ANALYSIS OF COMBUSTION AND HEAT TRANSFER IN A POROUS GRAPHITE 
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THESIS

ANALYSIS OF COMBUSTION AND HEAT TRANSFER IN A POROUS GRAPHITE MEDIUM

by

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June 1980

Thesis Advisor: D. Salinas

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The problem of a porous graphite fiber plate subject to combustion is formulated. The transient one-dimensional model leads to two heat transfer equations on the graphite and air, and a mass transfer equation on the oxygen. In addition to a combustion heat generation term, the heat transfer model includes the mechanisms of conduction, radiation between fibers, and convection due to induced air flow through the mat. The mass transfer model
includes diffusion and convection mechanisms, as well as a combustion consumption term. The temperature dependency of the system parameters is taken into account. The resulting nonlinear, coupled partial differential equations are solved by a Galerkin formulation of the finite element method. A number of computer analysis results are presented. The results show the effects of initial conditions, plate thickness, fiber diameter and air flow rate on ignition and extinction.
Analysis of Combustion and Heat Transfer in a Porous Graphite Medium

by

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Submitted in partial fulfillment of the requirements for the degrees of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING
and
MECHANICAL ENGINEER

from the
NAVAL POSTGRADUATE SCHOOL
June 1980

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ABSTRACT

The problem of a porous graphite fiber plate subject to combustion is formulated. The transient one-dimensional model leads to two heat transfer equations on the graphite and air, and a mass transfer equation on the oxygen. In addition to a combustion heat generation term, the heat transfer model includes the mechanisms of conduction, radiation between fibers, and convection due to induced air flow through the mat. The mass transfer model includes diffusion and convection mechanisms, as well as a combustion consumption term. The temperature dependency of the system parameters is taken into account. The resulting nonlinear, coupled partial differential equations are solved by a Galerkin formulation of the finite element method. A number of computer analysis results are presented. The results show the effects of initial conditions, plate thickness, fiber diameter and air flow rate on ignition and extinction.
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LIST OF SYMBOLS

A\(^{-1}\) - characteristic time of reaction
A - stiffness matrix
B - molecular diffusivity
B - Gram matrix
C - specific heat at constant pressure
D - ply thickness
d - filament diameter
E - activation energy
e - element number
F - excitation vector
f - stoichiometric ratio
G - global basis function
g - local basis function
Gr - Grashof number
G_o - pseudo mass velocity
h - convection heat transfer coefficient
\(\hat{h}\) - specific enthalpy
\(\Delta H\) - enthalpy of formation
J - Colburn j-factor
k - thermal conductivity
L - plate thickness
L* - reference length (= unity)
l_e - element length
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<tr>
<td>M</td>
<td>molecular weight</td>
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<td>$m$</td>
<td>specific permeability</td>
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<td>$\dot{m}$</td>
<td>mass flow rate</td>
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<td>$n$</td>
<td>number of nodal points</td>
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<td>$P$</td>
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<td>$Pr$</td>
<td>Prandtl number</td>
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<td>$q_g$</td>
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<td>$q_l$</td>
<td>heat loss</td>
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<td>Universal Gas constant</td>
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<td>gas constant for air</td>
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<td>$Re$</td>
<td>Reynolds number</td>
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<tr>
<td>$s$</td>
<td>spacing between fibers</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$\hat{T}$</td>
<td>absolute temperature</td>
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<tr>
<td>$t$</td>
<td>time</td>
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<tr>
<td>$U_\infty$</td>
<td>flow over exterior surface</td>
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<td>$u_p$</td>
<td>pore velocity</td>
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<td>$V$</td>
<td>molecular volume</td>
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<td>$w$</td>
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<td>$x$</td>
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<td>$y$</td>
<td>ratio of a fibers' surface area to its volume</td>
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<td>$z$</td>
<td>ratio of fiber surface area per unit volume</td>
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**Greek Symbols**

- $\Gamma$ - nondimensional air temperature
- $\Delta$ - difference
- $\delta$ - average pore diameter
- $\delta_{ij}$ - kronecker delta
- $\varepsilon$ - emissivity
- $\xi$ - local coordinates
- $n$ - nondimensional length
- $\Theta$ - nondimensional fiber temperature
- $\Lambda$ - spacial operator
- $\lambda$ - coefficient defined in Appendix A
- $\nu$ - dynamic viscosity of air
- $\nu$ - coefficient defined in Appendix A
- $\rho$ - density
- $\sigma$ - Stefan-Boltzmann constant
- $\tau$ - tortuosity
- $\phi$ - nondimensional oxygen concentration
- $\phi$ - oxygen concentration
- $\psi$ - approximate solution values
- $\psi_o$ - shape parameter for $h_i$
- $\omega$ - coefficient defined in Appendix A; dummy variable

**Subscripts**

- $a$ - air
- $B$ - diffusion
- $b$ - bulk property
c - convection
e - effective
f - fluid
g - fiber
i - internal
K - Kelvin
L - at x = L
O2 - oxygen
p - pore
r - radiation
s - solid
a - ambient condition
ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Professor David Salinas for his invaluable guidance, assistance and friendship as instructor and thesis advisor. In addition, the author wishes to thank Mr. John Fontenot and Mr. Joseph Mansfield for their interest, comments and thoughtful advice.

The author is obligated to Professor Richard Franke, Professor Matthew Kelleher, Professor David Netzer and Mr. Roger Hilleary for their assistance during the early stages of this work.

Finally, the author would like to thank his wife, Darlene, for her understanding and encouragement.
I. INTRODUCTION

The combustion of a composite laminate plate consisting of graphite fibers in an epoxy matrix is discussed. The combustion process for a graphite laminate takes place in two stages, epoxy combustion followed by combustion of the graphite. At the end of the first stage the epoxy has burned away exposing a porous graphite mat. Spacing between the fibers is maintained by the residue of the combustion products. This present work is concerned with the last stage of the combustion process, specifically, the thermal response of the remaining graphite fiber mat.

The selection of the second stage for the analysis was made as a result of observations during composite plate "burn" experiments at the Naval Weapons Center, China Lake [1]. The objective of the experiments was to assess damage of composite aircraft structures subjected to open deck fires on aircraft carriers. For the experiments, an epoxy-graphite laminate was mounted into the wall of a wind tunnel parallel to the flow. One side of the composite plate was exposed to the wind tunnel flow and the opposite side was open to the environment. This arrangement was to simulate a composite wing section or fuselage subjected to typical wind conditions present on a carrier flight deck. It was observed that after being subjected to a fire which resulted in the
epoxy burning away, the graphite fibers would continue to smolder or burn. In other instances, the fibers would cool depending on the flow velocity and plate thickness. To understand this behavior better, a mathematical model was formulated to predict the conditions for which the exothermic process is self-sustaining.

The air flow over one surface produces a pressure differential across the plate which induces a convective air flow through the porous medium governed by Darcy's Law. As a result, there is an enhancement of internal convection heat transfer, as well as a source of oxygen for combustion. The heat transfer mechanisms included in the model are (1) conduction, (2) convection, and (3) radiation. In addition, non-volatile combustion is included in the energy balance as a heat generation term of Arrhenius type. The oxygen concentration is accounted for by a molecule-mass balance which includes (1) molecular diffusion, and (2) mass transfer by convection. Consumption of oxygen due to combustion is accounted for by a term similar to the heat generation term.

The formulation of a one-dimensional model is presented. All properties are treated as temperature dependent in the transient analysis. To account for heat transfer mechanisms between the air and porous graphite medium, the air and graphite temperatures are treated as independent variables. The energy balance on the graphite and on the air, and the
molecule-mass balance on oxygen result in a system of three nonlinear, coupled partial differential equations.

Integration of the field equations is accomplished by a Galerkin formulation of the finite element method. Due to the inherent stiffness of the field equations in the time domain, a modified implicit-Gear integration scheme developed by Franke [2] is used for a more efficient solution.
II. THEORY AND BACKGROUND

A. INTERACTION OF HEAT TRANSFER AND COMBUSTION

In this work, the combustion model proposed by Semenov was adopted. It is described in the texts of Frank-Kamenetskii [3] and Vulis [4]. A brief discussion of those features of the model which relate to the present investigation will be given. Fundamental to the model is the relation of reaction rate to temperature, and the interaction of the heat generation and the heat transfer from the system. In general, the reaction rate, \( r \), may be shown by the Arrhenius law as,

\[
    r = A(T, \phi) \exp(-E/RT)
\]

where \( A^{-1} \) is the characteristic time of the chemical reaction, \( E \) is the activation energy, \( R \) is the universal gas constant, \( T \) is absolute temperature and \( \phi \) is the concentration of oxygen. The concentration appears as \( \phi^n \) in \( A(T, \phi) \) for an nth order reaction. The heat generated by the exothermic process is obtained by multiplying the reaction rate, \( r \), by the enthalpy of formation of the combustion products.

In Figure 1, the heat generation, \( q_g \), is plotted as a function of temperature. The curve is referred to as the S-curve for apparent reasons. The S-curves are distinguished
by two regions. Lower temperatures and reaction rates characterize region I, while region II is characterized by higher temperatures and reaction rates. In region I, for low reaction rates, there is an abundant supply of oxygen. The reaction is controlled by the temperature and the region is known as the kinetic regime. Here, the reaction rate increases exponentially with increasing temperature. In region II, the reaction is limited by oxygen concentration and is known as the diffusion regime. The reaction's weak dependence on temperature in region II is shown by the characteristic flattening of the S-curve in Figure 1. For higher concentrations of oxygen, the diffusion region is associated with higher reaction rates. The interaction of heat generation, \( q_g \), and heat loss, \( q_x \), will now be described.

In addition to the S-curve, Figure 1 shows two heat loss lines, \( q_{11} \) and \( q_{12} \). They represent the heat transfer by convection for air temperatures, \( T_1 \) and \( T_2 \), respectively. The intersection of the heat generation curve and the heat loss curve, \( q_g = q_x \), defines the stable quasi-stationary point, \( A \). As the air flow temperature increases from \( T_1 \) to \( T_2 \), the quasi-stationary temperature increases continuously from \( T_A \) to \( T_I \). For temperature \( T_I \), \( q_{12} \) is tangent to \( q_g \) and an infinitesimal increase in the air flow temperature results in a jump of the reaction temperature from \( T_I \) to \( T_B \). At point \( I \), which is defined as the "critical ignition condition", the reaction moves from the kinetic to the diffusion regime.
Temperature $T_i$ is referred to as the ignition temperature. Frank-Kamenetskii states the ignition temperature of graphite is approximately 1100 to 1300 degrees Celsius. In order to keep the reaction model simple, the present investigation was limited to the kinetic regime. Although the description of the heat transfer process just presented is for convection only, the underlying ideas are valid for the conduction and radiation heat transfer mechanisms as well. The slope of the heat loss lines, $q_x$, is equal to the product of the internal heat transfer coefficient and the surface area of fiber per unit volume. The slopes would not appear constant, and slight curvature in the heat loss lines, $q_x$, is apparent when radiation effects are included. As shown in Figure 1, an increase in the heat transfer rate resulting, for example, by an increase in the internal flow rate would result in a lower graphite temperature for a given air temperature.

One reason for limiting the model to the kinetic regime was due to the jump in temperature and reaction rate at ignition. Though the discontinuity may be accounted for, the results show that for the region of temperatures approaching the diffusion regime, the heat generation will dominate the process. The system temperatures then rise rapidly. Other factors that limit the model to the lower temperatures are (1) the chemical reaction fixing the heat of combustion (complex at higher temperatures) and (2) the rapid consumption of fibers after ignition.
B. HEAT TRANSFER EQUATIONS FOR POROUS MEDIA

In their investigation of oil recovery from underground reservoirs, Green and Perry [5] developed heat transfer equations for the solid and fluid phases in a porous medium. Their model included the three basic mechanisms: (1) physical movement of the fluid which carries its own heat capacity, (2) conduction of heat through the solid and fluid phases, and (3) convective heat transfer between the solid and fluid phases. Radiation heat transfer was assumed to be negligible, and combustion was not a consideration. The differential equations for the fluid and solid phases

\[
\text{Fluid: } \rho_f c_f \frac{\partial T_f}{\partial t} = -u \rho_f c_f \frac{\partial T_f}{\partial x} + k_f \frac{\partial^2 T_f}{\partial x^2} - h(T_f - T_s) \quad (II.1)
\]

\[
\text{Solid: } \rho_s c_s (1 - p) \frac{\partial T_s}{\partial t} = k_s (1 - p) \frac{\partial^2 T_s}{\partial x^2} + h(T_f - T_s) \quad (II.2)
\]

were solved numerically by the finite difference method. In equations II.1 and II.2, \(c\) is the specific heat at constant pressure, \(u\) is the fluid velocity through the medium, \(h\) is the coefficient of heat transfer, and \(k\) is a pseudothermal conductivity. The model did not include change of properties with temperature. Their numerical results showed good agreement with their experimental results. Two useful conclusions derived from their analysis were: (1) both conduction and convection heat transfer mechanisms are important for internal Reynolds numbers less than one, and (2) for values of the
dimensionless parameter, \( \xi = (hz/k_f)^{1/2}(k_f/\rho_f c_f u) \) greater than .342, the fluid temperature is approximately the solid temperature. Riaz [6] proposed the following equation resulting from the equal fluid-solid temperature assumption,

\[
\rho_s c_s \frac{\partial^2 T}{\partial t^2} + \rho_f c_f u \frac{\partial T}{\partial x} = k_s \frac{\partial^2 T}{\partial x^2}
\]  

(II.3)

where the coefficients are defined as those in equations II.1 and II.2. For the present model, the more general approach (i.e., Green and Perry) will be taken in the formulation of the energy equations.
III. FORMULATION OF THE FIELD EQUATIONS

A. INTERNAL FLOW MODEL

Referring to Figure 2, the porous mat may be represented by a flat plate. One side is exposed to still air, and the other to an air flow of velocity, \( U_a \). The environment on both sides is at ambient temperature, \( T_a \).

The first step in developing the model was to derive an expression for the flow through the porous plate. Effects of the combustion process on the physical properties were neglected. Specifically, these were (1) variations in the porosity due to fiber consumption, and (2) effects on the viscosity and density of air from the introduction of combustion by-products into the flow. Preliminary calculations showed that the porosity and density of the material did not change appreciably until temperatures reached approximately 2000 degrees Fahrenheit. Further, since eighty percent of air by volume is essentially inert, it was reasonable that its properties would not be significantly altered during the combustion process. Assuming a heterogeneous material of constant porosity, a Reynold's number for the interior flow is defined as

\[
Re_1 = \frac{\rho u_p d}{\mu} \quad \text{(III.1)}
\]
where $\rho_a$ is the mass density of air, $u_p$ is the pore velocity, $d$ is the diameter of the fiber, and $\mu$ is the dynamic viscosity. Extensive experimental work shows that Darcy's law for flow through a porous medium is valid for a limited range of Reynolds numbers. Muskat [7] proposed that Darcy's flow is valid for $Re_i < 1$. Scheidegger [8] points out that a number of investigators give a much higher upper limit. Preliminary calculations of $Re_i$ for the model showed typical magnitudes on the order of 1. Neglecting the gravity term, Darcy's law for flow in porous media is,

$$ u_p = -\frac{m}{\mu} \frac{dP}{dx} \tag{III.2} $$

assuming a constant pressure gradient, equation III.2 becomes,

$$ u_p = -\frac{m}{\mu} \frac{\Delta P}{L} \tag{III.3} $$

where $\Delta P$ is the pressure differential across the plate thickness, $m$ is the specific permeability of the medium, and $\mu$ is the dynamic viscosity. In equations III.1 and III.3, $\rho_a$ and $\mu$ were treated as temperature dependent.

Specific permeability is a measure of a porous medium's ability to allow fluid to flow through. It is dependent only on geometry and the physical nature of the medium. It has dimensions of length squared. Specific permeability is often
called the hydraulic conductivity due to the similarity of Darcy's law to that of Fourier's law for heat conduction. The permeability for a particular medium is normally measured by experimentation. However, there are several empirical models that may be used to obtain values for permeability that are in agreement with experimental results. For the idealized geometry of the porous medium shown in Figure 2, the permeability was obtained by a serial type model, equation III.4, proposed by Scheidegger [8],

\[ m = \frac{1}{96}p(\delta/\tau)^2 \]  

where the porosity, \( p \), is defined for a prous material comprised of fibers with cylindrical shape as,

\[ p = 1 - \frac{\pi}{4}(d/D)^2 \]  

and \( D \) is the thickness of a ply. Porosity has units of volume of void per unit volume of medium. The average pore diameter, \( \delta \) was defined as,

\[ \delta = (s + D)/2 \]

for the geometry in Figure 2. The tortuosity, \( \tau \), is a measure of length of travel for a fluid particle per unit thickness of the medium. Referring to the geometry of Figure 2, the
tortuosity depends on the ratio, $d/D$. The lower limit occurs when $d/D$ equals zero and for the upper limit, $d/D$ equals 1. Carman [9] presents a table of tortuosities for given geometries and particle shapes. For the model, Carman's suggested value of $\tau = 1.4$ was used.

To obtain the pressure differential, $\Delta P$, across the porous plate, Bernoulli's equation was used. The following observations showed this to be a valid assumption. Schlichting [10] points out that for the ratio of $u_p/U_\infty$ in the range of .0001 to .01, the effects of "blowing" or "suction" on the potential flow over the porous plate may be neglected. A typical value of $u_p/U_\infty$ for the model at which $U_\infty = 25$ kt. was .0028. For steady flow over a flat plate, the flow field outside the boundary layer may be described by Bernoulli's equation. This is a direct result of the Navier-Stokes equation. In addition, the pressure gradient across the boundary layer may be taken equal to zero. Therefore, Bernoulli's equation (eq. III.6) may be used to obtain the pressure differential across the plate,

$$\int \frac{P}{\rho_a} \, dx + \frac{U^2}{2} = \text{constant.} \quad (III.6)$$

The parameters in equation III.6 are defined as follows: $P$, pressure; $\rho_a$, the density of air; $U$, velocity of the air over the flat plate. For the model, the density of air was approximated by the Ideal Gas law as
\[ \rho_a = \frac{P}{R_a T_a} \]  \hspace{1cm} (III.7)

where \( R_a \) is the gas constant for air, \( T_a \) is the absolute temperature of air, and \( P \) is the pressure. Substituting this into equation III.6, gives

\[ \int \left( \frac{R_a T_a}{P} \right) dP + \frac{U_2^2}{2} = \text{constant} \]

and upon integrating, the expression becomes

\[ R_a T_a \ln P + \frac{U_2^2}{2} = \text{constant} \]

or,

\[ R_a T_a \ln P_1 + \frac{U_1^2}{2} = R_a T_a \ln P_2 + \frac{U_2^2}{2} \]

Letting

\[ P_1 = P_\infty, \ U_1 = 0, \ T_{a1} = T_{a2} = T_\infty, \ P_2 = P_L, \ U_2 = U_\infty \]

and substituting these in above, yields

\[ R_a T_\infty \ln P_\infty = R_a T_\infty \ln P_L + \frac{U_\infty^2}{2} \]

Rearranging the previous expression gives,
\[ \frac{R_a T_a}{U_a} \ln \left( \frac{P_\infty}{P_L} \right) = \frac{U_a^2}{2} \]

Taking the exponential of both sides and solving for \( P_L \) results in

\[ P_L = P_\infty \exp \left( -\frac{U_a^2}{2R_a T_a} \right) \]

From the above expression and noting that \( \Delta P = P_L - P_\infty \), \( \Delta P \) may be expressed as

\[ \Delta P = P_\infty \left[ \exp \left( -\frac{U_a^2}{2R_a T_a} \right) - 1 \right] \]

Darcy's law in approximate form (eq. III.3) for the model is

\[ u_p = -\frac{m}{\mu} \frac{\Delta P}{L} \]

and substituting for \( \Delta P \), it follows that,

\[ u_p = \left( \frac{m}{\mu} \frac{P_\infty}{L} [1 - \exp (-U_a^2/2R_a T_a)] \right) \quad \text{(III.8)} \]

where

\[ m = \frac{1}{96} \left( \frac{\Delta}{\mu} \right)^2 \]
The pore velocity, \( u_p \) of equation III.8 is for an isothermal medium at ambient temperature, \( T_a \). Since the model is to investigate the transient problem, a general expression to obtain pore velocity at different temperatures had to be developed.

Consideration of the continuity equation for one-dimensional steady flow,

\[
d(\rho_a u_p)/dx = 0
\]

becomes

\[
u_p (d\rho_a/dx) + \rho_a (du_p/dx) = 0
\]

or

\[
u_p (d\rho_a/dx) = -\rho_a (du_p/dx)
\]

Multiplying through both sides by \( dx \), the expression becomes,

\[
u_p d\rho_a = -\rho_a du_p
\]

Separating variables and integrating both sides, gives
\[
\int_{u_p}^{u_{p0}} \frac{du}{\rho_u} = - \int_{u_p}^{u_{p0}} \frac{du}{\rho_u}
\]

and the integral becomes

\[
\ln\left(\frac{\rho_u}{\rho_{u0}}\right) = \ln\left(\frac{u_{p0}}{u_p}\right)
\]

Taking the exponential of both sides, gives

\[
\frac{\rho_u}{\rho_{u0}} = \frac{u_{p0}}{u_p}
\]

Rearranging, the pore velocity is expressed as

\[
u_p = P_u \frac{u_{p0}}{\rho_u}
\]

(III.10)

Referring to the Ideal Gas law for the density of air, and substituting this into equation III.10 gives

\[
u_p = \frac{(\rho_u u_{p0} R_a T_a)/P}{P_u}
\]

(III.11)

The pore velocity is now a function of air temperature and pressure. However, in preliminary calculations, the pressure difference, \(\Delta P\) was small and for the range of parameters of interest, it did not exceed five percent of the magnitude of ambient pressure. Therefore, as a simplification, it was assumed that \(P\) equals the ambient pressure in equation III.11,
and the expression becomes

\[ u_p = \left( \rho_\infty u_{p,\infty} R_a T_a \right)/P_\infty \]

Noting that \( \rho_\infty = P_\infty/R_a T_\infty \), the pore velocity may be expressed as

\[ u_p = u_{p,\infty} T_a / T_\infty \]  \hspace{1cm} (III.12)

where \( u_{p,\infty} \) is obtained from equation III.8. \( T_a \) and \( T_\infty \) are the absolute temperatures of the air at a point in the interior of the medium and of the environment, respectively.

B. ENERGY EQUATIONS

Here we consider the development of the energy equations. However, before continuing, discussion must be made as to the approach taken for energy balances in porous media. In previous work by Denbigh and Turner [11], and by Colladay and Stepka [12], it was suggested that the temperature of the porous solid and of the fluid be considered equal. This greatly simplifies the formulation of the problem. However, under certain conditions, as shown by Green and Perry [5], the assumption of equal temperatures may not be valid. With recent developments in numerical techniques permitting the solution of non-linear systems, the development of the present model for heat transfer in a porous medium was not restricted
to the assumption of equal temperatures. Therefore, the temperatures of the air and of the porous solid are treated as independent variables in the present formulation.

The model is based on two assumptions. First, due to the small thickness of the graphite fibers, the temperature across each individual fiber was assumed to be constant. Second, for a similar size consideration, the temperature of the air in each individual pore was assumed constant. However, the temperature from fiber to fiber and from pore to pore was not restricted, and was allowed to vary according to the heat transfer mechanisms described next.

To perform energy balances on both the porous solid and on the air, a differential volume of porous material was segregated into respective volumes of constituents, that is, graphite fibers and air (Figure 3). An energy balance may be done on each volume separately. The convention used for the balance is:

\[
\text{Heat into } dV + \text{Heat Generation of } dV = \text{Heat out of } dV + \text{Increase in Internal Energy}
\]

where

\[
dV = dx \, dy \, dz
\]
1. **Fiber Heat Transfer Equation**

   Considering the differential volume, \((1-p)dV\) of graphite fibers, and "smearing" the fibers to form a macroscopically homogeneous material, the heat transfer mechanisms are shown for the \(x\)-direction in Figure 4. The heat transfer mechanisms are: \(q_{\text{cond}}\), heat conduction through the medium; \(q_{\text{conv}}\), convection from the fibers to the air; \(q_{\text{rad}}\), radiation transfer from fiber to fiber; \(\Delta H_r(T_g, \phi)\) heat generation rate per unit volume. \(\Delta H\) is the enthalpy of formation, and \(r(T_g, \phi)\) is the fiber mass consumption rate. This term will be discussed in detail later.

   Representing the terms in Figure 4 by Taylor series expansions and combining them with respect to the convention previously stated, the energy balance is

   \[
   q_{\text{cond}} + q_{\text{rad}} + \Delta H_r(T_g, \phi)(1-p)dxdydz = q_{\text{cond}} + \frac{3q_{\text{cond}}}{3x} \frac{dx}{x} 
   \]

   \[
   + q_{\text{rad}} + \frac{3q_{\text{rad}}}{3x} \frac{dx}{x} + q_{\text{conv}} + \frac{3E}{3t}(1-p)dxdydz 
   \]

   Subtracting terms, \(q_{\text{cond}}\) and \(q_{\text{rad}}\) from each side and rearranging, the expression becomes (neglecting the higher order terms)

   \[
   - \frac{3q_{\text{cond}}}{3x} \frac{dx}{x} - \frac{3q_{\text{rad}}}{3x} \frac{dx}{x} - q_{\text{conv}} + \Delta H_r(T_g, \phi)(1-p)dxdydz = \frac{3E}{3t}(1-p)dxdydz 
   \]

   \[
   \text{(III.13)} 
   \]
The heat transfer terms in equation III.13 will now be considered.

a. Thermal Conduction

Fourier's law for conduction may be written for the incremental volume as

\[ q_{\text{cond}} = -k_g \frac{\partial T}{\partial y} \text{dydz} \]  

where \( k_g \) is the effective conductivity of the porous solid, and \( T_g \) is the temperature of the fibers. There are a variety of models for the effective conductivities of porous media. They depend largely on the geometry and on the nature of the constituents. For the cylindrical shape of the fibers, the empirical relation proposed by Rohsenow [13] was used

\[ k_g = k_a \left[ 1 - \beta \left( 1 + \frac{\gamma}{(1-\gamma)^{1/2}} \ln \left( \frac{1-(1-\gamma)^{1/2}}{\gamma} \right) \right) \right] \]  

for \( \gamma^2 < 1 \). The parameters of equation III.15 are defined as follows: \( \beta = d/D \) (refer to Figure 2), \( \gamma \) is

\[ \gamma = \frac{1}{\beta} \left[ \frac{\Lambda}{(1-\Lambda)} \right] \]

where \( \Lambda = k_a/k'_g \), \( k_a \) is the conductivity of the air; and \( k'_g \) is the actual conductivity of the graphite in bulk form. Since \( k_a \) depends on temperature, \( k_g \) was also treated as temperature dependent.
b. Radiation Heat Transfer in Fibers

Radiation heat transfer in the model was represented by an analog to Fourier's law of conduction as

$$q_{\text{rad}} = -k_r \frac{\partial T}{\partial x} \ dy \ dz \quad (\text{III.16})$$

where $k_r$ is an equivalent conductivity of the fibers due to radiation. Simple assumptions and approximations may be made to show this. Assuming air to be transparent to radiation, and treating a ply of graphite fibers as an infinite wall, the net heat flux between plies may be written as

$$q_{\text{rad}}^n = \frac{\sigma e}{(2-\epsilon)} (\hat{T}_{gx}^4 - \hat{T}_{gx+dx}^4) \quad (\text{III.17})$$

where $\hat{T}_{gx}$ and $\hat{T}_{gx+dx}$ are the respective absolute ply temperatures, $\epsilon$ is the emissivity of the graphite, and $\sigma$ is the Stefan-Boltzmann constant. Treating $T_{gx}$ as the variable temperature, $q_{\text{rad}}^n$ may be expanded in a Taylor series about $\hat{T}_{gx+dx}$. Neglecting the higher order terms, the series expansion may be written as

$$q_{\text{rad}}^n = \frac{\sigma e}{2-\epsilon} (\hat{T}_{gx+dx}^4 - \hat{T}_{gx+dx}^4) + \frac{4\sigma e}{2-\epsilon} \hat{T}_{gx+dx}^3 (\hat{T}_{gx} - \hat{T}_{gx+dx})$$

Simplifying, the above equation becomes,

$$q_{\text{rad}}^n = \frac{4\sigma e}{2-\epsilon} \hat{T}_{gx+dx}^3 (\hat{T}_{gx} - \hat{T}_{gx+dx}) \quad (\text{III.18})$$
Taking Fourier's law for heat transfer,

\[ q_{\text{rad}} = -k_r \frac{dT}{dx} \]

and approximating \( \frac{dT}{dx} \) with

\[ \frac{dT}{dx} = \frac{\Delta T}{\Delta x} = \frac{T_{gx+dx} - T_{gx}}{\Delta x} \]

\( q_{\text{rad}} \) becomes

\[ q_{\text{rad}} = -k_r \frac{(T_{gx+dx} - T_{gx})}{\Delta x} \]  \hspace{1cm} (III.19)

Multiplying the right side of equation III.18 by \( \Delta x/\Delta x \), equation III.18 and equation III.19 may be equated

\[ q_{\text{rad}} = -k_r \frac{(T_{gx+dx} - T_{gx})}{\Delta x} = -\frac{4\varepsilon \Delta x}{2-\varepsilon} \frac{T_{gx+dx}^3}{\Delta x} \frac{(T_{gx+dx} - T_{gx})}{\Delta x} \]

Cancelling terms, \( k_r \) becomes

\[ k_r = \frac{4\varepsilon \Delta x}{2-\varepsilon} \frac{T_{gx+dx}^3}{\Delta x} \]  \hspace{1cm} (III.20)

where \( \Delta x \) is now equal to \( \delta \), the average pore diameter.

From the close spacing of the fiber layers, the temperature difference will be small as compared to the magnitude of the temperature. Noting this, the average absolute temperature of
the fibers, \( T_g \) may be substituted for \( T_{g(x+dx)} \). The equivalent radiation conductivity expression becomes

\[
k_r = \frac{4\pi \delta}{2-\varepsilon} T_g^3 \tag{III.21}
\]

and the radiation heat transfer from fiber to fiber may be represented by,

\[
q_{rad} = -k_r \frac{\partial T_g}{\partial x} = -\frac{4\pi \delta}{2-\varepsilon} T_g^3 \frac{\partial T_g}{\partial x} dy \, dz \tag{III.22}
\]

If the temperature gradient in the fibers is not large, then equation III.22 will be a good approximation of the radiation heat transfer between the fibers.

c. Convection Heat Transfer

The convection heat transfer term in Figure 4 was treated in a similar manner to that of the one-dimensional fin equation. \( q_{conv} \) was introduced as

\[
q_{conv} = h_i (T_g - T_a) \, dA \tag{III.23}
\]

where \( T_g \) is the temperature of the graphite, \( T_a \) is the temperature of the air, \( dA \) is the surface area of graphite in the differential volume, and \( h_i \) is the internal convection heat transfer coefficient. An empirical expression in the form of a Colburn j-factor was developed by Yoshida, Ramaswami, Hougen [14]. This was used to determine \( h_i \),
\[
J = \frac{h_i c_a^{\frac{2}{3}}}{c_b G_o k_a} = 0.91 Re^{-0.51} \theta_o
\] (III.24)

for \(Re < 50\). \(c_b\) is the effective specific heat of the porous medium and is defined as,

\[
c_b = \rho c_a + (1-p) c_g
\]

where \(c_a\) is the specific heat of air at constant pressure; \(c_g\) is the specific heat of graphite; and \(p\) is the porosity as defined by equation III.5. \(G_o\) is a pseudo mass velocity defined as \(G_o = \rho_a u_p p\). Also in equation III.24, \(u\) is the viscosity of air, and \(k_a\) is the conductivity of air. The Reynolds number appearing in equation III.24 is defined as

\[
Re = \frac{G_o}{z} \mu \theta_o
\]

and is not the same as \(Re_i\) defined previously for Darcy's law. \(z\) is the surface area of graphite fibers per unit volume, and is determined from geometrical considerations which will be described shortly. The parameter, \(\theta_o\), is a dimensionless shape factor which depends on the geometry of the fibers. For a cylindrical fiber shape, \(\theta_o\) is equal to .91. From the assumption of constant porosity, \(z\) remains constant. However, the air properties are temperature dependent; therefore, \(h_i\) is allowed to vary with temperature.
d. Heat Generation Rate

The reaction rate, \( r(T_g, \phi) \), whether based upon the Collision Theory or on the Theory of Absolute Reaction Rates, results in an expression of Arrhenius type,

\[
r = A(T, \phi) \exp(-E/RT)
\]

where \( E \) is the activation energy, \( R \) is the universal gas constant, and \( \hat{T} \) is absolute temperature. Although \( E \) is well defined for the combustion of graphite, \( A(T, \phi) \) is a function of temperature and concentration and depends upon chemical kinetic theory. Development of a general model for the chemical kinetics is beyond the scope of this work. A less elegant, but useful approach of using a relation obtained experimentally was taken. A relation derived from the work of Parker and Hottel [15] was used to predict the combustion rate of graphite fibers,

\[
r = 4.014 \times 10^7 R_a T_g^{1/2} \phi \exp\left(-\frac{39,883}{T_g}\right) \frac{1\text{bm}}{\text{ft}^3\cdot\text{hr}} \quad \text{(III.25)}
\]

The development of equation III.25 from the original work may be found in the appendix. This expression assumes a simple first-order reaction for the combustion of graphite in air. However, Frank-Kamenetskii [3] has further refined Parker and Hottel's expression to yield for the present model,
The combustion is now characterized as an nth order reaction, where n is in the range of 1/3 to 2/3. This form of the reaction rate expression better approximates Parker and Hottel's experimental data. For the analysis, n was chosen to be 1/2. However, it should be pointed out that the behavior of the reaction varies significantly within the range of n proposed by Frank-Kamenetskii.

In limiting the model to the kinetic regime of combustion, it was assumed that the dominant chemical reaction is

\[ C + O_2 \rightarrow CO_2 \]

This is an idealized treatment of the reaction that actually takes place. Kanury [16] states that, the actual reaction is more complex, yielding various concentrations of carbon dioxide and carbon monoxide, depending on the temperature. Noting the abundant oxygen supply present in the kinetic regime, it is reasonable that the leaner oxide is the prevalent by-product of combustion. The fuel-oxygen ratio, f, for this mechanism is 12/32. Carbon monoxide may be formed when there is little free-oxygen present, such as, during combustion in the diffusion regime. The stoichiometric
fuel-oxygen ratio, \( f = 12/16 \), for this mechanism is naturally greater. To obtain the heat generation rate, the reaction rate, \( r \), was multiplied by the enthalpy of formation, \( \Delta H \), of carbon dioxide. \( \Delta H \) has units of Btu per lbm of graphite consumed.

e. Change in Internal Energy

The last term of the graphite energy equation, the rate of change of internal energy, \( \frac{dE}{dt} \), results by setting \( E = \frac{\rho g c_g T_g}{\frac{dE}{dt}} \). Thus,

\[
\frac{dE}{dt} (1-p) dx dy dz = (1-p) \rho_g c_g \frac{dE}{dt} dx dy dz \quad (III.27)
\]

where \( \rho_g \) is the density of graphite in bulk; \( c_g \) is the specific heat of graphite; and \( p \) is the porosity.

The heat transfer mechanisms for the graphite fibers, equations III.14, III.16, III.23, III.26, III.27 are substituted into equation III.13 which upon dividing through by \( dV = dx dy dz \) yields the heat transfer equation for the fibers,

\[
(1-p) \frac{\partial}{\partial x}(k_g \frac{dT_g}{\partial x}) + (1-p) \frac{\partial}{\partial x}(k_r \frac{dT_g}{\partial x}) - h_i \frac{dA}{dV}(T_g - T_a) + (1-p) \Delta H r(T_g, \phi) = (1-p) \rho_g c_g \frac{dT_g}{dt}
\]

The coefficient, \( (1-p) \), accounts for the fact that not all of the volume of porous medium is comprised of graphite.
Dividing through by \((1-p)\), combining the first two terms, and defining \(dA/dV\) as \(z\), the surface area of graphite per unit volume, the expression becomes

\[
\frac{\partial}{\partial x} [(k_g + k_r) \frac{\partial T_g}{\partial x}] - \frac{h}{(1-p)}(T_g - T_a) + \Delta H_r(T_g, \phi) = \rho C_g \frac{\partial T_g}{\partial t}
\]

(III.28)

The parameter, \(z\), is obtained from geometry considerations. For a cylindrical fiber shape, the expression for \(z\) is,

\[
z = \frac{\pi d}{2D^2}
\]

(III.29)

The one-half factor in equation III.29 accounts for the uncertainty of the actual amount of exposed fiber surface area, which resulted from the initial combustion of epoxy. The non-dimensionalization of equation III.28 is presented in Appendix A.

2. Internal Flow Heat Transfer Equation

The energy balance on the internal flow is obtained by a procedure similar to the energy balance on graphite. Figure 5 presents a "smeared" differential volume of air, \(pdV\), which shows the heat transfer mechanisms. The heat transfer mechanisms are: \(q_{\text{cond}}\), heat conduction through the air; \(q_{\text{conv}}\), convection from the fibers to the air; \(m_a h\),
energy transport due to the air flow, where $\hat{h}$ is the enthalpy of the air.

Performing the energy balance (neglecting the higher order terms) yields,

$$q_{\text{cond}} + \dot{m}_a \hat{h} + q_{\text{conv}} = q_{\text{cond}} + \frac{\partial q_{\text{cond}}}{\partial x} \, dx$$

$$+ \dot{m}_a \hat{h} + \dot{m}_a \frac{\partial \hat{h}}{\partial x} \, dx + \frac{\partial E}{\partial t} \, pdx dy dz \quad (\text{III.30})$$

Cancelling terms and rearranging, the expression becomes,

$$- \frac{\partial q_{\text{cond}}}{\partial x} \, dx - \dot{m}_a \frac{\partial \hat{h}}{\partial x} \, dx + q_{\text{conv}} = \frac{\partial E}{\partial t} \, pdx dy dz \quad (\text{III.31})$$

a. Thermal Conduction

As before, $q_{\text{cond}}$ was replaced by Fourier's law of heat conduction in the form,

$$q_{\text{cond}} = - k_a \frac{\partial T_a}{\partial x} \, dy dz \quad (\text{III.32})$$

where $k_a$ is the conductivity of air.

b. Energy Transport by Flow

Using the perfect gas assumption for air, the enthalpy $\hat{h}$ was expressed as,

$$d\hat{h} = c_a \, dT_a$$
where $c_a$ is the specific heat of air at constant pressure. The specific heat is treated as constant since it does not vary appreciably over the range of temperatures associated with the problem under investigation. It follows that,

$$\frac{\partial h}{\partial x} = c_a \frac{\partial T_a}{\partial x} \quad (III.33)$$

Assuming the effects of the combustion by-products on air are negligible, continuity considerations give the mass flow as,

$$\dot{m}_a = \rho_a u \rho_0 dydz \quad (III.34)$$

where the density of air and the pore velocity are evaluated at ambient conditions.

c. Convection Heat Transfer

The convection heat transfer, $q_{conv}$ given by equation III.23, with $h_i$ obtained from equation III.24, becomes

$$q_{conv} = h_i (T_g - T_a) dA \quad (III.35)$$

d. Change in Internal Energy

For a perfect gas, the internal energy is,

$$dE = \rho_a c_a dT_a$$
where \( c_a \) is the specific heat of air at constant pressure.

From this it follows that

\[
\frac{\partial E}{\partial t} p \, dx\,dy\,dz = \rho_a c_a \frac{\partial T_a}{\partial x} p \, dx\,dy\,dz \quad (III.36)
\]

Substituting equations III.32-III.36 into equation III.31, the heat transfer equation for the internal flow becomes,

\[
\frac{\partial}{\partial x} \left( k_a \frac{\partial T_a}{\partial x} \right) p \, dx\,dy\,dz - \dot{m}_a c_a \frac{\partial T_a}{\partial x} p \, dx\,dy\,dz
\]

\[
+ h_i \frac{\partial A}{\partial x} (T_g - T_a) = \rho c_a \frac{\partial T_a}{\partial t} p \, dx\,dy\,dz
\]

As in the energy equation for the fibers, the porosity, \( p \), accounts for the fact that some of the volume of the porous medium is not air. Dividing the last equation through by \( p \, dx\,dy\,dz \) and letting \( z \) denote \( \frac{dA}{dx\,dy\,dz} \), yields

\[
\frac{\partial}{\partial x} \left( k_a \frac{\partial T_a}{\partial x} \right) - \dot{m}_a c_a \frac{\partial T_a}{\partial x} + \frac{h_i z}{p} (T_g - T_a)
\]

\[
= \rho c_a \frac{\partial T_a}{\partial t} \quad (III.37)
\]

Non-dimensionalization of equation III.37 may be found in Appendix A.
3. Oxygen Transport Equation

The final consideration in the formulation of the model is the mass transport of oxygen. The mass transport equation is obtained by a mass balance on a differential volume of porous medium. In accordance with convention, the mass balance is given by

\[
\text{Oxygen into } \frac{\text{d}V}{\text{d}V} = \text{Oxygen out of } \frac{\text{d}V}{\text{d}V} + \text{Consumption} + \text{Accumulation}
\]

Figure 6 presents a differential volume, pdV, showing the relevant molecule-mass transport mechanisms. The mass transport mechanisms are: \( \dot{m}_B \) is the molecular diffusion; \( \dot{m}_C \) is the convective transport due to mass flow; \( \frac{1}{r}(T_g, \phi) \) is the consumption of oxygen due to combustion; \( r \) is the reaction rate, and \( f \) is the stoichiometric ratio of the reaction. Representing the terms in Figure 6 by Taylor series expansions, the molecule-mass balance becomes (neglecting higher order terms),

\[
\dot{m}_B + \dot{m}_C = \dot{m}_B + \frac{\partial \dot{m}_B}{\partial x} \text{d}x + \dot{m}_C + \frac{\partial \dot{m}_C}{\partial x} \text{d}x
\]

\[
+ \frac{1}{r}(T_g, \phi) \text{pdx} \text{dxdydz} + \frac{\partial f}{\partial T} \text{pdxdydz}
\]

(III.38)

Upon cancelling terms and rearranging, equation III.38 becomes
- \frac{\partial \hat{m}_B}{\partial x} \, dx - \frac{\partial \hat{m}_C}{\partial x} \, dx - \frac{1}{2} \mu (T_g, \phi) \rho dx \, dy \, dz = \frac{\partial \phi}{\partial t} \, dx \, dy \, dz \quad (III.39)

where \( \phi \) is the concentration of oxygen. The individual mass transfer terms in equation III.39 will now be developed.

a. Molecular Diffusion

Molecular diffusion \( \hat{m}_B \), is given by Fick's law,

\[
\hat{m}_B = - B \frac{\partial \phi}{\partial x} \, dy \, dz \quad (III.40)
\]

where \( B \) is the diffusivity of oxygen into the porous medium.

The diffusion of gases in porous media is limited by two factors. One is the collision of the gas molecules with other molecules, and the second is the collision of the gas molecules with the solid pore walls. This second phenomenon is discussed by Bennett and Myers [17] and is referred to as Knudsen diffusion. To establish which behavior dominates, the mean free path of the gas (in this case, oxygen), must be calculated. Using the expression given by Treybal [18],

\[
w = \left(3.2u/P\right)(RT/2\pi g_C M)^{1/2}
\]

where \( g_C \) is the gravitational constant, the value obtained for mean free path, \( w \), is smaller than 0.16. Thus, the collision of the oxygen molecules with those of air will restrict the diffusion process. On the contrary, if the pore
diameter is smaller than the mean free path, then the collision of the molecules with the solid walls will be the limiting factor. In the intermediate case, the mean free path and the pore diameter are of the same order of magnitude. In this case, Bennett and Myers [17] proposed the following expression be used to obtain a value of diffusivity,

\[
\frac{1}{B_R} = \frac{1}{B_g} + \frac{1}{B_w}
\]  

(III.41)

where \(B_R\) is the resulting diffusivity; \(B_g\) is the diffusivity based on molecule-molecule collisions; and \(B_w\) is the diffusivity based on molecule-pore wall collisions.

For the problem under investigation here, preliminary calculations of the mean free path of the oxygen showed that diffusion is primarily of the inter-molecular collision type. With respect to this, and knowing that molecular diffusion is highly temperature dependent, the diffusivity, \(B^*\) was obtained from an expression presented by Gilliland [19].

\[
B^* = 435.7 \frac{\hat{T}_{a}^{3/2}}{P} (\frac{v_{a}^{1/3}}{V_{02}^{1/3}})^{-2} (\frac{M_{a}^{-1}}{M_{02}^{-1}})^{1/2} \text{ cm}^2/\text{s}
\]

(III.42)

where \(P\) is the absolute pressure; \(V_{a}\) and \(V_{02}\) are the molecular volumes of air and oxygen respectively; \(M_{a}\) and \(M_{02}\) are the molecular weights of air and oxygen, respectively, and \(B^*\) is the diffusivity. For this expression, \(\hat{T}_{a}\) is in degrees
Kelvin, and \( B^* \) has the units of \( \text{cm}^2/\text{s} \). As a result of tortuosity and porosity affecting the process, Denbigh and Turner [11] proposed that the effective molecular diffusivity for a porous medium is given by

\[
B_e = \frac{pB^*}{\tau}.
\]

The porosity \( p \) accounts for the void in a differential cross-section.

b. Convective Transport

The convective term in equation III.39 is given by

\[
\dot{m}_c = pu_p \phi \text{dydz} \tag{III.43}
\]

The pore velocity \( u_p \) is a temperature dependent variable, and is so treated.

c. Consumption Rate

The reaction rate term, \( r(T_g, \phi) \) was discussed previously as equation III.26. The consumption rate of oxygen is determined by multiplying \( r \) by \( 1/f \).

Substituting equations III.40 and III.43 into equation III.39, the oxygen transport equation becomes,

\[
\frac{2}{\tau \phi} \left( \frac{\partial B^*}{\partial x} \phi \right) \text{pdxdydz} - \frac{2}{\tau \phi} \left( u_p \phi \right) \text{pdxdydz}
\]

\[
- \frac{1}{f} r(T_g, \phi) \text{pdxdydz} = \frac{\partial \phi}{\partial t} \text{pdxdydz}
\]

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Dividing both sides by \( p dxdydz \), and letting \( B = B^* / \tau \), the expression becomes

\[
\frac{3}{3x} (B \frac{3}{3x} \phi) - \frac{3}{3x} (u_p \phi) - \left( \frac{1}{\tau} \right) r(T_g, \phi) = \frac{3}{3t} \phi . \tag{III.44}
\]

Expanding the second term of equation III.44,

\[
\frac{3}{3x} (u_p \phi) = u_p \frac{3}{3x} \phi + \frac{3}{3x} \frac{3}{3x} u_p \phi
\]

and noting that \( u_p \) depends on \( T_a \) from equation III.12, the expression yields

\[
\frac{3}{3x} (u_p \phi) = u_p \frac{3}{3x} \phi + \frac{3}{3x} \frac{3}{3x} u_p \phi \tag{III.45}
\]

\( \frac{3}{3x} u_p / \frac{3}{3x} T_a \) is obtained from equation III.12.

Substituting equation III.45 into equation III.44, the oxygen transport equation becomes,

\[
\frac{3}{3x} (B \frac{3}{3x} \phi) - u_p \frac{3}{3x} \phi \frac{3}{3x} u_p \frac{3}{3x} T_a \phi \frac{3}{3x} \phi - \left( \frac{1}{\tau} \right) r(T_g, \phi) = \frac{3}{3t} \phi \tag{III.46}
\]

In contrast with the heat transfer equations, mass balance does not depend on porosity. However, porosity does influence the oxygen concentration by appearing in the boundary conditions. Non-dimensionalization of equation III.46 is presented in Appendix A.

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IV. BOUNDARY CONDITIONS

It was intended that boundary conditions be imposed which simulate the composite plate "burn" experiments of Fontenot [1]. However, for a one-dimensional model, the boundary conditions can only be approximated at best. The difficulty is associated with the existence of momentum and thermal boundary layers on the exterior surface of the porous plate. Complications also arise from the "blowing and suction" effects which are caused by the flow through the plate.

The following boundary conditions provide a reasonable model for the one-dimensional problem,

\begin{align*}
(k_g + k_r) \frac{\partial T_g}{\partial x} &= h_1(T_g - T_\infty) + \sigma c(T_g^4 - T_\infty^4) \quad \text{at} \quad x = 0 \quad (IV.1) \\
(k_g + k_r) \frac{\partial T_g}{\partial x} &= -h_2(T_g - T_\infty) - \sigma c(T_g^4 - T_\infty^4) \quad \text{at} \quad x = L \quad (IV.2) \\
T_a &= T_\infty \quad \text{at} \quad x = 0 \quad (IV.3) \\
\frac{\partial T_a}{\partial x} &= \frac{\partial T_g}{\partial x} \quad \text{at} \quad x = L \quad (IV.4) \\
B \frac{\partial \phi}{\partial x} &= u_p(\phi - p\phi_\infty) \quad \text{at} \quad x = 0 \quad (IV.5) \\
\frac{\partial \phi}{\partial x} &= 0 \quad \text{at} \quad x = L \quad (IV.6)
\end{align*}
Boundary conditions IV.1 and IV.2 account for convection and radiation heat transfer from the porous solid. With an influx of air at \( x = 0 \), the heat transfer coefficient, \( h_1 \), will depend on the magnitude of the flow entering the plate. Neglecting "blowing and suction" effects, several empirical expressions based on free convection were used to obtain a heat transfer coefficient at \( x = 0 \). These expressions yielded a range of values for \( h_1 \) from 1 to 3 Btu/hr*ft\(^2\)*F. As an approximation, \( h_1 \) was taken as the average value, 2.0 Btu/hr*ft\(^2\)*F. However, choosing \( h_1 \) as any value in the above range did not affect the results of the analysis.

The magnitude of the forced convection heat transfer coefficient at \( x = L \), \( h_2 \), varies in a direction parallel to that of the external flow, \( U_\infty \). In addition, \( h_2 \) depends on the efflux of the gas at the surface. To simplify the analysis, \( h_2 \) was approximated by the relation for a smooth flat plate given by Holman [20] as

\[
h_2 = \frac{k_a}{L^*} \frac{Pr^{1/3}}{Re^{1/2}} \quad \text{(IV.7)}
\]

for laminar flow, where \( Pr \) is the Prandtl number and \( L^* \) is a reference length, arbitrarily taken as unity. For turbulent flow,

\[
h_2 = \frac{k_a}{L^*} Pr^{1/3} (0.037 Re^{0.8} - 850). \quad \text{(IV.8)}
\]
Kays [21] provides an alternate scheme for treating boundary layers and "blowing and suction" effects.

Boundary condition IV.3 accounts for the air entering the porous plate at ambient temperature. The Dankwerts boundary condition proposed by Riaz [6] which accounts for conduction-convection interaction, is a more reasonable simulation.

The boundary condition at \( x = L \) for the air heat transfer equation appears to be a reasonable one. It follows the behavior observed in the interior of the plate and does not fix the temperature of the air at the wall. The boundary condition, \( \partial T_a / \partial x = 0 \), used initially provided results similar to those obtained using equation IV.4.

Boundary conditions IV.5 and IV.6 are Dankwerts conditions [22] for flow through porous media. A convincing discussion for the Dankwerts conditions is given by Bischoff [23]. A brief summary of the discussion is presented in Appendix D.
V. FINITE ELEMENT METHOD

A. GALERKIN FORMULATION

A Galerkin formulation of the Finite Element Method was used to obtain solutions of the field equations. A convenient form of the field equations was used in the formulation. These equations are presented in Appendix A.

graphite energy equation

\[
\frac{2}{\pi} (\lambda_1 \frac{\partial \phi}{\partial n} - \lambda_2 (\phi - r) + \lambda_3 r(T_g, \phi) = \lambda_4 \frac{\partial \phi}{\partial t} \quad (V.1)
\]

air energy equation

\[
\frac{2}{\pi} (\nu_1 \frac{\partial r}{\partial n}) - \nu_2 \frac{\partial r}{\partial n} + \nu_3 (\phi - r) = \nu_4 \frac{\partial r}{\partial t} \quad (V.2)
\]

oxygen diffusion equation

\[
\frac{2}{\pi} (\omega_1 \frac{\partial \phi}{\partial n} - \omega_2 \frac{\partial \phi}{\partial n} - \omega_3 \frac{\partial \phi}{\partial n} - \omega_4 r(T_g, \phi) = \omega_5 \frac{\partial \phi}{\partial t} \quad (V.3)
\]

where the coefficients are defined in Appendix A.

Field equations III.28, III.37 and III.46 subject to boundary conditions, IV.1 - IV.6, and initial conditions define the problem. The closed domain (0,L) was partitioned into (n-1) contiguous elements of variable length \( l_i \), \( i = 1, \ldots, (n-1) \). This defines an n nodal point model. The three
non-dimensional field variables \( \Theta, \Gamma, \Phi \) were approximated by

\[
\Theta(n,t) = \psi_1(n,t) = \sum G_i(n) \Theta_i(t) \quad (V.4)
\]

\[
\Gamma(n,t) = \psi_2(n,t) = \sum G_i(n) \Gamma_i(t) \quad (V.5)
\]

\[
\Phi(n,t) = \psi_3(n,t) = \sum G_i(n) \Phi_i(t) \quad (V.6)
\]

where \( G_i \), for \( i = 1, \ldots, n \), is a set of specified basis functions with local support, and the sets \( \{ \Theta_1, \Gamma_1, \Phi_1; i = 1, \ldots, n \} \) are the solution coefficients to be determined. The \( G_i \) were selected to satisfy the condition \( G_i(n_j) = \delta_{ij} \), where the kronecker delta \( \delta_{ij} \) is defined by \( \delta_{ij} = 1 \) for \( i = j \), and \( \delta_{ij} = 0 \) for \( i \neq j \). As a result, \( \Theta, \Gamma, \) and \( \Phi \) are the values \( \psi_1, \psi_2, \psi_3 \) at the nodal points (i.e., \( \Theta_i(t) = \psi_1(n_i,t) \)).

Linear interpolation functions (Figure 7) were used as the basis functions. These are the lowest polynomial functions which provide the necessary function continuity.

As a measure of error, a residual function, \( r_i \), is defined for each field equation by

\[
r_i(n,t) = \Lambda_i(n) - \psi_i \quad i = 1, 2, 3 \quad (V.7)
\]

where \( \Lambda_i \) denotes the spatial operator of the \( i \)th equation.

For convenience, the following convention for differentiation
is adopted,

\[
\frac{\partial^3}{\partial r} = (\cdot)
\]

\[
\frac{\partial^2}{\partial r^2} = (\cdot)''
\]

\[
\frac{\partial}{\partial t} = (\cdot)
\]

For field equations V.1, V.2, and V.3 the residuals are:

\[
\begin{align*}
\mathbf{r}_1 &= \left[ \lambda_1(\Theta, \Gamma) \sum_{i=1}^{n} G_i^\theta \delta_i \right] - \lambda_2(\Gamma) \sum_{i=1}^{n} G_i(\delta_i - \Gamma_i) \\
&\quad + \lambda_3 \tau(\Theta) - \lambda_4 \sum_{i=1}^{n} G_i^\delta_i \\
&= (V.8)
\end{align*}
\]

\[
\begin{align*}
\mathbf{r}_2 &= \left[ \nu_1(\Gamma) \sum_{i=1}^{n} G_i^\Gamma \Gamma_i \right] - \nu_2 \sum_{i=1}^{n} G_i^\Gamma \Gamma_i + \nu_3(\Gamma) \sum_{i=1}^{n} G_i(\delta_i - \Gamma_i) \\
&\quad - \nu_4(\Gamma) \sum_{i=1}^{n} G_i^\Gamma \\
&= (V.9)
\end{align*}
\]

\[
\begin{align*}
\mathbf{r}_3 &= \left[ \omega_1(\Gamma) \sum_{i=1}^{n} G_i^\Omega \phi_i \right] - \omega_2(\Gamma) \sum_{i=1}^{n} G_i^\phi_i - \omega_3(\Gamma) \sum_{i=1}^{n} G_i^\Gamma \Gamma_i G_i^\phi_i \\
&\quad - \omega_4 \tau(\Theta, \phi) - \omega_5 \sum_{i=1}^{n} G_i^\phi_i \\
&= (V.10)
\end{align*}
\]
where the coefficients multiplying the response variables are themselves functions of the response variables, and thus the equations are nonlinear. In accordance with the Galerkin method, the final system of ordinary differential equations was obtained by setting each residual, $r_j$, orthogonal to each basis function, $G_i$, that is

$$\int_0^1 G_i r_j \, dn = 0 \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n \quad (V.11)$$

The $3n$ ordinary differential equations given by equations V.11 retain the character of the original set of partial differential equations. Thus linear field equations transform to matrix operators and nonlinear, coupled field operators become nonlinear, coupled algebraic operators. Incorporation of the boundary conditions resulted in $3n$ nonlinear, coupled ordinary differential equations

$$A(t) \psi(0, \Gamma, \phi) + F(t) = B(t) \psi \quad (V.12)$$

subject to initial conditions, where $B$ is a $3n \times 3n$ matrix, $A$ is the operator associated with the field operator $\Lambda_i$ in expression V.7, and $F(t)$ is an excitation vector.

Adopting the convention,

$$<G_i> \{\psi_i\} = \sum_{i=1}^{n} G_i \psi_i$$

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and applying the operation of expression V.11 with an integration by parts on the second order derivatives gives,

\[
\{G_i\} \lambda_1 <G_j> '|(\theta) \left| 1 - \frac{1}{\lambda_1} \int_0^1 \{G_i\}'<G_j> 'd\eta(\theta)
\]

\[
- \lambda_2 \int_0^1 \{G_i\}<G_j>d\eta(\theta) + \lambda_2 \int_0^1 \{G_i\}<G_j>d\eta(\Gamma) \quad (V.13)
\]

\[
+ \lambda_3 \int_0^1 \{G_i\}d\eta(\theta,\phi) = \lambda_4 \int_0^1 \{G_i\}<G_j>d\eta(\theta)
\]

\[
\{G_i\} v_1 <G_j> '|(\Gamma) \left| 1 - \frac{1}{\nu_1} \int_0^1 \{G_i\}'<G_j> 'd\eta(\Gamma)
\]

\[
- \nu_2 \int_0^1 \{G_i\}<G_j> 'd\eta(\Gamma) + \nu_3 \int_0^1 \{G_i\}<G_j>d\eta(\theta) \quad (V.14)
\]

\[
- \nu_3 \int_0^1 \{G_i\}<G_j>d\eta(\Gamma) = \nu_4 \int_0^1 \{G_i\}<G_j>d\nu(\Gamma)
\]

55
\[ (G_i)^{\phi} \omega_1 <G_j>\phi' \left|_0^1 \right. - \bar{\omega}_1 \int_0^1 (G_i)^{\phi} <G_j>\phi' \, d\eta(\phi) \]

\[ - \bar{\omega}_2 \int_0^1 (G_i) <G_j>\phi' \, d\eta(\phi) - \bar{\omega}_3 \int_0^1 (G_i) <G_j>\phi' \, d\eta(\phi) \]

\[ - \bar{\omega}_4 \int_0^1 (G_i) d\eta(\phi) = \bar{\omega}_5 \int_0^1 (G_i) <G_j>\phi' \, d\eta(\phi) \quad (V.15) \]

The first term in each of the above expressions is a boundary term which permits the incorporation of natural boundary conditions which will be shown in Section V.B. Coefficients \( \bar{\lambda}, \bar{\nu}, \bar{\omega} \) are variable-dependent properties, and were taken as the average value of the properties over an element. In the limit, as the elements get smaller (i.e., \( n \to \infty \)), the average values converge to the exact values.

Inspection of expression V.13, V.14, V.15 shows the five operators,

\[ \int_0^1 (G_i)^{\phi} <G_j>\phi' \, d\eta \quad (V.16) \]

\[ \int_0^1 (G_i) <G_j>\phi' \, d\eta \quad (V.17) \]

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\[
\int_0^1 \{G_i\} <G_j> d\eta \quad (V.18)
\]

\[
\int_0^1 \{G_i\} <G_j> '({\tilde{\eta}})<G_j> d\eta \quad (V.19)
\]

\[
\int_0^1 \{G_i\} d\eta \quad (V.20)
\]

To formulate these operators, the global shape function, \( G_i \), was defined on the local level by

\[
G_i = g^{(i-1)} \oplus g^{(i)} \quad (V.21)
\]

where \( g_1 \) and \( g_2 \) were defined by

\[
g_1^{(e)} (\xi) = \begin{cases} 
(1 - \frac{\xi}{\xi_e}) & \text{for } \xi \text{ in element } (e) \\
0 & \text{for } \xi \text{ not in element } (e)
\end{cases}
\]

\[
g_2^{(e)} (\xi) = \begin{cases} 
\frac{\xi}{\xi_e} & \text{for } \xi \text{ in element } (e) \\
0 & \text{for } \xi \text{ not in element } (e)
\end{cases}
\]
and \( l_e \) is the length of the eth element. The \( \oplus \) notation in expression V.21 means that \( G_i \) is the union of \( g_{(i-1)}^1 \) and \( g_{(1)}^2 \). The local (element) shape functions have the following properties,

\[
(i) \quad \int_0^{l_e} g_i^{(j)} g_k^{(m)} = 0 \quad \text{if} \quad j \neq m
\]

\[
= \begin{cases} 
1 & \text{if} \quad i = j \\
0 & \text{if} \quad i \neq j
\end{cases}
\]

(ii) \( g_i^{(e)}(\eta_j) = \delta_{ij} = \)

Having defined the local shape functions, the elemental matrix operators contributing to the global matrix operators V.16 through V.20 are,

\[
\int_0^{1} (G_i')<G_j> \, d\eta \quad \rightarrow \quad \frac{1}{l_e} \left[ \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right] \quad \text{(V.16')}
\]

\[
\int_0^{1} (G_i)^<G_j> \, d\eta \quad \rightarrow \quad \frac{1}{2} \left[ \begin{array}{cc} -1 & 1 \\ -1 & 1 \end{array} \right] \quad \text{(V.17')}
\]
The derivations of these operators are shown in Appendix E.

Since there is extensive coupling in the field equations and there are three degrees of freedom at each nodal point, the numbering scheme for the system matrices was important. To minimize the bandwidth of the matrices, the numbering scheme represented in Figure 8 was used.

Figure 9 represents the matrix \( A(t) \) of expression V.12 for any three successive nodal points. The distribution of the elemental matrices for the FEM operators is shown, as well as the possible locations for the coupling of the field equations over an element. In addition, Figure 9 shows the bandwidth that would be observed for any n-1 element solution. For this scheme, the bandwidth is nine. If coupling is not present, the bandwidth is seven. Figure 9 reflects the extensive coupling that is present in the model. All the
matrix elemental locations are filled except those which are shaded in.

B. IMPLEMENTATION OF BOUNDARY CONDITIONS

Having formulated the system matrices for the field equations, treatment of the boundary conditions will now be discussed. Each field equation is considered individually.

1. Fiber Heat Transfer Equation

The fiber heat transfer boundary conditions in non-dimensional form from Appendix A are

\[ n = 0 \quad \lambda_1 \frac{\partial \theta}{\partial n} = h_1 L (\theta - 1) + \frac{\sigma L}{T_\infty} (T_0^4 - T_\infty^4) \] (V.22)

\[ n = 1 \quad \lambda_1 \frac{\partial \theta}{\partial n} = -h_2 L (\theta - 1) - \frac{\sigma L}{T_\infty} (T_0^4 - T_\infty^4) \] (V.23)

Since the first term in expression V.13 is

\[ \{G_i \} \lambda_1 <G_j> ' (\theta) \bigg|_0^1 \]

or in analogous form

\[ \lambda_1 \frac{\partial \theta}{\partial n} \bigg|_0^1 \] (V.24)

natural boundary conditions V.22 and V.23 may be directly substituted in equation V.24.
The response dependent parameters, \( h_1', h_2', T_g' \), changing with time, are evaluated at the previous time step. Thus, the boundary conditions are incorporated in the system matrices as follows,

1. \(-h_1 L\): added to the stiffness matrix \( A(t) \) at location \( A_{1,1} \)
2. \( h_1 L - \frac{\sigma e L}{T_\infty} (T_g'^4 - T_o'^4) \): added to the excitation vector \( F(t) \) at location \( F_1 \)
3. \(-h_2 L\): added to the stiffness matrix \( A(t) \) at location \( A_{3n-2,3n-2} \)
4. \( h_2 L - \frac{\sigma e L}{T_\infty} (T_g'^4 - T_o'^4) \): added to the excitation vector \( F(t) \) at location \( F_{3n-2} \)

2. Internal Flow Heat Transfer Equation

The non-dimensional boundary conditions presented in Appendix A for the air energy equation are

\[
\begin{align*}
n & = 0 \quad \Gamma = 1 \quad (V.25) \\
n & = 1 \quad \frac{\partial \Gamma}{\partial n} = \frac{\partial \Theta}{\partial n} \quad (V.26)
\end{align*}
\]

The essential boundary condition at \( n = 0, \Gamma(n = 0) = 1 \) is imposed in the Galerkin equation as follows. The \( A_{2,i} \) row of the \( A(t) \) matrix, the \( B_{2,i} \) row and the \( B_{1,2} \) column of the \( \Omega \) matrix, and the \( F_2 \) location of the excitation vector, \( F(t) \),
are all set equal to zero. The B_{2,2} location of the B_{2} matrix is then set equal to one. The natural boundary condition at \eta = 1 for the air heat transfer equation was treated in the same manner as that of the fiber heat transfer equation.

Foregoing the individual steps the boundary condition is implemented by

\begin{align}
(1) & \quad - \frac{h_{2}L_{v_{1}}}{\lambda_{1}} : \text{added to the stiffness matrix } A(t) \\
& \quad \text{at location } A_{3n-1,3n-2} \\
(2) & \quad \frac{h_{2}L_{v_{1}}}{\lambda_{1}} - \frac{\sigma \varepsilon v_{1}}{T_{\infty}^{\lambda_{1}}} (T_{g}^{4} - T_{\infty}^{4}) : \text{added to the excitation vector } F(t) \text{ at location } F_{2}.
\end{align}

3. Oxygen Transport Equation

For the oxygen diffusion equation, the boundary conditions in non-dimensional form from Appendix A are:

\begin{align}
\eta = 0 & \quad \omega_{1} \frac{\partial \phi}{\partial \eta} = \omega_{2} (\phi - p) \quad (V.27) \\
\eta = 1 & \quad \frac{\partial \phi}{\partial \eta} = 0 \quad (V.28)
\end{align}

Since these are both natural boundary conditions, they were substituted for the first term in expression V.15 at \eta = 0 and \eta = 1, respectively. Coefficients \omega_{1} and \omega_{2} are evaluated at the previous time step. The boundary conditions are implemented by adding
(1) $-\omega_2$: to the stiffness matrix $A(t)$ at location $A_{3,3}$

and

(2) $\omega_2p$: to the excitation vector $F(t)$ at location $F_3$

This concludes the discussion for the implementation of the boundary conditions. A word of caution is in order. After each time step integration, the time-dependent coefficients of the boundary conditions (i.e., $h_1$, $h_2$, $\lambda$, $\nu$, $\omega$) are reevaluated. Before incorporating the updated coefficients into the stiffness matrix $A(t)$, the previous values must be subtracted out. The results of not taking this into account will be obvious.

The implicit system of ordinary differential equations was integrated numerically by a modified implicit-Gear method developed by Franke [2]. The time-dependent coefficients of the FEM operators in expression V.13, V.14, and V.15 were updated at the previous time. The reaction rate term (III.26) was also evaluated at the previous time, and appears in the excitation vector $F(t)$. In this way, the final system of ordinary differential equations was

$$A(t)\ddot{y} + F(t) = B \ddot{y}$$

where $A$ and $B$ are $(3n \times 3n)$ matrices of temperature dependent...
coefficients. \( F \) is a \((3n \times 1)\) vector arising partly from the reaction rate terms and partly from the boundary conditions. As noted, the elements of \( F \) are dependent on both temperature and oxygen concentration.
VI. RESULTS AND CONCLUSION

A. NUMERICAL CONSIDERATIONS

The input parameters which were varied in the computer analysis include (1) plate thickness, L, (2) fiber diameter, d, and (3) air flow rate, $U_a$. Other parameters which could be varied are the ply thickness, D, ambient temperature, $T_a$, and fiber emissivity, $\varepsilon$. One set of initial conditions was used for the analysis. These are shown in Figures 10 and 11. For this initial effort, the actual initial conditions were not of prime consideration. The selection of initial conditions of Figures 10 and 11 will be discussed in Section VI.C. The finite element program calculates the remaining system parameters such as permeability, porosity, pressure differential, and the response variables as functions of time and position. Parameters which are functions of temperature, such as $k_a$, $k_g$, $k_r$, $h_i$, $\rho_a$, $u_p$, and $u$ are continuously updated during the transient analysis.

The preliminary solution effort showed the system of equations to be very stiff (refer to Shampine/Gordon [24] and Gear [25] for a discussion of "stiffness"). Changing the integration algorithm from a sixth-order Runge-Kutta (IMSL subroutine DVERK) to a modified implicit-Gear method for stiff systems, developed by Franke [2], resulted in a significant reduction in CPU time. The computational effort
was carried out on an IBM 360/70. Typical runs required 20-30 minutes CPU time for problems for which ignition occurred, and 6-25 minutes CPU time for extinction problems. Ignition problems were terminated when the temperature at any nodal point exceeded 2200 degrees Fahrenheit, or when the graphite at a nodal point was totally consumed. Extinction problems were carried out to steady state.

A twenty five nodal point model (75 o. d. e.) provided results which differed less than five percent from the results of a 32 nodal point model (96 o. d. e.) and was adopted for all computer runs. For the twenty five nodal point model, approximately 275K bytes of core was required. The computer program which includes the FEM formulation and the integration routine as well as a sample input file is presented at the end of the appendices.

The numerical solution produced satisfactory results with one exception which will now be discussed. For a particular range of temperature and oxygen concentration (approximately 1500 degrees Fahrenheit and 0.001 lbm/ft$^3$, respectively), there is a significant increase in reaction rate. This accelerated reaction rate produced a negative/positive oscillation for the nodal values of oxygen concentration. The oscillation occurred in a region of the plate where the oxygen appeared to be totally consumed. As discussed by Frank-Kamenetskii [3], nth order reactions may yield zero values for the oxygen concentration and gradient within the
medium. The cause of the oscillation was as follows. Although the reaction rate is a function of temperature and oxygen concentration, numerically, it is treated as constant during an integration interval. As a result, the concentration in the region of high reaction rate becomes negative. The reaction rate was updated for the next time interval by setting negative concentrations to zero. During this time interval, for the region of zero concentration, there is no reaction. Without consumption, the oxygen concentration will increase and the cycle repeats itself. The oscillation was aggravated by large plate thickness and low permeability.

The instability occurring for the high reaction rate may be corrected by (1) numerically integrating the rate term in the time domain, (2) iterating until convergence is obtained between consecutive values of concentration, or (3) decreasing the time step and the length of the elements. As a matter of expediency, measure (3) was used to minimize the instability. The fiber and air temperatures were essentially unaffected by the oscillations in the oxygen concentration.

B. RESULTS AND OBSERVATIONS

Table 1 presents the results of fourteen problems. In Table 1, the parameters which depend upon temperature \(u_p\), \(\text{Re}_i\), and \(h_i\) are evaluated at ambient conditions for comparison purposes only. These parameters, in fact, will vary during the course of the transient analysis. In all cases,
the initial condition on graphite temperature was 1050 degrees Fahrenheit.

Figures 10, 11, and 12 show the transient behavior of Case 1. For the initial conditions shown, ignition occurred in approximately eight seconds. The reaction rate and oxygen concentration responses for Case 1 are typical of the problems for which ignition resulted. Figure 13 shows that the air and fiber temperatures for Case 1 do not differ significantly. This behavior is typical for most problems. However, for plates with high porosity (greater than .8) and subjected to relatively high flow rates, significant differences in the air and fiber temperatures do occur (see Figure 14 for Case 9). In a previous effort by Vatikiotis and Salinas [26], linearly varying initial conditions were investigated. Although the reaction rate used in that investigation differed from the one used in the present investigation, the overall results remain valid. The main observation was that ignition is less likely to occur for thin plates with the ambient air entering the hotter plate surface. Since temperature controls the reaction in the kinetic regime, cooler air entering the hotter plate surface enhances the heat loss. This reduces the fiber temperature, thereby decreasing the reaction rate. In effect, the cooler air enters the plate where it is most needed.

In the present analysis, observations were made by varying one input parameter (i.e., L or d or $U_\infty$) while keeping
all others fixed. The effects of each parameter on the behavior will be discussed individually.

1. **Effects of Exterior Velocity**

   As shown in Figures 15, 16 and 17 (cases 4, 1 and 2, respectively) for an initial condition of 1050 degrees Fahrenheit, three distinct regions of combustion were observed (extinction-ignition-extinction). These became apparent as \( U_m \) was increased from low velocities (10 knots) to higher velocities (120 knots). Vulis [4] discusses an experiment of a heated carbon rod with an air jet impinging upon it. For a certain range of flow velocities, ignition was observed. However, extinction did occur for velocities less than and greater than the velocity for which ignition occurred. The behavior of the carbon rod and that of the graphite mat is similar. This behavior may be explained by the Semenov model of Figure 1. For high \( U_m \), the internal heat transfer coefficient, \( h_i \), will be large as a result of an increase in the pore velocity. Since the slopes of the heat loss lines, \( q_x \), increase with increasing \( h_i \) for a constant \( T_{a*} \), the graphite temperature decreases from the critical ignition point \( I \). This causes extinction for the higher exterior flow velocities, \( U_m \). In contrast, for low \( U_m \), the slope of \( q_x \) will be small and ignition is more likely to occur. However, the heat generation curve, \( q_y \), changes position due to a decrease in oxygen concentration. This reduction in oxygen concentration is caused by the lower induced flow rate through the
plate. The overall shape of \( q_g \) is flattened such that the \( q_g \) line crosses \( q_g \) behind the new critical point \( I \). Thus extinction is observed for this region of low \( U_\infty \).

2. Effects of Fiber Diameter

Varying the fiber diameter with the remaining input parameters fixed, also yields three regions of combustion (extinction-ignition-extinction) for a fixed initial temperature. The results are shown in Figures 16, 18 and 19 (Cases 1, 9 and 10, respectively). The fiber diameter and ply thickness determine the permeability of the plate. Similar to \( U_\infty \), permeability affects the pore velocity (i.e., low permeability \( \rightarrow \) low velocity). In turn, this affects the convective heat transfer and the amount of oxygen entering the plate. These three regions of combustion are explained by the Semenov model for the same reasons as those of varying \( U_\infty \).

Porosity is a measure of void space and denotes the space available for oxygen. Porosity is associated with the internal geometry of the medium, as is permeability. However, they are basically different parameters. Permeability (hydraulic conductivity) is associated with convective air flow through the medium. Thus, an \( n \)-fold increase in the fiber diameter and ply thickness will not affect a change in porosity, but will affect an \( n^2 \) order change in the permeability. For an \( n \) greater than one, the result is a decrease in pore velocity without a change in porosity. Porosity
determines the maximum oxygen concentration per unit volume of fibrous mat. This indirectly affects the reaction rate. Thus, for a given $u_p$, the lower the porosity, the less oxygen there is for combustion. It appears that high porosity would enhance combustion by the presence of more oxygen. However, higher porosities also provide more fluid per unit volume resulting in greater heat transfer. This is observed in Case 9, Figure 18.

3. Effects of Plate Thickness, $L$

For the given initial temperature of 1050 degrees Fahrenheit, varying the plate thickness yielded two regions of combustion (extinction-ignition). Extinction was observed for thin plates (Case 2, Figure 17). As shown by Figures 20 and 21 (Cases 7 and 11, respectively), ignition was observed when plate thickness was increased. Plate thickness also affects the pore velocity. Decreasing $L$, increases the pressure gradient across the plate, thus increasing the pore velocity. The result is that extinction is more likely for thin plates because of the enhanced convection heat transfer resulting from the increase in pore velocity. Whereas pore velocity is inversely proportional to $L$, it is an exponential function of $U_\infty$. For the range of parameters in this investigation (i.e., $1" < L < 3"$, and 10 knots $< U_\infty < 100$ knots), an n-fold change in $U_\infty$ will produce a greater change in $u_p$, then will an n-fold change in $L$. Compare Cases 1 and 2 to Cases 7 and 8.
A characteristic feature of ignition for thick plates was observed. As the thickness increased, the ignition region (the spatial location) moved closer to the entrance surface of the ambient air. Compare Figures 20 and 21.

C. DISCUSSION

At this point, discussion is made of ignition temperatures for porous graphite plates. The ignition temperature for Case 1 was 1050 degrees Fahrenheit. This temperature was obtained by varying the initial temperature until ignition occurred. In all subsequent cases, 1050 degrees Fahrenheit was adopted as the initial condition, and we observed whether extinction or ignition occurred. The ignition temperatures given here are the result of taking $n = \frac{1}{2}$ in the nth order reaction rate. Significantly different ignition temperatures will result for other $n$. For example, for $n = 1$ (1st order reaction), the ignition temperature for Case 1 was approximately 1600 degrees Fahrenheit. As Frank-Kamenetskii [3] has suggested values of $n$ between $1/3$ and $2/3$, the average value of $1/2$ was adopted for this analysis. The ignition temperatures obtained using $n = \frac{1}{2}$ agree favorably with the experiments of Fontenot [1]. For fiber combustion, ignition temperatures for porous graphite plates are on the order of 1100 degrees Fahrenheit.

Consider the results of varying $U_\infty$. Extinction occurred for Case 2 and Case 4; thus, the ignition temperature
for each case is greater than 1050 degrees Fahrenheit. Since ignition does occur at 1050 degrees Fahrenheit for intermediate values of $U_\infty$ (Case 1, Case 7), ignition temperature is not a monotonic function of $U_\infty$. The precise determination of ignition temperature for a specific value of air velocity can be obtained by a large number of computer runs. A similar discussion can be made for the ignition temperatures dependence on fiber diameter. The general shape of these functions is represented by the shaded region on Figure 22. These suppositions may only be valid in the limited range of parameters associated with the present investigation.

It is interesting to observe that changing the initial condition of the oxygen concentration only affects the early part of the problem. The response of the concentration is fast compared to that of temperature. After approximately one second, the transient behavior is the same regardless of the initial condition on the oxygen concentration.

A brief description of the Semenov model was given in Section II.A. That discussion was for a lumped parameter model. The actual combustion process is more complex since there are spatial variations in temperature and in oxygen concentration. The results of this analysis show that both kinetic and diffusion regimes may exist simultaneously in a porous medium. Hence, it is not reasonable to restrict an
analysis of combustion in a porous medium to the kinetic regime.

D. CONCLUDING REMARKS

The model has provided considerable insight as to the behavior of fibrous composites subject to combustion. Improvements may be realized by extending the model to include:

1. The combustion of the epoxy matrix (i.e., start problem with first stage of combustion process).
2. The change in geometry of the porous medium due to combustion (affects $p$, $m$ and $z$).
3. A more complex reaction (i.e., secondary combustion of gaseous by-products, such as carbon monoxide).
4. The effect of gaseous by-products on fluid properties.
5. Generalization to two and three dimensional models (results in anisotropic properties).

The results show that certain properties, such as plate thickness, permeability and porosity, have significant effects on combustion. Therefore, one could select these design parameters to improve the flame resistance of composite materials. This would be beneficial to the survivability of a composite structure. A comprehensive set of analyses could be undertaken to couple the combustion problem to the strength problem. In this way, one could achieve a design which maximizes strength and minimizes combustion.
Another use of the present model would be in the study of energy systems utilizing particulate fuels (i.e., coal, biomass). In this case, the design parameters could be selected to maximize the performance of the system.
APPENDIX A

NONDIMENSIONALIZATION OF FIELD EQUATIONS

The field equations and boundary conditions are:

\[
\frac{\partial}{\partial x} \left[ (k_g+k_r) \frac{\partial q}{\partial x} \right] - \frac{h_1}{(1-p)} (T_g - T_a) + \Delta H r (T_g, \phi) = \rho_g C_g \frac{\partial q}{\partial t} \quad \text{(III.28)}
\]

\[
\frac{\partial}{\partial x} \left( k_a \frac{\partial T_a}{\partial x} \right) - mc_a \frac{\partial T_a}{\partial x} + \frac{h_1}{p} (T_g - T_a) = \rho_a c_a \frac{\partial T_a}{\partial t} \quad \text{(III.37)}
\]

\[
\frac{\partial}{\partial x} \left( E \frac{\partial \phi}{\partial x} \right) - u \frac{\partial E}{\partial T_a} (\frac{\partial T_a}{\partial x}) \phi - \frac{1}{\rho} r (T_g, \phi) = \frac{\partial \phi}{\partial t} \quad \text{(III.46)}
\]

at \( x = 0 \)

\[
(k_g+k_r) \frac{\partial q}{\partial x} = h_1 (T_g - T_a) + \sigma (T_g - T_\infty)^4 \quad \text{(IV.1)}
\]

\[ T_a = T_\infty \quad \text{(IV.3)} \]

\[ B \frac{\partial \phi}{\partial x} = u_p (\phi - p\phi_\infty) \quad \text{(IV.5)} \]

at \( x = L \)

\[
(k_g+k_r) \frac{\partial q}{\partial x} = -h_2 (T_g - T_\infty) - \sigma (T_g - T_\infty)^4 \quad \text{(IV.2)}
\]

\[
\frac{\partial T_a}{\partial x} = \frac{\partial T_g}{\partial x} \quad \text{(IV.4)}
\]
The nondimensional variables are defined as:

\[ \theta = \frac{T_g}{T_\infty} \quad \text{nondimensional fiber temperature} \]

\[ \gamma = \frac{T_a}{T_\infty} \quad \text{nondimensional air temperature} \]

\[ \phi = \frac{\phi}{\phi_\infty} \quad \text{nondimensional oxygen concentration} \]

\[ n = \frac{x}{L} \quad \text{nondimensional distance} \]

The time variable, \( t \), will not be nondimensionalized.

Using the temperature of the fiber, \( T_g \), to demonstrate the technique of transformation, we have,

\[ T_g = T_\infty \theta \]

It follows that,

\[ \frac{\partial^2 T_g}{\partial x^2} = T_\infty \frac{\partial^2 \theta}{\partial x^2} \]

\[ \frac{\partial T_g}{\partial x} = T_\infty \frac{\partial \theta}{\partial x} \]

Using the chain rule to transform \( x \) to \( n \), gives

\[ \frac{\partial T_g}{\partial x} = T_\infty \frac{\partial \theta}{\partial n} \frac{\partial n}{\partial x} \]
Since \( n = x/L \), the partial derivatives become

\[
\frac{\partial T_a}{\partial x} = \frac{T_a}{L} \frac{\partial}{\partial n}
\]

and

\[
\frac{\partial^2 T_a}{\partial x^2} = \frac{T_a}{L^2} \frac{\partial^2}{\partial n^2}
\]

Therefore,

\[
\frac{\partial^2 T_a}{\partial x^2} = \frac{T_a}{L^2} \frac{\partial^2}{\partial n^2}
\]

Similar transformations for \( T_a \) and \( \phi \) yield,

\[
\frac{\partial T_a}{\partial x} = \frac{T_a}{L} \frac{\partial}{\partial n} \quad \frac{\partial \phi}{\partial x} = \frac{\phi}{L} \frac{\partial}{\partial n}
\]

\[
\frac{\partial^2 T_a}{\partial x^2} = \frac{T_a}{L^2} \frac{\partial^2}{\partial n^2} \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{\phi}{L^2} \frac{\partial^2}{\partial n^2}
\]

Substituting these relations into the field equations and removing the constants from the differential operators gives,

\[
\frac{T_a}{L^2} \frac{\partial}{\partial n} \left( k \frac{\partial}{\partial n} \right) - \frac{h_z}{(1-p)(\Theta - \Gamma)} + \Delta Hr(T_g, \phi) = \rho \frac{\partial}{\partial t}
\]

\[
\frac{T_a}{L^2} \frac{\partial}{\partial n} \left( k \frac{\partial}{\partial n} \right) - \frac{\dot{m}c_a}{L} \frac{\partial}{\partial n} + \frac{h_z}{p(\Theta - \Gamma)} = \rho \frac{\partial}{\partial t}
\]

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Upon rearrangement, the equations become,

\[
\frac{\partial}{\partial n} \left[ \frac{h_i zL^2}{(1-p)} \right] - \frac{\partial}{\partial n} \left[ \frac{\Delta HL^2}{T_\infty} \right] r(T_g, \phi) = \frac{\rho g c g L^2 \partial}{\partial t} \left[ \Theta \right]
\]

\[
\frac{\partial}{\partial n} \left[ \frac{h_i zL^2}{p} \right] - \frac{\partial}{\partial n} \left[ \frac{\rho c a L^2 \partial}{\partial t} \right] = \frac{\rho c a L^2 \partial}{\partial t} \left[ \Theta \right]
\]

\[
\frac{\partial}{\partial n} \left[ \frac{B\partial}{\partial n} \right] - \frac{\partial}{\partial n} \left[ \frac{\partial u}{\partial n} \right] T_\infty \left[ \frac{\partial u}{\partial n} \right] - \frac{\partial}{\partial n} \left[ \frac{1}{T_\infty \partial} \right] r(T_g, \phi) = \frac{L^2}{T_\infty} \frac{\partial}{\partial t} \left[ \Theta \right]
\]

Letting,

\[
\lambda_1 = (k_g + k_r) \quad \lambda_2 = \frac{h_i zL^2}{(1-p)} \quad \lambda_3 = \frac{\Delta HL^2}{T_\infty} \quad \lambda_4 = \frac{\rho g c g L^2}{T_\infty}
\]

\[
\nu_1 = k_a \quad \nu_2 = \frac{\dot{m} c a L}{\partial n} \quad \nu_3 = \frac{h_i zL^2}{p} \quad \nu_4 = \frac{\rho c a L^2}{p}
\]

\[
\omega_1 = B \quad \omega_2 = \frac{\partial u}{\partial n} \quad \omega_3 = T_\infty \frac{\partial u}{\partial n} \quad \omega_4 = \frac{1}{T_\infty} \frac{L^2}{\partial n}
\]

the field equations may be written as,

\[
\frac{\partial}{\partial n} \left[ \frac{\lambda_1 \partial}{\partial n} \right] - \lambda_2 \left[ \Theta - \Gamma \right] + \lambda_3 r(T_g, \phi) = \lambda_4 \frac{\partial}{\partial t} \left[ \Theta \right] \quad (A.1)
\]

\[
\frac{\partial}{\partial n} \left[ \frac{\nu_1 \partial}{\partial n} \right] - \nu_2 \frac{\partial}{\partial n} \left[ \Theta - \Gamma \right] + \nu_3 \left[ \Theta - \Gamma \right] = \nu_4 \frac{\partial}{\partial t} \left[ \Gamma \right] \quad (A.2)
\]
Noting that the coefficients $\lambda$, $\nu$, $\omega$ are response dependent variables, equations A.1, A.2, and A.3 are considered in their final form in Section III.

Substituting the nondimensional variables into the boundary conditions gives,

$$\frac{\partial}{\partial n}(\omega_1 \frac{\partial \theta}{\partial n} - \omega_2 \frac{\partial \psi}{\partial n} - \omega_3 \frac{\partial r}{\partial n} - \omega_4 r(T_g, \psi) = \omega_5 \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (A.3)

$$\theta(L) = T_g - T_\infty \hspace{2cm} (A.4)$$

$$\phi(L) = \theta - T_\infty \hspace{2cm} (A.5)$$

The dimensional form of the absolute temperature will be retained for convenience.
APPENDIX B

POLYNOMIAL APPROXIMATIONS OF THERMAL PROPERTIES

Relations giving the viscosity and thermal conductivities of air for varying temperature were required. A simple way to obtain values for these properties is to fit empirical data with 2nd order Lagrange polynomials.

The general form of the 2nd order Lagrange polynomials for thermal conductivity and viscosity are

\[ k_a = \frac{(T_a-T_{a2})(T_a-T_{a3})}{(T_{a1}-T_{a2})(T_{a1}-T_{a3})} k_{a1} + \frac{(T_a-T_{a3})(T_a-T_{a1})}{(T_{a2}-T_{a3})(T_{a2}-T_{a1})} k_{a2} \]

\[ + \frac{(T_a-T_{a1})(T_a-T_{a2})}{(T_{a3}-T_{a1})(T_{a3}-T_{a2})} k_{a3} \]

(B.1)

\[ \mu = \frac{(T_a-T_{a2})(T_a-T_{a3})}{(T_{a1}-T_{a2})(T_{a1}-T_{a3})} \mu_1 + \frac{(T_a-T_{a3})(T_a-T_{a1})}{(T_{a2}-T_{a3})(T_{a2}-T_{a1})} \mu_2 \]

\[ + \frac{(T_a-T_{a1})(T_a-T_{a2})}{(T_{a3}-T_{a1})(T_{a3}-T_{a2})} \mu_3 \]

(B.2)

Choosing three points from a range of temperatures that would be representative of those observed during the analysis, the corresponding values of the properties are:
Substituting these values into expressions B.1 and B.2 gives,

\[
k_a = \frac{(T_a - 700)(T_a - 1500)}{(0-700)(0-1500)}(0.0131) + \frac{(T_a - 1500)(T_a - 0)}{(700-1500)(700-0)}(0.0284)
\]

\[
+ \frac{(T_a - 0)(T_a - 700)}{(1500-0)(1500-700)}(0.0423)
\]

\[
\mu = \frac{(T_a - 700)(T_a - 1500)}{(0-700)(0-1500)}(0.0394) + \frac{(T_a - 1500)(T_a - 0)}{(1500-0)(1500-700)}(0.0765)
\]

\[
+ \frac{(T_a - 0)(T_a - 700)}{(1500-0)(1500-700)}(0.107)
\]

These expressions reduce to,

\[
k_a = -3.232 \times 10^{-9} T_a^2 + 2.412 \times 10^{-5} T_a + 0.0131 \text{ Btu / lbm-hr-ft}
\]

\[
\mu = -1.041 \times 10^{-8} T_a^2 + 6.029 \times 10^{-5} T_a + 0.0394 \text{ lbm / ft-hr}
\]
Both expressions give property values which are within five percent of the data for temperatures to 2000 degrees Fahrenheit.
APPENDIX C

TRANSFORMATION OF THE REACTION RATE TERM

The expression for the oxidation rate of graphite fibers taken from Parker and Hottel [15] is

\[ r_g = \frac{9.55 \times 10^6}{T_{kg}^{1/2}} P_{O_2} \exp\left(-\frac{44000}{R \ T_{kg}}\right) \ \frac{gm}{cm^2 \cdot \text{sec}} \quad (C.1) \]

where \( R \) is the universal gas constant (1.958 cal/gmole-K) and \( P_{O_2} \) is the partial pressure of oxygen (atm.). The partial pressure may be represented in terms of the temperature, \( T \), and oxygen concentration, \( \phi \), by the Ideal Gas law in the form given by Kanury [16]

\[ P_{O_2} = \left( \frac{M_a}{M_{O_2}} \right) R_A \ T \ \phi \quad (C.2) \]

where \( M_a/M_{O_2} = 0.9 \) is the ratio of the molecular weights of air and oxygen; \( R_A \) ( = 53.34 ft.-lbf/lbm R) is the gas constant for air. Substituting \( M_a/M_{O_2} \) into equation C.2 and converting \( P_{O_2} \) to atmospheric units the expression becomes

\[ P_{O_2} = 4.25 \times 10^{-4} \ R_A \ T \ \phi \ \text{atm.} \]
Converting \( r_g \) to \( \text{lbm/ft}^2\text{-hr} \) from \( \text{gm/cm}^2\text{-sec} \), equation C.1 becomes

\[
r_g = \frac{7.04 \times 10^{10}}{T_g^{1/2}} P_{02} \exp\left(-\frac{44000}{R T_g k}\right) \frac{\text{lbm}}{\text{ft}^2\text{-hr}}
\]

Substituting in for \( P_{02} \) and converting the absolute temperatures from Kelvin to Rankine, the rate expression becomes,

\[
r_g = 4.014 \times 10^7 \frac{R \hat{T} k^{1/2}}{T_g^1} \exp\left(-\frac{39883}{T_g}\right) \frac{\text{lbm}}{\text{ft}^2\text{-hr}}
\]

The absolute temperature at the fiber surfaces is \( \hat{T}_g \). Thus, \( T \) in the numerator becomes \( T_g \), and the expression becomes

\[
r_g = 4.014 \times 10^7 \frac{R \hat{T}}{T_g^{1/2}} \exp\left(-\frac{39883}{T_g}\right)
\]

To transform the reaction rate to \( \text{lbm/ft}^3\text{-hr} \), \( y \), the ratio of a fibers surface area to its volume is placed in the numerator of \( r_g \). The expression for \( y \) is,

\[
y = \frac{1}{2} \frac{d}{\pi d^2} = \frac{2}{d}
\]

(C.3)

where \( d \) is the fiber diameter, and the \( \frac{1}{2} \) factor accounts for the uncertainty of the actual amount of fiber surface after the epoxy has burned away. The reaction rate of the graphite
fibers per unit volume is expressed as

\[ r = 4.014 \times 10^7 \gamma R_A T_g^{1/2} \phi \exp\left(-\frac{39883}{T_g}\right) \text{ lbm ft}^{-3}\text{-hr} \]

To obtain the heat liberated per lbm of fiber, \( r \) is multiplied by \( \Delta H \), the enthalpy of formation of carbon dioxide per unit mass of graphite consumed. This value is approximately 14,085 BTU per pound-mass of fiber and was obtained from Kanury [16]

\[ \text{Heat generation rate} = \Delta H \cdot r(T_g, \phi) \]

To obtain the oxygen consumption rate, \( r \) is multiplied by the inverse of the fuel-air stoichiometric ratio,

\[ \text{Oxygen consumption rate} = \left(\frac{1}{\phi}\right) r(T_g, \phi) \]
APPENDIX D

JUSTIFICATION OF THE DANKWERTS' BOUNDARY CONDITIONS

The Dankwerts' boundary conditions for the oxygen diffusion equation are:

\[
\frac{d\phi}{dx} = \frac{u_p}{B}(\phi - \phi_{\infty}) \quad x = 0
\]

\[
\frac{d\phi}{dx} = 0 \quad x = L
\]

Bischoff [23] presents a discussion of these equations. A brief summary is provided for completeness.

Figure 10 shows the one-dimensional oxygen transport region considered in this analysis.

\[
x = 0 \quad x = L
\]

Entrance Section | Porous Plate | Exit Section

Flow

\(\phi_1, B_1, u_1 \quad \phi, B, u_p \quad \phi_2, B_2, u_2\)

The concentration of oxygen, \(\phi\), for each region is distinguished by subscripts; \(B\) and \(u\) are the diffusivity and the velocity, respectively, for each region. The oxygen diffusion equations for each section are as follows,
\[ \frac{B_1}{u_1} \frac{d^2 \phi_1}{dx^2} - \frac{d \phi_1}{dx} = 0 \quad x \leq 0 \]  
(D.1)

\[ \frac{B}{u_p} \frac{d^2 \phi}{dx^2} - \frac{d \phi}{dx} - r(T, \phi) = 0 \quad 0 \leq x \leq L \]  
(D.2)

\[ \frac{B_2}{u_2} \frac{d^2 \phi}{dx^2} - \frac{d \phi_2}{dx} = 0 \quad x \geq L \]  
(D.3)

with the general boundary conditions,

\[ \phi_1(-\infty) = \phi_\infty \]  
(D.4)

\[ p \phi_1(0^-) = \phi(0^+) \]  
(D.5)

\[ p \phi_1(0^-) - p \frac{B_1}{u_1} \frac{d \phi_1(0^-)}{dx} = \phi(0^+) - \frac{B}{u_p} \frac{d \phi_1(0^+)}{dx} \]  
(D.6)

\[ p \phi(L^-) = \phi_2(L^+) \]  
(D.7)

\[ p \phi(L^-) - p \frac{B}{u_p} \frac{d \phi(L^-)}{dx} = \phi_2(L^+) - \frac{B_2}{u_2} \frac{d \phi_2(L^+)}{dx} \]  
(D.8)

\[ \phi_2(\pm \infty) = \text{finite} \]  
(D.9)
For convenience, the parameters $u_i$, $B_i$ will be treated as constant. In addition, the porosity, $p$, has been included to account for the discontinuity that exist for the concentration. The discontinuity is caused by the oxygen from a totally gaseous environment to one that is partly occupied by solid.

An analytical closed-form solution for this set of equations is not possible because of the nonlinearity of equation D.2. However, the solutions of equations D.1 and D.3 are,

$$
\phi_1 = A_1 + A_2 \exp(u_1 x/B_1) \tag{D.10}
$$

$$
\phi_2 = A_3 + A_4 \exp(u_2 x/B_2) \tag{D.11}
$$

Applying boundary conditions D.4 and D.9, the following results are obtained,

$$
A_1 = \phi_0
$$

$$
A_4 = 0
$$

The solutions D.10 and D.11 become

$$
\phi_1 = \phi_0 + A_2 \exp(u_1 x/B_1) \tag{D.12}
$$
\[ \phi_2 = A_3 \] (D.13)

It would be necessary to have the solution for the non-linear equation D.2 to solve for \( A_2 \) and \( A_3 \). However, the constants need not be known to continue with the analysis.

From equation D.12,

\[ \phi_1(0^-) = \phi_\infty + A_2 \]

and,

\[ \frac{d\phi_1(0^-)}{dx} = \frac{u_1}{B_1} A_2 \]

Substituting these into equation D.9 gives

\[ p\phi_\infty + pA_2 - p \frac{B_1}{u_1} \frac{u_1}{B_1} A_2 = \phi(0^+) - \frac{B}{u_p} \frac{d\phi(0^+)}{dx} \]

Cancelling terms yields,

\[ p\phi_\infty = \phi(0^+) - \frac{B}{u_p} \frac{d\phi(0^+)}{dx} \]

Rearranging, the Dankwerts' boundary condition at \( x = 0 \) is

\[ \frac{d\phi}{dx} = \frac{u_p}{B}(\phi - p\phi_\infty) \quad \text{at} \quad x = 0 \]
From equation D.13, and noting that $A_3$ is a constant, we have

$$\frac{d\phi_2}{dx} = 0$$

and

$$\frac{d\phi_2(L^+)}{dx} = 0$$

Substituting these expressions and equation D.7 into equation D.8,

$$p\phi(L^-) - p \frac{B}{u_p} \frac{d\phi(L^-)}{dx} = p\phi(L^-) - \frac{B_2}{u_2}(0)$$

Upon cancelling terms, the second Dankwerts' boundary condition becomes,

$$\frac{d\phi}{dx} = 0 \quad x = L$$

An important underlying consideration for using the Dankwerts' boundary conditions is that it simplifies the analysis since equation D.2 may be solved independently without having to consider entrance and exit regions.
APPENDIX E

DERIVATION OF THE FEM OPERATORS

In the section on the finite element formulation, the following five differential operators were identified,

\[ \int_0^1 \{G_i^r\} \langle G_j^r \rangle dn \]  \hspace{1cm} (E.1)

\[ \int_0^1 \{G_i \} \langle G_j \rangle dn \]  \hspace{1cm} (E.2)

\[ \int_0^1 \{G_i \} \{G_j \} \{G \} \langle G \rangle dn \]  \hspace{1cm} (E.3)

\[ \int_0^1 \{G_i \} \langle G_j \rangle \{G \} \langle G \rangle dn \]  \hspace{1cm} (E.4)

\[ \int_0^1 \{G_i \} dn \]  \hspace{1cm} (E.5)

where the \( G_i \) are the global basis functions. These operators are constructed on the element level by introducing the corresponding element basis functions, \( g_i \). The global and element basis functions are related by,
\[ G_i = g_1^{(i-1)} \oplus g_2^{(i)} \]

where \( g_1 \) and \( g_2 \) are defined by

\[
g_1^{(e)} = \begin{cases} 
(1 - \frac{\xi}{l_e}) & \text{for } \xi \text{ in element } (e) \\
0 & \text{for } \xi \text{ not in element } (e)
\end{cases} \]

\[
g_2^{(e)} = \begin{cases} 
\frac{\xi}{l_e} & \text{for } \xi \text{ in element } (e) \\
0 & \text{for } \xi \text{ not in element } (e)
\end{cases} \]

and \( l_e \) is the length of the \((e)\)th element.

The derivation of the local elemental matrices according to the Galerkin method for the global operations E.1 through E.5 proceeds as follows:

For operator E.1,

\[
g_{\text{global}} \begin{bmatrix} \int_0^1 (G_1^i) <G_j^i> d\eta \end{bmatrix} \rightarrow U \left[ \int_0^{l_e} \begin{bmatrix} g_1^i \\ g_2^i \end{bmatrix} <g_1^j g_2^j> d\xi \right]
\]

Noting that,

\[ g_1^i = - \frac{1}{l_e} \]
\[ g_2^i = \frac{1}{\frac{e}{e}} \]

the elemental matrix becomes

\[
\mathbf{J}_e \begin{bmatrix} \frac{1}{e}^2 & -\frac{1}{e}^2 \\ -\frac{1}{e}^2 & \frac{1}{e}^2 \end{bmatrix} \left[ \frac{1}{e} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right]
\]

For operator E.2,

\[
\int_0^1 \langle G_i \rangle <G_j^\prime \rangle d\xi \mathbf{U} \left[ \begin{array}{c}
\int_0^{\mathbf{J}_e} <g_1^i \rangle \\
\int_0^{\mathbf{J}_e} <g_2^i \rangle
\end{array} \right] \left[ \begin{array}{c}
1 \\
-1
\end{array} \right]
\]

Substituting in the local shape functions gives,

\[
\int_0^{\mathbf{J}_e} \left[ \begin{array}{c}
1 - \frac{\xi}{\mathbf{J}_e} \\
\frac{\xi}{\mathbf{J}_e}
\end{array} \right] \left[ \begin{array}{c}
1 - \frac{\xi}{\mathbf{J}_e} \\
\frac{\xi}{\mathbf{J}_e}
\end{array} \right] \left[ \begin{array}{c}
-\frac{\xi}{\mathbf{J}_e} \\
\frac{\xi}{\mathbf{J}_e}
\end{array} \right] d\xi
\]

Carrying out the operations gives
For operator E.3,

\[
\begin{align*}
\int_0^1 & \left[ \begin{array}{cc}
\frac{-1}{\xi_e} + \frac{\xi}{\xi_e} & \frac{1}{\xi_e} - \frac{\xi}{\xi_e} \\
-\frac{\xi^2}{2\xi_e^2} & \frac{\xi^2}{2\xi_e^2}
\end{array} \right] d\xi \quad \rightarrow \quad \frac{1}{2} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}
\end{align*}
\]

\[
\int_0^1 \{G_1\rangle G_2\rangle d\eta \quad \rightarrow \quad \frac{\xi_e}{2} \int_0^{\xi_e} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} <g_1\ g_2\rangle d\xi
\]

Substituting in for the local shape functions

\[
\int_0^{\xi_e} \begin{bmatrix} 1 - \frac{\xi}{\xi_e} \\ \left( \frac{\xi}{\xi_e} \right) \end{bmatrix} <(1 - \frac{\xi}{\xi_e})(\frac{\xi}{\xi_e})> d\xi
\]

the elemental matrix becomes

\[
\int_0^{\xi_e} \begin{bmatrix}
\frac{1 - 2\xi_e}{\xi_e} + \frac{\xi_e^2}{\xi_e^2} & \frac{\xi_e - \xi_e^2}{\xi_e} \\
\frac{\xi_e - \xi_e^2}{\xi_e} & \left( 1 - \frac{\xi_e}{\xi_e^2} + \frac{\xi_e^2}{\xi_e} \right)
\end{bmatrix} d\xi \quad \rightarrow \quad \frac{\xi_e}{3} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}
\]

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For operator E.4,

\[
\int_0^{\xi_e} \{G_i\}<G'>\{\psi}\langle G_j\rangle d\eta \uparrow U\{ \int_0^{\xi_e} \langle g_1'g_2'\rangle \begin{bmatrix} \psi' \\ \psi \end{bmatrix} \langle g_1g_2\rangle d\xi \}
\]

Multiplying out the matrices, the expression becomes,

\[
\int_0^{\xi_e} \begin{bmatrix}
(g_1g_1'g_1'_{i-1} + g_1g_1'g_2'_{i-1}) & (g_1g_1'g_2'_{i-1} + g_1g_2'g_2'_{i}) \\
(g_1g_2'_{i-1} + g_1g_2'_{i}) & (g_1g_2'_{i-1} + g_2g_2'_{i})
\end{bmatrix} d\xi
\]

Integrating each term separately,

\[
\int_0^{\xi_e} g_1g_1'g_1' d\xi = \int_0^{\xi_e} -\frac{1}{\xi_e} + \frac{2\xi}{\xi_e^2} - \frac{\xi^2}{\xi_e^3} d\xi = -\frac{1}{3}
\]

\[
\int_0^{\xi_e} g_1g_1'g_2' d\xi = \int_0^{\xi_e} \frac{1}{\xi_e} - \frac{2\xi}{\xi_e^2} + \frac{\xi^2}{\xi_e^3} d\xi = \frac{1}{3}
\]

\[
\int_0^{\xi_e} g_1g_1'g_2' d\xi = \int_0^{\xi_e} -\frac{\xi}{\xi_e^2} + \frac{\xi^2}{\xi_e^3} d\xi = -\frac{1}{6}
\]
\[ \int_0^\xi g_1 g_2 g_2 \, d\xi = \int_0^\xi \frac{\xi^2}{2} - \frac{\xi^3}{3} \, d\xi = \frac{1}{6} \]

\[ \int_0^\xi g_1 g_1 g_2 \, d\xi = -\frac{1}{6} \]

\[ \int_0^\xi g_1 g_2 g_2 \, d\xi = \frac{1}{6} \]

\[ \int_0^\xi g_1^2 g_2 \, d\xi = \int_0^\xi -\frac{\xi^2}{3} \, d\xi = -\frac{1}{3} \]

\[ \int_0^\xi g_1^2 g_2 \, d\xi = \int_0^\xi \frac{\xi}{3} \, d\xi = \frac{1}{3} \]

Substituting the values into the expression

\[
\begin{pmatrix}
(-\frac{1}{3} \psi_{i-1} + \frac{1}{3} \psi_i) & (-\frac{1}{6} \psi_{i-1} + \frac{1}{6} \psi_i) \\
(-\frac{1}{6} \psi_{i-1} + \frac{1}{6} \psi_i) & (-\frac{1}{3} \psi_{i-1} + \frac{1}{3} \psi_i)
\end{pmatrix}
\]

factoring out a \(-1/3\), the local matrix for the global operator becomes

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\[
\int_0^1 \{G_i\} \langle G_j \rangle \langle \psi \rangle \langle G_j \rangle \, d\xi \quad \Rightarrow \quad - \frac{1}{3} \begin{bmatrix}
(\psi_i - 1 - \psi_i) \\
(\frac{\psi_i - 1 - \psi_i}{2}) \\
(\frac{\psi_i - 1 - \psi_i}{2})
\end{bmatrix}
\]

For operator E.5,

\begin{align*}
\text{global} & \quad \text{local} \\
\int_0^1 \{G_i\} \, d\xi & \quad \Rightarrow \quad U[ \int_0^1 \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \, d\xi]
\end{align*}

Substituting in the local shape function, and integrating, the expression becomes

\[
\int_0^1 \begin{bmatrix} (1 - \frac{\xi}{L_e}) \\ \frac{\xi}{L_e} \end{bmatrix} \, d\xi \quad \Rightarrow \quad \frac{L_e}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

This last operator is used for the excitation vector as described in the FEM formulation.
TABLES AND FIGURES

**FIGURE 1** Semenov model of combustion.

**FIGURE 2** Idealized geometry of a fibrous graphite plate.
FIGURE 3 Separating a differential volume of porous medium into respective volumes of fiber and air.

![Diagram of separating a differential volume of porous medium into fiber and air volumes.]

\[
\begin{align*}
\text{FIBERS} & : q_{\text{cond}} \bigg|_x \quad q_{\text{rad}} \bigg|_x \\
\text{AIR} & : (1-p) dV \\
\text{ENERGY BALANCE} & : \Delta H_f(T_g, \phi) \\
\end{align*}
\]

FIGURE 4 Energy balance on the fibers.

![Diagram of energy balance on fibers.]

\[
\begin{align*}
\text{FIBERS} & : q_{\text{cond}} \bigg|_x \\
\text{AIR} & : p dV \\
\text{ENERGY BALANCE} & : \Delta H_f(T_g, \phi) \\
\end{align*}
\]

FIGURE 5 Energy balance on the internal flow.

![Diagram of energy balance on the internal flow.]

\[
\begin{align*}
\text{OXYGEN} & : m_B \bigg|_x \\
\text{ENERGY BALANCE} & : \frac{1}{T} r(T_g, \phi) \\
\end{align*}
\]

FIGURE 6 Molecule-mass balance on the oxygen.
FIGURE 7 Linear shape functions for the Galerkin formulation of FEM.

Element

<table>
<thead>
<tr>
<th>Local nodal point</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global nodal point</td>
<td></td>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
<td>n-2</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\psi_1 &= \Theta_1, \quad \psi_2 = \Theta_2, \quad \psi_3 = \Theta_3, \\
\psi_4 &= \Gamma_1, \quad \psi_5 = \Gamma_2, \quad \psi_6 = \Gamma_3, \\
\psi_7 &= \Phi_1, \quad \psi_8 = \Phi_2, \quad \psi_9 = \Phi_3, \\
\psi_{3n-2} &= \Theta_{3n}, \\
\psi_{3n-1} &= \Gamma_{3n}, \\
\psi_{3n} &= \Phi_{3n}
\end{align*}
\]

FIGURE 8 Numbering scheme used in system matrices.

FIGURE 9 Shows coupling in system of p. d. e.
### TABLE 1

**SUMMARY OF RESULTS**

<table>
<thead>
<tr>
<th>Case</th>
<th>$U_0$ kt.</th>
<th>$d$ in.</th>
<th>$L$ in.</th>
<th>$u_p$ ft/s.</th>
<th>$\Delta P$ psi</th>
<th>$Re_i$</th>
<th>$m$ ft$^2 \times 10^3$</th>
<th>$P$ blu/lbm-sec</th>
<th>$h_1$ ft/sec$^2$</th>
<th>$z$ ft/sec$^3$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.</td>
<td>.004</td>
<td>.25</td>
<td>.3745</td>
<td>.0597</td>
<td>.4487</td>
<td>.1652</td>
<td>.497</td>
<td>19.19</td>
<td>3016</td>
<td>ignition @ 8.3s</td>
</tr>
<tr>
<td>2</td>
<td>90.</td>
<td>.004</td>
<td>.25</td>
<td>1.208</td>
<td>.1925</td>
<td>1.447</td>
<td>.1652</td>
<td>.497</td>
<td>34.06</td>
<td>3016</td>
<td>extinction</td>
</tr>
<tr>
<td>3</td>
<td>120.</td>
<td>.004</td>
<td>.25</td>
<td>2.140</td>
<td>.3405</td>
<td>2.56</td>
<td>.1652</td>
<td>.497</td>
<td>45.04</td>
<td>3016</td>
<td>extinction</td>
</tr>
<tr>
<td>4</td>
<td>20.</td>
<td>.004</td>
<td>.25</td>
<td>.060</td>
<td>.0096</td>
<td>.0719</td>
<td>.1652</td>
<td>.497</td>
<td>7.82</td>
<td>3016</td>
<td>extinction</td>
</tr>
<tr>
<td>5</td>
<td>70.</td>
<td>.004</td>
<td>.25</td>
<td>.7235</td>
<td>.1168</td>
<td>.8778</td>
<td>.1652</td>
<td>.497</td>
<td>26.70</td>
<td>3016</td>
<td>ignition @ 6.7s</td>
</tr>
<tr>
<td>6</td>
<td>10.</td>
<td>.004</td>
<td>.25</td>
<td>.015</td>
<td>.0024</td>
<td>.0180</td>
<td>.1652</td>
<td>.497</td>
<td>3.97</td>
<td>3016</td>
<td>extinction</td>
</tr>
<tr>
<td>7</td>
<td>90.</td>
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<td>.5</td>
<td>.6039</td>
<td>.1925</td>
<td>.7236</td>
<td>.1652</td>
<td>.497</td>
<td>24.25</td>
<td>3016</td>
<td>ignition @ 4.9s</td>
</tr>
<tr>
<td>8</td>
<td>90.</td>
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<td>1.0</td>
<td>.3019</td>
<td>.1925</td>
<td>.3618</td>
<td>.1652</td>
<td>.497</td>
<td>17.27</td>
<td>3016</td>
<td>ignition @ 6.0s</td>
</tr>
<tr>
<td>9</td>
<td>50.</td>
<td>.002</td>
<td>.25</td>
<td>.6657</td>
<td>.0597</td>
<td>2.80</td>
<td>.5163</td>
<td>.8743</td>
<td>23.85</td>
<td>1508</td>
<td>extinction</td>
</tr>
<tr>
<td>10</td>
<td>50.</td>
<td>.0048</td>
<td>.25</td>
<td>.2813</td>
<td>.0597</td>
<td>.156</td>
<td>.0689</td>
<td>.276</td>
<td>13.62</td>
<td>3619</td>
<td>extinction</td>
</tr>
<tr>
<td>11</td>
<td>90.</td>
<td>.004</td>
<td>1.5</td>
<td>.2013</td>
<td>.1925</td>
<td>2.412</td>
<td>.1652</td>
<td>.497</td>
<td>14.16</td>
<td>3016</td>
<td>ignition @ 6.6s</td>
</tr>
<tr>
<td>12</td>
<td>90.</td>
<td>.004</td>
<td>2.0</td>
<td>.151</td>
<td>.1925</td>
<td>1.809</td>
<td>.1652</td>
<td>.497</td>
<td>12.29</td>
<td>3016</td>
<td>ignition @ 8.0s</td>
</tr>
<tr>
<td>13</td>
<td>90.</td>
<td>.004</td>
<td>4.0</td>
<td>.0755</td>
<td>.1925</td>
<td>.0905</td>
<td>.1652</td>
<td>.497</td>
<td>8.75</td>
<td>3016</td>
<td>ignition</td>
</tr>
<tr>
<td>14</td>
<td>90.</td>
<td>.004</td>
<td>6.0</td>
<td>.0503</td>
<td>.1925</td>
<td>.0603</td>
<td>.1652</td>
<td>.497</td>
<td>7.18</td>
<td>3016</td>
<td>ignition</td>
</tr>
</tbody>
</table>

* For D=.005" and initial conditions of Figure 10
FIGURE 11 Oxygen concentration response of Case 1 (u=50kt., d=.004", L=.25").
Flow temperature follows the fiber temperature.

CASE 1
AIR - FIBER

TEMPERATURE (deg F)

X/L
FIGURE 14  Flow and fiber temperature difference at high porosity and flow rate (for Case 9, $p=0.87$, $U_\infty=90$ kt.).
FIGURE 15 For Case 4, \( U_0 = 20 \text{kt} \), \( d = 0.004" \), \( L = 0.25" \) (for effect of varying exterior velocity, \( U_0 \), compare Figures 15, 16 and 17).
FIGURE 18 For Case 9, $U=50$kt, $d=0.002''$, $L=0.25''$ (for effect of varying fiber diameter, $d$, compare Figures 16, 18 and 19).
FIGURE 19 FOR CASE 10, \( \nu = 50 \text{kN}, d = 0.0048", I = 25".

FIBER TEMPERATURE (deg F)
FIGURE 20 For Case 7, $u = 90$ kt., $d = 0.004''$, $L = 0.5''$ (for effect of varying plate thickness, $L$, compare Figures 17, 20 and 21).
FIGURE 21

For Case 11, $U_{90}^t$, $d=0.004''$, $L=1.5''$.

FIBER TEMPERATURE (deg F)

$X/L$
FIGURE 22  Shaded region represents shape of ignition temperature curve as a function of $U_\infty$ or $d$. 
PARAMETER DEFINITIONS:

NNP=NUMBER OF NODAL POINTS
TAMB=AMBIENT TEMPERATURE (F)
TORT=TORTUOSITY
SPACEX=FLATNESS (IN.)

XLENT=FLAT THICKNESS (IN.)
GCOND=CONDUCTIVITY OF BULK GRAPHITE (BTU PER FT per DEGREE-F)
SPHTG=SPECIFIC HEAT OF GRAPHITE (BTU PER LB per DEGREE-F)

HSURF=SPECIFIC CONV. HEAT TRANSFER COEFF. AT X=C (BTU PER LB per DEGREE-F)
T=TIME STEP AT WHICH PRINT OUT OF VARIABLES DESIRED ALLOWABLE (SEC.)

TSTART=START TIME FOR PROBLEM (SEC.); USED FOR BOOKKEEPING SUCH AS A RESTART FROM PREVIOUS PROBLEM (INPUT 0.0 IF INITIAL RUNT).

X(I)=NODAL POINT LOCATIONS (VALUES BETWEEN 0.0-1.0)

CH(I)=ARRAY HOLDING INITIAL CONDITIONS (DEG-F, LBM PER FT**3);

FUEL-8 ENTERED INTO LOC (1,1), (1,4), ..., (1,3*NNP-2), TEMP-A ENTERED INTO LOC (1,2), (1,5), ..., (1,3*NNP-1); CONCA-ENTERED INTO LOC (1,3), ..., (1,6), ..., (1,3*NNP-3); CATA FILE STACKED WITH TEMP-A VALUES IF TEMP-A ENTERED INTO CATA FILE, FUEL-8 ENTERED INTO CATA FILE; CONCA ENTERED INTO CATA FILE STACKED WITH TEMP-A VALUES IF CONCA ENTERED INTO CATA FILE, FUEL-8 ENTERED INTO CATA FILE; AND CONCA ENTERED INTO CATA FILE STACKED WITH TEMP-A VALUES IF CONCA ENTERED INTO CATA FILE.

IELE=CCCE FOR N.P. DISTRIBUTION (0=ENTER NODAL POINT, X/L:1=AUTOMATICO DIVIDES XLENT1 INTO NNP NODAL POINTS)

FCF INPUT OF CATA LOCK AT SAMPLE INPUT FILE IN THESIS.

PROGRAM CALCULATES FOLLOWING PROPERTIES FOR AMBIENT CONDITIONS AT START OF PROGRAM:

DELPA=PRESSURE DIFF. ACROSS PLATE (PSI)

SURF AREA OF FIBER PER UNIT VOLUME (FT**2 PER FT**3)
PCRCS=PCRCSITY CF PLATE
FERM=FERMABILITY OF PLATE (SQ. FT.)
FDEP=DEPTH OF PLATE PER UNIT SURFACE AREA (PER SQ. FT.)
V=S=AVG. FLOW VELOCITY (FT PER SEC)
DELV=AVG. FLOW VELOCITY IN (IN.)
OCA=DENSTY CF AIR (LEW PER FT**3)
COND=CONDUCITIVITY OF AIR (BTU PER FT*HR*DEG-F)
WISC=VISCOSITY CF AIR (LEW PER FT**2HR)
CCND=CONDUCITIVITY OF PLATE (BTU PER FT*HR*CEG-F)
RACON=RADIATION COEFF. TO MULTIPLY T-CUBED FOR EQUIVALENT
CONDUCTIVITY (BTU PER FT*HR*CEG-R**4)
RG1=REYNOLDS CF POROS. WIDEIN (LNM PER FT**3)
REG=REYNOLDS NUMBER FOR EXTERIOR FLOW
REIN=INTERNAL REYNOLDS NUMBER
CONV=INTERNAL CONVECTION HEAT TRANSF COEF. (BTU PER HR*FT**2*F)
SURFR=HEAT TRANSFER COEF. AT RIGHT SURFACE (BTU PER HR*FT**2*F)

FCLLWING FFTCERTIES ARE EVALUATED AT THE PREVIOUS TIME STEP IN
S.R. FCRM2 FOR EACH NCAL PCINT:
1. VISCSITY CF AIR (VISC2)
2. CONDUCTIVITY CF AIR (ACCAD2I)
3. DENSITY CF AIR (ROA2I)
4. AVG. FLOW VELOCITY (L2I)
5. INTERNAL REYNOLDS NUMBER (REIN)
6. INTERNAL CONVECTION HEAT TRANSF. COEFF. (CCNV2I)
7. CONDUCTIVITY OF PLATE (CCND2I), BOTH RAD. AND T-ERM. CCNC.
8. EQUVALENTR CONDUCTIVITY DUE TO RADIATION (CCND2I)
9. CONDUCTIVITY OF PLATE (CCND2I), BOTH RAD. AND T-ERM. CCNC.
10. DIFFUSIVITY COEFFICIENT (TVGAI)
11. HEAT GENERATION TERM (RATHE)
12. OXYGEN CONSUMPTION TERM (RATEN)

MAIN PROGRAM

************

INPUT DATA

REAL*8 ZDCT, GAMA, GAMAC
EXTERNAL CLETH, JACMAT, LCASUB, COPYZ, NUTSL, PERVER
COMMON ZI75, 75I, ZCCTI75, 75I, XLENHT, CCNC, RACON, CONV, 2, ROG,
XCEL, PCRC5, CHI75, 75I, TAME, CAMB, F75I,
XPEM, ALCAC75, 75I, BCTI75, 75I, BSH75, 75I, E(75I), CDN75T, 75I,
XRGAS, BBEC75, 75I, CC75, 75I, ADCT75, 75I, BETA, CCND, DSPTG, ROA, SPHTA,
CC CGNOD2(25), TANT, CC2(25,4), SPHT, ZMASS(75), PERM, VISC,
XU2(25), ANN, NPA, NIP, NAF, NL, ICOR(75,2), IJK
DIMENSION CC(75), VL(7500), VLI(11), XZI(11), VY(25), Y2(25), Y3(25),
XX(25), FACH(75), CF(75)
READ(S,5) IELEM
5 FORMAT(II)
WRITE(C,1) IELEM
1 FORMAT(1,411)
CC C Geometry CF CCMAIN (1-1C1'L)****
READ(S,10) NNP
10 FORMAT(12)
NEL=NNP+1
IF(NEL.EQ.11) GO TO 20
WRITE(C,11) (X(I),I=1,NNP)
11 FORMAT(12.7)
GO TO 20
20 XX(I)=X(I)+1.0/FLOAT(NEL)
WRITE(C,40)
40 FORMAT(12,21,'NODAL FIT, X-POSITION')
WRITE(C,44)
44 FORMAT(1,4x,12.9x,F5.3)
CC C Physical CCFFFTIES****
READ(S,20) TMX,START, UINF, TAMB, PANB1, TCRT, D1, SPACE1, HLNGTH,
XLEN1, EAS, CCORD, EPS, RCG, SPHTG, SPHTA, HSTRFL, CAMB1
20 FORMAT(12,6)
UINF=UAPIX1.688
PANB=RAP1*144.0
CEL1=CEL1/SPACE1-D1/2.0
CEL=CEL1/12.0
C=CEL/12.0
SPACE1=SPACE1/12.0
XLEN1=1.0/12.0
tabs=TAP1+459.7
BETA=CSPACE
FORCS1=3.1416/4.0*(BETA**2)
PORS1=1.0-FCRCS1
96 FORMAT('DENSITY OF AIR=','E10.4', ' LB PER CU.FT.)
98 WRITE(6,51) ACCRD
99 FORMAT('CONDUCTIVITY OF AIR=','E10.4', 'BTU PER FT*HR*DEG-F')
100 WRITE(6,52) VISC
101 FORMAT('VISCOSITY OF AIR=','E10.4', ' LB PER FT*HR')
102 FORMAT('CONDUCTIVITY OF POROUS MEDIUM=','E10.4', 'BTU PER FT*HR*DEG-F')
104 WRITE(6,103) RADCON
106 FORMAT('RADIATION CCE. OF MEDIUM FOR T-CUBED TERM=','E10.4', '
108 FORMAT('CONDUCTIVITY OF POROUS MEDIUM=','E10.4', 'LB PER CU.FT.')
110 FORMAT('REYNOLDS NUMBER FOR EXTERIOR FLCW=','E10.4')
112 FORMAT('INTERNAL CONVECTION HEAT TRANSFER COEF=','E10.4', 'BTU P
115 FORMAT('OF 2X1.3X, THE FOLLOWING INTERNAL PROPERTIES WILL VARY WITH TE
XPERATCFE=','E5.2', '1. PERE VELOCITY=','E5.2', '2. DENSITY OF AIR=','E5.2
X,5X,4. CONVECTIVITY OF AIR=','E5.2', '5. INTERNAL CONVECTION HEAT
X,5X,6. DENSITY OF AIR=','E5.2', '6. DIFFUSION X CCEFFICIENT=','E5.2', '7. CONDUCTIVITY OF POROUS MEDIUM')
C
CC INITIAL CONDITIONS******
C
ANFA=3*NNP-2
ANFB=ANFA+1
ANFC=ANFA+1
READ(5,125) {CHI{1,1),I=1,NNPA,3}
125 FORMAT('125)
127 READ(5,127) {CHI{1,2),I=1,NNPB,3}
129 READ(5,129) {CHI{1,3),I=1,NNPC,3}
131 WRITE(6,131)
133 FORMAT('133)
135 WRITE(6,135)
137 FORMAT('137)
139 WRITE(6,139)
141 FORMAT('141)
143 WRITE(6,143)
145 WRITE(6,145) N,(CHI{1,1),I=NN,NNN)
147 CONTINUE
CO 147  I=1, NAP
II=3*J-
JJ=3*J-
KKK=3*J-
V1(I)={P(J,J,J)}
V2(I)={F(J,J,J)}
V3(I)={F(J,J,J)}

CONTINUE

LCCAL-GLCEAL CCRRESPOCNCE**

J=0
CO 155 IEL=1, NEL
DD  IEC. I=3.2
ICCR[IEL,1]=J+1

CONTINUE

J=J-1

CONTINUE

WRITE(6, 160)
WRITE(6, 165)
WRITE(6, 170) (IEL, (ICCR[IEL,1], I=1,2), IEL=1, NEL)

ACR-CINICINALIZE I.C.****

CO 190  I=1, NNPA+3
CHI(I,1)=-CF(I,1)/TAKE

CONTINUE

CO 195  I=1, NNPA+3
CHI(I,1)=-CF(I,1)/TAKE

CONTINUE

CAPB=FCFC5*CAMB
CC 200  I=3, NNPC+3
CHI(I,1)=-CF(I,1)/CAMB

CONTINUE

FORMACLA CF PRELIMINARY SYSTEM MATRICES****

CALL FCP1
SET COMBUSTION RATE PARAMETER****

DO 205 NC+K=1,NNPC
RAFRI(NCFK)=ROGI
202 CCATIALE

PRINT BOUNDARY CONDITIONS****

WRITE(6,225) HSRFL, EWS
225 FORMAT(1X, 'THE LEFT NATURAL B.C. FOR THE GRAPHITE ENERGY EQ. HAS A X\ nH-SURF. CONVECTION^15F6.2,^1 AND EMISSIVITY^15F4.2)
WRITE(6,225) FSURFR, EPS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

WRITE(6,225) HSRFL, EWS

INTEGRATION EC-LOOP****

TIME=0.0
TCL=0.0
TEND=C.C
IJK=0
AL=0
JSKF=0
MAXDER=€
f=1E-4
MIN=1E-6
RMSEFS=1.0E-2
CO 350: I=1,NNP
I11=301
J11=0
KKK=II1+2
Y1(I)=C(I,II1)
Y2(I)=C(J,JJ1)
3(I)=KKK
IF(J11.GT.2200.0) GO TO 501
501: CONTINUE
V4(I)=TME
V1(I)=TME
IF(I/11.EQ.(31.-TAMB).LT.1.0) GC TO 501
IF(I/11.EQ.600.0) GC TC 501
501: CONTINUE

SLICERUTINE FCP1****

SUBRoutines FCP1
REAL*E 2K-2DDT
CCPMAA 2K(75,76),XCC(75,76),XLENGTH,COND,RAECN,CONVZ,ROG,
XDELPH.FCCS,CHI(7,76),TAE1,CAFP,F(75),
XPAPB,ACTACX(75,4),BDT(25,4),BPSH(25,4),X(75),L,CD(25,4),
XRGAS,ECFCX(75,4),CC(25,4),ACDT(25,4),BETA,GCNO,D,SPAN,DHRA,SPHTA,
XCOND(25,1),TERT,CC(25,4),PVT,MASS(75),PERF,VISCI,
XUZ(25),NNP,NNPA,NNPE,ANPC,NEI,ICOR(75,2),IJK

C ZERO OUT THE MATRICES****
I2=3*NNP
CO 11C I=1,12
F(1)=0.0
CC 110 =1,1Z
2K(I)=J=1,CC
4K(I)=J=1,CC
IF(I/11.EQ.6) CONTINUE
110: CONTINUE

C FCPM ELEMENTAL COEFFICIENTS****
CO 500 IEL=1,NEL
IEL=IEL+1
I=ICOR(IEL)132
J=ICOR(IEL)132
KK=II+J
LL=J+1
MM=I+1
NN=J+1
KL=2*(IEL-1)*X(IEL)
AK11=1.0/KL
AK12=-AK11
AK22=AK11
ACACC(IEL,1)=Z*ZL*XLENTH**2/(3.0*(1.0-FCRCS))
ACACC(IEL,2)=ACACC(IEL,1)/2.0
ACACC(IEL,3)=ACACC(IEL,1)
ACACC(IEL,4)=ACACC(IEL,1)
ADCT(IEL,1)=ECH*SF*TEL/3.0*XLENTH**2*3600.0
ADCT(IEL,2)=ADOT(IEL,2)/2.0
ADCT(IEL,3)=ADOT(IEL,2)
ADCT(IEL,4)=ADCT(IEL,1)
BK11=1.0/KL
EK12=-EK11
EK21=BK11
BK22=BK11
BSH(IEL,1)=.5*XLENTH/SPHTA
BSH(IEL,2)=BSH(IEL,1)
BSH(IEL,3)=BSH(IEL,1)
BSH(IEL,4)=BSH(IEL,2)
BCBC(IEL,1)=2.0*ZL*XLENTH**2/(3.0*POROS)
BCBC(IEL,2)=BCBC(IEL,1)/2.0
BCBC(IEL,3)=BCBC(IEL,1)
BCBC(IEL,4)=BCBC(IEL,1)
BDOT(IEL,1)=BDCT(IEL,1)/2.0
BDOT(IEL,2)=BDOT(IEL,1)
BDCT(IEL,4)=BDCT(IEL,1)
CD(IEL,1)=CD(IEL,1)
CD(IEL,2)=CD(IEL,2)
CD(IEL,3)=CD(IEL,2)
CD(IEL,4)=CD(IEL,2)
CC(IEL,1)=XLENTH/2.0
CC(IEL,2)=CC(IEL,1)
CC(IEL,3)=CC(IEL,1)
CC(IEL,4)=CC(IEL,1)
CC2(IEL,1)=-(XLENTH*7APE)/3.0
CC2(IEL,2)=CC2(IEL,1)/2.0
DISTRIBUTE ELEMENT COEFFICIENTS INTO SYSTEM MATRICES****

ZK(II,II)+ZK(II,II)-AK11
ZK(IJ,II)+ZK(IJ,II)-AK12
ZK(JJ,II)+ZK(JJ,II)-AK22
ZK(KK,II)+ZK(KK,II)-AK12
ZK(KK,II)+ZK(KK,II)-AK12
ZK(LL,II)+ZK(LL,II)-AK22
ZK(II,LL)+ZK(II,LL)-AK22
ZDCT(PP,PP)=ZDCT(PP,PP)+CDT11
ZDCT(NP,NA)=ZDCT(NP,NA)+CDT12
ZDCT(NP,NA)=ZDCT(NP,NA)+CDT12
CGRT|JE| RETLRA ENC

SLERCUTINE FCRM2****

SUBRCUTINE FCRM2

DO 500 II=1,NEL
II=ICOR|EL,1|+3-2
IUI

C4C 444a

In

TP

UM~rMUWUUAUr~qL

G

UM~rMUWUUAUr~qL

G

X -9

O94.

u9uww w

0

*JJJ

VWOWWtp.Wtv~jj-jJ-J.-J KUSUA %J 6

Z

Ilo

ow O

bc

LL

ow

P

w.9

%~ft

%W%% JP%

.a

2w;

L

&aw

IM1W Z

QLL

Mww sC

P4~w-

.w

Wfr" -S

ZZZZ~i~:4aZ

*jM

q.Js-Mun'.&N

+CCOI I

3WWI.X.L)

ZmA

*J~nCYIetfl

430x288
da

LW

0Z.

LS.

i

401x263

~~~~~~~c

266x263

**9,..x94

458x263

U

x4OZ.

192x258

www~ w~

247x258


489x257

1A

336x246

0

356x246

].

497x246

4

335x239

aJI£~UIL

335x233

tn

464x232

q

331x203

131W

477x203$

9

000

SLERGUTINE CIIFUN****

C

SUBRUTINIE CIIFUN(\gamma, \gamma L , T, f, INV, CV)

REAL 18 ZK, IOT

CCPMCA 2R175, 75), ZECT(75, 75), XLENH, CCNC, RACCGN, CONV, Z, ROG,

XELP, FCFC, CHIT(75, 75), TAPB, CAMB, F(75);

XFAH, ACACC(25, 4), BCLT(25, 2), BSH(25, 4), X(75), CDOT(25, 4);

XRGA, BC etc(25, 25); CL(25, 4), ACET(25, 11), BETA, ECCND, D, SPHTG, ROA, SPHTA,

XCOND(25, 25), TONT, CC(25, 4), SPHT, ZMASS(75), PERP, VISC,

XL2(25), AN, XAPA, EFE, XAF, NEL, ICOR(75, 21, 1JK)

DIMENTICA \gamma(7,1), \gamma L(1), CV(1)

IA = 1
```
IB=3
JA=1
JB=6
CO 300 K=1 A=P
CO 200 IA, IB
CO(3) C=C
DO 1 CC J=JA, JB
CC(I)=CC(I)+DOT(I,J)*Y(2,J)*HINV-ZK(I,J)*Y(1,J)
CONTINUE
CC(I)=CC(I)-F(I)
CONTINUE
IA=IA+1
IB=IB+1
IF(K,EC,1) GC TO 5
JA=JA+1
IF(K, EC, AEL) GO TO 300
JB=JB+1
CONTINUE
RETURN
END

SUBROUTINE JACMAT(Y, VL, T, HINV, A2, N, NY, EPS, D, F, P, W)
REAL AB, ZK, ZCT
CPCACA ZK(15, 75), ZCT(75, 75), XLENGTH, CONC, RACCCN, CENV, RCG,
XCELPC, FCFCSC, CT(17, 15), TAPIE, CAMB, F(75)
XAPBF, ACACC(25, 41), ECT(25, 41), EPS(25, 41), X(75), UCOD(25, 41)
XGAS, RBCC(25, 41), CC(25, 41), ACCT(25, 41), BETA, GCCD(10), SPHTG, RCA, SPHTA,
XCCDA(25, 10), TICT, CC(25, 41), SPHT, ZMASS(75), PERP, VISC,
XUZ(25), KAF, KAPA, KAFB, ANFC, AEL, ICOR(75, 21), TJK
CINAEKCN, Y(7, 1), VL(I1), F(1), D(1), P(I), W(N, 1)
IB = A2 + 1
CO 7 I = 1, A
CO 7 I = 1, A
CONTINUE
CC(I) = 10
CONTINUE
IA = 1
IB = 3
JA = 1
JB = 6
DO 1 CC K=1 A=P
CO 200 IA, IB
CO 100 = JA, JB
```
FM(1,K)=AI+2COT(1,K)-2K(1,K)
CONTINUE
IF(1,K)=1 GO TO 5
IF(1,K)=N GO TO 5
CONTINUE
RETURN
END

SUBROUTINE SDESCL(Y,YT,TEND,NY,NL,JSNF,MAXDER,IPRT,H,HMIN,HMAX,MRSEPS)

SDEGUTINE SDESCL IS A DRIVER ROUTINE FOR SUBROUTINE LDASUB. ITS PURPOSE IS TO SET UP THE NECESSARY REFERENCES TO A LARGE BLOCK OF AUXILLARY STORAGE, AND OBTAIN INITIAL VALUES OF DERIVATIVES. THE CALLING SEQUENCE FOR SDESCL IS
CALL SDESCL(Y,YT,TEND,NY,NL,JSNF,MAXDER,IPRT,H,HMIN,HMAX,MRSEPS)

WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.


I-TH VARIABLE AT T=E IS JS SUM Y(J+1,I)*S**J
J=0

THE VALUE OF JS IS OBTAINED IN THE CALLING PROGRAM BY JS = JAES*(JSNF/10)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>YL</td>
<td>ARRAY OF NL VARIABLES WHICH APPEAR LINEARLY</td>
<td></td>
</tr>
<tr>
<td>NTTEC</td>
<td>CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)</td>
<td></td>
</tr>
<tr>
<td>NY</td>
<td>NUMBER OF DIFFERENTIAL EQUATIONS AND ACNLINAR</td>
<td></td>
</tr>
<tr>
<td>NL</td>
<td>NUMBER OF LINEAR VARIABLES</td>
<td></td>
</tr>
<tr>
<td>PM</td>
<td>NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST</td>
<td></td>
</tr>
<tr>
<td>JSKFR</td>
<td>AN INDICATOR USED BOTH ON INPUT AND OUTPUT</td>
<td></td>
</tr>
<tr>
<td>SCAPE</td>
<td>JKF = -1 INDICATES A RESTART CALL TO SCAPE</td>
<td></td>
</tr>
<tr>
<td>SCESOL</td>
<td>JKF = 0 INDICATES AN INITIAL CALL TO SCESOL</td>
<td></td>
</tr>
<tr>
<td>SCESOL</td>
<td>JKF = 0 INDICATES A CONTINUATION OF THE PREVIOUS CALL TO SCESOL</td>
<td></td>
</tr>
<tr>
<td>JKF</td>
<td>JKF &lt; -1 MAY HAVE RESULTED FROM THE USER NEGLECTING TO TEST FOR ERROR RETURNS</td>
<td></td>
</tr>
<tr>
<td>JKF</td>
<td>JKF &lt; 0 BECAUSE OF THIS FOCUSIBILITY, JKF &lt; -1</td>
<td></td>
</tr>
<tr>
<td>JSKFR</td>
<td>JKF &lt; 0 IS THE ORDER OF THE FORMULA CURRENTLY BEING USED. F INDICATES THE TYPE</td>
<td></td>
</tr>
<tr>
<td>JSKFR</td>
<td>OF RETURN, AS FOLLOWS:</td>
<td></td>
</tr>
<tr>
<td>JSKFR</td>
<td>P = 1 IS THE NORMAL RETURN</td>
<td></td>
</tr>
<tr>
<td>JSKFR</td>
<td>JSKFR &lt; 0 IS AN ERROR RETURN, WITH THE FOLLOWING MEANINGS:</td>
<td></td>
</tr>
<tr>
<td>MAXC</td>
<td>MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN THE METHOD. IT MUST BE NO</td>
<td></td>
</tr>
<tr>
<td>MAXC</td>
<td>GREATER THAN SIX.</td>
<td></td>
</tr>
<tr>
<td>IFRT</td>
<td>INTERNAL PRINT CONTROL INDICATOR FOR DASUB.</td>
<td></td>
</tr>
<tr>
<td>IFRT</td>
<td>IFRT = 0 PRINT COUNTERS, STEPSIZE, CURRENT TIE AND VALUES OF DEPENDENT</td>
<td></td>
</tr>
<tr>
<td>IFRT</td>
<td>VARIABLES AT EACH STEP</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>CURRENT STEPSIZE. AN INITIAL VALUE MUST BE SUPPLIED</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>BUT KEEP ACT THE ONE WHICH MUST BE USED, SINCE THE SUBROUTINE WILL CHOOSE</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>A SMALLER ONE IF NECESSARY TO KEEP THE ERROR PER STEP SMALLER THAN THE</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>SPECIFIED VALUE. IT IS BETTER TO UNDERESTIMATE THE INITIAL STEPSIZE THAN TO</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>OVERESTIMATE IT. THE STEPSIZE IS NORMALY CHANGED BY THE USER.</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>MAXIMUM STEPSIZE ALLOWED</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>MAXIMUM STEPSIZE ALLOWED.</td>
<td></td>
</tr>
<tr>
<td>RPEF</td>
<td>THE ERROR TEST CONSTANT. THE 80TH MEAN-SQUARE OF THE SINGLE STEPSIZE ERROR</td>
<td></td>
</tr>
<tr>
<td>RPEF</td>
<td>ESTIMATES: ERROR/ DIVIDED BY VMAX(1)</td>
<td></td>
</tr>
<tr>
<td>RPEF</td>
<td>VMAX(1) = (MAXIMUM TO CURRENT TIME OF Y(1)) MUST BE</td>
<td></td>
</tr>
</tbody>
</table>

Note: The text appears to be a continuation of a previous message, possibly from a manual or manual-like document, discussing a mathematical or scientific topic. The table format seems to summarize various parameters or conditions with their meanings or implications.
LESS THAN EPS+ THE STEPSIZE AND/OR THE ORDER
ARE VARYING ACROSS MINIMUM STEPSIZE. IT MAY BE AT LEAST 13*NY + 5*NL
LOCATIONS. PLUS THOSE REQUIRED FOR STORAGE OF THE
MATRIX PW (SEE DESCRIPTION OF SUBROUTINE JACMAT).
THE STORAGE OF PW WILL NORMALLY REQUIRE NO MORE THAN
NY*2 + 2*NY LOCATIONS. AND IF CONTINUOUS STORAGE TECH-
NIQUES ARE USED, CAN BE MUCH FEWER.

DIMENSION WI (1,1), W(1,1)
IF (JSKF.GT.1) GC TC 120
IF (JSKF.LT.1) GC TC 140
N = NY4AL
IF (JSKF.LT.0) GC TC 110

IF THIS IS THE FIRST ENTRY, OBTAIN VALUES OF THE DERIVATIVES.
CALL CERVAL (Y,YL,T,A,NY,W,KRETR)
IF (KRETR.AND.C) GC TC 130

NOW SET LF STORAGE BLOCKS IN THE W ARRAY. THIS NEEDS TO BE DONE
ONLY INITIALLY AND ON RESTARTS.

THE ARRAY SAVE STARTS AT LOCATION 1 IN THE W ARRAY
THE ARRAY LSV STARTS AT LOCATION KS VL IN THE W ARRAY
THE ARRAY YMMA STARTS AT LOCATION KNYX IN THE W ARRAY
THE ARRAY ESV STARTS AT LOCATION NEXS IN THE W ARRAY
THE ARRAY FI STARTS AT LOCATION NSF1 IN THE W ARRAY
THE ARRAY DY STARTS AT LOCATION NCKY IN THE W ARRAY
THE MATRIX PW STARTS AT LOCATION NP W IN THE W ARRAY

ASV = 7*NY
NYPX = NY4AL
AY = NY4AL
NEX = NER*NW

NPR = NER*NV
AP = NC4A

110 JSF = JSF
CALL LCASE (Y, YL, T, TEND, N, NY, M, JS, KS, MAXCER, IPRT, HW, HWMX, HW
MAX, HMAX, KMSEP, W, WNSV, W(KF), WFI, WNY, WNPW)

120 JSF = JSF
CODE JSF CAN RETURN FROM LCSUB
JSEF = 1SIGMA(JSF+1*ABS(KF),KF)
RETURN
130 JSF = -6
RETURN
14C PRINT 1, JSKF
STOP
C
1 FORMAT ('CIT IS AN ERROR TO ENTER SDESCL WITH JSKF = ',I10//
1 ' RLH HAS BEEN TERMINATED.'
END
SUBROUTINE LCASUB (Y, YL, T, TEND, N, NY, M, JSTART, KFLAG, MAXOR, IPRT, H,
1 MIN, FMAX, RPSEPS, SAVE, YLSV, YMAX, ER, ESV, FI, DY, PW)

WHICH IS DUE TO R. L. BROWN AND C. W. GEAR. DFASUB IS DOCUMENTED
IN THE REPORT
DECLARATION FCF CFASUB--
BY R. L. BROWN AND C. W. GEAR
REPORT UIUCCS-R-73-578, JULY 1973
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
URBANA, ILLINOIS 61801
THIS REPORT IS AVAILABLE FROM THE NATIONAL TECHNICAL INFORMATION
SERVICE OF THE U. S. DEPARTMENT OF COMMERCE UNDER ACCESSION NUMBER
CCC-1465-225.

THE DOCUMENTATION HERE IS DOCUMENTED IN THE REPORT
A PROGRAM FOR THE NUMERICAL SOLUTION OF LARGE SPARSE SYSTEMS OF
ALGEBRAIC AND IMPLICITLY DEFINED STIFF DIFFERENTIAL EQUATIONS
BY RICHARD FRANK
REPORT AFFS-3F76051, MAY 1976
NAVY, POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93940

THE CALLING SEQUENCE FOR LCASUB IS
CALL LCASLEIV(Y, YL, T, TEND, N, NY, M, JSTART, KFLAG, MAXOR, IPRT, H, MIN,
1 FMAX, RPSEPS, SAVE, YLSV, YMAX, ER, ESV, FI, DY, PW)
WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS
Y -- ARRAY DIMENSIONED (T, NY). THIS ARRAY CONTAINS THE
DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES. Y(J+1, I) CONTAINS THE
J-TH DERIVATIVE OF THE I-TH VARIABLE TIMES H**(J-J!)-FACTORIAL, WHERE M IS THE CURRENT
STEP SIZE. ON FIRST ENTRY THE CALLER SUPPLIES THE
INITIAL VALUES OF EACH VARIABLE IN Y(1, I) AND AN
ESTIMATE OF THE INITIAL VALUES OF THE DERIVATIVES
IN Y(2, I). ON SUBSEQUENT ENTRIES IT IS ASSUMED THAT
THE ARRAY HAS NOT BEEN CHANGED. TO INTERPOLATE TO
NON-MESH POINTS, THESE VALUES CAN BE USED AS FOLLOWS.
IF H IS THE CURRENT STEPSIZE AND VALUES AT TIME T+E
KEEP EW; LET S = E/H AND THEN

I-T VARIABLE AT T+E IS SUM Y(J+1,I)*H**J
J=0

THE VALUE OF NC IS OBTAINED IN THE CALLING PROGRAM
BY NC = JSTART.

YL - ARRAY OF ALL = N - NY VARIABLES WHICH APPEAR LINEARLY
THE USER SUPPLIES INITIAL VALUES FOR THESE VARIABLES.
T - CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)
TYC - END TIME
NY - TOTAL NUMBER OF VARIABLES
NY - NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR
VARIABLES
P - NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST; THIS
NUMBER CAN BE NO GREATER THAN NY. IF IT IS
GREATER THAN NY, NY VARIABLES ARE USED IN THE ERROR
TEST.

JSTART - INPUT AND OUTPUT INDICATOR
ON INPUT JSTART HAS THE FOLLOWING MEANINGS:
<0 THIS INDICATES A RESTART FROM PREVIOUS
POINT FOLLOWING TERMINATION OF THE RUN OR
SOLUTION OF ANOTHER PROBLEM DURING THE SAME
RUN. PARAMETERS IN THE CALLING SEQUENCE
MUST HAVE BEEN PRESERVED FROM THE PREVIOUS
USE, PARTICULARLY THE ARRAYS
SAVE, VLSV, ESV, AND PW.
THESE ARRAYS MUST BE SAVED AFTER A CALL
to SUBROUTINE LSANS, WHICH ALSO SAVES
NECESSARY PARAMETERS INTERNAL TO LCSUB.
=0 ATTIC AS AN INITIAL CALL TO LSANS; THE
RCLTIME INITIALIZES ITSELF, SCALES THE
DERIVATIVES IN Y(1:1) AND THEN PERFORMS THE
INTEGRATION UNTIL J > TEND
>0 INDICATES THE SOLUTION IS TO BE CONTINUED.
AFTER THE INITIAL ENTRY IT IS NEITHER
DESIRABLE NOR NECESSARY TO RE-ENTER
WITH JSTART = 0, SINCE THIS RE-INITIALIZES
THE CODE, BEGINNING WITH A FIRST ORDER
METHOD AGAIN.

CN OUTPUT JSTART IS SET TO THE VALUE OF NC, THE
ORDER OF THE FORMULA CURRENTLY BEING USED.

KFLAG - THE COMPLETION CODE INDICATOR, WITH THE FOLLOWING
MEANINGS:
+1 THE INTEGRATION WAS SUCCESSFUL
-1 ERROR TEST FAILURE FOR H > HMIN
-2 CCORRECTOR FAILED TO CONVERGE FOR H > HMIN
-4 CCORECTOR FAILED TO CONVERGE FOR FIRST
-5 RGB ERROR METHOD
-6 ERROR RETURN FROM SUBROUTINE NUITS
MAXF - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN THE METHOD. IT MUST BE NO GREATER THAN SIX. IF IT IS GREATER THAN SIX, THE MAXIMUM ORDER USED WILL BE SIX.
IFRT - INTERNAL PRINT CONTROL INDICATOR
+0 NO PRINT
>0 PRINT COUNTERS, STEPSIZE, CURRENT TIME AND VALUES OF DEPENDENT VARIABLES AT EACH STEP.

I CURRENT STEPSIZE. AN INITIAL VALUE MUST BE SUPPLIED BUT NEXT ONE WILL BE THE ONE WHICH WILL BE USED. SINCE THE SUBROUTINE WILL CHOOSE A SMALLER ONE IF NECESSARY TO KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED VALUE, IT IS BETTER TO UNDERESTIMATE THE INITIAL STEPSIZE THAN TO OVERESTIMATE IT. THE STEPSIZE IS NORMALLY NOT CHANGED BY THE USER.

HMNA - MINIMUM STEPSIZE ALLOWED
HMNX - MAXIMUM STEPSIZE ALLOWED
RASEFS - THE CCORF TEST CONSTANT, THE ROOT-MEAN-SQUARE OF THE SINGLE STEP ERROR ESTIMATE, ER(1), DIVIDED BY YM1(1) = MAXIMUM AND CURRENT TIME OF Y(1)) MUST BE LESS THAN RASEFS. THE STEPSIZE AND/OR ORDER ARE VARIED TO ACHIEVE THIS.

SAVE - AN ARRAY OF LENGTH AT LEAST 7*N
YMA - AN ARRAY OF LENGTH AT LEAST NL
NL - THE LENGTH OF THE VALUE WHICH CONTAINS THE MAXIMUM OF EACH Y SEEN SO FAR. ON THE FIRST CALL, THESE WILL BE INITIALIZED AS YM1(1) = MAX(1, Y(1, 1))

ESV - A VECTOR OF LENGTH NY
ESV - A VECTOR OF LENGTH NY

PV - A VECTOR IN WHICH THE J MATRIX COMPUTED IN SUBROUTINE JACMAT WILL BE STORRED. SIZE WHICH MUST BE ALLOCATED IS DETERMINED BY THE STORAGE TECH-

DIMENSIONS (7,1), YL(1), SAVE(7,1), YMAM(1), ER(1), YLSV(1), F(1)
1, PER(1, 2), COF(1), ES(1), R(1), P(1), SAVE, A(29)
EQUIVALENT (A(1), ENC), (A[5], R) (A(10), R) (A(1), ENC)
1(1), ENC1, (A[13], ENC2), (A(14), ENC3), (A(15), EPS1), (A(16), ENC)
THE COEFFICIENTS IN THE FERM ARRAY ARE USED FOR ERROR TESTING AND
CHANGING STEPSIZE AND NEED TO BE ACCURATE TO ONLY A FEW DIGITS.

THE ENTRIES IN THE CCF ARRAY ARE THE COEFFICIENTS FOR THE STIFFLY
STABLE FEM COS USED IN THIS PROGRAM AND ARE TO BE THE MACHINE
PRECISION EQUIVALENTS OF THE FOLLOWING CONSTANTS.

-1/2, -1/2
-3/8, 1/2, -1/6
-3/16, -15/24, -7/24, -1/24
-9/16, -15/24, -7/24, -1/24
-147/160, -203/240, -45/4, -35/144, -7/240, -1/720

IF THIS IS A RESTART ENTRY, RESTORE Y AND YL FROM THE SAVE AND
YLSV ARRAYS, WHERE THEY WERE SAVED BY A PREVIOUS CALL TO LDASAV.

CALL CCPY1 (YSAVE, LCCPY1)
CALL CCPY2 (YLSAVE, LCCPY2)
GO TO 150

IF THIS IS THE FIRST CALL, INITIALIZE Y, SCALE DERIVATIVES, AND
INITIALIZE INDICATORS, AND SET ORDER TO CCF.
FCF ECFLE PRECISION; SET LCCPY = 1 4NY AND LCCPYL = 2*NL IF
SUBCLASSIC CCPY2 IS IN SINGLE PRECISION.

AL = A/N
LCCPYV = 1/N
LCCPYVL = AL
F = F ICCPY2(Y)
EPS = SCF(90*11)*RSEPS
DAXCER = PINC(MAXCER,6)
IF (IPRT=LENC) GO TO 120
PRINT 5, A, X, RMSEPS, IENC, H
120 AS = 0
AW = 0
C
DO 130 J=1, NY
YMAX(J) = AMAX1 (1., AFS(Y(1,J)))
130 Y(2, J) = Y(2, J)*H
C
AC = 1
BR = 1.
ASSIGN 190 TO IRET
C
SET COEFFICIENTS FOR THE ORDER CURRENTLY BEING USED.
E IS A TEST FOR EFPCPS IF THE CURRENT ORDER NO
EUP IS TC TEST FOR INCREASING THE ORDER, EDWN FOR DECREASING THE
C
C
140 K = AC*(AC-1)/2
CALL CCFF2 (A(2), COF(K+1), NO)
K = AC+1
IDCLB = AC
ENC1 = AC
ENC2 = AC/K
ENC3 = AC/(NO+2)
EFST = EFST**2
EO = EFST*(AC-1)*EFST
EUF = EFST*(NO+2)*EFST
EDWN = EFST*(AC+1)*EFST
BND = EFST**NO**2
MEVAL = 3
GO TO IRET, (190, 200, 490, 570)
150 IF (F, EC= FNEH) GO TO TC 150
C
IF CALLER HAS CHANGED F, RESCALE DERIVATIVES TO REFLECT THAT HNEW
WAS USED ON THE LAST CALL.
C
R = F/FNEH
ASSIGN 190 TO IRET
GO TO TC 160
C
SET JSTART TO NO, THE CURRENT ORDER OF THE METHOD, BEFORE EXIT,
AND SAVE THE CURRENT STEPSIZE IN HNEW.
C
160 JSTART = AC
FNEH = F
RETURN
AS = AS + 1
IF (IPR1.LT.2) GO TO 100
PRINT DATA IF DESIRED BY USER
PRINT 1, TS, NW, AQ + T, (Y(J1, I), I = 1, NY)
IF (NL.CE.GT.1) PRINT 2, (YL(J1, I), I = 1, NL)
CONTINUE
IF (KFLAG.LT.0) GO TO 160
IF (T.LT.TEND) GO TO 16C
TAKE ANOTHER STEP IF T < TEND
JSTART = 1
SAVE DATA FOR TRIAL WITH A SMALLER Timestep IF THIS STEp FAILS
CALL CCFY2 SAVE (Y, LCDP(Y))
CALL CCFV2 (VLSV, YL, LCDPY)
CALL = 1
KFLAG = 1
FCLC = +
ACLD = AC
TOLD = T
T = T +
FINV = 1.0 /H
ECEFTE PREDICTED VALUES BY EFFECTIVELY MULTIPLYING DERIVATIVE
VECTOR BY PASCAL TRIANGLE MATRIX
GO 210 H = J, K
J3 = K + J - 1
GO 210 J1 = J, K
J2 = J2 + 1
GO 210 I1 = 1, NY
Y(J2, I) = Y(J2, I) + Y(J2 + 1, I)
GO 220 I = 1, NY
ER(I) = C
GO IF TC THREE CORRECTOR ITERATIONS, CONVERGENCE IS OBTAINED WHEN
CHANGES ARE LESS THAN ENC WHICH IS DEPENDENT ON THE ERROR TEST
**C**

STEP IS REDUCED TO TRY AND GET CONVERGENCE.

1. IF (I.LE.280,300,250) GC TO 310
2. IF (I.E.280,300,250) GC TO 310
3. GACUP = FACUP + 0.25
4. GC TO 360

**C**

RESTORE Y AND YL AFTER CONVERGENCE FAILURE

**C**

CALL CCPY2 (Y,SAVE,LCCPY2)
CALL CCPY2 (Y,YLSV,LCCPY1)
H = HCC
KO = NCCC
GO TO 170

**C**

THE CCPYCR CONVERGES, SO NOW THE ERRCR TEST IS MADE.

**C**

C = C

GO 340
YH = APX1(ABS(Y(I,I)),YMAX(I))
340 C = (C+1E6*Y(I,YM)*Y2

**C**

C陀EYL = 8

CC TC 380

**C**

THE ERRCR TEST IS OKAY, SO THE STEP IS ACCEPTED. IF IDCLB
NOW BECOMES NEGATIVE, A TEST IS MADE TO SEE IF THE SIZE
CAN BE INCREASED AT THIS ORDER OR ONE HIGHER OR CAN LOWER.
THE CHANGE IS MADE ONLY IF THE STEP CAN BE INCREASED BY AT
LEAST 10%. IDOLB IS SET TO NO TO PREVENT FURTHER TESTING
FOR A WHILE. IF NO CHANGE IS MADE, IDOLB IS SET TO 9.

**C**

IF (K.EQ.2) GO TO 36C

**C**

CO 350 = 2, K

CO 350 = 1, NY

**C**

VY,11 = Y(J,1)+A(J)*ER(I)

**C**

KFLG = 1
IDCLB = IDCLB-1
IF (I.EQ.LCL) 410, 370, 372C
370 CALL CCP12 (ESV,ER,FI)
GO TO 51C
GO 470 J=1:N
47C VIK(,J) = E1(J1*PI)
C
480 CONTINUE
C
IF THE STEP WAS OKAY, SCALE THE Y VARIABLES IN ACCORDANCE
WITH THE NEW VALUE OF K. IF KFLAG < 0, FOREVER, USE THE
SAVED VALUES (IN SAV AND VL5), IN EITHER CASE, IF THE ORDER
HAS CHANGED IT IS NECESSARY TO FIX CERTAIN PARAMETERS BY CALLING
THE FFCEFA SEGMENT AT STATEMENT NUMBER 140.
C
ICUE = AC
IF (NE6 < EC,NC) GC TC 490
NQ = NE6C
ASSIGN 490 TC IRET
GO TC 74C
490 IF (KFLAG GT 0) GO TC 50C
RACL = F1LPR
GO TC 55C
50C F = AMAX1(AMIN1(VMAX/F,R1,FMIN/H)
F = F+F
INEVAL = 1
ASSIGN 510 TC IRET
GO TO 31C
C
51C GC 520 I=1,N
52C VMAX[I] = AMAX1(ABS(V(1,N)),VMAX[I])
C
GO TC 17C
C
THE ERROR TEST HAS NOW FAILED THREE TIMES, SO THE DERIVATIVES ARE
IN BAD SHAPE. RETURN TO FIRST ORDER METC AND TRY AGAIN. OF
COURSE, IF NO = 1 ALREADY, THEN THERE IS NO HOPE AND WE EXIT WITH
KFLAG = -4.
C
530 IF (NC.NE.1) GO TO 540
NQ = 1
ICUE = 1
ASSIGN 570 TC IRET
GO TC 14C
540 NQOLF = 1
KFLAG = -4
GO TC 15C
550 KFLAG = -7
GO TC 17C
C
THIS SECTION RESTORES THE SAVED VALUES OF Y AND VL, SCALING THE
Y DERIVATIVES AS NECESSARY, AND THEN RETURNS TO THE PREDICTOR COCP
C
ENTRY LEASEV(SAV)
LCEFS = 25
CALL CCFY2 (SAV,A,LCCFYS)
CALL CCFY2 (YLYS,YL,LCCFLY)
RETURN

ENTRY LEASEE(SAV)
LCEPS = 25
CALL CCFY2 (A,SAV,LCCFYP)
RETURN

---

1 FORMAT (2I5,1F,192E10.2,1E14.6/(32X,7E14.6))
3 FORMAT (I3,1X,01,15.4,6/, NL = 'i3,' RMSEPS = '1PES.2,' TEND = '1
4 FORMAT (1NS NL C H',8X,'T', TE, 'IY(1,s) AND YL(*)/')
ENC
SUBROUTINE CCFY2 (5,Y,L)
DIMENSION 5(1),Y(1)

THIS SUBROUTINE CCFIES THE ARRAY Y, OF LENGTH L, INTO THE ARRAY S

IF(L .LE. 0) RETURN
CC 100
S(1) = 1
LCC
RETURN
END

SUBROUTINE CERVAL (Y,YL,T,N,NX,W,KERET)

THIS SUBROUTINE CALCULATES THE INITIAL VALUES OF THE DERIVATIVES
IN THE GENERAL CASE. IT IS WRITTEN SO THAT IT SHOULD WORK IF THE
FIRST N EQUATIONS ALL INVOLVE DERIVATIVES. IT ATTEMPTS TO SOLVE
TO THE FIRST N EQUATIONS USING NEWTON'S METHOD, BUT SINCE IT TRIES
TO EVALUATE DP/DY BY CALLING JACMAT IN SUCH A WAY AS TO MAKE THE
DP/DY TERM INSIGNIFICANT, IT IS POSSIBLE THAT IT MAY FAIL FOR THAT
REASON. IT MAY FAIL FOR OTHER REASONS, AS WELL. IF IT DOES FAIL
THE USER CAN SUPPLY HIS OWN VERSION OF CERVAL, OR MODIFY THIS
ROUTINE IN SUITABLE FASHION. THIS ROUTINE ASSUMES THAT VALUES OF
THE LINEAR VARIABLES HAVE BEEN SUPPLIED PREVIOUSLY. IF THESE
MUST BE SOLVED FOR SIMULTANEOUSLY WITH THE DERIVATIVES, THE USER
MUST SUPPLY HIS OWN VERSION OF CERVAL.

---
THE CALLING SEQUENCE FOR THIS SUBROUTINE IS

CALL CEFVAL(Y,YL,T,T, NY, NYK, KERCT)

WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS

Y       - SAME AS IN LOASUB AND SDESCL. Y(1,1) CONTAINS THE
         INITIAL VALUES OF THE DEPENDENT VARIABLES. THE
         VALUES ON THE DERIVATIVES ARE RETURNED IN Y(2,1).
         YL      - SAME AS IN LOASUB AND SDESCL. THE INITIAL VALUES OF
         THE LINEAR VARIABLES MUST BE SUPPLIED TO THIS VERSION.
         T       - INITIAL TIME
         N       - SAME AS IN LOASUB, TOTAL NUMBER OF VARIABLES
         NY      - SAME AS IN LOASUB, NUMBER OF DIFFERENTIAL EQUATIONS
         AND NONLINEAR VARIABLES
         W       - SCRATCH ARRAY W FROM THE CALLING SEQUENCE OF SDESOL.
         KERCT   - RETURN INDICATOR
                     = 0 ALL VALUES RETURN
                     = 1 ERFOR RETURN

DIMENSION Y(7,11), YL(11), W(11)

GO TO 10C  J=1, NY
         W(Z+N+1) = AMAX1(ABS(Y(1,1)),1.)
          10C  Y(2,1) = C.
         FINV = 16.4*20
         KERCT = C
         EPS2 = NY/1.E6
         EPS = SCFT(EPS2)
   C
   C
   C
   C
   C
   C
   C
   C
   C
   C

C

DO 14C  IT=1,5

   14C  Y(2,1) = Y(2,1)/FINV
   C
   DO 11C  I=1, NY

   11C  Y(I,1) = Y(I,1)/FINV

   CALL CEFVL (Y,YL,T,T, INV, KERCT)
   CALL CEFVL (Y,YL,T,T, INV, -1, NY, NY, EPS, W, W(N+1), W(3*N+1))
      NEWP = 1
   C
   C
   C
   C
   C
   C

   12C  IT = W(1,1)*FINV

   CALL ALITSL (W(2*N+1), W(N+1), NY, NY, EPS, W(2*N+1), NEWP, KERCT)
   IF (KERCT .EQ. C) GO TO 170
C ER = 0.
C C0 170 I=1, NY
C Y(3,I) = Y(3,I)-X(N+I)
120 ER = ER+V(I+I)/MAX(ABS(Y(3,I)),1.1)**2
130 WRITE(6,131) ER
131 FORMAT(2X,G12.5)
C IF (ER.LT.EPS2) GO TO 150
C CONTINUE
C GC TC 170
C 150 C0 160 I=1, NY
160 Y(2,I) = Y(3,I)
C C RETURN
C KRET = 1
C RETURN
ENC
SUBROUTINE ALITSL (Ff, EY, F1, NY, EPS, YMAX, NEWP, KRET)
C ---------------------------------------------------------------
C THE PURPOSE OF THIS SUBROUTINE IS TO SOLVE A
C LINEAR SYSTEM OF EQUATIONS FOR THE NEWTON ITERATES WHEN THE
C CORRECTED EQUATION IS BEING SOLVED. UPON ENTRY TO THIS SUBROUTINE
C THE SYSTEM OF EQUATIONS TO BE SOLVED IS J W = -F , WHERE
C J IS STORED IN F(1) LFCC ENTRY
C W IS RETURNED IN F(1)
C -F IS STORED IN LFCC ENTRY
C
C THIS SUBROUTINE IS GENERALLY SUPPLIED BY THE USER, ALTHOUGH THERE
C ARE SOME STANDARD FORMS AVAILABLE. FOR EXAMPLE, THIS VERSION
C ASSUMES THAT PW IS STORED IN FULL STORAGE MODE IN AN N*N MATRIZ.
C IF NEWP = 1, AN LL DECOMPOSITION IS DONE, NEWP IS SET TO ZERO
C AND FORWARD AND BACKWARD SUBSTITUTION FOR THE SOLUTION IS DONE.
C IF NEWP = 0, ONLY FORWARD AND BACKWARD SUBSTITUTION FOR THE
C SOLUTION IS NECESSARY.
C
C NOTE THAT THIS VERSION OF ALITSL REQUIRES THAT PW HAVE N**2 + 2*N
C LOCATIONS SINCE 2*N LOCATIONS ARE USED BY THE IMSL LINEAR EQUATION
C SOLVER.
C
C NOTE THAT THE PARAMETERS EPS AND YMAX ARE USEFUL IF AN ITERATIVE
C TECHNIQUE IS USED TO SOLVE THE SYSTEM OF EQUATIONS.
C
C THE CALLING SEQUENCE FOR THIS SUBROUTINE IS
C CALL ALITSL(FF, EY, F1, NY, EPS, YMAX, NEWP, KRET)
WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.

FW - THE J MATRIX CALCULATED IN SUBROUTINE JACMAT
DY - THE RIGHT HAND SIDE OF THE LINEAR SYSTEM TO BE SOLVED
F1 - THE SOLUTION IS RETURNED IN THE ARRAY F1
A - SAME AS IN LEASUB, TOTAL NUMBER OF VARIABLES
NV - SAME AS IN LEASUB, NUMBER OF DIFFERENTIAL EQUATIONS
AND NONLINEAR VARIABLES
EPS - L2 ERRFC OR CONSTANT USED IN LEASUE
YMAX - MAXIMUM VALUES OF Y(I), I SEE IF IT GTO THE CURRENT TIME
NEWP - INDICATES WHETHER A NEW J MATRIX HAS BEEN COMPUTED
  =1 INDICATES A NEW J MATRIX HAS BEEN COMPUTED
  SINCE THE LAST ENTRY TO NLISSL. NEWP
  SHOULD BE SET TO ZERO IF SOME PREPROCESSING
  SUCH AS LU DECOMPOSITION MUST BE DONE ON A
  NEW J MATRIX
  =0 INDICATES THE J MATRIX IS THE SAME AS WHEN
  NLISSL WAS LAST ENTERED
KRET - RETURN INDICATOR
  =0 NORMAL RETURN
  =1 ERROR RETURN. SOLUTION OF EQUATIONS COULD
  NOT BE OBTAINED.

DIMENSION FW(1), CY(1), F1(1), YMAX(1)
NL = A-A
IF (NEWFM.EQ.0) GO TO 100
NEWP = C
NN = N+42+1
NNA = N+44
CALL LLCATF (PW,PB,1,N,0,D1,D2,FW(NNA),FW(NNA),F1,IER)
IF (IER.EQ.0) GO TO 100
KRET = 1
RETURN
100 CALL LELEF (PW,DY,PW(NN),N,N,F1)
KRET = C
RETURN
END
SAFEGE INPUT FILE FOR CASE 1

I (ZERO CELFA LOCATION)

1ELEM = 0, must input N.P. locations; 1, automatically divides domain into NNP-1 equilength elements.

NNP = number of nodal points.
T = time step of integration (sec.)
HMAX = maximum allowable time step (sec.)
TSTART = start time of problem, normally equal zero (sec.)
UINFI = exterior surface velocity (kt.)
TAMB = ambient temperature (deg F)
PAMB = ambient pressure (p.s.i.)
TORT = tortuosity
D1 = filament diameter (inches)
SPACE1 = ply thickness (inches)
HLENGT = reference length (equal to 1 ft.)
XLENT1 = plate thickness (inches)
RGAS = gas constant for air (lbf*ft.per lbm*deg-R)
GCOND = bulk conductivity of graphite (btu per hr*ft*deg-F)
EMIS = fiber emissivity
RG = bulk density of graphite (lbm per cu.ft.)
SPHTG = specific heat of graphite (btu per lbm*deg-F)
SPHTA = specific heat of air at constant pressure (btu per lbm*deg-F)
HSURF = heat transfer coefficient at x=0 (btu per sq. ft*hr*deg-F)
CAMBI = ambient concentration of O2 (lbm per cu.ft.)

Initial temperature of fibers (deg-F)
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