NEARSHORE WAVE CHARACTERISTICS GENERATED BY VARIABLE WINDS
NORTH CAROLINA STATE UNIV. RALEIGH DEPT OF MARINE SCI-ETC
Nearshore Wave Characteristics Generated by Variable Winds.

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depths into the formulation, governing equations are derived to predict the development of a wind-wave field in terms of integral properties of the spectrum such as mean period, direction, and significant wave height. Under deep water conditions, governing equations admit simple closed form solutions which are shown to compare very favorably with observational data as well as various wind-wave prediction formulæ currently available. Consequently, they are numerically solved first for the case of wind-waves in uniform water depth and then for a circular basin with variable bathymetry as an illustrative general case.
NEARSHORE WAVE CHARACTERISTICS GENERATED BY VARIABLE WINDS

By

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ABSTRACT

The prediction of wave characteristics generated by variable winds under fetch-and duration-limited conditions is considered by using a parametric approach and a simplified form of the radiative transport equation. The wind-generated sea surface is characterized in terms of an equilibrium range spectrum with a well-defined low wave number cut-off. By relating the local wind velocity to the rate of increase in wave energy and incorporating the refractive and frictional effects of transitional water depths into the formulation, governing equations are derived to predict the development of a wind-wave field in terms of integral properties of the spectrum such as mean period, direction, and significant wave height. Under deep water conditions, governing equations admit simple closed form solutions which are shown to compare very favorably with observational data as well as various wind-wave prediction formulae currently available. Consequently, they are numerically solved first for the case of wind-waves in uniform water depth and then for a circular basin with variable bathymetry as an illustrative general case.
INTRODUCTION

The field of wind-generated waves has been the subject of intensive research effort over the last three decades and, as a natural consequence, a great deal of progress has been made on various aspects relevant to understanding the physics of wind waves, their generation, propagation and dissipation. An excellent review of recent advances in the theory of wind waves, their observation, measurement and prediction is given by Barnett and Kenyon (2). It appears that, in spite of these advances achieved, the present state of knowledge is still in need of considerable improvement, and various gaps in theory, observation and instrumentation remain to be filled through future research.

One of the primary applications of wind wave research is wave forecasting. The complete problem is a field radiation problem in which the directional energy spectrum of a wave field is governed spatially and temporally by an energy balance or, radiative transfer equation (1,2,3,10,11,21,22,23). The solution of the radiative transfer equation requires integration of various non-linear source terms representing the action of local winds, wave-wave interactions, dissipative forces, etc. along wave rays which are the propagation paths given by the Hamilton-Jacobian equations. However, the integration is not straightforward, since the source terms are non-linear. This means that different rays are coupled, making the solution of the problem quite difficult and impractical under general conditions. As a
result, most of the explicit applications such as those demonstrated by Hasselmann and Collins (11), Resio and Vincent (22, 23), and others (1,2,3,7,14,19) are either based on simplified approximations of various source terms or consider deep water conditions. At present, there is a large number of such computer-based discrete spectral models for forecasting waves (see, e.g., Refs. 7,14,19).

A second approach which avoids the intricate complexity of discrete spectral models requiring large computers and complex programming is the parametric specification and forecasting of wind waves. This approach is based on one-dimensional generation models which parameterize some integral property of the wave spectrum such as wave height, or wave period in terms of wind speed, fetch and wind duration. Included in this group are the original Sverdrup and Munk model (28), and its subsequent versions improved by Bretschneider (4, 25), and Ijima and Tang (12,25) based on quasi-empirical, quasi-theoretical arguments, and the methods developed by Pierson, Neuman and James (20) and Wilson (30). More recently, Hasselmann et al. (10) have proposed a parametric model based on the assumption of the invariant spectral shape of the JONSWAP spectrum. The validity of this model depends on the dominance of the self-stabilizing effects of wave-wave interactions as compared to external sources of energy. A comparison of various parametric models is given by Resio and Vincent (22).

Parametric and discrete spectral models have been experimented with mostly under open ocean conditions. Except for the recent applications of discrete spectral models to the Great Lakes
by Resio and Vincent (22,23), and the parametric procedures developed by Bretshneider (4,25), and Ijima and Tang (12,25) for conditions of unlimited wind duration in uniform water depth, there has been a lack of effort to extend either of these approaches to wave field predictions involving variable winds in areas bounded by irregular shorelines. Various pragmatic procedures and definitions of effective fetch and effective duration have been used to incorporate such effects into the nearshore applications (see e.g., Ref. 24).

The principal motivation here is to develop a method of forecasting nearshore wave characteristics generated by variable winds under fetch-and duration-limited conditions by using a parametric approach and a simplified form of the radiative transfer equation. It is assumed that the wind sea is characterized in terms of an equilibrium range spectrum with a well-defined low wave number cut-off. By relating the local wind velocity to the rate of increase in wave energy and incorporating the refractive and frictional effects of transitional water depths into the formulation, governing equations are derived to predict the development of a wind-wave field in terms of the integral properties of the spectrum such as mean period and significant wave height. Various incompletely understood or complex mechanisms such as wave-wave interactions, turbulent dissipation due to wave breaking, wave-current interactions, percolation, etc. are neglected. Under deep water conditions, governing equations admit simple closed form solutions which
are shown to compare very favorably with observational data as well as various wind-wave prediction formulae currently available. Consequently, they are numerically solved first for the case of wind-waves in uniform water depth and then for a circular basin with variable bathymetry as an illustrative general case.

ASSUMPTIONS AND FORMULATION

With the neglect of current-wave interactions, the energy balance of wind generated waves can be expressed as (see, e.g., Ref. 21, page 147)

\[
\frac{d\Psi}{dt} = \left( \frac{\partial}{\partial t} + \hat{v} \cdot \hat{v} \right) \Psi = \Gamma \tag{1}
\]

where \( \Psi(k) \) = directional spectral density over a two-dimensional wave number space \( \hat{k} = (k_1, k_2) \); \( \hat{v} \) = group velocity associated with the spectral component with wave number vector \( \hat{k} \); \( \hat{v} \) = horizontal gradient operator; and, \( \Gamma \) represents various source terms from any process that transfers energy into and out of the spectral component \( \Psi(k) \). For a detailed discussion of the general form of \( \Gamma \), the reader is referred to Refs 10 and 23. Eq. (1) implies that the rate of change of spectral density along the propagation path of a wave group is \( \Gamma \), which is in general a complex non-linear function of the local wind and the spectral density \( \Psi \) itself. The propagation path is given by the Hamilton-Jacobian equations

\[
\frac{dx_i}{dt} = \frac{\partial \sigma}{\partial k_i} = V_i \tag{2}
\]
\[
d \frac{d \kappa_i}{dt} = -\omega / \partial x_i_i \\
(3)
\]
where \( x_i_i (i=1,2) \) represents a horizontal coordinate system in the still water plane;
\[
\omega = (gk \tanh kd)^{1/2}
(4)
\]
is the radian frequency with \( g = \) gravitational constant; and, \( d = \) local still water depth. Eqs. 2 and 3 describe, respectively, the wave ray associated with the spectral component \( \psi(\kappa) \) and the variation of \( \kappa \) along that ray. Therefore, assuming that the functional form of \( \Gamma \) is known, the solution of the wind-wave problem involves solving first the refraction problem, i.e., Eqs. 2 and 3. Once the rays are determined, Eq. 1 is integrated along each ray. As previously mentioned in the introduction, the integration is not straightforward, since \( \Gamma \) is non-linear and wave rays are coupled. Consequently, in order to overcome some of these difficulties, we will appeal to various simplifying approximations as follows.

**Functional Form of \( \psi(\kappa) \).** - For wind generated waves, the spectral density function \( \psi \) is observed to have an equilibrium range with a well-defined low-wave-number cut-off (13, 21, 29). Combining the dimensional reasonings of Phillips (21), Kitaigorodskii et al. (13), and Thornton (29), it can be shown that
\[
\psi(\kappa) = (\beta / 2\pi) n \kappa^{-4} ; \ k > k_m, \ |\alpha| \leq \pi
\]
where \( \beta = 1.31 \times 10^{-2} ; \alpha = \) direction of the saturated components; \( k_m = \) low-wave-number cut-off; and,
\[ n = \frac{1}{2} \left\{ 1 + \frac{2kd}{\sinh 2kd} \right\} \] (5)

The spectral density as a function of wave number moduli or, the distribution of the mean square surface displacement \( \eta^2 \), regardless of direction, is given by

\[ F(k) = \int_{-\pi}^{\pi} \Psi(k) \, k \, d\alpha = \beta n k^{-3} ; \, k \geq k_m \] (6)

If it is assumed that the spectral density \( \Psi \) is mostly concentrated in the direction of the spectral peak wave number \( k_m \), \( F(k) \) is approximately equal to the integral of \( \Psi(k) \) along lines perpendicular to the direction of \( k_m \) (see, e.g., Ref. 21, page 130).

**One-Dimensional Form of Energy Balance.** - Considering the direction of the spectral peak wave number \( k_m \) as a reference, a contracted form of the energy balance equation can be obtained by integrating Eq. 1 along lines perpendicular to \( k_m \). Provided that \( \Psi(k) \) is mostly concentrated around \( k_m \), the contracted form can be approximated as

\[ \frac{dF}{dt} = \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) F(k) = S \] (7)

**Functional Form of Source Term S.** - Among various possible processes that contribute to the source term, most of the discussion will be centered on the generative action of overwater winds and the dissipative effects of turbulent bottom friction. Guided by previous studies by Miles (17), Snyder and Cox (26),
Manton (15, 16), Barnett (1), Barnett and Wilkerson (3), and Hasselmann and Collins (11), the functional form of $S$ is taken as

$$S = F(k_m) \left( \Omega - \phi_f \right)$$  \hspace{1cm} (8)

where

$$\Omega = \gamma \left( \frac{\rho_a}{\rho_w} \right) \left( \frac{k_m}{\sigma_m} \right) \hat{w} - \sigma_m$$  \hspace{1cm} (9)

with $\left( \frac{\rho_a}{\rho_w} \right)$ = ratio of air to water density; $\hat{w}$ = wind velocity at 10 m above still water; and $\gamma$ = a constant (<1) representing the fraction of wind input energy that is transferred down the spectrum to build up the lower wave number end of the spectrum $F(k)$. Hence, it is assumed that an equilibrium exists such that wind input balances the energy losses due to breaking away from the spectral peak towards higher wave numbers. The function $\phi_f$ represents the dissipative effect of turbulent bottom friction. For a Gaussian sea state in which the spectrum is narrow-beam centered around $k_m$, the general form of this term (see, e.g., Refs. 11 and 23) can be replaced with the unidirectional approximation

$$\phi_f = 2 C_f \left( g k_m^2 / \sigma_m^2 \cosh^2 k_m d \right) \langle |\hat{u}| \rangle$$  \hspace{1cm} (10)

where $C_f$ = an empirical drag or friction coefficient; and,

$$\langle |\hat{u}| \rangle = \left\{ \frac{2}{\pi} \int (gk/\sigma \cosh kd)^2 F(k) \, dk \right\}^{1/8}$$  \hspace{1cm} (11)
GOVERNING EQUATIONS

Substituting Eq. 6 into the left hand side of Eq. 7, we obtain

\[
\frac{dF}{dt} = \beta n_k^{-3} \left(-3k_m^{-1} \frac{dk_m}{dt} + n^{-1}(\partial n/\partial k_m) \right)
\]

\[
\left(\frac{dk_m}{dt}\right) + n^{-1}(\partial n/\partial d) \left(\frac{dd}{dt}\right)
\]

(12)

where

\[
\partial n/\partial k_m = d(1-2k_m d \text{ctnh } 2k_m d) / \sinh 2k_m d
\]

(13)

\[
\partial n/\partial d = k_m(1-2k_m d \text{ctnh } 2k_m d) / \sinh 2k_m d
\]

(14)

Similarly, substitutions of Eqs. 4 and 6 into Eqs. 6, 10, and 11 yield

\[
\Omega = \gamma(\rho_a/\rho_w) k_m W_o \{w \cos(\theta - \theta_w) - (\sigma_m/k_m W_o)\}
\]

(15)

\[
\phi_f = (8C_f k_m/\sinh 2k_m d) \left(\beta g d \pi^{-1} \int_{2k_m d}^{\infty} (\sinh v + v) \right)
\]

\[
(v \sinh v)^{-2} dv \right)^{\frac{1}{2}}
\]

(16)

where \(W_o\) = a reference 10 m - wind speed at a fixed \(\hat{x}\) and \(t\); \(w = W/W_o\), a dimensionless function representing spatial and temporal variation of wind field; and, \(\theta, \theta_w\) = directions of \(k_m\) and \(\hat{W}\), respectively, relative to a fixed reference axis.
We can now introduce the dimensionless variables

\[ k = k_m W_o^2 / g \]

\[ a = a_m W_o / g \]  \hspace{1cm} (17)

\[ t = g t / W_o \]

\[ t = g \dot{X} / W_o^2 \]

\[ d = g d / W_o^2 \]

so that

\[ C = C_m / W_o = a_m / k_m W_o = (\tanh k d / k)^{\frac{1}{2}} \]  \hspace{1cm} (18)

and

\[ T = g T_m / 2\pi W_o = (k \tanh k d)^{-\frac{1}{2}} \]  \hspace{1cm} (19)

represent, respectively, the dimensionless wave speed and period associated with the spectral peak. Likewise, the dimensionless significant wave height can be shown to be

\[ H = g H_s / W_o^2 = 4 \left( \int_0^\infty F(v) \, dv \right)^{\frac{1}{2}} \]  \hspace{1cm} (20)

Finally, by substituting Eqs. 12 through 17 into Eq. 7 and solving for \( \frac{dk}{dt} \), we obtain
\[
\frac{dk}{dt} = (\partial/\partial \tau + \hat{\nabla} \cdot \hat{\nabla}) k = (S_1 - S_2 - S_3) S_4^{-1}
\]  \hspace{1cm} (21)

where \( \hat{\nabla} = \text{dimensionless horizontal gradient operator} \);

\[
\hat{\nabla} = n \, C \, \hat{k}
\]  \hspace{1cm} (22)

with \( \hat{k} = \hat{k}/k = (\cos \theta, \sin \theta) \); and,

\[
S_1 = \gamma (\rho_a/\rho_w) \, n \, k^2 \{w \cos(\theta - \theta_w) - (\tanh k \, d/k)^{1/2}\}
\]
\[
\sinh 2 \, k \, d
\]  \hspace{1cm} (23)

\[
S_2 = 8 \, C_f \, n \, k^2 \{\beta d \, \pi^{-1} \int_{2k \, d}^{\infty} (\sinh v + v) (v \sinh v)^{-2} \, dv\}^{1/2}
\]  \hspace{1cm} (24)

\[
S_3 = k^2 \,(1 - 2 \, k \, d \, \text{ctnh} \, 2 \, k \, d) \, d(d)/dt
\]  \hspace{1cm} (25)

\[
S_4 = k \, d \,(1 - 2 \, k \, d \, \text{ctnh} \, 2 \, k \, d) - 3n \, \sinh 2 \, k \, d
\]  \hspace{1cm} (26)

Hence, the variation of the modulus \( k \) is described by Eq. 21 along the wave ray given by

\[
\frac{dk}{d\tau} = \hat{\nabla} = n \, C \, \hat{k}
\]  \hspace{1cm} (27)

What remains, therefore, is the derivation of an expression which will describe the variation of \( \hat{k} \) or \( \theta \) along the same ray.
due to refraction. This easily follows if we note from Eqs. 3,4 and 17 that

\[ \frac{dk}{dt} = \frac{d(k \hat{k})}{dt} = - \left( \sigma \frac{k}{\sinh 2k d} \right) \hat{y} \cdot \hat{d} \]  

(28)

Solving this for \( \frac{dk}{dt} \) and taking the dot product of the resulting expression with \( \hat{m} = (-\sin \theta, \cos \theta) \) immediately gives

\[ \frac{d\theta}{dt} = -\left( \frac{\sigma}{\sinh 2k d} \right) \hat{m} \cdot \hat{d} \]  

(29)

In summary, the complete problem requires the solution of the system of Eqs. 21, 27, and 29 for a specified wind field, i.e., \( W_o, W, \) and \( \theta^w \), with appropriate initial and boundary conditions on \( \hat{k} \). Then, the estimation of wind-wave parameters \( C, T \) and \( H \) follows immediately from Eqs. 18, 19, and 20, respectively.

**COMPARISON WITH EXISTING THEORIES IN DEEP WATER**

Under deep water conditions with constant wind direction \( \theta^w, \phi = \theta^w, \) and no refraction, it can be verified that Eqs. 21 and 27 become, respectively

\[ \frac{dC}{dt} = \gamma_o C^{-1} \left[ W(x, t) - C \right] \]  

(30)

\[ \frac{dx}{dt} = \frac{1}{C} C \]  

(31)

where \( \gamma_o = (1/6) \gamma (\rho_a/\rho_w) \); and use has been made of \( k = C^{-2} \). In this special case, the reduced Eqs. 30 and 31 are
identical with the deep water relations previously obtained by Manton (15).

The dimensionless significant wave height follows readily from Eq. 20 as

$$H = 2 \beta^2 C^2 \tag{32}$$

Hence, by specifying functional form of $w(x,t)$, various special cases can be examined and compared with other available parametric forecasting theories and observational data. In particular,

$$w(x,t) = \psi(x, t) \Phi(t) \tag{33}$$

where $\psi(.) = \text{Heaviside unit step function}$, corresponds to a duration-and fetch-limited generation case with the invariant solution

$$C + \ln (1-C) = C_0 + \ln (1-C_0) - \gamma_o (t - t_o) \tag{34}$$

$$x = x_o + \frac{1}{2} \int_{t_o}^{t} C(x, \tau) \, d\tau \tag{35}$$

where

$$x \geq x_o ; t \geq t_o ; \text{and}, C_0 = C(x_o, t_o) < 1 \tag{36}$$

Wind-wave generation under conditions of uniform wind with unlimited fetch, i.e., $w(t) = \Phi(t)$, is represented by the solution
of Eq. 30 with \( \frac{d}{dt} = \frac{\partial}{\partial t} \)

\[
C + \ln (1 - C) = C_o + \ln (1 - C_o) - \gamma_o (t - t_o)
\]

(37)

where

\( t > t_o \); and, \( C_o = C(t_o) < 1 \) \hspace{1cm} (38)

Finally, the solution under conditions of uniform wind with unlimited duration, i.e., \( w(x) = W(x) \), also follows easily from Eq. 30 with \( \frac{d}{dt} = \frac{C}{2} \frac{d}{dx} \) as

\[
C (1 + \frac{1}{2} C) + \ln (1 - C) = C_o (1 + \frac{1}{2} C_o) + \ln (1 - C_o) - 2\gamma_o (x - x_o)
\]

(39)

where

\( x > x_o \); and, \( C_o = C(x_o) < 1 \) \hspace{1cm} (40)

Noting that all of these solutions are identical to Manton's (15), we can proceed now to compare them with various other forecasting theories and observational data readily available in the literature. First, the fetch-limited case with the null boundary condition \( C_o = 0 \), i.e.,
\[ C (1 + \frac{k}{3} C) + \ln (1 - C) = -2\gamma_o x \quad (x > 0) \quad (41) \]

\[ H = 2 \beta^2 C^2 \quad (42) \]

where, following Manton (15), \( \beta = 1.31 \times 10^{-2} \) and \( \gamma_o = 2.5 \times 10^{-5} \), will be compared with the parametric relations of Hasselmann et al. (10):

\[ C = 5.5 \times 10^{-2} \frac{x^{0.3}}{} \quad (43) \]

\[ H = 1.54 \times 10^{-3} \frac{x^{0.5}}{} \quad (44) \]

the Sverdrup-Munk-Bretschneider (SMB) formulae in the Shore Protection Manual (SPM) (25):

\[ C = 1.2 \tanh (0.077 \frac{x^{0.25}}{}) \quad (45) \]

\[ H = 0.283 \tanh (0.0125 \frac{x^{0.42}}{}) \quad (46) \]

and the relations by Wilson (30):

\[ C = 5.7 \times 10^{-3} \frac{x^{0.3}}{} \quad (47) \]

\[ H = 2.5 \times 10^{-3} \frac{x^{0.4}}{} \quad (48) \]

The wind-wave parameters represented by Eqs. 41 through 48 are shown in Figure 1 together with the region of field data.
obtained from Figure 15 of Wilson (30). Evidently, the general character of the results generated here compares favorably with the field data and other theories or empirical curves in question. So far as the principal purpose of any parametric approach is concerned, apparent differences observed between various forecasting relations can be considered to be within acceptable limits. There are, however, certain points of concern about each forecasting relation. Specifically, the equations of the present study provide estimates with inherent upper bounds such that \( C < 1 \), and \( H < 0.229 \). These bounds appear to be slightly underestimated as compared to some field data. On the other hand, parametric relations of Hasselmann et al. (10) and Wilson's empirical curves are likely to overestimate the wind-wave parameters at large fetches; and, SMB curves overestimate the same parameters at small fetches.

The duration-limited case with the null initial condition \( C_0 = 0 \), i.e.,

\[
C + \ln (1 - C) = -\gamma_0 t \quad (t \geq 0)
\]  
(49)

\[
H = 2 \beta \frac{C^2}{C}
\]  
(50)

can also be compared with the relations based on the parametric approach of Hasselmann et al. (10):

\[
C = 9.17 \times 10^{-3} t^{0.43}
\]  
(51)

\[
H = 7.77 \times 10^{-5} t^{5/7}
\]  
(52)
and the forecasting curves of Sverdrup and Munk (28), which are reproduced here in Figure 2 together with the region of data given in Figure 7 of Ref. 28. Also shown in Figure 2 are the wind-wave predictions based on Eqs. 49 through 52. Again, the overall comparison is favorable. Particular differences observed are of the same nature as those just mentioned in the fetch-limited case, i.e., $C < 1, H < 0.229$ for the predictions of the present study, which are likely to be underestimates for long durations; the parametric relations of Hasselmann et al. (10) overpredict $C$ and $H$ for long durations; and, the Sverdrup and Munk theory is likely to overestimate the same parameters for short durations.

**FORECASTING IN FINITE WATER DEPTH**

**Uniform Depth Case.**—Letting $S_3 = 0$, corresponding to a uniform water depth $d = \text{const.}$, and with $\theta = \theta_w$, being coincident with one of the reference axes, the governing equations take the form

$$\frac{dk}{dt} = (\partial/\partial x + V \partial/\partial z) k = (S_1 - S_2) S_4^{-1} \quad (53)$$

$$\frac{dx}{dt} = V = n C \quad (54)$$

where $S_1$, $S_2$, $S_4$, and $C$ are defined as before. The wind-wave parameters follow from Eqs. 19 and 20 as

$$T = (k \tanh k d)^{-\frac{1}{2}} \quad (55)$$

$$H = 4 \left( \beta \int_k^{-} n v^{-3} dv \right)^{\frac{1}{2}} \quad (56)$$
We can now proceed to consider the numerical simulation of a set of three wind-wave generation situations corresponding, respectively, to fetch-, duration-, and fetch- and duration-limited wind fields with invariant direction. In all these simulations, we take \( C_f = 0.01 \) as a characteristic value which allows us to compare the results generated, in particular, under fetch-limited conditions with those presented in SPM (25). For the remaining cases here and in the following section where we will examine the more general case of a variable bathymetry, there is no parametric results available for comparison.

The fetch-limited situation corresponds to setting \( \frac{d}{dt} \) in Eq. 53. Hence, the equation to be solved becomes

\[
\frac{dk}{dx} = \frac{(S_1 - S_2)}{(n \cdot C \cdot S_4)}
\]

where \( w(x) = \Psi(x) \); and, \( T \) and \( H \) are given by Eqs. 55 and 56. In this case and in all subsequent simulations, null boundary and/or initial conditions will be employed so that \( C_o = k_o^{-k} \), \( T_o \), and \( H_o \) are all zero at \( x \) and/or \( t = 0 \). Hence, the duration-limited case with \( w(t) = \Psi(t) \) corresponds to the solution of

\[
\frac{dk}{dt} = \frac{(S_1 - S_2)}{S_4^{-1}}
\]

and the duration-and fetch-limited case with \( w(x,t) = \Psi(x) \cdot \Psi(t) \) is given by the solution of the full set of Eqs. 53 and 54. The latter can also be expressed in terms of the integral quadratures.
\[ k = k_0 + \int_{t_0}^{t} (S_1 - S_2) S_4^{-1} \, d\tau \] (59)

\[ x = x_0 + \int_{t_0}^{t} n(k, \tau; d) C(k, \tau; d) \, d\tau \] (60)

Many sophisticated numerical algorithms exist for the numerical simulation of these nonlinear ordinary differential equations. However, the results to be discussed in the following were generated with the simplest possible scheme by approximating the left-hand sides by forward finite differences, and the right-hand sides with the values of variables \( k, x, t \), evaluated at a preceding step. Illustrating this with Eq. (58) as an example, we have then

\[ k_{i+1} = k_i + \left[ (S_1 - S_2) S_4^{-1} \right]_i \Delta t \] (61)

Since the null initial and/or boundary conditions imply that \( k_0 \rightarrow \infty \) (or, \( C_0 = 0 \)), each case is initiated with the corresponding closed-form solution in deep water, i.e., Eqs. 34 through 40 whichever is applicable, and the computation via closed-form solutions are continued until the parameter \( k_d \) becomes small enough for the effects of transitional water depths to be accounted for. In all cases here, the transitional criterion used is \( k_d = 10\pi \), after which the solution is continued numerically. It is noted that for the duration- and fetch-limited case described by Eqs. 53 and 54 or, Eqs. 59 and 60, it is expected that the local field is initially duration-limited and hence grows until it becomes fetch-limited. The boundary between the fetch-limited region and the duration-limited region is given by Eqs. 59 and 60...
with $x_0 = \xi_0 = 0$ or, equivalently, from a comparison of the solutions corresponding to the fetch-limited and duration-limited cases. In other words, once the solutions under fetch-limited and duration-limited conditions are obtained, the fetch-and duration-limited case follows from the combination of the two separate cases by patching them together in a straightforward manner.

The computations for the fetch-limited case for various $d$ values are shown in Figure 3 together with the corresponding results obtained from Eqs. 3-25 and 3-26 of SPM (25). The general agreement between the two sets of predictions is favorable except for short fetches where, just as mentioned previously, the SPM curves tend to overestimate the parameters $T$ and $H$. Illustrated in the subsequent Figures 4 and 5 are the computations for the duration-limited case for various $d$ values and the fetch-and duration-limited case for $d = 0.3$, respectively. There is no readily available observational data or parametric theories with which these latter results could be compared.

**Variable Depth Case.** - In order to illustrate the application of the proposed scheme to a more general situation, consider an artificial example of wind-wave generation over a circular basin with variable depth

$$d = 1 + 10^{-2} (x^2 + y^2)^{\frac{1}{3}} \quad (x^2 + y^2 \leq 95) \quad (62)$$

Under conditions of unlimited wind-duration with $w(x) = \theta(x)$ and invariant wind direction $\theta_w = 0$ with respect to $+x$ - axis, the governing equations follow from Eqs. 21, 27, and 29 as
\[
dk/dt = \hat{V} \cdot \hat{k} = (S_1 - S_2 - S_3) S_4^{-1} \tag{63}
\]

\[
d\theta/dt = - (a/\sinh 2k d) \hat{m} \cdot \hat{v} d \tag{64}
\]

\[
d\hat{x}/dt = \hat{v} = n C \hat{k} \tag{65}
\]

where \(S_1, S_2, S_4\) are defined by Eqs. 23, 24, 26, respectively; 
\(S_3\) is as in Eq. 25 with \(d(d)/dt = \hat{v} \cdot \hat{v} d\); and

\[
\hat{k} = (\cos \theta, \sin \theta) \tag{66}
\]

\[
\hat{m} = (-\sin \theta, \cos \theta) \tag{67}
\]

\[
a = T^{-1} = (k \tanh k d)^{1/2} \tag{68}
\]

The parameters \(T\) and \(H\) are given by Eqs. 19 and 20 as before, and 
\(\theta\) is with respect to the \(+x\) axis.

The solution in this case proceeds by integrating Eqs. 63 and 64 along the rays described by (65). Hence, using forward differences and setting \(\Delta s = n C \Delta t\) as a stepsize along a ray, the computation is performed in the following order:

\[
\theta_{i+1} = \theta_i - (a \hat{m} \cdot \hat{v} d/ n C \sinh 2k d) \Delta s \tag{69}
\]

\[
X_{i+1} = X_i + \frac{1}{2} (\cos \theta_i + \cos \theta_{i+1}) \Delta s \tag{70}
\]

\[
Y_{i+1} = Y_i + \frac{1}{2} (\sin \theta_i + \sin \theta_{i+1}) \Delta s \tag{71}
\]
\[ k_{i+1} = k_i + \frac{1}{2} \left\{ (S_1 - S_2 - S_3)_i \left( n \cdot S_4 \right)_i^{-1} + (S_1 - S_2 - S_3)_{i+1} \left( n \cdot S_4 \right)_{i+1}^{-1} \right\} \Delta s \]  

(72)

The boundary conditions consist of the null values, i.e., \( \theta = \bar{H} = \bar{T} = 0 \) for any ray originating along the semicircular arc \( x^2 + y^2 = 95 \) (\( x < 0 \)) corresponding to the \( d = 0.05 \) depth contour. As in the previous uniform depth situations, the computation is initiated through the closed-form solution under fetch-limited conditions, i.e., Eqs. 41 and 42, until \( k_d = 10\pi \), after which the solution proceeds numerically via Eqs. 69-72. The results obtained in the manner just described with \( \Delta s = 0.2 \) are presented in Figure 6 through 8. In particular, Figure 6 illustrates the geometry of the basin, depth contours, wind direction, and computed wave rays, which remain indifferent to refraction over most of the basin except towards the fetch-end shoreline. The contours of constant \( T \) and \( \bar{H} \) are shown in Figures 7 and 8, respectively.

SUMMARY AND CONCLUDING REMARKS

Prediction of nearshore wave characteristics generated by variable winds was considered by using a parametric approach based on a contracted form of the radiative transport equation. It was assumed that the wind-disturbed sea surface can be described in terms of a depth-dependent equilibrium range spectrum with a well-defined low wave number cut-off. Among all possible processes at work in wind-wave generation, propagation, and dissipation, only the generative effect of winds and the dissipative
influence of turbulent bottom friction were taken into account, and governing equations were derived to describe the development of a wind-wave field in terms of the integral properties of its surface spectrum such as significant wave height and characteristic period. The governing equations were solved first for various simple cases under deep water conditions, obtaining results which compare very favorably with other parametric theories and observational data available. Consequently, the solutions were numerically extended first to the case of wind-waves in uniform water depth and then to the more general case of a circular basin with variable bathymetry. Except for the case of wind-waves generated under fetch-limited conditions in uniform water depths, where the solutions of the present study compare well with those referenced in SPM (25), there is no other corresponding theory or readily available field data to verify the validity of the remaining set of numerical results presented in Figures 4 through 8. Hence, these results are of a theoretical nature and remain to be substantiated with observational data or, by generating the corresponding integral properties through the application of a proven discrete spectral model to the same cases.

Finally, it is emphasized that the proposed parametric approach is an approximate one involving various inherent simplifying assumptions as to processes which operate in wind-wave generation. Therefore, the proposed approach is intended for operational uses and engineering design purposes to forecast or hindcast the wind-wave climatology in a nearshore zone for short-
or long-term applications or, to fill gaps in wave data base for extreme value statistics. Its application to complex situations such as hurricanes involving wind fields with rapidly varying directions and magnitudes should be considered only with extreme caution.
Figure 1. Comparison of Observational Data of Wind-Wave Parameters with Various Forecasting Theories for Conditions of Unlimited Duration in Deep Water.
Figure 3. Forecasting of Wind-wave Parameters for Conditions of Unlimited Duration in Uniform Water Depths: A Comparison of SPM Theory with the Present Study.
Figure 4. Forecasting of Wind-Wave Parameters for Conditions of Unlimited Fetch in Uniform Water Depths in Uniform Water Depths Based on the Present Study.
Figure 5. Forecasting of Duration and Fetch-Limited Wind-Wave Parameters in Uniform Water Depth ($d = 0.3$) Based on the Present Study.
Figure 7. Contours of Scaled Wave Period.
Figure 8. Contours of Scaled Significant Wave Height $H$. 
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