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DAAK30-78-C-0022

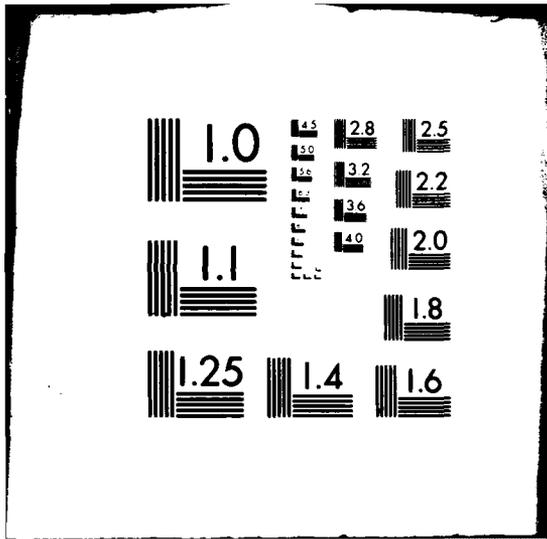
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COUNTERWEAPON  
SYSTEM DEVELOPMENTS  
IN TERMS OF CONTRIBUTIONS  
TO FORCE EFFECTIVENESS**

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R. L. Farrell

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## PREFACE

This is the final report of Vector Research, Incorporated, (VRI) efforts under contract number DAAK30-78-C-0022. Under this contract, VRI designed methods for the comparative analysis of weapon system developments which could be used in analyzing counterweapon designs. The methods were demonstrated with analyses of artillery-related problems. The work was conducted for and in conjunction with the Battlefield Systems Integration Directorate (DBSI) of USADARCOM. The project staff are particularly grateful for the guidance and participation of Dr. Gerald Andersen of DBSI, who acted as contract monitor, and for the support and assistance in artillery-related areas which was provided by LTC William Breen, also of DBSI.

The *methods and conclusions* of this report are those of the author and project staff, and do not represent the conclusions of DBSI, USADARCOM, or any other US Government agency.

## 1.0 INTRODUCTION AND BACKGROUND

### 1.1 Introduction

In order to compare the relative value of weapon system developments that is, changes in weapon system designs - analysts require methods to evaluate the potential costs of alternative developments and their potential contributions to some overall measures of value. Exhibit 1-1 shows conceptually the information which the analyst wishes to develop.<sup>1</sup> For many weapon system developments, it is convenient to consider overall value to be some measure of resulting force effectiveness. The exceptions are typically developments designed to affect future force costs.<sup>2</sup> The work described here is restricted to the analysis of developments affecting force effectiveness.

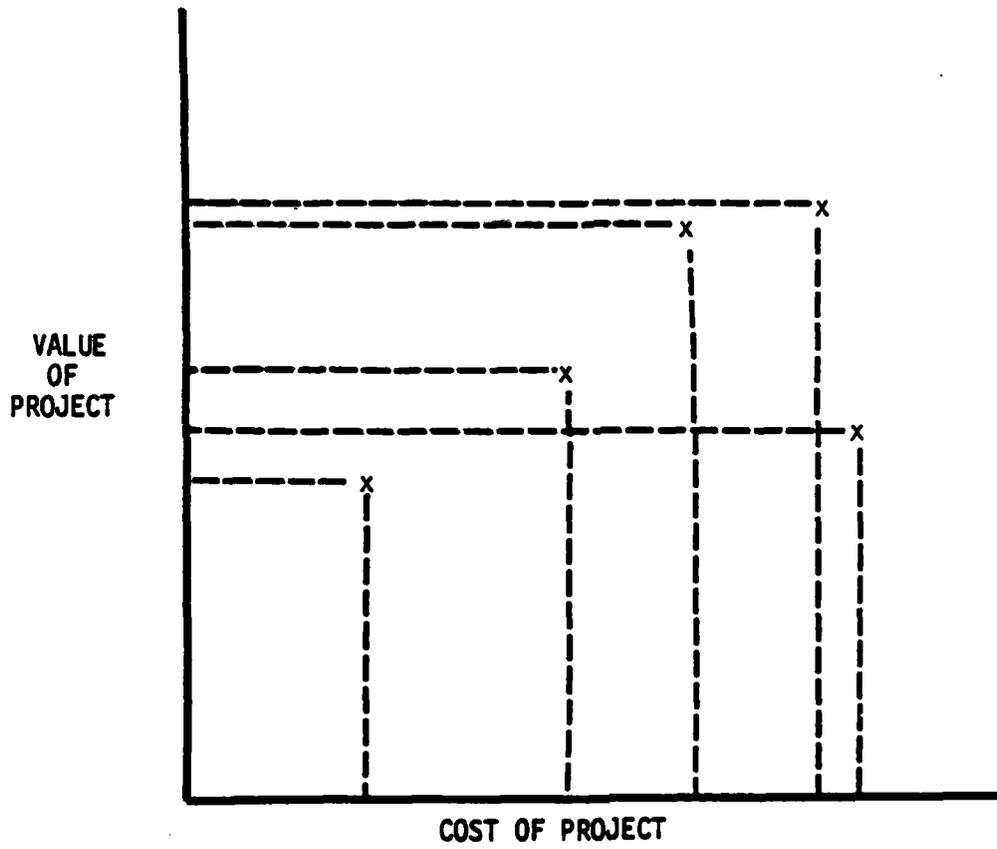
To give a concrete example of the type of problem with which such an analysis is concerned, consider an analyst asked to provide information on the relative cost and values of:

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<sup>1</sup>Exhibit 1 and all of the discussions of this report omit any explicit treatment of the stochastic nature of the development programs. In fact, development programs under analysis typically have only estimated results, and may have significant uncertainties concerning both their costs and their effects on weapon performance. This problem can be treated analytically (see [VRI, 1976] and [U/M, 1971]), but such treatment is not relevant to the particular portions of the overall analysis problem examined in this report and the potential complexities which may arise with information on alternative developments are omitted here for clarity of presentation.

<sup>2</sup>An additional minor set of development programs may have other goals including, for example, control of environmental contaminants, energy consumption, noise, or safety of peacetime forces.

EXHIBIT 1-1: INFORMATION REQUIREMENTS  
IN ANALYZING WEAPONS PROJECTS



- (1) two projects to improve meteorological information for artillery forces,
- (2) a project to improve the warhead lethality for some tank rounds,
- (3) a project to improve the armor protection of TOW vehicles, and
- (4) an increase in the number of DRAGONS fielded with mechanized infantry forces.

These are only examples of the kinds of alternatives for which defense analysts must attempt to provide understandable, comprehensive supporting information which will allow management decisions on program priorities and budgets. A clear presentation of the probable costs and results in terms of force effectiveness would provide significant support to the overall decision making process.

Force effectiveness is a complex concept having no one best measure for its quantification in formal analyses. Examination of many proposed weapons system developments, however, shows that a high percentage of potential developments are designed to have their principal impacts on force effectiveness through effects on the pattern in which weapon system attrition would be exchanged by opposing forces in a potential combat situation. Thus, a weapon system development may be designed to:

- (1) provide increased firepower which increases this weapon's attrition of enemy weapon systems,
- (2) provide decreased vulnerability which decreases the enemy weapon's attrition of this weapon, and
- (3) provide suppression or other synergistic effects which enhance *other* friendly systems' firepower or decrease their vulnerability, or which decreases the effective firepower of enemy systems or increases their vulnerability in relation to *other* friendly systems.

The methods of this project are designed for, and restricted to, the comparative evaluation of force changes designed to affect force effectiveness through such attrition-oriented results.

In a total methodology to compare such potential developments, three types of analyses are necessary:

- (1) cost and engineering analyses of particular developments, designed to provide the basic data concerning the feasibility, potential costs, and effects on basic system performance characteristics of alternative development efforts,
- (2) system- and small-force-level effectiveness analyses designed to relate the potential changes in weapon performance to the resulting changes in operational system firepower and/or vulnerability for this or other systems, and
- (3) force effectiveness analysis, relating the resulting changes in attrition effects to overall force effectiveness.

This decomposition of the overall analysis is shown in exhibit 1-2. As exhibit 1-3 goes on to show, the analysis tools and methods used in the three areas have differing degrees of generality. Cost and engineering analysis methods and models are typically very system-specific, and only the most abstract principals of cost accounting and attribution and the most basic engineering fundamentals will be used in common across the analyses of different developments. In system- and small-force-level effectiveness, a greater degree of methodological commonality exists, but even here, commonality is limited to classes of basically similar systems, such as mechanized or armored combat systems, artillery systems, aircraft weapon systems, logistic systems, etc. At the highest level of aggregation,

EXHIBIT 1-2: SCHEMATIC ANALYSIS STRUCTURE

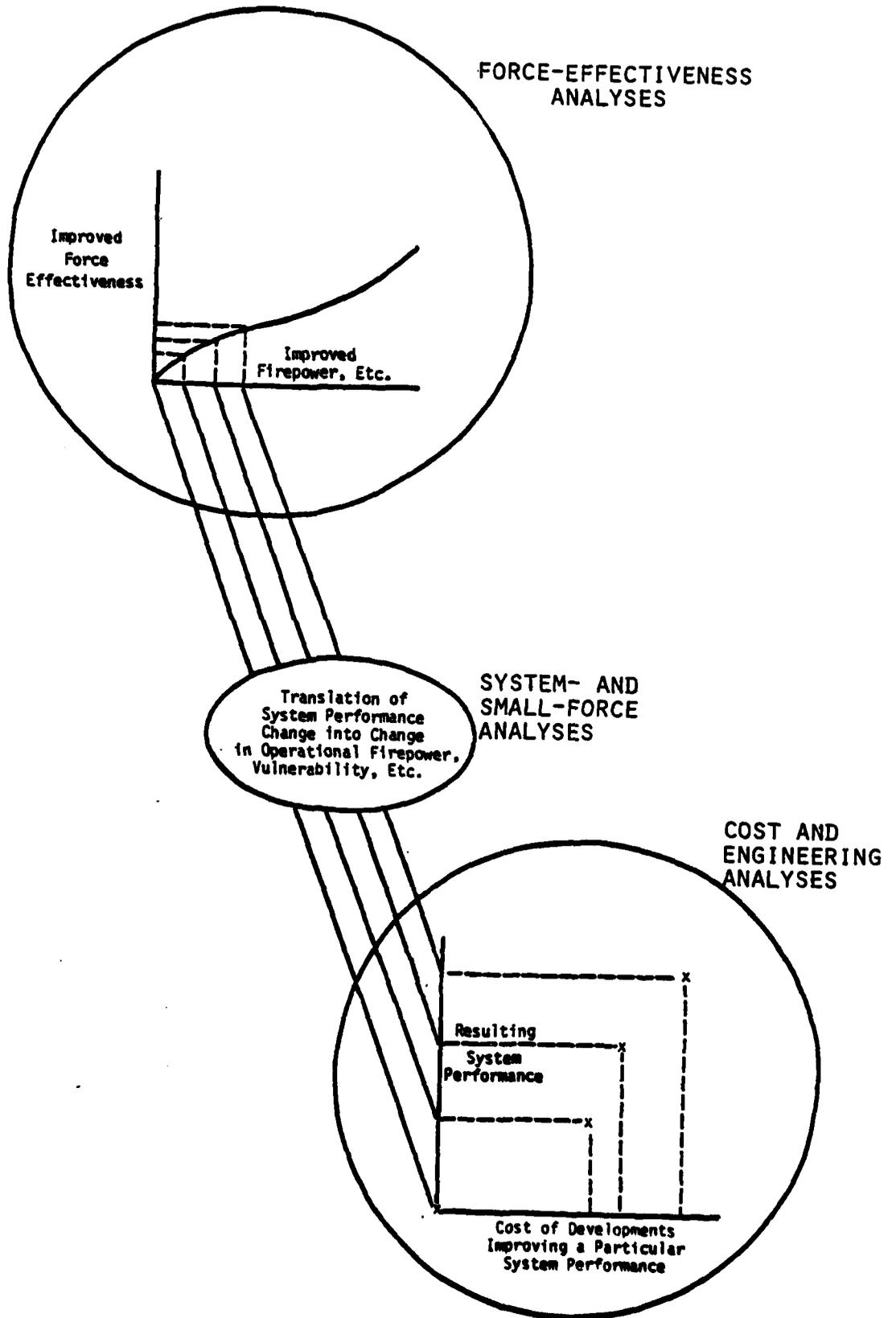
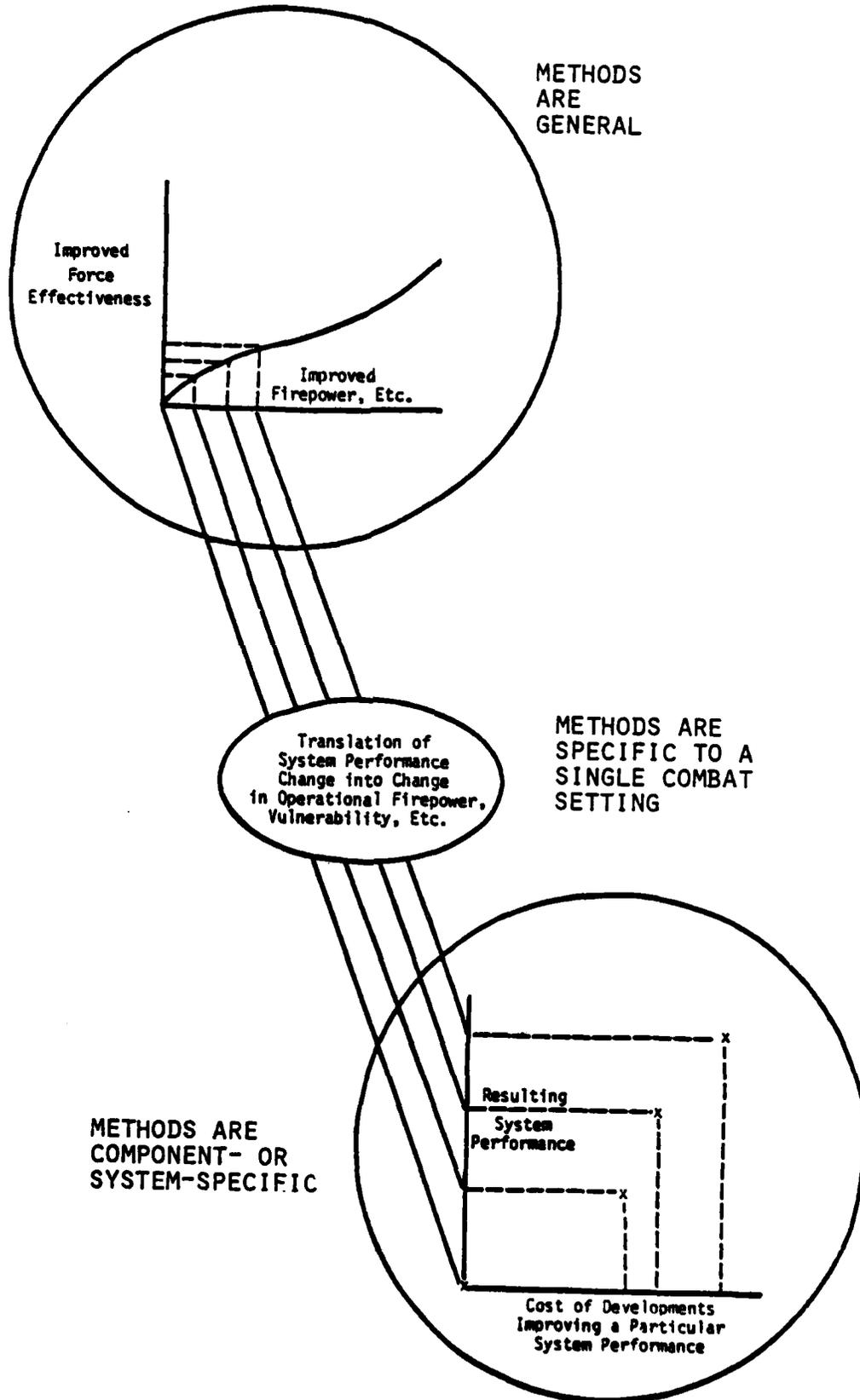


EXHIBIT 1-3: GENERALITY OF METHODOLOGIES



force-effectiveness-level analyses of weapon developments can share a single methodology, at least for ground and air tactical systems.

The research project reported in this document is the second of two projects intended to develop and demonstrate with parametric examples such a uniform method of performing force-effectiveness comparisons among alternative weapon system development projects.<sup>1</sup> In the first project, methods were designed and demonstrated which were suitable for the analysis of the highly-interactive weapons of the maneuver unit central duel [VRI, 1977a]. These methods were not fully applicable to problems involving artillery or aircraft systems for reasons that will be described in some detail in the remainder of this chapter. This project was undertaken to design extensions to the original methods which could be applied to both central duel systems and counterweapons - that is, weapons which attrit central duel systems but are not significantly attrited by them.

Of course, many detailed methodological tools, principally combat simulations or field tests, experiments, or exercises, exist to deal with force effectiveness analysis. However, the detailed analysis of the potential contribution of a developmental military system requires major commitments of time and resources. For many purposes during the decision-making processes involved in the design and modification of an overall Army development program, it is important to examine potential contributions of

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<sup>1</sup>Specifically, as described above, among development projects intended to impact force effectiveness rather than only future force costs or other measures, and within this class, among those developments intended primarily to affect the exchange of attrition in a potential combat.

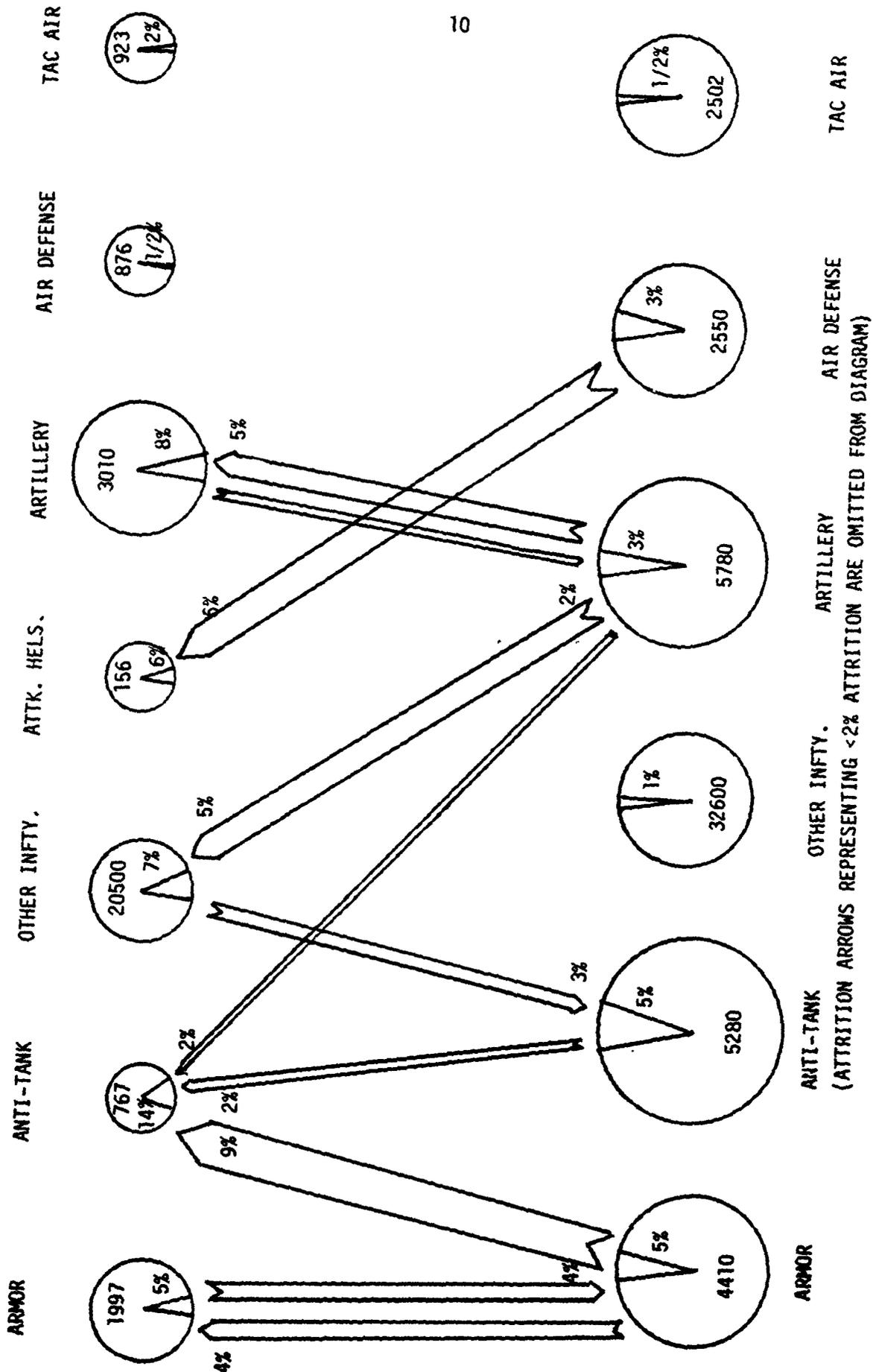
conceptual developments in significantly reduced times and with limited resources. The methods studied in this research have been designed to complement detailed studies with a simple, flexible, methodology which can use as inputs such data from detailed studies as the "killer-victim scoreboard" reporting the systems attrition which might be expected in hypothetical battles or sequences of battles, and which can combine such data with systematic analytic parametric or judgmental inputs as to the possible firepower performance areas as acquisition rates or probabilities, accuracy of fire, rate of fire, and lethality of fire, and such weapon survivability areas as enemy acquisition rates or probabilities, accuracy of enemy fire, rate of enemy fire, and vulnerability to enemy fire in order to extrapolate and/or bound the probable contribution of such changes to force effectiveness. The methods are not intended to replace or compete with more detailed analysis methodologies for accuracy, but to provide quick, limited answers to questions.

### *1.2 Structures for Representing Attrition*

In order to further describe the specific work of this research project, it is necessary to review the work of its predecessor. As described above, the two projects together are designed to provide methods to perform force-effectiveness comparisons of development projects for which system-level (or small-force level) analyses have provided information on the effects which the new system development would have in terms of potential changes in the pattern of attrition exchange in a combat action.

A graphical presentation of what is meant by a pattern of attrition exchange is shown in exhibit 1-4. Here, various weapon types are represented by nodes (circles) of a directed graph. One force is at the top (generally the friendly force) and its opponent is at the bottom. The force strengths of the various weapons - that is, the numbers of each weapon type present and participating in a (possibly hypothetical) combat - are shown by a numerical value in or just outside the appropriate circle (above for friendly forces, below for enemy). A wedge is shown in each circle to represent the total losses (attrition) of that weapon in the combat. Each wedge is labelled with the percent losses of the weapon system involved. The various circles are connected by arrows running upwards or downwards to portray individual attrition relations. Each arrow represents the attrition caused to a single kind of weapon system target by a single type of firer. The arrow may be labelled with the number of targets attrited or with the percent of the target total strength attrited by this type of firer. The precise graphical presentation and format is not critical, but the sample display is designed to show what is meant by a pattern of attrition: it is the complex of data described in such a display.

EXHIBIT 1-4: EXAMPLE ATTRITION PATTERN \*



ARMOR ANTI-TANK OTHER INFNTY. ARTILLERY AIR DEFENSE TAC AIR  
 ANTI-TANK OTHER INFNTY. ARTILLERY AIR DEFENSE TAC AIR  
 (ATTRITION ARROWS REPRESENTING <2% ATTRITION ARE OMITTED FROM DIAGRAM)

An alternative presentation is the tabular killer-victim scoreboard (KVS), shown schematically in exhibit 1-5. All the values in either form are assumed to be the mean values obtained in a series of similar combats differing only due to random effects. The statistical problems associated with the measurement of such data were not addressed in this project. This presentation contains the equivalent information to the graphical display, but it is presented in a tabular or matrix pattern of attrition, in the sense that it has a unique pattern of total force strengths and consequent losses.

An alternative presentation of an attrition pattern is created by reducing the actual attrition to attrition "rates". This may be done in several ways:

- (1) each KVS entry  $s_{ij}$  ( $s_{ji}$  for Red firing at Blue) may be divided by the total firer strengths  $m_i$  (or  $n_j$ ),
- (2) each KVS entry  $s_{ij}$  ( $s_{ji}$  for Red firing at Blue) may be divided by the average of the total (initial) firer strength and the final firer strength  $m_i - 1/2 \sum_j s_{ji}$ , and
- (3) each KVS entry  $s_{ij}$  ( $s_{ji}$  for Red firing at Blue) may be divided by the average number of firers in the combat, with the average taken continuously through time.<sup>1</sup>

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<sup>1</sup>On the assumption that the battle evolves according to a non-time-homogeneous, non-weapon-homogeneous form of the Lanchester square law differential equations, this third solution is the Lanchester attrition rate. Further, it can be obtained by a sequence of successive approximations of which the first two attrition rate forms are the first two approximations.

## EXHIBIT 1-5: KILLER-VICTIM SCOREBOARD TABLEAU

## RED KILLER/BLUE VICTIM SCOREBOARD (By weapon type)

## KILLER

Victim	type = $R_1$ # = $n_1$	type = $R_2$ # = $n_2$	...	type = $R_N$ # = $n_N$
type = $B_1$	$S'_{11}$ = # kills of $B_1$ by fires from $R_1$	$S'_{21}$ = # kills of $B_1$ by fires from $R_2$	...	$S'_{N1}$ = kills of $B_1$ by fires from $R_N$
type = $B_2$	$S'_{12}$ = # kills of $B_2$ by fires from $R_1$	$S'_{22}$ = # kills of $B_2$ by fires from $R_2$	...	$S'_{N2}$ = # kills of $B_2$ by fires from $R_N$
...	...	...	...	...
type = $B_i$	$S'_{1i}$ = # kills of $B_i$ by fires from $R_1$	$S'_{2i}$ = # kills of $B_i$ by fires from $R_2$	...	$S'_{Ni}$ = # kills of $B_i$ by fires from $R_N$
...	...	...	...	...
type = $B_M$	$S'_{1M}$ = # kills of $B_M$ by fires from $R_1$	$S'_{2M}$ = # kills of $B_M$ by fires from $R_2$	...	$S'_{NM}$ = # kills of by fires from $R_N$

-- Continued on next page--

EXHIBIT 1-5: KILLER-VICTIM SCOREBOARD TABLEAU  
(Concluded)

BLUE KILLER/RED VICTIM SCOREBOARD (By weapon type)

KILLER

Victim	type = $B_1$ # = $m_1$	type = $B_2$ # = $m_2$	...	type = $B_M$ # = $m_M$
type = $R_1$	$S_{11}$ = # kills of $R_1$ by fires from $B_1$	$S_{21}$ = # kills of $R_1$ by fires from $B_2$	...	$S_{M1}$ = # kills of $R_1$ by fires from $B_M$
type = $R_2$	$S_{12}$ = # kills of $R_2$ by fires from $B_1$	$S_{22}$ = # kills of $R_2$ by fires from $B_2$	...	$S_{M2}$ = kills of $R_2$ by fires from $B_M$
...	...	...	...	...
type = $R_{N-1}$	$S_{iN-1}$ = # kills of $R_{N-1}$ by fires from $B_1$	$S_{2n}$ = # kills of $R_{N-1}$ by fires from $B_2$	...	$S_{Mi-1}$ = # kills of $R_{N-1}$ by fires from $B_M$
type = $R_N$	$S_{iN}$ = # kills of $R_N$ by fires from $B_1$	$S_{2n}$ = # kills of $R_N$ by fires from $B_2$	...	$S_{MN}$ = # kills of $R_N$ by fires from $B_M$

While the third form appears analytically preferable, the second form is often quite close to it in practice. Attrition descriptions in terms of such attrition rates have been regularly found to be more robust than killer-victim scoreboard type information. That is, predicted attrition rates often remain very similar, even when simulated battle conditions change. (There has been insufficient analysis of real combat data to address the robustness of actual, as opposed to predicted, attrition rates.) Exhibit 1-6 gives the symbolic or mathematical notation which will be used throughout the remainder of this report to represent attrition rate data. Exhibit 1-7 shows this data in matrix form.

Since the proposed analysis methods call for the use of attrition rate data for both base cases and variations, it is necessary to briefly discuss sources of such data.

Killer-victim scoreboards or equivalent data are typically available from many major Army and other DoD studies. For example, in the preceding project VRI used base case data from more than six past Army and DoD studies, including data generated by the following models and wargames: CEM, DBM, JIFFYGAME (as used in TRADOC SCORES analyses), BLDM (as used in CACDA's analyses supporting the Anti-Armor Systems Program Review), and IDAGAM. It was felt that the use of these several bases would prevent conclusions being drawn based on any idiosyncrasies of a single model or group of related models. The combat results predicted for similar forces in similar periods of battle in the various models showed very great differences. One of the questions that was addressed in this project was whether, in the face of such differences in basic beliefs about combat outcomes, the values of different development programs

## EXHIBIT 1-6: BASIC MATHEMATICAL NOTATION

- $a_{ij}$  = the average attrition rate of type-j Red systems (targets) by a single type-i Blue system (firer);
- $l_j$  = the overall loss rate of Red systems of type-j (which =  $\sum_i m_i a_{ij}$ );
- $b_{ji}$  = the average attrition rate of type-i Blue systems (targets) by a single type-j Red system (firer);
- $k_i$  = the overall loss rate of Blue systems of type-i (which =  $\sum_j n_j b_{ji}$ );
- $m_i(t)$  = the number of type-i Blue systems at a point t in the combat;
- $m_i$  =  $m_i(0)$  = the total number of type-i Blue systems;
- $n_j(t)$  = the number of type-j Red systems at a point t in the combat;
- $n_j$  =  $n_j(0)$  = the total number of type-j Red systems;
- t = a measure of the total amount of battle that has occurred (although this may occasionally be proportional to time, indicating a constant intensity of battle, it will generally not be simply related to clock time or calendar time).

EXHIBIT 1-7: VECTOR-MATRIX FORMS FOR  
THE BASIC COMBAT SITUATION

$$m = \begin{bmatrix} m_1 \\ m_2 \\ \dots \\ m_M \end{bmatrix}$$

$$n = \begin{bmatrix} n_1 \\ n_2 \\ \dots \\ n_N \end{bmatrix}$$

$$a = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{M1} \\ a_{12} & \dots & \dots & a_{M2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & \dots & \dots & a_{Mn} \end{bmatrix}$$

$$b = \begin{bmatrix} b_{11} & b_{21} & \dots & b_{N1} \\ b_{12} & \dots & \dots & b_{N2} \\ \dots & \dots & \dots & \dots \\ b_{1M} & \dots & \dots & b_{NM} \end{bmatrix}$$

predicted using the different bases might not be so diverse that substantive conclusions as to relative value would differ for the various models. In fact, substantial agreement concerning the relative value of various developments was found, even though predictions of the absolute combat outcomes were extremely variable.

While data on alternatives may occasionally be available in similarly detailed form (in cases where a detailed study has been made concerning a question), the method can also be useful in cases where more limited data is available. In most cases, some level of detailed data is available on changes in basic system performance parameters, such as warhead lethality, probability of hit, probability of target acquisition, etc. From this data and an understanding of the structure and determination of attrition rates (see, for example, [Bonder and Farrell, 1970]), it is generally possible to closely approximate the effect of a new system on the attrition exchange. (This was confirmed in the investigations of the previous study [VRI, 1977a].) Other sources of data such as field experiments or historical combat data may also be usable in generating attrition rates.

### 1.3 Lethality-Weighted Force Ratios and Force Effectiveness

The summarization of the force effectiveness data represented in a pair of attrition rate matrices into a one-dimensional measure on which results may be compared is a complex task. One natural method which has been extensively examined is the use of weighted force strength ratios

$$\frac{\sum v_i m_i}{\sum u_j n_j} ,$$

where the weights depend in some fashion on the system capabilities. One particular suggestion has been that the weight used for a particular Blue system should be proportional to the rate at which such a Blue system (on the average) attrits the (weighted) Red Force, and that the weight for a Red system be proportional to the rate at which such a system (on the average) attrits the (weighted) Blue Force. Throughout the remainder of this report, such a system of weights will be termed lethality weights. Translating the conditions given above for lethality weights into equations, with  $v_i$  for the Blue weights and  $u_j$  for the Red weights, one obtains

$$v_i = k \sum_j a_{ij} u_j \quad i = 1, 2, \dots$$

$$u_j = k \sum_i b_{ji} v_i \quad j = 1, 2, \dots ,$$

or, considering the row vectors

$$v = [v_1, v_2, \dots]$$

and

$$u = [u_1, u_2, \dots],$$

the matrix equations

$$v = K u a$$

and

$$u = K v b,$$

where  $a$  and  $b$  are the matrices with elements  $a_{ij}$  and  $b_{ji}$  written in a form which is the transpose of most standard matrix notation, so that the first subscript denotes the column, not the row.

These equations, or minor variations of them, have been investigated and discussed by several authors.<sup>1</sup> The basic mathematical properties, of which proofs are available in the cited papers, are as follows:

- (1) there are always a finite number of solution vector pairs  $v_i$  and  $u_j$ , except for an arbitrary constant of proportionality;<sup>2</sup>
- (2) there is always at least one of these solutions which has all the  $v_i$  and  $u_j$  non-negative; and
- (3) in many, but not all, cases,<sup>3</sup> there is a unique non-negative solution for  $v_i$  and  $u_j$  (unique, that is, except for a proportionality constant).

These properties for the  $v_i$ ,  $u_j$  lethality weights suggested to several investigators that the unique non-negative solutions (when they existed) be considered as "values" for weapon systems. While still in some use for some purposes there are significant problems with this view (see, for instance,

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<sup>1</sup>See, for instance, [Anderson, 1974], [Dare and James, 1971], [Farrell, 1970], [Holter, 1973], [Spudich, 1968], and [Thrall, 1972].

<sup>2</sup>That is, if  $v_i$  and  $u_j$  satisfy the conditions, so do  $cv_i$  and  $cu_j$ , for an arbitrary  $c$ .

<sup>3</sup>A unique solution is guaranteed when the attrition system is irreducible or indecomposable: that is, when every weapon attrits weapons which attrit weapons which eventually, through a chain of attrition, attrit all friendly systems. Even in reducible cases, there may be a unique non-negative solution.

[Anderson, 1974] and [Farrell, 1970]). The major problems are in the following general areas:

- (1) there may be multiple non-negative solutions to the lethality-weight equations, and/or no solutions involving positive "values" for all weapons, even though each weapon causes attrition. Any battle including counterweapons has one or both of these problems;
- (2) changes in weapon systems vulnerability do not cause differences in its "value"; and
- (3) the value solutions show erratic and inappropriate sensitivities, in which *improvements* in weapon lethality *and* vulnerability lead to *decreases* in "value".

Because of the heuristically appealing definition of the lethality weights and the existence of these problems, other methods of valuation using the lethality weights in indirect ways have been examined. As outlined in the various papers cited above, a lethality weighted force ratio

$$R = \frac{\sum_i m_i v_i}{\sum_j n_j u_j}$$

can be defined and has natural interpretations in terms of Lanchester and related combat models. Given the problems with the direct use of  $v_i$  (and  $u_j$ ) as "values" for weapons (and thus with changes in  $v_i$  as values for changes in weapon designs), the possibility of using changes in  $R$  to evaluate changes in weapon design (or weapon mix) has been examined (See [VRI, 1977a] and [VRI, 1977b]). This approach was shown in the cited work with the USADARCOM Directorate of Battlefield Systems Integration, (DBSI) and in

separate work at DBSI, to have partially solved the major problems with the direct use of lethality weights as values. Specifically, the ratio

- (1) may still have non-unique or non-meaningful weights, although in a few practical cases R may show little variation or effect from this;
- (2) responds properly to changes in system vulnerability; and
- (3) shows only limited inappropriate sensitivities in a wide variety of real test cases, even where the lethality weights themselves may be behaving inappropriately.

Thus, the major problem remaining is with the reducible cases: these are precisely the cases with counterweapons (or with two or more entirely separate battles). The previous work has shown that the approach using R and evaluating a weapon system design change in terms of its effect on R seems generally satisfactory (within the limitations of any attrition-based methodology) for zone-1 combats between mechanized or armored forces, as long as counterweapons are not included.

The problems with evaluating counterweapon developments are not entirely related to the mathematical difficulties with the lethality weighting procedure. An additional problem is related to suppression: while methods are known which can extrapolate battle results and battle matrices for changes in weapon system vulnerability firepower (lethality) or numbers, any treatment of counterweapons, and particularly artillery, should also deal with the effects of various kinds of suppression.<sup>1</sup> Given these problems in treating counterweapon problems, the purpose of this project has been to define and demonstrate

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<sup>1</sup>The absence of this treatment may also limit the degree to which the original methods could reasonably be extended to non-mechanized infantry combat.

with examples a methodology for the comparative evaluation of counter-weapon developments, and of counterweapon developments with central duel weapon developments. These methods were to be consistent with the earlier methods [VRI, 1977a] developed and demonstrated for the analysis of central duel weapon developments.

## 2.0 EXTENDING THE METHODS TO COUNTERWEAPON CASES

In order to describe the particular methodological extension designed for analyses including counterweapons, it is necessary to review some technical details of the mathematics behind the lethality weights. The first section of this chapter describes the relation between lethality-weighted force ratio analysis and lethality-weighted force exchange ratio analysis. This is required since the extended methodology for counter-weapon cases is based on the force exchange ratio.

$$\frac{\text{Percentage or proportion of Red force lost}}{\text{Percentage or proportion of Blue force lost}}$$

This force exchange ratio is a natural indicator of the degree of win or loss. If the numerator exceeds the denominator, Blue is improving his situation -- i.e., winning. If it is less, Red is improving his situation. The technical relations between this exchange ratio approach and the force ratio approach which justify the use of the exchange ratio as a consistent extension of the force ratio are reviewed in section 2.1. Section 2.2 then takes up the explicit definition of the extended methodology, in terms of a particular weighted force exchange ratio.

2.1 *The Equivalence of Force Exchange Ratios and Force Strength Ratios*

A lethality-weighted force strength ratio, as described in the preceding section, is exactly equivalent to the square root of a weighted force exchange ratio

$$\sqrt{\frac{\text{proportion (weighted) Red force lost}}{\text{proportion (weighted) Blue force lost}}}$$

or, in our earlier terms,

$$\sqrt{\frac{\sum u_j l_j / \sum u_j n_j}{\sum v_i k_i / \sum v_i m_i}}$$

The proof of this is straight forward (and can be constructed from arguments in Thrall's, Dare and James's, Spudich's, and Holter's papers, although this precise theorem is not given by any of the previous authors writing on this topic). Specifically, take the equations defining the lethality-weighted force strength ratio.

$$R = \frac{\sum_i m_i v_i}{\sum_j n_j u_j},$$

with

$$\begin{aligned} v_i &= K \sum_j a_{ij} u_j \\ u_j &= K \sum_i b_{ji} v_i \end{aligned}$$

Then, by substitution

$$R = \frac{K \sum_i m_i \sum_j a_{ij} u_j}{K \sum_j n_j \sum_i b_{ji} v_i}$$

which, on reversing the order of summation and noting that  $\sum_i m_i a_{ij} = 1_j$  and  $\sum_j n_j b_{ji} = k_i$ , gives

$$R = \frac{\sum_j 1_j u_j}{\sum_i k_i v_i} ,$$

multiplying by the original expression for R gives

$$R^2 = \frac{\left(\sum_j 1_j u_j\right)\left(\sum_i m_i v_i\right)}{\left(\sum_i k_i v_i\right)\left(\sum_j n_j u_j\right)} ,$$

which, on algebraic re-expression, becomes

$$R^2 = \frac{\left(\sum_j 1_j u_j\right) / \left(\sum_j n_j u_j\right)}{\left(\sum_i k_i v_i\right) / \left(\sum_i m_i v_i\right)} ,$$

which completes the proof.

It is worth noting that this ratio of fractional weighted force losses is also expressible as a weighted average of the fractional losses of various weapons. Specifically,

$$\begin{aligned} & \left(\sum_j 1_j u_j\right) / \left(\sum_j n_j u_j\right) \\ &= \sum_j \left(\frac{n_j u_j}{\sum_k n_k u_k}\right) \left(\frac{1_j u_j}{n_j u_j}\right) \end{aligned}$$

where  $n_j u_j / \sum_k n_k u_k$  is the weight for the j type fractional losses and  $1_j u_j / n_j u_j$  is the fractional losses of system j. The weights obviously sum to 1.0. This and the parallel form for Blue forces give

$$R^2 = \frac{\sum_j \left( \frac{n_j u_j}{\sum_k n_k u_k} \right) \left( \frac{l_j u_j}{h_j u_j} \right)}{\sum_i \left( \frac{m_i v_i}{\sum_k m_k v_k} \right) \left( \frac{k_i v_i}{m_i v_i} \right)}$$

as was asserted.

As with all of the prior results concerning lethality-weighted ratios, the attrition coefficients  $a_{ij}$  and  $b_{ji}$  and the loss coefficients  $k_i$  and  $l_j$  may be defined in any of the several ways outlined earlier. This general equivalence between lethality-weighted force exchange ratios and force exchange ratios and lethality-weighted force strength ratios is important for several reasons:

- (1) the force exchange ratio,

$$\frac{\text{fraction of Red force lost}}{\text{fraction of Blue force lost}},$$

has a natural, heuristic interpretation as an indicator of relative force effectiveness;

- (2) the lethality-weighted force exchange ratio may be compared naturally with other heuristic weighted force exchange ratios in order to confirm the results of lethality-weight analysis and control the impact of the potentially anomalous behavior of lethality weights and lethality-weighted ratios.
- (3) the correspondence between the lethality-weighted force strength ratio and the heuristically more meaningful force exchange ratio suggested the use of the marginal analysis of system developments on force-ratios, rather than the effects of changes on the lethality weights themselves. This approach, which showed significant benefits in the preceding work on central duel problems, is also at the heart of the methods developed in this project. The preceding project work, following the lines suggested by this comparison, did show that in a wide spectrum of practical cases the measurement of the contributions of design changes or force structure changes in terms of effects on force ratios were much more stable and robust than direct measurements of lethality weights. (See [VRI, 1977 a] and 1977 b]).

A slightly more compact notation for systems of attrition rates and lethality weights can be used when convenient. By considering the system status vector composed of Blue strengths followed by Red strengths, we can obtain a single attrition matrix, having zeroes for all Blue-Blue or Red-Red interactions.

We will call this resultant matrix, composed of the two attrition-rate matrices and two rectangular matrices of zeroes, a battle matrix. Any single

row vector composed of lethality weights for the force followed by that for the Red force will be called a lethality-weight vector for the battle. This notation will allow a more compact expression of the mathematics of our situation: it does not change the substance of the extended, two-sided form, and both the compact and extended forms will be used interchangeably, with the choice generally depending on expositional convenience. Specifically, the symbol  $B$  will be used for the battle matrix (with  $B_{ij}$  a typical term) and  $W$  for a lethality-weight vector. In terms of the extensive-notation symbols  $a$ ,  $b$ ,  $u$ , and  $v$ ,  $B$  is the composite matrix.

$$\left[ \begin{array}{c|c} 0 & b \\ \hline a & 0 \end{array} \right],$$

where the zeroes are actually rectangular zero matrices, and  $W$  is the composite row-vector  $[v | u]$ . The matrix  $B^2$  is

$$\left[ \begin{array}{c|c} ba & 0 \\ \hline 0 & ab \end{array} \right],$$

and  $W$  is a left-eigenvector of  $B^2$  with the additional properties which make  $u$  and  $v$  a set of lethality weights, as described above.

The Lanchester differential equation

$$Z' = B Z,$$

with  $Z$  the composite vector of Blue and Red strengths, can be used to provide an estimate of the battle's evolution as the amount of combat progresses. The relation of this differential equation to the lethality weighting techniques is discussed in the references.

## 2.2 Extensions to Counterweapon Analyses

The first necessity in describing the extensions which make this method applicable to problems including counterweapons is the adoption of appropriate notation. Assume a battle with  $M + 1$  Blue and  $N + 1$  Red weapon types. Let the  $M$ -by- $N$  battle of Blue weapons 1 through  $M$  and Red weapons 1 through  $N$  be an irreducible (central) duel. Let the  $(M + 1)$ st and  $(N + 1)$ st weapons be pure counterweapons, which may attrit all weapons, including counterweapons, on the opposing side, but which are not attrited by the central duel weapons. Exhibit 2-1 shows the structure of the attrition rate matrices for this case. (Results for cases with additional counterweapons are straightforward algebraic extensions of those for the one counterweapons-per-side case.) An example of the type of problem for which this case is designed is one with (1) tanks, (2) lightly armored vehicles mounting anti-tank weapons, (3) infantry squads with anti-tank weapons, and (4) artillery systems on each of two opposing sides.

The irreducible central duel for such a battle is that involving the first three weapon types on each side, all of which attrit all three enemy types. (This condition guarantees irreducibility, but is not required for irreducibility.) The artillery, which we assume attrits enemy artillery and some or all of the central-duel weapons but is not attrited by any, is the counterweapon.

A basic mathematical theorem governs the existence and properties of lethality weights in a battle with counterweapons:

EXHIBIT 2-1: ATTRITION RATE MATRICES  
IN THE COUNTERWEAPON CASE

Irreducible Central Duel			
$a_{11}$	...	$a_{M1}$	$a_{M+1 1}$
	...		...
$a_{1N}$	...	$a_{NM}$	$a_{M+1 N}$
0		0	$a_{M+1 N+1}$

← Counterweapon  
Fires at Central  
Duel Weapons

← Counterweapon  
fires at  
Counterweapon

Irreducible Central Duel			
$b_{11}$	...	$b_{N1}$	$b_{N+1 1}$
	...		...
$b_{1M}$	...	$b_{NM}$	$b_{N+1 M}$
0		0	$b_{N+1 M+1}$

← Counterweapon  
Fires at Central  
Duel Weapons

← Counterweapon  
Fires at  
Counterweapon

Theorem: If a simple battle matrix containing one or more counter-weapons and an irreducible duel has a lethality-weight vector with all positive weights, this vector is an extension of the unique lethality-weight vector for the irreducible duel. Further, there is only one such extension. This extension, when it exists, will be referred to as the extended lethality-weight vector.

The proof of this theorem is obtained by straightforward algebraic manipulations. Specifically, group the attrition-rate matrices into block matrices

$$\left[ \begin{array}{c|c} a_1 & a_2 \\ \hline 0 & a_3 \end{array} \right]$$

and

$$\left[ \begin{array}{c|c} b_1 & b_2 \\ \hline 0 & b_3 \end{array} \right]$$

in the same general way as was done in the one-counterweapon case. Consider lethality weight vectors in block form also,

$$v = [v_1 | v_2]$$

$$u = [u_1 | u_2]$$

Then by the definition of a lethality-weight vector

$$[v_1 | v_2] = k [u_1 | u_2] \left[ \begin{array}{c|c} a_1 & a_2 \\ \hline 0 & a_3 \end{array} \right],$$

and

$$[u_1 | u_2] = k [v_1 | v_2] \left[ \begin{array}{c|c} b_1 & b_2 \\ \hline 0 & b_3 \end{array} \right]$$

for some  $k$ .

Multiplying the block forms, one obtains

$$v_1 = k u_1 a_1 \quad (1)$$

$$u_1 = k v_1 b_1 \quad (2)$$

$$v_2 = k (u_1 a_2 + u_2 a_3) \quad (3)$$

$$u_2 = k (v_1 b_2 + v_2 b_3). \quad (4)$$

The first two equations are precisely the conditions which make  $v_1$  and  $u_1$  lethality-weight vectors for the irreducible portion of the battle. This proves that any extended lethality-weight vector is an extension of that for the irreducible portion. Since there is a unique non-negative solution in the irreducible portion, any non-negative pair  $v, u$  must be extensions of this non-negative solution. The impossibility of multiple solutions for  $v_2$  and  $u_2$  is simply a matter of eigenvalue analysis of equations (3) and (4).

Interpreting the theorem in words, one obtains as a principal result that the relative lethality weights for the central duel portion of the battle are precisely those for the central duel standing alone.

It is possible to go well beyond this result, however, and show that the reduced two-dimensional battle -- in which the central duel forces of each opponent are viewed in terms of the lethality-weight, one-dimensional summary

$v_1 \begin{bmatrix} m \\ \dots \\ m_M \end{bmatrix}$  ( $u_1 \begin{bmatrix} n \\ \dots \\ n_N \end{bmatrix}$  for Red) and the single counterweapons remain unaggregated -- obeys a two-dimensional Lanchester law of battle whenever the original, unaggregated battle obeyed an  $(M + 1)$  by  $(N + 1)$  dimensional Lanchester Law.<sup>1</sup>

<sup>1</sup>This point follows the general line of the Dare-James treatment of the lethality weights, although it is a slight extension of their results [Dare and James, 1971].

Specifically, one reduced case is

$$\frac{d \left( \sum_{i=1}^M v_i m_i(t) \right)}{dt} = -k \left( \sum_{j=1}^N u_j n_j(t) \right) \left\{ \sum_{i=1}^M b_{N+1 i} v_i \right\} n_{N+1}(t),$$

$$\frac{d \left( \sum_{j=1}^N u_j n_j(t) \right)}{dt} = -k \left( \sum_{i=1}^M v_i m_i(t) \right) \left\{ \sum_{j=1}^N a_{M+1 j} u_j \right\} m_{M+1}(t)$$

$$\frac{d m_{M+1}(t)}{dt} = - b_{N+1 M+1} n_{N+1}(t),$$

$$\frac{d n_{N+1}(t)}{dt} = - a_{M+1 N+1} m_{M+1}(t).$$

Alternative cases, differing only by scaling factors, can produce two-dimensional cases in which the aggregated central duel has differing attrition rates and the initial forces are measured on different scales (for example, on a lethality-weighted percent survival scale, so that the initial force strengths are 1.0 Blue force units for Blue and 1.0 Red force units for Red). In this case, the aggregated central-duel force ratio is entirely displayed in the aggregated attrition rates, and as we have seen above, the central-duel force ratio  $R$  in this case will be the square root of the reduced-form attrition rate ratio.

One specific equation for this case is

$$\begin{aligned}
 & d \left( \frac{\sum_{i=1}^M v_i m_i(t)}{\sum_{i=1}^M v_i m_i(0)} \right) \\
 & \quad \quad \quad d(kt) \\
 & = - \left( \frac{\sum_{j=1}^N u_j n_j(0)}{\sum_{i=1}^M v_i m_i(0)} \right) \frac{\sum_{j=1}^N u_j n_j(t)}{\sum_{j=1}^N u_j n_j(0)} \\
 & \quad \quad \quad - \frac{\sum_{i=1}^M b_{N+1 i} v_i n_{N+1}(0)}{k \sum_{b=1}^M v_i m_i(0)} \frac{\sum_{i=1}^M b_{N+1 i} v_i n_{N+1}(t)}{\sum_{o=1}^M b_{N+1 i} v_i n_{N+1}(0)},
 \end{aligned}$$

with analogous equations for the other terms.

This transformation reduces the study of any functions of the lethality-weighted central duel surviving strengths

$$\sum_{i=1}^M v_i m_i(t) / \sum_{i=1}^M v_i m_i(0) \quad \text{and} \quad \sum_{j=1}^N u_j n_j(t) / \sum_{j=1}^N u_j n_j(0) \quad \text{and the}$$

counterweapon surviving strengths  $m_{M+1}(t) / m_{M+1}(0)$  and  $n_{N+1}(t) / n_{N+1}(0)$ ,

to the study of two-dimensional battles with initial force strengths all 1.0 and attrition rate matrices

$$\begin{bmatrix} \alpha_1 & \alpha_2 \\ 0 & \alpha_3 \end{bmatrix}$$

and

$$\begin{bmatrix} \beta_1 & \beta_2 \\ 0 & \beta_3 \end{bmatrix}.$$

As was noted above, the lethality-weight concept, when used in such a setting may lead to

- (1) a unique non-negative solution, with all weight on the counter-weapons and zero weight for the central duel weapons (when  $\alpha_3\beta_3 \geq \alpha_1\beta_1$ ) and
- (2) two non-negative solutions, one of which weights the counter-weapons only and one of which gives positive weight to both the weapon types.

In fact, the following theorem shows some of the poor behavior of the lethality weights for counterweapons:

**Theorem:** A battle matrix of the form being discussed has a positive extended lethality-weight vector if and only if the geometric average counter-counterfire force loss rate is less than the geometric average force loss rate of the irreducible portion of the battle.

The proof is as follows: consider a set of lethality weights  $v_1, v_2, u_1, u_2$ .

Then, by the defining properties of lethality weights,

$$v_1 = k u_1 \alpha_1$$

$$u_1 = k v_1 \beta_1$$

$$v_2 = k u_1 \alpha_2 + k u_2 \alpha_3$$

$$u_2 = k v_1 \beta_2 + k v_2 \beta_3.$$

Simple algebraic substitutions of one relation into another give

$$v_1 = k^2 v_1 \alpha_1 \beta_1,$$

which implies

$$k^2 = 1 / \alpha_1 \beta_1.$$

Other substitution transformations give

$$v_2 = k^2 v_2 \alpha_3 \beta_3 + k^2 v_1 (\beta_1 \alpha_2 + \beta_2 \alpha_3),$$

but substituting for  $k^2$  gives

$$v_2 = \frac{\alpha_3 \beta_3}{\alpha_1 \beta_1} + v_1 \left( \frac{\beta_1 \alpha_2 + \beta_2 \alpha_3}{\beta_1 \alpha_1} \right)$$

or

$$\begin{aligned} v_2 &= v_1 \frac{\left( \frac{\beta_1 \alpha_2 + \beta_2 \alpha_3}{\beta_1 \alpha_1} \right)}{\left( 1 - \frac{\alpha_3 \beta_3}{\alpha_1 \beta_1} \right)} \\ &= v_1 \left( \frac{\beta_1 \alpha_2 + \beta_2 \alpha_3}{\alpha_1 \beta_1 - \alpha_3 \beta_3} \right), \end{aligned}$$

which cannot be positive and finite when all the variables are positive and

$$\alpha_3 \beta_3 \geq \alpha_1 \beta_1.$$

Because of the problems of non-unique and non-meaningful solutions, some alternative to the direct use of the lethality weight techniques is required. This alternative must be consistent with the use of lethality weights in the central duel.

Such an alternative can be constructed in terms of the general equivalence of weighted loss-ratios and the lethality-weighted force ratio discussed above. The measure of force effectiveness is then taken to be the lethality-weighted force exchange ratio of the central duel weapons or the square root of this measure.<sup>1</sup> This method of measuring force effectiveness for forces including counterweapons:

- (1) treats all battles as long as there are not multiple, separate central duels (as in a composite of strategic nuclear exchange and tactical warfare, or of completely independent naval and land battles);
- (2) provides a well-defined meaningful unique measure consistent with the lethality-weighted force ratio for central duel analyses;
- (3) gives non-zero, positive weight to any weapon which causes attrition; and
- (4) shows the value of changes in design characteristics which ultimately affects the attrition exchange through such performance areas as firepower, vulnerability, etc.

As with the original lethality-weighted force ratio techniques, the general steps in the extended methodology are:

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<sup>1</sup>Other weighted loss ratios might also be used to provide additional assurance that anomalies of the eigenvalue weights are not affecting the answer.

- (1) obtain from one or more sources a description of the attrition in a base case battle and a battle with a change in one or more weapon designs<sup>1</sup> (or in force mix);
- (2) determine the (unique non-negative) lethality-weighted force-exchange ratio for central-duel weapons for each case;
- (3) measure the value of the weapon system in terms of the difference in the two results;
- (4) repeat the procedure for different sources of base case data to ensure that the conclusions are robust and not driven by the specific original data source: it should be noted that this method, as was pointed out in the earlier project, may provide a method for robust evaluation or comparison of force changes even when sources differ significantly about the absolute performance of various weapons or a force as a whole [VRI, 1977b].

Two remaining points must be addressed concerning this procedure before we may turn to examples and experiment to further describe and validate it. The first of these is the fact that there is one degree of freedom left in the methodology -- the amount of battle to be considered before the exchange-ratio is computed has not been specified. In fact, this introduces an additional requirement for sensitivity analysis -- such analyses must not only address the robustness of the conclusions to changes in data sources, but also

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<sup>1</sup>In some cases, the second battle is created parametrically from the base battle.

to changes in assumptions about the total amount of attrition to occur before measurements are made. This point will be further discussed in the example analyses presented.

The remaining point that must be addressed at this juncture is the definition of units of measurement for the evaluation of system design changes. As defined in the general method, every measurement of value is in terms of an achieved improvement (or degradation) in force effectiveness. There is no direct way to compare values unless they involve identical base cases, in which case one can determine which system change caused the greater improvement in force ratio. Since it is desirable to be able to compare system changes even when different base cases have been used -- as, for instance, in the required sensitivity analyses -- it is useful to define one or more common scales of measurement on which to compare system design changes.

A simple way to accomplish this is to use a standard unit of force, for example, a standard US armored division, a US armor battalion, or a standard Soviet tank army, etc. Since real base cases may not all involve this standard force, an analyst cannot use the force exchange ratios directly to determine such a measure. Rather, the analyst must include with whatever changes are being examined an additional change -- one in which some standard force amount is added to the battle. For example, an analyst whose base battles were of approximately brigade-to-division scale might use as a standard of measurement the effect of adding one percent of a standard US armored division to all the battles. Alternatively, he might adopt two scales, comparing all division results to the addition of one percent of a standard division and all brigade results to the addition of one percent of some nominal brigade force.

As can be seen, these methods offer various scales of measurement with no guarantees of consistency and no clear preference among them. Rather, the choice must be made by the analyst for each problem examined. An alternative, in which no judgment of choices are necessary, but which may be more difficult to interpret meaningfully, is a purely relative scale. In this approach, every improvement in force effectiveness is measured in percent relative to the base case from which it was produced. The justification and interpretation of such an approach rests on the assumption that the base case or base cases used are in fact completely representative of potential future combats. If the bases are representative, then the average percent improvement corresponds to the average percent in improvement in total US fighting strength or US NATO fighting strength or other specific theater forces (as reflected in weapon system attrition, friendly and enemy). Such a scale is as good as and perhaps better than those scaled to smaller forces. However, the assumption of representativeness cannot be expected to be met with any frequency. The next chapter's analyses will demonstrate these and other measurement scales and comparisons.

The overall approach to the counterweapon problem which has been described in this section can be summarized as follows:

- (1) Counterweapons become valuable by affecting the exchange of attrition in the central duel.
- (2) The exchange of attrition in the central duel can be measured by a lethality-weighted force exchange ratio.
- (3) Accordingly, the value of changes in counterweapon designs can be determined by estimating their effects on the lethality-weighted force exchange ratio of central duel weapons.

This is the basic meaning of the mathematical derivations of this section. Chapter 3 contains demonstrations of the use of this type of methodology.

## 3.0 EXAMPLES OF ANALYSES

## 3.1 Base Case Definition

In presenting examples of the use of the techniques described above, this report will concentrate on artillery system examples. Using the techniques described in chapter 2, these cases may be reduced to two-dimensional forces for each side, with one weapon type on each side an aggregated maneuver force element constructed by the methods of section 2.2, and one weapon type an artillery system.

A base case is defined in such a system by six numbers,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  composing the attrition matrices

$$\begin{bmatrix} \alpha_1 & \alpha_2 \\ 0 & \alpha_3 \end{bmatrix}$$

and

$$\begin{bmatrix} \beta_1 & \beta_2 \\ 0 & \beta_3 \end{bmatrix}.$$

We assume the units of measurement of the force elements to be scaled to the forces involved, so that  $m_1(0) = m_2(0) = n_1(0) = n_2(0) = 1.0$  in the base case. Further, we may consider the Blue maneuver forces to be those of an armored division and the Red forces those of a tank army. Maneuver

force results produced by typical studies for such situations show lethality-weighted force ratios ranging from .6 to 1.1, depending on the assumptions and tools used [VRI, 1977b]. Because the methods and data used in the detailed maneuver force analyses are available from earlier work, they will not be further addressed in the detailed examples. Instead, the examples will concentrate on the counterweapon-specific elements of the analysis.

Since these example analyses are intended to be general, rather than specific to one particular situation or model, we will consider several base cases in this range. The central case will be one with which is equivalent to a force ratio of about .9. Since there is one degree of freedom left in the measurement units (the amount-of-battle scale), it may be fixed by assuming that  $\alpha_1 = .8$  and  $\beta_1 = 1.0$  for the central base case. (This properly has  $\alpha_1/\beta_1 \cong (.9)^2$ .)

Assuming appropriate artillery forces for both opponents, studies suggest a range of values for artillery-caused attrition. As with maneuver force assumptions, several base cases will be used in the examples. The central base will be one in which  $\beta_2 = .05$ ,  $\beta_3 = .40$ ,  $\alpha_2 = .15$ , and  $\alpha_3 = .60$ .

As with the maneuver force data, these data are based on actual studies, particularly the 1985 portion of the LEGAL MIX V DIWAG analyses, using the reported performance of the preferred mix. The  $\alpha_3$  and  $\beta_3$  counterbattery effectiveness numbers include both the reported effects of artillery system damage and destruction and the effects of personnel casualties in artillery units. Artillery units determined to be partially effective were treated as having been partially attrited in determining  $\alpha_3$  and  $\beta_3$ . As with the maneuver force data, which were selected to be representative of data from CEM, DBM, JIFFYGAME, BLDM, and IDAGAM analysis, the artillery

data is merely intended to be representative and not exact: in the example problems, sensitivity analyses will be used to show that this possible inexactness is not critical to most conclusions.

Exhibit 3-1 shows the resulting attrition rate matrices. These matrices portray a battle in which the Blue maneuver force is slightly outweighed by the Red maneuver force (with a maneuver force strength ratio of approximately .89). The total Blue artillery destroys or otherwise makes Red maneuver force weapons ineffective at an initial rate which is 15 percent of the rate at which the total Blue direct fire weapons accomplish such attrition. (The term initial rate is used because as the attrition proceeds, the relative strengths of the maneuver forces and artillery forces will change, so that the total attrition rates will change.) The remaining entries have similar meanings. These scaled data were, of course, produced by the examination of fully dimensioned killer-victim scoreboards, and the use of the scaling transformations of section 2.2. All of the example analyses could be performed on the fully-dimensioned attrition rate matrices without scaling, and identical conclusions would result. The scaling is simply a convenient mathematical technique to reduce the battle matrix to a particular canonical form.

Exhibit 3-2 shows the evolution of the central base forces as combat occurs.<sup>1</sup> A limitation of the reduction to two dimensions is that data on individual maneuver force weapons is not available. While this does not affect the examples, real analyses should include the full available

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<sup>1</sup>Heterogeneous Lanchester square-law differential equations have been used, as discussed in chapter 2, to interpolate and extrapolate from the base killer-victim scoreboard. This technique was shown in the previous study to be robust and quite accurate in predicting the actual results of more detailed models.

EXHIBIT 3-1: BASE CASE ATTRITION RATES, MEASURED IN PERCENT  
OF FORCE ATTRITED PER UNIT TIME

	RED	:	MF	ARTY
	FIRER			
BLUE	MF	:	1.	.05
TARGET	ARTY	:	0	.40
	BLUE	:	MF	ARTY
	FIRER			
RED	MF	:	.8	.15
TARGET	ARTY	:	0	.60

dimensionality so that the effect of the amount-of-battle parameter is completely understood. Under some circumstances, it would be possible for the model to show tank forces, for example, reaching zero at some point while the maneuver force aggregate is still comfortably positive. Since the methods are not usable for cases in which weapon system types are annihilated, amounts of combat usable in analyses may be limited by fully-dimensioned result information.

## EXHIBIT 3-2: RESULTS OF COMBAT FOR BASE CASE

<u>Blue Mvr. Fce. Fraction Losses</u>	<u>Blue Artillery Fraction Losses</u>	<u>Red Mvr. Fce. Fraction Losses</u>	<u>Red Artillery Fraction Losses</u>	<u>Maneuver Force Exchange Ratio</u>
0.195	0.076	0.176	0.116	0.904
0.356	0.143	0.321	0.224	0.902
0.489	0.202	0.440	0.324	0.900
0.599	0.253	0.538	0.417	0.897
0.691	0.297	0.618	0.504	0.895
0.766	0.334	0.683	0.587	0.891
0.829	0.365	0.736	0.665	0.887
0.881	0.389	0.778	0.740	0.883

### 3.2 Improving Artillery Firepower or Vulnerability

As a first example of this valuation analysis, consider a possible change in artillery weapons or ammunition which would make it more effective in fires at maneuver weapon targets. (As will be discussed later, the basic improvement might be in accuracy of target location, timeliness of meteorological data, lethality of warhead, etc.) Such a change would affect the  $\alpha_2$  element of our data. In the base case

$$\alpha_2 = .15,$$

suppose we consider a specific change that would make

$$\alpha_2 = .20,$$

and would have no direct effect on other attrition rates. (Later discussions will show how this estimate of the quantitative effect of the design change might be made.) Exhibit 3-3 shows the results which would be predicted for the new combat.

Comparing these results with those of the base case, we see that this change in artillery system performance has significantly affected the results of the combat. Looking at different amounts of combat, the comparison is

<u>Base Case Maneuver Force Exchange Ratio</u>	<u>New Case Maneuver Force Exchange Ratio</u>	<u>Percentage Improvement</u>
.902	.965	7.0%
.897	.984	9.7%
.891	1.01	13.6%
.883	1.05	19.4%

EXHIBIT 3-3: RESULTS OF COMBAT WITH INCREASED ARTILLERY  
FIREPOWER AGAINST MANEUVER TARGETS

<u>Blue Mvr. Fce. Fraction Losses</u>	<u>Blue Artillery Fraction Losses</u>	<u>Red Mvr. Fce. Fraction Losses</u>	<u>Red Artillery Fraction Losses</u>	<u>Maneuver Force Exchange Ratio</u>
0.194	0.076	0.186	0.116	0.957
0.353	0.143	0.340	0.224	0.965
0.481	0.202	0.468	0.324	0.973
0.585	0.253	0.575	0.417	0.984
0.668	0.297	0.665	0.504	0.996
0.733	0.334	0.742	0.587	1.012
0.783	0.365	0.807	0.665	1.030
0.820	0.389	0.864	0.740	1.054

As one would expect, the impact of the artillery change on the maneuver force battle increases with time. If we believe that external conditions will limit the amount of battle, the improvement of artillery performance will have a limited impact on maneuver force attrition results; as the potential duration of battle increases, the impact of possible artillery changes is greater. (Although of greater magnitude in the case of counterweapons, the same effect appears in the evaluations of central duel weapons [VRI, 1977b].)

The methods are intended to compare alternative design changes. To this point, the discussion has concentrated on a single design change. In order to demonstrate the methods as they were designed for use, consider an alternative artillery system change which would change the vulnerability of the artillery force, rather than its firepower. The vulnerability of the Blue artillery is reflected in the  $\beta_3$  coefficient, which describes Red counterbattery effectiveness. Assume that the artillery change one is considering would improve Blue survivability by a factor of two, whether through an increase in protection or other changes. Then, since in the base case

$$\beta_3 = .40,$$

the adjusted case would have

$$\beta_3 = .20.$$

Exhibit 3-4 shows the results of this case. Summarizing the results from the two possible developments after various amounts of combat, one obtains

<u>Percentage Gain From Change 1</u>	<u>Percentage Gain From Change 2</u>
7.0%	0.7%
9.7%	2.0%
13.6%	3.9%
19.4%	6.9%

EXHIBIT 3-4: RESULTS OF COMBAT WITH DECREASED  
ARTILLERY VULNERABILITY

<u>Blue Mvr. Fce. Fraction Losses</u>	<u>Blue Artillery Fraction Losses</u>	<u>Red Mvr. Fce. Fraction Losses</u>	<u>Red Artillery Fraction Losses</u>	<u>Maneuver Force Exchange Ratio</u>
0.195	0.038	0.177	0.118	0.906
0.356	0.072	0.323	0.232	0.908
0.488	0.100	0.445	0.342	0.911
0.597	0.125	0.546	0.449	0.915
0.686	0.145	0.631	0.553	0.920
0.758	0.162	0.702	0.654	0.926
0.816	0.174	0.762	0.754	0.934
0.861	0.182	0.813	0.853	0.944

Assuming that one wishes to consider battles which are not artificially terminated, the first alteration, improving firepower, has about three times the effect of the survivability change. A complete examination of the problem would require, of course, examination of the relative costs of such changes -- including the research and development, procurement, and operating costs. Additional examination would also be required to look at any impacts in areas other than the combat attrition area, and to examine the sensitivity of the conclusions to the base case situation and data sources. (Some of these issues will be addressed in further sections of this chapter.)

So far, the discussion has used the relative, or percentage gains, scaled to describe the effects. In order to consider the use of a force unit scale of procurement, consider the effect of adding ten percent of a standard division to the base force.<sup>1</sup> Exhibit 3-5 shows the resulting combat in the same format as the previous results exhibit. The gains from the potential artillery changes are shown below as percentages of the gain from the force expansion, at varying amounts of battle.

<u>Gain from Change 1 As a Percentage of Gain from Adding Force</u>	<u>Gain from Change 2 As a Percentage of Gain from Adding Force</u>
27%	2.6%
29%	5.9%
29%	8.5%
28%	10.4%

---

<sup>1</sup>Such a force addition is, of course, a mathematical artifact. There is no implication intended that such a force -- ten percent of a division -- could exist. However, the mathematical methods permit the assumption that each weapon system is increased by ten percent in numbers, and that the new systems are precisely as effective as the original ones in causing attrition. It is this assumption that is meant when the term ten percent of a division is used.

EXHIBIT 3-5: RESULTS OF COMBAT WITH  
INCREASED BLUE FORCES

<u>Blue Mvr. Fce. Fraction Losses</u>	<u>Blue Artillery Fraction Losses</u>	<u>Red Mvr. Fce. Fraction Losses</u>	<u>Red Artillery Fraction Losses</u>	<u>Maneuver Force Exchange Ratio</u>
0.176	0.069	0.195	0.128	1.110
0.317	0.129	0.360	0.248	1.134
0.430	0.180	0.500	0.360	1.163
0.517	0.224	0.620	0.466	1.200
0.583	0.260	0.725	0.566	1.245
0.629	0.288	0.819	0.663	1.302
0.658	0.310	0.905	0.755	1.375
0.672	0.325	0.987	0.846	1.470

These results are the same as the earlier results, except for the scale of measurement. In terms of the new scale, the first change is worth about 28 percent as much as adding ten percent of a division -- *in terms of the exchange of attrition alone*. That is, the firepower change under consideration would be worth -- *in terms of attrition exchange alone* -- about 2.8% of a division. It must be emphasized that there is no intention in the use of this mathematical scale to imply that the worth of additional forces in a division is completely or even primarily reflected in potential attrition exchanges.

There are other significant elements of overall value to be considered in assuming the value of force changes -- the use of a measurement technique scaled to force units is simply to provide a uniform scale for comparing weapon system changes which are designed principally to alter attrition exchanges. The applicability of any implied comparisons between weapon improvements and force levels has not been examined or tested. Thus, the value of the first change should be thought of as 2.8 percent of  $x$  times a division, where  $x$  is an unknown coefficient (less than one) taking other factors into consideration. This prevents a comparison of total force size changes with weapon system changes by these methods without further research. However, one can compare the first weapon change with the second on this scale, since  $x$  appears identically in both cases. (Further, one-sided comparisons with force strength changes are possible. Since  $x$  must be  $\leq 1.0$  -- that is, other factors must generally have some positive impact -- we may conclude that the firepower change is worth *no more than* 2.8 percent of a division.)

The technique can compare artillery changes with maneuver force changes. The first case we have considered involved an artillery firepower change.

Consider comparing this with a change in tank firepower. A fully detailed comparison would involve use of the disaggregated battle matrix. However, in the previous study [VRI, 1977a], it was shown that a 33 percent increase in tank firepower<sup>1</sup> would cause a change in the maneuver force aggregated /firepower  $\alpha_1$  of about five to ten percent with the value depending on the base case data used. Assuming that these changes cause no change in the other aggregated force attrition rates  $\alpha_2$ ,  $\beta_1$ , and  $\beta_2$ , we would find that a tank firepower increase of 33 percent would give

$$\alpha_1 \approx .85$$

with no other changes.<sup>2</sup>

The results of such a case are shown in exhibit 3-6. Comparing them (using the percentage gain scale) with the artillery firepower improvements, one obtains, at increasing amounts of combat,

<u>Gain from Increase in Artillery Firepower</u>	<u>Gain from Increase in Tank Firepower</u>
7%	6.2%
9.7%	7.8%
13.6%	10.1%
19.4%	13.4%

The gains from the artillery improvement are of the same order of magnitude as those from the tank improvement and are slightly greater (with the precise data used -- as remarked earlier, however, there is some variability in the data depending on the study sources used). The artillery improvement increases

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<sup>1</sup>Chosen to compare with the 33 percent increase in artillery firepower considered.

<sup>2</sup>Use of the data from the previous study and the aggregation formulae from chapter 2 shows only minor effects on  $\alpha_2$ ,  $\beta_1$  and  $\beta_2$ , which are neglected for simplicity of exposition.

EXHIBIT 3-6: RESULTS OF COMBAT WITH  
INCREASED TANK FIREPOWER

<u>Blue Mvr. Fce. Fraction Losses</u>	<u>Blue Artillery Fraction Losses</u>	<u>Red Mvr. Fce. Fraction Losses</u>	<u>Red Artillery Fraction Losses</u>	<u>Maneuver Force Exchange Ratio</u>
0.194	0.076	0.185	0.116	0.955
0.353	0.143	0.338	0.224	0.958
0.482	0.202	0.464	0.324	0.962
0.587	0.253	0.568	0.417	0.967
0.672	0.297	0.654	0.504	0.973
0.740	0.334	0.725	0.587	0.981
0.794	0.365	0.785	0.665	0.990
0.835	0.389	0.836	0.740	1.001

in value as the duration of combat assumed increases to the feasible limits (as weapon system annihilation is approached). In general, we would expect that priorities between these weapon system developments would depend on relative costs (as well as any effectiveness issues unrelated to attrition). Insofar as the costs of artillery improvements would be less than (or even slightly greater than) the cost of comparable tank improvements, the higher survivability and longer time to apply the increased firepower of the artillery would make artillery improvements preferable in improving the attrition exchange. If the artillery costs were significantly greater than the tank costs, this conclusion would be reversed.

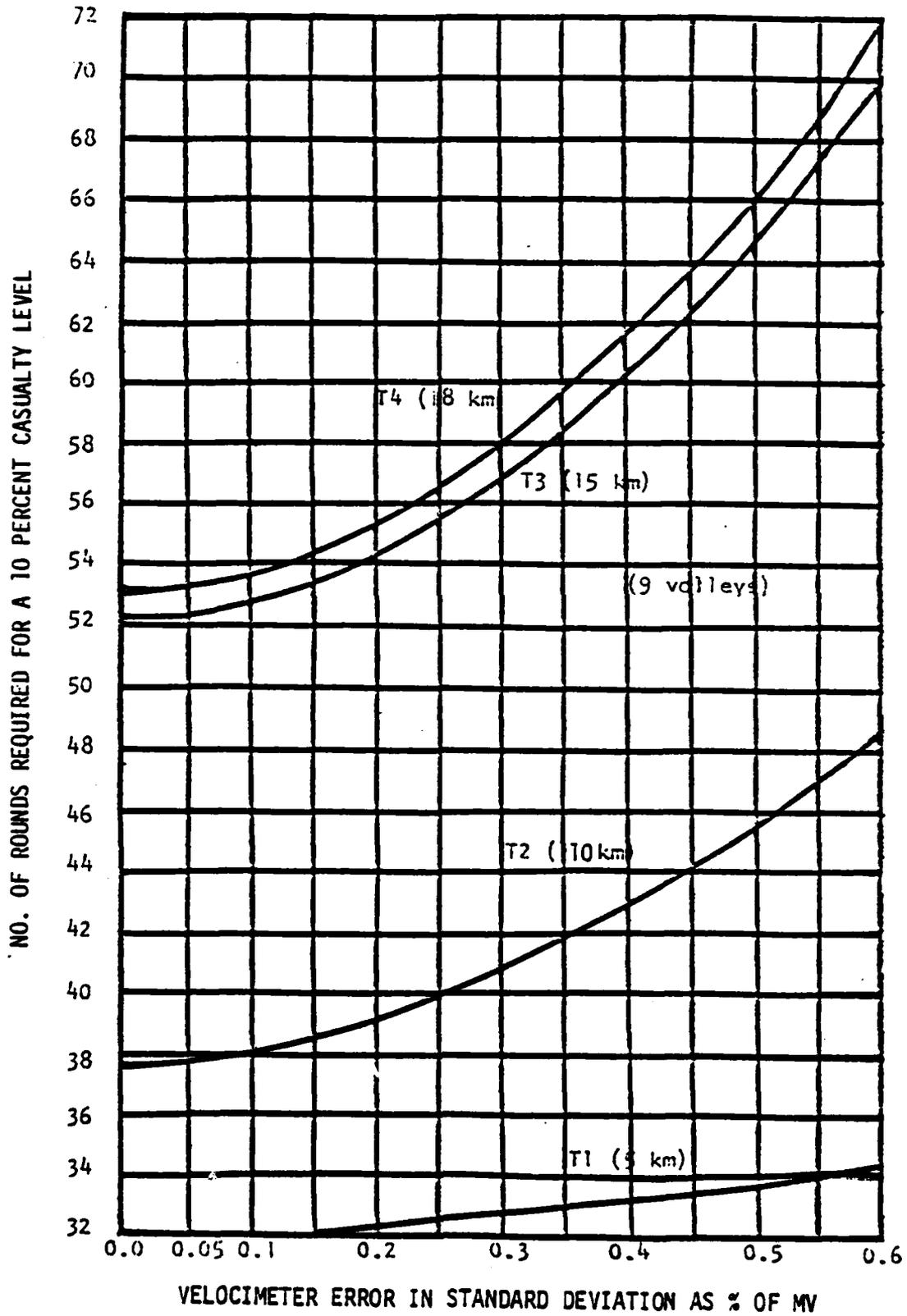
While these analyses have been presented as demonstrations and examples of the techniques involved, it should be remembered that the parameter values have been selected as falling in the range of those which would be obtained from actual contemporary Army studies, including the LEGAL MIX V study of artillery mixes and several studies of armor and mechanized force structure and weapon system issues. Accordingly, subject to sensitivity analysis (see the next section) and/or to any questions concerning the merits of the studies and models which served as data sources, the observations are realistic and applicable to the questions addressed.

At this point, all the changes examined have been completely parametric or hypothetical. In order to verify that this technique can be used on specific problems, it is necessary to demonstrate that available data for possible artillery or other weapon system changes can be used to generate quantitative estimates of the resulting changes in the  $\alpha$ 's and/or  $\beta$ 's.

Consider some typical changes and the associated data which would be available, either directly or through the use of existing artillery accuracy and effects models (including those used or referenced in [VRI, 1975] and [MITRE, 1977]). For changes in system protection, some estimate of the change in the kill probabilities -- against both personnel and systems -- of incoming rounds would be required. This data could be generated from estimates of the effects of the changes or vulnerable areas and/or personnel exposure probabilities. These data are precisely what is normally generated and used in detailed design trade-off studies of the systems, so that the methodology can be applied directly. Similar arguments apply to changes in round lethality.

For changes in system components which affect the target location accuracy or the accuracy of fires, detailed systems' accuracy models and fire effects models must be used together. A good system accuracy model is available in [VRI, 1975] and is available on TRADOC computers. It has been used in demonstration studies with other TRADOC computer routines analyzing fire effects. A published example of such analyses is reproduced in exhibit 3-7. This figure, taken from [VRI, 1975], shows the relation between the velocimeter or other velocity-estimation error and the resulting attrition per round for several fire missions. For example, consider the possible alternatives with two percent and three percent standard deviations in their muzzle velocity estimates. The rounds-to-kills ratio for these two cases is approximately five percent different for all the missions examined. (A more comprehensive examination would possibly include more fire mission scenarios, but for this

EXHIBIT 3-7: RELATING BASIC PERFORMANCE TO FIRE EFFECTIVENESS



demonstration, let these scenarios be considered representative.) This five percent difference would produce a five percent difference in the appropriate  $\alpha$ . The same techniques can be used to analyze all other accuracy-related parameters. This example demonstrates the capability of the methods designed in this project to be used in available data in their analyses.

### 3.3 *The Effect of Variations in the Base Case*

One can summarize some of the major points made in the preceding evaluations as follows:

- (1) a 33 percent improvement in artillery firepower against maneuver unit targets improves force effectiveness about 13.6 percent;<sup>1</sup>
- (2) a factor of two improvement in artillery system survivability improves force effectiveness about 3.9 percent;
- (3) addition of ten percent of a division to the Blue force, considered only in terms of its mathematically estimated effects on the attrition exchange, improves force effectiveness about 46 percent; and
- (4) a 33 percent improvement in tank firepower improves force effectiveness about ten percent.

Sensitivity analyses can examine the degree to which these conclusions are robust -- that is, how much they may change if the base case data changes.

As an initial sensitivity examination, consider the form these conclusions would have taken in the following five sensitivity cases:

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<sup>1</sup>For this sensitivity analysis, the amount-of-combat level has been selected to provide greater than 50 percent attrition to maneuver force, but not greater than 90 percent. (This led to selecting the point  $t = 1.2$  in the scaled units, corresponding to *about* one day's combat in most of the study sources.)

- (A) the Red-to-Blue maneuver force firepower ratio is 20 percent worse:  
 $\alpha_1 = .64$ ;
- (B) the Red-to-Blue maneuver force firepower ratio is 20 percent better;  
 $\alpha_1 = .96$ ;
- (C) the threat is greater by ten percent in all weapon strengths;
- (D) the threat is smaller by ten percent in all weapon strengths;  
and
- (E) the Blue artillery is only 50 percent as effective in attriting  
the Red forces:  $\alpha_2 = .075$ ,  $\alpha_3 = .3$ .

Each of these cases is within the realm of realistic variability in study estimates. Case E, which involves a large change in artillery effectiveness, would result, for example, from studies that assume artillery fires are significantly more constrained by ammunition availability than the studies generating the base data. (Such studies do exist.) Exhibit 3-8 summarizes the results of these sensitivity analyses. (The individual case results have been omitted, and only the relevant comparative data presented. Each case was generated, of course, using data equivalent to the full set of exhibits from section 3.2.)

As can be seen, the relative results are highly similar except for the artillery-related changes in Base E. Here, since the changes are expressed as proportions of the base capabilities and the base capabilities have changed drastically (by a factor of two), one must expect a similar change in the values given the improvements (and, in fact, this change is approximately a factor of two also). Even with this drastic change, one should note that the values of the example artillery firepower improvements, are, *in either case*, of the same order of magnitude as those for the example tank firepower improvements. Thus, independent of reasonable variations in the base case, choices

EXHIBIT 3-8: SENSITIVITY ANALYSIS RESULTS

	Gain with Original Base	Gain with Sensitivity Base A	Gain with Sensitivity Base B	Gain with Sensitivity Base C	Gain with Sensitivity Base D	Gain with Sensitivity Base E
Artillery Firepower Increase (see text for details)	13.6%	14.7%	13.0%	13.3%	14.2%	7.0%
Artillery Vulnerability Decrease (see text for details)	3.9%	4.3%	3.6%	4.4%	3.4%	2.4%
Additional Force Increment (see text for details)	46%	42%	49%	46%	48%	45.2%
Tank Firepower Increase (see text for details)	10.1%	10.4%	10.0%	9.0%	11.3%	10.5%

between artillery-related and tank-related anti-maneuver-force firepower improvements of compatible percentage magnitudes will be based heavily on cost and other non-attrition related considerations.

An alternative method for examining the sensitivity of various conclusions to base case data is to vary a base case parameter continuously and plot the gains from a single alternative or the ratio of a selected pair of alternatives against this parameter variation. High slopes and strong variations in the resulting plots indicate a sensitivity to the base case parameter involved; low slopes and limited variations correspond to highly robust estimates of relative values.

This sensitivity technique will be used here to display the sensitivity of the value of a ten percent increase in artillery firepower and survivability (or in the amount of artillery, since this is mathematically equivalent). Exhibit 3-9 shows such a sensitivity analysis. The base case parameter varied is  $\alpha_3$ , the effectiveness of Blue counterbattery fires. The value of the improvement is plotted using a division-scaled comparison as described above. Two cases are shown, one in which the changes in Red artillery fires due to changes in the attrition of Red artillery have no effects on maneuver force fires, and one in which it is assumed that as Red artillery fires decrease, Blue maneuver forces fire more or become more effective. In order to estimate this effect in quantitative terms, it is necessary to estimate the rough quantitative suppressive effects of Red artillery fires in the base case: for this demonstration, it has been assumed either that there is no suppressive effect or that the  $\alpha_1$  observed in the central base case ( $\alpha_3 = .6$ ) is ten percent lower than it would have been without Red artillery fire suppression. For each other value of  $\alpha_3$ , the value of  $\alpha_1$  is then linearly adjusted between the original value and 110 percent of that value, based on the total Red artillery fires (which are, of course, proportional to the average Red unattrited artillery over the course of the combat).

data is merely intended to be representative and not exact: in the example problems, sensitivity analyses will be used to show that this possible inexactness is not critical to most conclusions.

Exhibit 3-1 shows the resulting attrition rate matrices. These matrices portray a battle in which the Blue maneuver force is slightly outweighed by the Red maneuver force (with a maneuver force strength ratio of approximately .89). The total Blue artillery destroys or otherwise makes Red maneuver force weapons ineffective at an initial rate which is 15 percent of the rate at which the total Blue direct fire weapons accomplish such attrition. (The term initial rate is used because as the attrition proceeds, the relative strengths of the maneuver forces and artillery forces will change, so that the total attrition rates will change.) The remaining entries have similar meanings. These scaled data were, of course, produced by the examination of fully dimensioned killer-victim scoreboards, and the use of the scaling transformations of section 2.2. All of the example analyses could be performed on the fully-dimensioned attrition rate matrices without scaling, and identical conclusions would result. The scaling is simply a convenient mathematical technique to reduce the battle matrix to a particular canonical form.

Exhibit 3-2 shows the evolution of the central base forces as combat occurs.<sup>1</sup> A limitation of the reduction to two dimensions is that data on individual maneuver force weapons is not available. While this does not affect the examples, real analyses should include the full available

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<sup>1</sup>Heterogeneous Lanchester square-law differential equations have been used, as discussed in chapter 2, to interpolate and extrapolate from the base killer-victim scoreboard. This technique was shown in the previous study to be robust and quite accurate in predicting the actual results of more detailed models.

EXHIBIT 3-9: SENSITIVITY ANALYSIS TO INPUT  
COUNTERBATTERY EFFECTIVENESS

VALUE OF 10%  
IMPROVEMENT  
IN  $\alpha_2$  AND  $\alpha_3$  and  
10% DECREASE  
IN  $\beta_3$

.02 DIVISION

ASSUMING RED  
ARTILLERY HAS  
10% SUPPRESSIVE  
EFFECT ON BLUE  
MANEUVER FORCES  
IN BASE CASE

.01 DIVISION

ASSUMING RED  
ARTILLERY HAS  
NO SUPPRESSIVE  
EFFECT ON BLUE  
MANEUVER FORCES

.6

$\alpha_3$

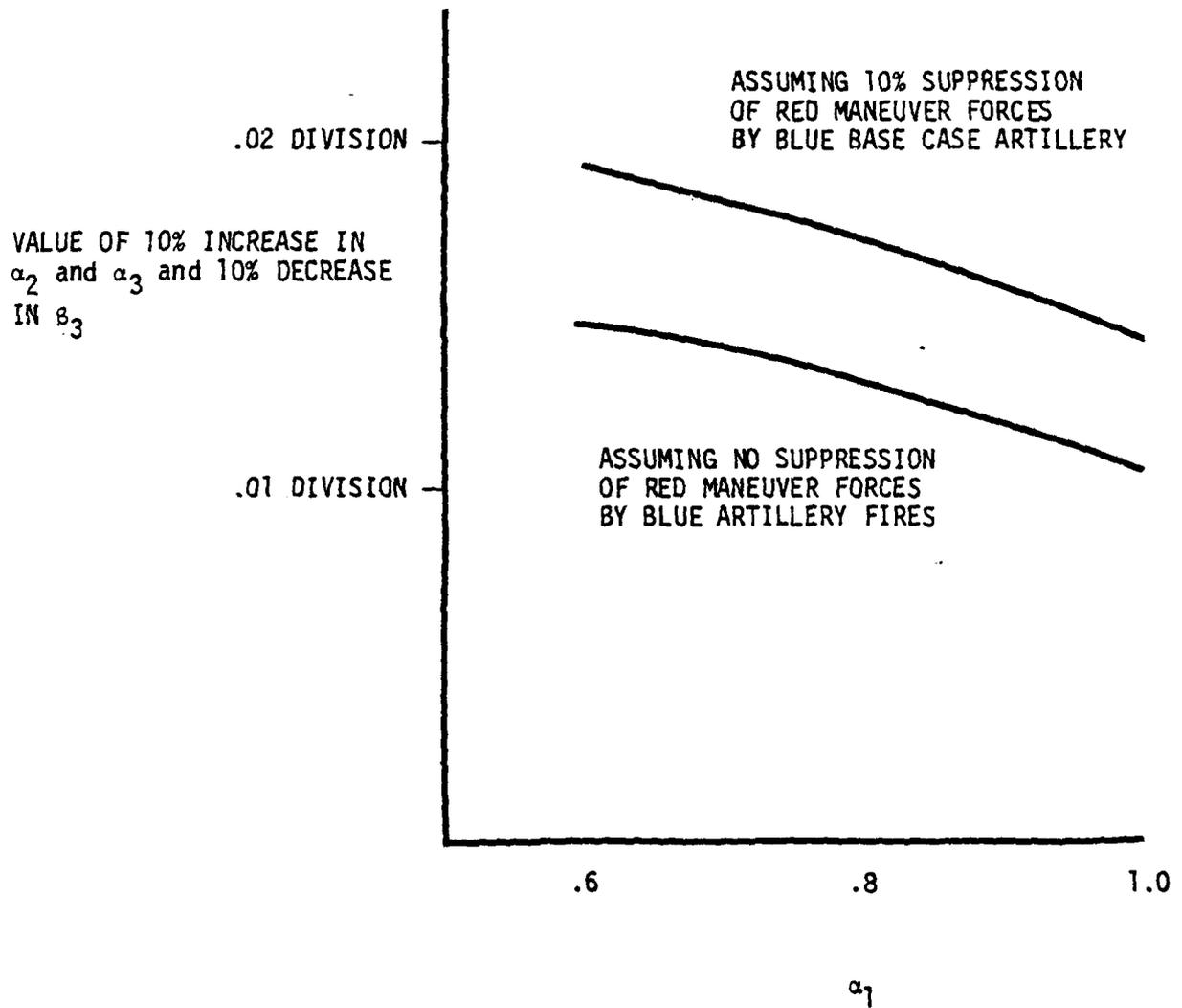
Exhibit 3-10 shows a similar sensitivity analysis of the effects of varying the input parameter  $\alpha_1$ , representing Blue maneuver force firepower. In both cases, the graphical percentations confirm the earlier impressions that the effects of base case variations on results are not severe.

As a further demonstration of the capabilities of this method, consider the analysis of the effects of the distribution of artillery fires between maneuver unit targets and counterbattery targets. In the studies providing the base case data, about one third of the rounds fired were fired at counterbattery targets. A natural question, even though not precisely directed at a weapon system design change, is: "What would be the improvement or degradation in force effectiveness if this fraction were changed?" The methods described here can address this problem.

Consider decreasing counterbattery fires. Assuming the availability of appropriate ammunition, this would result in possible increases in fires against maneuver unit targets. Taking this as a straight round-for-round tradeoff for this analysis, one would use, for example, the possible firing patterns

$$\begin{array}{ll} \alpha_2 = .15, & \alpha_3 = .6 \quad (\text{base case}) \\ \alpha_2 = .16, & \alpha_3 = .52 \\ \dots & \\ \alpha_2 = .225 & \alpha_3 = 0.0, \end{array}$$

and for similar increases in counterbattery fires, again assuming a round-for-round tradeoff, with all rounds fired in each role providing the original average effectiveness

EXHIBIT 3-10: SENSITIVITY ANALYSIS TO INPUT MANEUVER  
FORCE EFFECTIVENESS

$$\begin{array}{ll}
 \alpha_2 = .14, & \alpha_3 = .68 \\
 \alpha_2 = .13, & \alpha_2 = .76 \\
 \dots & \\
 \alpha_2 = .075 & \alpha_3 = 1.2 \\
 \dots &
 \end{array}$$

As analysis cases, take

$$\begin{array}{ll}
 \alpha_2 = .20, & \alpha_3 = .20 \quad (\text{decreased counterbattery}) \\
 \alpha_2 = .10, & \alpha_3 = 1.0 \quad (\text{increased counterbattery}).
 \end{array}$$

Consider each case under both the no-suppression and ten percent suppression assumptions used in the earlier analyses. Computing the maneuver force exchange ratio, one obtains

CASE	RESULTS
Decreased counterbattery, assuming no suppression	9.4 percent better than base case
Decreased counterbattery, assuming 10 percent suppression	0.2 percent worse than base case
Increased counterbattery, assuming no suppression	9.5 percent worse than base case
Increased counterbattery, assuming 10 percent suppression	0.2 percent worse than base case

On the basis of these results, two major conclusions can be reached:

- (1) the appropriate level of counterbattery fire, considering its overall effects on the total exchange of attrition, is highly

dependent on the degree to which artillery fires result in suppression of maneuver unit firepower -- that is, in reductions of maneuver unit attrition rates.

- (2) at reasonable levels of suppression effectiveness, a counterbattery fraction of one-third is near optimal, in terms of the effects on the total attrition exchange. As Blue force protection is increased and Red artillery fires become less suppressive, Blue counterbattery allocations should decrease in favor of fires on maneuver units.
- (3) this optimum, in cases with reasonable suppression assumptions is a wide, flat optimum. Significant increases or decreases in the allocation of fires away from the optimum cause very limited decreases (of the order of tenths of a percent) in force effectiveness.

This example concludes the sample problems addressed in this study. At this point, it is worth discussing the degree to which these sample problems may themselves be useful analyses, as well as examples of the methods developed in this project. As has been discussed, the data used in this sample work has been based on detailed analyses from the Department of Defense games and simulations. Further sensitivity analyses have showed that the results in the examples are reasonably robust and invariant as the data is changed. Accordingly, the sample analyses must be considered accurate studies of the problems addressed. Although other analysts will wish to check the conclusions using their own data sources and methods,

it must be anticipated that as long as their data agrees generally with that in the spectrum of data sources used in these examples, their conclusions will also agree.

These example analyses have shown how the methods developed in this project can be useful in analyzing the comparative value of diverse research and development programs including those involving counter-weapons and also in addressing other planning and doctrinal issues related to counterweapon design and use. The next chapter discusses the major areas in which the methodology is limited.

#### 4.0 LIMITATIONS AND REQUIREMENTS FOR ADDITIONAL RESEARCH

The major limitation of the methodology developed in this research, as has been made clear throughout this report, is that it only treats force changes designed to impact force effectiveness through changes in weapon system attrition results. It, therefore, cannot compare the values of developments which could reduce the procurement or operations costs of possible future forces with developments affecting attrition. It also omits all other components of force effectiveness, including political and foreign relations areas, ground control and FEBA movement measures, assessments of risk, evaluation of personnel casualties, etc. Within these limitations, however, it provides a broad, general methodology which can be used on the wide spectrum of decisions which are within its scope.

A practical limitation should also be mentioned; the methods have not been applied in conjunction with supporting cost analyses. Until true cost and effectiveness analyses are conducted together, much of the potential utility of the methods will remain only potential. In fact, in this area, one methodological problem is still unsolved. That problem is the design of proper, distinct treatments for research and development, procurement, and operating costs for use with this type of evaluation, which is intimately related to the problem of scope mentioned above. The techniques, without the design of special development-cost-versus-future-procurement-cost (or operating cost) as well as force effectiveness analysis methods, will not be capable of comparing cost-reduction projects with

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effectiveness-improvement projects, or of properly dealing with projects with combined goals (as when a new system may have increased firepower -- affecting attrition results -- and lower maintenance costs).

As was remarked earlier in chapter 1, many detailed methodological tools, principally combat simulations or field tests, experiments, or exercises, exist to deal with force effectiveness analysis. However, the detailed analysis of the potential contribution of a developmental military system requires major commitments of time and resources. For many purposes during the decision-making processes involved in the design and modification of an overall Army development program, it is important to examine potential contributions of conceptual developments in significantly reduced times and with limited resources. The methods studied in this research have been designed to complement detailed studies with a simple, flexible, methodology which can use as inputs such data from detailed studies as the "killer-victim scoreboard" reporting the systems attrition which might be expected in hypothetical battles or sequences of battles, and which can combine such data with systematic analytic parametric or judgmental inputs as to the possible firepower performance areas as acquisition rates or probabilities, accuracy of fire, rate of fire, and lethality of fire, and such weapon survivability areas as enemy acquisition rates or probabilities, accuracy of enemy fire, rate of enemy fire, and vulnerability to enemy fire in order to extrapolate and/or bound the probable contribution of such changes to force effectiveness. The methods are not intended to replace or compete with more detailed analysis methodologies for accuracy, but to provide quick, limited answers to questions.

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