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NATURAL FREQUENCIES AND MODE SHAPES OF CABLES WITH ATTACHED MAS--ETC(U)
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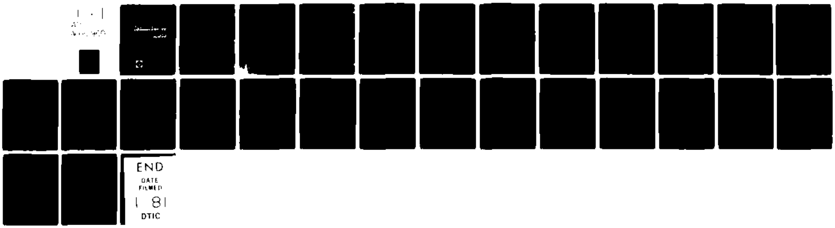

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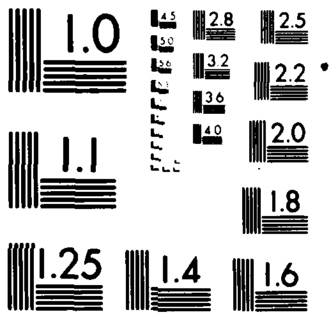

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date: August 1980

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An algorithm has been developed to calculate mode shapes and natural frequencies of taut cables with attached masses. The transcendental equations of motion are solved by an iterative technique that allows accurate calculation of extremely high mode numbers. The algorithm has been implemented as a FORTRAN program primarily as a tool in determining drag coefficients of submerged strumming cables; however, any taut cable can be analyzed.

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1. Strumming cables 2. Drag coefficients I. YF59.556.091.01.402

An algorithm has been developed to calculate mode shapes and natural frequencies of taut cables with attached masses. The transcendental equations of motion are solved by an iterative technique that allows accurate calculation of extremely high mode numbers. The algorithm has been implemented as a FORTRAN program primarily as a tool in determining drag coefficients of submerged strumming cables; however, any taut cable can be analyzed. To assess the accuracy of the program, a simple experiment was conducted to determine the natural frequencies and mode shapes of a wire with attached masses driven sinusoidally by a shaker. The algorithm shows close agreement with the experimental data.

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INTRODUCTION

Many ocean-founded structures rely on cables for support or anchoring. Static and dynamic modeling of the interaction of a structure with the sea requires calculation of the loads due to cable drag. Strumming (vortex-induced vibration) of the cables increases the cable drag coefficient and causes a proportional increase in the total drag force. Several methods have been developed to predict the amplification of drag due to strumming. These methods require a calculated cable mode shape as an input to algorithms that determine drag amplification. The algorithms have been developed from experimental data and represent the state of the art in drag amplification prediction. In the past, matrix methods have been used to determine cable mode shape, but these are costly techniques, subject to inaccuracy, particularly for higher modes.

The Civil Engineering Laboratory (CEL), under sponsorship of the Naval Facilities Engineering Command, has been developing computer models for analyzing the response of cable structures. As part of this project, CEL has developed an iterative solution of the transcendental equations describing cable mode shape. The iterative approach is fast, accurate, and can easily accommodate a variety of system configurations, including bodies attached to the cable.

One of the initial cases analyzed by the new iterative technique was a long cable with 380 attached bodies. This cable is representative of those encountered in ocean engineering applications. Such cables are often excited by ocean currents to vibrate at very high mode numbers. The calculated mode shape for mode 162 is shown in Figure 1. It is apparent that the mode shape is complex and much beyond the range of intuition.

Since the mode shape calculation has a significant influence on the drag calculation, it is considered essential that the determination scheme for the mode shape be verified by comparison to experimental data. Therefore, an experiment was carried out to measure natural frequencies and mode shapes. The description of the iterative algorithm, the measured results of the experiment, and a comparison between the experimental data and the computed results are presented.

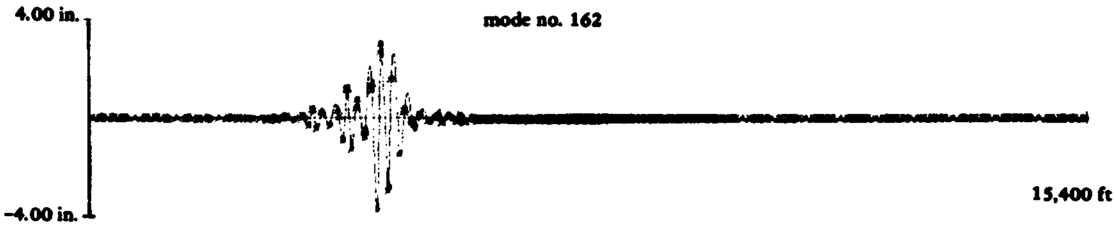

Figure 1. High mode number for a real cable system.

STRUMMING OF MULTISEGMENT CABLE SYSTEMS

The amplitude of oscillation and the effective drag coefficient of a strumming cable system are determined in the following analysis. The natural frequencies and mode shapes of cable oscillation are obtained by an iterative substitution algorithm that finds the solution satisfying the imposed boundary conditions of the problem.

Cable Dynamics

The cable system considered is shown in Figure 2. It consists of n cable segments attached to n-1 masses. The cable segments have an effective mass per unit length, ρ_i, and a tension, T_i, which is assumed to be constant over the length, ℓ_i, of the segment. The effective attached mass (including added mass) is M_i.

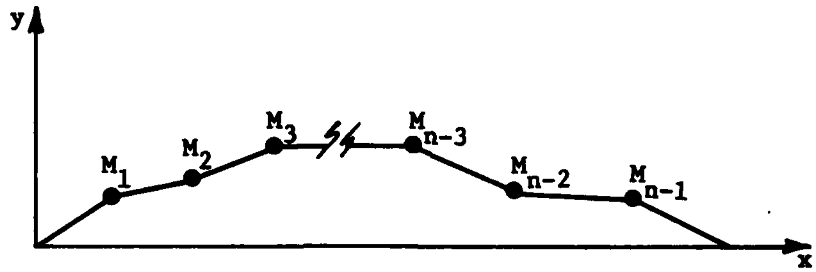

Figure 2. System of masses and cable segments.

Equations of Motion

The equation of motion for the displacement y_i of the i^{th} cable segment, assuming no bending rigidity, is (see Figure 3)

$$\rho_i \frac{\partial^2 y_i}{\partial t^2} = T_i \frac{\partial^2 y_i}{\partial x^2} \qquad i = 1,\ldots,n \tag{1}$$

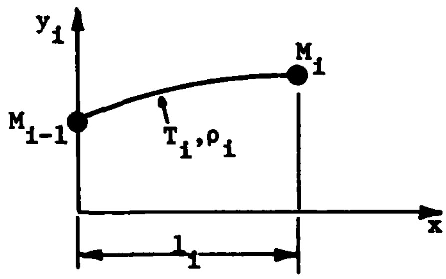

Figure 3. Displacement of the i^{th} segment.

The harmonic solution of this equation has the form

$$y_i(x,t) = Y_i(x)\, e^{j\omega t} \tag{2}$$

where $Y_i(x)$ gives the shape of the deformation, and ω is the frequency of oscillation. Substituting Equation 2 into Equation 1 gives

$$Y_i(x) = A_i \sin \alpha_i \omega x + B_i \cos \alpha_i \omega x \qquad 0 \leq x \leq \ell_i \qquad (3)$$

$$i = 1, \ldots, n$$

where $$\alpha_i = \sqrt{\rho_i / T_i} \qquad (4)$$

Boundary Conditions

In addition to satisfying the equation of motion, the deflection of each cable segment must satisfy certain boundary conditions. These conditions result from the geometric conditions imposed on the ends of the cable assembly, the continuity of displacement at each attached mass and the balance of forces at each attached mass.

Continuity of Displacement. Displacement must be continuous at each attached mass. Therefore,

$$Y_{i+1}(0) = Y_i(\ell_i) \qquad (5)$$

Substituting from Equation 3 yields

$$B_{i+1} = A_i \sin \alpha_i \omega \ell_i + B_i \cos \alpha_i \omega \ell_i \qquad (6)$$

If A_i and B_i are known, Equation 6 can be used to find B_{i+1}.

Force Balance. Figure 4 shows the forces acting at each attached mass. The balance of forces at each of these points gives

$$T_{i+1} \left. \frac{\partial y_{i+1}}{\partial x} \right|_{x=0} - T_i \left. \frac{\partial y_i}{\partial x} \right|_{x=\ell_i} = M_i \left. \frac{\partial^2 y_{i+1}}{\partial t^2} \right|_{x=0} \qquad (7)$$

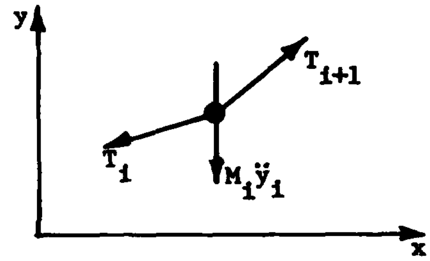

Figure 4. Force balance.

Substituting from Equation 3 yields

$$T_{i+1}\,\alpha_{i+1}\,A_{i+1} - (\alpha_i\,A_i \cos \alpha_i\,\omega\,\ell_i - \alpha_i\,B_i \sin \alpha_i\,\omega\,\ell_i)\,T_i = -M_i\,\omega\,B_{i+1} \qquad i = 1,\ldots,n \tag{8}$$

Using Equation 6 and solving Equation 8 for A_{i+1} gives

$$A_{i+1} = \frac{1}{\alpha_{i+1}\,T_{i+1}}\Big[(\alpha_i\,T_i \cos \alpha_i\,\omega\,\ell_i - M_i\,\omega \sin \alpha_i\,\omega\,\ell_i)\,A_i - (\alpha_i\,T_i \sin \alpha_i\,\omega\,\ell_i + M_i\,\omega \cos \alpha_i\,\omega\,\ell_i)\,B_i\Big] \tag{9}$$

If A_i and B_i are known, Equation 9 can be used to find A_{i+1}.

Geometric Boundary Conditions. At the left-hand end of the cable assembly, the displacement is assumed to be zero. Thus,

$$Y_1(0) = 0 \tag{10}$$

Substituting from Equation 3 thus implies that

$$B_1 = 0 \tag{11}$$

Since the scale of the deflected shape of the cable (mode shape) is, at this point, arbitrary, let

$$A_1 = 1 \tag{12}$$

With Conditions 11 and 12 on A_1 and B_1 and Equations 6 and 9 for A_{i+1} and B_{i+1}, all subsequent A's and B's can be determined provided ω (the natural frequency) is known.

The system must satisfy one additional boundary condition at the right-hand end of the assembly, where the displacement is again assumed to be zero. This gives

$$Y_n(\ell_i) = 0 = Y_{n+1}(0) \tag{13}$$

Equation 13 in turn implies that

$$B_{n+1} = 0 \tag{14}$$

The values of ω that give solutions satisfying Condition 14 are the natural frequencies of the problem.

Solution Algorithm for Mode Shapes and Frequencies

If ω is varied from zero to some large value and the corresponding values of B_{n+1} are calculated, the result will be as shown in Figure 5. Each point for which $B_{n+1} = 0$ represents a valid solution of the free oscillation problem. The ω_k so obtained are the natural frequencies of the system. There are an infinite number of such frequencies.

The mode shape associated with each natural frequency ω_k will be denoted by $Y_i^{(k)}(x)$. Then,

$$Y_i^{(k)}(x) = A_i^{(k)} \sin \alpha_i \omega_k x + B_i^{(k)} \cos \alpha_i \omega_k x \tag{15}$$

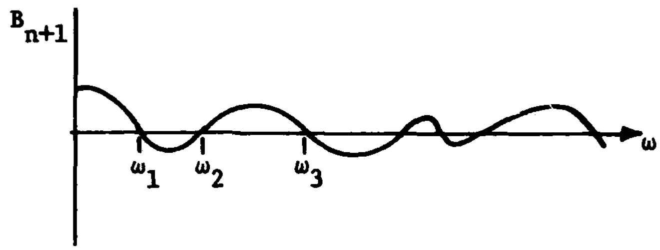

Figure 5. System natural frequencies.

Let ω_k be the k^{th} natural frequency. Then the deflected shape of the cable system will be such that the number of internal zero crossings (nodes) is equal to k - 1. The mode number of a particular mode shape can therefore be determined by counting the number of internal zeros associated with the function $Y_i^{(k)}$; $i = 1,\ldots,n$.

Summary of Solution Procedure

The solution process for the mode shapes and frequencies is summarized as follows:

1. Assume a value for ω_k.

2. Let $B_1 = 0$, $A_1 = 1$.

3. Solve for B_2,A_2; B_3,A_3;...; B_n,A_n; B_{n+1} from Equations 6 and 9.

4. Check for $B_{n+1} = 0$. If $B_{n+1} \neq 0$, compare with previous value and estimate a new trial value for ω.

5. Go to step 2 and repeat until B_{n+1} is less than some prescribed value or the change in ω_k is less than some prescribed limit.

6. Determine the mode number by calculating the number of internal zeros of the mode shape $Y_i^{(k)}(x)$; $i = 1,\ldots,n$.

EXPERIMENTAL COMPARISON

In order to verify the accuracy of the algorithm, a simple, easily conducted experiment was designed. A taut piano wire was fixed at one end and excited sinusoidally at the other end by a shaker; natural frequencies were determined by varying the excitation frequency. The experimental configuration is shown in Figure 6; wire properties are given in the figure. Two cases of attached bodies were considered:

Case 1: Four equally spaced bodies, each with a mass of 18.3×10^{-5} slug

Case 2: Six unequally spaced bodies, each with a mass of 12.9×10^{-5} slug

The objective of the experiment was to find the first 8 to 10 natural frequencies and mode shapes for each case for comparison with calculations.

Tension was measured by shaking the unloaded wire (no attached masses) in the 5^{th} mode ($n = 5$). Period τ_5 (sec) was measured by a digital counter accurate to 0.01 ms. Tension T (lbf) was then calculated by the natural frequency formula for an ideal cable:

$$T = \left(\frac{2\,\ell}{n\,\tau_n}\right)^2 \mu$$

Split-shot lead weights (the type used in fishing) were attached at appropriate locations with a small amount of contact cement.* When ten samples of shot were weighed, the mass variation (standard deviation) for a given size of shot was found to be about ±1% of the mean value cited. The size of the masses was approximately 0.32 inch in the x-direction.

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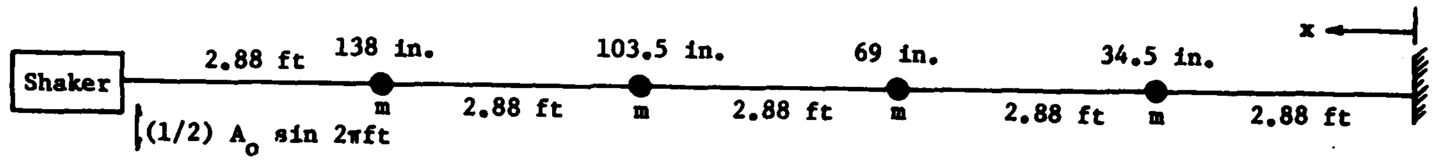
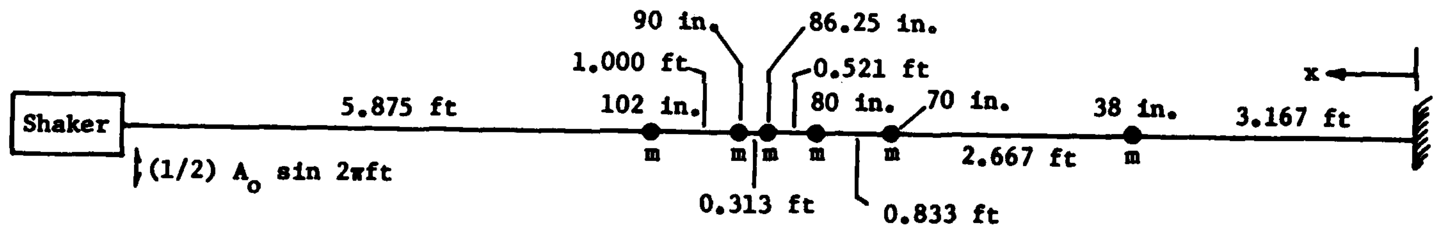
*The cement was necessary to prevent the masses from shaking loose during the tests; its weight is assumed negligible.



<u>Case 1</u>: Four masses, equally spaced; $m = 18.3 \times 10^{-5}$ slug.
$wt = 5.893 \times 10^{-3}$ lb ea.



<u>Case 2</u>: Six masses, unequally spaced; $m = 12.9 \times 10^{-5}$ slug.
$wt = 4.154 \times 10^{-3}$ lb ea.

NOTES:

- Cable parameter (same for both cases): Length, $\ell = 172.5$ in.
  Diameter, $d = 0.033$ in.
  Mass/ft, $\mu = 8.71 \times 10^{-5}$ slug/ft
  Wt/ft $= 2.805 \times 10^{-3}$ lb/ft
- Locations x measured from the right.
- A_o is peak-to-peak shaker amplitude; excitation is perpendicular to wire.

Figure 6. Experimental test cases.

Except for some very high frequency noise caused by air rushing through the shaker's air bearing, excitation was harmonic, as measured by an accelerometer mounted directly on the shaker. For each experimental run, the shaker's displacement amplitude, A_o, was reckoned from

$$\text{Displacement Amplitude} = \frac{1}{(2\pi f)^2}\,[\text{Acceleration Amplitude}]$$

The antinode amplitudes recorded in the data have not been corrected for the shaker amplitude (the correction is usually negligible). However, if the correction is desired, the following formula should be used:

$$A_{corrected} = A_{measured} - A_o\left(\frac{x_{antinode}}{\ell}\right)$$

The shaker frequency, f, was monitored by a digital counter capable of measuring the period to within ±0.01 ms. Experimentally, the shaker frequency, f, was decreased from some arbitrary frequency, f_a, lying well above f_n, to f_b, at which point a sudden, highly visible increase in amplitude occurred. The frequency, f_b, was then recorded as the "natural frequency."

The location of nodes was measured by eye or, in the rare case of very small amplitudes, with the help of a small strip of paper against the vibrating wire. Obviously, the uncertainty in measurement is least where the nodes are well defined; i.e., when the surrounding antinodes have large amplitude and/or are close together. Thus, depending on the mode shape, accuracy varied from about ±0.1 inch to about ±0.75 inch. In general, of all the quantities measured, node location was the most accurate.

The location of the antinode was also measured by eye. Judgment of antinode location by this method is very difficult, inasmuch as the antinodes are not sharply defined. Every effort was made to make the measurement as accurate as possible without resorting to sophisticated equipment. A rule was placed a short distance below the wire and observed from a fixed distance above the wire. A strobe light running slightly slower or faster than the vibration frequency was used to illuminate the vibrating wire; this made the wire appear to oscillate slowly in its mode shape. In this way, the amplitude, $A_{measured}$, which is reported in the data tables, was read to an estimated accuracy of 0.01 inch.

Three runs of mode 5, Case 1 were made in succession. Two runs were as identical as possible, with four masses equally spaced. For the third, the mass at x = 138.0 inches was moved to x = 138.1 inches. On the basis of these runs, it was concluded that the repeatability of the antinode amplitude measurements was of the order of 5%, and could be as large as 10% under circumstances less controlled than those in the tests described. It was also observed that the amplitude ratios were fairly critically dependent upon the exact location of the masses.

To judge the bias in the measurements caused by the asymmetrical excitation (shaker at one end), several cases were rerun with the configuration of the masses mirrored about $x = \ell/2$. The predominant effect of asymmetrical excitation is a preference for certain modes over others. For example, in one run, mode 10 was easily excitable, but not excitable at all in the mirrored run. The reverse was true for mode 8.

EXPERIMENTAL VERSUS COMPUTER RESULTS

From Tables 1 and 2 it is observed that the agreement between the calculated and experimentally determined natural frequencies is well within the range of the expected experimental error. The node and antinode locations also show good agreement, especially for the lower mode numbers. Modes of order eight and higher were rather difficult to observe accurately with the experimental setup. However, the results for the higher modes still appear to verify the computer model.

Response amplitude was compared on a normalized basis. To compare on an absolute basis would have required that both internal and external damping be known and modeled and that the precise nature of the excitation be known. Since this information was not readily available, the normalized response was used for comparison. As shown in the tables, the normalized amplitudes of response compare favorably.

The nature of the mode shapes of the first 15 modes for the six mass cases is indicated in Figure 7. Modes 1 through 10 are those presented in Table 2. The additional modes shown illustrate that the non-uniformly distributed masses cause the mode shapes to become increasingly complex. It is interesting to note that the sinusoidal portions of the mode shapes, shown clearly in mode 15, but also evident in other modes, represent the free response of the wire unaffected by the attached masses. This figure was generated by the computer algorithm.

CONCLUSION

A new computer algorithm for computing the natural frequencies and mode shapes of cable systems has been presented. This algorithm is computationally efficient and shows excellent convergence, even for very high mode numbers. The algorithm is also quite flexible since it has the capacity for treating a wide range of different system configurations.

A comparison of the results of the computer model with the results of an experimental study of a vibrating cable system in air has shown that the model accurately predicts the natural frequencies, node and antinode locations, and relative response amplitudes for all modes obtainable experimentally. The extremely close agreement obtained for modes up to number 10 does not insure that higher mode shapes are correctly calculated, but it does give increased confidence that the solution technique can be validly extrapolated as required by the users needs.

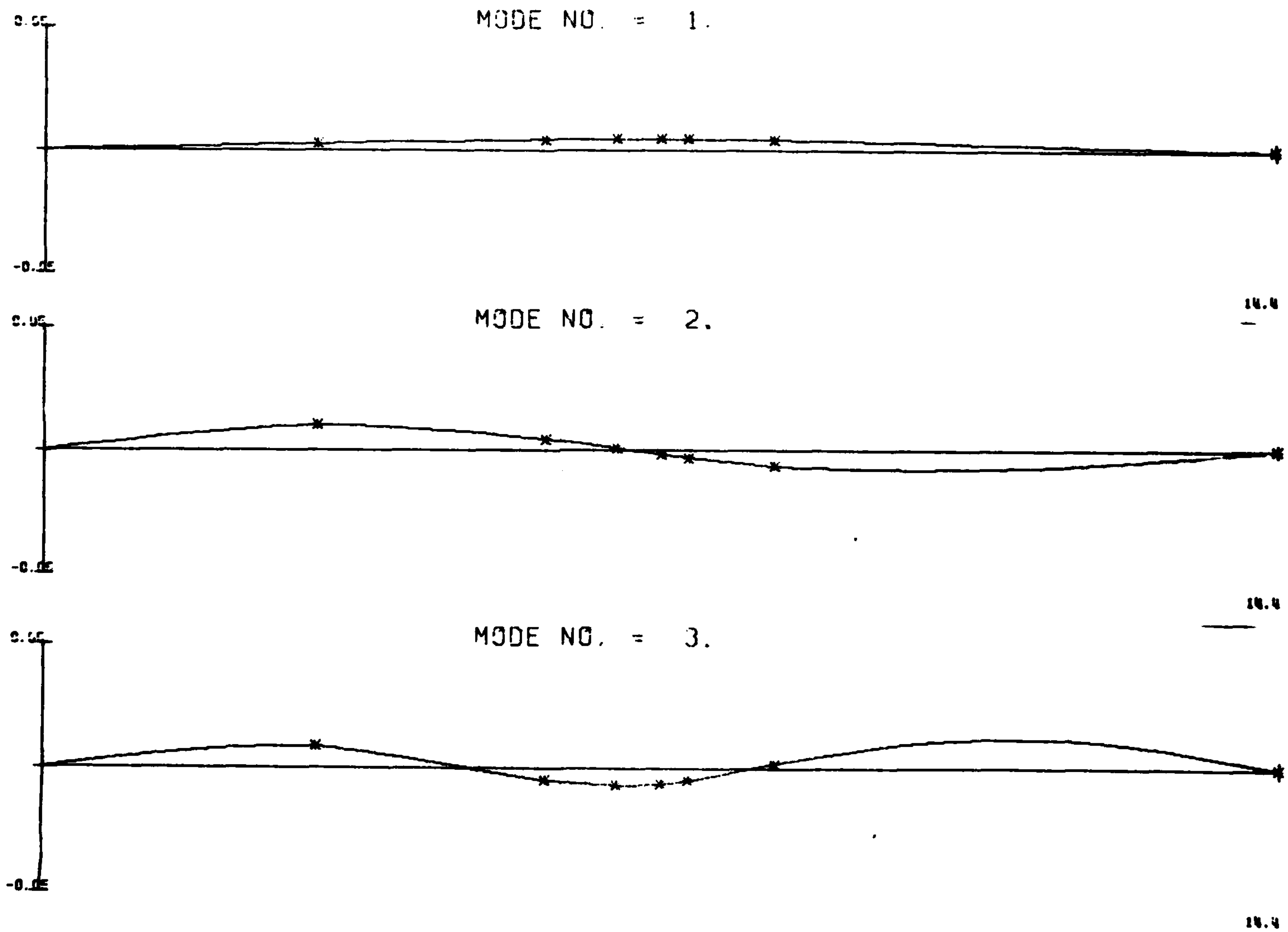


Figure 7. First 15 mode shapes for Case 2.

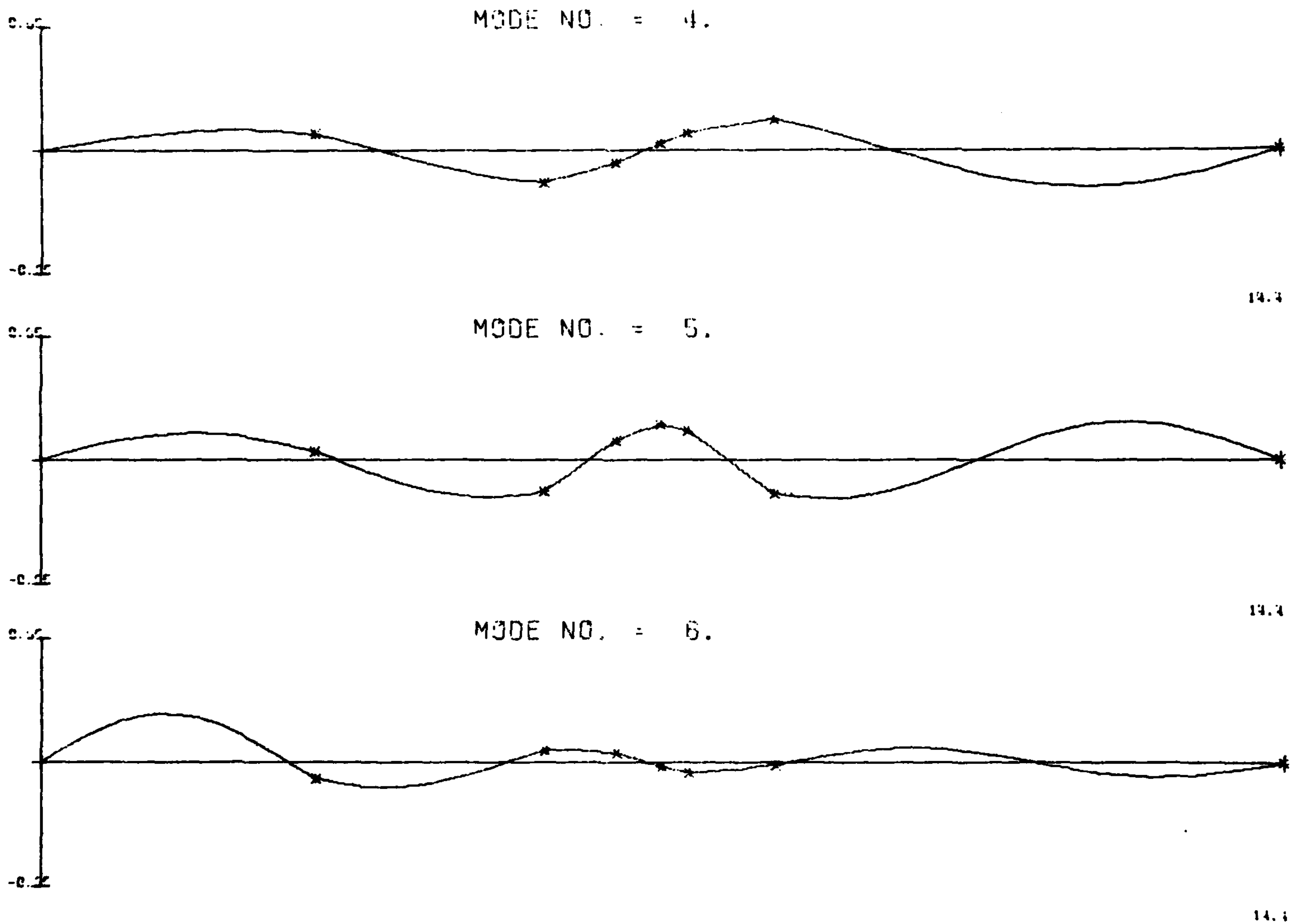
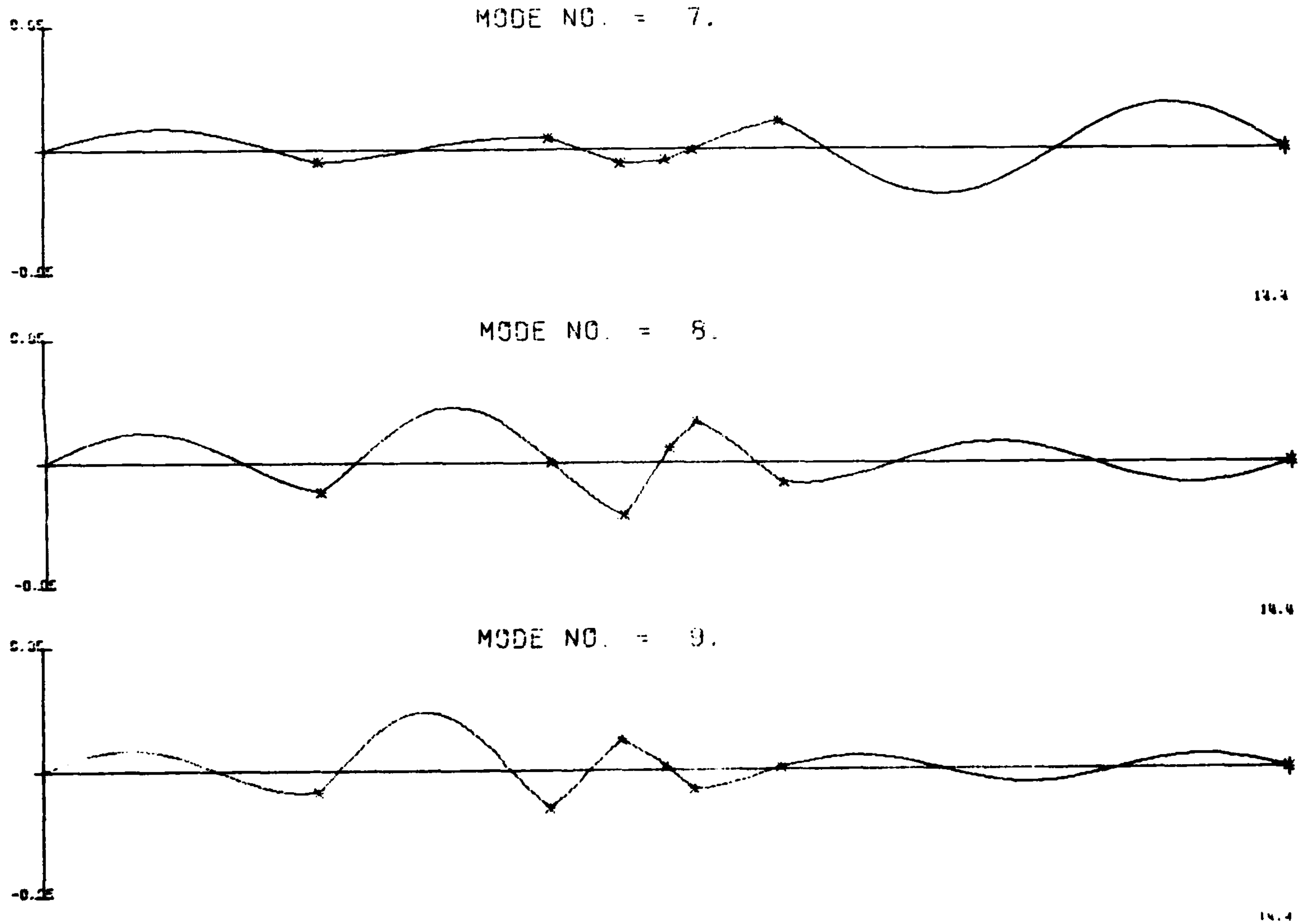


Figure 7. Continued.



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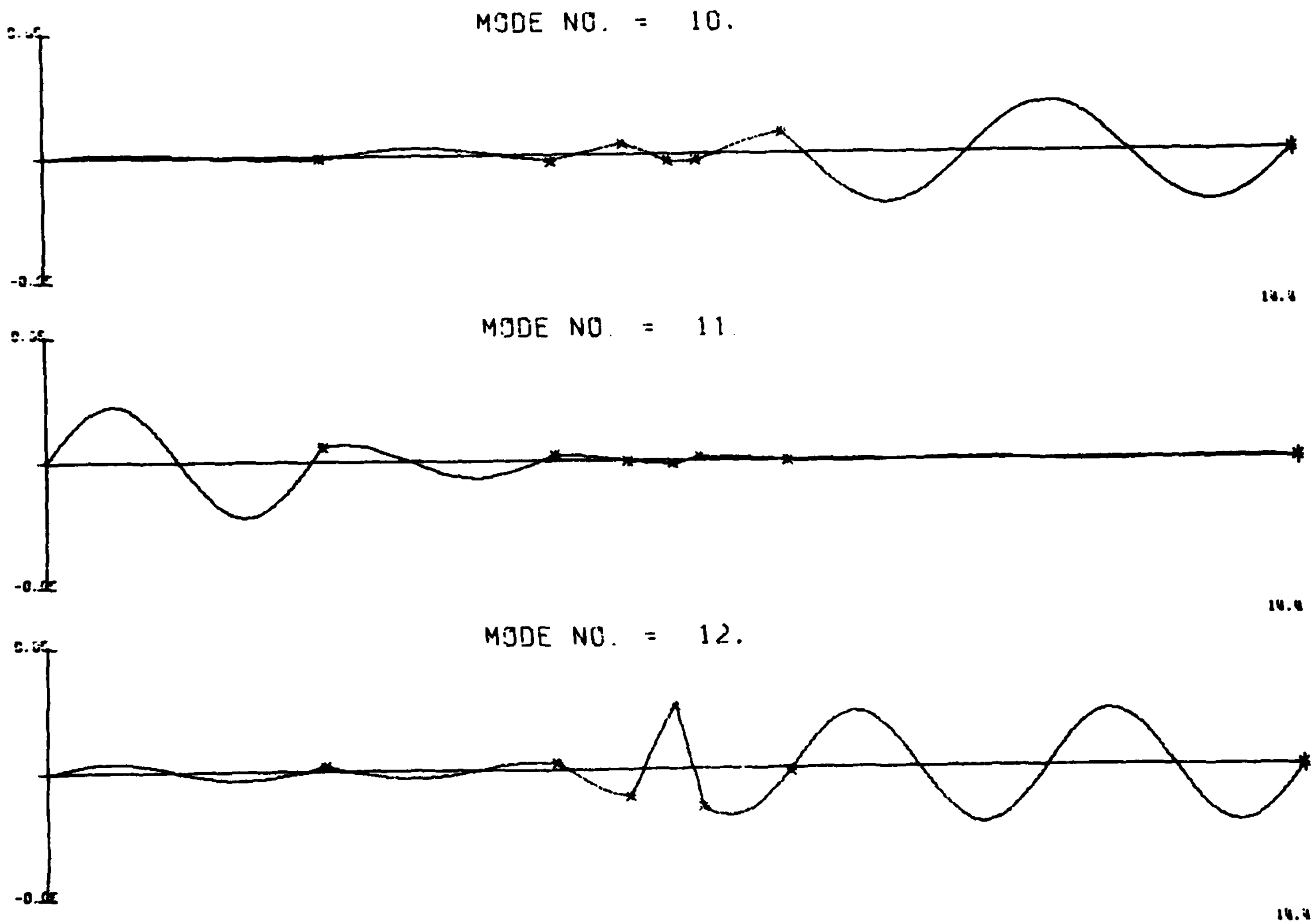

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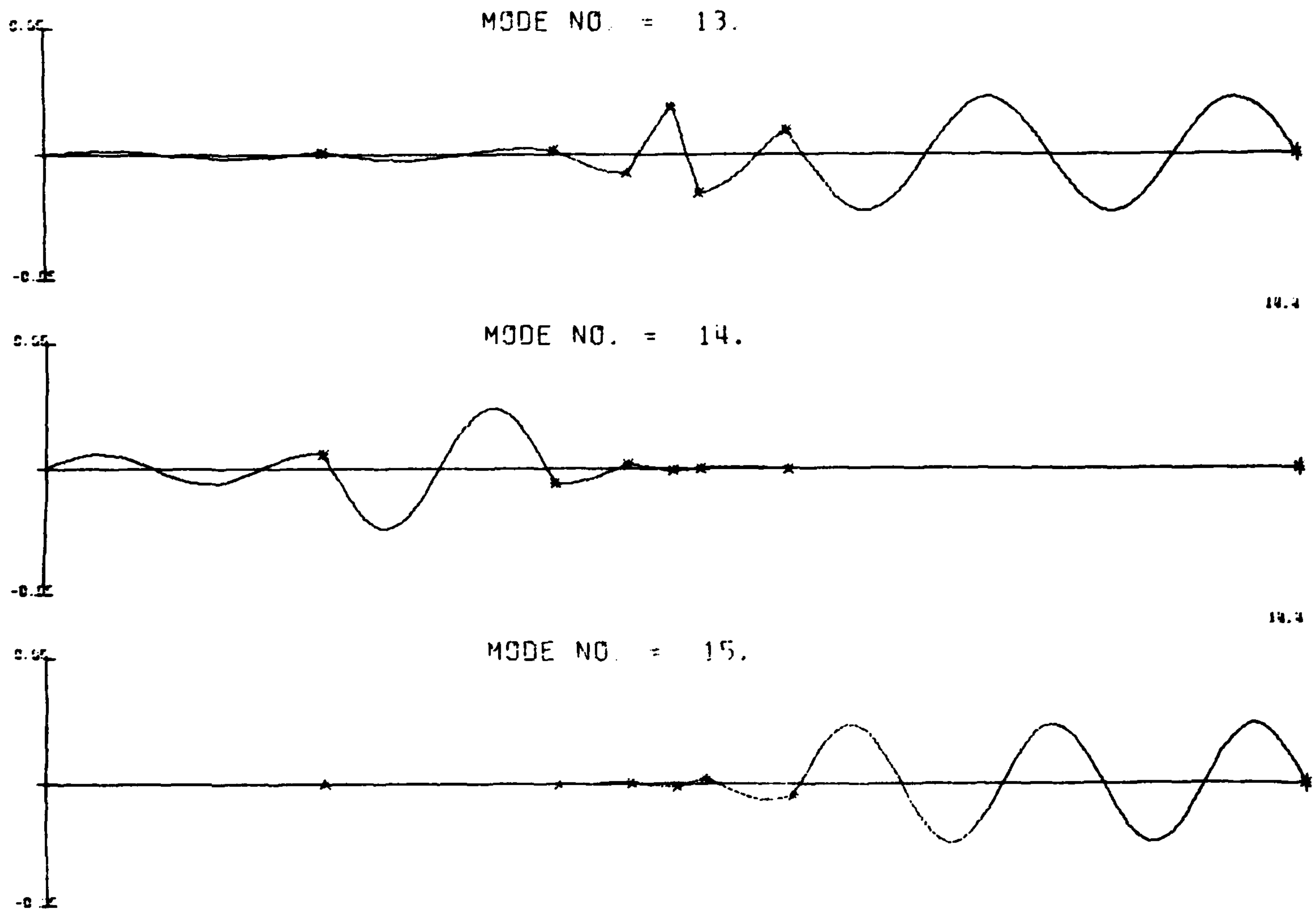

Figure 7. Continued.

Table 1. Case 1: Four Masses, Equally Spaced

Case	Mode No., n	Tension, T (lbf)	Peak-to-Peak Shaker Amplitude, A_o (in.)	Natural Frequency, f_n (Hz)		Node Locations, x_{node} (in.)		Antinode Locations, $x_{antinode}$ (in.)		Peak-to-Peak Antinode Amplitudes, A		
				Exper.	Comp.[a]	Exper.	Comp.[a]	Exper.	Comp.[a]	Exper. (in.)	Exper. (norm)	Comp.[a] (norm)
1.1	1	5.95	0.060	7.0	6.9	-	-	86.2	b	1.84	1	1
1.2	2	5.95	0.020	13.4	13.6	86.2	87.3	40.7 132.0	39.2 133.5	1.16 1.12	1 0.97	1 0.83
1.3	3	5.95	0.078	20.1	20.0	56.7 115.8	57.4 114.1	33.7 86.2 139.0	34.4 86.3 138.0	0.78 0.62 0.72	1 0.79 0.92	0.92 0.69 1
1.4	4	5.95	0.055	25.5	25.3	46.3 86.2 126.3	46.4 85.9 127.1	30.7 68.7 103.7 141.7	33.3 69.1 103.4 138.1	0.47 0.68 0.69 0.42	0.68 0.99 1 0.61	0.65 0.94 1 0.58
1.5	5	5.95	0.020	44.8	45.4	34.5 69.0 103.5 138.0	34.4 68.4 103.8 131.1	17.2 51.7 86.2 120.7 155.2	17.0 51.5 85.9 120.3 155.0	0.38 0.36 0.34 0.34 0.40	0.95 0.90 0.85 0.85 1	0.97 0.94 1 0.91 1
1.6	6	5.95	0.019	48.4	47.5	32.8 64.5 87.0 108.7 140.0	32.6 64.3 86.6 107.2 139.9	17.7 48.7 69.7 102.7 123.7 154.7	16.2 49.1 71.0 101.4 123.0 156.0	0.44 0.32 0.10 0.10 0.24 0.42	1 0.73 0.23 0.23 0.55 0.95	1 0.63 0.26 0.21 0.68 0.97

continued

Table 1. Continued

Case	Mode No., n	Tension, T (lbf)	Peak-to-Peak Shaker Amplitude, A_o (in.)	Natural Frequency, f_n (Hz)		Node Locations, x_{node} (in.)		Antinode Locations, $x_{antinode}$ (in.)		Peak-to-Peak Antinode Amplitudes, A		
				Exper.	Comp.[a]	Exper.	Comp.[a]	Exper.	Comp.[a]	Exper. (in.)	Exper. (norm)	Comp.[a] (norm)
1.7	7	5.95	0.018	52.4	52.0	30.2	29.9	14.7	14.6	0.35	1	
						45.0		35.0		0.14	0.40	
						71.3		57.7		0.14	0.40	
						101.2		86.2		0.34	0.97	
						127.7		114.8		0.18	0.51	
						142.5		137.5		0.13	0.37	
								157.8		0.28	0.80	
1.8	8	5.95	0.016	59.0	56.7	28.0	27.5	13.7	13.8	0.28	0.74	0.67
						39.2	38.6	34.0	34.4	0.18	0.47	0.45
						66.1	66.5	52.7	48.9	0.38	1	1.0
						86.2	84.7	73.7	72.3	0.12	0.32	0.29
						106.2	106.4	98.7	99.5	0.13	0.34	0.33
						133.3	133.2	119.8	119.8	0.38	1	0.95
						144.8	145.1	138.5	141.2	0.16	0.42	0.48
								158.8	158.7	0.25	0.66	0.67

[a] Computed values.

[b] Could not be determined accurately.

Table 2. Case 2: Six Masses, Unequally Spaced

Case	Mode No., n	Tension, T (lbf)	Peak-to-Peak Shaker Amplitude, A_o (in.)	Natural Frequency, f_n (Hz)		Node Locations, x_{node} (in.)		Antinode Locations, $x_{antinode}$ (in.)		Peak-to-Peak Antinode Amplitudes, A		
				Exper.	Comp.[a]	Exper.	Comp.[a]	Exper.	Comp.[a]	Exper. (in.)	Exper. (norm)	Comp.[a] (norm)
2.1	1	5.52	0.062	6.0	6.0	-	-	86.2	b	0.90	1	1
2.2	2	5.52	0.074	15.0	14.8	81.8	84.0	40.0 127.0	37.9 122.2	0.77 0.57	1 0.74	1 0.70
2.3	3	5.52	0.046	20.4	20.4	59.5 98.6	60.9 96.8	35.5 83.0 135.0	35.8 80.2 137.0	0.60 0.42 0.77	0.78 0.55 1	1 0.67 0.83
2.4	4	5.84	0.053	29.7	28.7	46.7 84.2 118.3	47.5 83.0 119.8	25.7 69.7 101.7 143.7	27.4 69.9 101.9 145.6	0.44 0.58 0.59 0.62	0.65 0.85 0.87 1	0.64 0.86 1 1
2.5	5	5.84	0.035	37.2	36.8	41.1 76.3 95.0 129.8	41.3 75.6 96.1 129.1	20.7 62.7 86.7 109.7 148.7	21.4 62.0 86.1 110.2 151.2	0.27 0.37 0.30 0.35 0.34	0.73 1 0.81 0.95 0.92	0.69 0.88 0.94 0.88 1
2.6	6	5.52	0.028	45.8	44.1	34.3 64.7 84.2 104.5 138.7	34.8 63.0 85.7 98.8 142.2	17.7 47.7 73.0 91.0 122.0 155.7	16.2 47.5 71.6 89.9 120.9 154.9	0.57 0.31 0.14 0.11 0.21 0.20	1 0.54 0.49 0.19 0.37 0.35	1 0.46 0.31 0.13 0.41 0.21

continued

Table 2. Continued

Case	Mode No., n	Tension, T (lbf)	Peak-to-Peak Shaker Amplitude, A_o (in.)	Natural Frequency, f_n (Hz)		Node Locations, x_{node} (in.)		Antinode Locations, $x_{antinode}$ (in.)		Peak-to-Peak Antinode Amplitudes, A		
				Exper.	Comp.[a]	Exper.	Comp.[a]	Exper.	Comp.[a]	Exper. (in.)	Exper. (norm)	Comp.[a] (norm)
2.7	7	5.52	0.024	49.0	47.3	32.2	34.8	16.7	16.2	0.35	0.51	0.49
						52.0	50.6	38.7	37.9	0.19	0.28	0.24
						75.0	75.1	66.7	68.5	0.19	0.28	0.30
						90.3	89.7	81.7	80.9	0.19	0.28	0.27
						108.7	108.8	101.7	101.9	0.40	0.58	0.65
						140.5	140.5	124.7	124.6	0.64	0.93	0.97
								155.7	156.3	0.69	1	1
2.8[c]	8											
2.9[c]	9											
2.10	10	5.52	0.012	68.9	67.2	d	d	11.0	10.7	0.06	0.10	0.07
								31.7	33.1	0.06	0.10	0.01
								51.7	52.0	0.16	0.26	0.20
								69.7	69.9	0.12	0.19	0.10
								79.7	79.9	0.18	0.29	0.24
								89.3	87.8	0.14	0.23	0.10
								101.3	101.9	0.26	0.42	0.46
								116.0	116.0	0.59	0.95	0.95
								138.7	139.1	0.58	0.94	1
								160.7	161.5	0.62	1	0.95

[a]Computed values.

[b]Could not be determined accurately.

[c]Not excitable by experimental setup.

[d]Not measured.

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