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Reconstructed images of Alpha Orionis using stellar speckle interferometry

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INTRODUCTION

The application of stellar speckle interferometry to astronomical purposes was done by Labeyrie and Gerari et al.1 The basic principle used in that method consists of measuring the mean-squared modulus of short-exposure astronomical photographs. This is generally accomplished by summing the infinity diffraction patterns of these photographs optically produced using a laser. A variety of theoretical analyses of this method2-9 have shown that this result contains recoverable information on scales out to the telescope diffraction limit. However, since the mean-squared modulus is produced, phase information is lost and a diffraction-limited image cannot be reconstructed.

The information needed to completely define a diffraction-limited image is present within the speckle data. Methods have been proposed and simulated to extract this information.10 In such techniques the phase information is obtained from the statistical autocorrelation of speckle photographs in a manner similar to that used to extract the modulus information. However, for objects with sizes near the telescope resolution limit we have developed an empirical technique for reconstructing nearly diffraction-limited images.11 This method has been used to study the possibility of surface structure on the bright star α Orionis (Betelgeuse).

Since the detailed procedures and results are presented in this paper we will present only a summary here concentrating on the limitations and subsequent results.

EXPERIMENTAL METHOD

A single speckle pattern is composed of individual fringes or speckles which for a point source object have scales near that of the telescope Airy disk.2 In Fig. 1 speckle exposures obtained on the Kitt Peak National Observatory 4 m telescope for three different types of objects are shown. As stated above, the individual speckles for the point source object α Orionis have scales near the Airy disk size for the 4 m telescope. Since a more complex object, such as a binary star system, exhibits a speckle pattern which is the convolution of the object intensity with a point source speckle pattern, the speckles in the image of the binary star α Auriga are each doubled. Consequently, if a point source speckle pattern can be observed close enough to a program object so that the light from both objects passes through the same column of turbulent atmosphere (i.e., within the isoplanatic patch) then the speckle pattern of the point source object may serve as the instantaneous point spread function and recon-

![Fig. 1. Speckle exposures taken on the Kitt Peak National Observatory 4 m telescope. Each photograph covers a 3 arcsec square area. (a) Alpha Orionis (Betelgeuse), a resolved supergiant star with an angular diameter of ~0.05 arcsec; (b) Gamma Orionis (Bellatrix), a point source star; (c) Alpha Auriga (Capella), a binary star with an angular separation of ~0.06 arcsec.](image-url)
structed images may be made using standard Fourier deconvolution methods. But there are very few suitable point source objects near enough interesting objects (about 2 arc sec). Consequently, the Lynds et al. technique relies on extracting point spread function information directly from the observed object speckle patterns themselves. This is accomplished by identifying the brightest local maxima (i.e., the location of the brightest speckles) within the resolved object speckle photographs. If this set of positions is then assumed to represent the locations where a point source would also exhibit bright speckles, then it can be used as a synthetic “point spread function” to recover a seeing-free image.

In practice we use the locations of bright speckles to construct the profile of an average speckle for both a point source and resolved object. We then assume that the average object speckle \( o(x,y) \) is the convolution of the average point source speckle \( p(x,y) \) with the actual object intensity profile \( I(x,y) \), just as the entire object speckle pattern is the convolution of a point source speckle pattern with the real object intensity distribution.

\[
o(x,y) = p(x,y) * I(x,y).
\]

Consequently, by deriving \( o(x,y) \) and \( p(x,y) \) observationally, we may recover \( I(x,y) \). The key practical limitation to this technique is the ability to identify individual speckles. In Fig. 1(c), the Betelgeuse image, the key limitation is evident. The fainter speckles overlap to such a degree that it is difficult to identify individual ones. However, the brightest speckles seem to be widely enough separated so that they are individually identifiable. For this reason speckle identification is restricted to the brightest speckles in each photograph.

A digital reduction scheme is used to identify and extract the desired mean speckle profiles. Photographic speckle data is obtained using the Kitt Peak National Observatory 4 m telescope. With an image scale of 2.2 arc sec/cm, these 35mm photographs have exposure times of 0.01 s. To study the red supergiant star Alpha Orionis (Betelgeuse) 40 exposures of it and a comparison point source star, Gamma Orionis, were obtained on each of five nights in March 1974. Two bandpasses of 100 A each were chosen for these exposures. One at 5180 A covers the TiO absorption band in the spectrum of cool stars like a Orionis. Light from this band represents a temperature-sensitive level high within the stellar atmosphere. For comparison purposes a second band at 5100 A was used. The data are digitized using a PDS microdensitometer, with the recorded photographic densities converted to intensities using a standard conversion curve.

To identify the location of the bright speckles each speckle photograph is fit with a two-dimensional Gaussian profile which is then removed from the data. This serves to delete a background which consists of the unresolved faint speckles and noise from a variety of sources such as scattered light within the system. To unambiguously identify the center of the bright speckles, a low-pass digital filter is applied to the data to remove high-frequency film grain noise. The cutoff of this filter is chosen to lie near the telescope cutoff. It is then easy to pick out the bright speckles as the peaks of local intensity maxima brighter than a pre-chosen threshold. A field of impulses, \( \delta(x,y) \), is constructed which represents these locations.

In order to co-add the now located speckles in each image we consider each photograph to be represented by a simple model where the image, \( O(x,y) \), is the sum of discrete speckles:

\[
O(x,y) = \sum a(x,y) = \sum p(x,y) \ast f(x,y);
\]

as mentioned previously, \( p(x,y) \) is the profile of a point source speckle. This relationship can now be re-written using the impulse field \( \delta(x,y) \) which was constructed to represent the location of the bright speckles:

\[
O(x,y) = \{ \delta(x,y) \ast p(x,y) \} \ast f(x,y).
\]

We obtain our mean speckle profile by deconvolving \( \delta(x,y) \), which we have derived from the observed image \( O(x,y) \). \( p(x,y) \) is obtainable from the point source star observations. To insure that systematic effects are minimized the point source is chosen to lie relatively near to the object on the sky and the exposures are obtained in time as closely preceding and following the object exposures as possible. Resulting mean speckle exposures averaged over all exposures obtained during one of the nights are shown in Fig. 2.

Using these data several types of information may be obtained. By comparing the images of \( \alpha \) Orionis reconstructed in both wavelength regions, an indication of surface structure is present which persisted in all five nights' observations. Since the results are discussed at length by Lynds et al. they will not be further discussed here. Another parameter, useful for the study of conditions within stellar atmospheres, is the limb darkening \( f(r) \), which represents the change of brightness on the stellar surface out from the center of the stellar disk: It usually takes a form

\[
f(r) = f_c [1 - x (1 - \cos \theta)],
\]

where \( f_c \) is the intensity at disk center, \( \theta \) is the angle between the point at \( r \) and disk center, and \( x \) is the limb darkening parameter taking values \( 0 < x < 1 \). To derive this and the stellar radius, both assumed to be radially symmetric quantities, the two-dimensional speckle images are summed radially to produce the one-dimensional curves shown in Fig. 2. It is to be noted that on theoretical grounds, although a point source speckle is not expected to exactly resemble an Airy disk, the observed profile bears a remarkable similarity to one, especially the presence of secondary rings. This point source profile is used to derive the above parameters by convolving it with a variety of stellar disk profiles varying the limb darkening and radius to provide the best fit to the observed \( \alpha \) Orionis profile. Estimates of accuracy are acquired by deriving these parameters using different subsets of the
data. In this manner very high accuracy (~± 1%) results have been obtained from a small amount of data.

LIMITATIONS

This method is subject to several sources of systematic errors. The basic requirements of this method may be summarized: (i) Discrete bright speckles must be clearly identifiable on individual exposures, and (ii) a comparison point source profile must be obtained which represents as closely as possible the function which is convolved with the object to produce the speckles in the object’s speckle pattern.

For this technique to work there must be enough photons to produce a speckle pattern in which the 20 or so brighter speckles are identifiable. We found that speckles would not be appreciable smeared by the atmospheric motions for exposure times as long as 0.08 s. We also found that a spectral bandpass of as large as 500 Å centered at 5500 Å could be used. For larger bandpasses the higher-order speckles near the edge of the pattern were badly elongated, showing the effects of incoherence over the approximately 20 orders of interference present. With these limitations it was found that a speckle pattern showing recognizable speckles could be obtained for stars as faint as 7th magnitude using the Kitt Peak photographic system. With a more efficient TV recording system this limit may be extended to about 9th magnitude.

With larger objects even the brightest speckles overlap so that individual speckles cannot be identified. Since there does not exist a wide selection of objects with similar brightness and varying angular diameter it is difficult to determine when this limitation becomes critical. However, for σ Orionis, with an angular diameter of about twice the 4 m Airy disk size, accurate diameters which agreed well with other deter-
TABLE I. Derived angular diameters (uniform disk) for Alpha Orionis obtained using differencing reduction parameters.

<table>
<thead>
<tr>
<th>Diameter for a uniform disk (units of 0.001 arc seconds)</th>
<th>Low-pass filter cutoff frequency (units of airy disk size)</th>
<th>Threshold for identification of bright speckles (units of the brightest point in each photograph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>53 x 4</td>
<td>0.43</td>
<td>0.85</td>
</tr>
<tr>
<td>52 x 3</td>
<td>1.0</td>
<td>0.85</td>
</tr>
<tr>
<td>49 x 3</td>
<td>1.5</td>
<td>0.65</td>
</tr>
<tr>
<td>53 x 4</td>
<td>1.0</td>
<td>0.75</td>
</tr>
<tr>
<td>48 x 2</td>
<td>1.0</td>
<td>0.55</td>
</tr>
</tbody>
</table>

minutions of the diameter were obtained. Recently, angular diameters and limb darkening coefficients were obtained for α Ceti (Bira) which were about three times the Airy disk size. Clearly identifiable speckles are present for both of these stars. No speckles are visible in speckle photographs of the Jovian moons which have angular diameters of about 20 Airy disks (~0.6–1 arc sec). However, for asteroid Vesta with a diameter of about 5–10 Airy disks, individual speckles may be just identifiable. Based on these data we conclude that objects up to 3–5 Airy disk diameters in angular extent may have images reconstructed using this method.

Implicit in our technique is the assumption that each speckle is an approximation to an average speckle. However, by restricting the method to bright speckles we must consider the possibility that speckles systematically differ as a function of their brightness. We have therefore determined the sensitivity of our results to the method used to locate bright speckles as well as the criteria under which bright speckles are defined. To accomplish this a set of data obtained on January 15, 1976 was reduced to determine angular diameters while varying the parameters used to identify speckle locations. The observations consisted of 10 speckle photographs each of α Orionis. These data were reduced as described previously.

We varied both the cutoff frequency for the low-pass filter used to smooth that data and the threshold intensity above which speckles were defined as bright. The low-pass filter used in the speckle algorithm was set at values corresponding to 1/2, 1, and 1 1/2 times the Airy disk size (0.020, 0.030, and 0.045 arc sec, respectively). The threshold above which speckles were called “bright” was altered between 55%, 65%, and 75%. The results of these tests are listed in Table I.

It is apparent that all of the derived diameters agree to within the quoted errors. However, the values derived using a higher cutoff in the filter, as well as that result which included fainter speckles, may be slightly smaller than the others. This may be attributable to the effects of high-frequency noise in the photographic data. With a filter that admits frequencies higher than the telescope diffraction cutoff, substantial noise may be passed causing some noise peaks to be identified as speckles. This would lower the size of a mean resolved object speckle proportionately more than the compari-sun point source profile causing a decrease in the derived diameter. Similarly, the lowering of the intensity threshold for the identification of bright speckles will admit more of the fainter speckles with lower signal-to-noise. Consequently, including these speckles would also lower the derived diameter for the same reason.

It should be noted that in all cases where we obtained data on a number of different nights, the diameters derived agreed from night to night within the errors of the result. It is concluded that our results are not sensitive to atmospheric conditions.

We have shown that our method provides nearly diffraction limited images for objects up to several times the telescope Airy disk size. Although there is some sensitivity to high frequency noise in the results, it does not appear to be serious. Current efforts are underway to improve the data acquisition system add a low-noise photoelectric system which will extend our capabilities to fainter objects.

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