DRSAR/PE/N-86

PROGRAM ANALYSIS AND EVALUATION DIRECTORATE

ACTIVITIES SUMMARY

SEPTEMBER 1980

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US ARMY ARMAMENT MATERIEL READINESS COMMAND
PROGRAM ANALYSIS AND EVALUATION DIRECTORATE
ROCK ISLAND, ILLINOIS 61299

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

M110A2 SP Howitzer	Testing Methodology	Sequential Testing
Loader-Rammer Tests	Operations Research	Multi-Stage Sampling
Human Factors Studies	Allocation Methodology	Cost-Effective Testing (CET)
Environmental testing	Reliability Assessment	155mm Propelling Charges
Statistical Analysis	Computer Programs	Training/Ramming Projectile

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report contains Memoranda for Record (MFR's) describing some of the activities of the Program Analysis and Evaluation Directorate, US Army Armament Materiel Readiness Command, Rock Island, IL 61299. Subjects dealt with are the systems analysis of cannon damage in the M110 systems, evaluation of rotating bands for M106 training projectile, logistic implications of the distribution and type of 155mm propelling charges, TDP cancellations, ammunition lot acceptance and lot demil criteria, review of COEA for 155mm propelling charges, and cost-effective testing (CET).

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19. Key Words:

Projectile Fallback in M110A2
Technical Data Package (TDP) Cancellations
Procurement Request Order Number (PRON)
Product Improvements to M110 Howitzer
Plastic Rotating Bands for Artillery Projectiles
Logistics of Artillery Ammunition and Propelling Charges
Operational Distribution of Propelling Charges
Gun Tube Wear and Erosion
Cost-Effectiveness Analysis
Lot Acceptance Test Procedures
Statistical Quality Control
Statistical Confidence Bounds
Field Tests of M106 8-inch Training Projectile
Categorical Assignment via Testing

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Memorandum for Record

ANALYSIS

OF

RESULTS

OF

M106 TRAINING PROJECTILE TESTS

AND

OF

M110A2 LOADER-RAMMER

COLD ENVIRONMENT TESTS

George J. Schlenker

DRSAR-PEL 6 March 1980

MEMORANDUM FOR RECORD

SUBJECT: Analysis of Results of M106 Training Projectile Tests and of M110A2 Loader-Rammer Cold Environment Tests

1. Reference:

a. Loader-Rammer Annex to TPR 2-MU-003-106-026, DRSAR-PE, HQ, ARRCOM, Jul 79.

b. MFR, DRSAR-PEL, HQ, ARRCOM, 29 Aug 79, subject: Observations Concerning the M110A1 Loader-Rammer Performance Test at YPG, Aug 79.

c. Displays Used in IPR of Cannon Damage Study, DRSAR-PEL, HQ, ARRCOM, 10 Sep 79.

2. Background

The subject tests were carried out during the period 28 Jan through 15 Feb 1980 at Yuma Proving Ground (YPG) in accordance with paragraphs 5q and 5m of the Reference a Test Program Request (TPR). The recent tests completed the list of tests outlined in Reference a. Other reports related to this project include References b and c. All of this work is related to the problem of land damage in the M201 cannon. As a consequence of investigations into this problem, it was discovered that the M106 projectile does not always become and/or remain fully seated prior to firing the M110 system. Firing of a projectile which has fallen back upon the propelling charge has been shown to be capable of causing the type of cannon damage which originally provoked the investigation.

3. The Blue Ribbon Panel which conducted the initial inquiry recommended that the conditions giving rise to fallback be identified. As a result of this injunction, Reference a tests of loader-rammer (L/R) were devised. Additionally, the Blue Ribbon Panel recommended certain changes to system hardware among which were the development of a training projectile and the modification of the L/R to preclude short cycling and ramming out of time. The tests described in this memorandum specifically concern the performance of the training projectile at ambient and low (-25 deg F) temperatures and of the loader-rammer, in two configurations, at -25 deg F.

4. Purpose

The purpose of this memorandum is to present the data included in the daily logs and to analyse and interpret these data for the benefit of the ARRCOM Heavy Artillery Office, the test sponsor, and for other interested organizations. The data presented here were obtained visually from pressure gages and digital voltmeters and are considered preliminary. A final report should be prepared by YPG based upon transducer output which was recorded on magnetic tape. These data ought to be free of most of the quantization effects* and reading errors which characterize data obtained manually.

5. The purposes of the tests reported here fall into two sets -- those concerned with the training projectile and those concerned with the loader-rammer. With respect to the former, one wishes to know the probability distribution of extraction force at ambient and at very low temperatures. Does the plastic rotating band used with the training projectile behave properly at low (-25 deg F) temperature? Further, if trends in extraction force with multiple rams exist, at what point does the replaceable rotating band become unusable? What is the fallback sensor output, proportional to tube strain, produced by using the training projectile? With respect to the loader-rammer (L/R), one wishes to know the reliability of the modified L/R (L/R MOD) in a cold environment. Using the M106 projectile, how does the probability distribution of extraction force using L/R MOD compare with that using the standard loader-rammer (L/R STD)? How does the distribution of extraction force change when the temperature of the environment changes from 90 deg F to -25 deg F? What functional relationship exists between fallback sensor output and peak extraction force? Finally, does the extraction force vary measurably with the means of extraction?

6. Anecdotes

Two highlights of the anecdotal record are provided here due to their importance. The first incident occurred because the rotating bands sent to YPG to support the tests of the training projectile were oversize. On 1 Feb the first test ram of a training projectile occurred. This projectile had a carbon-filled (black) nylon "supertuf"** rotating band which had been temperature-conditioned to -25 deg F.

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* Resolution and "favorite-number" effects which produce quantization of analog data are discussed later in this memorandum.

** The name "supertuf" is a copywrited designation for Dupont Zytel. An alternative plastic was also used -- white, 66 nylon.

This projectile failed to seat properly in three consecutive attempts. The interpretation of this result was confounded somewhat by the inconsistent performance of the rammer which was experiencing intermittent interruptions during ramming at this phase of the program.* Then the rotating band outside diameter (OD) of the test projectile was measured and found to vary from 8.260 to 8.290 inches. At this point I discovered that, although this dimension was too large, no one knew the "correct" band dimension for this kind of plastic. Accordingly, I had the band OD machined to 8.220 inches and the projectile replaced in the temperature conditioning chamber. After a 12 hour cold soak this projectile was retested. Again the band would not seat properly. At this point I had the band removed and replaced with a band made of white "66 nylon" which was machined to an OD of 8.170 inches. Another training projectile was equipped with a black "supertuf" nylon band machined to 8.170 inches OD. This projectile was tested at 70 deg F and found to yield satisfactory results. Subsequent testing of the white nylon at -25 deg F produced extremely low extraction forces. Additionally, when the white nylon band was removed while still cold, it fractured producing small blade-like splinters. For these reasons further testing of the white 66 nylon material was suspended.

7. A second incident of importance pertains to the power rammer in its modified configuration. After thoroughly checking the timer, solenoids, and microswitches of L/R MOD following the training projectile tests, the weapon (carriage no. 12FJ05) was placed in the LMPEC environmental chamber for conditioning to -25 deg F. After soaking in excess of 48 hours, the weapon was tested starting 13 Feb 80. At this time it was discovered that the rammer would not operate. After starting, the rammer head would advance approximately 6 inches and immediately retract to its seated position. Checks were performed independently on the solenoids and microswitches to assess their function, with satisfactory results. However, only after the out-of-time microswitch was effectively removed from the circuit did the rammer function properly. Apparently, the action of the solenoids in the L/R MOD occurred so slowly that the electrical connections (in the timer), switched in by their action, could not be made before the out-of-time microswitch lost contact with the projectile, causing a retract signal to occur. This incident is evidence that the L/R MOD configured as tested would be unsatisfactory for fielding. However, after removal of the out-of-time microswitch, rammer operation proved reliable at -25 deg F, although slower, and the force required to extract the M106 projectile was quite large, averaging 10,403 lbs.

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* The rammer would occasionally start and stop several times during a ram. This behavior produced very low extraction forces.

8. Extractors

The process of extracting projectiles for these tests employed two tools. One type (A) is a modified XM753 extraction tool for rearward extraction. This is a hydraulic, manually pumped unit equipped with a pressure dial gage and with a pressure transducer for magnetic recording. When using this tool, the projectile must be drilled and tapped to accept the threaded shaft of the extractor. The other type of tool (B), designed by DRDAR-LC, is positioned forward of the projectile for pushing, using a hydraulic jack. The hydraulic pressure is also recorded by dial gage and the force by load cell, which is recorded on magnetic tape. Although both types of tools were substantially the same as those used in the M110 tests of July - Aug 79, new pressure gages were employed. Calibration curves, relating hydraulic pressure to force, are shown in Figures 1 and 2. Quadratic fits to the calibration points are also shown here.

9. Results

As mentioned above, generally acceptable extraction performance was shown by the black "supertuf" nylon rotating band with 8.170 inch OD at 70 deg F. However, as shown in Figures 3 and 4, there is an apparent gradual decrease in the extraction force with successive ramming. With continued ramming this decrease would eventually produce an unacceptably low extraction force and associated tube strain. In this case one can infer that the fallback sensor, which measures tube strain near the origin of rifling, might give an indication of an improperly seated projectile even though rammer operation is satisfactory. This effect clearly limits the useful life of the band. An estimate for the probability distribution of extraction force over a limited portion of the lifetime of the 8.170 inch OD band is shown in Figure 5. To determine whether a larger band diameter of the same material would give better performance, I had an additional band machined to 8.180 inches OD. (Here is another departure from the TPR (Ref a)) A projectile with this band was tested for endurance by ramming and extracting 75 times at 70 deg F (approx.). On the first ram the projectile failed to remain seated. This was similar to the behavior exhibited by the 8.220 inch OD band tested earlier. As a consequence I was first led to believe that the 8.180 inch diameter was excessive. However, subsequent rams of this projectile produced adequate extraction forces. These forces increased progressively for 8 rams and thereafter remained (nearly) stochastically stationary. The extraction history for rams 9 through 75 is shown in Figure 6. Actually, there is also a weak downtrend in the extraction force for the 8.180 inch OD band. The average decrease of extraction force per ram (slope of a linear regression) was about 6 lb/ram. This is substantially smaller than the unit decrease with the 8.170 inch OD band which was 32 lb/ram. Both linear regressions of extraction force versus ram sequence number are statistically significant at the 0.005 level or better although only 12% of the total sample variance is explained by the regression for the 8.180 inch OD band, indicating substantial variability about the downtrend.

10. The extraction force history over 30 rams for a cold (-25 deg F) projectile in a relatively warm tube (70 deg F) is given in Figure 4. An estimate of the probability distribution of extraction force using these data is given in Table 1. As with all probability estimates made here, the ordered data values are displayed together with their associated median ranks, the best unbiased, nonparametric estimate of the cumulative probability. It is evident from these data that the extraction forces using the 8.170 inch OD band at -25 deg F are quite low and may not give acceptable performance with the developmental fallback sensor. For this reason a somewhat larger band OD is preferable.

11. During the ramming tests of the training projectile with the 8.180 inch OD band the L/R system pressure was permitted, with one exceptional interval, to assume the value permitted by the normal operation of the hydraulic pump. Due to a nitrogen leak and the occurrence of random delays following restoration of full pressure there occurred various initial L/R system pressures. These are shown in Figure 7. Consequently, the sequence of L/R system pressures can be compared with the corresponding sequence of extraction pressures. These comparisons are shown in Figures 8 and 9. It is noteworthy that the extraction pressure (or force) is positively correlated with the L/R system pressure although the correlation is not large, namely, 37%. This result is based upon 67 data points with system pressures between 1650 to 2350 psi. The average increment in extraction force per unit increase in L/R system pressure is estimated to be 0.693 -- nearly 0.7 lb/psi. Thus, a change of 700 psi in system pressure, from one operating limit to the other, would change the average extraction force for the training projectile by 485 pounds, or about 15% of the mean for these tests.

12. Previous experience (Ref c) with extracting M106 projectiles in the M110A2 system led to the expectation that the probability distribution of extraction force using the training projectile would also be gaussian. However, as shown in Figure 10, this distribution exhibits quantization effects and, in fact is rejected as gaussian by a Lilliefors (modified Kolmogorov-Smirnov) test at a 5% risk level. In spite of this result, I adhere to the belief that the extraction force is essentially a gaussian random variable. I digress at this point to explain this position. During the tests I observed that the portion of the range of pressure indicated by the dial gage was quite small, that extraction pressure

readings tended to be multiples of 5 psi (a practical resolution interval), and that readings tended to cluster at the scribe marks indicated on the dial, which were multiples of 25 psi. Because of the unusually high frequency of these "favorite numbers", one may conjecture that the reader rounds dial readings at the "hash" marks whenever the pointer falls within the resolution interval of a mark. Elsewhere the reading is given moduluo the resolution interval. To test that this hypothesis would produce a distribution similar to that observed, I conducted a series of Monte-Carlo simulations. Briefly, my hypothesis was confirmed! The simulations involved generating gaussian random variables and subsequently transforming them, using the quantization procedure described above. A single numerical experiment consisted of drawing a sample of N from a distribution having mean (μ) = 1 and SD (σ) = 0.1 using a given resolution interval, then ordering the sample and performing the modified Kolomogorov-Smirnov (K-S) test at the 5% risk level. This experiment is repeated 30 times to estimate a probability of passing the K-S test of normality. This experimental set is, then, repeated for various values of the ratio of resolution interval, R, to standard deviation, σ, to generate an operating characteristic (OC) for the K-S test operating on this quantized gaussian distribution. The results of these experiments are shown in Figure 16 for N = 30 and 60. The 90% two-sided confidence intervals for points on the OC are indicated as well as the best Normal approximation. These approximations are:

$P_A(N)$ = Probability of accepting K-S (0.05) Test, given a sample size N;

$$P_A(30) \cong 1 - \Phi[(R/\sigma - 0.346)/0.098]$$

and

$$P_A(60) \cong 1 - \Phi[(R/\sigma - 0.220)/0.085],$$

where $\Phi[\ ]$ is the standard Normal integral.

13. The situation that generally existed with the pusher extractor indicated a resolution of about 5% of the mean and a standard deviation of about 10% of the mean. Thus, the resolution was typically one-half of a standard deviation. On this basis one can readily understand why the K-S test rejected the Normality hypothesis for the extraction data taken in these tests. Additional inferences from the Monte Carlo experiments are that the quantization does not affect the estimate of the mean and insignificantly underestimates the variance.

14. Returning to the main theme of the analysis of tests, I refer the reader to Tables 1 and 2. One of the objectives of the TPR was to compare the magnitude of the peak extraction force generated in pulling the projectile with that generated in pushing. One would not expect these forces to differ on the average if both forces acted along the same axis throughout

the extraction. However, as seen in the results of these tables, the force required to push out the projectile -- either the M106 training projectile or the M106 -- is 8 to 9% greater than the extraction force produced by a puller. Although the sample size of 8 in these tests is small, Monte Carlo simulation indicates that the observed result would occur by chance only 10% of the time if the mean extraction force by pulling actually was the same as the mean extraction force by pushing. Therefore, it is improbable that the average extraction force by pulling is the same as that by pushing.

15. During the ramming tests of the M106 training projectiles, ram-and-extraction experiments were carried out at the beginning and end of each experimental treatment using a previously rammed M106 projectile, which functioned as an experimental control. Thus, two sets of data were obtained: a large sample of data pertaining to the training projectile and a small sample pertaining to the M106. Both fallback sensor output* and extraction pressure were recorded. By crossplotting fallback sensor output and extraction pressure data for a subsample of the 8.180 inch OD training projectile ramming tests, one obtains the result shown in Figure 11. Two separate clusters of points identify the two different projectiles. The numbers placed at various grid locations indicate the frequency of occurrence of an experimental result located there. A straight line is drawn between the centroid of the cluster belonging to the training projectile and that belonging to the M106. Whereas there is considerable dispersion orthogonal to this line, the greatest variation occurs along it, supporting the validity of this linear relationship. Note that the line passes through the origin indicating a direct proportionality between fallback sensor output and extraction pressure.

16. Cold (-25 deg F) Temperature Tests

Thirty M106 projectiles were temperature conditioned to -25 deg F in the LMPEC along with the weapon. Fifteen of these were used initially in ramming tests with the L/R MOD configuration and the balance were subsequently used with the L/R STD configuration. As anecdotally described above, the L/R MOD configuration had to be modified to commence testing in this environment. However, nearly all components of this system were tested. The values of extraction pressure and of fallback sensor output

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* For this experiment the strain gage transducers in the fallback sensor were glued directly to the tube with a circumferential pair at the origin of rifling and an axial pair located 5 inches muzzleward. Excitation for the Wheatstone bridge was $\pm$ 5 volts with a DC amplifier gain of 200.

for both configurations are plotted versus ram sequence number in Figure 12. Interestingly the L/R MOD exhibited nonstationary stochastic properties. A gradual decline in extraction pressure was evident throughout the 15-ram test. However, the fallback sensor output did not mimic this decline. In fact, this output rose during both the tests of L/R MOD and of the L/R STD. I have no explanation of this phenomenon.

17. In the tests of the L/R STD configuration, the extraction pressure is stochastically stationary (Fig 12) and nearly gaussian as seen in Figure 13. Using the baseline extraction data obtained with the same tube (No. 9710) at about 90 deg F (Ref c), one can determine the effect of environmental temperature on extraction force. The estimated distributions of extraction force for the M106 projectile at these two environmental temperatures are shown in Figure 14. The ranked data and marginal statistics are tabulated in Table 4. The distribution of extraction force in a cold environment is dramatically shifted to larger values relative to that at ambient temperature. Note that the mean increases by 53%. Incidentally, note that L/R STD produces a slightly larger mean force than does L/R MOD -- 11,148 versus 10,403 pounds. Therefore, the effect of cold temperatures -- uniformly felt by both the weapon and projectile -- would actually <u>reduce</u> the likelihood of fallback in a properly maintained* system. To assess the effect of projectile temperature when ramming into a cold tube, a single M106 projectile, which had been rammed once, was removed from the LMPEC and warmed to approximately 60 deg F for a final ramming test. The extraction pressure for this projectile was 425 psi, 7.24 standard deviations <u>below</u> the mean of 882 psi for cold projectiles in a cold tube. The corresponding extraction force for the last ram, 5460 lb, suggests that warm projectiles rammed in a cold weapon produce even lower extraction forces than warm projectiles in a warm system.

18. Although the data in Figure 12 do not seem to indicate a correlation between fallback sensor output and extraction force for the cold-environment tests, the crossplot of these variables, shown in Figure 15, indicates a monotonic relationship when data for both new M106 and previously rammed M106 (check) projectiles are included.

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* During the cold environment tests it was found to be necessary to frequently replace nitrogen lost from the accumulator. The vehicle (no. 12FJ05) used in these tests had previously been used in other firing programs at YPG. Maintenance personnel indicated that a slow leak was present but could not be located within the time permitted for test setup. This leak worsened considerably when the temperature was reduced to -25 deg F. In fact, during the tests in the LMPEC the pressure loss rate averaged about 12%/hr. Although this condition was tolerable at YPG, given the convenient logistics, a similar condition would create severe equipment support problems in a field setting.

19. Summary

Ramming and extraction tests were conducted on M106 training projectiles having nylon rotating bands to determine the adequacy and endurance of this type of band and to identify an optimum design. Additionally, the M110A2 loader-rammer was environmentally tested at -25 deg F in two configurations: L/R STD and L/R MOD. With both configurations, a fallback sensor was mounted on the gun tube to provide a relative measure of tube strain following a projectile ram. The conclusions derived from these two tests are summarized below.

20. Using black nylon "supertuf" rotating bands at ambient temperature, one observes a gradual decrease in the expected extraction force as a function of ram sequence number. The magnitude of the declining trend is dependent upon the max outside diameter (OD) of the band. Although substantial variability occurs about the trend, a band having a 8.180 inch OD appears to give higher and more consistent extraction forces. A band of this dimension using the optimum "supertuf" material is expected to have a life of (100 rams per band edge X 2 edges) 200 rams. It is noteworthy that the extraction force is positively correlated with the L/R system pressure, although the correlation coefficient of 37% is not particularly large for these tests. Average fallback sensor output was found to be proportional to average extraction force.

21. A comparison of the forces required to extract projectiles by alternative means -- pulling or pushing -- indicates that the expected force exerted by a pusher exceeds that for a puller by 8 to 9%. This result is based upon tests with two types of projectiles: (1) the M106 training projectile with a "supertuf" nylon rotating band having a 8.170 inch OD, and (2) the standard M106 projectile subjected to multiple rams.

22. The cold (-25 deg F) environment tests of the loader-rammer shows that the L/R MOD, as tested, is unsatisfactory. However, the L/R STD configuration produces a distribution of extraction force with cold M106 projectiles which exceeds (at any percentile) that produced by that system at 90 deg F. Therefore, actually a superior retention of this projectile occurs in a colder environment even though rammer speed decreases. Given proper maintenance and operation, the loader-rammer of the M110A2 is less likely to produce an inadequate ram at a low temperature than at ambient temperature.

George Schlenker

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Operations Research Analyst

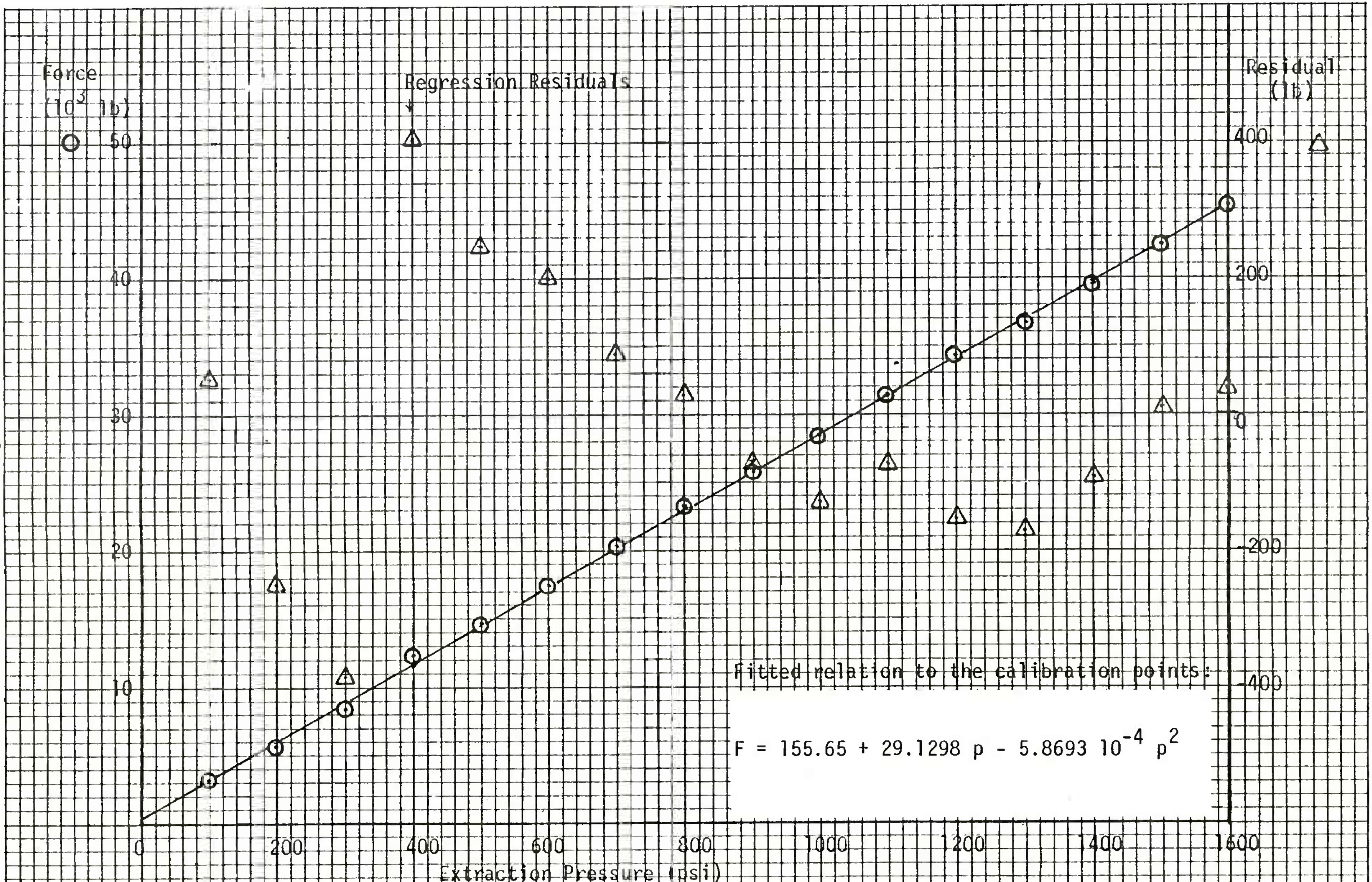

Figure 1. Calibration Relation of Extraction Force to Extraction Pressure for the Forward Pusher Extractor (Gage No. 009-15)

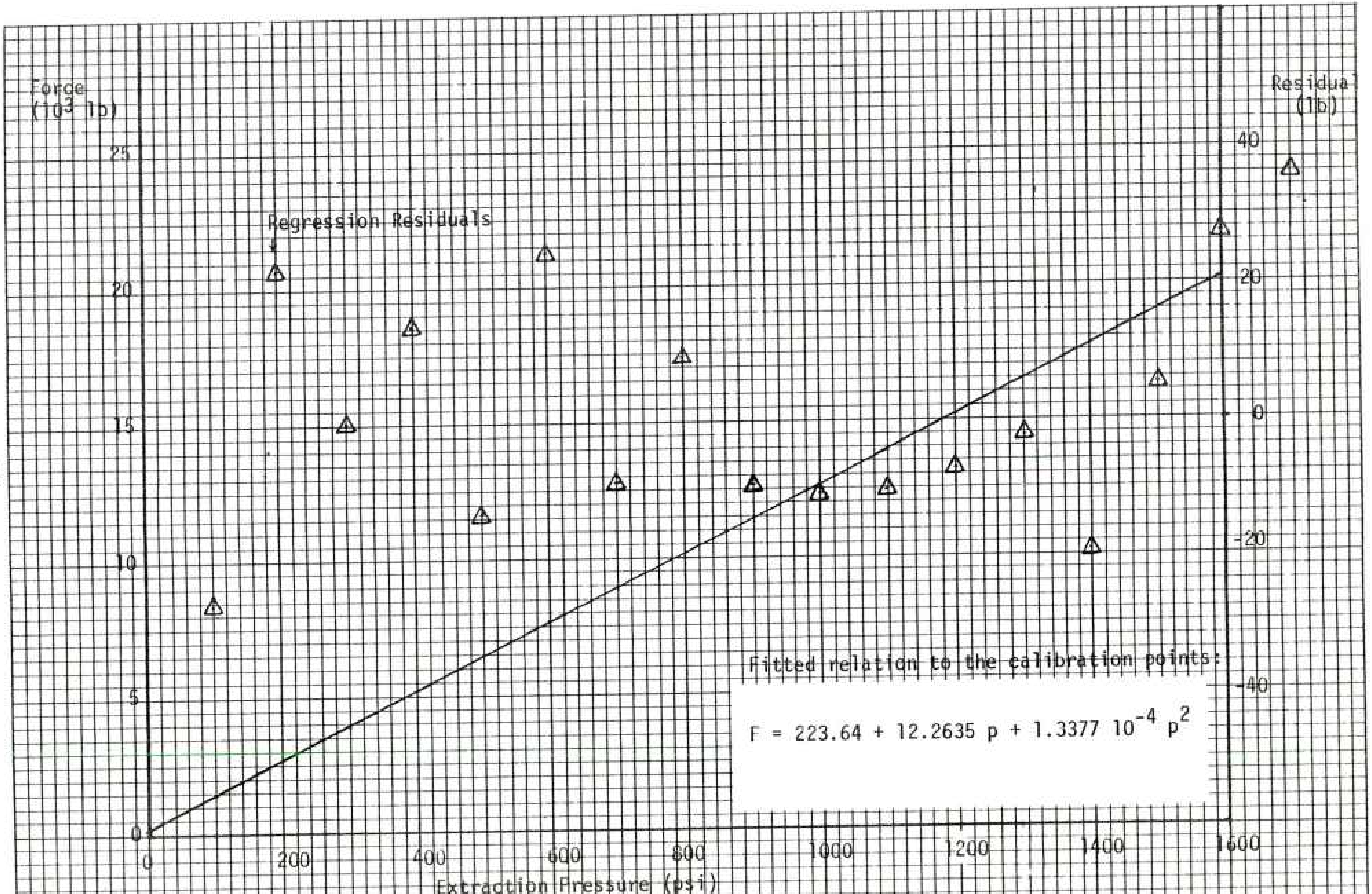

Figure 2. Calibration Relation of Extraction Force to Extraction Pressure for the XM753 Puller Extractor (Gage No. 009-12)

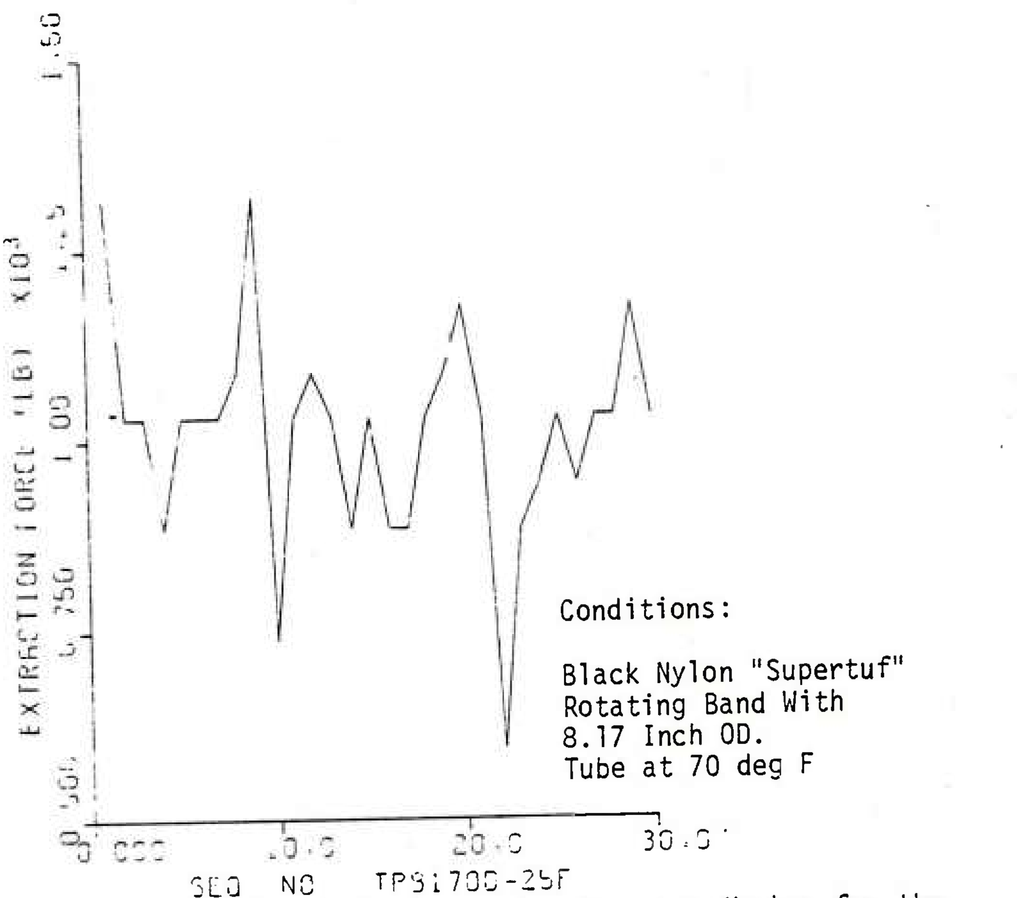

Fig 4. Extraction Force Versus Ram Sequence Number for the Training Projectile Temperature-Conditioned to -25 deg F

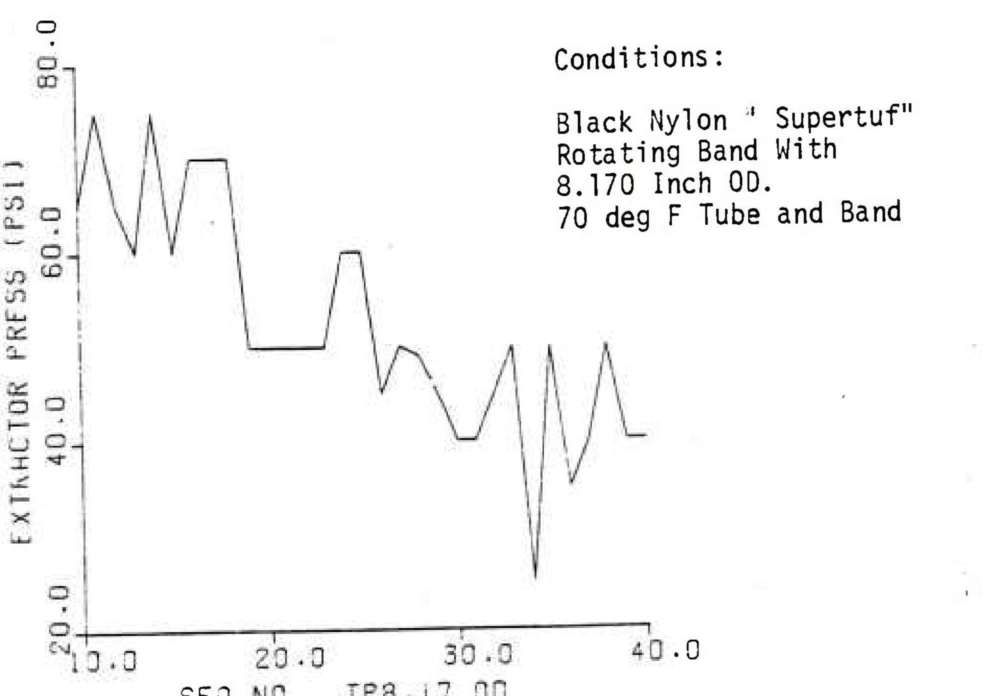

Fig 3. Pusher Extractor Pressure Versus Ram Sequence Number for the Training Projectile

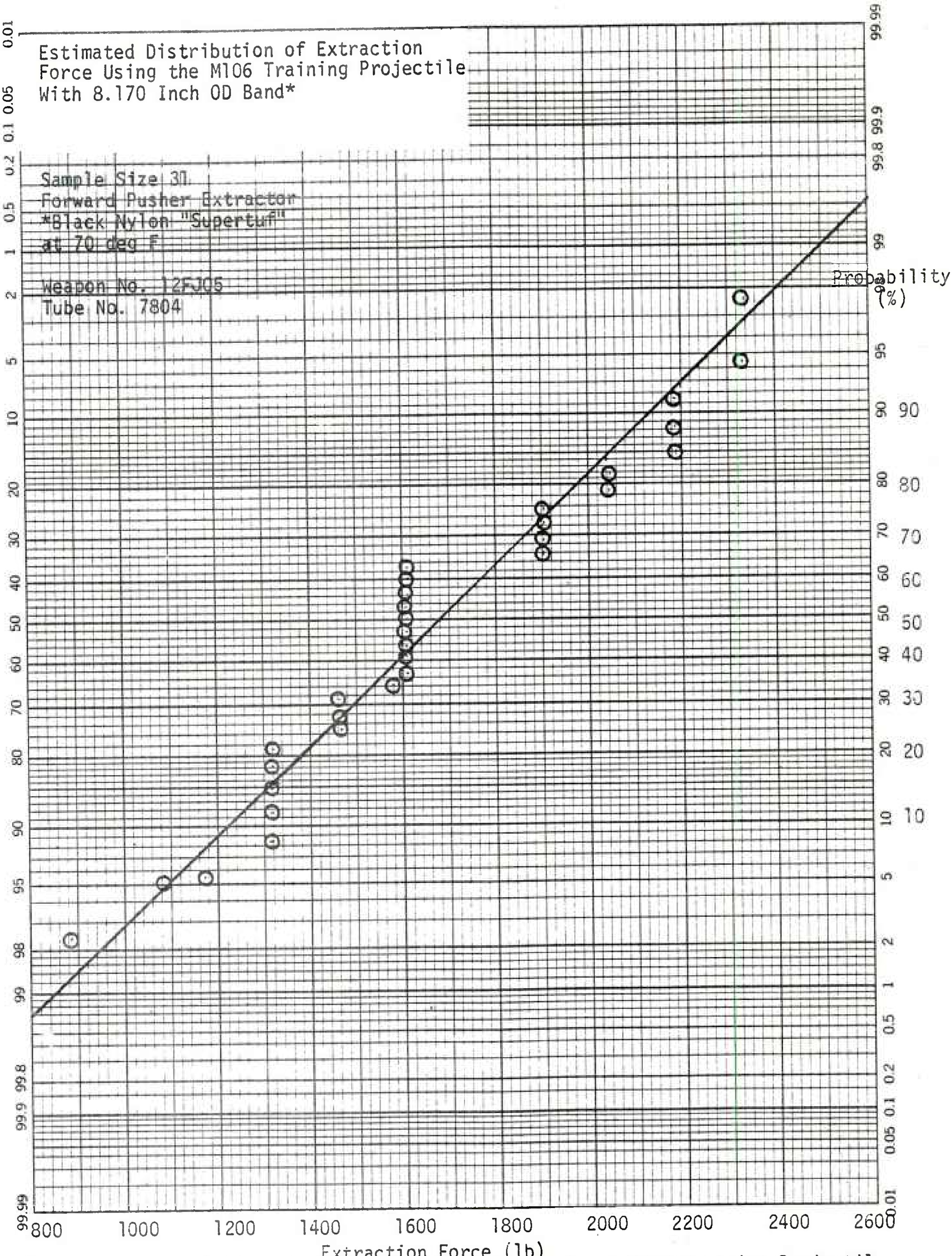

Figure 5. Probability Distribution of Extraction Force With Training Projectile

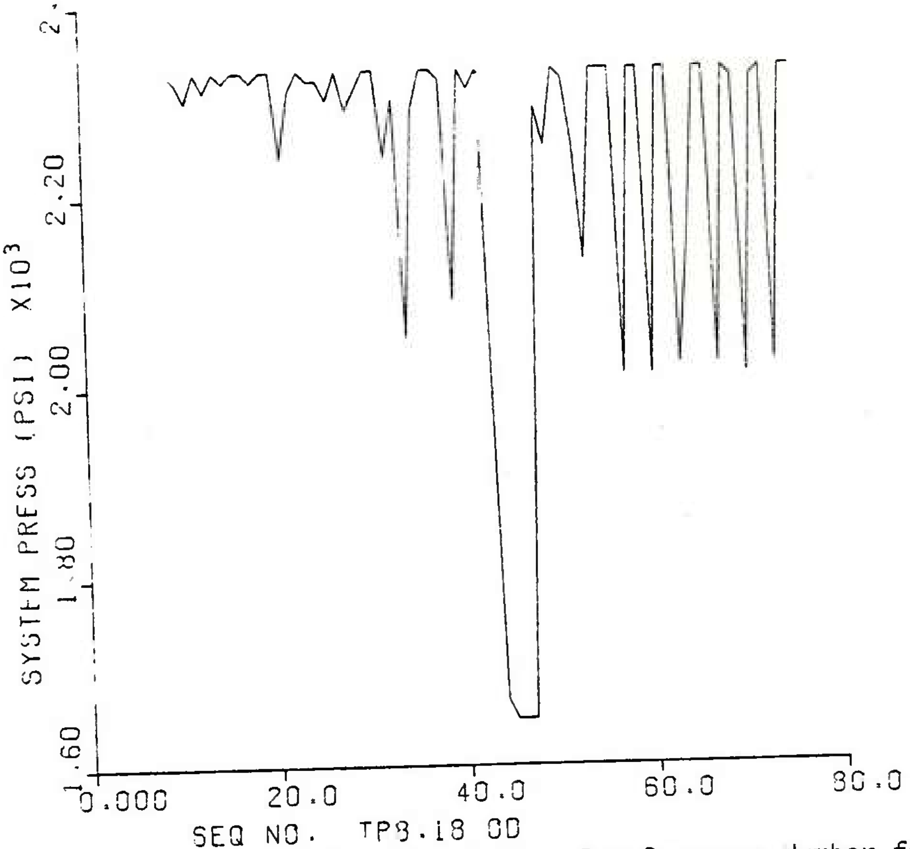

Fig 7. L/R System Pressure Versus Ram Sequence Number for Tests of the Training Projectile Having 8.180 inch OD

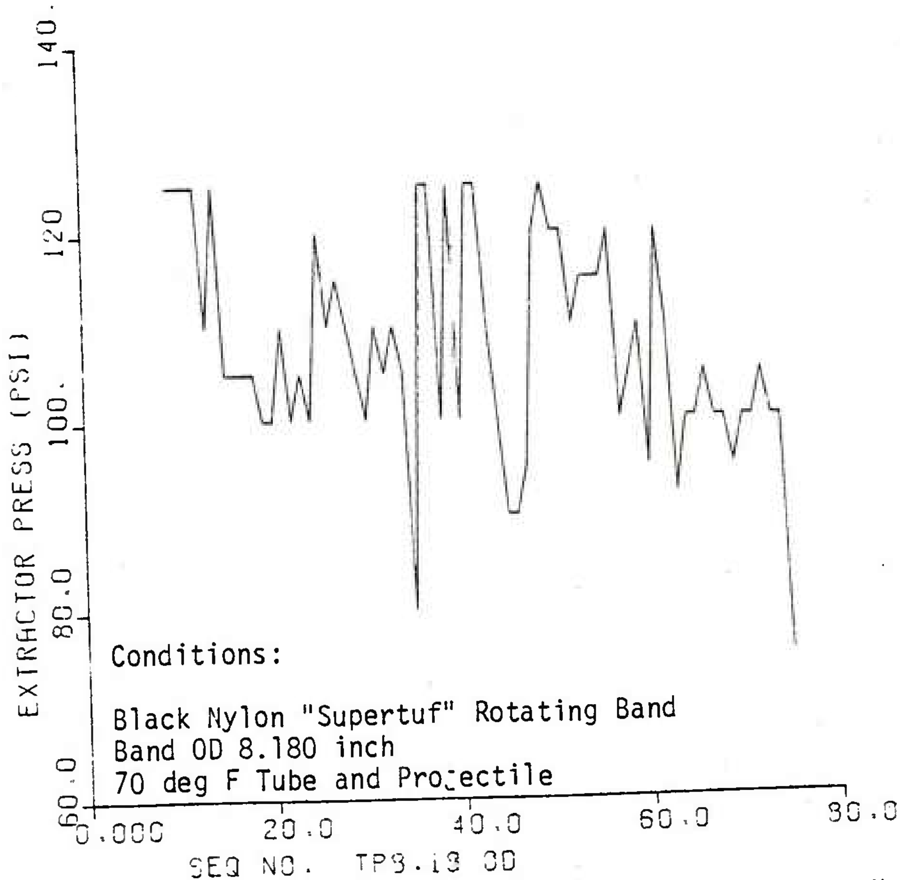

Fig 6. Pusher Extractor Pressure Versus Ram Sequence Number for the Training Projectile

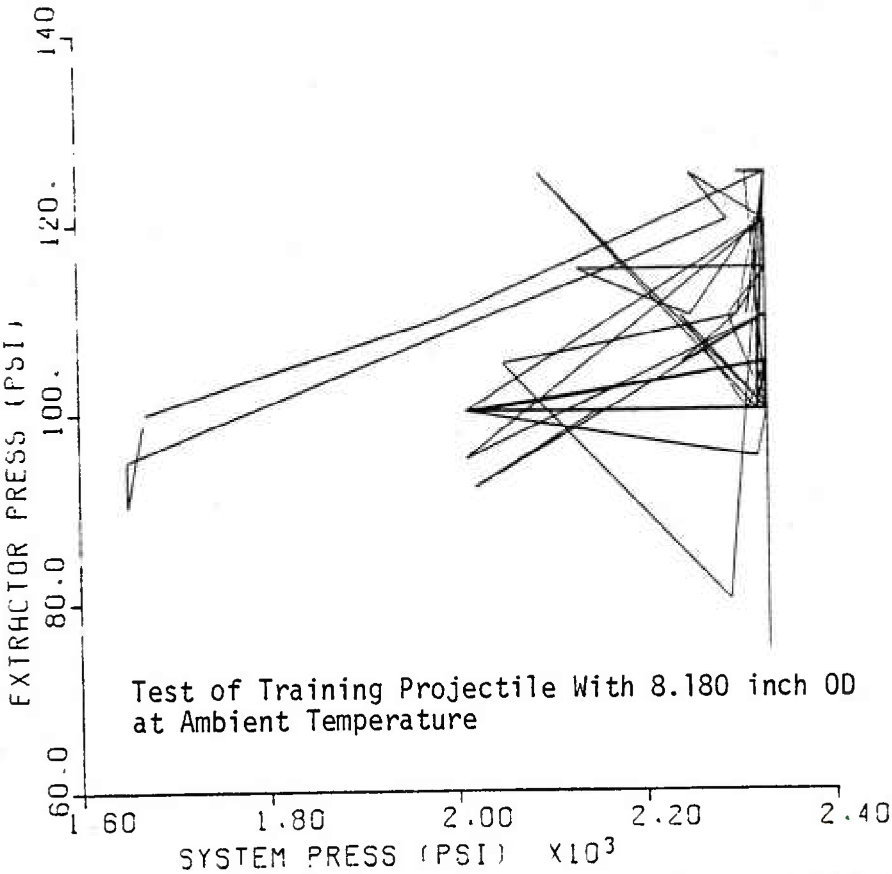

Fig 9. Crossplot of Pusher Extractor Pressure and L/R System Pressure

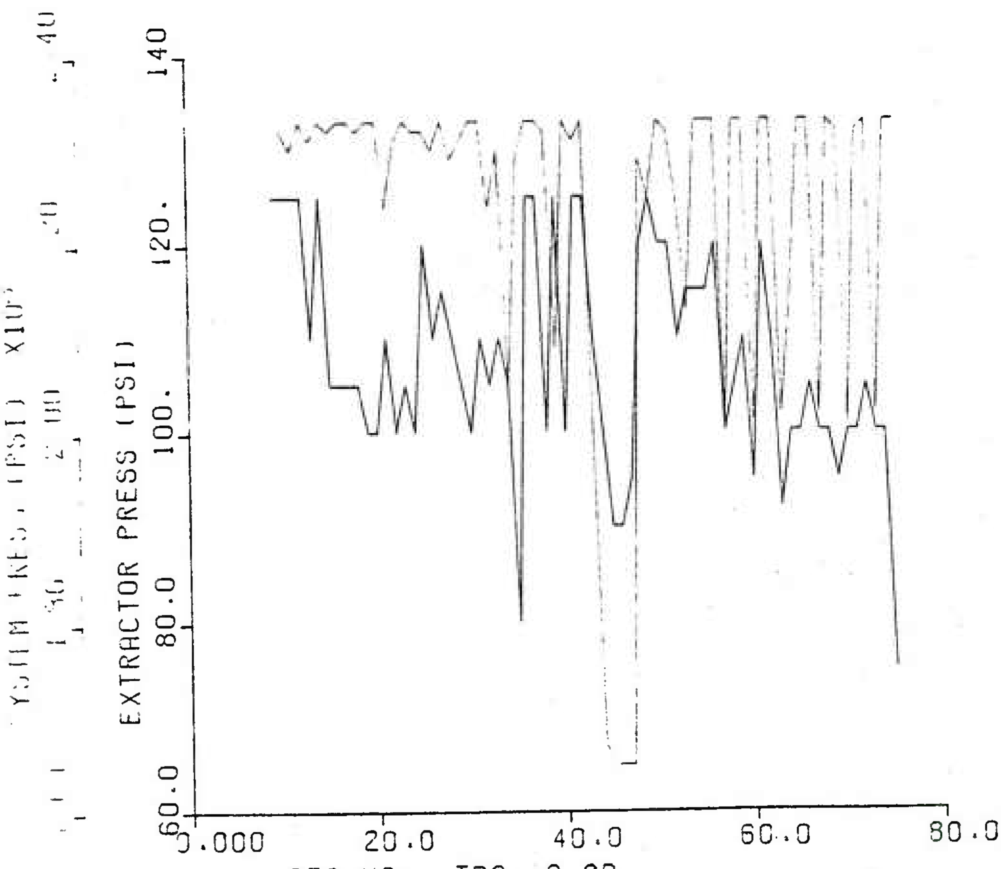

Fig 8. Comparison of L/R System Pressure and Extractor Pressure Sequences

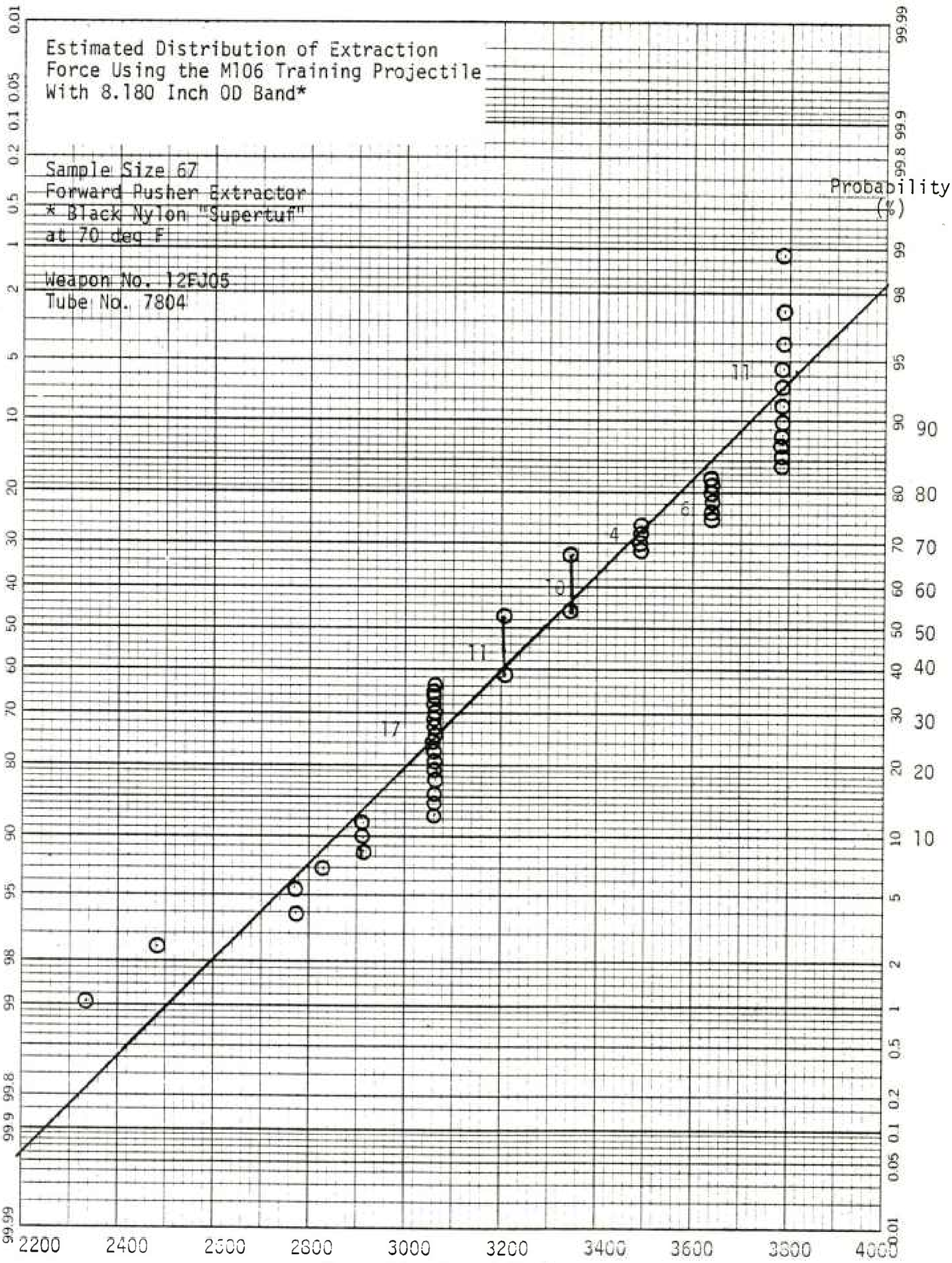

Figure 10. Probability Distribution of Extraction Force With Training Projectile

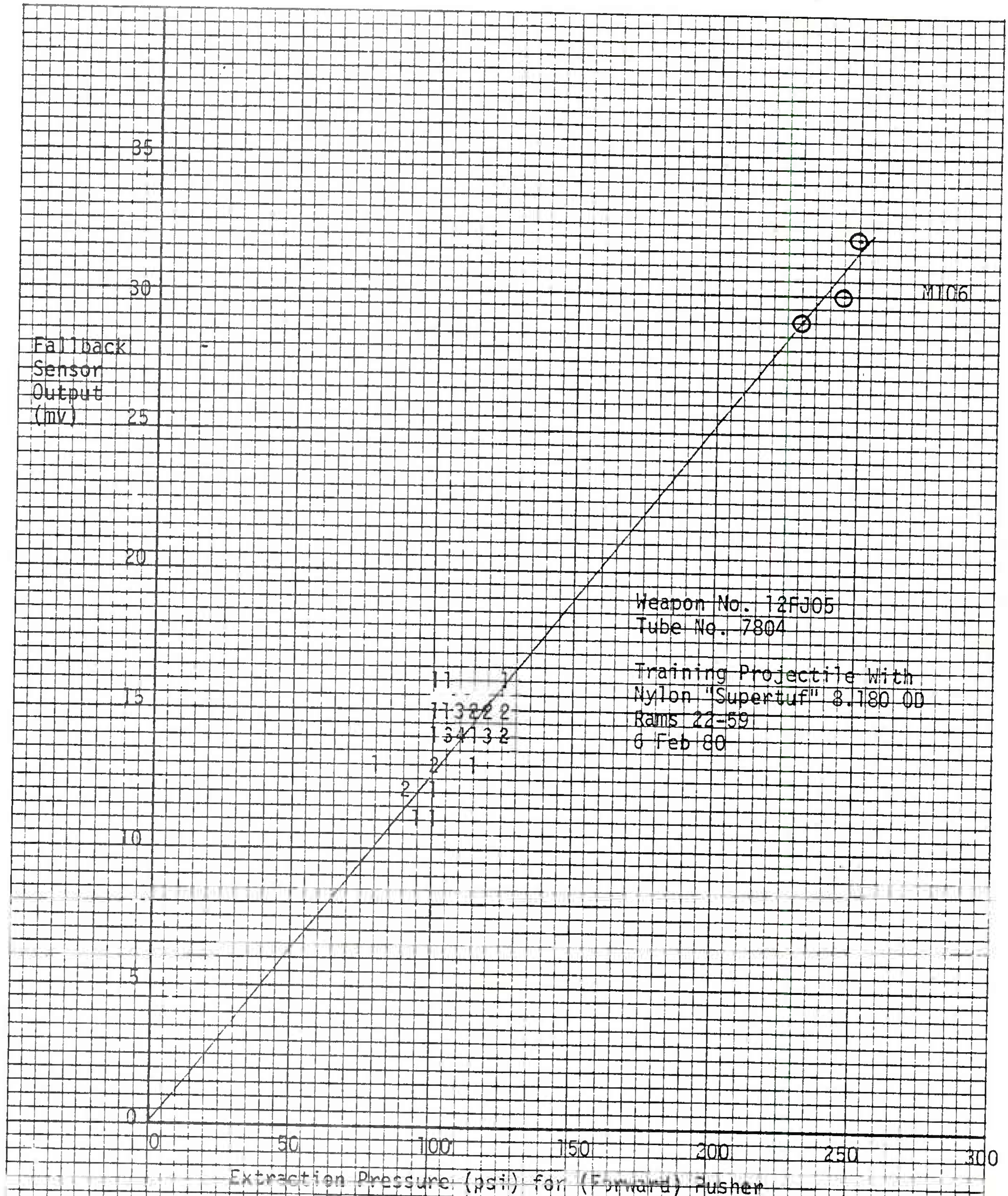

Figure 11. Relationship of Fallback Sensor Output to Extraction Pressure Using Two Projectiles Each Rammed Several Times

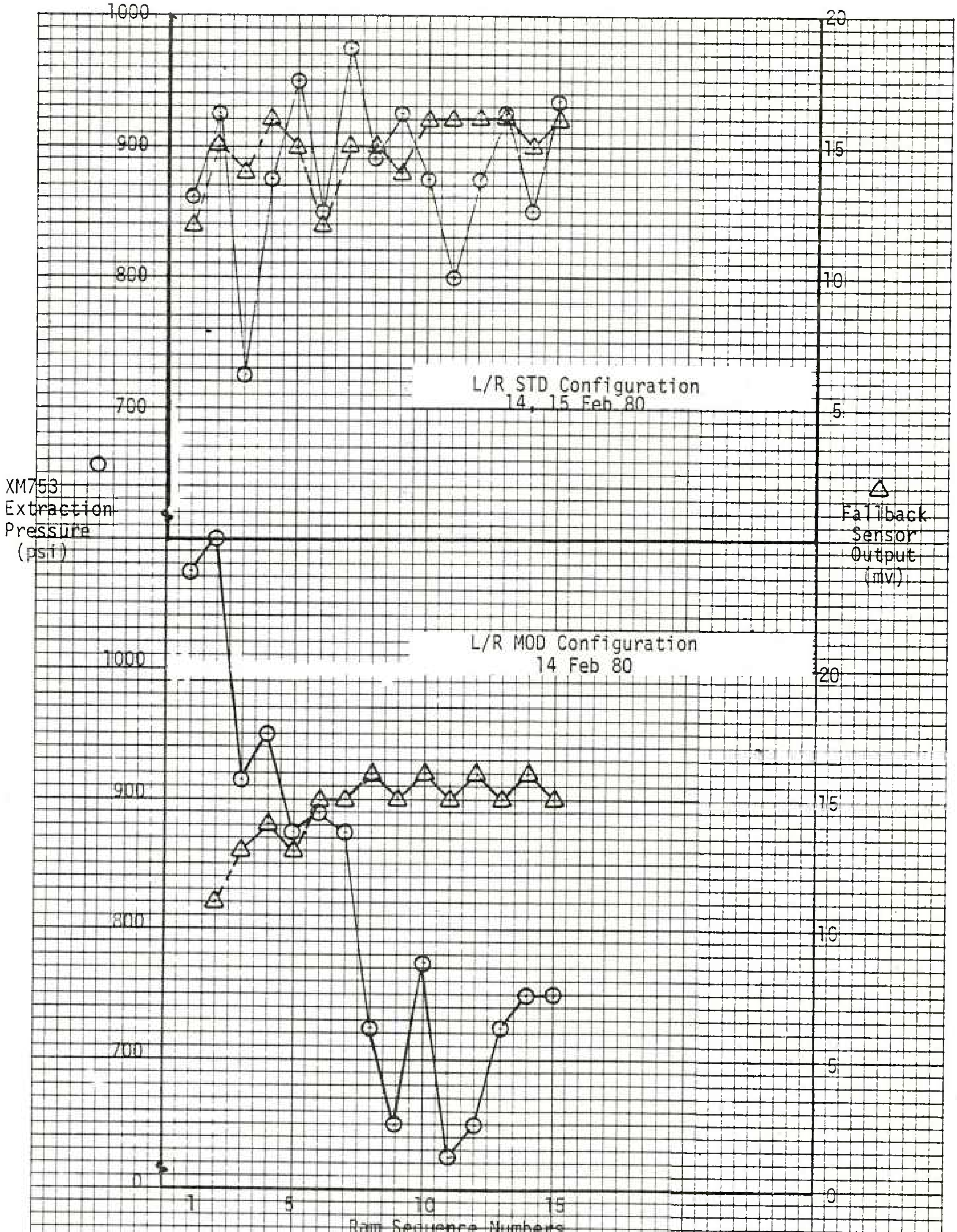

Figure 12. Comparison of Extraction Pressure and Fallback Sensor Output During Extraction of M106 Projectiles in a -25 deg F Environment

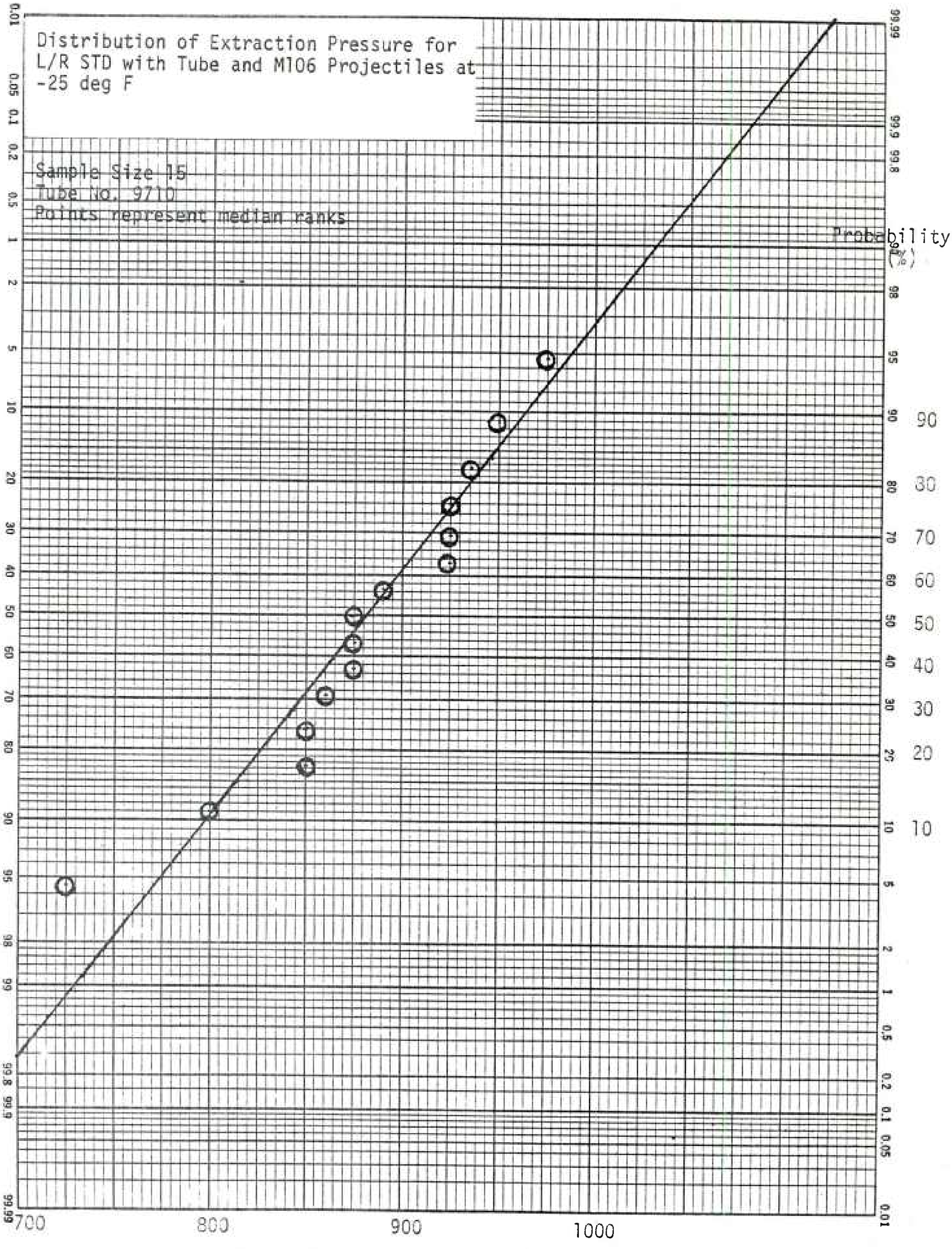

Figure 13. Distribution of Extraction Pressure in a Cold Environment

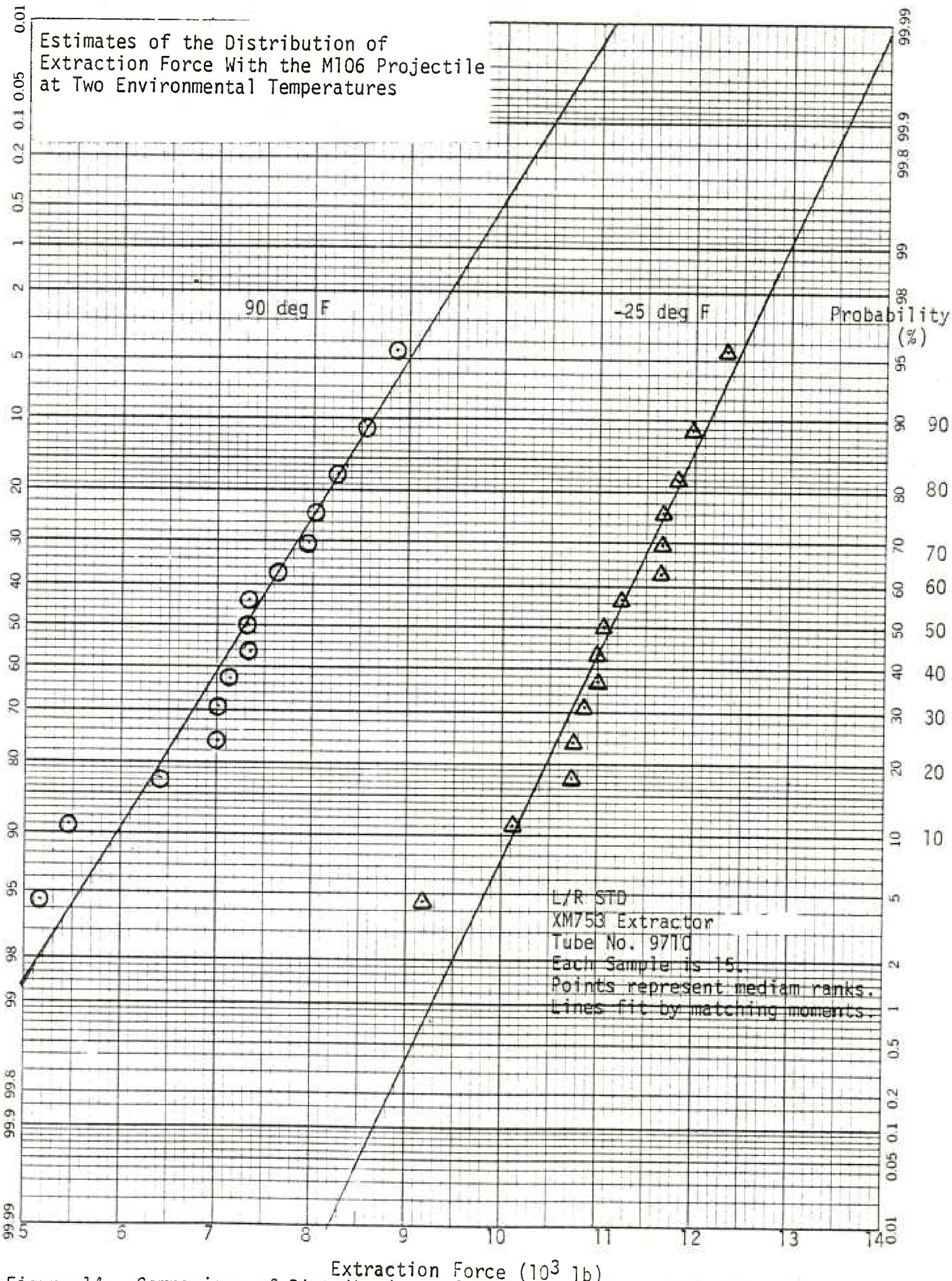

Figure 14. Comparison of Distributions of Extraction Force at Two Temperatures

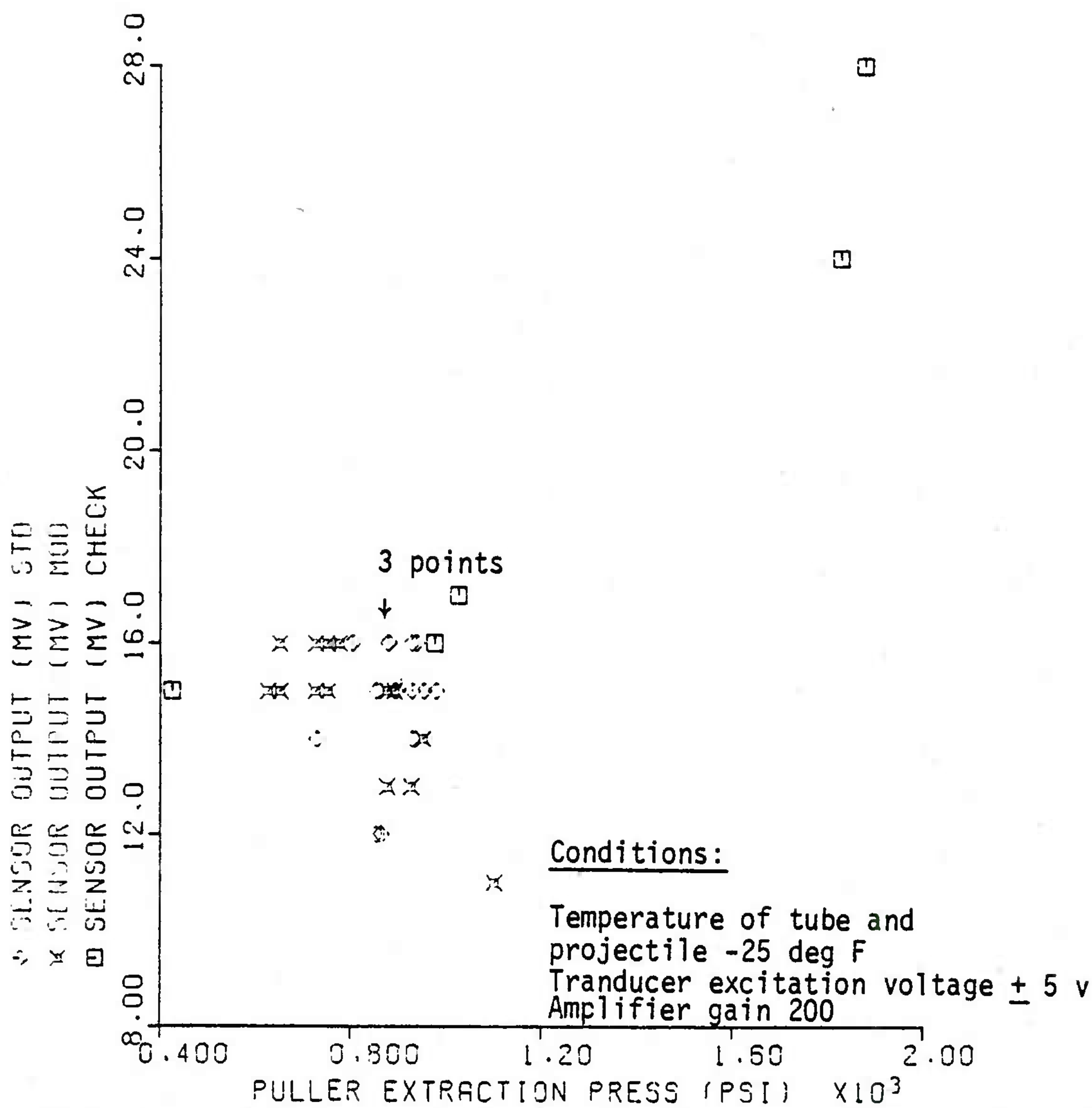

Fig 15. Fallback Sensor Output Versus Extraction Pressure with M106 Projectiles

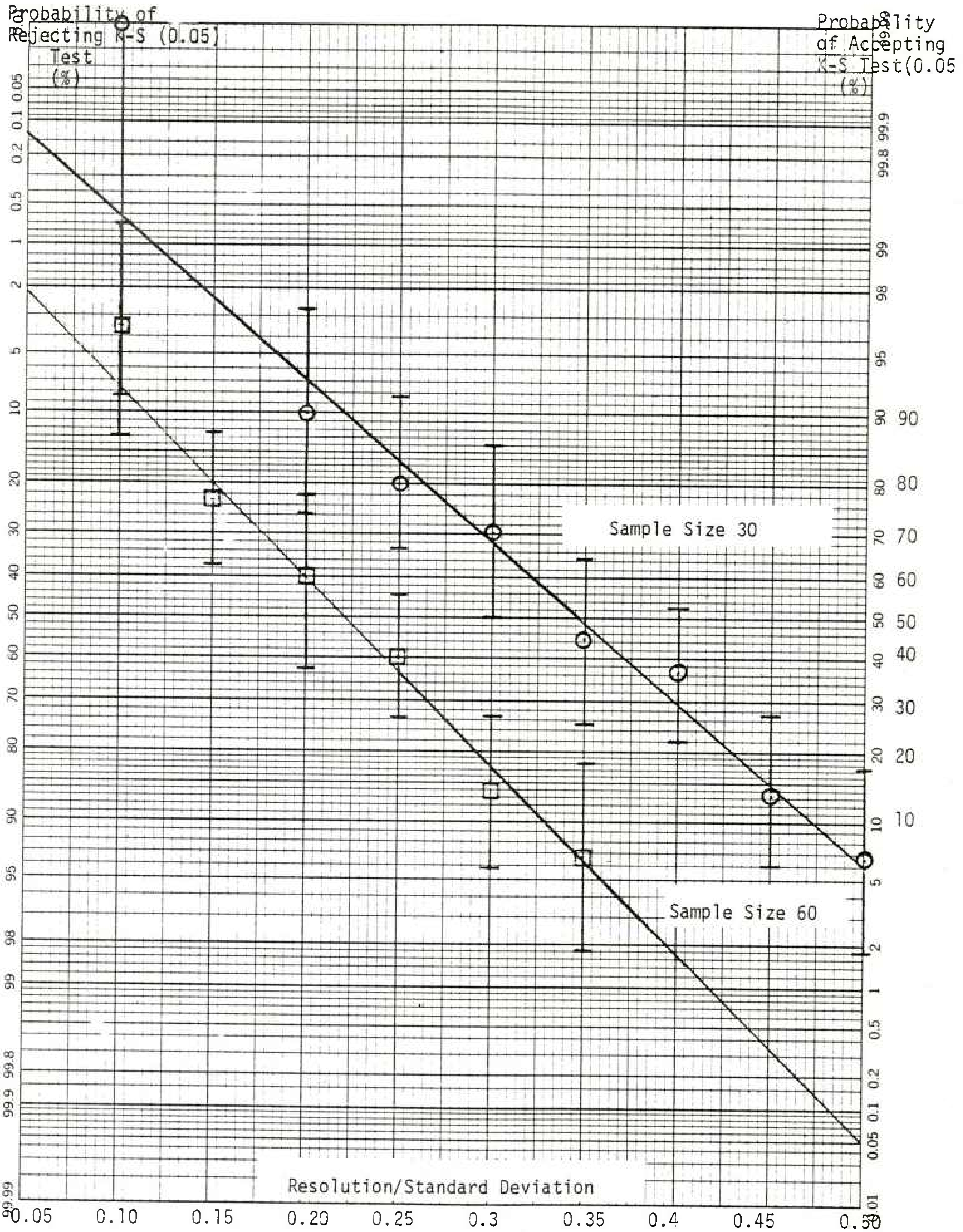

Fig. 16. Operating Characteristic for K-S Test of a Quantized Normal Distribution

TABLE 1

ESTIMATED PROBABILITY DISTRIBUTION OF EXTRACTION FORCE FOR THE M106 TRAINING PROJECTILE HAVING 8.170 INCH OD ROTATING BAND AT A PROJECTILE TEMPERATURE OF -25 DEG F

Rank Order Number	Extraction Force (lb)	Median (Probability) Rank
1	592	0.023
2	738	0.056
3	883	0.088
4	883	0.122
5	883	0.155
6	883	0.188
7	883	0.220
8	942	0.253
9	942	0.286
10	1029	0.319
11	1029	0.352
12	1029	0.385
13	1029	0.418
14	1029	0.450
15	1029	0.484
16	1029	0.516
17	1029	0.549
18	1029	0.582
19	1029	0.615
20	1029	0.648
21	1029	0.681
22	1029	0.714
23	1029	0.747
24	1087	0.780
25	1087	0.813
26	1087	0.845
27	1174	0.878
28	1174	0.911
29	1320	0.944
30	1320	0.977
Mean	1010	
Standard Dev	145	

TABLE 2

COMPARISON OF EXTRACTION FORCE FOR THE M106 TRAINING PROJECTILE[1] USING ALTERNATIVE MEANS[2] OF EXTRACTION

Ram Sequence Number	Extractor Type[2]	Extr. Press. (psi)	Extraction Force[3] (lbf)
1	A	125	1759
2	B	50	1611
3	B	65	2047
4	A	130	1820
5	B	65	2047
6	A	125	1759
7	A	125	1759
8	B	65	2047

(1) With black nylon "supertuf" (c) rotating band having an 8.170 inch OD. Tests were performed 4 Feb 80 at 70 deg F (approx).

(2) Notationally,

A => XM 753 puller extractor
B => forward pusher extractor

(3) Calibrations used:

$F = 223.64 + 12.2635\ p + 1.3377\ 10^{-4}\ p^2$ for the XM753 puller extractor (A), and
$F = 155.65 + 29.1298\ p - 5.8693\ 10^{-4}\ p^2$ for the forward pusher extractor (B).

Average force for type A = F_A = 1774 lbf

Average force for type B = F_A = 1938 lbf

$F_B/F_A = 1.09$

TABLE 3

COMPARISON OF EXTRACTION FORCE FOR THE M106 PROJECTILE USING ALTERNATIVE MEANS OF EXTRACTION

Test Date	Extractor Type(1)	L/R System Pressure (psi)	Extr. Press. (psi)	Adjusted Extr. Force(2) (lbf)
4 Feb 80	A	2040	250	3434
	B	2250	150	4512
6 Feb 80	B	2050	250	7690
	A	2320	625	7829

(1) Notationally,

A => XM753 puller extractor
B => forward pusher extractor
F_A= average force for type A

F_B= average force for type B

(2) Extraction forces are adjusted to account for the effect of L/R system pressure. A standard L/R system pressure of 2250 psi is adopted. Forces are adjusted in proportion to deviations from standard using the correction factor 0.021%/psi.

For the above data $F_B/F_A = 1.08$.

TABLE 4

ESTIMATE OF THE DISTRIBUTION OF EXTRACTION FORCE FOR THE M106 PROJECTILE AT AMBIENT TEMPERATURE (90 DEG F) AND AT -25 DEG F

Rank Order	Extr. * Press @90°F (psi)	Force(1) @ 90°F (lbf)	Extr. * Press @-25°F (psi)	Force(2) @ -25°F (lbf)	Median Rank
1	450	5169	725	9185	0.0452
2	475	5478	800	10120	0.1101
3	550	6406	850	10744	0.1751
4	600	7025	850	10744	0.2401
5	600	7025	860	10869	0.3051
6	610	7149	875	11056	0.3700
7	625	7334	875	11056	0.4350
8	625	7334	875	11056	0.5000
9	625	7334	890	11244	0.5650
10	650	7644	925	11682	0.6300
11	675	7953	925	11682	0.6949
12	680	8015	925	11682	0.7599
13	700	8262	935	11807	0.8249
14	725	8572	950	11995	0.8899
15	750	8881	975	12308	0.9548
Mean		7305		11149	
SD		1033		788	
90% CI		(6835, 7775)		(10790, 11508)	

* Pressures were obtained on different gages. Ambient temperature data were obtained at YPG, Jul 79; and -25 deg F data were obtained in the LMPEC, YPG, Feb 80.

(1) $F = -400 + 12.375\ p$

(2) $F = 223.6 + 12.264\ p + 1.3377\ 10^{-4}\ p^2$

Ratio of average forces is 1.526.

Memorandum for Record

INDEPENDENT EVALUATION OF M106 TRAINING PROJECTILE

George J. Schlenker

DRSAR-PEL 6 March 1980

MEMORANDUM FOR RECORD

SUBJECT: Independent Evaluation of M106 Training Projectile

1. Reference:

a. FONECON between Mr. Stan Smith, DRSAR-HA, HQ, ARRCOM and Mr. George Schlenker, DRSAR-PEL, HQ, ARRCOM, 3 Mar 80, SAB.

b. Loader-Rammer Annex to TPR 2-MU-003-106-026, DRCPM-M110E1, July 1979.

2. The following observations are given in response to the Ref a FONECON requesting subject evaluation. Supporting detailed analyses will be provided later in a memorandum report. The conclusions given here are based upon the results of tests of the training projectile, prescribed in Ref b, and carried out at YPG over the period 28 Jan - 6 Feb 80 with Mr. Schlenker as the ARRCOM Test Advisor. The principal conclusion of our evaluation is that the training projectile having the optimum plastic rotating band is expected to perform satisfactorily over a useful band life of 200 rams.

3. Several replaceable plastic rotating bands were tested with the training projectile. The ones found to give satisfactory performance were made of carbon-filled (black) "supertuf" (c) nylon having outside diameters (OD) in the range 8.170 to 8.180 inches.

4. A band having an 8.170 inch OD was tested at 70 deg F by ramming and extracting 40 times. Another band of this dimension was tested at -25 deg F in a 70 deg F tube for 30 extractions. At both band temperatures, the bands exhibited good abrasion resistance, satisfactory ductility, and adequate extraction force. A band having an 8.180 inch OD was given an endurance test by ramming and extracting 75 times at 70 deg F. No unusual damage was observed on the leading edge of the band and extraction forces remained within an acceptable range throughout.

5. One can infer from this test that this type of band is capable of sustaining at least 100 rams before removing and reversing the band. In the judgment of the ARRCOM Test Advisor, a 200 round band life is possible using both edges of the band, provided reasonable care is given in handling the projectile and in aligning the rammer tray and trough. If the tray and trough are misaligned, the rear or trailing edge of the band

may catch at the junction of tray and trough during extraction and suffer damage by nicking.

6. With repeated ramming of these projectiles, the extraction force exhibited a slight downward trend. For example, with the 8.180 inch OD band, the best linear fit to the extraction force versus ram sequence number, when evaluated at 100 rams, gives an expected force of 2949 pounds. This evaluation constitutes an acceptable extrapolation beyond the test range: 75 to 100 rams. Allowing two standard errors below the mean as a margin of safety yields 2345 pounds which is well above the 1000 pound threshold set to indicate the occurrence of an inadequate ram. However, with the 8.170 inch OD band, the extraction force at 100 rams is likely to be marginally low. A large extrapolation to 100 rams, based on the 40 tested, indicates an expected extraction force of 1033 pounds.

7. In terms of moisture absorption by the "supertuf" nylon, the limited test experience fails to indicate a problem. A 80 hour soak in 70 deg F water of the 8.180 inch OD band showed no measurable increase in band dimensions.

8. In summary, it is concluded that the training projectile with the 8.180 inch OD band should perform satisfactorily during its lifetime of 100 rams per band edge.

GEORGE SCHLENKER
Operations Research Analyst

Memorandum for Record

LOGISTIC IMPLICATIONS OF THE DISTRIBUTION AND TYPE OF PROPELLING CHARGES FIRED IN 155MM ARTILLERY

George J. Schlenker

MEMORANDUM FOR RECORD

SUBJECT: Logistic Implications of the Distribution and Type of Propelling Charges Fired in 155mm Artillery

1. Reference:

References are indicated throughout this memorandum by square bracketed numbers. These citations are tabulated at Attachment 1.

2. Background

It is evident that the quantity of spare parts required as a war reserve (and supplied during wartime) depends upon the expected (ultimately, actual) manner in which a weapon system is used in combat. For example, the requirement for spare artillery tubes depends upon the frequency with which each propelling charge zone is fired. The relative frequency, or probability density function, depends, in turn, upon the gun-to-target (GT) range distribution experienced. Additionally, if there are alternative propelling charges which can be used to reach the same range, the decision as to which to use will, in general, affect the requirement (and cost) of tube spares.

3. Because of incentives (pressures) to develop alternatives to some of the present 155mm propelling charges, there is considerable interest within the armaments community concerning the logistic impact of substituting one charge for another. One quantitative aspect of this impact is the change in expected tube life due to the introduction of a new propelling charge. This expected life, in a statistical sense, is a function of the scenario-dependent range distribution which affects the distribution of firing zones. This memorandum will examine two distinct approaches to determine the distribution of firing zones of 155mm artillery. Results of these approaches will be compared. The logistic implications of a parametric change in this distribution will be determined. Additionally, the logistic and economic consequences of substituting one type of charge for another will be calculated.

4. Probability Distribution of 155mm Artillery Ammunition

According to CAA and DCS Ops. [1], the anticipated distribution of 155mm artillery usage in a European war scenario by type of projectile and by propelling charge is given in Table 1. For absolute (classified) values consult [1]. These results, while considered correct, are provisional.*

[1] Page 2-32, Concepts Analysis Agency, AMMO P-86/WARF P-86, February 1980.

* FONECON between Major J. Green (DAMO-RQR) and G. Schlenker (DRSAR-PEL).

TABLE 1A. FORECASTED DISTRIBUTION OF 155MM ARTILLERY AMMUNITION

Projectile Type	Fraction of Total Usage
M107/XM795	0.396
M483	0.601
M549	0.003

TABLE 1B. FORECASTED DISTRIBUTION OF PROPELLING CHARGES FOR 155MM ARTILLERY AMMUNITION

Tabular entries are values of fractional usage of charge types, given the type of projectile.

Projectile Type	Charge Type			Fract. Proj. Usage
	Low Zones*	M119	M203	
M107/XM795	0.29	0.53	0.18	0.396
M483	0.31	0.47	0.22	0.601
M549	0.00	0.25	0.75	0.003
Overall Average	0.30	0.49	0.21	1.000

* Either M3A1 and M4A2 or XM211, spanning zones 1 through 7.

The overall average distribution of charges used in a European-type scenario, given in Table 1, is, of course, a rough estimate, derived by simulated firings against a postulated target array. However, using WW II and Korean War artillery experience as precedent and by taking account of the change in weapon systems which has occurred, I will show that the above estimates are quite reasonable. Further, I will provide a detailed estimate of the distribution of firings by zone. This distribution will also provide a baseline for sensitivity analysis.

5. One of the interesting results of the study of artillery usage in Europe and Korea is the similarity in the form of the probability distributions which characterize the gun-to-target (GT) range for both DS and GS artillery. Both types of artillery systems have GT range distributions which are well approximated by a Weibull distribution with nearly the same shape parameter β. This

distribution has the form

$$F(R) = 1 - \exp[- (R/\eta)^{\beta}] \quad ,$$

where

β is the shape parameter having a value of 4.3 to 5 and where

η is the scale parameter, having the value of 70 to 72% of the maximum range of the most commonly used weapon system. Thus, as noted most recently in [2], the value of η for the 155mm GS artillery used in WW II is 10.8 km.

Assume that a similar probability law will characterize the distribution of GT range of 155mm artillery systems in the future, with η selected on the basis of maximum range at zone 8 in the M109A1 SP howitzer. This establishes a particular Weibull distribution for range with $\beta = 5$ and $\eta = 13.6$ km. To convert this range distribution to a distribution associated with propelling charge zones, one employs the known range characteristics of each firing zone and the operational rule-of-thumb: switch to the next higher zone at 75 to 80% of max range. I employed this rule and the above range distribution, obtaining the results shown in Table 2.

TABLE 2. ESTIMATED DISTRIBUTION OF PROPELLING CHARGES BASED ON GUN-TARGET RANGE ESTIMATES

Zone i	Range Interval [km,km)	Max Useful Range (km)	Probability for Zone i	Cumulative Probability
1	0.0, 3.0	4.0	0.000	0.000
2	3.0, 3.8	5.0	0.001	0.001
3	3.8, 4.9	6.5	0.004	0.005
4	4.9, 6.2	8.3	0.015	0.020
5	6.2, 7.5	10.0	0.030	0.050
6	7.5, 9.0	12.4	0.069	0.119
7	9.0,11.0	14.8	0.173	0.292
8	11.0,14.8	18.1	0.491	0.783
9	14.8,30.0	30.0	0.217	1.000

Note that this distribution agrees very closely with the overall aggregate given in Table 1b. Comparison of this distribution with those for WW II howitzers is shown in Figure 1.

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[2] p. 137, Systems Analysis Directorate Activities Summary, June 1976, DRSAR/SA/N-50, AD B012850L, ARMCOM, July 1976.

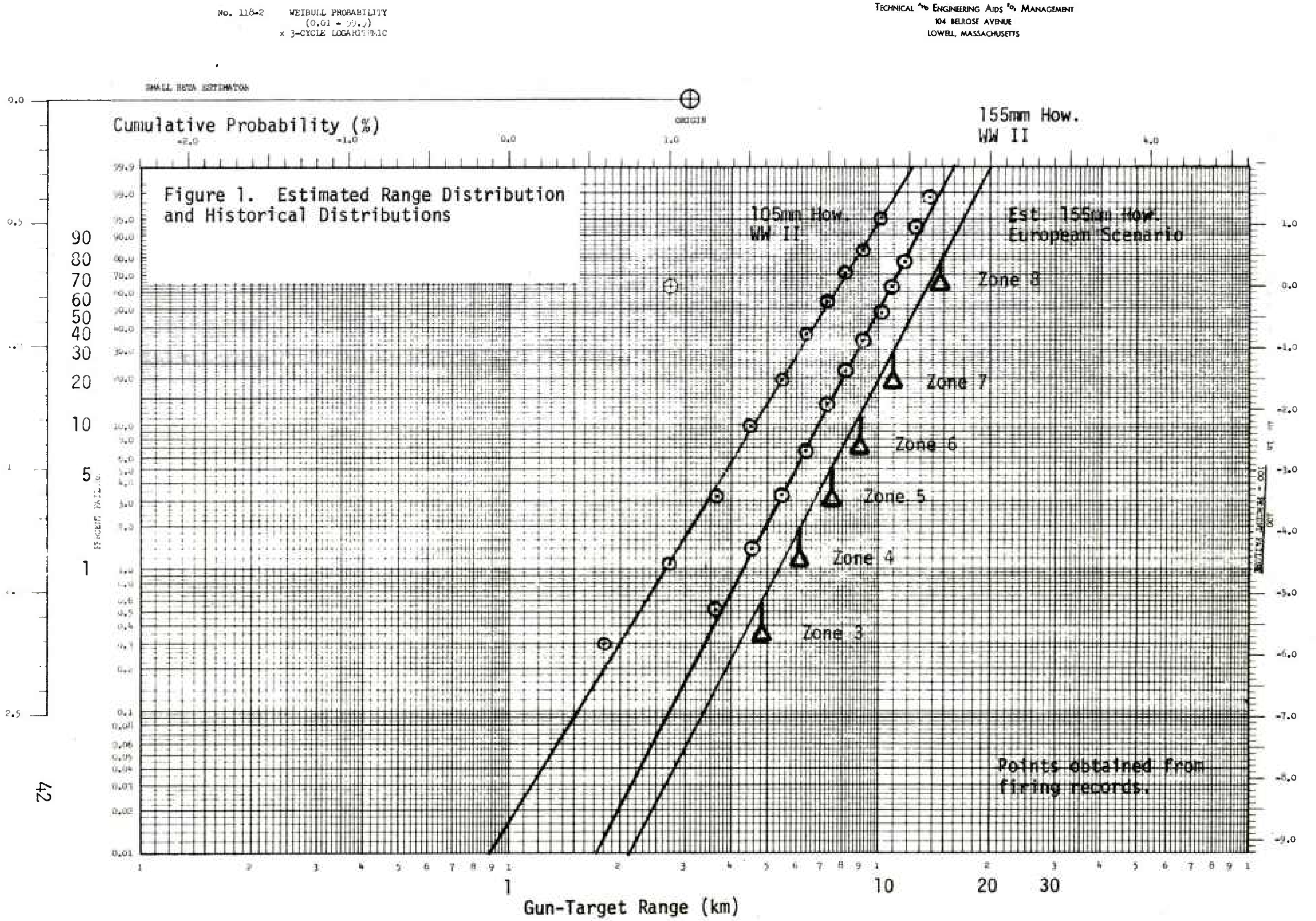

Figure 1. Estimated Range Distribution and Historical Distributions

6. Expected Firing Life

Using the distribution of zones fired and the EFC factors and tube condemnation limit established for the M185 cannon, one can calculate the expected number of shots between tube replacements. The present condemnation limit for the M185 cannon is 5000 (zone 8) EFC rounds. The expected wear at a point near the origin (40 inches RFT) at 5000 EFC rounds is 0.124 inches. The number of rounds equivalent in erosion to one zone 8 full charge is tabulated here from [3]. Due to some uncertainty in the erosiveness of zone 9, an alternative calculation for zone 9 is given below.

TABLE 3. WEAR CHARACTERISTIC OF 155MM M185 CANNON

Zone	No. of Rounds Equivalent to One (Z8) Full Charge
1-6	4.00
7	1.43
8	1.00
9*	0.36

* Wear estimate of M203 charge provided in [4], p. II-349.

The expected number of rounds, E(N), fired from an M185 cannon during its life can be calculated from the data in Tables 2 and 3.

$$E(N) = (0.119*4 + 0.173*1.43 + 0.491*1 + 0.217*0.36)*5000$$
$$E(N) = 6462 \text{ rounds.}$$

7. Sensitivity Analyses

The sensitivity of expected tube life to operational uncertainties -- target opportunities and our own use concept -- can be examined parametrically by permitting the average GT range to vary by ± 10% around the baseline**

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[3] FT 155-AM-1, Firing Tables for Cannon, 155mm How., M185 on Howitzer, Med. SP, 155mm M109A1 Firing Projectile, HE M107, HQ, Dept of Army, Sep 1972.

[4] Wurzel, E. "Cannon Wear Single-Shot Testing Method", Proceedings of the Tri-Service Gun Tube Wear and Erosion Symposium, ADA046600, March 1977.

** The shape parameter (β) was held constant at 5.

and by recalculating the probability distribution of firing zone. These results are tabulated below.

TABLE 4. SENSITIVITY OF TUBE LIFE TO OPERATIONAL UNCERTAINTY

Avg GT Range (km)	n (km)	Probability of Using Zone:				E(N) (rounds)
		≤6	7	8	9	
12.5 (base)	13.60	0.119	0.173	0.491	0.217	6462
11.2 (-10%)	12.24	0.193	0.250	0.481	0.076	8189(+27%)
13.7 (+10%)	14.96	0.075	0.118	0.419	0.388	5137(-21%)

A substantial sensitivity of E(N) to operational uncertainty is exhibited here since a small change in the average GT range produces a more than twofold change in E(N).

8. One may ask what the logistic impact would be if the wear life at zone 9 could be doubled by the use of a propelling charge with a significantly lower flame temperature*. In this case the number of zone 9 rounds wear-equivalent to one zone 8 would be 0.72. As a consequence of this change, the expected life would increase from 6462 to 6853, a 6% increase. In terms of economic impact, a 6% savings in M185 tube replacements can be estimated roughly as follows. Assume a lifetime requirement of 3,000 tubes for the system based on forecasted usage in a European scenario. At a current replacement cost of approximately $13,000 per tube**, the lifetime savings in current dollars is $2.3 million! Even though only 22% of the wartime usage is expected at zone 9,

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* A comparable situation existed in 1976 with propelling charges in the 8-inch M110A2 SP howitzer. By changing from M30A2 propellant, having a flame temperature of 3025 deg K, to M31A1 propellant, which has a flame temperature of 2600 deg K, the rate of wear has been halved at zone 9 in the M188A1 charge [5].

[5] Firing Report No. 14616, Product Improvement Test (Wear Test), U.S. Army Yuma Proving Ground, December 1979.

** The choice of $13K per tube understates the effective unit cost somewhat since the M199 cannon tube, used with the M198 howitzer, is more expensive than the M185 cannon tube. Current (1980) cost of the M199 tube is $14,600. For the estimate of savings we have neglected the additional unit cost for that fraction of the inventory which employs the M199 cannon.

halving the wear rate in this zone would have a significant logistic and economic impact.

9. Considerable historical experience with propelling charge developments has shown that the effort to improve a propelling charge in one way or another is often counterproductive. An example of this is the decade-long effort to develop a cheaper version of the M119 charge -- from the XM123 series through the XM201 to the XM201E5 and the XM119E4, all of which flash more frequently and are considerably more erosive than the M119. Pursuing the line of inquiry introduced above, suppose that the M119 is replaced with the XM119E4 (M30A1 propellant). Present wear experience with this charge indicates that a reduction in tube life would occur: from 5000* to 2100 zone 8 rounds [4, loc. cit.]. Using the distribution of firing zones presented above, the impact of the change would be a reduction of the expected tube life from 6462 to 5039, a factor of 0.78. Therefore, a 28% greater inventory would be required, costing an additional $11 million over the life of the system**, under previous assumptions. Evidently the greatest cost and logistic impact would occur in changing the zone 8 charge.

10. Theoretical Estimation of Average Wear Rate

As previously mentioned, it is possible to independently estimate the wear rate of the M185 cannon at zone 9 using a semi-empirical formula developed by workers at the NSWC, Dahlgren, VA. This relationship is called the Smith-O'Brasky formula and is given on p. 6 of [6]. The formula relates the average*** wear rate, w, of the lands diameter measured at the origin of rifling to certain thermochemical and physical parameters for the particular charge. The formula applies to unplated guns employing projectiles with copper rotating bands. The formula is repeated here as follows.

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* Tube life of the M185 at zone 8 (M119 charge) is limited by metal fatigue.

** No estimate is included here of the additional cost associated with increased weapon vulnerability when using charges which flash more frequently than the M119.

*** Average taken over the useful cannon life.

[6] Samos, G., Grollman, B. B. and Ward, J. R. Barrel Erosion Rate of a 60mm Gun, ARBRL-MR-02857, Ballistic Research Lab, Aberdeen, MD, August 1978.

Let

w = wear rate (10^{-4} inches/round)

T_v = propellant flame temperature (deg K)

T_c = temperature decrease ascribed to boundary-layer coolants (deg K)
$300 \leq T_c \leq 500$ with coolants
$T_c = 0$ without coolants

T_w = effective surface temperature of the chamber wall (deg K)

d = weapon caliber (inches)

c = charge mass (pounds)

p = maximum chamber pressure (ksi)

Then,

$$T_w = 0.0763\ (T_v - T_c - 600)(cp)^{1/2}/d$$

$$w = 0.0166 \exp\ (4.9\ 10^{-3}\ T_w) \quad .$$

Although some uncertainty is involved in the meaning of "average" wear rate and in the assignment of a temperature decrease T_c due to cooling by additives, it can be useful at least to gage the relative erosiveness of various propelling charges and, as a consequence, their relative wear lives.

11. Using data which characterize 155mm propelling charges, I have calculated the "theoretical" rates of wear for each charge and zone with the above formula. The results are shown in Table 4. Similar results were obtained for zone 9 charges in the 8-inch M201 cannon. These results are compared with experimental estimates in Table 5. It appears that the formula provides reasonable estimates in this range of the input parameters. Comparing the values of number of EFC rounds from the M109A1 Firing Tables (Table 3) with the calculated estimates (Table 5), one notes that somewhat fewer zone 6 rounds are declared equivalent to zone 8 by the FT than calculated -- 4 versus 5.1. Also, fewer zone 7 rounds -- 1.43 versus 2.6. However, the no. EFC at zone 9 provided by [4] -- 0.36 -- is somewhat higher than that calculated, namely, 0.30. Thus, the calculated life at zone 9 suggests an even greater economic and logistic advantage were the wear life to be improved in the manner indicated.

TABLE 5. ESTIMATED* AVERAGE WEAR RATES FOR THE 155MM M185 CANNON FIRING THE M107 PROJECTILE AT SEVERAL ZONES

Charge Nom.	Zone	Flame Temp (deg K)	Chg Mass (lb)	Max Press (ksi)	Wear Rate (10^{-4} in/ round)	No./ EFC
M3A1	1G	2417	1.77	4.9	0.023	14.1
	2G		2.29	5.9	0.025	13.0
	3G		3.09	7.9	0.029	11.2
	4G		4.02	10.6	0.034	9.6
	5G		5.47	15.4	0.046	7.1
M4A2	3W		3.89	6.2	0.029	11.3
	4W		5.12	7.8	0.034	9.7
	5W		6.86	10.6	0.043	7.6
	6W		9.66	15.1	0.064	5.1
	7W		13.15	25.1	0.126	2.6
M119	8W	2570	20.34	29.9	0.326	1.00
XM201E1	8W	3040	17.25	32.1	0.364	0.90
XM203E2	9W		26.19	47.5	1.695	0.19
M203**	9W		26.19	47.5	1.100	0.30

* Using the Smith - O'Brasky formula [6, p. 27] with the tabulated input data and with $T_c = 300^0K$ for the XM201E1 and XM203E2 charges.

** As above with $T_c = 500^0K$.

TABLE 6. COMPARISON OF THEORETICAL* AND EXPERIMENTAL AVERAGE WEAR RATES IN THE EIGHT-INCH M201 CANNON USING THE M106 PROJECTILE

Propellant Type	Charge Nom.	Zone	Flame Temp (deg K)	Charge Mass (lb)	Max Press (ksi)	Wear Rate (10^{-4} in/rnd) Calc.	Wear Rate (10^{-4} in/rnd) Experi.**
M30A2	XM188E1	9	3025	42.99	38.7	0.953	
M31A1	M188A1	9	2599	47.48	39.4	0.515	0.5 [5]
M30A1	XM188	9	3040	43.20	39.0	1.01	1.2 [7]

Notes found on next page .

* Using the Smith - O'Brasky formula [6, p. 27] with the tabulated input data and with consideration given to the boundary layer coolants by assigning the parameter T_c the value 300°K.

** Wear rates are calculated after chrome plating has broken down.

[7] "XM201 Cannon Wear Life Characterization," Firing Report No. 12239, Dept. of Army Yuma Proving Ground, September 1972.

12. Conclusions

On the basis of the analytic results presented here, it is possible to draw several conclusions:

a. Two independent approaches to estimating the distribution of use of 155mm charges yield essentially the same result. The distribution presented by CAA is consonant with extrapolation of historical experience.

b. Whichever method is used, one would expect that nearly half of the rounds would be fired at zone 8 (M119 charge). This result is fairly insensitive to changes of ± 10% in average GT range due to operational uncertainties.

c. The (statistically) expected number of rounds fired from the M185 cannon during its lifetime is, at present, 6462, 29% greater than the 5000 EFC round life. However, this value of E(N) is quite sensitive to operational (scenario-dependent) uncertainties.

d. The expected tube life is fairly sensitive to the wear at zone 9. A technically feasible decrease in the wear rate at zone 9 by 50% would reduce the requirement for tube replacements by 6%.

e. The greatest sensitivity of the requirement for tube replacements is shown at zone 8 where a 28% increase in the inventory would be necessitated by a change from the M119 to the XM119E4 (or comparable) propelling charge. Over the life of the M109A1 system the estimated cost of such extra tube replacements (alone) would be $11 million (FY 80). No estimate was made of the cost of additional weapon replacements required because of the increased incidence of counterfire associated with more muzzle flash and smoke. Therefore, product "improvements" to the M119 propelling charge should not be such as to incur the less obvious costs associated with increased erosive wear and greater weapon vulnerability.

f. Calculations of the relative erosive wear at zone 7 and below indicate that the M109A1 Firing Tables may slightly exaggerate the relative wear of these zones. Some confidence in the formula used for these calculations is afforded by the favorable comparison with experimental results, using two types of propellant, in the 8-inch M188 propelling charge with the M110A2 SP howitzer.

George Schlenker

GEORGE SCHLENKER
Operations Research Analyst

Attachment 1

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2. Systems Analysis Directorate Activities Summary, June 1976, DRSAR/SA/N-50, AD B012850L, ARMCOM, July 1976.

3. Firing Tables for Cannon, 155mm Howitzer, M185 on Howitzer, Med. SP, 155mm M109A1 Firing Projectile, HE M107, FT 155-AM-1, HQ, Dept. of Army, September 1972.

4. Wurzel, E. "Cannon Wear Single-Shot Testing Method", Proceedings of the Tri-Service Gun Tube Wear and Erosion Symposium, AD A046600, March 1977.

5. Firing Report No. 14616, "Product Improvement Test (Wear Test)," US Army Yuma Proving Ground, December 1979.

6. Samos, G. Grollman, B. B. and Ward, J. R. Barrel Erosion Rate of a 60mm Gun, ARBRL-MR-02857, Ballistic Research Lab, Aberdeen, MD, August 1978.

7. Firing Report No. 12239, "XM201 Cannon Wear Life Characterization," Dept. of Army Yuma Proving Ground, September 1972.

Memorandum for Record

TDP CANCELLATIONS

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4 April 1980

MEMORANDUM FOR RECORD

SUBJECT: TDP Cancellations

1. OBJECTIVE.

Assignment was made to investigate the feasibility for developing a standard for the TDP cancellation rate and if feasible to derive such a standard or develop the methodology for the future development of such a standard. This assignment was in response to a request by the CG ARRCOM to develop a standard for TDP cancellations based upon the number of TDP requests.

2. STUDY METHODOLOGY.

Contact was made with the concerned areas within ARRCOM (MM, LE, PD, and DA) to gain an understanding of the TDP process, the TDP cancellation causes and the benefits that would be achieved if the TDP cancellation rate could be reduced. Evaluation was made using the data gathered.

3. FINDINGS.

a. At this time, the TDP cancellation documentation is not in a format that makes it possible to either quantify the current TDP cancellation rate or to investigate the relationship between TDP cancellation and PRON cancellation. Because of the causal relationship between PRON's and TDP's, such a relationship would appear to exist. Personnel in PD who work with Procurement Appropriation (PA) dollar items state that they could compute their rate for TDP cancellations if about a manweek's effort was expended. LE is currently collecting data on the overall TDP cancellation rate; however, at this time, the software is not available to query this data.

b. The TDP cancellations for items using PA dollars for weapons and ammunition appear to be minimal. DA estimates a 1% cancellation rate on a volume of about 100 ammunition item requests per year and personnel involved with PA dollars for weapons say that they have had only one cancellation this year thus far on an expected yearly volume of about 30 requests. TDP's on PA items, in general, dependent upon their size, require a much greater expenditure of effort than those on Army Stock Fund (ASF) items and therefore, the impact is much greater when a PA dollar item TDP is cancelled.

c. DA, PD and LE all state that the PA dollar item cancellation rate for ammunition and weapons is low and currently at an acceptable level. Furthermore, PA funded item cancellations which result in the cancellation of the associated TDP's appear to be caused normally by influences external to ARRCOM control. Human errors do occur in the PD handling of the paper work and can cause a resubmission to be necessary; however, this normally does not affect the TDP requirement. Changes in quantity levels of items, or in the time frame of buying, both of which frequently occur, do not affect TDP requirements unless the item is totally cancelled or the buy is moved beyond the 13 month validity time of the TDP.

d. PRON cancellations on the ASF and PA funded secondary weapon items are more frequent, both relatively and absolutely, than the cancellations of TDP's for PA funded ammunition and weapons. The current cancellation rate is estimated for ASF items by MM at 13%. This was computed by evaluating the over 6,000 PRON's generated between October 1979 and March 1980. For PA funded secondary items the rate was 8% for 2,500 PRON's generated in FY 1979. Each ASF item TDP may be supported by several PRON's and PRON amendments and a TDP is only cancelled when all of its associated justifying PRON's or amendments have been cancelled. They are thus often utilized for PRON's that are submitted subsequent to the original justifying PRON's cancellation but prior to the expiration date.

e. When it is necessary to cancel a TDP before it is completed by the design agency, LE makes no effort to investigate whether it is likely that additional PRON's requiring the same TDP may be forthcoming. This could result in waste of effort on the part of the design agency if a subsequent justifying PRON for the same TDP is forthcoming.

f. The PRON cancellation rate within MM appears to have varied causes, many of which probably lie outside ARRCOM control. There are, however, two which appear to result in a significant number of cancellations and which may be of a nature that they could be affected by some type of additional management. The first is human error in processing. This usually results in the resubmission of paper work and would not in most cases affect a TDP requirement. The second is the existence of poor quality data in the ARRCOM Data Base. One of the uses for this data is to determine the type of source to utilize in filling an item requirement. Also, incorrect or delayed inputs by the field as to inventories, numbers of rebuilds, etc., can lead to an improper decision concerning the source for a required item. Because the data in this data base come from a great number of sources it may be difficult, expensive or even impossible to improve it by the placement of additional quality controls.

g. A major burden seems to be put on LE by MM in that requests for a single item may be received as often as every month. This requires a large amount of paper work which would be reduced if such requests could at least be received in batches, say quarterly (this is currently under investigation).

4. CONCLUSIONS

a. A very important aspect of this assignment is determining whether or not there is a need for any type of standard. This basically relates to the questions, "Is there a problem"? It appears that there is not a significant problem with the TDP cancellation rate. Although the rate is not quantified, LE says that they are able to handle the current rate and there is no indication that the current rate is atypical. They do state, however, that if the PA item TDP cancellation requests were to increase greatly for an extended time period, the amount of wasted labor would become significant and perhaps the request would become difficult to handle without delays. Because PA item TDP cancellations have causes that are external, it would be difficult to remedy this situation beforehand.

b. There does appear to be a problem with the amount of wasted labor associated with the PRON cancellation rate. MM states that their rate is similar to or even less than many of the other commands; however, the current cancellation rate (13%) because of the volume of PRON's involved (over 22,000 initiated PRON's and amendments per year) is a burden on LE and the reduction of the cancellation requests would allow LE to more quickly accomplish their other responsibilities such as TDP review.

c. An evaluation of the types of data in the ARRCOM Data Base that result in PRON cancellations and the resulting frequency of occurrence would yield information so that the feasibility of controlling such data could be determined. Furthermore, similar analyses should be made concerning the sources of human error in all other organizations involved in this process in order to evaluate whether controls could reduce the cancellation rates of the various concerned documents (PRON, FL337, MIPR, TDP, etc.,) and what the cost of such controls would be.

5. RECOMMENDATION

It is recommended that any kind of a TDP cancellation standard should be based upon the optimum value of a selected benefit parameter. The

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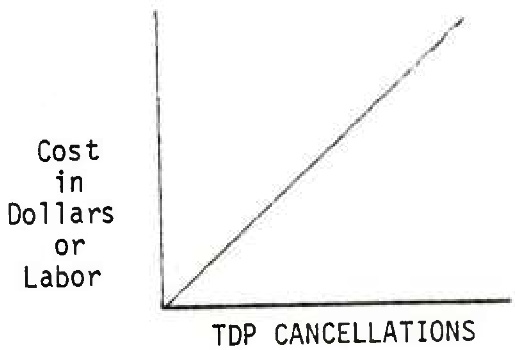
benefit parameter is most likely cost of manpower. In other words, some type of linear function could be derived relating the TDP cancellation rate for a given volume of TDP requests or the absolute value of TDP cancellations against the manpower cost or manhour waste that results from the cancellation rate or volume. Such a function is depicted below.



After the function is derived, a tradeoff would then be made between the cost or hours saved by lowering the rate or volume of cancellations and the cost of the controls that would have to be added in order to realize an increased benefit. It may well be that the cost of additional control may be greater than the cost saving derived and in fact greater than the current waste costs of the cancelled TDP's. Since the redundant waste costs in either money or manhours is not available at this time such a determination cannot now be done.

The first step in developing a standard for the cancellations would be to compute the associated average cost (manhour or money) per TDP cancellation for each of the involved offices (MM, PD, LE, QA, DA, Design Agencies, etc.,). Thus, the total cost of a TDP cancellation can be understood. This probably should be done separately for the ASF and the PA items. At this point it can be decided whether it is desirable to expend effort to try to achieve a savings given the current TDP cancellation rates. Next, the causes of cancellation must be examined to see which are under local control. Thus far, two that appear to be such are human error and inaccuracies in the ARRCOM Data Base. Other causes such as customer cancellations, budget zeroing, failures in R&D programs and decisions to utilize different major items appear to be beyond ARRCOM control. Finally the relationship between the potential benefits and associated cost could be evaluated to find an optimum benefit or cancellation rate with respect to the manpower or cost that would be required to obtain it.

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Memorandum for Record

LOT ACCEPTANCE CRITERIA

FOR

AMMUNITION

EXHIBITING

CRITICAL DEFECTS

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13 May 1980

MEMORANDUM FOR RECORD

SUBJECT: Lot Acceptance Criteria for Ammunition Exhibiting Critical Defects

1. Background

The problem addressed in this MFR concerns the design of procedures for functionally testing samples from lots of ammunition items, having critical defects, for the purpose of accepting or rejecting the lot. It appears that there is a need for better discrimination between "bad" and "good" lots of ammunition when performing these acceptance tests. When the existence of a defect in a round of ammunition poses a safety problem for the user or, otherwise, has catastrophic economic consequences, it is prudent to require that the fraction lot defectives (LD) which is accepted, with fixed probability, following a test be quite low. Increasing the sample taken for a test helps avoid accepting bad lots and reduces the LD for accepted lots. Unfortunately, simply increasing the sample size has the undesirable effect of rejecting marginally good product more often. Moreover, testing costs increase in a nearly proportional manner with sample size. More discriminating, multi-stage sampling plans should be used where possible since -- as is shown in this memorandum -- they can be more discriminating at fixed test cost or, alternatively, they cost less at a particular level of discrimination.

2. An example of the need for more discriminating lot acceptance testing is found in the 105mm M392 projectile retrofit program. Breakup of metal parts in bore can cause condemnable tube damage. Following such an incident, the firing of other rounds may produce an inbore premature explosion. Apart from this safety hazard, there is a substantial economic loss in tube replacements due to rounds that severely damage the tube without prematuring. One would imagine, therefore, that the fraction lot defectives which is accepted, say, at even odds is low. Quite the contrary! Presently, lots are accepted (on waiver) if zero or (one) defective is discovered during the test of a sample of 20*. This test procedure implies acceptance with probability 0.5 of lots whose fraction defectives is 8.3%. Further, at a risk of 0.05 one accepts lots with a fraction defectives of 21.6%! Clearly, this is an unacceptable test characteristic for testing for critical defectives.

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* This waiver option departs from the original plan to reject if one (or more) defectives is found in a sample of 20.

3. Notation and Terminology

For convenience and brevity we will use some standard quality assurance terms. The operating characteristic (OC) of a test procedure is the probability of accepting a lot having a particular fraction defectives, π. This functional dependence on π is denoted by $P_A(\pi)$. A specific test parameter is the acceptance number, c, defined as the maximum allowable number of defectives in a sample of size N for lot acceptance. The number of defectives in such a sample which causes immediate rejection is the rejection number, r. When denoting the operating characteristics of several tests, the parameters will be displayed explicitly in the notation. For example: $P_A(\pi;N;r)$. Conventionally, the designer of a lot acceptance test is asked to specify a value of fraction lot defectives, π_1, considered "good" and a larger value, π_2, considered "bad". For a particular plan, there are risks associated with these values of π. The probability of rejecting a lot with $\pi = \pi_1$ is termed the producer's risk, α. Clearly, from the OC,

$$\alpha = 1 - P_A(\pi_1) \quad .$$

At $\pi = \pi_2$ the probability of (the consumer) accepting the lot is called the consumer's risk, β, where

$$\beta = P_A(\pi_2) \quad .$$

4. Purposes of this Memorandum

This MFR has several purposes and is addressed in part to both technical and managerial audiences. One of the objectives is to display the risk consequences of changing the acceptance number, using the present M392 lot acceptance plan as an example. Another purpose is to present two alternative approaches to designing lot acceptance plans -- (1) a method based upon specifying producer's and consumer's risks and associated values of lot quality, and (2) a method based upon minimizing the quantifiable expected economic consequences of selecting a particular sample size, N, for the acceptance test. This approach uses the concept of average accepted lot quality (AAQ). Both of these approaches are illustrated by examples. However, the main purpose of this MFR is to compare a single-stage lot acceptance plan with a sequential, two-stage plan meeting the same required (α and β) risks. The intention here is to demonstrate the relative efficiency of multi-stage plans relative to comparable single-stage plans.

5. Results

Various mathematical derivations have been placed in Annexes. The derivations of formulas for the OC of various lot acceptance plans are found in Annex 1. Also found there are derivations of formulas for the expected or statistical average costs of executing the test procedures, given incoming lot quality, π. This variable is termed the expected test cost or ETC, which is a function of π and of sample size N. A concept of importance to quality assurance is the average (over incoming lot quality) fraction defectives of lots which are accepted. This average accepted quality (AAQ) is functionally dependent upon the lot acceptance procedure employed. Formulas for AAQ are derived in Annex 2. The utility of a lot acceptance test for items having critical defects resides mainly in reducing the economic loss or penalty associated with the defectives, in accepted lots, which are encountered in a future operational setting. The costs associated with this benefit are the expected test cost and any costs related to actions taken on rejected lots -- storage, rework, etc*. One can formulate the problem of choosing an optimum lot acceptance sample size for a given type of test in terms of maximizing the net utility. This is done here using the classical economic method of finding the point of zero (net) marginal utility. The derivation of the formulas which utilize this approach are found in Annex 3. The economic method is illustrated for the particular case of the 105mm M392 projectile retrofit program.

6. Operating Characteristic (OC) of the Lot Acceptance Test for the M392 Projectile

As indicated above, one objective of this memorandum is to illustrate the risk consequences of choosing different acceptance numbers, c, in a single-stage lot acceptance test. The probability of lot acceptance, P_A, for lots having a given lot fraction defectives, i.e., the OC, is the best way to consider risk. The original M392 acceptance test called for an exhaustive (fixed sample) test of 20 randomly selected projectiles. Only lots having c = 0 defectives were accepted. Subsequently, lots having c = 1 and, then, 2 were accepted on waiver. The OCs for a fixed sample of 20, with c as a parameter, is shown in Figure 1. One aspect of increasing c is to increase the steepness of the OC thereby improving discrimination. However, another consequence of increasing c, at fixed N, is to increase the risk of obtaining a lot having an unacceptably large fraction defectives, π. At a given level of π, say, 0.1, the value of the OC drastically increases with c. For example, $P_A = 0.12$ at c = 0; $P_A = 0.39$ at c = 1; and $P_A = 0.68$ at c = 2. An alternative to direct examination of the OC, as above, is to examine a crossplot in which the lot fraction defectives is a function of c, with consumer's risk a parameter. This type of presentation is made in Figure 2. At a fixed risk of 5%, note how the value of π increases with c: π = 13.9% (already large!) at c = 0; π = 21.6% at c = 1; π = 28.3% at c = 2; and π = 34.4% at c = 3.

* The reader should recognize that costs and benefits are felt at different times. Thus, it may be appropriate to use discounting and inflating to express all costs as present values. This additional sophistication was not employed here.

Figure 1. Operating Characteristics for Single-Stage Lot Acceptance Tests Having Sample Size 20 With Acceptance Number as a Parameter

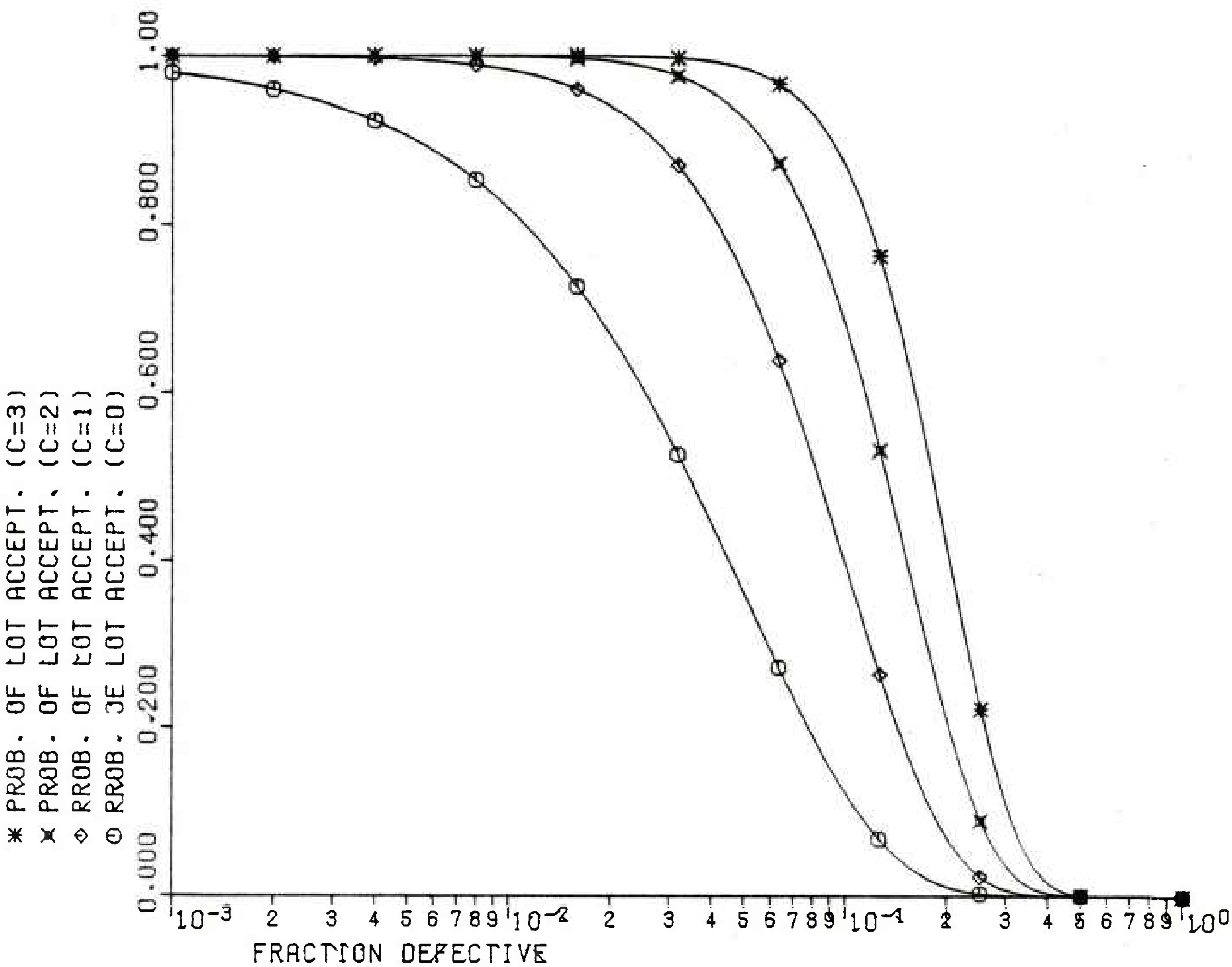


Figure 1a. Operating Characteristics (Normal Prob. Scale) for Single-Stage Lot Acceptance Tests Having Sample Size 20 With Acceptance Number as a Parameter

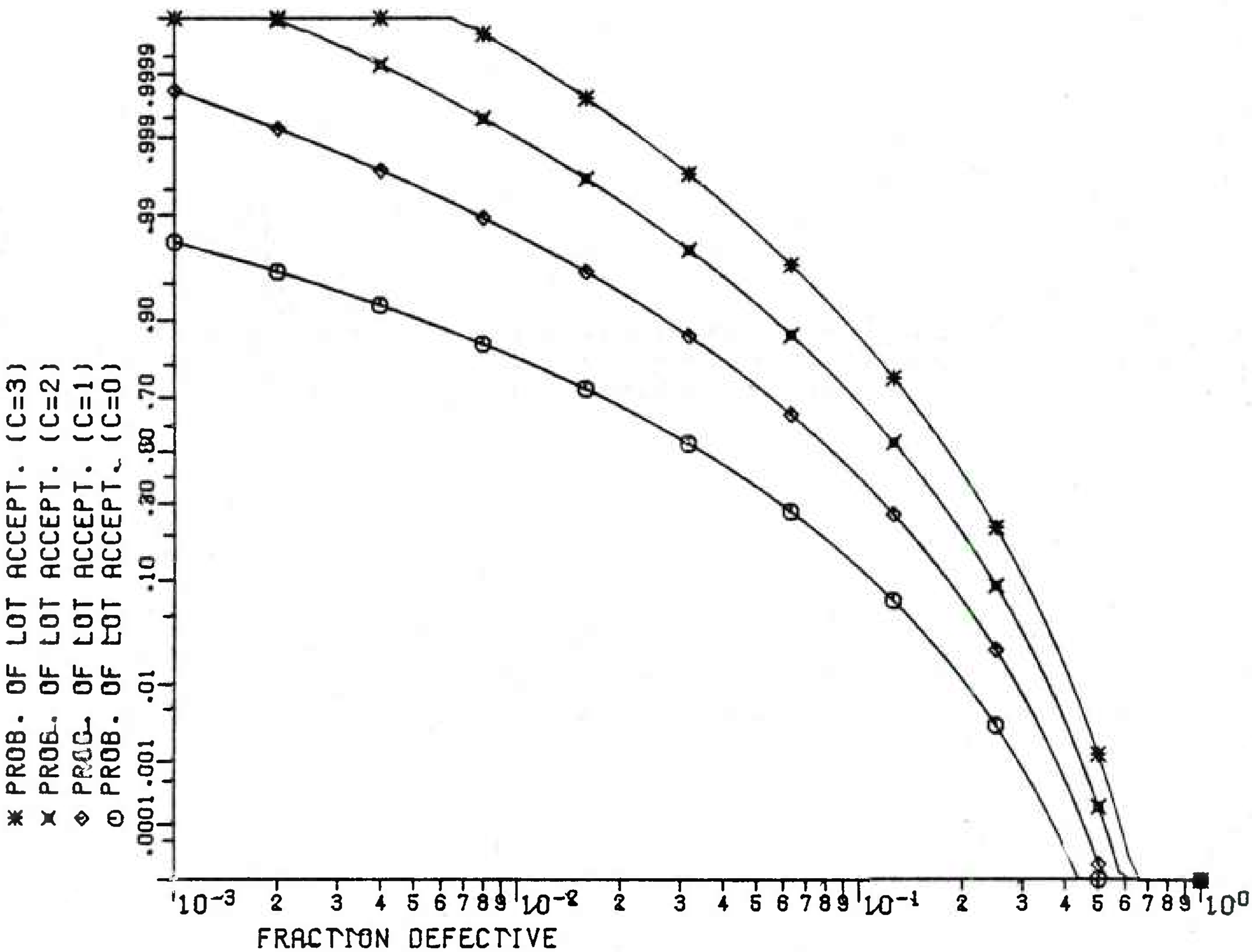
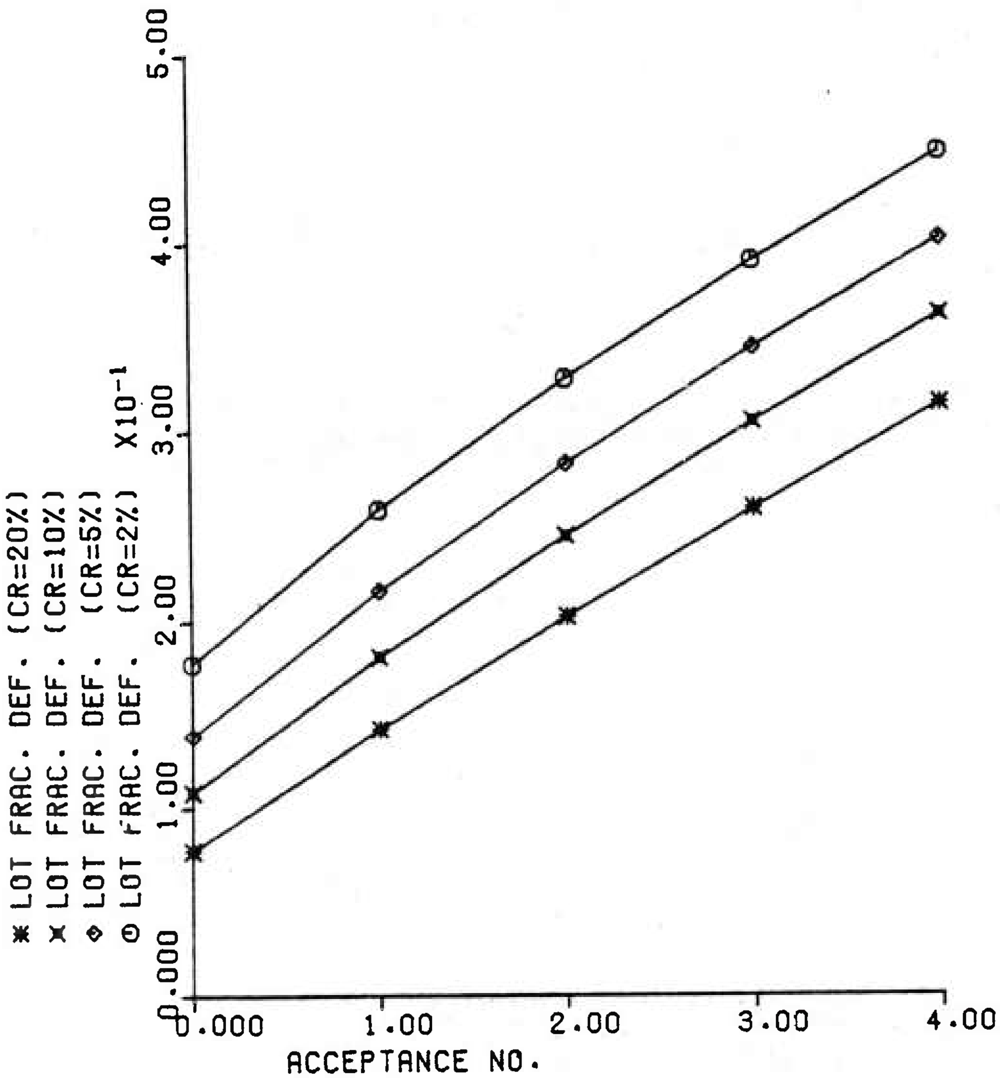


Figure 2. Lot Fraction Defectives Versus Acceptance Number With Consumer's Risk as Parameter for a Single-Stage Test With Sample Size 20



7. Acceptance Tests Designed to Given Risks

The traditional approach of statistical quality control to designing an acceptance test is to require "the decision maker" to specify a producer's (α) risk and a consumer's (β) risk and associated values of π. With this specification the sample size -- or sizes, for multi-stage tests -- and acceptance and rejection numbers are selected so that the α and β risks are both satisfied without excessive sampling. This method is not difficult to use once the risk specification has been stated. Unfortunately, the burden of producing a reasonable (and cost-effective) risk statement may call for unusual intuition. This is a weakness of this approach. An example of this approach is given now for a particular risk specification. Two different test procedures will be generated, each of which satisfies the same risk specification. One purpose of pursuing this example is to demonstrate the superiority of a multi-stage sequential test over a single-stage counterpart.

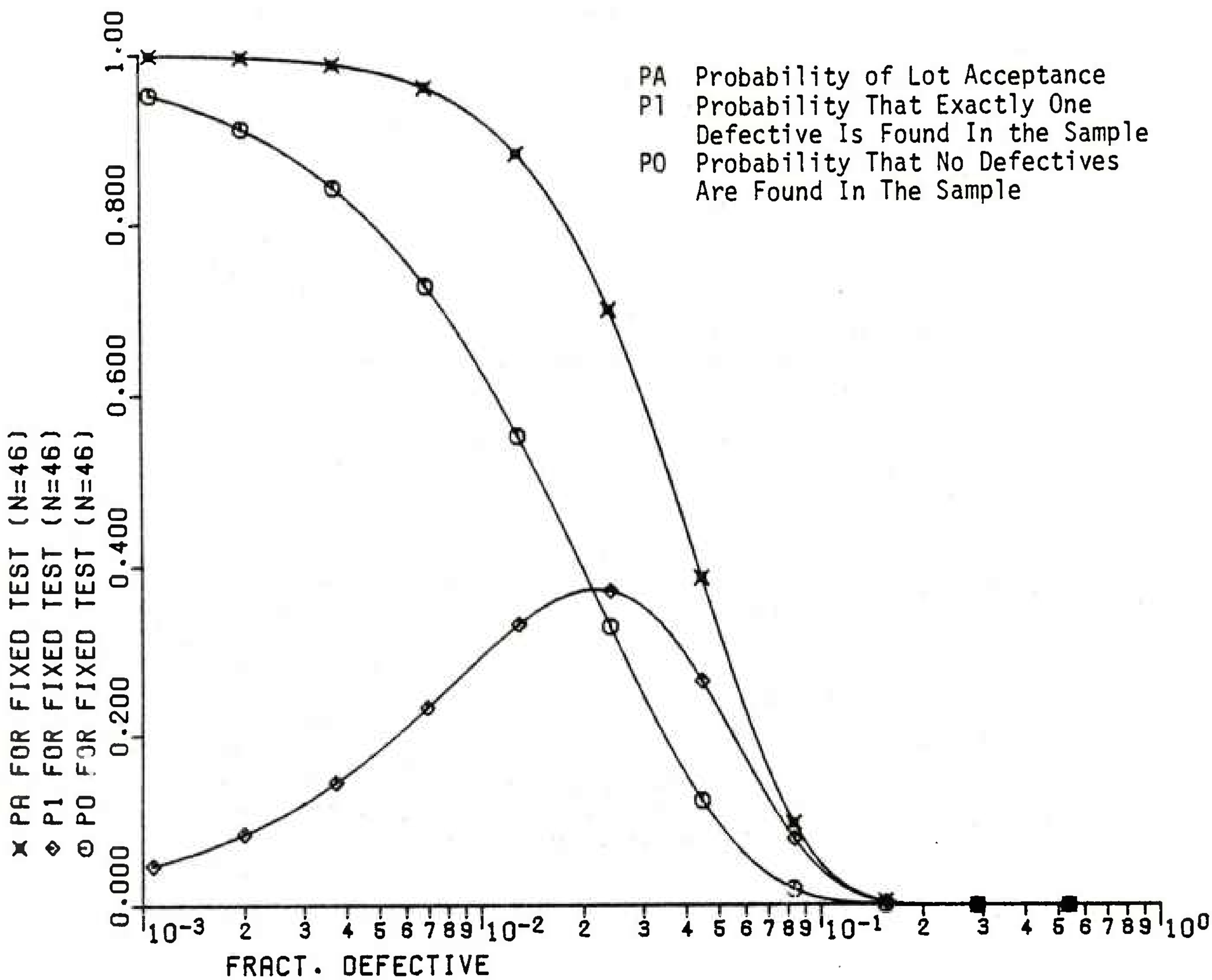
8. Risk Specification

A reasonable objective for an acceptance test might be the following: Take a 5% or less (α) risk of rejecting lots having π = 1/2% and a 5% or less (β) risk of accepting lots having π = 10%. Incidentally, we note (Figure 1) that the original M392 lot acceptance plan had α and β risks of 9.5% and 12.2%, respectively! A single-stage plan (whether sequential or fixed sample) which satisfies the above specification at minimum test cost would have an acceptance number, c = 1 and a sample size, N = 46. Several properties of this particular procedure are shown in Figure 3. The OC for this test is displayed together with the probabilities of finding zero and one defective item in the sample. Of course, the occurrence of a second defective would stop the testing if the test was sequential.

9. A Comparable Two-Stage Plan

A two-stage sequential test satisfying the same risk specification, indicated above, satisfied by the single-stage plan can be described as having equal sample sizes of 33 at each stage with first-stage acceptance and rejection numbers c = 0 and r = 2, respectively, and with second-stage rejection number r = 2. The formulas related to this plan are derived in Annex 1. Table 1 compares the properties of this procedure and those of the single-stage procedure. In several respects the two procedures are nearly the same. For example, by design, both have about the same consumer's risk -- β = 4.8% for the single-stage versus 4.7% for the two-stage plan -- and average accepted quality (AAQ) -- 3.13% versus 3.14%. However, the two-stage procedure has a smaller producer's risk (α) at π = 1/2% -- 1.4%

Figure 3. Characteristics Associated With A Single-Stage Lot Acceptance Test Having N=46 and c=1



versus 2.2%, for the single-stage procedure. A smaller producer's risk is also evident at π = 1% -- 5.3% versus 7.8%. Thus, the two-stage procedure is more discriminating. However, the most conspicuous advantage of the two-stage procedure lies in having a lower expected test cost (ETC). Even when both procedures are sequential, i.e., terminating at the r th failure, the ETC for the two-stage procedure is 10% less with π = 1%; is 18% less with π = 1/2%; and is 28% less with π = 0%! Furthermore, as shown in Table 2, the two-stage procedure has greater sensitivity of AAQ to increases in N.

10. A comparison of the operating characteristics of the two procedures is shown in Figure 4, where the steeper decline of the two-stage OC is evident. A complete picture of the ETC as a function of π is shown for both procedures in Figure 5 and for the two-stage procedure alone in Figure 6. Only if the incoming fraction defectives were extremely large would the two-stage procedure generally cost more than the sequential single-stage. Irrespective of lot quality, the sequential two-stage procedure is less costly than a fixed-sample, single-stage procedure.

11. <u>Additional Properties of the Two-Stage Procedure</u>

Additional interesting properties of this two-stage procedure are the following:

a. The probability that the first-stage will terminate early, i.e., before testing all 33 items (Figure 7).

b. The expected number of items tested in a truncated sequential test with sample size 33 and r = 2 (called EN_2 in Annex 1). This is shown in Figure 8. One of the virtues of a sequential test is that the test is stopped as soon as sufficient information is at hand to make a decision, saving test time. Thus, lots having a large fraction defectives tend to display that fact early by reaching the rejection number before N items are tested. Figure 7 shows that as π exceeds about 1% the probability of early termination of the first stage becomes appreciable. Another advantage of this type of procedure is that the conditional expected value of the items actually subjected to test in a single stage -- the second, in this case -- declines as lot quality declines. This is illustrated by Figure 8.

12. Some results of a sensitivity analysis are shown in Figures 9 and 10. Increasing N by steps of one from 30 to 33 in the two-stage procedure changes the operating characteristic, slightly, in the manner shown in Figure 9. Good lots are not affected by this change as much as are poor. The expected test costs as functions of π, with N a parameter, are shown in Figure 10. Note that increasing N does not significantly increase the cost of testing poor lots, for this type of test.

TABLE 1

COMPARISON OF CHARACTERISTICS OF SINGLE-STAGE WITH TWO-STAGE LOT ACCEPTANCE TEST PROCEDURES, EACH HAVING THE SAME CONSUMERS RISK OF ACCEPTING TEN-PERCENT DEFECTIVE LOTS

Parameter	Acceptance Test Procedure Single Stage	 Two Stage
Acceptance Number(s)	1	0, 1
Rejection Number(s)	2	2, 2
Sample Size(s)	46	33, 33
Consumers Risk at 10% Defectives	4.80%	4.72%
Producer's Risk at 1% Defectives	7.75%	5.33%
Producer's Risk at 1/2% Defectives	2.24%	1.36%
Average Accepted Defectives	3.13%	3.14%
Expected Test Cost (Test Items), Given:		
1/2% Defectives	45.66	37.49
1% Defectives	44.77	40.31
10% Defectives	19.44	20.32
Expected Test Cost, Given Lot Acceptance	46.0	33.0

TABLE 2

SENSITIVITY OF CONSUMER'S RISK
TO SAMPLE SIZE FOR SINGLE VERSUS
TWO-STAGE TEST PROCEDURES

Type of Procedure					
Single Stage			Two-Stage		
Sample Size	Prob.† of Acceptance (%)	Avg. Accept. Defectives (%)	Sample Size	Prob.† of Acceptance (%)	Avg. Accept. Defectives (%)
43	6.22	3.33	30	6.84	3.43
44	5.71	3.26	31	6.04	3.33
45	5.24	3.19	32	5.34	3.23
46*	4.80	3.13	33*	4.72	3.14
47	4.40	3.06	34	4.18	3.05
48	4.03	3.00	35	3.69	2.96
49	3.69	2.94	36	3.27	2.88

† Consumer's risk when lot percent defectives is 10%.

* Samples having comparable consumer's risk. For these samples the single-stage plan has the same average percent defectives for accepted lots.

Figure 4. Comparison of the Operating Characteristics of the Two-Stage Sequential Procedure (N=33) With an Equivalent Single-Stage Procedure (N=46)

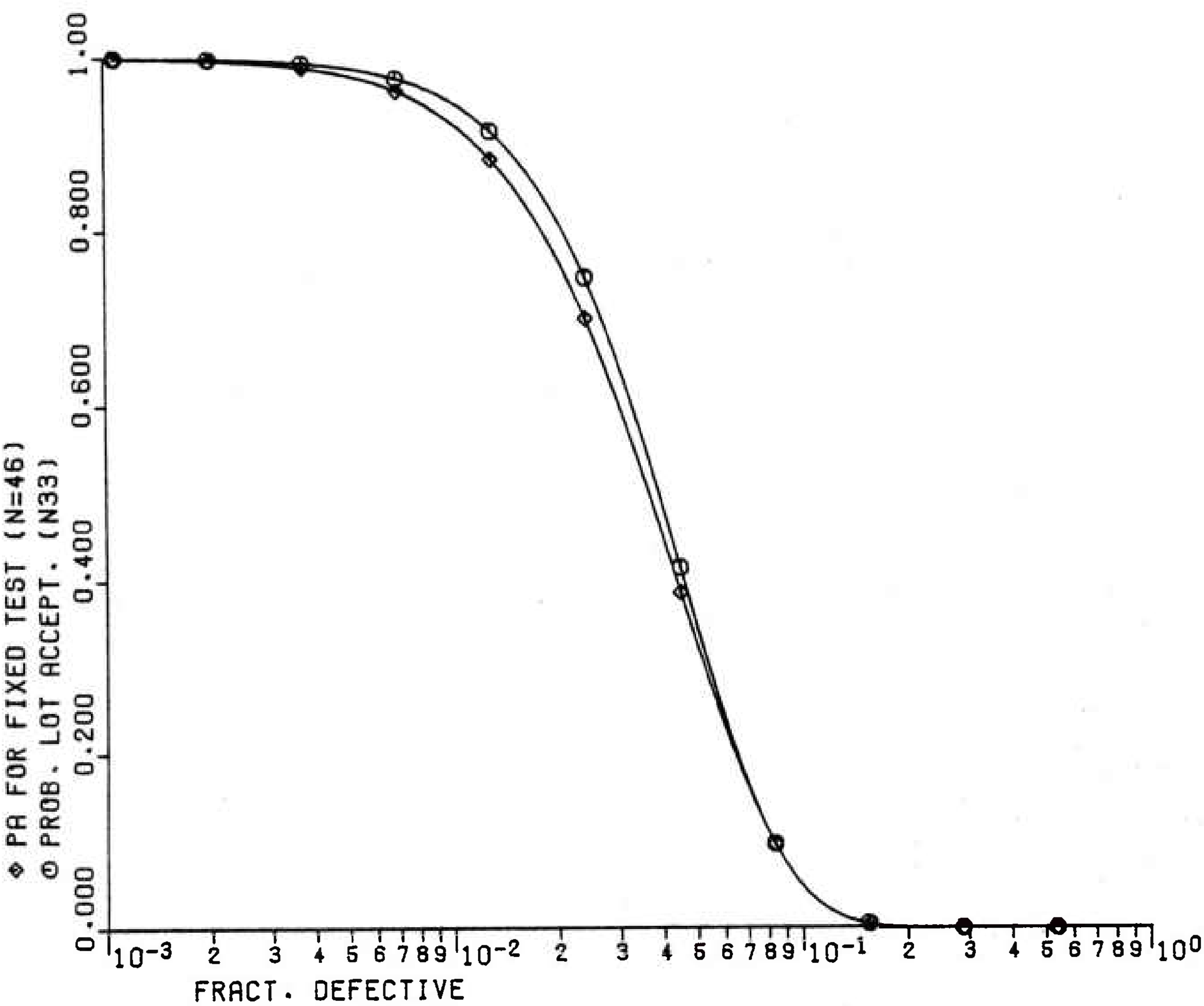


Figure 4a. Comparison of the Operating Characteristics of the Two-Stage Sequential Procedure (N = 33) With an Equivalent Single-Stage Procedure (N = 46) (Normal Probability and Log Scales)

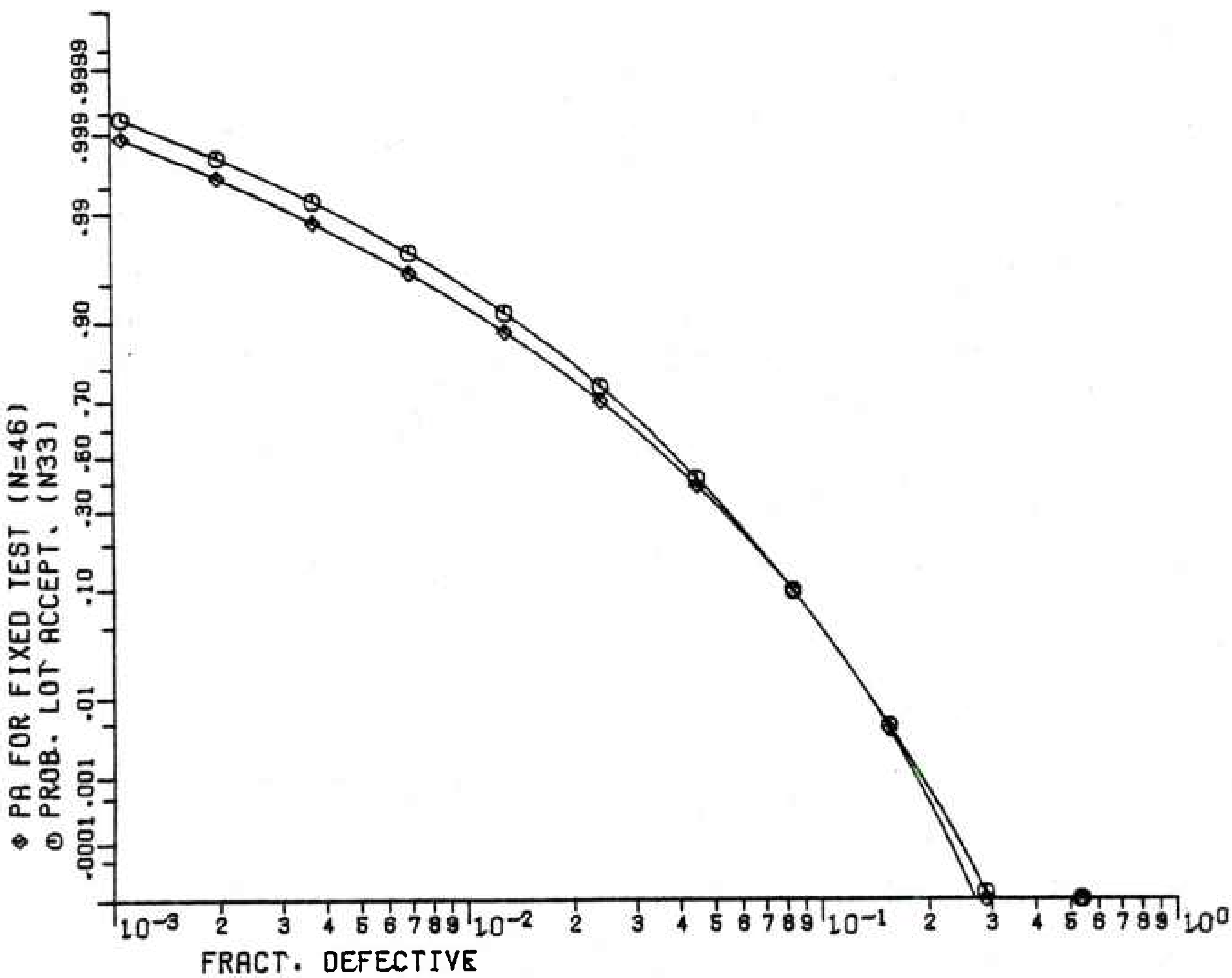


Figure 5. Comparison of the Expected Test Cost for Single-Stage and Two-Stage Sequential Procedures Having Nearly the Same Risks

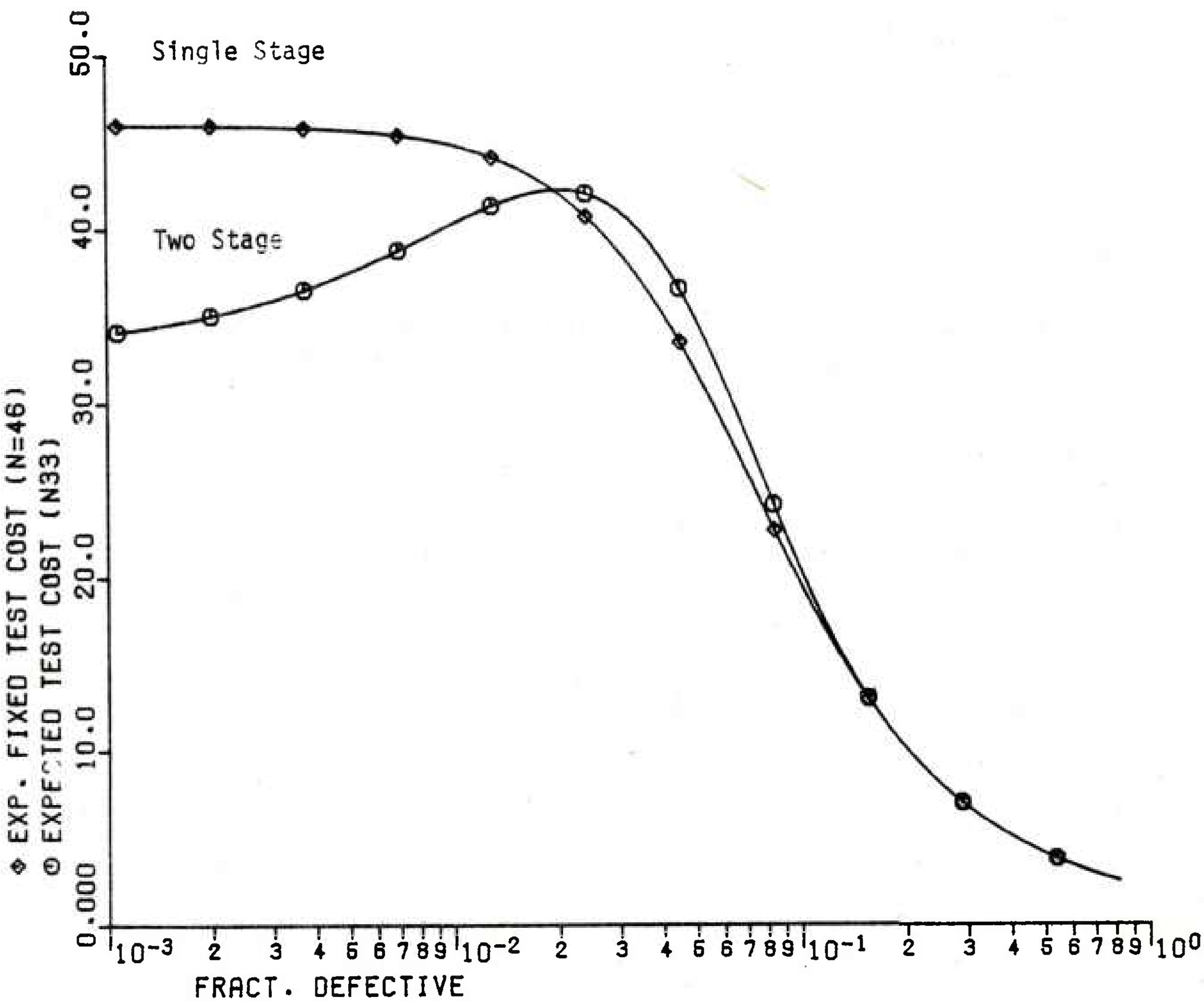
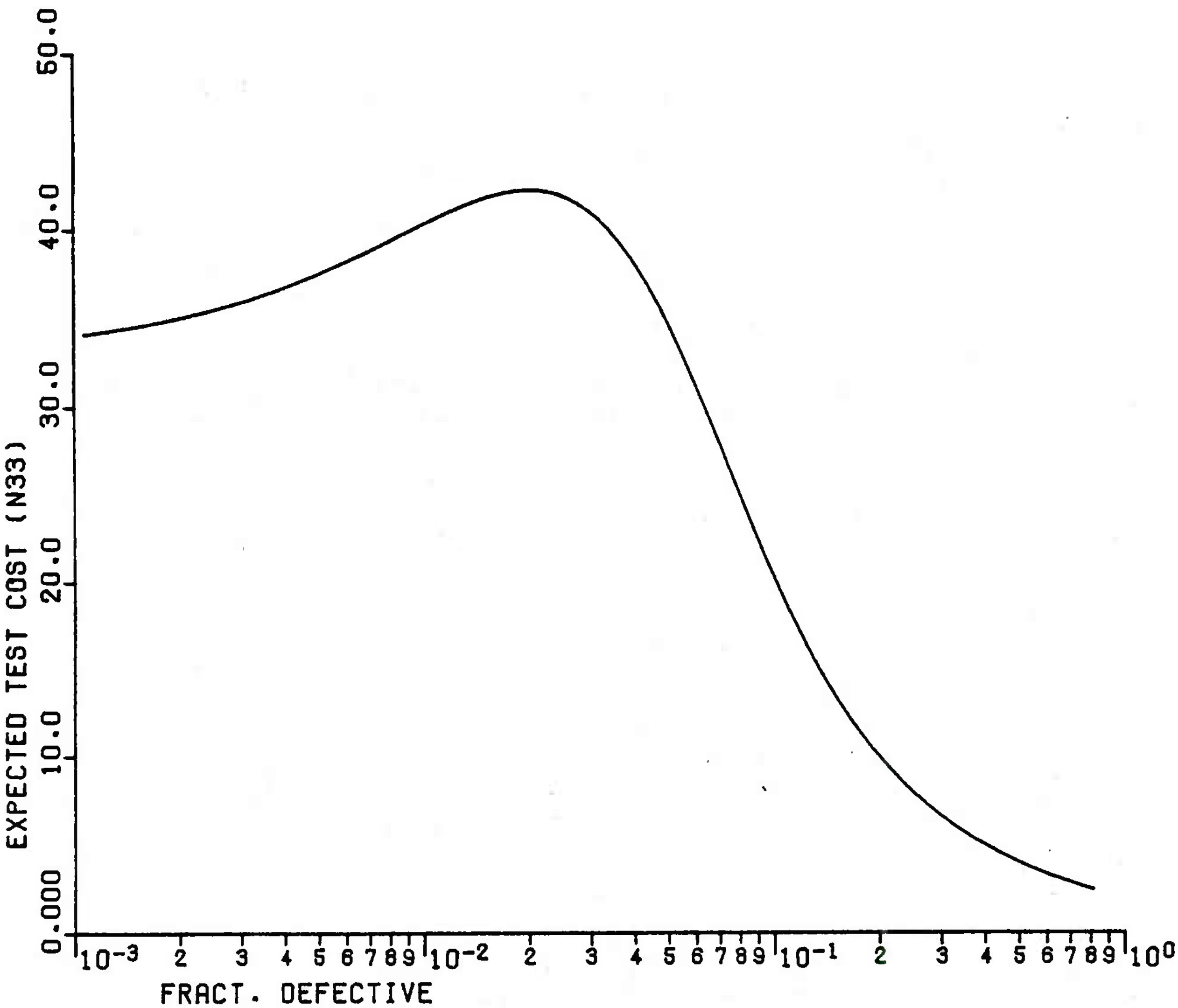


Figure 6. Expected Test Cost Versus Lot Fraction Defectives for a Two-Stage Sequential Procedure (N = 33)



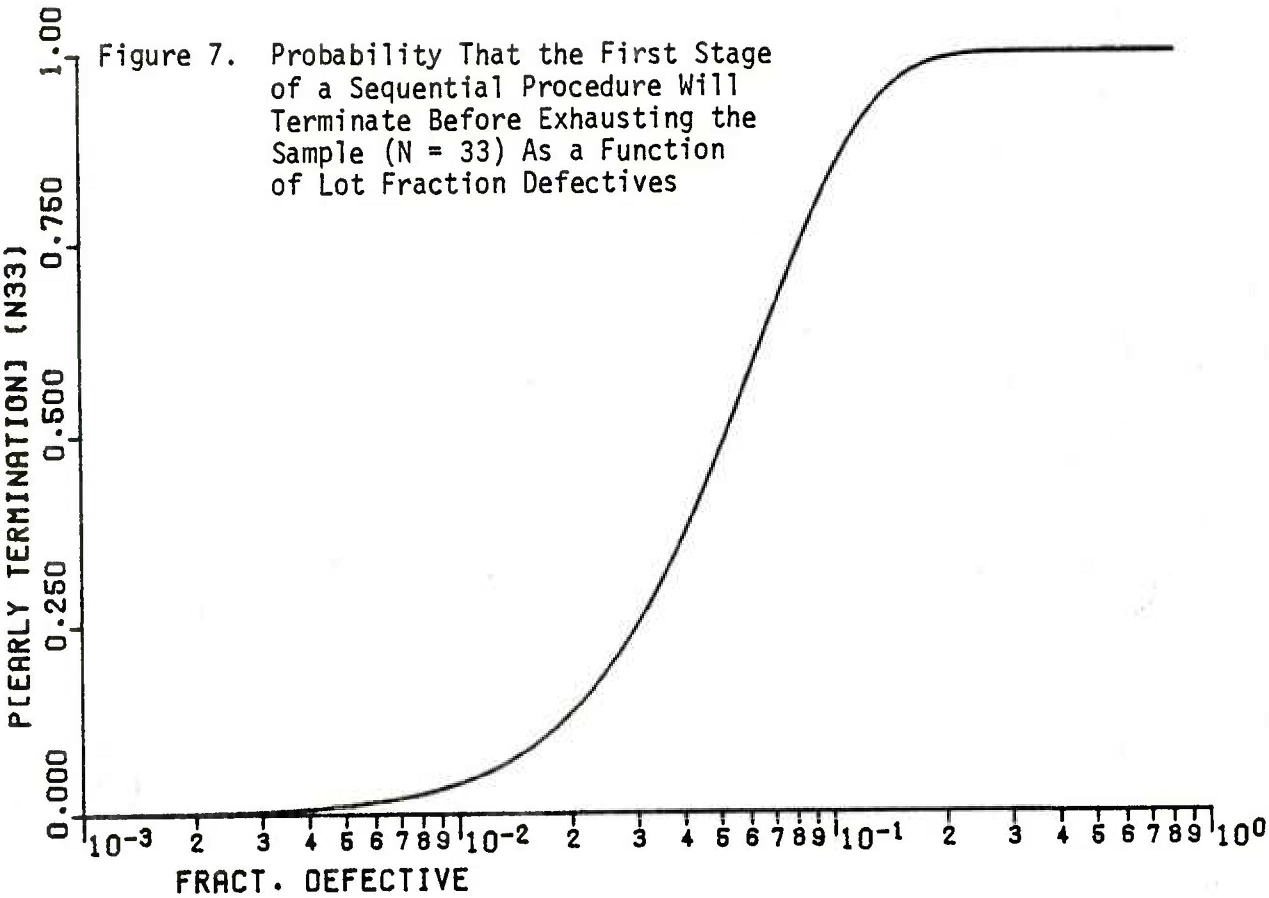


Figure 7. Probability That the First Stage of a Sequential Procedure Will Terminate Before Exhausting the Sample (N = 33) As a Function of Lot Fraction Defectives

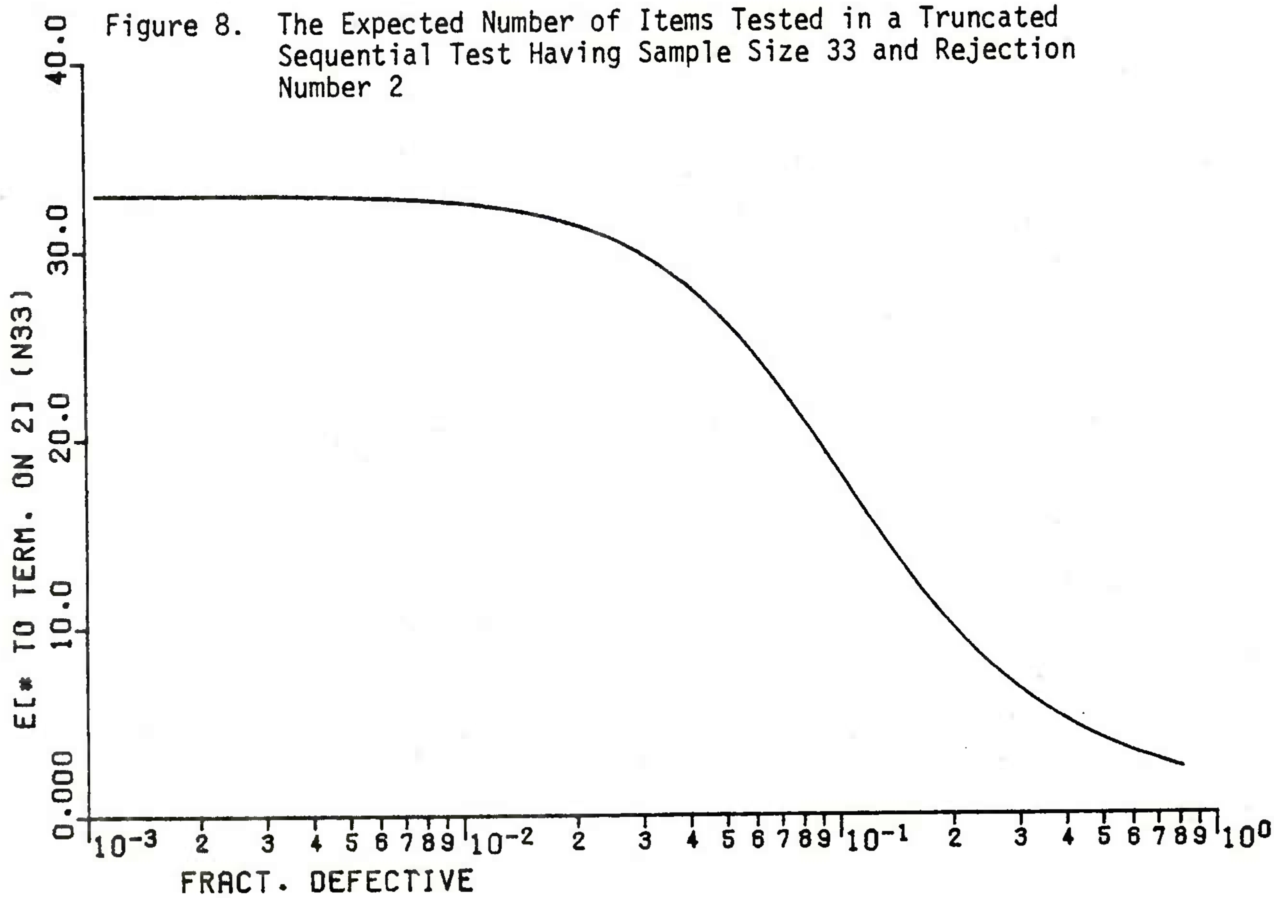


Figure 8. The Expected Number of Items Tested in a Truncated Sequential Test Having Sample Size 33 and Rejection Number 2

Figure 9. Illustration of the Effect of Sample Size on the OC of a Two-Stage Sequential Test Procedure

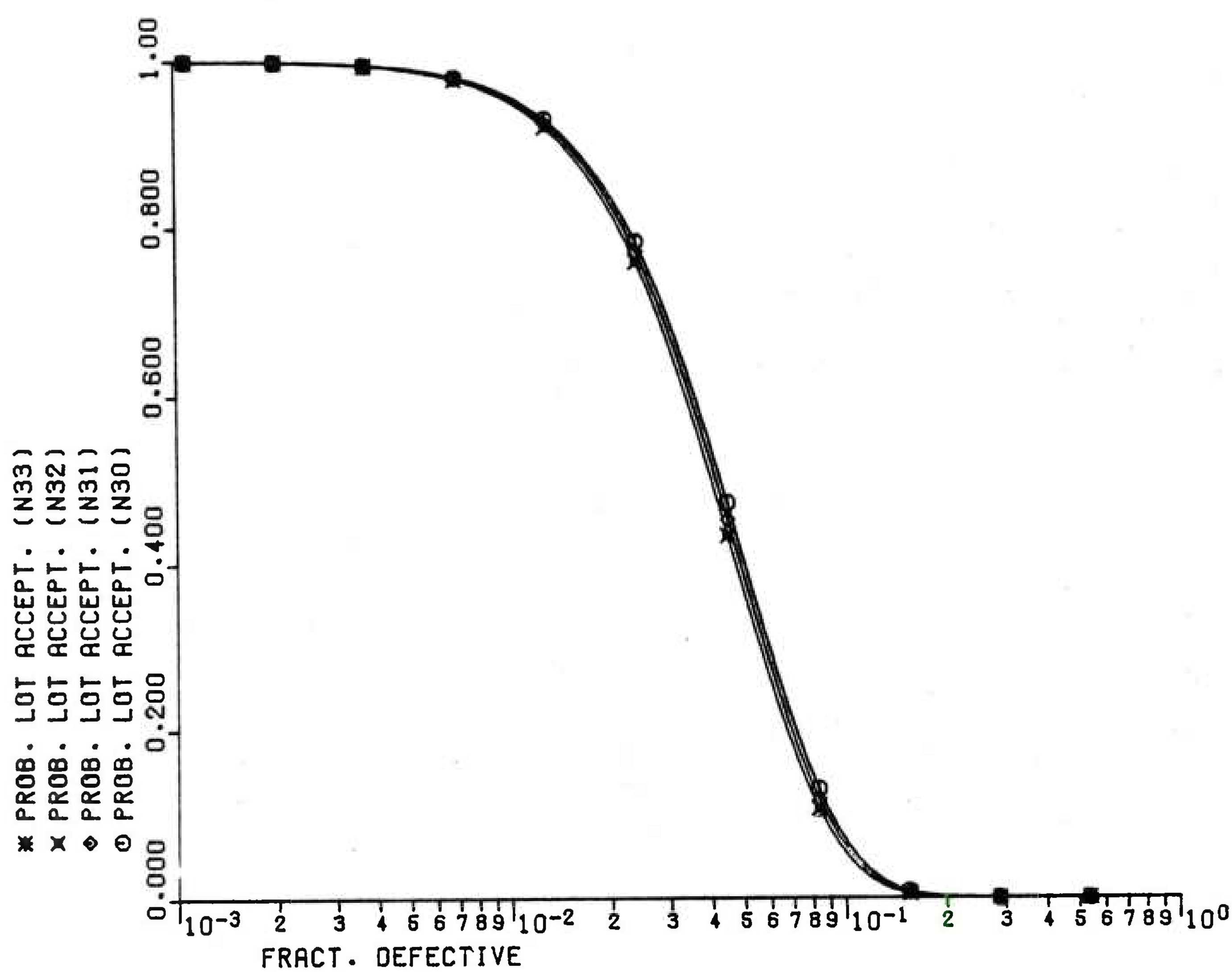
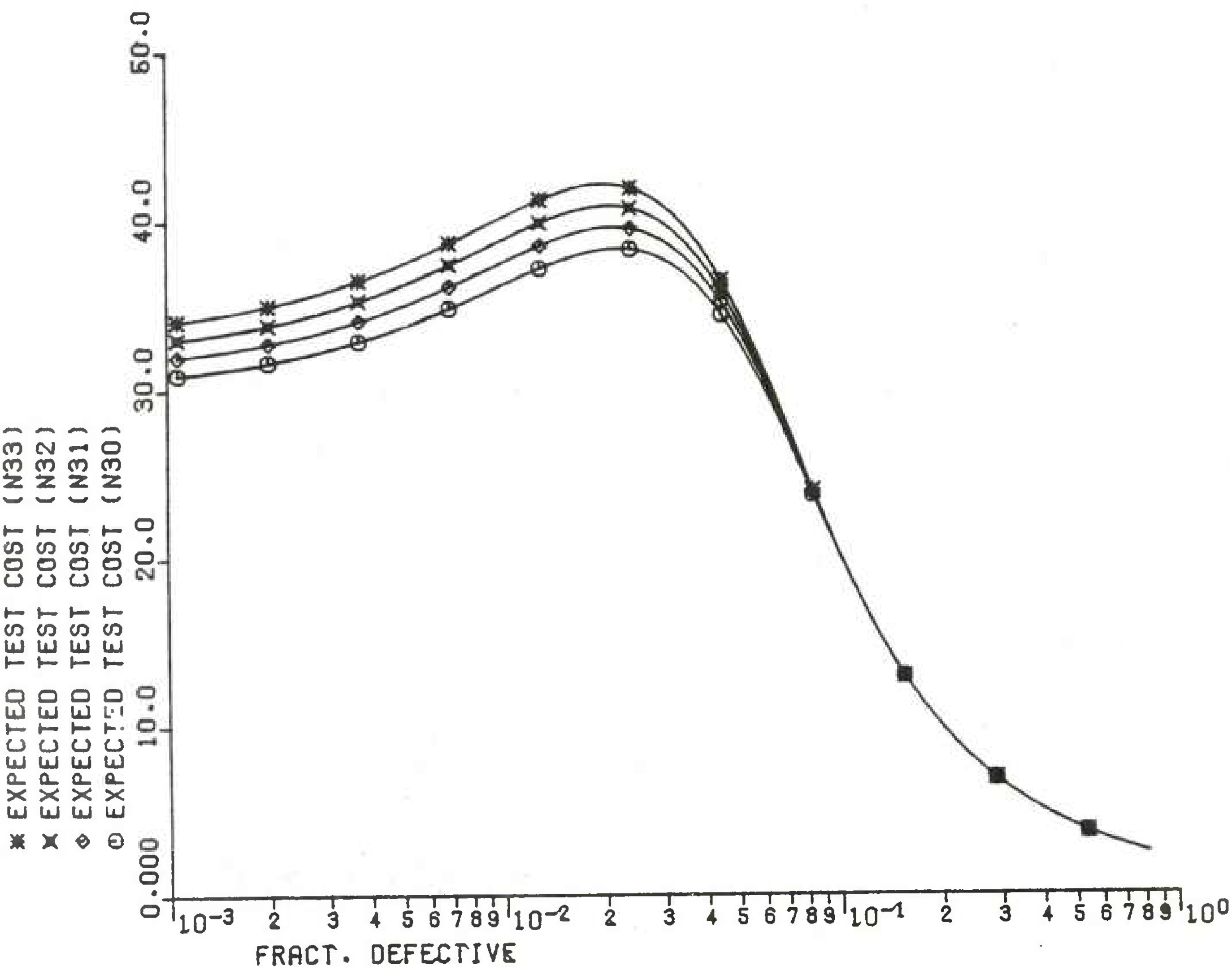


Figure 10. Expected Test Cost Versus Lot Fraction Defectives With Sample Size as a Parameter for a Two-Stage Sequential Test



13. Observations on Economic Sample Size Determination

An objective of this MFR is to submit the general concept of economic sample size determination as an object for consideration and to illustrate this by example. The principle of this method involves the selection of one or more parameters of a particular type of test procedure as independent variables for minimizing the cost of decision. The decision cost will, generally, incorporate the cost of testing, the operational costs associated with accepted product, and the costs incurred by rejecting lots. Because of the stochastic nature of these costs, a summary statistic of each of these must be used in the objective function. For example, one can write the objective function using expected values, given π, of these costs as follows:

$$ETC(\pi) \; + P_A(\pi)N_L\pi C_P + (1 - P_A(\pi))N_L C_R \quad ,$$

where

N_L is the lot size,

C_P is the unit penalty cost associated with using a defective item,

C_R is the item unit cost of treating -- perhaps, inspecting and retro-fitting -- rejected lots.

This function must also be averaged over incoming lot quality. A very simple algorithm for finding the minimum of this expression was found for the special case of a two-stage sequential test with N the independent variable and with C_R zero. This case is delineated in Annex 3. In a particular instance one may find certain input requirements of this methodology difficult to evaluate or estimate. For example, the probabilities associated with various contingencies of operation of the test item may be rather difficult to estimate. In the case of the M392 such parameters as the following required evaluation in order to estimate the operational penalty associated with encountering a defective item:

a. Probability that the cannon is damaged, given M392 projectile failure.

b. Probability that the tank escapes destruction in combat, given disablement of its major armament.

c. Probability that a lot of M392 projectiles is used in combat (as opposed to training).

This illustrates the difficulty of implementing the method of economic sample size.

14. <u>Summary</u>

In this memorandum we have demonstrated that changing the acceptance number for the M392 lot acceptance test poses an incrementally large risk of accepting bad product. It is suggested that multi-stage, sequential test procedures possess significant advantages over single-stage procedures. Using the traditional method of specifying α and β risks and associated π's, we designed alternative test procedures -- a single-stage and a comparable two-stage sequential procedure. The quantitative properties of these tests were calculated and displayed. Finally, an alternative to the traditional QA approach to acceptance test design was presented. This method has an economic orientation and does not require a prior risk specification but does require the evaluation of cost parameters, which may be difficult to estimate in a particular application.

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ANNEX 1

DERIVATION OF THE EQUATIONS FOR THE OPERATING CHARACTERISTIC AND EXPECTED TEST COST FOR A TWO-STAGE SEQUENTIAL LOT ACCEPTANCE TEST

In this annex a particular two-stage sequential test will be delineated whose intended application is lot acceptance for critical defects via functional tests. The probability of accepting a lot, given the proportion lot defectives, π, is called the operating characteristic of a test procedure. This probability of acceptance, $P_A(\pi)$, as a function of π, will be derived for large lots. The test procedure is viewed as sequential with the test terminating whenever the r th defective is discovered in sequence. Because this is a random event, a probabilistic measure of test cost, given π, will be derived. This quantity is designated the expected test cost or ETC(π) and is defined as the mathematical expectation of the number of rounds (samples) tested to discover the r th defective, truncated by the test sample size N.

<u>Test Description</u>

Briefly, this particular two-stage test proceeds as follows:

(1) Take a random sample of N from the lot.

(2) Proceed sequentially with functional tests of the sample.

(a) If no failures or defectives are found after testing N, immediately accept the lot.

(b) If only one (1) defective occurs after N tests, take another random sample of N.

(c) If two (2) defectives occur, stop testing immediately and reject the lot.

(3) In the event a second sample of N is drawn, test this sample sequentially and reject the lot if two defectives are encountered; otherwise accept.

This test can be described simply as a two-stage sequential test with first-stage acceptance and rejection numbers c = 0 and r = 2, respectively, and with second-stage rejection number r = 2.

Notationally, let

$$q = 1 - \pi \qquad (1.1)$$

and

$$p_j = P\{\text{exactly } j \text{ defectives are encountered in a sample of } N\} \qquad (1.2)$$

Then,

$$p_0 = q^N \qquad (1.3a)$$

$$p_1 = N\,\pi\,q^{N-1} \qquad (1.3b)$$

$$p_2 = \tfrac{1}{2}N(N-1)\,\pi^2\,q^{N-2} \quad . \qquad (1.3c)$$

From the test description, lot acceptance can occur in two distinct ways:

(1) No defectives in first stage, or

(2) One defective in first stage and zero or one in second stage.

Thus,

$$P_A(\pi) = P\{\text{accept, given } \pi\}$$

$$P_A(\pi) = p_0 + p_1(p_0 + p_1) \quad . \qquad (1.4)$$

By contrast, a single-stage plan with acceptance number c would have an operating characteristic

$$P_A(\pi) = \Sigma_{j=0}^{c}\, p_j \quad . \qquad (1.5)$$

Probability of Terminating

During an unconstrained sequential test with rejection number r, the probability of terminating on the j th sample (after the occurrence of the r th failure) is the negative binomial density:

$$p_t(j;r) = \binom{j-1}{r-1} p^r q^{j-r}, \; j \geq r \quad , \qquad (1.6)$$

where, notationally, $p \equiv \pi$, the prior probability of a failure (defective).

Also, let

$$P_t(n;r) = P\{\text{terminating the test on or before the } n \text{ th sample with the occurrence of } r \text{ failures}\} \qquad (1.7)$$

$$P_t(n;r) = \Sigma_{j=r}^{n} \binom{j-1}{r-1} p^r q^{j-r} \quad . \qquad (1.8)$$

Let

$$k = j - r + 1 \quad , \quad 1 \leq k < \infty \; . \qquad (1.9)$$

Then,

$$P_t(n;r) = p^r \Sigma_{k=1}^{n-r+1} \binom{k+r-2}{r-1} q^{k-1} . \qquad (1.10)$$

As an example with r equal to 1,

$$P_t(n;1) = p\ \Sigma_{k=1}^{n}\ q^{k-1} \quad .$$

$$= p\ (1-q^n)/(1-q)$$

$$P_t(n;1) = 1 - q^n \quad . \tag{1.11}$$

Also, for r equal to two a convenient, closed form expression for P_t is available. From (1.10),

$$P_t(n;2) = p^2\Sigma_{k=1}^{n-1}\ kq^{k-1}$$

$$P_t(n;2) = p^2q^{-1}\Sigma_{k=1}^{n-1}\ kq^k$$

Notationally,

$$S_1 = \Sigma_{k=1}^{n-1}\ kq^k$$

where

$$S_1 = (1 - q^n)p^{-2} - p^{-1}(1 + (n-1)q^n) \quad . \tag{1.12}$$

Therefore,

$$P_t(n;2) = (1 - q^n)q^{-1} - pq^{-1}(1 + (n-1)q^n) \quad . \tag{1.13}$$

The expression for $P_t(n;3)$ is also instructive to derive.

From (1.10),

$$P_t(n;3) = p^3\Sigma_{k=1}^{n-2}\ \binom{k+1}{2}q^{k-1}$$

$$= \frac{p^3}{2q}\ \Sigma_{k=1}^{n-2}\ (k+1)\ kq^k$$

$$P_t(n;3) = \frac{p^3}{2q}[\Sigma_{k=1}^{n-2}\ k^2q^k + \Sigma_{k=1}^{n-2}\ kq^k] \quad . \tag{1.14}$$

But

$$Q_1 = \Sigma_{k=1}^{n-1}\ k^2q^k = q\frac{dS_1}{dq} \quad . \tag{1.15}$$

Then, from (1.12) and (1.15),

$$Q_1 = -\ nq^np^{-2} + 2q(1 - q^n)p^{-3}$$

$$- n(n - 1)q^np^{-1} - qp^{-2}(1 + (n-1)q^n) \quad . \tag{1.16}$$

The sums in (1.14) are related to S_1 and Q_1 as follows:

$$S = \Sigma_{k=1}^{n-2}\ kq^k = S_1 - (n - 1)q^{n-1} \tag{1.17a}$$

$$Q = \Sigma_{k=1}^{n-2}\ k^2q^k = Q_1 - (n - 1)^2q^{n-1} \quad . \tag{1.17b}$$

From (1.14),

$$P_t(n;3) = \frac{p^3}{2q}(Q + S) \quad . \qquad (1.18)$$

A strategy similar to that used in deriving $P_t(n;3)$ can be used to obtain closed-form expressions for $P_t(n;4)$, but the result is inelegant*. However, the preceding results from (1.6) through (1.18) will suffice for present purposes. Before abandoning the unconstrained sequential test, it is noted that the expected value of the negative-binomial, random variable j in (1.6) is given by

$$E[j] = r/p \quad , \qquad (1.19)$$

a particularly simple result.

Expected Test Cost for the Two-Stage Sequential Test

A general, standardized way to measure test cost is in units of test items required to complete the test. For a two-stage sequential test, such as the one being discussed, the expected test cost involves two types of conditional expectations, given π. The first, EN_1 , is the expected number of items contributed by experimental realizations which terminate with two defectives at or before the N th sample.

Thus,

$$EN_1 = \Sigma_{j=2}^{N} jP\{2 \text{ nd failure occurs at the j th sample}\} \qquad (1.20)$$

Using the notation of equation (1.6) ,

$$EN_1 = \Sigma_{j=2}^{N} jP_t(j;2) \quad . \qquad (1.21)$$

The second type of conditional expectation, EN_2 , is the expected value, given π , of the number of samples tested before encountering the 2 nd defective, limited by the sample size N. Formally,

$$EN_2 = \Sigma_{j=2}^{N-1} jP\{2 \text{ nd failure occurs at the jth sample}\}$$
$$+ NP\{\text{a 2 nd failure does not occur by the N-1 st sample}\}. \qquad (1.22)$$

---

* Generally,

$$P_t(n;r) = 1 - \Sigma_{j=0}^{r-1} \binom{n}{j} p^j q^{n-j} \quad .$$

Using previous notation,

$$EN_2 = \Sigma_{j=2}^{N-1} jp_t(j;2) + N(1 - P_t(N-1;2)) \quad , \tag{1.23}$$

or, from (1.6) with

$$k = j - 1$$

and

$$q = 1 - \pi \ ,$$

$$EN_2 = \pi^2 q^{-1} \Sigma_{k=1}^{N-2} (k+1)kq^k + N(1 - \pi^2 q^{-1} \Sigma_{k=1}^{N-2} kq^k) \quad . \tag{1.24}$$

and,

$$EN_2 = \pi^2 q^{-1} [\Sigma_{k=1}^{N-2} k^2 q^k + \Sigma_{k=1}^{N-2} kq^k] + N(1 - \pi^2 q^{-1} \Sigma_{k=1}^{N-2} kq^k) \quad . \tag{1.25}$$

Apart from other interpretations, EN_2 is also the expected test cost for a sequential, single-stage procedure having a rejection number of 2.

Referring to (1.17), where the above sums have been evaluated, one can write

$$EN_2 = \pi^2 q^{-1}(Q + S) + N(1 - \pi^2 q^{-1} S) \quad . \tag{1.26}$$

Similarly, for EN_1 , from (1.21) ,

$$EN_1 = \pi^2 q^{-1} \Sigma_{k=1}^{N-1} (k + 1) kq^k \tag{1.27}$$

or

$$EN_1 = \pi^2 q^{-1}(Q_1 + S_1) \quad , \tag{1.28}$$

where Q_1 and S_1 are displayed in (1.16) and (1.12), respectively.

Having developed expressions for the auxilliary expectations, we return to the expected test cost for the particular two-stage procedure under consideration. Referring to the procedure, note that cost N is incurred <u>if</u> no failures are encountered at the first stage; and cost $N + EN_2$ is incurred, given exactly one at the first stage. The contribution to the ETC from failures to finish the first stage with less than two defectives is EN_1 .

Thus,

$$ETC = p_0 N + p_1(N + EN_2) + EN_1 \quad . \qquad (1.29)$$

This expression has been evaluated as a function of π with N as a parameter. Results are displayed in the figures contained in the body of this memorandum.

ANNEX 2

DERIVATION OF THE EQUATIONS FOR AVERAGE ACCEPTED LOT QUALITY (AAQ) FOR SEVERAL ACCEPTANCE TEST PROCEDURES

In this annex the formulas for AAQ, used elsewhere in this memorandum, are derived. The AAQ is somewhat of a misnomer since what is meant is the average fraction defectives for accepted lots. This average is taken over all incoming lots. Therefore, the probability density of lot proportion defectives, π, for incoming lots is required. Generally, this density function is unknown; so that, conventionally, a uniform density for π is assumed, implying complete ignorance of incoming lot quality (Laplace's principle).

By definition, the AAQ equals

$$\int \pi P\{\text{accept lot}|\pi\} P\{\pi<\Pi<\pi + d\pi\}d\pi \Big/ \int P\{\text{accept lot}|\pi\} P\{\pi<\Pi<\pi + d\pi\}d\pi \qquad (2.1)$$

Assume

$$f(\pi) = P\{\pi<\Pi<\pi + d\pi\}/d\pi \quad ,$$
$$= \xi^{-1} \quad , \quad 0<\pi<\xi<1 \quad . \qquad (2.2)$$

Then,

$$AAQ = U/D \qquad (2.3a)$$

with

$$U = \int_0^\xi \pi P_A(\pi)d\pi \qquad (2.3b)$$
$$D = \int_0^\xi P_A(\pi)d\pi \quad . \qquad (2.3c)$$

I. <u>Single-Stage Procedures</u>

Functional values, AAQ(N), are derived below for tests having acceptance numbers,

c = 0, 1, and 2, respectively.

<u>c = 0</u>

$$P_A(\pi) = (1 - \pi)^N \qquad (2.4)$$

Then,

$$D(N) = \int_0^\xi (1 - x)^N dx \quad . \qquad (2.5)$$
$$D(N) = [1 - (1-\xi)^{N+1}]/(N + 1) \quad . \qquad (2.6)$$

However, with N typically greater than 30 and $\xi \ll 1$,

$$(1 - \xi)^N \sim 0 .$$

Approximately,

$$D(N) \cong 1/(N + 1) \quad . \tag{2.7}$$

Also, from (2.3) and (2.4) ,

$$U(N) = \int_0^{\xi} x(1 - x)^N dx \quad . \tag{2.8}$$

After integration by parts,

$$U(N) \cong 1/[(N + 1)(N + 2)] \quad . \tag{2.9}$$

Then,

$$AAQ = U/D = 1/(N + 2) \quad . \tag{2.10}$$

This result applies for c = 0.

<u>c = 1</u>

For a single-stage acceptance test with acceptance number equal to 1,

$$P_A(\pi) = (1 - \pi)^N + N\pi(1 - \pi)^{N-1} \quad . \tag{2.11}$$

Then,

$$D(N) = \int_0^{\xi}(1 - x)^N dx + N\int_0^{\xi} x(1 - x)^{N-1} dx \quad . \tag{2.12}$$

$$D(N) = 1 - (1 - \xi)^N - [1 - (1-\xi)^{N+1}](N - 1)/(N + 1) \quad . \tag{2.13}$$

However, with N large and $\xi \ll 1$,

$$(1 - \xi)^N \sim 0 \quad .$$

Approximately,

$$D \cong 2/(N + 1) \quad . \tag{2.14}$$

For example, with

$D = 46$

and

$\xi = 0.25$,

$D = 4.255269 \ 10^{-2}$, exactly;

$\cong 4.255319 \ 10^{-2}$, from (2.14).

From (2.3) and (2.4),

$$U(N) = \int_0^{\xi} x(1 - x)^N dx + N\int_0^{\xi} x^2(1 - x)^{N-1} dx \quad . \tag{2.15}$$

After some manipulation,

$$U(N) = \frac{3}{(N + 1)(N + 2)} \quad . \tag{2.16}$$

Then,

$$AAQ = U/D = \frac{3}{2(N + 2)} \quad . \tag{2.17}$$

This result applies for c = 1, and may be compared with (2.10).

c = 2

For a single-stage acceptance test with acceptance number equal to 2,

$$P_A(\pi) = (1 - \pi)^N + N\pi(1 - \pi)^{N-1} + \frac{N(N - 1)}{2}\pi^2(1 - \pi)^{N-2} \quad . \tag{2.18}$$

$$D(N) = \int_0^\xi (1 - x)^N dx + N\int_0^\xi x(1 - x)^{N-1} dx + \frac{N(N - 1)}{2}\int_0^\xi x^2(1 - x)^{N-2} dx \quad . \tag{2.19}$$

For large N,

$$D(N) \cong 3/(N + 1) \quad . \tag{2.20}$$

With (2.18),

$$U(N) = \int_0^\xi x P_A(x) dx$$

becomes

$$U(N) = \int_0^\xi x(1 - x)^N dx + N\int_0^\xi x^2(1 - x)^{N-1} dx + \frac{N(N - 1)}{2}\int_0^\xi x^3(1 - x)^{N-2} dx \quad . \tag{2.21}$$

And,

$$U(N) = 6/[(N + 1)(N + 2)] \quad . \tag{2.22}$$

Therefore,

$$AAQ = U/D = 2/(N + 2) \quad . \tag{2.23}$$

This result applies for c = 2, and may be compared with (2.10) and (2.17).

II. A Two-Stage Procedure

For a two-stage, sequential procedure having equal samples and with acceptance numbers 0 for the first stage and 1 for the second, and with rejection numbers 2 for the first and 2 for the second, the operating characteristic is given by

$$P_A(\pi) = (1 - \pi)^N + N\pi(1 - \pi)^{2N-1} + N^2\pi^2(1 - \pi)^{2N-2} \quad . \tag{2.24}$$

As above for single-stage procedures,

$$D(N) = \int_0^{\xi} P_A(x)dx$$

$$D(N) = \int_0^{\xi}(1 - x)^N dx + N\int_0^{\xi} x(1 - x)^{2N-1}dx$$
$$+ N^2\int_0^{\xi} x^2(1 - x)^{2N-2}dx \quad . \qquad (2.25)$$

For large N,

$$D \cong \frac{1}{N + 1} + \frac{N}{2N} - \frac{N}{2N + 1} + N^2\left[\frac{1}{2N + 1} - \frac{1}{N} + \frac{1}{2N - 1}\right] ,$$

which simplifies somewhat to

$$D \cong \frac{1}{N + 1} + \frac{1}{2(2N + 1)} + \frac{N}{(2N + 1)(2N - 1)} \quad . \qquad (2.26)$$

With (2.24) ,

$$U(N) = \int_0^{\xi} xP_A(x)dx$$

$$U(N) = \int_0^{\xi} x(1 - x)^N dx + N\int_0^{\xi} x^2(1 - x)^{2N-1}dx$$
$$+ N^2\int_0^{\xi} x^3(1 - x)^{2N-2}dx \quad . \qquad (2.27)$$

After some manipulation,

$$U(N) \cong \frac{1}{(N + 1)(N + 2)} + \frac{5N - 1}{(2N - 1)(2N + 1)(2N + 2)} \quad . \qquad (2.28)$$

Finally,

$$AAQ = U/D$$
$$= \left[\frac{1}{(N + 1)(N + 2)} + \frac{5N - 1}{2(2N - 1)(2N + 1)(N + 1)}\right] /$$
$$\left[\frac{1}{N + 1} + \frac{1}{2(2N + 1)} + \frac{N}{(2N - 1)(2N + 1)}\right]$$

$$AAQ = \frac{13N^2 + 9N - 4}{(N + 2)(12N^2 + 3N - 3)} \quad . \qquad (2.29)$$

This result applies to this two-stage sequential procedure. Asymptotically, for large N,

$$AAQ \sim (13/12)N^{-1} \quad .$$

ANNEX 3

ECONOMIC CRITERION FOR OPTIMAL SAMPLE SIZE FOR THE TWO STAGE SEQUENTIAL TEST APPLIED TO LOT ACCEPTANCE OF AMMUNITION EXHIBITING CRITICAL DEFECTS

The consequences of increasing the test sample from N to N + 1 are twofold: the expected test cost increases by ΔC_T , where

ΔC_T = average over proportion lot defectives of [ETC(N + 1) - ETC(N)]† times the unit test cost

$\cong$ unit test cost, C_T ; (3.1)

and the average accepted lot defectives decreases*. The average penalty cost associated with encountering a defective during combat is C_p , which is calculated below for the M392 projectile. Thus, the average differential penalty cost affected by increasing N by unity for lots of size N_L is

$$\Delta C_p = N_L C_p[AAQ(N - 1) - AAQ(N)] \quad . \qquad (3.2)$$

For the optimal sample size N* the cost of adding an additional unit should just exceed the reduction in the expected penalty cost. Mathematically, this statement is equivalent to

$$C_T \leq \Delta C_p(N^*) \qquad (3.3a)$$

and

$$C_T > \Delta C_p(N^* + 1) \quad . \qquad (3.3b)$$

Forming the ratio

$$\rho = \frac{C_T}{C_p N_L} \qquad (3.4)$$

and the function

$$\phi(N) = AAQ(N - 1) - AAQ(N) \quad , \qquad (3.5)$$

equations 3.3a and b can be compactly written as

$$\rho \leq \phi(N^*) \qquad (3.6a)$$

and

$$\rho > \phi(N^* + 1) \quad . \qquad (3.6b)$$

---

† See Annex 1 for definition of ETC(N).

* In the development here costs related to actions taken on rejected lots are neglected.

The expression for AAQ(N) for the sequential test, derived in Annex 2, is repeated here as

$$AAQ(N) = \left[\frac{1}{(N+1)(N+2)} + \frac{5N-1}{(2N+2)(2N+1)(2N-1)}\right] \Big/ \left[\frac{1}{N+1} + \frac{1}{2(2N+1)} + \frac{N}{(2N+1)(2N-1)}\right] . \qquad (3.7)$$

Then, equations (3.4 thru 3.7) suffice to determine the cost-optimal sequential test sample. Values of $\phi(N)$ are tabulated below for N = {20,(1),60} for use in an example.

Example

We take the case of the 105mm M392 projectile used in the M68 tank cannon. Breakup of metal parts of the M392 in bore can cause damage such that subsequent rounds will render the tube non-functional. It is estimated that, approximately, one disabled cannon occurs for eight such incidents of metal parts failure. Other pertinent assumptions are the following:

(1) The unit test cost, C_T , for small samples is about \$1.1K.

(2) The cost of replacing a M68 cannon tube is \$8.0K and the cost of tube and breech is about \$12.0K.

(3) Given a disabling incident, about one-half the time both tube and breech require replacement. Therefore, the expected cost of a cannon replacement is \$10K.

(4) During combat, the tank rendered fire-inoperable by the occurrence of metal parts breakup of the M392 will be in the process of engaging enemy weapon systems. With probability 0.75 the tank in this situation will be able to avoid being defeated by enemy weapon systems.

(5) The probability that an accepted lot of projectiles will actually be used in combat is 0.02.

(6) The average cost of tanks in combat which fire the M392 projectile is \$900K.

(7) The average lot size is 2000.

With these assumptions the expected penalty cost associated with a critical defect in the M392 can be calculated as follows.

C_p = expected penalty during combat * P{lot used in combat} + expected penalty during training * P{lot used in training},

with the expected penalty during combat = average replacement cost of cannon times probability that cannon is damaged, given M392 projectile failure, times probability that tank escapes destruction plus average replacement cost of tank times probability that cannon is damaged, given critical failure of M392, times probability that tank is destroyed. And, the expected penalty during training = average replacement cost of cannon times probability that cannon is damaged, given M392 projectile failure. (3.8)

Then,

$$C_p(\$K) = (10 * 0.125 * 0.75 + 900 * 0.125 * 0.25) * 0.02 + 10 * 0.125 * 0.98 = 1.8 .$$

Therefore,

$$\rho = C_T/C_p/N_L$$

$$\rho = 1.1/1.8/2000 = 3.05\ 10^{-4}$$

From Table 3.1 ,

$$\phi(58) > \rho$$

$$\phi(59) < \rho .$$

Therefore, the optimal value of N in this example is 58.

Due to the uncertain nature of the parameter values used in this example, a parametric sensitivity analysis is provided in Table 3.2. Generally, a change of 20% in each of the parameters implies a change of about six units or 10% in the optimal value of N.

TABLE 3.1

VALUES OF AVERAGE ACCEPTED QUALITY (AAQ) AND ITS FIRST BACKWARD DIFFERENCE (ϕ) AS FUNCTIONS OF SAMPLE SIZE (N) FOR A TWO-STAGE SEQUENTIAL TEST

N (Units)	AAQ (10^{-2})	ϕ (10^{-4})	N (Units)	AAQ (10^{-2})	ϕ (10^{-4})
20	5.0312	24.536	40	2.6076	6.434
21	4.8076	22.353	41	2.5463	6.131
22	4.6031	20.450	42	2.4878	5.849
23	4.4153	18.779	43	2.4320	5.586
24	4.2423	17.306	44	2.3785	5.341
25	4.0823	15.999	45	2.3274	5.111
26	3.9339	14.835	46	2.2785	4.896
27	3.7960	13.794	47	2.2315	4.694
28	3.6674	12.859	48	2.1865	4.505
29	3.5472	12.016	49	2.1432	4.326
30	3.4347	11.253	50	2.1016	4.158
31	3.3291	10.561	51	2.0616	4.000
32	3.2298	9.930	52	2.0231	3.851
33	3.1362	9.355	53	1.9860	3.710
34	3.0480	8.828	54	1.9502	3.576
35	2.9645	8.344	55	1.9158	3.449
36	2.8855	7.899	56	1.8825	3.330
37	2.8106	7.489	57	1.8503	3.216
38	2.7395	7.110	58	1.8192	3.108
39	2.6720	6.759	59	1.7892	3.005
			60	1.7601	2.907

TABLE 3.2

SENSITIVITY OF THE COST-OPTIMUM M392 PROJECTILE ACCEPTANCE TEST SAMPLE SIZE TO SEVERAL PARAMETERS

Parameter	Value	Change (%) Re: Nominal	Optimum Sample Size, N*	Change in N*
Probability that defective proj. disables cannon	0.15	20	64	+6
Probability that fire-disabled tank retreats successfully	0.90	20	52	-6
Probability that lot of projectiles is used in combat	0.04	100	67	+9
Avg. unit cost of tank	$1.08M	20	60	+2
Avg. unit test cost	$1.32K	20	53	-5
Avg. lot size	2400	20	64	+6

Memorandum for Record

REVIEW

OF

AMSAA

TECHNICAL

REPORT

NO. 242

George J. Schlenker

DRSAR-PEL 7 July 1980

MEMORANDUM FOR RECORD

SUBJECT: Review of AMSAA Technical Report No. 242

1. Reference:

a. Message, DRDAR-CG, HQ, ARRADCOM, 201915Z Jun 80, subject: 155mm XM211 Propelling Charge.

b. Technical Report No. 242, AMSAA, 1978, title: 155mm Propellant Charge Cost and Operational Effectiveness Study.

2. Background

In an attempt to challenge the requirement for the 155mm XM211 propelling charge, DRSAR-AS had raised the issue of whether it was cost-effective to continue to develop and, subsequently, to procure the XM211 charge. This charge is intended to replace the M3 and M4 charges in the M109A1 SP howitzer and M198 artillery systems. In view of the large prospective procurement cost exceeding $300 million to obtain a minimum required inventory of XM211 charges, it seemed reasonable to request that a COEA be performed to support continued development of the XM211 versus the alternative of retaining the current system. In response to a proposal to that effect by the ARRCOM CG, the ARRADCOM Commander sent the Reference a message wherein it is stated that a COEA (Reference b) had already been performed for 155mm propelling charges. In the context of the foregoing events I have reviewed the COEA, AMSAA TR 242.

3. General Comments

Seven (7) propelling charge options or mixes are considered in TR 242 all of which contain developmental charges. These charges satisfy the NATO Memorandum of Understanding (MOU) with respect to projectile muzzle velocity for new artillery developments. Using the MOU velocities for each charge and zone the relative frequency of charge use was calculated in an artillery combat simulation set in Europe in the 1980's. In this connection several assumptions were made which seem contrary to fact:

a. Although the artillery combat model treated a European scenario in the early 1980's, it was implicitly assumed that M198 howitzers, which could employ the M203 propelling charge, were in respectable densities. (The M203 appears in nearly every option.) This is contrary to fact. Actually, units in Europe are principally Mechanized Infantry and Armor which receive direct artillery support from the M109A2/A3 system. This system is not compatible with the M203 charge. Only a few M198's would appear in Europe with airmobile units flown from CONUS with the advent of hostilities.

b. The family of 155mm projectiles -- XM795, M483, M549 -- used with each mix of propelling charges does not include the M107 which will be used in Europe during the time period under study. Similarly the 8-inch, DS reinforcing and general support artillery in the weapons mix did not employ the M106 projectile. Therefore, lethality and accuracy estimates in that caliber are based only on the XM711, XM650E4, and M509 projectiles.

4. It is noted that AMSAA found that for zones corresponding to the M4A2 6W and lower zones only 4% of the rounds were fired under a "max charge" algorithm (for zone assignment) and only 13% under a "min charge" algorithm. Further, under the "max charge" algorithm only 8% of the missions used the XM211 charge with min QE = 200 mils and only 21% with min QE = 300 mils in heavy divisions. On this basis it is questionable whether the artillery system mix effectiveness is very sensitive to the choice of XM211 versus M4 charge.

5. Costing of Options

It appears that AMSAA TR 242 was exclusively intended to provide information for choosing between developmental propelling charge alternatives -- not between the present set of 155mm charges and an alternative set with the XM211 simply replacing the M3 and M4 charges. Actually, none of the seven (7) charge options examined in detail addresses the issue: XM211 versus M3 and M4. The only reference to a charge option containing M3 and M4 is found on page 27 in which these charges are combined with the M119 and M203 to be costed as option number 8. However, the weighted average unit cost of option 8 cannot be compared with developmental options to evaluate the cost contribution of the XM211 charge since its effect is confounded with the much larger effect of zone 7.

Using the cost data for these charges provided in TR 242, I have calculated the weighted average unit charge cost for the option (no. 1) which includes the XM211, M119, and M203. Using AMSAA's relative frequencies (weights) for each charge, the average unit cost for this option is $95.7 (FY 77). This is slightly larger than the average unit cost of the current option -- $95.4 (FY 77). An example of the costing of charge options is shown in Table 1 in which AMSAA's option 1 is compared with the current option. Based upon the distribution of 155mm propelling charges with zone used in the CAA P-86 Combat Rates Study (and confirmed by an ARRCOM MFR of 25 March 1980), the overall fractional use of XM211 charges in a European scenario is 30% with 49% M119 usage and 21% M203 usage. Applying these weights to the unit charge costs yields a weighted average unit cost of

$$\$88.4 = \$58(0.30) + \$91(0.49) + \$126(0.21)$$

for the current system and

$$\$89.6 = \$62(0.30) + \$91(0.49) + \$126(0.21)$$

for ARRCOM option no. 1. Regardless of the costing assumptions, the necessity to establish a new production base for the XM211 and the cost of acquiring a minimum inventory of about four million XM211 charges renders ARRCOM option no. 1 quite expensive relative to staying with the current system of propelling charges.

It is noted that the above costs do not include a per round cost for tube replacement. Whenever charge options differ significantly with respect to erosion and wear, it is important to include prorated tube costs to accurately reflect the true differential cost between options. For example, it is recognized that the life of the M185 cannon using the M107 projectile is 5000 rounds with the M119 charge, which is limited by fatigue in this case. However, the XM201E5 produces a wear-limited life of 2100 rounds. Thus, a $13,000 tube would prorate to $2.60 if fired exclusively with the M119 charge and $6.19 if fired exclusively with the XM201E5. The difference of $3.59 in favor of the M119 somewhat reduces the $17.70 unit cost differential in favor of the XM201E5 (Table 1).

6. In comparing the XM211 with the M4A2 with respect to gun tube life one does not have a satisfactory data base and, therefore, must fall back upon analytic estimates. A formula with reasonably good predictive value is the Smith - O'Brasky law. This result uses charge weight, maximum chamber pressure, and flame temperature to calculate average wear rate. The input parameters and calculated wear rates for comparable zones of the M4A2 and XM211 charges are shown in Table 2. It is noted that the predicted wear rate with

the XM211 charge is 20% greater than with the M4A2 charge, although both rates are quite small relative to, say, the M203 -- 1.7 10^{-4} inches/round. The XM211 charge contains a combination of cool-burning M1 propellant (like the M4A2 charge), having a flame temperature of 2417 deg K and a rather hot propellant, the M10, having a flame temperature of 3000 deg K. There are 2.5 pounds of M10 in increment 2 and 5.5 pounds of M1 in increments 3 through 5. On this basis the average flame temperature of the combustion products of the XM211 charge is nearly 2600 deg K, almost 200 deg K higher than that for M1 propellant. This is the principal reason that the XM211 is expected to be more erosive. Notwithstanding the greater wear of the XM211, one can reasonably assert that because its erosivity is much less than the high zones and its frequency of use in an operational setting is quite small, any difference in erosivity between the XM211 and the M4A2 charges would pass unnoticed.

7. Other Measures of Effectiveness

At zone 6 of the XM211 charge or zone 7 of the M4A2, which would be used most frequently at the low zones, a typical range of engagement is 12km. For this situation, the range precision probable error is about 32m for the XM211 charge with the XM795 projectile and about 30m for the M4A2 charge with the M107 projectile. As shown in Table 3, this magnitude of error is only 1/3 to 1/4 of the total circular probable error (CPE), making the delivery error quite insensitive to the choice of charge. AMSAA TR 242 concluded in this connection: "Poorer precision levels have little effect on force performance for most conditions of artillery employment. For employment conditions in which engagement error is relatively small, such as for observed fire [against relatively small targets], good precision is more desirable." Because of the insignificant difference in total delivery error using the XM211 or the M4A2 charges, the terminal effect is not a discriminating basis for choosing between these charges.

8. Summary

Apart from minor shortcomings related to the weapons densities assumed in the artillery effectiveness simulation, the main defect of AMSAA TR 242 is its failure to address an important issue: to replace the M3 and M4 charge families with the XM211, other things held constant. Evidence from this report shows no significant effectiveness benefit in changing from existing low-zone charges to the XM211. Further, unit production costs are shown to be about 7% greater. The difficulty and cost of establishing a production base for the XM211 is not addressed. Since this charge is considerably more complicated than either the M3 or M4, one would expect some initial production problems and higher initial production costs. In addressing a variety of developmental propelling charge options, the weighted average cost of each option is calculated. Because of the large value of the weight used for MOU zone 7 (70%), the weighted average cost for an option depends quite strongly upon the cost of the charge used for zone 7. Thus, the cost of each option is insensitive to the low zones and is dominated by the cost of the high zones (7 and 8). This result makes difficult the selection of a low-zone charge using the method of analysis of AMSAA TR 242.

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TABLE 1

COMPARISON OF AVERAGE UNIT COSTS FOR SETS
OF 155MM PROPELLING CHARGES

Charge Types and Estimated Unit Production Costs

Low Zone Charges	Unit Cost ($) FY77	High Zone Charges	Unit Cost ($) FY77
M3A1	36.56	M119	91.08
M4A2	58.12	XM201E5	73.35
XM211	61.63	M203	125.74

Calculation of Weighted Average Unit Cost

MOU Vel Zone	V_0 (f/s)	Fract. Opnl. Use*	AMSAA Option 1**		Current System	
			Type	Unit Cost($)	Type	Unit Cost($)
2	850	0.000	XM211	62	M3A1/3G	37
3	975	0.000	XM211	62	M4A2/3/4	58
4	1160	0.004	XM211	62	M4A2/4/5	58
5	1510	0.032	XM211	62	M4A2/6	58
6	1980	0.043	XM211	62	M4A2/7	58
7	2245	0.720	XM201E5	73	M119	91
8	2710	0.201	M203	126	M203	126
Wtd. Avg.		1.000		$ 82.8		$ 95.4

* AMSAA estimate for Heavy Units in Europe (early 80's) using the max charge selection algorithm.

** ARRCOM Option No. 1 substitutes the M119 for the XM201E5 in AMSAA Option 1 to yield a weighted average unit cost $95.7.

TABLE 2

ESTIMATES OF RELATIVE GUN TUBE WEAR USING THE 155MM M4A2 AND XM211 CHARGES

Parameter (Dimension)	Charge Type	
	M4A2/6W	XM211/Z5
V_o in M109A1 (f/s)	1545	1510
in M114 (f/s)	1520	not compatible
Charge mass (lb) using		
M1 propellant	9.8	5.5
M10 propellant	0.0	2.5
Total charge, c, (lb)	9.8	8.0
Avg. flame temp (T_v) (deg K)	2417	2599
Max. pressure, p, in M109A1 (ksi)	15.5	20.0
Caliber, d, (inches)	6.1	6.1
T_w* (deg K)	280	316
Wear rate (10^{-4} in/rnd), w	0.065	0.078

Note: Wear rate is 20% greater for the XM211 charge than for the M4A2.

* Wall temperature parameter in the Smith - O'Brasky formula for wear rate:

$$T_w = 0.0763\ (T_v - 600)(cp)^{1/2}/d$$

$$w = 0.0166\ \exp\ (4.9\ 10^{-3} T_w)\quad .$$

TABLE 3

TOTAL DELIVERY ERROR USED IN AMSAA TR 242 VERSUS RANGE PROBABLE ERROR USING THE M4A2 PROPELLING CHARGE

Range (km)	CPE (m)	Range PE (m)*
	for the Charge and Projectile:	
	XM211/Z6 and XM795	M4A2/Z7 and M107
9	71	25
10	82	26
11	95	28
12	110	30
13	123	32
14	140	34

*Source: FT 155-AM-1, Firing Tables for Cannon, 155mm Howitzer, M185 on Howitzer, Medium, Self-Propelled, 155mm, M109A1 Firing Projectile, HE, M107, September 1972.

Memorandum for Record

REVIEW

OF A

REPORT

ON

COST-EFFECTIVE TESTING (CET)

George J. Schlenker

DRSAR-PEL 15 July 1980

MEMORANDUM FOR RECORD

SUBJECT: Review of a Report on Cost-Effective Testing (CET)

1. Reference:

a. Technical Report, Maximus, Inc., 2 June 80, title: Development and Implementation of a Statistical Methodology for Testing Defense Weapons Systems Cost Effectively, with Appendix entitled: Handbook for the Calculation of Lower Statistical Confidence Bounds on System Reliability.

b. MFR, DRSAR-PEL, HQ, ARRCOM, 13 May 80, subject: Lot Acceptance Criteria for Ammunition Exhibiting Critical Defects.

c. MFR, DRSAR-PEL, HQ, ARRCOM, 13 June 80, subject: Lot Acceptance and Lot Demil Criteria for Lots of 105mm M392 Projectiles.

2. Background

At the request of MG Eicher DRSAR-PE has conducted a review of the Reference a technical report. This memorandum provides the details of that review. As part of the review I was particularly attentive to the possible applicability of the results to ARRCOM's quality assurance and product improvement programs. The perceived useful results as well as the shortcomings are highlighted.

3. Objectives of the Report

Working under a DARPA contract, awarded in October 1978, the MAXIMUS Corporation investigated various ideas that would lead to increasing the amount of useful information derived from tests of weapons systems within a fixed budget. The principal focus of their efforts were complex, Army nuclear weapons. A specific system used as an example of their methodology is a subsystem, code named Alpha, of a nuclear missile. During the conduct of the study, the MAXIMUS Corporation joined the activities of the Joint Ad Hoc Methodology Working Group on Nuclear Weapons Reliability Assessment to establish lower confidence bounds on system reliability when failure data are available on subsystems and components. At this point in the study the MAXIMUS Corporation served the work of the Ad Hoc Committee. The contribution of all participants in this activity has been published as the Reference a Appendix to the basic report reviewed here.

4. Basic Concepts

Among the useful outcomes of the subject study are some concepts associated with measuring the effectiveness of testing complex weapon systems. The uncertainty concerning the reliability of a single component is directly related to the number of pass-fail or attribute tests conducted on the component. By analogy a concept advanced in this report for complex systems is the equivalent number of system tests. This concept is defined as follows:

"The equivalent number of tests is that number of tests, in conjunction with the point estimate of system reliability, which would yield, for a single component, the same variance or lower confidence bound on system reliability which was obtained by using the component test data."

Another contribution of the subject study is to show how to calculate an approximate probability distribution for the estimated system reliability using component failure data. This distribution (approximately beta) is then used to calculate a lower confidence bound on system reliability. It is noted that this procedure is the methodology recommended by the Ad Hoc Methodology Working Group (mentioned previously). Whether this approximation will prove useful to the armaments community for application to non-nuclear systems is an open question.

5. Perspectives

The study takes the point of view that an effective test program is one which maximizes the effective, or equivalent, number of system tests derived from component and higher level test data, given a fixed test budget. The report does not address the issue of how much testing is optimal or, stated differently, at what point does the cost of additional testing exceed the incremental value of information derived therefrom. Parenthetically, it is noted that the latter issue was addressed for a special application in a recent ARRCOM MFR (Reference b).

6. As a direct consequence of the objective of cost-effective testing (CET), the MAXIMUS study provides the following guidelines:

a. Do relatively more testing on serial than on parallel components.

b. Combine test data, when applicable, from different levels of testing and from different test programs.

c. Test sequentially so as to exploit reliability information as soon as possible.

I am particularly sympathetic to the last principle since it is consonant with the methods advocated in Reference b and Reference c.

7. Costs

The subject study finds that "in general, cost data in the detail necessary for cost-effective testing decisions are not available in the Services..." The nuclear subsystem Alpha was found to be particularly deficient in accurate cost data. Because of this finding, the study described the type, i.e., hierarchy of costs necessary to a CET program and outlined a cost collection and estimation program for acquiring these data with particular emphasis on the life cycle of a nuclear weapon system. It is noted that because of the division of responsibilities between DoD and DoE activities with respect to nuclear weapons development and acquisition, the acquisition of good cost data for carrying out a CET program may be more difficult for a nuclear than for a non-nuclear weapons system.

8. Some of the categories of the Army testing program do not seem to be properly placed in the hierarchy of testing presented in this report. For example, operational testing is placed under "Laboratory Testing of Components" and, again, under "Laboratory Testing of Subsystems." Also, the categorization of a facility cost as exclusively a laboratory fixed cost is, I believe, a misrepresentation of that cost element and would underestimate the cost of laboratory testing.

9. A number of observations pertinent to cost categorization and to the tradeoff between types of tests are specific to nuclear systems. This substantially reduces the utility of the section on costing in this report for those interested in testing non-nuclear systems.

10. Some Oversights

One of the shortcomings of the philosophy of cost-effective testing (CET) enunciated in this report is the focus on only one testing objective: to reduce the uncertainty in the system-level reliability. Actually, throughout development, testing seeks to identify weak components so that they can be replaced with redesigned versions having greater longevity. This goal and the complementary goal of identifying unsafe operating conditions are not necessarily consistent with the goal: minimize the variance of the reliability estimate.

11. Other Limitations of the Study

Although the subject study methodology purports to be applicable to all system configurations, in fact, only systems having a combination of series and parallel elements with respect to function are addressed. The accomplishment of a mission by a system often depends upon elements which are in standby from a reliability point of view. These standby elements do not become operational until a failure of the primary element occurs. The probability of mission success (reliability) for standby systems is not generally the same as that for parallel systems, which presume continuous operation of two or more components performing the same function. An additional type of reliability system diagram not covered in the report is the Wheatstone bridge arrangement, in which components cannot be decomposed into serial and parallel arrangements. A third arrangement of components not treated is, for example, one in which at least m of n common elements must function for successful system operation. Clearly, the methodology of the report is intended only to be a convenient approximation of the estimate of the probability distribution of system-level reliability. Presently, it is unclear whether or not the above limitations will substantially affect the utility of this approach as applied to certain ARRCOM conventional weapon systems. Another aspect of ARRCOM systems which may limit the utility of this methodology is the fact that all ARRCOM systems are in the operational phase of their life cycle. In this connection the report notes: "Opportunities for applying CET may be more plentiful in R&D systems, where procurement decisions have not yet been made."

12. Summary

This review of the MAXIMUS study highlights the study contributions to the goal of cost-effective testing (CET). The subject study views CET strictly as a means of obtaining the greatest amount of system-level reliability information under a fixed test budget. The study does not address the question: How much testing is enough? The study has been found to provide useful guidance for allocating test resources among component and subsystem tests. Apart from certain technical limitations, the study offers some principles to the test developer which will produce testing economies.

George Schlenker

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Memorandum for Record

LOT ACCEPTANCE

AND

LOT DEMIL

CRITERIA

FOR

LOTS OF

105MM M392 PROJECTILES

George J. Schlenker

Lanny D. Wells

DRSAR-PEL 24 July 1980

MEMORANDUM FOR RECORD

SUBJECT: Lot Acceptance and Lot Demil Criteria for Lots of 105mm M392 Projectiles

1. This memorandum supercedes MFR, DRSAR-PEL, HQ, ARRCOM, 13 Jun 80, SAB.

2. Background

Employing considerations of safety and economics, it is desired to define the condition code of lots of retrofitted M392 projectiles which have previously exhibited critical defects. Lots will be identified as satisfactory for immediate use (Code A) if the lot fraction defectives is 5% or better, at a 10% risk level. Lots will be condemned for demil (Code H) if one can be 90% confident (at each step in the test process) that the lot fraction defectives exceeds a critical value, say 10%. Lots which are not placed in Code A or Code H will be placed in Code N -- combat emergency use only. The above risk specification for Code A does not include a producer's risk. Implicitly the producer's risk will be whatever is determined by the operating characteristic of the minimum cost test procedure satisfying the above consumer's risk.

3. Test Procedure

Based on the prescribed risk statement, the criterion for placing a lot in Code A is that no (0) defectives are found in a sample of 45. The occurrence of the first defective item removes the lot from further consideration as Code A. The lot will be condemned (Code H) if at any round, n, the number of defectives equals a particular rejection number r(n), which is a function of n. Derivation of r(n) is found in Annex 1 with computer source program listed in Annex 2. The rejection boundary for $2 \leq n \leq 45$ is tabulated in Table 1 and displayed graphically in Figure 1. If testing exhausts the sample of 45 without reaching the rejection boundary and with at least one defective, the lot will be classified Code N. An alternative sequential procedure for declaring a lot Code H is one which satisfies the less conservative requirement: demil if 90% confident (at each step) that the lot fraction defectives exceeds 5%. This choice of critical value -- 5% as opposed to 10% -- is consistent with the average lot fraction defectives for all retrofitted M392 lots tested to date. The rejection boundary for this alternative procedure is shown in Figure 6.

4. Statistical Aspects of the Test

The operating characteristic (OC) associated with acceptance of a lot as Code A is shown in Figure 3. Note that the probability of accepting lots having 5% lot defectives is less than 0.1, as prescribed. Continuing the sequential test beyond the n th specimen with probability $P_c(n)$ is contingent upon not reaching the rejection boundary, defined for Code H. But $P_c(n)$ is also a function of the incoming lot proportion defectives, π. The function $P_c(n)$ is shown in Figure 2 for the fixed value of π equal to 0.1. The parametric effect of π upon $P_c(n)$ is shown in Figure 2a.

5. The event -- reaching the maximum sample, N = 45 -- implies not declaring the lot Code H. As shown for $P_c(n)$, the probability of not declaring the lot Code H is a function of π. This relationship is shown in Figure 4. We note in passing that this function is analogous to an OC for the procedure to accept a lot as Code A. Additionally, note that the probability of not Code H for π equal to 0.1 is about 0.77, which differs from the stepwise confidence level of 90%. The expected test cost (ETC) measured in number of rounds used is, of course, a function of π. For example, for $\pi = 0.05$ ETC = 43.6 units; and for $\pi = 0.10$ ETC = 38.0 units. The general functional relationship is shown in Figure 5.

6. An Alternative Procedure

For the alternative procedure to declare a lot Code H the critical lot fraction defectives is 0.05. The rejection boundary for this plan, shown in Figure 6, is smaller than for the first procedure (Figure 1). The statistical characteristics previously displayed for the first procedure are displayed for the alternative procedure in Figures 7 through 10. Figure 7 shows the probability of continuing the test, $P_c(n)$, versus n for a 0.05 lot fraction defectives (π). Figure 8 displays $P_c(n)$ with π as a parameter, and may be compared with Figure 2a. Figure 9 shows the probability of not declaring a lot Code H versus π. This figure may be compared with Figure 4. Finally, the expected test cost (ETC) function is shown in Figure 10. Comparing this ETC with that for the first procedure (Figure 5) indicates that a considerable cost saving is likely for the alternative procedure. For example, for a 10% lot defectives the ETC is only 25 units compared with 38 units for the first procedure.

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TABLE 1

REJECTION BOUNDARY FOR SEQUENTIALLY DETERMINING
THAT LOT FRACTION DEFECTIVES EXCEEDS 10% WITH 90% CONFIDENCE

Sample Number, N	Rejection Number, R	Sample Number, N	Rejection Number, R
2	2	24	5
3	"	25	"
4	"	26	6
5	"	27	"
6	3	28	"
7	"	29	"
8	"	30	"
9	"	31	"
10	"	32	"
11	"	33	7
12	4	34	"
13	"	35	"
14	"	36	"
15	"	37	"
16	"	38	"
17	"	39	"
18	"	40	"
19	5	41	8
20	"	42	"
21	"	43	"
22	"	44	"
23	"	45	"

Figure 2. Probability of Continuing a Sequential Test Beyond Specimen n, Given a 0.1 Fraction Defectives

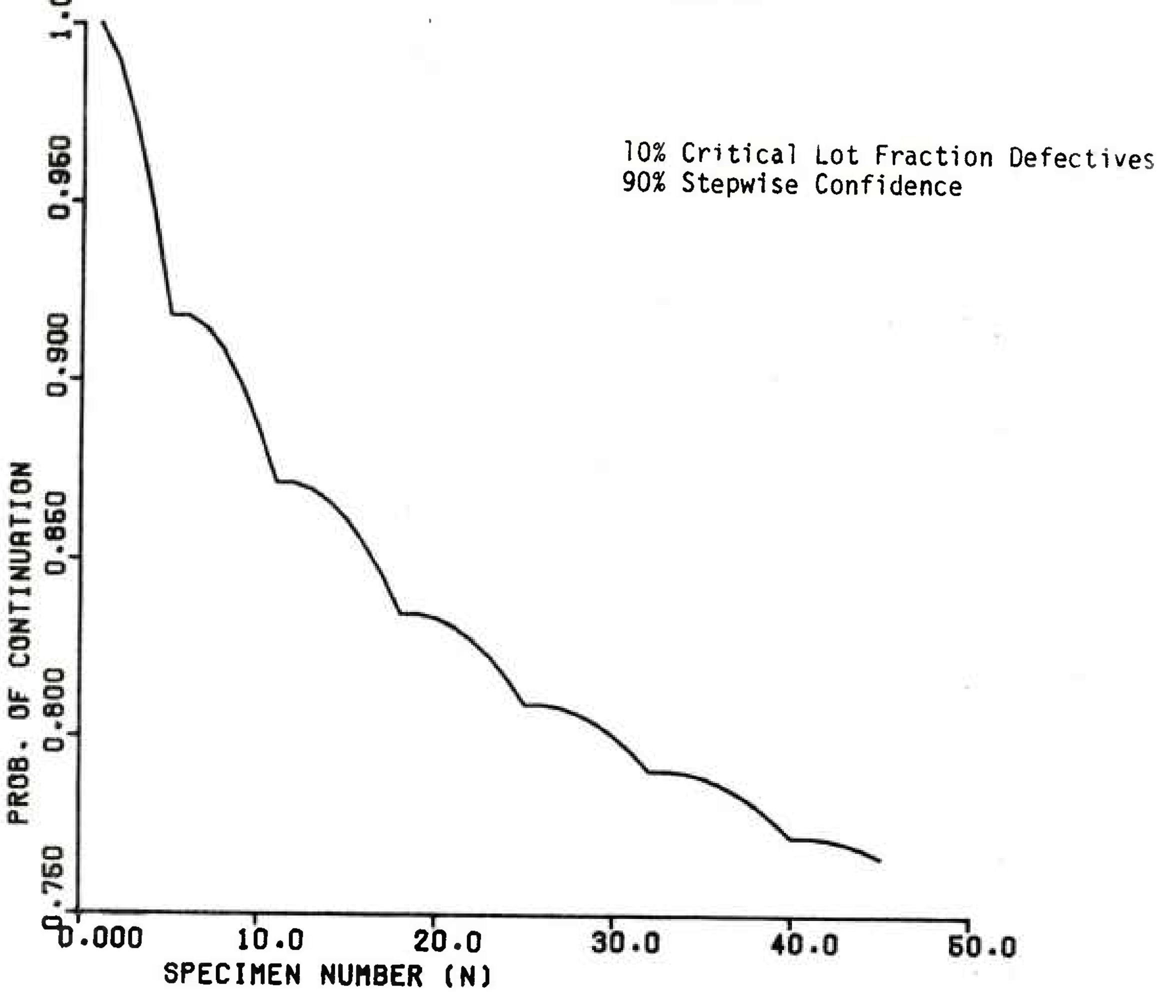


Figure [illegible]. Rejection Boundary for a Sequential Procedure to Condemn a Lot (Code H)

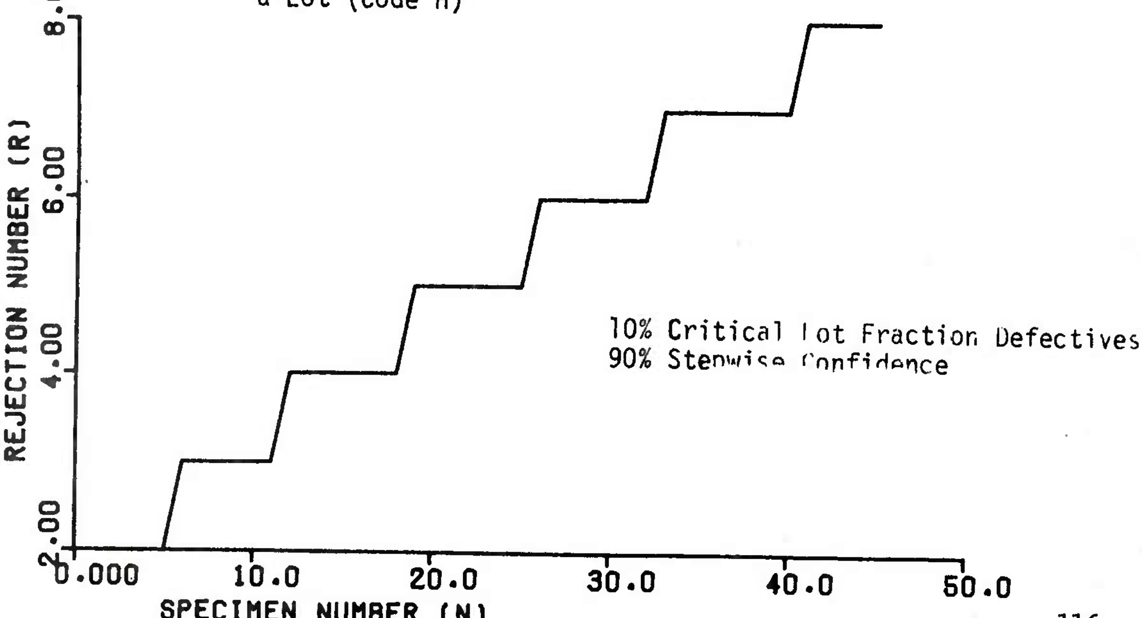
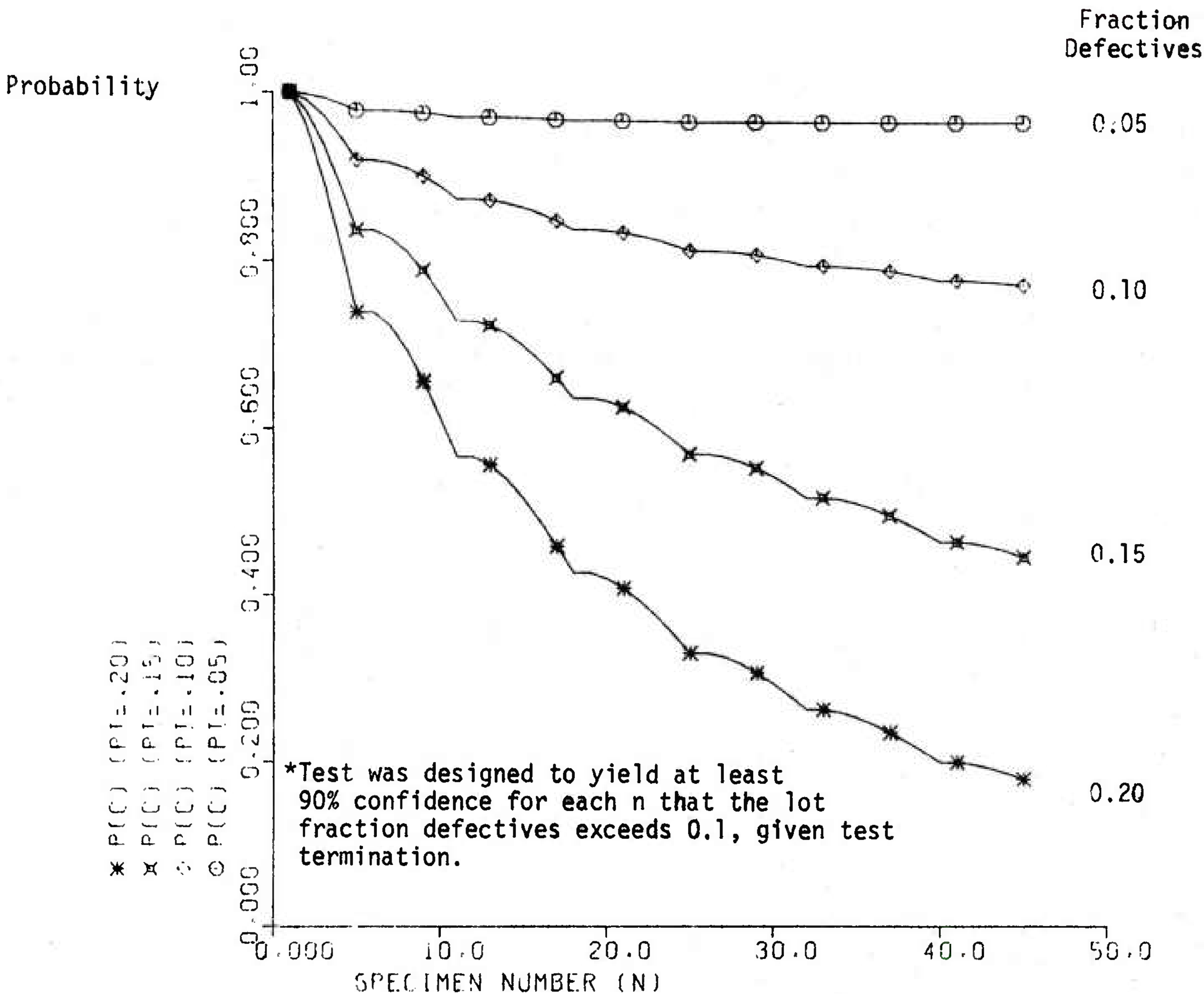


Figure 2a . Probability of Continuing the Sequential Test* Beyond Specimen n, Given the Lot Fraction Defectives



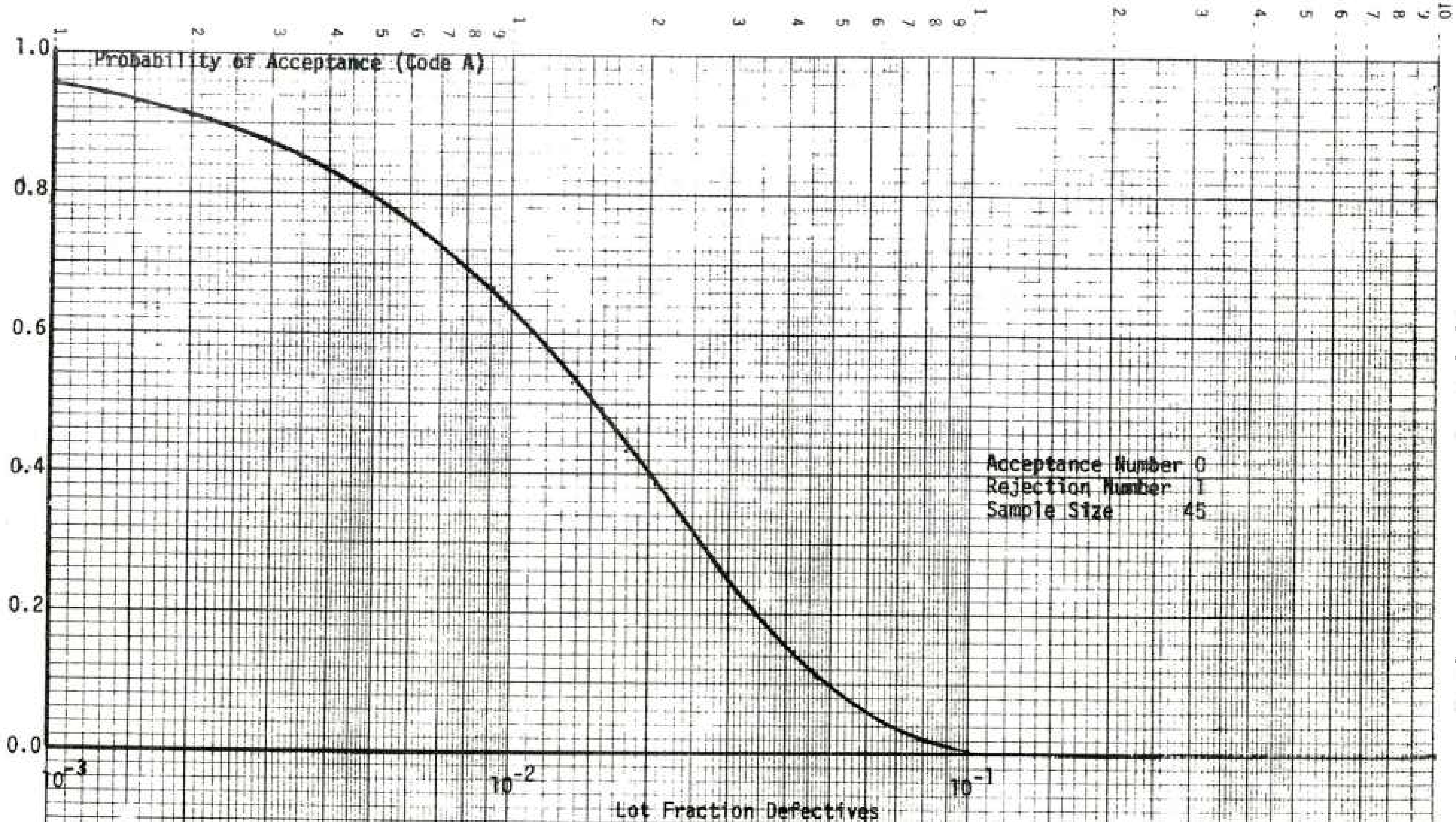


Figure 3. Operating Characteristic for a Single-Stage Lot Acceptance Test Procedure

Figure 4. Probability of Not Declaring a Lot Code H as a Function of the Lot Fraction Defectives

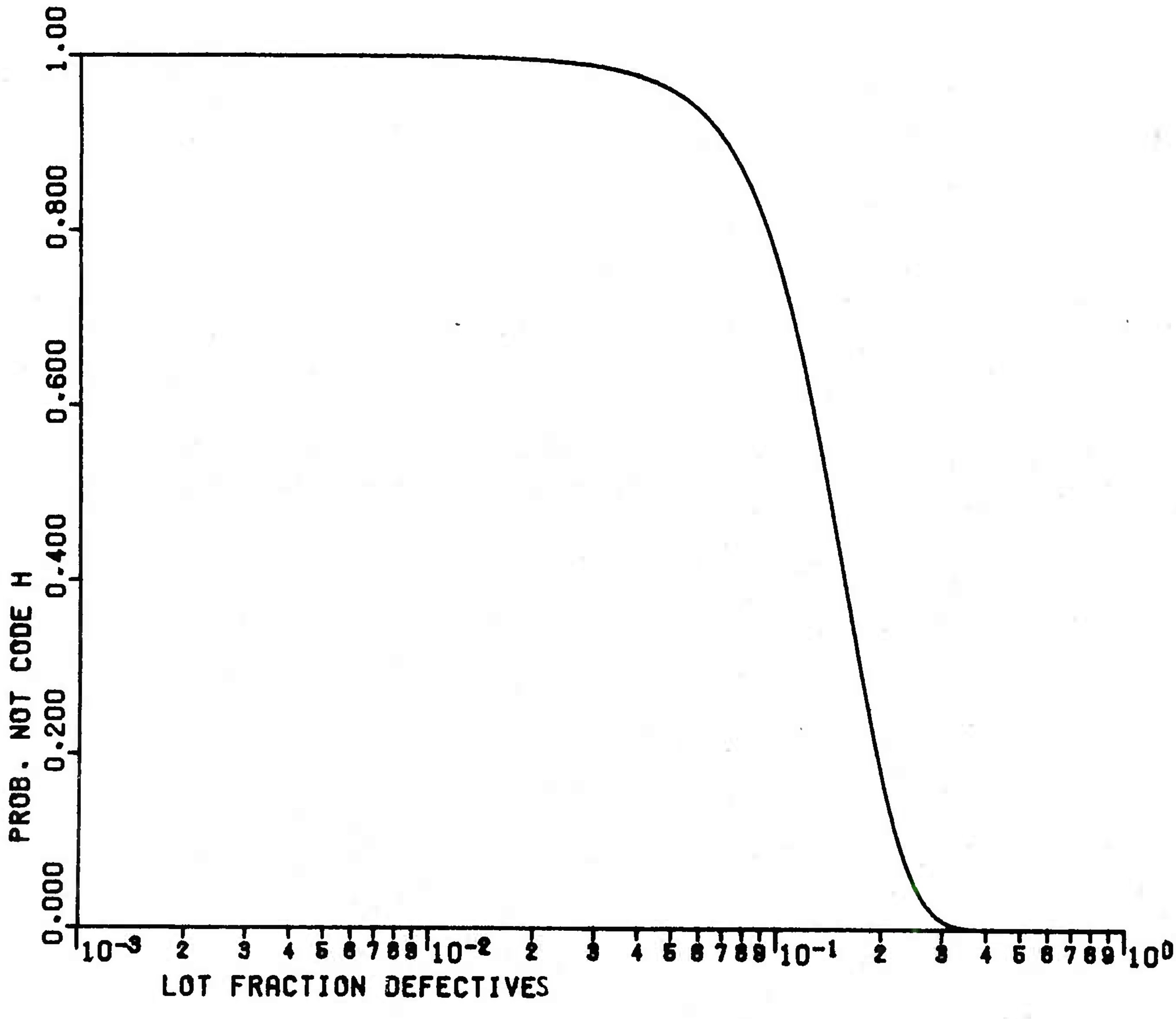


Figure 5. The Expected Test Cost Versus Lot Fraction Defectives for a Sequential Test to Condemn the Lot

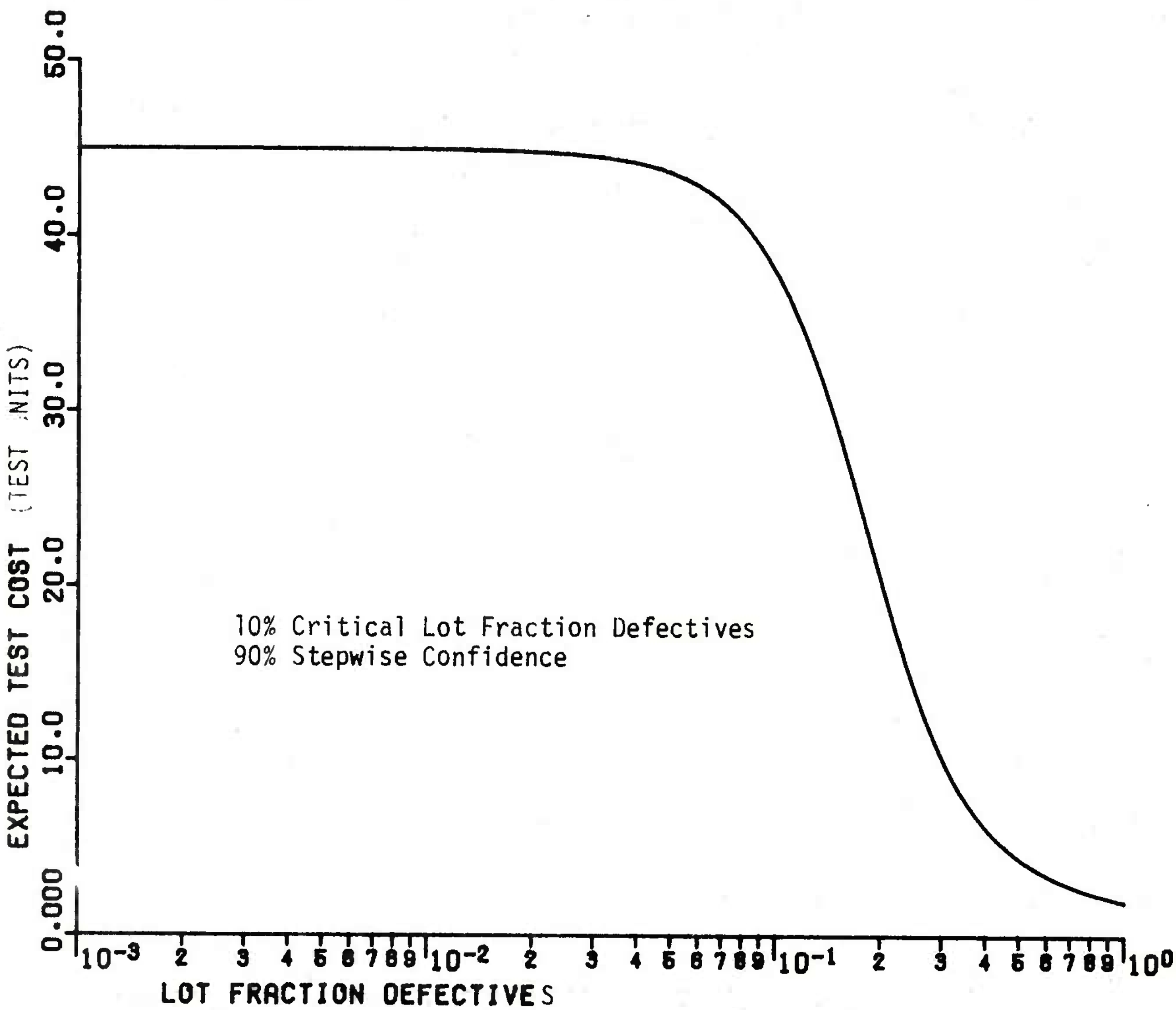


Figure 6. Rejection Boundary for a Sequential Procedure to Condemn a Lot (Code H)

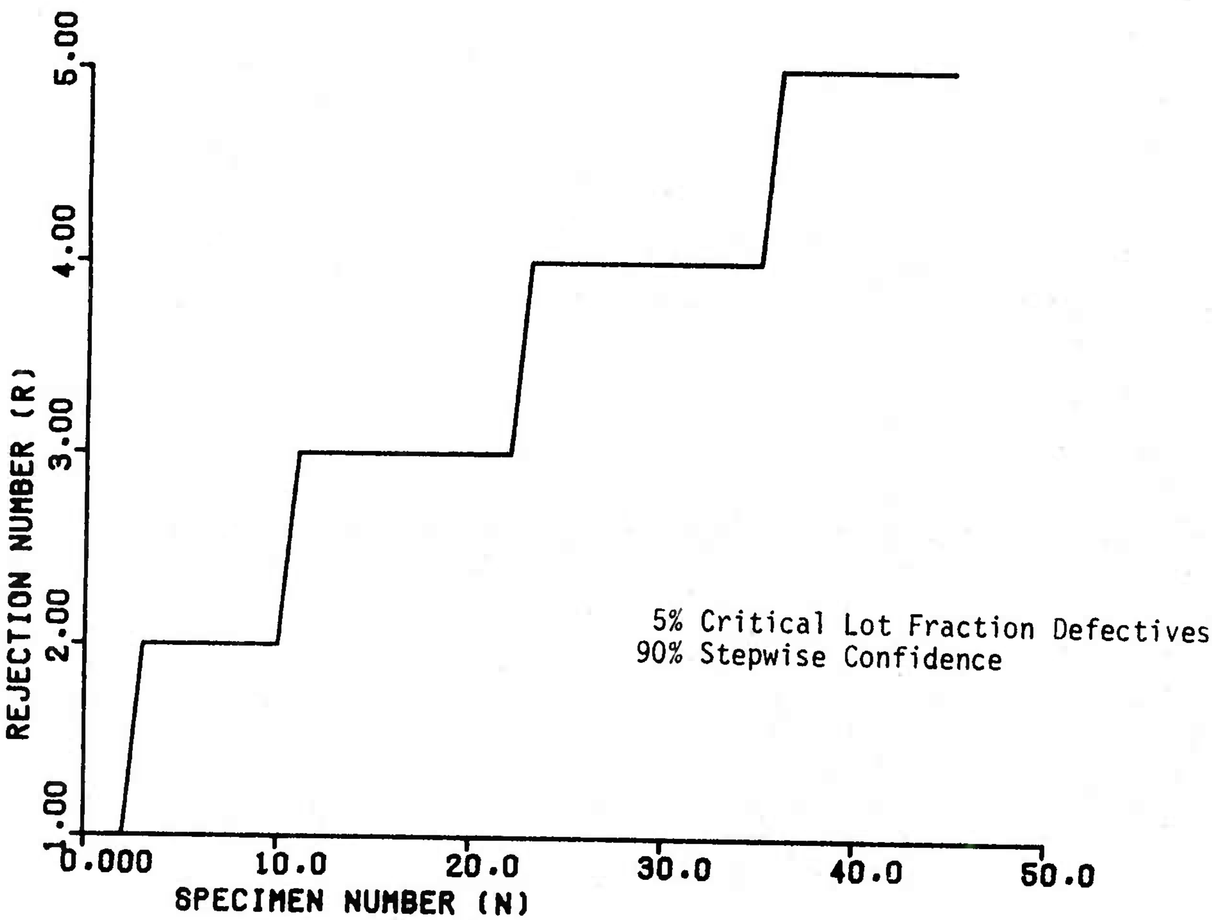


Figure 7 Probability of Continuing a Sequential Test Beyond Specimen n, Given a 0.05 Fraction Defectives

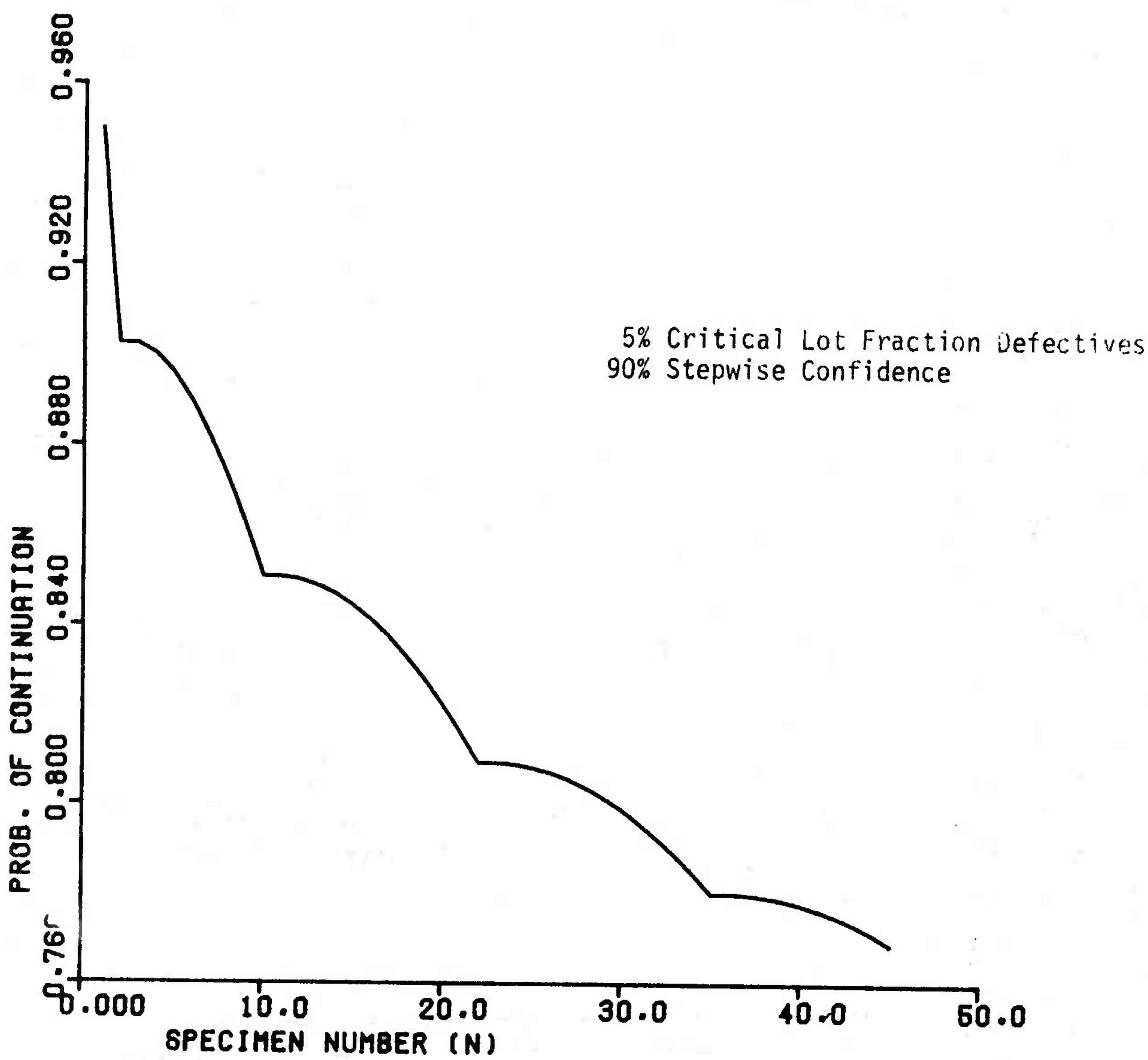


Figure 8. Probability of Continuing a Sequential Test Beyond Specimen n With Fraction Defectives as a Parameter

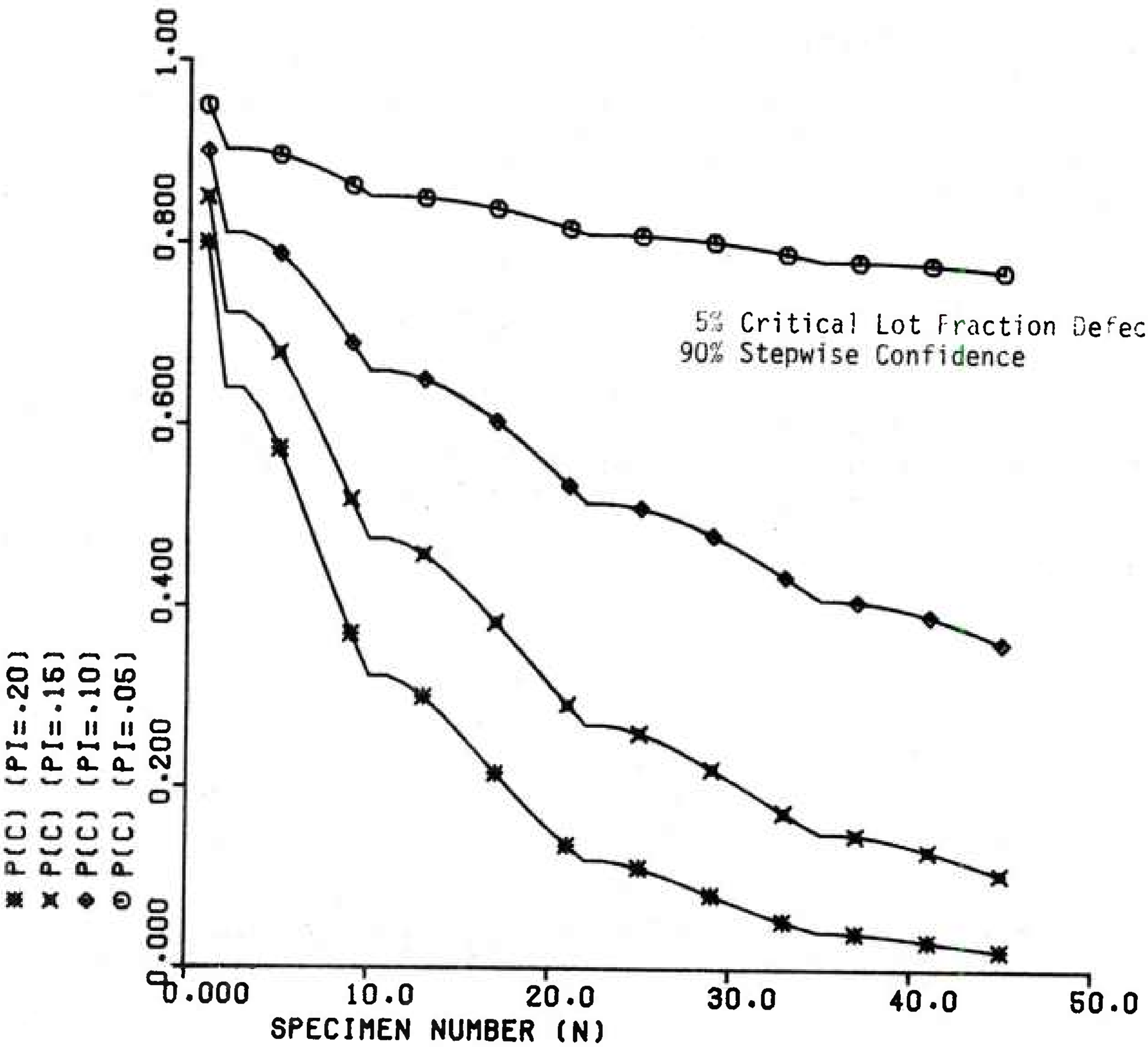


Figure 9. Probability of Not Declaring a Lot Code H as a Function of the Lot Fraction Defectives

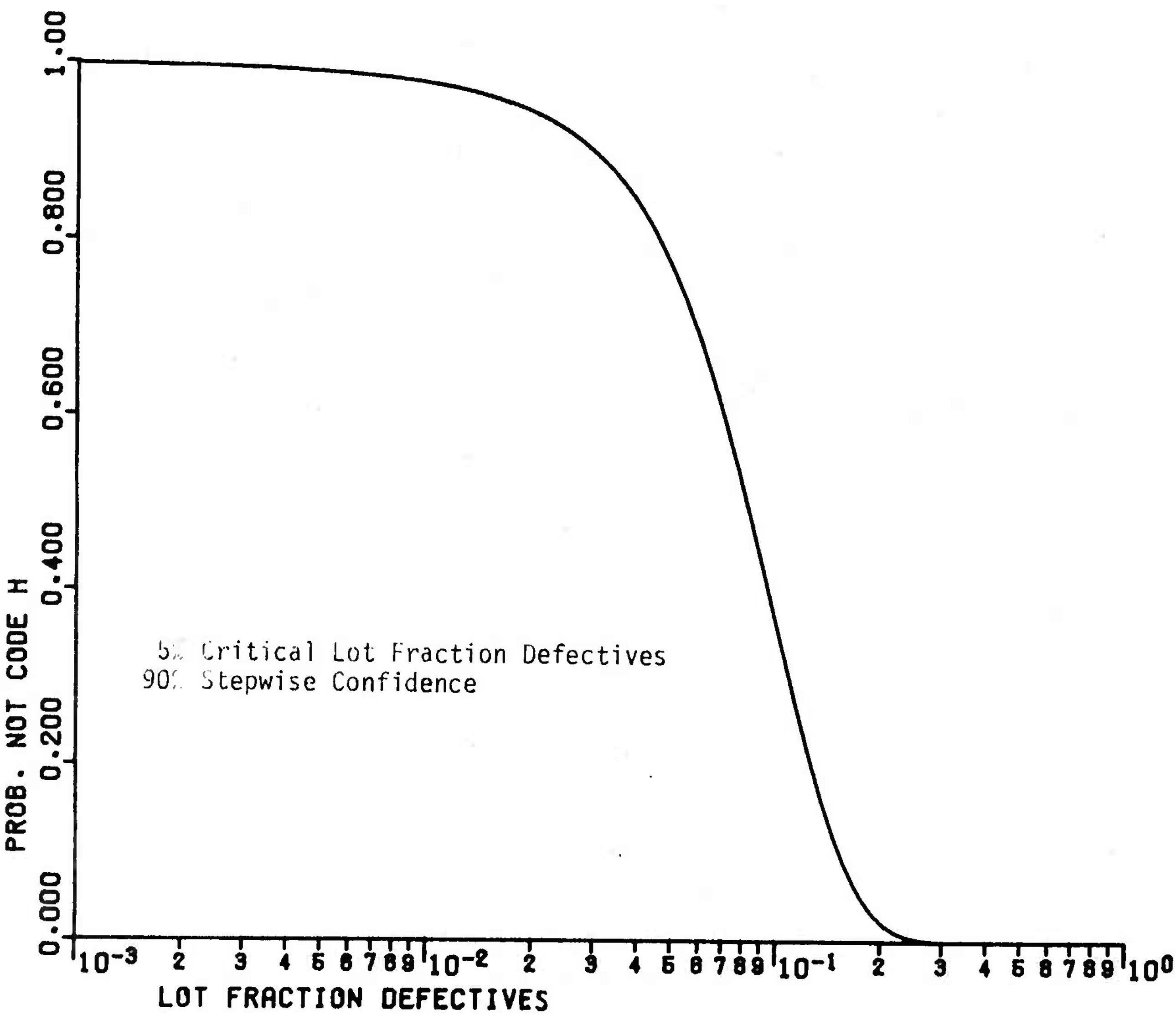
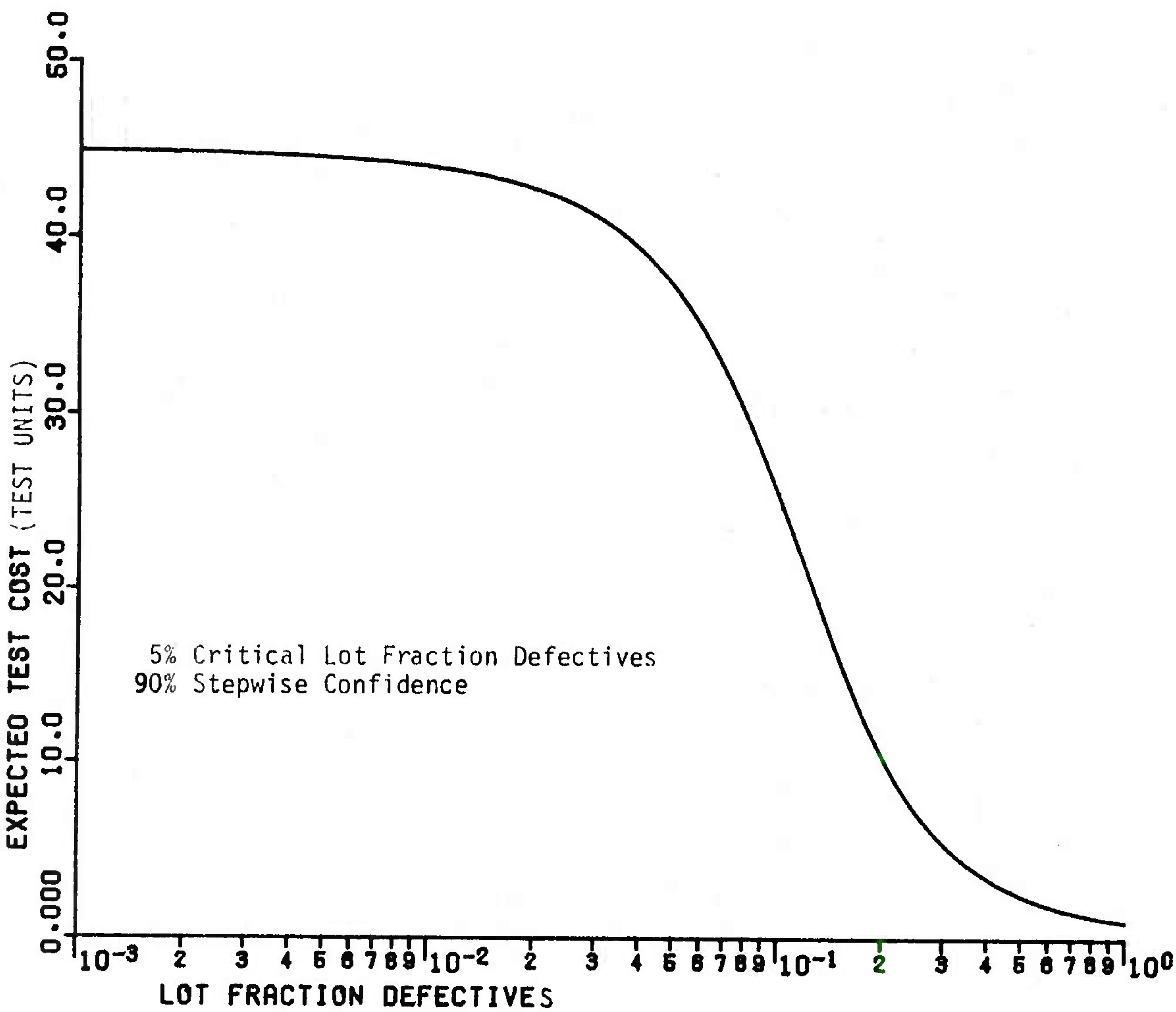


Figure 10. The Expected Test Cost Versus Lot Fraction Defectives for a Sequential Test to Condemn the Lot



ANNEX 1

DERIVATION OF CONDITIONS FOR TERMINATING A SEQUENTIAL LOT ACCEPTANCE TEST

Section 1

Termination With the Conclusion that the Fraction Lot Defectives Exceeds a Prescribed Critical Value

Notationally, let

$p_j(n,\pi)$ = P{exactly j defectives occur in a sample of n, given prior probability of a defective π}

For large lots,

$$p_j(n,\pi) = \binom{n}{j} \pi^j (1-\pi)^{n-j} \quad , \qquad 0 \le j \le n \quad . \tag{1.1}$$

Also let

$P_k(n,\pi)$ = P{k or more defectives are observed, given n and π}

$$P_k(n,\pi) = \Sigma_{j=k}^{n} p_j(n,\pi)$$

$$P_k(n,\pi) = 1 - \Sigma_{j=0}^{k-1} p_j(n,\pi) \quad . \tag{1.2}$$

To define a rejection boundary or terminating number of defectives, r, we wish to be $100(1-\alpha)\%$ confident that a test which reaches the boundary does have a lot fraction defectives which is no smaller than a critical value π^*. This stipulation can be restated as follows: Given a sample of n with lot fraction defectives exactly equal to π^*, the probability of observing r or more defectives shall not exceed α.

Formally, we seek a value of k,

$$r = k^*(n,\pi^*,\alpha) \quad ,$$

such that k^* is the smallest integer for which

$$P_k(n,\pi^*) \le \alpha \quad . \tag{1.3}$$

The value k^* represents the rejection number after n specimens with critical value of π, π^*, at a stepwise risk level α. For example, with

$\pi^* = 0.1$ and $\alpha = 0.1$ (90% confidence) at the n = 5 th step,

$$P_1(5,0.1) = 0.4095$$

$$P_2(5,0.1) = 0.08146 \quad .$$

Therefore,

$k^*(5,0.1,0.1) = 2,$

since 2 is the smallest value of k for which $P_k(5,0.1)$ is less than $\alpha = 0.1$.

An alternative way of calculating the rejection number is to require that the lower $100(1-\alpha)\%$ confidence bound on π exceeds the prescribed critical value π^*, given n and r. For example, the lower 90% confidence bound on π, given n = 5 and r = 2 (as above) is 0.112. Since this bound exceeds the critical value of 0.1, the value r = 2 is the rejection boundary at this step.

Section 2

Termination With the Conclusion that the Fraction Lot Defectives is Less Than a Prescribed Critical Value

The null hypothesis, H_0, is that the fraction lot defectives is less than or equal to π^*. Previously, the rejection boundary r(n) was defined for this hypothesis such that the producer's risk of rejection for each value of n, given H_0, is α. In an analogous manner we can define an acceptance boundary, c(n), for H_0. The set of integers c(n), given n, is defined so that if the number of observed defectives equals c(n) at n, H_0 is accepted with consumer's risk β and the test is terminated. Mathematically, we seek the maximum integer c such that

$$P\{c \text{ or less defectives are observed, given } n, \pi^*\} \leq \beta, \quad (1.4)$$

or

$$\sum_{j=0}^{c} \binom{n}{j} \pi^{*j} (1-\pi^*)^{n-j} \leq \beta . \quad (1.5)$$

Alternatively, we require that the upper $100(1-\beta)$ confidence bound on π is less than π^*, given n and c. As an example, for $\pi^* = 0.1$ and for $\beta = 0.1$ (90% confidence), the values of n at which c(n) makes unit jumps are:

n = 22 for c = 0

n = 38 for c = 1

n = 52 for c = 2 .

Since the determination of Code A must also be made, termination of the sequential test to determine Code H will not occur with c = 0.

Thus, if the number of specimens tested reaches 22 without a defective, one can accept the hypothesis that $\pi \leq 0.1$ at a 10% consumer's risk.

Section 3

Derivation of Equations for the Expected Test Cost for a Truncated Sequential Test

To facilitate the derivations some additional notation is required.

Let

$$p_c(n,k) = P\{\text{continuation of test beyond } n \text{ samples with } k \text{ defectives}\} \quad , \quad 0 \leq k < r(n). \tag{1.6}$$

The array of values of p_c represents the probability of occupying each of the possible states of the test process.

And,

$$p_t(n) = P\{\text{test termination occurs at the } n \text{ th sample}\} \quad ,$$

and

$$P_c(n) = P\{\text{continuation beyond } n \text{ samples}\} \quad .$$

Clearly, the unconditional probability of continuation beyond n samples is given by

$$P_c(n) = \Sigma_{k=0}^{r(n)-1} \; p_c(n,k) \quad . \tag{1.7}$$

Since the test either continues beyond the n th sample or terminates there,

$$p_t(n) = P_c(n-1) - P_c(n) \quad . \tag{1.8}$$

The expected test cost (ETC) with truncation at N samples is given by

$$ETC(N) = \Sigma_{n=1}^{N-1} \; np_t(n) + NP_c(N-1) \quad . \tag{1.9}$$

For the moment, consider test termination which occurs only by encountering a rejection boundary, i.e., for which the number of defectives at n is r(n). Since r(n) is a step function of n with unit jumps, this function can be conveniently described in terms of the values of n -- n_i -- at which unit jumps (break points) occur. Thus,

$$r(n_i-1) = r(n_i) - 1 \quad , \qquad 1 \leq i \leq i_{max} \quad . \tag{1.10}$$

An example of this discrete representation of the rejection boundary in terms of its points of discontinuity is shown in Table 1.1.

Table 1.1

DISCRETE FORM OF THE REJECTION BOUNDARY FOR THE SEQUENTIAL TEST HAVING A CRITICAL FRACTION DEFECTIVES OF 0.1 AND A PRODUCER'S RISK of 0.1 (90% STEPWISE CONFIDENCE)

Index i	Break Point n_i	Rejection Number r_i
1	6	3
2	12	4
3	19	5
4	26	6
5	33	7
6	41	8
7	48	9

The values of $p_c(n,k)$ can be calculated recursively. The recursive formula is derived from the fact that there are at most two exclusive ways of reaching the test state: being at the n th step (having taken n specimens) with k defectives:

(1) Being at the n-1st step with k-1 defectives and observing another defective at the n th step, or

(2) Being at the n-1st step with k defectives and not observing a defective at the n th step.

Events (1) and (2) occur with prior probabilities

$$p_c(n-1,k-1)\ \pi$$

and

$$p_c(n-1,k)(1-\pi), \text{ respectively.}$$

Thus,

$$p_c(n,k) = p_c(n-1,k-1)\ \pi + p_c(n-1,k)(1-\pi) \quad , \qquad n > 1,\ 1 \le k < r(n) \ . \tag{1.11}$$

Computationally, it is convenient to define the sample (or step) index n in terms of n_i and an auxillary index j. Thus,

$$n = n_i + j \quad , \qquad \begin{array}{l} 1 \le i \le i_{max} \\ 1 \le j \le n_{i+1} - n_i - 1 \end{array} \quad . \tag{1.12}$$

For all n,

$$\begin{aligned} p_c(n,0) &= p_0(n,\pi) \\ p_c(n,0) &= (1-\pi)^n \quad . \end{aligned} \tag{1.13}$$

Further, if the smallest value of r exceeds 1, for all n,

$$\begin{aligned} p_c(n,1) &= p_1(n,\pi) \\ p_c(n,1) &= n\pi(1-\pi)^{n-1} \quad . \end{aligned} \tag{1.14}$$

In general, at the points where $n = n_i$,

$$p_c(n_i, r_i - 1) = p_c(n_i - 1, r_i - 2)\ \pi \quad . \tag{1.15}$$

This equation expresses the fact that the only feasible way to reach the state in which n is one of the break points and the number of observed defectives is one less than the associated rejection number is to have one fewer defectives at the previous step and, then, observe a defective at the n th step. Having r_i - 1 defectives at the previous step is impossible since that state results in immediate rejection.

For points between n_i and n_{i+1} the value of $p_c(n,k)$ is calculated recursively by stepping through the index j in $p_c(n_i+j,k)$, using equation (1.11), and with the index k nested most deeply:

$$p_c(n_i+j,k) = p_c(n_i+j-1,k-1)\pi + p_c(n_i+j-1,k)(1-\pi)$$

for

$$\begin{array}{l} 1 \le i \le i_{max} - 1 \\ 1 \le j \le n_{i+1} - n_i - 1 \\ 1 \le k \le r_i - 1 \end{array} \quad . \tag{1.16}$$

Also, by procedural definition,

$$p_c(n,k) = 0 \quad , \qquad k \ge r(n) \quad . \tag{1.17}$$

Treatment of an Acceptance Boundary

When testing can terminate on an acceptance boundary as well as on a rejection boundary, a slight modification of the above procedure is required. First, the acceptance boundary must be defined in terms of the break points, n_i', at which the acceptance number, c_i, is incremented by unity. For example, for 10% critical lot defectives at a 10% consumer's risk, the acceptance boundary is defined as follows:

Table 1.2

EXAMPLE OF A DISCRETE ACCEPTANCE BOUNDARY

i	n_i'	c_i
1	38	1
2	52	2
3	65	3

$$c(n) = c_i \quad \text{for} \quad n_i' \leq n < n_{i+1}' \quad .$$

Any test which reaches a value of n with number of defectives equal to c(n) is terminated. Thus, the probability for continuing the test at this state is zero. To reflect this in the calculations, the $p_c(n,k)$ array is initially filled with zeros for values of $p_c(n,k)$ for $k \leq c(n)$. Following this initialization, the recursive calculation of $p_c(n,k)$ proceeds using equation (1.11).

ANNEX 2

COMPUTER PROGRAMS

Computer source programs, written in standard FORTRAN for the PRIME computer, are furnished in this annex. There are two main programs with associated subroutines. The first program is listed on pages A-2-2 through A-2-8. This program calculates the rejection boundary for a sequential lot acceptance test with prescribed critical fraction defectives and producer's risk. Selected characteristic statistics are also calculated and may be plotted by invoking other PRIME programs.

A second program is listed on pages A-2-9 through A-2-12. This program calculates and stores several variables which characterize a general, sequential lot acceptance test which is truncated after N specimens. The rejection and acceptance boundaries are specified in terms of vectors of the values of n at which unit steps (or increments) occur in the boundaries. These are program inputs which define the test. Probabilities for all of the possible states of the test process are calculated and stored in an array for display. The probability of terminating the test with the n th specimen and the expected test cost, given the fraction defectives, are also calculated and displayed.

```
C
C     ..................................................................
C
C     THIS PROGRAM IS USED TO DEFINE A REJECTION BOUNDARY FOR A SEQUEN-
C     TIAL ACCEPTANCE TEST AND DISPLAY THE OPERATING CHARACTERISTICS.
C
C     THE FOLLOWING LIBRARIES ARE REQUIRED:
C
C        1. LWRLIB
C        2. PLOTLB (CALCOMP PLOT LIBRARY)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      INTEGER FF,NAME(16,6),INFO(40),NR(100)
      REAL*4 PARRAY(101,3),S(9)
      DIMENSION SPV(20)
C
$INSERT SYSCOM>A$KEYS
C
      DATA FF/:106000/               /* ASCII FORM FEED CHARACTER
C
C     THE ARRAY NAME CONTAINS NAMES FOR PLOT LABELS.
C
      DATA NAME/
     1'F LOT FRACTION DEFECTIVE        ',
     2'P PROB. NOT CODE H              ',
     3'E EXPECTED TEST COST            ',
     4'N SPECIMEN NUMBER (N)           ',
     5'R REJECTION NUMBER (R)          ',
     6'C PROB. OF CONTINUATION         '/
C
C     ARRAY INFO CONTAINS INFORMATION FOR SUBROUTINE PPLOT AND/OR CPLOT.
C     INFORMATION ON THESE ROUTINES MAY BE OBTAINED BY DOING THE FOL-
C     LOWING:
C
C        SPOOL PEXLOW>A>PLOT-INFO
C
      DATA INFO/1,:040000,2,0,:100000,101,0,0,1,18,
     $          1,:040000,2,0,:100000,101,0,0,1,34,
     +          1,:100000,50,0,:100000,0, 0,0,1,66,
     *          1,:100000,50,0,:100000,0, 0,0,1,82/
C
```

```
C     ARRAY S CONTAINS SIZE ANO SCALE INFORMATION FOR PPLOT AND CPLOT.
C
      OATA S/6.0,3*0.0,5.0,3*0.0,0.098/
C
C     THE FOLLOWINO STATEMENT WRITES OUT PRINTER CONTROL CHARACTERS TO
C     PUT THE PRINTER IN A NO CONTROL MODE.  IT IS RECOMMENDEO THAT A
C     COMO FILE BE USED TO LIST THE PROGRAM PRIOR TO EXECUTION SO THE
C     PROGRAM LISTING ANO THE OUTPUT COME OUT TOGETHER.
C
      CALL TNOU(:000400,2)
C
C     GET THE CRITICAL PI (PC) AND THE PROOUCER'S RISK.
C
    1 CALL TNOUA('OEMIL CRITERION, BETA, PC?...> ',31)
      REAO(1,*) BETA,PC
C
C     REAO THE MAXIMUM NUMBER OF SPECIMENS TO BE TESTED BASEO ON THE
C     APPROPRIATE CONSUMER'S RISK CRITERION.
C
      CALL TNOUA('N(MAX) FOR TRUNCATED TEST?...> ',31)
      READ(1,*) NMAX
C
C     INITIALIZE
C
      INFO(26)=NMAX                /* NUMBER OF POINTS TO BE PLOTTED.
      INFO(36)=NMAX
      Q=1.00 00-PC
      PNT1=1.00 00                 /* PROBABILITY OF NOT TERMINATING
      ETC1=0.00 00                 /* EXPECTED TEST COST
      L=55                         /* LINE COUNTER
C
C     SPV IS A STATE PROBABILITY VECTOR THAT CONTAINS THE PROBABILITY
C     OF OBSERVING 0, 1, ... , R-1 DEFECTIVES AS A FUNCTION OF THE GIVEN
C     PC AND THE NUMBER OF SPECIMENS TESTEO (N).  NOTE THAT THE INITIAL
C     PROBABILITY OF ZERO OEFECTS IS 1.0 (100%) ANO ALL OTHER PROBABIL-
C     ITIES ARE ZERO.
C
      SPV(1)=1.00 00
      00 2 I=2,20
    2 SPV(I)=0.00 00
C
      00 9 N=1,NMAX
```

```
C
C     THE FOLLOWING SUBROUTINE RETURNS THE REJECTION NUMBER (NR) AS A
C     FUNCTION OF PC AND THE BETA RISK TAKEN AT EACH STAGE.
C
      CALL RN(N,PC,BETA,AR,NR(N))
C
C     COMPUTE THE PROBABILITY OF TERMINATION ON THE N'TH SPECIMEN.
C
      J=NR(N)
      PT1=SPV(J)*PC
C
C     UPDATE THE STATE PROBABILITY VECTOR.
C
    3 IF(J-1) 5,5,4
    4 SPV(J)=SPV(J-1)*PC+SPV(J)*Q
      J=J-1
      GO TO 3
    5 SPV(1)=SPV(1)*Q
C
C     UPDATE THE PROBABILITY OF THE NOT TERMINATING AFTER THE N'TH
C     SPECIMEN.
C
      PNT1=PNT1-PT1
C
C     UPDATE THE EXPECTED TEST COST.
C
      ETC1=ETC1+DBLE(FLOAT(N))*PT1
C
C     CHECK LINE COUNT.
C
      IF(L-55) 8,6,6
    6 L=0
C
C     WRITE PAGE HEADER.
C
      WRITE(1,7) FF
    7 FORMAT(A1/T5,1HN,T10,1HR,T16,2HPT,T26,2HPC,T37,4HP(0),T47,4HP(1),
     $T57,4HP(2),T67,4HP(3),T77,4HP(4),T87,4HP(5),T97,4HP(6),T107,
     &4HP(7),T117,4HP(8),T127,4HP(9))
    8 L=L+1
      IMAX=NR(N)
C
```

```
C     SAVE DATA FOR PLOTTING.
C
      PARRAY(N,1)=FLOAT(N)
      PARRAY(N,2)=FLOAT(IMAX)
      PARRAY(N,3)=SNGL(PNT1)
C
C     WRITE OUT SPECIMEN NUMBER, PROBABILITY OF TERMINATION, PROBABILITY
C     OF CONTINUATION, AND THE STATE PROBABILITY VECTOR.
C
    9 WRITE(1,10) N,IMAX,PT1,PNT1,(SPV(I),I=1,IMAX)
   10 FORMAT(2I5,2F10.6,2X,10F10.6)
C
C     ADD ADDITIONAL TEST COST DUE TO PROBABILITY OF TRUNCATION OF THE
C     TEST ON THE NMAX'TH SPECIMEN.
C
      ETC1=ETC1+PNT1*DBLE(FLOAT(NMAX))
      WRITE(1,11) ETC1
   11 FORMAT(//'EXPECTED TEST COST = ',F10.4)
C
C     PLOT THE REJECTION NUMBER (R) AND THE PROBABILITY OF CONTINUATION
C     PAST THE N'TH SPECIMEN AS FUNCTIONS OF N.
C
      CALL TNOU(FF,1)
      CALL PPLOT(PARRAY(1,1),PARRAY(1,2),NAME,INFO(21),S)
      CALL CPLOT(PARRAY(1,1),PARRAY(1,2),NAME,INFO(21),S)
      CALL TNOU(FF,1)
      CALL PPLOT(PARRAY(1,1),PARRAY(1,3),NAME,INFO(31),S)
      CALL CPLOT(PARRAY(1,1),PARRAY(1,3),NAME,INFO(31),S)
C
C     CONPUTE THE OPERATING CHARACTERISTIC AND THE EXPECTED TEST COST
C     AS FUNCTIONS OF PI.
C
C     WRITE HEADER.
C
      WRITE(1,12) FF
   12 FORMAT(A1/T7,2HPC,T17,3HPLA,T26,3HETC)
C
      DO 14 I=1,101
      PI=10.0D 0D**(0.03D 0D*DBLE(FLOAT(I))-3.03D 0D)
      CALL SAT(PI,SPV,NR,NMAX,PLA,ETC)
C
C     WRITE RESULTS
```

```
C
      WRITE(1,13) PI,PLA,ETC
   13 FORMAT(3F10.4)
C
      PARRAY(I,1)=SNGL(PI)
      PARRAY(I,2)=SNGL(PLA)
   14 PARRAY(I,3)=SNGL(ETC)
C
C     PLOT THE RESULTS.
C
      CALL TNOU(FF,1)
      CALL PPLOT(PARRAY(1,1),PARRAY(1,2),NAME,INFO(1),S)
      CALL CPLOT(PARRAY(1,1),PARRAY(1,2),NAME,INFO(1),S)
      CALL TNOU(FF,1)
      CALL PPLOT(PARRAY(1,1),PARRAY(1,3),NAME,INFO(11),S)
      CALL CPLOT(PARRAY(1,1),PARRAY(1,3),NAME,INFO(11),S)
C
      CALL TNOU(FF.1)
C
      IF(YSNO$A('NEW CASE',8,A$NDEF)) GO TO 1
C
C     TO OBTAIN CALCOMP PLOTS REMOVE THE C FROM THE STATEMENTS ABOVE
C     WHICH CALL CPLOT AND THE FOLLOWING STATEMENT.
C
      CALL PLOT(S(4)+4.0,0.0,999)
C
      CALL EXIT
      END
```

```
C
C     THIS ROUTINE RETURNS THE REJECTION NUMBER (NR) AS A FUNCTION OF
C     THE SPECIMEN NUMBER (N), THE CRITICAL PI (PC), AND THE SPECIFIED
C     RISK (BETA).  ALSO RETURNED IS THE ACTUAL RISK  (AR) BEING
C     TAKEN AT THE N'TH STAGE WHICH WILL GENERALLY BE LESS THAN BETA.
C
      SUBROUTINE RN(N,PC,BETA,AR,NR)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      FN=DBLE(FLOAT(N))
      Q=1.0D 00-PC
      I=0
      FX=Q**N
      S=FX
    1 IF(1.0D 00-S-BETA) 2,3,3
    2 AR=1.0D 00-S
      NR=I+1
      RETURN
    3 X=DBLE(FLOAT(I))
      FX=PC*(FN-X)/((X+1.0D 00)*Q)*FX
      S=S+FX
      I=I+1
      GO TO 1
      END
```

```
C
C     THIS ROUTINE COMPUTES THE PROBABILITY OF LOT ACCEPTANCE (PLC)
C     AND THE EXPECTED TEST COST (ETC) FOR A SEQUENTIAL ACCEPTANCE TEST
C     FOR A GIVEN FRACTION LOT DEFECTIVE (PI), MAXIMUM NUMBER OF
C     SPECIMENS TO BE TESTED, AND A GIVEN REJECTION BOUNDARY WHICH IS
C     SPECIFIED BY ARRAY (NR).  ARRAY SPV IS USED FOR A WORK ARRAY AND
C     DOES NOT HAVE TO BE INITIALIZED.
C
      SUBROUTINE SAT(PI,SPV,NR,N,PLA,ETC)
C
      IMPLICIT REAL*8 (A-H.O-Z)
C
      DIMENSION NR(1),SPV(1)
C
C     INITIALIZE THE STATE PROBABILITY VECTOR
C
      SPV(1)=1.0D 00
      DO 1 I=2,20
    1 SPV(I)=0.0D 00
C
      Q=1.0D 00-PI
      ETC=0.0D 00
      PLA=1.0D 00
      DO 4 J=1,N
      K=NR(J)
      PT=SPV(K)*PI
      PLA=PLA-PT
      ETC=ETC+PT*DBLE(FLOAT(J))
C
C     UPDATE THE STATE PROBABILITY VECTOR.
C
    2 IF(K-1) 4,4,3
    3 SPV(K)=SPV(K-1)*PI+SPV(K)*Q
      K=K-1
      GO TO 2
    4 SPV(1)=SPV(1)*Q
C
      ETC=ETC+PLA*DBLE(FLOAT(N))
      RETURN
      END
```

```
C
C     THE PROGRAM CALCULATES SEVERAL VARIABLES WHICH CHARACTERIZE A
C     GENERAL SEQUENTIAL LOT ACCEPTANCE TEST TRUNCATED AT NMAX
C     SPECIMENS.
C
C     AMONG THE VARIABLES CALCULATED AND STORED FOR PLOTTING ARE:
C       1. PCA(N,K+1).  THE ARRAY OF PRIOR PROBABILITIES OF CONTINUING
C          THE TEST WITH K DEFECTIVES AT N SPECIMENS.
C
C       2. PCU(N).  THE UNCONDITIONAL PRIOR PROBABILITY OF CONTINUING
C          THE TEST BEYOND N SPECIMENS.
C
C       3. PT(N).  THE PRIOR PROBABILITY OF TERMINATING THE TEST AT THE
C          N'TH SPECIMEN.
C
C       4. ETC.  THE EXPECTED TEST COST MEASURED IN NUMBER OF TEST UNITS.
C
C     TEST TERMINATION OCCURS WHEN EITHER THE REJECTION BOUNDARY, R, OR
C     ACCEPTANCE BOUNDARY, C, IS REACHED.  THESE BOUNDARIES ARE STEP
C     FUNCTIONS OF N AND ARE SPECIFIED BY VECTORS OF THE VALUES OF N AT
C     WHICH THE JUMPS (UNIT STEPS) OCCUR: NV FOR THE REJECTION BOUNDARY
C     AND NCV FOR THE ACCEPTANCE BOUNDARY.  THE ASSOCIATED REJECTION
C     NUMBERS AT THE JUMP POINTS ARE SPECIFIED IN THE R VECTOR.
C
C     ALL OF THE OUTPUT VARIABLES ARE FUNCTIONS OF THE ASSUMED LOT
C     FRACTION DEFECTIVES, PI.  RESULTS ARE OBTAINED FOR A SET OF VALUES
C     OF PI CONTAINED IN THE VECTOR PIV.  CHANGES TO THE REJECTION AND
C     ACCEPTANCE BOUNDARIES REQUIRE PROGRAM EDITING.
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
C     DEFINE VECTORS AND MATRICES
C
      REAL*4 DATA(4,45)
      INTEGER R(7),NV(7),NCV(3)
      DIMENSION PCU(50),PCA(50,10),PT(50),PIV(4)
C
C     FOLLOWING DATA APPLIES TO REJECTION FOR 0.10 CRITICAL DEFECTIVES
C
C     DATA NV/6,12,19,26,33,41,48/,R/3,4,5,6,7,8,9/,IMAX/6/
C
C     FOLLOWING DATA APPLIES TO REJECTION FOR 0.05 CRITICAL DEFECTIVES
```

```
C
      DATA NV/3,11,23,36,50,64,79/,R/2,3,4,5,6,7,8/,IMAX/4/
C
      OATA NCV/52,52,65/     /* ACCEPTANCE FOR 0.10 CRITICAL OEFECTIVES
C
C     THE ACCEPTANCE NUMBERS ASSOCIATEO WITH THE SPECIMENS NCV(1),
C     NCV(2), ETC. ARE 1, 2, ETC.
C
      OATA PIV/0.05,0.10,0.15,0.20/
C
      CALL TNOU(:000400,2)
C
C     READ NMAX,PI
C
C     CALL TNOUA(' NMAX, PI?...> ',15)
C     REAO(1,*) NMAX,PI
      NMAX=45
      NC=0
C
C     LOOP OVER LOT FRACTION OEFECTIVES
C
      00 190 L=1,4
      PI=PIV(L)
C
C     ZERO THE PCA ARRAY
C
      00 100 N=1,50
      PCU(N)=0.00 00
      PT(N)=0.00 00
      00 100 K=1,10
  100 PCA(N,K)=0.00 00
      KPI=R(1)
C
C     FILL THE FIRST KPI-1 COLUMNS OF THE PCA ARRAY
C
      IMP1=IMAX+1
      NFIN=NV(IMP1)-1
      00 110 N=1,NFIN
      IF(N.GE.NCV(1)) GO TO 110
      FN=DBLE(FLOAT(N))
      PCA(N,1)=(1.00 00-PI)**N
      IF(KPI.EQ.3) PCA(N,2)=FN*PI*(1.00 00-PI)**(N-1)
```

```
  110 CONTINUE
C
C     FILL THE REMAINDER OF PCA
C
      DO 120 I=1,IMAX
      PCA(NV(I),R(I))=PI*PCA(NV(I)-1,R(I)-1)
      JMAX=NV(I+1)-NV(I)
      DO 120 JJ=1,JMAX
      J=JJ-1
      N=NV(I)+J
C
C     FIND ACCEPTANCE NUMBER--C(N)
C
      IF(N.GE.NCV(1)) NC=1
      IF(N.GE.NCV(2)) NC=2
      IF(N.GE.NCV(3)) NC=3
      KPMAX=R(I)
      DO 120 KP=KPI,KPMAX
      PCA(N,KP)=PCA(N-1,KP-1)*PI   +
     $PCA(N-1,KP)*(1.DD DO-PI)
      IF(KP.LE.NC+1) PCA(N,KP)=D.DD DD
  120 CONTINUE
C
C     CALCULATE THE PROBABILITY OF CONTINUING THE TEST WITH N SAMPLES
C     TAKEN, I.E. FILL THE PCU VECTOR. AND CALCULATE THE PROBABILITY
C     OF TERMINATING AT N SAMPLES
C
      PT(1)=PI
      IF(KPI.EQ.3) PT(1)=0.DD 00
      PCU(1)=1.DD OD-PT(1)
      DATA(L,1)=PCU(1)
      DO 13D N=2,NMAX
      SUM=D.OD DO
C
C     FIND NR
C
      DO 132 I=1,IMP1
      IF(N.GE.NV(I)) GO TO 132
      NR=R(I)-1
      GO TO 134
  132 CONTINUE
      NR=R(I)-1
```

```
  134 CONTINUE
C
      KPMAX=NR
C
      OO 136 KP=1.KPMAX
  136 SUM=SUM+PCA(N.KP)
      PCU(N)=SUM
      DATA(L,N)=PCU(N)
  130 PT(N)=PCU(N-1)-PCU(N)
      NF=NMAX-1
C
C     CALCULATE THE EXPECTEO TEST COST
C
      PT(NMAX)=PCU(NMAX-1)
      SUM=0.0D 00
      DO 140 N=1,NF
  140 SUM=SUM+OBLE(FLOAT(N))*PT(N)
      ETC=SUM+OBLE(FLOAT(NMAX))*PCU(NMAX-1)
C
C     WRITE ARRAYS OF PCU(N).PT(N). AND THE EXPECTEO TEST COST
C
      CALL TNOU(:106000,1)
      WRITE(1,170) ETC,PI,NMAX
  170 FORMAT(///,T25,'EXPECTED TEST COST = ',F10.5.'   FOR FRACTION DEFE
     &CTIVES = ',F10.4/T25,'AND SAMPLE SIZE = ',I4)
      WRITE(1,150) PCU
  150 FORMAT(///,T25,'PCU VECTOR',//,(1P5G15.6))
      WRITE(1,160) PT
  160 FORMAT(///,T25,'PT VECTOR',//,(1P5G15.6))
      CALL TNOU(:106000,1)
      WRITE(1,175)
  175 FORMAT(T35,'STATE PROBABILITY ARRAY')
      WRITE(1,180) (I,(PCA(I,J),J=1,10),I=1,50)
  180 FORMAT(/,T4,'SAMPLES',T35,'DEFECTIVES'/T7,'N',T16,'0',T26,'1',
     &T36,'2',T46,'3',T56,'4',T66,'5',T76,'6',T86,'7',T96,'8',T106,'9'/
     +,' ',109('-')/(I7,' 1',10F10.5))
  190 CONTINUE
      CALL TNOU(:106000,1)
      WRITE(1,191) PIV
  191 FORMAT(T10,'SUMMARY OF PC VECTORS',/T10,'FRACTION DEFECTIVES',/
     $T6,1HN,4F10.2)
      WRITE(1,200) (J,(DATA(I,J),I=1,4),J=1,45)
  200 FORMAT(/,(I6,4F10.5))
      CALL EXIT
      END
```

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