THE THERMODYNAMICS OF THE CARBONATE SYSTEM IN SFAWATER (U)
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The thermodynamics of the carbonate system in seawater

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Abstract—The apparent constants \( K \) for the ionization of carbonic acid in seawater at various salinities \((S, \ldots)\) have been fit to equations of the form

\[ \ln K_i = \ln K'_i + A_i S + B_i S \]

where \( K_i \) is the thermodynamic ionization constant in water, \( A_i \) and \( B_i \) are adjustable parameters. The temperature dependence \((T_K)\) of \( K_i \), \( A_i \), and \( B_i \) were of the form \( a_0 + a_1 T + a_2 \ln T \). Equations of similar forms have been used to analyze the ionization constants for water and boric acid and the solubility product of calcite in seawater. The effect of pressure on the apparent constants \( K_i' \) has been fit to equations of the form

\[ \ln \left( \frac{K_i'}{K_i} \right) = -(\Delta V P + 0.5 \Delta K P^2) + RT \]

where the volume \((\Delta V)\) and compressibility \((\Delta K)\) changes are polynomial functions of temperature.

The equations generated for various acids in seawater have been used to examine the carbonate system in seawater. Equations relating the NBS and Tris pH scales have been derived as well as equations of \( pH \) as a function of temperature and pressure. The equations from HANSSON (1972, Ph.D. Thesis, University of Göteborg, Sweden) and MEHRBACH et al. (1973, Limnol. Oceanogr. 18, 891-907) have been used to examine the components of the carbonate system. At a fixed total alkalinity and total carbon dioxide, differences of \( \pm 0.01 \text{ m-equiv kg}^{-1} \) in \( \text{HCO}_3^- \) and \( \text{CO}_3^{2-} \) were found; however, the \( [\text{CO}_3^{2-}] \) and \( \text{P}_{\text{CO}_2} \) are nearly the same. The contribution of borate ion, \( [\text{BO}_3] \), determined from the equations of HANSSON (1972, Ph.D. Thesis, University of Göteborg, Sweden) and LYMAN (1957, Ph.D. Thesis, University of California, Los Angeles) differ by \( \pm 0.01 \text{ m-equiv kg}^{-1} \) for waters with the same salinity and temperature.

INTRODUCTION

Recently we had a need to use apparent constants for boric and carbonic acid in seawater to determine the parameters of the carbonate system in the Mediterranean Sea (MILLERO et al., 1978, 1979). An examination of the equations presently available (EDMOND and GIESKES, 1970; TAKAHASKI et al., 1970; ALMGREN et al., 1975; MEHRBACH et al., 1973) for the salinity and temperature dependence of the apparent constants revealed a wide discrepancy in the form and number of parameters used. EDMOND and GIESKES (1970) have reviewed the earlier work of LYMAN (1957) and BUCH (1932, 1933, 1938, 1951). In recent years two independent studies on the carbonate system have appeared (HANSSON, 1972; MEHRBACH et al., 1973). The measurements of HANSSON (1972) are based on a Tris buffer scale (HANSSON, 1973) while the measurements of MEHRBACH et al. (1973) are based on the NBS buffer scale (BATES, 1964).

The purpose of the present paper is to thoroughly analyze the measurements of these workers and to provide a statistically valid representation of the apparent constants as a function of salinity, temperature and pressure.

EQUATIONS FOR APPARENT CONSTANTS AT ONE ATMOSPHERE

Two general criterion were used to examine the salinity and temperature dependence of the apparent constants for the ionization of acids in seawater: (1) the temperature derivatives of the constants should yield reliable thermodynamic quantities (\( \Delta H \) and \( \Delta C_p \), enthalpies and heat capacities); (2) the equations should be extrapolated to pure water when the salinity goes to zero. As shown by CLARK and GLEW (1966) reliable thermodynamic functions can be derived from equilibrium constants \( (K_i) \) by using an equation of the form \((T_K)\)

\[ \ln K_i = A + B/T + C \ln T + DT + ET^2 + \ldots \] (1)

where \( A, B, C, \ldots \) are adjustable parameters. The differentiation of eqn (1) with respect to temperature gives

\[ \Delta H_i = RT^2(\delta \ln K_i/\delta T) = R[-B + CT + DT^2 + 2ET^3 + \ldots] \] (2)

\[ \Delta C_p = (\delta \Delta H_i/\delta T) = R[C + 2DT + 6ET^2 + \ldots] \] (3)

To fit equilibrium constants for the ionization of acids over a temperature range of 50° only the \( A, B \) and \( C \) terms are necessary. HARNED and OWEN (1958) have used a slightly different equation

\[ \ln K_i = A + B/T + CT \] (4)

to fit the ionization constants of acids in water. The statistical analysis of the ionization constants for carbonic acid in water shows that eqn (1) is slightly better than eqn (4) when three parameters are used.
Li and are more than sufficient.

The temperature dependence of the ionization of acids in seawater data is represented by various estimates and measurements, we select the value of $K_w$ (aragonite)/$K_w$ (calcite) = $1.48 \pm 0.14$ at 25°C and $1.43 \pm 0.14$ at 5°C. These values can be compared with the ratio calculated from the free energies of the two solids. LAMCMUR (1964) gives $\Delta G$(calcite - aragonite) = $-230 \pm 30$ cal mol$^{-1}$ at 25°C, while CHRIST et al. (1974) give $-218 \pm 30$ cal mol$^{-1}$ at 25°C and $-276 \pm 40$ cal mol$^{-1}$ at 5°C. The value of $K_w$ (aragonite)/$K_w$ (calcite) = $1.46 \pm 0.08$ at 25°C obtained from these two studies are in good agreement with the measured value of BERNER (1976). At 5°C a value of $1.65 \pm 0.12$ is obtained from CHRIST et al. (1974) which is slightly higher than the experimental value of BERNER (1976). Within the error of the various estimates and measurements, we select the value of $K_w$ (aragonite)/$K_w$ (calcite) = $1.5 \pm 0.1$ from 5 to 25°C. Combining this estimate with the temperature coefficients, we have estimated the effect of temperature on the solubility of aragonite [the coefficient A (aragonite) = A (calcite) + ln(1.5) or 0.405].

The equation selected to represent the salinity dependence of apparent constants is given by

$$\ln K_i = A' + B'S^{1/2} + CS$$  \hspace{1cm} (5)

This equation was arrived at by examining the value of $K_i$ of LYMAN (1957), HANSON (1972), MEHRBACH et al. (1973), CULBERSON and PYTKOWICZ (1973), and INGLE et al. (1973, 1975) at a given temperature. Figures

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### Table 1. Parameters for the temperature dependence of the ionization of acids and the solubility of CaCO$_3$ in water

<table>
<thead>
<tr>
<th>Acid</th>
<th>$A$</th>
<th>$-B$</th>
<th>$-C$</th>
<th>$\sigma$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O</td>
<td>148.9002</td>
<td>13847.26</td>
<td>23.6521</td>
<td>0.0014</td>
<td>HARNED and OWEN (1958)</td>
</tr>
<tr>
<td>B(OH)$_3$</td>
<td>148.0248</td>
<td>8966.90</td>
<td>24.4344</td>
<td>0.0027</td>
<td>OWEN (1934);</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Manov et al. (1944)</td>
</tr>
<tr>
<td>H$_2$CO$_3$</td>
<td>290.9097</td>
<td>14554.21</td>
<td>45.0575</td>
<td>0.0024</td>
<td>HARNED and DAVIS (1943)</td>
</tr>
<tr>
<td>HCO$_3^-$</td>
<td>207.6548</td>
<td>11843.79</td>
<td>33.6485</td>
<td>0.0033</td>
<td>HARNED and SCHOLES (1941)</td>
</tr>
<tr>
<td>Solid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calcite</td>
<td>300.1036</td>
<td>13348.09</td>
<td>48.7537</td>
<td>0.035</td>
<td>JACOBSON and Langmuir (1974);</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>BERNER (1976)</td>
</tr>
<tr>
<td>Aragonite</td>
<td>300.5363</td>
<td>13348.09</td>
<td>48.7537</td>
<td>0.05</td>
<td>BERNER (1976)</td>
</tr>
</tbody>
</table>

$^a$ Temperature range $0 \leq T \leq 50^\circ$.

We have selected eqn (1) to fit the thermodynamic ionization constants of water (HARNED and OWEN, 1958), boric acid (OWEN, 1934; MANOV et al., 1944), and carbonic acid (HARNED and SCHOLES, 1941; HARNED and DAVIS, 1943) in water from 0 to 50°C. The values of $A$, $B$ and $C$ (eqn 1) along with the standard errors are given in Table 1. Values of $\Delta H$ and $\Delta C_p$ for the ionization of H$_2$O, boron and carbonic acids at 25°C determined from eqns (2) and (3) are given in Table 2 along with directly measured values (HALE et al., 1963; VANDERZEE and SWANSON, 1963; MILLERO, 1979b; LARSON and Hepler, 1969; BERG and VANDERZEE, 1978). The agreement in the measured and calculated values of $\Delta H$ are good. For $\Delta C_p$, the agreement is not as good due to the errors in differentiating eqn (1) twice. As discussed by LARSON and HEPLER (1969) a more reliable temperature function for the ionization of acids can be obtained by integrating eqn (3) than the differentiation of eqn (1). As more reliable heat capacity data becomes available (for a recent review see MILLERO, 1979a), this should become possible in the near future. For the analysis of seawater data our representation of the temperature dependence of the ionization of acids are more than sufficient.

We have also fit the solubility product of calcite (JACOBSON and Langmuir, 1974 and BERNER, 1976) in water from 0 to 50°C to eqn (1). The coefficients $A$, $B$ and $C$ and the standard error are given in Table 1.

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### Table 2. Values of $\Delta H$ and $\Delta C_p$ for the ionization of boric and carbonic acids in water at 25°C

<table>
<thead>
<tr>
<th>Acid</th>
<th>$\Delta H$ (kcal mol$^{-1}$)</th>
<th>$\Delta C_p$ (cal mol$^{-1}$ K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This study Literature</td>
<td>This study Literature</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>13.50</td>
<td>13.34*</td>
</tr>
<tr>
<td>B(OH)$_3$</td>
<td>3.34</td>
<td>3.37*</td>
</tr>
<tr>
<td>H$_2$CO$_3$</td>
<td>2.23</td>
<td>2.19*</td>
</tr>
<tr>
<td>HCO$_3^-$</td>
<td>3.60</td>
<td>3.51*</td>
</tr>
</tbody>
</table>

$^a$ HALE et al. (1963); VANDERZEE and SWANSON (1963).
$^b$ Calculated from the heat capacity data tabulated by MILLERO (1979a).
$^c$ LARSON and Hepler (1969).
$^d$ BERG and VANDERZEE (1978).
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lated by Manny Mehr) Successive higher order terms were dropped until the standard error increased sufficiently to indicate by an F-test that the term was significant at the 95% level. The $h_1$ and $h_2$ terms were not needed for any of the systems studied. The $a_2$ term was needed only for the ionization of water. It should be pointed out that Hansson's measured values of $\ln K_p$ and $\ln K'$ at various temperatures were made only at $S = 35%$.

The derived parameters represent the best statistically representation of the experimentally measured apparent constants. The validity of the constants for seawater diluted with pure water below 20°C needs to be demonstrated experimentally.

The new carbonate results of Mehrbach et al. (1973) and Hansson (1972) have similar standard errors and appear to be more reliable than the earlier work of Lyman (1957). The results of Hansson (1972) for boric acid appear to be more reliable than the

![Fig. 1. The log of the apparent ionization constant of boric acid ($K_a$) in seawater plotted versus the square root of the salinity.](image1)

![Fig. 2. The log of the apparent ionization constant of carbonic acid ($K_1$) in seawater plotted versus the square root of the salinity.](image2)

1-5 show the plots of $\ln K'_i$ vs $S^{1/2}$ for the ionization of water, boric and carbonic acids, and the solubility of calcite. As is quite apparent from these plots the values of $\ln K_i$ for boric acid and $H_2CO_3$ from the work of Lyman and Mehrbach are linear functions of $S^{1/2}$ and the intercepts approach the thermodynamic values for pure water. For the ionization of $H_2O$, $HCO_3^-$, the solubility of calcite, and all the constants measured by Hansson (1972) a second degree function is needed. The smooth curves in Figs 1-5 represent the least squares best fit of the data.

The temperature dependence of the terms in eqn (5) have been determined by fitting the directly measured apparent constants to the equation

$$\ln K'_i - \ln K_0 = A'S^{1/2} + B'S$$

(6)

where the parameters $A'$ and $B'$ have the same temperature form as $\ln K_i$ (the thermodynamic constant)

$$A' = a_0 + a_1/T + a_2 \ln T + \ldots$$

(6a)

$$B' = b_0 + b_1/T + b_2 \ln T + \ldots$$

(6b)

The parameters $a_0$ to $a_1$ and $b_0$ to $b_1$ for the ionization of $H_2O$, $B(OH)_3$, $H_2CO_3$, $HCO_3^-$ and the solubility of $CaCO_3$ are given in Table 3 (along with the standard errors). The least squares computer program used to determine the adjustable parameters was the same as used in our equation of state work (formu-
The differences between the values of $\ln K'_w$, $\ln K'_1$, and $\ln K'_2$ obtained by Hansson and other workers at $25^\circ C$ are given in Table 4. The values of $\ln K'_1$ and $\ln K'_2$ from Hansson's work are, respectively, $0.32 \pm 0.03$ and $0.37 \pm 0.04$ higher than the values of Lyman and Mehrbach et al. from 5 to 40°, salinity. This is equivalent to a pH difference of 0.14-0.16 which is in good agreement with the measured values of 0.14-0.16 (HANSSON, 1973) and is related to the differences in the NBS and Tris buffer scales. For boric acid the differences in $\ln K'_2$ (between Hansson and Lyman) are $0.22 \pm 0.04$ which is 0.1 lower than expected. The recent measurements and calculations of CARBONIC ACID in seawater are shown in Figs 6-8. The smoothed values of $\ln K'_1$ and $\ln K'_2$ for carbonic acid from the results of Lyman and Mehrbach are within the sum of the standard errors of the fits (0.06 for $\ln K'_1$ and 0.13 for $\ln K'_2$) over the entire temperature and salinity range of the measurements. The differences in $\ln K'_1$ at $25^\circ C$ are quite small (0.004) and are randomly distributed. For $\ln K'_2$ the differences are independent of temperature and nearly a linear function of salinity. The errors at high salinities are slightly larger (0.21 at $S = 40$) than the sum of the standard errors (0.13).

From the comparisons with Hansson's work at these high salinities, Lyman's work appears to be in error.
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Table 3. Parameters for the temperature dependence of apparent constants in seawater

<table>
<thead>
<tr>
<th>Solute</th>
<th>$\ln K'_1 - \ln K_1 = (a_0 + a_1/T + a_2 \ln T) S^2 + h_2 S$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$h_210^2$</th>
<th>$\sigma$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O</td>
<td>$-97.9429$</td>
<td>$4149.915$</td>
<td>$14.8269$</td>
<td>$-2.3694$</td>
<td>$0.019$</td>
<td>HANSSON (1972)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-79.2447$</td>
<td>$3298.720$</td>
<td>$12.0408$</td>
<td>$-1.9813$</td>
<td>$0.020$</td>
<td>CULBERSON and PYTKOWICZ (1973)</td>
<td></td>
</tr>
<tr>
<td>B(OH)$_3$</td>
<td>0.0473</td>
<td>49.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.049</td>
<td>LYMAN (1957)</td>
</tr>
<tr>
<td>H$_2$CO$_3$</td>
<td>-0.1110</td>
<td>74.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.035</td>
<td>LYMAN (1957)</td>
</tr>
<tr>
<td>HCO$_3$</td>
<td>0.0251</td>
<td>84.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.007</td>
<td>MEHRBACH et al. (1973)</td>
</tr>
<tr>
<td>CaCO$_3$</td>
<td>1.6233</td>
<td>-118.64</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.008</td>
<td>HANSSON (1972)</td>
</tr>
</tbody>
</table>

* In the salinity range $0 \leq S_{ow} \leq 40$ and temperature range $0 \leq ^\circ{C} \leq 50$.

of BYRNE and KESTER (1974) at 25°C give a value of $\Delta \ln K_B = 0.27$ which is slightly higher (the earlier work of BUCH, 1933, gives $\Delta \ln K_B = 0.18$). These comparisons tend to indicate that the results ofハンソン for boric acid are too low by 0.15 in $\ln K_B$. Further studies are needed to clarify these differences.

The differences in $\ln K_B$ of HANSSON (1972) and CULBERSON and PYTKOWICZ (1973) are $0.04 \pm 0.01$ at 25°C from 5 to 40°C, salinity which is nearly within the sum of the standard error of the fits (0.03).

Another method that can be used to examine the reliability of the apparent constants for the ionization of carbonic acid is to examine the ratio of $K'_1/K_1$ (SKIRROW, 1975). Values of $\ln (K'_1/K_1)$ obtained from the derived equation are shown at various salinities and temperatures in Fig. 9. The results of Lyman et al. are in good agreement and within the standard error of the measurements. Larger differences in $\ln (K'_1/K_1)$ are found in the temperature dependence than in the salinity dependence.
The values of \( K'_{\text{op}} \) for the solubility of calcite in seawater of INGLE et al. (1973, 1975) have been selected because their results cover a wide salinity and temperature range. These results have recently been confirmed at 5 and 25°C by the measurements of MORSE et al. (1979). The recent measurements of MORSE et al. (1979) yield values for the solubility of aragonite in seawater that are \( \sim 1.5 \) times their value for calcite. Thus, the equations for the effect of temperature and salinity on the solubility of calcite in seawater can also be used for the solubility of aragonite.

EQUATIONS FOR THE EFFECT OF PRESSURE ON APPARENT CONSTANTS

The equation selected to represent the effect of pressure on the apparent ionization constants of acids in seawater (CULBERSON and PYTKOWICZ, 1968) is

\[
\ln (K'/K^0) = -(AV_/RT)P + (0.5 AK_/RT)P^2 \tag{7}
\]

where \( \Delta V_1 \) and \( \Delta K_1 \) are the volume and compressibility change for the ionization. The values of \( \Delta V_1 \) and

<table>
<thead>
<tr>
<th>Table 4. Comparisons of the values of ( \ln K ) obtained by various workers for various acids in seawater at 25°C</th>
</tr>
</thead>
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<tr>
<td>( S(\text{eq}) )</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>40</td>
</tr>
</tbody>
</table>

* HANSson CULBERSON and PYTKOWICZ.
* HANSson Lyman.
* HANSson MEHRBACH et al.
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Table 5. Coefficients for eqn (7a) and (7b)

<table>
<thead>
<tr>
<th>Solute</th>
<th>a₀</th>
<th>a₁</th>
<th>a₂</th>
<th>a₃×10⁴</th>
<th>b₀</th>
<th>b₁</th>
<th>b₂</th>
<th>σ(ln K⁺/K⁰)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(OH)₃</td>
<td>29.48</td>
<td>-0.295</td>
<td>-0.1622</td>
<td>2.608</td>
<td>2.84</td>
<td>-0.354</td>
<td>--</td>
<td>0.0030</td>
</tr>
<tr>
<td>H₂CO₃</td>
<td>25.50</td>
<td>0.151</td>
<td>-0.1271</td>
<td>--</td>
<td>3.08</td>
<td>0.578</td>
<td>-0.0877</td>
<td>0.0043</td>
</tr>
<tr>
<td>HCO₃⁻</td>
<td>15.82</td>
<td>-0.321</td>
<td>0.0219</td>
<td>--</td>
<td>-1.13</td>
<td>0.314</td>
<td>0.1475</td>
<td>0.0042</td>
</tr>
<tr>
<td>CaCO₃</td>
<td>48.76</td>
<td>-0.5304</td>
<td>--</td>
<td>11.76</td>
<td>-0.3692</td>
<td>0.054</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Determined from the results of CULBERSON and PYTKOWICZ (1968).

From the work of INGLE (1975). Valid only at S = 35 and for calcite. The value of –ΔVₐ should be 2.8 cm³/mole lower and ΔKₐ can be assumed to be equal to ΔK_c.

The above results of the calculations of a different nature than those of the previous authors should be possible.

where ΔVₐ and ΔKₐ are the volume and compressibility change for the ionization. The values of ΔVₐ and ΔKₐ have been fit to equations of the form

\[-ΔV = a₀ + a₁(S - 34.8) + a₂t + a₃t^² \]  
\[-10^3ΔK = b₀ + b₁(S - 34.8) + b₂t + \ldots \]  

Coefficients of these equations generated from the results of CULBERSON and PYTKOWICZ (1968) are given in Table 5. Results obtained from the work of INGLE (1975) for the solubility of calcite are also given in Table 5.

As discussed elsewhere (MILLERO and BERNER, 1972; WARD and MILLERO, 1975) the values of ΔVₐ for the ionization of boric and carbonic acids determined from molal volume measurements are in good agreement with those generated from the work of CULBERSON and PYTKOWICZ (1968) (see Table 6). Values of ΔKₐ for the ionization of boric and carbonic acids determined from the measurements of CULBERSON and PYTKOWICZ (1968) are also in good agreement (Table 6) with the values calculated from partial molal compressibility data (MILLERO, 1979a). No attempt was made to force the seawater of AV and ΔKₐ to the pure water values because direct measurements are not available at low salinities and molal volume calculations (MILLERO et al., 1972; WARD and MILLERO, 1974) indicate that AVₐ and ΔKₐ are strongly dependent on concentration in dilute solutions. The coefficients a₁ and b₁ were obtained from S = 38.5°C and 13.5°C results of CULBERSON and PYTKOWICZ (1968).

The values of ΔVₐ and ΔKₐ for the solubility of calcite in seawater are in reasonable agreement with the values determined from molal volume data (MILLERO, 1976, 1979b). The value of –ΔVₐ = 31.8 ± 1.0 cm³/mol·¹ determined for foraminifera at 25°C by PYTKOWICZ and FOWLER (1967) is 5.0 cm³/mol·¹ higher than the value obtained by INGLE (1975). These differences are probably related to differences in the solid phases. The AVₐ for the solubility of aragonite should be 2.8 cm³/mol·¹ lower and ΔKₐ should be approximately equal to ΔK_c (CRC CHEMICAL HANDBOOK, 1975). The value of ΔVₐ determined in this manner does not agree with the measurements of INGLE (1973) and HAWLEY and PYTKOWICZ (1969) for oolites at 2°C. The work of Pytkowicz and coworkers yields a ΔV difference of 2.9 cm³/mol·¹ for foraminifera and oolites at 2°C while the work of Ingle yields a difference of 10.5 cm³/mol for calcite and oolites. Part of this difference could be due to the larger temperature coefficient found by INGLE (1975) for calcite compared with the value found by HAWLEY and PYTKOWICZ (1969) for oolites. Further measurements are needed to elucidate these apparent discrepancies. Until this is done the effect of pressure on the solubility of calcite and aragonite can be estimated from the results given in Table 5.

**CALCULATION OF PARAMETERS FOR THE CARBONATE SYSTEM IN SEAWATER**

The various components of the carbonate system in seawater can be characterized by measuring at least two of the following: pH, A₁, ΣCO₂ and PICO₂. The equations relating the selected parameters to the concentration of HCO₃⁻, CO₃²⁻, CO₂(H₂CO₃) are given by SKIRROW (1975). The classical way of determining the components of the carbonate system is to make measurements of the pH and total titration alkalinity, A₁, given by

\[ A₁ = [\text{HCO}_₃⁻] + 2[\text{CO}_₃²⁻] + [\text{B(OH)}₄⁻] \]

\[ \]}

\[ + [\text{OH}⁻] - [\text{H}⁺] + \sum B⁻ \]  

where [B(OH)₄⁻], [OH⁻], [H⁺] are, respectively, the total concentrations of borate ion, hydroxide ion and the proton, and \( \sum B⁻ \) represents other bases in seawater that can accept a proton (H₂PO₄²⁻, PO₄³⁻, etc., EDMOND, 1970). At the pH of natural seawater the carbonate alkalinity (A_c) can be determined from

\[ A_c = A₁ - [\text{B(OH)}₄⁻] = [\text{HCO}_₃⁻] + 2[\text{CO}_₃²⁻] \]  

The concentration of the borate ion can be determined from

\[ [\text{B(OH)}₄⁻] = Kₐ[B] / (Kₐ + a₀) \]  

where the total boron, \( [B] = 1.212 \times 10^² \) S (CULKIN, 1965). \( Kₐ \) is the apparent composition constant for boric acid, and \( a₀ \) is the apparent activity of the proton. From the pH and \( A_c \), the parameter of the
carbonate system are determined from

\[
[HCO_3^-] = A_c(1 + 2K_2/ao_t) \\
[CO_2^-] = A_cK_2'(ao_t + 2K_1) \\
[CO_3^{2-}] = (A_c ao_t/K_1)(1 + 2K_2/ao_t) \\
\Sigma CO_2 = [HCO_3^-] + [CO_2^-] + [CO_3^{2-}] = A_c(1 + ao_t/K_1 + K_2/ao_t)(1 + 2K_2/ao_t)
\]

(11) (12) (13) (14)

where the values of \( K_i \) are the apparent constants for carbonic acid. The values of \( K_i \) and \( ao_t \) used in these equations are for the \textit{in situ} temperature and pressure of sample. The effect of temperature, pressure and salinity on the apparent constants were given earlier. The effect of temperature, pressure and salinity on the pH needed to calculate \textit{in situ} values of \( ao_t \) can be determined in two ways: (1) an iterative computer technique, and (2) by using explicit function of pH as a function of S, T and P. The iterative method (Edmond and Gieskes, 1970) requires the solving of the cubic equation (Skirrow, 1975).

\[
ao_t^6 + ao_t^5[K'_1(A - 1) + K'_2(A - B)]/A + ao_t[K'_1K'_2(A - B - 1) + K'_1K'_3(A - 2)]/A + K'_1K'_2K'_3(A - 2)/A = 0
\]

which is derived by combining eqns (10) to (14). The values of A and B in eqn (15) are given by

\[
A = A_c/\Sigma CO_2 \\
B = [B]_c/\Sigma CO_2
\]

(16) (17)

Values of A and B are first estimated from eqns (8) to (12) using the measured \( ao_t \) and the \textit{in situ} apparent constants. Equation (15) is solved for \( ao_t \) using the solutions for a cubic equation (CRC Chemical Handbook, 1975) or by iterative solutions (Benn, 1970; Almgren et al., 1975). Once the \textit{in situ} \( ao_t \) is determined the various parameters of the carbonate system can be determined from eqns (8) to (12). Since many workers do not have computer systems available, we have generated values of pH from eqn (15) as a function of temperature, pressure and salinity. The values have been generated using the constants of Hansson and Mehrbach et al. The effect of temperature and pressure on the pHNSB is shown in Fig. 9. The effect of salinity on the temperature coefficients from \( S = 30 \) to \( 40^\circ \), is quite small and can be neglected. The least squares fit of the generated values of pH on the NBS and Tris scales are given by

\[
\text{NSB Scale} \\
\phi_h = \phi_{H_{25}} + Alt - 25t + Bt - 25 \quad (18)
\]

\[
10^A = -9.702 - 2.378(\phi_{H_{25}} - 8) + 3.885(\phi_{H_{25}} - 8)^2 \quad (18a)
\]

\[
10^B = 1.123 - 0.003(\phi_{H_{25}} - 8) + 0.933(\phi_{H_{25}} - 8)^2 \quad (18b)
\]

(\( \sigma = 0.002 \))

\[
\text{Tris Scale} \\
\phi_h = \phi_{H_{25}} + Alt - 25t + Bt - 25 \quad (19)
\]

\[
10^A = -9.296 + 32.505(\phi_{H_{25}} - 8) + 63.806(\phi_{H_{25}} - 8)^2 \quad (19a)
\]

\[
10^B = 3.916 + 23.000(\phi_{H_{25}} - 8) + 41.637(\phi_{H_{25}} - 8)^2 \quad (19b)
\]

(\( \sigma = 0.003 \))

where \( \phi_{H_{25}} \) is the measured pH at \( t = 25^\circ \). These equations are valid from \( t = 0 \) to 40, \( C = 30 \) to 40, and \( \phi_{H_{25}} = 7.6 - 8.2 \). The generated values of pH have also been used to relate the two scales at various temperatures

\[
\phi_{H_{25}} = \phi_{H_{25}} + A + Bt \quad (20)
\]

\[
10^A = 5.93 - 3.6(S - 35) \quad (20a)
\]

\[
10^B = 3.381 + 0.058(S - 35) \quad (20b)
\]

The differences between the two buffer scales are

<table>
<thead>
<tr>
<th>Solute</th>
<th>( -\Delta V \text{ (cm}^3\text{ mol}^{-1})</th>
<th>Meas</th>
<th>Calc</th>
<th>( -\Delta K \text{ (cm}^3\text{ mol}^{-1} \text{ atm}^{-1})</th>
<th>Meas</th>
<th>Calc</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(OH)(_2)</td>
<td>27.1</td>
<td>25.7</td>
<td></td>
<td>2.8</td>
<td></td>
<td>2.9</td>
</tr>
<tr>
<td>H(_2)CO(_3)</td>
<td>22.3</td>
<td>21.7</td>
<td></td>
<td>0.9</td>
<td></td>
<td>1.7</td>
</tr>
<tr>
<td>HCO(_3)</td>
<td>15.4</td>
<td>15.9</td>
<td></td>
<td>2.4</td>
<td></td>
<td>3.6</td>
</tr>
<tr>
<td>CaCO(_3)</td>
<td>35.5</td>
<td>39.7</td>
<td></td>
<td>2.5</td>
<td></td>
<td>8.2</td>
</tr>
</tbody>
</table>

* From Ward and Millero (1975). A higher value of \( -\Delta V = 26.4 \text{ cm}^3\text{ mol}^{-1} \) is obtained for 0.725 m NaCl.

* From Millero and Berner (1972) using \( f(H_2O) = 18.1 \text{ cm}^3\text{ mol}^{-1} \) instead of the value of 18.8 used in the original paper.

* From Millero and Berner (1972) using \( f(CO_2^-) = 19.2 \text{ cm}^3\text{ mol}^{-1} \) (Dudell, 1972).

* From K data of Millero (1979a).

* From Millero (1976).
Table 7. Comparisons of the pH$_{bas}$ and pH$_{Tris}$ scales in seawater

<table>
<thead>
<tr>
<th>Temp</th>
<th>S = 30</th>
<th>S = 25</th>
<th>S = 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°C</td>
<td>0.073</td>
<td>0.057</td>
<td>0.038</td>
</tr>
<tr>
<td>5</td>
<td>0.092</td>
<td>0.077</td>
<td>0.059</td>
</tr>
<tr>
<td>10</td>
<td>0.109</td>
<td>0.095</td>
<td>0.079</td>
</tr>
<tr>
<td>15</td>
<td>0.125</td>
<td>0.113</td>
<td>0.099</td>
</tr>
<tr>
<td>20</td>
<td>0.141</td>
<td>0.130</td>
<td>0.117</td>
</tr>
<tr>
<td>25</td>
<td>0.155</td>
<td>0.147</td>
<td>0.136</td>
</tr>
<tr>
<td>30</td>
<td>0.170</td>
<td>0.162</td>
<td>0.153</td>
</tr>
<tr>
<td>35</td>
<td>0.184</td>
<td>0.178</td>
<td>0.169</td>
</tr>
<tr>
<td>40</td>
<td>0.198</td>
<td>0.192</td>
<td>0.185</td>
</tr>
</tbody>
</table>

$\Delta$pH = pH$_{bas}$ - pH$_{Tris}$

$^\dagger$ Experimentally measured by HANSSON.

given in Table 7. The calculated value of pH$_{bas}$ - pH$_{Tris}$ = 0.147 at 25°C and S = 35 is in reasonable agreement with the directly measured values of 0.159 by HANSSON (1973). Further measurements as a function of salinity and temperature are needed to confirm eqn (20). The effect of applied pressure on the pH (NBS scale) of seawater is a linear function

$$\text{pH}^\circ = \text{pH}^0 + \Delta P$$  (21)

The slopes are functions of t and S. A least squares fit of the generated values of pH$^\circ$ - pH$^0$ give

$$-10^3A = 0.424 - 0.0048(S - 35) - 0.00282t - 0.0816(\text{pH}^0 - 8)$$  (21a)

which is valid from S = 32-38 and t = 0-25°C.

A comparison of the various parameters of the carbonate system calculated from the apparent constants based on the two pH scales are given in Table 8. These calculations were made at $t = 25°C$ and $S = 35$ for waters of the same $A_c = 2.400$ m-equiv kg$^{-1}$ and $\Sigma CO_2 = 2.111$ mmol kg$^{-1}$. The differences of $K_b$ determined from the results of LYMAN (1957) and HANSSON (1973) cause a difference in [B(OH)$_2^+$] of 0.011 m-equiv kg$^{-1}$. This difference is larger than what one would expect (0.004) from the standard errors in ln $K_b$. Until these discrepancies are clarified, it is not possible to determine the carbonate alkalinity, $A_c$, to better than $\pm 0.01$ m-equiv kg$^{-1}$. The differences in [HCO$_3^-$] and [CO$_3^{2-}$] are 0.010 mmol kg$^{-1}$. For waters with the same $A_c$ (the values in parentheses in Table 8) the difference in [HCO$_3^-$] increases to 0.019 mmol kg$^{-1}$, while the difference in [CO$_3^{2-}$] is nearly the same (0.009 mmol kg$^{-1}$). These differences in [HCO$_3^-$] and [CO$_3^{2-}$] are about what one would expect from the standard errors obtained from the constants of HANSSON (1972) and MEHRBACH et al. (1973). The calculated values of [CO$_2^+$] and $P_{CO_2}$ using the two sets of apparent constants are in good agreement and within the expected errors due to the standard errors of $K_b$ and $K_{eq}$. These calculations indicate that the presently available apparent constants expressed on the NBS and Tris pH scales cause maximum differences of $A_c = 0.011$ m-equiv kg$^{-1}$, $\delta[HCO_3^-] = 0.019$ mmol kg$^{-1}$, $\delta[CO_3^{2-}] = 0.010$ mmol kg$^{-1}$, $\delta[CO_2^+] = 0.0001$ mmol kg$^{-1}$, and $\delta P_{CO_2} = 4 \times 10^{-5}$ atm. It is interesting to compare the errors due to the apparent constants to those obtained from the experimental errors in pH, $A_c$, and $\Sigma CO_2$. The precision of the measurements of these parameters are $\pm 0.003$ in pH (PYTKOWICZ et al., 1966; ZIRINO, 1975); $\pm 0.002$ m-equiv kg$^{-1}$ in $A_c$ (HANSSON, 1972; ALMGREN and FONSELUS, 1976) and $\pm 0.003$ mmol kg$^{-1}$ in $\Sigma CO_2$ (HANSSON, 1972; ALMGREN and FONSELUS, 1976). The errors of various parameters of the carbonate system due to experimental measurements of pH and $A_c$, $A_c$, and $\Sigma CO_2$ are given in Table 9. The uncertainty in the various carbonate parameters due to experimental errors are within the errors due to the apparent constants, except for the errors in $P_{CO_2}$ (due to errors in $\Sigma CO_2$). If one set of constants are used the errors become comparable except for HCO$_3^-$ and CO$_3^{2-}$. To improve our ability

Table 8. Comparisons of the parameters of the carbonate system determined at 25°C and S = 35 using various pH scales

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NBS Scale</th>
<th>Tris Scale</th>
<th>$\Delta \delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$, m-equiv kg$^{-1}$</td>
<td>2.400</td>
<td>2.400</td>
<td>0</td>
</tr>
<tr>
<td>$\Sigma CO_2$, 10$^6$ mmol kg$^{-1}$</td>
<td>2.111</td>
<td>2.111</td>
<td>0</td>
</tr>
<tr>
<td>pH</td>
<td>8.151</td>
<td>8.007</td>
<td>0.144</td>
</tr>
<tr>
<td>B(OH)$_2^+$</td>
<td>0.095 ± 0.003</td>
<td>0.064 ± 0.001</td>
<td>0.011 ± 0.003</td>
</tr>
<tr>
<td>$A_c$, m-equiv kg$^{-1}$</td>
<td>2.305 ± 0.003</td>
<td>2.316 ± 0.001</td>
<td>0.011 ± 0.003</td>
</tr>
<tr>
<td>[HCO$_3^-$], mmol kg$^{-1}$</td>
<td>1.890 ± 0.011</td>
<td>1.880 ± 0.011</td>
<td>0.010 ± 0.018</td>
</tr>
<tr>
<td>[CO$_3^{2-}$], mmol kg$^{-1}$</td>
<td>0.208 ± 0.005</td>
<td>0.217 ± 0.007</td>
<td>0.009 ± 0.009</td>
</tr>
<tr>
<td>[CO$_2^+$], mmol kg$^{-1}$</td>
<td>0.01340 ± 0.0002</td>
<td>0.01347 ± 0.00004</td>
<td>0.00003 ± 0.00004</td>
</tr>
<tr>
<td>$P_{CO_2}$, 10$^6$ atm</td>
<td>472 ± 7$^a$</td>
<td>471 ± 16</td>
<td>1 ± 16</td>
</tr>
</tbody>
</table>

$^a$ The errors are only related to the errors in the apparent constants.

$^b$ The ± deviations have been determined from $(\delta A_c + \delta \delta)^{1/2}$.

$^c$ Calculated using the Henry's law a from WISS (1974).
Table 9. Comparisons of the experimental errors in the values of parameters for the carbonate system with errors due to the apparent constants

<table>
<thead>
<tr>
<th>Parameter</th>
<th>From pH and $A_1$ $\Delta pH^a$</th>
<th>From $A_1$ and $\Sigma CO_2$ $\Delta A_1^a$</th>
<th>From constants$^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[OH$^-$, m-equiv kg$^{-1}$]</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.001</td>
</tr>
<tr>
<td>$A_1$, m-equiv kg$^{-1}$</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>$[HCO_3^-]$, mmol kg$^{-1}$</td>
<td>0.002</td>
<td>0.001</td>
<td>0.0005</td>
</tr>
<tr>
<td>$[CO_3^{2-}]$, mmol kg$^{-1}$</td>
<td>0.002</td>
<td>0.001</td>
<td>0.0005</td>
</tr>
<tr>
<td>$P_{CO_2}$, 10$^3$ atm</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$\Sigma CO_2$, 10$^6$ mmol kg$^{-1}$</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>$\Delta pH$</td>
<td></td>
<td>0.0003</td>
<td>0.005</td>
</tr>
</tbody>
</table>

$^a$ Error due to $\Delta pH = \pm 0.003$ (pH = 8.15 and $A_1 = 2.4$).
$^b$ Error due to $\Delta A_1 = \pm 0.002$ (pH = 8.15 and $A_1 = 2.4$).
$^c$ Error due to $\Delta CO_2 = \pm 0.003$ ($A_1 = 2.4$ and $\Sigma CO_2 = 2.111$).
$^d$ The maximum errors related to the uncertainty of the apparent constants on NBS and Tris scales.

To summarize, I have attempted to provide a statistically valid representation of the apparent ionization constants of water, boric and carbonic acids, and the solubility of CaCO$_3$ in seawater as a function of $t$, $S$ and $P$. At present, I feel that the apparent constants on the NBS pH scale (LYMAN, 1957; MEHRBACH et al., 1973; INGLE et al., 1975; CULBERSON and PYTKOWICZ, 1968) should be used to characterize the parameters of the carbonate system in seawater. Further measurements for all of the constants are needed below $S = 20^\circ_0$ as well as measurements for $K_p$ from $S = 20-40^\circ_0$, on the NBS scale. High pressure measurements on the solubility of various forms of CaCO$_3$ are also needed.

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REFERENCES


The thermodynamics of the carbonate system in seawater


