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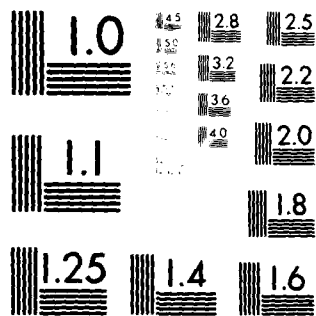UNCLASSIFIED   NPRDC-TR-80-33   NL

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OPTIMAL ACCESSION REQUIREMENTS (OAR) MODEL

Alan W. Whisman

Reviewed by
Joe Silverman

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Released by
James F. Kelly, Jr.
Commanding Officer

Navy Personnel Research and Development Center
San Diego, California 92152

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This report is the third in a series describing manpower flow models that may be used to determine accession plans that prevent large surpluses and shortages in various force categories. Previous reports described the development of quarterly optimization models to determine the allocation of recruits over a 5-year planning horizon. In this effort, a similar optimization technique was used to develop an annual accession model that, while retaining much of the comprehensiveness of the quarterly models, is small enough to allow it to be linked with other personnel forecasting models to form a more comprehensive personnel planning system.

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FOREWORD

The effort described in this report supports the development of Structured Accession Planning (STRAP), an exploratory development objective under Task Area ZF63-521-001-010, Manpower Management Decision Technology, Work Unit 3.16, Accession Planning Models. The objective of the task area is to develop techniques to improve the Navy's managerial decision-making capabilities; the objective of the work unit, to develop quantitative techniques to analyze tradeoffs between enlisted manpower requirements, personnel policies, and available enlisted manpower supply.

Two earlier reports (NPRDC Tech. Reps. 80-12 and 80-32) described the development of optimization models to determine quarterly accession plans to meet petty officer requirements over a 5-year planning horizon. This report describes a similar annual model that can be linked with larger personnel forecasting systems, such as the STRAP system currently under development. A prototype version of the STRAP system was installed in OP-01 (under the sponsorship of OP-12) in November 1979. Experience gained in the use of the prototype will be used in the progressive design and redefinition of STRAP.

JAMES F. KELLY, JR.   JAMES J. REGAN
Commanding Officer   Technical Director

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SUMMARY

Problem

In the past, Navy enlisted accession planning has been oriented primarily toward attaining a particular total enlisted force end-year strength. This approach fails to consider the resultant effects several years down the line on other force objectives, such as future requirements for petty officers, trained strength, and careerists (persons with more than 4 years of service). An accession policy designed to meet total end strength year by year can lead to large surpluses or shortages in critical force categories in future years.

Purpose

The purpose of this effort was to develop an accession planning model that would allow a user to determine an accession plan for each year of a planning period, considering both the constraints on the recruiting process (e.g., recruit quality and boot camp capacities) and objectives concerning the size and structure of the enlisted force (e.g., total end strength, end strength by grade, number of careerists, number of trained personnel, etc.).

Model Formulation

The principal mathematical technique employed in this research was linear goal programming, a special case of the well-known technique of linear programming. The objective is to minimize a weighted sum of surpluses and shortages from the desired goals. Constraint equations control the flow of personnel by pay grade and time in service, and incorporate restrictions on the number of recruits available for each year of the planning horizon.

Model Operation

The Optimal Accession Requirement (OAR) model was developed on an IBM 3032 computer system using the MPSX/370 Mathematical Programming System. Test scenarios were run for planning horizons up to 10 years in length to examine the sensitivity of the solution to changes in model parameters. A sensitivity analysis of the flow rates was performed by constructing paired scenarios using identical force goals, goal penalties, and recruit supply estimates from the RAND supply model. One scenario of each pair used FY77 flow rates for the entire 10-year planning horizon while the other used FY78 rates. Scenarios using FY77 rates generally yielded higher numbers of accessions and greater shortages in certain force categories, which corresponded to the higher attrition rate and different promotion policy in effect in FY77. Similar tests were performed to examine the effect of alternative planning horizons, goal values, and goal penalties on the number of accessions required.

Conclusions

1. Linear goal programming provides a flexible technique for analyzing tradeoffs between manpower requirements, enlisted manpower supply, and accession policy. Reasonable accession policies were obtained for several test scenarios analyzing the effects of changes in flow rates, length of the planning horizon, and goal priorities.

2. All inputs required by the OAR model are currently available. Data sources include the Survival Tracking File (STF) and the output of the Enlisted Cohort (ECO) model, which is used to project enlisted forces.

Future Direction

1. The OAR model will be used as a major component of the Structured Accession Planning System (STRAP) currently under development. In this context, OAR will account for the effects of both long-term manpower requirements and the supply of available recruits. Additionally, OAR can be run as a "stand-alone" model using recruit supply estimates and personnel flow rates obtained from other sources.

2. Further research will investigate the sensitivity of the solution to changes in the penalty function used to prioritize goals. This will provide guidelines to users in constructing scenarios. Research will also be performed to determine the feasibility of extending this approach to model the flow of personnel to ratings.

3. The OAR model will be linked with the ECO model to yield better estimates of the gain, loss, and promotion flows of enlisted personnel. Restrictions on recruit supply can be provided by the Enlisted Personnel Supply Model (EPSUM), developed at NAVPERS-RANDCEN or the supply models developed by the RAND Corporation.

4. The feasibility of extending the model to include the disaggregation of personnel by rating will be examined. To solve this much larger problem efficiently, the linear programming formulation may be modified to incorporate efficient network flow algorithms or simulation procedures.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Problem

In the past, Navy enlisted accession planning has been oriented primarily toward attaining a particular total enlisted force end-year strength. This approach fails to consider the resultant effects several years down the line on the other force objectives, such as future requirements for petty officers, trained strength, and careerists (persons with more than 4 years of service). An accession policy designed to meet total end strength year by year can lead to large surpluses or shortages in critical force categories in future years.

The strength levels of the various force categories is a complex function of personnel variables (such as attrition and reenlistment rates) and their interaction via Navy policy (such as training and promotion policies). In terms of promotion policy alone, it is worth noting that the number of personnel available for promotion into a pay grade depends on the number and experience of the personnel at the pay grade below. Thus, personnel planners must consider the experience or time in service (TIS) distribution of the force and the time lag between the access of new recruits and their availability to fill petty officer positions.

Although more personnel can be promoted if there are fewer restrictions on their experience level, a reasonable level of experience must be maintained to perform the Navy's work. Consequently, the need for adequate experience may conflict with the need for promotions to satisfy petty officer vacancies. In any event, personnel planners must plan Navy accessions over a planning horizon in a manner that best meets these sometimes conflicting and interrelated goals.

Purpose

The purpose of this effort was to develop an accession planning model that would allow a user to determine an accession plan for each year of a planning period, considering both the constraints on the recruiting process (e.g., recruit quality and boot camp capacities) and objectives concerning the size and structure of the enlisted force (e.g., total end strength, end strength by grade, number of careerists, number of trained personnel, etc.). Since it will generally not be possible to meet all of these force objectives simultaneously, a technique is required that relaxes each of the requirements slightly to allow for deviations from the desired strengths. With this technique, the user can alter the relative importance of failing to meet the various goals, and thus examine the effects of different goal priorities on the accession forecasts.

Background

This report is the third in a series describing the development of long-range accession planning models for the Navy enlisted force. The first two reports describe the development of the Recruit Input Optimization (RIO) model (Yen, 1980) and the Accession Gaming Model (AGAM) (Whisman, Yen, & Chipman, 1980). These models determine optimal accession plans by quarter for 5 years, but are too large to be linked with other personnel forecasting models to form more comprehensive personnel planning systems. The annual model--called the Optimal Accession Requirements (OAR) model--is designed to greatly reduce the amount of computer time and storage required for solution, yet retain many of the features of the larger quarterly models.

As shown in Table 1, which compares the three models, the OAR model requires much less computation time to solve the linear programming system of equations than do the

1

earlier models (390 vs. 4000+ equations). This increase in computational efficiency is gained at the cost of losing the detail of quarterly planning periods in favor of annual planning periods. The other major differences among the models occur in the manner in which strength goals and supply goals are handled. AGAM and OAR expand on RIO's goals of total strength by pay grade to allow goals to be specified by trained and structured strength as well. OAR also includes a supply shortage penalty function based on econometric estimates of recruit supply in place of the simple upper and lower bounds used in RIO and AGAM.

Table 1

Characteristics of Three Accession Planning Models

Characteristics	RIO	AGAM	OAR
Equations (for 5-year model)	4200	4400	390
Strength Goals (by pay grade)	Total	Total Structured Trained	Total Structured Trained
Planning Periods	Quarter	Quarter	Year
Constraint on Recruit Supply	Upper Bound	Upper and Lower bounds	Penalty Function
CPU Time (seconds)	650-1000	650-1000	12-27

MATHEMATICAL FORMULATION

The principal mathematical technique employed for this research was linear goal programming, a special case of the well-known technique of linear programming (Charnes & Cooper, 1977). This technique minimizes deviations from the planning goals, while allowing the capability to place greater emphasis on attaining some goals than others. In this way, a wide variety of accession policies can be compared, all of which will be efficient with respect to the set of goals.

The planning goals are represented by linear constraints, with variables that represent the surplus or shortage relating to that goal. The objective function to be minimized is represented by a weighted sum of these surplus and shortage variables. Additional constraints control the flow of personnel by pay grade and time-in-service (TIS) from one time period to the next, and incorporate additional restrictions on the number of recruits available. Model outputs include the optimal number of recruits for each year of the planning horizon, the personnel force structure by TIS and pay grade in each year, and information on each planning goal (e.g., the desired value, amount of deviation, and the "cost" incurred in the objective function due to missing the goal).

As noted above, constraint equations are required in the model to properly flow individuals from cell to cell over time. Individuals in a particular pay grade/TIS category

at a point in time are restricted by Navy promotion policy as to which categories they can occupy at a later point in time. The model uses time periods and TIS measured in years, and assumes fixed fractional flows between categories from one year to the next (see Yen, 1980). In a fixed fractional flow model, the proportion of the personnel force in pay grade i, TIS j, at the end of year t that moves to pay grade k, TIS j+1, at the end of year t+1 is assumed to be a fixed fraction $f_{i,j,k}(t+1)$. The linear programming constraints that represent the conservation of flow for this type of model are described in Appendix A.

The data from which the flow rates $f_{i,j,k}(t)$ are estimated were taken from the Navy Survival Tracking File (STF). Personnel movements for fiscal years 1977 and 1978 were analyzed to develop the yearly flow rates required for model testing. Work is currently underway to relate the model to the Enlisted Cohort model (ECO), a much modified version of MINIFAST, the Navy's enlisted personnel simulation model (Butterworth, 1976). ECO forecasts personnel losses, promotions, and demotions in a given year utilizing a specified Navy promotion policy. It does this at a level of detail that includes such characteristics as time-in-grade, mental category, sex, and educational level, as well as TIS and pay grade. Flow rates derived from ECO would thus provide the accession model with flow rates based on current or planned Navy policy rather than historical policy.

The other major constraint equations of the linear programming formulation of the model are the goal equations that calculate deviations from desired end strengths and the "cost" or "penalty" for deviation, and the recruit supply and year-to-year fluctuation restrictions, which control unusually large oscillations in the number of recruits processed each year. The mathematical form of these constraint equations is given in Appendix A. Special consideration of methods for incorporating recruit supply forecasts in the model is given in Appendix B.

MODEL OPERATION

The Optimal Accession Requirements (OAR) model has been set up to use the IBM linear programming package MPSX/370. The linear programming problem set-up and solution procedure are written in MPSX/370's Extended Control Language (ECL) (Slate & Spielberg, 1978), and linked to FORTRAN routines, which process the input data into linear programming format and then write the solution outputs into an easily readable report format. Inputs that are under direct user control include the number of accessions from sources other than normal recruitment (e.g., personnel with prior service in the Navy or other Armed Forces), the desired strength levels in the various "goal" categories, and the "costs" or "penalties" incurred for being either over or under each goal.

Goals can also be specified for categories other than total strength in the pay grade. For example, "structured space" goals may be used to specify the number of persons required to fill billets in operational units, known as structured billets. The model uses historical data to determine the portion of personnel in a TIS/pay grade cell who fill a structured billet, and the personnel engaged in other, "unstructured" billets, such as students and trainees, as well as transients, prisoners, and patients (TPP). Likewise, goals may be specified according to the number of trained personnel desired in a pay grade. Enlisted personnel are considered trained if they have a TIS greater than 2 years, or have been assigned to a structured billet. Using techniques similar to those used for the structured space goals, the model will arrive at a force strength with a mix of trained and untrained personnel based on analysis of the historical mix in each TIS/pay grade cell. The classification of the enlisted force into the trained and untrained categories of structured and unstructured billets is shown in Figure 1.

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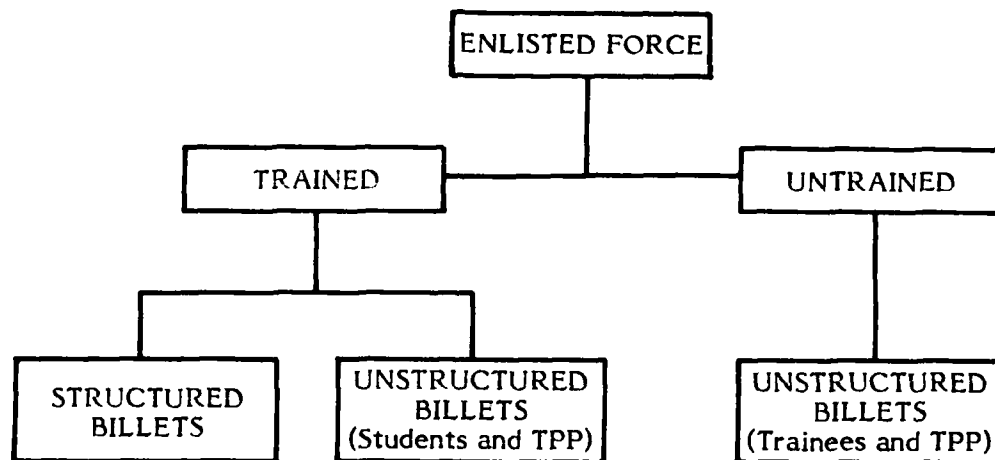
Figure 1. Trained and untrained categories of structured and unstructured billets.

Regardless of the type of goal specified, information on total end strength, trained strength, and structured strength are all provided in the solution report, as well as accession levels, deviations from goals, and personnel force structures arrayed by TIS and pay grade. Depending on the length of the planning horizon, the model takes from 12 to 27 CPU seconds to solve on an IBM 3032 computer. This rapid turnaround allows the user to compare several scenarios quickly and analyze the effects of important variables.

Test Scenarios

The results of several test runs of the OAR model are presented below to give some indication of the scope and flexibility of the linear goal programming approach. Although the three scenarios described use the set of desired end strengths, oscillation limits, and recruit supply estimates presented in Table 2, they differ in the relative priorities given to each of the types of goals. To avoid the "cut-off" effects of terminating the model suddenly at the end of year 5, each scenario was run for 8 years, with goal values and goal penalties in years 6 through 8 identical to those used in year 5 of each run. The actual scenarios used are described below.

● Scenario 1--This scenario simply attempts to bring in enough recruits to fill total end strength in each year, irrespective of the end strength by pay grade, careerist, recruitment oscillation, or recruit supply goals. It can thus be thought of as a baseline from which the effects of adding increasingly complex goals can be observed.

● Scenario 2--This scenario, like the first one, places no constraints on recruit supply or year-to-year oscillations. It includes all of the careerist and pay grade goals specified in Table 2, however, weighted equally and at a slightly higher level than the total end strength goal.

● Scenario 3--This scenario is similar to scenario 2, except that it also includes a high priority goal of minimizing expected recruitment shortages (supply goal). This will, of course, have the effect of causing shortfalls in some of the other goal categories, due to the added restrictions on the numbers of recruits available.

Table 2

Goals for the Model Scenarios

Goal	Desired End Strength				
	Year 1	Year 2	Year 3	Year 4	Years 5-10
E-1	43642	36890	—	—	—
E-2	56695	53012	—	—	—
E-3	83519	90053	—	—	—
E-4	87972	91502	96532	98270	99456
E-5/9	189743	190819	189559	189301	189873
Total	461571	462276	451361	454863	454604
Supply--expected value	103657	82251	77021	80270	81419
Standard deviation	5000	5000	4722	4291	4177
Careerists	192019	190330	186863	188313	188206

Prior service gains (fixed)--11160 in each year.

Table 3 presents the results obtained for several of the more important model goals and the percentage deviation from those goals. As shown, different accession policies do not strongly affect the size of the petty officer force until the third or fourth year of the planning period. Total end strength, however, is greatly altered by the addition of pay grade goals and constraints on the supply of recruits. The decrease in accessions brought about by these extra constraints also acts to increase the shortage at pay grade E-4 in the last 2 years. The importance of using a restriction on the supply of recruits can be seen by examining Table 4. In scenarios 1 and 2, where no restrictions on supply were included, the model yielded accession levels that were unrealistically high and oscillated widely from one year to the next. The supply restrictions and oscillation constraints allow the user to plan a steady flow of recruits that does not exceed training capacity or realistic estimates of enlistments.

Effect of Penalties

To examine the effect of changes in the relative penalties placed on certain goals, modifications were made to scenario 3 to examine two effects: (1) weighting the goals differently in different time periods, and (2) changing the relative weighting of the petty officer, total strength, and supply penalties. This yielded the following four scenarios:

● Scenario 4--Petty officer and total strength goals have a penalty three times as large in years 3, 4, and 5 as the other years. The supply penalty is twice its value in scenar 3.

● Scenario 5--Similar to scenario 4, except that petty officer and total strength goals in years 3, 4, and 5 have 15 times as large a penalty as other years.

● Scenarios 6 and 7--Analogous to scenarios 4 and 5 except that the supply penalty declines over time, decreasing by 10 percent of its original value each year.

5

Table 3

Model Results--Initial Scenarios

| | End Strength | | | | | |
| | Scenario 1 | | Scenario 2 | | Scenario 3 | |
Goal	N	% Deviation	N	% Deviation	N	% Deviation
E-4						
Year 1	87436	-0.61	87357	-0.70	87357	-0.70
Year 2	91918	0.46	91502	0.00	91324	-0.20
Year 3	95494	-1.08	93354	-3.29	93698	-2.94
Year 4	94150	-4.19	89815	-8.60	90114	-8.30
Year 5	90149	-9.36	83731	-15.81	84433	-15.11
E-5/9						
Year 1	189944	0.11	189944	0.11	189944	0.11
Year 2	189945	-0.46	189933	-0.46	189933	-0.46
Year 3	190728	0.62	190609	0.55	190583	0.54
Year 4	192821	1.86	192181	1.52	192110	1.48
Year 5	193554	1.94	191577	0.90	191813	1.02
Total						
Year 1	461571	0.00	459681	-0.41	459681	-0.41
Year 2	462276	0.00	457331	-1.07	453798	-1.83
Year 3	451361	0.00	427523	-5.28	441871	-2.10
Year 4	454863	0.00	442670	-2.68	433612	-4.67
Year 5	454604	0.00	454604	0.00	427153	-6.04

Table 4

Model Results--Required Numbers of Recruits and Expected Recruiting Shortfalls

Item	Year 1	Year 2	Year 3	Year 4	Year 5
Scenario 1					
Recruits	104706	90313	79657	95602	93041
Expected Shortfall	2585	8127	3469	15332	11622
Scenario 2					
Recruits	102454	86393	56682	105082	105143
Expected Shortfall	1474	4708	0	24812	23724
Scenario 3					
Recruits	102454	82251	77021	80270	81419
Expected Shortfall	1474	1960	1851	1682	1638

Table 5 summarizes the results of scenarios 4 through 7. As shown, the extremely high penalties on petty officer and total end strength in scenarios 5 and 7 have a profound effect on accession patterns and the degree to which goals are met. Notice that deviations from the E-5/9 goal increase in these scenarios, despite the higher E-5/9 penalties. This is due to the need to reduce some very large shortages in E-4 and total end strength, which can be done only at the cost of slightly increasing the E-5/9 surplus. Using a supply penalty that decreases over time in place of a constant one also affects the accession pattern, but the effect is not as great as the effect of changing strength penalties.

Effect of Flow Rates

Although the user is allowed great flexibility in choosing goals and priorities, the degree to which the goals can be attained depends largely on the fractional flow rates used to simulate the loss, gain, and promotion flows through the enlisted system. These parameters differ somewhat from year to year, reflecting the impact of both changes in Navy policy and external factors on attrition, promotion, reenlistment, etc. In order to observe the effects of using various flow rates in the model, several scenarios were run in pairs, using FY77 historical rates for all years of the planning horizon in one run, and FY78 rates in the other. The results of one such pair, otherwise identical to scenario 3 above, are shown in Table 5 as scenarios 8 and 9. The higher number of accessions and lower end strength totals in the FY77 scenario reflect the higher attrition rate that occurred in that year. Also, the E-4 and E-5/9 end strength figures reflect a major difference in promotion flows during the 2 years.

These results indicate that the 5- to 10-year forecasts generated by OAR can be improved by attempting to incorporate changes in attrition and retention behavior and Navy promotion policy in the development of flow rates. It is for this purpose that work is now underway to relate OAR to the output of a personnel policy simulation model (such as MINIFAST or ECO), so that the sensitivity of the solution to these factors can be explored more fully.

7

Table 5

Model Results--Scenarios 4 through 9

| | Alternative Penalty Values | | | | | | | | Alternative Flow Rates | | | |
| | Scenario 4 | | Scenario 5 | | Scenario 6 | | Scenario 7 | | Scenario 8 (FY77 Rates) | | Scenario 9 (FY78 Rates) | |
Year	N	% Deviation	N	% Deviation	N	% Deviation	N	% Deviation	N	% Deviation	N	% Deviation
End Strength E-4												
1	87224	-0.85	87399	-0.65	87224	-0.85	87340	-0.72	87366	-0.69	94541	7.47
2	90691	-0.89	91671	0.18	90691	-0.89	91492	-0.01	83666	-8.56	99152	8.36
3	91760	-4.94	94944	-1.65	91759	-4.94	94523	-2.08	78639	-18.54	96470	-0.06
4	86677	-11.80	92989	-5.37	86767	-11.71	93088	-5.27	72506	-26.22	90199	-8.21
5	80217	-19.34	83475	-11.04	80580	-18.98	90126	-9.38	66887	-32.75	83919	-15.62
End Strength E-5/9												
1	189942	0.11	189942	0.11	189942	0.11	189942	0.11	189942	0.11	185082	-2.46
2	189911	-0.48	189939	-0.46	189913	-0.47	189930	-0.47	195565	2.49	188401	-1.27
3	190391	0.44	190665	0.58	190392	0.44	190596	0.55	197477	4.18	191579	1.07
4	191327	1.07	192553	1.72	191327	1.07	192341	1.61	198185	4.69	193948	2.46
5	190224	0.19	193054	1.68	190236	0.19	192921	1.61	195761	3.10	193392	1.85
End Strength Total												
1	456492	-1.10	460691	-0.19	456496	-1.10	459268	-0.50	459809	-0.38	463009	1.40
2	446835	-3.34	458919	-0.73	446835	-3.34	457865	-0.95	445016	-3.73	453905	-0.73
3	431865	-4.32	451361	0.00	431862	-4.32	451361	0.00	425703	-5.69	442614	-1.94
4	419607	-7.75	447238	-1.68	421431	-7.35	454863	0.00	414129	-8.96	433230	-4.76
5	410606	-9.68	436749	-3.93	413937	-8.95	448895	-1.26	404169	-11.09	427099	-6.05
Accessions												
1	98657		103657		98657		101960		102739		98657	
2	77251		87251		77251		87429		84088		82251	
3	72299		83022		72299		84107		77021		74661	
4	73834		86707		75979		95601		80270		79535	
5	75153		79331		77242		85596		79331		81419	

8

Figures 2 through 6, which compare the degree of goal attainment for some of the scenarios discussed above, show the effect of the supply penalty increases, which, in turn, lead to shortages in various manpower categories in later years. Hence, to reduce the risk of falling short of recruiting goals, planners must be prepared for possible shortages in certain manpower categories in future years. For the scenarios given, the critical shortage is at pay grade E-4, with a smaller but significant shortage in total force strength. Figures 5 and 6 show how changes in promotion and retention factors can change this distribution of surpluses and shortages under a given set of penalty weights.

CONCLUSIONS AND FUTURE DIRECTION

The linear goal programming formulation provides an accurate representation of enlisted personnel flows over time, while allowing a great deal of flexibility in analyzing the tradeoffs between manpower requirements, external manpower supply, and policies controlling the personnel inventory. The scenarios tested have provided reasonable accession policies for a variety of flow rates, planning horizons, and goal priorities. All inputs required by the model are currently available. Flow rates have been obtained from the Navy's Survival Tracking File (STF) for FY77 and FY78 and will soon be available for FY79.

The OAR model has the capability of interfacing with the Enlisted Personnel Supply Model--EPSUM--and the personnel policy simulator--ECO--as part of the structured accession planning system (STRAP) now under development. In this way, the model will aid in deriving a set of manpower requirements and personnel policies which are consistent. Additionally, OAR can be run as a "stand-alone" model using recruit supply estimates and personnel flow rates from other sources.

The OAR model can be used in Navy recruit planning to account for the effects of both long-term manpower requirements and forecasts of recruit supply in determining annual recruiting goals. It is desirable, however, to study the sensitivity of the solution to changes in the penalty function used to prioritize goals. This will allow guidelines to be set up to assist users in constructing scenarios before implementing the model operationally.

The direction of further research at NAVPERSRANDCEN will be to examine the process of personnel flows by rating in addition to pay grade and TIS. As there are over 100 ratings in the Navy, the computational problems of solving such a large linear program will require a different approach to the problem. Network flow algorithms and other optimization procedures will be examined to find a procedure for solving the rating problem with a computationally efficient model.
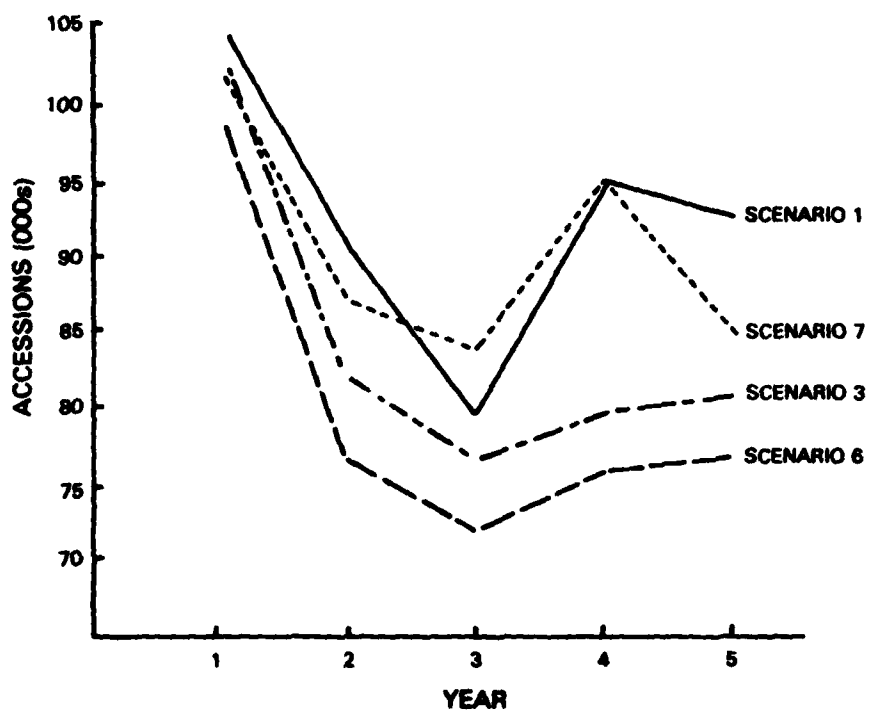
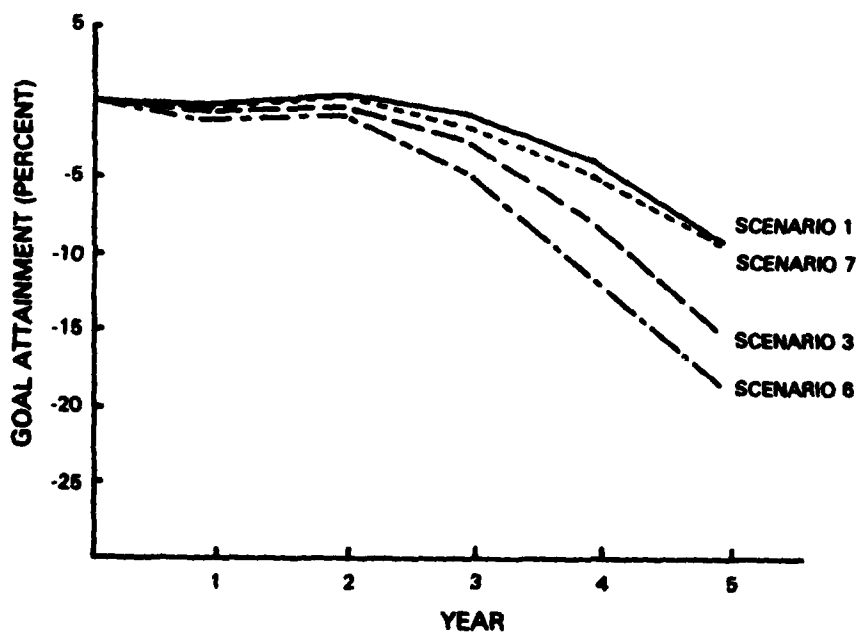Figure 2. Numbers of accessions for OAR scenarios 1, 3, 6, and 7.

Figure 3. Strength at pay grade E-4 for OAR scenarios 1, 3, 6, and 7.
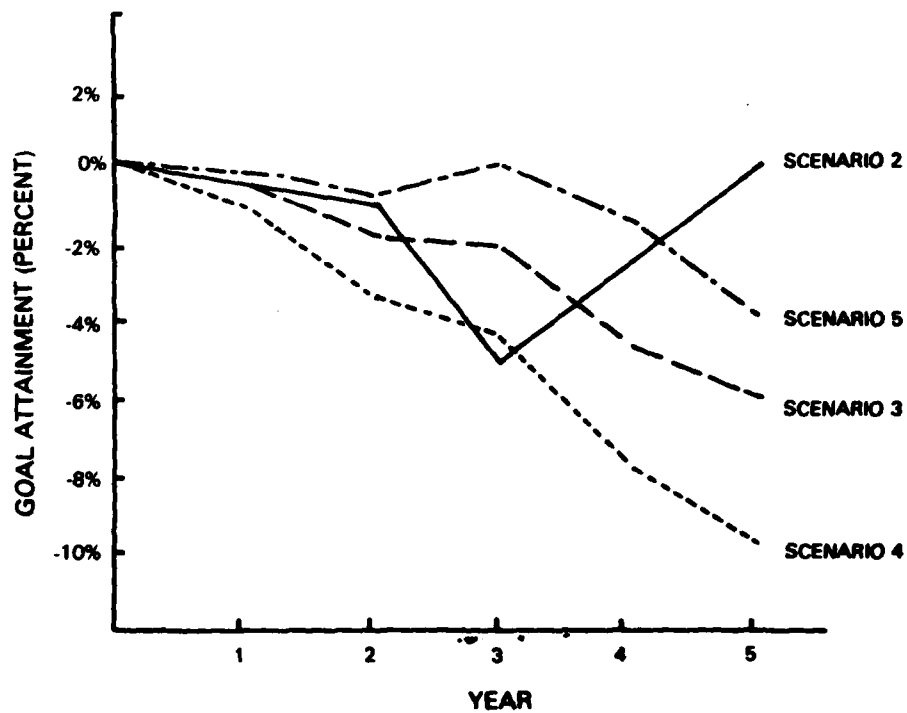
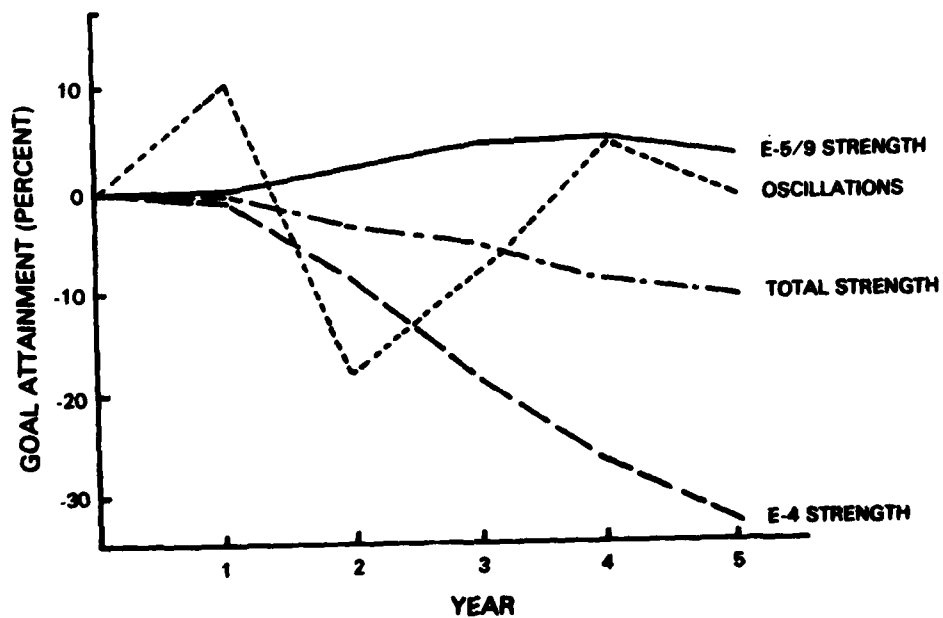Figure 4. Total end strength patterns for OAR scenarios 2, 3, 4, and 5.

Figure 5. Attainment of various goals when using FY77 flow rates (scenario 8).
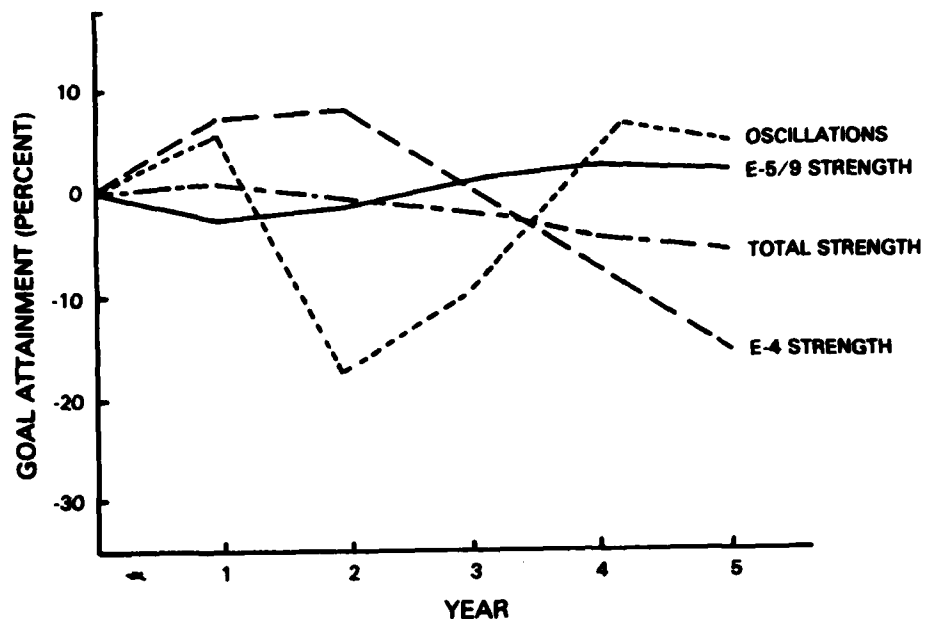
11

Figure 6. Attainment of various goals when using FY78 flow rates (scenario 9).

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APPENDIX A

MATHEMATICAL MODEL

MATHEMATICAL MODEL

Definition of Variables

$X_{i,j}(t) \geq 0$ = The number of personnel in pay grade i, TIS between j-1 and j years, at the end of year t where i = 1,2,3,4,5 and TIS category j = 11 represents all personnel with more than 10 years of service; and i = 5, all personnel in pay grades 5-9.

$f_{i,j,k}(t)$ = Fraction of the force in the inventory $X_{i,j}(t-1)$ that is found in pay grade k at the end of year t.

$G_{i,j}(t)$ = Gains during year t from sources other than normal recruitment (such as prior service gains) who end year t as part of the inventory $X_{i,j}(t)$.

$P_i(t)$ = Fraction of regular nonprior service recruits in year t who end the year in pay grade i.

$R(t) \geq 0$ = The number of recruits during year t.

H = The number of years in the planning horizon.

Goals in Each Time Period

1. End Strengths--Pay grades E-1, 2, 3, 4, 5/9, and total.
2. Trained Strengths--Pay grades E-1, 2, 3, 4, 5/9, and total.
3. Structured Spaces--Pay grades E-1, 2, 3, 4, 5/9, and total.
4. Careerists--Personnel with a TIS > 4 years.
5. Limit oscillations in the number of recruits from one year to the next.
6. Reduce the probability of demanding recruits in excess of the available supply.

Objective Function

Solve for values of variables $X_{i,j}(t)$ and $R(t)$ to minimize the weighted sum of deviations from each goal (D_m^- for understrength deviations, D_m^+ for over strength) in time period t.

Minimize

$$\sum_t \sum_m W_m(t) \, D_m^-(t) + V_m(t) D_m^+(t)$$

where the Ws and Vs are weights applied to under- and overstrength deviations, respectively.

Flow Constraints

$FEQ_{i,j,t}$ = Equation of personnel flowing into pay grade i, TIS j, during year t, t=1, . . ., H.

$FEQ_{i,1,t}$:

$$X_{i,1}(t) - P_i(t) R(t) = G_{i,1}(t).$$

$FEQ_{i,j+1,t}(1 \leq j \leq 9)$:

$$X_{i,j+1}(t) - \sum_k f_{k,j,i}(t) X_{k,j}(t-1) = G_{i,j+1}(t).$$

$FEQ_{i,11,t}$:

$$X_{i,11}(t) - \sum_{j=10}^{11} \sum_k f_{k,j,i}(t) X_{k,j}(t-1) = G_{i,11}(t).$$

Fixed Starting Inventory

$X_{i,j}(0)$ given: $i = 1, \ldots, 5$ and $j = 1, \ldots, 11$.

Goal Equations

1. For trained, structured, careerist, and end strength goals in a particular category at end of year t, the form of the constraint is

(Inventory in goal category m, time t) + $D_m^-(t) - D_m^+(t)$ =

(desired strength in category m, time t) $t = 1, \ldots, H$.

2. Oscillation goals--If recruits are allowed to vary up to α percent above the previous year's level or β percent below it without penalty, then the oscillation goal constraints can be written

$$R(t) - \left(1 + \frac{\alpha}{100}\right) R(t-1) - D_{oscill}^+ \leq 0$$

$$R(t) - \left(1 - \frac{\beta}{100}\right) R(t-1) - D_{oscill}^- \geq 0 \quad t=1, \ldots, H.$$

3. Supply goals--If we are given a discrete probability distribution representing a forecast of available supply in period t, with supply taking on n possible values

$$a^1(t) \leq a^2(t) \leq \ldots \leq a^n(t)$$

with probabilities

$$p^1(t), p^2(t) \ldots p^n(t),$$

then the supply constraint can be written as

$$R(t) - \sum_{k=1}^{n} D_{supp}^{k+}(t) \leq a^1(t) \qquad\qquad t=1,\ldots,H$$

with bounds on the goal variables of

$$D_{supp}^{\ell+}(t) \leq a^{\ell+1}(t) - a^{\ell}(t) \qquad\qquad \ell=1,\ldots,n-1.$$

Under this formulation, the terms of the objective function for the deviation variables $D_{supp}^{\ell+}(t)$ will represent the <u>expected shortage</u> in period t if the weight $V_{supp}^{\ell}(t)$ on variable $D_{supp}^{\ell+}(t)$ is chosen so that

$$V_{supp}^{\ell}(t) = \sum_{k=1}^{\ell} p^k(t) \qquad \text{(Dantzig, 1963).}$$

The procedure to derive this expected shortage function from various forecasts of recruit supply is detailed in Appendix B. The minimization procedure will attempt to reduce the expected shortage, just as it reduces the deviation levels of the other goals.

APPENDIX B

**INCORPORATING RECRUIT SUPPLY FORECASTS
IN THE MODEL**

INCORPORATING RECRUIT SUPPLY FORECASTS
IN THE MODEL

In Appendix A, a technique was described whereby limitations on the supply of new recruits could be incorporated in the goal programming framework. When an unrealistically large number of accessions is required, realism dictates that a "cost" or "penalty" be incurred by including it in the sum of terms to be minimized by the model. This formulation explicitly accounts for the fact that the supply of available recruits at some point in the future is not known with certainty and is usually expressed in a forecast by a distribution of possible values and their probability of attainment. If S is a random variable representing these possible values of recruit supply and D is the number of recruits desired by the Navy, then we can define the shortage to be the value D - S when $D \geq S$, and zero when D < S, or,

shortage = max (D - S, 0).

This quantity will also be a random variable and will increase as D increases.

To penalize an excessive (i.e., unrealistically high) demand for recruits in the goal programming problem, we include a cost term that is proportional to the average value of the shortage, or expected shortage, that results from demanding D = R(t) recruits in year t. When the probability distribution of the number of available recruits in year t is given as a discrete distribution taking on a finite number of values, the linear programming representation given in Appendix A yields the correct penalty function. Supply forecasts, however, are generally derived by econometric forecasting methods that yield continuous supply distributions, such as normal, lognormal, etc. In this case, the expected shortage function will be a nonlinear function, which cannot be used in a linear optimization formulation. There are methods by which a linear approximation of the expected shortage function can be derived, however, yielding a close approximation to the original penalty function.

The first method involves simply approximating the cumulative distribution function (c.d.f.) of supply with the cumulative distribution function of a discrete random variable, and computing the approximate expected shortage as in Appendix A. One such method is to take a finite number of possible supply values $a_1 \leq \ldots \leq a_n$, and compute the corresponding values of the c.d.f. $p_i = F(a_i) = \text{Prob}(S \leq a_i)$. Then, the c.d.f. Q given by

$$Q(x) = 0, \qquad\qquad x < a_1$$
$$ = \tfrac{1}{2}(p_i + p_{i+1}), \quad a_i \leq x < a_{i+1}, \; 1 \leq i \leq n-1$$
$$ = 1 \qquad\qquad x \geq a_n$$

will be an approximation of the function F, which represents a discrete probability distribution.

A second technique, which can be used when the supply follows a normal distribution with mean μ and variance σ^2, is to compute the actual expected shortage function and then replace it with a function made up of linear segments that closely approximate the actual function and can be solved within the linear programming framework. The expected shortage function when supply is normally distributed with mean μ and variance σ^2 can be written as

$$(D-\mu)Z(\frac{D-\mu}{\sigma}) + \sigma z(\frac{D-\mu}{\sigma})$$

where D is the demand for recruits, and Z and z are the c.d.f. and density function, respectively, of the standard normal distribution with mean zero and variance one.

This function is a convex, increasing function of D and can be approximated by choosing n values of D, $d_1 \leq d_2 \leq \ldots \leq d_n$, and creating a function that has the same value as the expected shortage function at these points, and is linear between adjacent pairs of points (see Figure B-1). By representing each linear segment by a bounded variable in the linear programming model, the problem can be solved using this approximate shortage function in a manner similar to that described in Appendix A. This method has the advantage of allowing the actual and approximate shortage functions to be computed and compared, so that the approximating function can be chosen to achieve any desired level of accuracy.
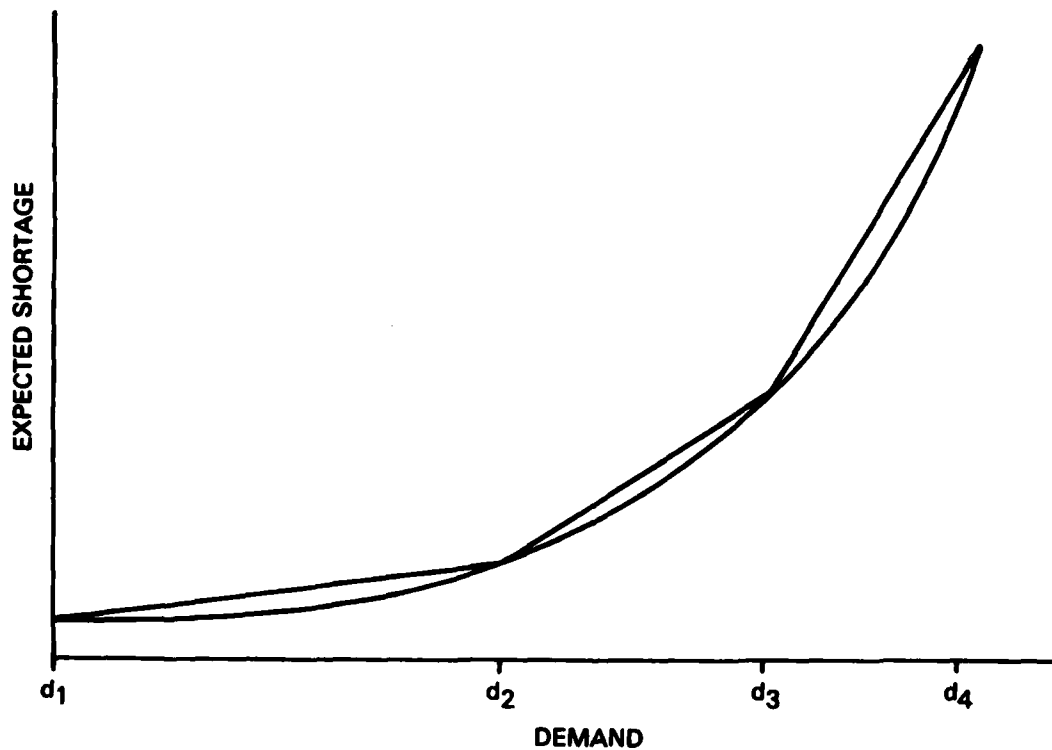

Figure B-1. Piecewise linear approximating function.

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