COMPETITION EFFECTS IN LASERS

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FINAL REPORT
Grant No. AFOSR 76-2918
1 October 1975 - 30 September 1980
November 1980

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Research Sponsored by
Air Force Office of Scientific Research
Building 410
Bolling AFB, D.C. 20332

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### Report Documentation Page

<table>
<thead>
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<th>Report Date: November 1979</th>
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<td>Number of Pages: 15</td>
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**Title:**

**(and Subtitle)**

**COMPETITION EFFECTS IN LASERS.**

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AFOSR(NP)
Building 410
Bolling AFB, D.C. 20332

**Title of Report & Period Covered:**

Final

**Type of Report & Period Covered:**

**Performing Organization Report Number:**

AFCOSR-76-2918

**Report Date:**

November 1979

**Security Class. (of this Report):**

UNCLASSIFIED

**Distribution Statement:**

Approved for public release; distribution unlimited.

**Summary:**

This is the Final Report on research carried out under AFOSR Grant No. 76-2918 during the period October 1, 1975 to September 30, 1980. The research dealt principally with the ring laser and the coherence properties of laser light. Among the most interesting discoveries was the fact that a ring laser can simultaneously emit one beam of coherent light and one beam of incoherent light, and when the laser is homogeneously broadened, as is the dye ring laser, microscopic quantum fluctuations have macroscopic consequences in that they cause switching between two modes. The research resulted in 33 papers published.
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TABLE OF CONTENTS

SECTION PAGE

Abstract ................................................. 1
1 Introduction ........................................ 3
2 A Brief Review of Research Accomplishment ......... 3
3 Research Publications and Abstracts ................ 3
4 Lectures and Papers Presented at Meetings .......... 12
5 Personnel ............................................. 15

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Abstract

This is the Final Report on research carried out under AFOSR Grant No. 76-2918 during the period October 1, 1975 to September 30, 1980. The research dealt principally with the ring laser and the coherence properties of laser light. Among the most interesting discoveries was the fact that a ring laser can simultaneously emit one beam of coherent light and one beam of incoherent light, and when the laser is homogeneously broadened, as is the dye ring laser, microscopic quantum fluctuations have macroscopic consequences in that they cause switching between two modes. The research resulted in 33 papers published or in press and 1 book, and in addition, 40 lectures describing various aspects of the work were presented at scientific meetings.
1. Introduction

This final report covers research carried out under the general heading "Competition Effects in Lasers" in the period October 1, 1975 to September 30, 1980, supported by AFOSR Grant No. 76-2918.

During this five-year period the work resulted in 33 research publications in technical journals (or in press) in which AFOSR support was acknowledged, and one book. These are listed together with abstracts in Section 3 below. Also during this time, 40 lectures describing various aspects of the research were presented at scientific meetings and research seminars, and these are listed in Section 4. Five graduate students completed Ph.D. degrees with partial research support from the AFOSR grant.

2. A Brief Review of Research Accomplishments

A major part of the research was devoted to an understanding of the ring laser, and to answering the question how competition between counterpropagating traveling wave modes affects the character of the emitted light. A thorough theoretical treatment of the ring laser was developed (refs. 1, 7, 14, 15, 18, 20, 25, 27) and this was tested by photoelectric measurements of the photon statistics (refs. 9, 10, 13, 26, 27, 29) and the beam divergence (ref. 22). It was shown that a ring laser can simultaneously emit one beam of coherent light and one beam of incoherent light. When the gain medium is homogeneously broadened, as in a dye ring laser, the excitation switches spontaneously between the two modes under the influence of quantum fluctuations, and the nature of the switching phenomenon was investigated (refs. 18', 20, 25, 26, 27, 29, 30). The phenomenon of optical bistability in lasers was studied (refs. 19, 20, 21, 23, 26, 27), together with the coherence properties of dye lasers (refs. 2, 3, 11, 15, 17, 30). The question how cavity length affects the fluctuation properties of a single-mode laser was also investigated (refs. 8, 12). In addition, contributions were made to optical coherence theory (refs. 4, 16, 28, 31, 32, 33), to photoelectric detection (refs. 4, 6, 24), and to the problem of optical communication (ref. 5).

3. Research Publications and Abstracts

The following papers were published or are now in press as a result of the research carried out under the AFOSR grant:


It is shown that the mode coupling constant for a ring laser is not arbitrary but, to a first approximation, is completely determined by the detuning of the laser cavity from line center. In the limit far above threshold, it becomes equal to the negative of the intensity cross-correlation coefficient, and its functional dependence on detuning should therefore be demonstrable by photoelectric correlation measurements.

A method to stabilize the output frequency and intensity of a cw dye laser for periods of several hours is described. The dye laser frequency is controlled by a servo system which locks it to a Lamb-dip stabilized He:Ne laser to an accuracy better than 1 MHz with the help of a scanning Fabry-Perot interferometer. The dye laser intensity is controlled to better than 1% by direct control of the pumping source.


The fluctuation properties of a cw dye laser, pumped by an argon-ion laser, have been investigated by photon counting and two-time correlation measurements. The results show significant departures from the usual single-mode laser theory in the region of threshold and below. However, there are indications that the departures may be due to extraneous rather than intrinsic effects, and when these effects are subtracted out, the results are in substantial agreement with the predictions of the usual theory.


It is pointed out that photoelectric counting correlations in an electromagnetic field that are associated with induced and spontaneous emission at the source, and the so-called Hanbury Brown-Twiss effect, are indistinguishable effects. It is shown by examples that interference effects involving the stimulated emission may be regarded as a source of the counting correlations. The results reported in a recent experiment of Scarl and Smith are therefore readily understandable.


It is shown that, in principle, there exists a light source giving a higher information rate per symbol in an optical communication channel than a single-mode laser. Such a source emits a definite number of photons per symbol. A possible form for this source is suggested, and the corresponding information rates are compared.


It is pointed out that it is impossible to distinguish between photons and classical wave packets by measurement of photoelectric counting statistics for any state of a light beam for which a classical description exists. Therefore, an experiment with a thermal light source recently proposed by Karp as a test for the existence of photons can yield no information on this question, even if it should be able to distinguish between two different statistical models of a light beam. The quantum nature of the optical field can, however, be inferred from other photoelectric measurements, some of which are briefly discussed.

Expressions for the fluctuations and correlations of the light emitted by a ring laser with counter-rotating traveling wave modes are derived theoretically. It is shown that the competition of the two modes for the excited atomic population leads to negative correlations between their intensity fluctuations, whose magnitude depends on the detuning of the laser cavity from the atomic line center. As a result of the mode competition, the relative intensity fluctuations do not die out in general as the working point of the laser is raised above threshold at line center, and the emitted light does not become fully coherent as in a conventional laser. The two-time amplitude and intensity correlation functions of the light are calculated at line center, and are shown to be expressible in terms of the eigenfunctions and eigenvalues of a certain Schrödinger equation, in complete analogy with the theory of the single-mode laser. However, the intensity correlation time increases with pump parameter above threshold, unlike that for the single-mode laser.

8. "Laser Statistics and Cavity Length",

The effect on the optical field of changing the cavity length of a laser, while keeping the resonance frequency constant, is examined. It is shown that the fully quantized laser theory gives explicit expressions for the length dependence of the light intensity distribution, that are not shared by the semiclassical theory, and are quite different above, at, and below threshold. Some possible experiments are suggested.

9. "Mode Competition in a Ring Laser at Line Center",

The fluctuations and correlations of the light intensities associated with the two counter-rotating modes of a ring laser have been investigated by measurement of the photoelectric counting statistics. With the laser tuned to line center, it is found that the weaker mode intensity does not grow with increasing pump parameter above threshold and that its relative intensity fluctuations do not die out, as in a conventional laser, but become thermal instead. The cross-correlation coefficient tends toward a constant negative value. The results are all in substantial quantitative agreement with the theory.

10. "Intensity Fluctuations in a Two-Mode Ring Laser",

The light intensities of the two counter-rotating modes of a ring laser have been investigated by photoelectric counting measurements near the laser threshold and for various detunings. The results are found to be in good quantitative agreement with the theory. They confirm that, at the line center, competition effects prevent the light output in one mode from growing and becoming coherent as the pump parameter is increased. Instead the relative intensity fluctuations of this mode become unity, as for thermal light. Negative cross correlations between the intensity fluctuations of the two modes are also measured. The competition effects virtually disappear when the laser is detuned three natural linewidths from line center, and an intermediate situation is found for smaller detuning.

By an analytic examination of the photon probability distribution derived by Schaefer and Willis it is shown that, under the conditions for a second order phase transition, the dye laser statistics may be indistinguishable from those of a conventional laser, despite the presence of the triplet states. Under the conditions for a first order phase transition the statistics change, and for some combinations of parameters the change is manifest in the behavior of the normalized second factorial moment of the photon number. Curves are presented that should be directly verifiable by photoelectric counting experiments.


The question of how the light output of a laser and its fluctuations depend on the optical-cavity length at a given frequency is investigated theoretically for a laser oscillating in a single Gaussian mode. The treatment is quantum mechanical, and is based on the Scully-Lamb laser model, except that a perturbation expansion is used and possible cooperative atomic effects are included. It is shown, with the help of some reasonable approximations, that the probability distribution of the photon-occupation number can be cast into a form that is similar to the Scully-Lamb formula, but with coefficients that depend on cavity length in a more complicated way. Curves are presented that illustrate the behavior and should lend themselves to experimental test.


The ring laser has the interesting property, when it is tuned close to the center of the atomic resonance, that it can simultaneously emit both coherent and incoherent light beams. This phenomenon can be attributed to the competition between the two counter-rotating modes for the same inverted atomic population, a competition that one mode always wins in the end. The coherence and fluctuation properties of a two-mode ring laser have recently been investigated both theoretically and experimentally. The theory is based on the solution of a Fokker-Planck equation and is a generalization of an earlier treatment by Grossmann and Richter. For a laser operating at line center, the analysis is carried as far analytically as the well known treatments of the single-mode laser. Experiments that have been carried out are all in quantitative agreement with the theory, and show that, at the line center, atomic competition effects prevent the light output in one mode from growing and becoming coherent as the pump parameter is increased. Instead, the intensity fluctuations of this mode become thermal, and the correlation coefficient between the intensity fluctuations of the two modes tends towards -1. The competition effects die out when the laser is detuned a few natural linewidths from the atomic line center.


The intensity and amplitude correlation functions of the optical field of a ring laser at line center are calculated, under conditions when the two pump parameters corresponding to the counter-rotating traveling wave modes are not necessarily equal. This represents a generalization of earlier treatments by M-Tehrani and Mandel (1978) and Hioe (1978). The laser is assumed to be at rest. A perturbative technique is used to express the correlation functions for a small difference $\epsilon$ of pump parameters in terms of the solutions for $\epsilon = 0$. It is found that the cross correlations are unchanged to the first order in $\epsilon$, whereas the autocorrelations both of the light amplitude and of the light intensity are modified. Curves are presented to illustrate the behavior.


The photon-number distributions for a laser with a saturable-absorber and a dye laser are obtained by extensions of the Scully and Lamb optical-maser theory. The results are shown to be in correspondence with those of Lugiato et al. for the saturable-absorber laser and with those of Schaefer and Willis for the dye laser. Moreover, they are in a form which clearly shows the relation between the saturable-absorber laser and the dye laser. The thermodynamic potentials for the first-order phase-transition analogy are calculated and are found to be of the form predicted by Scott, Sargent and Cantrell using semiclassical equations.


It is shown that the total number of photons per unit cell of phase space, either within a blackbody radiation field or in the far-field of a blackbody source, is always of order unity.


It is shown that the general time-dependent equation for the matrix elements of the dye laser field is identical in form to that for the laser with a saturable absorber to order $1/n_0$, where $n_0$ is the mean number of photons at threshold. The parameters of the saturable absorber, have, of course, to be replaced by the appropriate parameters of the dye molecule.


The equations previously derived for the intensity fluctuations of a two-mode ring laser are applied to a homogeneously broadened laser. It is shown that the mean light intensity of the more lossy mode passes through a maximum and then tends to zero as the excitation is increased. The probability distribution of the intensity of each mode may exhibit two peaks, and spontaneous switching between the intensities associated with the peaks may occur. This is manifest in relative intensity fluctuations of the more lossy mode that can be much greater than unity. Estimates are given for the characteristic switching time.

The problem of optical bistability in a standing wave cavity in the steady state leads to a pair of coupled, nonlinear, ordinary differential equations for the forward and backward waves. Only numerical solutions have so far been presented for these equations. We give their exact analytic solutions and find good agreement with the numerical results. The exact solutions are shown to reduce to the mean field equation for the input and output fields in the double limits $T \to 0$ and $\alpha L \to 0$ for the mirror transmission and the linear absorption coefficient, respectively.

20. "Quantum Theory of a Two-Mode Laser with Coupled Transitions",

The quantum statistical properties of the optical field of a two-mode laser with two coupled transitions have been studied using a generalization of the Scully and Lamb (1967) theory. The photon number distribution and the mode intensity distribution are obtained in the steady state for a system of homogeneously broadened atoms in resonance with the laser field. It is shown that the mode coupling constant $\xi$ is unity and that near threshold the results of earlier treatments are recovered. Furthermore, it is shown that certain limit measures for the relative intensity fluctuations predicted by the semi-classical Fokker-Planck treatments based on the third order theory are valid even in the limit of arbitrarily high gain levels.

21. "Beyond the Mean Field Theory of Dispersive Optical Bistability",

The problem of dispersive optical bistability has so far been treated only in the mean-field approximation. A rigorous justification of the mean-field theory can only be obtained from exact solutions of the steady state Maxwell-Bloch equations which retain the spatial dependence of the field. In this paper, we present exact analytic solutions to these equations. We demonstrate that the mean field equation connecting the input and the output fields follows naturally from these solutions in the limits $T \to 0$, $\delta \to 0$ and $\alpha L \to 0$ for the mirror transmission coefficient, the detuning of the field from the cavity resonance and the linear absorption respectively, with $\frac{\alpha L}{2T}$ and $\frac{\delta L}{2cT}$ remaining finite. No constraint is placed on the detuning of the laser field from the atomic resonance frequency. We illustrate our results with the help of graphs showing the output intensity vs. the input intensity for different values of the relevant parameters. The effect of these parameters on the phase shift of the output field is also displayed.

22. "Effect of Mode Competition on Beam Divergence in a Ring Laser",

Beam profiles of the two light beams produced by a ring laser have been measured, corresponding to waves traveling in opposite directions around the ring. It is found that the two beam profiles do not change significantly as the cavity detuning from line center is reduced from several natural linewidths to zero, even though this is accompanied by large changes in the intensities and the fluctuations of the two light beams.

It is shown that the fluctuation properties of certain Zeeman lasers and amplifiers are expected to exhibit a sudden change when the coupling constant $\xi$ is varied near $\xi = 1$. The possibility of a phase transition at $\xi = 1$ is discussed and fluctuation properties both above and below this point are investigated using some previously derived equations for two-mode lasers.


It is shown that the formula for the probability of photoelectric counting of light in a short time in the presence of nonparalyzable counter dead-time effects may be inverted exactly. This allows the counting probability for an ideal dead-time-less counter to be derived from the probability measured with a real counter, without prior knowledge of the light intensity fluctuations.


A unified quantum treatment is presented for bidirectional two-mode ring lasers using the approach of Haken and co-workers. Equations of motion for the reduced density matrix of the field have been derived and used to study the fluctuation properties of the radiation field in different kinds of active media. The steady state photon number distribution for an inhomogeneously broadened gas ring laser is derived. It is shown that for equal losses the relative fluctuations and normalized cross-correlation of the photon numbers approach values $1/3$ and $-1/3$, respectively, for large excitations. For unequal losses the laser may emit typical single-mode laser radiation in the direction of the favored mode and typical equilibrium blackbody radiation in the direction of the weaker mode. Our results generally agree with the predictions of earlier treatments when the losses are equal. However, for unequal losses we predict different behavior for large excitations. Effects of detuning, spatial and temporal variation of atomic inversion on the fluctuation properties have been investigated in the coherent state diagonal representation of the density matrix, and steady state intensity distributions have been derived for various kinds of active atoms. It is shown that for inhomogeneously broadened solid state ring lasers spatial inhomogeneities may couple the modes strongly and the statistical properties of such lasers, characterized by a double-peaked intensity distribution and large relative intensity fluctuations, may be similar to those of a homogeneously broadened ring laser. Curves are presented to illustrate the behavior.


The probability distribution of photoelectric counting has been measured for each of the two counter-propagating travelling-wave modes of a ring dye laser. Each distribution is found to be double-peaked. The system exhibits quasi-bistability, with spontaneous switching between states, and the relative intensity fluctuation of the weaker mode is greater than unity. These features are characteristic of a first order phase transition.

The theory of a homogeneously broadened two-mode ring laser is discussed. Because the mode coupling constant can be twice as great as under conditions of inhomogeneous broadening, new effects like quasi-bistability appear, and the radiation field undergoes a phase transition near threshold that is of the first order. This is reflected in a double-peaked probability distribution of each mode intensity, and in alternate switching of the excitation between modes. A first passage time calculation shows that the dwell times should increase rapidly with excitation. These predictions are tested by photoelectric counting and other measurements on a dye ring laser. Although the observed photoelectric counting distributions closely resemble the theoretical ones, measurements of the dwell times reveal significant discrepancies between theory and experiment, that are believed to be due to backscattering. The adequacy of a theory based on two-level atom models in accounting for the behavior of a dye ring laser is briefly discussed.


Photoelectric correlation measurements of optical fields are now becoming commonplace and, as is well known, they require fourth order correlation functions of the field for a description. Nevertheless, the fourth order theory of optical coherence has not received a great deal of attention. This is the subject of the present paper. When interference effects are combined with intensity correlations, we encounter correlation functions of the general form

$$\Gamma^{(2,2)} = \langle V^{*}(r_1,t_1) V^{*}(r_2,t_2) V(r_3,t_3) V(r_4,t_4) \rangle,$$

which are functions of four space-time arguments. Some properties of such fourth order correlations, as they relate to interference and intensity correlation effects, are derived. The space arguments are generally suppressed in the calculation. The correlation functions are expressible as multiple Fourier transforms of certain fourth order spectral densities, which are functions of three frequencies in the stationary state.

The superposition law for fourth order spectral densities is derived for independent light beams, and it is shown that the fourth order spectral densities exhibit the reproducing property under superposition only when the light obeys thermal statistics. Next the superposition law is found for the spectral densities of the photoelectric signals generated by light beams falling on photodetectors. Investigation of the photoelectric signal spectrum yields information on the spectrum of the light intensity fluctuations, and removes the measurement from the domain of optics to the domain of electronics. It is shown that the photoelectric spectral densities reproduce under superposition when the two light beams in question obey a certain factorization condition, that is analogous to the cross-spectral purity condition for second order correlations. Finally, it is shown that, under certain circumstances, all the relevant fourth order correlation properties of the optical field are expressible in terms of simpler two-frequency spectral functions.

The effect of backscattering on the behavior of a dye ring laser is investigated by photoelectric counting measurements. It is found that there exist two laser regimes, a low backscattering regime in which the laser exhibits meta-bistability and random mode switching, and a high backscattering regime in which switching is suppressed. In the latter case the photon statistics are similar to those of a single-mode standing wave dye laser.


The distribution of first passage times is calculated for a homogeneously broadened two-mode laser, that is characterized by a bistable potential. As a consequence of quantum fluctuations, such a system tends to switch spontaneously between the two metastable states. The results of the calculation are compared with first passage time measurements of a two-mode dye laser.


A measure of the time for which intensity correlations persist in a stationary optical field, suitable for both classical and quantum fields, is proposed, and illustrated by several examples.


An expression is derived for the general form of the cross-spectral density of a field that is spatially fully coherent, throughout some region of space, at some particular frequency \( \omega \).


Two commonly used envelope representations of an electromagnetic wave are compared. The analytic signal representation is shown to be preferable, in the sense that it leads to a more slowly varying envelope.


4. Lectures and Papers Presented at Meetings

The following lectures dealing with the research were presented at various scientific meetings and research seminars:


2. "Do We Need Photons in the Optical Domain?", L. Mandel, Department of Physics Colloquium, S.U.N.Y. at Buffalo, Amherst, N.Y., October 30, 1975.

3. "Do We Need Photons in the Optical Domain?", L. Mandel, Department of Physics Colloquium, S.U.N.Y. at Stony Brook, Stony Brook, N.Y., November 12, 1975.


8. "Do We Need QED in Optics?", L. Mandel, Physics Division Colloquium, Argonne National Laboratory, Argonne, Ill., June 4, 1976.


   L. Mandel, Department of Physics Colloquium, Rochester Institute of Technology, Rochester, N.Y., November 3, 1976.

14. "Fundamental Limits of an Optical Communication Channel",
   L. Mandel, Seminar at Wright-Patterson AFB, Dayton, Ohio, March 10, 1977.

15. "Correlation Functions of the Light Emitted by a Two-Mode Ring Laser",


22. "Laser Photon Statistics and Cavity Length",
   C.-Y. Huang, Quantum Optics Seminar, University of Rochester, Rochester, N.Y., February 14, 1978.


31. "Are Atomic Quantum Jumps for Real?", L. Mandel, Colloquium at Department of Physics, Dalhousie University, Halifax, N.S., Canada, January 16, 1980.


33. "Experiments on Resonance Fluorescence from a Single Atom, or Are Atomic Quantum Jumps for Real?", L. Mandel, Seminar at Department of Physics, University of Toronto, Toronto, Canada, March 25, 1980.

34. "Length Dependence of Laser Photon Statistics", Chour-Yih Huang, Seminar at Department of Physics, Emory University, Atlanta, Ga., April 8, 1980.


38. "Photoelectric Correlations and Fourth Order Coherence Properties of Optical Fields",
L. Mandel, International Commission for Optics Conference, Ensenada, Baja California, Mexico, August 5, 1980.

39. "What's Going on in Quantum Optics?",
L. Mandel, Graduate Research Seminar, Dept. of Physics and Astronomy, University of Rochester, Rochester, N.Y., September 19, 1980.

40. "Spontaneous Switching in Homogeneously Broadened Lasers",
Rajarshi Roy, Quantum Optics Seminar, Department of Physics and Astronomy, University of Rochester, Rochester, N.Y., September 23, 1980.

5. Personnel

The following people contributed to the research carried out under the AFOSR grant:

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