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TIME SERIES IN M DIMENSIONS: SPATIAL MODELS. (U)

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**ABSTRACT**

The general theory of stationary spatial models is developed: namely MA, moving average; AR, autoregressive; and ARMA, autoregressive moving average processes. As compared to the time series in m dimensions, spatial models may be one-sided, two-sided, or mixed. Free use is made of the previous results of Aroian and his associates in time series in m dimensions. The main theoretical properties of the models in the univariate case are established. The multivariate case is even more important than the univariate. Estimation by minimum variance and simulation of the models are included.

1. INTRODUCTION

The results of time series in m dimensions by Aroian and his coauthors are used to establish the results of spatial models in m dimensions. If m=1 the results apply to events on a line such as ecological distribution of a plant, the average rainfall for a mine, pollution is space or distribution of a mineral in a mine.

Important assumptions are outlined: the characteristic of an event in space is given by

\[ z_{x} = \{ x_{1}, x_{2}, \ldots, x_{m} \} \]

\[ x_{x} = \{ x_{1} - 1, x_{2} - 2, \ldots, x_{n} - m \} \]

Weak stationarity is assumed in space as a minimum assumption:

\[ z_{x} = \{ x_{1}, x_{2}, \ldots, x_{m} \} \]

\[ x_{x} = \{ x_{1} - 1, x_{2} - 2, \ldots, x_{n} - m \} \]

All second order moments exist. Note x may be in any coordinate system, and \( \xi \) may be plus or minus; the results are in m dimensions, only the time coordinate has been dropped from consideration. Although time is a variable, it is not spatial, so new theory must be developed.

Two results, in general, follow from time series. For m=1, one-sided spatial models are covered by Box and Jenkins (1976) if the variable \( t \) in their models is replaced by \( x_{i} \). Isotropic models in space m=2, where \( x_{i} \) is the radius of a circle, are models of m=1, time series in m variables, and are discussed briefly in a later section.

2. MA, AR, AND ARMA MODELS

The two-sided theoretical spatial MA model is defined by

\[ z_{x} = \phi_{0} + \phi_{1} \cdot z_{x-1} + \phi_{2} \cdot z_{x-2} + \cdots + \phi_{p} \cdot z_{x-p} + \epsilon_{x} \]

(2.1)

\[ n = (n_{1}, n_{2}, \ldots, n_{m}) \]

\[ \epsilon_{x} \text{ is a random variable with } \mu = 0, \sigma^{2} > 0 \]

More usually n is finite:

\[ z_{x} = \phi_{0} + \phi_{1} \cdot z_{x-1} + \phi_{2} \cdot z_{x-2} + \cdots + \phi_{p} \cdot z_{x-p} + \epsilon_{x} \]

(2.2)

\[ \phi_{0}, \phi_{1}, \phi_{2}, \ldots, \phi_{p} \text{ are the \( \phi \) parameters} \]

An MA model of spatial order \( p \) in each spatial variable \( x_{i} \) is defined:

\[ z_{x} = \psi_{0} + \psi_{1} \cdot z_{x-1} + \psi_{2} \cdot z_{x-2} + \cdots + \psi_{q} \cdot z_{x-q} + \epsilon_{x} \]

(2.3)

\[ \psi_{0}, \psi_{1}, \psi_{2}, \ldots, \psi_{q} \text{ are the } \psi \text{ parameters} \]

and \( \epsilon_{x} \cdot x_{i} \cdot x_{j} = 0 \), unless \( i = 0 \).

For m=1:

\[ z_{x} = \phi_{0} + \phi_{1} \cdot z_{x-1} + \epsilon_{x} \]

(2.4)

\[ \phi_{0}, \phi_{1} \text{ are the } \phi \text{ parameters} \]

For m=2:

\[ z_{x_{1}, x_{2}} = \phi_{0} + \phi_{1} \cdot z_{x_{1}, x_{2}} + \epsilon_{x_{1}, x_{2}} \]

(2.5)

\[ \phi_{0}, \phi_{1} \text{ are the } \phi \text{ parameters} \]

Usually n is finite: \( n_{1}, n_{2}, \ldots, n_{m} \) of spatial order \( p \) in each \( x_{i} \), is defined:

\[ z_{x} = \psi_{0} + \psi_{1} \cdot z_{x-1} + \epsilon_{x} \]

(2.6)

\[ \psi_{0}, \psi_{1} \text{ are the } \psi \text{ parameters} \]

The two-sided theoretical spatial AR model is defined by:

\[ z_{x} = \phi_{0} + \phi_{1} \cdot z_{x-1} + \epsilon_{x} \]

(2.7)

\[ \phi_{0}, \phi_{1} \text{ are the } \phi \text{ parameters} \]

The theoretical ARMA model for time series in m dimensions is, Voss et al (1980):

\[ z_{x} = \phi_{0} + \phi_{1} \cdot z_{x-1} + \epsilon_{x} \]

(2.8)

\[ \phi_{0}, \phi_{1} \text{ are the } \phi \text{ parameters} \]

The corresponding two-sided ARMA spatial model is

\[ z_{x} = \phi_{0} + \phi_{1} \cdot z_{x-1} + \epsilon_{x} \]

(2.9)

\[ \phi_{0}, \phi_{1} \text{ are the } \phi \text{ parameters} \]

Examples for \( m = 1 \) and 2 respectively are:

\[ z_{x_{1}, x_{2}} = \phi_{0} + \phi_{1} \cdot z_{x_{1}, x_{2}} + \epsilon_{x_{1}, x_{2}} \]

(2.10)

\[ \phi_{0}, \phi_{1} \text{ are the } \phi \text{ parameters} \]

**TIME SERIES IN M DIMENSIONS: SPATIAL MODELS**

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The simplest MA model
\[ z_t = \theta_1 z_{t-1} + \theta_2 z_{t-2} + \epsilon_t \]  
(3.1)
is obtained from (2.3) by replacing \( \psi \)'s by \( \theta \)'s and setting \( P_1 = -1 \), \( \beta = 0 \), and \( \theta = 0 \), \( \theta_1 \neq 0 \). Given
\[ \{ \theta_1, \theta_2 \} \text{, } \{ \theta_1, \rho_2 \} \text{ are given by (3.2). Conversely} \]
\[ \text{given } \{ \rho_1, \rho_2 \} \text{ a sample estimate of } \{ \theta_1, \theta_2 \}, \{ \theta_1, \theta_2 \} \]
may be found from the set replacing population values by estimates:
\[ \rho_1 = \frac{\rho_1}{1 - \theta_1^2}, \rho_2 = \frac{\rho_2}{1 - \theta_1 \theta_2} \]
(3.3)
Set \( \theta_1 = \mu \), \( \theta_2 = \nu \), then
\[ 1 + 2(\mu - \mu')^2 = -2(\mu - \mu')^2 \]
(3.4)
whose solution involves the intersection of a circle and hyperbola. This may lead to four possible sets, but the condition
\[ | \theta_1 | + | \theta_2 | < 1 \]
(3.5)
limits the results to one set.
If \( \theta_1 = 0 \), \( \rho_1 = 0 \), and if \( \theta_1 \), or \( \theta_2 = 0 \), \( \rho_2 = 0 \).
Table 1 lists the values of \( \{ \theta_1, \theta_2 \} \) given \( \{ \theta_1, \theta_2 \}, \{ \theta_1, \theta_2 \} \)
and conversely. The manipulations of (3.4) show
\[ | \rho_1 | \leq 1/3, \text{ and } (1 + 2 \rho_2^2) \geq 4 \rho_2^2 \]
(3.6)
Only the values of \( \rho_1, \rho_2 \) for \(-1 \leq \theta_1 \leq 1 \), and \( \theta_2 = 0 \) \( \theta_2 = 0 \) are tabulated, since the remaining values of \( \{ \theta_1, \theta_2 \} \) may be found from the skew symmetry implied by (3.3).

The characteristic equation of (3.1) is
\[ 1 - \theta_1 B - \theta_2 B^2 = 0 \]
and the corresponding AR representation of the MA model is:
\[ a_t = \sum_{i=0}^{\infty} \phi_i a_{t-i} \]
(2.11)
where \( \phi_i \) are the roots of the characteristic equation.

Note that if \( \theta_1 \neq 1 \), and \( \theta_2 \neq 0 \), then the variance \( \sigma^2 \) converges for the values of \( \{ \theta_1 \} \text{ and } \{ \theta_2 \} \).

Given a sample of \( \{ \theta_1, \theta_2 \} \) is estimated from (3.3) using sample estimates \( \{ \hat{\theta}_1, \hat{\theta}_2 \} \) and the methods of solution already indicated. The approximate variance of \( \hat{\theta} \) is \( \hat{\theta}^2 = (1-\hat{\theta}^2)/n \), and confidence intervals may be obtained if it is assumed that the errors are distributed normally. Another method of estimation is to vary \( \hat{\theta} \), and choose the \( \hat{\theta} \) which minimizes the variance of the error of prediction, \( \epsilon^2 = \epsilon^2 \).

Another simple MA model \( n = 2 \), of the first order is:
\[ z_{1,2} = -\theta_1 a_1 + \theta_2 a_2 + \epsilon_t \]
(3.7)
The characteristic function is:
\[ \phi(z) = \frac{1}{1 - \theta_1 z - \theta_2 z^2} \]
(3.8)
and the approximate variances are:
\[ \sigma^2 = \frac{\epsilon^2}{1 - \theta_1^2 - \theta_2^2} \]
(3.9)

Simulation of MA(1,1) Model
Let \( \theta_1 = -2, \theta_2 = -2 \) in (3.1), \( a_t \) being distributed as \( N(0,1) \). This model is simulated with 100 observations. First random shocks \( a_t \) are
generated, and $z_i$'s are found from (3.1). The estimated correlation coefficients $r_1$, $r_2$ are obtained. The $(\hat{\theta}_1, \hat{\theta}_1)$ is found from (3.4) assuming $r_1$, $r_2$. Minimum error prediction of $\hat{\theta}_1$ and $\hat{\theta}_2$ are also found by using $x_i$'s obtained from $z_i$'s by minimizing $e_2=(z-x)^T z$, this estimate is $\hat{\theta}=(10^-1) = (.330, .340)$, note, that given $z_i$, $x_i = k^2 \hat{\theta}_1 x_i + k^2 \hat{\theta}_2 x_i$. With twenty-five simulation runs it is found that $\hat{\theta}_1 = .284$, $\hat{\theta}_2 = .239$, while minimum variance estimates are $\hat{\theta}_1 = .285$, $\hat{\theta}_2 = .237$. From $\hat{\theta}_1^2 = (1-\hat{\theta}_1^2)/n$, $\hat{\theta}_2^2 = \hat{\theta}_1^2\hat{\theta}_2^2 = .0384$. The approximate formula for the covariance $\gamma(\hat{\theta}_1, \hat{\theta}_1)$ is $\gamma(\hat{\theta}_1, \hat{\theta}_1)$, $\gamma(\hat{\theta}_1, \hat{\theta}_1) = .00784$ for the simulated case versus the actual of $.0032$.

4. AR MODELS

Some simple AR models $m=1$ and $m=2$ are analyzed to show how results may be obtained. Note that in all fully two-sided AR models $\phi_1 = \phi_1$.

The two-sided simplest AR model, $m=1$, from (2.6) is

$$z_i = \phi_1 x_{i-1} + \phi_2 x_{i-2} + \phi_3 (x_{i-4} + x_{i+1}) + \epsilon_i.$$  

Since

$$\hat{\phi}_1^2, \hat{\phi}_2^2, \hat{\phi}_3^2$$

for all $\epsilon$, and given $\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3$, a parabola. (4.2) Note $|\hat{\phi}_1| < \epsilon$, and given $\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3$.

The values of $(\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3)$ are given in Table 2.

The MA model written as an MA model is:

$$x_i = \{ \sum_{l=0}^{\infty} \phi_l z_i \}$$

if $x_i = \{ \sum_{l=0}^{\infty} \phi_l z_i \}$, $\hat{\phi}_1 = \hat{\phi}_1 \{ \sum_{l=0}^{\infty} \phi_l \}$, $\hat{\phi}_2 = \hat{\phi}_2 \{ \sum_{l=0}^{\infty} \phi_l \}$, $\hat{\phi}_3 = \hat{\phi}_3 \{ \sum_{l=0}^{\infty} \phi_l \}$, and $\hat{\phi}_4 = \hat{\phi}_4 \{ \sum_{l=0}^{\infty} \phi_l \}$. (4.3)

Given $\hat{\phi}_1, \hat{\phi}_0 = [2, 2, 2]$, $\hat{\phi}_1$, $\hat{\phi}_2$.

For $m=2$, a simple two-sided model is:

$$z_{i-1}x_i = \hat{\phi}_1 z_{i-1}x_{i-1} + \hat{\phi}_2 z_{i-1}x_{i-2} + \hat{\phi}_3 x_{i-1}x_{i-2}$$

and $\hat{\phi}_1 = .20871 = \hat{\phi}_1$, which checks Table 2.

The autoregressive model given in (4.1) is simulated given that $\hat{\phi}_1 = .2$, and $\hat{\phi}_2, \hat{\phi}_3$'s are $N(0,1)$.

The number of observations in each simulation is 100. First $\hat{\phi}_2, \hat{\phi}_3$'s are obtained and $\hat{\phi}_2, \hat{\phi}_3$'s are obtained by using (4.1) to do this consecutive forward and backward substitutions are performed until convergence is assured. Results of the twenty-five simulation runs are: $\hat{\phi}_1 = .2282, \hat{\phi}_2 = .0143$, $\hat{\phi}_1 = .2091, \hat{\phi}_2 = .0099$. For one particular run when $\hat{\phi}_1 = .2$, theoretically $\hat{\phi}_1 = .2087$ and $\hat{\phi}_2 = .0143$ $\hat{\phi}_1 = .2091, \hat{\phi}_2 = .0099$. For one particular run when $\hat{\phi}_1 = .2$, theoretically $\hat{\phi}_1 = .2087$ and $\hat{\phi}_2 = .0143$.
with \( \phi_{11} = \phi_{-1-1} = \phi_1 \) and \( \phi_{1-1} = \phi_{-11} = \phi_2 \),

\[
\begin{align*}
\text{hence } & z_{x_1} x_2 = \phi_1 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) \\
& + \phi_2 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) x_1 x_2 \\
& + \phi_3 z_{x_1-1} x_2 + a' z_{x_1} x_2,
\end{align*}
\]

then \( z_{x_1} x_2 = \phi_1 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) + \phi_2 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) x_1 x_2 + \phi_3 z_{x_1-1} x_2 + a' z_{x_1} x_2. \)

The characteristic function is:

\[
1 - \phi (B) (B_x + B_x^{-1}) - \phi (B) (B_x + B_x^{-1}) = \phi (B_x + B_x^{-1}) = 0, \quad \phi \neq 0, B_x \neq 0.
\]

The Yule-Walker equations are:

\[
\begin{align*}
\rho_{01} &= \phi_1 (1+\rho_{02}) + \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2,
\end{align*}
\]

with solution:

\[
\begin{align*}
\phi_1 (1+\rho_{02}) - \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2,
\end{align*}
\]

and

\[
\phi_2 (1+\rho_{02}) - \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2.
\]

Estimation, the approximate variance-covariance matrix, and confidence limits as well as minimum variance estimates are found as before.

It should be mentioned that the partial autocorrelation function of the AR models have a cutoff property, for the first model \( m = 1 \), all \( \phi_i \), \( i > 1 \) are zero, and \( \phi_i \) is a partial coefficient of correlation. Where there are two \( \phi_i 's \), \( \phi_1 \) and \( \phi_2 \) both are partial coefficients of correlation, and \( \phi_1 > 1 \) are zero. This property is helpful in determining how far to proceed in the \( \phi_i 's \); alternatives are the analysis of variance methods, and that in which the variance of the error of prediction falls as the \( \phi_i 's \) increase. Other possible alternatives still unexplored are the \( \chi^2 \) test and the Kolmogoroff-Smirnov test. An important point to remember is, as the number of \( \phi_i 's \) increase, so does the variance of the forecast errors.

5. ARMA MODELS

The two models considered are simplifications of (2.10) and (2.11):

\[
\begin{align*}
z_{x_1} x_2 &= \phi_1 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) + \phi_2 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) + a_{x_1} x_2, \quad (5.1)
\end{align*}
\]

\[
\begin{align*}
z_{x_1} x_2 &= \phi_1 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) + \phi_2 (z_{x_1-1} x_2 - 2 z_{x_1} x_2 + 1) + a_{x_1} x_2, \quad (5.2)
\end{align*}
\]

For (5.1) the equations for \( \phi_1^2, \phi_1, \) and \( \phi_2 \) are:

\[
\begin{align*}
\phi_1 &= \phi_1 (1+\rho_{02}) + \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2, \quad (5.3)
\end{align*}
\]

\[
\begin{align*}
\phi_1 &= \phi_1 (1+\rho_{02}) + \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2, \quad (5.4)
\end{align*}
\]

\[
\begin{align*}
\phi_2 &= \phi_1 (1+\rho_{02}) + \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2, \quad (5.5)
\end{align*}
\]

From these:

\[
\begin{align*}
\phi_1^2 = \phi_1 (1+\rho_{02}) + \phi_2 (1+\rho_{01}) \rho_{01} - \phi_2 (1+\rho_{01}) \rho_{01} + \phi_3 z_{x_1} x_2 + a' z_{x_1} x_2.
\end{align*}
\]

From (5.6) may be solved for \( \phi_1, \rho_1 \) using \( (\phi_1, \rho_1) \). The restrictions on \( \phi_i, \phi_i^0 \) for the AR and MA model apply. Both (5.1) and (5.2) may be written in the equivalent AR or MA model since:

\[
\begin{align*}
\phi_1 (1+\rho_{02}) (1+\rho_{01}) x = (1-\phi_1 (1+B_x+1)) x, \quad \phi_2 (1+\rho_{02}) (1+\rho_{01}) x = (1-\phi_1 (1+B_x+1)) x,
\end{align*}
\]

\[
\begin{align*}
\phi_1 (1+\rho_{02}) (1+\rho_{01}) x = (1-\phi_1 (1+B_x+1)) x, \quad \phi_2 (1+\rho_{02}) (1+\rho_{01}) x = (1-\phi_1 (1+B_x+1)) x.
\end{align*}
\]

with similar results for (5.2).

A more general model than (5.1) is:
\[ z = \psi_1(z_{t-1} + z_{t-2}) + \theta_1 \epsilon_t - \theta_0 \epsilon_{t-1} \]  
which reduces to (5.1) if \( \theta_0 = 1 \).

Now
\[ 0 \phi z = a \phi_{z}^{2} + \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) = 1 - \phi_{1} \phi_{z_{1}}^{-1} \]
\[ \phi_{z} = \psi_{1}(1 + c) = 0 \phi_{z}^{2} + \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = \psi_{1}(1 + c) + 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = \psi_{1}(1 + c) + 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = \psi_{1}(1 + c) + 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = \psi_{1}(1 + c) + 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]

Solve (5.9) using \( 0 \phi_{z} \phi_{z} \) from the first equation, substitute this into the other three and solve for \( \phi_{z} \phi_{z} \phi_{z} \) using the sample values \( t_{1}, t_{2}, t_{3}, t_{4} \) for the \( \phi_{z} \). Write (5.8) as
\[ \phi_{z} = (1 - \phi_{z} \phi_{z}^{-1}) \phi_{z} = 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]
\[ \phi_{z} = 0 \phi_{z}^{2} \phi_{z_{1} \phi_{z_{1}}}(1 - 2 \phi_{1} \phi_{z_{1}}^{-1}) \]

For nonstationary processes differences in two variables may be used or transformations. Another alternative to transformations or to differing direct treatment of nonstationarity is feasible and will be investigated subsequently.

6. CONCLUSIONS

The methods of time series in \( m \) dimensions are applied to two-sided spatial models in one and two dimensions: MA, AR, and ARMA models illustrate the techniques including estimation. These results presented in the paper are based on the second order moments, and MA, AR, and ARMA models as developed in time series in \( m \) dimensions.

BIBLIOGRAPHY


Table 2

Values of \( \{\phi_1, \rho_1, \rho_2, \rho_3\} \)

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