Magnetospheric and Interplanetary Electrostatics: A Simple but Explicit Model

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**14. ABSTRACT (Continue on reverse side if necessary and identify by block number)**
This is a generalization of Dungey's 1961 model for the prototype of an "open" magnetosphere. The magnetic-field lines are traced by superimposing a uniform B-field parallel to the geomagnetic dipole axis. There results a quasi-cylindrical separatrix (the "magnetopause") between field lines that intersect the earth and those that do not. However, the magnetic field outside the magnetosphere can be multiplied by an arbitrary factor (positive or negative) without altering the field-line geometry. Any discontinuity in B at the "magnetopause" is supported by azimuthal currents there. The electrostatic problem is...
asymptotically analogous to that of a dielectric cylinder in an asymptotically uniform electric field (which owes its origin to the solar wind in this case). It is easy to map from the asymptotic solution inward and equatorward, since the magnetic field lines are treated as electric equipotentials. The tangential component of the electric field is continuous at the magnetopause but varies between limits that correspond to the open and closed models of the magnetosphere, as defined by the access of interplanetary plasma.
Dungey (1961) and Stern (1973; 1977) have shown how the reconnection of magnetospheric B-field lines to "moving" interplanetary magnetic-field lines can generate a cross-tail electric field and thus a magnetospheric electric field that is consistent with the commonly inferred plasma-convection pattern. Alfvén (1975) has emphasized instead the mapping of an interplanetary electric field into the magnetosphere to produce equivalent results that are possibly easier to calculate. The purpose of the present work is to generalize somewhat the concepts put forward by the above authors and to provide an explicit mapping of the interplanetary, cross-tail, and magnetospheric electric fields for a simple model of the magnetospheric B field, viz., a modest generalization of that introduced by Dungey (1961).

The presence of magnetic reconnection at the magnetopause distinguishes the "open" model of the magnetosphere from the "closed" model. Interplanetary field lines in the "closed" model are draped around an identifiable magnetospheric surface and slide over it, so that none can be traced to an intersection with the earth. Conversely, tail field lines in the "open" model ultimately become interplanetary field lines, with the consequence that those emanating from the appropriate polar cap of the earth can be traced to an intersection with the sun.

Although the foregoing discussion tends to emphasize the tracing of magnetic field lines, the difference of major dynamical significance between the "closed" and "open" models involves the mapping of interplanetary and magnetospheric electric fields. A certain model magnetosphere introduced by Dungey (1961), although unrealistic in certain respects, serves to illustrate important relationships in the electrostatic phenomenology of reconnection. The model magnetosphere is based on the expression

$$B = - g_0 \nabla \left[ \left( \frac{a^3}{r^2} \right) \cos \theta - \left( \frac{a^3 r^2}{2 b^3} \right) \cos \theta \right],$$

(1)
in which $a$ is the planetary radius, $a^3 g_0$ is the planetary magnetic moment, $r$ is the distance from the center of the planet, $\theta$ is the colatitude, and $b$ is the radius of the circle of vanishing $B$ that is found in the equatorial ($\theta = \pi/2$) plane of this model field. The circular neutral line of the model field corresponds (in some sense) to the dayside neutral points and nightside field cusp characteristic of real magnetospheres. Moreover, the neutral line serves to separate field lines belonging to the three classical topologies: closed, tail, and interplanetary (Vasyliunas, 1975). The equation of a field line derived from (1) is such that
\[ (r/\sin^2 \theta) \left[ 1 + \frac{1}{2} (r/b)^3 \right]^{-1} = \text{constant}. \]  

Convenient labels for field lines are derived from values of the constant ascertained by taking the limits \( r \rightarrow 0 \) and \( z = r \cos \theta \rightarrow -\infty \). One thus identifies either an \( L \) parameter (motivated by the similarity of \( B \) to a dipole field for \( r \ll b \)) or a value of \( \rho_\infty \) (asymptotic distance from the field axis):

\[ \frac{r/\sin^2 \theta}{1 + \frac{1}{2} (r/b)^3} = \begin{cases} L_a, & r \rightarrow 0 \\ 2b^3/\rho_\infty^2, & z \rightarrow -\infty \end{cases} \]

Since both labels are valid simultaneously for tail field lines, one finds that \( L = 2b^3/\rho_\infty^2 a \) for these. The separatrix bears the field-line labels of the circle \( r = b \) on the equatorial (\( \theta = \pi/2 \)) plane, viz., \( L^* = 2b/3a \) and \( \rho_\infty^* = \sqrt{3} b \). This corresponds to the magnetopause and nightside neutral sheet.

The foregoing development follows that of Dungey (1961), as elaborated by Stern (1973), Hill and Rassbach (1975), and Alfvén (1975). To these various authors it seemed natural to identify the tail field \( B_t = (a/b)^3 g^0_1 \) with the interplanetary magnetic field \( B_i \). This would be a valid identification if one took (1) to apply everywhere. However, it is clear that the configuration of interplanetary field lines would be unaffected if one assigned an entirely different value of \( g^0_1 \) to field lines that do not intersect the planet. Thus, it is entirely proper to derive the magnetospheric field from (1) and the interplanetary field from the expression

\[ \mathbf{B} = -\mathbf{g}^0_1 \nabla \left[ \frac{(b^3/r^2) \cos \theta - r \cos \theta}{r} \right] \]

with no expectation that \( \mathbf{g}^0_1 = (a/b)^3 g^0_1 \) nor even that \( \mathbf{g}^0_1 \) and \( g^0_1 \) have the same sign. This is the course pursued in the present work, which otherwise constitutes an implementation of the mapping proposed by Alfvén (1975), at least in certain limits. Any resulting discontinuity in \( \mathbf{B} \) can be supported by an azimuthal (\( \varphi \)) current flowing on the magnetospheric surface. Thus, it is natural to identify \( B_\varphi = \mathbf{g}^0_1 \) and \( B_r = (a/b)^3 g^0_1 \) with no expectation that the interplanetary field and tail field are equal, nor even of the same sign (Schulz, 1976).

It is convenient to postulate an asymptotically uniform (cf. Alfvén, 1975) interplanetary electric field \( \mathbf{E}_i = -\nabla \mathbf{B}_i \) as \( \rho \rightarrow -\infty \) (\( \mathbf{E}_i \) being the asymptotic solar-wind velocity) and an asymptotically uniform cross-tail electric field \( \mathbf{E}_t = -\mathbf{E}_t \) as \( z \rightarrow -\infty \) for \( \rho_\infty < \rho_\infty^* \), where \( y = r \sin \theta \sin \varphi \). These postulates are made consistent with Maxwell's equations by introducing the electrostatic potentials

\[ V = \mathbf{E}_i \rho_\infty \left[ 1 - (\rho_\infty^*/\rho_\infty)^2 \left( 1 - (\mathbf{E}_t/\mathbf{E}_i) \right) \right] \sin \varphi \]

for interplanetary field lines (\( \rho_\infty > \rho_\infty^* \)).

\[ V = \mathbf{E}_t \rho_\infty \sin \varphi = \mathbf{E}_t b (2b/L_a)^{1/2} \sin \varphi = \mathbf{E}_t b (3L^*/L)^{1/2} \sin \varphi \]
for tail field lines \( (p_\infty < p_\infty^*, \ L > L^*) \), and

\[
V = (3)^{1/2} E_t b (L/L^*)^{n+1} \sin \varphi
\]  

(7)

for closed field lines \( (L < L^*) \). The forms of (6) and (7) were suggested by Volland (1975) under the assumption that \( B \) was dipolar. The present magnetospheric model (cf. Hill and Rassbach, 1975) offers a rationale for the auroral discontinuity of \( E = -\nabla V \) at \( L = L^* \). The parameter \( n \) in (7) is arbitrary, but Volland (1975) recommends the value \( n = 1 \). The value \( n = 0 \) had been introduced by Brice (1967) and used by many subsequent investigators.

The particular form of (5) is dictated by (a) the asymptotically cylindrical geometry attained as \( z \to \infty \) and (b) the requirement that the tangential \( (\varphi) \) component of \( E = -\nabla V \) be continuous across the magnetopause in a static model. The other tangential component \( (\hat{B} \cdot \hat{E}) \) is also continuous; in fact, it vanishes everywhere since \( V \) is a function only of the field-line labels \( (L, p_\infty, \varphi) \). Kennel and Coroniti (1975) define the reconnection efficiency \( \epsilon \) as \( \epsilon = E_\varphi/E_t \). The relevance of this definition is well illustrated by (5). For \( \epsilon = 0 \) the tangential component of \( E \) vanishes, and so there is no transport of interplanetary plasma \( (\text{via} \ E \times \hat{B} \text{ drift}) \) into the tail of the magnetosphere. This case corresponds to the "closed" model, since the drift of plasma attached to interplanetary field lines is diverted around the magnetopause. For \( \epsilon = 1 \) the interplanetary electric field is "uniform" even up to the magnetopause itself. There is no diversion of the drift of plasma attached to interplanetary field lines, and so the interplanetary plasma drifts quite freely across the magnetopause. This case corresponds to the "open" model of the magnetosphere and illustrates quite well the definition of reconnection proposed by Vasyliunas (1975) as "the process whereby plasma flows across a surface that separates regions containing topologically different magnetic field lines." No reference to the "motion" or to the "breaking open" of such field lines is really necessary (Alfvén, 1975).

The above equations admit intermediate values \( (0 < \epsilon < 1) \) of the reconnection efficiency and thus permit a continuous transition between the "closed" and "open" models. The equations also admit values of \( \epsilon \) that lie outside this range. However, values of \( \epsilon > 1 \) tend to focus the flow of interplanetary plasma, thus diverting the drift velocity toward normal incidence on the magnetopause. Values of \( \epsilon < 0 \) create a "forbidden zone" outside the magnetosphere, a buffer zone that truly interplanetary plasma is not allowed to enter. The present model is not a dynamical model but a phenomenological one. The dynamically unrealistic possibilities \( \epsilon > 1 \) and \( \epsilon < 0 \) should probably be excluded from the model on physical grounds. It is interesting that the case \( \epsilon = 0 \) would likely correspond to the case of a northward interplanetary \( \hat{B} \) field outside the earth's magnetosphere, since it is in this case that the interplanetary electric field \( E_i = -(1/c) u \times \hat{B} \) would be directed from dusk toward dawn. Perhaps one should set \( \epsilon = 0 \) in this case. The equations proposed in the present work assure in all cases a continuous \( E_t \) at the magnetopause. However, the normal component of the drift velocity (and therefore the plasma density) may well be discontinuous across the magnetopause, since there is no requirement between (1) and (4) that the tail field and the interplanetary \( \hat{B} \) field be equal in magnitude or direction there.
Figure 1 (not cited in text). Meridional (noon-midnight) section of magnetospheric model used here (see also Dungey, 1961; Stern, 1973; Alfvén, 1975; Hill and Rassbach, 1975; Schulz, 1976) for mapping $E$ and $B$. Open field lines correspond to $L > L^*$. The earth's angular velocity $\Omega$ and the azimuthal ($\varphi$) component of $E$ are directed as shown.
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