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The behavior of the pulse of a high intensity laser beam is investigated as the beam penetrates into a low temperature high density plasma. Given the physical characteristics of the target plasma a study is made of the evolution of the incident laser pulse as it propagates through the plasma. A study is made of the dynamics of the strongly turbulent plasma generated by the interaction of an intense laser beam and the background plasma. The strongly turbulent plasma can be viewed as a collection of localized highly nonlinear electric field structures.
20. (solitons). The behavior of a laser heated aluminum exploding wire plasma is investigated by means of a radiation transport model for line radiation. MHD computer codes with nonequilibrium ionization dynamics are developed for the analysis. The physics of a strongly coupled plasma is studied via an appropriate kinetic model. The dynamic structure factor which enables one to interpret the scattered light from the plasma due to laser radiation is obtained. This is a very essential tool for diagnostics of such strongly coupled plasmas.
THEORETICAL STUDIES OF LASER LIGHT INTERACTION 
AND OPTICAL AND X-RAY EMISSION 
FROM DENSE METALLIC PLASMAS

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This report describes the achievements of the research program sponsored by AFOSR under Grant AFOSR-80-0029 during the period 1 October 1979 to 1 October 1980. The object of the research program is to investigate laser interaction with dense metallic plasmas and its optical as well as X-ray emissions. We study the dynamics of a high density, relatively low temperature (20 eV) plasma when illuminated by a high intensity CO\textsubscript{2} laser beam. The investigation is carried out both theoretically and by means of an advanced numerical simulation code. The following sections give a description of the various efforts and accomplishment.

A. THERMAL SELF-FOCUSBING OF A HIGH INTENSITY LASER BEAM IN A HIGH DENSITY, LOW TEMPERATURE PLASMA

Our analysis begins by constructing the governing equation for the propagation of a pulse of electromagnetic wave through a plasma. We start with the Maxwell equation in a general dielectric medium (scalar dielectric constant \(\varepsilon\))

\[c^2 \nabla^2 \mathbf{E} = 4\pi \frac{\partial \mathbf{j}}{\partial t} + \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\omega^2 \varepsilon \mathbf{E}\]

where \(\mathbf{E}\) is the electric field in the radial direction and \(\mathbf{j}\) is the corresponding current density. By means of the transformation

\[E = \psi(x, t) \exp(-ikz + i\omega t) + c.c.\]
we remove the last space and time dependence and obtain the following
equation for the wave envelope \( \psi \):

\[
(A-2) \quad 2i \frac{\partial \psi}{\partial t} + 2i \frac{\partial \psi}{\partial z} = \sqrt{1} \psi + f\psi.
\]

Equation (A-2) has been written in non-dimensional coordinates \((k_0, \epsilon_0 \psi t)\); the nonlinear term \( f \) is given by

\[
(A-2a) \quad f = \frac{\epsilon (|\psi|^2, x, t) - \epsilon_0}{\epsilon_0} = \frac{\omega_0^2}{\epsilon_0} \left( \omega - \frac{n_e (|\psi|^2, x, t)}{\eta_0} \right).
\]

Equation (A-2) can be put into standard form by means of a transformation
to wave frame coordinates \((\xi, t)\), where

\[
\xi = z - t.
\]

We then obtain

\[
(A-3) \quad 2i \frac{\partial \psi}{\partial t} = \nabla_\perp \psi + f\psi.
\]

In Eq. (A-3) the nonlinear term \( f\psi \) is both space and time dependent.

We apply the "moment" method to the analysis of Eq. (A-3). Its
success depends, in part, upon identifying at least two invariants associated with Eq. (A-3). The first constant of the motion is

\[
(A-4) \quad I_0 = \int dx \, dy \, |\psi|^2.
\]

The second constant of the motion is

\[
(A-5) \quad I_2 = \int dx \, dy \left\{ |\nabla_\perp \psi|^2 - \int dt' \frac{\partial |\psi|^2}{\partial t'} \right\}.
\]

Using these constants of the motion we can construct from Eq. (A-3)
a relation governing the behavior of the pulse radius. First we define
the radius based upon the second moment of the radial distribution

\[ <a^2> \equiv \int \int dx\, dy\, r^2 \, |\psi|^2 / \int \int dx\, dy\, |\psi|^2. \]

The governing relation is then found to be

\[ \frac{d^2<a^2>}{dt^2} = \frac{21/2}{1/0} - \left( \frac{2/2}{1/0} \right) \int \int dx\, dy \left\{ \frac{1}{2} f \nabla_r |\psi|^2 + f |\psi|^2 - \int dt' J(t', a') \right\}. \]

In order to progress we need to assume a radial shape function for the
pulse intensity:

\[ |\psi|^2 = \frac{a^2 s(\xi)}{a^2(\xi, t)} \exp \left( -\frac{r^2}{a^2(\xi, t)} \right) \]

where \( s(\xi) \) is a parameter. Then we can write Eq. (A-7) in the form

\[ \frac{d^2(a^2)}{dt^2} = 2a^2 - 4 J(t, a) - 8 \int_0^t dt' J(t', a') \frac{1}{a'} \frac{\partial a'}{\partial t'} \]

where \( J \) is a convolution integral

\[ J(t, a) = \int_0^\infty dr f(r, t) \left( 1 - \frac{r^2}{a^2} \right) \exp \left( -\frac{r^2}{a^2} \right). \]

Since \( f \geq 0 \) at \( r = 0 \), and \( f \to 0 \) as \( r \to \infty \), we can show that \( J \) is always
positive. Thus, as the pulse begins to penetrate the plasma, it interacts
with it and creates a density depression \( (f \geq 0) \). At early times the pulse
radius is determined by the relative magnitude of the two terms

\[ 2a^2 - 4 J(t, a). \]

If the first term dominates, the pulse diffracts
\( (d^2(a^2)/dt^2 > 0) \), otherwise it will self-focus \( (d^2(a^2)/dt^2 < 0) \).
The general behavior as indicated by Eq. (A-8) conforms to our expectations. We expect the front edge of the pulse to be diffracted while the later parts (experiencing the nonlinearity created by the front part) undergo more focusing.

To proceed further we need to specify the nature of the interaction which causes the nonlinearity. The response of the plasma density to the electromagnetic field needs to be analyzed.

The laser field can influence the plasma target in several ways: through the ponderomotive force effect and through heating the plasma. In our low temperature high density plasma the collision frequency is sufficiently large as to make plasma heating the dominant mechanism. At the same time, the collision frequency is sufficiently small as to make electron thermal conduction (rather than electron-ion collisional energy transfer) the dominant energy dissipation mechanism. Thus the temperature equilibrates at a much faster time scale than the hydrodynamic flow time. Hence the plasma interacts with the plasma mainly through the temperature equilibration process described by the energy balance equation:

\[(A-9) \quad \nabla_\perp \cdot (KE_\perp T_e) = eE \cdot j = \frac{e^2 n e v e i}{m_e \omega} |\psi|^2.\]

We linearize this equation and combine it with the continuity and momentum equation for a one-fluid plasma. The electron density is then found to satisfy the equation:
We approximate the time dependence in a reasonable manner such that this enables us to consider the radial variation separately. Such an approach yields

\[
(A-10) \quad \left( \frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2 \right) \frac{n_1}{n_0} = \frac{k_B}{m} \frac{\omega P_0 \nu_e i}{4 \pi \omega^2 K_e} |\psi|^2. 
\]

\( \tau_R \) is a characteristic relaxation time of the flow, e.g.,

\( \tau_R \approx a_o / c_s \). The convolution integral \( J \) in Eq. (A-8) is first written in the form (integrate by parts twice):

\[
(A-11) \quad \frac{c_s^2}{s} \nabla^2 \frac{n_1}{n_0} = \frac{k_B}{m} \frac{\omega P_0 \nu_e i}{4 \pi \omega^2 K_e} \int_0^t dt' |\psi|^2 e^{-(t-t')/\tau_R}. 
\]

Using (A-11) we now find

\[
J = \frac{\omega P_0}{4 \pi \omega^2} \int_0^\infty dr (\nabla n) r \exp(-r^2/a^2). 
\]

Here \( \xi' = z - t' \), and the integral with respect to the time must be performed at constant \( z \) (rather than at constant \( \xi \)). The final form of the self-focusing equation then becomes:
\[ \frac{d}{dt} \left( \frac{a^2}{2} \right) = 2a_0^2 - a^4 \Omega \int \frac{s(t')}{a^2 + a'^2} \exp \left\{ (t-t')/\tau_R \right\} \, dt' \]

\[ - 2\Omega \int_0^t dt' a' \frac{\partial a'}{\partial t'} \int_0^{t''} \frac{s(t'')}{(a' + a'')^2} \exp \left\{ (t'' - t)/\tau_R \right\} \]

where

\[ \Omega = \omega_p^4 e_0^2 \psi_0^2 \frac{2^2}{8\pi} \epsilon_0^4 \omega^4 KeT_{eo} \]

and \( \int \) denotes integration with respect to time with \( z \) kept constant.

This last equation will be solved by numerical techniques. It will then yield the variation of the radius of the laser pulse as it penetrates through the plasma target. Thus, knowing the properties of the target plasma we can predict the behavior of the laser pulse as it penetrates through the plasma.

B. DYNAMICS OF THE STRONGLY TURBULENT PLASMA GENERATED BY LASER-PLASMA INTERACTIONS

A high intensity laser beam disturbs a plasma in such a way that instabilities appear. These unstable waves grow to form very intense, spatially localized electric fields. Consequently, the plasma is rendered strongly turbulent. Moreover, such localized nonlinear structures can interact with themselves as well as with individual electrons and ions, thus leading to a significant modification of the zero order dynamic as well as thermodynamic properties of the plasma itself (e.g., density, temperature, flow velocity, etc.)
A model has been developed to study the large amplitude kinetic Alfvén wave in more than one dimension. The model incorporated the effects of finite electron inertia and finite ion Larmor radius. No amplitude expansion is used and the results can be extended to arbitrary amplitudes consistent with the assumption of low $\beta$ in the plasma.

Solutions to the model equations have been analyzed in a cylindrical geometry. These solutions take the form of a waveguide elongated in the direction of the ambient magnetic field. The azimuthal magnetic field is described by a radially varying envelope containing sinusoidal oscillations along the ambient field line. The solution is localized transverse to the background magnetic field. The width of the radial profile is given as a function of the parameter $K_z (= V_A / V_{\text{phase}})$. As $K_z$ increases, the width of the envelope decreases and the solution amplitude increases correspondingly. In the opposite limit, as $K_z$ decreases, towards unity, the envelope spreads and the soliton amplitude decreases until $K_z$ equals unity, when the nonlinearity goes to zero. At this point the solution spreads toward infinity and the localization effect vanishes. We call this the plasma wave limit. Therefore, the parameter $K_z$ characterizes the nonlinearity. As this nonlinearity increases, wave steepening is observed. However the width of the envelope reaches an asymptotic limit of approximately three sonic ion Larmor radius. This limit suggests that there is a stabilization effect against collapse.
A stability analysis is performed for the two-dimensional cylindrical solitary wave solution found above. First, perturbations preserving the symmetry of the solution are considered. A method following the work of Lyapunov is developed. The advantage of this method is that it can be applied to problems where an explicit analytical solution for the "equilibrium" state cannot be found. A criterion for the stability is obtained and, in principle, can be computed to arbitrary accuracy. This criterion is then applied to our waveguide solution and it is shown that it is stable against symmetry preserving perturbations.

This part of the research constitutes the Ph.D. dissertation of Mr. J. Sheerin.
C. MODELS OF RADIATION TRANSPORT IN DENSE PLASMAS

We have developed a radiation transport model for line radiation which fits into a one-dimensional, cylindrically symmetric Lagrangian, MHD computer code with nonequilibrium ionization dynamics. A laser heated aluminum exploding wire plasma was considered.

The model combines escape and transmission probability theory with Eddington theory and is henceforth referred to as the EPDM model. Empirical functions were developed to relate cylindrical shell probabilities to slab probabilities by use of two sets of model problems: emission from an infinite homogeneous purely absorbing cylinder due to an axial line source and a uniform volumetric source. The EPDM model was compared with the variable Eddington theory on a set of problems typical of the ones observed in the plasma simulation. The EPDM model showed similar trends for all sets of problems. In the optically thin or purely absorbing regimes the EPDM model predicted higher absorption and less emission than the variable Eddington calculation. However, as the optically thick, scattering dominated regime was approached, the EPDM model showed lower absorption and higher emission than the Eddington method. This was to be expected since the approximations made in the EPDM formulation invalidate it in this regime.

The EPDM model was then incorporated in the magneto-hydrodynamic code. The results of a standard run were compared to those of a transparent plasma. The most dramatic effect was that the addition of
self-absorption decreases the radiation emission from the plasma (see Figure for a typical case). The emission decreased anywhere from a small fraction for an optically thin plasma to as much as two to five times for the optically thick plasma. There was also some redistribution of radiant energy due to the fact that some of the excited levels could decay by several paths. There were also some minor effects on plasma dynamics. The electron temperatures, plasma radius, and the effective charge were greater with self-absorption while the ion temperatures and the densities were less than their respective values for the transparent case.

The EPDM model is now being used within the MHD code to analyze the results of X-ray measurements of the emission from laser-irradiated exploding aluminum wire plasmas being made in our laser-plasma interaction laboratory.

D. KINETIC MODELS OF ULTRA HIGH DENSITY PLASMAS

In the theory of neutral systems such as gaseous liquids, the hard sphere fluid model has served as a calculational basis for research into the nature of real fluids. The classical one component plasma (OCP) is the simplest mathematical model of a system incorporating Coulomb interactions and serves the same purpose for real strongly coupled plasmas that the hard sphere model has served for real fluids. The model consists of $N$ point nuclei of charge $Ze$ and mass $m$ embedded in a charge-neutralizing fluid ($NZ$ points of charge
Figure: Total Emission vs. Laser Energy Deposition
-e and mass $m_e$ smeared into a continuous bath).

To further specify the OCP and relate the model to real physical plasmas, we need to define a few parameters. For an ion fluid of density $n = N/V$ ($V$ is the plasma volume), an appropriate scale of length, the ion sphere radius (or mean ion separation distance), is given by

$$a = \left( \frac{4\pi}{3} n \right)^{-1/3}. $$

Because of the $1/r$ nature of the Coulomb potential, the equilibrium properties of the OCP with temperature $T (= T_1 = T_e)$ which deviate from ideal gas behavior are governed by

$$\Gamma = (Ze)^2 \beta / a$$

where $\beta = (k_B T)^{-1}$; $k_B$ is Boltzmann's constant.

$\Gamma$ is a measure of the amount of correlational (potential) energy in the plasma relative to the amount of kinetic energy present. With this relation we can see that the system will be strongly coupled if $\Gamma$ is of order one or greater.

The one component plasma (OCP) is a benchmark problem in the study of systems interacting via long range potentials. The model has been the subject of much investigation, especially in the last decade, not only in regard to the understanding of abstract systems, but also with respect to a more accurate description of real physical objects. In particular white dwarf stars, neutron stars and heavy planet cores are objects in which matter is in a state similar to that typified by
the OCP model. The plasma produced in inertial confinement fusion experiments can be dense and "cool" enough to be considered strongly coupled.

The OCP model should be useful in interpreting diagnostic data from such experiments.

One of the primary tools in plasma diagnostics is the scattering of laser radiation from the plasma. The direct interpretation of the scattered light data is accomplished via the Fourier transformed density-density time correlation function, which in this case describes the differential cross-section of electromagnetic wave scattering from charged particles.

There are various methods available for calculating this dynamic structure factor. However, few of these methods are suited for strongly coupled plasmas. The generalization of the equation of hydrodynamics to include Coulomb interactions in the system assumes that the non-hydrodynamic plasma modes are collisionally dominated, an assumption that ignores the independence of plasma oscillations from conserved variables. Hence a microscopic kinetic description of the full phase space density is required to develop a first principle theory applicable to a plasma in a strongly coupled state.

The well known perturbation expansions and low density kinetic approaches to obtaining a kinetic equation for the phase space density are inadequate for strongly coupled systems. A somewhat better suited approach involves transforming the Liouville equation into a form of a
Figure Comparison of the JCP model with systems of physical interest.
generalized Langevin equation that provides a point of departure for further approximation. The result is a kinetic equation involving a memory function or kernel containing the static correlation information as well as the difficult collisional dynamics.

The separation of the static and collisional parts of the memory function and the ability to choose a collision term that dynamically evolves from the exact static correlations statics allow one to utilize the recently available "exact" Monte Carlo (MC) equilibrium data directly in the solution. In the cases where the computer data was compared with a Debye-Hückel equilibrium correlation, exact in the long wavelength and small coupling regimes, the consequent dynamic structure factor was found to be in closer agreement with "exact" particle dynamics simulation results when the MC correlations were used. In view of the short wavelength difference between the Debye-Hückel and MC tabulations, this conclusion should be expected. But the point is that since equilibrium data is fairly easy to generate (as opposed to wavelength-frequency-coupling specific molecular dynamics simulations), a kinetic representation should make full use of such information.

A variety of collision models have been developed within the kinetic equation framework and applied to the analysis of the OCP. All are capable of using the computer-evaluated static correlations, and all allow the kinetic equation to be solved exactly. In the course of these solutions we found that the mean field or Vlasov approximation, which completely ignores the dynamics of the memory function, is not
adequate for long wavelengths or large coupling ($\Gamma$). The Fokker-Planck-like Lenard-Balescu model allows the inclusion of dynamics in the simple form of a collision time. As long as small wave numbers are avoided and the collision frequency is derived from available molecular dynamics data, the spectrum of $S(k, \omega)$ is improved dramatically. The Fokker-Planck approximation, however, does not satisfy the collisional invariant of momentum, hence the anomalous results at long wavelength.

The exact calculation of the collision term to second order in frequency modified by the inclusion of two unknown relaxation times—a model that does satisfy conservation of momentum—is relatively easy to accomplish. This modified Duderstadt-Akcasu-Linnebur (DAL) model makes use of more known computer information (via the "exact" shear viscosity) and gives results that are in very good agreement with simulation calculations over a wide range of wave number and coupling.

The Jhon-Forster (JF) model is an extension of the DAL model to include the energy collision invariant. Due to the fact that the OCP is dominated by the plasma oscillations and that charge and mass fluctuations in the OCP are equivalent, the energy modes do not couple efficiently to the mass density modes. The consequence is that the JF model provides only slight improvement over the DAL model except at very large coupling and very small wave number. The amount of calculational effort expended in procuring this slight advantage along with the need to obtain yet another transport coefficient and still unavailable static correlations indicate that the JF solution is more extensively
Figure $S(k, \omega)$ from DAL, FP, and Vlasov models using MC statics at $\Gamma = 2$, $q = 1.85$. 
Figure $S(k, \omega)$ from DAL, FP, Vlasov models using MC statics at $\Gamma = 110$, $q = 6.19$. 

$\Gamma = 110$

$Q = 6.19$
Figure S(K,\omega) from Dal, FP, and Vlasov models using MC states

at $\omega = 155$, $\vartheta = 3.09$ and $\vartheta = 1.85$.
approximated and not as efficient as the DAL calculation. The results can even be worse for shorter wavelengths.

The dynamical collision models require known presumed exact transport coefficients as input to insure correct behavior or to cover the low frequency, low wave number limit of an interpolative model. Thus, no information on these coefficients can be gained by utilizing these terms in a kinetic approach toward a calculation of transport properties. In addition, for the strongly coupled OCP we found that the long wavelength, long time values of $S(k, \omega)$ calculated from the four collision models were not always in good agreement with the temporal simulation.
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