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FOR GEODESY

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This paper reviews the development status of gravity gradiometer instrumentation and the utilization of these gradiometers in earth orbiting artificial satellites for geodesy. Recent studies have shown that gravity gradiometers in artificial satellites operated in a low altitude polar orbit could produce in 20 to 30 days a complete map of the higher order harmonics (25 to 75 degree and order) of the earth's gravity field provided the instrumentation flown has a sensitivity of 0.01 EU (10^{-11} \text{sec}^{-2} = 1 \text{ngal/m}) for a 35 sec integration time. During the past decade gravity gradiometer development efforts have produced instrumentation which have measured gravity gradients in the laboratory and in orbit and which can be made to operate at the 0.01 EU sensitivity level in an orbital environment. Special emphasis is placed on the instrument concepts that can easily be increased in size from their present configuration to obtain the desired sensor signal-to-noise and which use spacecraft rotation modulation of the gravity gradient signal to aid in reducing externally generated noise to below the 0.01 EU level.
INTRODUCTION

This paper reviews the present status of the concept of using artificial satellites containing on-board gravity gradiometer instrumentation for the direct measurement of the gravity field in orbit and the use of this gravitational gradient data for the purposes of geodesy.

First, papers will be examined that have investigated the characteristics of the gravity gradient field at orbital altitudes, especially the probable field strength as a function of harmonic order and altitude. This will determine some of the important spacecraft and instrument parameters. Next, the many extant spacecraft and mission design studies will be summarized to show the constraints on the data collection and the instrumentation produced by the practical considerations of spacecraft engineering and orbital dynamics. Then the papers summarizing the state of development of gravity gradient instrumentation will be examined, with special attention to those instruments that can be readily modified for orbital use and which have received preliminary study of their behavior in orbit.
GRAVITY GRADIENT DATA AT ORBITAL ALTITUDE

The possibility for the development of gravity gradient instrumentation suitable for orbital use in artificial satellites has lead to a number of studies whose purpose is to obtain estimates of the gravity gradient field strength at orbital altitudes as a function of the harmonic order and measurement altitude. Most of these studies were carried out by collecting the surface gravity anomaly data that are available at high resolution and accuracy over certain regions of the earth (America and Europe) and extrapolating it upward to the orbital altitude. The simulation studies then determine what spacecraft, mission, and instrument parameters would be necessary to obtain a sufficiently accurate measurement of these data so that the data can be reduced to a form usable in geodesy.

The pioneering paper on the subject was that by Köhnlein [1967] who calculated the global gravity gradient field in $10^0 \times 10^0$ blocks using the satellite orbital gravity data then available. During the intervening period there were some studies by Savet, et al. [1967], Ganssle [1967], Thompson [1970], and Bell [1970, 1971] estimating the lunar gravity gradient at orbital altitude but it has not been until recently that attention has returned to the earth.

The first of the recent estimates was based on extrapolations of the rule-of-thumb proposed by Kaula [1968] which states that the relative rms amplitude of an individual normalized harmonic of degree $l$ follows a curve given by $10^{-5}/l^2$. This rule was known to be followed closely out to degree 15 but for these estimates it was extrapolated out to degree 75.
Kaula [1971] has used the rule to estimate that the effect of all harmonics of degree \( I \) would produce about 0.01 EU (Eötvös Unit, 
\[ 1 \text{ EU} = 10^{-9} \text{ gal/cm} = 10^{-9} \text{ sec}^{-2} \approx 10^{-10} \text{ g's/m} \]) for \( I = 40 \) and 0.005 EU for \( I = 75 \) at 260 km altitude.

Glaser and Sherry [1972c], also using extrapolations of Kaula's rule, calculated the gravity gradient signal remaining in the higher orders as a function of harmonic degree and altitude. These data are shown in Figure 1. They also compared the relative accuracy of doppler, altimeter, and gradiometer techniques for obtaining gravity field data from orbital satellites (Figure 2). They show that each technique has its region of applicability and the three techniques should be considered complementary rather than competitive techniques.

Forward [1972a, b] also has used the Kaula rule to estimate the gravity gradient field of the earth at orbital altitude (Figure 3). He pointed out that the gravity gradient field, which is the spatial derivative of the gravity acceleration field, is more sensitive to the short wavelength components which have comparatively high density differences. He also showed that doppler satellite tracking techniques, since they utilize the relative velocity of the satellite, which is the square root of the spatial integral of the gravity acceleration field, are most suitable for measurement of the harmonics of lower order, which have lower density differences, but because of their longer wavelength allow a longer integration time.

Forward estimated that if the present levels of instrumentation technology were extrapolated to a doppler velocity measurement of 0.005 cm/sec at 30 sec and a gravity gradient measurement of 0.01 EU
Fig. 1. Gravity Gradient Signal Remaining by Harmonic Degree and Altitude (from Glaser & Sherry 1972c).
Fig. 2. Regions of Best Sensitivity for Altimeter, Doppler, and Gradiometer by Harmonic Degree and Altitude (from Glaser and Sherry, 1972c).
Fig. 3. Vertical Doppler Velocity, Gravity, and Gravity Gradient in 250-km Orbit. (Forward, 1972b).
at 30 sec, then the doppler technique would give better information at the lower harmonics and the gravity gradient technique would give better information at the higher harmonics (out to order 75) with the crossover being at order 35. This work agrees closely with that of Glaser and Sherry [1972 c].

After the first series of papers using extrapolations of Kaula's rule, another series appeared which used surface gravity data to calculate the gravity gradient field at orbital altitudes.

One of the first was by Sandson and Strange [1972] who calculated the gradient at an altitude of 300 km from $1^\circ \times 1^\circ$ surface gravity data and a 12th degree and order satellite gravity field. Their results (Figure 4) indicated that accuracies better than 0.1 EU were required to provide useful improvements to present geophysical knowledge.

Hopkins [1972] developed a mathematical model for the combination of gravity gradient data into gravitational models obtained from satellite tracking data to obtain a combined solution of greater scope. The supporting simulations show the feasibility of using gradiometry data either alone or in a comprehensive earth-model solution to obtain the values of the harmonic coefficients. Hopkins concluded that although satellite borne gradiometry and doppler tracking data are usually considered complementary data sources (doppler tracking being most useful at longer wavelengths, while gradiometry is most useful at shorter wavelengths) that there is useful information contained in the gradiometer signal that can also be used in combination solutions at the longer wavelengths. Hopkins work reiterates the need for a low, near-polar orbit for a satellite gradiometer system.
Fig. 4. Unmodelled Anomalous Gravity and Gravity Gradient at Satellite Altitude of 300 km (from Sandson and Strange, 1972).
Reed [1973] recently completed a geodesy oriented investigation to develop procedures for the use of satellite gradient measurements in obtaining gravity boundary values and to determine if gradiometry can provide, with sufficient accuracy, discrete geopotential information equivalent to harmonic degree 90. The investigation relied heavily on computer simulation experiments in the form of simulated least squares solutions. The simulations investigated the two presently viable generic types of satellite gradiometry systems: (1) hard mounted system consisting of six orthogonal low-g accelerometers capable of sensing five independent components of the gravity gradient tensor, mounted in a satellite utilizing active attitude control and gradient torque stabilizations and (2) rotating satellite gradiometer system which produces a twice-spin harmonic oscillating signal that can be analyzed in terms of signal amplitude and phase as a function of the three components of the gravity gradient tensor in the plane of rotation of the satellite and is attitude controlled by spin stabilization.

The reference background gravity field used by Reed was the spheropotential of a high order spherop defined by a spherical harmonic series to degree and order 14. The anomalous gravity field then consisted of mean gravity anomalies referred to this high order spherop.

Reed's simulation experiments strongly indicate that the measuring sensitivities required to obtain the cross-gradient terms in the hard-mounted system are probably beyond practical limits for accelerometers used in a satellite gradiometer system. The simulation experiments demonstrated, however, that a rotating gradiometer system with a sensitivity and accuracy of 0.01 EU can satisfactorily resolve
2° x 2° mean anomalies (equivalent to degree 90) with an accuracy of 0.5 to 1 mgal at 250 to 300 km altitude.

The most recent and most thorough study of gravity gradients at orbital altitudes is that of Chovitz, Lucas, and Morrison [1973] which extends the previous work by the authors [Chovitz, et al., 1972] and serves to confirm the other earlier studies. The authors first obtained all available (over 20,000) 1° x 1° gravity anomalies and then filled in the remainder of the globe with 5° x 5° mean anomalies. Satellite tracks were then simulated over those regions with dense 1° x 1° coverage and the gravity gradient computed at 30 sec intervals. The computation was carried out through ℓ, m = 88. However, only the data through the 75 degree and order were used. The first result of the simulation was the confirmation of the rule-of-thumb developed by Kaula [1968] out to the 75th degree and order.

The data obtained from the simulated orbital passes were then analyzed by differing techniques to determine the accuracy with which the gravity field information could be extracted from the simulated gravity gradient data. They first used a simple averaging technique which indicated that in order to distinguish the combined harmonics of a single degree in the range of 60 or 70 a sensitivity of better than 0.01 EU is required by the instrumentation. They then applied a more sophisticated analysis procedure which uses a detailed harmonic analysis making use of the maximum entropy technique. The harmonic analysis reveals the existence of a large number of gravity gradient components with amplitudes that are larger than 0.03 EU and which are well separated in spatial frequency from each other. The authors
conclude that this more refined analysis indicates that an instrument sensitivity of 0.01 EU will not be at just the threshold of yielding useful information on the short components of the geopotential but well beyond it.

Glaser [1972 a, b, c, d] has looked at the problem of reducing the large amount of data that would be obtained from a satellite gravity gradiometer system. The total data set for a complete global map is relatively large because the analysis is attempting a 75, 75 harmonic fit instead of the usual 14, 14 or 20, 20 fit that is presently possible with satellite tracking data. Glaser and Sherry [1972] expressed concern in the first look at this problem, but the recent thesis by Glaser (1972 b) describes a data reduction procedure that will produce a reasonable accuracy in the reduced data in the presence of errors caused by integrating over blocks of data and by digitizing the signal amplitude at the 0.01 EU level. These two error sources studied by Glaser do not arise from fundamental considerations, but are produced by engineering constraints on the satellite.

As is pointed out in the above papers and in the JPL Satellite Gravity Gradiometer Study lead by Sherry [1972], the actual data obtained from the satellite will be integrated by the gradiometer response time constant (usually chosen at 30 sec for optimum sensor signal-to-noise) and will not be digitized much oftener than that (typically every 6 sec) in order to lower the data storage requirements on the satellite. The gravity gradiometer analog output will have a range of about 10,000 EU (assuming reasonable estimates for bias inputs caused by satellite dynamics and drag). A measurement of this signal to 0.01 EU
requires a 19 bit analog-to-digital conversion. Such converters are pushing the state-of-the-art and it would be desirable not to require a 20 bit converter. Glaser's recent work shows that these error sources will not cause a significant decrease in the accuracy of the reduced data. Glaser [1972 c] also emphasizes the need for a circular polar orbit with as low an altitude as possible to obtain the complete coverage at high resolution and high signal-to-noise that is desired.

In summary, the work on estimation of gravity gradients at orbital altitude indicates that for orbital gravity gradiometry it would be desirable to have gravity gradient instrumentation with a sensitivity of 0.01 EU or better at an integration time of 35 sec or better and the mission should be designed to obtain the data in a circular polar orbit with as low an altitude as possible.

The data obtained can be combined with other data for an accurate determination of the lower harmonics of the earth's gravity potential and, for the first time, would give global coverage at the 0.5 to 1 mgal level of the higher harmonic components out to degree and order 75 (or optimistically 90).

**ORBITAL GRAVITY GRADIOMETER MISSIONS AND SPACECRAFT**

During the past seven years there have been a series of engineering feasibility and design studies, nearly all carried out under NASA sponsorship, on the design of a spacecraft, mission, and gravity gradient instrument for orbital gravity gradiometry. Because of the predominant influence of the Apollo program in the mid-1960's, most of these studies, such as those by Beil [1970, 1971], Canssle [1967],
Savet, et al. [1967], and Thompson [1965, 1966, and 1970] were based on a lunar orbiter mission. Only the studies by Forward [1973] and Sherry [1972] have concerned themselves specifically with the different problem of earth orbital gravity gradiometry.

The work by Forward [1973] primarily was concerned with the design, fabrication, and test of a gravity gradiometer suitable for earth orbital use. However, a number of preliminary mission and spacecraft design studies were carried out prior to the design of the sensor to fix some of the sensor parameters.

The mission described by Forward would use a Scout with a 12-inch payload shroud to launch two orthogonally oriented, spin stabilized satellites into a 330 km circular polar orbit 15 to 20 days before the vernal or autumnal equinox. Each satellite would carry an 8 kg, 80 cm diameter gradiometer with a sensitivity of 0.01 EU at 35 sec integration time. The orbital lifetime would be short, but during that time the satellites would obtain at least two complete maps of the vertical gradient and the horizontal gradients, both along and across the orbital track, with a resolution of about 270 km (540 km wavelength or degree 75).

Forward required a low orbit with orbital parameters in which the orbital tracks interleave so complete coverage is obtained in a period shorter than the orbital lifetime, and where the track spacing is matched to the swath width (equal to the altitude). A set of orbital parameters exists that fits these requirements reasonably well. Swenson [1971] and King [1972] have shown that at an orbital altitude of 270 km, there exists an "integer orbit." The orbital track repeats upon itself after exactly 16 orbits. This can be a polar orbit, with 16 orbits per sidereal day or a sun synchronous orbit (at a
slightly different altitude and inclination) with 16 orbits per solar day. If the altitude is slightly higher or lower, then the orbital track drifts so that the sixteenth orbit is displaced to one side or the other of the first track. These offset orbits finally begin to repeat after a number of days when the drift has caused the satellite track to overlap the second ground track. Two of these orbits are of interest. They repeat after about five days, and their track spacing is approximately equal to the altitude. One is a polar orbit at about 320 km which repeats after 79 orbits and the other is a polar orbit with altitude of 220 km which repeats after 81 orbits (see Figure 5). The track spacing between the half arcs for both orbits is approximately 250 km, so that there is a good match between the track spacing and the swath width.

In reality, the orbital altitudes will decay because of drag, so that these simple orbital path models will not be followed exactly. One possibility is launching into a 330 km polar orbit and allowing the altitude to decay through these two altitudes to get overlapping coverage. A choice of a polar orbit rather than a sun synchronous orbit would obtain full coverage of the earth and provide for calibration points twice per orbit at the two poles. The orbital lifetime estimated for the mission is 30 to 50 days. The time spent near 320 km would be long enough to obtain good coverage of the earth at that resolution (640 km wavelength or degree 62). As the altitude decreases, the resolution would improve steadily. There would be a substantial amount of coverage near 220 km altitude with excellent resolution (440 km wavelength or degree 90), but some coverage will be lost as a result of the rapidly decreasing altitude and because the track spacing at the equator of 250 km is slightly larger than the sensor resolution.
Fig. 5. Swath Pattern of Full Coverage Orbit (after King, 1971).
Forward also describes a preliminary design for the spacecraft. His spacecraft design is similar to, but less detailed, than one of the two designs developed later by Sherry [1972].

A Phase A study carried out by Sherry [1972] of JPL is the most recent and best of the spacecraft and mission design studies. Two mission flight profiles are discussed. Both cases utilize polar orbits (see Figure 6). In the first case, the orbital plane is positioned approximately perpendicular to the terminator (the earth-sun line is in the orbit plane). For a spinning cylindrical spacecraft, the angular momentum vector is perpendicular to the orbital plane, with the cylinder exposed to the sun. The spacecraft is in solar occultation approximately 40 min (depending on orbit altitude) per revolution.

The second case has the orbit plane positioned in the plane of the terminator (hence the name, terminator orbit) perpendicular to the earth-sun line. Just as in the first case, the angular momentum vector is positioned perpendicular to the orbit plane. However, in this mission a base end of the cylinder is constantly exposed to the sun. The sun rotates relative to the orbit plane and solar occultation will eventually occur, depending on orbital altitude.

The nominal mission sequence involves targeting for a 300 km circular orbit for both cases. Lifetime in this orbit is approximately 22 days.

Sherry [1972] reports two artificial satellite designs. Each design is compatible with either the rotating MESA gradiometer of Metzger and Allen [1972] or the rotating resonant torsional gravity gradiometer of Forward [1973], although the actual design assumed the torsional gradiometer for the instrument payload (see Figure 7).
Fig. 6(a). Eclipsing Orbit Mission Profile.

Fig. 6(b). Terminator Orbit Mission Profile.
Fig. 7(a). Rotating Resonant Torsional Gravity Gradiometer.

Fig. 7(b). Rotating MESA Gradiometer.
A cylinder was chosen as the basic shape of both designs to facilitate the mounting and spin requirements of the gravity gradiometer instrument. The design approach is a spin-stabilized vehicle whose spin axis is oriented normal to the orbital plane, and whose spin speed can be made compatible with the requirement of the rotating gradiometer which is securely mounted inside the cylinder. Carefully controlled spin speed provides the desired angular velocity to the gradiometer and the stability and inertial stiffness of the vehicle. The support structure and electronic components are arranged so that their mass will be distributed to maximize the moment of inertia about the spin axis. To enhance attitude stability of the satellite, the ratio of moment of inertia about the other two axes must be greater than one.

The design effort encompassed sufficient detail of the satellite to warrant confidence in the feasibility of the proposed configurations for both cases. Ample sensing and monitoring of system operational conditions and various engineering data were provided to assess the operation of the gradiometer. After release from the launch vehicle all subsystems are expected to operate continuously. Gradiometer sensor data will be taken continuously and stored in the onboard memory subsystem. Attitude sensor outputs and various engineering data will be sampled automatically at predetermined time intervals controlled by an onboard time clock and later telemetered to a ground station. The satellite is designed to perform for 40 days after having been put into a 300 km orbit circularized, in the case of Scout launch, by an orbit correction propulsion motor that will be jettisoned soon after completion of its burn.
One of the satellite designs configured in the JPL study is a right circular cylinder with overall dimensions of 1.22 m x 0.9 m, whereas the second (Figure 8) has cylindrical dimensions of 0.32 m x 0.9 m. Since length is the only major difference between the two designs, they had a nearly identical subsystem arrangement within the satellite. The packaging configuration was influenced primarily by the requirement to achieve a maximum value for the moment of inertia about the spin axis for a 0.9 m diameter vehicle to help maintain vehicle spin momentum and attitude stabilization.

As shown in the figure, the outer shell and the end covers are made of light gauge aluminum. The end covers along with the omni-antennas are removable to allow internal assembly operations. Attached to the outer cylindrical shell are four circular segments in each of two rings having channel cross sections. The gravity gradiometer is mounted and secured on four flanged supports, one for each arm of the sensor, by eight bolts and four dowel pins.

The thermal control subsystem is made up of both passive and active components. A multilayer thermal blanket is used to stabilize internal temperatures of the satellite. Passive methods, combined with electrical resistance heaters, are used to maintain the temperature requirements of the gradiometer.

The solar array cells are cemented on the cover of one end of the satellite, giving a total array area of approximately 0.6 m². As this type of satellite is flown in a terminator or near-synchronous sun orbit, the area of the array can be less than for the eclipse case. Since the solar array will be sun-oriented throughout the mission there is no need for a battery once the satellite is in proper orbital attitude.
Fig. 3. Earth Physics Gravity Gradiometer Satellite (Sherry, 1972).
The attitude control subsystem consists of sun and earth horizon sensors, nutation damper, and cold gas reaction jets. All of these components are mounted on the primary structure with cutouts in the cylindrical shell of the satellite to allow viewing of the sun and the earth's horizon. The reaction jets are mounted externally to the outside diameter of the satellite and are part of a one-piece assembly made up of nitrogen gas tank, manifold and regulator valves, and plumbing lines.

Attitude and spin speed determination and control is effected by ground control by computing and programming commands for the onboard reaction jet control system to provide the necessary corrections. For attitude reference a pair of redundant sun sensors and a pair of redundant earth horizon sensors were chosen to provide space references for vehicle attitude determination. The data acquired periodically from these sensors will be telemetered to the ground where computations will be made and commands sent to the satellite for corrective maneuvers. Additional information, such as satellite orbital position and velocity, will be derived from ground tracking data and used in the attitude determination process.

The nutation damper, a passive device, is part of the attitude control subsystem and provides nutation control. This device is mounted internally on the prime structure with its length dimension parallel to the spin axis. The device is a small diameter tube (about 25 mm), between 0.20 m and 0.30 m long and partially filled with liquid mercury.

The remaining electronics aboard the satellite are data handling and command and the telecommunications subsystems.
The study by Sherry [1972] also covers in detail the weight and power budgets, the data handling procedures (which are discussed in more detail by Glaser [1972 b and d] who was a member of the JPL team), and the launch and operational procedures.

In summary, Sherry concludes that a global gravity map obtained by a gravity gradiometer satellite in the near future seems not only an exciting possibility, but also is technically feasible given some technical advances that appear well within reach.

GRAVITY GRADIENT INSTRUMENTATION

The history of gravity gradiometry instrumentation goes back to Baron von Eötvös who in 1880 developed the torsion balance gravity gradiometer. Although used extensively for geophysical prospecting for many years, the balance was fragile and required a long setup and measurement time (typically one hour per station). The major difficulty with its use for geophysical prospecting, however, was the strong response of the Eötvös balance to local terrain effects. This strong response to nearby masses, although desirable in an orbital gravity gradiometer, was intolerable in field surveys and the Eötvös balance was replaced with sensitive gravity acceleration meters.

Further development in gravity gradiometers did not emerge until the beginning of the space age when spacecraft engineers started to investigate ways to determine "which way is up?" in a free fall orbit. Papers by Crowley, Kolodkin, and Schneider [1959], Streicher, Zehr, and Arthur [1959], Carrol and Savet [1959], Roberson [1961], and Savet [1962] pointed out that the gravity gradient field could provide
"direction" in space and suggested a number of instrumentation concepts to obtain this information.

In addition to the many papers from 1959 to 1962 on space navigation by gravity gradiometry, there has been a number of instrumentation papers on new concepts for surface gravity gradiometers. Some of these have been developed to the point of laboratory hardware.

Thompson [1970] developed a vertical gravity gradiometer based on the principle of a sensitive fused quartz balance with a capacitance pickoff. Working laboratory models have demonstrated the capability to measure gradients with a sensitivity of 1 EU. A portable field exploration model has been fabricated (Figure 9). Designs for use in spacecraft have been considered and should operate satisfactorily in an attitude stabilized spacecraft environment.

A vibrating string gradiometer was proposed by Thompson, Bock, and Savet [1965] and Thompson [1966] for the lunar exploration program. Of all the accelerometer principles, the vibrating string accelerometer is the only one that naturally lends itself to the design of a gravity gradient measuring instrument. In the vibrating string gradiometer (Figure 10) the two proof masses are identically suspended by their supports and restraining springs. The vibrating string between the two proof masses then senses the relative motion of the masses by changing its frequency of vibration as the tension in the string is changed. Although very promising in concept, and the obvious choice of many looking for orbital gravity gradient instrumentation in the early 1960's, the instrument was never developed. It is suspected that the relatively low sensitivity and the nonlinearity of the vibrating string as a transducer was a partial reason.
Fig. 9. Prototype Field Exploration Quartz Gravity Gradiometer (Thompson, 1970).
A cylindrical floated gravity gradiometer was described by Trageser [1971] for geophysical exploration applications. The sensing element of the gradiometer consists of a mass quadrupole in a precisely machined cylinder with a series of conducting lands (Figure 11). The cylinder is floated in a heavy oil as is done with a floated gyro. The gravity gradient field induces torques on the cylinder which are detected with capacitance pickoff fingers sensing the lands. A spherical version of this design suitable for planetary and asteroid applications in an attitude stabilized spacecraft was described in a review paper by Forward [1971]. The development of this instrument has been successful, and a recent paper by Trageser [1972] reports a sensitivity of a few EU. Although conceptually the floated gravity gradiometer could be increased in size to attain the sensitivity needed for earth geodesy, the requirement for precise machining and floatation of the entire sensor case makes very difficult any straightforward extrapolation of the present laboratory results to an orbital design.

Another recent gravity gradiometer design is that of Hansen [1971]. This uses four servocontrolled bubbles under a single quartz flat to measure the horizontal gradients of gravity (Figure 12). The gradiometer contains eight servomotors, controlled by currents sensing the motion of the four bubbles, which produce forces to level, bend, and twist the quartz flat until the bubbles stop moving. The three components of the gradient in the plane of the sensor can be read out from the currents in the force motors. Laboratory versions of the four-bubble gravity gradiometer have measured 10 EU. Although
Fig. 11. Floated Gravity Gradiometer Showing Float and Control Electrodes Inside Housing. (Forward 1971).
Fig. 12. Four Bubble Horizontal Gravity Gradiometer (Hansen, 1971).
possible free-fall versions of this gradiometer concept have been studied, the obvious gravity orientation of the device makes it unlikely that a suitable orbital version will be developed.

Many accelerometers could be considered for use as matched differential pairs in an orbital gravity gradiometer system. Some of the more recent ones are the freely falling sphere concept of Savet [1968, 1970], the quartz torsion fiber pendulous accelerometer of Block, Dratler, and Bartholomew [1972], and the Model VII pendulous sea gravity meter and the miniature electrostatic accelerometer (MESA) discussed by Metzger and Allen [1972]. The Model VII gravity meter has detected the low acceleration levels ($10^{-10}$ to $10^{-11}$ g's) required for surface gradiometry and a rotating gravity gradiometer system using these instruments has been proposed for airborne gravity gradient measurements. Of all the known accelerometers, the only one that has received sufficient development and study to warrant its serious consideration for orbital gravity gradiometry, with its extreme ($10^{-12}$ g) sensitivity requirements, is the MESA, operated in the rotating gravity gradiometer mode.

The MESA accelerometer has accumulated thousands of hours of orbital performance data on numerous flights. Data on most of these flights are classified and not available for general publication. The instrument, as presently configured for orbital use, was optimized for low level measurements of satellite drag and for calibration of the low level thrust of ion engines. It is notable that even though the orbital experiments were not optimized in the design of the instrument or in
placement of the instruments on the spacecraft, they still were able to measure the gravity gradient of the earth from an orbiting artificial satellite.

The data shown in Figure 13 are the acceleration in micro-g's in the output of a MESA during the SERT II ion engine tests before the engine was turned on. The MESA was positioned approximately \( l = 2.4 \text{ m} \) from the center of mass of the spacecraft, which was gravity gradient stabilized. In a gravity gradient stabilized spacecraft the spacecraft is rotating with respect to inertial space. For this case, Berman and Forward [1968] have shown that the gradient of the rotation must be added to the earth's gravity gradient. Because the two are related through the orbital equation the net vertical gradient is \( \Gamma = 3 \frac{GM}{R^3} \).

For the orbital altitude of about 1000 km the net vertical gradient was about 3000 EU (3\( \times 10^{-6} \text{ sec}^{-2} \)) which accounts for the accelerometer reading of

\[
a = \Gamma l \approx 7 \times 10^{-6} \text{ m/sec}^2 \approx 0.7 \mu \text{g}
\]

observed during the test.

The above measurement of 3000 EU to an accuracy of perhaps 100 EU is far from a desired measurement accuracy of 0.01 EU or better. However, it does show that even nonoptimized experiments

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\(^a\text{Miniature Electrostatic Accelerometer (MESA), Bell Aerospace Co., Buffalo, N.Y. 14240, USA. Unnumbered brochure, undated.}\)
Fig. 13. MESA Output Prior to SERT II Test Showing Earth Gravity Gradient Signal.
can produce gravity gradient data in orbit and give us confidence that an experiment utilizing instrumentation and spacecraft designed for orbital gravity gradient measurement should produce information of value to the science of geodesy.

Rotating Gravity Gradiometers

The rotating gravity gradiometer concept was independently developed by Forward [1963, 1965] and Diesel [1964] and is used in the earth geodesy orbital gravity gradiometer studies being conducted by Forward [1973], Metzger and Allen [1972] and Sherry [1972]. The basic concept is that the deliberate rotation at $\omega$ of a structure responsive to the gravity gradient will induce alternating differential forces in the gradiometer at twice the rotation frequency ($2\omega$).

The definitive work of Diesel [1964] showed that not only was the gravity gradient signal modulated by the rotation to produce an ac signal at $2\omega$, but that many of the error sources were either not modulated or were modulated at $\omega$ and thus could be separated from the desired gravity gradient signal by frequency filtering.

The rotating gravity gradiometer concept described by Forward [1963] goes one step further, in that mechanical resonance is used in the gradiometer. The mechanical resonance frequency of the gradiometer is chosen at exactly twice the rotation speed. Thus, in the resonant gradiometer concept, the gravity gradiometer is driven at its resonance frequency and the signal is amplified by the mechanical resonance before its detection by the piezoelectric strain
transducers. This resonance concept was also used by Chobotov [1968] who described a radially vibrating rotating gravity gradiometer which was stable, unlike the early radial vibrating designs of Forward [1963].

The development of the resonant rotating torsional gravity gradiometer has been in progress for almost a decade. After the initial studies by Forward [1963] on the general concept, the work proceeded to an experimental phase which developed the instrumentation concepts [Forward, et al., 1966]. A major milestone was a paper by Forward and Miller [1967b] which showed that the sensor structure, transducers, and electronics could detect actual gravity gradient forces. Later improvements such as the torsional mass quadrupole configuration of Bell, et al. [1965] lead to laboratory demonstrations [Bell, 1970 and Bell, et al., 1971] that showed that the rotating torsional gravity gradiometer could measure real gravity gradient fields of 1 to 2 EU in a time compatible with lunar orbit (10 sec). This work led to a development program for a gradiometer suitable for airborne gravity gradiometry [Forward, et al., 1967a; Ames, et al., 1970, 1972, and 1973, and Rouse, 1971] as well as a development program for a gradiometer suitable for earth geodesy [Forward, et al., 1973].

The gravity gradient instrumentation developments of the past decade have left us with two gradiometer concepts that have survived the scrutiny of the many studies and the many instrument development efforts. These are the rotating MESA gradiometer described by Metzger and Allen [1972] and the rotating resonant torsional gravity gradiometer described by Forward, et al. [1973].

Rotating MESA Gradiometer. The rotating MESA gradiometer of Metzger and Allen [1972] consists of four accelerometers arranged
tangentially around the perimeter of a spinning spacecraft (Figure 14) as in the method described by Diesel [1964]. The combined output of the accelerometers is given by

\[ a = 2l \left[ (\Gamma_{xx} - \Gamma_{yy}) \sin 2\omega t + 2\Gamma_{xy} \cos 2\omega t \right] \]

where \( \Gamma_{xx}, \Gamma_{yy}, \Gamma_{xy} \) are the components of the gravity gradient tensor and \( l \) is the radius arm to each accelerometer.

Since the measurement of 0.01 EU requires the detection of \( 10^{-12} \) g over a meter, the major concern of the MESA gradiometer system design is to insert adequate mechanisms to maintain the relative accelerometer scale factor and bias constant to \( 10^{-12} \) g's. Metzger and Allen [1972] propose feedback loops to null out the accelerometer pair outputs at the rotation frequency (see Figure 15) and at other induced modulation frequencies that are different than the \( 2\omega \) gravity gradient signal frequency. In this way they plan to maintain the necessary balance between the accelerometers.

Metzger and Allen have carried out an error analysis of the rotating MESA Gradiometer system. The results of their analysis are shown in Table 1. As can be seen, the major error source is the random mechanical thermal noise in the MESA masses. This will likely be true for any gravity gradiometer system attempting to reach the 0.01 EU level.

**Rotating Resonant Torsional Gravity Gradiometer.** The rotating gradiometer of Forward [1963] and Bell, et al. [1970], is a resonant cruciform mass-spring system with a torsional vibrational mode.
Fig. 14. Orbital MESA Gravity Gradiometer (Metzger and Allen, 1972).
Fig. 15. Typical Balance Loop for Automatic Scale Factor Balance Using \( \Omega \) Output of Summing Amplifier (Metzger and Allen, 1972).
Table 1. Error Analysis Summary for Rotating MESA Gradiometer

<table>
<thead>
<tr>
<th>Error Category</th>
<th>Error in EUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Noise</td>
<td></td>
</tr>
<tr>
<td>Mechanical</td>
<td>0.005</td>
</tr>
<tr>
<td>Electrical</td>
<td>0.002</td>
</tr>
<tr>
<td>Magnetic Sensitivity</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Temperature Sensitivity</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Linearity</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Variation of Drag at $\Omega$</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Variation of Drag at $2\Omega$</td>
<td>&lt;0.002</td>
</tr>
<tr>
<td>Coning of Spin Axis</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Variation of Solar Pressure at $2\Omega$</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>$2\Omega$ Modulation of Spin Speed</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

Data Handling

Phase Reference

[Metzger and Allen, 1972]
(Figure 16). In operation the sensor is rotated about its torsionally resonant axis at an angular rate $\omega$ which is exactly one half the torsional resonant frequency. Forward [1965] has shown that when a gravitational field is present, the differential forces on the sensor resulting from the gradients of the gravitational field excite the sensor structure at twice the rotation frequency. The differential torque $\Delta T$ between the sensor arms at the doubled frequency is coupled into the central torsional flexure. The strains in this flexure are sensed with piezoelectric strain transducers which provide an electrical output.

Since the rotating gravity gradiometer moves through the gravity gradient field and obtains a continuous sample of the field components in its plane of rotation, the output of the gradiometer contains two independent measurements of certain components of the gravity gradient field tensor. The two measurements appear as two sinusoidal signals in quadrature

$$\Delta T = \frac{m t^2}{4} \left[ (\Gamma_{xx} - \Gamma_{yy}) \cos 2\omega t + 2 \Gamma_{xy} \sin 2\omega t \right].$$

One output is a measurement of the difference between two of the diagonal components and the other measures the cross product component of the gravity gradient tensor in the coordinate frame of the sensor. The output of the resonant torsional gravity gradiometer is seen to be the same as that of the rotating MESA gradiometer (to within a $90^\circ$ phase shift).

The feasibility of the rotating resonant torsional gravity gradiometer concept was demonstrated in a series of laboratory experiments conducted from 1967 through 1969 [Bell, et al., 1970]. This work was directed toward demonstrating sensor performance under simulated lunar orbital conditions.
Fig. 16. Method of Operation of Torsional Gravity Gradiometer. (Bell, et al., 1971).
Because a gravity gradiometer measures the gradient of the gravitational force field, which falls off as $1/R^3$, it is found that large-sized geophysical masses can be simulated by direct scaling of the test mass parameters (with the sensor size and mass held constant). The configuration of the flyby simulation experimental equipment used is shown in Figure 17. The mechanical portion consists of a sensor and its isolation and drive mechanisms contained in a vacuum system, and a flyby track with drive motor, drive chain, mass carriages, and test masses.

The flyby track was placed 47 cm from the center of the sensor. The actual velocity of the masses on the flyby track was measured at just under 2.5 cm/sec. The simulated velocity for a 30 km lunar altitude therefore, would be 1.6 km/sec, which is the orbital velocity of a 30 km lunar orbit.

The frequency of the sensor used for the simulation was 32 Hz (rotation speed of 16 rps = 960 rpm). The $Q$ was 400, which gave a sensor time constant of approximately 4 sec. The output of the sensor was passed through a two-stage, 3 sec RC filter that increased the effective integration time to approximately 10 sec. Thus, these experimental simulations were run with signal processing parameters that were realistic, as there were definite effects on the data because of the finite delay and integration time of the sensor and its electronics. The computer program had to utilize these time constants to fit the experimental data.

In the data curve the solid line is a trace of the recorder plots showing the output of the sensor as a function of time. The first curve is the amplitude of the sensor response; the second, the sine or inphase
Fig. 17. Mass Flyby Drive and Track (Bell, et al., 1971).
response; and the third, the cosine or quadrature response. The dotted lines are a plot of the theoretical prediction of the sensor output with time. The bias levels used in the computer program to obtain the best fit to the experimental curves for the in-phase and quadrature channels are printed at the top of the plots. A difference of 1 EU usually gave a significant difference in fit to the experimental curve.

The curve was taken with two approximately equal masses (14.4 and 15.5 kg) that were flown by in the plane of the sensor with a horizontal separation distance of 62 cm (Figure 18). As expected, it was easy to identify the separate peaks when the two masses were separated by more than the sensor distance. The curve also shows that the sensor was able to discern the 2 EU difference in the gravity gradient caused by the 1 kg difference in mass.

After these initial laboratory results, a much larger and more sensitive version of this sensor was developed by Forward, et al. [1973] for earth orbital use. The desirability of obtaining 0.01 EU sensitivity dictated the requirement for a sensor arm length as long as possible. A sensor arm length of 76 cm from center to center of the end masses (86 cm overall) was selected as the largest arm diameter possible for the 96 cm spacecraft diameter, which, in turn, was dictated by the Scout payload envelope of 106.5 cm diameter. The chosen arm end masses were 2 kg each; this weight was considered reasonable for the size of the sensor.

A 35 sec sensor time constant was chosen for the sensor by using the time required for the spacecraft to pass through one resolution element at the nominal altitude of 270 km at the orbital velocity of
Fig. 18. Two-Mass Horizontal Separation Flyby Test (62 cm Separation Distance) (Bell, et al., 1971).
7.75 km/sec. With this size, weight, and time constant for the sensor, the thermal noise caused by the Brownian motion of the sensor structure had an equivalent noise level of 0.007 EU. This sensor system time constant is the smoothing time to be used in the sensor data preprocessing. The smoothed sensor output would be sampled approximately once every 5 sec to overcome digitalization noise, prevent aliasing, and pick up strong, short period signals resulting from dense localized anomalies.

The sensor frequency of operation is not critical and is set by conflicting requirements. This frequency should be as low as possible to ease the spin speed stress requirements on the satellite structure, and should be high as possible to avoid the low-frequency noise in the electronics and for ease in laboratory testing, where it is difficult to obtain adequate vibrational and acoustic isolation for mechanical structures below 10 Hz. The selected design frequency was 8 Hz, which implies a spin speed of 240 rpm (4 rps) for the satellite; although fast, this speed is not unreasonable.

A sensor based on the orbital design requirements was constructed (Figure 19) and tested. A list of the sensor parameters is given in Table 2.

In summary, the published literature and contract reports on gravity gradiometer instrumentation indicate that there are a number of different gravity gradiometer instrument concepts that can be considered for both attitude stabilized and spin stabilized vehicles. The most promising instrumentation techniques utilize spacecraft rotation to modulate the gravity gradient signal to aid in its
Fig. 19. Breadboard Model of an Earth Orbiting Rotation Resonant Torsional Gravity Gradiometer (Forward, 1973).
TABLE 2. Earth Orbiting Rotating Resonant Torsional Gravity Gradiometer Prototype Design Parameters

<table>
<thead>
<tr>
<th>Type</th>
<th>Rotating Resonant Doubly Differential Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm Diameter</td>
<td>76 cm</td>
</tr>
<tr>
<td>Spacecraft Diameter</td>
<td>96 cm (Scout Payload Envelope)</td>
</tr>
<tr>
<td>Resonant Frequency</td>
<td>8 Hz (Nominal)</td>
</tr>
<tr>
<td>Spacecraft Spin Rate</td>
<td>4 rps = 240 rpm</td>
</tr>
<tr>
<td>End Mass (4 required)</td>
<td>2 kg</td>
</tr>
<tr>
<td>Sensor Subsystem Weight</td>
<td>30 kg</td>
</tr>
<tr>
<td>Spacecraft Weight</td>
<td>140 kg</td>
</tr>
<tr>
<td>Sensor Q</td>
<td>360 (Nominal)</td>
</tr>
<tr>
<td>Sensor Time Constant</td>
<td>15 sec</td>
</tr>
<tr>
<td>Filter Time Constant</td>
<td>20 sec</td>
</tr>
<tr>
<td>System Integration Time</td>
<td>35 sec</td>
</tr>
<tr>
<td>Sensor Thermal Noise</td>
<td>0.007 EU, 1 σ, 35 sec</td>
</tr>
<tr>
<td>System Noise Goal</td>
<td>0.01 EU, 1 σ, 35 sec</td>
</tr>
</tbody>
</table>

[Forward, 1973]
separation from externally and internally generated noise. There are two detailed gradiometer system designs based on this concept, one of which has proceeded to the breadboard hardware phase, which should produce the desired 0.01 EU sensitivity in an earth orbital spacecraft.
CONCLUSION

A review of the published literature and contract reports on artificial satellite gravity gradiometer techniques for geodesy has resulted in the following conclusion:

An earth orbital geodesy mission using a spin stabilized artificial satellite in a low polar orbit containing an onboard gravity gradiometer with a sensitivity of 0.01 EU or better and an integration time of 35 sec or better is feasible. The mission will produce gravity gradient data suitable for obtaining a global gravity map at the 1 mgal level or better to degree and order 75 or better.

If such a mission were carried out, it would produce results which would significantly advance the science of geodesy. It is hoped that such a mission will be planned, funded, and flown soon.
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