ON THE USE OF FISHER'S LINEAR DISCRIMINANT FOR IMAGE SEGMENTATION--ETC (U)

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by

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DISCRIMINANT FOR IMAGE SEGMENTATION

1. Introduction

The application of supervised and unsupervised classification technique
for image segmentation by computer has received much attention within the last
few years (see e.g. [1],[2],[3]). In this report further segmentation results
are presented on infrared and reconnaissance images using statistical pattern
recognition method.

The available pictorial data are some pieces of infrared photographs and
one piece of aerial photograph for reconnaissance applications. Each infrared
image considered contains 64x64 points with 256 possible gray levels, and the
aerial photo contains 300x400 points with same possible gray levels. Each pic-
ture is stored in the magtape line by line. The computer segmentation is per-
formed at the PDP 11/45 minicomputer.

2. Algorithm and Definitions

A general introduction to the Fisher’s linear discriminant analysis is in
[3], [4]. Let \( \mathbf{x} \) be a matrix measurement and \( \mathbf{X}_i \), \( i = 1,2 \) be the collection of
\( n_i \) measurements belonging to the \( i \)th class. The two classes considered are the
target area and the background area. Define the sample mean matrix and the
scatter matrices as

\[
M_i = \frac{1}{n_i} \sum_{\mathbf{x} \in \mathbf{X}_i} \mathbf{x} ; \quad i = 1,2
\]

\[
S_i = \sum_{\mathbf{x} \in \mathbf{X}_i} (\mathbf{x} - m_i)(\mathbf{x} - m_i)^T
\]

\[
M = \frac{1}{n} \sum_{\mathbf{x} \in \mathbf{X}} \mathbf{x} = \text{mean of all samples}; \quad n = n_1 + n_2
\]

\[
S = S_1 + S_2 = \text{pooled scatter matrix}
\]
Then the Fisher's linear discriminant computes

\[ Y = W'X + W_0 \]

where \( W \) = weight matrix

\[ = \alpha n S_w^{-1} (m_1 - m_2); \alpha \) is an arbitrary constant.

\( W_0 \) = threshold weight

\[ = -W'W \]

Decision is to choose class 1 if \( Y > 0 \) and Class 2 otherwise.

Now consider the extension to noisy scenes. Let the noisy image be

\[ x(i,j) = S(i,j) + n(i,j) \]

where \( S(.) \) and \( n(.) \) are the original image and noise respectively. The presence of observation noise (assumed to be additive white Gaussian with variance 1) creates a probability distribution of gray levels in the background as well as in the object. That is the object and the background gray levels, instead of being constant, are normally distributed with common variance 1 and means \( M_s \) and 0, respectively. Definition of the theoretical error probability as given in [5] is

\[ P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} \, dy \]

where \( u = \text{norm of } [n(M_1 - M_2)'S_w^{-1} (M_1 - M_2)] \)

\[ = \text{norm of } A \]

\[ = (\text{dimension of } A). \max_{i,j} |a_{i,j}| \] (see e.g. [6])

The signal-to-noise ratio is defined as

\[ SNR = \frac{M_s}{\text{noise variance}} \]

where the noise is Gaussian distributed with variance 1 and zero mean.
Other definition of $\frac{S}{N}$, like,

$$\frac{S}{N} = \left(\frac{M - \text{noise standard deviation}}{\text{noise variance}}\right)^2$$

may not be appropriate to these image.

3. Experiments

3.1 Segmentation of the Alabama data base infrared images: Table 1a describes the number sequence of sub-pictures of Figures 1 & 2. Figure 1a is the original image and noisy images with different magnitude Gaussian noise added. Figure 1b is classification results using fixed weight matrix in [3]. Figure 1c is classification results using unfixed weight matrix which depend on each sub-picture. Figure 2 is same as Figure 1 except that targets are one tank and one jeep. Percentage of errors for noisy images and theoretical value are tabulated in Table 1b. Figure 3 is the relationship between the percentage (%) of errors and the signal-to-noise ratio $\left(\frac{S}{N}\right)$ for the one target (upper figure) and the two targets (lower figure). The object boundaries can be completely extracted from segmented images by using the cross (Robert) gradient and Sobel operator [7] as shown in Figure 4.

3.2 Segmentation of the reconnaissance images. A diagram that illustrates the selection of learning samples is given in Figure 5. Figure 6a is the original image (256 x 256) with threshold = 150. Figure 6b, 6c are classified results. In Figure 6c, $M_1$, $M_2$ samples are taken from regions III, I, respectively. Because of the gray level distribution in region II, the classified result given by Figure 6c is better than Figure 6b. Figure 7, 8 and 9 include additive white Gaussian noise with variance 1 and zero mean; $\frac{S}{N}$ are 0.2, 1.0, 2.0 respectively. In these three sets of figures, b and c are classified results by using fixed weight matrix, d and e are classified results by using weight matrix
which depends on each image. By following the same procedure to segment the whole image (300 x 400), the results are shown in Figure 10 to Figure 13.

4. Conclusion

These results suggest that the Fisher's linear discriminant performs very effectively in pixel classification and image segmentation. The classified results compare very favorably with theoretical analysis. However, the results are somewhat dependent upon the selection of learning samples. Further study will be made in using unsupervised classification techniques for image segmentation.

References


Table 1a

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Table 1b Percentage of errors for noisy data

<table>
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<th>Test image</th>
<th>Signal-to-noise ratio</th>
<th>Single tank Fixed W</th>
<th>Unfixed W</th>
<th>Theoretical</th>
<th>Tank and Jeep Fixed W</th>
<th>Unfixed W</th>
<th>Theoretical</th>
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<td>1</td>
<td>No noise</td>
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<td>-</td>
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<td>2</td>
<td>0.2</td>
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</table>
Fig. 3a

Fig. 3b

Notes:
(1) * fixed weight matrix
(2) o unfixed weight matrix
(3) q theoretical
Original Image  Classified Result  Sobel Operator Threshold = 4  Robert Gradient Threshold = 1

Fig. 4

Fig. 5
Fig. 6a
$\text{Th}=150$

Fig. 6b $M_1, M_2(\text{II, I})$

Fig. 6c $M_1, M_2(\text{III, I})$

Fig. 7a
$\text{Td}=150$
$s/n=.2$

Fig. 7b $M_1, M_2(\text{II, I})$

Fig. 7c $M_1, M_2(\text{III, I})$

Fig. 7d $M_1, M_2(\text{II, I})$

Fig. 7e $M_1, M_2(\text{III, I})$
Fig. 8a
Til = 150
s/n = 1.

Fig. 9a
Til = 150
s/n = 2.
Fig. 12a  
TH = 150  
s/n = 1.

Fig. 12b $M_1, M_2(II, I)$  
Fig. 12c $M_1, M_2(III, I)$

Fig. 12d $M_1, M_2(II, I)$  
Fig. 12e $M_1, M_2(III, I)$

Fig. 13a  
TH = 150  
s/n = 2.

Fig. 13b $M_1, M_2(II, I)$  
Fig. 13c $M_1, M_2(III, I)$

Fig. 13d $M_1, M_2(II, I)$  
Fig. 13e $M_1, M_2(III, I)$
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Fisher's linear discriminant; image segmentation; noisy images, infrared image; reconnaissance image; theoretical error probability.

Further experimental results are reported on the use of Fisher's linear discriminant for segmentation of infrared images and reconnaissance images. Theoretical error probability computed compares very favorably with the experimental percentage misclassification.