SUBMARINE HYDRAULIC CONTROL ANALYSIS
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by

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Jun 1980

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with individual elements in modular computer programs to facilitate design and alteration decisions.

The line model was developed and coupled with a submarine driving-system servo-loop in order to demonstrate the above premises. However, due to programming difficulties additional work is necessary to successfully complete the analysis.
Submarine Hydraulic Control Analysis

by

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Submitted in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the
NAVAL POSTGRADUATE SCHOOL
June 1980

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ABSTRACT

A mathematical model was developed to include line effects in the submarine hydraulic system dynamic performance analysis. The project was undertaken in an effort to demonstrate the necessity of coupling the entire hydraulic power network for an accurate analysis of any of the subsystems rather than the current practice of treating a component loop as an isolated system. It was intended that the line model could be coupled with individual elements in modular computer programs to facilitate design and alteration decisions.

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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area (in²)</td>
</tr>
<tr>
<td>BV</td>
<td>blocking valve</td>
</tr>
<tr>
<td>c or c₁</td>
<td>constant</td>
</tr>
<tr>
<td>Cₙd</td>
<td>orifice flow discharge coefficient</td>
</tr>
<tr>
<td>Cₑ₁</td>
<td>coefficient of external leakage (in³/sec/psi)</td>
</tr>
<tr>
<td>Cᵢ₁l</td>
<td>coefficient of cross-port leakage (in³/sec/psi)</td>
</tr>
<tr>
<td>cs</td>
<td>centstokes</td>
</tr>
<tr>
<td>d</td>
<td>differential operator</td>
</tr>
<tr>
<td>D</td>
<td>diameter</td>
</tr>
<tr>
<td>dₙl</td>
<td>land diameter of valve (in)</td>
</tr>
<tr>
<td>Dᵣp</td>
<td>piston tail rod diameter (in)</td>
</tr>
<tr>
<td>DV</td>
<td>deceleration valve</td>
</tr>
<tr>
<td>e</td>
<td>exponential (2.71828...)</td>
</tr>
<tr>
<td>F_L</td>
<td>load force</td>
</tr>
<tr>
<td>ꜞₚ₀</td>
<td>degrees Fahrenheit - temperature (lb)</td>
</tr>
<tr>
<td>K</td>
<td>loss coefficient due to abrupt area change</td>
</tr>
<tr>
<td>Kₑ₀</td>
<td>null flow pressure coefficient (in³/sec/psi)</td>
</tr>
<tr>
<td>L</td>
<td>length (in)</td>
</tr>
<tr>
<td>Lₙcy</td>
<td>total cylinder length (in)</td>
</tr>
<tr>
<td>Le</td>
<td>effective length (in)</td>
</tr>
<tr>
<td>L₀</td>
<td>length available for ram to travel with ram in center position (in)</td>
</tr>
<tr>
<td>mₒil</td>
<td>mass of oil (slugs)</td>
</tr>
<tr>
<td>mᵣr</td>
<td>mass of ram (slugs)</td>
</tr>
<tr>
<td>Mₜ</td>
<td>total mass (slugs)</td>
</tr>
<tr>
<td>MIL-L</td>
<td>military lubricant specifications</td>
</tr>
<tr>
<td>P</td>
<td>pressure (psi)</td>
</tr>
<tr>
<td>( \dot{P} = \frac{dP}{dt} )</td>
<td>change in pressure with respect to time (psi/sec)</td>
</tr>
<tr>
<td>P₀</td>
<td>reference pressure (psi)</td>
</tr>
<tr>
<td>psi</td>
<td>pounds per square inch - pressure</td>
</tr>
<tr>
<td>ΔP</td>
<td>differential pressure (psi)</td>
</tr>
</tbody>
</table>
\[ Q_{el} \text{ external leakage flow (in}^3/\text{sec)} \]
\[ Q_{il} \text{ internal cross port flow (in}^3/\text{sec)} \]
\[ Q_{in} \text{ supply flow from header (in}^3/\text{sec)} \]
\[ Q_r \text{ return flow to header (in}^3/\text{sec)} \]
\[ SAE \text{ Society of Automotive Engineers} \]
\[ SG \text{ specific gravity} \]
\[ SSU \text{ Saybolt Seconds Universal} \]
\[ t \text{ time (sec)} \]
\[ T \text{ temperature (°F)} \]
\[ T_0 \text{ reference temperature (°F)} \]
\[ t_p \text{ piston thickness (in)} \]
\[ \dot{V} = \frac{dV}{dt} \text{ changes in volume with respect to time (in}^3/\text{sec)} \]
\[ V_o \text{ reference volume (in}^3) \]
\[ w \text{ flow area coefficient (in}^2/\text{in)} \]
\[ X_r \text{ ram position (in)} \]
\[ \dot{X}_r \text{ ram velocity (in/sec)} \]
\[ \ddot{X}_r \text{ ram acceleration (in/sec}^2) \]
\[ X_v \text{ servovalve spool position (in)} \]
\[ X_{vm} \text{ maximum servovalve spool position (in)} \]
\[ \alpha \text{ cubical expansion coefficient (°F}^{-1}) \]
\[ \beta \text{ bulk modulus (psi)} \]
\[ \beta_l \text{ liquid bulk modulus (psi)} \]
\[ \lambda \text{ absolute viscosity temperature coefficient} \]
\[ \rho \text{ density (slug/in}^3) \]
\[ \rho_o \text{ reference density (slug/in}^3) \]
\[ \rho_{w60} \text{ density of water at 60°F (slug/in}^3) \]
\[ \mu \text{ absolute viscosity (lb sec/in}^2) \]
\[ \nu \text{ kinematic viscosity (in}^2/\text{sec)} \]
\[ \omega_h \text{ hydraulic natural frequency} \]
I. INTRODUCTION

A. GENERAL DISCUSSION

The vast majority of hydraulic systems currently in use are of a dedicated type. That is, they are designed for a specific function with consistent and well defined constraints. In most cases they are further limited to a loop in which the pump supplies power to a single power element. Such systems can be delivered as compact units with a minimum of connecting piping. Some examples of dedicated systems in Naval use are: surface ship steering units, gun mount train and elevation units, and ammunition conveyor systems.

Although submarines have employed dedicated systems in the past, all modern U.S. submarines use constant pressure systems [Ref. 1, Tab. 561-1]. The constant pressure systems contrast with the dedicated type in that the supply and return headers extend the entire length of the ship and serve a multitude of power elements. The total piping length of these systems may exceed the length of the ship by a factor of four or more.

B. THE NEED FOR A LINE MODEL

For the packaged configuration of the dedicated system, with its relatively short piping lengths, the current practice of neglecting line dynamics and lumping the system volume has produced acceptable analytical results. However, even in systems with low line-volume to total-volume ratios, the computed values of the hydraulic natural frequency ($\omega_h$)
are frequently in excess of 40% higher than measured values [Ref. 2]. Since $\omega_n$ sets the upper limit on system response and is affected by the volume as well as line dynamics, a model which includes these factors would undoubtedly produce more accurate results.

It was the intent of this project to demonstrate that the submarine hydraulic system, with its extremely long lines, requires that the line dynamics be included to achieve any degree of accuracy in analytical solutions.

C. POTENTIAL FOR SYSTEM MODEL

Discussions between the author and Mare Island Naval Shipyard Hydraulic Design Division personnel (code 260), as well as a Naval Sea Systems Command representative [Refs. 2 and 3], disclosed that there is currently no computer program to model the dynamic behavior of a submarine hydraulic system. Such a program would be useful in the design of new systems, both for sizing of lines and elements and for determining the response of components served. For existing systems, it would allow a determination of the effects of alterations prior to their implementation.

Additionally, a computer program of a dynamic system model would be advantageous in analyzing vibrations transmitted through the system. The "noise" generated by vibrations is a critical element of the "Submarine Sound Quieting Program," while the fatigue due to vibrations is important to the life of mechanical elements in the system, especially the seals [Refs. 5, 6, and 7].
The development of a dynamic line model which could be programmed in a modular format and coupled with models for the individual elements would facilitate the creation of such an overall system model.
II. SYSTEM MODELED

A. MAGNITUDE

It was recognized early in the undertaking that it would not be possible to develop a model for an entire submarine hydraulic system in the time available. For this reason, the stern diving planes servo-loop was chosen for the primary development (Fig. 1). This sub-system was considered to be representative of one of the more complex loops in the overall system. (a loop being defined as the fluid path through a control valve - to the power element - back through the control valve). Successful modeling of the stern planes servo-system would readily permit expansion of the system to include the fairwater planes and rudder loops in parallel operations. With these three sub-systems modeled, the remainder of the hydraulic system could be relatively easily developed.

B. REALISM

An operational ship's hydraulic system was deemed essential as the basis for the model, as it would allow the computed parameters to be compared with actual performance data. The hydraulic plant of USS PERMIT (SSN 594) was chosen as the system to be modeled. It was considered representative of a modern submarine hydraulic system and the technical information was readily available at Mare Island Naval Shipyard.
III. HYDRAULIC FLUID

A. TYPE

The hydraulic oil used in all U.S. submarine internal high pressure (3000 psi) systems is Navy symbol 2190TEP. This is a petroleum based fluid conforming to military specification MIL-L-17331 [Ref. 8]. It exhibits approximately the same characteristics as an SAE-30-Detergent motor oil.

B. PROPERTIES

1. Density

The density ($\rho$) of the oil was obtained from Ref. 9 in terms of specific gravity (SG). The values supplied were

$$SG = 0.896 \quad \text{at } 74^\circ\text{F}.$$  
$$SG = 0.0901 \quad \text{at } 60^\circ\text{F}.$$  

The density of water at 60 degrees Fahrenheit is $9.3475 \times 10^{-5}$ (slug/in$^3$) and is assumed to be the reference for specific gravity unless otherwise specified [Ref. 10]. Specific gravity is defined as the ratio of the density of the oil at the given temperature to that of water at 60 degrees Fahrenheit.

$$SG = \frac{\rho_o}{\rho_w}_{60} \quad (1)$$

Solving for the density of oil at temperature (T)
yields
\[ \rho = 8.4221 \times 10^{-5} \text{ slug/in}^3 \text{ at } 60^\circ F \]
\[ \rho = 8.3754 \times 10^{-5} \text{ slug/in}^3 \text{ at } 74^\circ F \]

2. **Cubical Expansion Coefficient**

The cubical expansion coefficient (\( \alpha \)) is the fractional change in volume of the fluid due to a change in temperature and is used in the determination of density, viscosity (\( \mu, \nu \)), and bulk modulus (\( \beta \)) values.

Density values arrived at in the previous section were used to obtain a value for the cubical expansion coefficient.

Using the form given in Ref. 11,
\[ \rho(P,T) = \rho_o [1 + (P-P_o)/\beta - \alpha(T-T_o)] \] (2)

where the subscript (\( o \)) denotes reference values of one atmosphere of pressure and 60°F temperature.

Since the values for the density of the oil were obtained at one atmosphere
\[ P = P_o \]
equation (2) reduces to
\[ \rho = \rho_o [1 + \alpha (T - T_o)] \] (2a)

Inserting appropriate values into (2a) and solving for \( \alpha \) yields
\[ \alpha = 3.9638 \times 10^{-4} \text{ } ^\circ F^{-1} \]

3. **Viscosity**

a. **Kinematic Viscosity**

Reference 12 specifies that 2190TEP oil shall have a kinematic viscosity (\( \nu \)) of 190SSU (Saybolt Seconds Universal) at a temperature of 130°F.
Using the standard conversion [Ref. 14] for kinematic viscosity, where the time \( t \) is greater than 100 seconds (SSU)

\[
v = 0.220 \, t - (135/t) \quad (3)
\]
yields

\[
\begin{align*}
v & = 41.09 \text{ cs at } 130^\circ F \\
v & = 92.08 \text{ cs at } 100^\circ F
\end{align*}
\]

Converting from centistokes (cs) to English units was accomplished with the conversion factor (Ref. 15) of \( 1.549 \times 10^{-3} \) \((\text{in}^2/\text{sec})/\text{cs}\). Thus the values obtained for kinematic viscosity were

\[
\begin{align*}
v & = 0.06367 \text{ in}^2/\text{sec at } 130^\circ F \\
v & = 0.1427 \text{ in}^2/\text{sec at } 100^\circ F
\end{align*}
\]

b. Absolute Viscosity

Knowing values of density and kinematic viscosity permits the determination of absolute viscosity \( \mu \).

\[
\mu = \rho v \quad (4)
\]

Using the form presented in Ref. 16,

\[
\mu = \nu_0 e - \lambda (T - T_o) \quad (5)
\]
at one atmosphere of pressure, where \( \lambda \) is the temperature coefficient of absolute viscosity with units of \( ^\circ F^{-1} \).

Combining equations (4) and (5) and then solving for lambda gives

\[
\lambda = \ln \left( \frac{\rho_0 \nu_o}{\rho v} \right) / (T_T_o) \quad (6)
\]

Arbitrarily setting \( T_o = 100^\circ F \) and \( T = 130^\circ F \)
in conjunction with values of density and kinematic viscosity at these temperatures yields
Substituting these values into equation (6) produced a value of

$$\lambda = 2.731 \times 10^{-2} \degree F^{-1}$$

Reference 17 presents the pressure conversion for absolute viscosity as

$$\log_{10}(\mu/\mu_0) = cF$$  \hspace{1cm} (7)

However, as the other functions are related by the difference in pressure divided by the bulk modulus ($\Delta P/\beta$), it would seem reasonable to assume that $c$ in equation (7) has a form of $(c_1/\beta)$. This would then yield an equation for absolute viscosity at constant temperature of

$$\mu = \mu_0 e^{-(c_1/\beta) \Delta P}$$  \hspace{1cm} (8)

Lacking any other information, $c_1$ was set equal to one, and a final form for absolute viscosity was formulated as

$$\mu = \mu_0 e^{(P/\beta) - \lambda(T-T_0)}$$  \hspace{1cm} (9)

where

- $P = P - P_0$
- $P$ = reference pressure (o psig)
- $P$ = system pressure (psig)
- $\beta$ = system effective bulk modulus (psi)
- $T$ = system temperature ($\degree F$)
- $T_0$ = reference temperature ($\degree F$)
4. **Bulk Modulus**

   Although the literature on hydraulic systems indicated that the bulk modulus ($\beta$) would be supplied with the oil, visual inspection of shipping containers and bills of lading proved futile. Likewise, the military specifications (MIL-L-17331) failed to produce any worthwhile information in this regard. A nominal value of bulk modulus for the oil was therefore assumed to be 220,000 psi, as recommended in Ref. 18.

C. **LIMITATIONS**

   In a further effort to maintain the scope of the project within reasonable constraints, it was decided to treat the temperature as constant. Inferences drawn from the Naval Ships' Technical Manuals previously cited and the author's own experience, indicated that a temperature of 130°F was a reasonable choice. With this value of temperature, the equations for the other properties were simplified.

   From equation (2)
   \[ p = 8.188 \times 10^{-5} + 8.4221 \times 10^{-5}(P/\beta) \quad (10) \]
   and equation (9) yields
   \[ \mu = 5.213 \times 10^{-6} e^{(P/\beta)}. \quad (11) \]
IV. MODEL DEVELOPMENT

A. SYSTEM OPERATION

Submarine steering and diving control surfaces (rudder and diving planes) are positioned through mechanical linkages by hydraulic rams. Each surface has normal as well as alternate sources of hydraulic and electrical power to ensure reliability. For the purpose of model development, only the normal mode of the stern planes was considered.

The normal mode of operation for the SSN 594 class of submarine, employs a position control electro-hydraulic servo-loop (Fig. 1). In this type of system (Fig. 2), control station operator manually positions a stick to achieve a desired angle of inclination of the planes. This produces an electrical signal which is summed with the feedback signal from the position output to create an error signal. The error signal is amplified and transmitted to the torque of the servovalve. The torque motor displaces the wand spring of the servovalve-pilot, which causes an imbalance in pilot pressure, which in turn shifts the main spool allowing the servovalve to pass oil to position the ram. As the desired position is attained, the error is reduced and the pilot returns the spool to a neutral positions, blocking the flow of oil.
B. SERVO-SYSTEM

The area of interest for the model development was the hydraulic fluid flow in the servo-loop as depicted in Figure 3. By inputting the valve spool motion and ram load force as explicitly defined functions, the influences of the external factors involved in the feedback loops (the servovalve torque-motor and pilot valve, and the ram loading) were minimized. This permitted the generation of flow and pressure equations around the loop with a minimum of complications due to the ancillary characteristics.

1. Servovalve

The two servovalves commonly in use in the Submarine Force are both manufactured by the Sargent Corporation and are designated Sanders Model SV 438-10P and SV 438-15P [Ref. 19]. Design and performance of both valves are similar. The SV 438-10P which is installed in the (SSN-594), was used for this simulation. Figure 4 is a representative drawing of the actual valve employed - a four-land four-port element. However, comparison with the valve depicted in Figure 3 - a three-land four-port model - will show that the flow is essentially the same in both valves. It was considered easier to follow the flow paths of the valve in Figure 3.

All technical data for the servovalve was obtained from Ref. 20.

a. Flow Paths

The pilot valve flow was disregarded, as it has its own supply and return lines.
Figure 3 Servovalve-RAM Hydraulic Loop
Designating the supply flow as $Q_{in}$, the return flow as $Q_r$, and the other flows by their respective nodal points (Fig. 3), the flow equations were derived. Using the form given in Ref. 21,

$$\sum Q_{in} - \sum Q_{out} = \frac{dV_o}{dt} + (V_o/\beta)\frac{dP}{dt} \quad (12)$$

the results were:

$$Q_{in} = Q_1 + Q_2 \quad (13)$$

$$Q_r = Q_3 + Q_4 \quad (14)$$

$$Q_1 = Q_3 + Q_5 + V_1\frac{\dot{\beta}_1}{\beta_1} + \dot{V}_1 \quad (15)$$

$$Q_2 = Q_4 + Q_6 + V_2\frac{\dot{\beta}_2}{\beta_2} + \dot{V}_2 \quad (16)$$

Here the volume across the lands of the valve was approximated as zero. The superscript (′) represents differentiation with respect to time.

Since the valve is manufactured from high strength alloys,

$$\dot{V}_1 = \dot{V}_2 = 0$$

$$V_1 = V_2,$$

as shown in Appendix A.

In addition to summing the flows through the valve, the flow rate was also defined through the lands for three different valve positions.

(1) **Null Positions.** In the centered (null) position, a critical-center servovalve will have leakage past the lands.¹ This flow is described in terms of null

¹For a comprehensive coverage of all terms used in this paper, the reader is referred to Ref. 2.
flow pressure coefficient \((K_{co})\), as
\[
Q = \frac{1}{2} K_{co} (\Delta P).
\] (17)

The four node flows for \(X_v = 0\) were
\[
Q_1 = \frac{1}{2} K_{co} (P_{in} - P)
\] (18)
\[
Q_2 = \frac{1}{2} K_{co} (P_{in} - P_2)
\] (19)
\[
Q_3 = \frac{1}{2} K_{co} (P_1 - P_r)
\] (20)
\[
Q_4 = \frac{1}{2} K_{co} (P_2 - P_r)
\] (21)

with
\[
P_3 = P_4 = P_r.
\]

See appendix B for values \(K_{co}\) used for the model.

(2) Valve Spool Off-Centered Left. The valve travel \((X_v)\) was arbitrarily defined as positive to the left (Fig. 3). Employing the orifice flow equation [Ref. 22] for the fluid passage through nodes one and four, and an empirical relationship for nodes two and three, the flow equations were determined to be
\[
Q_1 = C_d A \sqrt{2(P_{in} - P_1)}/\rho_1
\] (22)
\[
Q_2 = \frac{1}{2} K_{co} e^{-\lambda (X_v/X_{vm})^2} (P_{in} - P_2)
\] (23)
\[
Q_3 = \frac{1}{2} K_{co} e^{-\lambda (X_v/X_{vm})^2} (P_1 - P_r)
\] (24)
\[
Q_4 = C_d A \sqrt{2(P_2 - P_r)/\rho_r}
\] (25)

where \(C_d = 0.61\) for full periphery rectangular ports and
\[
A = \pi d_1 X_v
\] (26)

with \(d_1\) defined as the land diameter of the spool valve.

Since a value of leakage flow was not determinable from Ref. 20, the empirical relationships of Appendices C and D were developed using Ref. 23 as a guide.
Valve Spool Off-Centered Right. The equations for the "negative" flow direction were similar to those developed in the previous section, with the obvious exception that $Q_2$ and $Q_3$ became the principal flows. In order to maintain their flows as positive values, $X_v$ was squared and included in the radical.

The form for $X_v$ less than zero was

$$Q_1 = \frac{K_{co}}{2} e^{-\lambda (X_v/X_{vm})^2} (P_{in} - P_1)$$  \hspace{1cm} (27)

$$Q_2 = C_d W \sqrt{2X_v^2 (P_{in} - P_2)/\rho_2}$$  \hspace{1cm} (28)

$$Q_3 = C_d W \sqrt{2X_v^2 (P_1 - P_r)/\rho_2}$$  \hspace{1cm} (29)

$$Q_4 = \frac{K_{co}}{2} e^{-\lambda (X_v/X_{vm})^2} (P_2 - P_r)$$  \hspace{1cm} (30)

with

$$W = \pi d_1$$  \hspace{1cm} (31)

and all other variables as defined in previous sections.

2. Lines

The line equations were generated with the intention of accounting for all factors influencing the pressure and flow in the system. The schematic diagram of the actual system employed in this development is shown in Figure 5. For development purposes, only the odd numbered nodes are shown here as the even numbered side equations were identical.

a. Flow And Pressure

Disregarding all but the main line piping, the flow path from the servovalve to the ram was as depicted in Figures 3 and 5 were identified by the same numbers. The dynamics of these segments were then defined at each point when viewed as in Fig. 6.
Point A corresponds to nodal point (5) of Figure 3 and is considered to be located at the entrance of the pipe at its junction with the servovalve. Similarly, point B is located at the connection of the two different diameter pipes, in the entrance of the downstream pipe. It is equivalent to node (7) of Figure 3. Location C is just beyond the end of the line segment at the entrance to the cylinder, identified as node (9) in Figure 3.

Utilizing these definitions for the nodal points and viewing the direction of flow as always being from A to C, the dynamic equations for each element were developed.

(1) Node Five. The pressure and flow equations for node five were derived from Ref. 24.

\[ Q_5 = Q_7 + V_{57} \frac{P}{B_{57}} + \dot{V}_{57} \quad (32) \]

\[ P_5 = P_1 - \rho_5 K_5 \frac{Q_5^2}{2A_5^2} \quad (33) \]

The double-subscripted variables were employed to denote average values through the line segment, while the single-subscripted variables represent values at the particular node. Thus
\[ V_{57} = A_{5}L_{57} \]  \hspace{1cm} (34)

with \( L_{57} \) being the actual length of pipe from node (5) to node (7) and
\[ A_{5} = \pi D_{5}^{2}/4. \]  \hspace{1cm} (35)

As can be seen in Figure (6), the diameter of segment A-B (5-7) is constant and equal to the diameter at (5). The total volume and actual length will be developed further in later sections of this report.

The average pressure in the interval was
\[ P_{57} = (P_{5} + P_{7})/2 \]  \hspace{1cm} (36)
with
\[ P_{57} = \int P_{57} \, dt. \]  \hspace{1cm} (37)

The change in volume with respect to time \((V)\) was set equal to zero for all line segments, because the lines are made from high strength materials which do not permit any significant expansion or contraction. The exception to this would be in cases where there are flexible sections installed (Aeroquip -type hoses). There are numerous locations in the system where these are installed, but not in the servo-loop modeled. In the instances where flexible sections are incorporated in the system, an expression for the change in volume would have to be developed based on the properties of the material used.

The value of \( K_{5} \) was based on an abrupt change in the cross-sectional area of a pipe, and the development is shown in Appendix E. As used in the above equations, the sign of \( K_{5} \) changes with the direction of flow such that the second term of equation (33) will always reflect a pressure
loss for the referenced point. Thus, when flow is positive, as shown in Figure (6), \( P_1 \) will be greater than \( P_5 \), but when the flow is reversed (negative in Figure (6)) \( P_5 \) will be greater than \( P_1 \).

\( Q_7 \) is the flow at node (7) and will be defined in a subsequent section. \( P_1 \) is the pressure in the main servovalve cavity and is

\[
P_1 = \int \dot{P}_1 dt
\]

(38)

where \( \dot{P}_1 \) was introduced in section IV.B.I.a.

(2) **Node Seven.** The flow through node seven is exactly the same as that through node five, with subscripts changed to reflect the appropriate line segment.

\[
Q_7 = Q_9 + V_{79}\dot{P}_{79}/B_{79} + \dot{V}_{79}
\]

(39)

\[
P_{79} = \int \dot{P}_{79} dt.
\]

The pressure at point (7), on the other hand, must reflect the losses due to flow in line segment (57). This was accomplished with a combination of the equations in Ref. 24 with those in Appendix E:

\[
P_7 = P_5 - 32\mu_{57}L_{57}Q_{57}/A_5D_5^2 - 1.14\rho_{57}Q_{57}^2/A_5^2 - \rho_{7}K_{7}Q_{7}^2/2A_5^2.
\]

(40)

The second term on the right side of equation (40) is the Hagen-Poiseuille term for fully developed laminar flow in pipes. The third term accounts for developing laminar flow. The final term accounts for the entry/exit loss at cross-sectional area changes, as shown in Appendix E.

All three terms must reflect a loss of pressure for flow in either direction. Since \( Q_{57} \) assumes a negative value for flow reversal, it presents no problem.
However, \( Q_{57}^2 \) will always be a positive quantity unless it is defined as

\[
Q_{57}^2 = Q_{57} |Q_{57}|, \tag{41}
\]

in which case the sign of \( Q_{57}^2 \) will reflect the direction of flow. As noted previously, \( K_7 \) determines the sign of the final term.

\( A_5^2 \) is used in the last term of equation (40), because the area employed is always that of the smaller of the two at a junction.

Previously undefined variables are

\[
Q_{57} = (Q_5 + Q_7)/2 \tag{42}
\]

\[
L_{e57} = \text{effective length from (5) to (7)}. \tag{43}
\]

Effective line length will be discussed in a later section.

(3) Node Nine. The pressure at point nine has the same form as that at seven

\[
P_9 = P_7 - 32v_{79}L_{e79}Q_{79}/A_7D_7^2 - 1.14 \rho_{79}Q_{79}^2/A_7^2 - \rho_{9}K_9Q_9^2/2A_7^2 \tag{43}
\]

with

\[
Q_{79}^2 = Q_{79} |Q_{79}| \tag{44}
\]

for the reasons discussed in the previous section.

Flow through this junction is

\[
Q_9 = Q_{11} + Q_{e1}. \tag{45}
\]

\( Q_{11} \) will be defined with respect to ram flow in a following segment of the paper.

\[
Q_{e1} = C_{e1}P_9 . \tag{46}
\]

The coefficient of external leakage \((C_{e1})\) is the allowable leak rate past the cylinder tail rods to the atmosphere. Appendix F derives the values used, based on information contained in Ref. 25.
There were no volume terms included in equation (45) because the transition from the end of the line to the cylinder entrance was considered an infinitesimal distance.

b. Secondary Component Effects

It is readily apparent from a comparison of Figure (5) with Figure (3) that the simplified system omits numerous valves, fittings, and other miscellaneous elements which are installed between the servovalve and ram. Although these "secondary" components have a minor influence individually, when combined they contribute significantly to both static and dynamic characteristics of the system.

The approach taken in this development was to consider them in three categories: branch lines, minor components, and major elements.

(1) Branch Lines. Branch lines are the segments of pipe connected to the main line. They are subjected to system pressure but do not ordinarily allow fluid to flow. Gage lines, hand positioning branches, and capped sections were included in Figure (5).

These lines contribute only to the total system volume.

(2) Minor Components. The minor components were considered to be any elements installed in the system which contributed to the effective length and volume of the system but were not specifically listed in the list of major components [Ref. 26].
The elements in this group were ball valves, tees, and elbows. Reference 27 was used as the source for information on them and Appendix G explains their development for use in the overall analysis of the system.

(3) Major Components. The deceleration valves (decel) and the blocking valve were included as major components. These were listed in Ref. 26 with specific pressure and flow characteristics. The volumes employed were a "best estimate" based on diameter and the author's personal experience. The effective lengths were as determined from the procedure in Appendix H.

The function of the blocking valve is to transfer to an alternate source of hydraulic supply (the Vital System), in the event of a loss of main header pressure. As pressures in the two systems are ordinarily the same (within 100psi. of each other) and the valve has internal seals, cross-port leakage is negligible.

The deceleration valve is a cam operated full-flow valve which is installed to prevent over-travel of the ram. As the ram approaches its maximum limit of travel, a collar on the tail rod mechanically trips the decel can, shutting the deceleration valve. This permits a fluid cushion to arrest the ram travel rather than allowing it to a "hard-stop" (metal-to-metal) at maximum velocity. Motion in the reverse direction is permitted by flow through the internal check valve until the linkage allows the main deceleration valve to reopen.
(4) **Combined Effects.** The various elements were combined in tabular form for inclusion in the system analysis (Tables 1 and 2). These were input directly into the computer program to solve for the various parameters as discussed above.

3. **Cylinder and Piston**

The final element in the system is the cylinder and piston combination. Here the power is transmitted from the fluid to the stern planes by the piston and its associated linkages.

a. **Fluid Dynamics**

The pressure and flow equations assumed a form similar to those of the lines.

(1) **Flow.** The flow in the cylinder was described by

\[ Q_{11} = Q_{i\ell} + \dot{V}_{11} + V_{11} \frac{p_{11}}{\beta_{11}} \]  

where

\[ Q_{i\ell} = C_{i\ell}(P_{11} - P_{12}) \]  

with

\[ p_{11} = \int \dot{p}_{11} \, dt \]  

\( Q_{i\ell} \) is the internal leakage from the high pressure side to the low pressure side of the piston. The determination of \( C_{i\ell} \) is shown in Appendix F. Flow was defined as positive from node (11) to node (12); thus, when \( P_{12} \) is greater than \( P_{11} \), \( Q_{i\ell} \) will assume a negative value as required by the line equations.
<table>
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<th>IDENTIFICATION</th>
<th>OD</th>
<th>TH</th>
<th>L</th>
<th>NTT</th>
<th>NTB</th>
<th>NL45</th>
<th>NL90</th>
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OD - outside diameter  NTT - number of flow through tees  NL90 - number of 90° elbows
TH - wall thickness    NTB - number of branch flow tees   NVB - number of ball valves
L - length             NL45 - number of 45° elbows
<table>
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<th>V</th>
<th>Q_REF</th>
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</table>

**TABLE II MAJOR COMPONENT DATA**
\[ \dot{V}_{11} = A_{11} \dot{x}_r \quad (50) \]

where \( A_{11} \) is the effective piston area,
\[ A_{11} = \pi(D_{11}^2 - D_r^2)/4 \quad (51) \]

where \( D_{11} \) is the land diameter of the cylinder and \( D_r \) is the diameter of the piston tail-rod.

\( \dot{x}_r \) is the velocity of the ram.

\[ V_{11} = A_{11} L_{11} \quad (52) \]

where
\[ L_{11} = L_o + X_r \quad (53) \]

and
\[ L_o = \frac{1}{2}(L_{cy} - t_p) \quad (54) \]

\( L_{cy} \) is the length of the cylinder \( t_p \) is the thickness of the piston. Ram position was defined as positive to the left in Figure (5), so that motion to the left increases \( V_{11} \) while movement of the piston to the right decreases \( V_{11} \). For this reason, it was necessary to define
\[ V_{12} = L_o - X_r \quad (55) \]

and ultimately
\[ Q_{12} = -Q_{11} - V_{12} \dot{\rho}_{12} - V_{12} \beta_{12} \quad (56) \]

Thus, the various flows in the system would automatically assume the proper sign to reflect the direction of flow.

(2) **Pressure.** The pressure equation for the cylinder and piston arrangement became
\[ P_{11} = P_o - 32\eta u_{11} L_{11} Q_{11}/A_{11} D_{11}^2 - 1.14 \rho_{11} Q_{11}^2/A_{11}^2. \quad (57) \]

There was no entry loss term because it was taken into account with \( P_o \). The other difference, with regard to the previous line loss equations, is that \( D_{11} \) is the
land diameter of the cylinder while $A_{11}$ is the effective area of the piston. The effective area was used because $Q/A$ is a velocity term employed in defining viscous loss. On the other hand, $D_{11}$ was used to account for the surface area upon which the fluid is "dragging"; i.e., the walls of the cylinder. Since the tail rod is traveling at essentially the same velocity as the fluid, it contributes no losses due to viscosity.

As in previous sections, it was necessary to define

$$Q_{11}^2 = Q_{11} |Q_{11}|$$

(58)

to account for flow direction.

b. Piston Dynamics

Ram motion was defined in terms of position ($X_r$), velocity ($\dot{X}_r$), and acceleration ($\ddot{X}_r$).

$$X_r = \int \dot{X}_r \, dt$$

(59)

$$\dot{X}_r = \int \ddot{X}_r \, dt$$

(60)

$$\ddot{X}_r = \frac{(A_{11}P_{11} - A_{12}P_{12} - \dot{\rho}_{oil} \ddot{X}_r - F_L)/M_I}{(61)}$$

The load force ($F_L$), as described at the outset, was input as a ramp function dependent on ram position and velocity.

The flow of oil ($\dot{\rho}_{oil}$) term was included to account for the force required to maintain the momentum of the oil. It was determined to be

$$\dot{\rho}_{oil} = \rho_{12} \dot{V}_{12}$$

for $\dot{X}_r > 0$

(62)

and

$$\dot{\rho}_{oil} = \rho_{11} \dot{V}_{11}$$

for $\dot{X}_r < 0$.

(63)
The total mass of the system \( M_T \) was defined as the mass of the ram plus the mass of the oil from the low pressure side of the piston to the return tank, such that

\[
M_T = m_r + \sum_{\text{tank}} \rho_i V_i + \sum_{\text{ram}} \rho_i V_i \tag{64}
\]

where the subscript \((i)\) indicates each main line segment (flow path) from the ram face to the return tank.

C. PROGRAM

The foregoing equations were re-arranged to yield the form shown in the attached computer program. They were modeled in a Continuous System Modeling Program (CSMP III) in the belief that it was the best modeling method available. However, numerous attempts failed to accomplish the desired result.

The algebraic equations form a loop for which no solution was found.
V. SUMMARY

A. CONCLUSIONS

It is felt that the development of the line equations demonstrates the necessity of including these factors in any in-depth analysis of the submarine hydraulic system. It is also the author's belief that the approach taken in this presentation will yield worthwhile results for any system employing extensive quantities of piping.

As stated in the introduction, a generalized computer program of this type would have many beneficial aspects for both the submarine designer as well as the operator.

B. RECOMMENDATIONS

The first recommendation is a possible means of clearing the algebraic loop problem currently preventing the program from executing. This would require clearing all of the pressure and flow equations of reference to any variables except for themselves or ones which directly invoke an integral. That is

\[ P_7 = f(P_{79}, P_{57}, P_1, P_7, \text{etc.}) \]

However, it could not take the form where \( P_7 \) was set equal to \( P_9 \) or any of the flow terms \( (Q_i) \), as they do not invoke an integral function.

The other possibility would be to locate another program which could efficiently handle both integral as well as simultaneous algebraic equations.
APPENDIX A

Determination of Servovalve Volume

No values were obtainable for the volume of the servovalve. Therefore, the following method was employed to obtain a realistic approximation.

Using nominal dimensions given in the technical manual [Ref. 20] and the knowledge that the valve is made of aluminum alloy, a volume to weight ratio was determined.

Dimensions $18.5 \times 8.0625 \times 9.5$ in.
Specific Gravity of alloy approximately 2.8
Given weight 69 lb.
Density of water at $60^\circ F$. 62.4 lb/ft$^3$

$$V_{\text{body}} = 1417 \text{ in}^3$$
$$\gamma_{\text{alloy}} = 0.101 \text{ lb/in}^3$$
$$W = V\gamma = 143.3 \text{ lb for a solid block}$$

then

$$\frac{W_{\text{given}}}{W} = 0.48.$$  

Thus the valve body weighs approximately half of what a solid block would weigh. It was therefore assumed that the volume of metal in the valve was 708 in$^3$. Further, estimating that the passages for the pilot system occupy half of the void area yielded a main valve volume of about 350 in$^3$, and since this volume is that occupied by both the supply and the return segments of the valve, half was assigned to each.

$$V_1 = V_2 = 175 \text{ in}^3$$
APPENDIX B

Null Flow Pressure Coefficient Determination

The allowable null flow leakage was given as 600 cc/min. at 1200 psi. [Ref. 20]. This was converted to standard form of \((\text{in}^3/\text{sec})/\text{psi}\) as follows

\[ K_{co} = \frac{600(\text{cc.min})}{1200 \text{ psi}} \times \frac{1 \text{ in}^3}{16.39 \text{ cc}} \times \frac{1 \text{ min}}{60 \text{ sec}} \]

This yielded

\[ K_{co} = 5.0844 \times 10^{-4} \text{ (in}^3/\text{sec})/\text{psi}. \]

and

\[ K_{co} = 2.5422 \times 10^{-4} \text{ (in}^3/\text{sec})/\text{psi}. \]

APPENDIX C

Empirical Valve Leakage Coefficient

The leakage flow across a valve port decreases as the valve moves to block the port. Since a plot of leakage flow \((Q_1)\) versus valve position \((X_v)\) is symmetrical, with an exponential decay as shown in Fig. C-1, and exact data was not given for leakage other than the null position an empirical expression was developed.

\[ Q_1 = Q_{1, \text{MAX}} e^{-kX_v^2} \]

*Figure C-1  Leakage Flow vs Valve Position*
The most appropriate expression to fit this curve took the form of

\[ \frac{Q_\ell}{Q_{\ell \text{max}}} = e^{-\lambda \left( \frac{X_v}{X_{\text{vmax}}} \right)^2} \]

where

- \( Q_\ell \) = leakage flow at any valve position
- \( Q_{\ell \text{max}} \) = maximum leak rate = \( \frac{1}{2} K \cdot c_0 \)
- \( X_{\text{vmax}} \) = maximum valve travel from null.

Representative values were then chosen such that

\[ \frac{Q_\ell}{Q_{\ell \text{max}}} = 0.02 \text{ at } \frac{X_v}{X_{\text{vmax}}} = 0.25 \]

That is, at 25% of maximum valve travel, leakage through the "blocked" port of the valve decreases to 2% of the null position leakage.

Solving for \( \lambda \)

\[ \lambda = \left( \ln \frac{Q_{\ell \text{max}}}{Q_\ell} \right) \left( \frac{X_v}{X_{\text{vmax}}} \right)^2 \]

\[ \lambda = 62.6. \]

In reality \( \lambda \) could vary from near zero to about 100. Realistically it would have a value between 50 and 100.

\( \lambda = 65 \) was chosen as the value to employ in this presentation.

The form used was

\[ Q_\ell = \frac{1}{2} K \cdot c_0 e^{-\lambda \left( \frac{X_v}{X_{\text{vmax}}} \right)^2} \]

with the determination of maximum valve travel (\( X_{\text{vm}} \)) shown in Appendix D.
Determination of Maximum Valve Travel

Reference shows that

\[ Q_{\text{max}} = 80 \text{ GPM} = 80 \times 231/60 \text{ in}^3/\text{sec} \]

at

\[ T = 115^\circ \text{F} \]

\[ \Delta P_{\text{total}} = 1200 \text{ psi} \]

\[ \Delta P_{\text{port}} = 600 \text{ psi} \]

as depicted in Figure D-1.

Using the orifice equation

\[ Q = C_d A (2 \Delta P / \rho)^{1/2} \]

with

\[ C_d = 0.61 \]

\[ \rho = 8.442 \times 10^{-5} \]

\[ A = \pi d_1 X_v \text{ where } d_1 = 1 \text{ in.} \]

\[ X_v = X_{vm} \]

\[ Q = Q_m \]

46
such that

\[ X_{vm} = \left( \frac{Q_m}{\pi C_d d_1 (2 \Delta P_p / \rho)^{\frac{1}{2}}} \right) \]

resulting in \( X_{vm} = 0.0426 \text{ in.} \)

This value was used in spite of the fact that it seemed large for a "standard" pilot controlled servo valve.
The standard form of the pressure loss due to abrupt piping diameter change is given as

\[ H_1 = \frac{KV^2}{2g} \]

where

- \( H_1 \) is head loss, expressed in feet of fluid
- \( V \) is fluid velocity
- \( g \) is acceleration due to gravity.

Head loss can also be expressed as

\[ H_1 = \frac{\Delta P}{\gamma} \]
\[ \gamma = \rho g \]

and

\( \gamma \) is weight density

while velocity is

\[ V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \]

combining the above equations

\[ \Delta P = \frac{8\rho KQ^2}{\pi^2 D^4} \]

\( K \) is defined from Figure D-1

\[ K = \begin{cases} \frac{1}{2}(1-D_1/D_2) & \text{for } +Q \\ -(1-(D_1/D_2)^2)^2 & \text{for } -Q. \end{cases} \]

**NOTE** \( D_1 \) is always the smaller of the two diameters being considered.

\[ \delta_2 \quad D_1 \]

Figure E-1 Flow at an Abrupt Area Change
APPENDIX F

Seal Leakage Coefficients

Seal leakage was based on permissible values listed in Naval Ship's Technical Manual, [Ref. 25.] The values used in this presentation were for "used" seals with maximum allowable leakage before replacement is required. This was felt to be reasonable since the operating temperature of the oil is normally greater than the test temperature, which will produce slightly higher leak rates.

The leak rates were given as one milliliter per ten cycles per inch of rod diameter for external leakage, and five milliliter per five minutes per inch of rod diameter for internal leakage. These were based on normal operating pressures and a fluid temperature of 100°F.

Operating pressures were assumed to be

\[ P_{el} = 2000 \text{ psi} \]
\[ P_{il} = 1000 \text{ psi} \]

where \( P_{el} \) is the pressure for external leakage and \( P_{il} \) is the pressure for internal leakage. A cycle was defined as the time to go from neutral (zero plane angle) to full rise to full dive and back to neutral position. For the system modeled, stern planes travel is plus or minus 27°. Thus, a full cycle would be four times this value, or 108°. The normal rate of travel is about five degrees per second. Thus, \( 108/5 = 21.6 \text{ sec/cycle} \) or approximately

3 cycles/min.
Since the tail rod used, had a diameter of four inches, the allowable external leakage was four milliliters per ten cycles.

Then

\[
\frac{4\text{ml}}{10\text{cy}} \times \frac{1\text{cc}}{1\text{ml}} \times \frac{3\text{cy}}{1\text{min}} = 1.2\text{cc/min}
\]

and

\[
C_{el} = \frac{1.2\text{ cc}}{\text{min}} \times \frac{1\text{ min}}{60\text{ sec}} \times \frac{1\text{ in}^3}{16.39\text{cc}} \times \frac{1}{2000\text{ psi}}
\]

\[
C_{el} = 6.1 \times 10^{-7} \text{ (in}^3/\text{sec)/psi}
\]

Similarly the internal leakage for a land diameter of 8.25 inches, rounded to nine inches, was nine cubic centimeters per minute.

And

\[
C_{il} = \frac{9}{60 \times 16.39 \times 1000}
\]

\[
C_{il} = 9.15 \times 10^{-6} \text{ (in}^3/\text{sec)/psi}
\]
APPENDIX G

Minor Component Analysis

The values used in the derivation of the minor component effects, considered herein, were obtained from Crane Technical Paper No. 410, [Ref. 27].

The volume was determined to be

\[ V = A L \]

where \( A \) was the cross-sectional area of the pipe entering or leaving the element and \( L \) was a value determined for each type of element. The values for \( L \) were chosen as

- Ball valves \( L = 3 \times D \)
- Tees \( L = 2 \times D \)
- Elbows \( L = 1.6 \times D \)

In cases where the element has differing entry and exit diameters, \( D \) is the larger of the two.

The values for the effective lengths were

- Ball valves \( L_e = 3 \times D \)
- Flow through tees \( L_e = 20 \times D \)
- Branch flow tees \( L_e = 60 \times D \)
- 45° elbows \( L_e = 16 \times D \)
- 90° elbows \( L_e = 30 \times D \)

where the diameters were as above. An additional factor of \( K \) (appendix E) would have to be included for diameter differences in the main flow path.

The value of effective length cited above, were determined for Reynolds numbers (\( R_e \)) greater than 1000. Therefore,
it was necessary to modify them

Since \[ R_e = \frac{4Q}{\pi D v} \]

and \[ L_{ed} = \frac{L_{ec} R_e}{1000} \quad \text{for } R_e < 1000 \]

with \[ L_{ed} = \text{the desired effective length} \]

\[ L_{ec} = \text{the calculated effective length from above} \]

then \[ L_e = L_{ec} \left( \frac{4Q}{\pi D v} \right) \times 10^{-3} \]

If \[ R_e \geq 1000 \] then \[ L_e = L_{ec} \]

so that for \[ \frac{4Q}{\pi D v} > 1000 \]

or \[ \frac{Q_p}{D u} > 785.4 \]

\[ L_e = L_{ec} \]

and for \[ \frac{Q_p}{D u} < 785.4 \]

\[ L_e = 1.273 \times 10^{-3} L_{ec} \frac{Q_p}{D u} \]
The standard head loss equation
\[ H_l = K_v^2/2g \] was rearranged in the form
\[ \Delta P/\rho g = KQ^2/2gA^2 \]
This yielded
\[ K = 2\Delta PA^2/\rho Q^2 \]
since
\[ A = \pi D^2/4 \]
\[ K = \pi^2 \Delta PD^4/8\rho Q^2 \]
and since
\[ K = L_e/D \]
\[ L_e = \pi^2 \Delta PD^5/8\rho Q^2 \]
To allow \( Q \) to be entered in the form presented (GPM),
a conversion was included.

231 in\(^3\)/gal x 1 min/60 sec

so that
\[ Q \text{ (GPM)} x 231/60 = Q \text{ (in}\(^3\)/sec) \]
and
\[ L_e = 0.0832 \text{ PD}^5/\rho Q^2 \]

where
\[ D = \text{diameter of valve (in)} \]
\[ P = \text{given pressure loss (psi)} \]
\[ Q = \text{given flow (GPM)} \]
\[ \rho = \text{density of fluid at given temperature (slug/in}^3\) \]
APPENDIX I

TO EMPLOY:
SUPPLY LINES; DL IS ALWAYS THE POINT CLOSEST TO THE PUMP - THE UPSTREAM POINT
RETURN LINES; DL IS ALWAYS THE POINT FARDEST FROM THE TANK - THE UPSTREAM POINT
DUAL DIRECTION (REVERSING) LINES; DL IS THE POINT NEAREST THE CONTROL ELEMENT (VALVE), VIEWED AS IF THE LINE ALWAYS SUPPLIES PRESSURE TO THE POWER ELEMENT

ENDMAC

INITIAL

CONSTANT KCCCC=600., FT=1200., LAMB=65.

VALVE LEAK RATE OF MAIN SPOOL
NOTE: LAMB IS STRICTLY EMPIRICAL AND BASED ON AN AVERAGE CURVE OF LEAK-RATE VS VALVE DISPLACEMENT.

CONSTANT XVM=0.0425, DLV=1.0, CD=0.61, V1=175., V2=175.

CONSTANT OD5=1.90, TH5=0.25, L5=48.0, NTB5=0., NTT5=0. . .
   NL45=0., NL95=1., NV5=0.
CONSTANT OD6=1.9, TH6=0.25, L6=48.0, NTB6=0., NTT6=0. . .
   NL46=0., NL96=1., NV6=0.
CONSTANT CD7=2.0, TH7=0.27, L7=228.0, NTB7=2., NTT7=1., NL47=0. . .
   NL97=6., NV7=1.
CONSTANT OD8=2.0, TH8=0.27, L8=216.0, NTB8=3., NTT8=0., NL48=0. . .
   NL98=5., NV8=1.

CONSTANT OD71=0.75, TH71=0.109, L71=8.0, NTB71=1., NTT71=0. . .
   NL471=0., NL971=0., NV71=0.
CONSTANT OD72=0.375, TH72=0.058, L72=120.0, NTB72=0., NTT72=0. . .
   NL472=0., NL972=1., NV72=1.
CONSTANT OD73=1.5, TH73=0.203, L73=12.0, NTB73=0., NTT73=0. . .
   NL473=0., NL973=0., NV73=0.

CONSTANT OD81=0.75, TH81=0.109, L81=8.0, NTB81=1., NTT81=0. . .
   NL481=0., NL981=0., NV81=0.
CONSTANT OD82=0.375, TH82=0.058, L82=120.0, NTB82=0., NTT82=0. . .
   NL482=0., NL982=2., NV82=1.
CONSTANT OD83=1.5, TH83=0.203, L83=12.0, NTB83=0., NTT83=0. . .
   NL483=0., NL983=0., NV83=0.

CONSTANT PDR7=60.0, QDR7=50.0, RDR7=8.68E-05, VDV7=60., DDV7=1.094
CONSTANT PER7=50., QBR7=50., RBR7=8.68E-05, VBV7=20., DBV7=1.46
CONSTANT E=3.0E07, P01S=0.3
CONSTANT BO=200000.
CONSTANT PR=50., PIN=3000., FA=0.1, ODRE=2.0, THRE=0.148, VR=2750.
CONSTANT OILCC=9.0, CELCC=1.2, PCEL=2000., Pcil=1000.
CONSTANT DLR=8.255, DHR=4.0, ODR=11.5, LCY=39.875, TR=5.125, MR=12.1
CONSTANT PI=3.141593
KCC = 0.5 * KCOCC / (PT * 983.4)

The value of (0.5) is used because the tech. manual lists a leak-rate for the whole valve; the value sought is that for half the valve (one set of ports).

KCOCC is tech. manual leak-rate in (cc/min).
PT is test pressure used to establish the leak-rate (psi).
KCO is leak-rate in ((in**3/SEC)/psi).

AG = CD * PI * DLV * SQRT(2)

AG is the product of the constant coefficients used in the servo-valve equation.

CFA = FA / 1.4

FA is the fraction of air entrapped in the system, expressed as a percentage of the total system volume.
CFA is the coefficient of the air term in bulk modulus calculations.

K5P, K5M = ENTCO(D1, D5)
K6P, K6M = ENTCO(D2, D6)
K7P, K7M = ENTCO(D5, D7)
K8P, K8M = ENTCO(D6, D8)
K9P, K9M = ENTCO(D7, D11)
K10P, K10M = ENTCO(D8, D12)

These coefficients account for exit losses only; entry losses are considered as exit losses for the next element upstream.

VP5, A5, LEP5, D5LA5, BCS5 = VALE(P1, E, POIS, OD5, TH5L5, NTT5, NTB5, ...
NL45, NL95, NV5)
VP6, A6, LEP6, D6LA6, BCS6 = VALE(P1, E, POIS, OD6, TH6, L6, NTT6, NTB6, ...
NL46, NL96, NV6)
VP7, A7, LEP7, D7, LA7, BCS7 = VALE(P1, E, POIS, OD7, TH7, L7, NTT7, NTB7, ...
NL47, NL97, NV7)
VP8, A8, LEP8, D8, LA8, B(S8 = VALE(P1, E, POIS, OD8, TH8, L8, NTT8, NTB8, ...
NL48, NL98, NV81)

V&B1, A7B1, LE7B1, D7B1, LA7B1, BCS71 = VALE(P1, E, POIS, OD71, TH71, ...
L71, NTT71, NTB71, NL471, NL971, NV71)
V7B2, A7B2, LE7B2, D7B2, LA7B2, BCS72 = VALE(P1, E, POIS, OD72, TH72, ...
L72, NTT72, NTB72, NL472, NL972, NV72)
V7B3, A7B3, LE7B3, D7B3, LA7B3, BCS73 = VALE9PI, E, POIS, OD73, TH73, ...
L73, NTT73, NTB73, NL473, NL973, NV73)
V8B1, A8B1, LE8B1, D8B1, LA8B1, BCS81 = VALE(P1, E, POIS, OD81, TH81, ...
L81, NTT81, NTB81, NL481, NL981, NV81)
V6B2, A8B2, LE8B2, D8B2, LA8B2, BCS82 = VALE9PI, E, POIS, OD82, TH82, ...
L82, NTT82, NTB82, NL482, NL982, NV82)
V8B3, A8B3, LE8B3, D6B3, LA6B3, BCS83 = VALE(PI, POIS, OD83, TH83, ...
L83, NTT83, NTB83, NL483, NL983, NV83)

BO1 = 1.0 * BO
BCII = 2.0 * (1.0 + POIS) / E
BI1 = BOI + BC1I
BI2 = BI1
BI5 = BI1
BI6 = BI1
BI7 = BI1
BI8 = BI1
BI9 = BI1
B110 = BI1
B111 = BI1
B112 = BI1
BCDV8 = BI1 * VDV8
BCDV7 = BI1 * VDV7
BCBV8 = BI1 * VBV8
BCBV7 = BI1 * VBV7

BI57 = BOI + BCS5 / V57
BI68 = BOI + BCS6 / V68
BI79 = BOI + (BCS7 + BCS71 + BCS72 + BCS73 + BCS74 + BCDV7 + BCBV7) / V79
BI81 = BOI + (BCS8 + BCS81 + BCS82 + BCS83 + BCS84 + BCDV8 + BCBV8) / V81

CQ57, CQ57S = LINCO(A5, D5, O5D5)
CQ58, CQ58S = LINCO(A6, D6, O6D6)
CQ79, CQ79S = LINCO(A7, D7, O7D7)
CQ81, CQ81S = LINCO(A8, D8, O8D8)
CQ11, CQ11S = LINCO(A11, D11, ODR)
CQ12, CQ12S = LINCO(A12, D12, ODR)

P1Z = (PIN - PR) / 2.0
P2Z = 0.1Z
P57Z = P1Z
P68Z = P1Z
P79Z = P1Z
P81Z = P1Z
P11Z = P1Z
P12Z = P1Z

Q7Z = 0.0
Q8Z = 0.0
Q9Z = 0.0
Q10Z = 0.0
Q11Z = 0.0
Q12Z = 0.0
Q57Z = 0.0
Q68Z = 0.0
Q79Z = 0.0
Q81Z = 0.0

A11 = PI * (D11 * D11 - D11R * D11R) / 4.0
A12 = PI * (D12 * D12 - D12R * D12R) / 4.0
D11=DLR
D12=DLR
D11R=DRR
D12R=DRR

A5S=A5*A5
A6S=A6*A6
A7S=A7*A7
A8S=A8*A8

L57=LA5
A68=LA6
L79=LA7
L81=LA8

LEP57=LEP5
LEP79=LEP7+LDV7+LBV7
LEP68=LEP6
LEP81=LEP8+LDV8+LBV8

V57=VP5
V68=VP6
V79=VP7+V7B1+V7B2+V7B3+VDV7+VBV7
V81=VP8+V8B1+V8B2+V8B3+VDV8+VBV8

LDV7=SPECV(PDR7, QDR7, DDV7)
LDV8=LDV7
LBV7=SPECV(PBR7, RER7, DBV7)
LBV8=LBV7
VBV8=VBV7
VDV8=VDV7

DRE=ODRE-2.0*THRE
BIR=BOI+2.0*((1. + POIS) * ODRE * ODRE + (1. - POIS) * DRE * DRE) / ...
    (E*(ODRE*ODRE-ORE*ORE))

CEL=CELCC/(PCEL*983.4)
CIL=CILCC/(PCIL*983.4)

XRZ=0.0
ZRDZ=0.0

D1=3.*DLV
D2=D1

LG=0.5*(LCY-TR)

DYNAMIC
** PROCEDURE SERVO-VALVE ***

PROCED Q1,Q2,Q3,Q4=SERVO(AG,KCO,LAMB,PIN,PR,P1,P2,...
XV,XVM,RHO1,RHO2,RHOR)

CEX=KCO*EXP(-LAMB*(XV/XVM)**2)

IF(XV)10,20,30

Q1=CEX*(PIN-P1)
Q2=AG*SQRT(XV*XV*(PIN-P2)/RHO2)
Q3=AG*SQRT(ZV*ZV*(P1=PR)/RHOR)
Q4=CEX*(P2-PR)

GO TO 40

Q1=CEX*(PIN-P1)
Q2=CEX*(PIN-P2)
Q3=CEX*(P1-PR)
Q4=CEX*(P2-PR)

GO TO 40

Q1=AG*XV*SQRT((PIN-P1)/RHO1)
Q2=CEX*(PIN-P2)
Q3=CEX*(P1-PR)
Q4=AG(XV*SQRT((P2-PR)/RHOR)

CONTINUE
ENDFRO

** PROCEDURE KPRO ***

PROCED K5,K6,K7,K8,K9,K10=KPRO(P1,P2,P5,P6,P7,P8,P9,P10,K5P,...

IF(P1.GE.P5)GO TO 100
K5=K5M
GO TO 101

100 K5=K5P
101 IF(P2.GE.P6)GO TO 102
K6=K6M
GO TO 103

102 K6=K6P
103 IF(P5.GE.P7)GO TO 104
K7=K7M
GO TO 105

104 K7=K7P
105 IF(P6.GE.P8)GO TO 106
K8=K8M
GO TO 107

106 K8=K8P
107 IF(P7.GE.P9)GO TO 108
K9=K9M
GO TO 109

108 K9=K9P
109 IF(P8.GE.P10)GO TO 110
K10=K10M
GO TO 111

110 K10=K10P
111 CONTINUE
ENDFRO
**PROCEDURE EFFECTIVE LINE LENGTH**

PROCED LE57=PRO57(LEP57,L57,Q57,D5,RH057,MU57)
CLE57=Q57*RH057/(MU57*D5)
IF(CLE57.LT.785.4)GO TO 200
LE57=L57+LEP57
GO TO 201
200 LE57=L57+1.273E-03*CLE57*LEP57
201 CONTINUE
ENDPRO

PROCED LE68=PRO68(LEP68,L68,Q68,D6,RH068,MU68)
CLE68=Q68*RH068/(MU68*D6)
IF(CLE68.LT.785.4)GO TO 202
LE68=L68+LEP68
GO TO 203
202 LE68=L68+1.273E-03*CLE68*LEP68
203 CONTINUE
ENDPRO

PROCED LE79=PRO79(LEP79,L79,Q79,D7,RH079,MU79)
CLE79=Q79*RH079/(MU79*D7)
IF(CLE79.LT.785.4)GO TO 204
LE79=L79+LEP79
GO TO 205
204 LE79=L79+1.273E-03*CLE79*LEP79
205 CONTINUE
ENDPRO

PROCED LE81=PRO81(LEP81,L81,Q81,D8,RH081,MU81)
CLE81=Q81*RH081/(MU81*D8)
IF(CLE81.LT.785.4)GO TO 206
LE81=L81+LEP81
GO TO 207
206 LE81=L81+1.273E-03*CLE81*LEP81
207 CONTINUE
ENDPRO

**PROCEDURE MASS OF OIL**

IF(XRD.GE.0.)GO TO 400
MOIL=RH011*V11+RH079*V79+RH057*V57+RH0*V1+RHOR*VR
GO TO 401
400 MOIL=RH012*V12+RH081*V81+RH068*V2+RHOR*VR
MDOIL=RH012*V12D
401 MT=MR+MOIL
ENDPRO
P1D=(Q1-Q3-Q5)/(V1*B1)
P2D=(Q2-Q4-Q5)/(V2*B2)
P57D=(Q5-Q7)/(V57*B57)
P68D=(Q6-Q8)/(V68*B68)
P79D=(Q7-Q9)/(V79*B79)
P81D=(Q8-Q10)/(B81/V81)
P11D=(Q11-QIL-V11D)/(V11I)*V11)
P12D=(Q12+QIL+V12D)/(B12I)*V12)

P5=2.0*P57-P7
P7=2.0*P79-P9
P9=(Q9-Q11)/CEL
P6=2.0*P68-P8
P8=2.0*P81-P10
P10=(Q10-Q12)/CEL

*** LINE AND RAM FLOWS ***

Q5=A5*SQRT(2.0*(P1-P5)/(RHO5*K5))
Q6=A6*SQRT(2.0*(P2-P6)/(RHO6*K6))

THE FOLLOWING EQUATIONS IN IMPLICIT FORM WERE INSERTED IN AN EFFORT TO CLEAR THE ALGEBRAIC LOOP PROBLEM. IT DID NOT WORK.

THE NEXT LOGICAL STEP WOULD BE TO COMPACT ALL EQUATIONS SUCH THAT EACH VARIABLE IS EQUATED ONLY TO ITSELF OR A VARIABLE WHICH IS DIRECTLY COUPLED TO AN INTEGRAL.

Q7=IMPL(Q7Z,0.001,Q7CC)
Q7C=((P5-P7-CQ57*RH057*ABS(Q57(-0.5*RH07*K7Q7/Q7S)... /(CQ57*MU57*LE57)))*2.0
Q7CC=Q7C
Q8=IMPL(Q8Z,0.001,Q8CC)
Q8C=((P6-P8-CQ68*RH068*ABS(Q68)-0.5*RH08*K8*Q8/A6S)... /(CQ68*MU68*LE68)))*2.0
Q8CC=Q8C
Q9=IMPL(Q9Z,0.001,Q9CC)
Q9C=((P7-P9-CQ79*RH079*ABS(Q79)-0.5*RH09*K9*Q9/A7S)... /(CQ79*MU79*LE79)))*2.0
Q9CC=Q9C
Q10=IMPL(Q10Z,0.001,Q10CC)
Q10C=((P8-P10-CQ81*RH081*ABS(Q81)-0.5*RH010*K10*Q10/A8S)... /(CQ81*MU81*LE81)))*2.0
Q10CC=Q10C
Q11=IMPL(Q11Z,0.001,Q11C)
Q11C=((P9-P11-CQ11S*RH011*ABS(Q11)/(CQ11*MU11*L11)
Q12=IMPL(Q12Z,0.001,Q12C)
Q12C=((P10-P12-CQ12S*RH012*ABS(Q12)/(CQ12*MU12*L12)

60
Q57 = IMPL(Q57Z, 0.0001, Q57CC)
Q57c = (P5 - P7 - CQ57S*RHO57*Q57*ABS(Q57) - 0.5*RHO57*K7*Q7*Q7/A57) /
       (CQ57*MU57*LE57)
Q57CC = Q57C

Q68 = IMPL(Q68Z, 0.001, Q68CC)
Q68c = (P6 - P8 - CQ68S*RHO68*ABS(Q68) - 0.5*RHO68*K8*Q8*Q8/A68) /
       (CQ68*MU68*LE68)
Q68CC = Q68C

Q79 = IMPL(Q79Z, 0.001, Q79CC)
Q79c = (P7 - P9 - CQ79S*RHO79*Q79*ABS(Q79) - 0.5*RHO79*K9*Q9*Q9/A79) /
       (CQ79*MU79*LE79)
Q79CC = Q79C

Q81 = IMPL(Q81Z, 0.001, Q81CC)
Q81c = (P8 - P10 - CQ81S*RHO81*ABS(Q81) - 0.5*RHO81*K10*Q10*Q10/A81) /
       (CQ81*MU81*LE81)
Q81CC = Q81C

QIL = CIL*(P11 - P12)

B1I = CFA/P1 + B1I
B2I = CFA/P2 + B2I
B3I = CFA/PR + B3I
B5I = CFA/P5 + B5I
B6I = CFA/P6 + B6I
B7I = CFA/P7 + B7I
B8I = CFA/P8 + B8I
B9I = CFA/P9 + B9I
B10I = CFA/P10 + B10I
B11I = CFA/P11 + B11I
B12I = CFA/P12 + B12I
B57I = CFA/P57 + B57I
B68I = CFA/P68 + B68I
B79I = CFA/P79 + B79I
B81I = CFA/P81 + B81I

**** DENSITY VALUES ****

RHOR = 8.256E-05 + 8.423E-05*PR*BRI
ROH1 = 8.256E-05 + 8.423E-05*P1*B1I
ROH2 = 8.256E-05 + 8.423E-05*P2*B2I
ROH5 = 8.256E-05 + 8.423E-05*P5*B5I
ROH6 = 8.256E-05 + 8.423E-05*P6*B6I
ROH7 = 8.256E-05 + 8.423E-05*P7*B7I
ROH8 = 8.256E-05 + 8.423E-05*P8*B8I
ROH9 = 8.256E-05 + 8.423E-05*P9*B9I
ROH10 = 8.256E-05 + 8.423E-05*P10*B10I
ROH11 = 8.256E-05 + 8.423E-05*P11*B11I
ROH12 = 8.256E-05 + 8.423E-05*P12*B12I
ROH57 = 8.256E-05 + 8.423E-05*P57*B57I
ROH68 = 8.256E-05 + 8.423E-05*P68*B68I
ROH79 = 8.256E-05 + 8.423E-05*P79*B79I
ROH81 = 8.256E-05 + 8.423E-05*P81*B81I
*** VISCOSITY EQUATIONS ***

\[
\begin{align*}
\mu_{57} &= 8.198 \times 10^{-6} \exp(p_{57} - B_{57}) \\
\mu_{68} &= 8.198 \times 10^{-6} \exp(p_{68} - B_{68}) \\
\mu_{79} &= 8.198 \times 10^{-6} \exp(p_{79} - B_{79}) \\
\mu_{81} &= 8.198 \times 10^{-6} \exp(p_{81} - B_{81}) \\
\mu_{11} &= 8.198 \times 10^{-6} \exp(p_{11} - B_{11}) \\
\mu_{12} &= 8.198 \times 10^{-6} \exp(p_{12} - B_{12})
\end{align*}
\]
LIST OF REFERENCES


9. Wilcox, W., op cit.


13. Ibid., Fig. 9450-1.


22. Ibid., p. 81.

23. Ibid., p. 89.

24. Ibid., p. 32-46.


27. Crane Company, op. cit.
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