I. Research Objectives

The objective of the supported research was a continuation of the principal investigator's analysis of decision processes with arbitrary decision sets, with special emphasis on two classical payoff functions. The first was a process with single fixed goal in which the objective is to maximize the probability of reaching the goal, and to minimize the expected time to the goal. A specific objective was to determine whether one can do as well with stationary strategies as he can with strategies which take the whole past into account. The second object of study was the average reward payoff in finite state decision processes, a specific objective being to determine whether or not strategies based only on the current time and state are as good as those which take the whole past into account.

II. Status of the Research

A complete, affirmative answer was found to the first question; in every dynamic programming or gambling problem with single fixed goal and finite state space, there exists a stationary strategy which not only uniformly (nearly) maximizes the probability of reaching the goal, but also uniformly (nearly) minimizes the expected time to the goal. Techniques in the proof of this result were considerably generalized to answer several questions about decision processes with arbitrary state spaces and total-cost criteria. It was shown that in a countable state decision process with non-negative costs depending on the current state, the action taken, and the following state, there is always available a Markov strategy which uniformly (nearly) minimizes the expected total cost. If the costs are strictly positive and depend only on the current state, there is even a stationary strategy with the same property.

Investigations of these results led peripherally to several results in optimal stopping theory, and in classical probability theory. Universal, best possible constants were found which compared the optimal expected return of a decision maker with the expected supremum of a sequence of random variables. For example, it was found that for every sequence of independent random variables taking only values between zero and one, the difference between the optimal stop rule expectation and the expected value of the supremum of the random variables is no more than one-fourth. Results in classical probability stemming from this research include a stronger form of the Borel-Cantelli Lemma, and a very general conditioning principle for strong laws which conclude the partial sums converge almost surely.

For the question of existence of good Markov strategies in decision processes with average reward criteria, various partial results have been obtained, including examples showing that the limit of good strategies for the discounted reward payoff is not necessarily average-reward good. This research is still in progress and it is hoped that a complete answer to the finite state case is not far away.
III. List of Publications


IV. Spoken Papers

A. Presented

1. "Betting Against a Prophet," seminar in Mathematics Department at University of California at Berkeley, August 1979.


6. , Probability Seminar, Mathematics Department, Georgia Institute of Technology, October 1980.


B. Scheduled


The principal investigator carried out research in probability theory, and, in particular, in the theory of decision processes. First, it was shown that in every finite state decision process (gambling problem) with single fixed goal, there always exists a stationary strategy which not only (nearly) maximizes the probability of reaching the goal, but (nearly) minimizes the expected time to the goal. This result was considerably generalized to include decision processes with arbitrary state spaces and total cost criteria.
Investigations of these processes led to results in optimal stopping theory, and in classical probability theory. Universal, best possible constants were found which compared the optimal expected return of a decision maker with the expected supremum of a sequence of independent random variables. A generalization of the classical Borel-Cantelli Lemma was found, as was a very general conditioning principle for strong laws of several forms.

The question of existence of good Markov strategies in finite state decision processes with average reward criteria was addressed, and various partial results were obtained, although the general case was not settled.