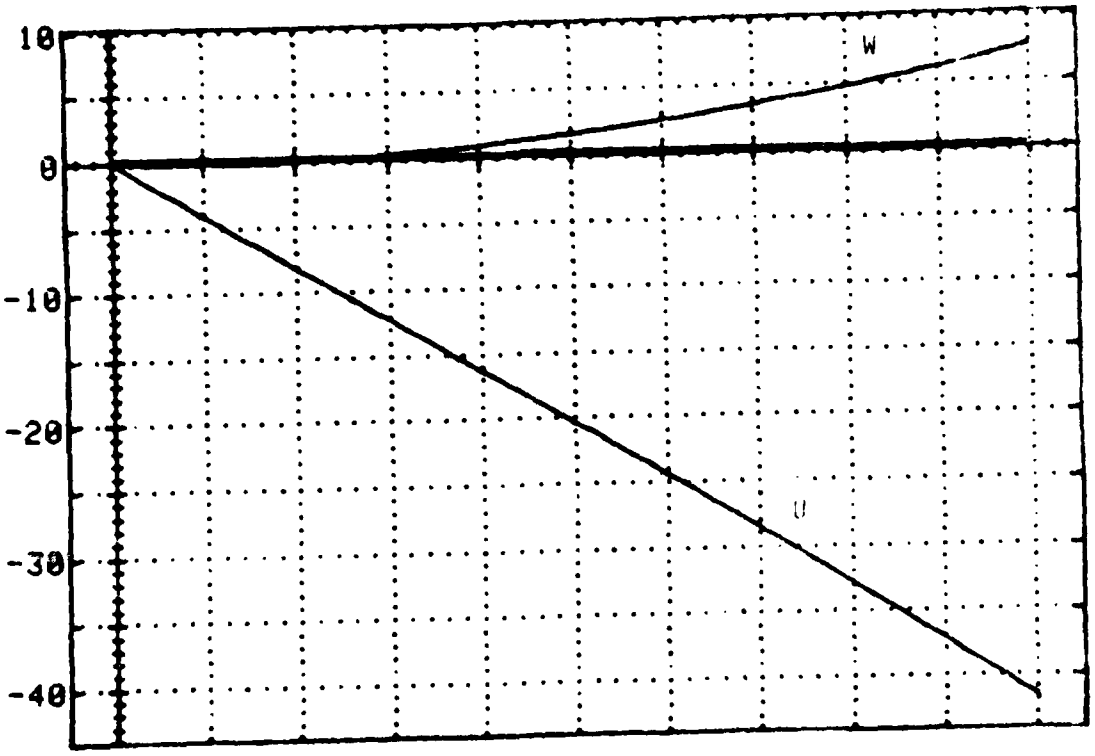
III.B) LONGITUDINAL; VELOCITY = 30; CONV



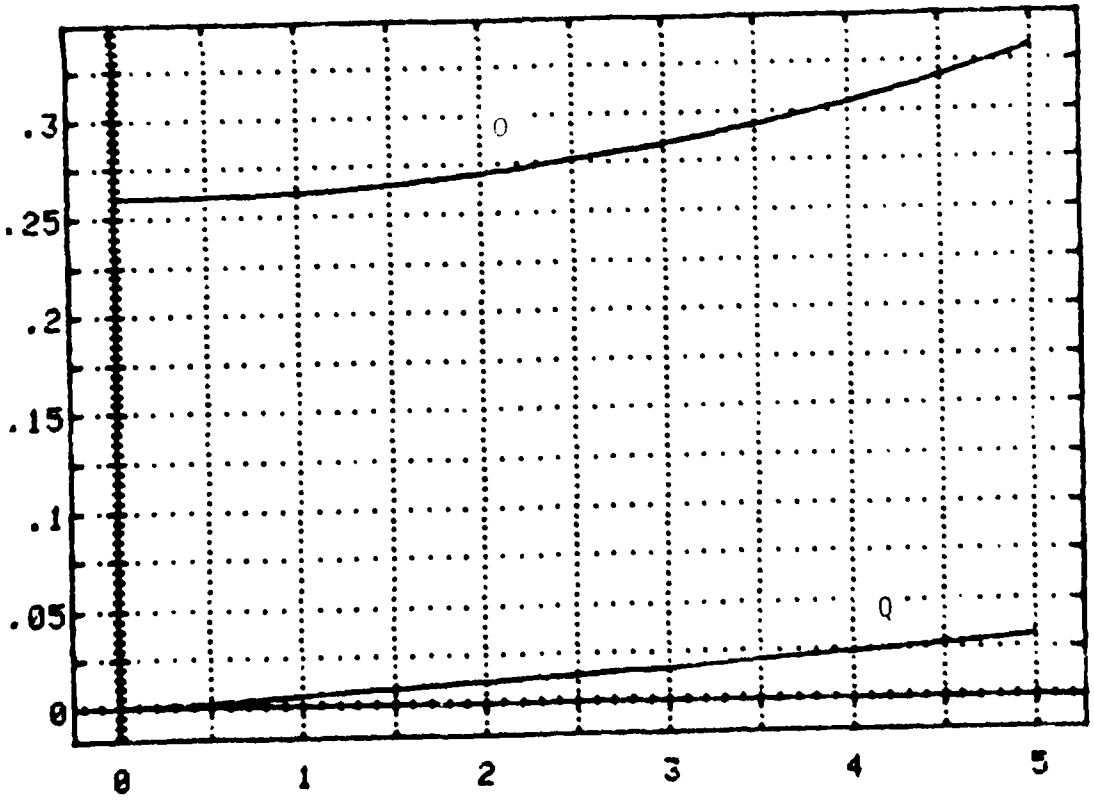


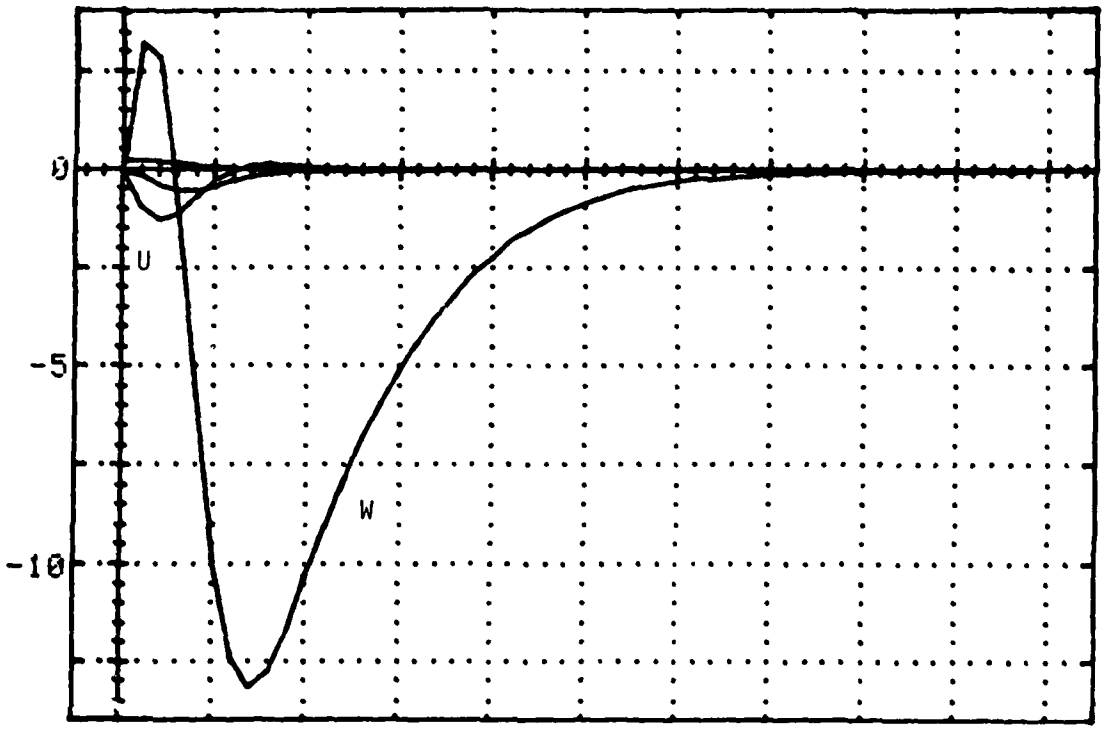
III.C) LONGITUDINAL; VELOCITY = 50; SFB



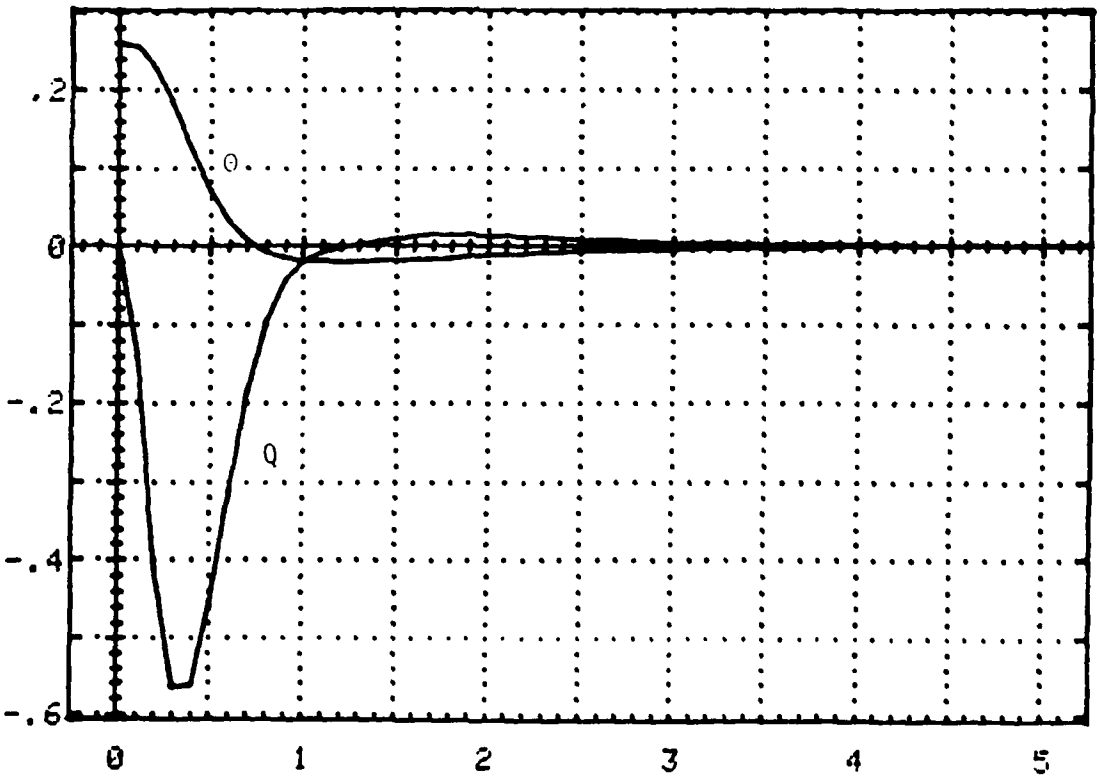


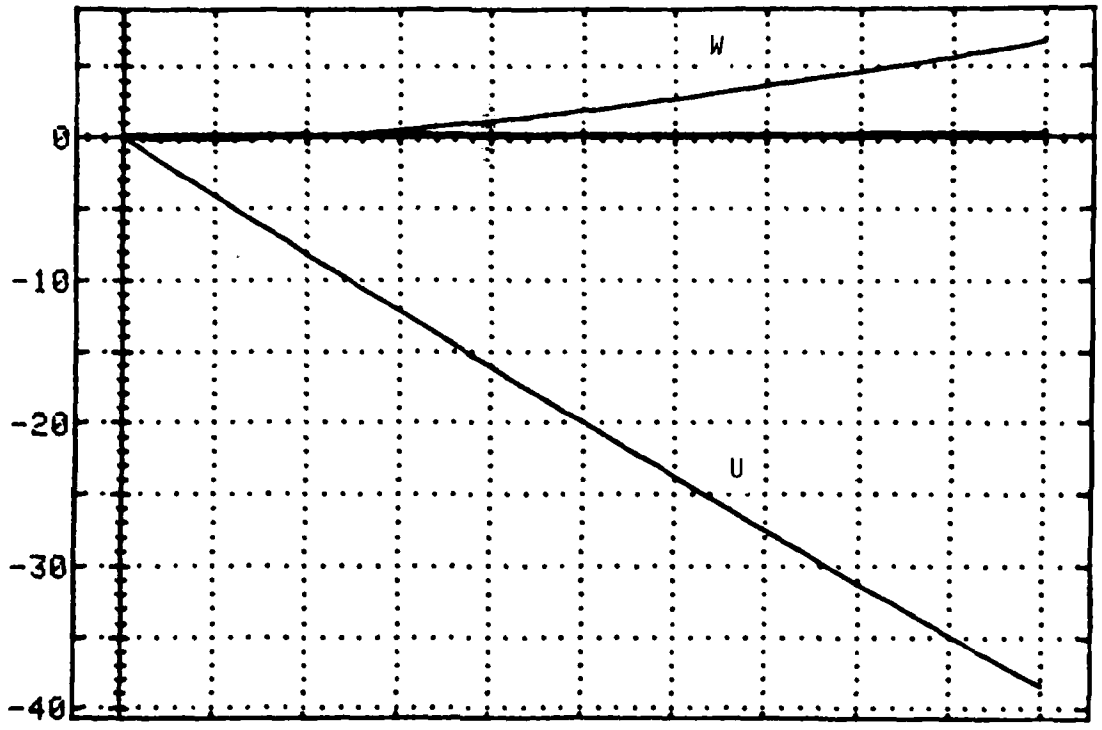
III.C) LONGITUDINAL; VELOCITY = 50; CONV



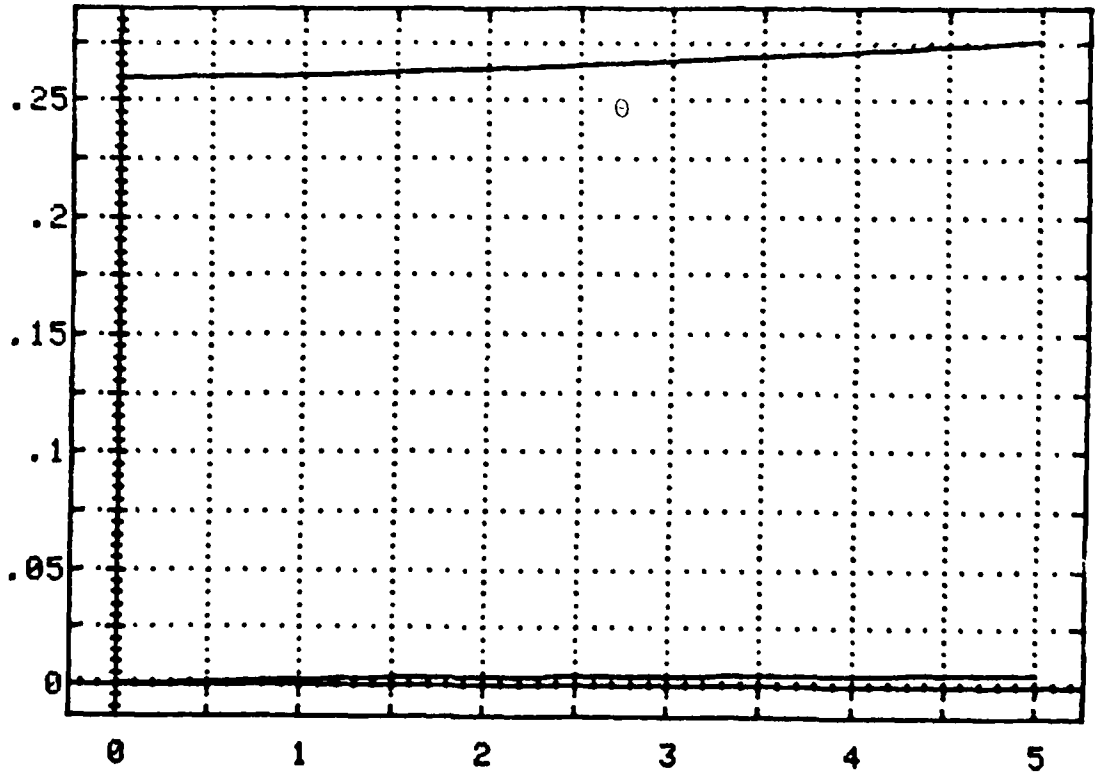


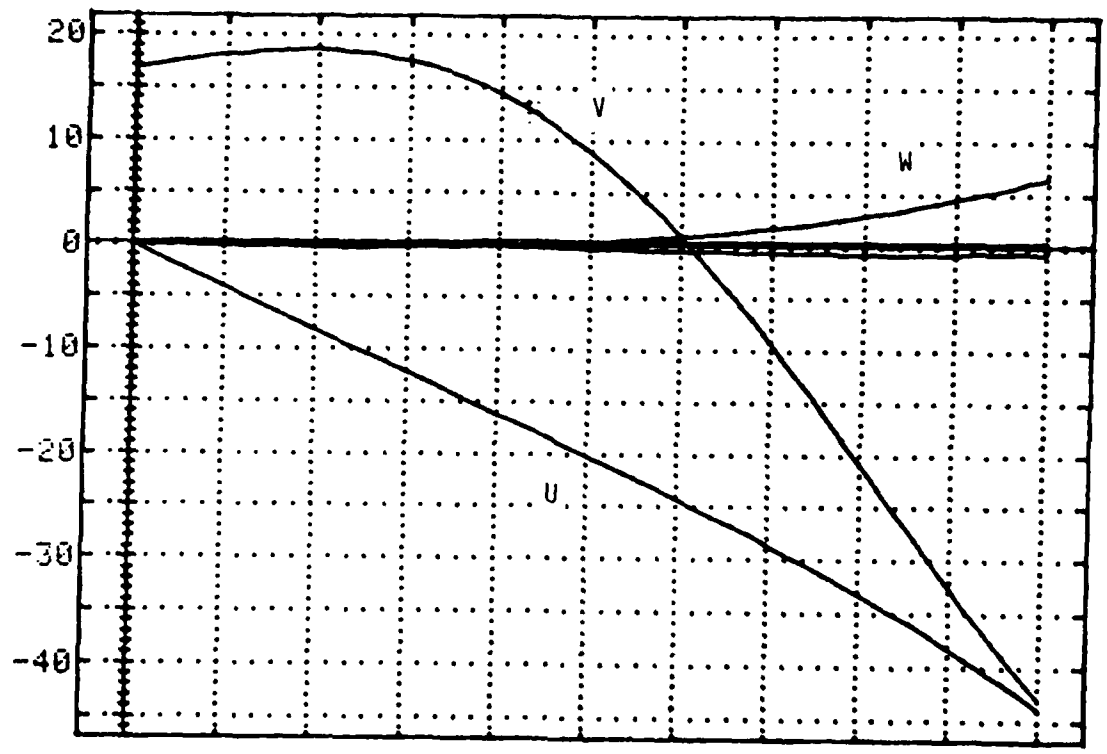
III.D) LONGITUDINAL; VELOCITY = 80; SFB



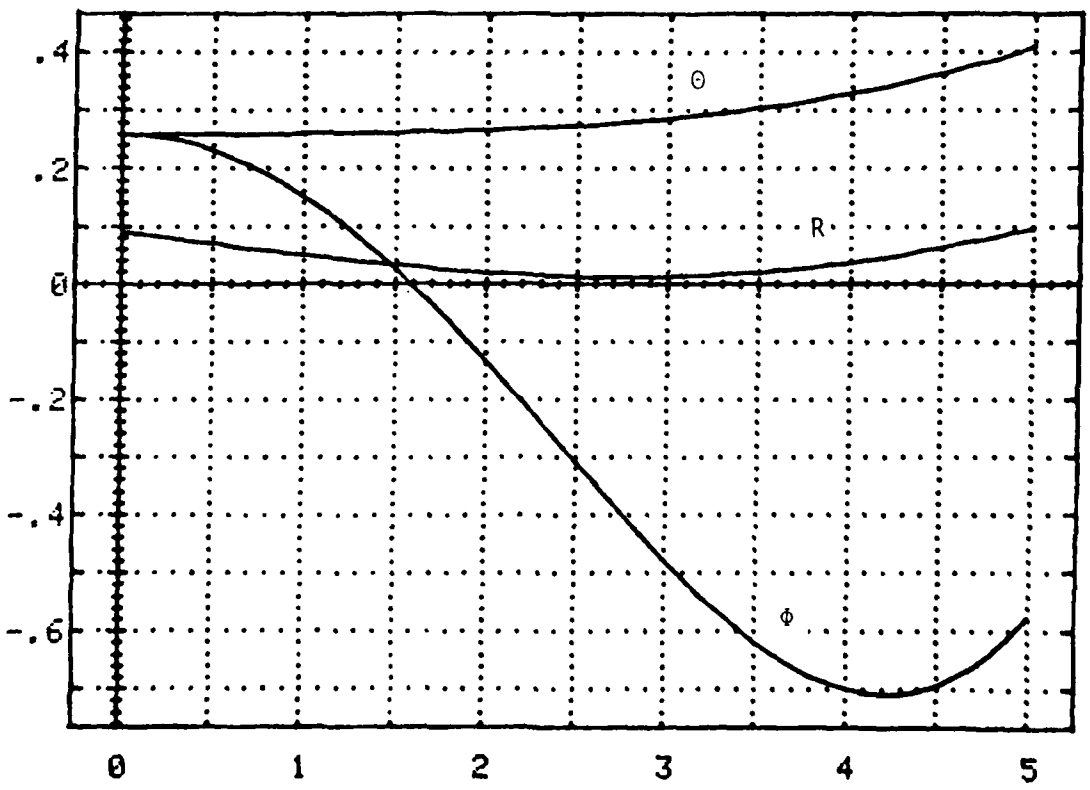


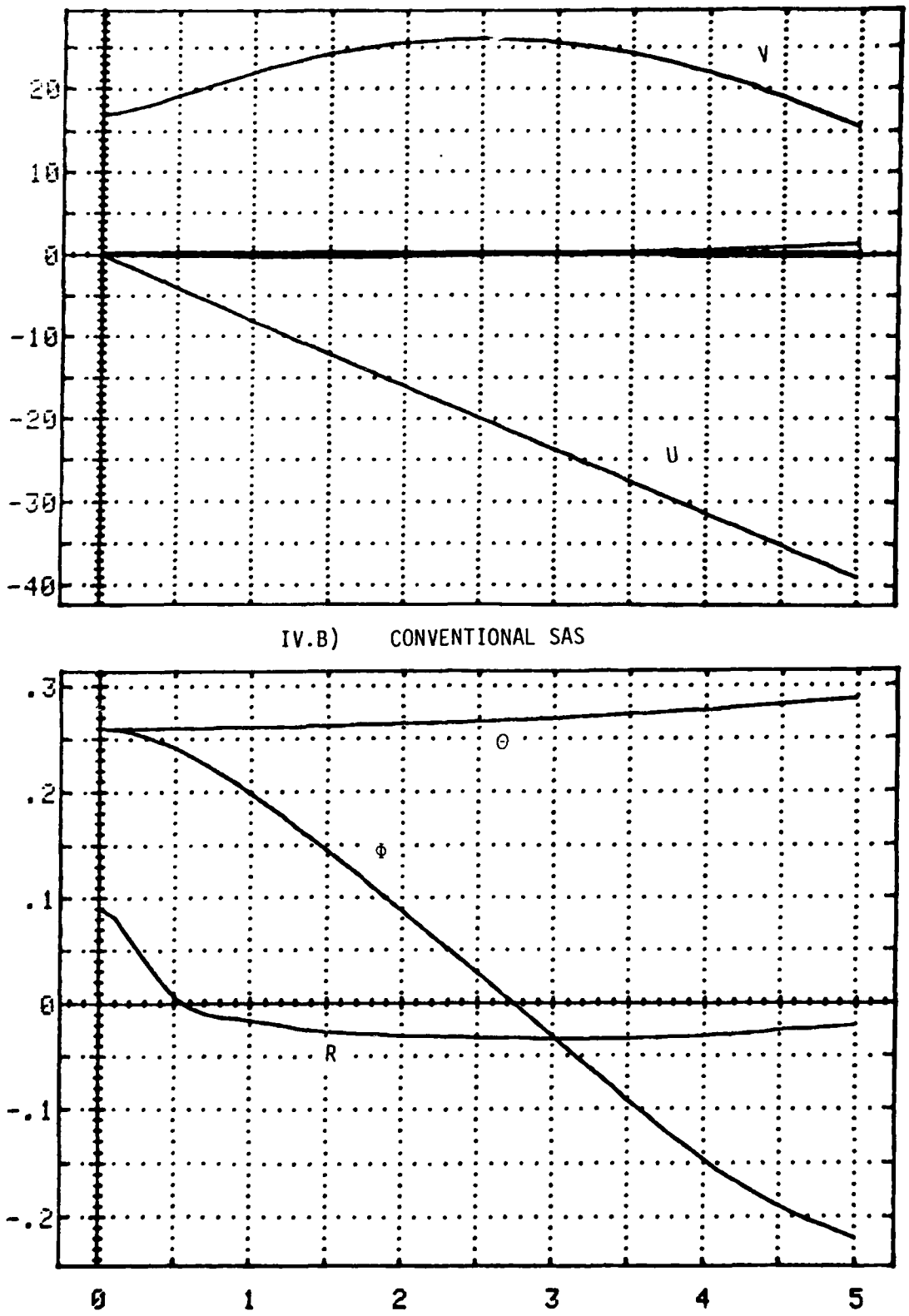
III.D) LONGITUDINAL; VELOCITY = 80; CONV



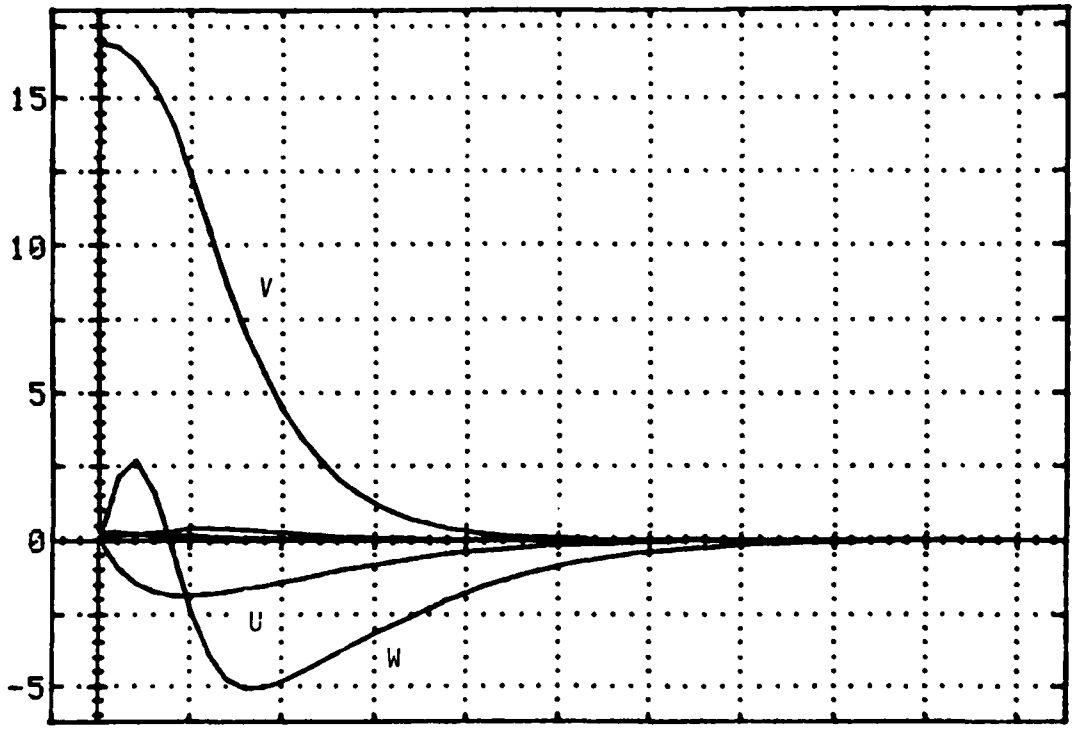


IV.A) BARE AIRFRAME - NO STAB AUG

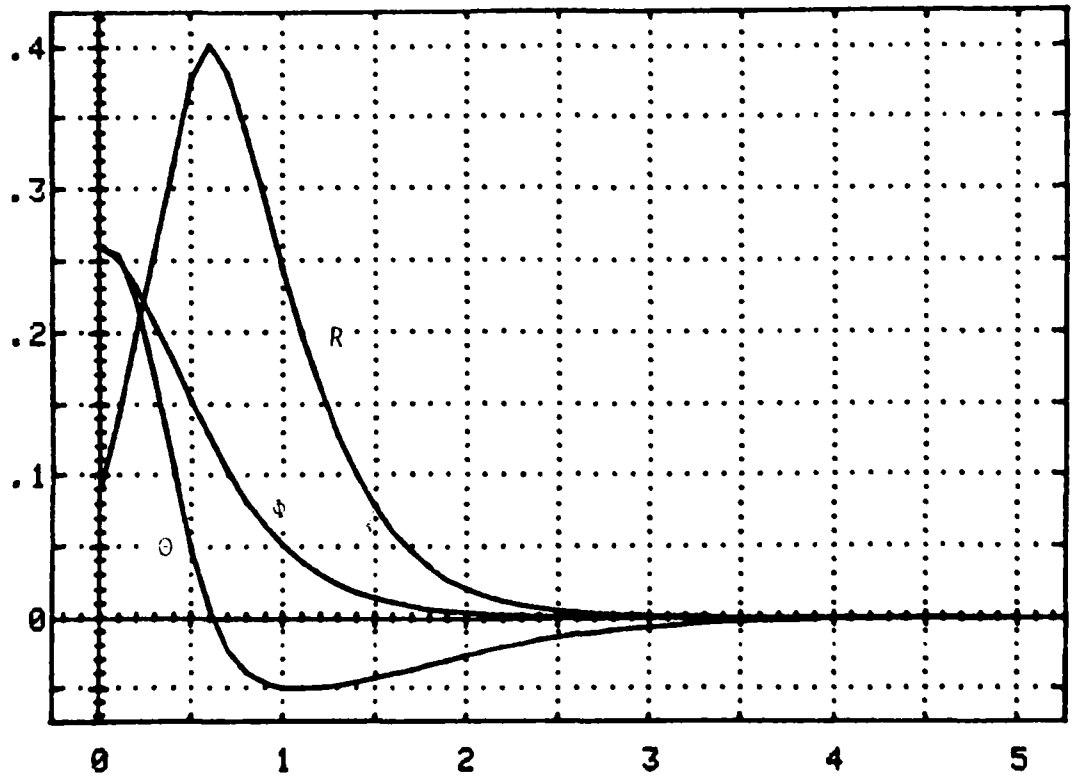


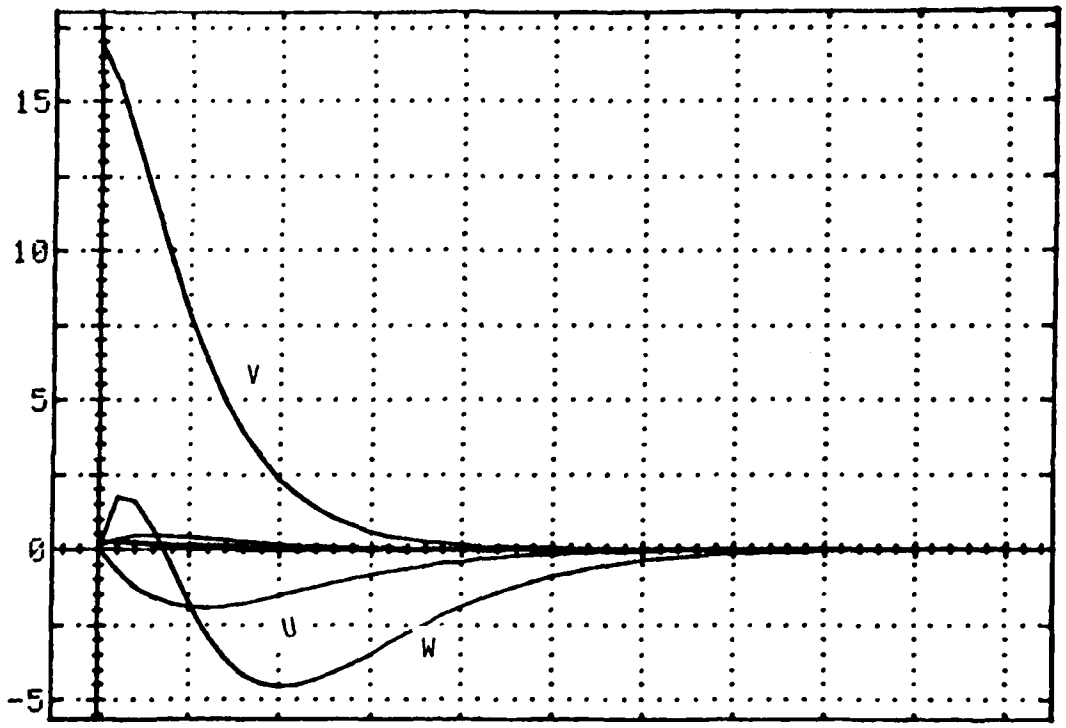


IV.B) CONVENTIONAL SAS

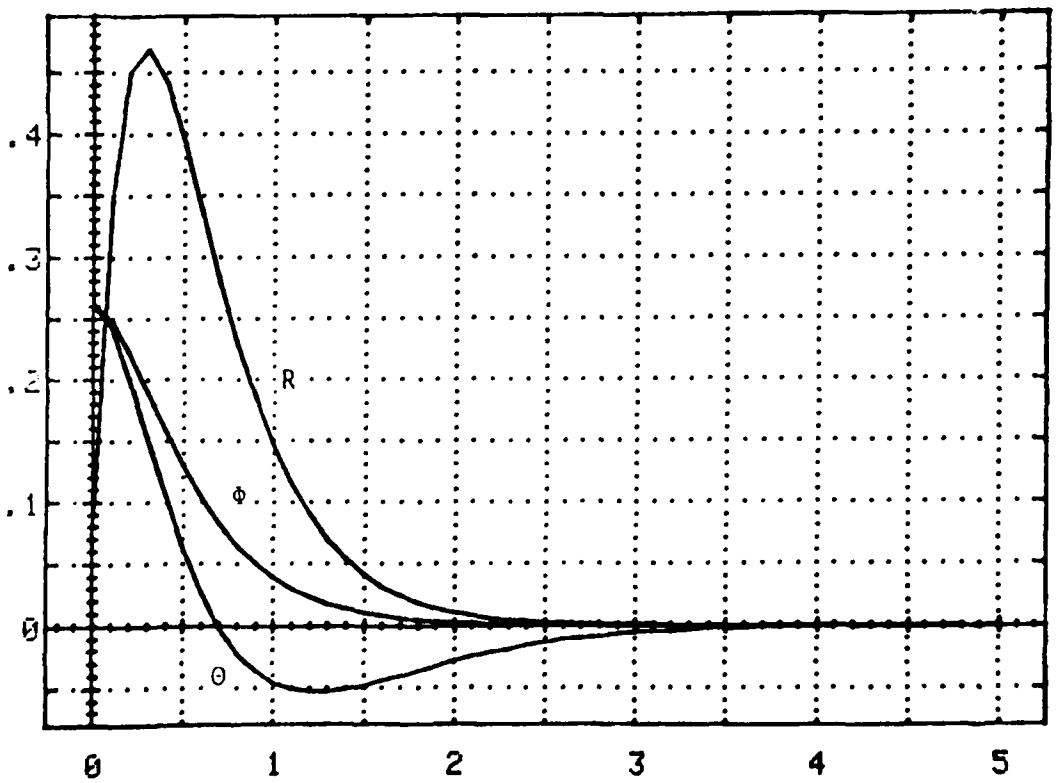


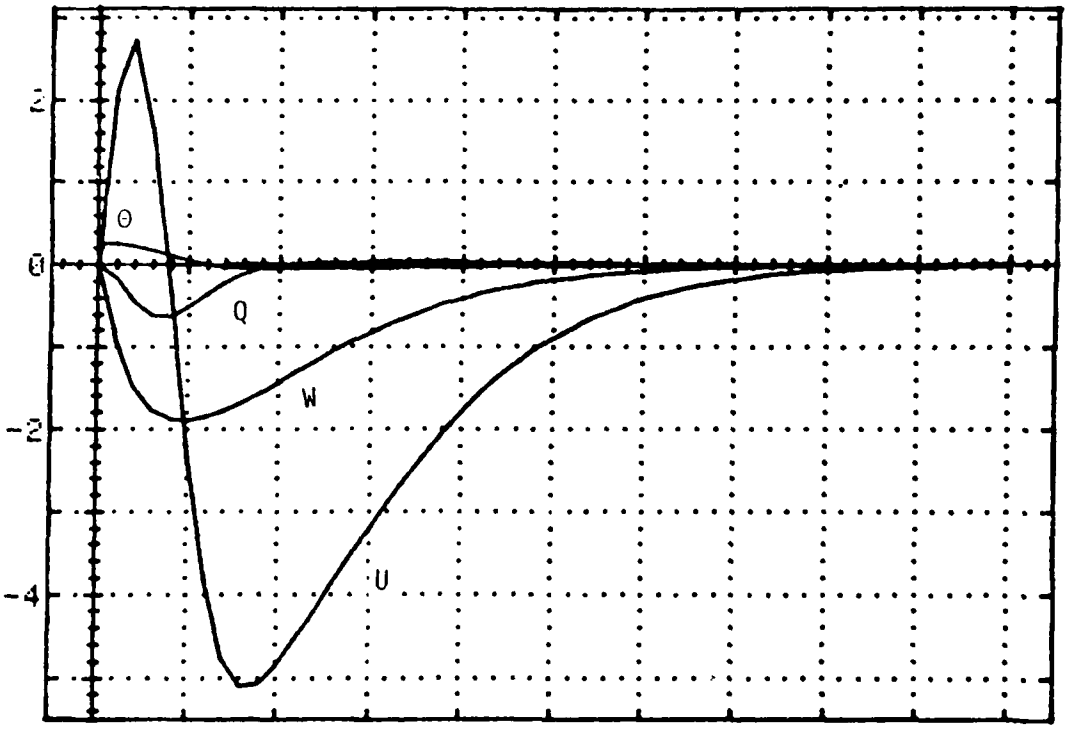
IV.C) STATE FEEDBACK CONTROLLER (SFC)



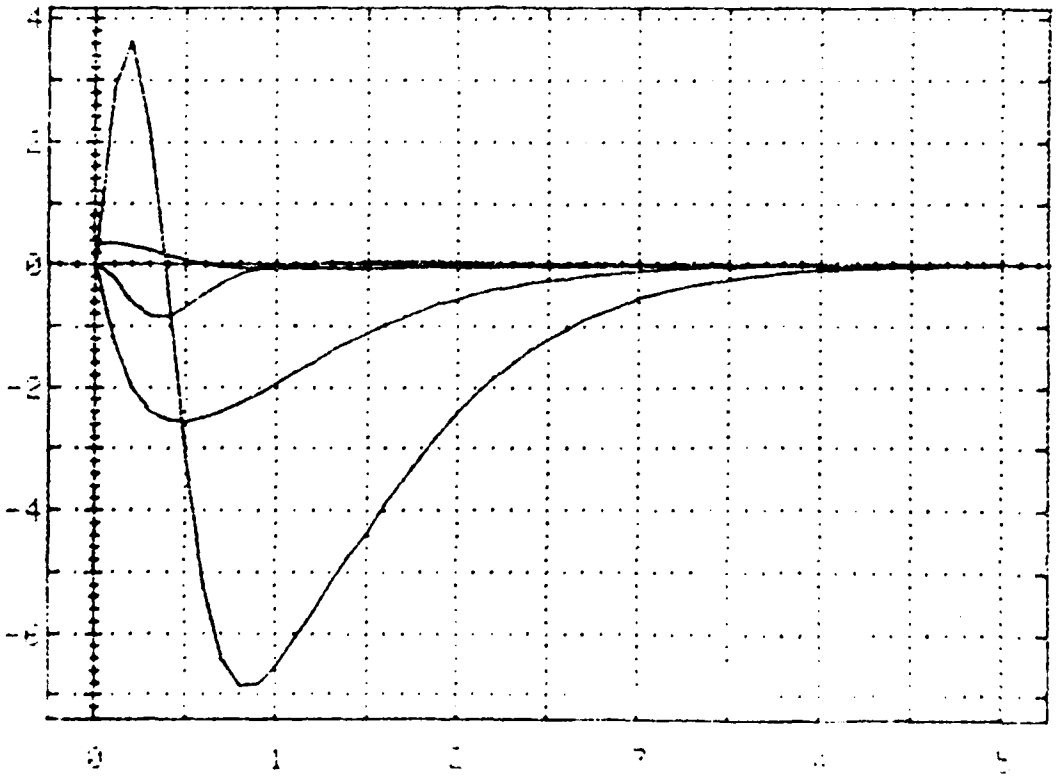


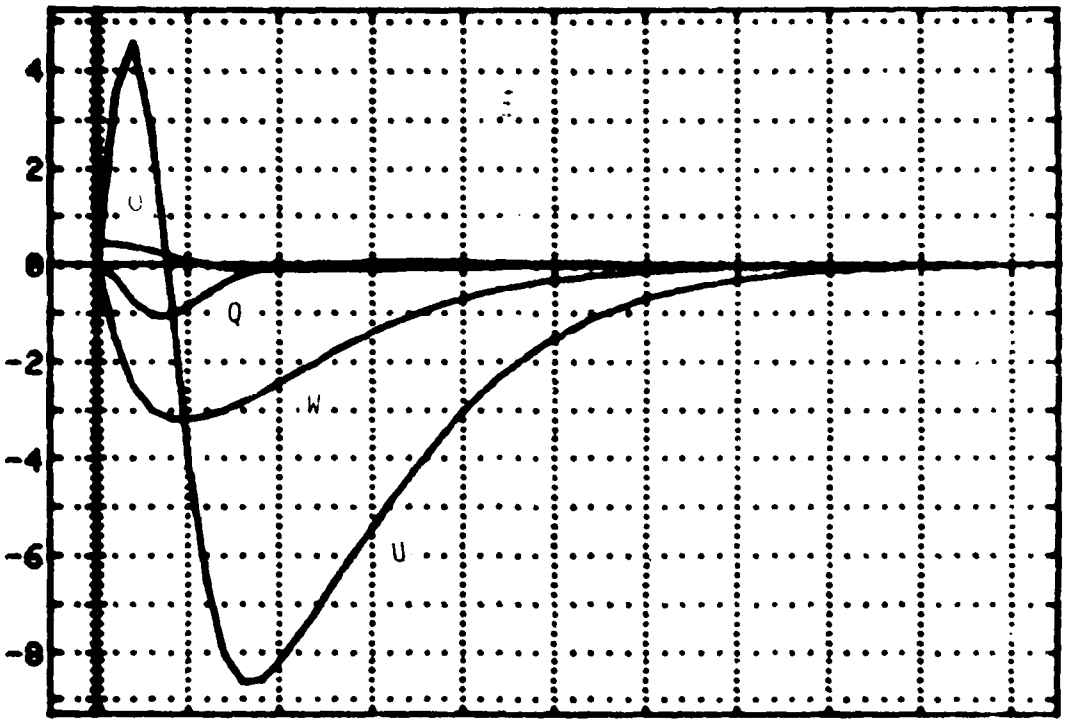
IV.D) SFC, No Actuator Dynamics



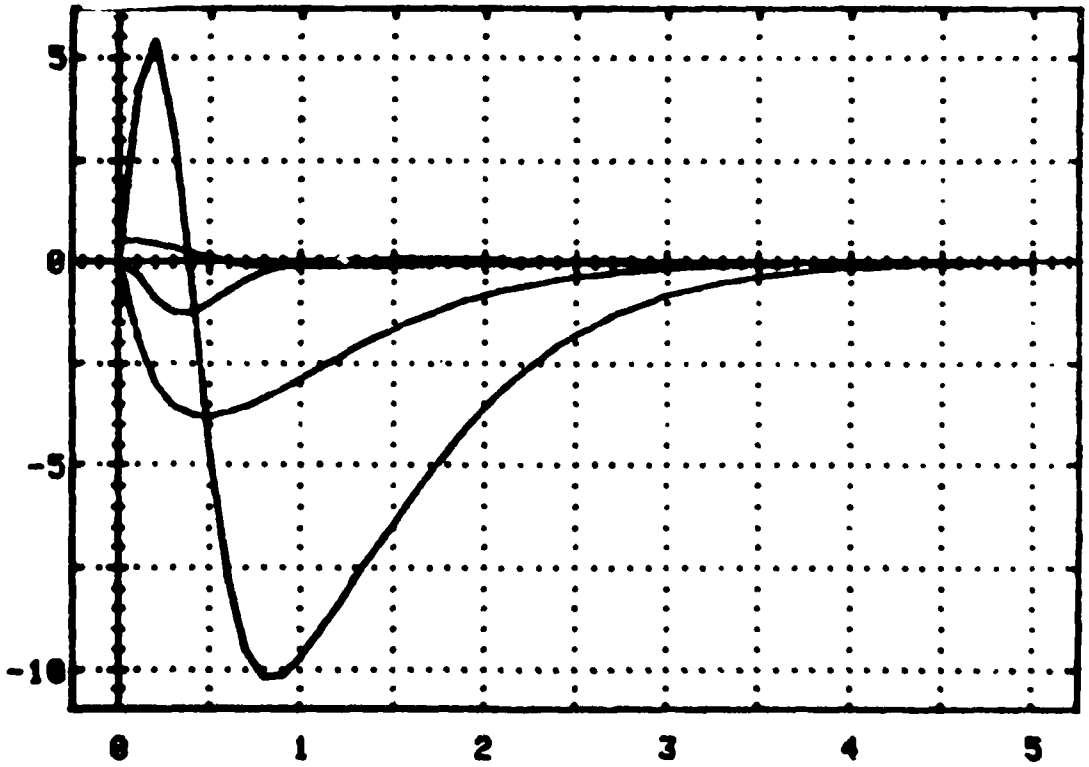


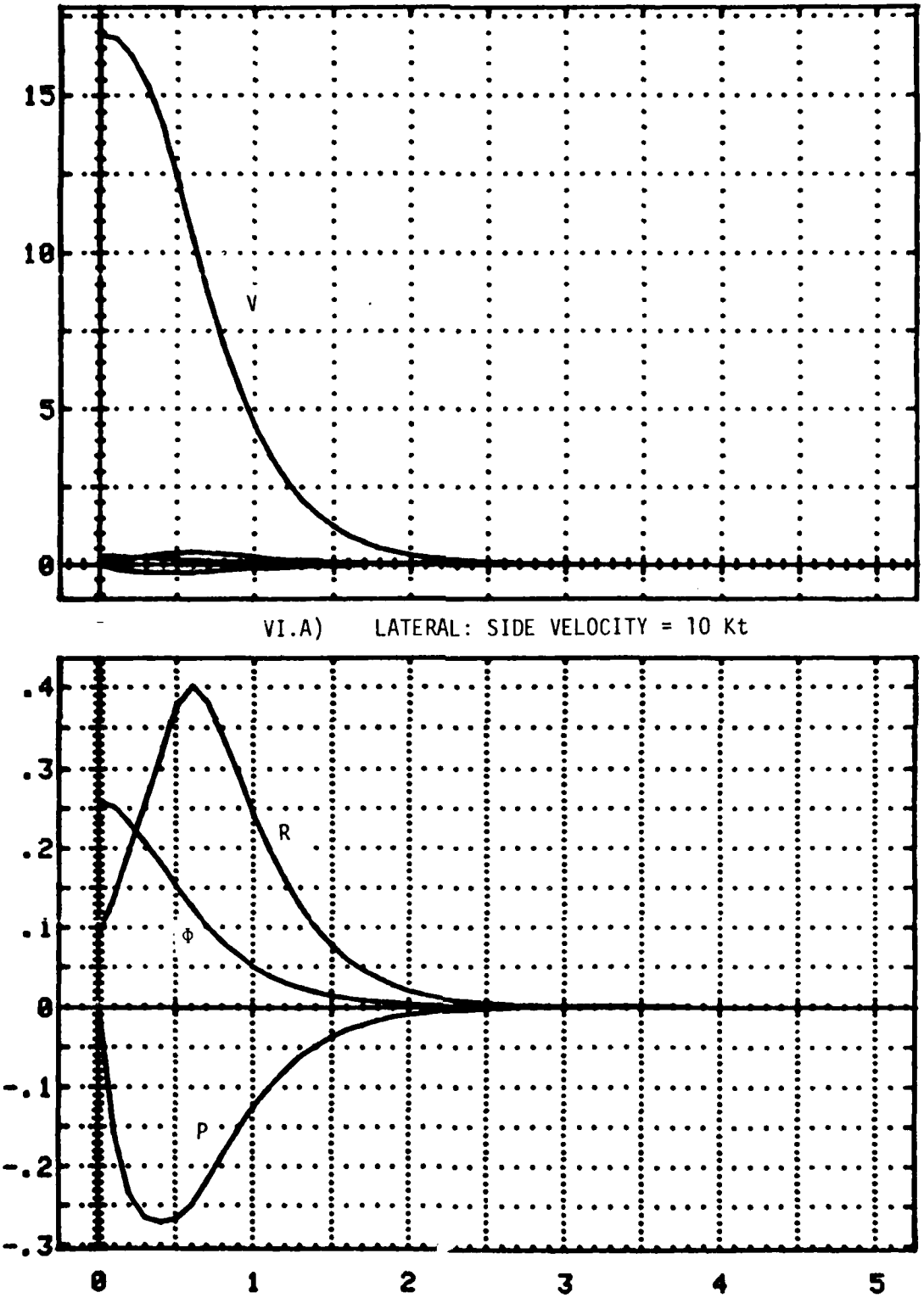
V.A) LONGITUDINAL: PITCH ANGLE = 15° (Top), 20° (Bottom)



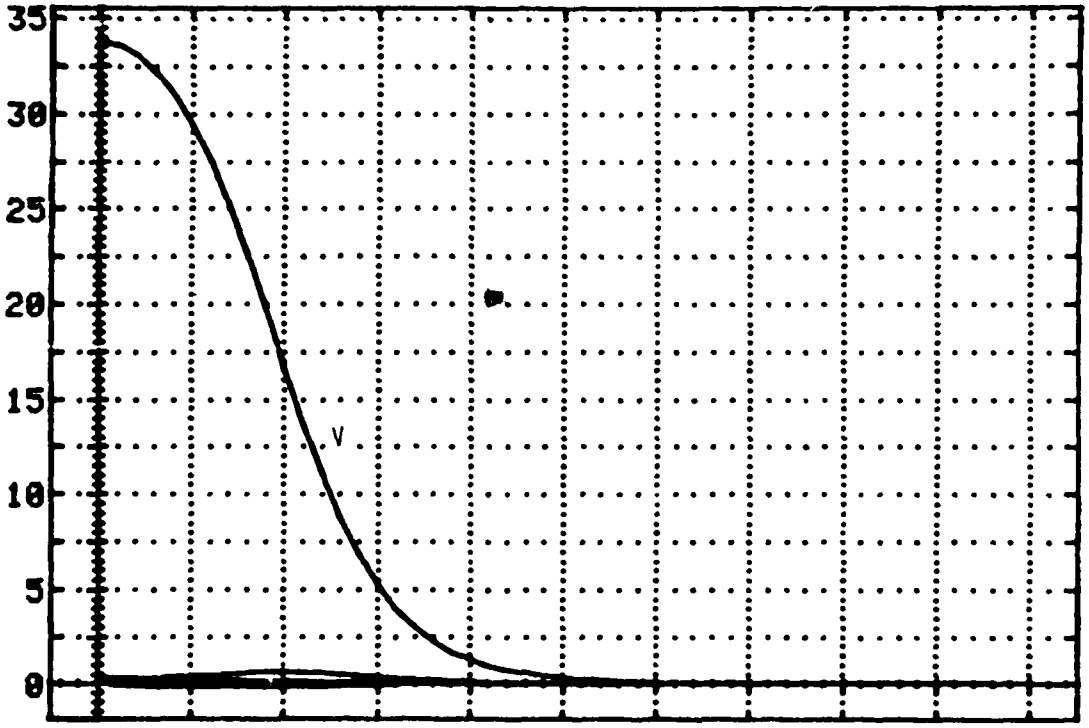


V.B) LONGITUDINAL: PITCH ANGLE = 25° (Top), 30° (Bottom)

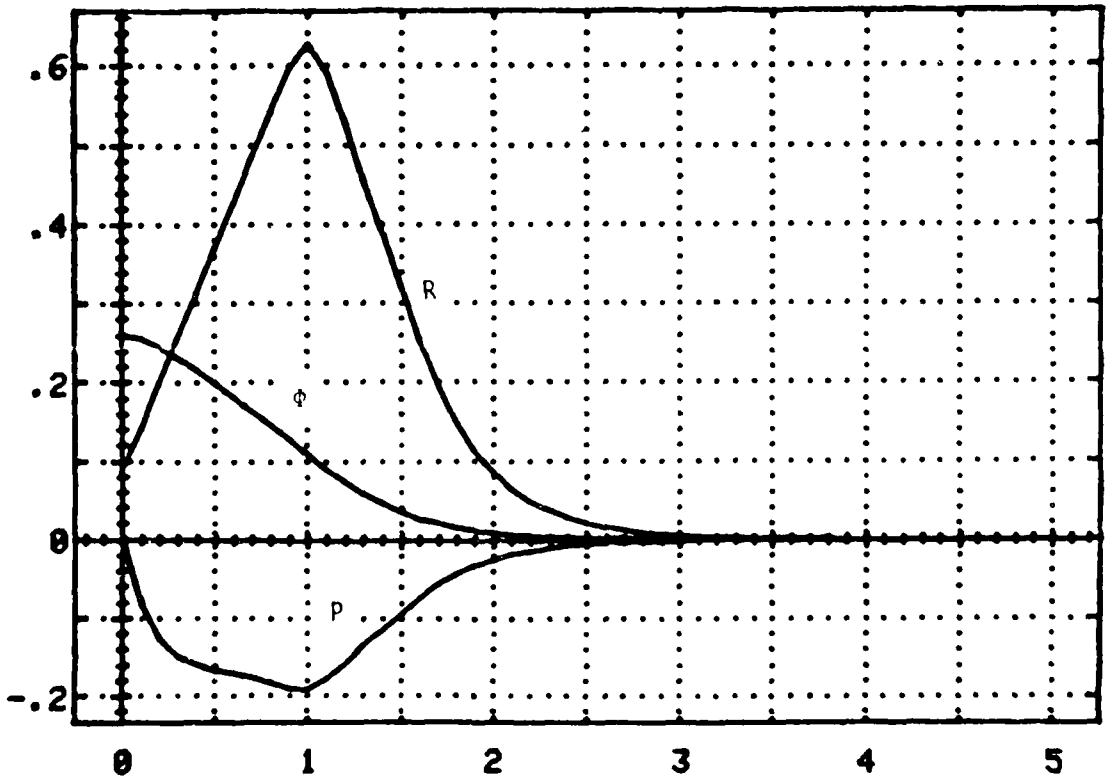




VI.A) LATERAL: SIDE VELOCITY = 10 Kt



VI.B) LATERAL; SIDE VELOCITY = 20 Kt



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MICROPROCESSOR CONTROL OF LOW SPEED VSTOL FLIGHT.(U)
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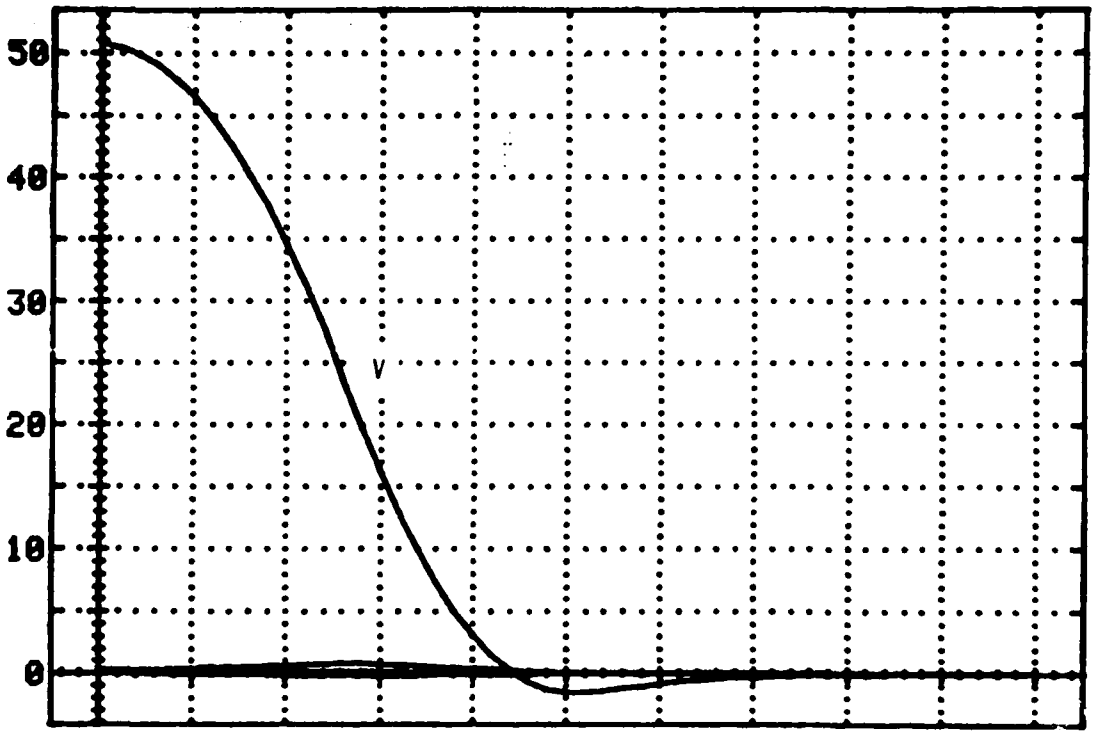
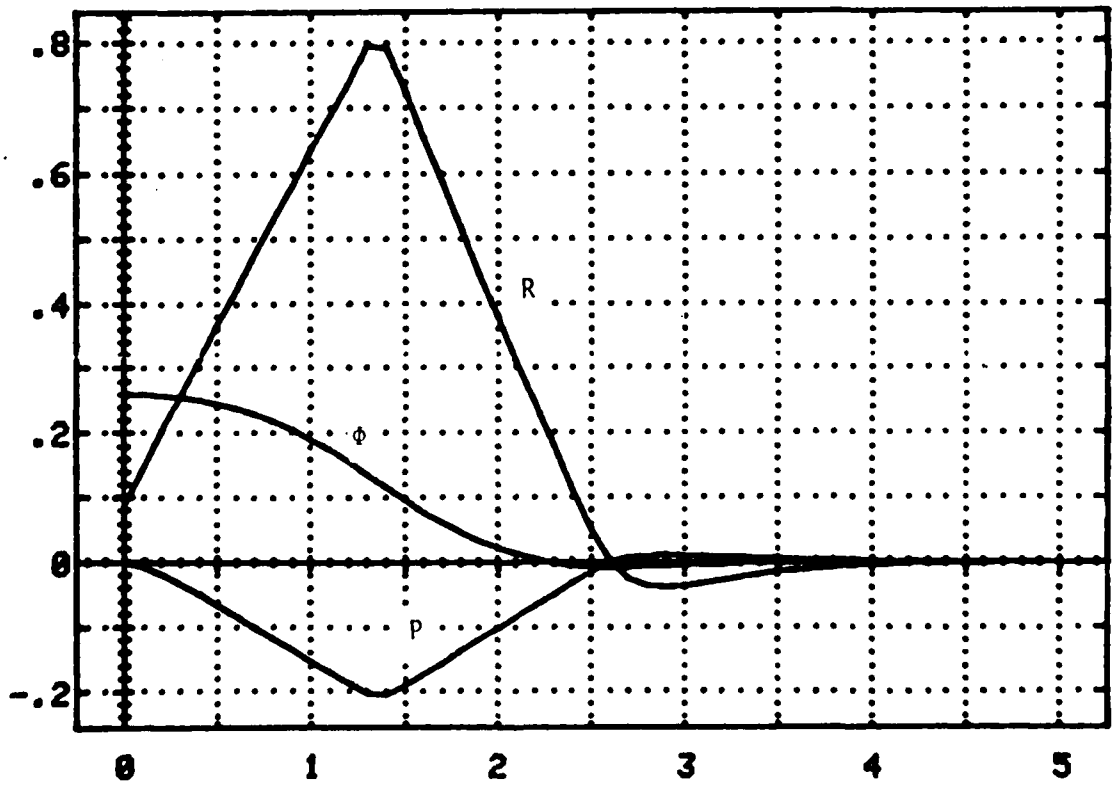
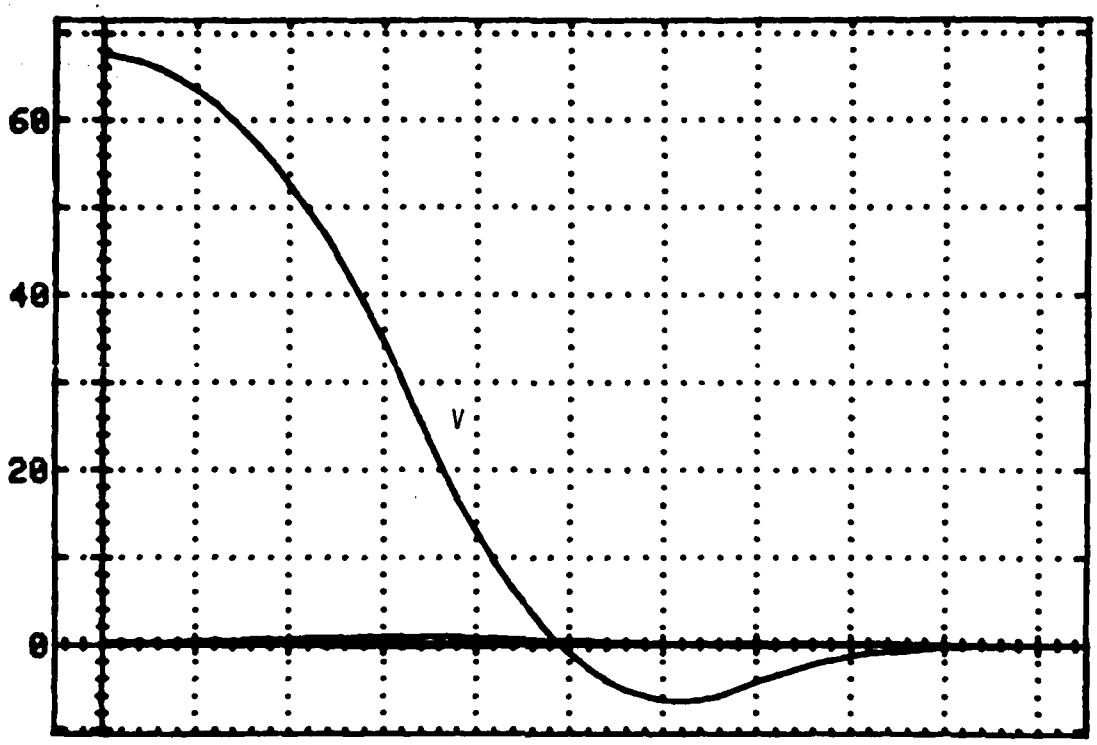
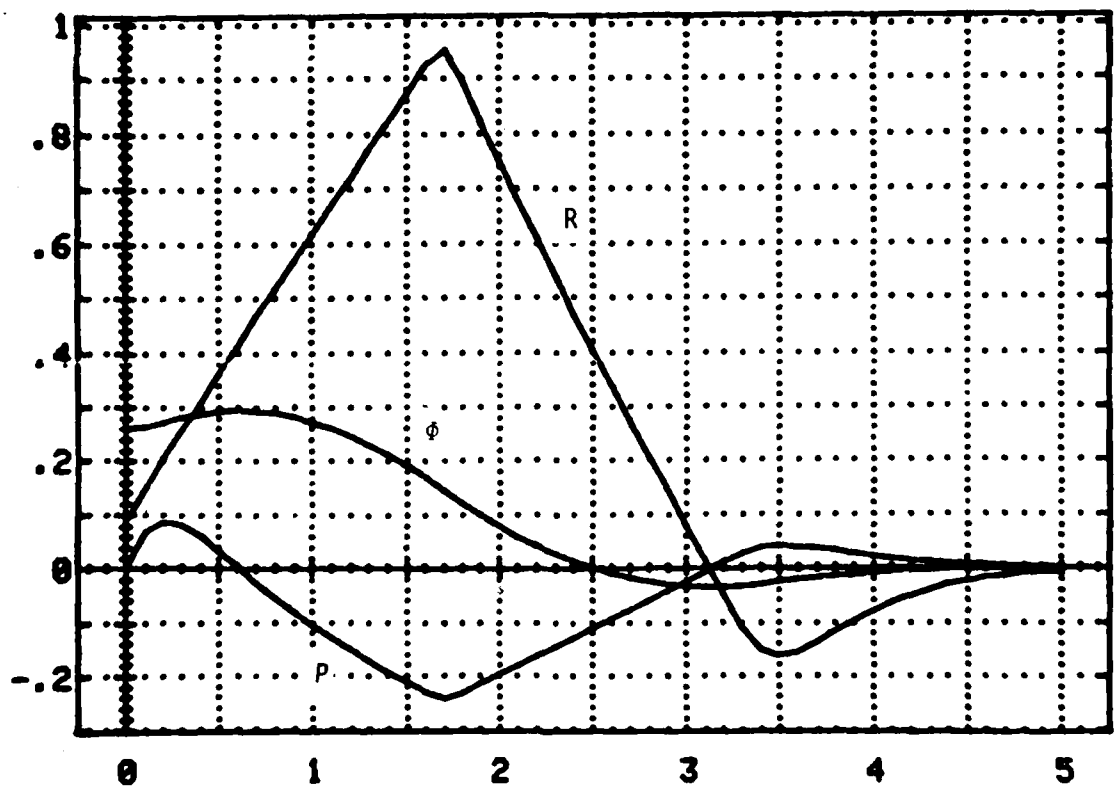


VI.C) LATERAL; SIDE VELOCITY = 30 Kt





VI.D) LATERAL: SIDE VELOCITY = 40 Kt

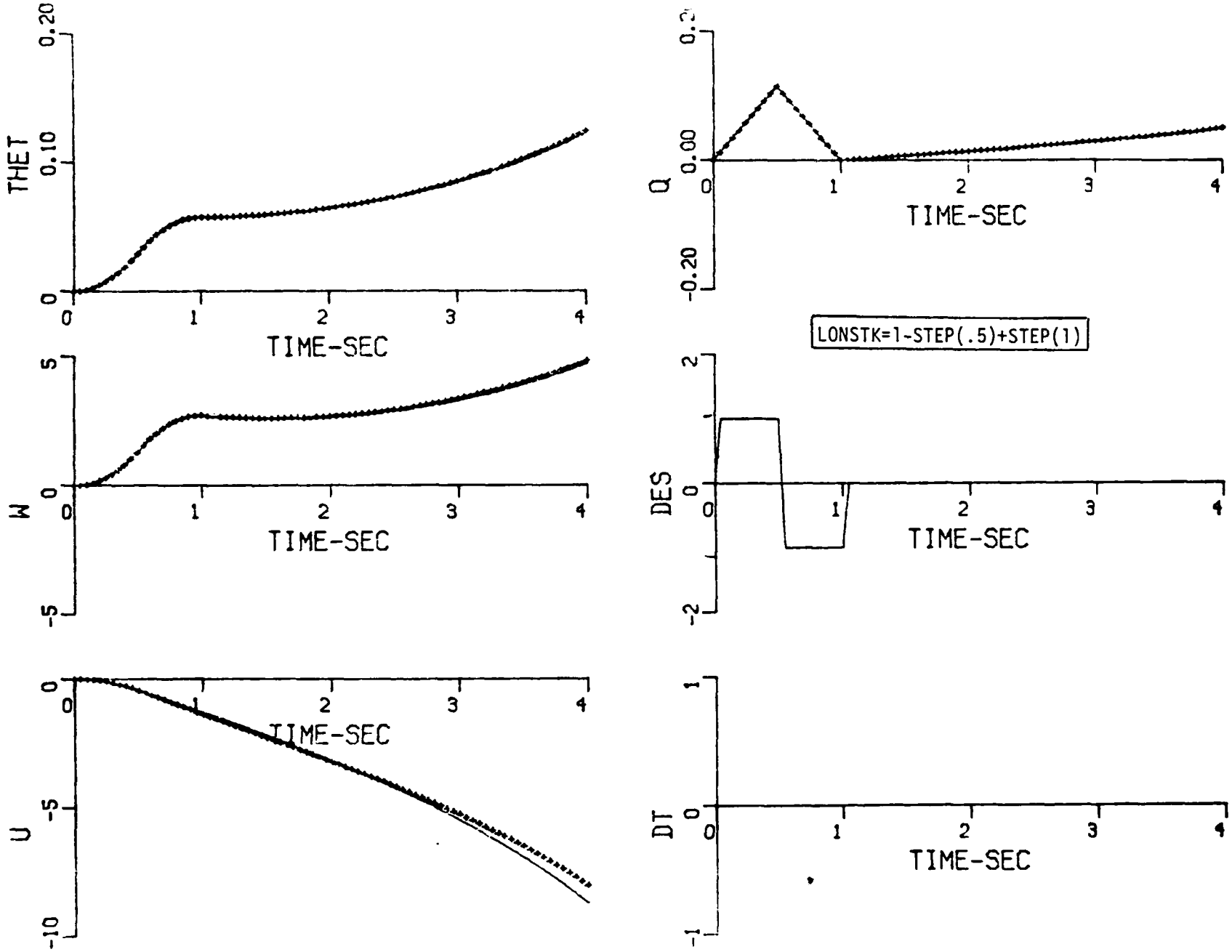


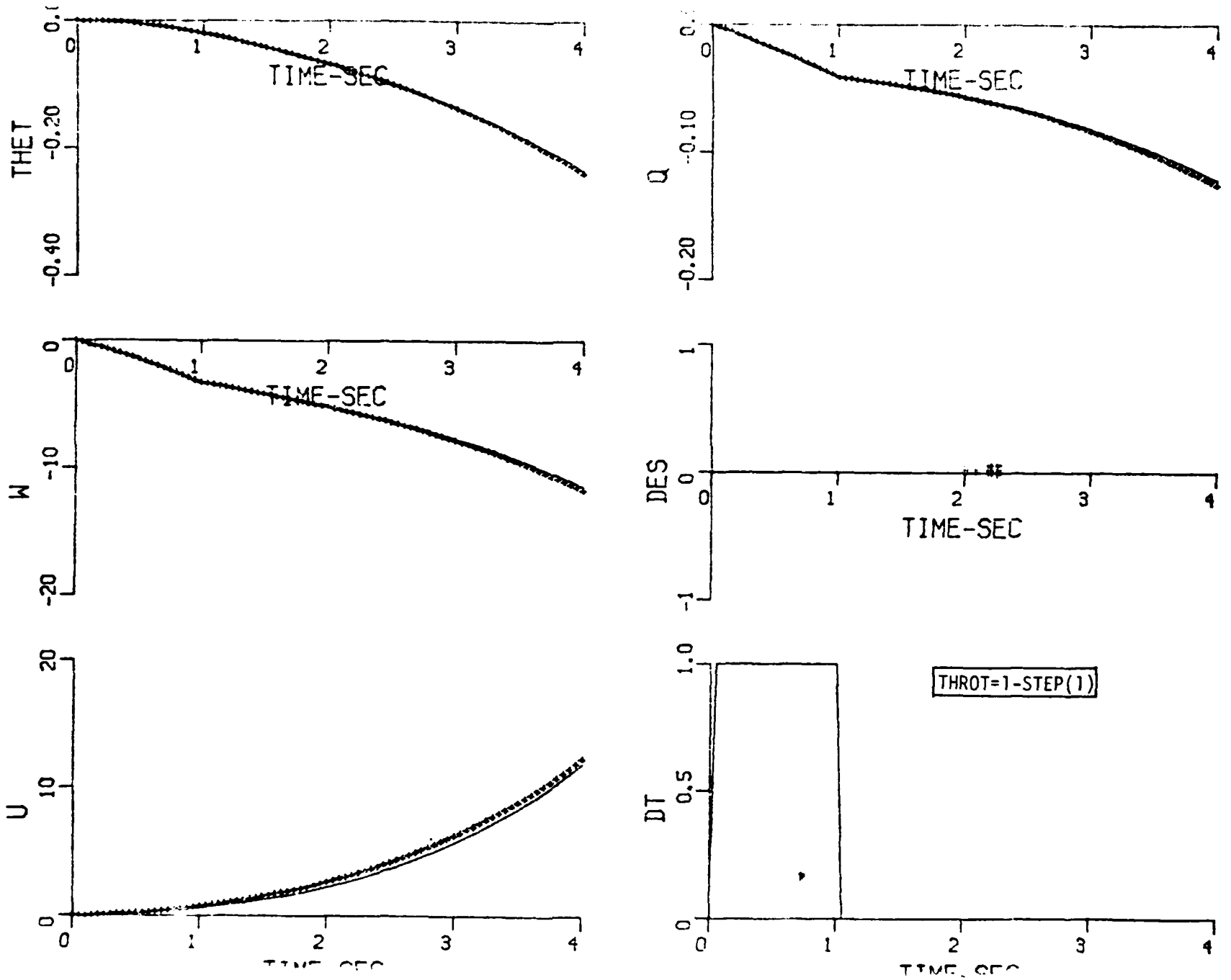
APPENDIX M

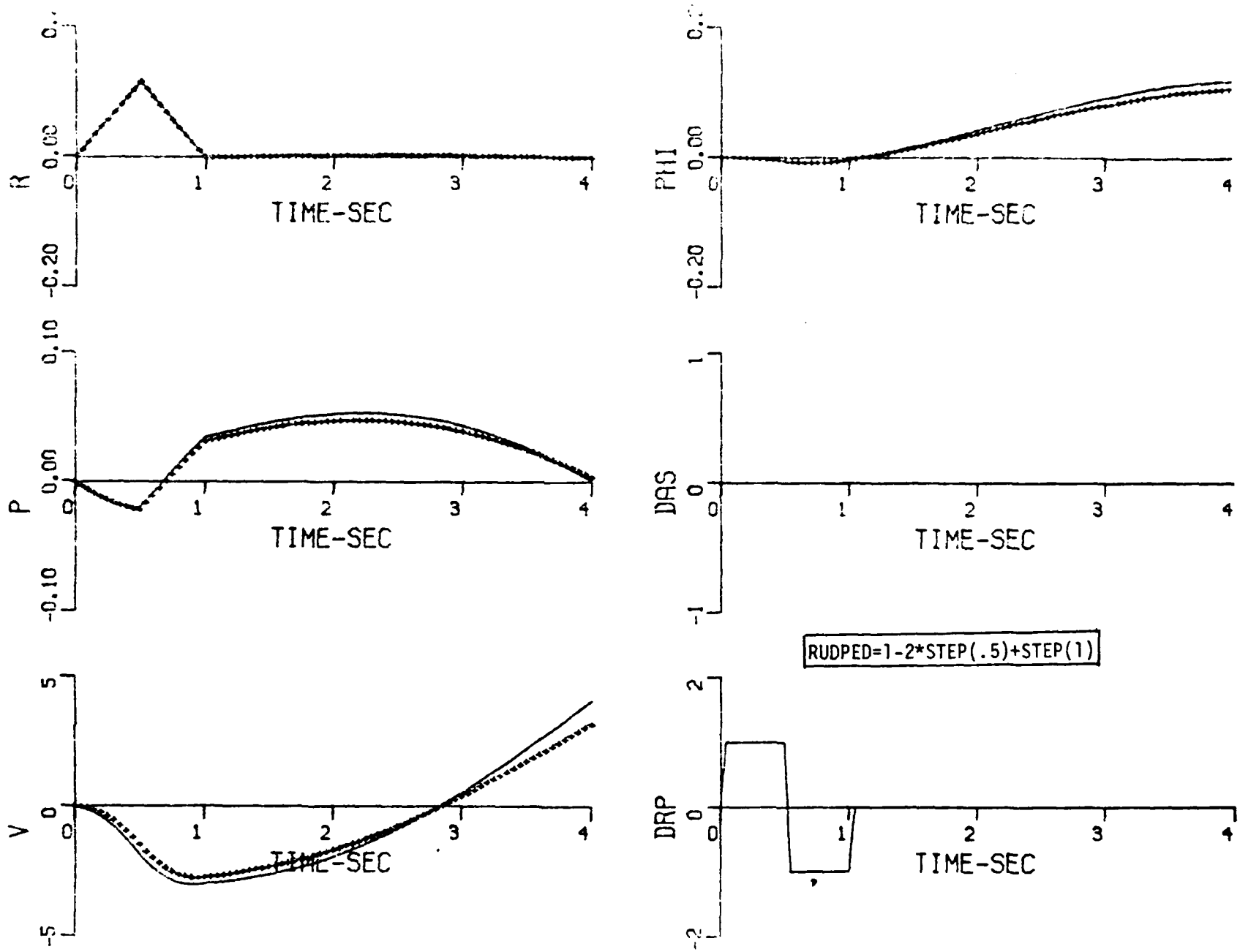
CALSPAN TIME RESPONSES

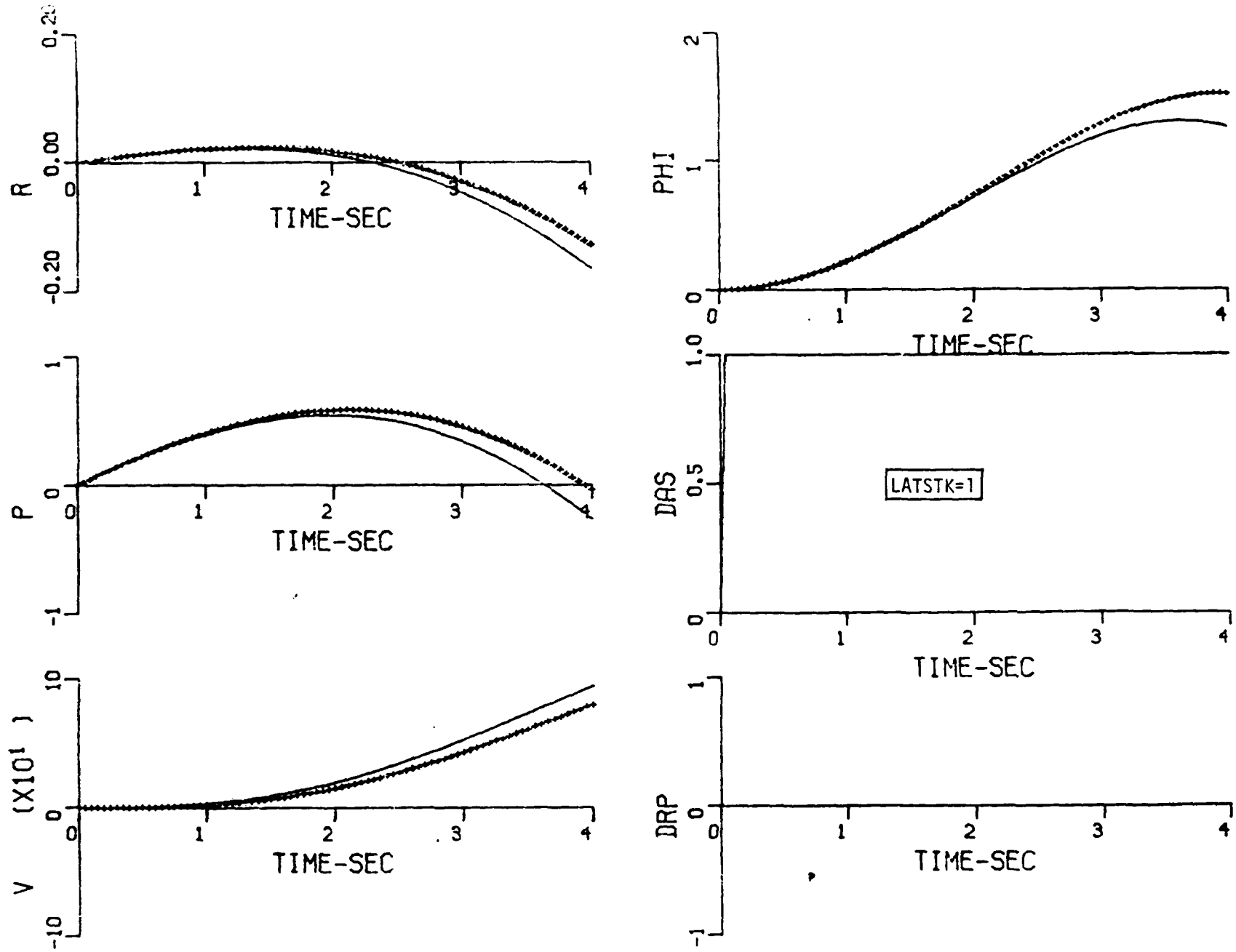
(EXTRACTED FROM TM NO. 98)

This appendix contains Calspan results to control inputs used in Appendix L, Part I. Comparison of the two enables assessment of simulation fidelity to be made.


THET
0.20
0.10
0
0
1
2
3
4
TIME-SEC
W
5
0
-5
0
1
2
3
4
TIME-SEC
U
0
-5
-10
0
1
2
3
4
TIME-SEC
Q
0.20
0.00
-0.20
0
1
2
3
4
TIME-SEC
LONSTK=1-STEP(.5)+STEP(1)
DES
2
0
-2
0
1
2
3
4
TIME-SEC
DT
1
0
-1
0
1
2
3
4
TIME-SEC


THET
TIME-SEC
Q
TIME-SEC
W
TIME-SEC
DES
TIME-SEC
U
DT
THROT=1-STEP(1)


R
0.00
-0.20
TIME-SEC
PHI
0.00
-0.20
TIME-SEC
P
0.10
0.00
-0.10
TIME-SEC
DAS
1
0
-1
TIME-SEC
RUDPED=1-2*STEP(.5)+STEP(1)
V
5
0
-5
TIME-SEC
DRP
2
0
-2
TIME-SEC
0
1
2
3
4


R
0.20
0.00
-0.20
TIME-SEC
P
1
0
-1
TIME-SEC
V (X10¹)
10
0
-10
TIME-SEC
PHI
2
1
0
TIME-SEC
DAS
1.0
0.5
0
LATSTK=1
TIME-SEC
DRP
1
0
-1
TIME-SEC
0
1
2
3
4

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18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aircraft Flight Control; Digital Control; Aircraft Flying Qualities; Microprocessor Control Systems; Stability Augmentation Systems; Aircraft Stability and Control; Simulation; AV-8 Harrier; Harrier; VSTOL Flight

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The objective of this project has been to design an improved three-axis stability augmentation system (SAS) for the AV-8B Advanced Harrier VSTOL aircraft using microprocessor-based digital control. The research focuses on improving the handling qualities of the airplane through SAS redesign in the low speed flight regime. Particular attention is paid to the so-called "weather-cocking" instability encountered in transition (hover ↔ conventional) flight. Until quite recently, there has been a dearth of information

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about the flight characteristics of the Harrier. A major breakthrough in this field was achieved by the development of MCAIR's X-22A AV-8B mathematical model, which yielded a set of linearized stability derivatives for the aircraft. The first step toward the improvement of the AV-8B SAS requires the utilization of these coefficients in the development of an analog/hybrid model of both the aircraft itself and of the unmodified SAS. The controller design employs digital state feedback control to relocate the system closed loop poles. The conclusion reached is that this method of control represents a valid approach to the final solution of the Harrier's stability and control problems. A long range goal of this research is that this controller design concept be applied to future aerospace vehicles.

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