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ARMY WEAPON SYSTEMS ANALYSIS, PART TWO

TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

CHAPTER 27
DETECTION PHENOMENA, CHANCES OF TARGET DETECTION, AND SEARCH STRATEGIES

CHAPTER 28
INTRODUCTION TO COMBAT THEORY AND ITS APPLICATIONS — HOMOGENEOUS FORCES

TABLE OF CONTENTS (cont'd)

CHAPTER 29

LANCHESTER COMBAT THEORY — MODEL EXTENSIONS; HETEROGENEOUS FORCES; AND COMMAND, CONTROL, AND INTELLIGENCE EFFECTS

TABLE OF CONTENTS (cont'd)

CHAPTER 30
WEAPON EQUIVALENCE STUDIES

CHAPTER 31
OPTIMAL FIRING POLICIES FOR SINGLE AND MULTIPLE WEAPONS

CHAPTER 32
WEAPON-TARGET ALLOCATION PROBLEMS

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

CHAPTER 35
WEAPON SYSTEM LIFE CYCLE COST ESTIMATION (LCCE)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

CHAPTER 39
COUNTERMEASURES AND THEIR ANALYTICAL TREATMENT

CHAPTER 40
INTRODUCTION TO WAR GAMES AND COMPUTERIZED SIMULATIONS OF COMBAT

TABLE OF CONTENTS (cont'd)

CHAPTER 41
AN EVALUATION OF SOME MIXES OF INFANTRY SMALL ARMS WEAPONS

TABLE OF CONTENTS (cont'd)

CHAPTER 42
AN EXAMPLE OF TANK WARFARE IN THE AGE OF THE GUIDED MISSILE

CHAPTER 43
AN EVALUATION METHOD FOR ARTILLERY OR SUPPORT TYPE WEAPON FAMILIES

CHAPTER 44
THE MODERN GUN EFFECTIVENESS MODEL (MGEM) FOR EVALUATING AIR DEFENSE GUNS

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

CHAPTER 46
COST AND OPERATIONAL EFFECTIVENESS ANALYSIS OF THE WICV-WOW ARMORED INFANTRY FIGHTING VEHICLE

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (cont'd)

LIST OF TABLES

LIST OF TABLES (cont'd)

LIST OF TABLES (cont'd)

PREFACE

The *Army Weapon Systems Analysis* (AWSA) Handbook consists of two parts and has been prepared to record an extensive field in a condensed form, i.e., some of the highlights of weapon evaluations developed by the Army since about 1943. The need to describe and to standardize weapon evaluation methodologies insofar as possible also existed — the AWSA Handbook attempts to satisfy this need. *Parts One* and *Two* are in separate parts — *Part One* contains Chapters 1-24, and *Part Two* contains Chapters 25-46. A brief but fairly informative description of *Part One* is given at the beginning of Chapter 25.

Although *Part Two* covers some of the more advanced topics of the field of Army Weapon Systems Analysis, it starts with the definition of and concepts relating to measures of effectiveness (MOE), and describes in some detail many MOE's. The aim is to point out that MOE's are not universal but may depend on particular evaluations, and the Army analyst is introduced to the relation between the problem of modeling processes and MOE's. After an introduction to target detection phenomena and to the development of target detection probabilities, the important topics of Lanchester type combat theory for homogeneous and heterogeneous forces are given in much depth since these topics lead up to weapon equivalence concepts and studies. For the present-day analyst, the fields of optimal firing policies, weapon-target allocation problems, human factors, and cost analysis estimation must be rather thoroughly covered — at least to the extent herein. Moreover, it was felt important to include also an introduction to cost-effectiveness evaluations, the concepts of survivability, and an introduction to countermeasures and their analytical treatment. We describe some of the prime topics in the history of war games and combat simulations, including developments and uses, and brief descriptions of some of the key war games or computer simulations of combat. The last chapters of *Part Two* cover evaluation techniques for infantry weapons, tank weapon systems, artillery families, air defense (Modern Gun Effectiveness Model), and the principles and an illustration of cost and operational effectiveness analyses.

With this brief but cursory explanation of the contents of the AWSA Handbook, we believe it should be clear that both parts contain sufficient depth of subject matter to provide both an appropriate background for the young analysts entering the field of Army military operations research, and a valuable source of reference material for the practicing systems analysts. An attempt has been made to prepare the Handbook in somewhat of an elementary manner; derivations were kept to a minimum by citing pertinent references in the operations research literature. Nevertheless, it is realized that some suitable background in the way of military operations research theory and symbolic representation is necessary to record many key results of which the Army analyst should have occasional use. We believe that the cited references and the bibliographies within the chapters will suffice to give much valuable source material for those who desire to acquire a more extensive knowledge of the subjects discussed herein, or to provide a base on which interested and capable analysts could perform further research on the methodology. For those readers primarily interested in applications, we have provided many pertinent examples or illustrations to indicate just how theories or models may be used.

As was the case for *Part One*, *Part Two* is also predominantly the contributions and work of Dr. Frank E. Grubbs — formerly Chief Operations Research Analyst of the US Army Ballistic Research Laboratories — who prepared both parts for the Engineering Design Handbook Office of the Research Triangle Institute, Research Triangle Park, NC, prime contractor to the US Army Materiel Development and Readiness Command (DARCOM). Some of the material of Chapter 27 on detection phenomena et al. has been based on a draft prepared by the ARINC Corporation in the early 1970's, and we have leaned on the contributions of many military operations research analysts for their fine publications in the literature. In connection with the preparation of this handbook, we are indebted to Dr. Robert J. Eichelberger, Director of the US Army Ballistic Research Laboratory, for his support which contributed in a major way to the accomplishments recorded herein.

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CHAPTER 25
INTRODUCTION TO PART TWO, ARMY WEAPON SYSTEMS ANALYSIS HANDBOOK

This introductory chapter gives a brief account of the contents of PART ONE (*Chapters 1-24 of the* ARMY WEAPON SYSTEMS ANALYSIS *Handbook (AWSA) in the form of chapter titles and abstracts or summaries, thus recording in this* PART TWO *an account of the topics which have been discussed and covered in the separate volume.*

PART TWO *of the AWSA, covering Chapters 25-46, is then described in pertinent discourse for the remainder of this chapter so that the reader may have a reference guide and obtain a suitable understanding of the extended methods of weapon systems analysis covered in this separate and final volume.*

25-1 INTRODUCTION

The *Army Weapon Systems Analysis* Handbook (AWSA) consists of forty-six chapters which cover the methods of military operations research used in the evaluations of weapons or weapon system potential. The Handbook is divided into two parts. *Part One* covers twenty-four chapters of the more or less introductory material which the young or practicing weapon systems analyst will often have need of in his work. Chapters 1-24 of *Part One* are described in par. 25-2 in the form of chapter titles and abstracts or summaries. This coverage of chapter contents should provide the reader with a useful account of the methods and techniques which will have rather wide applications to many problems in the analysis of weapon performance.

Part Two of the *Army Weapon Systems Analysis* Handbook consists of Chapters 25-46, inclusive. These chapters cover some of the more advanced topics in the field of weapon systems analysis, including measures of effectiveness (MOE); target detection phenomena and probabilities; combat theory for homogeneous and heterogeneous forces; weapon equivalence studies; optimal weapon firing policies; weapon-target allocation problems; human factors and human engineering; analysis of costs; cost-effectiveness studies; survivability considerations; countermeasures and their analytical treatment; war games; computerized combat simulations; some example evaluations of small arms, tank and antitank weapons, field artillery, and air defense weapons; and cost and operational effectiveness analyses (COEA's). In order to orient the reader properly and indicate the coverage more precisely, a brief discussion of these topics of *Part Two* is given in par. 25-3.

25-2 BRIEF DESCRIPTION OF THE *ARMY WEAPON SYSTEMS ANALYSIS, PART ONE,* HANDBOOK

In the following, we present an informative and comprehensive account of Chapters 1-24 of the *Army Weapon Systems Analysis, Part One,* Handbook in the form of the various chapter titles and the abstracts or summaries. This type of presentation should provide the reader with a good synopsis of the analytical methods covered, as well as indicate just where to locate any material of possible interest.

PART ONE — ARMY WEAPON SYSTEMS ANALYSIS HANDBOOK

Chapter 1. Background and Purpose of the Army Weapon Systems Analysis Handbook

A brief sketch is given of the historical development and value of military operations research and systems analysis in the US Army. The purposes of the handbook are also outlined.

Chapter 2. What Is Operations Research/Systems Analysis?

Definitions are given for the relatively new fields of operations research and systems analysis (OR/SA), and some current OR/SA terminology is discussed.

Chapter 3. Handbook Content and Use

An overview of the content and use of this Handbook (PART ONE) *is presented.*

Chapter 4. Objectives and Applications of Weapon Systems Analysis

This chapter describes the objectives and benefits sought by the Army through the performance of weapon systems analysis. The chapter also addresses the progression of a weapon system from concept through development and deployment to disposal.

Chapter 5. Documentation and Management of Weapon System Resources

An account is given of some of the goals, documentation, and management of weapon systems resources in the Department of Defense (DOD) and the Army.

Chapter 6. Role of the Systems Analyst

The role of the weapon systems analyst is discussed in sufficient detail to indicate the character, scope, and boundaries of his general activities.

Chapter 7. Role of the Decision Maker

The key role of the decision maker in the review and implementation of the weapon systems analysis studies is characterized and highlighted.

Chapter 8. The Sphere of Conflict

The types of war, intensities of conflict, levels of commitment; army combat functions, objectives, operations, and trends; and the army combat organizations are discussed.

Chapter 9. The Physical Environment

The nature and effect of the physical environment in combat on general weapon employment is discussed.

Chapter 10. Some Fundamentals of Offense, Defense, and Target Damage Assessment

An introduction to offensive actions, defensive actions, and target damage assessment is given to enlighten the analyst in such phases of combat. The "shoot-look-shoot" tactic and methodology are discussed also.

Chapter 11. Factors Affecting Target Selection

Some of the problems of detecting, acquiring, locating, and engaging enemy targets by friendly weapon systems are discussed. Also, problems relating to target analysis, worth, assignment, reaction, and recovery are introduced. The *scope of the chapter is rather elementary and introductory since the overall target acquisition problem is of wide scope and must be covered in more detailed analyses elsewhere. The importance of timely detection, acquisition, and engagement of enemy targets cannot be stressed too much, however, since the efficient utilization of friendly weapons is critically dependent on target selection and engagement. Target detection chances are introduced.*

Chapter 12. The Scenario

The use of the scenario is examined as an important tool in the evaluation of weapon systems. The chapter explains how study objectives, assumptions, limitations, and specific guidance received from the sponsor of the study — in addition to operational factors — are used to simulate realistic conflict situations.

Chapter 13. Weapon Delivery Error Characteristics and Distributions

Described are delivery error distributions, for the impacts of rounds fired from a weapon, which are commonly used in evaluations, along with the concepts of probable error (PE), circular probable error (CEP), and some preliminary coverage of the probability of hitting. The problem of estimation of parameters of delivery error distributions also is considered briefly.

Chapter 14. Probability of Hitting for Single Rounds Single-Shot Hit Probabilities

A description is given of the methods of calculating the chances of hitting targets of different shapes for the case of single or individual rounds. The methodology includes both exact and approximate techniques for determining hit probabilities for the cases of the centered aim point and offset aim point.

Chapter 15. Vulnerability and Lethality

Vulnerability of targets to attack, and the lethality of warheads against personnel or soft targets are presented from the point of view of the weapon systems analyst. In particular, the analyst must deal with the basic concepts of vulnerable areas and lethal areas, or "mean areas of effectiveness", in his evaluation of weapon systems.

Chapter 16. Rates of Fire

In view of the importance of the rate of fire of weapons, this topic is introduced and explored in some preliminary detail for the weapon systems analyst.

Chapter 17. Introduction to Stochastic and Other Duels

Hit probabilities, conditional chances that hits are kills, and rates of fire are the basic parameters in the analysis of duels. These quantities are combined in models for stochastic duels, and the chances of winning can be determined for various firing strategies, thereby predicting weapon performance.

Chapter 18. Response Time

Some implications of weapon response times are discussed.

Chapter 19. Fuzing

Fuze action generally has some effect on both the delivery accuracy of projectiles or missiles along their trajectories and also on the terminal effectiveness of the warhead. Thus, the analyst must be familiar with the principles of fuze operation and evaluate fuze performance, including in particular random variations, which must be taken into proper account in the analysis of weapon systems. The reliability and safety of fuzing systems represent major considerations to be reckoned with.

Chapter 20. Multiple Round Hit Probabilities, Target Coverage, And Target Damage

Methods for calculating multiple round hit probabilities (usually the chance of at least one hit), the fractional coverage of targets for salvos of rounds, and the fractional damage to targets (casualties) for salvos are covered. The models used necessarily must take into account the round-to-round ballistic dispersion, the aiming errors for multiple rounds, the correlation between rounds for automatic or target tracking weapons, target sizes and characteristics, target vulnerability, and warhead lethality. Moreover, suitably accurate approximations must be used.

Chapter 21. Reliability, Life Testing, Reliability Growth, Availability, and Maintainability

Due to the ever increasing complexity of materiel and the demand for high quality, we can say that reliability, life-testing, maintainability, and availability now represent some of the more important characteristics of weapon systems requiring accurate evaluation. The weapon systems analyst must be thoroughly familiar with certain life-time or failure-time distributions. Therefore, we cover here the exponential, the lognormal, the Weibull, the gamma, and the binomial reliability distributions, and how they are applied to the evaluations of weapons. We cover also the estimation of population parameters, the system reliability, and how to determine confidence bounds on system reliability from component test data. Some considerations of the analytical aspects of high reliability are discussed for the analyst, as well as the concept of tolerance limits for distributions. On occasions, availability and maintainability analyses will be required of the analyst and are therefore introduced.

Chapter 22. Mobility, Maneuverability, and Agility

The concepts of mobility, maneuverability, and agility have defied definition, quantification, and adequate modeling for many, many years. Nevertheless, the weapon systems analyst must be thoroughly conversant with such measures of effectiveness and often take them into consideration in his evaluation process. The description given in this chapter should give the analyst a good introduction to some of the principles involved.

Chapter 23. Logistic Planning and Support

The design, development, production, and deployment of weapon systems must take into account the problems of logistic planning and support; therefore, the evaluation of weapons necessarily involves the quantification of logistical factors in the overall process. Some of the considerations for logistic planning and support type factors for the systems analyst are covered, indicating the need for the analysis of a complex stochastic area of endeavor.

Chapter 24. The WSEIAC Evaluation Model

An account is given of the Weapon Systems Effectiveness Industry Advisory Committee (WSEIAC) model or methodology for evaluating weapon systems. This study of methodology was performed for the US Air Force in the mid-1960's and attempts to evaluate weapon systems on the basis of three primary factors: (1) Availability (readiness), (2) Dependability (reliability), and (3) Capability (performance). These three factors are converted to a single measure of effectiveness which characterizes the overall performance of a weapon system. Examples illustrating the methodology are given.

25-3 BRIEF DESCRIPTION OF THE CHAPTERS OF THE *ARMY WEAPON SYSTEMS ANALYSIS, PART TWO,* HANDBOOK

In the paragraphs that follow, we outline the contents of Chapters 26-46 of the *Army Weapon Systems Analysis, Part Two,* Handbook, giving some guidelines on their usefulness and characteristic formulations.

Chapter 26 gives a rather broad and informative introduction to Measures of Effectiveness (MOE). A MOE is a criterion expressing the extent to which a combat system or a weapon performs its mission assignment under a spe[illegible]d set of conditions. Almost any characteristic of a weapon or weapon system (unfortunately) may be a measure of effectiveness, however. For example, weapon delivery error characteristics, such as the Circular Probable Error (CEP), hit probabilities, and kill chances, represent possible MOE's. The analyst is warned that the proper choice of a MOE for an evaluation is often difficult, but it is mandatory nevertheless. It is recommended that the practicing analyst attempt, if at all possible, to select that MOE which possesses an overall description of system effectiveness. In this connection, the reader may note, based on many topics covered in the Handbook, that kill rates appear to have a very central role. Otherwise, some extensive study may be required for various applications. Many examples of MOE's are given in Chapter 26, including an instructive one for the infantry rifle.

A critical problem during battles is that of timely detection, identification, and the bringing of effective fire on enemy targets. For this reason, *Chapter 27* presents a discussion of some of the basic phenomena that are employed in target detection devices or equipment. This leads to some of the models which are found to be useful in describing the probability of detecting a target. It is found that signal-to-noise ratio is important, and that the target range and atmospheric conditions also represent critical parameters. Terrain and vegetation also play an important role. The chapter is aimed at giving the young weapon systems analyst both a competent background and a proper respect for the problem of detecting targets on a timely basis, for otherwise our friendly weapon systems would not exhibit their potential effectiveness. Some accounts of search strategies are also covered.

In *Chapter 28*, we tackle one of the central problems of weapon systems analysis, i.e., the development of models or theories which describe combat accurately between opposing forces. Frederick William Lanchester is widely recognized as the pioneer who began to develop the theory of combat in about 1914, and many combat models carry his name. Our aim in this chapter is to present some of the more basic or preliminary combat laws which have been used rather widely or employed to advantage by weapon systems analysts. These include the famous Lanchester Linear Laws for direct fire or area fire, the Lanchester Square Law, the Logarithmic Law, the Mixed Law or Deitchman's Guerrilla Warfare model, and a new formulation of combat theory which analyzes target kill-times to predict the course of a battle. The validation of Lanchester Laws is discussed and the estimation of attrition coefficients covered. Also, the transition probabilities during battles and chances of a side's winning are formulated and discussed. In some cases, such as for exponential kill-times, stopping rules on when to stop a combat simulation may be developed which will control the risks of errone[illegible] judgments concerning the battle outcome. Many useful examples are displayed for the reader. [illegible]apter 28 is developed around the concept of homogeneous forces, i.e., for similar weapon systems on a side.

The combat type formulations for homogeneous forces having been covered, the next step is to extend models to cover heterogeneous forces or the combined arms. The central problem here is to describe the relative or potential effectiveness of combined arms teams, employing, for example, infantry, artillery, tanks, and antitank weapons, jointly and simultaneously. *Chapter 29* covers additional terms for the Lanchester equations whic[illegible] may involve, for example, resupply; additional production; and noncombat losses due to accidents, diseases, and epidemics. The idea of range-dependent attrition coefficients and the generalization of the Lanchester Laws for line-of-sight considerations are presented, and a basic theory for combined arms or heterogeneous weapon systems is discussed for the analyst. All of this material leads up to the problem of searching for methods which will lead to equivalence relations between weapons of different types.

The weapon systems analyst would surely be at a great loss if he were not properly equipped to handle the problem of weapon equivalence studies. Therefore, the next chapter, *Chapter 30*, is devoted to presenting methodology which can be used to determine equivalence relations for diverse or heterogeneous weapon systems. In fact, a Lanchester type relationship is found between opposing forces (1) employing heterogeneous weapon systems and (2) employing equivalent homogeneous type weapons. This is developed through the important and central parameter in all weapon systems analysis studies — namely, kill rates. The theory is developed to display the determination of relative weights or values of different weapon types in a conflict. Also, killer-victim scoreboards are discussed and analyzed, and the force ratio as a function of battle time is given. Several useful applications to typical weapon analysis problems are covered for the young or practicing analyst.

Although the course of battles may often take on more or less a random form of excursion, it is nevertheless worthwhile to consider optimal policies for the firing of weapons and the best allocation of weapons to targets that appear on the battlefield. These two topics are presented in Chapters 31 and 32. As will be seen, there are gains to be realized from either or both of the suggested practices of employment of weapons during combat.

Published literature on optimal firing policies for weapons is not very extensive, although there is nevertheless something to say for conserving ammunition and firing so that the effectiveness of a weapon may be maximized in some way. Thus, there is no point in firing rounds at the remote ranges for which hit and kill chances are very small; and, given a fixed number of rounds or some boundary conditions on amount of ammunition supplied per day, the firing procedures must be conducted to best defend friendly elements. Simply stated, it becomes desirable to know the opening range that Blue should open fire on approaching Red troops, and to allocate his rounds so that maximum effectiveness is attained in protecting Blue—especially for a given or limited number of rounds available for firing. This indeed is the type of problem approached in *Chapter 31*, and it is found that one must develop the concept of a "gain" function to be used. In other words, Blue must consider what can be gained; or better still the "value received" by Blue overall will depend on the distance at which Red can be annihilated, since Blue would neither like to have his position overrun nor would he like to risk too much to close-in fighting either. Optimal policies for firing a single weapon at a target are discussed in Chapter 31, and examples of problems which can be solved are given. Also, the problem of firing many or diverse types of weapons is discussed.

Ammunition often is wasted by firing at targets in an indiscriminate manner. Therefore, it becomes highly desirable, and in fact leads to improved weapons effectiveness, to allocate particular weapons to engage specific targets which the weapons are capable of defeating in some systematic fashion. This brings up the idea of investigating methods for development of the best allocations of weapons to targets as covered in *Chapter 32*. There are many different models and procedures available for allocating weapons to targets. Some are rather involved or intricate and, for example, use linear programming techniques, dynamic programming procedures, Lagrange multipliers, or other methods of allocation. Chapter 32 presents some of the more worthwhile and useful methods or models for best allocations, and also presents a number of illustrative examples, so that the weapon systems analyst may find his evaluation requirements available in a single location. Weapon-target allocation factors determined in accordance with the principles of Chapter 32 are also needed for the generalized Lanchester models or laws covered in Chapter 29 — for example as indicated in Eqs. 29-42 and 29-43, or Eq. 29-53.

Weapons or weapon systems should not be evaluated without paying critical attention to the performance of military personnel who operate the weapons, i.e., the effect of human factors or human engineering aspects of the problems of combat. Therefore, *Chapter 33* is devoted to an introduction to human factors and their interface problems with the analysis of weapons or weapon system performance. It is seen that the analyst often will have to analyze and quantify, usually on a statistical basis, the reliability and performance characteristics of the military personnel who employ the weapons. Therefore, we discuss some of the typical human engineering type problems the weapon systems analyst might face in his evaluations and give some examples of just how human factors problems can be handled. Chapter 33 aims at giving the young weapon systems analyst a good start at and a proper appreciation for human factors and weapon analysis interface activities.

Historically, analysis methods were initially developed primarily to evaluate or estimate the field performance (effectiveness) of weapons, and costs were not then of any major consideration. As time went on, however, it was found that methods to estimate, analyze, and model weapon system costs became mandatory indeed since available resources are very definitely of a finite character. Therefore, Chapters 34, 35, and 36 are devoted to the introduction of cost analysis problems. In order to provide the young or practicing analyst with proper background knowledge, a rather broad introduction to the problem of cost analyses for Army Weapon Systems is taken up in *Chapter 34* to provide some general guidelines. Indeed, the practicing weapon systems analyst must strive now to include costs as a major parameter which must be properly evaluated in his weapon systems analysis studies. The study of all possible costs of weapon systems becomes necessary, of course, in the cost-effectiveness studies of Chapter 37 and also for the cost and operational effectiveness analyses (COEA's) covered in Chapters 45 and 46.

Chapter 35 takes up the important and now central problem of life cycle cost estimation (LCCE) of weapon systems. Life cycle cost estimation must include costs for the research and development phase; the investment or procurement phase; and the entire operating and support phase for weapon systems, including its manned personnel structure. Both the "bottoms-up" or engineering type approach and the "top-down" or analytical and statistical approach to the problems of weapon systems cost estimation are covered. Techniques of using cost estimation relations (CER) are discussed, especially in terms of the regression approach which relates costs to the primary parameters of interest. Of course, the estimation of the useful life of a weapon system is important and includable in the cost analysis process. Chapter 35 covers an extensive example relating to life cycle cost estimation for the Utility Tactical Transport Aircraft System (UTTAS).

The weapon systems analyst must be acquainted with some rather special cost estimation techniques, and some useful ones are covered in *Chapter 36*. For example, the concept of the so called "learning curve" is important, and its derivation is covered in appropriate detail for the analyst. Also, the Program Evaluation and Review Technique (PERT) may often be encountered by the weapon systems analyst, so it too is introduced. Moreover, as should be expected, there are numerous design changes during development; accordingly, the cost analyst may well have to model the cost aspects of some of these types of occurrences. Finally, there is much interest in reliability growth of weapon systems since design changes and/or quality control production methods have to be considered in connection with improvement of system reliability. The problem of estimating costs in this connection is only beginning to be studied.

With appropriate background material involving models or methods for the analysis of weapon performance and techniques for estimating costs of weapon systems, the analyst is now ready to conduct some cost-effectiveness type studies. Cost-effectiveness type studies are introduced in *Chapter 37*, giving the analyst some general and specific guidelines. In a cost-effectiveness study, the systems analyst may have the opportunity either to fix the cost for competing weapons or systems and then estimate the performance of them, or he may first calculate the effectiveness of two or more different weapon systems and then develop their overall costs, perhaps on a life cycle basis, to determine which system would be best on a "cost-effectiveness" basis. Both procedures are covered in Chapter 37, and illustrative examples are given for the two different methods of analysis.

The concept of target vulnerability and some of its analytical treatment were introduced and discussed in Chapter 15. The less vulnerable a target is to attack, the higher its chance of "survival". In recent years, more and more emphasis has been placed on assuring the survivability of personnel and systems in the field. If one were to take the term "survivability" literally, it would involve very general and broad studies of all aspects of the problem of survivability of personnel and equipment in combat. In fact, would it not be natural to study all systems from the standpoint of their chance of surviving in the field under combat and other conditions of usage? It might be argued, for example, that survivability is about as broad and encompassing as the field of weapon systems analysis itself! Nevertheless, as it is turning out, the newer area of survivability is one that is currently being defined with reference to what now exists, and it would seem that survivability should probably be includable within the scope of weapon systems analysis and related activities. In *Chapter 38*, the term survivability is defined in line with some of its current trends, and the analyst is introduced to some of its more promising features. For example, systems should be designed and used so that they are not easily detected, and if and when they are in fact detected, it is best for the system to be small and compact so that it is not easily hit. Otherwise, taking cover may be necessary. Once a system is "hittable", then consideration should be given to its being as invulnerable as possible. Finally, some design considerations should be given to ease of repair of systems on the battlefield so that they may be returned to action as soon as possible. Chapter 38, by using these guidelines, develops several areas of interest to the analyst so that he will be cognizant of them in his evaluation problems. Much of the current interest in survivability appears to involve development or engineering details. It is seen in this connection that the field has a long way to go in terms of the overall analysis aspects of survivability.

More and more frequently, the weapon systems analyst finds himself in the midst of evaluation problems involving countermeasures. In fact, it can be said that combat itself is a series of measures and countermeasures, then counter-countermeasures, etc. As soon as either side places a new or more potent weapon on the battlefield, the other side has to learn to counter it in some way or reduce its effectiveness. Thus, it is natural and necessary that the weapon systems analyst sometimes will become involved in the analysis of countermeasures in the normal course of his duties. The purpose of *Chapter 39* is to acquaint the weapon systems analyst with some countermeasures in warfare and their analytical treatment. In fact, Chapter 39 provides some of the basic definitions and concepts and some of the simpler analytic framework for evaluating measures and countermeasures, along with some specific examples. It is found that statistical analysis procedures often aid in the Commander's decision processes, at least in some typical areas of interest where the analyst might become involved. An interesting feature of measures and countermeasures is that the concepts lead rather naturally to the play of "games" by opposing sides. Thus tactics and counter-tactics often come into consideration, thereby developing the basis for scenarios which become useful in the play of war games.

With *Chapter 40* on war games and computerized simulations of combat, the handbook takes a very decided turn in presentation and content. With the present state of the art in the analysis of weapon systems, all problems cannot be handled with available mathematical or operations research models since present theory is just too limited in scope. Indeed, there are many, many situations for which it is desirable or necessary to evaluate weapons when analytical models will not suffice at all. Therefore, it is necessary to resort to use of war games or simulations in order to determine the worth of weapons in a hypothesized combat environment. We introduce and cover many of the important aspects of war games and combat simulations in Chapter 40. It could well be said that war games are "as old as the hills", for historically they have been played for hundreds of years — the Prussians and Germans having placed much effort in developing and exploiting the advantages of simulations. Our coverage of war games in Chapter 40 gives some historical points of interest before World War I, then a brief account from World War I through World War II for some of the more significant developments, and some highlights since World War II. Then, we discuss some of the modern or present uses of war games and proceed to discussions concerning computer simulations of combat. Many of the more pertinent details of playing war games are discussed and the importance of time-sequenced scenarios developed — including target detection; terrain effects; the firing of weapons; assessment of casualties; command, control, and communication problems; and other considerations. With reference to war games and computerized simulations of combat, a large number of current combat simulations are summarized in Chapter 40 for the practicing analyst. These simulations include for example, CARMONETTE (which is rather extensively covered), the Army Small Requirements Battle Model, the Individual Unit Action, Bonder/IUA, Legal Mix IV, DYNTACS X, Division Battle Model, Division War Game, TARTARUS IV, Tank Exchange Model, ATLAS, and others. Finally, we discuss the problem of near real time casualty assessment. All of this coverage is given in order to lead up to combat simulations and studies covering the analysis of infantry weapons, tank warfare, artillery, air defense, and cost and operational effectiveness analyses in Chapters 41-46.

The remaining chapters of the handbook are devoted to combat simulations or war game type studies, and especially the analysis of results from hypothesized battles involving combined arms, for in current practice much dependence is placed on evaluations of this kind. Also, we approach and actually exhibit several different methods of attacking weapon evaluation problems so that the analyst may acquire an overall glimpse of the various possibilities.

Chapter 41 discusses in detail the evaluation of several different mixes of small arms or hand-held weapons for the situation where a Blue infantry company is involved in a defense against an attacking Red infantry company. The question to be settled concerns just which of several Blue mixes of hand-held weapons would be the most effective in attaining the maximum number of Red kills. Moreover, we stress the advantage of using statistical designs of experiments in the planning and analysis of results for the combat simulations played. In particular, for the infantry evaluation studied, we make good use of the Latin Square statistical design, and we show just how it would possess much superiority concerning the final analyses of the combat simulated data, and hence likely lead to better supported conclusions and recommendations.

Chapter 42 gives an example of a possible tank warfare situation in Western Germany for the age of the guided missile. Here, we hypothesize a Red breakthrough into Western Germany, and the main mission of Blue at the time is to stop the Red tank attack which might sweep across that country toward the Ruhr Industrial Basin. For this particular combat simulation, we suggest the use of CARMONETTE and extract from it the numbers of target kills by weapons on both sides. Then a killer-victim scoreboard can be set up at some key time of the conflict, and the results analyzed or projected. An advantage of this approach is that we may determine the relative effectiveness of both antitank guided missiles and tank armament against enemy tanks or guided missiles, and quantify the worths of all weapons in the engagement in accordance with the principles of Chapter 30 on weapon equivalence studies.

In *Chapter 43*, we cover a method of evaluating artillery or support type weapon systems. For this evaluation, interest centers around determining the particular mixture of weapons in an artillery family which can engage the most targets at a reasonable cost. The artillery or support weapons considered in the analysis include the 155-mm Howitzer, the 8-in. Howitzer, and the 175-mm Gun — the problem being to find the best numbers or percentages of each caliber to employ. The study is carried out through the aid and use of the Legal Mix simulation. The Legal Mix simulation technique establishes a target complex for support weapons to "attack" and determines the effectiveness of different mixes of artillery weapons in neutralizing the target complex established. Based on parameters — such as weapon delivery errors or CEP's, lethality of projectiles, availability of weapons, cost of complete rounds, the number of rounds required to attack targets successfully, and weapon response capability — it was found that a two-weapon mix consisting of the 155-mm Howitzer and the 8-in. Howitzer appears best and entirely sufficient.

One of the important current problems in air defense is that of finding a suitable replacement for the VULCAN 20 mm Gatling type gun Preliminary studies in recent years indicated that air defense guns of calibers of about 30-40 mm should be evaluated so that there would be a suitable range of lethality values and delivery errors covered to settle once and for all the proper choice of a gun type weapon against aerial targets, especially for the shorter engagement ranges. Various candidate air defense guns falling within the outline of this scope were evaluated using the Modern Gun Effectiveness Model (MGEM) or simulation to study the competing candidates. The MGEM simulation is described in some detail in *Chapter 44* and is considered to be a very useful means for evaluating guns against aerial targets, since some rather extensive efforts have been expended on validating the MGEM simulation. Various comments on this type of analysis are given also in Chapter 44.

Chapters 45 and 46 approach the problem of cost and operational effectiveness analyses (COEA's). It is expected that COEA type evaluations will perhaps be the primary or main systems analysis procedures for the immediate future. Recently, the Deputy Under Secretary of the Army for Operations Research has put forward some guidelines on the principles for conducting cost and operational effectiveness analysis studies. These are covered in *Chapter 45* and may be used as a reference by systems analysts to guide COEA type studies.

Finally, *Chapter 46*, the last chapter of the handbook, presents an example of a cost and operational effectiveness analysis for an armored infantry fighting vehicle called the "WICV-WOW". Hopefully, the evaluation procedure presented in Chapter 46 will serve to give the reader some insight into and a preliminary account of the problem of performing cost and operational effectiveness type analyses.

CHAPTER 26
MEASURES OF EFFECTIVENESS

Good evaluations of weapon systems depend very critically on proper choices of Measures of Effectiveness (MOE's). An MOE generally is a quantitative expression of the degree to which a system meets its objectives, and hence an analytical standard of comparison. In many applications, there may be more competing MOE's than are useful. Accordingly, the analyst has the problem of making some judicious selection; otherwise he must weight the pertinent MOE's in a proper manner for final evaluation judgments of a system. Proper choice of the MOE goes hand-in-hand with the appropriate choice of the overall evaluation model.

26-0 LIST OF SYMBOLS

$E(R)$ = average or expected range of engagement, m
$f(R)$ = probability density function for likely engagement ranges
$\overline{P}_k$ = average or expected chance of kill over all ranges
$p(k|h)$ = conditional probability that a hit is a kill
$p_h(R)$ = probability of hitting as a function of range
R = range to target, m
r_T = radius of target, m
β = parameter for a gamma distribution
β = $(\pi/4)$(mean engagement range)^2, m^2
σ_0 = total delivery standard deviation or error, mil

(Other symbols are defined as needed on Figs. 26-2 through 26-9)

26-1 DEFINITION

Ref. 1 of the former US Army Combat Developments Command defines the term "measure of effectiveness" as, "A criterion expressing the extent to which a combat system performs a task assigned to that system under a specified set of conditions. Thus, an individual MOE supplies a partial answer to the question: How well does System X perform assigned Task Y under a set of conditions Z?" Hence, we might keep in mind that for the purposes of this Handbook the performance of a weapon or weapon system must be measured against appropriate criteria which will indicate its combat potential; put simply, the MOE should be a "robust" quantitative expression of the degree to which the system under evaluation meets its objectives.

26-2 INTRODUCTION AND GENERAL GUIDELINES

It is not always easy to formulate a good MOE. In fact, it is often a subjective or "value" judgment—one that may vary considerably from one application to another—hence, the systems analyst must give much thought to the selection of an MOE (or MOE's) that describe the potential of a weapon or weapon system. As Leibowitz (Ref. 2) has so aptly pointed out, "It does little good to optimize an auto assembly line to provide the maximum number of coffee breaks per hour." Leibowitz (Ref. 2), in a letter to the Editor of *Operations Research*, presses his point by saying, "A measure of effectiveness resembles a moral principle in that its validity cannot be established by reason alone. We must make a value judgement. We must play it 'by feel'." Further, he argues that the process of selecting the proper MOE involves the making of four compromises, and hence might be called the "method of dynamic compromise". Then Leibowitz cites the case of "George" lazily fishing on a warm Sunday afternoon who sooner or later has to come to grips with the problem of just why is he there fishing, and hence just what is his MOE? George, apparently being smart, suddenly realizes that every system has, or is contained in, a "supersystem", and that the supersystem pertains to George's overall recreational program, and further through more supersystems to the universe! He then decides that a good approximate purpose for his fishing is to give him a pleasurable Sunday afternoon, and this is his first dynamic compromise—to get things down to earth. Now, therefore, George is going to maximize his amount of pleasure per Sunday afternoon, but he still needs a quantitative and practical MOE. After some soul searching, and even realizing that he really enjoys peace and quiet without catching any fish at all, he comes to the point that his brother-in-law will make disparaging remarks if he comes home empty-handed. Therefore, a practical and suitable MOE is to catch exactly four fish, and with minimum exertion. Thus, it is clear that George has just made his second dynamic compromise. But conflict begins to rage in the mind of George, the operations research analyst. Why four fish each Sunday, and won't his brother-in-law catch on, and why not a random number? Shouldn't the random number be between four and seven, especially since George's wife absolutely refuses to clean more than seven fish? And with a random catch between four and seven, George has reached his third dynamic compromise, but even this practical measure may fail to satisfy George, the operations analyst, for he must also satisfy the decision maker, who happens to be George, and the MOE should be mutually agreeable. By then, George reached the important step of compromising his choice in order to put forth an MOE which has the highly valuable asset of pleasing the decision maker, or at least taking his viewpoint into proper consideration, and he is now ready to start the analytical phase of his study. But, clearly, this brings on the need for much effort: "George will need a distribution function for fish weight versus worm diameter for various values of hook size, maybe obtainable by repeated samplings, and he will need data on this and that, resulting perhaps in some 20 man-years of effort!" Having such an estimate of his work program, George—the fisherman, the OR analyst, and the decision maker—terminates the whole process, arriving at the null state, a perfect picture of "relaxed gray matter". (Parenthetically, perhaps George may even have thought of the need for a "cost-effectiveness" study). What happened for George to make this fourth dynamic compromise and reduce the scope of the study? Well, he reasoned that for a desirable, but limited study, he may as well throw out any consideration of his brother-in-law's wisecracking tendency and even his wife's fish-cleaning capacity so that any number of fish caught would be acceptable, and furthermore, "It so happens that the number that can be caught with minimum effort is exactly zero, and that the most pleasurable technique for doing this is simply to put aside the rod and tackle, and doze off. This is just what George did."

Leibowitz continues and concludes his metaphysical considerations for an MOE with:

"In the dynamic compromise process, (1) we make use of our limited understanding of the supersystem to obtain an approximate measure of the system's effectiveness, (2) then adjust this measure so that it becomes possible to relate it to the system's elements, (3) we readjust the measure until it is satisfactory to the decision-maker, and (4) we re-readjust it until the projected study does not exceed the time-and-effort deadlines.

"We are not quite finished. We must examine the resulting fourth-order approximation to see if it is close enough to the 'true' measure of effectiveness to make the study worthwhile. This can only be done 'by feel'. If we decide that the approximate measure is too far off, then, depending on the situation, we have five courses of action: (1) learn more about the supersystem, (2) learn more about the system itself, (3) talk the decision-maker into revising his interpretation, (4) suggest an extension of the scope of the study, or (5) call the whole study off. However, in most cases, this last drastic step should not be necessary.

"The point is that regardless of how you finally select a measure of effectiveness, this measure must be reasonably close to representing the true purpose of the system. If it is not, then all the linear programming and all the game theory in the world will not save us from optimizing auto assembly lines so as to provide the maximum number of coffee breaks per hour. And, then we would soon find that no one was willing to sponsor (such) an operations-research study, with the possible exception of labor unions and coffee vendors."

Perhaps even though Leibowitz had to choose a "fishy" example, one might nevertheless get a clearer view of the role between the decision maker (Chapter 7) and the systems analyst (Chapter 6), and their negotiations to arrive at a good, useful or practical MOE, which will have an important bearing on the choice of the weapon.

The process of determining proper MOE's may not be very different actually from that of designing, or arriving at, the threat our weapons must defeat. For, example, Tombach (Ref. 3) attempts to "formalize" an approach to threat model design "that provides the model builder with criteria for selecting, from the universal set of all possible threat models, a limited subset of (threat) models that are realistic, usable, and useful." Tombach suggests doing this by comparing the likelihood of occurrence of various possibilities, and the general methodology is that of successive approximation. In fact, he suggests starting with the most general or universal set of all possible threats (U), then by successive eliminations he reduces the universal threat to a "phantasy" threat (P), then on to a "state-of-the-art" threat (S) for the enemy, on further to the "economic capacity" threat (E) for the enemy, and through our own intelligence capability the realm of the threat becomes the intelligence threat (I). Further refinements through a "matrix reduction" method rules out impossible threats, useless threats, unusable threats, and on to a conservative or practically valid threat for evaluating and designing weapons. Thus, the role and usefulness of a scientific type of approach to threat design are shown.

In case some readers may be a bit confused because of our sudden mixture of the terms—"models", "threats", and "measure of effectiveness"—perhaps a few words are in order. A *model* is a miniature or facsimile representation of a thing or process, and for our purposes here it is usually an analytical or computerized expression or description of the process or system (see Ref. 4, for example). Thus, the weapon systems analyst will model the major characteristics and expected performance (effectiveness) of a weapon under combat conditions. The *threat* consists of a group or complex of enemy targets, weapons, supply lines, etc., which our friendly weapons are required to defeat or neutralize, once in battle. The performance of weapons and the threat can both be "modeled", while on the other hand the *measure of effectiveness* should be a quantitative expression of the degree to which our weapon systems accomplish the mission of defeating the target threats.

Brooks (Ref. 5) also discusses the problem of MOE's, or choices of payoffs for military operations, and the everlasting competition between adversaries in fielding improved weapons through a sequential and conditional nature of the measure, countermeasure, and counter-countermeasure process for the two sides. We quote from his paper:

"Therefore, whereas measures of effectiveness dominate decisions on current or near-term weapons systems, measures of effort to reach some approximate level of effectiveness dominate (though of course not monopolize) decisions for long-term weapons developments.

"Specifically, it is suggested that effectiveness analysis emphasize two features:

"First, a hunt for measure, counter-measure dead-ends; that is, we can design a hypothetical measure, counter-measure, counter-counter-measure sequence to see whether the enemy might develop a decisive counter at some stage not counterable by a reasonable extension of our own system's growth potential.

"Second, determination of the system's 'off-design' capabilities; that is, its versatility in the face of unexpected enemy technical or strategic developments. Incidentally, it is in these effectiveness analyses that the familiar combat model-building and war gaming of operations research finds its application.

"These two measures of effectiveness can perform the negative function of screening out the least suitable weapons, among which could conceivably be the so-called 'optimum' under expected or average value assumptions about projected trends. They can also guide the future development of those which are favored by pin-pointing weaknesses and the key indicators of possible future enemy attempts at exploiting these weaknesses. We have recently put the POLARIS system through this twin wringer."

Hayward (Ref. 6) discusses the measurement of combat effectiveness. He suggests that the proper quantitative measure of combat effectiveness of a military force is the "probability of success" in combat, and he says:

"The probability of success depends not only on the capabilities of the specified force but also on the nature of the enemy, the combat environment, and the mission. Since it is impractical to measure combat effectiveness experimentally, i.e., in actual combat, military judgment must be called on to specify the relation between the probability of success and the parameters of force capability, environment, and mission."

Hopefully, these quotations give the analyst some appreciation for the concept of MOE's and some of the problems the analyst will face in arriving at suitable MOE's. In fact, it certainly seems desirable to try to arrive at a single MOE for each application of Army weapon systems analysis. Unfortunately, however, it is not always possible to establish a single MOE for each case, and moreover the task of arriving at good weights for calculating overall or representative MOE's may represent quite a problem also. Hence, the analyst may expect that some judgments will have to be made by purely qualitative processes of determination, which nevertheless will be of value to the decision maker.

The analyst is often the one to decide on the MOE or at least make recommendations to the decision maker. Also, his selection of the MOE, as we have seen, may be just as important or sometimes more important than the development or choice of a model. Thus, the keen analyst will not only give the most serious consideration to the choice of an MOE, but will keep the pertinence of the MOE in mind during and even at the end of his evaluation.

The analyst and the decision maker often are considered to have individual or mutually exclusive roles but, as we have seen, that cannot be the most desirable situation. Since the roles of the analyst and the decision maker overlap, at least partly, on most evaluation studies there exists the continuing problem of revising aims, goals, MOE's, etc. Thus it becomes clear that communication and compromises must be sought often during and especially toward the end of a study.

Finally, the concept of MOE's cannot ordinarily be separated from the more general area of cost-effectiveness studies and related criteria. The analyst would do well to keep this in mind because the costs incurred and the benefits derived in fielding systems cannot be measured in the same units, although the analyst can and should always strive to come up with a single, overall, "robust" measure of effectiveness.

26-3 COMBAT DEVELOPMENT MOE's

"The basic mission of combat developments is to formulate and document concepts, doctrine, materiel requirements, and organization pertinent to the Army in the field. Included in that responsibility is the design of land combat systems for at least 20 years into the future to facilitate the integration of new or improved doctrine, materiel, and organizations. The combat developments process includes studies, simulations, and testing and experimentation in which the final product is recommended doctrinal, organizational, and equipment changes for the immediate future and for long range planning programs. The recommendations involve estimates based upon the best available information indicating the impact of such recommendations. Doctrinal recommendations are applied in field manuals. Organizational recommendations are applied in Tables of Organization. Materiel recommendations are applied in Tables of Equipment and in materiel specifications. As such the credibility of MOE establishes the validity of such things as basis of issue of equipment, the establishment and maintaining of MOE, and the credibility of requirements for organizational and doctrinal changes." (Ref. 1)

USACDC Pamphlet No. 71-1 (Ref. 1) likens the combat developments process to the scientific process, as shown in Fig. 26-1, and thus indicates the thoroughness of the study procedure. This pamphlet also emphasizes that military judgment is just as important as scientific judgments in the process of measuring the effectiveness of combat systems, and that "practical criteria as well as academic and mathematical criteria are considered as the basis for selection of MOE's to compare systems". An approach to the development, formulation, and use of MOE's in the combat development process is thoroughly covered in Ref. 1, and a compendium of Ref. 1 contains some 207 typical combat development MOE's. They are divided into two categories: (1) Combat Development Functions, and (2) Land Combat Functions. The combat development function MOE's include examples divided into the subject matters of doctrine, organization, materiel, training, and logistics. The land combat function MOE's include examples concerning command-control-communications, firepower, mobility, intelligence, and combat service support. We have extracted and include herewith for the information of the analyst some typical MOE's, for some of the cited categories in Figs. 26-2 through 26-9. Appendix D of Ref. 1 gives a thorough and informative coverage of the principles of measurement of effectiveness for the combat development processes.

Going perhaps a step further, Table 26-1 outlines some of the possible thinking, during combat development studies, which might precede the final establishment of MOE's for some broadly defined Army systems. Here, the system mission is considered for the likely environment and begins to take sharp focus. Also, analysis objectives are considered, and some details of the probable methods of analysis given. The measures of effectiveness are divided into "primary" and "secondary" MOE's. One then begins to see that several or many MOE's will probably have to be considered and quantified in some suitable way by the analyst. Moreover, it also is made clear at this stage that problems of some difficulty arise, and some compromises will have to be made toward selecting the best single MOE (if possible), weighting several of them appropriately, or combining them properly.

26-4 HANDBOOK EXAMPLES OF MOE's

Obviously, we have already discussed and established many measures of effectiveness in preceding chapters. In Chapter 10, for example, and the example therein, the expected number of armor piercing rounds to rout the enemy tank attack may be considered as an MOE.

The time to detect a target, or the number of "glimpses" expected in a typical target detection, also represents MOE's for target identification and detection problems (Chapter 11).

PHASE	THE COMBAT DEVELOPMENTS PROCESS	PHASE ORDER IN THE PROCESS	THE SCIENTIFIC PROCESS
1.	Study Directive, Study Plan		Formulating the Problem
2.	War Gaming, Modeling		Constructing a model to represent the system under study
3.	Cost-Effectiveness Analysis		Deriving a solution from the model
4.	Test and Experimentation		Testing the model and the solution
5.	Materiel Needs and Personnel Needs Documentation		Putting the solution to work: Implementation

Figure 26-1. Analogy of the Combat Development Process With the Scientific Process, (Ref. 1)

FORCE EFFECTIVENESS INDICATOR

1. *DEFINITION OF THE MEASURE.* Force effectiveness indicator (*FEI*) is the ratio of the total value of the blue force (*TVB*) and total value of the red force (*TVR*):

$$FEI = \frac{TVB}{TVR}$$

The total force value for blue (*TVB*) is computed as the sum (Σ) of the number n_j of each type red weapon destroyed multiplied by the value v_j of that type weapon for all red weapons ℓ, and the total red force value is computed similarly for all blue weapons k:

$$TVB = \sum^{\ell} n_j v_j \qquad TVR = \sum^{k} n_i v_i .$$

The unique characteristic of this measure is that weapon values are computed as the fractional value of the enemy force destroyed by a given weapon. That is, the value v_i of a type blue weapon (i) is the ratio of all ℓ of the numbers n_{ij} of red kills by that type weapon multiplied by the values v_j of the destroyed red weapons to the total red value (*TVR*), and the value of blue weapons is computed similarly:

$$v_i = \frac{\sum^{\ell} n_{ij} v_j}{TVR} \qquad v_j = \frac{\sum^{k} n_{ji} v_i}{TVB}$$

The *FEI* does not have a closed form solution; it is usually calculated by assuming an initial finite value for all weapons and solving the equation in a series of iterations until final values converge reflecting losses inflicted.

2. *DIMENSION OF THE MEASURE.* Ratio—weighted by losses inflicted.

3. *LIMITS ON THE RANGE OF THE MEASURE.* The output value may be zero or any positive value. Since losses are a function of several factors in the scenario, the output value of the *FEI* cannot be disassociated from the circumstances under which it was derived. The measure has a weakness in that a force that completely destroys the other without taking any losses is zero effective because the weapons destroyed had not obtained any value by inflicting losses.

4. *RATIONALE FOR THE MEASURE.* This is a complex form of loss exchange ratio with the advantage that weighted values are based on actual performance.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* This measure is suitable for measuring overall effectiveness of a mixed weapons force. In the referenced studies it was used to evaluate candidate armor-infantry mixes in terms of combined force firepower and survivability.

6. *ASSOCIATED MEASURES.* Proportion forced destroyed. Loss exchange ratio.

Figure 26-2. An MOE for Doctrine (Ref. 1)

CROSS-COUNTRY RATE COMPATIBILITY

1. *DEFINITION OF THE MEASURE.* Cross-country rate compatibility is the difference between mean cross-country rate of all vehicle types in the organization and the cross-country rate of the slowest vehicle. Input data are the cross-country rates of each type of ground vehicle in the organization. The relation between output and input is:

$$\text{cross-country rate compatibility} = \frac{\sum_{i=1}^{n} R_i}{n} - R_s$$

where:

R_1 = cross-country rate of first vehicle type
R_2 = cross-country rate of second vehicle type
R_n = cross-country rate of last vehicle type
R_s = cross-country rate of slowest vehicle type.

2. *DIMENSION OF THE MEASURE.* Difference in two rates — Output value is a rate in terms of kilometers per hour or other suitable expression of rate.

3. *LIMITS ON THE RANGE OF THE MEASURE.* There is no limit on the output value; it may be zero or any positive number. Input values are not limited, but must be expressed in terms of the same definition of rate. The measure is most meaningful when measures are most refined, i.e., kilometers per hour is more meaningful than kilometers per day, because rounding off of cruder measures sacrifices some of the measure.

4. *RATIONALE FOR THE MEASURE.* This is a measure of one aspect of efficiency of organization. An organization's vehicle mix should be compatible in the sense that no one type vehicle should detract seriously from the overall movement rate of an organization. While movement rate itself is a measure of mobility, compatibility of movement rates is an indicator of soundness of organization between fastest and slowest vehicles, variation of rates, or some comparison of the slowest rate to others. The difference between the mean rate and the slowest rate is selected as the most meaningful in the military sense of identifying critical restraints.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* The measure is useful for comparing competing hypotheses of organization when mobility is one of the aspects of comparison.

6. *ASSOCIATION MEASURES*:

Movement rate	Turn-around time
Payload capacity	On-road movement rates compatibility.

Figure 26-3. An MOE for Organization (Ref. 1)

Similarly, and continuing, the following are examples of some of the measures of effectiveness in preceding chapters:

1. Chapter 13—For weapon delivery characteristics, the circular probable error (CEP) is a measure of effectiveness, and the CEP might involve both aiming errors and ballistic-round-to-round variations.

2. Chapter 14—Single-shot hit probabilities are very definitely measures of effectiveness. In fact, hit probabilities combine and summarize the delivery error characteristics of weapons, the effects of target

MOBILITY INDEX (WHEELED VEHICLES)

1. *DEFINITION OF THE MEASURE.* The mobility index for wheeled vehicles is a relative index used for comparing the ability of wheeled vehicles to traverse real estate without hinderance from obstacles, which include water barriers, steep slopes, soft soils, and dense vegetation. Input data are:

CPF = contact pressure factor expressed as: $\dfrac{\text{gross vehicle wt (lb)}}{\text{tire width, (in.)} \times \text{rim dia., (in.)} \times \text{no. of tires}}$

WF = weight factor (expressed in pounds)

TF = tire factor expressed as: $\dfrac{1.25 \times \text{tire width, in.}}{100}$

GF = grouser factor (expressed as a factor for vehicle with or without chains)

WLF = wheel load factor = $\dfrac{\text{gross vehicle weight}}{\text{no. of wheels (single or dual)}}$

CF = clearance factor = $\dfrac{\text{ground clearance, in.}}{10}$

EF = engine factor (hp/ton expressed as a factor). (Factors 0.6 and 20 are used to scale down the mobility indexes of wheeled vehicles for purposes of comparison.)

Relation of output to input is:

$$\text{mobility index} = 0.6\left[\left(\frac{CPF \times WF \times WLF}{TF \times GF} - CF\right) \times EF \times TF\right] - 20.$$

2. *DIMENSION OF THE MEASURE.* Index number.

3. *LIMITS ON THE RANGE OF THE MEASURE.* The output may assume any value but is ordinarily a large positive number driven by vehicle weight in pounds. The combination of factors makes it difficult to use the index for any other purpose than comparison of vehicles.

4. *RATIONALE FOR THE MEASURE.* This is a combination of most vehicle characteristics significant to wheeled vehicle mobility.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* Used to compare wheeled vehicles.

6. *ASSOCIATED MEASURES.* Mobility index (tracked vehicles).

Figure 26-4. An MOE for Materiel (Ref. 1)

size and shape, and include the important effects of range from weapon to target. Indeed, hit probabilities may often be the only needed basis for comparing competing weapons in certain applications.

3. Chapter 15—The vulnerable area of a target to specific attack and the lethality of a warhead to enemy targets represent MOE's that characterize and summarize terminal effects. In some cases, final judgments concerning weapon effectiveness may be arrived at by using vulnerability or lethality for competing weapons having about the same delivery errors or distributions. Moreover, the conditional probability that a hit is a kill constitutes somewhat of an overall terminal effectiveness MOE.

TIME TO ESTIMATE RANGE

1. *DEFINITION OF THE MEASURE.* Time to estimate range is the elapsed time from detection of a target to estimation of range. Input data are the moment of detection and the moment estimation of range is complete. Relation of output to input is:

$$\text{time to estimate range} = \text{time of estimation} - \text{time of detection.}$$

2. *DIMENSION OF THE MEASURE.* Interval—elapsed time in terms of seconds. If the measure is taken at different times or under varying circumstances, it can be used in the form of mean time to estimate range or median time.

3. *LIMITS ON THE RANGE OF THE MEASURE.* The output can be zero or any positive value. The resolution of the measure is limited by the precision of taking start time and end time. The data cannot be disassociated from the definition of computed estimation used, whether it is the first estimate stated regardless of accuracy or is the final in a series of estimates which is used for firing.

4. *RATIONALE FOR THE MEASURE.* This measure addresses a component of target acquisition time. Problems in estimation are assumed to contribute to the length of estimation time.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* This measure can be used to compare estimation times of means of range estimation (techniques, aids, rangefinders, trained personnel) to each other or to a standard. It would not ordinarily be used alone, but would be combined with accuracy of estimation or accuracy of firing in most cases.

6. *ASSOCIATED MEASURES*:
   - Accuracy of range estimation
   - Firing accuracy
   - Time to detect
   - Exposure time
   - Time to identify
   - Probability of hit
   - Probability of kill.

Figure 26-5. An MOE for Training (Ref. 1)

4. Chapter 16—Rates of fire can be of critical importance for surface-to-air weapons, or for close combat, for example. Thus, rate of fire represents a key MOE or characteristic parameter describing the weapon under consideration.

5. Chapter 17—The individual or "isolated" MOE's of hit probability, conditional chance that a hit is a kill, and rate of fire can be multiplied to obtain a kill rate which represents a more inclusive MOE for the weapon or a weapon system. The kill rates, along with various strategies of firing, determine chances of winning a duel. The chance of winning a duel, therefore, gives even a still more inclusive MOE since combat usage of the weapon is also taken into account in the analysis of duels.

6. Chapter 18—Weapon response time is another example of an MOE. System response time—including a proper combination of detection, acquisition, command-control-communication effectiveness, and weapon response time—describes yet a higher level of MOE.

REQUIRED AMMUNITION RESUPPLY RATE

1. *DEFINITION OF THE MEASURE*: Required ammunition resupply is the rate of ammunition need. Input is rounds required per day. Unit of measure of input is rounds, or alternatively tons or DOA. (Day of Ammunition—a specified number of rounds for a type weapon.) Relation of output to input is:

$$\text{required ammo resupply} = \frac{\text{total number of rounds required (or tons, or DOA)}}{\text{number of days in time period observed}}$$

2. *DIMENSION OF THE MEASURE*. Ratio—a rate in terms of rounds per day or tons per day. Unit of measure of output is rounds (or tons). In its most esoteric form it is the ratio between a predetermined "day of ammunition" which is meant to be the amount of ammunition required per day and the actual ammo per day. In this form it is "DOA per day".

3. *LIMITS ON THE RANGE OF THE MEASURE*. The measure must include at least one day's observation, and as the denominator gets larger the measure gets better. The output may assume any positive value. The measure is limited to a single type of round in the form "rounds per day". In the form of weight per day, it is more encompassing. For complete inclusion of different types of ammunition it is usually necessary to use the form "DOA per day".

4. *RATIONALE FOR THE MEASURE*. This measure addresses sustainability. It is reasoned that a good performance in other respects may be offset somewhat by difficulty in sustainability. If sustainability were difficult enough, it would affect performance and could be measured otherwise. This measure is meant to be sensitive enough to address sustainability before it is serious enough to affect performance of the mission.

5. *DECISIONAL RELEVANCE OF THE MEASURE*. This measure could be used to distinguish between firepower systems that are equal in productivity. Or it could be used as a further refinement in a more complete description of successful systems.

6. *ASSOCIATED MEASURES*:
   Resupply frequency
   Ammunition expenditure.

Figure 26-6. An MOE for Logistics (Ref. 1)

7. Chapter 19—Analysis of fuze performance toward optimizing the effectiveness of the terminal engagement and the inclusion of analyses of reliability and safety factors will involve the analyst in some rather complex problems of MOE's, especially since fuze action affects vulnerability and lethality.

8. Chapter 20—Although single-shot hit probabilities may be sufficient for some analyses, the chance of at least one hit as an MOE becomes important for cases where multiple rounds must be fired. The fraction of target coverage—or the expected fraction of target damaged—gives higher level more overall weapon or system effectiveness MOE's.

9. Chapter 21—The concept of MOE's and their usage in system evaluations transfer easily and properly to system reliability, a confidence bound on system reliability, the availability or readiness of

COMMUNICATIONS PERFORMANCE INDEX

1. *DEFINITION OF THE MEASURE.* This communication index is a weighted sum of a communication system's performance in relation to its requirements. Input data are the relative weights of each requirement ($W_1 \ldots W_n$) and the performance ($P_1 \ldots P_n$) observed in each requirement ($R_1 \ldots R_n$). Relation of output to input is:

$$\text{index} = W_1\left(\frac{P_1}{R_1}\right) + W_2\left(\frac{P_2}{R_2}\right) + \cdots W_n\left(\frac{P_n}{R_n}\right) = \sum_{i=1}^{n}\left[W_i\left(\frac{P_i}{R_i}\right)\right]$$

Examples of system requirements are: direct communication capacity, organic communication equipment, conference call capability, specific range, security, mobility, message hard copy, dependability, and vulnerability, each of which is measured directly or rated by evaluators on a common scale.

2. *DIMENSION OF THE MEASURE.* Index—A weighted sum.

3. *LIMITS ON THE RANGE OF THE MEASURE.* The values assumed by the output depend on the performance/requirements scale and weights. The maximum value is n times the maximum scale, times the total weight. The measure is limited by the selection of requirements and weights.

4. *RATIONALE FOR THE MEASURE.* The measure is intended to combine performance in all requirements to preclude over-valuing some requirements.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* The measure can be used to compare alternative communication systems.

6. *ASSOCIATED MEASURES:*
   Percent messages completed
   Communications system capacity.

Figure 26-7. An MOE for Command-Control-Communications (Ref. 1)

a weapon to start a mission, and the ease of maintainability of a system in combat. As may be clearly seen, sometimes an MOE involving only the system reliability may be sufficient for an inclusive analysis.

10. Chapter 22—Weapons in the field must possess the required characteristics of mobility, maneuverability, and agility—depending on actual usage. Therefore, suitable MOE's describing the level of performance of mobility, maneuverability, and agility are needed for evaluation purposes.

11. Chapter 23—Additional and extensive studies seem required to develop MOE's for logistic and planning purposes, insofar as the support for weapons is concerned. Many of the military operations research and statistical studies will, no doubt, aid in improvement of logistic MOE's.

12. Chapter 24—The WSEIAC evaluation model gives a prime example of combining lower level or individual measures of effectiveness into a systematic and inclusive analysis which can result in a single and overall summary of system effectiveness. The final MOE for a WSEIAC type analysis, when properly obtained, should be very satisfying to both the analyst and the manager because it encompasses system availability, reliability, and terminal performance in a unified manner.

MEAN TIME TO NEGOTIATE OBSTACLES

1. *DEFINITION OF THE MEASURE.* Mean time to negotiate obstacles is the arithmetic average of each elapsed time consumed in overcoming an obstacle to advance. Input data are the delay time for each obstacle and the number of obstacles. Relation of output to input is:

$$\text{mean time to negotiate obstacles} = \frac{\sum^{n} \text{(each elapsed obstacle delay time)}}{\text{number obstacles}}$$

2. *DIMENSION OF THE MEASURE.* Ratio—output is a mean time in hours and minutes.

3. *LIMITS ON THE RANGE OF THE MEASURE.* The output may assume any positive value. As it is stated, the measure makes no distinction among different types of obstacles. It would probably be better to break it down into measures for river crossings, minefields, barriers, barbed wire, and so forth.

4. *RATIONALE FOR THE MEASURE.* This measure addresses mobility performance in terms of times to negotiate obstacles based on the premise that shorter negotiation delay times mean better mobility.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* Since this is a measure of performance rather than a true measure of effectiveness, it is applied to comparing mobility systems under the same conditions. It could be converted to a measure of effectiveness by taking total move time into account with obstacle delay time as "percent delay", assuming that zero delay for obstacles is ideal performance.

6. *ASSOCIATED MEASURES*:
   Percent delay
   March rate
   Percent moves completed on time.

Figure 26-8. An MOE for Mobility (Ref. 1)

Need there be any more arguments about the role of MOE's in Army weapon systems analyses? Perhaps, with these examples, the reader is convinced of the somewhat central role of MOE's.

26-5 FURTHER COMMENT ON THE RELATION BETWEEN A MODEL AND AN MOE

It is important for the reader to understand that models and MOE's go hand-in-hand, so to speak. In fact, the MOE clearly depends on the status of development of the model describing the performance of a system or process. The better the overall model developed for the expected combat effectiveness of a weapon, then the better the MOE which gives a numerical value of performance expressing the relative degree to which the system accomplishes its mission. The MOE thus is obtained by substituting appropriate values of the parameters into a suitable model, and we can easily see that the best MOE's are most often obtained through the modeling process. The MOE depends on the system also.

MEAN TIME TO ACQUISITION

1. *DEFINITION OF THE MEASURE.* Mean time to acquisition is the arithmetic average of the elapsed times to complete all successful acquisitions. Acquisition is defined as including detection. recognition, identification, and location of the target. Input data are the elapsed times for each completed acquisition. Relation of output to input is:

$$\text{mean time to acquisition} = \frac{\sum^{n} \text{(elapsed time each successful acquisition)}}{\text{number successful acquisitions}}$$

2. *DIMENSION OF THE MEASURE.* Ratio—Output in terms of an average time in seconds, minutes, hours, or days as appropriate. Could also be used in the form of a "median time to acquisition".

3. *LIMITS ON THE RANGE OF THE MEASURE.* The number of successful acquisitions must be enough to average out large differences from chance factors in the conditions concerned. The output value cannot be disassociated from the circumstances under which it was derived. The output may assume any positive value.

4. *RATIONALE FOR THE MEASURE.* This measure directly addresses the timeliness of acquisition. It applies only to the case of completed, successful acquisitions and not to the expected time to acquisition of a target. Since it subsumes other time measures (such as time-to-detection) it is a grosser measure suitable to the evaluation of larger systems.

5. *DECISIONAL RELEVANCE OF THE MEASURE.* This measure may be used in any situation in which timeliness of target acquisition is a factor.

6. *ASSOCIATED MEASURES*:
   Time to detection
   Time to identification
   Expected time to acquisition.

Figure 26-9. An MOE for Intelligence (Ref. 1)

26-6 CAUTION ON AN OFTEN USED MOE

It is the job of the systems analyst to establish good MOE's and use them properly in each evaluation study. It is his daily task—it should be clear that "universal" MOE's do not exist, and that very frequently the analyst has a difficult job in establishing the best MOE. Lest one might take the job of arriving at a proper MOE as "duck soup", let us consider an MOE which has been widely used; namely, the "cost per kill" type MOE. On the surface, it seems certain that "cost per kill" often would be a most useful MOE and also should apply widely when we are evaluating the worth of competitive systems or components of a system. Indeed, it often is a good MOE, and in the case where systems are employed in such a manner or in an environment where they really do not affect each other, i.e., are "independent", then "cost per kill" may be quite adequate. On the other hand, if interactions among

TABLE 26-1. ILLUSTRATIVE EXAMPLES OF SYSTEMS ANALYSIS — PROBLEM ENVIRONMENTS AND MATCHING MEASURES OF EFFECTIVENESS

Ex. No.	THE PROBLEM ENVIRONMENT				MEASURES OF EFFECTIVENESS	
	Description Of System	The System Mission Under Analysis	Analysis Objective	Method of Analysis	Primary	Secondary
1	Man Portable Assault Weapon	Provide infantry units with organic firepower capability to assault and destroy enemy field fortifications.	From among a list of alternative, candidate designs of a man-portable assault weapon, select the best candidate.	Computer simulation of an infantry platoon assault against a reinforced, enemy squad deployed in and around a field fortification.	Probability of assault mission success.	• Number of enemy casualties • Number of friendly casualties • Probability that the assault weapon gunner will be a casualty • Number of rounds fired and hits scored by the assault weapon • Number of minutes to accomplish the mission
2	Armored Reconnaissance Scout Vehicle	Provide armored cavalry units with capability to move quickly about the battlefield to collect and report data about battlefield conditions and the disposition and activities of enemy forces.	From among a set of alternative, candidate designs of a new scout vehicle, select the candidate of greatest reconnaissance mission effectiveness.	Computer simulation of armored cavalry platoon reconnaissance missions in a variety of terrain, weather, and threat environments.	Intelligence Score	• Mission duration • Total distance traveled by scout vehicles • Final force ratio • Number of targets (active and passive) acquired by scout vehicles • Number of scout vehicles, US units, and enemy units killed • Ammo consumed by scout vehicles, US and enemy units • Mean range and mean time of scout vehicle kills
3	Armored Reconnaissance Scout Vehicle	Provide armored cavalry units with capability to conduct security and economy of force combat missions.	From among a set of alternative, candidate designs of a new scout vehicle, select the candidate of greatest security mission effectiveness.	Computer simulation of armored cavalry and scout platoon; screen security and economy of force missions in a variety of terrain, weather, visibility, and threat environments.	Final Force Ratio	• Mission outcome • Mission duration • Duration of actual engagement • Time and range of acquisition and discovered for each enemy unit acquired • Number of rounds fired at each unit, by each unit • For each unit killed, time and range of kill and killer identity

(continued)

TABLE 26-1. (cont'd)

Ex. No.	THE PROBLEM ENVIRONMENT				MEASURES OF EFFECTIVENESS	
	Description Of System	The System Mission Under Analysis	Analysis Objective	Method of Analysis	Primary	Secondary
4	Field Army Tactical Communication System	Provide voice communication capability in support of the full spectrum of field army missions.	From among a set of candidate tactical communication networks of varying geometries (but constant overall equipment cost), select the candidate which provides greatest voice communication effectiveness.	Computer simulation of voice message dynamic traffic throughout the network during various, typical, field army operations.	Grade of Service (i.e., Probability that a subscriber at any randomly chosen instant will be able to obtain a circuit connection to his party)	None
5	Field Army Tactical Communication System	Provide data communication capability in support of the full spectrum of field missions.	From among a set of candidate tactical communication networks of varying geometries (but constant overall grade of service) select the candidate that provides greatest data communication effectiveness.	Computer simulation of data message dynamic traffic throughout the network during various, typical, field army operations.	Quality of Service (i.e., Error rate)	None
6	Field Army Tactical Communication System	Provide data and voice communication capability in support of the full spectrum of field army missions.	Given a single, field army communication system design (i.e., physical plant and network geometry); and several candidate system control procedures; select the most effective procedure.	Computer simulation of data and voice message dynamic traffic throughout the network during various, typical, field army operations.	Grade of Service	• Balance of traffic throughout network, i.e., average utilization of each node and link • Level of control signalling requirements, i.e., overall ratio of control signalling to user message signalling.

(continued)

TABLE 26-1. (cont'd)

Ex. No.	THE PROBLEM ENVIRONMENT				MEASURES OF EFFECTIVENESS	
	Description Of System	The System Mission Under Analysis	Analysis Objective	Method of Analysis	Primary	Secondary
7	Tank Battalion	Provide capability for conduct of the full spectrum of combat missions required of armored units.	From among a set of alternative, candidate tank battalion configurations, select the most effective configuration. (A configuration is defined by the unit's weapons list and internal organization).	Computer simulation of tank/mechanized infantry battalion offensive and defensive operations in a single area of the world against tactically plausible threat forces.	Probability of Mission Success	• Number of tanks and other major weapons lost by each side • Number of casualties (friendly and enemy)
8	Tank Weapon System	Provide tank killing capability in offensive, defensive, and delay situations.	From among a set of candidate tanks, select the candidate which provides the most effective tank killing capability over a series of engagements.	Computer simulation of tank companies in offensive, defensive, and delay situations against a specified threat in the geographical area of interest.	Number of threat tanks killed at a fixed level of friendly losses	• Duration of engagement • Weight and cost of ammunition expended • Range at which kills were made
9	Small Arms Weapon System	Provide basic infantry elements and next echelon basic infantry elements with organic firepower capability to engage and destroy enemy personnel in offensive, defensive, delay, and meeting engagements.	From a list of candidate small arms weapons mixes, select the best combination of rifle, assault rifle, machine gun, and grenade launcher.	Computer simulation of squad and platoon elements in fire fights representative of offensive, defensive, delay, and ambush in a variety of terrain, illumination, and threat situations.	Ratio of kills	• Ammunition expended • Range at which kills occurred • Absolute numbers of friendly and enemy kills

forces or weapons exist, then the "cost per kill" MOE may be misleading, and hence may not describe some very important details or outcomes of battles. Thus, if we can in some way establish the minimum cost per kill for a weapon or military unit in judging overall effectiveness, then have we not reached a suitably optimum goal? Walsh (Ref. 7) has studied the use of minimum cost per "kill", determined independently of overall defense systems interactions, in a very detailed and unique way for area defense against an enemy attack. He sets up a mathematical expression for the fraction of enemy units entering defense sectors which survive attrition in the ensuing battle and shows that the criterion of minimum cost per kill does not necessarily represent the optimum strategy for the defenders. Walsh's analytical account of the ground battle turns out to be rather complex although his model for the fraction of enemy units which enter the defended sectors, and yet survive combat attrition, includes the total potential of the defender's weapons, number of defense units for each sector, and the number of attackers and their effectiveness. With this account of a "realistic" battle, Walsh shows in his illustrative example that when the optimum defense strategy is employed rather than the "minimum cost per kill" measure of effectiveness, the battle outcome for the defense is greatly improved. This, he explains, is due to involved interactions between weapons and forces. The analyst would do well, therefore, to take a very hard look at any such criteria of effectiveness, or MOE's, even though minimum cost per kill may often be a valid and useful criterion.

26-7 DEVELOPING A MODEL FOR MOE USE WITH THE INFANTRY RIFLE AS AN EXAMPLE

While we have indicated that a good MOE often will depend largely on the particular weapon application and that the usefulness of an MOE will also depend rather critically on the problem of modeling the performance of the weapon properly, an example of model development and the resulting value of the MOE is illustrative. As a simple and yet instructive example, we will consider the case of an infantry rifleman defending ground against the attack of oncoming enemy infantrymen. Suppose, for example, that the friendly rifleman has the job of neutralizing an attacking enemy rifleman who is making a frontal attack on the defended position. Suppose further that the enemy rifleman, being the attacker, is seen often and is in rather full view of the defender. Moreover, he approaches the defender in a fairly crouched manner, so that he presents a fairly "circular" or "square" target (for ease of computation). As the enemy soldier tries to overrun the defender, the defending rifleman will engage him at some range of engagement, depending on characteristics of the terrain, vegetation, perhaps chance of hitting by the defender, and other considerations. Also, assume as is often the case, that the conditional chance $p(k|h)$ that a hit is a "kill" or incapacitation is constant over the likely ranges of engagement. In such a situation, then just how would we develop a model for a good MOE, and what would it be?

It is certainly desirable to stop the enemy attacker with the first shot; hence we may view the problem as follows. The crouching attacker may appear at a randomly chosen range between some reasonable limits so that there is a distribution of ranges of engagement, and therefore the chance of a hit as a function of range will also be an important consideration. Hence, we may seek to find an overall measure of effectiveness; namely, the average chance over probable engagement ranges that the defending rifleman will incapacitate the enemy attacker. In this connection, the chance of a hit for any range R may be taken (Chapter 14) as

$$\left.\begin{aligned} p_h(R) &= 1 - \exp\{-r_f^2/[2\sigma_0^2(R/1000)^2]\} \\ &= 1 - \exp[-500{,}000 r_f^2/(\sigma_0^2 R^2)] \end{aligned}\right\} \quad (26\text{-}1)$$

where

r_T = radius of the target, m
σ_0 = total delivery error (a constant), mil
R = range to target, m.

For the probable distribution of the ranges of engagement for the defending rifleman firing at the attacker, we will use the gamma density

$$f(R) = (4\beta^{-3/2}/\sqrt{\pi})R^2\exp(-R^2/\beta) \tag{26-2}$$

where it can be shown that the parameter β is given by

$$\beta = (\pi/4)[E(R)]^2 = (\pi/4)(\text{Mean Range})^2, \quad \text{m}^2. \tag{26-3}$$

This seems to be a reasonable choice for rifle ranges of engagement since Eq. 26-2 peaks toward the closer ranges and has some positive skewness, tailing off to the longer ranges at which the attacker may possibly appear. In connection with Eq. 26-3, one determines the distribution of the range of engagement simply by selecting the mean or expected range of engagement, squaring it and multiplying by $\pi/4$, which gives the single parameter β for substituting in Eq. 26-2.

We are now ready to set up the model, taking the chance of any random engagement range, multiplying this by the probability of a hit and then by the conditional chance that a hit is a kill or incapacitation, and finally integrating over all ranges (zero to infinity here). Thus, we have that the average kill probability $\bar{P}_k$ will be determined by

$$\bar{P}_k = \int_0^\infty p(k|h)f(R)p_h(R)dR \tag{26-4}$$

This integration for constant $p(k|h)$ leads to

$$\bar{P}_k = p(k|h)\langle 1 - \{1 + [2\times10^6 r_T^2/(\sigma_0^2\beta)]^{1/2}\}\exp\{-[2\times10^6 r_T^2/(\sigma_0^2\beta)]^{1/2}\}\rangle \tag{26-5}$$

where, in summary,

r_T = target radius, m
σ_0 = total delivery error, mil
β = $(\pi/4)$(mean range)^2, m^2.

Thus, we take the average kill probability $\bar{P}_k$ over probable ranges of engagement as a useful measure of effectiveness. It will be informative to give a numerical example.

EXAMPLE 26-1:

Let us consider the typical tactical situation previously outlined between a friendly and an enemy rifleman, and assume that the delivery errors for the friendly rifleman amount to a round-to-round standard error of 0.5 mil and an aiming error of 1 mil. The conditional chance that a hit is a kill is 0.9. The target radius is ⅓ m, and the terrain and vegetation is such that the average or expected range of engagement is 150 m. What is the value of the MOE, taking it to be the average kill probability $\bar{P}_k$ over likely engagement ranges?

We have:

$r_T = 0.333$ m

$\sigma_0 = \sqrt{1^2 + (0.5)^2} = 1.12$ mils

$\beta = (\pi/4)(150)^2 = 17{,}671$ m^2 (Eq. 26-3).

The single-shot hit probability as a function of range is calculated from Eq. 26-1, and is plotted as the upper curve in Fig. 26-10. Taking the mean engagement range as 150 m, and the parameter $\beta = 17{,}671$ m^2 determined from Eq. 26-3, one can calculate by Eq. 26-2 (for illustrative purposes) the probability density function $f(R)$ which is plotted as the lower curve on Fig. 26-10, with the ordinate scale given at the right.

Since $p(k|h) = 0.9$ and is constant for ranges of interest here, then from Eq. 26-5 one may calculate

$$\overline{P}_k = (0.9)(0.824) = 0.742.$$

(Had the expected range of engagement been 300 m instead of 150 m, then $\overline{P}_k$ would have been $(0.9)(0.469) = 0.422$, showing the influence of the true mean engagement range.)

The MOE for this example was taken as the kill probability $\overline{P}_k$ which was determined by integrating over the range of engagement in order to get a more overall measure of effectiveness for the rifle system. The model given on the right-hand-side of Eq. 26-5 would be different if there were a change in the form of the distribution of the ranges of engagement or the hit probability model although the concept of $\overline{P}_k$ as a useful MOE may not necessarily change. Hence, it might be said that the best MOE should be based on the most appropriate or overall mode, so to speak, and the size of $\overline{P}_k$ would indicate the degree to which the system meets this objective in comparison with other rifles—hence, the relation between model development and the MOE.

Concerning small arms firings and some other related MOE's, Sterne (Ref. 8) gives analytical expressions for the lethal areas of small arms, carrying over that concept from the lethality of artillery projectiles to that for a rifle firing bullets. Groves (Ref. 9) discusses the effectiveness of unaimed small arms fire into a region by carrying forward the small arms lethal area concept of Sterne and obtaining a specific expression for rifle kill probability (not including the distribution of engagement ranges). Eq. 26-5 is, of course, for aimed fire and represents an attempt to develop a higher level of an MOE for such application. Nevertheless, much more can be said about generalized MOE's, which leads us to some further considerations.

26-8 SOME ADDITIONAL COMMENTS AND CONSIDERATIONS FOR GENERALIZING MOE's

An important function for an Army in combat zones is to seize and hold ground areas, or deny the enemy use or occupation of certain regions. Hence, a good measure of effectiveness of a weapon might be the area it can defend or the area it can preclude an enemy from taking. The rifle can reach out effectively to hundreds of meters and can also be aimed or fired in large angular sectors. Thus, there is the question of just how much ground area a rifleman can attack or defend, and to this we might even add the mobility of the rifleman for a more generalized MOE! An artillery projectile on the other hand can deliver a lethal spray of fragments over a fairly wide area when fired to thousands of meters in range, and the artillery weapon can fire in wide angular directions. Thus, this too would seem to bring up the question of perhaps more generalized concepts for MOE's involving attacked or defended ground areas and even related costs. Of course, for such considerations we invariably have to get away

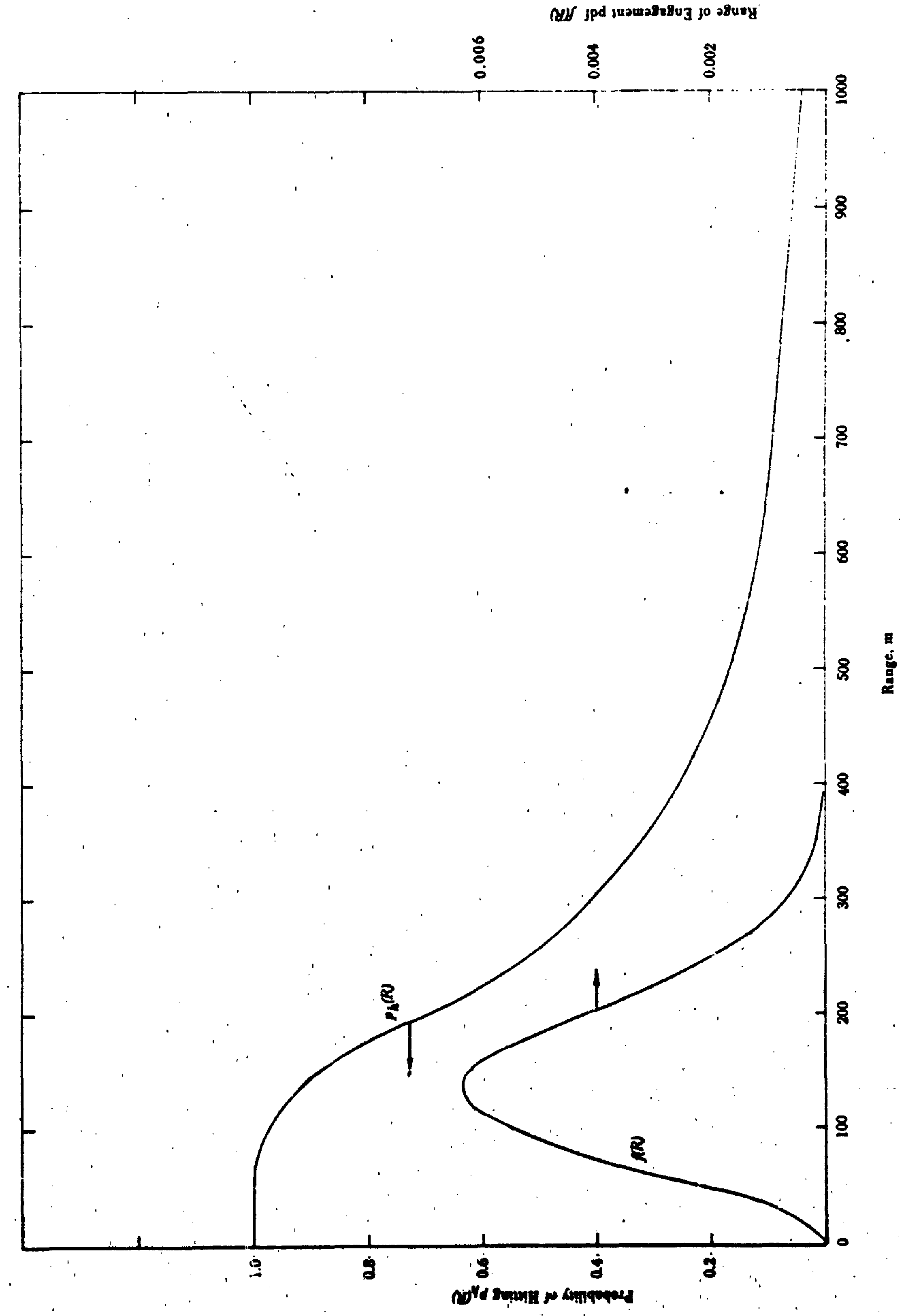

Figure 26-10. Probability of Hitting and Range of Engagement Probability Density as a Function of the Range

from dealing with individual weapons and evaluate small units or organizations. Nevertheless, the concept might well deserve more study and, as a matter of fact, such ideas are used in high level war games.

Finally, and of some interest is the "Concept of Opportunity", which has been proposed by Blum (Ref. 10) as a possible measure of effectiveness. Blum considers the opportunity presented to any system or the opportunity generated by the system itself, along with the system response to such opportunities. The product of these two factors, on a relative frequency basis, may show the net effectiveness of the system under study as compared to another one. Blum points out that "opportunity is dynamic", and indicates as an example:

"The number of targets presented to a combat system is a measure of that system's opportunity to engage. But if those targets are not benign with respect to the system being examined, then their increase presages an increase in the risk that the candidate system will be attacked and its marginal effectiveness reduced before it can effectively respond to the opportunities presented.".

Blum then points out that perhaps it would be appropriate to describe opportunity as the immediate precursor to the effectiveness of any given subsystem to the whole system under study. As an extension of the concept of opportunity, he considers a widened scope for a weapon system—including the opportunity to acquire a target, the opportunity to engage a target, the opportunity to hit, and the opportunity to kill or defeat the target.

Blum (Ref. 10) concludes his concept by suggesting two principles concerning where to begin the assessment of system effectiveness:

"First Principle: Thoroughly define the system being evaluated through all its major subsystems. Construct a diagram of the subsystem flow. Determine if opportunity is endogenous or exogenous to the system.

"Second Principle: If the system generates its own opportunity and there are no interactions between subsystems (series-parallel flow without feedback), then maximizing opportunity is a necessary condition to maximizing system effectiveness.".

26-9 SUMMARY

MOE is a quantitative expression of the degree to which a system under study meets its objective. The MOE, therefore, often will depend on the possibility of quantifying or modeling the weapon system objective. In fact, the value of the MOE selected to measure the worth of a system may well depend on the proper development of a useful and quantitative overall model of system performance. We have given several illustrations for MOE's in different military operations research areas of interest and developed an example for small arms which may apply elsewhere.

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CHAPTER 27
DETECTION PHENOMENA, CHANCES OF TARGET DETECTION, AND SEARCH STRATEGIES

Some of the basic physical phenomena and principles applicable to target detection devices are discussed for the general background information of the analyst. Some of the models to estimate probabilities of detecting targets are given and illustrated. In view of its importance, an introductory account is given of some available strategies for the target searching problems. References for further reading are listed.

27-0 LIST OF SYMBOLS

A = area in which target is randomly distributed or locatable, m^2
A_e = antenna size, m^2
A_t = target cross-sectional area, m^2
A_u = area of uncertainty
A_1, A_2 = typical areas
B_I = acceptance bandwidth of receiver, Hz
c_i = cost of searching ith region
c_1 = constant = 3.7415×10^{-12} W•cm^2
c_2 = constant = 1.4388 cm•K
E = elevation angle, deg
$E(n)$ = expected number of trials to a detection
$\vec{E}_0$ = electric field vector
E_p = total emissive power, W/(cm^2•hemisphere)
$E(t)$ = expected time (to detect)
$E(W)$ = mean or expected value of W
$F(R)$ = appropriate function of the lateral range R
f = false alarm rate
G = antenna gain, dimensionless
G_n = chance of not detecting target in $(n-1)$ glimpses but detecting it on the nth trial
g_i = chance of detecting target on ith trial
$g(W, \alpha, \nu)$ = gamma probability density, Eq. 27-33
$\vec{H}_0$ = magnetic field vector
h_1 = antenna height, ft or m
h_2 = target height, ft
I_0 = intensity of output signal, W/m^2 or W/cm^2
I_R = intensity of returned signal, W/m^2 or W/cm^2
$J(\lambda)$ = emissive power of unit area in wavelength interval λ, W/(cm^2•cm)
K = constant, dimensionless
K' = constant, m
K'' = constant, m^3
k = number greater than unity
k = Boltzmann constant = 1.38×10^{-23} W/(Hz•K)

L = system loss factor, dimensionless
L = length of path search, m
m = number of boxes (possibly searched)
m^* = optimal search policy
N = noise
NF_0 = noise factor, dimensionless
n = number
n = given number of glimpses
n = unknown number of target elements present
P_n = chance of detecting target in n glimpses
$P(R|R_m)$ = chance of target detection within range R, given maximum range R_m
P_t = transmitted power, W
p = unknown chance of seeing or detecting a target element $= 1 - q$
$\hat{p}$ = estimate of $p = 1 - s^2/\bar{x}$
p_i = prior probability target is in the ith region
$p(R)$ = chance of detecting target within the lateral range R
$p(u)$ = chance of detection
$p(t)$ = chance of detection in time t
q = chance of not seeing or detecting a target element $= 1 - p$
$\hat{q}$ = estimate of $q = s^2/\bar{x}$
R = range to target, m
R = lateral range, m
R_m = maximum range to target, mi
R_0 = limiting range, m
r = ratio (Eq. 27-31)
$r_{A(b\ell)}$ = area rate of search for background b and light level ℓ
r_{A1}, r_{A2} = reference rates of search
S = signal
s^2 = variance of the number of target elements detected in several trials
$s^2 = \hat{\sigma}^2$ = estimate of σ^2
T = absolute temperature, K
T_0 = ambient temperature, 290 K
T_u = number of time units apart for intermittent glimpses
t = time
t = time variable
u = search speed (constant)
V = target speed, tank speed, m/s
v = relative speed between target and observer, m/s
W = width of path (sweep) seerched by sensor, m
W = width of path, as before, except W is now a random variable
W' = effective width of path searched by sensor, m, Eq. 27-29
x_0 = interval length
$\bar{x}$ = observed mean number of target elements detected in several trials
$\bar{x} = \hat{\mu}$ = estimate of μ
$\mathrm{Var}(W)$ = variance of W

α = an optimal value for the iterated constant α determined from Eq. 27-32
α = reciprocal of scale parameter (Eq. 27-33)
α_i = chance target is in ith region
ϵ = total emissivity of a surface, dimensionless
θ = MTTD = mean time to detection
λ = wavelength, cm or m
μ = np = true average number of target elements detected per trial $\doteq n(1-q)$
ν = shape parameter
Σ = target size, m^2
Σ_t = target cross section, m^2
σ^2 = true unknown variance
σ_s $\doteq$ Stefan-Boltzmann constant
= 5.7×10^{-12} W/(cm^2•K^4)
τ = average time wasted for a false alarm

27-1 INTRODUCTION

In Chapter 11, we discussed many of the factors affecting target selection and presented some probabilistic techniques of analysis of continuous search or search by target detection devices which may involve distinct glimpses. The purposes of this chapter are to give the analyst some further introduction, especially to the general physical principles or characteristics of target detection devices used in the field, and also cover selected topics in elementary search theory. Concerning the first topic, we believe the Army weapon systems analyst should be aware of physical phenomena and functions related to target detection and location, the general types of detection devices used, something on the accuracy of the processes, including the effect of sensor-to-target range which represents the primary tactical variable.

Weapon systems cannot be employed properly in the field without the prompt or timely detection of enemy targets as they appear on the battlefield. Hence, the importance of superior target detection equipment.

As the reader is no doubt aware, much of the information on target detection and acquisition devices is classified. Hence, information on specific items must be obtained from appropriate Army publications and reports.

27-2 DETECTION

27-2.1 SENSORS

Before undertaking an analysis of search detection and the attendant errors with their range dependencies, it is advantageous to discuss briefly the physical and operational characteristics of sensing processes and devices.

27-2.1.1 Electromagnetic Sensors

By far the most important class of sensors are those capable of detecting and interpreting electromagnetic radiations. The spectrum of electromagnetic frequencies covers ranges from those in the "audio" range, although all frequencies here may not be audible, to those encountered in connection with the most energetic cosmic rays. The spectrum is shown in Fig. 27-1.

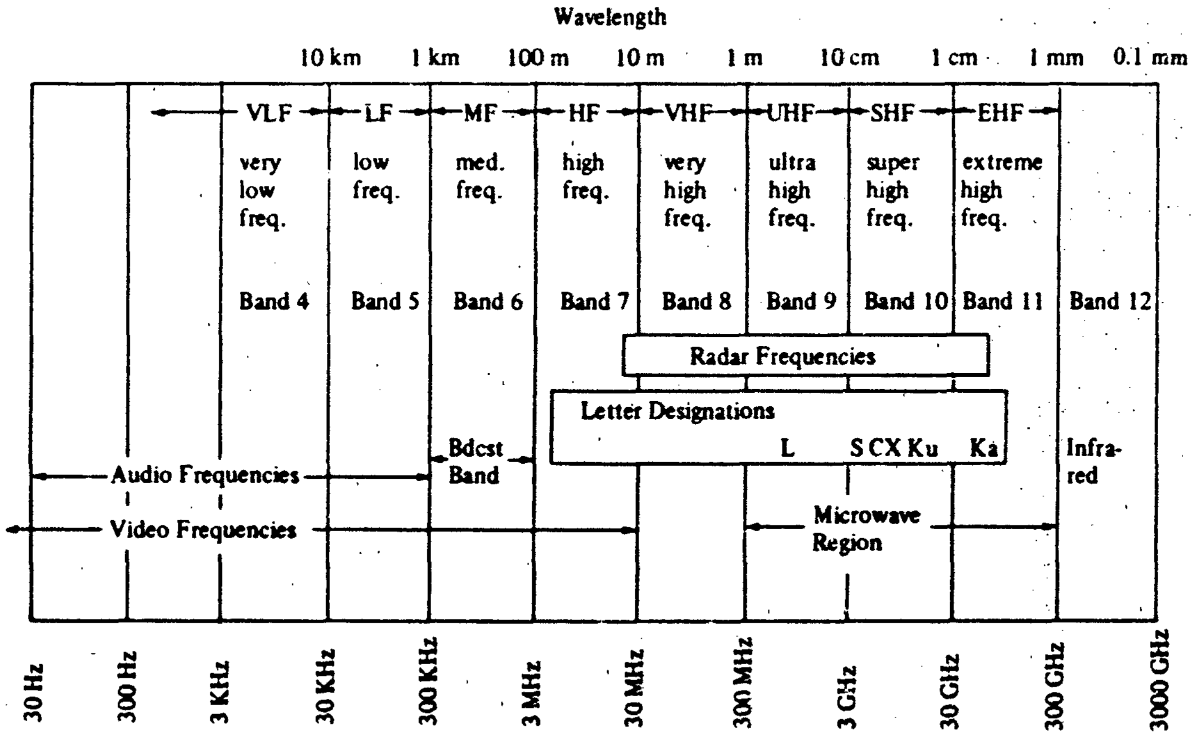


Figure 27-1. Radar Frequencies and the Electromagnetic Spectrum

While all electromagnetic waves are transmitted at the same speed in free space, 3×10^{8} m/s, the speed through other media is strongly frequency dependent, which accounts for many of the characteristics of these radiations. Refraction is a consequence of propagation velocity changes along the direction of wave front propagation; the magnitude of this effect is frequency dependent.

Electromagnetic absorption and scattering also are frequency dependent phenomena. Absorption is the frequency selective diminution of electromagnetic waves or signals by the medium through which propagation is taking place; the energy carried in the wave train is absorbed by the material of the medium. Scattering is precisely what the name implies; it is the diversion of radiant energy from its original propagation path to be scattered more or less uniformly through the solid angle 4π sr. Scattering is a characteristic of all wave-like phenomena. In general, this process is most pronounced when the wavelength of the radiation is the same order of magnitude as the scattering particle diameter.

Reflection also is frequency dependent, and dependent as well on the nature of the reflecting surface.

Another property of electromagnetic radiations is their susceptibility to polarization. When these waves are reflected from smooth plane surfaces, i.e., from sea water or grassy fields, two important changes take place: (1) the waves are polarized into a plane which is horizontal with respect to the line of propagation, and (2) a phase lag of approximately one-half wavelength is introduced. The effects of refraction, absorption, scattering, reflection, and polarization are important operationally for target detection devices.

27-2.1.1.1 Radar

The word radar is an acronym for radio detection and ranging. The main elements of a radar system are:

1. A transmitter which acts as a source of electromagnetic energy
2. An antenna to radiate this energy
3. A receiving antenna to collect energy reflected from a target (The receiving antenna may be the same as the radiating antenna.)
4. A receiver to process electronically the energy received through the antenna
5. A display element which makes the received energy available to a decision maker.

The transmitted signal may be uninterrupted continuous wave (CW), pulsed CW, phase modulated, amplitude modulated, or frequency modulated—depending on its tactical application. The emphasis here will be on pulsed CW systems.

The basic principle of pulsed radar is that the transmitter sends out radio waves in a series of short, powerful pulses and then rests during the remainder of its cycle. During the particular period or interval in which the transmitter is at rest, echo signals may be received and timed to determine the range to the reflecting surface. In CW radar, the transmitter sends out a continuous signal. If a nonmoving surface is in the path of the transmitted wave train, the frequency of the reflected signal will be the same as that of the transmitted signal. If the surface is moving, the frequency of the reflected echo will differ from that of the transmitted signal and the frequency difference can be used as an indicator of target motion. This target motion can also be detected by pulsed CW radar, but not as well.

For initial target detection, long-range search radar is used to evaluate targets before they can come within firing range. It differs in many features from a fire control radar, the function of which is to give accurate target location and tracking information. The search radar beam is broadly focused vertically and horizontally by the antenna to enable it to search a more extensive volume of space. It usually scans a full 360 deg around the installation. A search radar may be especially designed for air or surface search, or a single radar may be used for both.

Interference (including reflection), refraction, scattering, absorption, and polarization—all frequency dependent—are discussed in the paragraphs that follow in order to give the analyst some appreciation of the problems associated with the detection of targets by radar. Since frequency is inversely proportional to wavelength λ, these various phenomena affect the sensitivity of the signal-to-noise S/N ratio given by Eq. 27-1. This, in turn, affects the chance of target detection which depends greatly on the value of the S/N ratio.

27-2.1.1.1.1 Parameters

Frequency is an important radar parameter. Conventional radars have been operated at frequencies ranging from 25 MHz to 70 GHz.

The long wavelength region is used for ground radar; the center region is used for airborne radar; and the shortwave region is used for guided missile application. This short microwave region borders on the far infrared region, which is why components of radar and infrared systems (particularly antenna and lens systems) are similar.

Fig. 27-1 shows several different ways of identifying the frequency spectrum. During World War II, as a part of military security, radar frequency bands were given letter code designations. The practice, now a convenience, has continued. There is no universal agreement on the precise locations of these letter bands.

Other parameters are related to the characteristics of the type of radar used. (See Ref. 1, or other standard works, for a more complete discussion of the various radar types, their systems of modulation, employment, and associated parameters.)

The probability of radar target detection is a monotonic increasing function of the S/N ratio. This ratio is given by Eq. 27-1 which includes a significant number of the operationally meaningful parameters, i.e.,

$$\frac{S}{N} = \frac{P_t G^2 \lambda^2 \Sigma_t}{(4\pi)^3 R^4 k T_0 \overline{NF}_0 L B_l}, \text{ dimensionless} \qquad (27\text{-}1)$$

where

P_t = transmitted power, W
G = antenna gain, dimensionless
λ = wavelength, m
Σ_t = target cross section, m²
R = range to target, m
k = Boltzmann constant, 1.38×10^{-23} W/(Hz•K)
T_0 = 290 K
$\overline{NF}_0$ = noise factor of the receiver and which accounts for antenna and receiver noise inputs, dimensionless
L = system loss factor, dimensionless
B_l = acceptance bandwidth of receiver, Hz (If bandwidth of receiver is matched to bandwidth of transmitted pulse, B_l = 1.)

The antenna gain G is itself functionally related to the radar parameters:

$$G = 4\pi A_e/\lambda^2, \text{ dimensionless} \qquad (27\text{-}2)$$

where

A_e = antenna size, m².

It is important to note (Eq. 27-1) that S/N depends inversely on the fourth power of the range to the target and hence falls off very, very rapidly with range.

27-2.1.1.1.2 Targets

When a train of electromagnetic waves encounters a discontinuity in the propagation medium, a certain proportion of the energy is reflected (scattered) in many different directions. If the discontinuity is a finite body, then these reflections from the body enable it to be distinguished from its background and hence detected. From this quality or characteristic of reflectivity, which is in part frequency dependent and in part dependent on the electrical properties of the body radiated, a construct has been defined called the "radar cross section" of the target. This is also referred to simply as the cross section or target size. For most military radar targets, the cross section does not have a simple analytic relationship to the target except that the larger the target, the larger the cross section is apt to be. On page 40 of Ref. 1 cross section is defined in the following way: "The radar cross section of a target is the area intercepting that amount of power which, when scattered equally in all directions, produces an echo at the radar equal to that from the target.".

27-2.1.1.1.3 Propagation Path

The net detection effect produced by a radar depends not only on the radar itself and on its target, but also on the conditions of propagation. Of primary interest, as we have already mentioned, is the S/N ratio at the receiver given by Eq. 27-1. The five fundamental physical phenomena acting along the propagation path that affect the S/N ratio are:

1. Interference, including reflection
2. Refraction effects
3. Scattering
4. Absorption
5. Polarization.

The influence or impact of all of these on the signal to noise ratio depends on the carrier frequency.

27-2.1.1.1.3.1 Interference and Reflection

Electromagnetic waves can be given a vector representation, which is time and frequency dependent. If the same signal arrives at a point over two different paths so that one lags behind the other (which lag is termed a phase shift) by a time period equal to one-half cycle, and if the amplitudes of the two are the same, the two add vectorially and cancel each other. If their amplitudes are not the same, cancellation is less than complete. Conversely, the inphase signals reinforce each other.

When radar is used to detect air targets, a cancellation and reinforcement effect is produced as a result of signal reflection from the surface of the earth. This is known as the Lloyd's Mirror Effect. If E is the elevation angle of the air target, then the following relationships hold

$$\tan E = \frac{(2n + 1)\lambda}{4h_1}, \text{ cancellation} \tag{27-3}$$

where

n = 0, 1, 2
h_1 = antenna height, m
λ = wavelength, m.

Otherwise,

$$\tan E = (n + 1)\lambda/(2h_1), \text{ reinforcement.} \tag{27-4}$$

The importance of these relationships can scarcely be overemphasized. To the tactician who must be aware of the effect in planning his operations and to those involved in air defense, this is one of the sources of difficulty associated with detecting low flying aircraft. To an attacker, of course, this phenomenon offers the chance to approach a target closely with minimum chance of detection. The maximum cancellation occurs at zero elevation angle, since only in this region, which is known as the fade zone, are the signal amplitudes equal and out of phase. For higher elevations, the direct path signal is greater than the indirect path signal.

This effect is quite unrelated to line-of-sight propagation, and hence the weapon systems analyst should be aware of it in evaluating air defense systems.

27-2.1.1.1.3.2 Refraction

Electromagnetic waves are refracted on transmission through a nonisotropic medium, and the earth's atmosphere is such a medium. The velocity of propagation through the atmosphere is primarily dependent on air density. Density in turn is dependent on temperature and pressure. The primary refraction effects of the lower atmosphere are to introduce errors in range and elevation. Tracking noise is also introduced through small scale disturbances, such as turbulence.

Since the speed of propagation generally increases with altitude (decreased atmospheric density), there is a tendency for electromagnetic wave trains, or waves, to be bent downward. This phenomenon generally causes the radar horizon to be farther from the antenna than the optical horizon.

A simple approximation exists to calculate the distance or the maximum range to air targets, i.e.,

$$R_m \approx \sqrt{2h_1} + \sqrt{2h_2}\text{, mi} \tag{27-5}$$

where

R_m = maximum distance to target, mi
h_1 = antenna height, ft
h_2 = target height, ft.

Thus, if the radar antenna is 150 ft high and an air target is at 30,000 ft, the horizon-limited maximum range of detection would be approximately $\sqrt{300} + \sqrt{60,000} \approx 262$ mi.

If the propagation velocity increases more rapidly with height than normal, an anomolous condition—called super-refraction, trapping, or ducting effect—can cause the radar horizon to be greatly extended. It is possible to have ducts above the surface, but ground-based ducts are more common. In order for the energy to be trapped, the antenna must be in the duct.

Ground based ducts are created when the air temperature and/or the humidity decrease rapidly with height. Of the two factors, humidity is the more important one. These conditions frequently are found on the west coasts of continental land masses where ocean current upwelling brings cold water to the surface, and where high atmospheric pressure systems produce subsidence. This subsidence causes a high temperature gradient, which is accentuated at the plane of contact with the cold water.

27-2.1.1.1.3.3 Scattering

Scattering is another phenomenon that affects the S/N ratio. There are two main effects of scattering:

1. Loss of signal intensity
2. Loss of energy scattered back into the receiver.

Both of these effects are adverse. The loss of signal intensity diminishes the S/N ratio. Since backscatter appears as increased noise, it, too, decreases the S/N ratio.

Scattering targets are described as volume or area targets. Rain and clouds are volume targets, while the surface of the earth or sea are surface (area) targets. Fig. 27-2 displays the backscatter cross section, in square centimeters, of a cubic meter of air which is occupied by rain. Rain density is defined in terms of rainfall rate in millimeters per hour.

27-2.1.1.1.3.4 Absorption

Absorption causes a loss of signal intensity which is cumulative over distance. This signal attenuation is measured in decibels (dB) per km.* The process is caused essentially by the various gases which

*The decibel is a dimensionless number used to measure ratios. It is defined as 10 $\log_{10} R$, where R is the ratio to be measured. To illustrate, let $I_R = 5\times10^{-10}$ W·cm^{-2}, and $I_o = 10^{-6}$ W·cm^{-2}. Then for the ratio $R = I_R/I_o$, we have 10 $\log_{10} R = -43$, or I_R is 43 dB down. The decibel relation offers the convenience of the logarithmic compression of a number. It is important that I_R and I_o be expressed in the same units.

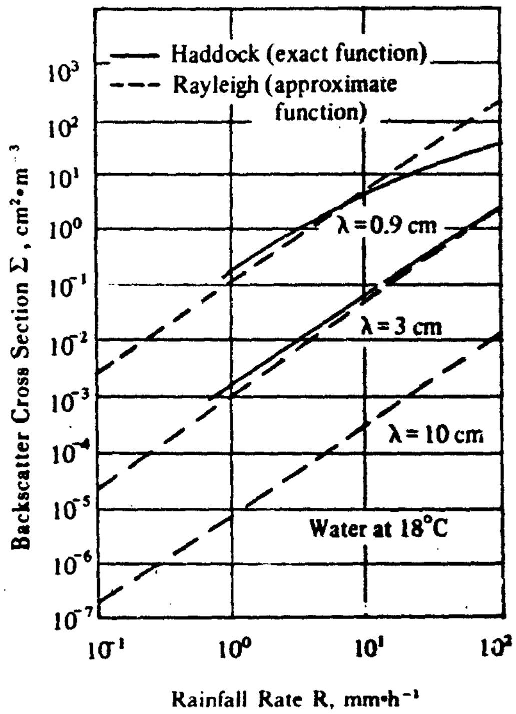


Figure 27-2. Exact and Rayleigh Approximate Backscattering Cross Sections Per Unit Volume of Rain-Filled Space, Plotted Against Rainfall Intensity*

*D. Atlas et al., *Air Force Surveys in Geophysics*, no. 23, Geophysics Research Directorate, Cambridge, Mass., 1952.

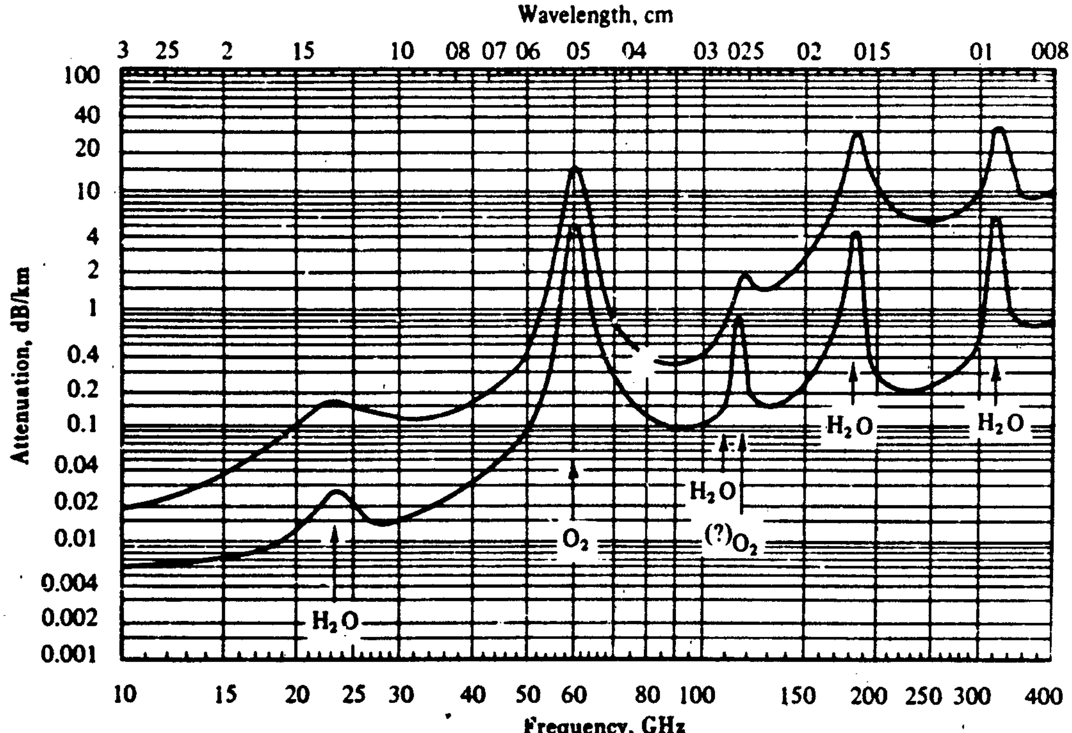
constitute the atmosphere. The molecules of these gases are excited electrically by the wave train. This excitation absorbs energy from the radar signal.

Fig. 27-3, which relates the gas density dependency on altitude, shows the effect of wavelength on attenuation.

27-2.1.1.1.3.5 Polarization

The electromagnetic wave comprises two sinusoidally varying field vectors, one electric $\vec{E}_0$, the other magnetic $\vec{H}_0$. These vectors are at all times perpendicular to one another and to the direction of energy propagation. The simplest case of polarization is that in which the electric field vector varies in magnitude in a plane having some fixed orientation with respect to the direction of propagation. This is shown in Fig. 27-4. Here the electric field vector $\vec{E}_0$ is constrained to the *YX*-(vertical) plane, and the wave is said to be vertically polarized. The plane of polarization could have any orientation. It may be that the plane of orientation of the electric field vector rotates in time. Naturally, the magnetic field vector $\vec{H}_0$ rotates in order to maintain their perpendicularity.

Fig. 27-5 illustrates a point in space, viewing the *YZ*-plane along a direction of increasing *X*. If the vector $\vec{E}_0$ at this point rotates in the *YZ*-plane in such a way that its magnitude describes an ellipse as shown, then the wave is said to be elliptically polarized. If $E_Z = E_Y$ for all time, then the wave is circularly polarized.



Upper curve: "Sea level"

P = 760 mm Hg
T = 20°C
ρH_2O = 7.5 g/m³

Lower curve: "4 km"

T = 0°C
ρH_2O = 1.0 g/m³

From "Atmospheric Absorption of 10-400 kMcps Radiation: Summary and Bibliography to 1961" by E. S. Rosenblum, *Microwave Journal*, Vol. 4, Copyright © 1961 by Horizon House-Microwave, Inc. Reprinted by permission of Horizon House-Microwave, Inc.

Figure 27-3. Attenuation Per Kilometer for Horizontal Propagation

27-2.1.1.2 Infrared Sensing

Electromagnetic waves lying in that region of the spectrum between 1−500×10⁶ MHz are classified as infrared (ir). Because the frequencies are in millions of megahertz, it is customary, for brevity, to refer to the wavelength rather than the frequency. The unit in common usage is the micron, which is one millionth of a meter (10^{-6} m). In this system, the ir region extends from 0.75 to 1,000 microns. The ir region itself is further divided into three subregions:

1. The near ir (nir): $0.75 \leq$ nir < 1.2 microns
2. The intermediate ir (iir): $1.2 \leq$ iir < 7.0 microns
3. The far ir (fir): $7.0 \leq$ fir $< 1,000$ microns.

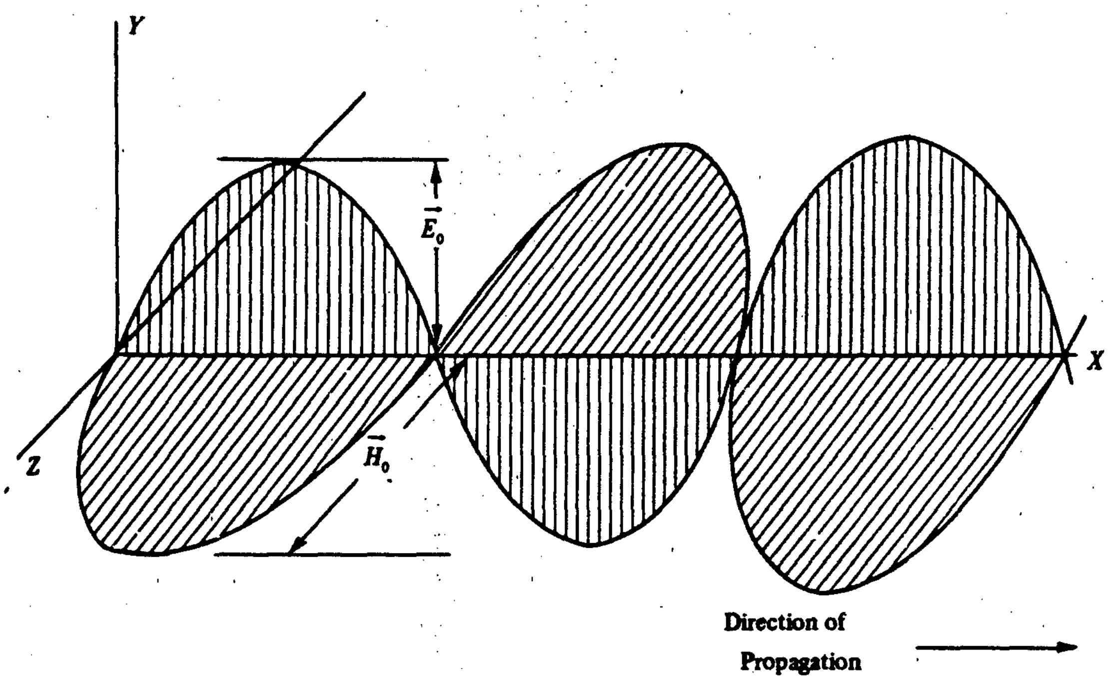


Figure 27-4. Electric and Magnetic Vectors

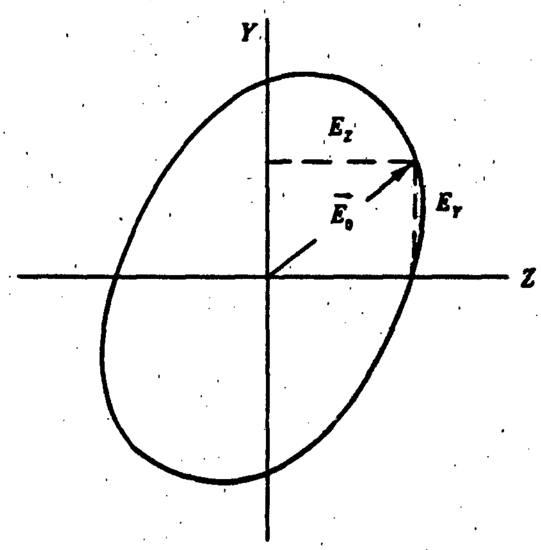


Figure 27-5. Elliptical Polarization

A further subclass of the fir region is the 8 to 30 micron region, termed the long wavelength infrared region (LWL). In recent years, many ir systems have been developed for operation in the LWL.

Qualitatively, the ir region lies between radio and the visible portions of the spectrum, with the fir shading off into visible red and the nir just above the highest radar frequencies.

Infrared radiation is a consequence of molecular agitation in translation and rotation. All matter consists of electrically charged particles. The movement of an orbital electron is equivalent to an electrical current. These currents create the ir radiation. Molecular movement occurs in all matter at temperatures above absolute zero. In fact, absolute zero is defined as the temperature at which such motion ceases. The origin of the Kelvin scale, at this point, is written 0 K which is equivalent to −273° C or −459° F. If one considers a plane surface of area 1 cm^2 at absolute temperature T K, then the total power radiated into one hemisphere is given by the Stefan-Boltzmann Law

$$E_p = \epsilon \sigma_s T^4 \tag{27-6}$$

where

E_p = total emissive power, W/(cm^2•hemisphere)
σ_s = the Stefan-Boltzmann constant = 5.7×10^{-12} W/(cm^2•K^4)
T = absolute temperature, K
ϵ = total emissivity of the surface (unity for a black body), dimensionless.

The emissivity constant is unity for a perfect black body, or in other words, a body with emissivity 1 is defined to be a black body. For the long wavelength portion of the ir spectrum, most bodies except metals can be considered black. A black body has the capacity to absorb totally all radiant energy falling on it. Any enclosure with constant temperature interior walls and a very small aperture can be considered a black body.

The Stefan-Boltzmann Law gives the total emissive power, but does not reveal anything about the frequency distribution of this power. This distribution is given by Planck's Radiation Law

$$J(\lambda) = \left(\frac{c_1}{\lambda^5}\right) \frac{1}{\exp[c_2/(\lambda T)] - 1}\ , \ \mathrm{W/(cm^2 \cdot cm)^*} \tag{27-7}$$

where

$J(\lambda)$ = the emissive power of unit area in the wavelength interval τ, W/(cm^2•cm)*
c_1 = 3.7415×10^{-12} W•cm^2
c_2 = 1.4388 cm•K
λ = wavelength, cm
T = temperature, K.

This relationship is graphed in Fig. 27-6.

The total emissivity is the integral of Eq. 27-7 for $0 < \lambda < \infty$ for any temperature in the graphs; the total emissivity is the total area under the curve, including that portion of the upper tails not shown in the figure.

Refs. 2 and 3 have a good introduction to these relationships and cite the scientific literature for further reference.

*W/cm^2 of surface area per cm wavelength of the radiation

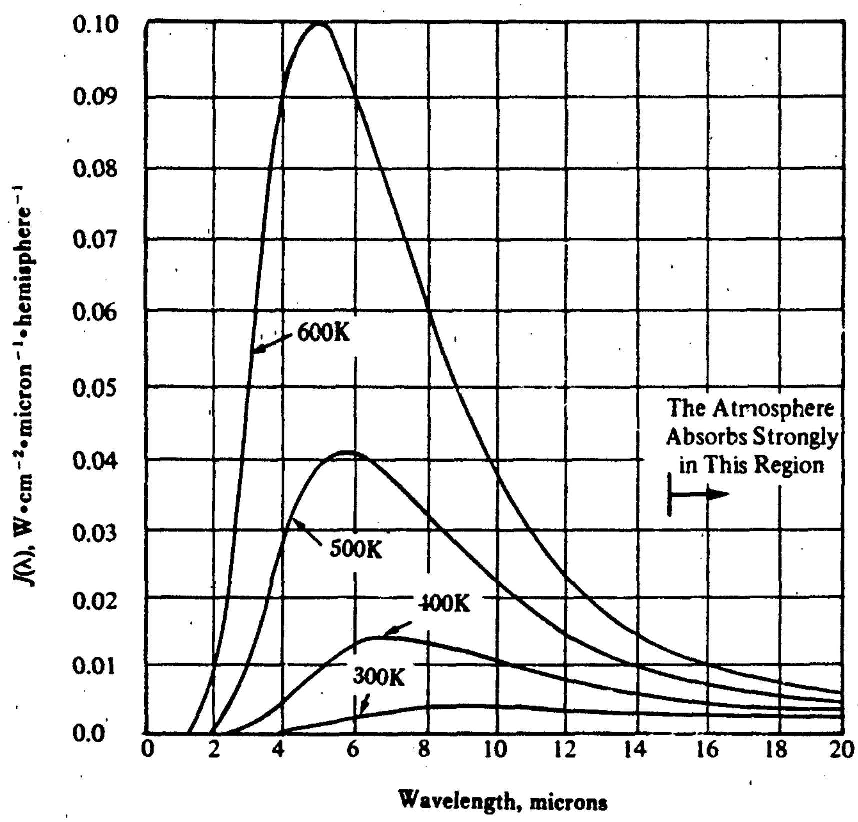


Figure 27-6. Spectral Distribution of Energy for Perfect Emitters

27-2.1.1.2.1 IR Detectors

The infrared part of the electromagnetic spectrum is important in military applications. This importance arises from the fact that these radiations produce heat when they fall upon a material object. Thus; the presence of infrared can be sensed directly without resort to electronic receiving devices or optical systems, although these classes of devices have been found to be extremely useful for enhancement of ir detection, perception, and analysis. The sidewinder, a pit viper of the southwest American desert, senses ir sources directly with his sense of feel.

The human eye is insensitive to ir. However, in the 1930's, RCA developed an image tube that converted ir radiation into visible light. This development made night visual surveillance possible, including such military applications as the sniper scope and battlefield surveillance.

IR image forming systems generally obey the same optical laws as do visual systems but, because the wavelength differences are so great, there is considerable variation of the refractive index for most common optical materials. Ref. 3 contains an extensive list of optical properties of ir-suitable materials.

A wide range of ir-sensitive devices has been used. There are two main categories, quantum detectors and thermal detectors. In an idealized quantum detector, each incident ir photon which is absorbed produces one or more countable electrons in an external measuring circuit. Thermal detectors measure some heat sensitive characteristic of the element—such as a change in electrical resistivity, physical dimension changes, or voltage.

Thermal detectors, measuring energy, have a fairly flat frequency response curve. It is a characteristic of radiation that the higher the frequency of radiation, the greater the energy of a photon. Since the number of electron excitations which take place depends on the energy of the absorbed photon, it can be seen that quantum detectors would be most sensitive in the near ir.

Quantum detectors can be classified as:

1. Photo emissive
2. Photo conductive
3. Photo voltaic
4. Photo electromagnetic.

Thermal detectors are of the following types:

1. Liquid thermometer
2. Golay cell
3. Calorimeter
4. Thermocouple
5. Thermopile
6. Bolometer.

Ref. 3 contains concise descriptions of these devices and their characteristics.

The two most commonly used modes of target detection by ir radiation can be classified as passive and active. Characteristic radiation ir detection is the term used to describe the passive mode of detecting emitted ir radiation which is characteristic of the target. The active mode operates on the principle that ir radiation is detected when a beam of ir is radiated by a special transmitter and then reflected by the target. A third mode, semiactive, is not generally used. It depends on the reflection of ir energy from a source independent of both the sensor and target.

27-2.1.1.2.2 Effectiveness

IR systems offer one great advantage over radar in that they can operate completely passively—thus not being susceptible to detection and countermeasures. Also, these passive ir systems are much simplified by not requiring complex, expensive, and bulky transmitters. Coupling these advantages with the fact that almost all physical objects are ir transmitters, we then see that the passive ir becomes a most attractive military detection system.

Ref. 3 contains the following list of advantages and disadvantages of ir systems.

"A summary of the advantages of ir systems includes:

1. Small size and light weight compared to comparable active systems
2. Low cost compared to active systems
3. Capable of passive or active operation
4. Effective against targets camouflaged in the visible region of the optical spectrum
5. Day or night operation
6. Greater angular accuracy than radar
7. No minimum range limitation
8. Minimum requirement for auxiliary equipment.

"The performance limitations of ir systems are imposed mostly by atmospheric conditions. Humid atmosphere, fog, and clouds present serious limitations. The problems can be briefly summarized as follows:

1. Lack of all-weather capability (in operation within the atmosphere)
2. Line-of-sight detection capability only
3. Requirements for cryogenic cooling during LWL operation."

27-2.1.1.3 Visual Sensing

The visible portion of the electromagnetic spectrum lies between 0.38 and 0.72 micron. The human eye interprets the long wavelength end of this window as red, which shades off to ir, and the short end as violet, which runs into the ultraviolet. The eye—the retina, lens, and iris—is the ultimate optical sensor since it is the connection between the world of visible images and the brain.

There are several optical aids, which can be classified as image enhancing, image preserving, and measuring devices. Image enhancing devices run the gamut from spectacles through microscopes, binoculars, to large astronomical reflecting telescopes. Image preserving devices are primarily photographic, although video tape and other technologies are also available. Typical measuring devices are optical range finders, surveyor's transits, and diffraction screens. Ref. 4 provides an excellent survey of optical principles.

27-2.1.1.4 Photography

Photography plays an extremely important role in the employment and analysis of military systems. The principles of image formation, magnification, resolution, etc., in photography are identical to those in visual optics. There is one minor difference of principle, however, which is important in application. The camera lens projects its image on a photographic film rather than on the retina of the human eye. Two results follow:

1. Film sensitivity to radiations of various wavelength in and near the visual window can be varied and controlled.

2. The image formed is fixed, i.e., it is preserved for later detailed analysis. It can be further processed by enlargement and chemical enhancement.

By appropriate chemical formulation, and when used in conjunction with special filters, color film can be made sensitive to portions of the electromagnetic spectrum which are not visible to the eye. The region which has been found to be most useful is the ir. Thus, the film, sensitive to radiations invisible to the eye, records these images by translating them into color images which are visible. The usefulness of this lies in the fact that two dissimilar subjects which may reflect light equally in the visible spectrum may not do so in the ir spectrum. Thus, a truck, for example, may be virtually invisible to the naked eye if it is painted green and viewed from above against a green background of foliage. However, the pigment of the truck paint and the pigment (chlorophyll) of jungle vegetation may differ greatly in ir reflectivity. Thus, on the ir color film, the two will produce strikingly different color images which are readily detectable to the eye.

Heat sources can be photographed using ir sensitive film, either black and white or color.

Many changes in the flora of an area which are not apparent to the unaided eye or conventional color film are detectable using ir sensitive color film. This fact is finding widespread scientific and military application. Fungi or plant disease incursions—with potential commercial, scientific, or military inference—are thus detectable in their earliest stages. See Ref. 5 for a good introduction to the uses of color aerial photography.

27-2.1.2 Acoustic Sensing

Whereas electromagnetic waves may be transmitted through a vacuum, acoustic waves require an elastic medium for their propagation. This difference, while essential, does not lead to analytic techniques which differ greatly, however. The same kind of relationships exists between frequency and such parameters as beam width of a transmitting device (transducer), gain of a receiving array, reflection, and refraction. One characteristic of electromagnetic waves totally absent from acoustic waves is polarization. Active acoustic devices—ones in which a sound signal is generated, transmitted, and an echo received from a target—are not often found in military systems other than the SONAR (sound navigation and ranging) used under water. Despite range and other limitations, sound waves are just about the only energetic waves which can be transmitted for any distance through water.

Passive acoustic or listening devices do have application in certain circumstances. Essentially, they involve a receiving transducer (microphone or hydrophone), an audio amplifier and some sort of signal processor and display arrangement. The receiving transducer can be made highly sensitive and directional so that unwanted off-axis noise can be sharply limited while desired signals are received unimpeded and amplified.

27-2.1.3 Chemical Sensing

Most substances at ordinary temperatures emit molecules, even if in the most minute quantity, into the air. The process is related to those of vaporization and sublimation. These molecules produce in living creatures the sensation of smell and can often be used to identify their source uniquely. Obvious military applications of this phenomenon exist and have been used since ancient times. The trained sentry dog and bloodhound are the most common applications.

Recently modern technology has been brought to bear, and electrochemical devices such as "sniffers" have been constructed which detect human beings by their scent. Internal combustion engines may be detected through emission of their exhaust products with great accuracy and sensitivity.

For the most part, scents are windborne, so the application of these devices is somewhat circumscribed thereby. Nevertheless, the weapon system designer and analyst should be alert to their potentialities as signatures.

27-3 SEARCH AND RELATED DETECTION PROBABILITIES

27-3.1 INSTANTANEOUS DETECTION

Search is the employment of detection devices or systems in an operational environment to discover targets. Search theory provides a variety of principles or plans which are suitable for detection of targets in a number of tactical situations. The analyst often will have some interest in chances of detecting targets with sensors of all types—including the unaided eye, binoculars, and the more sophisticated radars, etc.

For the weapon systems analyst, some of the more basic and important contributions in the mid-1950's are due to B. O. Koopman (Refs. 6 and 7) and Stone (Ref. 8). In Ref. 7, Koopman discusses the geometric and kinematic factors involved in search—i.e., the positions, motions, and contacts of observers and targets. Although probability models for detection of targets are introduced in Refs. 6 and 7, they are developed more fully in Ref. 8; the latter reference covering optimum procedures for the problem of search.

As presented in par. 11-9, two models of sensing were considered, depending on the nature of the sensor: (1) one in which the sensing is characterized by a succession of glimpses, as in the case of echo ranging wherein each sweep or scan constitutes a distinct glimpse of the target, and (2) the other involving continuous search or looking typified by visual search, for example. For the case of glimpses, the chance P_n of detecting a target at least once in n trials was given in Eq. 11-2 and Refs. 7 and 8 as

$$P_n = 1 - \prod_{i=1}^{n} (1 - g_i) \tag{27-8}$$

where g_i = chance of target detection on ith trial or glimpse, and this reduces to

$$P_n = 1 - (1 - g)^n \approx 1 - \exp(-ng) \tag{27-9}$$

if the chances of detection on all trials are equal ($g_i = g$), and g is sufficiently small.

The chance G_n of not detecting a target in the first $(n - 1)$ glimpses but detecting it on the nth glimpse was given in Eq. 11-4 by the negative binomial (or here the geometric) distribution

$$G_n = (1 - g)^{n-1}g \tag{27-10}$$

and the expected number $E(n)$ of trials to a detection is, from Eq. 11-5,

$$E(n) = 1/g. \tag{27-11}$$

For continuous scanning, the instantaneous chance $p(t)$ of detection in time t was determined to be

$$p(t) = 1 - \exp(-t/\theta) \tag{27-12}$$

where

θ = MTTD = mean time to a detection.

We note that the glimpse or discontinuous model, Eq. 27-9, and that for continuous search, Eq. 27-12, become equivalent for

$$ng = t/\theta \tag{27-13}$$

the expected number of detections (in time t).

For the simple models—Eqs. 27-9, 27-10, and 27-11—the parameters g and θ depend on the detection equipment, target characteristics, and background terrain and flora, although they may be estimated experimentally for different sensors, field conditions, and tactical considerations. In particular, the mean time to detect θ of Eq. 27-12—since it involves parameter estimation for the exponential distribution—may be calculated advantageously from equations such as Eq. 21-83, which involve either truncated or complete sample data. These simpler models, however, cannot possibly cover the more complex situations likely to be faced in the field, for, as we have seen, the S/N ratio given by Eq. 27-1 comes into importance,—depending on the detection equipment, target characteristics, terrain features, atmospheric conditions, background noise, etc.

In Ref. 7, Koopman develops some of the basic theory on detection probabilities (primarily for naval target search operations on the ocean surface) which are useful to the Army weapon systems analyst. Koopman considers key parameters involved in target detection, including in particular the range to the target, the solid angle subtended by the target at that range, target reflectivity characteristics (in the form of a constant of proportionality), target speed if applicable, type of sensor (glimpse or continuous), and effective search (or sweep) width and rate of the sensor. A tactical consideration of importance is whether or not a target is detected during the time it is in a detectable state. If not, the target may detect the sensor and hence bring fire upon the sensor unopposed.

Koopman (Ref. 7) shows that the chance $p(R)$ of target detection depends substantially on the lateral range R* to the target and, as a matter of fact, one may use the equation

$$p(R) = 1 - \exp[-F(R)] \tag{27-14}$$

where $F(R)$ is the appropriate function of the lateral range R. The graph of $p(R)$ versus R is called the lateral range curve and expresses the distribution in the lateral range. Koopman calls the area W under the lateral range curve the effective search or sweepwidth of the sensor.

For the case of intermittent glimpses, occurring T_u units of time apart, Koopman assumes the definite range law

$$p(0) = 0$$

when the lateral range R exceeds a limiting range R_0 and

$$p(R) = 1$$

when the total length $2\sqrt{R^2 - R_0^2}$ of the relative track during which the target is within range R_0 of the observer and farther away than VT_u, where V is the target speed. In this case, the lateral range curve is simply a rectangular, or a uniform distribution, where $p(R) = 1$ for $-R_0 \leq R \leq R_0$, and $p(R) = 0$ otherwise.

However, the chance of target detection within range R_0 is otherwise

$$p(R) = 2\sqrt{R_0^2 - R^2}/(VT_u) \tag{27-15}$$

where we must have

$$-\sqrt{R_0^2 - V^2T_u/4} \leq R \leq \sqrt{R_0^2 - V^2T_u/4} \tag{27-16}$$

Personnel of the ARINC Research Corporation showed that for continuous search and an assumption that the *pdf* for detection with time is proportional to the target cross section divided by the range squared, then the chance of detection, given the maximum range of detection R_m, may be expressed as

$$P(R|R_m) = 1 - \exp[-(K'/R)\arctan(\sqrt{R_m^2 - R^2}/R)] \tag{27-17}$$

where

$K' = 2KA_t/W$, m (27-18)

K = parameter determined by experiment for the sensor, or by more fundamental theoretical investigation, dimensionless

*The lateral range R is the minimum distance between the target and the line along which the sensor moves.

A_t = target cross-sectional area, m^2

W = width of path (sweep) searched by sensor, m.

Some typical lateral range curves for Eq. 27-17 as a function of the range R, where the maximum range R_m is taken to be 200 m, are shown in Fig. 27-7. We emphasize that Eq. 27-17 is for continuous looking and an inverse square law relationship.

If the sensor is a radar and the detection probability is a nondecreasing function of the S/N ratio, then the intensity I_R of the returned echo at the receiver is given by

$$I_R = P_t G\Sigma/[(4\pi)^2 R^4], \quad W/m^2 \tag{27-19}$$

where

P_t = transmitted power, W

G = antenna gain, dimensionless

Σ = target size, m^2

R = range, m.

For such a radar, ARINC research personnel have shown that the chance of target detection at range R then becomes

$$P(R|R_m) = 1 - \exp\{-[K''/(2R^2)][\sqrt{R_m^2 - R^2}/R_m^2 + (1/R)\arctan(\sqrt{R_m^2 - R^2}/R^2)]\} \tag{27-20}$$

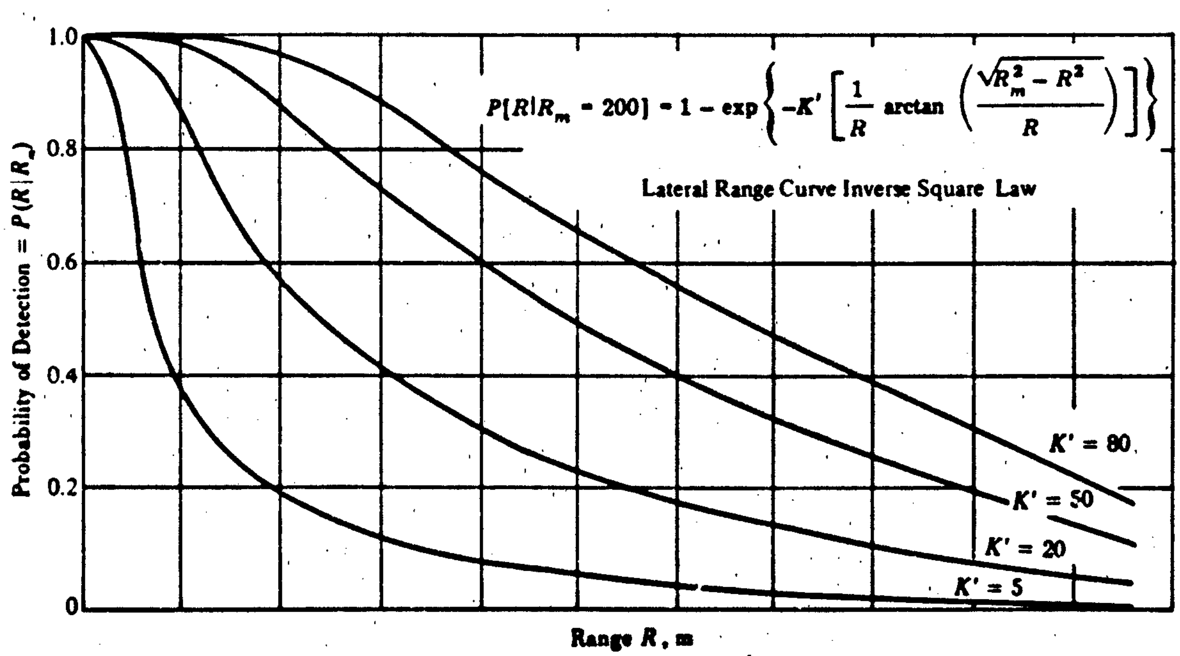


Figure 27-7. Typical Lateral Range Curves

where the coefficient K''—expressed in cubic meters—depends on the radar, and may be determined experimentally or perhaps from applicable theory. In the case of the S/N ratio depending on the inverse fourth power of range as in Eq. 27-1, the chance of target detection falls off very rapidly as compared to the inverse square law. The sharp differences are indicated in Fig. 27-8. In spite of such a rapid drop for the inverse fourth power law, most radars—and air search radars in particular—can operate at very high power with high gain antennas so that their detection capability is not so much limited by this sort of range dependency as by the truncation imposed by the radar horizon.

The principles examined so far are adaptable to almost any kind of sensor, provided its dependency on range is known, even approximately. Moreover, the analyst must frequently be satisfied with first order approximations because more exact information is hardly ever available. The lateral range curve may be viewed as an intermediate analytical, though perhaps not as an overall, measure of system detection capability. The system with the "highest" lateral range curve would ordinarily be the one selected, depending on perhaps other overpowering considerations.

The most comprehensive and authoritative current work on search theory is that of Stone (Ref. 8).

27-3.2 RANDOM SEARCH

Koopman (Ref. 7) also covers the case of random search in some area A of interest in combat. To determine chances of detection, the following assumptions are made (Ref. 7):

1. The position of the target is assumed to be uniformly distributed in A.
2. The observer's path is random in A in the sense that it can be thought of as having its different (not too near) portions placed independently of one another in A.

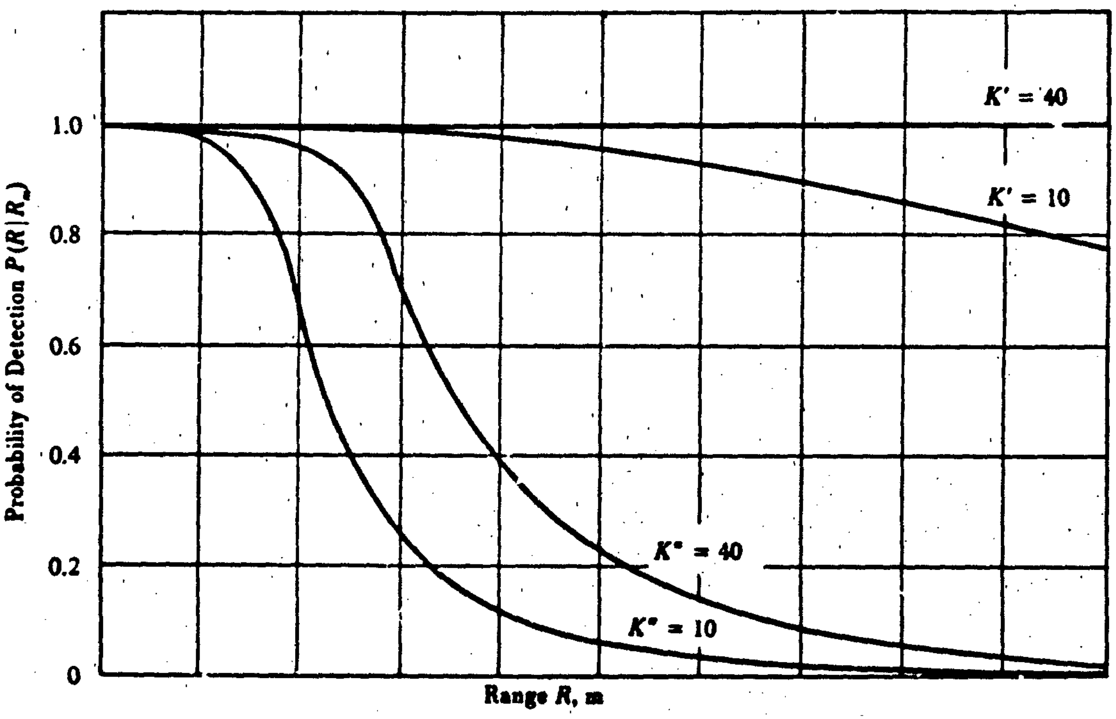


Figure 27-8. Comparison of Inverse Square and Inverse Fourth Law Lateral Range Curves

3. On any portion of the path which is small relatively to the total length of path, but decidedly larger than the range of possible detection, the observer always detects the target within the lateral range $W/2$ on either side of the path and never beyond.

Now consider an effective searchwidth W and the total length of the observer's path (or distance of the sensor) covered in area A. If the length L of the path of search is divided into n equal portions of length L/n, then the chance that along all of L there will be no detections is

$$1 - p = [1 - WL/(nA)]^n \tag{27-21}$$

and hence the probability of detection of the target is simply

$$p = 1 - [1 - WL/(nA)]^n \approx 1 - \exp(-WL/A) \tag{27-22}$$

for n sufficiently large. Thus, we arrive again at the exponential type of probability law, and the reader may note the close similarity of Eq. 27-22 to the coverage functions of Chapter 20.

If the region swept out consists of a straight line, or a path with practically no bending, then the total area swept is WL and the chance of the target being within this region is

$$p \approx WL/A. \tag{27-23}$$

Fig. 27-9 indicates the relative difference between Eqs. 27-23 and 27-21.

In case there is a target-observer relative speed v then the chance of detection in time t corresponding to Eq. 27-22 becomes

$$p(t) = 1 - \exp(-Wvt/A). \tag{27-24}$$

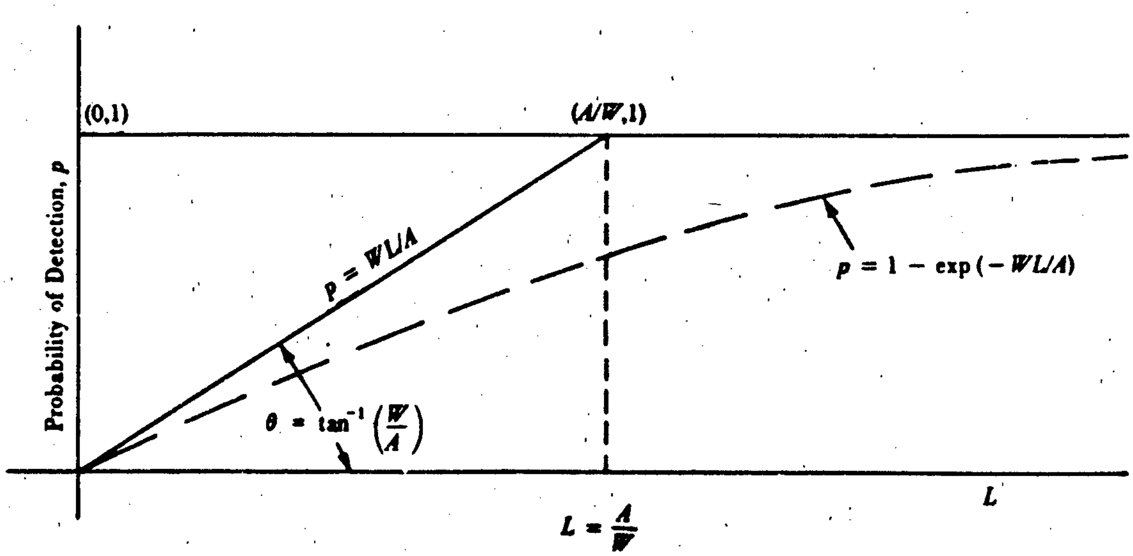


Figure 27-9. Detection Probability as a Function of Search Effort

For this case of "exponential-saturation", Koopman (Ref. 9) develops a theory for the optimum distribution of searching effort, and gives a very clever, useful, and rather simple graphical approach as the solution. He assumes the probabilities of the target being in various positions are known with sufficient accuracy for the particular case where the target searched for is a point on a line (the x-axis or range R, say). Then he plots the natural logarithm of the chance of detection as a function of the range and draws a horizontal line, keeping it always parallel to the x- or R-axis until the area above the line and under the curve has a value equal to the total available searching effort. Then, finally, the search should be made along the ground only for those intervals above which the drawn curve (natural logarithm of detection probability) has peaks above the effort line.

F. I. Hill (Ref. 10) shows that for objects on the terrain the probability of detection within time t of a target of presented area A lying on the terrain is determined by the area rate of search $r_{A(b\ell)}$ of the area of uncertainty A_u where $r_{A(b\ell)}$ is the visual rate of search for the target under background b at lighting level ℓ. In fact, the chance of detection is expressed as

$$p(t) = 1 - \exp[-r_{A(b\ell)}t/A_u]. \tag{27-25}$$

Hill also indicates that the search rate $r_{A(b\ell)}$ can be scaled directly with the presented area of the target—i.e., for constant background and lighting and target areas A_1 and A_2—then

$$r_{A2} = r_{A1}(A_2/A_1). \tag{27-26}$$

For n observers, then it is easy to see that the chance of detection in time t becomes

$$p(t) = 1 - \exp[-nr_{A(b\ell)}t/A_u] \tag{27-27}$$

as the rate of search is effectively increased from $r_{A(b\ell)}$ to $nr_{A(b\ell)}$. In his Table 2, Hill (Ref. 10) gives some search rates for the unaided eye (field of view of 62 deg) and 7×50 binoculars (field of view of 7.23 deg) when searching for trucks, jeeps, and man. He also gives search rates from a tank for these targets.

EXAMPLE 26-1:

Suppose an observer with binoculars is searching from a tank for a truck of presented area 160 ft^2 in moonlight. Then for an area of uncertainty of 4900 m^2, what is the chance of detection of the truck in 5 s?

We have A_u = 4900 m^2, t = 5 s, and $r_{A(b\ell)}$ = 1805 m^2/s, the latter figure being taken from Hill's Table 2, Ref. 10.

Hence,

$$p(t) = p(5) = 1 - \exp[-1805(5)/4900] = 0.52.$$

Larsken (Ref. 11) studied the impact of mine warfare on combat mobility, and converted Hill's Eq. 27-27 to the chance of tanks on the move detecting surface-laid mines. He indicated that the chance of detecting mines from n tanks is approximately

$$p(d) = 1 - \exp[-430nA/(WV)] \tag{27-28}$$

where

n = number tanks searching for mines
A = mined area, m^2
W = width of path searched, m
V = tank speed, m/s
430 = coefficient in units of reciprocal seconds.

Larsken also discusses a comparison of the theory with experiments in Ref. 11.

27-3.3 SOME RELATED INVESTIGATIONS OF SEARCH THEORY

Danskin (Refs. 12 and 13) discusses the theory of reconnaissance and search from the standpoint of information theory or the objective of information gain.

Dobbie (Ref. 14) gives a very useful survey of topics in search theory until 1968, and Pollack (Ref. 15) deals with the problem of search detection and subsequent action, and discusses interface problems. Pollack indicates that one should consider the probability of false alarms while searching. If, for example, τ is the average time wasted for each false alarm, and the false alarm rate while searching is f (the average number of false alarms per unit of time), then W (the sweepwidth of Eq. 27-24) should be replaced by W', i.e.,

$$W' = W/(1 + f\tau). \tag{27-29}$$

An excellent set of references for further study is also given by Dobbie (Ref. 14) and Pollack (Ref. 15). Dobbie extends the theory of search problems with false contacts in Ref. 16.

Mela (Ref. 17) points out that information theory and search theory should be regarded as special cases of the more general theory of statistical decisions and gives examples to back his point.

27-4 SEARCH STRATEGIES

A very important area of investigation in search considerations and related theory is related to optimal strategies of search in order to detect targets. This problem was mentioned briefly in par. 27-3.2 in connection with the contributions of Koopman (Ref. 9).

An observer often is interested in determining which of several regions should be searched for the problem of target detection. Blackwell and Ross (Ref. 18) have considered the problem of an object hidden in one of m boxes and how best to search for it. For our application here, we might consider dividing the terrain into some m regions in which we are required to locate a target. If the cost (or the effort expended) of searching region i is equivalent to c_i dollars, and the chance of finding the target is α_i if it is in region i, then for prior probabilities p_i that a target is hidden in region i the optimal strategy is to search only in that region which has the largest ratio

$$\alpha_i p_i / c_i. \tag{27-30}$$

Of course, the quantities, α_i, p_i, and c_i, must be estimated with whatever information may be available. Based on Bayes theorem, one may often have to take the p_i as being equal.

One begins to see that in the various strategies for searching it is likely that he must get involved rather deeply with subjective probabilities! (See Stone, Ref. 8.)

Cameron and Narayanamurthy (Ref. 19) studied efficient policies for a search in which it is desired to locate a point in an interval with uniform *a priori* probability density by repeated application of a test to determine whether the point lies to the left or right of the "test" point with different associated costs or efforts. To illustrate, consider that in one direction (on a line) we are interested in a search in an interval of length x_0, and desire to search the interval at test points until the last search or test point is within one unit of the "target" point. Suppose further that the "right" and "left" are designated so that the cost of testing (or the effort expended) is 1 if the target point lies to the left and the cost of testing is $k > 1$ if it is to the right. Then Cameron and Narayanamurthy show that a suboptimal policy is to test at the point that divides the interval x_0 in the ratio

$$r = \alpha/(1 - \alpha) \tag{27-31}$$

where α is determined from

$$\alpha^k + \alpha = 1. \tag{27-32}$$

For iteration, the reader should consult Ref. 19 for more details, and also the optimum search method, although on some practical grounds Eq. 27-31 might be adequate.

Pollack (Ref. 20) develops a model for the search of a moving target that moves between two regions in a Markovian fashion, i.e., involves conditional transition probabilities from one region to the other. Discrete amounts of search effort or "looking" may be allocated to one of the two regions at a time. Pollack uses a dynamic programming technique to develop equations that characterize the minimum expected number of looks to detect the target, and the maximum probability of detecting the target with a given number of looks. Schweitzer (Ref. 21) also studied this same problem of minimum-search policy. In Ref. 21, Schweitzer presents a fairly simple recursive procedure for calculating the "threshold" probability which is used to determine which region to search next. Again, one gets rather involved in subjective probabilities, and any information indicating chances concerning the whereabouts of the target may be very important and useful in such problems.

Tognetti (Ref. 22) discusses the concept of "whereabouts search" and indicates that the best strategies for target "detection search" on one hand and "whereabouts search" on the other are not the same. In "detection search", one wishes to maximize the chance of detecting the target using a search strategy which has a limited cost, budget, or effort. For a "whereabouts search", however, the primary objective is to maximize the probability of correctly stating which region the target is in after conducting a search of a given cost or effort—and usually having been unsuccessful in finding the target. The "whereabouts search" is more of a reconnaissance type mission, so to speak. In Ref. 22, Tognetti discusses optimal strategies for a "whereabouts search". Kadane (Ref. 23) generalizes the work of Tognetti and shows that, once some box (region) has been chosen to be guessed, the optimal "whereabouts search" is an optimal detection search involving all the other regions.

Finally, for our account of search strategies here, we return to the case of the continuous, sweeping type of search of an area. Instead of the assumption of a fixed width of path (sweep) searched by a sensor, Richardson and Belkin (Ref. 24) in a paper on "Optimal Search With Uncertain Sweep Width" allow for the treatment of path (sweep) width as a random variable. They indicate in this connection that path widths for target sensors are always subject to testing errors and the conditions of search may lead to randomly varying path widths. This leads to their assumption of a gamma prior path-width distribution. In other words, the path-width distribution is assumed to follow a gamma probability density function g

$$g(W,\alpha,\nu) = W^{\nu-1}\alpha^{\nu}\exp(-\alpha W)/\Gamma(\nu) \tag{27-33}$$

where

W = width of path (sweep) searched by sensor

α = scale parameter reciprocal

ν = shape parameter.

The mean of the distribution is

$$E(W) = \nu/\alpha \tag{27-34}$$

and the variance is

$$\mathrm{Var}(W) = \nu/\alpha^2. \tag{27-35}$$

Hence, we note that if we have an estimate of the mean path width and its variance, we can fit the two-parameter gamma density, Eq. 27-33. If we further assume that the target is likely to be uniformly distributed in the area A, then Richardson and Belkin (Ref. 24) show that the optimal (maximum probability of detection) search procedure m^* is to allocate the total search effort such that the track length per unit area over the region searched is

$$m^* = ut/A \tag{27-36}$$

where

u = the constant search speed

t = time allowed for the search

A = area of interest in which the target is located.

In this case of optimal searching the (maximum) chance $p(t)$ of detecting the target in time t is shown to be

$$\begin{aligned} p(t) &= 1 - [1 + ut/(\alpha A)]^{-\nu} \\ &\approx 1 - \exp[-\nu ut/(\alpha A)] \end{aligned} \tag{27-37}$$

for suitably large areas A.

The expected time $E(t)$ to detect the target for the uniform prior distribution (Ref. 24) turns out to be

$$E(t) = \alpha A/[u(\nu - 1)] \quad , \quad \nu > 1 \tag{27-38}$$

but is infinite if $0 < \nu \leq 1$.

Richardson and Belkin (Ref. 24) also cover the case of the target location following a bivariate normal probability distribution, which is more complex. They also compare their optimal search plans with Koopman's (Ref. 9). Belkin (Ref. 25) extends further the research of Richardson and Belkin (Ref. 24) on the gamma search plans.

Stone's book (Ref. 8) is recommended as the best available overall current reference.

27-5 OTHER APPLICATIONS OF SEARCH STRATEGIES

The reader should realize that the use of optimum search strategies is not limited to target detection problems. In fact, there are many other Army applications, and one, for example, has to do with locating faults or the causes of failure in a complex system (Refs. 26 and 27).

27-6 ESTIMATION OF TARGET POPULATION DENSITY

We round out this chapter with an important and useful statistical estimation procedure for determining the total number of target elements, or their density given an area of occupation, along with a technique for estimating the chance of seeing a target element.

Let

p = the unknown chance of seeing or detecting a target element

n = the unknown number of target elements present.

Now from a series of trials on several occasions, or with several sensors, we calculate the mean number $\bar{x}$ of elements detected and the variance s^2 of the number of elements seen. The resulting data are binomially distributed if p and n are fairly constant from trial to trial. Hence, the true average number μ of target elements detected per trial is approximately equal to the total number of elements in the target multiplied by the (unknown) chance of detecting an element, i.e.,

$$\mu = np. \tag{27-39}$$

Furthermore, for the binomial distribution we have that the true variance is

$$\sigma^2 = npq \tag{27-40}$$

where

$q = 1 - p =$ chance of not seeing an element.

Now clearly $\bar{x}$ is an estimate of μ, and s^2 is an estimate of σ^2. But we note that

$$\sigma^2/\mu = npq/(np) = q. \tag{27-41}$$

Thus q the chance of not seeing a target element may be estimated from

$$\hat{q} = s^2/\bar{x} \tag{27-42}$$

and p, the chance of detecting an element, from

$$\hat{p} = 1 - \hat{q} = 1 - s^2/\bar{x}. \tag{27-43}$$

Finally, to estimate the total number of target elements n, we may use Eq. 27-39 or Eq. 27-40, i.e., from Eq. 27-40 we see that for the estimate of n we have

$$\hat{n} = \bar{x}/\hat{p}. \tag{27-44}$$

For general probability calculations, the normal approximation to the binomial distribution may be used satisfactorily here—see almost any standard textbook on statistics.

27-7 SUMMARY

We have covered some of the important physical phenomena applicable to target detection sensors for Army applications. Also, we have introduced some of the analytical techniques which aid in calculating chances of target detection for various situations, and have covered some of the basic type strategies for searching regions in which targets may be located. The interested analyst should consult the references and bibliography for further information on search and detection.

Due to security classification, we have not covered the characteristics and detection capabilities of particular Army sensors. Nevertheless, the analyst responsible for any given applications naturally will have the required clearance for pertinent classified data he will use in his evaluations.

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CHAPTER 28
INTRODUCTION TO COMBAT THEORY AND ITS APPLICATIONS—HOMOGENEOUS FORCES

Investigations into combat theory represent attempts to model and summarize two-sided conflict in order that predictions may be made of battle outcomes in terms of the major identifiable parameters. Combat theory is applied often to weapon evaluation processes and to study the overall effectiveness of organizations. Here, we give an account of the Lanchester Linear Law for direct and area fire, the Lanchester Square Law, the Guerrilla Warfare model, and proceed to cover transition probabilities and chances of winning. Some approaches to both deterministic and stochastic models for homogeneous forces are developed, although the reader may easily see some complications in describing real situations, the problems of any complete stochastic treatment. Elementary combat theory involves kill rates and numbers of weapons, personnel, etc., or "key" systems, on each side. We develop a number of examples to illustrate to some extent the deterministic and the stochastic models of combat so that the analyst may proceed to more involved applications of two-sided conflict models, as opposed to firing against only passive targets. Finally, we discuss a new formulation based on target kill time.

28-0 LIST OF SYMBOLS

A_i = coefficient (see Mann, Ref. 24 or Table 28-8)

A_L = lethal area

A_R = total area occupied by Red forces or guerrillas

A_{R_i} = average area occupied by one member of the Red force

A_{vR} = vulnerable area of all the Red forces in the battle

A_{vR_i} = vulnerable area of one member of the Red force

B = $B(t)$ = number of Blue combatants, key elements, or weapon systems, etc., at any time t after the start of the battle

ΔB = small change in Blue numbers

(B,R) = "state" of B Blues and R Reds

B_e = Blue force size at the end of the battle

B_i = Blues at ith time interval

B_0 = $B(0)$ = initial size of Blue force at the start of a battle

B^*, R^* = battle cutoff points for Blue and Red, respectively

$B'(t)$ = derivative of Blue with respect to time

C, c = constants

C_i = coefficient (see Mann, Ref. 24 or Table 28-8)

C_1 = constant = $R_0 - [\rho/(2\beta)]B_0^2$ (Eq. 28-107)

C_2 = constant = $[\rho/(2\beta)]B_0^2 - R_0$ (Eq. 28-114)

$F(B,R,t)$ = function of the numbers of Blues and Reds, and the time t

$F(t)$ = function of time

$F(t)$ = rate of troop replacement for the U.S.

$F(t_m)$ = cumulative distribution function at mission time t_m

$f(t)$ = pdf of time t

H_0 = null hypothesis

H_A = alternative hypothesis
$I_x(y,z)$ = Karl Pearson's incomplete beta function (Eq. 28-82)
K_1 = $\sqrt{2\beta C_1/\rho}$ = a constant (Eq. 28-106)
K_2 = $\sqrt{2\beta C_2/\rho}$ = a constant (Eq. 28-113)
k = constant: −1, 0, or +1
$P(B,R)$ = chance of Blue winning the battle over Red
$1 - P(B,R)$ = chance of Red winning the battle over Blue
p_B = chance that first casualty after time t occurs for Blue
p_{B1} = chance of Blue losing an individual engagement
p_k = single-shot kill probability
p_{KR} = average kill probability per shot for a Red weapon against Blue
p_R = chance that first casualty after time t occurs for Red
$p_R(h)$ = chance of Red hitting Blue
$p_R(k|h)$ = chance of Red killing Blue given a hit
p_{R1} = chance of Red losing an individual engagement
R = $R(t)$ = number of Red combatants, key elements, or weapon systems, etc., at any time t
ΔR = small change in Red numbers
R_e = Red force size at the end of the battle
R_i = Reds at ith time interval
R_0 = $R(0)$ = initial size of Red force at the start of a battle
r = number of kills for truncated sample
r_B = rate of fire of a Blue weapon
r_R = rate of fire of a Red weapon
t = time of battle
Δt = small change in time
t_B = time at which Blue force is annihilated
t_i = ith time interval
t_m = mission time
t_R = time at which Red force is annihilated
u = B/R = force ratio of Blue to Red
$\hat{u}$ = $\Sigma A_i \ln t_i$ for Blue = estimate of $1/\beta^{\alpha}$* (Table 28-8)
α = shape parameter for the Weibull distribution
α = $\lambda' p_0 A/(\lambda A_L)$, or alternatively a shape parameter
$1/\hat{\alpha}$ = $\Sigma C_i \ln t_i$ for Blue = estimate of Weibull parameter $1/\alpha$ (Table 28-8)
α_1 = $\rho K_1/2$ = a constant (Eq. 28-108)
α_2 = $\rho K_2/2$ = a constant (Eq. 28-115)
β = attrition or kill rate of Blue forces by Red forces
β = Weibull scale parameter
βR_0 = fighting power of Red (Linear Law)
βR_0^2 = fighting power of Red (square Law)
$\beta(t,R,B)$ = function of time, Red and Blue forces
$\beta/(\beta+\rho)$ = chance of a Blue kill or loss
$1/\beta$ = Red mean time to kill a Blue
$1/\hat{\beta}$ = estimate of $1/\beta$

γ = Type I error, or chance of rejecting H_0 when true
δ = shape parameter for the Weibull distribution
$1/\hat{\delta}$ = $\Sigma C_i \ln t_i$ for Red = estimate of Weibull parameter $1/\delta$ (Table 28-8)
η_γ = lower γ probability level of the standard normal distribution
$\eta_{1-\nu}$ = upper ν probability level of the standard normal distribution
θ = $1/\beta$ or $1/\rho$
$\hat{\theta}$ = estimate of θ
θ_0 = mean time to kill when H_0 is true
θ_A = mean time to kill when H_A is true
κ = $\beta R_0/(\rho B_0)$ = ratio of fighting power of Red to Blue
λ = $(\eta_{1-\nu} - \mu\eta_\gamma)/(\mu - 1)$ = parameter used in Eq. 28-139 (not to be confused with exponential distribution parameter).
$1/\lambda', 1/\lambda$ = means of exponential distributed time interval
μ = $(\theta_0/\theta_A)^{1/\delta}$ = parameter used in Eq. 28-139
ν = Type II error, or chance of accepting H_0 when H_A is true
ρ = attrition or kill rate of Red forces by Blue forces
$1/\rho$ = Blue mean time to kill a Red
ρ = Weibull scale parameter also
$1/\hat{\rho}$ = estimate of $1/\rho$
ρB_0 = fighting power of Blue (Linear Law)
ρB_0^2 = fighting power of Blue (square Law)
ρ/β = attrition ratio
$\rho/(\beta + \rho)$ = chance of a Red kill or loss
χ^2_α = αth percentage point of the chi-square distribution. (This α is not to be confused with the shape parameter α. $\chi^2_{0.95}$ = 95% point of chi-square, for example.)
$\chi^2(2r)$ = chi-square with $2r$ degrees of freedom
cosh = hyperbolic cosine
sinh = hyperbolic sine
tanh = hyperbolic tangent

28-1 HISTORICAL BACKGROUND AND INTRODUCTION

An interesting historical sketch of the work of Lanchester on combat theory is given by Newman (Ref. 1) in the *World of Mathematics*, which we quote here. "Frederick William Lanchester was a very brilliant Englishman who died in 1948 at the age of 78. Although Lanchester was basically an engineer, he was interested in economic and industrial problems, the theory of relativity, aerodynamics, fiscal policies, and military strategy. Lanchester made a brilliant analysis of the inherent stability of model airplanes in 1897, long before there were real airplanes. His work was a little like a treatise on the dynamics of the automobile before any automobile existed. The Physical Society of London declined to print his paper, but some thirty years later Lanchester was awarded a gold medal for it by the Royal Aeronautical Society. Lanchester was also one of the foremost pioneers of the automobile design, and he built an experimental engine in 1895—probably the first to be made in England. The Lanchester automobile was put into production in 1900. It was an outstanding vehicle of the vintage period, incorporating then many unorthodox and advanced features.

"Lanchester was one of the first to recognize the extent to which aircraft would alter the character of warfare. He was the first to consider the matter quantitatively and set down his conclusions in his book

[Ref. 2], *Aircraft in Warfare: The Dawn of the Fourth Arm*, which consisted mainly of a series of articles which appeared in the British journal Engineering during 1914. Lanchester was convinced that most of the important operations hitherto entrusted to land armies could be executed 'as well or better by a squad or fleet of aeronautical machines. If this should prove to be true, the number of flying machines eventually to be utilized by any of the great military powers will be counted not by hundreds but by thousands, and possibly by tens of thousands, and the issue of any great battle will be definitely determined by the efficiency of the aeronautical forces.' "

Lanchester's analysis of the use of aircraft in warfare led him to be one of the first to apply mathematical modeling to land warfare. Moreover, it was Lanchester who showed analytically the importance of the *concentration* of firepower in battle to achieve victory. To prove his point, Lanchester found it necessary to make a mathematical analysis of the relation of opposing forces in battle. Under what circumstances can a smaller army (or naval fleet) defeat a larger one? Can a mathematical measure be assigned to concentrations of firepower and, if so, can equations in which such measures appear be set up to describe what happens and what may be expected to happen in military engagements? These were among the questions he considered and for which he devised the elegant Pythagorean equation later described. His so-called "*n*-square law" of the relative fighting strength of two armies is simple, but its implications are not. Scientists engaged in operations research have done a considerable amount of mathematical work to draw some of the consequences from Lanchester's equations. However, his equations are not recognizable in many of these later formidable elaborations. But then today's theories have become so elaborate that Mars himself would not recognize them and it was inevitable that mathematicians would have to advance from the basic theory. As indicated, much of our discussion here has been based on Newman's article (Ref. 1) on Lanchester in the *World of Mathematics*.

In Lanchester's mathematical theory of combat, it is evident in reading his papers that his primary interest related to the importance of concentration of forces in winning battles. As some background for studying the importance of concentration, we quote Lanchester (Ref. 2):

". . . In olden times, when weapon directly answered weapon, the act of defense was positive and direct, the blow of sword or battle-axe was parried by sword and shield; under modern conditions gun answers gun, the defence from rifle-fire is rifle-fire, and the defence from artillery is artillery. But the defence of modern arms is indirect: tersely, the enemy is prevented from killing you by your killing him first, and the fighting is essentially collective. . . Under the old conditions it was not possible by any strategic plan or tactical maneuver to bring other than approximately equal numbers of men into the actual fighting line; one man would ordinarily find himself opposed to one man. Even were a General to concentrate twice the number of men on any given portion of the field to that of the enemy, the number of men actually wielding their weapons at any given instant (so long as the fighting line was unbroken), was roughly speaking the same on both sides".

Thus, the situation here is that the assumption is made that man fights man in an engagement and then the winner goes on to fight another of the opposite side. The outcome of the individual combat depends on the skill of one individual versus the one he fights in a single engagement; furthermore, on an overall basis each side has an effective average attrition rate against the other. There is no concentration of a relatively large number of individuals on one side versus a much smaller number on the other—the principle of concentration of forces is not in effect—and the fighting line remains unbroken, so to speak. This type assumption leads us to the "Linear Law".

28-2 LANCHESTER'S FIRST LINEAR LAW (DIRECT, AIMED FIRE)

In this case, Blue fights Red in individual combats or single man-vs-man or weapon-vs-weapon type engagements, and the attrition rate for each side averages out to a constant figure. We have here the "Horatio-at-the-Bridge" analogy, or "Three Musketeers" taking on a larger size enemy but in individual engagements, one at a time (for example, fighting on a narrow-bridge or on stairs or in a hallway, etc.). For Lanchester's first linear law we define the following parameters:

$B = B(t)$ = the number of Blue troops, or weapons, or systems, i.e., the size of the Blue force at any time t

$R = R(t)$ = the number of Red troops, or weapons, or systems, i.e., the size of Red force at any time t

$B_0 = B(0)$ = initial size of the Blue force, i.e., number of Blue combatants (or weapons) at time $t = 0$

$R_0 = R(0)$ = initial size of the Red force, i.e., the number of Red combatants (weapon systems) at time $t = 0$

β = the constant rate at which Blue forces are attrited by Red, or the number of Blue forces lost per unit of time.

By way of further explanation, β is Red's kill rate against Blue forces. In accordance with Chapter 17 on duels, for example, β may be taken as the product

$$\beta = p_R(h) \cdot p_R(k|h) \cdot r_R \tag{28-1}$$

where

$p_R(h)$ = chance of Red hitting Blue

$p_R(k|h)$ = chance of Red killing Blue given a hit

r_R = rate of fire of Red weapons.

The quantity $1/\beta$ is Red's mean time to kill Blue, and hence may be estimated from the mean kill times of Blue in a simulation or war game.

Finally, we define

ρ = the constant rate at which Red forces are attrited by Blue.

We note that the kill or attrition rates, β and ρ, do not change for any engagement of one Blue versus one Red at a time during the battle.

Then with these definitions, Lanchester's first Linear Law may be written as

$$\frac{dB}{dt} = -\beta, \quad t, \beta, \rho \geq 0; \quad B_0 \geq B \text{ and } R_0 \geq R \tag{28-2}$$

and

$$\frac{dR}{dt} = -\rho. \tag{28-3}$$

In this case, the solution of the equations is very simple and we may integrate directly to find

$$B_0 - B = \beta t \quad \text{or} \quad B = B_0 - \beta t \tag{28-4}$$

$$R_0 - R = \rho t \quad \text{or} \quad R = R_0 - \rho t \tag{28-5}$$

for any time t.

These equations are used primarily to describe the "homogeneous" case of very similar weapons on a side.

We also call attention to the fact that a solution independent of the time t can easily be found, either by eliminating t in Eqs. 28-4 and 28-5, or obtaining a solution in B and R by taking the ratio of Eq. 28-2 to Eq. 28-3. Thus, it is easily seen that

$$\frac{dB}{dR} = \beta/\rho \quad \text{leads to} \quad \rho dB = \beta dR \tag{28-6}$$

and

$$\rho(B + C_1) = \beta(R + C_2). \tag{28-7}$$

But when $t = 0$, we have

$$\rho(B_0 + C_1) = \beta(R_0 + C_2) \tag{28-8}$$

and upon subtracting Eq. 28-7 from Eq. 28-8, we have finally that

$$\rho(B_0 - B) = \beta(R_0 - R) \quad \text{always.} \tag{28-9}$$

Eq. 28-9 is known as the "state" equation.

If we put

$$u = B/R \tag{28-10}$$

which is the force ratio of Blues to Reds at any time t, then the "force ratio" equation is easily seen from Eqs. 28-4 and 28-5 to be

$$u = (B_0 - \beta t)/(R_0 - \rho t) \tag{28-11}$$

or the rate of change of the force ratio u is

$$\frac{du}{dt} = (\rho B_0 - \beta R_0)/(R_0 - \rho t)^2. \tag{28-12}$$

The quantity ρB_0, which is equal to the attrition rate of Red forces multiplied by the initial number of Blues, has been referred to as the "fighting power", or total (initial) killing power of Blue. In a like manner βR_0 is the fighting or killing power of Red. When the initial fighting power of Blue is greater than that of Red, we have

$$\rho B_0 > \beta R_0 \quad \text{or} \quad \rho B_0 - \beta R_0 > 0 \tag{28-13}$$

which implies that

$$\rho B > \beta R \quad \text{always} \tag{28-14}$$

and hence that Blue wins. In this case, when Red has lost all its troops, we have—e.g., from Eq. 28-9—that the remaining number of Blues B_e is

$$B = B_e = (\rho B_0 - \beta R_0)/\rho. \tag{28-15}$$

On the other hand, if

$$\beta R_0 > \rho B_0 \quad \text{or} \quad \beta R_0 - \rho B_0 > 0 \tag{28-16}$$

then Red has the fighting power advantage and wins with

$$R = R_e = (\beta R_0 - \rho B_0)/\beta \tag{28-17}$$

combatants (weapons) remaining.

Finally, if $\rho B_0 = \beta R_0$, then we have parity and annihilation occurs for both sides! So, why even fight?

Lanchester referred to the Linear Law as "ancient conditions", indicating that if concentration of forces cannot be effected, then the battle consists of only man-vs-man engagements, the fighting line remains unbroken, encirclement does not occur, etc. In fact, such might well be the case in some situations of battle. For the Linear Law and the case of equal attrition coefficients, Lanchester gave the following example, and we quote him (Ref. 2).

"Taking first, the ancient conditions where man is opposed to man, then, assuming the combatants to be of equal fighting value, and other conditions equal, clearly, on an average, as many of the 'duels' that go to make up the whole fight will go one way as the other, and there will be about equal numbers killed of the forces engaged; so that if 1,000 men meet 1,000 men, it is of little or no importance whether a 'Blue' force of 1,000 men meets a 'Red' force of 1,000 men in a single pitched battle, or whether the whole 'Blue' force concentrates on 500 of the 'Red' force, and, having annihilated them, turns its attention to the other half; there will, presuming the 'Reds' stand their ground to the last, be half the 'Blue' force wiped out in the annihilation of the 'Red' force in the first battle, and the second battle will start on terms of equality—i.e., 500 'Blue' against 500 'Red'."

Nevertheless, we will see later that the principle of concentration, where attrition is not constant and depends on the number of opposing forces, will indeed lead to more startling results.

To determine how long such a battle lasts, we simply need to determine the time at which either side is annihilated. If Blue wins, i.e., Eq. 28-15 holds, then the battle lasts

$$t = R_0/\rho \tag{28-18}$$

units of time. If Eq. 28-17 holds, and hence Red wins, then

$$t = B_0/\beta \tag{28-19}$$

as would be expected.

EXAMPLE 28-1:

Twelve Blue riflemen engage 12 Red riflemen, and terrain features are such that the battle consists of one-vs-one "duels". Blue's kill rate averages one Red every 10 min, and Red with poorer rifles and less marksmanship, has a kill rate equal to one Blue lost per 15 min. Who wins? How many remain when one side is annihilated, and how long does the battle last? When is the winner's force twice the size of the loser's force?

We have:

$B_0 = 12$
$R_0 = 12$
$\beta = 0.067$ Blue kill/min
$\rho = 0.1$ Red kill/min.

Hence,

$$B_0\rho = (12)(0.1) = 1.2 \quad \text{and} \quad R_0\beta = (12)(0.067) = 0.8.$$

Thus, from Eq. 28-13 Blue wins and from Eq. 28-15, we have

$$B_e = (1.2 - 0.8)/(0.1) = 4$$

Blues remaining when Red is annihilated.

The battle lasts, using Eq. 28-18,

$$R_0/\rho = 12/0.1 = 120 \text{ min.}$$

The time at which Blue has twice as many combatants as Red is found by solving Eq. 28-11 for t when $u = 2$, and is

$$t = 90 \text{ min.}$$

As a summary for Lanchester's (first) Linear Law, we note that two-sided conflict is involved, that kill rates and numbers of weapons on both sides are accounted for, but that the outcome is completely deterministic. Chance does not really enter into battle procedures or results as in the case of stochastic duels (Chapter 17), but nevertheless application of the Linear Law brings forth the concept of "fighting power" or battle capability, and hence the principles studied may be informative.

The chance of a Blue loss or kill may be taken as

$$Pr(\text{Blue loss}) = \beta/(\beta + \rho) \tag{28-20}$$

and that for a Red as

$$Pr(\text{Red loss}) = \rho/(\beta + \rho) \tag{28-21}$$

except for near end conditions.

28-3 LANCHESTER'S (SECOND) LINEAR LAW FOR AREA FIRE (INVISIBLE FIRING)

The relation between $B = B(t)$ and $R = R(t)$ for the (first) Linear Law actually applies to a much more complex situation than the constant attrition coefficient case for direct fire. For example, consider longer range, unaimed fire, concentrated in an area known to be occupied by combatants with the size of that area taken to be rather independent of the numerical value of the force. Thus, the attrition rate for Blue will be proportional to R, the number of Red units firing at Blue; but the Blue losses will vary also with density of Blue troops which is proportional to $B(t)$, the number of Blue troops occupying its zone at any time. Thus, the same considerations also apply for Red, and we have

$$\frac{dB}{dt} = -\beta RB \tag{28-22}$$

$$\frac{dR}{dt} = -\rho BR. \tag{28-23}$$

Since dB/dR still equals β/ρ as in Eq. 28-6, the same solution (Eq. 28-9) for the first Linear Law still holds with the more complex models of Eqs. 28-22 and 28-23. Furthermore, if the right-hand sides of Eqs. 28-22 and 28-23 were even some common complex function, $F(B,R,t)$, aside from the constants β and ρ, the linear solution would still hold as in Eq. 28-9.

The solutions for the number of Blue forces, $B = B(t)$ and the number of Red forces $R(t)$ as a function of time, however, are much more complex than in Eqs. 28-4 and 28-5. In fact, for the area fire model, we have

$$B = \frac{-B_0(\kappa - 1)\exp[-\rho B_0(\kappa - 1)t]}{\exp[-\rho B_0(\kappa - 1)t] - \kappa} \tag{28-24}$$

and

$$R = \frac{-R_0(\kappa - 1)}{\exp[-\rho B_0(\kappa - 1)t] - \kappa}, \text{ for any time } t \tag{28-25}$$

where

$$\kappa = \beta R_0/(\rho B_0) \tag{28-26}$$

is the ratio of initial fighting power of Red to Blue.

Note here that in terms of the time solutions given in Eqs. 28-24 and 28-25 we have that the Blue to Red force ratio at any time t is

$$\frac{B}{R} = \frac{B_0}{R_0}\exp[-\rho B_0(\kappa - 1)t] \tag{28-27}$$

so that Blue wins when $\kappa = \beta R_0/(\rho B_0) < 1$, or Blue has the greater fighting power. On the other hand Red wins when $\kappa > 1$, or Red has the greater fighting power.

For parity, $B/R = B_0/R_0 = \beta/\rho$, which states that although the number of Blues and Reds vary throughout the battle, their ratio remains constant during the battle for this area fire model as before for direct fire.

Thus, with the two Lanchester "linear" type laws, we have one for "direct" fire or one vs one duels, and the other for area fire which more or less applies to artillery, for example.

EXAMPLE 28-2:

Blue and Red engage in an artillery exchange with 18 Blue artillery pieces firing into an area occupied by 18 Red artillery pieces which return counter battery fire. Blue's kill rate of Red artillery pieces put out of action is $\rho = 0.008$ per min and Red's corresponding kill rate of Blues is $\beta = 0.01$ per min. Who wins, and how many Red artillery pieces remain to fight after 30 min?

We have

$$B_0 = R_0 = 18$$
$$\beta = 0.01$$
$$\rho = 0.008$$

so that by Eq. 28-26

$$\kappa = \beta R_0/(\rho B_0) = 1.25$$

and Red wins.

From Eq. 28-25, we find $R = 4.94$, i.e., Red has 5 artillery weapons left. (Blue by Eq. 28-24 has only 1.68 weapons left.)

28-4 LANCHESTER'S SQUARE LAW

28-4.1 PRELIMINARIES

As contrasted to the Linear Law, Lanchester's Square Law seemed to fit "modern" fighting conditions better, and so said Lanchester (Ref. 2):

"With modern long-range weapons—fire-arms, in brief—the concentration of superior numbers gives an immediate superiority in the active combatant ranks, and the numerically inferior force finds itself under a far heavier fire, man for man, than it is able to return. The importance of this difference is greater than might casually be supposed, and, since it contains the kernel of the whole question, it will be examined in detail."

Lanchester did indeed examine this type of question in much detail.

For this kind of warfare, Lanchester said, "Each man will in a given time score, on an average, a certain number of hits that are effective; consequently, the number of men knocked out per unit time will be directly proportional to the numerical strength of the opposing force." (Ref. 2). Hence the idea of the "Square" law.

In the Linear Law assumption, each of the two rates of attrition was a constant due to individual fighting individual always. Now, however, a commander may throw a large force against a smaller—or against a weak part of the battle line—and concentrate, as it were, so that the attrition rate depends directly on the opposing numbers involved in the battle at that time. In view of this, and for the "Square Law", we now have (compare with Eqs. 28-2 and 28-3)

$$\frac{dB}{dt} = -\beta R, \qquad \beta, \rho, t > 0 \qquad (28\text{-}28)$$

and

$$\frac{dR}{dt} = -\rho B, \qquad B \leq B_0, R \leq R_0. \tag{28-29}$$

The solution of these differential equations for the Square Law is still relatively simple since the variables are separable. In fact, using Eqs. 28-28 and 28-29, we see that

$$\rho B dB = \beta R dR \tag{28-30}$$

which on integration leads to

$$(\rho/2)(B^2 + C_1) = (\beta/2)(R^2 + C_2). \tag{28-31}$$

But at time $t = 0$, $R = R_0$ and $B = B_0$, so that

$$(\rho/2)(B_0^2 + C_1) = (\beta/2)(R_0^2 + C_2) \tag{28-32}$$

and upon subtracting Eq. 28-31 from Eq. 28-32, we get

$$\rho(B_0^2 - B^2) = \beta(R_0^2 - R^2) \tag{28-33}$$

as Lanchester's Square Law (compare with Eq. 28-9).

The initial so-called "fighting powers" of Blue and Red now depend on the *squares* of the numbers of Blue and Red forces. The fighting power of Blue is now ρB_0^2 and that of Red is βR_0^2, which represents quite a gain over the linear laws. If $\rho B_0^2 > \beta R_0^2$, so that Blue has the advantage, then for some terminal B_e, we have

$$\rho(B_0^2 - B_e^2) = \beta R_0^2 \tag{28-34}$$

and the residual Blue force as a result of the battle is obtained from

$$B_e^2 = B_0^2 - (\beta/\rho)R_0^2 \tag{28-35}$$

when Blue wins (compare with Eq. 28-15).

Likewise, if $\rho B_0^2 < \beta R_0^2$, then Red has the advantage and

$$\rho B_0^2 = \beta(R_0^2 - R_e^2) \tag{28-36}$$

so that (compare with Eq. 28-17)

$$R_e^2 = R_0^2 - (\rho/\beta)B_0^2. \tag{28-37}$$

When $\rho B_0^2 = \beta R_0^2$, then we have parity and

$$\rho B^2 = \beta R^2 \quad \text{always.} \tag{28-38}$$

Then $B\sqrt{\rho} = \pm R\sqrt{\beta}$, where we must take the + sign since B, R, β, and ρ are all positive and the battle proceeds along a "standoff" line going into the origin as in Fig. 28-1.

For a battle starting off the standoff line, then one side has the greater fighting power and wins, the battle proceeding along the branch of a hyperbola. For example, if Red has the advantage, the battle goes as indicated in the Fig. 28-1.

Summarizing and confining our attention to the first quadrant, if the battle starts at a point just above the standoff line, Red has the advantage; while Blue will win for conditions of the initial fighting power starting below the parity (standoff) line. Furthermore, the greater the advantage of one side, the faster annihilation of the other side proceeds.

When $\beta = \rho$, we have perhaps a reasonable assumption, for then the opposing forces are technologically equal, so to speak, and the battle depends only on the numbers, i.e.,

$$B_0^2 - B^2 = R_0^2 - R^2 \tag{28-39}$$

or we have always that

$$B^2 - R^2 = B_0^2 - R_0^2.$$

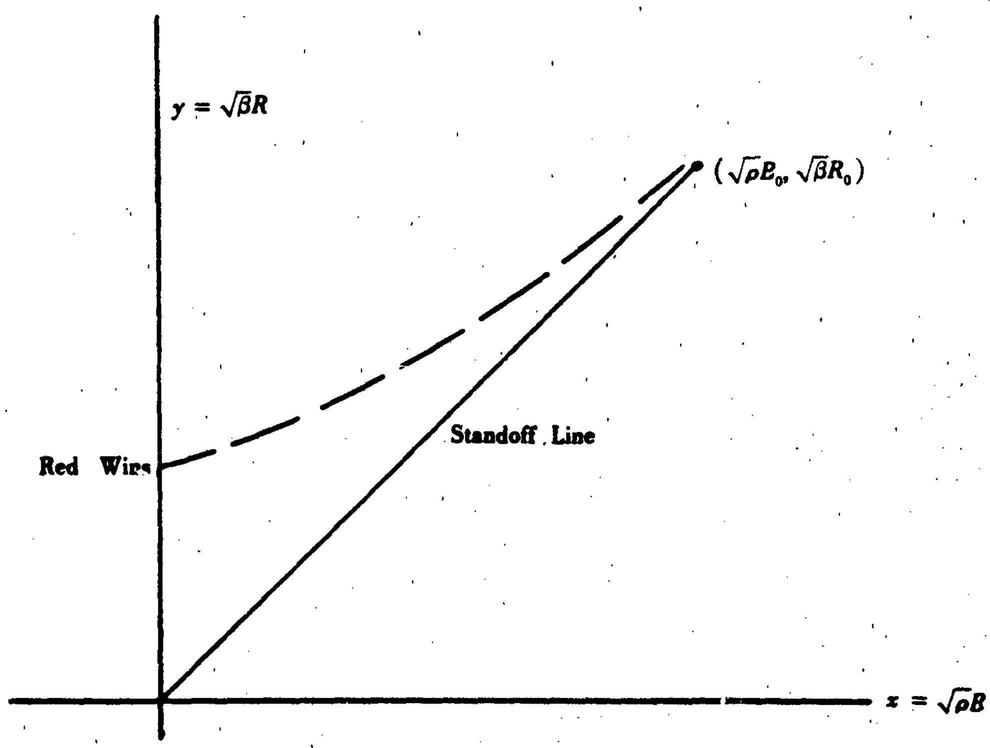

Figure 28-1. Graph of Square Law

As an example, let $B_0 = 1000$ initial Blue forces which takes on two successive forces of 500 Reds each. For $\beta = \rho$, we have:

1st Battle (by Eq. 28-35):

$$B_e^2 = (1000)^2 - (500)^2$$

or

$$B_{e1} = 866 \text{ Blue remaining, and 500 Red lost.}$$

2nd Battle:

$$B_{e2}^2 = (866)^2 - (500)^2$$

or

$$B_{e2} = 707 \text{ Blue remaining, and another 500 Red lost.}$$

Thus, by the principle of concentration, Blue has annihilated 1000 Red forces (two forces of 500 each) and has lost only 293 men! The principle of "divide and conquer" therefore works very well for the assumption of Lanchester's Square Law.

As a very startling example, consider an initial force of 1001 Blue and 1000 Red, still assuming $\beta = \rho$. Now Blue has an advantage of only 1 man, but the Square Law produces

$$B_e^2 = B_0^2 - R_0^2 = (1001)^2 - (1000)^2 = 2001$$

or

$$B_e \approx 45 \text{ remaining Blue forces (remarkable).}$$

But such a battle is not worth it to either side!

Let us now look at the difference in fighting power for any general time t:

$$\rho B^2 = \text{fighting power of Blue at time } t \neq 0$$

$$\beta R^2 = \text{fighting power of Red at time } t \neq 0$$

$$\rho B^2 - \beta R^2 = \text{difference in fighting power.}$$

The rate of change of the difference in fighting power is thus

$$\frac{d}{dt}(\rho B^2 - \beta R^2) = 2\rho B \frac{dB}{dt} - 2\beta R \frac{dR}{dt} \tag{28-40}$$

$$= 2\rho B(-\beta R) - 2\beta R(-\rho B) = 0$$

which says that the difference in fighting power is always a constant (or zero) for the Square Law. Thus,

$$\rho B^2 - \beta R^2 = C \quad \text{(constant), or zero,} \tag{28-41}$$

and when $C \neq 0$, the relation between Blue and Red remaining forces can be described by the positive branches of hyperbolas going into the Blue ($x = \sqrt{\rho}B$) or the Red ($y = \sqrt{\beta}R$) axes, depending on whether Blue or Red has the advantage. The asymptote $\sqrt{\rho}B = \sqrt{\beta}R$ represents the standoff or parity condition (Fig. 28-1).

28-4.2 DISCRETE CONSIDERATIONS

Following Clayton J. Thomas (Ref. 3), it is instructive to consider Eqs. 28-28 and 28-29 as an equivalent set of difference equations:

$$\frac{\Delta B}{\Delta t} = -\beta R \quad \text{or} \quad \frac{B_{i+1} - B_i}{t_{i+1} - t_i} = -\beta R_i \tag{28-42}$$

$$\frac{\Delta R}{\Delta t} = -\rho B \quad \text{or} \quad \frac{R_{i+1} - R_i}{t_{i+1} - t_i} = -\rho B_i \tag{28-43}$$

Now consider unit time intervals, so that $t_{i+1} - t_i = 1$ for $i = 0, 1, 2, \ldots, n$. Then, we get

$$B_{i+1} = B_i - \beta R_i \,; \quad i = 0, 1, 2, \ldots, n \tag{28-44}$$

$$R_{i+1} = R_i - \rho B_i \,; \quad i = 0, 1, 2, \ldots, n \tag{28-45}$$

and the battle proceeds as a function of the ith time period as follows:

i	t	B_i	R_i
0	$t = 0$	B_0	R_0
1	t_1	$B_1 = B_0 - \beta R_0$	$R_1 = R_0 - \rho B_0$
2	t_2	$B_2 = B_1 - \beta R_1$	$R_2 = R_1 - \rho B_1$
		$= B_0 + \beta\rho B_0$	$= R_0 + \beta\rho R_0$
		$-2\beta R_0$	$-2\rho B_0$
		etc.	

We may now use a numerical example of C. J. Thomas (Ref. 3) and construct Table 28-1 for $B_0 = 100$. $R_0 = 50$, $\rho = 0.05$, and $\beta = 0.10$.

Thus, after just more than 12 time units have passed, Blue has annihilated Red and lost only one-third of his (Blue) force. The ratio of Blue to Red increases from 2 to 45!

As can be seen from Table 28-1, the value of $\beta = 0.10$ is not enough to reach "parity". As can be seen from Table 28-2, parity is achieved when $\beta = 0.20$ is used in the computation (by Eq. 28-38, $\rho B^2 = \beta R^2$ or $0.05(100)^2 = \beta(50)^2$; $\beta = 0.20$), and at this stage Blue and Red proceed to annihilate each other. [Note at this last stage that the $(0.20)(32.805)^2 = (0.05)(65.61)^2 = 215.23$.]

28-4.3 FORCE RATIO CONSIDERATIONS

For the force ratio $u = B/R$, we record here that the rate of change of the force ratio equation for the Square Law is given by

TABLE 28-1. STATUS OF RED AND BLUE FORCES FOR $B_0 = 100$, $R_0 = 50$, $\rho = 0.05$, $\beta = 0.10$ AFTER 12 TIME UNITS

i	B_t	R_t	B_t/R_t	$B_t - R_t$	$B_t + R_t$
0	100.00	50.00	2.00	50.00	150.00
1	95.00	45.00	2.11	50.00	140.00
2	90.50	40.25	2.24	50.25	130.75
3	86.48	35.73	2.42	50.75	122.21
4	82.91	31.41	2.64	51.50	114.32
5	79.77	27.26	2.93	52.51	107.03
6	77.04	23.27	3.31	53.77	100.31
7	74.71	19.42	3.85	55.29	94.13
8	72.77	15.68	4.64	57.09	88.45
9	71.20	12.04	5.91	59.16	83.24
10	70.00	8.48	8.25	61.52	78.48
11	69.15	4.98	13.9	64.17	74.13
12	68.65	1.52	45.2	67.13	70.17

TABLE 28-2. PARITY ACHIEVED BETWEEN RED AND BLUE FORCES FOR $B_0 = 100$, $R_0 = 50$, $\rho = 0.05$, $\beta = 0.20$

i	B_t	R_t	B_t/R_t	$B_t - R_t$	$B_t + R_t$
0	100.00	50.00	2.00	50.00	150.00
1	90.00	45.00	2.00	45.00	135.00
2	81.00	40.50	2.00	40.50	121.50
3	72.90	36.45	2.00	36.45	109.35
4	65.61	32.805	2.00	32.805	98.415

$$\frac{du}{dt} = \rho u^2 - \beta \tag{28-46}$$

for any time t of the battle.

Also for the Square Law, the exchange ratio is easily seen to be, from Eqs. 28-28 and 28-29,

$$\frac{dB}{dR} = \frac{\beta}{\rho u} \tag{28-47}$$

EXAMPLE 28-3:

Given a Blue force of 1200 men, which through superior command and control, maneuvers and concentrates against 1900 Red troops such that three battles are fought which involve 500, 600, and 800 Red troops at a time. If Blue and Red have equally effective weapons, which side annihilates the other, and how many men are left?

Since $\beta = \rho$, we may therefore deal only with numbers in each battle.

1st Battle:

$$(1200)^2 - (500)^2 = 1{,}190{,}000$$

or

$$(1{,}190{,}000)^{1/2} = 1091 \text{ Blue troops remaining after 1st battle.}$$

2nd Battle:

$$(1091)^2 - (600)^2 = 830{,}281$$

or

$$(830{,}281)^{1/2} = 911 \text{ Blue troops remaining after 2nd battle.}$$

3rd Battle:

$$(911)^2 - (800)^2 = 189{,}921$$

or

$$(189{,}921)^{1/2} = 436 \text{ Blue troops remaining after 3rd battle.}$$

Hence, Blue wins with 436 men remaining.

One notes that had Red been able to pit all of his 1900 men against 1200 Blues in a battle, then Red would win with

$$[(1900)^2 - (1200)^2]^{1/2} = 1473 \text{ Red troops remaining!}$$

28-4.4 AN APPLICATION OF THE LANCHESTER SQUARE LAW TO CUSTER AT LITTLE BIGHORN*

One of the battle simplications of American history is that Colonel George Armstrong Custer blundered his way to defeat at Little Bighorn by splitting his force. Perhaps the Lanchester Square Law can aid in assessing this proposition.

Custer headed for his final battle with a 7th Cavalry force of about 600 men. He anticipated meeting an Indian force of between 1000 and 1500 warriors. The actual Indian fighting strength at Little Bighorn lay somewhere between 4000 and 5000.

On the day of the battle, 25 June, 1876, Custer divided his command into four parts. Captain Benteen was sent off with 125 men on what turned out to be a fruitless search; Major Reno was ordered to attack with an effective force of 115 men; a group of 130 men was assigned to the pack train; and Custer himself lead a column of about 225 soldiers.

The ensuing action can be viewed in three parts. First, Reno's abortive attack on the Indian village. [illegible]nd, the annihilation of Custer's column. Third, the successful defensive perimeter established on [illegible]ound by Reno's beaten force reinforced by Benteen's group. This last part will not be analyzed w[illegible] Lanchester Square Law, as it was mainly a matter of holding out in a fortified position until the Indians were driven off by the approach of General Terry's column. (It also seems fair to mention that after the stunning success they had in the attack on Custer, the Indians were not so much interested in attacking the Reno-Benteen defensive position as they were in celebrating.)

*Contributed by Mr. Thomas Nolan during a class on weapon systems analysis in 1968 at the BRL Ballistic Institute.

Now, what can the Lanchester Square Law tell us about Reno's attack on the Indian village? First, we need to recall Lanchester's Square Law as in Eqs. 28-28 and 28-29:

$$\frac{dB}{dt} = -\beta R \qquad \frac{dR}{dt} = -\rho B$$

which implies

$$\rho(B_0^2 - B^2) = \beta(R_0^2 - R^2)$$

where

B_0 = initial US strength
R_0 = initial Indian strength
B = size of the US force at any time t
R = size of the Indian force at any time t
ρ = constant rate at which a single US unit (man) kills an Indian unit
β = constant rate at which a single Indian unit (man) kills a US unit.

The hopelessness involved in Reno's attack with 115 men on an Indian force of 1000 or more is shown in Table 28-3 which shows the remaining Indian force resulting from an annihilation of Reno's command as a function of the ratio ρ/β of attrition coefficients. To have achieved parity, or $\rho B_0^2 = \beta R_0^2$, Reno's force would have needed an attrition coefficient or kill rate advantage of 76:1!!

The actual value of the ratio of the attrition coefficients is perhaps indicated by the engagement of a 1300-man force under General Crook on the upper Rosebud Creek on 17 June, 1876 by a force of 1000 to 1500 of the same Indians involved at Little Bighorn. From an optimistic US point of view this battle may be called a draw, indicating an attrition coefficient ratio near 1:1. (The battle was privately admitted to be an Indian victory by General Crook.)

So the Lanchester Square Law would indicate that Reno had little chance of success. He was fortunate to extract himself from the position to which his ill-conceived attack placed him.

Custer's column of 225 men was engaged by an Indian force which may have numbered as many as 5000. Its strength was probably in the 3000 to 5000 range. Table 28-4 illustrates the Lanchester Square Law analysis of this situation. Table 28-4 predicts an overwhelming Indian victory. Even an unrealistically small Indian force of 1000 (which may have been the size force Custer anticipated encountering) puts an attrition coefficient ratio advantage of 20:1 as the requirement for parity!

These results indicate that Custer's divided forces had little chance of achieving victory. But what if he had maintained his 600-man force intact? Table 28-5 addresses that possibility. The conclusion,

TABLE 28-3. RENO'S ATTACK

B_0 = 115	R_0 = 1000
Ratio of Attrition Coefficients ρ/β	**Size of Indian Force After Annihilation of US Force**
1	993
2	987
5	966
10	932
50	582
76	0 (PARITY)

TABLE 28-4. CUSTER'S LAST STAND

Ratio of Attrition Coefficients ρ/β	Size of Remaining Indian Force for Five Initial Indian Force Sizes				
	1000	2000	3000	4000	5000
1	974	1987	2992	3994	4995
2	948	1975	2983	3987	4990
5	864	1936	2958	3968	4975
10	703	1869	2914	3936	4949
20	0*	1728	2826	3871	4898
79		0*	2236	3464	4583
100			1984	3307	4465
178			0*	2644	3999
316				0*	3000
494					0*

*This is the point where parity is reached.

TABLE 28-5. CUSTER AT LITTLE BIGHORN WITH AN UNDIVIDED FORCE
$B_0 = 600$

Ratio of Attrition Coefficients ρ/β	Size of Remaining Indian Force for Five Initial Indian Force Sizes				
	1000	2000	3000	4000	5000
1	800	1908	2939	3955	4964
2	529	1811	2877	3909	4927
3	0*	1709	2814	3863	4891
5		1483	2683	3768	4817
10		632	2324	3521	4626
11		0*	2245	3470	4587
25			0*	2646	4000
44				0*	3027
69					0*

*This is the point where parity is reached.

applying Lanchester's Square Law, indicates that even with an undivided force of 600, Colonel Custer would have been unlikely to defeat the Indian force he met at Little Bighorn. An analysis such as that made here, however, may have given Custer improved guidance on not engaging so many Indians, waiting for additional help, etc. Finally, we might ask: Does not combat theory aid in some better judgments or in planning? Par. 28-4.5 attempts to answer this question.

28-4.5 LANCHESTER'S SQUARE LAW AS A FUNCTION OF TIME

As we have seen, the solutions of the differential equations for Lanchester's Square Law are straightforward. The solutions in terms of the time history of events, however, are somewhat more difficult. The remaining Blue forces $B(t)$ and remaining Red forces $R(t)$ at any time t are given by

$$B(t) = B_0 \cosh\sqrt{\rho\beta}\, t - \sqrt{\frac{\beta}{\rho}}\, R_0 \sinh\sqrt{\rho\beta}\, t \qquad (28\text{-}48)$$

$$R(t) = R_0 \cosh\sqrt{\rho\beta}\, t - \sqrt{\frac{\rho}{\beta}}\, B_0 \sinh\sqrt{\rho\beta}\, t. \qquad (28\text{-}49)$$

By use of Eq. 28-49, it can be shown (Ref. 4, for example) that, if Blue wins, the time t_R at which Red is annihilated (i.e., $R(t) = 0$) is given by

$$t_R = [1/(2\sqrt{\rho\beta})]\ln[(\sqrt{\rho}\,B_0 + \sqrt{\beta}\,R_0)/(\sqrt{\rho}\,B_0 - \sqrt{\beta}\,R_0)]. \qquad (28\text{-}50)$$

Similarly, by use of Eq. 28-48, if Red wins, then Blue's time of annihilation (i.e., $B(t) = 0$) is given by

$$t_B = [1/(2\sqrt{\rho\beta})]\ln[(\sqrt{\rho}\,B_0 + \sqrt{\beta}\,R_0)/(\sqrt{\beta}\,R_0 - \sqrt{\rho}\,B_0)]. \qquad (28\text{-}51)$$

EXAMPLE 28-4:

Given the initial data for Table 28-1, which involve a discrete type calculation, find the time at which Blue annihilates Red.

We have $B_0 = 100$, $R_0 = 50$, $\rho = 0.05$, and $\beta = 0.10$; and we know that Blue wins since

$$\rho B_0^2 = 500 \quad \text{and} \quad \beta R_0^2 = 250$$

Hence, from Eq. 28-50 we find

$$t_R = \frac{1}{2(0.0707)} \ln(38.17/6.55) = 12.47.$$

Furthermore, we note that the 12.47 time units calculated here agree with the discrete computation of Table 28-1.

Given the Lanchester Square Law for concentration of forces and assume that Blue and Red have equal kill rates, i.e., $\rho = \beta$, we can then determine the time required to reduce the Blue force to $1/n$ of its original size, i.e., $B(t) = B_0/n$, by Eq. 28-48

$$t = (1/\rho)\ln\left\{\left[(B_0/n) - \sqrt{R_0^2 - B_0^2(n^2 - 1)/n^2}\right]/(B_0 - R_0)\right\}. \qquad (28\text{-}52)$$

If the term under the radical is negative, then the Red force would be annihilated before the Blue force could reduce to $1/n$ of its original size.

Similarly, the time required to reduce the Red force to $1/n$ of its original size, i.e., $R(t) = R_0/n$, with $\rho = \beta$, by Eq. 28-49 is

$$t = (1/\rho)\ln\left\{\left[(R_0/n) - \sqrt{B_0^2 - R_0^2(n^2 - 1)/n^2}\right]/(R_0 - B_0)\right\}. \qquad (28\text{-}53)$$

28-4.6 HISTORICAL COMMENT

With regard to Lanchester's well-known and famous Square Law, Weiss (Ref. 5) indicates that Rear Admiral Bradley A. Fiske of the US Navy may have anticipated its implications some 10 years earlier than Lanchester. In Ref. 5, Weiss describes the numerical analyses of gunnery made by Fiske, which favor the Square Law. However, Fiske did not set down the differential equations as did Lanchester for modeling combat, and hence did not establish a general law.

28-5 VALIDATION OF LANCHESTER'S SQUARE LAW

It is of interest to point out that some effort has been made to verify Lanchester's combat theory through studies of actual battles. In this connection Engel (Ref. 6) studied records of the battle of Iwo Jima in World War II, and was able to analyze US casualties each day, the number of friendly troops put ashore each day, and some appropriate information on Japanese casualties and the reinforcement of Japanese forces. With such information, Lanchester's Square Law was fitted to attrition data on both sides for the battle of Iwo Jima, and appeared to give a good fit. Engel (Ref. 6) was also able to determine the attrition or kill rates for Blue (US) and Red (Japanese). In all, there were about 73,000 US troops involved and 21,000 Japanese troops. Engel (Ref. 6) estimated 0.0106 enemy casualty per day per effective friendly troop, and 0.0544 friendly casualty per day per effective enemy troop. Thus, these kill rate figures indicate that in the defensive role at Iwo Jima the Japanese were nearly 5 times as effective as the Americans. Perhaps one of the most impressive points is that a suitable mathematical model might be validated, and hence could be used for general inferences. Samz (Ref. 7) checks Engel's attrition rates.

J. R. Thompson (Ref. 8) appears to question the exact applicability of Lanchester's Square Law for the Iwo Jima battle. In fact, Thompson found some additional information which indicates the Japanese commander, General Kunibayashi, estimated that for the 21st day of combat there were only about 1500 Japanese remaining instead of the 8550 estimated from Engel's fit of the Square Law. (The Iwo Jima campaign lasted about 36 days.) Indeed, Thompson indicates that the area fire model

$$\frac{dB}{dt} = -\beta BR + F(t) \tag{28-54}$$

$$\frac{dR}{dt} = -\rho RB \tag{28-55}$$

where $F(t)$ is a reinforcement term or rate of troop replacement for the U.S., gives a Japanese strength on the 21st day as 2250—which is closer to Kunibayashi's estimate of 1500; thereby raising some doubt about the 8550 figure. Thus, we see the need for sufficiently accurate and detailed experimental or battle data to validate mathematical models of combat. Indeed, the problem of obtaining battle data accurately—and especially at precise time instants that the attrition on each side occurs—represents one of the basic considerations for and obstacles to validating the worth of any proposed combat model.

As a remark of some importance, we should point out the need for obtaining exact kill times for the combatants on each side in a battle or simulation, for these kill times can be used to determine kill rates experimentally. Thus, for example, the reciprocal of the average kill times for either Blue or Red gives the kill rates or attrition rates, which are useful in Lanchester's equations, and this would provide a check on (passively determined) kill rates as in Eq. 28-1.

28-6 THE LOGARITHMIC LAW (Weiss and Peterson)

In a study of the Civil War, H. K. Weiss (Ref. 9) found that the Lanchester Linear and Square Laws did not apply well for battles involving about 15,000 or more combatants. Rather, the rate of losses, or the losses at each time period, on each side appeared to be directly related to the number of combatants on that side. (To many readers, this is no doubt a surprising development, as it almost says "too much 'bureaucracy' gets in its own way." In the sequel, however, we will show that the "Logarithmic" Law may be derived from kill times). Also, R. H. Peterson (Ref. 10) in a study of tank combat during World War II in Western Europe, found that the first kills in tank engagements seemed to depend on the number of tanks on that same side, but that second kills seemed to be governed by the Lanchester Square Law. For first kills, the "Logarithmic Law" seemed to apply better. Perhaps, this has something to do with initial shots and their advantage in catching the enemy by surprise, as compared to subsequent, regular combat. (Also, the sparsity of suitably accurate data could be a problem.)

Weiss and Peterson indicate that the limited studies with the Logarithmic Law involved the form

$$dB/dt = -\beta B \ln R \tag{28-56}$$

showing rather weak dependence on the opposite side numbers, although here we will use Peterson's "simple form" in the following assumptions:

$$dB/dt = -\beta B \tag{28-57}$$

and

$$dR/dt = -\rho R. \tag{28-58}$$

Then

$$B = B_0 \exp(-\beta t) \tag{28-59}$$

and

$$R = R_0 \exp(-\rho t) \tag{28-60}$$

so that losses and remaining numbers of combatants depend on initial numbers on the *same side* and the attrition rate constants β and ρ. Otherwise, Blue's and Red's losses appear somewhat independent of each other!

We see also from Eqs. 28-59 and 28-60 that

$$B/R = (B_0/R_0)\exp(\rho - \beta)t. \tag{28-61}$$

If $\rho > \beta$, then B/R steadily increases with the time of battle and Red may soon have to give up; hence Blue wins. If $\rho < \beta$, the B/R steadily decreases with time and Blue may have to capitulate. If $\beta = \rho$, then Eq. 28-61 becomes

$$B/R = B_0/R_0 \tag{28-62}$$

and the fighting ratio stays constant, so that one could argue stalemate. On the other hand, battles may stop quickly as indicated in Peterson's tank combat studies.

For a relation between remaining Blue and Red not involving time, we may eliminate t from Eqs. 28-57 and 28-58 and obtain the ratio

$$dR/dB = \rho R/(\beta B) \tag{28-63}$$

and then arrive at

$$(1/\beta)(\ln B_0 - \ln B) = (1/\rho)(\ln R_0 - \ln R) \tag{28-64}$$

or rewrite this as

$$(B/B_0)^{1/\beta} = (R/R_0)^{1/\rho} \tag{28-65}$$

or finally as

$$B = B_0(R/R_0)^{\beta/\rho} \tag{28-66}$$

The so-called "Logarithmic Law", therefore, leads to the exponential type of decay or attrition of forces on each side. In addition, we note that if the Logarithmic Law fits the data, and if we know the initial number on each side and the number of survivors, then the ratio of attrition or kill rates may be determined, for example, from Eq. 28-66. Furthermore, Eqs. 28-59 and 28-60 may be linearized by taking logarithms and, if the times of casualties are known, then the kill rates may be determined experimentally rather than predicted "passively", as perhaps in Eq. 28-1.

Finally, we return to a potentially important point concerning analyses of combat or simulated battles--i.e., kill times may be of much interest since, the faster that losses occur on a side, the greater likelihood the decision to withdraw.

EXAMPLE 28-5:

Six Blue tanks engage in a battle with eight Red tanks until Red desires to withdraw with half of its tanks lost. If Blue also lost half of its tanks, then find the quantitative advantage of one side over the other.

Assuming that the Logarithmic Law is valid for this case, we have $B_0 = 6$, $B = 3$, $R_0 = 8$, and $R = 4$. Hence, using Eq. 28-66, we find that

$$\begin{aligned} \beta/\rho &= [\ln(B/B_0)]/[\ln(R/R_0)] \\ &= 1 \end{aligned} \tag{28-67}$$

or that the attrition rates are equal. However, it is obviously true that Blue has superior tanks, for with a loss of three tanks he has killed four Red tanks. Hence, the "trickiness" of the exponential decay combat law.

28-7 SOME CONSIDERATIONS ON TRANSITION PROBABILITIES

Some remarks on transition probabilities for the Linear, Square, and Logarithmic Laws are pertinent here. We are interested ultimately in one side, Blue or Red, winning through a sequence of losses or some total number of losses of combatants on a side. Blue wins if Red's force size goes to zero first (which is drastic), or if we state hypothetically that a side has lost when, for example, it loses 1/3 of its original size, and consequently disengages.

For the Linear Law the chance p_{B1} of Blue losing an individual engagement at any given state of the battle is found from

$$p_{B1} = \frac{dB}{dt}\Bigg/\left(\frac{dB}{dt} + \frac{dR}{dt}\right) = -\beta/(-\beta - \rho) = \beta/(\beta + \rho) = (1 + \rho/\beta)^{-1} \quad (28\text{-}68)$$

and thus the chance p_{R1} that a Red loses an individual engagement is

$$p_{R1} = \rho/(\beta + \rho) = (1 + \beta/\rho)^{-1} \quad (28\text{-}69)$$

as we have already indicated in par. 28-2.

In view of the foregoing, we may say that the "transition probability" from the "state" (B,R), i.e., of B Blues and R Reds, to "state" $(B - 1,R)$ is given by Eq. 28-68, and the transition probability from state (B,R) to state $(B,R - 1)$ is given by Eq. 28-69.

For the Square Law, since $dB/dt = -\beta R$ and $dR/dt = -\rho B$, then

$$p_{B1} = \beta R/(\beta R + \rho B) = [1 + \rho B/(\beta R)]^{-1} \quad (28\text{-}70)$$

and

$$p_{R1} = \rho B/(\beta R + \rho B) = [1 + \beta R/(\rho B)]^{-1}. \quad (28\text{-}71)$$

For the Logarithmic Law, since $dB/dt = -\beta B$ and $dR/dt = -\rho R$, then

$$p_{B1} = \beta B/(\beta B + \rho R) = [1 + \rho R/(\beta B)]^{-1} \quad (28\text{-}72)$$

and

$$p_{R1} = \rho R/(\beta B + \rho R) = [1 + \beta B/(\rho R)]^{-1}. \quad (28\text{-}73)$$

If we assume a constant or parameter $k = -1$ for the Logarithmic Law, $k = 0$ for the Linear Law and $k = 1$ for the Square Law, then Peterson (Ref. 10) uses the general form

$$p_{B1}(k) = [1 + (\rho/\beta)(B/R)^k]^{-1}, \quad k = -1, 0, \text{ or } +1 \quad (28\text{-}74)$$

to describe the chance of one Blue loss for the three laws.

In a like manner, we see that the chance of a Red loss may be expressed as

$$p_{R1}(k) = [1 + (\beta/\rho)(R/B)^k]^{-1}, \quad k = -1, 0, \text{ or } +1. \quad (28\text{-}75)$$

In order to estimate k from actual statistics on battles, or from a simulation, we observe that from Eq. 28-74

$$1 - p_{B1}(k) = (\rho/\beta)(B/R)^k[1 + (\rho/\beta)(B/R)^k]^{-1} \tag{28-76}$$

and it is thus easy to see that for Blue we have

$$\ln[p_{B1}(k)/\{1 - p_{B1}(k)\}] = \ln(\beta/\rho) + k\ln(R/B). \tag{28-77}$$

In a like manner, for Red we get

$$\ln[p_{R1}(k)/\{1 - p_{R1}(k)\}] = \ln(\rho/\beta) + k\ln(B/R). \tag{28-78}$$

Thus, knowing the number of combatants B and R at some stage of a battle, the attrition rates β and ρ, and having estimates of the transition probabilities from other sources; then k may possibly be determined, giving the form of the correct law to fit. Or, knowing the form of the law (Linear, Square, or Logarithmic) and having estimates of transition chances, then the intercepts of Eqs. 28-77 and 28-78 lead to the ratios of kill rates.

28-8 CHANCES OF WINNING A BATTLE

As a further consideration, now let $P(B,R)$ denote the probability that Blue finally wins over Red when Blue starts with any force size B, and Red starts with a force size R. In this connection, we may find all needed values of $P(B,R)$ by reasoning as indicated.

To begin with, it is clear that $P(B,0)$ for $B \geq 1$ is always 1; Red has no chance to win, being down to zero men already. On the other hand, $P(0,R)$ for $R \geq 1$ is always zero; since B has zero combatants, Red has already won, and Blue therefore cannot win. Next, consider $P(1,1)$. Here, Blue can win only if $P(1,1)$ changes from this state to the state $P(1,0)$, i.e., Red has to lose his only combatant for Blue to win at this state. The chance of this is p_{R1} and for the Linear Law, for example, we know from Eq. 28-69

$$p_{R1} = (1 + \beta/\rho)^{-1}, \text{ and this equals } 1/2 \text{ if } \beta = \rho.$$

Therefore,

$$P(1,1) = p_{R1}P(1,0) = (1 + \beta/\rho)^{-1}P(1,0) = (1 + \beta/\rho)^{-1}.$$

Next, what about $P(2,1)$ or $P(1,2)$? From state (2,1) Blue may win merely by Red losing 1 or the engagement going from state (2,1) to state (2,0), the chance of which is p_{R1}. Also, Blue can win by losing just one Blue, going to state (1,1), but then winning from state (1,1). The chance of these two mutually exclusive events is

$$P(2,1) = p_{R1}P(2,0) + p_{B1}P(1,1).$$

In a like manner,

$$P(1,2) = p_{R1}P(1,1) \text{ only.}$$

In general, however, $P(B,R)$ is the total chance that Blue wins either if he loses 1 in a battle, but still wins, and Red loses one but Blue still wins. This may be written quite generally as

$$P(B,R) = p_{B1}P(B-1,R) + p_{R1}P(B,R-1) \tag{28-79}$$

which for the Square Law, for example, would be

$$P(B,R) = \frac{\beta R}{\beta R + \rho B} P(B-1,R) + \frac{\rho B}{\beta R + \rho B} P(B,R-1). \tag{28-80}$$

For example, in C. J. Thomas' Table III, Page VII-25, (Ref. 3), $P(3,2)$ is listed as 0.7750 and, from Eq. 28-77, we have where $\rho = \beta$

$$P(3,2) = \frac{2}{2+3}P(2,2) + \frac{3}{2+3}P(3,1) = (0.4)(0.5000) + 0.6(0.9583) = 0.7750.$$

Thus, we are able to use the transition probabilities of par. 28-7 to determine overall chances of winning a battle. However, the reader may easily see that the computational details get to be very involved indeed.

EXAMPLE 28-6:

Given that Lanchester's Square Law applies to a certain battle and that the exchange ratio of Blue's loss rate to that of Red is $\beta/\rho = 3/5$. Then build up a table of the chances of Blue winning for one to five combatants on each side.

Now since $P(B,R)$ given by Eq. 28-80 is the chance that Blue wins if all the enumerations are carried out, then we see for $\beta/\rho = 3/5$ that we have

$$P(B,R) = \frac{3R}{3R + 5B} P(B-1,R) + \frac{5B}{3R + 5B} P(B,R-1).$$

The first few computations, starting with zero Blue and one Red, and one Blue and zero Red, are as follows:

$$P(0,1) = 0$$

$$P(1,0) = 1$$

$$P(1,1) = \frac{3 \cdot 1}{3 \cdot 1 + 5 \cdot 1} P(0,1) + \frac{5 \cdot 1}{3 \cdot 1 + 5 \cdot 1} P(1,0)$$

$$= \frac{3}{8} \cdot 0 + \frac{5}{8} \cdot 1$$

$$= \frac{5}{8} = 0.625$$

$$P(1,2) = \frac{3 \cdot 2}{3 \cdot 2 + 5 \cdot 1} P(0,2) + \frac{5 \cdot 1}{3 \cdot 2 + 5 \cdot 1} P(1,1)$$

$$= \frac{6}{11} \cdot 0 + \frac{5}{11} \cdot \frac{5}{8}$$

$$= \frac{25}{88} = 0.284091$$

$$P(1,3) = \frac{3 \cdot 3}{3 \cdot 3 + 5 \cdot 1} P(0,3) + \frac{5 \cdot 1}{3 \cdot 3 + 5 \cdot 1} P(1,2)$$

$$= \frac{5}{14} \cdot \frac{25}{88}$$

$$= \frac{125}{1232} = 0.101461.$$

$$P(1,4) = \frac{3 \cdot 4}{3 \cdot 4 + 5 \cdot 1} P(0,4) + \frac{5 \cdot 1}{3 \cdot 4 + 5 \cdot 1} P(1,3)$$

$$= \frac{5}{17} \cdot \frac{125}{1232}$$

$$= \frac{625}{20944} = 0.029841.$$

The resulting computations giving the chances of Blue winning are given in Table 28-6. In case the Lanchester Linear Law applies, then Eq. 28-80 simplifies to

$$P(B,R) = \frac{\beta}{\beta + \rho} P(B - 1,R) + \frac{\rho}{\beta + \rho} P(B,R - 1) \tag{28-81}$$

and Brown (Ref. 11) shows that the chance of Blue winning is the binomial sum

$$P(B,R) = \sum_{j=0}^{B-1} (B + R - 1)!/[j!(B + R - 1 - j)!]$$
$$\times [\beta/(\beta + \rho)]^j [\rho/(\beta + \rho)]^{B+R-1-j} \tag{28-82}$$

$= I_{\beta/\rho}(B,R)$, or Karl Pearson's incomplete beta function (e.g., par. 21-3.1).

TABLE 28-6. PROBABILITY OF BLUE WINNING (SQUARE LAW AND $\beta/\rho = 3/5$)

Blue Force Size \ Red Force Size	0	1	2	3	4	5
0	$P(0,0)$ Not Defined	$P(0,1)$ 0.0000	$P(0,2)$ 0.0000	$P(0,3)$ 0.0000	$P(0,4)$ 0.0000	$P(0,5)$ 0.0000
1	$P(1,0)$ 1.0000	$P(1,1)$ 0.6250	$P(1,2)$ 0.2841	$P(1,3)$ 0.1015	$P(1,4)$ 0.0298	$P(1,5)$ 0.0075
2	$P(2,0)$ 1.0000	$P(2,1)$ 0.9135	$P(2,2)$ 0.6774	$P(2,3)$ 0.4046	$P(2,4)$ 0.2002	$P(2,5)$ 0.0846
3	$P(3,0)$ 1.0000	$P(3,1)$ 0.9856	$P(3,2)$ 0.8975	$P(3,3)$ 0.7127	$P(3,4)$ 0.4849	$P(3,5)$ 0.2847
4	$P(4,0)$ 1.0000	$P(4,1)$ 0.9981	$P(4,2)$ 0.9749	$P(4,3)$ 0.8935	[illegible] 0.7403	$P(4,5)$ 0.5451
5	$P(5,0)$ 1.0000	$P(5,1)$ 0.9998	$P(5,2)$ 0.9950	$P(5,3)$ 0.9681	$P(5,4)$ 0.8942	$P(5,5)$ 0.7633

Now the binomial sum may be approximated by using the normal or Gaussian distribution (see Brown, Ref. 11), and we could put $B = B_0$ and $R = R_0$ for the start of the battle in Eqs. 28-80, 28-81, or 28-82.

Brown (Ref. 11) also gives a normal approximation for the chance of Blue (and hence Red) winning for the Lanchester Square Law, although we see that otherwise the calculations become very detailed because of the discrete nature of the battles.

Having covered the Lanchester type Linear, Square, and Logarithmic Laws, we now turn to the problem of modeling guerrilla warfare, which is also an important topic.

28-9 THE MIXED LAW OR DEITCHMAN'S GUERRILLA WARFARE MODEL

28-9.1 BASIC CONSIDERATIONS AND THEORY

S. J. Deitchman (Ref. 12) made a study of Lanchester type models to explore the force ratios—of "regulars" to guerrillas—that might be required for a side to win. He shows that an attacking guerrilla force, by using tactics which compensate for its weaknesses otherwise, can defeat a force of defending regulars which has overall superiority in number of men and weapons. On the other hand, the defenders or regulars can win by appropriate selection of weapons, counter-tactics, and rather high force ratios for individual engagements.

Table 28-7 gives the force ratios for some nine limited wars, along with the winner and is based on testimony of General Maxwell D. Taylor before the House Appropriations Committee (1960). We have added South Vietnam as the 10th limited war for additional information.

Table 28-7 shows that the "regulars" won only with rather overwhelming force ratios, and even for force ratios of about 9/1 the guerrillas won in Algeria and more recently in South Vietnam. (In the latter stages of the battle for South Vietnam, the North Vietnamese sent in large forces to take over as help from the U.S. dwindled.) Thus, the so-called "limited" wars bring forth some special considerations that require proper analyses, and hence the need for modeling guerrilla type warfare as

TABLE 28-7. LIMITED WARS, FORCE RATIOS, AND WINNERS

Limited War	Force Ratio (Reg/Guerr.)	Winner
Greece, 1946-49	9/1	Regular
Malaya, 1945-54	18/1	Regular
Kenya, 1953	10/1	Regular
Philippines, 1948-52	5/1	Regular
	Ave 10/1	
Indonesia, 1945-47	3/2	Guerrilla
Indochina, 1945-54	2/1	Guerrilla
Cuba, 1958-59	5.5/1	Guerrilla (Castro)
Laos, 1959-62	2/1	Guerrilla
Algeria, 1956-62	9/1	Guerrilla
	Ave 4/1	
South Vietnam	9/1	North Vietnam won in 1975

Deitchman did. (Deitchman was particularly interested in exploring the question, "Can a numerically very inferior force of guerrillas defeat a much larger Army in a complete war?")

In what follows, we will let Blue denote the defenders or regulars, and Red the guerrillas. Blue moves through an area searching for guerrillas, while the guerrillas counter the attack by preparing an ambush for the defenders. According to Deitchman, the "mixed" character of the model arises from the asymmetrical or unbalanced nature of the combat situation. Generally, Red or the guerrillas will fight only when the advantage seems to be decidedly theirs, although in some cases the guerrillas may be forced to fight when stumbled upon by the defenders or otherwise when forced into conflict.

In moving through an area searching for guerrillas, the defenders (Blue) are assumed to be in full view, and therefore Blue's losses are assumed to be directly proportional to the number of guerrillas (Red) who bring *aimed* fire to bear on the regulars. Thus,

$$\frac{dB}{dt} = -\beta R \tag{28-83}$$

On the other hand, Blue's return fire is rather ineffective since the guerrillas in ambush are hidden and Blue must fire blindly into the area occupied by Red, i.e., Blue's fire is "area" fire. Thus, for the guerrilla's loss rate, we have the area fire model

$$\frac{dR}{dt} = -\rho BR. \tag{28-84}$$

From Eqs. 28-83 and 28-84, we see that we have a mixture of the Lanchester Square and the Linear type Laws.

In Eqs. 28-83 and 28-84 we have a decidedly unbalanced type of model, where the attrition constants β and ρ take on somewhat different meanings than they did before in the Square Law. To begin with, Eq. 28-83 states that the rate of change of Blue depends on a constant attrition coefficient multiplied by the number of opposing Reds or guerrillas (the Square Law part). On the other hand, Eq.

28-84 indicates that the rate of losses for the guerrillas changes not only according to the constant attrition coefficient ρ and the number of opposing Blues or Regulars, but also the number of guerrillas in ambush (the Second Linear Law). Thus, the more guerrillas in ambush, the higher the rate of losses for the guerrillas—a perfectly reasonable assumption. Moreover, it is to the guerrillas' advantage to keep the number of ambushers small, or to bring about dispersement of troops, or better still to provide concealment, cover, and protection, which has the effect of keeping the total attrition to a low figure, relatively speaking. We should expect also, therefore, and in fact as Deitchman endeavors to point out, that the coefficient ρ should be kept as small as possible in order that the guerrillas and their "fighting power" can achieve parity, thereby prolonging the war, or achieve "local" superiority and hence wipe out larger and larger forces of regulars.

It is easy to solve Eqs. 28-83 and 28-84 for the number of Blue (regulars) $B = B(t)$, and the number of Red (guerrillas) $R = R(t)$, for any time t of the battle. In fact, eliminating the time variable, we see easily that

$$\rho B dB = \beta dR \tag{28-85}$$

$$\rho(B^2 + C_1) = 2\beta(R + C_2). \tag{28-86}$$

Now when $t = 0$, $B = B_0$ and $R = R_0$, then

$$\rho(B_0^2 + C_1) = 2\beta(R_0 + C_2) \tag{28-87}$$

or finally

$$\rho(B_0^2 - B^2) = 2\beta(R_0 - R). \tag{28-88}$$

The condition of parity occurs when the requirement

$$\rho B_0^2 = 2\beta R_0 \tag{28-89}$$

is met, for then

$$\rho B^2 = 2\beta R \tag{28-90}$$

always, and the regulars and guerrillas annihilate each other eventually if the battle is allowed to continue.

If $\rho B_0^2 > 2\beta R_0$, then

$$\rho B_0^2 - 2\beta R_0 = \rho B^2 - 2\beta R > 0 \tag{28-91}$$

always, and eventually R must go to zero before B so that Eq. 28-88 becomes

$$\rho B_0^2 - 2\beta R_0 = \rho B_e^2 \tag{28-92}$$

or the remaining number of Regulars is

$$B_e = [(\rho B_0^2 - 2\beta R_0)/\rho]^{1/2} \tag{28-93}$$

with the regulars winning the battle.

On the other hand, when $2\beta R_0 > \rho B_0^2$, meaning that the guerrillas can keep ρ relatively small and avoid fighting with too large a number of regulars, then the guerrillas can win and have R_e fighting units remaining:

$$R_e = (2\beta R_0 - \rho B_0^2)/(2\beta). \tag{28-94}$$

We note in this case, however, that we must have

$$\beta/\rho > B_0^2/(2R_0) \tag{28-95}$$

or a criterion depending on the square of the initial number of Blues.

One should note that the condition for parity, Eq. 28-89, for the guerrilla warfare model, imposes a stiffer requirement on the size of the Blue (defender) force than does the Square Law. In fact, for the Square Law we have for parity

$$\rho B_0^2 - \beta R_0^2 = \rho B^2 - \beta R^2 = 0 \text{ ~~always.~~} \tag{28-96}$$

Thus, for the right-hand side of Eq. 28-96, we see that

$$\begin{aligned} \rho B^2 - \beta R^2 &= -BR(\beta R/B - \rho B/R) \\ &= -BR\left[\frac{1}{B}\left(\frac{dB}{dt}\right) - \frac{1}{R}\left(\frac{dR}{dt}\right)\right] = 0 \end{aligned}$$

or for the Square Law (see par. 28-4.1) we have

$$\frac{1}{B}\left(\frac{dB}{dt}\right) = \frac{1}{R}\left(\frac{dR}{dt}\right) \tag{28-97}$$

i.e.,

$$\frac{\dfrac{dB}{B}}{dt} = \frac{\dfrac{dR}{R}}{dt} \tag{28-98}$$

always for parity. This says that fractional or percentage losses on both sides go at the same rate for the Square Law.

For the guerrilla warfare model, however, we may substitute the conditions of Eqs. 28-83 and 28-84 into Eq. 28-97 and obtain, $\beta R/B = \rho B$, or

$$\rho B^2 = \beta R. \tag{28-99}$$

Hence, we see from Eq. 28-90 for Deitchman's "Mixed" Law case that parity requires that

$$\rho B^2 = 2\beta R \tag{28-100}$$

so that Blue winning a guerrilla warfare engagement requires a factor greater than twice that for the Square Law, relatively!

We see that guerrilla warfare can indeed represent a very special type of fighting because the guerrillas can take full advantage of the terrain and canopy, and more or less make the regulars fight on the basis of the guerrillas' own terms, so to speak. Thus, as the guerrillas spread out—taking advantage of the terrain features for protection, remaining hidden, and preparing ambushes—they force the regulars to split their forces and hence violate the principle of concentration. The result is that the guerrillas often can easily achieve local superiority and trap the regulars in ambush. Thus, some further analysis of the attrition rates is of interest here.

For Deitchman's guerrilla warfare model, we have from Eqs. 28-89 and 28-90 relating to parity that the ratio $\rho/(2\beta)$ is of considerable interest, since for $R_0 > (\rho/2\beta)B_0^2$ then Red wins, and for $B_0 > \sqrt{2\beta R_0/\rho}$ Blue wins.

It is instructive therefore to consider the attrition constants in more detail, especially in terms of rates of fire of weapons and average kill probabilities per shot fired from Blue and Red weapons. Thus, we may take ρ as

$$\rho = r_B A_{vR}/A_R \tag{28-101}$$

where the loss or kill rate constant ρ against the guerrillas depends on

r_B = rate of fire of a Blue weapon

A_{vR} = vulnerable area of all the Red forces in the battle

A_R = total area occupied by the Red forces or guerrillas.

(The ratio A_{vR}/A_R is really the single shot kill probability of Blue against Red.)

For Blue's loss rate constant β on the other hand, we have that since Blue initially is in full view of Red, then we may take β as

$$\beta = r_R p_{KR} \tag{28-102}$$

where

r_R = rate of fire for a Red weapon

p_{KR} = average kill probability per shot for a Red weapon against Blue.

Finally, the ratio $\rho/(2\beta)$ is thus given by

$$\rho/(2\beta) = r_B A_{vR}/(2A_R r_R p_{KR}) \tag{28-103}$$

which equals $A_{vR}/(2A_R p_{KR})$ if rates of fire on opposing sides are equal.

We note that Red can effectively decrease ρ by increasing A_R, or in other words by spreading out his forces so that for the same number guerrillas his density becomes lower—a very worthwhile tactic. Blue may increase his effectiveness by going to more devastating weapons—rifles to mortars, mortars to artillery, etc. Again, the guerrillas compensate by spreading out, effecting concealment, taking cover for protection, etc., until he has local superiority to win over the split Blue force or attrite him badly. In summary, therefore, an inferior numerical force of guerrillas might be able to fight very effectively by proper choice of tactics, splitting the opposition, and taking good advantage of the terrain features.

We next turn to the time solutions for the guerrilla warfare model.

Deitchman points out that the time solutions for the "Mixed" Law in case Red wins, i.e., $2\beta R_0 > \rho B_0^2$, are:

$$R = R(t) = C_1\{1 + [(B_0 - K_1 \tan \alpha_1 t)/(K_1 + B_0 \tan \alpha_1 t)]^2\} \quad (28\text{-}104)$$

and

$$B = B(t) = K_1[(B_0 - K_1 \tan \alpha_1 t)/(K_1 + B_0 \tan \alpha_1 t)] \quad (28\text{-}105)$$

where

$$K_1 = \sqrt{2\beta C_1/\rho} \quad (28\text{-}106)$$

$$C_1 = R_0 - [\rho/(2\beta)]B_0^2 \quad (28\text{-}107)$$

$$\alpha_1 = \rho K_1/2 \quad (28\text{-}108)$$

$$t\alpha_1 < \pi/2 \quad (28\text{-}109)$$

and

$$R_e = R(\text{final}) = R_0 - [\rho/(2\beta)]B_0^2 \quad (28\text{-}110)$$

On the other hand, if Blue wins, i.e., $\rho B_0^2 > 2\beta R_0$, then

$$R = R(t) = C_2\{[(B_0 + K_2 \tanh \alpha_2 t)/(K_2 + B_0 \tanh \alpha_2 t)]^2 - 1\} \quad (28\text{-}111)$$

and

$$B = B(t) = K_2[(B_0 + K_2 \tanh \alpha_2 t)/(K_2 + B_0 \tanh \alpha_2 t)] \quad (28\text{-}112)$$

where

$$K_2 = \sqrt{2\beta C_2/\rho} \quad (28\text{-}113)$$

$$C_2 = [\rho/(2\beta)]B_0^2 - R_0 \quad (28\text{-}114)$$

$$\alpha_2 = \rho K_2/2 \quad (28\text{-}115)$$

and

$$B_e = B(\text{final}) = \sqrt{B_0^2 - (2\beta/\rho)R_0}. \quad (28\text{-}116)$$

Thus, the time solutions for guerrilla warfare are obtainable in analytic form, although they are somewhat complex

EXAMPLE 28-7:

200 Vietcong (VC) armed with rifles of about 0.4 single-shot kill probability, prepare and occupy an ambush area of some 80,000 ft². A Regular force of 600 riflemen, looking for guerrillas in the area, are suddenly fired upon from the ambush. If the rate of fire on each side is about 12 rd per min and the vulnerable area of a VC is about 1.5 ft², then who wins the battle and how many men are left? Indicate how parity could be achieved, if possible, by using reasonable numerical values in the analysis. Assumptions must be quite explicit.

Given:

R_0 = 200 VC guerrillas
B_0 = 600 Regulars
p_{KR} = 0.4 (single shot kill probability of VC rifles)
A_R = area occupied by the guerrillas
A_{vR_1} = 1.5 ft² (vulnerable area of one VC guerrilla)
r_B = r_R = 12 rd/min (rate of fire of weapons on both sides).

The average area A_{R_1} occupied by one guerrilla is

$$A_{R_1} = A_R/R_0 \quad (28\text{-}117)$$

or

$$A_{R_1} = 80{,}000/200 = 400\ \text{ft}^2.$$

Then (Eq. 28-102)

$$\beta = r_R p_{KR} = (12)(0.4) = 4.8 \quad (28\text{-}118)$$

and (Eq. 28-101)

$$\rho = r_B\left(\frac{A_{vR_1}}{A_{R_1}}\right) = 12\left(\frac{1.5}{400}\right) = \frac{18}{400} = 0.045. \quad (28\text{-}119)$$

How does ρB_0^2 compare with $2\beta R_0$?

$$\rho B_0^2 = (0.045)(600)^2 = 16{,}200$$

$$2\beta R_0 = 2(4.8)(200) = 1920.$$

Since $\rho B_0^2 > 2\beta R_0$, Blue (Regulars) will win, and

$$B_e = \sqrt{(\rho B_0^2 - 2\beta R_0)/\rho}$$

$$= \sqrt{(16200 - 1920)/0.045}$$

$$= \sqrt{317333.33}$$

$$= 563 \text{ Regulars remaining.}$$

To achieve parity we need $\rho B_0^2 = 2\beta R_0$. Since we do not have the option of changing B_0 or R_0, and β for all practical purposes is constant (for example, we could increase p_{KR} by giving him mortars, but he also suffers a reduction in r_R), we must look toward changing ρ. So for parity

$$\rho = \frac{2\beta R_0}{B_0^2} = \frac{1920}{(600)^2} = 0.005333.$$

This means to achieve parity we must somehow reduce ρ by an order of magnitude! Since (Eq. 28-101)

$$\rho = r_B\left(\frac{A_{vR}}{A_R}\right)$$

we must change these factors to achieve parity. We can hardly decrease r_B from 12 rd per min. If the guerrillas take cover behind trees and rocks, stay in ditches, or behind other natural or man-made barriers, it is not unreasonable to assume they can decrease their vulnerable area from 1.5 ft^2 to 0.5 ft^2. This means that

$$A_{R_1} = r_B\left(\frac{A_{vR_1}}{\rho}\right) = 12\left(\frac{0.5}{0.005333}\right) = 1125 \text{ ft}^2.$$

If we take A_R/R_0 to be the average area occupied by one guerrilla, then the total area occupied by the guerrillas must be increased from 80,000 ft^2 to

$$A_R = R_0 \cdot A_{R_1} = (200)(1125) = 225{,}000 \text{ ft}^2. \tag{28-120}$$

Thus, the guerrillas must realize that their salvation lies in hiding and occupying a larger area (spreading out), especially to split the regulars as much as possible.

28-9.2 WINNING CHANCES FOR GUERRILLA WARFARE

The stochastic treatment of guerrilla warfare, or the probability of winning in an ambush engagement, has been studied by Kisi and Hirose (Ref. 13) and Smith (Ref. 14). In fact, Kisi and Hirose (Ref. 13) developed an approximation for Blue or Red winning based on the work of Brown (Ref. 11) in par. 28-8. Kisi and Hirose (Ref. 13) set up the following formulation:

1. Every unit of Red (Guerrilla) fires at Blues (Regulars or Counterguerrillas) with an exponentially distributed time interval with mean $1/\lambda'$, and single shot kill probability p_R.

2. The distribution of time intervals between successive firing of Blue (Regulars) follows an exponential distribution with mean $1/\lambda$.

3. Blue firing is distributed uniformly over an area A, and the effective lethal area per Blue shot is A_L, and is considered so small that two or more guerrillas are never killed by a single Blue shot. Then the chance p_B that the first casualty after time t occurs for Blue is

$$p_B = \alpha/(B + \alpha) \tag{28-121}$$

and p_R for Red is

$$p_R = B/(B + \alpha) \tag{28-122}$$

where

$$\alpha = \lambda' p_k A/(\lambda A_L). \tag{28-123}$$

As a final definition, Kisi and Hirose select the cutoff point—or withdrawal from battle number—for Blue to be a designated number B^* which may be zero, and that for Red to be R^*.

With the given formulations, the approximate chance $P(B,R)$ that Blue wins is

$$P(B,R) \approx \sum_{i=R-R^*}^{\infty} \{[B^2 - (B^*)^2]/(2\alpha)\}^i \exp\{-[B^2 - (B^*)^2]/(2\alpha)\}/i! \tag{28-124}$$

and that for Red is

$$1 - P(B,R) = \text{sum of Eq. 28-124 from 0 to } (R - R^* - 1).$$

For a sample computation, Deitchman's analysis indicates that the α of Eq. 28-123 is about 500 for guerrilla warfare. Thus, if we take $\alpha = 500$, $B^* = R^* = 0$, and $B = 100$ and $R = 10$, then the chance that Blue annihilates Red using Eq. 28-124 is found to be 0.546, whereas the exact value is 0.542—a negligible difference.

We record here that Deitchman's "deterministic" model for guerrilla warfare gives equality of strength or fighting power at any stage to be

$$[B^2 - (B^*)^2]/(2\alpha) = R - R^*. \tag{28-125}$$

Moreover, if the strength of the Regulars were reduced from B to B^* at some time t, say, then there would be

$$[B^2 - (B^*)^2]/(2\alpha) \tag{28-126}$$

guerrilla casualties at the same time, and Eq. 28-124 may be referred to as the stochastic counterpart.

We see that this analysis allows for a breakpoint B^* for Blue such that Blue would withdraw from battle, and a breakpoint R^* for Red—indicating a more reasonable and more practical requirement than complete attrition on a side. (Battle breakpoints are discussed in par. 28-11.)

Finally, Kisi and Hirose point out, as one might expect, that the number of casualties for the guerrillas is a random variable having a Poisson distribution with mean equal to Eq. 28-126.

For some further enlightening study on guerrilla type warfare, the reader should consult Schaffer (Ref. 15).

28-10 SOME CONSIDERATIONS OF THE ATTRITION COEFFICIENTS

We now come to one of the critical issues concerning Lanchester type models of combat, and that is the realistic determination of the attrition coefficients or the kill rates. In this connection, we have already indicated that the attrition coefficients may be estimated, somewhat "passively", by the use of equations such as Eq. 28-1. We see easily that in using kill rates so determined one is dealing primarily with three components—i.e., the chance of a hit upon the target, the conditional chance that a hit is a kill, and the rate of fire of the weapon. Thus, the chance of hitting will vary drastically with range to the target, whereas the conditional probability that a hit is a kill will not necessarily vary so drastically, although it will vary some. The rate of fire of a weapon will not be dependent on target range, although it is true that the rapid fire weapons are used predominately at the shorter ranges, and large artillery or missile warheads fired for the long ranges will naturally have relatively slow rates of fire.

From this discussion, therefore, we see that the attrition coefficients cannot possibly be constant, as we have more or less used them heretofore. Moreover, it may not be proper or realistic to use average kill rates over the ranges of engagement which two opposing forces fight each other. In other words, we may be dealing with a very complex problem indeed, although some of the simpler models could be adequate for some applications or fighting conditions. Therefore, the problem of modeling combat adequately may become quite complex indeed—even for the kill rates or attrition coefficients alone—to say nothing about the best choice of model otherwise, trying to model the terrain features encountered, weather, command and control, etc. Then again, many of us are in agreement that combat is bound to involve stochastic considerations which may often turn out to be very influential in all types of warfare.

Bonder (Ref. 16) has made a study of the Lanchester attrition rate coefficients by hypothesizing that such coefficients are random variables following some probability distribution, since the concept of an "average value implies a distribution". Bonder's treatment (Ref. 16) is conditioned on the number of rounds that must be fired to destroy a target, and involves some of the more basic considerations of the time to acquire targets, time to fire the first round, time to fire subsequent rounds given a hit or a miss on the preceding round, projectile time of flight, and other events of firing. Bonder derives an expression for the probability density of the attrition rate and indicates that it is the reciprocal of the total time to defeat a target, as one would surmise, and as we have brought out heretofore. The criticality of the ranges to targets is brought out in Ref. 16 only in terms of such implication as range affects the factors just mentioned.

Barfoot (Ref. 17) is somewhat critical of Bonder's analysis and argues that a valid prediction of the average attrition should be obtained by using the harmonic mean of the variable attrition rates rather than the arithmetic mean. He points out that this change results in a constant Lanchester attrition rate coefficient being defined as the reciprocal of the expected time to kill a target, and he gives an alternate method for obtaining the coefficients. Bonder (Ref. 18) then shows, nevertheless, that his methodology of Ref. 16 leads to an average or expected time to kill a target, the reciprocal of which may be used as the average attrition rate, and hence takes care of Barfoot's objections—at least for the case of a single target kill probability.

One of the original and profound treatments of the target range problem is no doubt that of Weiss (Ref. 19) who extended Lanchester type combat theory to the relative movement of forces, combat among small groups of forces in the presence of large areas of effectiveness for weapons, and combat between heterogeneous forces with consideration of the problem of target assignment. Indeed, Weiss' paper (Ref. 19) and some of his subsequent studies represent several major contributions to combat theory (see par. 29-3).

Taylor (Ref. 20) shows how to obtain a solution to Lanchester type equations for combat between two homogeneous forces when the attrition rates are variable, but their quotient is nevertheless a constant throughout the battle. Also, Taylor's solutions are developed for either time or force separation as the independent variable. Indeed, one may easily appreciate the dependence between time of battle and the closing of opposing forces in range against each other.

In summary, we might say that the whole matter of determining attrition rates for Lanchester's combat equations needs much more research. Also, we should not forget that the analysis of available data from actual battles such as that by Engel for Iwo Jima (Ref. 6) remains quite relevant for study of the attrition rate problem and the validation of models.

28-11 BREAKPOINTS OF BATTLES

Another critical issue concerning the analysis or modeling of combat has to do with breakpoints, or when and under just what circumstances will one of the opposing forces withdraw? Clearly, and especially as time marches on, it now seems very unrealistic to assume that sides will fight to annihilation as in olden times perhaps. Hence, stopping criteria for battle disengagement need considerable study if analyses are to be used for prediction purposes. Some investigators have suggested that a side might withdraw or disengage from battle when it has suffered some 25% or 30% casualties, for example, or a tank unit might disengage when, say, 40% of its tanks are lost, etc. Helmbold (Ref. 21) has studied various reasons for breaking battle, and apparently found nothing very systematic for battle breakpoint criteria that could really be depended upon as anything approaching universality. In fact, many commanders withdrew for unexplained reasons. Blakeslee (Ref. 22) reviewed and analyzed all available studies on battle breakpoint casualty criteria. He found that percent casualties may still be the best criterion to use, and that the breakpoint in percent casualties for the attacker was only one-half of that for the defender. Moreover, a considerable amount of randomness should be expected, especially for the defender.

In spite of the limitations and state of the art of analyses on attrition coefficients and battle breakpoints, however, there recently has been developed some new thoughts and a fresh approach to the analysis of combat type data, both of which might help to circumvent some of the old problems. This, we take up next as the final topic of this chapter.

28-12 A NEW FORMULATION OF LANCHESTER TYPE COMBAT THEORY

28-12.1 BACKGROUND AND BASIC APPROACH

Up to this point, the reader will no doubt have acquired some appreciation for many of the difficulties of modeling combat or obtaining suitably accurate predictions in studying new weapons and tactics for a future conflict. Our account of combat theory here so far has been concerned primarily with numbers of opposing elements, or weapons, etc., on each side for several Lanchester type models of combat and the attrition coefficients or kill rates. However, any realistic representation of combat must involve many other considerations such as terrain, line of sights to targets, target detection probabilities, command and control procedures, and other characteristics. Thus, we face an enormous and rather complex problem, as the reader will appreciate no doubt. Hence, is there another approach to the problem of analyzing combat data in some way that might be helpful? Or, is it perhaps possible to approach the battle description problem in terms of fewer variables or parameters? Recalling that attrition rates are tied in with the reciprocals of kill times at which targets are deleted from the battlefield, then one might possibly consider the problem of analyzing only kill time data. Hence, suppose that we have at hand target kill time data from an actual battle of the past, or kill time data from a "realistic" battle simulation, or a war game played on a computer, or even such data as they occur from an actual battle (especially in the early stages) in the field. This latter consideration is one of some importance, for commanders in the field now have sufficient intelligence resources to gather such critical information, and they also have sufficient computer capability to analyze data rather rapidly. Analyzing the situation a bit further, one may see that the side which loses too much of his combat capability first, or before the other side does, will of necessity have to come to grips with the problem as to whether he should withdraw, or break battle—perhaps hoping for reinforcements, or to fight later under much improved conditions. Hence, the faster a side loses his key elements, weapons, etc., as compared to the other side, then the closer he comes to defeat. This background brings forward the idea of Grubbs and Shuford (Ref. 23), who examine the problem of working with kill times in battles or simulations for the key elements, weapons, targets, etc., of interest. They point out that Lanchester's differential equations of combat are inherently deterministic in nature, although considerable effort has been devoted in recent years to introducing stochastic treatments into the theory, for example, by dealing with transition probabilities and "variable" attrition coefficients. They also advance the advantageous idea that the time to kill, or time to neutralize key opposing targets, is the more logical random variable to be treated on a probabilistic basis, and hence that the fraction of remaining combatants on each side should properly be estimated from the time-to-kill probability distributions sampled—in other words, from principles of the statistical theory of reliability and life testing. The advantages of such treatment include the possibility that the future course of a battle may be predicted from data on casualties in the early stages of an engagement, and therefore that field commanders will have available information on which to base critical decisions—for example, either to withdraw or to augment fighting forces—in order to bring about more desirable future courses of combat for a given mission. Also, commanders may even use the analyses suggested independently of information on enemy losses to decide whether the course of combat is proceeding satisfactorily or according to plan by comparing data on early casualties observed in an engagement with standards that have been determined from experience or specified in advance. Another advantage of the suggested method is that available Weibull theory leads to placing confidence bounds on the fractions of survivors for any specified mission times. The degree of confidence on final predictions depends, as would be expected, on the number of targets put out of action in an engagement or simulation, the nature of the time-to-kill distributions encountered or sampled, the degree of accuracy or confidence desired, and the number of runs or the size of the war game.

In other words, when a Blue force meets a Red force, or one stumbles upon the other, then the ensuing battle involves changing decisions on the part of commanders, many human variables, the random effects of terrain, weather conditions, the selected or available weapon mixes, timely deployment and use of weapons, accidental occurrences relating to the reliability and maintainability of equipment, resupply, etc. Thus it is perhaps unnecessary to argue further that many conditions leading to various degrees of randomness are ever-present, that the variable logically treated on a probabilistic basis should be the time to kill opposing targets, and therefore that other Lanchester parameters should depend in a probabilistic manner on elapsed times in battles, in particular, when kills or other forms of attrition occur. As a matter of fact, if in a battle one were to tabulate the times from zero at which targets are destroyed or combatant losses occur on both sides, then he might well develop a better understanding of applied combat theory; such data could well help to develop general Lanchester-type theory further, or extend our knowledge of its validity—but such data are now unfortunately hard to acquire since they have not been demanded. Why not work the time to kill concept into the Lanchester-type theory nevertheless to see where it might lead? This we now proceed to do along lines similar to the ones covered in some detail by Grubbs and Shuford (Ref. 23).

28-12.2 THE NEW FORMULATION

We begin with the concepts of par. 28-12.1 and a simple argument. As before, let B_0 and R_0, respectively, represent the initial numbers of Blue and Red combatants, targets, or fighting units, etc., that are deemed appropriate as key elements or key targets in an engagement; and let B and R be the numbers remaining on each side at any time t after combat has begun. Thus, the fractions of survivors, B/B_0 and R/R_0, each represent quantities that will vary in a random manner from unity at the start of a battle down to some fraction (or perhaps to zero), at the time the engagement ceases, or a side withdraws. Moreover, the proportions B/B_0 and R/R_0 clearly vary in a random manner with time: i.e., $B = B(t)$ and $R = R(t)$, and indeed they are the fractions of survivors on the two sides at any time t, including also perhaps the projected or assigned "mission" time t_m to reach some objective. Therefore, it can be argued that these proportions, or a function thereof could be related to various forms of probability distributions of time to kill. These probability distributions of time must involve meaningful physical definitions; criteria; or descriptions for time to kill, time to incapacitate, time to failure of equipment, etc.; and their parameters should in some way describe the "fighting power" or capability of a side at the random times required to kill opposing targets. To win a battle, one must kill or incapacitate before his opponent disables him. In this connection, it is well known that the two-parameter Weibull distribution (actually a probability distribution of R. A. Fisher discovered independently by Weibull) can be used to represent a very wide variety of time to fail (or, in this case, time to kill) probability distributions. Moreover, the fraction of survivors at given times in life tests of equipment is now rather widely recognized as the reliability of the equipment. In general, such percentage or fractions of survivability could be equated to reliability which depends upon the random time-to-kill variables in combat. For continuous distributions the reliability or fraction surviving with respect to a mission time t_m may be defined as the integral of an appropriate probability density function (pdf) from t_m to ∞. Thus, immediately we have the following approximations or relations for remaining fractions of Blue and Red at any time t after the battle starts:

$$B/B_0 = \exp(-\beta t^{\alpha}), \quad B = B(t), \quad (\alpha,\beta > 0;\ t \geq 0) \tag{28-127}$$

$$R/R_0 = \exp(-\rho t^{\delta}), \quad R = R(t), \quad (\rho,\delta > 0;\ t \geq 0) \tag{28-128}$$

where $\beta = \beta(t,R,B)$ is an "attrition" coefficient for Blue, i.e., the loss or failure rate, or scale parameter; and $\alpha = \alpha(t,R,B)$ a shape parameter for the time-to-kill probability distribution encountered. These parameters represent the capability of Red forces to destroy Blue targets, Blue to protect himself, etc. In combination, we might say that α and β represent in perhaps an obscure way the "total fighting power" of Red against Blue, but including also various attrition accidents that occur to Blue in battle. Similar arguments apply to Eq. 28-128. By the notation $\beta = \beta(t,R,B)$, for example, we mean that β is the parameter (constant) of a life-time chance distribution that is statistically estimable from the probabilistic relation between the remaining Blue and Red forces with time. Note that Eqs. 28-127 and 28-128 may be interpreted as the chances of survival of a Blue or a Red, respectively, and these proportions may approach zero theoretically, but not practically, in most battles.

The suggested use of the Weibull distributions in Eqs. 28-127 and 28-128, along with the proposed method of analyzing data from combat or simulations, requires some discussion and characterization for further justification. To begin with, we do not take the approach often used in the past in which one is interested in changes in the numbers of combatants or key targets for individual engagements, battles, etc. Rather, we visualize the concept of sampling populations—that is to say, a very large number of similar engagements or battles—and we concentrate on studying a sample engagement that is "representative" of the hypothesized general characteristics of many battles in the supposed environment. We regard the outcome of individual engagements or battles as being accidental in character, and—except for superior weapons, tactics, favorable weather, terrain, etc.—one side may sometimes "win" over the other due to chance. If it were possible to fight out many such engagements or carry out a very large number of simulations, we could obtain the desired characteristics of the population in great detail. However, since there will rarely ever be time for this and the cost would be high, it seems of value to make inferences from samples representing combat situations to the populations of such engagements or battles. This appears to us to be precisely what we should be getting at in individual, or a few, simulations of combat. In particular, for example, we may be interested in running a sample simulation of a combat situation in order to see whether or not it is likely that our choice of weapons, the tactics employed in using them, and certain command-and-control principles would overcome and defeat an enemy with somewhat different weapon capabilities in the same hypothesized battle environment.

For our purposes here, therefore, we regard the problem of analyzing combat as that of sampling two-sided mutually interacting failure situations or games in which time is of the essence, since one side, if he expects to win, must put targets on the other side out of action before his own fighting capability is destroyed. The quicker Blue's weapons and tactics bring the Red side "to its knees", then the better for Blue. He will have gained command of the battle situation before Red, who must now withdraw, lose further men and equipment, or go down in defeat. Thus, the lifetimes at which combat elements are put out of action seems to be of such importance that we must concentrate on analyzing the random lifetimes, times to kill, or random times to failure of combat elements. Moreover, such an analysis would give a summary of the battle conditions under study. In this connection, it is now rather widely known that the class of probability functions known as Weibull distributions possesses a very attractive capability for treating (positive) data for times to fail, cycles to failure, mileages to failure, etc. Thus, it seems evident that Weibull theory may be applied to combat data, especially lifetimes for the combat elements. We also note that Engel's analysis (Ref. 6) of combat data for the Iwo Jima campaign appears to support an exponential type of decay, which is a special case of Weibull probability distributions. Furthermore, in order to have some confidence that our choice of a class of probability distributions will be "robust" enough to cover many of the different forms of time-to-kill distributions that might occur in simulations or in combat, we might well consider the two-parameter Weibull distributions for which the *pdf*'s appear in Fig. 21-7.

Of course, we do not claim that Weibull theory will apply to all combat situations or that only Weibull distributions should be used; for in fact the idea advanced here of analyzing time-to-kill data should be more general than this. Indeed, other forms of probability distributions such as the Pearson Type III or gamma distributions, or the four-parameter Pearson Type I or beta distributions, could also be considered, or even some very special probability distributions that accurately describe particular combat engagements. Nevertheless, we do point out that there are some rather distinct advantages to using Weibull theory at the present time. In particular, statistical investigators have concentrated on and worked out a considerable volume of useful theory concerning the Weibull two-parameter probability distributions, and have been able to place confidence bounds on the true, unknown fraction of survivors in the populations by using either a complete random sample, or truncated data. Although we believe this accomplishment could be attained eventually for many classes of *pdf* 's, it may nevertheless take many years, and we feel that the presently available Weibull theory can be used immediately and to considerable advantage in the analysis of simulated or combat data. Also, much computer time can be saved. The analyses suggested here apply only to the two-parameter Weibull theory, of course. Also, we believe, as apparently does Engel, that the parameters should properly be estimated from two-sided conflict engagement data. It may be possible to use weapon rates of fire, lethality, delivery accuracy, mobility characteristics, target vulnerability, etc., to estimate the parameters in advance, but we do not see now just how this can be done from one-sided or "passive" weapon characteristics not including enemy return fire.

We might derive Eqs. 28-127 and 28-128 somewhat formally from the consideration that $B/B_0 = B(t)/B_0$ is the fraction of Blue forces remaining at time t; or the chance that a Blue combatant, tank target, or fighting unit, etc., will survive to time t; and hence that $(B_0 - B)/B_0$ is the chance of a Blue combatant being lost by time t. Thus, we may hypothesize that $(B_0 - B)/B_0$ is the cumulative chance of kill for Blue within the random time t and that furthermore the time derivative of this quantity can be equated to a probability density function of times to kill or lifetimes. In summary, we say, for example, that

$$dF/dt = (1/B_0)[d(B_0 - B)/dt] \approx \alpha\beta t^{\alpha-1}\exp(-\beta t^{\alpha}) \tag{28-129}$$

where the left-hand side is the fractional rate of losses for Blue and the right-hand side is the two-parameter Weibull *pdf* for the time to kill Blue targets. Integrating Eq. 28-129, we obtain immediately

$$B = B_0\exp(-\beta t^{\alpha}). \tag{28-130}$$

The Weibull *pdf* has been suggested here because of its inherent generality in describing accurately the various possible shapes of time-to-kill distributions occurring in combat.

Also, we could argue that, since $(B_0 - B)/B_0$ is the fraction of losses for Blue, then the conditional failure rate for Blue, given survival to some time t, may be described somewhat generally in the form

$$[B'(t)/B_0]/[B(t)/B_0] = -\alpha\beta t^{\alpha-1} \tag{28-131}$$

where the right-hand side depends on the time of battle and $B'(t)$ is the time derivative of B. That is to say, the conditional failure or loss rate of Blue forces may vary with some power of time, possessing the generality of an increasing, constant, or decreasing kill rate. Hence, we get immediately that $\ln[B(t)/B_0] = -\beta t^{\alpha}$ or, as before, $B(t)/B_0 = \exp(-\beta t^{\alpha})$.

Now the fractions of survivors, or the "reliabilities" given by Eqs. 28-127 and 28-128, as we have already indicated, can really encompass a wide range of probability distributions on time for combat engagements. In fact, the two-parameter Weibull *pdf* given by

$$f(t) = \alpha\beta t^{\alpha-1}\exp(-\beta t^{\alpha}) \tag{28-132}$$

is somewhat of a natural choice, for it can—by proper selection of the shape and scale parameters α and β—vary from the subexponential to the exponential (in which case $\alpha = 1$, and the conditional failure or kill rate is constant and equal to β) to the super-exponential models of time to kill. Indeed, various combinations of α and β even include the normal or Gaussian *pdf*, as well as skew, platykurtic, and leptokurtic probability distributions. We can, therefore, through the use of the Weibull model, equate the random fractions of Blue and Red survivors with time to any of a wide variety of realistic probability distributions for remaining lives, which in some way will depend on the "fighting powers" or combat capabilities of the opposing sides. If, for roughly equal forces, the chance of survival for Blue forces (i.e., the proportion of survivors at various times t) consistently exceeds that of Red, then Blue obviously has the advantage in an engagement.

28-12.3 PARAMETER ESTIMATION FOR THE WEIBULL KILL TIME DISTRIBUTIONS

We have already discussed methods for estimating the Weibull scale parameters, β and ρ, and the Weibull shape parameters, α and δ, in Chapter 21. In particular, if the kill times are distributed exponentially, then the reciprocal or the mean time to kill for either Blue or Red may be estimated from Eq. 21-83. The recommended methods for estimating the Weibull scale and shape parameters generally are covered in par. 21-8, and hence need not be repeated. Nevertheless, we will illustrate the matter of parameter estimation and the details of analysis in an instructive example which follows.

28-12.4 AN INSTRUCTIVE EXAMPLE

EXAMPLE 28-8:

In a study of the effectiveness of antitank missiles as the main armament of tanks, it was decided to simulate a "typical" engagement in Western Europe for a certain version of the Chief battle tank (CBT) versus the R10 tank. One of the main purposes of the simulation was to determine whether missiles could successfully engage opposing tanks at longer ranges than guns, and hence obtain an early advantage in killing enemy tanks, thereby neutralizing the enemy tank force and obtaining a given objective on schedule. In particular, a mission time of about 90 min was suggested for accomplishing the objective.

In a valley, 20 R10's were in position near the bottom of an inclining ground area leading up to a town of key importance in the hills of the general battle zone. The R10's were initially in defilade and hence not easily in view of the friendly task force of 20 CBT's approaching them. At about 2500 m, however, the R10's opened fire on the approaching CBT's, but the latter were out of range for very accurate fire from the R10's. As a result, and as the battle proceeded, the first tank knocked out by the missile armament of approaching CBT's was an R10 at 4 min after the engagement had started. In 8 min, one CBT had come within range of the R10's and was killed. In summary, five R10's were knocked out at 4, 9, 15, 23, and 40 min elapsed time from the beginning of the engagement. On the other hand, three CBT's were killed at 8, 13, and 24 min, and later at 60 min another CBT was finally knocked out. During the period of 40-60 min, it was thought that another R10 had been put out of action, but a heavy fog had set in, making such determination uncertain, and the battle was stopped just before night. With these data on times to kill targets on each side, and assuming no major changes in the commanders' tactics, resupply, etc., what can be said about the progress and outcome of such a battle in general had it continued to 90 min, assuming the available data represent a valid sampling for a population of such engagements?

We assume that the time-to-kill distributions for tank targets on each side follow two-parameter Weibull probability distributions because of the wide variety of possible shapes for fitting such data,

TABLE 28-8. COMPUTATIONS FOR THE INITIAL SAMPLE SIZES (B_0 and R_0 for Example 28-8)

CBT Data (B_0 = 20)				R10 Data (R_0 = 20)			
t_i	$\ln t_i$	A_i	C_i	t_i	$\ln t_i$	A_i	C_i
8	2.079	−0.408	−0.244	4	1.386	−0.273	−0.193
13	2.565	−0.386	−0.239	9	2.197	−0.259	−0.191
24	3.178	−0.346	−0.223	15	2.708	−0.234	−0.181
60	4.094	2.141	0.706	23	3.136	−0.200	−0.166
				40	3.689	1.965	0.732

CBT Data:
$\Sigma A_i \ln t_i = 5.828 = \hat{u}$
$\Sigma C_i \ln t_i = 1.061 = 1/\hat{\alpha}$
Thus, $\hat{\alpha} \approx 1/1.061 \approx 0.943$, and
$\hat{\beta} = \exp(-\hat{\alpha}\hat{\mu}) \approx 1/244 = 0.0041$
$1/\hat{\beta} = 244$

R10 Data:
$\Sigma A_i \ln t_i = 5.040 = \hat{u}$
$\Sigma C_i \ln t_i = 1.002 = 1/\hat{\delta}$
Thus, $\hat{\delta} \approx 1/1.002 \approx 0.998$, and
$\hat{\rho} = \exp(-\hat{\delta}\hat{\mu}) \approx 1/154 = 0.0065.$
$1/\hat{\rho} = 154$

(The constants A_i and C_i are taken from Mann, Ref. 24.)

and we proceed to estimate the parameters so that an appropriate fit can be obtained to describe the probable remainder of such an engagement. For quickness and convenience, we use the theory and tables of Mann (Ref. 24 or par. 21-8.2.3) to estimate α, β, δ, and ρ, although other methods of estimation could be used [for example, the maximum-likelihood estimates of Cohen (Ref. 25, or par. 21-8.2.2) or of Billman, Antle, and Bain (Ref. 26)]. In order to use Mann's estimates, i.e., the linear invariant statistics, it is convenient to tabulate the computations for the initial sample sizes B_0 and R_0 as shown in Table 28-8.

From these results, we note that, since the estimates of the shape parameters α and δ are each practically equal to one, exponential time-to-kill distributions may be used to describe the battle, i.e., the losses on each side. In fact, the estimated true mean time to kill an R10 is estimated to be about 154 min*. To put this result another way, since the exponential failure distribution involves a constant conditional failure rate at any time t, the instantaneous kill rate for CBT's is predicted to be 0.0041/min. and that for R10's to be 0.0065/min.

Since the single-parameter negative-exponential distribution seems to be a suitable hypothesis from these estimates of shape parameters ($\alpha \approx 1$) for the small numbers of kills, we can estimate the scale parameters β and ρ (i.e., the conditional failure rates) from Eq. 21-83. We have, in fact,

$$1/\hat{\beta} = \text{est}(1/\beta) = \left[\sum_{i=1}^{i=r} t_i + (B_0 - r)t_r\right]/r \quad (r = \text{number of kills})$$

$$= [105 + (16)(60)]/4 = 266 \quad (\text{vs } 244)$$

$$1/\hat{\rho} = \text{est}(1/\rho) = [91 + (15)(40)]/5 = 138 \quad (\text{vs } 154)$$

so that the agreement is surprisingly good in this case.

*This example is for illustrative purposes, and hence we do not imply that 154 min is a typical or average combat kill time.

An interesting and important feature of our method is that we may easily place confidence limits on the fractions of survivors for each side. For example, for the assumption of an exponential distribution, it is known from the theory of Epstein and Sobel (Ref. 27) that

$$2r\hat{\theta}/\theta = \chi^2(2r) \tag{28-133}$$

where

$\theta = 1/\beta$ or $1/\rho$

$\hat{\theta} = 1/\hat{\beta}$ or $1/\hat{\rho}$

$\chi^2(2r)$ = chi-square with $2r$ degrees of freedom.

That is to say, $2r\beta/\hat{\beta}$ and $2r\rho/\hat{\rho}$ are each distributed in probability as the well known chi-square, and hence—since the true unknown fraction of Blue survivors is $\exp(-\beta t)$, and that for Red is $\exp(-\rho t)$—we may determine confidence limits for the true fractions of survivors as follows.

We start with

$$Pr[\chi^2_\alpha(2r) \le \chi^2(2r) = 2r\hat{\theta}/\theta \le \chi^2_{1-\alpha}(2r)] = 1 - 2\alpha \tag{28-134}$$

where χ^2_α is the lower α probability level and $\chi^2_{1-\alpha}$ the upper α probability level of the chi-square distribution for $2r$ degrees of freedom. Hence, for a mission t_m we can convert this probability statement to

$$Pr[t_m\chi^2_\alpha(2r)/(2r\hat{\theta}) \le t_m/\theta \le t_m\chi^2_{1-\alpha}(2r)/(2r\hat{\theta})]$$
$$= Pr[\exp\{-t_m\chi^2_{1-\alpha}(2r)/(2r\hat{\theta})\} \le \exp(-t_m/\theta) \le \exp\{-t_m\chi^2_\alpha(2r)/(2r\hat{\theta})\}] = 1 - 2\alpha. \tag{28-135}$$

But $\exp(-\beta t_m) = B/B_0$ and $\exp(-\rho t_m) = R/R_0$ for any mission t_m, and thus we have lower and upper confidence limits on the true unknown fractions of Blue and Red survivors. Thus, had the tank battle gone to 1.5 hr (90 min), we could state for the assumption of an exponential distribution that

$$Pr[B/B_0 \ge \exp\{-t_m\hat{\beta}\chi^2_{1-\alpha}(2r)/(2r\hat{\theta})\}] = 1 - \alpha$$

or

$$Pr[B/B_0 \ge \exp\{-(90)(1/266)\chi^2_{0.95}(8)/8\} = 0.52] = 0.95.$$

In other words, we state with 95% confidence that at least 52% (10.4) of the CBT's will survive after 90 min of such a battle. On the other hand, we can only say that at least 30.4% (6.0) of the R10's will survive after 90 min, again with 95% confidence.

With two-sided confidence limits based on $\chi^2_{0.025}(8) = 2.18$, $\chi^2_{0.975}(8) = 17.53$, $\chi^2_{0.025}(10) = 3.25$, and $\chi^2_{0.975}(10) = 20.48$, we can state with 95% confidence that at 90 min the fraction of surviving CBT's will be between 0.48 and 0.91, while for the same confidence level the fraction of surviving R10's will lie between 0.26 and 0.81. Of course, the widths of the confidence intervals depend markedly on the number of kills, the conditional failure rate, the mission time, and the confidence level; in this illustration, we are dealing with data from a rather limited engagement too sparse to allow us to infer very precise statements about the general population. For the mission time of 90 min, the estimated fractions for point estimates of surviving CBT's and R10's are, respectively, $\exp(-90/266) = 0.71$ and $\exp(-90/138) = 0.52$ for further population inferences. Should more precise information be desired, the simulation could be carried further, repeated, or the problem enlarged in consonance with the importance of the decision to be made.

In this example we have concentrated properly on placing confidence bounds on fractions of survivors; however, if desired, relevant statistical literature is available for comparing the Blue and Red population parameters.

We have indicated that confidence bounds can be estimated also for the reliability or proportion of survivors, $R(t_m) = B(t_m)/B_0 = 1 - F(t_m) = \exp(-\beta t^{\alpha})$ related to the Weibull distributions of time to kill, i.e., for $\alpha \neq 1$ and $\delta \neq 1$ in Eqs. 28-127 and 28-128. This recent work has been carried out by N. R. Mann (Ref. 28), and should prove to be most useful indeed to the weapon systems analyst.

28-12.5 SOME REFLECTIONS

As mentioned earlier, it is difficult under ordinary circumstances to obtain times at which casualties occur in actual battles—especially such data for the opposing side. Nevertheless, in realistic simulations of battles or computer games, etc., one can acquire the needed data and hence have at hand information to judge the probable future outcomes of engagements by using the method suggested here. Also, data obtained in a natural manner on the friendly side, with no such information at all on enemy casualties, may be of considerable importance. For example, as we have indicated the Army in the field carries computers as part of its equipment at the present time. Hence, if Blue were in a battle and had been allocated a certain time, say 3 h, to accomplish an objective, then computations could be made in the field and during the battle to arrive at estimates from the Blue casualties occurring, say, during the first 30, 45, or 60 min of battle. From these data the shape of the appropriate Weibull *pdf* could be determined and, hence, the remaining Blue survivors at the mission time of three hours could be predicted. (We remark in this connection that truncating a simulation or battle at some predetermined fixed time as compared to that of a fixed number of casualties would lead to somewhat different methods of estimation.) If this estimated fraction of survivors is expected or is satisfactory, then Blue proceeds; otherwise, higher headquarters would be so advised and hence have important information on which to base any decision to withdraw, throw additional units into the battle, etc. Furthermore, standard values of the Weibull parameters β and α can be developed from experience, and hence computed casualties as a function of time could be compared with observed rates in a simulation or actual battle to determine whether requirements are satisfactorily met, or various alternative actions should be taken by commanders. Moreover, confidence bounds may be placed on the predictions.

Finally, other forms of probability distributions could, of course, be fitted to observed time-to-kill data on targets in a battle or simulation—for example, the gamma, lognormal, or especially the extreme-value distribution—although it is believed that the two parameter Weibull model suggested here represents a single form of distribution that will be sufficient for many battle situations of interest.

For an application using Lanchester type combat theory to study armor protection, firepower, and mobility for tanks see Ref. 29.

28-12.6 STOPPING RULES TO CONTROL RISKS FOR EXPONENTIAL LIFETIME WAR GAMES OR SIMULATIONS

A problem of considerable interest and importance in military operations research is that of providing appropriate stopping rules for war games and computerized simulations of combat. Past practice has been to run many simulations in order to study the variation in outcomes of a stochastic game and hence arrive at some idea of the confidence that might be placed on the results. The new formulation of Lanchester combat theory in par. 28-12, makes possible the analyses of results in terms of random times-to-kill in battle. Hence, in accordance with the statistical theory of reliability and life-testing, our new procedure has the advantage that stopping rules may be found for games with *exponential life times* of combat elements simply by using statistical decision theory. Shuford and Grubbs (Ref 30) have solved this military operations research problem and an example of the analytical solution is presented here.

Consider, for example, what actually may be a typical problem faced in the weapon acquisition process. Should Blue forces equip its new main battle tank (say, the XM1) with missiles or guns to oppose effectively Red's new battle tank (call it the R10), which is equipped with guns? When Blue tanks are equipped with guns, we might assume that the Blue force would normally lose about 25% of its tanks on the average in the first 90 min of combat. (This 25% loss could have been predicted by using a detailed computer simulation model or verified from historical records.) The proponents of the missile armament for the Blue XM1 might claim that the Blue force would lose only 10% of its tanks in 90 min. How, therefore, may we settle the issue?

A study team decides that if it can be reasonably sure that the fraction of XM1's surviving after 90 min of battle is in fact as high as 90% when armed with missiles, the change should be made. If, however, the Blue fraction surviving after 90 min appears to be close to 75% the change would not be "cost-effective". The study team, therefore, decides to test the following battle hypothesis for Blue's missile armament:

H_0 = The fraction of Blue XM1's surviving at mission time t_m = 90 min is 0.90, against the alternative hypothesis

H_A = The fraction of Blue XM1's surviving at mission time t_m = 90 min is only 0.75.

The team also decides that the acceptable risk of rejecting H_0 when it is actually true should be about 5% (chance of a Type I error is $\gamma = 0.05$); and that an assurance level of 90% is required for rejecting H_0 when it is false and H_A is true, or chance of a Type II error is put at $\nu = 0.10$.

Since pretests with the simulation model and analyses of actual tank battles show that the life-times of tanks in combat can be approximated with an exponential distribution, then the hypotheses to be tested for a typical mission can be restated as follows:

$$H_0:\ B/B_0 = \exp(-t_m/\theta_0) = 0.90 \qquad (28\text{-}136)$$

$$H_A:\ B/B_0 = \exp(-t_m/\theta_A) = 0.75. \qquad (28\text{-}137)$$

With the mission time t_m = 90 min, the problem reduces to determining whether the fraction of survivors or the "reliability," $\exp(-90/\theta)$, is 0.90, or as low as 0.75; i.e., whether in an engagement the mean-time-to-kill the XM1 armed with missiles is θ_0 = 854.2 min, or is as low as θ_A = 312.8 min. These values are found from Eqs. 28-136 and 28-137, respectively, for t_m = 90.

Our hypotheses now can be written equivalently as $H_0{:}\theta_0$ = 854.2 min versus $H_A{:}\theta_A$ = 312.8 min. Our problem is to determine the number of kills that we must observe before we can truncate the simulation to perform our test of significance and control risks as previously indicated.

Grubbs (Ref. 31) has shown that for exponential life-testing and the case where $\theta_A < \theta_0$, the power function of the test, or the operating characteristic curve of the significance test given in Eq. 28-138, implies that

$$\frac{\theta_0}{\theta_A} = \frac{\left[1 - \frac{1}{9r} + \eta_{1-\nu}\left(\frac{1}{9r}\right)^{0.5}\right]^3}{\left[1 - \frac{1}{9r} + \eta_\gamma\left(\frac{1}{9r}\right)^{0.5}\right]^3} \tag{28-138}$$

Here, η_γ is the lower γ probability level of the standard normal distribution, $\eta_{1-\nu}$ is the upper ν probability level, and r is the required number of data or tank kill times required. Solving Eq. 28-138 for r, we find that

$$r = \frac{4(1 - \mu)^2}{9\{\mu\eta_\gamma - \eta_{1-\nu} + [(\mu\eta_\gamma - \eta_{1-\nu})^2 + 4(\mu - 1)^2]^{0.5}\}^2}$$

$$\approx \frac{\lambda^2}{9} \tag{28-139}$$

where $\mu = (\theta_0/\theta_A)^{1/3}$ and $\lambda = (\eta_{1-\nu} - \mu\eta_\gamma)/(\mu - 1)$.

Our stopping rule then is analytically to find r, the number of kills required before stopping the simulation, that will fit the operating characteristic curve as nearly as possible through the risks, $\gamma = 0.05$ and $\nu = 0.10$, for the acceptable and unacceptable true mean times-to-fail, $\theta_0 = 854.2$ min and $\theta_A = 312.8$ min, respectively. We can find such an r from Eq. 38-139.

For $\gamma = 0.05$, and $\nu = 0.10$, then, from a table of values for the standard normal distribution we find $\eta_\gamma = -1.645$ and $\eta_{1-\nu} = 1.282$. Then we compute

$$\mu = (\theta_0/\theta_A)^{1/3} = 1.40 \tag{28-140}$$

and

$$\lambda = (\eta_{1-\nu} - \mu\eta_\gamma)/(\mu - 1) = 8.9625. \tag{28-141}$$

Finally,

$$r \approx \lambda^2/9 \approx 8.9 \text{ or } 9 \text{ kills required.} \tag{28-142}$$

Our stopping rule tells us that we need 9 Blue tank kills before we stop our simulation and perform our test at the risk levels $\gamma = 0.05$ and $\nu = 0.10$.

To complete our test, we run the simulation with some initial numbers, B_0 and R_0, of tanks on each side (much greater than 9, say $B_0 = 20$ or so) until we have obtained 9 Blue tank kills. We record the time from the start of the battle at which each tank kill occurred. Next we compute our estimate of θ from Epstein and Sobel (Ref. 27) as

$$\hat{\theta} = [\sum_{i=1}^{i=r} t_i + (B_0 - r)t_r]/r = [\sum_{i=1}^{i=9} t_i + (B_0 - 9)/t_9]/9 \tag{28-143}$$

for the ordered kill times $t_1 \leq t_2 \leq \cdots \leq t_{B_0}$, with the battle being truncated immediately at $r = 9$ Blue tank kills. Since $2r\hat{\theta}/\theta = \chi^2(2r)$ is distributed in probability as chi-square with $2r$ degrees of freedom, we will accept the hypothesis that

$$H_0: B/B_0 = \exp(-90/\theta) = 0.90$$

and hence that missiles are very effective, if the observed

$$\hat{\theta} \geq \theta_0 \chi^2_{0.05}(2r)/(2r) = 445.6. \qquad (28\text{-}144)$$

If $\hat{\theta} < 445.6$ we reject the hypothesis that $B/B_0 = 0.90$ and accept the alternative hypothesis that missiles are not so effective.

Thus, with the technique developed by Shuford and Grubbs (Ref. 30), risks of erroneous judgments in war games or simulations may be controlled for exponentially distributed combat life-times.

28-13 SUMMARY

We have described the two Lanchester Linear Laws of combat, the Lanchester Square Law, the logarithmic or exponential decay law of Weiss and Peterson, and the Guerrilla Warfare model of Deitchman, along with some methods of estimating chances of a side winning a battle. The determination or estimation of attrition rates or kill rates was discussed and some of the problems of verification of the laws of combat brought out. Finally and for proper stochastic analyses, we recommend for future applications the matter of treating *survival times* of key targets, or battlefield elements, and show that such an analytical approach would have some very decided advantages, including the capability of being able to place confidence bounds on the proportion of survivors at an extrapolated mission time. Several instructive examples are given to indicate various types of applications. The considerations of this chapter apply primarily to homogeneous forces, whereas heterogeneous forces are treated in Chapter 29.

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CHAPTER 29

LANCHESTER COMBAT THEORY—MODEL EXTENSIONS; HETEROGENEOUS FORCES; AND COMMAND, CONTROL, AND INTELLIGENCE EFFECTS

Lanchester type models of combat theory to describe and predict outcomes of battles are extended from the homogeneous force concepts of Chapter 28 to include additional terms for resupply; production and resupply rates; some noncombat type of losses due to the environment, disease, or accidents; and the scale of operations. Attrition of forces due to the critical factor of range or distance of separation of Blue and Red forces, i.e., range-dependent attrition rates are introduced, and the concept of trading off time or range for combat losses brought out. Also, the generalization of Lanchester laws to include the effect of line-of-sight problems is covered, along with criteria to establish whether the Lanchester Linear Law or the Square Law is likely to apply. Then the highly important concept of combat between heterogeneous forces is introduced and illustrated with some useful examples of applications. Then, we indicate a model for a battle which is assumed to be made up from many individual duels between Blue and Red forces. Finally, in a new topic involving Lanchester combat theory, a derivation due to Schreiber (Ref. 12) points out the key importance of command, control, and intelligence effects on battles, especially as compared to numbers of weapons or forces on each side.

29-0 LIST OF SYMBOLS

a = aircraft availability rate

a = constant of proportionality in Eqs. 29-27 and 29-28

B = number of Blue forces or units at any time t

$B(r)$ = number of Blue forces as a function of separation distance r

dB/dr = instantaneous rate of change in the number of Blue forces as a function of separation distance r

dB/dt = rate of change in the number of Blue forces with respect to time t

B_0 = initial number of Blue forces

B_i = $B_i(t)$ = remaining number of i-type Blue units at any time t of the battle

B_{i0} = initial number of i-type Blue units or weapons

$\bar{B}$ = average number of surviving Blue forces at time t of battle

B_1 = number of Blue tanks at time t

B_2 = number of Blue infantry at time t

B_3 = number of Blue artillery weapons at time t

B_4 = number of Blue aircraft at time t

b_a = Blue aircraft attrition rate per sortie

c_B = casualty rate Blue will accept

c_R = casualty rate Red will accept

E = parameter given by Eq. 29-5

$E(R)$ = expected number of Red forces killed by Blue

e = final or top level of command and intelligence efficiency

e_0 = initial level of command and intelligence efficiency

e_B = command and intelligence efficiency of Blue forces

e_R = command and intelligence efficiency of Red forces
F = parameter given by Eq. 29-6
F = $\sqrt{(2 - e_0)/(2 - e)} - 1$ = parameter defined in Eq. 29-71
f = fraction of Blue aircraft sorties employed against Red tanks
$1 - f$ = fraction of Blue aircraft employed against Red artillery
$f(t)$ = probability density function of number of duels
G = $\sqrt{\beta}R - K/\sqrt{\beta}$ = parameter defined by Eq. 29-10
G_0 = initial value of G
g = fraction of Blue artillery used against Red tanks
$1 - g$ = fraction of Blue artillery used against Red artillery
$g(r)$ = monotonically decreasing function of the separation distance r to indicate the dependence of attrition rates on r
H = $\sqrt{\rho}B$ = parameter defined by Eq. 29-11
H_0 = initial value of H
$h(t)$ = probability density function of duration of duels
$[I]$ = identity matrix
i = $1, 2, \ldots, I$ represents the ith type of Blue units or weapons for heterogeneous forces
j = $1, 2, \ldots, J$ represents the jth type of Red units or weapons for heterogeneous forces
K = replacement rate for Blue forces
L = replacement rate for Red forces
M = $\sqrt{\beta\rho}$ = parameter defined by Eq. 29-9
n = number of duels
P = chance that Blue will kill a Red in view of target detection chances and time to fire
$P(B,R,t)$ = chance that B Blue and R Red forces survive at time t
p = chance that a Blue wins a duel against a Red
p_k = single-shot kill probability for Blue against Red
p_k' = single-shot kill probability for Red against Blue
$[Q]$ = matrix of the product of kill rates and allocation factors (Eq. 29-51)
q = $1 - p$ = chance that a Red attrits a Blue
R = number of Red forces or units at any time t
$R(r)$ = number of Red forces as a function of separation distance r
dR/dr = instantaneous rate of change in the number of Red forces as a function of separation distance r
dR/dt = rate of change in the number of Red forces with respect to time t
R_0 = initial number of Red forces
R_j = $R_j(t)$ = remaining number of j-type Red units at any time t of the battle
R_{j0} = initial number of j-type Red units or weapons
$\bar{R}$ = average number of surviving Red forces at time t of battle
R_1 = number of Red infantry at time t
R_2 = number of Red tanks at time t
R_3 = number of Red artillery weapons at time t
r = $|s_B - s_R|$ = separation distance between Blue and Red forces
r_0 = separation distance between Blue and Red forces at the start of the battle
r_0 = replacement rate for Red tanks

s = $(s_B + s_R)/2$ = distance from the reference line to a line (LOB)' which is parallel to and moves with the same speed as the LOB

s = sortie rate, per available aircraft

s_B = distance of Blue forces from a reference line well within Blue's ground area and parallel to the line of battle (LOB)

s_R = distance of Red forces from the same reference line used in the definition of s_B

t = time

$t_1 \leq t_2 \leq \cdots \leq t_n$ = ordered times

v = dr/dt = closing speed of Blue and Red forces

$[Z]$ = row vector representing all Blue and Red units (Eq. 29-48)

$[dZ/dt]$ = time derivative of the vector $[Z]$ (Eq. 29-49)

$[Z_0]$ = initial values for the vector $[Z]$ (Eq. 29-50)

α = reciprocal of mean time to detect a target for Blue (rate of detection)

α' = Red's rate of detecting Blue targets

β = combat attrition rate of Blue forces

β_{ji} = attrition rate of Blue elements, i.e., the rate at which an individual weapon or element of the jth type Red weapon attrits ith type Blue elements or targets when firing on Blue

γ = noncombat loss rate of Blue forces

γ_{ji} = allocation of Red weapons against Blue targets, i.e., the proportion (or probability) of the jth type Red weapon firing against the ith type of Blue target

δ = noncombat loss rate of Red forces

δ_{ij} = allocation of Blue weapons against Red targets, i.e., the proportion (probability) of the ith type Blue weapon firing against the jth type of Red target

$$\theta = \theta(r) = -(\beta/\rho)^{1/2}\int_{r_0}^{r} g(r)dr = \text{angular value for Eq. 29-25}$$

λ = rate of duels between Blue and Red forces

μ = rate at which duels are completed

$1/\mu$ = mean or expected time of a duel

ρ = combat attrition rate of Red forces

ρ_{ij} = attrition rate of Red elements, i.e., the rate at which an individual weapon or element of the ith Blue group attrits jth type Red elements or targets when firing on Red

σ^2 = variance of number of Red forces killed = $E[R - E(R)]^2$

τ = time to fire for Blue

τ' = Red's time to shoot

29-1 INTRODUCTION

Chapter 28 dealt primarily with Lanchester's equations for homogeneous forces, which generally were kept simple enough to introduce some of the rather basic concepts. Nevertheless, the idea of reinforcements for either side was brought out during the discussion of validating the Lanchester Square Law model for the Iwo Jima campaign. One can only begin to model more complex battle situations with such simple concepts because battle results are not only a function of the numbers of forces and weapons on each side, but also the various types of weapon systems (taking into account their diversity), the capabilities of the various types of weapon systems in combined arms roles, range, doctrine of employment (tactics, organization, etc.), intelligence of enemy activities, the environment of employment, logistic considerations, industrial capability, and other factors. Thus, there is a need to include more parameters in any realistic models which attempt to provide sound inferences for future conflicts. Therefore, it is the purpose of this chapter to cover some of the more inclusive models of combat and to see just how they may be used for the purpose of evaluating weapons.

In particular, we will discuss the matter of additional terms in Lanchester type models of combat, along with some of the recent developments of theory and application relating to force separation, the value of intelligence, and some account of heterogeneous forces. Moreover, one can see that if we are able to deal with the case of heterogeneous forces in terms of combined arms effects or the "equivalent" homogeneous models, then some very useful simplifications will have been accomplished. Finally, we need to indicate the relation between deterministic models and that of the probabilistic models.

We first discuss some additional terms (par. 29-2) for Lanchester's homogeneous equations for the case of the Square Law. (We have already introduced a term for troop replacement in Eq. 28-51 of Chapter 28.)

29-2 ADDITIONAL TERMS IN LANCHESTER'S EQUATIONS

Lanchester's differential equations for either the Linear or Square Laws may be extended to involve additional terms such as replacement rates and noncombat losses due to accidents, diseases, epidemics, etc. For example, for the Square Law we might add additional terms, bringing about the following:

$$\frac{dB}{dt} = -\beta R - \gamma B + K \qquad (29\text{-}1)$$

$$\frac{dR}{dt} = -\rho B - \delta R + L \qquad (29\text{-}2)$$

where

K = replacement rate for Blue forces
L = replacement rate for Red forces
γ = noncombat loss rate of Blue forces
δ = noncombat loss rate of Red forces
β = combat attrition rate of Blue forces
ρ = combat attrition rate of Red forces
B = number of Blue forces or units at any time t
R = number of Red forces or units at any time t
t = time.

In these extended Square Law equations, the constants K and L can be considered to be replacement rates for the Blue and Red forces, respectively, whereas the terms $-\gamma B$ and $-\delta R$ for Blue and Red represent noncombat or nonoperational type losses, or losses dependent on the scale of each side's activity, or in accidents, etc., and are not related directly to the size of the opponent. Combat losses are such that β and ρ dominate γ and δ, however, as would be expected.

When the opposed forces are equally effective, i.e., for $\beta = \rho$ and $\gamma = \delta$, but the replacement rates are different, for example, then the solutions of Eqs. 29-1 and 29-2 are:

$$B = \frac{L\beta - K\gamma}{\beta^2 - \gamma^2} + E\exp[(\beta - \gamma)t] + F\exp[-(\beta + \gamma)t] \tag{29-3}$$

and

$$R = \frac{K\beta - L\gamma}{\beta^2 - \gamma^2} - E\exp[(\beta - \gamma)t] + F\exp[-(\beta + \gamma)t] \tag{29-4}$$

where

$$E = \frac{1}{2}\left[\left(B_0 + \frac{K}{\beta - \gamma}\right) - \left(R_0 + \frac{L}{\beta - \gamma}\right)\right] \tag{29-5}$$

$$F = \frac{1}{2}\left[\left(B_0 + \frac{K}{\beta + \gamma}\right) + \left(R_0 - \frac{L}{\beta + \gamma}\right)\right] \tag{29-6}$$

B_0 = initial number of Blue forces

R_0 = initial number of Red forces.

We note that the size of the constant E, which is fixed by the initial conditions and the production and resupply rates, determines which of the forces goes to zero. The total strength on a side is the initial fighting force plus the replacement rate (or it could be the production rate) divided by $\beta \div \gamma$.

The equations take a very special and interesting form when $\gamma = \delta = L = 0$, for then only Blue replaces troops in battle, or adds to them. In this particular case, we have

$$\frac{dB}{dt} = -\beta R + K = -\sqrt{\beta}(\sqrt{\beta}R - K/\sqrt{\beta}) = -MG/\sqrt{\rho} \tag{29-7}$$

$$\frac{dR}{dt} = -\rho B = -\sqrt{\rho}(\sqrt{\rho}B) = -MH/\sqrt{\beta} \tag{29-8}$$

where

$$M = \sqrt{\beta\rho} \tag{29-9}$$

$$G = \sqrt{\beta}R - K/\sqrt{\beta} \tag{29-10}$$

$$H = \sqrt{\rho}B \tag{29-11}$$

and we immediately see that

$$dH/dt = -MG \tag{29-12}$$

$$dG/dt = -MH. \tag{29-13}$$

But Eqs. 29-12 and 29-13 are in precisely the same form as Eqs. 28-28 and 28-29 for Lanchester's Square Law as a function of time, and we may write down immediately (see Ref. 1 for example) that

$$H = \sqrt{\rho}B = \sqrt{\rho}B_0\cosh(\sqrt{\beta\rho}t) - (\sqrt{\beta}R_0 - K/\sqrt{\beta})\sinh(\sqrt{\beta\rho}t) \tag{29-14}$$

and

$$\begin{aligned} G &= \sqrt{\beta}R - K/\sqrt{\beta} \\ &= (\sqrt{\beta}R_0 - K/\sqrt{\beta})\cosh(\sqrt{\beta\rho}t) - \sqrt{\rho}B_0\sinh(\sqrt{\beta\rho}t). \end{aligned} \tag{29-15}$$

We recall from the Square Law that for Blue to win we must have $\rho B_0^2 > \beta R_0^2$, and hence such condition here means

$$MH_0^2 > MG_0^2 \tag{29-16}$$

$$H_0 > G_0 \equiv \sqrt{\rho}B_0 > \sqrt{\beta}R_0 - K/\sqrt{\beta} \tag{29-17}$$

or Blue wins if

$$\sqrt{\rho}B_0 + K/\sqrt{\beta} > \sqrt{\beta}R_0. \tag{29-18}$$

If $H_0 = G_0$ at time $t = 0$, then H and G approach zero asymptotically; thus, B approaches zero while R approaches a limiting value $K/\sqrt{\beta}$ which really just permits Red to destroy Blue's replacements at a rate equal to their arrival rate!

Engel (Ref. 2) has applied these very equations to the battle of Iwo Jima in an attempt to validate the Square Law as we indicated in par. 28-5.

Thus, the reader can see that additional terms can be included in the basic Lanchester type combat equations to represent a variety of considerations, although it can be seen also that the solutions could become somewhat complex. An important problem is to develop the best model for a given application.

EXAMPLE 29-1:

Given that Blue and Red have equally effective forces in a battle, but that Blue's replacement rate of forces is at the rate of 15 per unit of time while that for Red is 5 per time unit. Suppose that Blue has only 100 men while Red has 200 men, and the kill rates of Blue and Red are equal at the value $\beta = \rho = 1.5$ per time unit, while noncombat losses for both sides are at the rate $\gamma = \delta = 0.3$. (1) Does Red have enough men initially to overcome Blue's resupply rate, and who wins? (2) Assume that there are only losses on each side due to combat and that Blue has an artillery advantage which prevents Red from any resupply; then determine how the battle will go.

From Eq. 29-5, we note that E may be positive, in which case Blue will win; E may be zero and bring about a stalemate; or E could be negative, in which case Red wins (as may be noted from Eq. 29-4). Hence, the computation of E is of central interest. For (1), from Eq. 29-5

$$E = -45.83$$

and hence Red's 200 combatants initially is much more than that required to win.

For (2), $\gamma = \delta = 0$, and $L = 0$, and hence now

$$E = -45$$

so that Red still wins easily.

29-3 RANGE-DEPENDENT ATTRITION COEFFICIENTS

Appropriate models for describing ground combat should account for the fact that the attrition coefficients or kill rates will depend on ranges of engagement. In fact, we have already brought out this point several times, for example, by indicating that the probability of hitting drops off rather rapidly with increased range to target. A first step concerning this matter was taken by Weiss (Ref. 3), who extended Lanchester-type warfare equations of combat between two homogeneous forces to include relative movement of forces and hence allowed for a trade-off between time and space in generating casualties. Weiss' formulation considered that the attrition coefficients depended upon force separation in such a manner that the ratio of Blue to Red kill rates was a constant. This is probably a fairly reasonable assumption; otherwise, one would have to introduce much more complexity into the modeling process. Later, Bonder (Ref. 4) used Weiss' extension technique to study the effects of mobility and range dependent attrition rates on the number of surviving forces. Bonder (Ref. 4) also developed a second-order differential equation for the purpose of relating average force strength to the force separation distance, and he obtained a solution for the number of Blues and Reds at any time after combat had begun for the case of constant relative closing speed of the two sides. Taylor (Ref. 5) has given a rather compact treatment of this very problem. Hence, it is of interest to record some of the accomplishments of these investigators here.

Following the notation of Chapter 28, let:

B = number of Blue forces at any time t

R = number of Red forces at any time t

B_0 = initial number of Blue forces

R_0 = initial number of Red forces

β = rate at which Blue forces are killed by Red forces

ρ = rate at which Red forces are killed by Blue forces

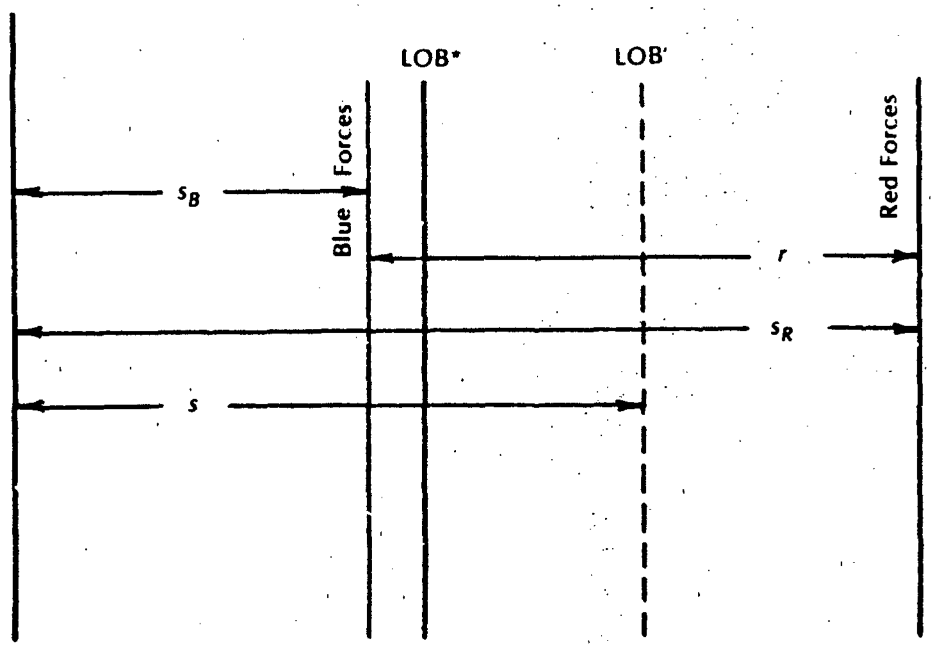
s_B = distance of Blue forces from a reference line well within Blue's ground area and parallel to the line of battle (LOB)

s_R = distance of Red forces from the same reference line used in the definition of s_B

$s = (s_B + s_R)/2$ = distance from the reference line to a line (LOB)' which is parallel to and moves with the same speed as the LOB

$r = |s_B - s_R|$ = separation distance between Blue and Red forces.

A representation of s_B, s_R, s, and r is shown on Fig. 29-1.



* The line of battle is between the Red and Blue forces; but the location of it depends on the rate of advance of these forces.

Figure 29-1. Location of Red and Blue Forces

Then Weiss (Ref. 3) sets up the Lanchester combat equations taking account of the force separation distance r as

$$\frac{dB}{dt} = -\beta R g(r) \tag{29-19}$$

and

$$\frac{dR}{dt} = -\rho B g(r) \tag{29-20}$$

where

$g(r)$ = monotonically decreasing function of the distance or separation r.

One may easily note that upon comparing Eqs. 29-19 and 29-20 with Lanchester's basic Square Law, Eqs. 28-28 and 28-29, the attrition rates now become $\beta g(r)$ and $\rho g(r)$, and hence the rates may depend markedly on the separation distance r. Nevertheless, the effective ratio of the time derivatives, Eq. 29-19 to Eq. 29-20, leads to

$$\rho B dB = \beta R dR. \tag{29-21}$$

Eq. 29-21 is precisely equal to Eq. 28-30, so that as Weiss (Ref. 3) showed, the usual Square Law given by Eq. 28-33, or

$$\rho(B_0^2 - B^2) = \beta(R_0^2 - R^2) \tag{29-22}$$

still holds for force levels in spite of the dependence on the separation distance r between Blue and Red forces. Obviously, this seems to be a rather unique outcome, which reproduces and generalizes the Lanchester Square Law.

Since Eq. 29-22, the ordinary Lanchester Square Law, for this much more complex case does not involve the separation distance r, the rate of change of Blue forces (and Red) with respect to time may be transformed to the equivalent rate of change of Blue forces (and Red) with respect to distance r. For Blue, for example, we have

$$\frac{dB}{dt} = v\frac{dB}{dr}, \quad v = \frac{dr}{dt} \tag{29-23}$$

so that Taylor (Ref. 5) and Bonder (Ref. 4) give force levels as a function of the closing range r for Blue as

$$B = B(r) = B_0 \cosh\theta + R_0(\beta/\rho)^{1/2}\sinh\theta \tag{29-24}$$

where

$$\theta = \theta(r) = -(\beta/\rho)^{1/2}\int_{r_0}^{r} g(r)dr \tag{29-25}$$

with r_0 = the separation distance between Blue and Red at the start of the battle.

A similar expression for Red as a function of separation distance is

$$R = R(r) = R_0 \cosh\theta + B_0(\rho/\beta)^{1/2}\sinh\theta. \tag{29-26}$$

Thus, the numbers of remaining Blue and Red forces for any closing range r can be determined also, or we may say that the effect of mobility is to trade casualties for control of ground.

Weiss' clever analysis (Ref. 3) also gives some interesting equations for the closing speeds of the Blue and Red forces. In terms of the closing speed of opposing Blue and Red forces, Weiss (Ref. 3) shows that for the distances s_B and s_R of Blue and Red from the reference line, one finds the speed relations

$$\frac{ds_B}{dt} = a\left(\frac{dB}{c_B dt} - 1\right) \tag{29-27}$$

and

$$\frac{ds_R}{dt} = a\left(1 - \frac{dR}{c_R dt}\right) \tag{29-28}$$

where a is a constant of proportionality, and c_B and c_R are the casualty rates Blue and Red, respectively, are willing to accept before withdrawing or retreating from battle.

Weiss (Ref. 3) indicates that a method of obtaining the solution to Eqs. 29-27 and 29-28 is to consider first the initial part of the action and assume that Blue and Red reach an equilibrium at some closing range r before large losses have been received by either. (This is reasonable since only rarely will a whole force be annihilated without withdrawal or retreat.) The complete action is then solved under the assumption that the time lag in change of this equilibrium is small compared with the time for the whole engagement. In fact, at this stage we may say that the closing speed dr/dt could be set equal to zero, and the function $g(r)$ in Eqs. 29-19 and 29-20 solved for, so that the attrition equations become

$$\frac{dB}{dt} = 2\beta R/(\beta R/c_B + \rho B/c_R) \qquad (29\text{-}29)$$

and

$$\frac{dR}{dt} = 2\rho B/(\beta R/c_B + \rho B/c_R). \qquad (29\text{-}30)$$

Moreover, the corresponding speed of the LOB is given by

$$\frac{ds}{dt} = a(c_R\beta R - c_B\rho B)/(c_R\beta R + c_B\rho B) \qquad (29\text{-}31)$$

from which we see that due to the chosen sizes of the casualty rates c_R and c_B, the speed ds/dt may be positive, zero, or negative; or that an inferior force may "hold the line" against a superior force by accepting a higher casualty rate, so to speak. Clearly, this would seem to help with the validation of this particular model, and as a matter of fact we have already remarked in par. 28-11 concerning breakpoints of battles that once a force is on the defensive it will then often suffer nearly double the offensive casualty rate.

29-4 A GENERALIZATION OF LANCHESTER'S LAWS FOR LINE OF SIGHT CONSIDERATIONS

While we are discussing additional terms for Lanchester's basic linear and square laws, it is of some interest to consider a somewhat different kind of generalization due especially to line-of-sight problems or chances of seeing targets. The particular formulation that follows is due to Owen (Ref. 6), and approaches establishment of the Square Law of Eqs. 28-28 and 28-29, and the area fire model of Eqs. 28-22 and 28-23 in a different manner. Owen (Ref. 6) indicates that the Square Law should be valid for close combat, whereas the Linear Law for area fire should be valid for combat "at a distance" and proceeds to establish this with a rather clever analytical development. He assumes that an individual, for example on the Blue side, takes a certain time τ to fire, and he will fire only when he sees a Red target or has detected one. Hence, the chance that a Blue does not see or detect a particular Red target in a given time τ is $\exp(-\alpha\tau)$, where α is a "visibility" parameter, and in fact is the detection rate (or its reciprocal $1/\alpha$ is the mean time to detect). Clearly, α may be relatively large for open terrain and small in the dark, or for trees, ambushes, etc. Now if Red has R men with weapons, or there are R targets, the chance that a Blue sees none of them will be given by $\exp(-\alpha R\tau)$, so that the chance that a single Blue sees at least one target to shoot at is $[1-\exp(-\alpha R\tau)]$. Hence, if the kill probability per shot is p_k (which would consist of the chance of hitting multiplied by the conditional chance that a hit is a kill), then in the time interval τ, a Blue member has the chance P or

$$P = p_k[1 - \exp(-\alpha R\tau)] \tag{29-32}$$

of killing an opponent. Moreover, since each Blue has a probability of killing given by Eq. 29-32, then for B Blues the expected number $E(R)$ of Reds killed by Blue will be

$$E(R) = Bp_k[1 - \exp(-\alpha R\tau)] \tag{29-33}$$

where we have ignored the "small" chance that two Blues kill the same Red. In addition, it is easily seen that due to binomial probability theory, then the variance of the number of Reds killed is simply

$$\begin{aligned}\sigma^2 &= E[R - E(R)]^2 \\ &= Bp_k[1 - \exp(-\alpha R\tau)]\{1 - p_k[1 - \exp(-\alpha R\tau)]\}.\end{aligned} \tag{29-34}$$

In conclusion, we see that the number of Red elements killed is a random variable with mean and variance given by Eqs. 29-33 and 29-34, respectively, and perhaps for many applications such distributions may be approximately described by the normal fit.

Owen (Ref. 6) at this stage replaces the random number of Red kills by its mean value, Eq. 29-33, thereby disregarding the stochastic element or variance, and obtains the deterministic form

$$\frac{dR}{dt} \approx -E(R)/\tau = -(Bp_k/\tau)[1 - \exp(-\alpha R\tau)] \tag{29-35}$$

and similarly

$$\frac{dB}{dt} \approx -(Rp'_k/\tau')[1 - \exp(-\alpha' R\tau')] \tag{29-36}$$

where α', p'_k, and τ' now have similar definitions for Blue side kills, or they are respectively the visibility parameter, the kill probability, and firing times for Red.

Now suppose that the visibility is poor, i.e., $\alpha R\tau$ is small, or we have distance firing, or concealment, in which cases

$$\exp(-\alpha R\tau) \approx 1 - \alpha R\tau \tag{29-37}$$

or from Eq. 29-35 we then establish that

$$\frac{dR}{dt} \approx -\alpha p_k BR. \tag{29-38}$$

But this is precisely the form of the linear law for area fire, or Eqs. 28-22 and 28-23. Similarly, we get that

$$\frac{dB}{dt} \approx -\alpha' p_k' BR \tag{29-39}$$

for the poor visibility case of Red seeing Blue.

On the other hand, if the visibility is good, or $\alpha R\tau$ is large (or Red has a "large" Army, or a very low rate of fire), then clearly the exponential approaches zero, i.e.,

$$\exp(-\alpha R\tau) \approx 0 \tag{29-40}$$

and from Eq. 29-35 one sees that in this case

$$\frac{dR}{dt} \approx -(p_k/\tau)B \tag{29-41}$$

which for any "fixed" τ is of the form of the Lanchester Square Law, Eqs. 28-28 and 28-29. Thus, with this more general approach to combat, we are still able to validate, on some practical grounds, the Lanchester Linear and Square Laws.

In this more general formulation of Owen, which involves the chance of finding the target, there is more to be said for the now three combat parameters—α the visibility parameter, p_k the single-shot kill chance, and τ the time available for target detection. In particular, we note that the single-shot kill probability appears in both models, i.e., in Eqs. 29-38 and 29-41, or these limiting cases. Thus, for either law, advantages are gained by always trying to increase kill probability per shot, i.e., whether for the linear law for area fire, or for close combat and the square type law. (The reader is no doubt aware that for small arms type weapons it may be difficult to increase single-shot kill chances for area or ambush fire, whereas increasing the rate of fire will be an advantage for the weapon, such as use of a machine gun. On the other hand, the use of artillery and large lethal areas per projectile or warhead payload becomes very much in order in this case.) Moreover, it is clearly to Red's advantage here to decrease Blue's single-shot kill chances, and he may do this by hiding or hardening his units, or by cover protection.

For the sighting or visibility parameter α, we note that the attrition rate in Eq. 29-38 is directly dependent on it, i.e., for area fire; whereas for high visibility and the square law of close combat it is completely missing in Eq. 29-41. We conclude then that should the enemy be "hard to find", it becomes of critical importance to increase visibility or improve on target detection. On the other hand, if the product $\alpha R\tau$ is or can be made sufficiently large, then increasing the size of the visibility parameter may be of relatively little importance indeed.

Finally, we might take a look at the firing time τ available for Blue. We note here that the kill rate in Eq. 29-41, or for the limiting square law, is inversely proportional to τ or hence the rate of fire. Thus, the machine gun may be extremely valuable in such conditions of combat. On the other hand, for the model of Eq. 29-38 or area fire, very little, if anything at all, is gained through rate of fire; and even the machine gun, for example, may be of little value against ambush, as we are well aware.

These arguments make considerable sense in the analyses of combat, but of more importance is the fact an appropriate theoretical development has been carried out which will aid in more precise quantification of combat than otherwise would have been possible. In addition, we gain considerably more appreciation for the basic work Lanchester originally performed for us.

EXAMPLE 29-2:

A Blue force of company size (≈ 200 men) believes that a much smaller Red force, estimated to be 50, is in the general area just ahead. If this be true, Blue hypothesizes that Red's single-shot kill probability per weapon may be as large as 0.075, and the chance of a Red seeing a Blue is about 0.6. Blue, on the other hand from immediate past experience has been detecting about one Red target every 30 min and Blue's p_k is about 0.05. Considering that Blue may be caught by surprise and must fire immediately, within 6 s while Red may use as much as 5 min to detect and fire on Blue, what can be said about any appropriate choice of a combat law which might be applicable to such a situation?

It is easy for the Blue commander to analyze these available data in the following terms or parameters:

$$B = 200 \qquad R = 50$$
$$p_k = 0.05 \qquad p_k' = 0.075$$
$$\alpha = 1/30 = 0.033 \qquad \alpha' = 0.6/5 = 0.12$$
$$\tau = 6/60 = 0.1 \qquad \tau' = 5.$$

Hence, we find that

$$\alpha R\tau = (0.033)(50)(0.1) = 0.165, \text{ while } \alpha' B\tau' = (0.12)(200)(5) = 120.$$

Thus, there is some evidence that since $\alpha R\tau$ is relatively small and $\alpha' B\tau'$ is large, then Deitchman's mixed or guerrilla warfare model of par. 28-9 would be appropriate for the commander to predict casualties or infer outcomes of such an engagement.

We note that the determination of the model to fit depends on the size of $\alpha R\tau$ and $\alpha' B\tau'$, and not on the single-shot kill chances p_k or p_k'.

29-5 HETEROGENEOUS FORCES OR COMBINED ARMS

29-5.1 PRELIMINARIES

By heterogeneous forces, we mean the employment and "mixture" of weapons of different types on a side for various firing missions in combat. Through long and past bitter experience, we have learned that combat against any current or potential enemy must involve infantrymen with their rifles and machine guns; artillery to attack targets at longer ranges, or for counterbattery, or to deny the enemy the use of key areas of the terrain; and tanks to aid in breakthroughs or fight enemy tanks, or carry out mopping-up actions, etc. Thus, modern war depends on the wisest use of combined arms or heterogeneous type forces to get and keep an advantage over the enemy in combat actions.

Lanchester's original investigations into combat theory involved primarily the analysis of "homogeneous" forces on each side, and he touched very lightly on the problem of evaluating combat between heterogeneous forces. Nevertheless, we must discuss some of the problems involving the analysis of heterogeneous forces, for this is actually the case in practice, even though this is obviously a rather involved and difficult area of analysis.

Obviously, for heterogeneous forces, there is a problem in allocating weapons to targets on both sides. This was not too involved a problem for the homogeneous case, in which several riflemen, tank crews, etc., could fire at single similar targets on the opposite side. Needless to say, the allocation problem becomes very important indeed for combat between heterogeneous forces since, for example, artillery can successfully engage enemy infantry, riflemen may waste bullets against tanks, some man-portable weapons can destroy tanks, aircraft may attack some ground targets without coming under fire, etc. Hence, it becomes clear that, for combat between forces, appropriate parameters must be considered to take account of weapon-target allocation problems. Moreover, there is the problem of determining the "best" weapon-target allocation modes in order to conserve ammunition, or to maximize effectiveness of weapons for a given logistical supply or other criteria of importance.

It is not difficult to establish appropriate notation and the general model for the case of combat between heterogeneous forces on each side. We consider $i = 1,2,\ldots,I$ different types of Blue weapons, men, key elements, etc., and $j = 1,2,\ldots,J$ distinct types of Red weapons, men, key elements, etc. Then define:

$B_i = B_i(t)$ = remaining number of i-type Blue elements at any time t of the battle
B_{i0} = initial number of i-type Blue elements
$R_j = R_j(t)$ = remaining number of j-type Red elements at any time t of the battle
R_{j0} = initial number of j-type Red weapons or forces
β_{ji} = the attrition rate of Blue elements, i.e., the rate at which an individual weapon or element of the jth type Red weapon attrits ith type Blue elements or targets when firing on Blue
γ_{ji} = allocation of Red weapons against Blue targets, i.e., the proportion (or probability) of the jth type Red weapon firing against the ith type of Blue target
ρ_{ij} = the attrition rate of Red elements, i.e., the rate at which an individual weapon or element of the ith Blue group attrits jth type Red elements or targets when firing on Red
δ_{ij} = the allocation of Blue weapons against Red targets, i.e., the proportion (probability) of the ith type Blue weapon firing against the jth type of Red target.

With these definitions, it is easy to see that the rate of change (i.e., decrease) in i-type Blue targets and j-type Red targets can be expressed as:

$$\frac{dB_i}{dt} = -\sum_{j=1}^{J} \beta_{ji}\gamma_{ji}R_j \ , \quad i = 1,2,\ldots,I \tag{29-42}$$

and

$$\frac{dR_j}{dt} = -\sum_{i=1}^{I} \rho_{ij}\delta_{ij}B_i \ , \quad j = 1,2,\ldots,J. \tag{29-43}$$

Thus, Eqs. 29-42 and 29-43 represent the generalization of Lanchester type differential equations of combat to describe the course of battle for heterogeneous forces or combined arms on each side. We note in particular, as previously stated, that there is the problem of estimating the I and J distinct attrition coefficients and also the allocation proportions, or "probabilities of assignment" of every weapon on each side against targets on the other.

As a particular example, and to illustrate further, suppose that we consider that Blue and Red have only infantry and artillery on their sides. Then, Eqs. 29-42 and 29-43 simplify to the following considerations:

$i = 1$ represents Blue infantry
$i = 2$ represents Blue artillery weapons

B_1 = number of Blue infantrymen at time t of the battle
B_2 = number of Blue artillery weapons at time t of the battle
j = 1 represents Red infantry
j = 2 represents Red artillery
R_1 = number of Red infantrymen at time t of the battle
R_2 = number of Red artillery weapons at time t of the battle

and hence that the basic equations become

$$\frac{dB_1}{dt} = -\beta_{11}\gamma_{11}R_1 - \beta_{21}\gamma_{21}R_2 \tag{29-44}$$

$$\frac{dB_2}{dt} = -\beta_{12}\gamma_{12}R_1 - \beta_{22}\gamma_{22}R_2 \tag{29-45}$$

$$\frac{dR_1}{dt} = -\rho_{11}\delta_{11}B_1 - \rho_{21}\delta_{21}B_2 \tag{29-46}$$

and

$$\frac{dR_2}{dt} = -\rho_{12}\delta_{12}B_1 - \rho_{22}\delta_{22}B_2. \tag{29-47}$$

Continuing, if we put, for example, $\beta_{21} = \rho_{21}$, then Red artillery kills Blue infantry at the same rate that Blue artillery attrits Red infantry. If we were to put $\gamma_{12} = \delta_{12} = 0$, then no Red infantry is assigned to kill Blue artillery, and likewise no Blue infantry is allocated to fire against Red artillery. Indeed, this might make sense, for on the other hand the attrition rate β_{12} for Red infantry kills of Blue artillery could be practically zero, or the kill rate ρ_{12} of Blue infantry against Red artillery may likely be quite small. These points or examples should illustrate the relation between attrition rates and allocation factors. Attrition rates may be estimated from realistic hit probabilities, rates of fire, and conditional chances that hits are kills, as before; or they could be estimated from the reciprocals of kill times in a simulation, computer played battle, etc. The allocation factors, on the other hand, may be varied to determine the best or optimum weapon target engagement procedures; or perhaps, in some cases, they may even be known roughly from combat experience.

Concerning weapon-target allocation studies, Bonder and Honig (Ref. 7) indicate that based on some research of Dr. Stanley Sternberg there are some findings of much interest to determine the characteristics of good or optimal allocation strategies. It was assumed in this connection that the battle dynamics of heterogeneous force battles could be described by coupled sets of constant attrition coefficient differential equations, and also that:

1. Zero time is required to switch from one target to another.
2. Projectile flight times are small.
3. Blue and Red forces have perfect control and intelligence.

Based on these assumptions, the research results (Ref. 7) indicate that, "For linear payoff functions, it is ineffective for individual weapon groups to distribute their fire over different target groups. That is, all (Blue) i-group weapons should engage all (Red) j-group targets with no splitting of fire allocation within a group (or type of weapons). The optimal assignment strategies are such that all weapons of a single group (type) should be assigned to a single group in the opponent's arsenal." Moreover, "It has also been shown that the choice of (an enemy) group (target) to be fired upon is independent of the number of weapons in the group (performing) the firing." The targets to be fired upon are selected by determining the maximum attrition rates on the marginal utilities of the opposing sides and not directly on number of weapons available. "A good strategy seems to be to assign Blue group (weapon type) i to Red type j targets for which the product $\beta_{ji}\rho_i$, of attrition coefficients is a maximum, or vice versa." (These findings on the allocation problem seem to agree with those of Weiss in Ref. 3, who studied the case of heterogeneous forces where Blue and Red had only one "primary group" or "men" and one supporting weapon system or "air" on each side. Weiss points out that if Blue "Does attack ground unilaterally with his air, it is best for him to use all of his air rather than a fraction, and to commit it to ground attack as long as it exists rather than switching back to attacking enemy air.")

As some points of further consideration, we should emphasize that there is also a problem in the actual designation of Blue i-type weapons or systems which will be used against the j-type Red targets, and the j-type Red weapons which will be firing on i-type Blue targets, or vice versa. This will naturally depend to some extent on the analyst and the particular problem he faces in a given application. Moreover, the analyst must often arrive at some rather clever selections of what actually constitutes the best i- and j-type elements, weapons, or targets. For example, it could be argued that men are most important of all, and that B_1 and R_1, for example, should represent the number of Blue men and number of Red men, respectively, for if men are put out of action, then they cannot man weapons to fire at the enemy. Likewise, B_2 and R_2 might represent the number of Blue and Red artillery pieces, respectively, and B_3 and R_3 could represent the number of rifles on the two sides, etc. Alternatively, it might be appropriate to deal with "entire" weapon systems as key elements in an analysis. Thus, a tank and crew with its armament could be considered as one of the B_i's or R_j's, etc. Finally, for larger scale operations, then more aggregation would be in order, depending on the systems analysis application.

With this background, we now turn to the problem of solving the Lanchester type extended square law for heterogeneous forces.

29-5.2 GENERAL SOLUTION OF HETEROGENEOUS FORCE EQUATIONS

In spite of the increased complexity of Lanchester type equations, Eqs. 29-42 and 29-43, of combat for heterogeneous forces, fortunately, there exists a unique method of solution for the remaining numbers of Blue and Red type elements at any time t after the battle has started. We note that for the extension of the Lanchester Square Law type of analysis for heterogeneous forces as in Eqs. 29-42 and 29-43, we are dealing with a set of I plus J simultaneous differential equations. Moreover, as might be evident, a matrix theory approach will lead to a solution, and we proceed as follows to facilitate the study and solution of Eqs. 29-42 and 29-43. Consider the row vector $[Z]$ for remaining Blue and Red forces, $[dZ/dt]$ for time derivatives on both sides, and $[Z_0]$ for initial conditions, given by

$$[Z] = [B_1, B_2, \ldots, B_I, R_1, R_2, \ldots, R_J] \qquad (29\text{-}48)$$

$$[dZ/dt] = -\left[\frac{dB_1}{dt}, \frac{dB_2}{dt}, \ldots, \frac{dB_I}{dt}, \frac{dR_1}{dt}, \frac{dR_2}{dt}, \ldots, \frac{dR_J}{dt}\right] \qquad (29\text{-}49)$$

$$[Z_0] = [B_{10}, B_{20}, \ldots, B_{I0}, R_{10}, R_{20}, \ldots, R_{J0}] \qquad (29\text{-}50)$$

and the matrix of products of attrition and allocation coefficients given by

$$[Q] = \begin{bmatrix} & & & \rho_{11}\delta_{11} & \rho_{12}\delta_{12} & \cdots & \rho_{1J}\delta_{1J} \\ & 0 & & \rho_{21}\delta_{21} & \rho_{22}\delta_{22} & \cdots & \rho_{2J}\delta_{2J} \\ & & & \cdot & & & \\ & & & \cdot & & & \\ & & & \rho_{I1}\delta_{I1} & \rho_{I2}\delta_{I2} & \cdots & \rho_{IJ}\delta_{IJ} \\ \beta_{11}\gamma_{11} & \beta_{12}\gamma_{12} & \cdots & \beta_{1I}\gamma_{1I} & & & \\ \beta_{21}\gamma_{21} & \beta_{22}\gamma_{22} & \cdots & \beta_{2I}\gamma_{2I} & & 0 & \\ \cdot & \cdot & & & & & \\ \cdot & \cdot & & & & & \\ \beta_{J1}\gamma_{J1} & \beta_{J2}\gamma_{J2} & \cdots & \beta_{JI}\gamma_{JI} & & & \end{bmatrix}. \qquad (29\text{-}51)$$

Then, it becomes clear that we may convert Eqs. 29-42 and 29-43 to a new single system of matrix differential equations given by the schematic notation

$$[dZ/dt] = -[Z][Q] \qquad (29\text{-}52)$$

which has the matrix exponential solution

$$[Z] = [Z_0]\exp\{-[Q]t\} \qquad (29\text{-}53)$$

and where the matrix exponential is to be expanded as

$$\exp\{-[Q]t\} = [I] - [Q]t + [Q]^2t^2/2! + \cdots \qquad (29\text{-}54)$$

and $[I]$ is the identity matrix. Thus, through matrix notation and a generalization which combines both Blue and Red forces, we have succeeded in finding a rather simple mathematical form of the matrix differential equations, which handles the problem of Lanchester heterogeneous or combined arms type of weapon forces. Although the solutions of Eq. 29-53 may appear complex, it is nevertheless clear that solutions can be found, perhaps especially with modern computers. The problem is that of evaluating the matrix exponential, $\exp\{-[Q]t\}$, and Sternberg (pp. 389-436, Ref. 8) establishes an analytical method for writing equations of the type of Eq. 29-53 in closed form which hopefully will lend itself to "rapid" computation.

One may note that when $I = J = 1$, then Eqs 29-42, 29-43, and 29-52 simplify to the ordinary homogeneous type of Lanchester models for the Square Law in Chapter 28. To illustrate, we have from Eqs. 29-48, 29-49, and 29-50:

$$[Z] = [B, R]$$

$$[dZ/dt] = [dB/dt, dR/dt]$$

$$[Z_0] = [B_0, R_0]$$

and Eq. 29-51 gives

$$[Q] = \begin{bmatrix} 0 & \rho \\ \beta & 0 \end{bmatrix}$$

Moreover,

$$[Q]^2 = \begin{bmatrix} \beta\rho & 0 \\ 0 & \beta\rho \end{bmatrix}, \quad [Q]^3 = \begin{bmatrix} 0 & \beta\rho^2 \\ \beta^2\rho & 0 \end{bmatrix}, \quad [Q]^4 = \begin{bmatrix} \beta^2\rho^2 & 0 \\ 0 & \beta^2\rho^2 \end{bmatrix}, \text{ etc.,}$$

and finally

$$[Z] = [B, R] = [B_0, R_0] - [\beta R_0, \rho B_0]t + [\beta\rho B_0, \beta\rho R_0]t^2/2! - \cdots .$$

Thus, the remaining number of Blues as a function of time t is given by the fastly converging series

$$B = B_0 - \beta R_0 t + \beta\rho B_0 t^2/2! - \beta^2\rho R_0 t^3/3! + \beta^2\rho^2 B_0 t^4/4! - \cdots^*.$$

For the data of Table 28-1 of

$$B_0 = 100, \quad R_0 = 50, \quad \beta = 0.10, \quad \rho = 0.05$$

and time $t = 2$,

$$B = 100 - (0.10)(50)(2) + (0.10)(0.05)(100)(2) - (0.10)^2(0.05)(50)(4/3)$$

$$= 90.97$$

to four terms versus the discrete value of 90.5 in Table 28-1.

We will not go any further into the solution of the generalized Lanchester type models for heterogeneous forces or weapons, i.e., Eq. 29-53 here, but in Chapter 30, "Weapon Equivalence Studies", we will have much interest in converting values of heterogeneous weapons in a conflict to equivalent homogeneous weapon values, which represents a somewhat different approach but a very useful one indeed. Occasionally, the practicing analyst may have to solve equations such as Eq. 29-53 for the remaining numbers of Blue and Red weapon systems for the case of heterogeneous forces.

Finally, we remark that the allocation factors γ_{ji} and δ_{ij} in the previous equations for heterogeneous forces may be determined by applying the procedures of Chapter 32 on weapon-target allocation problems, this indicating the extent of our analytical treatment of generealized Lanchester type combat models in this handbook.

*The reader may have some interest in comparing this expression for the remaining number of Blues as a function of time versus that of Eq. 28-48.

29-5.3 ILLUSTRATIVE HETEROGENEOUS FORCE EXAMPLES

In recent years, there has been increased emphasis on the extended Lanchester type equations and applications of Eqs. 29-42 and 29-43. Weiss (Ref. 3) carried out some of the earlier investigations and, in particular, gives a rather extensive account of the case of men with support by aircraft. He assumes that men can attack only men, whereas aircraft can attack both men and enemy aircraft. He sets up a "value" function of the difference in numbers of Blue and Red men, and develops analytical criteria for the analysis of this type of simple battle.

Bonder and Honig (Ref. 7) further develop analytical models of ground combat theory for heterogeneous forces and consider in their model the use of attrition or kill rates, allocation factors, and also intelligence factors. In addition, they consider firing doctrine, terrain interactions, the comparison of analytical and Monte Carlo simulation results, and they also give some account of the Army's use of their analytical models or development. In connection with the study of battalion task force activities, Bonder, Farrell, *et al.* (Ref. 8) contribute many significant findings to the broad subject of analyzing combat between heterogeneous forces. Although we cannot delve extensively into these subjects here, systems analysts having some interest in similar applications will want to study Refs. 7 and 8, and also the various contributions of Thrall *et al.* (Ref. 6).

For our illustrative purposes here, we sketch some of the work of Willis (Ref. 9). In connection with his studies of mathematical models of weapon systems and tactics in land combat, Willis (Ref. 9) establishes Lanchester type (heterogeneous) square laws involving Blue tanks B_1, Blue artillery B_2, Blue aircraft B_3, Red tanks R_1, and Red artillery R_2 to illustrate the generality of possible applications of available theory. He then uses the following attrition equations:

Blue tanks:

$$\frac{dB_1}{dt} = -\beta_{11}R_1 \tag{29-55}$$

where β_{11} = rate at which Red tanks can kill Blue tanks. (Only Red tanks attack Blue tanks.)

Blue artillery:

$$\frac{dB_2}{dt} = -\beta_{22}R_2 \tag{29-56}$$

where β_{22} = rate at which Red artillery can kill Blue artillery. (Only Red artillery attacks Blue artillery.)

Blue aircraft:

$$\frac{dB_3}{dt} = -\beta_{13}saR_1 \tag{29-57}$$

where we see that only Red tanks attack Blue aircraft and

β_{13} = Blue aircraft attrition rate by Red tanks, per sortie flown
s = sortie rate, per available Blue aircraft
a = Blue aircraft availability rate.

Red tanks:

$$\frac{dR_1}{dt} = -\rho_{11}B_1 - \rho_{21}gB_2 - \rho_{31}asfB_3 + r_0 \tag{29-58}$$

where we see that Blue tanks, Blue artillery, and Blue aircraft all have the capability to attack Red tanks, and

ρ_{11} = rate at which Blue tanks can kill Red tanks
ρ_{21} = rate at which Blue artillery can kill Red tanks
ρ_{31} = rate at which Blue aircraft kill Red tanks
g = fraction of Blue artillery employed against Red tanks (the rest being used against Red artillery)
a = aircraft availability rate
s = sortie rate, per available aircraft
f = fraction of Blue aircraft sorties employed against Red tanks (the rest being used against Red artillery)
r_0 = replacement rate for Red tanks.

Red artillery:

$$\frac{dR_2}{dt} = -\rho_{22}(1 - g)B_2 - \rho_{32}sa(1 - f)B_3 \tag{29-59}$$

where we see that Blue artillery and Blue aircraft, but not Blue tanks, attack Red artillery, and

ρ_{22} = rate at which Blue artillery can kill Red artillery
ρ_{32} = rate of attrition of Red artillery by Blue aircraft
$1 - g$ = fraction of Blue artillery employed against Red artillery (the rest against Red tanks)
s = sortie rate
a = aircraft availability rate
$1 - f$ = fraction of Blue aircraft employed against Red artillery.

One may easily see and appreciate not only the generality but also the considerable amount of flexibility that can be incorporated into the Lanchester type equations describing combat of heterogeneous forces.

Finally, we remark concerning the solutions of (linear differential) equations such as Eqs. 29-55 through 29-59 that even though the general approach of the matrix exponential Eq. 29-53 may be used, one might be able to employ a "trick" or rather straight-forward solution. By this we mean that often one or more of Eqs. 29-55 through 29-59 could be differentiated, thereby giving second-order differential equations. Since the first derivatives, i.e., Eqs. 29-55 through 29-59 exist, they may be substituted into the second-order equations. Now since the second-order differential equations are in standard form, one has only to refer to a textbook on the subject such as Ref. 10 for their solution.

With the given definitions of coefficients, Willis (Ref. 9) provides some informative examples based on the certain values of the coefficients. In particular, suppose we omit terms B_2 and R_2 involving Blue and Red artillery for sake of computation, so that Blue tanks and aircraft attack Red tanks, but Red tanks can attack Blue tanks and not Blue aircraft. Further, take the values of the remaining parameters as

$\rho_{11} = 0.003$ $\quad s = 0.004$

$\beta_{11} = 0.001$ $\quad b_a = 0.05$

$a = 0.70$ $\quad \rho_{31} = 2$

where b_a = Blue aircraft attrition rate per sortie. Then for a Blue goal of killing either 400 or 300 Red tanks in 16-2/3 h of combat, Table 29-1 indicates different combinations of Blue aircraft and tanks needed to do the job for the attrition rates given. We see from Table 29-1 combinations upon which to make a selection based on total cost or other criteria.

A very interesting feature of this type analysis concerns the *trade-off* between Blue tanks and Blue aircraft and, as Willis (Ref. 9) points out, this depends on two major uncertainties: (1) the time duration of combat, and (2) the ratio of Blue tank effectiveness ρ_{11} to Red tank effectiveness β_{11}. In this connection, and for the given assumed numerical values of the parameters, Willis (Ref. 9) calculates Table 29-2. Hence, one may appreciate the importance of such analyses to the weapon decision-making process.

TABLE 29-1. COMBINATION OF BLUE AIRCRAFT AND BLUE TANKS REQUIRED TO KILL A GIVEN NUMBER OF RED TANKS

Number of Red Tanks Blue Must Kill	Combinations of Blue Aircraft and Tanks Needed	
	Aircraft	Tanks
400	100	133
	75	200
	50	267
300	100	33
	75	100
	50	167

TABLE 29-2. TRADE-OFF BETWEEN BLUE TANKS AND BLUE AIRCRAFT AS A FUNCTION OF ρ_{11}/β_{11}

Ratio of Blue Tank Effectiveness to Red Tank Effectiveness ρ_{11}/β_{11}	Combat Time, min	Number of Blue Tanks Equivalent to One Blue Aircraft
1 to 1	2000	8.5
	1000	6.2
	500	5.8
3 to 1	2000	5.3
	1000	2.7
	500	2.2

As a matter of fact, the trade-off equivalence between tanks and aircraft brings forward the concept of weapon equivalence studies, or equating the effectiveness of diverse weapons, which we will discuss in Chapter 30.

We now cover some other extensions of Lanchester's basic laws of combat, and in particular discuss battles of many individual duels and the relative value of intelligence, command, and control efficiency.

29-6 LANCHESTER'S LINEAR LAW AND THE BATTLE OF MANY DUELS

In Chapter 17 we discussed and presented models and techniques for analyzing several types of stochastic duels, and we brought out the possibility that some battles, especially for homogeneous forces, might well be treated in terms of many individual duels. Hellman (Ref. 11) made a study of a stochastic model of military engagements which are assumed to involve individual duels between combatants. Hellman assumes that the duels between individual Blue forces and individual Red forces start at the ordered (random) times, $0 < t_1 < t_2 < \cdots < t_n < \cdots$, where this sequence of times follow a Poisson process of density λ. This means that the probability density function $f(t)$ of n duels begun during the time interal zero to t will be given by

$$f(t) = [\exp(-\lambda t)](\lambda t)^n/n!. \tag{29-60}$$

Thus, the parameter λ may be estimated from any typical data on target detection and engagement times or otherwise hypothesized realistically. Hellman (Ref. 11), as a special case of interest in his more general theory, also assumes that the probability density function $h(t)$ of the duration of a duel is stochastic and follows an exponential law given by

$$h(t) = \mu \exp(-\mu t) \tag{29-61}$$

where μ is the rate at which duels are completed and the mean or expected time of an individual duel is $1/\mu$. Now a duel will always result in the elimination of one or the other combatants, and we define

p = chance that a Blue wins a duel against a Red
$q = 1 - p$ = chance that a Red attrits a Blue
B_0 = initial number of Blue forces
R_0 = initial number of Red forces
B = (random) number of remaining Blue forces at time t
R = (random) number of remaining Red forces at time t.

Finally, for this type of battle, we will be interested in

$\bar{B}$ = average number of surviving Blue forces at time t of battle

and

$\bar{R}$ = average number of surviving Red forces at time t of battle.

Hellman gives $\bar{B}$ and $\bar{R}$ as

$$\bar{B} = B_0 - \lambda q\{t - [1 - \exp(-\mu t)]/\mu\} \tag{29-62}$$

and

$$\bar{R} = R_0 - \lambda p\{t - [1 - \exp(-\mu t)]/\mu\}. \tag{29-63}$$

Clearly, these are relatively simple equations for a battle consisting of a series of stochastic duels, and p and q may easily be estimated from the principles of Chapter 17. Also, we remark that Hellman's theory gives us the opportunity to compare the results of the Lanchester deterministic linear type laws (Chapter 28) with that of the stochastic battle results of Eqs. 29-62 and 29-63.

Hellman (Ref. 11) also shows that the same form of the familiar Lanchester Linear Law, or

$$p(B_0 - \overline{B}) = q(R_0 - \overline{R}) \tag{29-64}$$

still holds even for this stochastic case.

Finally, Hellman is able to derive expressions for $P(B,R,t)$ or the chance that B Blue and R Red forces survive at time t.

EXAMPLE 29-3:

A computerized simulation of many Blue versus Red tank battles, involving 20 tanks on each side, indicates that on the average Blue had 15 tanks remaining after 60 min of battle time and Red had only 10. Assume the tank battle may be described as a series of duels; what can be said about the chance that a Blue tank will win in a single engagement against a Red tank?

We note from Eqs. 29-62 and 29-63 that our solution for p and hence q is quite independent of the parameters λ and μ, and even also of the time t. In fact, the ratio p/q depends only on B_0, R_0, $\overline{B}$ and $\overline{R}$—or on proper division of Eqs. 29-62 and 29-63—with the result

$$q/p = (B_0 - \overline{B})/(R_0 - \overline{R}) \tag{29-65}$$

which is Eq. 29-64 also. Thus,

$$q/p = (20 - 15)/(20 - 10) = 0.5.$$

That is to say, since $p + q = 1$,

$$p = 0.67 \text{ and } q = 0.33.$$

Thus, we see that Blue's chance of winning an individual duel against Red is 0.67, and indeed this figure may easily be estimated from rather cursory data on the initial and remaining numbers of forces.

29-7 THE VALUE OF COMMAND EFFICIENCY IN COMBAT

It is well known that intelligence of enemy activities, surprise, and good command and control of an Army will offset enemy advantages and indeed may often result in superiority. Schreiber (Ref. 12) studied this very problem in connection with the use of Lanchester's Linear Law for area fire. He considers two opposing homogeneous forces in an engagement which consists of a sufficiently large number of similar weapons on a side, although Blue and Red weapons could be different types. During the battle, every unit of each force is within range of and can be fired upon by every unit of the opposing force, and the battle ends when one force annihilates the other. Schreiber (Ref. 12) says:

"When the battle starts each force has complete information of the locations of the enemy units. During the battle each force employs an intelligence system to provide information on the effect of its fire on enemy units, and the effect of enemy fire on its own units. This information is used by a command and control system to redirect fire with the object of always distributing it uniformly over surviving enemy units, and in particular, avoiding wasteful fire on targets already destroyed.

"The effectiveness of the intelligence and command and control systems in this type of battle can be measured by the fraction of the enemy's destroyed units from which fire has been redirected. If this fraction is one, fire is always directed only at the enemy's surviving units and no 'overkilling' results; if it is zero, fire is directed all during the battle against the original enemy positions, and much of it is wasted in 'overkilling'. This fraction will be called the 'command efficiency', and is assumed to be constant throughout the battle.

"Assuming that the battle lasts long enough for some units to fire at least several rounds, and that the initial number of units on either side is sufficiently large, the following equations hold:"

$$\frac{dB}{dt} = -\beta BR/[B_0 - \epsilon_R(B_0 - B)] \tag{29-66}$$

$$\frac{dR}{dt} = -\rho BR/[R_0 - \epsilon_B(R_0 - R)] \tag{29-67}$$

where B, R, B_0, R_0 and time t are as previously defined, and

ϵ_B = command and intelligence efficiency of Blue

ϵ_R = command and intelligence efficiency of Red.

Note, for example, in Eq. 29-66 that if Red's intelligence and command efficiency is $\epsilon_R = 0$, then

$$\frac{dB}{dt} = -(\beta/B_0)BR \tag{29-68}$$

which is of the form of Lanchester's Linear Law for area fire, and Red's effectiveness is so limited.

On the other hand, if $\epsilon_R = 1$, i.e., Red has complete intelligence and conducts a most efficient battle against Blue, then Eq. 29-66 reduces to

$$\frac{dB}{dt} = -\beta R \tag{29-69}$$

which is none other than the ordinary Lanchester Square Law.

Hence, the outcome of the battle may well depend on the relative intelligence and command efficiencies of the Blue and Red sides.

Schreiber shows that for a draw or parity, then

$$\beta R_0^2/(2 - \epsilon_R) = \rho B_0^2/(2 - \epsilon_B). \tag{29-70}$$

Thus, an increase in the value of the intelligence and command efficiency from, say, an initial value ϵ_0 to some final or top level ϵ will increase the combat power by the equivalent amount as an increase in numerical strength given by the fraction F, or

$$F = \sqrt{(2 - \epsilon_0)/(2 - \epsilon)} - 1. \tag{29-71}$$

Hence, Schreiber constructs Table 29-3 to indicate the value of good intelligence and command control.

Needless to say, the advantages of target intelligence and superior command and control are very substantial indeed and up to a maximum of about 41%!

As a point of some particular interest, Schreiber shows that if, for example, Blue wins by annihilating Red, but, of course, suffers the fractional loss $(B_0 - B)/B_0$ in doing so, then this quantity is given by

$$(B_0 - B)/B_0 = \{1 - [1 - (2 - e_B)e_R\beta R_0^2/(\rho B_0^2)]^{1/2}\}/e_R. \qquad (29\text{-}72)$$

Thus, it is noted in Eq. 29-72 that the fractional loss of the winning force, Blue, is the same for all values of ρ, B_0, and e_B which result in the same relative "combat power" for Blue given by Schreiber as the right hand side of Eq. 29-70. This, however, means that the absolute loss is minimized by the smallest value of B_0, the initial starting force. Therefore, as Schreiber points out, if minimizing battle losses is the required criterion, and assuming the kill rate ρ fixed, then the "combat power" should be attained by increasing command efficiency rather than by increasing the number of Blue weapon units! In summary, the modified Lanchester Eqs. 29-66 and 29-67 throw much light on the relative importance of intelligence, command, and control as they affect combat capability along with the effectiveness of weapons.

29-8 SOME ADDITIONAL CONSIDERATIONS

In our account of Lanchester combat theory we have presented "deterministic" theory primarily but nevertheless have also introduced stochastic or probabilistic considerations in connection with changes of state from the numerical forces on each side to a smaller number of combatants. Also we have used the random times at which targets on each side have been killed, or put out of action, to establish appropriately fitted (Weibull) kill-time distributions and advantageously estimate future casualties after some point of truncating the battle or simulation.

For the deterministic approach, and as pointed out by Taylor (Ref. 1) in his Figs. 22 through 27, for example, the battle time results for the more complex stochastic transition probability equations approach those of the simpler deterministic models for sufficiently large numbers of forces. Hence, in such cases the systems analyst will naturally make his inferences from applying deterministic models, whenever possible. This also suggests that random effects, nevertheless, may be particularly important for combat among small numbers.

TABLE 29-3. EQUIVALENT PERCENT INCREASE IN NUMERICAL STRENGTH CORRESPONDING TO AN INCREASE IN COMMAND EFFICIENCY FROM e_0 to e

e_0	e				
	0.2	0.4	0.6	0.8	1.0
0	5.4	11.8	19.5	29.1	41.4
0.2		6.1	13.4	22.5	34.2
0.4			6.9	15.5	26.5
0.6				8.1	18.3
0.8					9.6

Clark (Refs. 13 and 14) has made a rather extensive investigation of this problem and found, for the Lanchester Square Law attrition process and the same values of the attrition coefficients, that the force level results were greater for the stochastic analysis than the corresponding deterministic model. Taylor's Figs. 25 through 27 of Ref. 1, which is readily available, give some of Clark's numerical findings on this subject.

Concerning the analysis of random kill times for targets on each side in a battle, the models for this situation were discussed in par. 28-12 for homogeneous forces on each side. A suggestion to extend this type of analysis to heterogeneous forces is advanced by Grubbs and Shuford (p. 939, Ref. 15).

For a discussion of a variety of Lanchester type differential equation models and some seven different measures of effectiveness, the reader is referred to a paper by Willis (Ref. 16). In fact, Willis discusses measures of effectiveness which include the loss rate difference, the ratio of percent losses, the loss ratio, differences in losses, surviving force ratio (effectiveness of a force), fractional reduction in force ratio, and percent losses of a side at the time of a battle when losses on the other side reach a specified value.

Weiss (Ref. 17) discusses optimum tactics for a combat model which includes a primary and supporting weapon system on each side, and covers the implications on force structure depending upon the weapon range, cost, and parametrically specified performance.

Some other topics of interest associated with this chapter are given in the Bibliography.

29-9 SUMMARY

The aim of Chapters 28 and 29 has been that of presenting to the analyst a rather comprehensive base of useful Lanchester type combat models which should aid in numerous applications requiring management decisions on weapons and forces. The basic models for homogeneous forces in Chapter 28 and the extensions in this Chapter—including additional terms of analysis, along with the important case of heterogeneous forces on each side, and the value of command efficiency—should give the systems analyst a considerable amount of expertise so that he can extend his knowledge and applications to a wide variety of combat theory type problems or uses. Some of the typical examples presented herein also should indicate some of the many areas of application.

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CHAPTER 30
WEAPON EQUIVALENCE STUDIES

A discussion is given of one of the fundamental and currently key studies in the evaluation of weapon systems; namely, the problem of quantifying, comparing, or equating in some way the relative performance of diverse weapon systems. Thus, can the effectiveness of infantry antitank weapons be equated to that of tank armament against common targets, or can infantry effectiveness measures be "equated" through some deterministic scale to equivalent measures of effectiveness (MOE) for artillery? Also, in general can one determine a useful overall value or worth of a military unit with combined arms, so that a proper comparison can be made in judging chance of success in defeating some hypothesized enemy unit with perhaps the same or different set of heterogeneous weapons? A survey is given of recent analytical work on this important and useful topic to convert combined arms or heterogeneous force MOE's to equivalent homogeneous values for all weapons of the same, identical type. Obviously, it can be seen that some introductory matrix applications are very useful, or even needed, to handle problems of this type, and indeed the use of eigenvalue techniques help in obtaining the solutions on suitable and common grounds.

The killer-victim scoreboard is discussed and examples of equivalence type studies are given for indicated applications.

We note also that the methods of this chapter can be used to develop combat unit values as accurately quantified inputs for theater-level war games.

30-0 LIST OF SYMBOLS

$A = \sum_{i=1}^{b} \beta_{1i}\rho_{i1} = K_1$ of Eq. 30-48 also

a_{ij} = number of j type weapons eliminated by i type weapons in some time t for a "killer-victim" scoreboard

$B = \sum_{i=1}^{b} \beta_{1i}\rho_{i2}$

$B_i = B_i(t)$ = number of Blue type weapons remaining at any time t of the battle $(i = 1, 2, \ldots, b)$

$[B]$ = column vector of numbers of Blue weapons in Eq. 30-4

$[dB/dt]$ = time derivative vector of Blue weapon numbers

$\overline{B}$ = weighted average Blue force $= \sum_{i=1}^{b} w_{Bi}B_i = V_0(B)$

B_{it} = number of Blue type i weapons at time t

B_{im} = initial number of Blue type i weapons in the mth engagement

b = number of types of Blue weapons employed in a battle

$b_{ij} = \rho_{ij}$ for $i = 1, \ldots, b$ and $j = b + 1, \ldots, b + r$ of Eq. 30-67

$b_{ij} = 0$ for $i = 1, \ldots, b$ and $j = 1, 2, \ldots, b$ of Eq. 30-7

$b_{ij} = \beta_{ij}$ for $i = b + 1, \ldots, b + r$ and $j = 1, 2, \ldots, b$ of Eq. 30-67

$b_{ij} = 0$ for $i = b + 1, \ldots, b + r$ and $j = b + 1, \ldots, b + r$ of Eq. 30-6

$C = \sum_{i=1}^{b} \beta_{3i}\rho_{i3}$

c = particular constant of proportionality = $1/\beta_B = 1/\rho_R$

D = $\sum_{i=1}^{b} \beta_{1i}\rho_{i1}$

$[E_b]$ = $\frac{1}{b}\begin{bmatrix}1\\ \cdot\\ \cdot\\ \cdot\\ 1\end{bmatrix}$ = column vector for Blue

$[E_r]$ = $\frac{1}{r}\begin{bmatrix}1\\ \cdot\\ \cdot\\ \cdot\\ 1\end{bmatrix}$ = column vector for Red

F = $F(t)$ = force ratio of Blue to Red at general time t

F_0 = $V_0(B)/V_0(R)$ = initial force ratio, Blue to Red

F_R = $V_t(R)/V_0(R)$ = fraction of relative Red initial strength remaining at time t

$[I]$ = identity matrix

K_{Bjim} = total number of kills by Red type j weapons of Blue type i weapons in the mth engagement

K_{Rijm} = total number of kills by Blue type i weapons of Red type j weapons in the mth engagement

k = superscript for number of iterations in Eqs. 30-37, 30-38, and 30-39

k = largest eigenvalue of a matrix in Johnsrud's notation (see Eqs. 30-68)

ℓ = integer = 1, 2, ...

M = total number of small unit engagements considered

$[M]$ = general matrix

$[M_1]$ = submatrix of $[M]$ in Eq. 30-30

$[M_{21}]$ = submatrix of $[M]$ in Eq. 30-30

$[M_{21}]$ = submatrix of $[M]$ in Eq. 30-30

n_i = Johnsrud's notation for number of type i weapons

n_1 = B_1 type 1 Blue weapons

n_2 = B_2 type 2 Blue weapons

⋮

n_b = B_b type b Blue weapons

n_{b+1} = R_1 type 1 Red weapons

n_{b+2} = R_2 type 2 Red weapons

⋮

n_{b+r} = R_r type r Red weapons

$[P_b]$ = $[\rho][\beta]$ = product of kill-rate matrices

$[P_r]$ = $[\beta][\rho]$ = product of kill-rate matrices

$[R]$ = column vector of numbers of Red weapons in Eq. 30-5

$[dR/dt]$ = time derivative vector of Red weapon numbers

$\bar{R}$ = weighted average Red force = $\sum_{j=1}^{r} w_{Rj}R_j = V_0(R)$

R_j = $R_j(t)$ = number of Red type j weapons remaining at any time t of the battle ($j = 1,2,\ldots,r$)

R_{jm} = initial number of Red type j weapons in the mth engagement

R_{jt} = number of Red type j weapons at time t

r = number of types of Red weapons employed in a battle

t = time

Δt_m = time duration of mth engagement

$V_0(B)$ = $\sum_{i=1}^{b} w_{Bi}B_i$ = initial value of all Blue weapons = $\bar{B}$ (ave)

$V_0(R)$ = $\sum_{j=1}^{r} w_{Rj}R_j$ = initial value of all Red weapons = $\bar{R}$ (ave)

$V_t(B)$ = value of equivalent Blue strength at time t

$V_t(R)$ = value of equivalent Red strength at time t

$[W]$ = new vector defined in Eq. 30-37

w = w_{R2}/w_{R1} (see Eq. 30-59)

$[w_B]$ = column vector for weights or values of Blue weapons

$[w_R]$ = column vector for weights or values of Red weapons

w_{Bi} = value or weight of Blue type i weapon ($i = 1,2,\ldots,b$)

w_{Rj} = value or weight of Red type j weapon ($j = 1,2,\ldots,r$)

W_i = relative worth or value of type i weapon (Johnsrud's notation)

$W_1, W_2, \ldots, W_b$ = $w_{B1}, w_{B2}, \ldots, w_{Bb}$, respectively

$W_{b+1}, W_{b+2}, \ldots, W_{b+r}$ = $w_{R1}, w_{R2}, \ldots, w_{Rr}$, respectively

$[\ ]^T$ = transpose of a vector or matrix

α_{ij} = detectability/availability factor or chance for Blue type i weapons engaging Red type j weapons

$[\beta]$ = Red's kill rate matrix against Blue in Eq. 30-6

β_B = constant of proportionality in Eq. 30-23

β_{ji} = constant rate at which a Red type j weapon kills a Blue type i weapon, unit, etc.

γ = eigenvalue (Howes-Thrall) = k^2 (Johnsrud)

λ = c^2

K_1 = $\sum_{i=1}^{b} \beta_{1i}\rho_{i1}$

$[\rho]$ = Blue's kill rate matrix against Red in Eq. 30-7

ρ_{ij} = constant rate at which a Blue type i weapon kills a Red type j weapon, unit, etc.

ρ_R = constant of proportionality in Eq. 30-22

30-1 INTRODUCTION

In a battle, it is likely that friendly infantry will fight enemy infantry; tanks will fight tanks; and artillery will attack artillery some of the time but often will be used against personnel targets as well. Nevertheless, in recent military history infantry personnel have been provided weapons to attack hard targets such as tanks, and hence man portable antitank weapons are becoming a real threat as tank killers, especially on a smaller cost basis. In addition, tanks have the capability not only to attack other tanks, but can be used advantageously to attack enemy personnel such as infantry or, in breakthroughs, may overrun many types of enemy positions including those of artillery. Artillery also has the capability to attack tanks, trucks, or personnel in protected positions. Thus, it is seen that the trend has been toward weapon systems which have the advantage of being "multipurpose" in character. There is no such military item as an "all-purpose" weapon nor is it very likely that future weapons can be used for "universal" applications. Hence, the different types of weapons in a fighting unit will naturally have the capability to take on the same or some common types of targets, and in this way will compete with each other, even on a cost-effectiveness, or some other basis. In addition, there is the need as we pointed out at the end of Chapter 8, to determine the overall effectiveness or "fire power" of any combat unit. Finally, we see from Chapter 29 that studies involving combined arms or heterogeneous weapons invariably run into much analytical complexity. Therefore, there exists much motivation not only to compare competing weapons, but also to try and equate their relative worth on some common scale. Clearly, this should be possible—especially since different weapons under study are employed to attack the same targets—and hence it might be expected that a useful scale of relative values could be established. Moreover, such an equivalence study of different type weapons could be used perhaps to simplify the complex problems of combined arms studies, i.e., the placing of values of heterogeneous weapon systems on a common or homogeneous basis. This, therefore, is the goal of weapon equivalence studies.

To approach this type of problem, it is seen that either key or meaningful measures of effectiveness (MOE's) must be used in any useful analytical development. In this connection, we have seen that perhaps one of the most important and "key" measures of effectiveness is the kill rate of each weapon on a side. Thus, we must find some analytical means of converting kill rates to useful quantities on a convenient scale of measurement. Some authors employ the "value" of Blue and Red combined arms or forces, and use the sum of products of the individual weapon "values" and numbers of weapons on each side as "good" measures of the relative strengths of opposing forces. Others may use the concept of "killer-victim" scoreboards which may easily be obtained from computer simulations of battles. The "killer-victim" scoreboard is a matrix of elements showing how many of each type of weapon were eliminated in a battle by each type of weapon on the opposing side. Thus, this represents a very useful measure to analyze, especially in view of the fact that currently there are wide-spread uses of computer simulated battles, and also there is an urgent need to analyze the type of data which may be obtained therefrom. Finally, other investigators approach the problem by determining "ideal linear weights" which are used to develop a "weighted average" of the effects of a given weapon on a side against each of the enemy's weapon types. All may lead to the same results.

In our account here, we will follow in particular the work of Holter (Ref. 1), that of Howes and Thrall (Ref. 2), Dare and James (Ref. 3), Anderson (Ref. 4), Spudich (Ref. 5), and Johnsrud (Ref. 6) since these authors seem to have accomplished some of the more significant results relative to the problem of weapon equivalence studies.

30-2 BASIC ASSUMPTIONS AND THE ANALYTICAL APPROACH

30-2.1 PRELIMINARIES

We consider a battle or combat situation between two opposing forces, Blue and Red. We define or hypothesize the following:

$B_i = B_i(t)$ = number of Blue type i weapons remaining at any time t of the battle ($i = 1,2,\ldots,b$)

$R_j = R_j(t)$ = number of Red type j weapons remaining at any time t of the battle ($j = 1,2,\ldots,r$).
(Hence, there are b different types of Blue weapons and r different types of Red weapons employed in the battle. Also, Blue and Red may employ the same types of weapons, or they may be different ones of their own choice or availability.)

β_{ji} = constant rate at which a Red type j weapon kills a Blue type i weapon, unit, etc.

ρ_{ij} = constant rate at which a Blue type i weapon kills a Red type j weapon, unit, etc.

(With regard to β_{ji} and ρ_{ij} and their definitions, we remark that for some applications they may be defined more generally than just in terms of kill rates. In fact, these might be extended to include also chances of seeing and engaging targets on the opposing side and in addition could include the allocation type factors γ_{ji} and δ_{ij} of Eqs. 29-42 and 29-43, for example, or chances of weapon-target engagements for heterogeneous forces, etc. Such will depend on the application at hand and the need for such generality.)

The problem of weapon equivalence studies is to find the weights for or "values" of Blue type i weapons, i.e.,

$$w_{B1}, w_{B2}, \ldots, w_{Bi}, \ldots, w_{Bb}$$

and Red type j weapons, i.e.,

$$w_{R1}, w_{R2}, \ldots, w_{Rj}, \ldots, w_{Rr}$$

which are then used to determine the linear combinations

$$V_0(B) = w_{B1}B_1 + w_{B2}B_2 + \cdots + w_{Bb}B_b{}^* = \sum_{i=1}^{b} w_{Bi}B_i \tag{30-1}$$

and

$$V_0(R) = w_{R1}R_1 + w_{R2}R_2 + \cdots + w_{Rr}R_r{}^* = \sum_{j=1}^{r} w_{Rj}R_j \tag{30-2}$$

where $V_0(B)$ and $V_0(R)$ are good overall measures or "values" of the relative strengths of Blue and Red forces, respectively, at time zero. Then, for example, the initial "effectiveness" or force ratio F_0, usually for attacker to defender strength, given by

$$F_0 = V_0(B)/V_0(R) \tag{30-3}$$

can be used as an index of relative force strengths. Moreover, a solution for the individual w_{Bi}'s and w_{Rj}'s provides a set of weapon effectiveness values (WEV's), which can be multiplied by the corresponding numbers of weapons, or force levels, on a side to give the overall unit or total force effectiveness values as in Eqs. 30-1 and 30-2.

*Actually, no confusion should result from making the B_i and R_j either the initial numbers of weapons in a battle or the numbers at time t.

It will be convenient and more useful to outline this problem in terms of vectors and matrices. Thus, the Blue force level column vector $[B]$, or numbers of different Blue i type weapons, is given by

$$[B] = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_b \end{bmatrix} \tag{30-4}$$

Correspondingly, the Red force level column vector $[R]$ is

$$[R] = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_r \end{bmatrix} \tag{30-5}$$

The kill rate matrix $[\beta]$ for Reds killing Blues is

$$[\beta] = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1b} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2b} \\ \vdots & & & \\ \beta_{r1} & \beta_{r2} & \cdots & \beta_{rb} \end{bmatrix} \tag{30-6}$$

an $r \times b$ matrix, and the kill rate matrix $[\rho]$ of Blues against Reds is

$$[\rho] = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1r} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2r} \\ \vdots & & & \\ \rho_{b1} & \rho_{b2} & \cdots & \rho_{br} \end{bmatrix} \tag{30-7}$$

a $b \times r$ matrix of elements.

The weight column vector $[w_B]$ to be determined for the Blue weapons is

$$[w_B] = \begin{bmatrix} w_{B1} \\ w_{B2} \\ \vdots \\ w_{Bb} \end{bmatrix} \tag{30-8}$$

and the weight column vector $[w_R]$ for Red is

$$[w_R] = \begin{bmatrix} w_{R1} \\ w_{R2} \\ \vdots \\ w_{Rr} \end{bmatrix} \tag{30-9}$$

Thus, with the definitions, Eqs. 30-4 through 30-9, we see that Eq. 30-1 may be expressed as

$$V(B) = [w_B]^T[B] \tag{30-10}$$

where the superscript T means transpose of the column vector to a row vector, and also we have for Eq. 30-2 that

$$V(R) = [w_R]^T[R]. \tag{30-11}$$

In summary, therefore, the problem is to establish methodology to determine some sort of "good" or "best" column vectors, $[w_B]$ and $[w_R]$. These vectors, established by using the kill rate matrices of Eqs. 30-6 and 30-7, then give us the needed weights for determining overall force effectiveness.

For Lanchester's Square Law model for the attrition of heterogeneous forces on each side, we see that with the definitions enumerated, we have also the differential vectors

$$[dB/dt] = \begin{bmatrix} dB_1/dt \\ dB_2/dt \\ \vdots \\ dB_b/dt \end{bmatrix} \tag{30-12}$$

and

$$[dR/dt] = \begin{bmatrix} dR_1/dt \\ dR_2/dt \\ \vdots \\ dR_r/dt \end{bmatrix} \tag{30-13}$$

Then the appropriate equations in matrix form are

$$[dB/dt] = -[\beta]^T[R] \tag{30-14}$$

and

$$[dR/dt] = -[\rho]^T[B]. \tag{30-15}$$

These Lanchester type heterogeneous force equations are similar in form to the basic model of Eqs. 29-42 and 29-43, except for convenience here we have used symbolically single coefficients.

Before proceeding to the problem of converting the Lanchester heterogeneous case to an equivalent form for the Lanchester combat theory homogeneous case, it will be useful and informative to indicate some particular contributions of Holter (Ref. 1) and some illustrative examples of the kill rate matrices of Howes and Thrall (Ref. 2). In fact, we follow initially much of Holter's contributions (Ref. 1) before proceeding to the results of others.

30-2.2 ESTIMATION OF KILL RATES

In Ref. 1, Holter indicates that his developments on the weapon equivalence problem were performed in connection with a study entitled "NATO Combat Capabilities Analysis II" (COMCAP II) under the sponsorship of the Deputy Chief of Staff for Operations and Plans, Headquarters Department of the Army. As Holter indicates, "One of the principal objectives of the study was to develop weapon effectiveness values (WEV's) and unit effectiveness values (UEV's) for representative US and Soviet forces engaged in mid-intensity combat in Western Europe, circa 1976. The objectives of the study were attained by analyzing killer/casualty data generated by an exercise of the Division Battle Model (DBM) over some six days of simulated warfare in the European theater." His paper (Ref. 1) presents the mathematical description and justification of the methodology we will cover here, and appears as Appendix D of the COMCAP II final Report of the General Research Corporation's Operations Analysis Division.

Concerning the kill rate matrices, $[\beta]$ and $[\rho]$, the elements β_{ji} and ρ_{ij} are measures of the killing powers of individual firers on one side against different types of targets on the other. Although the analyst may try to estimate such quantities realistically from weapon performance values of rate of fire, and hit and kill probabilities, Holter gives equations for their "realistic" determination in COMCAP II by grouping Division Battle Model killer/casualty data into discrete sets of small unit engagements according to Blue posture: delay, defense, or counterattack. The estimates of kill rates are:

$$\beta_{ji} = \sum_{m=1}^{M} K_{Bjim} / \sum_{m=1}^{M} (R_{jm})(\Delta t_m) \tag{30-16}$$

and

$$\rho_{ij} = \sum_{m=1}^{M} K_{Rijm} / \sum_{m=1}^{M} (B_{im})(\Delta t_m) \tag{30-17}$$

where

M = total number of small unit engagements considered
K_{Bjim} = total number of kills by Red type j weapons of Blue type i weapons in the mth engagement
R_{jm} = initial number of Red type j weapons in the mth engagement
Δt_m = time duration of mth engagement
K_{Rijm} = total number of kills by Blue type i weapons of Red type j weapons in the mth engagement
B_{im} = initial number of Blue type i weapons in the mth engagement.

Thus, the more realistic the Division Battle Model (DBM) is played, the more accurate the estimates of attrition rates brought about from the two-sided conflict simulated.

30-2.3 SOME NUMERICAL EXAMPLES OF KILL RATE MATRICES

As a preamble to understanding weapon equivalence studies and their use of "ideal linearized weights", Howes and Thrall (Ref. 2) give some rather striking numerical examples of Blue and Red side potential for two different classes of weapons, i.e., $b = r = 2$, and this should be informative to illustrate here. They consider for the sake of simplicity only an infantry weapon class and an artillery weapon class, for which $i = j = 1$ indicates infantry and $i = j = 2$ indicates artillery. Recalling that the first subscripts of β_{ji} and ρ_{ij} refer to rows and the second to columns of matrices, Howes and Thrall (Ref. 2) illustrate the following numerical effectiveness measures for the killing power of kill rate matrices:

$$[\rho] = \begin{bmatrix} 0.5 & 0 \\ 0.7 & 0.2 \end{bmatrix} \quad \text{and} \quad [\beta] = \begin{bmatrix} 0.6 & 0 \\ 0.6 & 0.1 \end{bmatrix} \tag{30-18}$$

Here, the numerical kill rate matrices indicate that

1. In infantry combat Red is more effective than Blue, i.e., 0.6 vs 0.5.
2. Neither Blue nor Red infantry has any capability against opposing artillery, i.e., 0 vs 0.
3. Blue artillery is superior to Red artillery, 0.7 vs 0.6, when artillery attacks infantry.
4. Blue and Red artillery have a positive effect against each other, with Blue being a bit superior to Red—i.e., 0.2 vs 0.1.

Now if the effectiveness matrices are changed to be

$$[\rho] = \begin{bmatrix} 0.5 & 0.1 \\ 0.7 & 0.2 \end{bmatrix} \quad \text{and} \quad [\beta] = \begin{bmatrix} 0.6 & 0.2 \\ 0.6 & 0.1 \end{bmatrix} \tag{30-19}$$

then Blue and Red are given some capability for infantry to attack opposing artillery, with Red having a slight upper hand—0.2 vs 0.1.

The following kill-rate matrices show that Blue (0.8) and Red (0.5) have both increased their artillery vs artillery capability, but their infantry units nevertheless have zero effect against artillery:

$$[\rho] = \begin{bmatrix} 0.5 & 0 \\ 0.7 & 0.8 \end{bmatrix} \quad \text{and} \quad [\beta] = \begin{bmatrix} 0.6 & 0 \\ 0.6 & 0.5 \end{bmatrix} \tag{30-20}$$

Finally, as Howes and Thrall (Ref. 2) point out, if the artillery units of Blue and Red are concealed, or are under good cover, or are out of each other's range, then the effectiveness matrices may become

$$[\rho] = \begin{bmatrix} 0.5 & 0 \\ 0.7 & 0 \end{bmatrix} \quad \text{and} \quad [\beta] = \begin{bmatrix} 0.6 & 0 \\ 0.6 & 0 \end{bmatrix} \tag{30-21}$$

for example. Hence, we see that the analyst certainly has the opportunity to use much flexibility and generality in setting up weapon systems analyses of the kind we are illustrating here. In fact, these numerical examples should give us a good background to study the relation of heterogeneous force analyses and the "equivalent" homogeneous forces type of Lanchester Square Law.

30-2.4 RELATION BETWEEN METHODOLOGY FOR LANCHESTER HETEROGENEOUS AND HOMOGENEOUS FORCE LAWS

We now establish one of the major results on which weapon equivalence values are based. This important and interesting result was apparently noticed first by Dare and James (Ref. 3) and subsequently elaborated on by Thrall (Ref. 7) and Anderson (Ref. 4). For our purpose here, we follow the development or proof given by Holter (Ref. 1). Holter highlights a "major premise" concerning this development:

"The total value of a number of weapons of a given type on a side is directly proportional to the total value of the opposing forces destroyed by those weapons per unit of time.".

This means that, as a direct consequence of the premise, positive constants β_B and ρ_R exist and can be found such that the relationships between Blue and Red weapon effectiveness values (WEV's), or weights w_{Bi} and w_{Rj}, can be written

$$\rho_R[w_B] = [\rho][w_R] \tag{30-22}$$

and

$$\beta_B[w_R] = [\beta][w_B] \tag{30-23}$$

where the vectors $[w_B]$ and $[w_R]$ are defined by Eqs. 30-8 and 30-9, respectively; the kill rate matrices $[\beta]$ and $[\rho]$ by Eqs. 30-6 and 30-7, respectively; and β_B and ρ_R are constants of proportionality, also to be determined in some "optimum" manner. This means that aside from constants of proportionality the kill rate matrix for attriting a side multiplied by the "values" (or weights) of the weapons or forces put out of action gives a vector of relative values for the weapons on the opposite side.

We can now transform Eq. 30-22 successively by using the three key Eqs. 30-15, 30-10, and 30-11. First, in view of Eq. 30-22 we have also that

$$[w_R]^T[\rho]^T = \rho_R[w_B]^T. \tag{30-24}$$

Then, multiply both sides of the equation by -1 and on the right by $[B]$, so that we get

$$-[w_R]^T\{[\rho]^T[B]\} = -\rho_R\{[w_B]^T[B]\}. \tag{30-25}$$

Finally, using Eqs. 30-15, and 30-10, we see that

$$\left.\begin{aligned} [w_R]^T[dR/dt] &= -\rho_R V(B) \quad \text{for equivalence} \\ \text{or} \qquad &= -\rho_R[w_B]^T[B]. \end{aligned}\right\} \tag{30-26}$$

But this last equation is a weighted average of time derivatives for all of the Red type weapons equated to a negative constant multiplied by a weighted average of all Blue type weapons, i.e., the weighted averages are representative single values for the different types of weapons. Expanded, Eq. 30-26 becomes

$$\frac{d}{dt}(w_{R1}R_1 + w_{R2}R_2 + \cdots + w_{Rr}R_r) = -\rho_R(w_{B1}B_1 + w_{B2}B_2 + \cdots + w_{Bb}B_b) \tag{30-27}$$

which is of the form of the Lanchester Square Law for homogeneous forces in terms of an "average" Red force $\bar{R}$ and "average" Blue force $\bar{B}$, i.e.,

$$\frac{d\overline{R}}{dt} = -\rho_R \overline{B} = -\rho_R \Sigma w_{Bi} B_i. \tag{30-28}$$

Similarly, for Blue attrition, we get

$$\frac{d\overline{B}}{dt} = -\beta_B \overline{R} = -\beta_B \Sigma w_{Rj} R_j. \tag{30-29}$$

Thus, if we can find unique values of β_B, ρ_R, and the "value" or weight vectors $[w_B]$ and $[w_R]$, for Eqs. 30-22 and 30-23, then Eqs. 30-10, 30-11, 30-22, and 30-23 imply that one may go from the heterogeneous Lanchester model, Eqs. 30-14 and 30-15, to an equivalent homogeneous Lanchester model represented by Eqs. 30-28 and 30-29. This very striking, remarkable, and important result, apparently first worked out by Dare and James (Ref. 3), can be used to convert the heterogeneous force overall effectiveness of a side to an equivalent value in terms of an average homogeneous force, so to speak.

30-3 DETERMINATION OF VALUES OR WEIGHTS

We next face the problem of determining the components of the weight vectors $[w_B]$ and $[w_R]$, and the scaling factors β_B and ρ_R. It is not ordinarily possible to determine unique values of all of these unknowns, and hence in order to make this possible some additional assumptions must be made or some criteria must be set in advance. Studies of this problem by Dare and James (Ref. 3), Thrall (Ref. 7), and Anderson (Ref. 4) would indicate in most cases that the product $\beta_B\rho_R$ may be determined uniquely, and the components of $[w_B]$ and $[w_R]$ may be determined to within an arbitrary scaling factor. Holter (Ref. 1) points out that if the matrices $[\beta][\rho]$ and $[\rho][\beta]$ determined from products, as indicated, of the kill-rate matrices of Blue and Red are "irreducible", then there is one and only one value of the product of factors $\beta_B\rho_R$ that leads to nonnegative values of components of $[w_B]$ and $[w_R]$, and this is the maximum eigenvalue of $[\beta][\rho]$ or $[\rho][\beta]$. (This maximum eigenvalue is the same for both products of kill-rate matrices). A nonnegative square matrix $[M]$ is said to be "reducible" if it has the form

$$[M] = \begin{bmatrix} M_1 & 0 \\ M_{21} & M_2 \end{bmatrix} \tag{30-30}$$

where $[M_1]$ and $[M_2]$ are also square matrices, or more generally, if the form of $[M]$ given by Eq. 30-30, can be obtained by a reordering of the rows followed by the same reordering of the columns. Otherwise, the matrix is irreducible. An example of the reducible matrix is that of Eq. 30-18. We will not go any further into the subject or details of "reducible" and "irreducible" matrices here, although the reader may consult Thrall (Ref. 7), or Howes and Thrall (Ref. 2) for further information. Holter (Ref. 1) points out that the matrices $[\beta][\rho]$ and $[\rho][\beta]$ are reducible (i.e., not irreducible) if at least two opposing weapon types are not interacting directly with the other participants in the battle—and in his COMCAP II DBM study previously referred to the problem of reducibility did not arise—so that irreducibility is usually to be expected for many heterogeneous force analyses.

Some of the assumptions or criteria adopted by various authors in their studies of heterogeneous forces to estimate weights include the following:

1. Dare and James (Ref. 3) take

$$\sum_{i=1}^{b} w_{Bi} = \sum_{j=1}^{r} w_{Rj} = 1 \tag{30-31}$$

or in other words the weights or values must add up to unity. Hence, the relative worths of the different types of weapons are placed on a relative frequency basis, and the vectors in Eqs. 30-8 and 30-9 may be referred to as "probability" vectors.

2. In Ref. 7, Thrall suggested taking

$$\rho_R = \sum_{j=1}^{r} w_{Rj} \tag{30-32}$$

and

$$\beta_B = \sum_{i=1}^{b} w_{Bi} \tag{30-33}$$

so that the weights for the different weapon types on one side are tied in with the scaling factor for the other side.

3. Howes and Thrall (Ref. 2) discuss several different methods for determination of the relative weights or values, and give examples of their recommended "ideal" weights. The interested reader should study their paper. We sketch their ideal linear weight procedure in par. 30-6.

With reference to 1, 2, and 3, a possible disadvantage—as pointed out by Holter (Ref. 1)—is that the weights or values need to be cross-structured so that the overall representative strengths or equivalent fighting powers are determinable on the same homogeneous scale and in terms of the same weapon. It is apparently for this reason that Holter suggested the following relationships be used

$$\beta_B = \rho_R = 1/c \tag{30-34}$$

where c is a single scaling factor for convenience, and

$$w_{B1} = 1 \tag{30-35}$$

for the designated Blue type 1 weapon. (For the COMCAP II study, Holter designated the M60A3 tank as the Blue type 1 weapon, and the results would be in terms of relative effectiveness values for the M60A3.) Another, and perhaps simpler procedure, which apparently gives equivalent results, is that of Johnsrud (Ref. 6) which will be discussed later. The advantage of both the Holter procedure and that of Johnsrud is that all Blue and Red weapon relative weights will be expressed or measured in terms of the worth of the same weapon, although we recognize that other techniques to determine the "best" or "optimum" weights may vary for different applications perhaps.

We now turn to the solutions of Eqs. 30-22, 30-23, and 30-34. These equations may easily be combined (Holter, Ref. 1) to obtain

$$[w_B] = c^2[\rho][\beta][w_B] \tag{30-36}$$

which involves only the scaling factor c and the Blue weight vector or weapon effectiveness values. Holter (Ref. 1) gives a rapidly converging algorithm (e.g., Ref. 8), which leads to unique values of $\lambda = c^2$ and the weight components of both $[w_B]$ and $[w_R]$. Let the superscript k be used to denote values obtained at the end of the kth iteration. Then Holter's iteration is:

Step 1. Put $k = 1$ (for the first iteration).

Step 2. Set all components of $[w_B]^{(k)}$ for $k = 1$ equal to unity to start the iterative process.

Step 3. Calculate in succession:

(a) $[W]^{(k)} = [\rho][\beta][w_B]^{(k)}$ (30-37)

where $[W]^{(k)}$ is a new vector.

(b) $\lambda^{(k)} = 1/W_1^{(k)}$ (30-38)

where $W_1^{(k)}$ is the first component of $[W]^{(k)}$ at that iteration, and then calculate

(c) $[w_B]^{(k+1)} = \lambda^{(k)}[W]^{(k)}$. (30-39)

Step 4. Now repeat Step 3, incrementing k by unity at each iteration, until a value $\lambda^{(k+1)} = \lambda^{(k)}$, at some stage or iteration k to within a specified degree of accuracy. The iterations converge to a unique value of λ (see Hildebrand, Ref. 9) and the vector $[w_B]$ desired with $w_{B1} = 1$, assuming the matrix $[\rho][\beta]$ is irreducible.

Step 5. Calculate:

(a) $c = \sqrt{\lambda}$ (30-40)

(b) with the value of c from Eq. 30-40, calculate

$$[w_R] = c[\beta][w_B] \quad (30\text{-}41)$$

where $[w_B]$ is the final iterated value in Eq. 30-39.

The final vectors, $[w_B]$ and $[w_R]$, give all the weights or relative values of the Blue and Red weapon types, and the total force values or strengths are found from Eqs. 30-1 and 30-2 or, that is, Eqs. 30-10 and 30-11.

We will illustrate the methodology with a very simple example involving only two different types of weapons on each side. The principles extend to any number of weapon types, and for such applications the analyst will no doubt want to program the calculations on a computer.

EXAMPLE 30-1:

A Blue infantry-mortar "team" meets a similar Red "team" in the jungle. Blue has 80 infantrymen with rifles and 24 mortars for the attack. Red, on the other hand, is estimated to have about 60 defending riflemen and 18 mortars. Blue's kill-rate matrix $[\rho]$ against Red is

$$[\rho] = \begin{bmatrix} 0.10 & 0.04 \\ 0.20 & 0.10 \end{bmatrix}$$

and Red's kill-rate matrix $[\beta]$ against Blue is

$$[\beta] = \begin{bmatrix} 0.15 & 0.10 \\ 0.20 & 0.05 \end{bmatrix}$$

Let the subscript 1 refer to riflemen and subscript 2 to mortars. Determine:

1. Relative values of all weapons employed in the engagement
2. Relative strengths of the forces

3. Equivalent force ratio
4. Percentage of losses for the winner assuming the loser breaks battle when he has sustained 30% casualties.

From the matrix $[\rho]$ we see, for example, that $\rho_{11} = 0.1$, or each Blue rifleman kills 0.1 Red rifleman per time unit of say one hour; and $\rho_{21} = 0.2$, or each Blue mortar can kill 0.2 Red rifleman per hour; etc. Similar statements may be made for $[\beta]$, or Red's kill rates of Blues. These kill rate matrices may have been estimated from calculations of hit and kill probabilities, and rates of fire for the weapons used under jungle conditions. Or perhaps, better still, they might have been determined from combat experience records of a similar engagement or from a realistic computer-simulated battle in jungle canopy—the better the kill rates, the better the final analysis. Accurate and relatively precise kill rates are necessary for any good rankings of weapon values or worths.

In line with Holter's assumptions, we will take the relative value of a Blue infantryman with rifle to be 1, i.e.,

$$w_{B1} = 1$$

and determine all other weapon values or weights with respect to a Blue rifle.

Following the step-by-step procedure previously given (Eqs. 30-37, 30-38, and 30-39), we initially set

$$[w_B] = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [w_B]^{(1)}$$

and calculate the product of the kill rate matrices

$$[\rho][\beta] = \begin{bmatrix} 0.1 & 0.04 \\ 0.2 & 0.10 \end{bmatrix} \begin{bmatrix} 0.15 & 0.10 \\ 0.20 & 0.05 \end{bmatrix} = \begin{bmatrix} 0.023 & 0.0120 \\ 0.050 & 0.0250 \end{bmatrix}$$

Then the iterations proceed as follows:

<u>1st Iteration</u>:

$$[W]^{(1)} = [\rho][\beta][w_B]^{(1)} = \begin{bmatrix} 0.023 & 0.0120 \\ 0.050 & 0.0250 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.0350 \\ 0.0750 \end{bmatrix}$$

$$\lambda^{(1)} = 1/W_1^{(1)} = 1/0.0350 = 28.5714$$

$$[w_B]^{(2)} = \lambda^{(1)}[W]^{(1)} = 28.5714 \begin{bmatrix} 0.0350 \\ 0.0750 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 2.1429 \end{bmatrix}$$

<u>2nd Iteration</u>:

$$[W]^{(2)} = [\rho][\beta][w_B]^{(2)} = \begin{bmatrix} 0.023 & 0.0120 \\ 0.050 & 0.0250 \end{bmatrix} \begin{bmatrix} 1.0000 \\ 2.1429 \end{bmatrix} = \begin{bmatrix} 0.0487 \\ 0.1036 \end{bmatrix}$$

$$\lambda^{(2)} = 1/W_1^{(2)} = 1/0.0487 = 20.5339$$

$$[w_B]^{(3)} = \lambda^{(2)}[W]^{(2)} = 20.5339\begin{bmatrix}0.0487\\0.1036\end{bmatrix} = \begin{bmatrix}1.0000\\2.1273\end{bmatrix}$$

<u>3rd Iteration</u>:

$$[W]^{(3)} = [\rho][\beta][w_B]^{(3)} = \begin{bmatrix}0.0485\\0.1032\end{bmatrix}.$$

$$\lambda^{(3)} = 1/W_1^{(3)} = 1/0.485 = 20.6186$$

$$[w_B]^{(4)} = \lambda^{(3)}[W]^{(3)} = \begin{bmatrix}1.0000\\2.1278\end{bmatrix}.$$

<u>4th Iteration</u>:

$$[W]^{(4)} = [\rho][\beta][w_B]^{(4)} = \begin{bmatrix}0.0485\\0.1032\end{bmatrix}$$

$$\lambda^{(4)} = 1/W_1^{(4)} = 1/0.0485 = 20.6186$$

$$[w_B]^{(5)} = \lambda^{(4)}[W]^{(4)} = \begin{bmatrix}1.0000\\2.1278\end{bmatrix}$$

Thus, three iterations were actually necessary, the fourth reproducing the third one. Hence, the final value of λ is

$$\lambda = 20.6186$$

and the weight or value vector for the Blue side is

$$[w_B] = \begin{bmatrix}1.0000\\2.1278\end{bmatrix}$$

or a Blue mortar is more than twice as valuable as a Blue rifle.

To obtain Red's weight or value vector, we calculate from Eq. 30-40

$$c = \sqrt{\lambda} = \sqrt{20.6186} = 4.5408$$

and from Eq. 30-41

$$[w_R] = \sqrt{\lambda}\,[\beta]\,[w_B] = 4.5408 \begin{bmatrix} 0.15 & 0.10 \\ 0.20 & 0.05 \end{bmatrix} \begin{bmatrix} 1.0000 \\ 2.1278 \end{bmatrix} = \begin{bmatrix} 1.6473 \\ 1.3913 \end{bmatrix}.$$

In summary, we have

$$w_{B1} = 1.00$$

$$w_{B2} = 2.13$$

$$w_{R1} = 1.65$$

$$w_{R2} = 1.39$$

and we see that a Red rifleman is 65% "more valuable" than a Blue rifleman (1.65 vs 1.00), that Blue's mortars are considerably more effective than Red's mortars (2.13 vs 1.39), and each Red mortar is only 39% more effective than a Blue rifleman (1.39 vs 1.00).

Continuing, Blue's overall unit or team effectiveness, measured in units of Blue riflemen, is

$$[w_B]^T[B] = [1.00 \quad 2.13] \begin{bmatrix} 80 \\ 24 \end{bmatrix} = 131.12$$

and that of Red is given by

$$[w_R]^T[R] = [1.65 \quad 1.39] \begin{bmatrix} 60 \\ 18 \end{bmatrix} = 124.02.$$

Hence overall, Blue is only slightly superior to Red, i.e., about (131.12 − 124.02)/124.02 = 6%.

The total value of all weapons on both sides is

$$(1)(80) + (2.13)(24) + (1.65)(60) + (1.39)(18) = 255.14$$

(summing Eqs. 30-1 and 30-2) or the equivalent of "255 Blue rifles" were involved in the conflict.

Holter (Ref. 1) shows that the "state equation", which relates Blue's and Red's equivalent strengths at any instant after the start of the battle [and given by his equation (24)] is

$$\{[1 - V_t^2(R)/V_0^2(R)]/[1 - V_t^2(B)/V_0^2(B)]\}^{1/2} = V_0(B)/V_0(R) = F_0 \qquad (30\text{-}42)$$

where:

$V_t(R)$ and $V_t(B)$ = strengths, respectively, of Red and Blue at time t

$V_0(R)$ = Red's worth at time zero = 124.02 from Eq. 30-2

$V_0(B)$ = Blue's worth at time zero = 131.12 from Eq. 30-1

and the fraction or force ratio F_0 is such that if one specifies the proportion of initial strength remaining on one side, i.e., a battle breakpoint, the corresponding fraction remaining on the opposing side may be determined from Eq. 30-42. In our example, F_0 = 131.12/124.02 = 1.057.

Thus, we see that Blue will be the victor and, if Red breaks battle after 30% casualties, then from Eq. 30-42 we get (since $V_t(R)/V_0(R) = 0.7$)

$$\{[1 - (0.7)^2]/[1 - V_t^2(B)/V_0^2(B)]\}^{1/2} = 1.057$$

or solving for the fraction $V_t(B)/V_0(B)$, of remaining Blues, we obtain

$$V_t(B)/131.12 = 0.737$$

or that is to say Blue has already suffered 26.3% equivalent casualties when Red breaks battle at 30% equivalent casualties. At this stage, Blue has left the equivalent of (0.737)(131.12) = 96.6 riflemen and Red has left (0.70)(124.02) = 86.8 equivalent Blue riflemen.

Finally, as another significant contribution, Holter established in Ref. 1 that the battle time t as a function of the fraction of initial Red strength remaining, is given by the expression

$$t = c \ln\left(\frac{\sqrt{F_R^2 + F_0^2 - 1} - F_R}{F_0 - 1}\right) \tag{30-43}$$

where

F_0 = relative force ratio as in Eq. 30-42, and

$F_R = V(R)/V_0(R)$ = fraction of Red relative initial strength remaining at time t. (30-44)

For the battle analyzed here, we use $c = 4.5408$, $F_R = 0.70$, $F_0 = 1.057$, and obtain the total engagement time of

$$t = 1.50 \text{ h or } 1 \text{ h and } 30 \text{ min.}$$

As a somewhat separate approach, suggested by Mr. Roger Willis of the US Army Training and Doctrine Command Systems Analysis Activity (TRASANA), we might return to Eqs. 30-28 and 30-29 and let Blue have b weapon types, but Red have only $r = 1$ weapon type. In this case, it can be shown that the weight for the ith Blue type of weapon is

$$w_{Bi} = w_{R1}\rho_{1i}/\rho_R, \quad \text{for each i.} \tag{30-45}$$

Also,

$$w_{R1}\beta_B = \sum_{i=1}^{b} \beta_{1i} w_{Bi}. \tag{30-46}$$

Eqs. 30-45 and 30-46 are still equivalent to Eqs. 30-22 and 30-23.

From Eqs. 30-45 and 30-46, it is also seen that the product $\rho_R\beta_B$ of positive constants becomes

$$\rho_R\beta_B = \sum_{i=1}^{b} \beta_{1i}\rho_{1i} \tag{30-47}$$

where for convenience we may take

$$K_1 = \sum_{i=1}^{b} \beta_{1i}\rho_{i1} \qquad (30\text{-}48)$$

for known or estimated kill rates β_{1i} and ρ_{i1}. Thus, this means that ρ_R is constrained to be

$$\rho_R = K_1/\beta_B \qquad (30\text{-}49)$$

and in fact Holter's c of Eq. 30-34 then becomes

$$c = 1/\sqrt{K_1} \qquad (30\text{-}50)$$

and Thrall's suggestions in Eqs. 30-32 and 30-33 become, in effect,

$$w_{R1} = K_1/\beta_B \qquad (30\text{-}51)$$

where β_B is still determined from Thrall's Eq. 30-33 or

$$\beta_B = \sum_{i=1}^{b} w_{Bi} \, . \qquad (30\text{-}52)$$

Finally, if we set for convenience

$$\beta_B = 1 \qquad (30\text{-}53)$$

then

$$\rho_R = K_1 \qquad (30\text{-}54)$$

and if we also take as a convenience—similar to Holter's assumption Eq. 30-35—that $w_{R1} = 1$, then from Eq. 30-46

$$\sum_{i=1}^{b} \beta_{1i} w_{Bi} = 1 \qquad (30\text{-}55)$$

and for each i we have from Eq. 30-45 that

$$w_{Bi} = \rho_{i1}/K_1 \qquad (30\text{-}56)$$

or that is to say each weight w_{Bi} for the Blue weapon types is uniquely determined from

$$w_{Bi} = \rho_{i1}\left(\sum_{i=1}^{b} \beta_{1i}\rho_{i1}\right)^{-1} \qquad (30\text{-}57)$$

We have a further useful inference of Willis; even if Red has r different weapon types, then Eqs. 30-14, 30-15, 30-28, and 30-29 imply for each $j = 1, 2, \ldots, r$ that

$$\rho_R \beta_B = \sum_{i=1}^{b} \sum_{\ell=1}^{r} \rho_{i\ell} \beta_{ji} w_{R\ell} / w_{Rj}. \tag{30-58}$$

In particular, if Red has two weapon types, i.e., $r = 2$, then the relative weight or worth of Red type 2 weapon to Red type 1 weapon is

$$w = w_{R2}/w_{R1} \tag{30-59}$$

where w is found from the quadratic equation

$$Bw^2 + (A - C)w - D = 0 \tag{30-60}$$

where

$$A = \sum_{i=1}^{b} \beta_{1i} \rho_{i1} \tag{30-61}$$

$$B = \sum_{i=1}^{b} \beta_{1i} \rho_{i2} \tag{30-62}$$

$$C = \sum_{i=1}^{b} \beta_{2i} \rho_{i2} \tag{30-63}$$

$$D = \sum_{i=1}^{b} \beta_{2i} \rho_{i1}. \tag{30-64}$$

Moreover, if for convenience concerning relative values or weights we put $w_{R1} = 1$, then w_{R2}, and hence w, becomes the positive solution of the quadratic Eq. 30-60. Furthermore, the worth or weight of the ith Blue type weapon is

$$w_{Bi} = (1/\rho_R)(\rho_{i1} + w_{R2}\rho_{i2}) \tag{30-65}$$

and Holter's λ becomes

$$\lambda = (A + Bw_{R2}/w_{R1})^{-1}. \tag{30-66}$$

Hence, for some particular cases it is possible to calculate c, or λ, of Eq. 30-40 directly without any need for iterations. The analyst may well, therefore, keep such results ready for reference when needed.

The methods of analysis we have just discussed are clearly of much importance in weapon systems analysis studies, and especially for battle analyses, or simulations or war games involving heterogeneous forces. For example, suppose that a computer simulation of opposing heterogeneous forces is carried out, and it is found that Blue lost 25 tanks, 8 howitzers, 40 machine guns, 125 rifles, 21 antitank weapons, and 15% of its personnel. In this same simulation, suppose that Red lost 22 of its tanks, 9 artillery pieces, 45 machine guns, 100 rifles, 19 hand-held antitank weapons, and 18% of its personnel. Then a simple question, but a very important one is, "Who has really won?" Recall that such a simulation was not only expensive, but also was conducted to settle important questions concerning some new weapon mix or force structure, and moreover that such (perhaps confusing) results are to be expected! One easily may see that it certainly seems reasonable to measure the worth of weapons on a side in terms of their potential or capability to destroy or attrit the enemy's weapons and personnel. Now especially, if the numbers of kills on each side are relatively equal—thereby bringing about some confusion—then there is indeed a rather formidable problem of reaching the right decision concerning overall force effectiveness. Hence, one easily should see that the use of weapon equivalence studies are certainly convenient for overall judgments.

These considerations are amplified more fully in the analysis in par. 30-4 of "killer-victim" scoreboards and Johnsrud's treatment (Ref. 6) of it.

30-4 ANALYSIS OF KILLER-VICTIM SCOREBOARDS

The killer-victim scoreboard is a matrix showing how many of each type of weapon were eliminated in an engagement by each weapon on the opposing side. The killer-victim scoreboard may be determined at specified or truncated times of a battle, or it may be obtained as a summary for the results of the entire battle. A typical killer-victim scoreboard is given in Table 30-1.

In Table 30-1, the killer-type weapons for Blue and Red are listed on the left as rows, and the numbers of Blue and Red weapons attrited from the battle are listed as elements of the matrix (columns). Thus, during battle time t a Blue type 1 weapon attrited a_{14} Red type 4 weapons and a_{15} Red type 5 weapons. There are three different types of Blue weapons, e.g., rifles, tanks, and artillery, and only two types of Red weapons, i.e., machine guns and tanks. The different types of Blue and Red are numbered together, with the smaller integer subscripts identifying the Blue side and the larger integer subscripts identifying types of Red weapons. This is a matter of convenience to group Blue and Red weapon types together. Note in Table 30-1 that there are no Blue victims from Blue weapons, as should be expected ordinarily, and no Red victims due to Red weapons. In general,

a_{ij} = number of j type weapons eliminated by weapons of type i in some time t.

For example,

a_{43} = number of Blue type 3 weapons (artillery) eliminated by Red type 4 weapons (machine guns)

TABLE 30-1. KILLER-VICTIM SCOREBOARD

	Killer Weapons	Victim Weapons				
		Blue			Red	
		1	2	3	4	5
Blue	1	0	0	0	a_{14}	a_{15}
	2	0	0	0	a_{24}	a_{25}
	3	0	0	0	a_{34}	a_{35}
Red	4	a_{41}	a_{42}	a_{43}	0	0
	5	a_{51}	a_{52}	a_{53}	0	0

a_{34} = 0, or Red tanks don't kill Red machine guns

a_{ij} = 0, when the subscripts i and j both refer to either Blue weapons alone or Red weapons alone.

The killer-victim scoreboard may be converted easily to a (combined) kill-rate matrix of elements, such as we have already discussed in connection with Eqs. 30-6 and 30-7. Suppose we let

n_i = number of weapons of type i

and

t = time of battle (completed or truncated).

Then, if we take any element a_{ij} of the killer-victim matrix and divide it and other elements of that row ("killer" weapon type i) by the product of the number of killer weapons and time t of the battle considered, we obtain new elements b_{ij}, where

$$b_{ij} = a_{ij}/(n_i t) = \text{the relative kill rates of type } i \text{ weapons against type } j \text{ targets, or kills per shooting weapon } i \text{ per unit of time against type } j \text{ targets.} \quad (30\text{-}67)$$

Hence, if we now replace the a_{ij} in Table 30-1 with the b_{ij} of Eq. 30-67, we get a new matrix or "kill-rate scoreboard" which has zeros in the upper left b-rows and b-columns for Blue weapons, zeros in the lower right r-rows and r-columns for Red weapons, whereas the upper right b-rows by r-columns represent the kill-rate matrix $[\rho]$ of Eq. 30-7 for Blues against Reds, and the lower left r-rows by b-columns represent the kill-rate matrix $[\beta]$ of Eq. 30-6 for Reds killing Blues. Further correspondence between the combined Blue and Red or killer-victim scoreboard and the treatment of weapon equivalence studies in pars. 30-1 through 30-3 indicate also for cross-reference purposes that we may equate

$n_1 = B_1$ type 1 Blue weapons

$n_2 = B_2$ type 2 Blue weapons

$\vdots$

$n_b = B_b$ type b Blue weapons

$n_{b+1} = R_1$ type 1 Red weapons

$n_{b+2} = R_2$ type 2 Red weapons

$\vdots$

$n_{b+r} = R_r$ type r Red weapons.

$b_{ij} = \rho_{ij}$ for $i = 1,\ldots,b$ and $j = b + 1,\ldots,b + r$ of Eq. 30-67

$b_{ij} = 0$ for $i = 1,\ldots,b$ and $j = 1, 2,\ldots,b$ of Eq. 30-7

$b_{ij} = \beta_{ij}$ for $i = b + 1,\ldots,b + r$ and $j = 1, 2,\ldots,b$ of Eq. 30-67

$b_{ij} = 0$ for $i = b + 1,\ldots,b + r$ and $j = b + 1,\ldots,b + r$ cf Eq. 30-6.

We now follow the analysis of Johnsrud (Ref. 6), which is aimed at the problem of using the killer-victim scoreboard for fighting among heterogeneous forces, and showing how to uncouple the interactions among weapons and derive an equivalent force ratio which will accurately reflect the course of the battle. Thus, he answers the question, "How does one analyze results from heterogeneous force battles?". As pointed out by Johnsrud (Ref. 6), "To date such time-dependent (killer-victim) scoreboards are not being used in weapon effectiveness studies because nobody knows how to extract all the information they contain.". His paper, therefore, is a contribution toward helping in this regard.

Johnsrud's analysis (Ref. 6) converts the killer-victim scoreboard to a matrix of kill rates b_{ij} as previously discussed and then sets up relative value equations for each weapon as follows, where we illustrate for the particular example of Table 30-1 with a_{ij} now replaced by b_{ij}:

$$
\left.\begin{aligned}
kW_1 &= b_{14}W_4 + b_{15}W_5 \\
kW_2 &= b_{24}W_4 + b_{25}W_5 \\
kW_3 &= b_{34}W_4 + b_{35}W_5 \\
kW_4 &= b_{41}W_1 + b_{42}W_2 + b_{43}W_3 \\
kW_5 &= b_{51}W_1 + b_{52}W_2 + b_{53}W_3.
\end{aligned}\right\} \qquad (30\text{-}68)
$$

Here, k is a constant of proportionality. Thus, one has a set of $n = 5$ homogeneous linear equations in $n = 5$ unknowns, and it is known that a solution for the values—weapon relative worths or weights W_i—is suitable when k is the largest eigenvalue of the ($n \times n$, or here) 5×5 matrix. This leads to Johnsrud's "worth" matrix, or really the expanded matrix of k's and kill rates for both sides given by

$$
\begin{matrix} \underline{W_1} & \underline{W_2} & \underline{W_3} & \underline{W_4} & \underline{W_5} \end{matrix}
$$

$$
\begin{bmatrix}
 & 0 & 0 & b_{14} & b_{15} \\
0 & -k & 0 & b_{24} & b_{25} \\
0 & 0 & -k & b_{34} & b_{35} \\
b_{41} & b_{42} & b_{43} & -k & 0 \\
b_{51} & b_{52} & b_{53} & 0 & -k
\end{bmatrix} \qquad (30\text{-}69)
$$

Due to the zeros, for which it is assumed Blue weapons don't kill other Blues and Red weapons don't kill other Reds, it is easy to see in general—for any number of weapon types on each side—that the corresponding matrix of any size may be expressed as

$$
\begin{bmatrix}
-k[I] & [\rho] \\
[\beta] & -k[I]
\end{bmatrix} \qquad (30\text{-}70)
$$

where $[I]$ is the identity matrix, and $[\beta]$ and $[\rho]$ are the usual kill-rate matrices of Eqs. 30-6 and 30-7 of Red vs Blue and Blue vs Red, respectively. Furthermore, and as Johnsrud (Ref. 6) points out, the largest eigenvalue, or characteristic root k, may be found from the determinantal equation

$$
|k^2[I] - [\beta][\rho]| = 0 \qquad (30\text{-}71)
$$

where the left-hand side determinant is the difference between two determinants, one consisting of only k^2 and the other one involving elements obtained simply from the product of the two kill-rate matrices of Blue and Red weapons—in either order, incidentally. The order of the determinant in Eq. 30-71 is the same as that for the product of the kill-rate matrices.

For Example 30-1, the product of the kill-rate matrices taken there as $[\rho][\beta]$ is

$$
[\rho][\beta] = \begin{bmatrix} 0.023 & 0.012 \\ 0.050 & 0.025 \end{bmatrix}
$$

and hence for that example, Eq. 30-70 becomes the determinantal equation

$$|k^2[I] - [\rho][\beta]| = \begin{bmatrix} k^2 - 0.023 & -0.012 \\ -0.050 & k^2 - 0.025 \end{bmatrix} = 0$$

or

$$k^4 - 0.048k^2 + (0.023)(0.025) - (0.050)(0.012) = 0$$

or

$$k^4 - 0.048k^2 - 0.000025 = 0.$$

The solution of this quadratic equation of k^2 yields two values of k^2, i.e.,

$$k^2 = 0.048515 \text{ and } k^2 = -0.0005153,$$

and the largest eigenvalue is

$$k = 0.22026.$$

Since Eqs. 30-68 are the same as Eqs. 30-22 and 30-23, with $\rho_R = \beta_B = k$, we have consistent with Eq. 30-34 that

$$c = \sqrt{\lambda} = 1/k = 4.5401. \tag{30-72}$$

Recall that with the iterations of par. 30-3, we obtained

$$c = 4.5408$$

there being a difference of 7 in the fourth decimal place due no doubt to not carrying enough significant figures at each stage of the iterations. For our example, the value of $c = 4.5401$ is likely the more accurate determination.

We may now proceed as before to obtain the relative values or weights for Blue and Red weapons. Another procedure, however, using the largest eigenvalues of k is to return to the full matrix Eq. 30-69 and triangularize it, as does Johnsrud (Ref. 6), i.e., we start with k replaced by its largest eigenvalue

$$\begin{bmatrix} -0.22026 & 0 & 0.1 & 0.04 \\ 0 & -0.22026 & 0.2 & 0.10 \\ 0.15 & 0.10 & -0.22026 & 0 \\ 0.20 & 0.05 & 0 & -0.22026 \end{bmatrix}$$

and use the first row to produce zeros elsewhere in the first column, obtaining

$$\begin{bmatrix} -0.22026 & 0 & 0.1 & 0.04 \\ 0 & -0.22026 & 0.2 & 0.10 \\ 0 & 0.14684 & -0.22343 & 0.04 \\ 0 & 0.05507 & 0.1 & -0.20257 \end{bmatrix}$$

Then we use the second row to produce zeros for the elements in the second column below the diagonal, etc., obtaining

$$\begin{bmatrix} -0.22026 & 0 & 0.1 & 0.04 \\ 0 & -0.22026 & 0.2 & 0.10 \\ 0 & 0 & -0.13515 & 0.16 \\ 0 & 0 & 0.59996 & -0.71021 \end{bmatrix}$$

Finally, the triangularized matrix is:

$$\begin{matrix} \underline{W_1} & \underline{W_2} & \underline{W_3} & \underline{W_4} \end{matrix}$$

$$\begin{bmatrix} -0.22026 & 0 & 0.1 & 0.04 \\ 0 & -0.22026 & 0.2 & 0.10 \\ 0 & 0 & -0.13515 & 0.16 \\ 0 & 0 & 0 & 0.00001 \end{bmatrix}$$

The lower right element 0.00001 indicates a suitably accurate computation due to its smallness. Hence,

$$W_3 = (0.16/0.13515)W_4 = 1.1839W_4$$

$$W_2 = (0.2W_3 + 0.1W_4)/0.22026 = 1.5290W_4$$

$$W_1 = (0.1W_3 + 0.04W_4)/0.22026 = 0.7191W_4.$$

Therefore, in terms of the first Blue weapon, we get

$$W_4 = 1.39W_1$$

$$W_3 = 1.65W_1$$

$$W_2 = 2.13W_1$$

which are the same values obtained by iteration for Example 30-1.

In summary, we see that Holter's procedure of Ref. 1 and that of Johnsrud (Ref. 6) lead to the same results.* For general application, however, the Johnsrud procedure is easier to apply, since it is

*It can be shown through matrix manipulations, as proven by Mr. Ralph Shear of the BRL, that Johnsrud's analysis is really identical to that of Holter, although Johnsrud's method is more convenient computationally. (See also Ref. 10, p. 66.)

relatively straight-forward and easy to remember. Of course, once the killer-victim scoreboard is determined and the kill-rate matrices of Blue and Red known (whether from transformation of the killer-victim scoreboard or estimated otherwise), then the relative worths or values of all Blue and Red weapons can be found. For large scale problems, involving many types of weapons, one may desire access to a high-speed computer programmed for making such calculations.

Next, we give a further example indicating a quantity-quality type of trade-off.

EXAMPLE 30-2:

Under what conditions would parity have been achieved using the data of Example 30-1?

The question we are asking here is, "Under what conditions is the equivalent initial effectiveness or force ratio unity?" This may be determined by using Eq. 30-3 equated to unity. Now we know that Blue would win from the data of Example 30-1. However, equivalence means for this example that

$$w_{B1}B_1 + w_{B2}B_2 = w_{R1}R_1 + w_{R2}R_2$$

or

$$(1)(80) + (2.13)(24) = w_{R1}R_1 + w_{R2}R_2$$

or

$$w_{R1}R_1 + w_{R2}R_2 = 131.12\text{, as we already know.}$$

Thus, if Red were to use weapons of the same relative values, i.e., $w_{R1} = 1.65w_{B1}$ and $w_{R2} = 1.39w_{B1}$, then the numbers of Red weapons R_1 or R_2, or both, could be changed. Thus, for example, we might keep $R_1 = 60$ Red riflemen as before and add to the number of Red mortars. In this case

$$(1.65)(60) + 1.39R_2 = 131.12$$

or

$$R_2 = 23.11.$$

Thus, Red could employ 23 mortars in the battle instead of 18 to achieve equality. Alternatively, he could use more riflemen, or employ tactics which make both rifles and mortars more effective, etc., since a unique solution does not exist for this problem.

30-5 FORCE RATIO AT ANY BATTLE TIME

The equivalent force ratio of Blue to Red at time t of the battle may be found from

$$F = F(t) = \sum_{i=1}^{b} w_{Bi}B_{it} \Big/ \sum_{j=1}^{r} w_{Rj}R_{jt} \tag{30-73}$$

i.e., an expression similar to Eq. 30-3, but where now we designate specifically that

B_{it} = number of Blue type i weapons at time t

R_{jt} = number of Red type j weapons at time t

and w_{Bi} and w_{Rj} are as before the relative values or weights for Blue and Red type weapons, assumed to be constant over the battle.

Johnsrud (Ref. 6) also points out that the eigenvalue k, which obviously has the dimension of reciprocal time, i.e., t^{-1}, can be used—with the assumption of the Lanchester Square Law and constant worth matrix during the battle—to obtain the force ratio also as

$$F(t) = \frac{(F_0 + 1)\exp(-kt) + (F_0 - 1)\exp(kt)}{(F_0 + 1)\exp(-kt) - (F_0 - 1)\exp(kt)} \tag{30-74}$$

where

F_0 = initial force ratio, i.e., Eq. 30-3, for example

k = largest eigenvalue

t = time.

Johnsrud points out the importance of using the force ratio plotted as a function of time over the battle and in his example gives graphs describing the changes in it as the battle proceeds. Indeed, for any battle involving many different types of weapons on each side or unbalanced forces, the relative force ratio would seem to be a key and perhaps one of the simplest effectiveness values to study, and hence place final judgments upon.*

30-6 ADDITIONAL CONSIDERATIONS

Some of the fundamentals of weapon equivalence studies having been covered, there remain some particular points of interest to the analyst, and which are worthy of mention here.

Whereas many heterogeneous weapon systems studies may naturally be conducted or programmed on computers because of analytical complexity or to introduce more realism, the establishment of data relating to killer-victim scoreboards will often represent the basic information to analyze. Nevertheless, Johnsrud (Ref. 6) points out that sometimes the methods of weapon equivalence studies will lead to very peculiar results if the simulations are not realistic, or one permits the use of Lanchester type Square Law attrition all the way to extinction. He suggests nevertheless that sharp, and hence unrealistic, discontinuities may be eliminated by considering target availability. In fact, attrition rates might be generated which include target availability by using equations of the form

$$\frac{dR_j}{dt} = -\sum_{i=1}^{b} b_{ij}B_i[1 - \exp(-\alpha_{ij}R_j)] \tag{30-75}$$

where, as previously,

R_j = number of Red type j weapons

t = combat time

b_{ij} = kill rate of Blue type i weapons against Red type j weapons for "available" targets to shoot at

B_i = number of Blue type i weapons

and, in addition,

α_{ij} = detectability/availability factor for Blue-type i weapons attacking Red type j targets. (Hence, α_{ij} is the chance or fraction of Red type j target weapons detected and brought under fire, while $\alpha_{ij}R_j$ is the expected number.)

Similar equations hold for the Blue side.

*Other measures of effectiveness in certain situations might be more meaningful—such as loss ratio, difference in percent losses, ratio of percent losses, time required for Blue to obtain a given objective, percent reduction (or increase) in force ratio, or Red percent losses by the time Blue losses have reached a critical level.

Hence, as easily seen from Eq. 30-75, when Red targets are very plentiful, i.e., R_j is large, then the loss rate dR_j/dt (for Red) approaches the negative sum of terms for the Lanchester Square Law. On the other hand, if targets are not detected or are very sparse, i.e., $\alpha_{ij}R_j$ is small, then the term in brackets of Eq. 30-75 becomes approximately $\alpha_{ij}R_j$, and we see that Lanchester linear type attrition takes over. Thus, as we have noticed before, combat often proceeds along the lines of either the Lanchester square or linear type of attrition, and hence Eq. 30-75 or some similar law tends to generalize what might happen. Johnsrud (Ref. 6) suggests solving Eq. 30-75 numerically by advancing time in small increments (dt) and calculating the corresponding decrements dR_j in the R_j. Such a procedure may clearly lead to worthwhile studies of rather complex battles, once fairly realistic kill rates and target detection chances are available for typical engagements of interest.

Whereas we have indicated the problem of "reducibility" for the kill-rate matrices may not ordinarily be involved, especially since many analyses will involve competing or interacting weapon systems, there will be occasions for which one weapon system will have absolutely no kill potential against some weapons on the opposite side. For example, rifle bullets usually would have zero effect against armor. One way out of this difficulty perhaps is to redefine targets or weapon systems for the battle studied so that some interaction is likely. Hopefully such a technique might not affect any major conclusions of the study otherwise. Failing this, however, then zeros will appear in kill-rate matrices and still the problem demands proper analysis and must be treated, nevertheless. Howes and Thrall (Ref. 2) treat the problem of reducibility in a thorough manner, so we only sketch their use of recommended "ideal linear weights" here. By way of summary in fact, their ideal linear weights for the different Blue weapons against Red weapons, i.e., the column weight or value vector $[w_B]$ of Eq. 30-8, turn out to be

$$\begin{aligned} [w_B] &= [\rho][w_R]/([E_r]^T[w_R]) \\ &= [\rho][\beta][w_B]/([E_b]^T[w_B][E_r]^T[w_R]) \end{aligned} \tag{30-76}$$

where

$[\rho]$ = kill rate matrix of Blue against Red

$[w_R]$ = column weight vector for Red weapons

$$[E_r] = \frac{1}{r}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \tag{30-77}$$

= an equally weighted probability column vector with r elements of $1/r$ each

$[\beta]$ = kill rate matrix of Red against Blue

$$[E_b] = \frac{1}{b}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \tag{30-78}$$

= an equally weighted probability column vector with b elements of $1/b$ each.

Likewise, the column vector $[w_R]$ of Eq. 30-9 for the ideal linear weights of the different types of Red weapons turns out to be

$$\begin{aligned}[w_R] &= [\beta][w_B]/([E_b]^T[w_B]) \\ &= [\beta][\rho][w_R]/([E_r]^T[w_R][E_b]^T[w_B])\end{aligned} \tag{30-79}$$

Although Eqs. 30-76 and 30-79, for determining the relative weights or worths of Blue and Red weapons might seem "circular"—especially since Blue's weights in the top expression of Eq. 30-76 depend on Red's weights, or indeed, in the bottom expression of Eq. 30-76 Blue's weights depend on Blue's weights, so to speak (!)—Howes and Thrall (Ref. 2), nevertheless, show that a unique solution is obtainable. They define

$$[P_b] = [\rho][\beta] \tag{30-80}$$

$$[P_r] = [\beta][\rho] \tag{30-81}$$

$$\gamma = [E_b]^T[w_B][E_r]^T[w_R]. \tag{30-82}$$

This means that the ideal linear weights must satisfy the following equations and also be nonnegative and nonzero:

$$[P_b][w_B] = \gamma[w_B] \tag{30-83}$$

$$[P_r][w_R] = \gamma[w_R]. \tag{30-84}$$

These vectors can be used to compare or relate elements on each side of the expressions in Eqs. 30-83 and 30-84, so that the ideal linear weights are obtainable.

The desired weights are easily calculated, and we illustrate the process for our Example 30-1. To begin with, we know that the products of the kill-rate matrices, both $[\rho][\beta]$ and $[\beta][\rho]$, have the same eigenvalues. Moreover, for the data of our Example 30-1, we found that the eigenvalues of $[P_b] = [\rho][\beta]$ were

$$\gamma = k^2\,(\text{Johnsrud}) = 0.048515 \text{ and } -0.0005153.$$

Then, by substituting in both sides of Eq. 30-83, we see that from

$$[P_b][w_B] = \gamma[w_B]$$

we have

$$\begin{bmatrix} 0.023 & 0.012 \\ 0.050 & 0.025 \end{bmatrix} \begin{bmatrix} w_{B1} \\ w_{B2} \end{bmatrix} = 0.048515 \begin{bmatrix} w_{B1} \\ w_{B2} \end{bmatrix}$$

or

$$\begin{bmatrix} 0.023w_{B1} + 0.012w_{B2} \\ 0.050w_{B1} + 0.025w_{B2} \end{bmatrix} = \begin{bmatrix} 0.048515w_{B1} \\ 0.048515w_{B2} \end{bmatrix},$$

and since corresponding elements on the two sides must be equal, we have two equivalent linear equations in two unknowns, from which w_{B2} may be found (from either equation) in terms of w_{B1}. The solutions yields

$$w_{B2} = 2.1263w_{B1}.$$

Thus, the relative weight of w_{B1} to use may be determined fr[illegible] [illegible] action

$$w_{B1}/(w_{B1} + 2.1263w_{B1}) = 0.320$$

and that for the relative weight of w_{B2} is given by

$$2.1263/(1 + 2.1263) = 0.680$$

and hence Blue's ideal weights or relative value or weight (probability) vector for his weapons is

$$\begin{bmatrix} 0.32 \\ 0.68 \end{bmatrix}$$

and "circularity" is thus avoidable. However, Howes and Thrall (Ref. 2) give also other methods of solution, including an iterative technique starting with all weights for a side being equal.

In a corresponding manner, the appropriate and unique ideal linear weight vector for Red may be found to be

$$\begin{bmatrix} 0.542 \\ 0.458 \end{bmatrix}$$

Finally, we see that relatively all weapon values of each side may be converted to values in terms of the Blue type 1 weapon, obtaining

$$w_{B2} = (0.68/0.32)w_{B1} = 2.13w_{B1}$$

$$w_{R1} = (0.542/0.32)w_{B1} = 1.69w_{B1}$$

$$w_{R2} = (0.458/0.32)w_{B1} = 1.43w_{B1}.$$

In summary, the ideal linear weights are somewhat (slightly) different from those derived by Holter or Johnsrud—which were 1.00, 2.13, 1.65, and 1.39—although consistency of proportionality for each side, Blue or Red, does indeed exist. In addition, we must point out in particular that the Howes-Thrall ideal linear weight approach may be used either for irreducible or reducible matrices; and hence it possesses generality in determining useful relative weapon worths, based especially on the largest eigenvalues of matrices.

It is well to point out to the cautious reader and analyst that the killer-victim scoreboards are very sensitive to variations in the weapon-target allocations whether fixed throughout a model run of simulation or whether varying with time during the run. Moreover, relative weapon values should really be very dependent on terrain, or on tactics or the command function, or on a host of possibly other factors not usually represented in combined arms simulations or models such as darkness, smoke, and electronic countermeasures. Also, there might just be some interest in expanding the methods presented herein to the possibility of taking into account the stochastic nature (!) of combat and therefore using probability distriputions of outcomes (Chapter 28) as well as constraints involving the number of replications used and associated confidence limits, perhaps.

30-7 SUMMARY

We have covered some of the current approaches concerning analyses pertaining to weapon equivalence or relative weapon values, and have seen that the techniques involved are not only very useful but are needed to make judgments on the effectiveness of heterogeneous forces. Moreover, there now exists many Army weapon systems study requirements which could well make good use of the concepts covered herein since improved accuracy of decision is bound to result from the theories of Holter, Johnsrud, Howes and Thrall, *et al.* It is realized the kill rates (which continue to be the key analysis parameters) may be very time dependent for some applications; hence, this particular problem should be explored in some future weapon equivalence studies. Nevertheless, the analyst should seek out and apply the techniques discussed herein to his particular problems, and hence advance the knowledge and usefulness of his weapon systems analyses as an aid to the decision process for selecting improved weapons.

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CHAPTER 31
OPTIMAL FIRING POLICIES FOR SINGLE AND MULTIPLE WEAPONS

The problem of just how Blue should allocate the firing of his rounds when a Red enemy is approaching his position is discussed in this chapter. The optimal policy depends on the characteristics of Blue's weapons, including especially his hit and kill probability as a function of range to Red's position in front of Blue. The optimal policy of allocating rounds fired by Blue at different ranges depends also on the use of an appropriate "gain" function, which indicates relative values of stopping Red at various ranges in front of Blue's position. It is found that a good strategy is for Blue to allocate his firing in such a manner that he will reduce Red's survival chance to as low a value as possible as Red approaches and attacks him. Because of this, the survival chance of Red is derived, and the allocation of Blue's rounds determined on the basis thereof. The results attained are based on some research of Karlin, Pruitt, and Madow (Ref. 1).

Several examples are given, and it is found, perhaps somewhat surprisingly, that Blue should commence firing at engagement ranges somewhat before he "sees the white of Red's eyes".

31-0 LIST OF SYMBOLS

$a(s)$ = infinitesimal accuracy of fire function
b = constant in Eq. 31-43
c = constant in Eq. 31-19
$E(k)$ = expected number of kills for Blue against Red up to range s in front of Blue
e = Naperian logarithm base = 2.71828 . . .
$f(s)$ = functional condition required for an optimal firing allocation (see Eq. 31-26)
$f_i(s)$ = optimal firing policy for the ith Blue weapon
$g(s)$ = "gain" or value function for Blue killing Red at range s
$g'(s)$ = first derivative of $g(s)$
$g''(s)$ = second derivative of $g(s)$
h = Δs = small change in range s (Ref. 1 notation)
$h(s)$ = derivative in Eq. 31-25
$h_i(s_i)$ = function defined by Eq. 31-60
$h_{i-1}(t_{i-1})$ = function defined by Eq. 31-58
k = constant in Eq. 31-23
$k_i(s)$ = particular function of s for the ith Blue weapon (see Eq. 31-54)
M_i = upper finite bound on $\rho_i(s)$
m = constant in Eq. 31-24
m = designation for the mth hit
n = number of weapons
n_i = number of Blue weapons of type i
$P_\lambda(0)$ = Red's overall survival chance from a very remote range and all the way to Blue's position
$P_\lambda(s)$ = Red's chance of survival from a very remote range to distance s in front of Blue
$P'_\lambda(s)$ = first derivative of Red's survival chance
$P_\lambda^m(s)$ = probability of Red surviving m hits
$p_h(s)$ = probability of hitting at range s

$p_k'(s)$ = first derivative of $p_k(s)$
$p_k''(s)$ = second derivative of $p_k(s)$
$p_k(s)$ = Blue's combination hit and kill probability against Red at range s
$p_k'(s)$ = first derivative of $p_k(s)$
$p_k''(s)$ = second derivative of $p_k(s)$
$p_{ki}(s)$ = kill chance for Blue's ith type weapon at distance s
r = total number of different types of Blue weapons
s = distance from Blue's position
Δs = small change in s
s_0 = particular range from Blue's position, selected in some optimal manner
s_1 = cutoff range
s_2 = cutoff range
s_3 = cutoff range
(s_i, t_i) = interval in range for Blue to fire his ith weapon ($s_i < t_i$)
t = time
Δt = small change in time t
v = speed
Δv = small change in speed v
α = parameter for the exponential distribution, or mean distance to a hit or kill
α_i = exponential parameter for the ith Blue weapon
β = parameter of a distribution (see Eq. 31-12)
γ = parameter of a distribution (see Eq. 31-12)
δ = upper finite bound on the integral of the firing allocation function over all ranges (see Eq. 31-5)
δ_i = upper finite bound for an integral (see Eq. 31-51)
$\lambda(s)$ = kill "rate" of Blue's weapons against Red, as a function of the range s to Red's forward line of attack. $\lambda(s)$ is actually the "intensity" of firing for Blue at range s.
$\lambda_i(s)$ = $\rho_i(s) p_{ki}(s)$ = kill "rate" of Blue's ith weapon type against Red
$\rho(s)$ = firing "rate" policy for Blue, or the manner that Blue should allocate the firing of rounds at Red for range s
$\rho_0(s)$ = optimal firing policy for Blue
$\rho_i(s)$ = firing allocation factor for Blue's ith type weapon at range s
$\rho_i^0(s)$ = optimal firing policy for the ith weapon

31-1 INTRODUCTION

Our problem here is to develop a method for finding the best combination of weapons, or weapon systems, which should be employed by Blue in a rather general combat situation somewhere in the world. The approach in this chapter is different from that of studying two-sided conflicts of homogeneous or heterogeneous forces, as in Chapters 28 and 29, or that of using computer simulations of typical battles. In fact, and by way of contrast, the criterion adopted for consideration here is that of selecting a single weapon, or some group of different weapons, which will have the highest chance of defeating an enemy force as it approaches our position. Thus, there is the rather critical problem of determining the optimal or best firing policy for employing our weapons as the enemy approaches and attacks our position. Clearly, efficient use or employment of weapons could be of great importance. In particular, there is no point in wasting rounds on enemy forces at too great a range; accordingly, the policy for defense will depend strongly on engagement ranges. We now describe the approach to this problem and the analytical parameters of major interest.

We first visualize a Blue force in position, protecting some ground area, which will soon be under attack by a Red force moving into a posture for assault. The Red force, starting well out of range of fire of friendly Blue troops, approaches our Blue force position and begins to come within the range of fire of some Blue weapons. Later, and as time moves forward the Red forces come within the ranges of many of our Blue weapons which will take them under fire with the mission of trying to destroy as many Reds as possible before they overrun our position.

Blue has at its disposal for study and use various weapon mixes to accomplish its mission; the objective is to decide on the optimal mix of weapons and just how to fire them in the situation just described. Some of the weapons may consist of artillery for long range, but which naturally will have relatively low rates of fire. Other weapons will be designed for close-in fighting with less lethality per round but, nevertheless, will have higher rates of fire and are easily handled for short ranges of engagement. Thus, our interest centers around the best firing policies and the selection of the optimal mix, regardless of the different weapons involved or their particular physical characteristics.

The central and most important weapon parameters we consider here will be the delivery accuracy, or probability of hitting, the warhead lethality or weapon wounding power, and the rate of fire. Delivery accuracy of a weapon depends markedly on range to the target. The enemy often will approach our defended area from some remote location or range for which the chance of a hit will be near zero initially and, as he comes closer, the probability of hitting may get as high as unity. Thus, the whole range of hit probabilities may be experienced and therefore must be taken into account in the analysis. Lethality or wounding power will depend on the weapons used and the targets attacked. Considerations of target vulnerability must be adequately accounted for, and the weapon type and warhead selected to destroy approaching targets. Although for many point or hard targets we will employ the concept of the conditional chance that a hit is a kill, we also permit the single shot kill probability concept, including the delivery accuracy function as covered in Chapter 20. Rate of fire of the weapon is very important and, as we know, is a major factor determining the weapon kill rate for the engaged targets. We are accustomed to thinking about rate of fire as so many rounds per minute, for example, but here there is involved the critical problem of tactics which should tell us at what range to open fire and how many rounds proportionately should be fired upon the enemy as he closes to assault our position. Thus, we will be particularly interested in a possible optimal policy concerning what range to start firing and just how to best allocate rounds fired as a function of any given distance the enemy is from our position.

Our approach now will be that of trying to minimize the chance that enemy forces survive as they attack our position. Therefore, appropriate weapon employment as a function of range of engagement is clearly the major problem to study here. We will follow the approach of Karlin, Pruitt, and Madow (Ref. 1) with some modifications that are brought out clearly. Also, it is natural to study initially the problem for a single Blue weapon and develop the required methodology for this simplest case before proceeding to weapon mixes or the employment of combined arms.

31-2 ANALYSIS FOR A SINGLE BLUE WEAPON

We consider first a single Blue weapon used to defend our position, and postulate an engagement for which the enemy is approaching the Blue position at a constant rate of speed. Thus, we may be able to interchange the battle time t for the distance s the enemy is from us, or to "trade-off" time and distance, so to speak.

Karlin, Pruitt, and Madow (Ref. 1) develop methodology first for the problem of determining an optimal policy for firing a single weapon of known accuracy against the attacking enemy. The effectiveness in firing a single weapon is described by a nonhomogeneous Poisson process with variable intensity rate $\lambda(s)$ where s represents the distance of the Red enemy forces from Blue's position. The quantity $\lambda(s)$ amounts to an integral density function of the kills per unit distance advanced by the Red forces, and hence it is not a probability density function for our purposes here. Also note that the ordinary kills per unit of time are converted here since time for a constant speed may be transformed to an equivalent distance. Thus, if the forward speed of Red is v miles per hour and Δt is a small or moderate change in time, then the equivalent change in distance is Δs, where

$$\Delta s = v\Delta t. \tag{31-1}$$

Thus, for each increment of time Δt, the Red force has moved some distance Δs toward Blue. Since for equal intervals of distance along the ground, the kills of Red per unit distance will increase from near zero at large separation distances and approach unity for close in fighting; thus the $\lambda(s)$ is not a constant but an increasing variable. Hence we have a nonhomogeneous Poisson process for the expected number of Red kills within the intervals on range separation instead of a homogeneous Poisson process.

Karlin, Pruitt, and Madow (Ref. 1) decompose the variable kill rate $\lambda(s)$ into two parts. That is, they put

$$\lambda(s) = \rho(s) \cdot a(s) \tag{31-2}$$

where as they say, "$\rho(s)$ is associated with a firing policy and usually signifies the rate of firing", and "$a(s)$ is the infinitesimal accuracy function as a function of the distance s for the weapon being considered, i.e., the probability of hitting the enemy at the distance between $(s + h)$ and s, when engaging the weapon system at unit rate, is $a(s)h + O(h)$. By its very meaning, $a(s)$ is naturally assumed to be decreasing as the distance s increases and could possibly vanish for $s \geq s_*$." [$O(h)$ means "order of h" or a small distance, and their h = our Δs]. The intensity function $\lambda(s)$, depending on range, could however be built up from another approach that we now present.

For some weapons, it is natural and easy for some applications to treat the hit probability problem and the conditional chance that a hit is a kill separately. We know that the chance of a hit in a single round will vary drastically with target range and in fact will decrease almost in an "exponential" manner with increasing range to target. Hence, the hit probability $p_h(s)$ may be described for target distance s by an exponential type falloff law or

$$p_h(s) = \exp(-s/\alpha) \tag{31-3}$$

where

s = distance to target, or engagement range

α = measure of "mean" distance to a hit.

Eq. 31-3 gives a chance of a hit equal to unity at zero range and decreases to a hit probability of zero at infinite range. Thus, it may suffice for some of the applications within the scope of this chapter.

Also, for some hit probabilities the following form of the law depending on the square of target range may be used:

$$p_h(s) = \exp(-s^2/\alpha). \tag{31-4}$$

Other types of falloff laws giving a reasonable approximation to the hit probability over the distance separating forces also may be used. With regard to Eqs. 31-3 and 31-4, or some other hit probability functions decreasing with increasing distance to target, it is well to keep in mind that such a fit or law must be "analytically tractable".

For the kill function, it may be that the conditional chance that a hit is a kill is one, i.e., a "hit" is always a "kill", for engagement ranges of interest. On the other hand, for some targets such as tanks there may be some part of the presented area which is not penetrable due to "headon" armor, and the conditional chance that a hit is a kill will be less than one, say 0.8, and the value may not change very much for target ranges of interest.

For the attack of other targets, such as artillery attacking personnel groups or units, the fractional kill in the target area depends on the random fall of shot over the target, for which hit and kill chances must necessarily be combined, as covered in Chapter 20. (Moreover, the lethal area of an artillery round depends on angle of fall of the projectile.) In any event, and for either case, the hit and kill functions may be described analytically by a single function which will decrease to zero at the longer ranges, since a hit will have a very low chance of occurrence.

Now, for the rate of fire function, $\rho(s)$ of Ref. 1, some comments are also in order. Clearly, there will be a maximum value that the rate of fire can obtain physically. The number of rounds fired per unit of time is discussed in Chapter 16. However, as we have already brought out, in this chapter rate of fire is to be treated as an allocation of rounds over probable engagement ranges in a battle, so that the defender or Blue takes on Red targets at ranges he can kill them and hence reduces Red's chance of survival. The firing rate, or allocation factor, $\rho(s)$ is an ordinary density function and *not* a *probability* density function. Thus, the integral of $\rho(s)$ over all all possible ranges of engagement may be described by

$$\int_0^\infty \rho(s)ds \leq \delta \tag{31-5}$$

which indicates a constraint on the quantity of total firepower and where the constraint or limit δ is the total number of rounds available to fire at the enemy. The larger δ is, the greater the range at which Blue opens fire on Red in an optimal firing policy. If we put $\delta = 1$, we get the policy for firing only a single round within optimal range limits.

In summary, and with a somewhat different argument, we are supporting the formulation of Karlin, Pruitt, and Madow (Ref. 1) to deal with a variable kill-rate function $\lambda(s)$ depending on the separation distance s, which may be decomposed into two major parts, i.e.,

$$\lambda(s) = \rho(s)p_k(s) \tag{31-6}$$

where we replace the $a(s)$ of Ref. 1 by the function $p_k(s)$ defined as

$p_k(s)$ = combination hit and kill probability function.

Moreover, we note in particular that the combination hit and kill function or $p_k(s)$ is determined completely by the weapon-target characteristics and the engagement range. That is to say—and we stress the point also—that for an optimal firing policy only $\rho(s)$ is at our disposal to adjust for the purpose of allocating rounds at different firing ranges s, especially once $p_k(s)$ is fixed by use of a given weapon and the targets it attacks.

With this background, we consider a general firing policy $\rho(s)$ and note that its choice really determines $\lambda(s)$ of Eq. 31-6. Further, let us define $P_\lambda(s)$ as the probability that the enemy, Red, survives up to a distance s in front of Blue. (We assume that Red starts at infinite distance ∞ and may approach $s = 0$, our Blue position.) Then, as seen clearly (Ref. 1), we have the following relation for the decrease in Red's survival chance from $(s + \Delta s)$ to s:

$$P_\lambda(s) = P_\lambda(s + \Delta s)[1 - \lambda(s)\Delta s] \tag{31-7}$$

to order Δs. That is to say,

$$\frac{P_\lambda(s + \Delta s) - P_\lambda(s)}{\Delta s} = P_\lambda(s + \Delta s)\cdot\lambda(s) \tag{31-8}$$

and as $\Delta s \to 0$, the left-hand side of Eq. 31-8 becomes the derivative of $P_\lambda(s)$, i.e.,

$$P_\lambda'(s) = \lambda(s)P_\lambda(s). \tag{31-9}$$

Upon integrating Eq. 31-9, and using the fact that the enemy survives at infinite distance, i.e., $P_\lambda(\infty) = 1$, we get the chance of survival for Red up to distance s from Blue's position is

$$P_\lambda(s) = \exp\left[-\int_s^\infty \lambda(s)ds\right].^* \tag{31-10}$$

We reiterate that $\lambda(s)$ is the kills per unit distance or the intensity of kills at engagement range s, and the integral in the exponent of Eq. 31-10 gives the expected number of kills over the distances or engagement ranges up to point s traversed by Red in attacking Blue's defense. If to a distance s, the expected number of kills $E(k)$, given by

$$E(k) = \int_s^\infty \lambda(s)ds \tag{31-11}$$

is large, then Red's chance of survival becomes small, whereas if the expected number of kills $E(k)$ is small, in which case s would ordinarily be a remote distance from Blue's position, then Red's survival chance would be high up to the particular value of s considered. Our problem, therefore, is that of finding the relative allocation of rounds for all distances s, determined by the factor $\rho(s)$, in view of the existing hit-kill function $p_k(s)$, for Blue's weapon which will keep Red's chance of survival as low as possible. This should give some idea of just how the model considered here works, and the importance of Eq. 31-10.

*In calculating $P_\lambda(s)$, one should keep in mind that the integration really is from a very remote distance ∞ to the distance s from Blue's position as in Eq. 31-10, and not from Blue's position toward Red forces.

Although we have considered only a single weapon for Blue so far, it becomes clear, nevertheless, that if Blue employs n similar weapons, and they fire independently at the approaching Red forces, then $\lambda(s)$ in Eqs. 31-10 and 31-11 would be replaced by $n\lambda(s)$. That is to say, Red's survival chance would be reduced very significantly with increasing numbers n of Blue's weapons.

To fix ideas a bit, we now give two illustrative examples involving only a single weapon, then proceed to the determination of an optimal allocation function $\rho(s)$ for one weapon, and finally the use of weapons of different types.

EXAMPLE 31-1:

Consider a Blue rifleman taking on a Red rifleman, and the Blue rifleman has a hit function given by Eq. 31-3, i.e.,

$$p_h(s) = \exp(-s/\alpha)$$

which falls off exponentially with increasing range. (We consider only hit chances here and ignore kill chances for the moment, assuming that a hit is a kill.) Now consider also a rate of fire or allocation function for Blue which uses the policy of practically no firing at the longer ranges but increases firing substantially as the enemy rifleman approaches Blue's position. Clearly, for sake of illustration, an exponential function will also satisfy this requirement. Thus, we may take the allocation function as having the form

$$\rho(s) = \gamma\exp(-s/\beta) \tag{31-12}$$

where we may use two parameters, γ and β, for proper scaling.

Then, we see that Red's chance of survival up to a distance s from Blue is determined from

$$P_\lambda(s) = \exp\left[-\int_s^\infty \rho(s)p_h(s)ds\right]$$

$$= \exp\left\{-\int_s^\infty \gamma\cdot\exp[-(s/\beta) - (s/\alpha)]ds\right\}$$

$$= \exp\{-[\gamma\beta\alpha/(\alpha + \beta)]\cdot\exp[-(\alpha + \beta)s/(\alpha\beta)]\}. \tag{31-13}$$

To get Red's chance of survival over all engagement ranges, zero to infinity, then we may put $s = 0$ in Eq. 31-13 and obtain

$$P_\lambda(0) = \exp[-\gamma\alpha\beta/(\alpha + \beta)]. \tag{31-14}$$

Thus, Red's overall survival chance depends on the three parameters—α, β, and γ. Now if we normalize the allocation function, Eq. 31-12, to make it a relative frequency function for allocating rounds fired over all ranges, i.e., make it a probability distribution, then we must have

$$\gamma = 1/\beta \tag{31-15}$$

and Eq. 31-13 becomes a proper probability given by

$$P_\lambda(s) = \exp\{-[\alpha/(\alpha + \beta)]\bullet\exp[-(\alpha + \beta)s/(\alpha\beta)]\}. \quad (31\text{-}16)$$

Moreover, the value of Eq. 31-16 over all engagement ranges reduces to

$$P_\lambda(0) = \exp[-\alpha/(\alpha + \beta)] \quad (31\text{-}17)$$

and we see, for example, that if the parameters α and β are practically equal, then Red's survival chance over all engagement ranges becomes approximately

$$e^{-0.5} = 0.61$$

so that Blue's hit probability parameter α has to be many times that of β for the allocation function $\rho(s)$ to reduce Red's survival chance. But in any event, if Blue adopts the policy of defending against Red for all ranges, then he cannot reduce Red's survival chance below

$$e^{-1} = 0.37$$

for the assumptions considered here. Therefore, and more importantly, one observes that the exponential type allocation of Eq. 31-12 may be relatively poor indeed because it seems that Blue may be firing too many rounds close in, relatively speaking, and needs a different type of firing policy. The optimal firing policy for the hit probability function, Eq. 31-3, will be given in Example 31-3. By way of contrast, however, let us now give an example (Example 31-2) involving very different hit(kill) and allocation functions.

EXAMPLE 31-2:

Consider a simple hit and kill function that falls off linearly to zero with increasing distance to a point $s = s_0$ to Blue's front. Such a hit and kill function may be taken as

$$p_k(s) = \begin{cases} (1/s_0)(s_0 - s), & \text{for } 0 \le s \le s_0 \\ 0, & \text{for } s > s_0 \end{cases} \quad (31\text{-}18)$$

and note that

$$\int_0^\infty p_k(s)ds = \int_0^{s_0} \frac{1}{s_0}(s_0 - s)ds = s_0/2$$

and thus the integral of the hit and kill function cannot be regarded as a probability distribution function over ranges of interest. (The point $s = s_0$ may represent a sighting, detection, or hit probability limitation.)

Suppose further that the proportion of rounds fired per unit distance is a constant c (i.e., uniform) and given by

$$\rho(s) = \begin{cases} c\ , & \text{for } 0 \le s \le s_0 \\ 0\ , & \text{otherwise.} \end{cases} \tag{31-19}$$

Then we have

$$\int_s^\infty \lambda(s)ds = \int_s^{s_0} \rho(s)p_k(s)ds = \int_s^{s_0} (c/s_0)(s_0 - s)ds$$

$$= \begin{cases} [c/(2s_0)](s_0 - s)^2\ , & \text{for } 0 \le s \le s_0 \\ 0\ , & \text{for } s > s_0. \end{cases} \tag{31-20}$$

Red's chance of survival for these assumptions is

$$P_\lambda(s) = \begin{cases} \exp[-c(s_0 - s)^2/(2s_0)]\ , & \text{for } 0 \le s \le s_0 \\ 1\ , & \text{for } s > s_0. \end{cases} \tag{31-21}$$

Hence, for distances beyond the point $s = s_0$, Red's survival chance is unity, and at distances less than s_0 it is the first part of Eq. 31-21, while at $s = s_0/2$ it is

$$P_\lambda(s) = \exp(-cs_0/8) \tag{31-22a}$$

thereby depending only on the rounds fired per unit distance and the factor 8. Finally, Red's survival chance over all ranges of engagement

$$P_\lambda(0) = \exp(-cs_0/2) \tag{31-22b}$$

and hence depends on the constant level c of firing and s_0, as one would expect. It can be said that the point $s = s_0$ is rather arbitrary here, and simply for illustrative purposes, although it may still have some physical significance pertaining to the terrain or weapon, and hence be useful for analyses. In the sequel, we will find the optimal method of allocation for this case also, as determined in Ref. 1. However, this determination depends on the concept and use of a "gain" function.

31-3 OPTIMAL FIRING POLICY FOR A SINGLE WEAPON

In order to determine an optimal type of firing policy, Karlin, Pruitt, and Madow (Ref. 1) make use of the concept of a "gain" function. In other words, the amount gained or "value received" by Blue depends on the distance at which Red can be annihilated—since Blue would not like to have his position overrun nor would he like to risk too much to close-in fighting. Hence, it would appear that a gain function should be nonnegative, starting at zero for Blue's position and increasing to some finite and constant limit for the longer engagement ranges from Blue's position. The gain function used will clearly have a decided effect on the optimal firing policy or allocation function $\rho(s)$. In Ref. 1, two different and useful gain functions are suggested and illustrated, although the results of Ref. 1—and in particular their Theorems 1, 2, and 3—are quite general and apply to any reasonable gain functions which may be adopted in a given evaluation as we will see in par. 31-3.1.

The particular gain functions $g(s)$ used merely for illustration in Ref. 1 are:

$$g(s) = \begin{cases} ks\,, & \text{for } s \le s_0 \\ ks_0\,, & \text{for } s > s_0 \end{cases} \qquad (31\text{-}23)$$

and

$$g(s) = \begin{cases} m(s/s_0)^{\beta}\,, & \text{for } s \le s_0 \\ m\,, & \text{for } s > s_0 \end{cases} \qquad (31\text{-}24)$$

where the slope k and values m and β are positive constants chosen for given applications, and s_0 is some remote or key engagement range of interest to Blue at or before which he will try and stop Red.

It is easily seen that the gain function Eq. 31-23 increases linearly at a slope of positive inclination to the constant ordinate value ks_0 at the point or distance before Blue's position $s = s_0$ as in Fig. 31-1.

On the other hand, the gain function (Eq. 31-24) increases in a concave or convex manner, depending on whether β is less than one or greater than one, to a constant level m at $s = s_0$ and beyond. It is seen that one or the other of these two gain functions should satisfy some general practical requirements. Alternatively, gain functions may be used involving costs of not stopping the enemy approaching one's position, or some proper function of risks incurred by not stopping Red, etc. Such considerations will depend on the particular problem of analysis and the analyst's use of the best type of gain functions for the application.

Once the hit and kill function for the weapon has been determined, and the appropriate gain function selected for the particular analysis, then the allocation function or rate $\rho(s)$ is found based upon techniques of the calculus of variations (Ref. 1). Thus, calculus of variation theory determines the *analytical form* of $\rho(s)$ for Blue's firing policy to reduce Red's survival chance to near minimum as he approaches Blue in the attack.

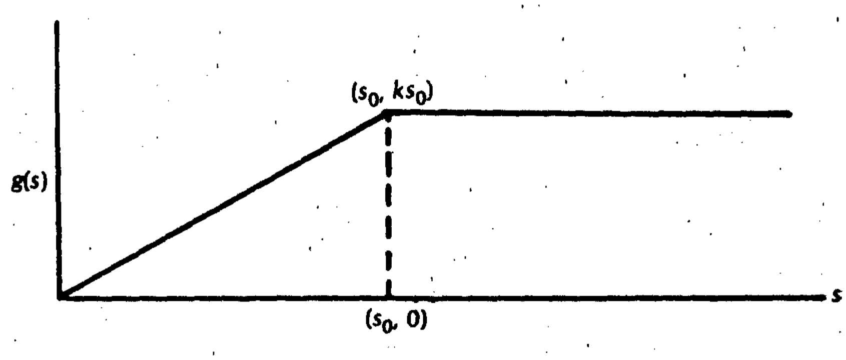

Figure 31-1. The Gain Function

A very useful theorem that applies to problems of the type given in our Examples 31-1 and 31-2 is the Theorem 2 of Karlin, Pruitt, and Madow (Ref. 1)—which we reword here somewhat for completeness and usefulness as follows:

Theorem 2 (Ref. 1):

Given the hit (and kill) probability function $p_k(s)$ and a gain function $g(s)$ for which there exists an engagement range $s = s_0$ such that the following conditions:

$$(1) \qquad \frac{d}{ds}[p_k(s)g(s)] = h(s) > 0 , \quad \text{for } s < s_0 \text{ and } h(s_0) = 0 \tag{31-25}$$

and

$$(2) \qquad f(s) = \frac{1}{p_k(s)} \left\{ \frac{p_k''(s)}{p_k'(s)} - 2\left[\frac{p_k'(s)}{p_k(s)}\right] - \frac{g''(s)}{g'(s)} \right\} > 0 \quad \text{for } s > s_0 \tag{31-26}$$

where the prime means the first derivative and the double prime means the second derivative, then the optimal firing policy has the form

$$\rho_0(s) = \begin{cases} 0 , & \text{for } 0 \le s \le s_0 \\ f(s) , & \text{for } s_0 < s \le s_1 \\ 0 , & \text{for } s_1 < s \end{cases} \tag{31-27}$$

and the value of s_1 is determined by the condition

$$\int_{s_0}^{s_1} f(s)ds = \delta. \tag{31-28}$$

(Recall that δ is based on a finite limit on the total available quantity of firepower expressed in Eq. 31-5.)

In order to illustrate this theorem we return to Example 31-1 and find the optimal $\rho(s)$ for a selection of the gain function (Eq. 31-23).

EXAMPLE 31-3:

Use the hit probability function of Example 31-1 and the gain function given in Eq. 31-23, then find the optimal allocation of Blue's rounds to be fired at Red, and contrast the optimal $\rho_0(s)$ with the negative exponential falloff law of Example 31-1.

We have, using Eqs. 31-3, 31-23, and 31-25 that

$$h(s) = \frac{d}{ds}[p_h(s)g(s)] = k\exp(-s/\alpha)(1 - s/\alpha) \tag{31-29}$$

which is positive for all values of $s < \alpha$, and becomes zero for the selection $s = s_0 = \alpha$. Further, by differentiation we get

$$p_h'(s) = -(1/\alpha)\exp(-s/\alpha), \quad g'(s) = k$$

$$p_h''(s) = (1/\alpha^2)\exp(-s/\alpha), \quad g''(s) = 0$$

so that the quantity $f(s)$ determined from Eq. 31-26 is

$$f(s) = (1/\alpha)\exp(s/\alpha) \tag{31-30}$$

and turns out to be, perhaps surprisingly, a *positive* exponential, or a predominant allocation of rounds at longer ranges instead of shorter ones as in Eq. 31-12.

Next we use Theorem 2, Ref. 1, and determine the optimal firing policy from Eq. 31-27 to be

$$\rho_0(s) = \begin{cases} 0, & \text{for } 0 \le s \le s_0 = \alpha \\ (1/\alpha)\exp(s/\alpha), & \text{for } \alpha < s \le s_1 \\ 0, & \text{for } s_1 < s \end{cases} \tag{31-31}$$

and where s_1 is determined from Eq. 31-28 for the total rounds available δ, i.e.,

$$\delta = \int_{\alpha}^{s_1} (1/\alpha)\exp(s/\alpha)ds = \exp(s_1/\alpha) - e$$

Thus, solving this equation for s_1, we get

$$s_1 = \alpha\ln(\delta + e). \tag{31-32}$$

The policy to fire no rounds at distances $0 \le s \le \alpha$ as indicated in Eq. 31-31 is no doubt surprising to many readers. However, this is not to say one would never fire if the enemy were there! Rather, it is to Blue's advantage to start firing at longer ranges where he has some reasonable chance of hitting Red.

In summary, all of the rounds available for Blue would be fired between the distance α in front of Blue, which is the parameter in Blue's hit probability function depending on the range s, and the opening engagement or firing range s_1 determined from Eq. 31-32. The fraction or percentage of rounds to be allocated and fired between two distances s_2 and s_3 bounded by $s = \alpha$ and $s = s_1 = \alpha\ln(\delta + e)$, i.e.,

$$\alpha < s_2 \le s \le s_3 < s_1 \tag{31-33}$$

may be determined from

$$\frac{1}{\delta}\int_{s_2}^{s_3} (1/\alpha)\exp(s/\alpha)ds = [\exp(s_3/\alpha) - \exp(s_2/\alpha)]/[\exp(s_1/\alpha) - e]. \tag{31-34}$$

Finally, the related probabilities of survival for Red at all distances s may be found with the aid of Eqs. 31-10 and 31-31, and are

$$P_\lambda(s) = \begin{cases} \exp\{-[\ln(\delta + e) - 1]\} & , \text{ for } 0 < s < \alpha \\ \exp\{-[\alpha \ln(\delta + e) - s]/\alpha\} & , \text{ for } \alpha \le s \le s_1 \\ 1 & , \text{ for } s_1 < s \end{cases} \tag{31-35}$$

and $s_1 = \alpha \ln(\delta + e)$ from Eq. 31-32.

As a matter of comparison, let us return to Example 31-1 using the negative exponential allocation function (Eq. 31-12), and for which we put $\delta = 1$, and hence found $\gamma = 1/\beta$. At the range $s = \alpha$, Red's survival probability for the negative exponential allocation may be determined from Eq. 31-13 and is

$$P_\lambda(\alpha) = \exp\left\{-\left(\frac{\alpha}{\alpha + \beta}\right)\cdot\exp\left[-\left(\frac{\alpha + \beta}{\beta}\right)\right]\right\} \tag{31-36}$$

No matter what the value of $\beta > 0$ used in Eq. 31-36, i.e., any positive multiple of α, it can be shown that the survival chance of Red to the distance α in front of Blue is never less than about

$$P_\lambda(\alpha) \approx 0.93.$$

On the other hand, from either the top or middle right-hand side of Eq. 31-35 for the optimum positive exponential allocation of Eq. 31-31, we get

$$P_\lambda(\alpha) = 0.73$$

and hence Red's survival probability is considerably less for the optimal firing policy (Eq. 31-27). In addition, one notes that the maximum range at which Blue starts the optimal firing policy, or shoots a round, is $s_1 = \alpha \ln(1 + e) \approx 1.31\alpha$, and the probability of hitting Red there is by Eq. 31-3

$$\exp(-1.31\alpha/\alpha) = 0.27$$

whereas for the distance $s = \alpha$ in front of Blue, where the firing allocation stops, Blue's hit chance against Red is

$$\exp(-\alpha/\alpha) = \exp(-1) = 0.37.$$

Thus, it can be said that the optimal policy is for Blue to start firing at Red before he "sees the white" of his enemy's eyes! Red's survival chance to $s = \alpha$ is by Eq. 31-35 $\exp(-0.31) = 0.73$ for $\delta = 1$ round, but is reduced to 0.58 for $\delta = 2$ rounds, etc.

The reader might wish to make other such comparisons for full appreciation of optimal firing policies. Of course, the actual form of the hit and kill functions, which depends on the weapon and target range, will have quite an influential effect on the optimal firing policies. Also, the choice of the gain function may require some skill for particular applications. For this reason, it will be informative to find next the optimal firing policy for the linear falloff hit and kill function of Example 31-2.

EXAMPLE 31-4:

Use the hit and kill function of Example 31-2. Find the optimal rate of fire policy for the gain function Eq. 31-23 and comment on it as compared to the uniform firing of rounds over engagement ranges used.

Using Theorem 2 (Ref. 1) again, we find that

$$h(s) = \frac{d}{ds}\{p_k(s)g(s)\} = \frac{d}{ds}\left[\frac{1}{s_0}(s_0 - s)ks\right] = k(s_0 - 2s)/s_0 \quad (31\text{-}37)$$

and since $h(s_0/2) = 0$ then $s_0/2$ is the lower cutoff range.

Further,

$$p_k'(s) = -1/s_0 \ , \quad g'(s) = k$$

$$p_k''(s) = 0 \quad , \quad g''(s) = 0$$

and from Eq. 31-26

$$f(s) = \left(\frac{s_0}{s_0 - s}\right)\left[-2\left(-\frac{1}{s_0}\right)\left(\frac{s_0}{s_0 - s}\right)\right]$$

or

$$f(s) = 2s_0/(s_0 - s)^2. \quad (31\text{-}38)$$

The upper bound or range for Blue to start firing at Red is

$$\int_{s_0/2}^{s_1} f(s)ds = \delta = 1, \text{ for illustration here,}$$

or

$$\left.\frac{2s_0}{s_0 - s}\right|_{s_0/2}^{s_1} = 1.$$

Hence, solving this latter equation for the bound s_1, we obtain

$$s_1 = 3s_0/5.$$

Hence, the optimal firing policy for this case is

$$\rho_0(s) = \begin{cases} 0 & , \text{ for } s < s_0/2 \\ 2s_0/(s_0 - s)^2, & \text{for } s_0/2 \leq s \leq 3s_0/5 \\ 0 & , \text{ for } 3s_0/5 < s. \end{cases} \quad (31\text{-}39)$$

Further, the chance of survival for Red is then found with the aid of Eqs. 31-10 and 31-18 to be

$$P_\lambda(s) = \begin{cases} 16/25 & , \text{ for } s < s_0/2 \\ 4s_0^2/[25(s_0 - s)^2] & , \text{ for } s_0/2 \le s \le 3s_0/5 \\ 1 & , \text{ for } 3s_0/5 < s. \end{cases} \quad (31\text{-}40)$$

In summary, we see that the optimal firing policy is not uniform as illustrated in Example 31-2, but for the reasonable gain function (Eq. 31-23), all of Blue's firing—as Red approaches and attacks Blue—should be between

$$s_0/2 \le s \le 3s_0/5$$

and according to proportions found from the integration of $\rho_*(s)$ in Eq. 31-39 over ranges of interest. In fact, it can be seen from Eq. 31-39 that for a single round, or $\delta = 1$, Blue should fire between $0.5s_0$ and $0.6s_0$! For $\delta = 2, 3$, etc., then s_1 increases to larger and larger values, i.e., Blue opens fire at greater and greater ranges.

It is of interest to plot Red's survival chances for Examples 31-3 and 31-4. This is done in Fig. 31-2, where it is seen for the same gain function it would be better for Blue to have weapons with hit and kill

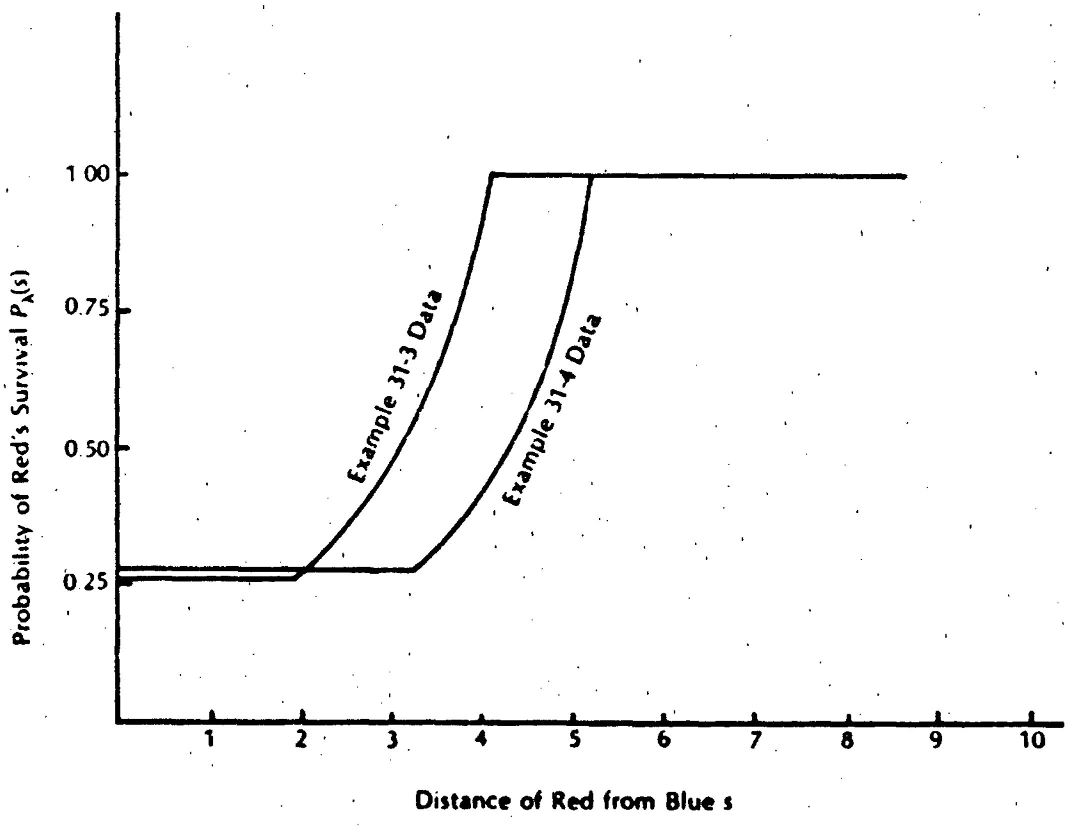

Figure 31-2. Survival Chance Curves for Red (Examples 31-3 and 31-4)

probability function falling off linearly rather than in a negative exponential manner. Of course, our coverage here is to be regarded mostly as an introduction, and hence that there remains much to be explored on the problem of optimal allocation of rounds for all weapon firings.

31-3.1 COMMENT ON UNBOUNDED HIT AND KILL FUNCTIONS

In our treatment of hit probability and kill probability functions so far in this chapter, we have, unlike Ref. 1, exhibited the use of hit probability and kill probability functions which start at the value of unity just in front of Blue's position, and then decrease to zero for the longer engagement ranges at which Red forces may begin to approach Blue. By observing the integral of the exponential in Eq. 31-10, but not including the negative sign, it can be seen that the value of the integral must be large to bring about a low or zero chance of survival for Red as he approaches and assaults Blue's position. Also, we have seen some of the limitations that might be involved with survival chances in Examples 31-1 and 31-2. This brings us to the concept of unbounded hit and kill functions, which are also included in Ref. 1, so that Eq. 31-10 may approach zero. However, it becomes of considerable interest first to discuss the physical units involved in the $\lambda(s)$, the $\rho(s)$, and the $a(s)$, remembering especially that so far in this chapter we have been using the concept of strictly a hit and kill function $p_h(s)$ or $p_k(s)$ in place of Karlin, Pruitt, and Madow's $a(s)$ in Ref. 1. Based on some queries about the paper of Karlin, Pruitt, and Madow (Ref. 1), the author of this handbook has, in a private communication (Ref. 2), received some clarification of the basic concepts these authors had in mind originally concerning their paper, and we give them here for the benefit of the user of this chapter. (We express our appreciation to Professor William E. Pruitt of the University of Minnesota for these clarifications.) To begin with, the units of $\lambda(s)$ are in 1/meters. The units for $\rho(s)$ are in rounds/meter, and those of $a(s)$ are in 1/rounds, which may be expressed also in terms of 1/meters divided by rounds/meter, if we so desire. Hence, we see that $\rho(s)a(s)$ multiplies properly to the units of 1/meters. However, this possible use of the units of (1/meters)/(rounds/meter) for $a(s)$ seems a bit strange, but it can be seen that the product $a(s)\cdot\Delta s$ could nevertheless be interpreted as "a probability per given unit firing rate". Then, for example, if the distance scale is changed from meters to centimeters then Δs will be multiplied by 100, but the new unit rate will be one round/centimeter, i.e., 100 times the old rate. Also, the probability per unit rate should then be 100 times as large as formerly. Hence, one may therefore see that there is much robustness in use of the product $\rho(s)\cdot a(s) = \lambda(s)$ of Ref. 1, as it is general enough to encompass a wide variety of situations. In particular, $a(s)$, for a very high weapon hit or kill rate, may even be an unbounded function near Blue's position and, for example, one may use $a(s) = 1/s$, in which case we then see that

$$\int_0^{\Delta s} a(s)ds = \infty \tag{31-41}$$

or that is the integral diverges near zero, regardless of how small the constraints on the firing policy $\rho(s)$ may be. Further, we get $P_\lambda(0) = 0$, or that is to say Red's survival chance is zero when he approaches all the way to Blue's position. Thus, the unbounded $a(s)$, if it fits, instead of our usual kill function $p_k(s)$, will guarantee "certain kills" On the other hand, high "firepower" or many rounds, in which case $\rho(s)$ is large, may give sufficiently small survival probabilities for Red also. In summary, Karlin, Pruitt, and Madow (Ref. 1), indicate that their theory is of sufficient robustness to use an unbounded $a(s)$ in place of our hit and kill probability functions, and suggest using, if they fit, either

$$a(s) = 1/s \tag{31-42}$$

or

$$a(s) = b/s^{\alpha}, \quad \alpha > 1 \tag{31-43}$$

along with the gain function (Eq. 31-23). Indeed, they show that if Eqs. 31-42 and 31-43 are adopted for use, then the optimal firing policy has the form

$$\rho_0(s) = \begin{cases} 0, & \text{for } s < \sqrt{\delta^2 + s_0^2} - \delta \\ 1, & \text{for } \sqrt{\delta^2 + s_0^2} - \delta \leq s \leq \sqrt{\delta^2 + s_0^2} \\ 0, & \text{for } \sqrt{\delta^2 + s_0^2} < s. \end{cases} \tag{31-44}$$

In other words, the optimal policy calls for the firing to be uniform over the interval of s given by the middle inequality of Eq. 31-44, and zero otherwise. Thus, all of the available rounds would be fired between

$$\sqrt{\delta^2 + s_0^2} - \delta \leq s \leq \sqrt{\delta^2 + s_0^2} \tag{31-45}$$

there being one round in each $1/\delta$ subinterval.

The chance of survival of Red in his attack of the Blue position for this particular case from Ref. 1 is

$$P_\lambda(s) = \begin{cases} [\sqrt{\delta^2 + s_0^2} - \delta]/\sqrt{\delta^2 + s_0^2}, & \text{for } s < \sqrt{\delta^2 + s_0^2} - \delta \\ s/\sqrt{\delta^2 + s_0^2}, & \text{for } \sqrt{\delta^2 + s_0^2} - \delta \leq s \leq \sqrt{\delta^2 + s_0^2} \\ 1, & \text{for } \sqrt{\delta^2 + s_0^2} < s. \end{cases} \tag{31-46}$$

Some other examples are given in Ref. 1

31-3.2 ADDITIONAL POINTS OF INTEREST

Concerning the general nature of all optimal firing policies, Karlin, Pruitt, and Madow (Ref. 1) list the following two properties, which may surprise some readers:

"1. If (the derivative) $g'(s)$ is bounded for $0 \leq s < s_0$, then the optimal policy never calls for firing at the maximum rate M in the neighborhood of $s = 0$.

"2. If $p_k(s)$ [or $a(s)$] is integrable at $s = 0$, and $g'(s)$ is bounded on $0 \leq s \leq s_0$, the optimal policy requires no firing in a neighborhood of $s = 0$."

31-4 OPTIMAL FIRING POLICIES FOR MIXTURES OF WEAPONS (HETEROGENEOUS SYSTEMS)

We now consider Red's chance of survival as he approaches Blue based on the assumption that Blue will, as in fact is often the case, fire different weapons at him. Thus, and more generally, let us assume that Blue has r different types of weapons and moreover may have several of each type to employ, there being n_i of type i. Then the different hit and kill functions may be defined as

$$p_{k1}(s), p_{k2}(s), \ldots, p_{kr}(s)$$

and the rate of fire or allocation functions as

$$\rho_1(s), \rho_2(s), \ldots, \rho_r(s).$$

It is easy to see, based on Eq. 31-10 for a single weapon, that Red's survival chance for the r different weapons now becomes

$$P_{\lambda}(s) = \exp\left[-\int_s^{\infty} \sum_{i=1}^{r} n_i\lambda_i(s)ds\right]$$

$$= \prod_{i=1}^{r} \exp\left\{-n_i\left[\int_s^{\infty} \lambda_i(s)ds\right]\right\} \quad (31\text{-}47)$$

since Red now has to survive all of Blue's weapons that bring fire on him.

Aside from the question of optimality of the firing policy as a function of ranges of engagement, Eq. 31-47 may be used for any group of Blue weapons when the kill rates

$$\lambda_i(s) = \rho_i(s)p_{ki}(s) \quad (31\text{-}48)$$

are known for each weapon system of Blue. In fact, a series of computations which show the kill rates of each type of weapon with range, and the number n_i, of each type may help to establish a near or suitable minimum for Red's survival chance.

Moreover, and as pointed out specifically by Ref. 1, a possible generalization of the basic theory could involve the concept that the damage done by a hit on Red is a random variable with different degrees of lethality. Hence, the damage done on successive hits may be cumulative, and the second, third, etc., hit by a weapon would bring about correspondingly higher degrees of damage to a target. In fact, the probability density of the mth hit on a target occurring in the interval of range from s to $(s + \Delta s)$ for a single weapon is a Poisson event, and may be described by

$$P_{\lambda}^{m}(s) = (1/m!)\left[\int_s^{\infty} \lambda(s)ds\right]^{m-1} \cdot \exp\left[-\int_s^{\infty} \lambda(s)ds\right] \quad (31\text{-}49)$$

This equation could be substituted for Eq. 31-10 to evaluate the gain accrued from each successive hit, especially when more than one hit would be necessary for a kill, as is the case currently for some well armored tanks. This type of concept brings forward a new and different area of interest for further study, but it will not be explored further here. Rather, we return to the basic problem of allocating rounds fired by different weapons for the case involving Eq. 31-10, but not Eq. 31-49, for multiple hits to obtain a kill.

Following our discussion of the optimal policy for firing a single weapon, it will be of some interest to make a few remarks about enemy targets, how they may approach Blue's position, and how Blue may fire on them. To begin with, and so far in this chapter, it has been more or less assumed that Red would move only one target forward at a time, so to speak, and that the firing policy we use would hopefully eliminate that particular target from the battle. In this connection, it could be considered that as far as Blue shooting a single weapon is concerned, then the problem could be looked at as if a single Red target is out front, or nearest Blue's position, and Blue would naturally engage that particular Red target. If that first target is eliminated under Blue's firing policy of allocating rounds, then Blue turns his attention to the next target that Red presents, and so on. In this way, Blue's single weapon attacks only one Red target at a time. Obviously there are certain weaknesses in this formulation. For example, as targets change, then the hit and kill function would also change, and this would surely complicate the analysis, although some switching of models might be feasible. Thus, we are merely suggesting some warnings concerning the possible applications of the methodology presented; we cannot develop this area of investigation any further here.

Now for the case of firing multiple weapons. Clearly, any worthwhile analysis involving multiple Blue weapons against multiple Red targets would immediately bring forward a host of many new problems. In fact, there is obviously the very important problem of allocating Blue weapons of different types to the various Red targets that are attacking the Blue position. In this connection, Ref. 1 does not introduce an analysis of this important allocation problem, and we will address only the problem of multiple Blue weapons which will bring fire on some "typical" Red target that will be closing in on Blue. Thus, the particular enemy force is somewhat hazy in concept. In Chapter 32, we do address the weapon-target allocation problem, but simply as a separate issue, so that the practicing analyst may apply it to a host of different problems he might face perhaps. Nevertheless, for the mixture of different types of weapons for Blue's defense, some general observations are in order and can be made here.

To begin with, if there are a number of different kinds of weapons which Blue will use against Red, then whenever two or more different types are being fired simultaneously by Blue, all but one type must necessarily be fired at the maximum intensity or rate. Moreover, the tendency will in fact be to fire different types of weapons at different times and different ranges because of their capability. This is simply because some types will be best for the longer ranges. Other types will be most suitable for the intermediate ranges, and finally others for the very short ranges. In this way, Blue will allocate the firing of his weapons so that their full potential will be used advantageously and effectively.

Another general observation concerns the firing of several weapons of the same type and is also pertinent here. A way of handling this type of problem is to note that one can approach the analysis be saying that the bounds in Eqs. 31-50 and 31-51 may be multiplied by the number of weapons n. In this way it is noted that, for the firing policy developed—call it $\rho(s)$—the firing policy for each weapon of that same type is taken as $\rho(s)/n$, a lower intensity but longer period.

Although much further research should be carried out for the multiple weapon problem, including especially the weapon-target allocation, there is a class of problems for which specific answers can be obtained. Again, it is necessary that the rate of fire or allocation functions, which depend on the firing range, be bounded, i.e.,

$$0 \le \rho_i(s) \le M_i \tag{31-50}$$

and

$$\int_0^\infty p_i(s)ds = \delta_i. \tag{31-51}$$

Then, Karlin, Pruitt, and Madow (Ref. 1) first assume that only one weapon of each type will be fired, and based on this have proved a theorem of some use. It is their Theorem 4, which uses the assumption of the unbounded hit and kill function of Eq 31-42. It also makes use of a common gain function $g(s)$ for all of the different Blue weapons fired at Red.

Theorem 4 (Ref. 1)

Suppose the hit and kill function designated as 1 for the first Blue weapon is unbounded at zero range, i.e.,

$$p_{k1}(0) = \infty^* \tag{31-52}$$

and tnat

$$(1) \quad p'_{k1}(s)/p_{k1}(s) < p'_{k2}(s)/p_{k2}(s) < \cdots < p'_{kr}(s)/p_{kr}(s) \tag{31-53}$$

and

$$(2) \quad k_i(s) = \frac{p''_{ki}(s)}{p'_{ki}(s)} - 2\left[\frac{p'_{ki}(s)}{p_{ki}(s)}\right] - \frac{g''(s)}{g'(s)} > 0 \tag{31-54}$$

for all values of the weapon type i and the range s. Also, and as before in Eq. 31-26, let us define $f_i(s)$ as

$$f_i(s) = k_i(s)/p_{ki}(s). \tag{31-55}$$

Then, if the $f_i(s)$ are specifically bounded, i.e.,

$$f_i(s) \le M_i \tag{31-56}$$

on the interval of range (s_i,t_i) given and determined by Eqs. 31-57 and 31-58, the optimal firing policy $p_i^0(s)$ for the ith weapon consists of firing that particular weapon with intensity $f_i(s)$ of Eq. 31-55 on the interval of range (s_i,t_i), where the ranges s_i and t_i are determined recursively from the equations

$$\int_{s_i}^{t_i} f_i(s)ds = \delta_i\,, \quad i = 1,2,\ldots,r \tag{31-57}$$

$$h_i(s_i) = h_{i-1}(t_{i-1})\,, \quad i = 2,\ldots,r \tag{31-58}$$

*The unbounded $p_{k1}(0)$ is not a necessary condition for Theorem 4, but is admissible even if it is unbounded.

and where also we take

$$s_1 = 0 \tag{31-59}$$

and

$$h_i(s) = g(s) + p_{ki}(s)g'(s)/p'_{ki}(s). \tag{31-60}$$

These relations are sufficient to determine the firing intervals (s_i,t_i), and firing allocation functions $f_i(s)$, uniquely (Ref. 1). We will illustrate this with an example.

EXAMPLE 31-5:

Blue's intelligence is such that he decides to stop a Red unit approaching his position by firing a 105-mm howitzer at the longer ranges and an 81-mm mortar at closer ranges if Red successfully gets through the area of artillery fire. Suppose that the mean distance to a kill for the 105-mm howitzer is 8000 m, that for the 81-mm mortar is 2000 m, and the hit and kill functions may be taken as approximately exponential. Consider also a gain function which starts at zero value just in front of Blue's position and increases exponentially to 0.99 at 10,000 m. What then would be Blue's allocation of rounds fired from the howitzer and mortar, and what would be Red's survival chance up to 1000 m in front of Blue's position?

To solve this problem, we will first set it up generally. Thus, we will use the subscript 1 for the mortar, and the subscript 2 for the howitzer. Then, take as in Eq. 31-3 the hit and kill functions for the mortar (1) and howitzer (2):

$$p_{k1}(s) = \exp(-s/\alpha_1) \qquad p_{k2}(s) = \exp(-s/\alpha_2).$$

Further, the derivatives are

$$p'_{k1}(s) = -\exp(-s/\alpha_1)/\alpha_1 \quad , \quad p'_{k2}(s) = -\exp(-s/\alpha_2)/\alpha_2$$

$$p''_{k1}(s) = \exp(-s/\alpha_1)/\alpha_1^2 \quad , \quad p''_{k2}(s) = \exp(-s/\alpha_2)/\alpha_2^2.$$

For the gain function, it is seen that a reasonable description meeting our requirements is given by

$$g(s) = 1 - \exp(-s/\beta) \tag{31-61}$$

and since the value of gain is to be equal to 0.99 at a range of 10,000 m for Blue killing Red there, then the parameter β is found from

$$\beta = \frac{s}{-\ln[1 - g(s)]} \tag{31-62}$$

or

$$\beta = \frac{10000}{-\ln(1 - 0.99)} = 2171.5 \text{ m}.$$

For the assumption (1), i.e., inequality (Eq. 31-53) of Theorem 4 (Ref. 1), we have

$$p'_{k1}(s)/p_{k1} = -\frac{1}{\alpha_1} = -1/(2000) = -0.0005$$

and

$$p'_{k2}(s)/p_{k2}(s) = -\frac{1}{\alpha_2} = -1/(8000) = -0.000125$$

so that assumption (1) is satisfied.

Now assumption (2), i.e., Eq. 31-54, gives $k_i(s)$ on substitution to be

$$k_i(s) = -\frac{1}{\alpha_i} + (2)\left(\frac{1}{\alpha_i}\right) + \frac{1}{\beta} = \frac{1}{\alpha_i} + \frac{1}{\beta} \tag{31-63}$$

for $i = 1$ and 2, and is greater than zero, where we have used

$$g'(s) = \exp(-s/\beta)\beta \quad \text{and} \quad g''(s) = -\exp(-s/\beta)\beta^2.$$

Thus, both assumptions (1) and (2) of Theorem 4 (Ref. 1) are satisfied.

Next, we find the $f_i(s)$ for $i = 1$ and 2 from Eq. 31-55, which are

$$f_1(s) = k_1(s)/p_{k1}(s) = \left(\frac{1}{\alpha_1} + \frac{1}{\beta}\right)\exp(s/\alpha_1) \tag{31-64}$$

and

$$f_2(s) = k_2(s)/p_{k2}(s) = \left(\frac{1}{\alpha_2} + \frac{1}{\beta}\right)\exp(s/\alpha_2). \tag{31-65}$$

Now, we start with $i = 1$ in Eq. 31-60 and find

$$\left.\begin{aligned} h_1(s) &= 1 - \exp(-s/\beta) + \exp(-s/\alpha_1)[\exp(-s/\beta)/\beta]/[-\exp(-s/\alpha_1)/\alpha_1] \\ &= 1 - \exp(-s/\beta)(1 + \alpha_1/\beta). \end{aligned}\right\} \tag{31-66}$$

Then using $s_1 = 0$ in Eq. 31-57, where we will take $\delta_1 = 1$ for illustrative purposes, one finds that the value of t_1 from

$$\int_0^{t_1} f_1(s)ds = \exp(t_1/\alpha_1)(1 + \alpha_1/\beta) - (1 + \alpha_1/\beta) = \delta_1 \tag{31-67}$$

generally, and solving finally for t_1 gives

$$t_1 = \alpha_1 \ln[(\delta_1 + 1 + \alpha_1/\beta)/(1 + \alpha_1/\beta)]. \quad (31\text{-}68)$$

Now since $\alpha_1 = 2000$, and $\beta = 2171.5$, the value of t_1 which makes the two sides of Eq 31-67 equal when $\delta_1 = 1$ is

$$t_1 = 838 \text{ m}.$$

Next, we use $i = 2$ in Eq. 31-58, which gives the relation

$$h_2(s_2) = h_1(t_1) \quad (31\text{-}69)$$

and with the aid of Eqs. 31-60 and 31-66 we see that

$$1 - \exp(-s_2/\beta)(1 + \alpha_2/\beta) = 1 - \exp(-t_1/\beta)(1 + \alpha_1/\beta). \quad (31\text{-}70)$$

On solving for s_2, we obtain generally that

$$s_2 = t_1 + \beta \ln[(1 + \alpha_2/\beta)/(1 + \alpha_1/\beta)]. \quad (31\text{-}71)$$

Substituting $\alpha_1 = 2000$, $\alpha_2 = 8000$, $\beta = 2171.5$, and $t_1 = 838$ into Eq. 31-71 yields

$$s_2 = 2773.5 \text{ m}$$

for the minimum range for the howitzer. This value of $s_2 = 2773.5$ then is finally used in Eq. 31-57 for $i = 2$, which gives

$$\int_{s_2}^{t_2} f_2(s)ds = \int_{s_2}^{t_2} \exp(s/\alpha_2)\left(\frac{1}{\alpha_2} + \frac{1}{\beta}\right)ds = \delta_2. \quad (31\text{-}72)$$

Upon integration, one easily finds solving for t_2 that

$$t_2 = \alpha_2 \ln[\exp(s_2/\alpha_2) + \delta_2/(1 + \alpha_2/\beta)]. \quad (31\text{-}73)$$

Upon substituting for all values into Eq. 31-73 including $\delta_2 = 1$, then we determine

$$t_2 = 3898 \text{ m}.$$

Hence, the optimal firing policy for Blue is to open fire on Red with his artillery weapon at a range no greater than about 3898 m, and firing a round between that range and no closer than 2774 m. Then Blue uses his mortar to fire a round from about a range of 838 m or soon thereafter as Red approaches Blue's position. Red's survival chance will depend on Blue's firing range.

The reader may verify that had two rounds been fired from the mortar, then the shooting of those two rounds may have commenced at $t_1 = 1427$ m instead of 838 m. Furthermore, had the howitzer shot three rounds, then the lower limit on range for firing the howitzer would have been at s_2 = 3362 m, and firing of the howitzer could have commenced at t_2 = 6171 m; a much longer range.

Thus, we have calculated the intervals or bounds on range for firing the mortar and the howitzer, either for single rounds each, or for two rounds for the mortar and three rounds for the howitzer. The optimal intensities for firing are determined by $f_1(s)$ of Eq. 31-64 for the mortar and $f_2(s)$ of Eq. 31-65 for the howitzer. By the use of Eqs. 31-47 and 31-48, the values of $p_{ki}(s)$ on page 31-21, and Eqs. 31-64 and 31-65 for $f_i(s)$, we are able to determine the survival chance of Red up to 1000 m in front of Blue's position to be 0.3%. Moreover, for the firing of two mortar rounds, the reader could put t_1 = 1427 m in Eq. 31-67 and show that δ_1 = 2; and he could substitute s_2 = 3362 m and t_2 = 6171 m in Eq. 31-72 to show that δ_2 = 3.

31-5 SUMMARY

We have given an introduction to the rather important problem of developing optimal firing procedures for a single weapon and for multiple weapons; these policies of allocating rounds being dependent on the range to enemy forces. Also, several examples have been given to acquaint the analyst with some of the types of applications he might face.

REFERENCES

1. Samuel Karlin, William E. Pruitt, and William G. Madow, "On Choosing Combinations of Weapons," Naval Research Logistics Quarterly **10**, pp. 95-119 (1963).
2. William E. Pruitt (University of Minnesota), personal communication to Dr. Frank E. Grubbs, 31 March 1977.

CHAPTER 32
WEAPON-TARGET ALLOCATION PROBLEMS

Weapons should be properly assigned to neutralize targets on the basis of their capability to do so. We therefore introduce and exhibit some of the better techniques for allocating weapons to targets in order to optimize effectiveness in some way. This may include criteria which minimize the potential threat of Red targets, or improve the worth of the attack to Blue, or reduce the logistic burden or cost. Weapon-target allocations involve operations research techniques such as Bellman's dynamic programming procedure, linear programming studies, the use of Lagrange multipliers, and other methods for finding maxima or minima subject to various constraints. We realize that weapon vs target allocations cannot be calculated in the field under normal combat conditions; however, the results of such studies can be used advantageously for improved training of commanders, or efforts to field better families of weapons, or for planning for the development and procurement of new weapons. Finally, we discuss random target allocation briefly.

We give a sufficient number of examples, which illustrate various applications, and the reader may extend his knowledge in appropriate directions by studying the References and Bibliography given herein.

32-0 LIST OF SYMBOLS

A_j = "common" factor of several values of $q_{ij} = (q_{1j})^{1/p_i}$

$[A]$ = $[r_{ij}]$ = coefficient matrix of the number of Blue i type rounds required to kill the jth Red target (type)

a_k = chance Blue destroys exactly k Red targets

a_0 = chance that both of two targets survive

c_1 = chance that exactly one of two targets is killed

a_2 = chance that both (of two) targets are killed

B = Blue force designation, or number of Blues

B_i = number of rounds (or sorties) available for the ith type of Blue weapon. (This may be a combat day of expenditure.)

B_j = number of Blue missiles assigned to the jth Red target

$\hat{B}_j$ = optimum value of B_j

$B_0 = \sum_{i=1}^{B} B_i$ = total number of Blue rounds, or weapons

$[b]$ = column vector of constants

b_{jk} = symbol in Eq. 32-31

c_{ij} = x_{ij}/t_j = commitment ratio of the ith type Blue weapon rounds to the jth Red target complex

$E(R_k)$ = expected number of Red targets killed

$f(R,B) = \sum_{j=1}^{R} t_j (w_j \prod_{i=1}^{B} p_{ij})^{-1/(B-1)}$

f_{ij} = relative frequency with which, or the chance that, the ith Blue weapon (type) takes on or is assigned to defend against the jth Red weapon (type). These relative frequencies may be zero in some cases or several Blue weapons may attack one Red target. $i = 1,2,\ldots,B$ and $j = 1,2,\ldots,R$.

f_{ij} = f = special case for f_{ij}

f_i, f_j = special cases of f_{ij} also

f_j = number of targets in the jth Red target class

$g_r(x,B)$ = minimum value of a sum in Bellman's dynamic programming technique (see Eq. 32-60)

$g_1(x,B)$ = $h_B^1(x)$ = first value of $g_r(x,B)$ attained when r = 1

$h_k^j(x_j)$ = certain minimum in dynamic programming (see Eq. 32-56)

$h_k^j(z_j)$ = minimum in dynamic programming based on weights z_j

$h_1^j(x_j)$ = $w_j(q_{ij})^{x_j}$ = initial value of $h_k^j(x_j)$ for i = 1

K = maximum number of times the Red threat may be defeated with Blue's available rounds

k = subscript, often used for an optimum condition

L_i = ρ_i/ρ_1 = normalization of the ρ_i with respect to the first ρ_1

L_i = slack variables (see Eq. 32-70)

P_k = chance Blue destroys at least k Red targets = $\sum_{j=k}^{R} a_j$

p_d = chance any Blue (the ith) searcher detects any Red (the jth) target

p_{ij} = conditional or average probability, for the engagement, that the ith Blue weapon (type) will destroy the jth Red target (type), given that the jth Red target survives all other Blue weapons. The p_{ij} may be determined or estimated by means of the methods in Chapters 14, 17, and 20.

p_{ij} = p = special case for p_{ij}

p_j = chance that a Blue missile kills the jth Red target

p_j = p, $j = 1, 2, \ldots, R$ is a special value of p_j

p_{ssk} = single-shot kill probability of Blue against Red

Q = $p_{ssk}[1 - (1 - p_d)^R]$ as in Eq. 32-82

q_{ij} = $1 - p_{ij}$ = chance that the jth Red target survives the ith Blue weapon

q_j = $1 - p_j$ = chance the jth Red target survives a Blue missile

q_j = q = special value of q_j

R = Red force designation, or number of Reds

R_k = number of Red targets killed

r = number of Red targets actually attacked

r_{ij} = number of rounds (sorties) of ammunition required to defeat the jth Red target with the ith Blue weapon

S_j = number of weapons (rounds) allocated to the jth target

S_j^* = optimum value of S_j (see Eq. 32-44)

T = threat, as measured by its worth or value

T_j = jth Red target class, referring to type of target, size, and location

t_j = number of target elements in the jth type Red target complex

V = total value to Blue, which is to be maximized, and which is given by (see Eq. 32-20)

$$V = \sum_{k=1}^{R} V_k a_k$$

V_k = value, or reduction in threat, to Blue for destroying exactly k Red targets. The Red targets are assumed to be more or less similar, although they could be of somewhat different types if the V_k and a_k take care of that possibility. (Note that V_1 = value of destroying one target, V_2 = value of destroying two targets, etc., so that the assumed targets are just about restricted to being of equal value, or that is $V_* = k\omega$, where ω is the value of a single target.)

$V_* = kv$ = special value of V_*

W = total worth

W = total allowable weight

$W(B)$ = total worth with respect to B

W_i = weight (in pounds) of one Blue i type round

W_V = particular worth given by Eq. 32-29

w_j = worth or value of the jth Red target. This may be estimated on a cost basis, measure of threat, risk, or by any other means of interest.

$w_j = w$ = special case for w_j

$w_j = v$ = special value of w_j

w_j = threat or value potential of the jth Red type target complex. (as before for simply the jth Red target)

$[x]$ = column vector to be solved giving the x_{ij}

x_{ij} = shortened designation for $x_{ij}(x_j)$ (see Table 32-2)

x_{ij} = number of rounds allocated for firing at the jth type Red target from the ith type Blue weapon

x_{ij} = number of targets in the jth Red class of a single threat assigned to the ith Blue weapon

x_j = integer value of x_{ij} (see Eq. 32-54)

x_* = total allocation of the x_j's

x_2 = computation for $x_{12}(x_1)$ (see Table 32-3)

$[y]$ = $[z_j/W_*]$ = greatest integer in [ ]

$y_j = \sum_{i=1}^{B} f_{ij}\ln(1 - p_{ij})$ = transformation given by Eq. 32-18

z_j = weight constraint (see Eq. 32-66)

$\ln\lambda = (\beta - B_*)/\gamma$ = "key" or parameter quantity given in Eq. 32-43

$[\ ]^T$ = transpose of a matrix

α = integer, 1, 2, ..., for number of Red kills specified

β = "key" quantity or parameter given in Eq. 32-41

γ = "key" quantity or parameter given in Eq. 32-42

$\eta_i = \sum_{j=1}^{R} t_j/p_{ij}$ (see Eq. 32-14)

ρ_i = exponent

ω = value of a single similar target

32-1 INTRODUCTION

Since the Army has a host of weapons, designed to attack various targets possessing different degrees of protection and located at different ranges, there naturally arises the problem of how best to employ different weapons in attacking the enemy or defending a position. Obviously, targets must be taken under fire as they appear on the battlefield and become a threat to friendly forces. Indeed, it often has been said that the target assignment problem is handled more or less through random occurrences; however, such random engagements often may result in much inefficiency, and it becomes of considerable importance to assign different weapons to particular targets, and hence not waste ammunition or effort unnecessarily. There is no point, for example, in firing small arms weapons that cannot penetrate armor at tanks, nor is it desirable to fire antitank rounds at large area targets which can better be neutralized by appropriate artillery fire. Then again weapons may be range-limited and hence cannot possibly take on targets beyond their range capability. Moreover, the delivery accuracy of weapons may be a limiting parameter, for so many rounds might be required to guarantee any high degree of assurance of killing a target that unwise firing policies would be the result if range-accuracy-lethality considerations are not taken into proper cognizance. Thus, completely random selection of and firing at targets cannot possibly be adopted as sound policy, although such may sometimes be necessary due perhaps to surprise. Planning of either the attack or the defense is always a worthwhile activity. In summary we might well point out that it is the function of command to conserve resources wherever possible, but at the same time to employ weapons in such a manner that the enemy will be denied ground and be annihilated promptly, once any conflict starts.

There are many ways or criteria that might be used to allocate weapons against targets, and the analyst will have to choose the more appropriate methods of approach for the particular problem at hand. Some very valuable bases which have been used in the past include allocation on the basis of maximizing the effectiveness of some family of weapons in damaging targets, or minimizing the cost or weight of ammunition required, or minimizing the total cost of the weapon family, and others. Then again, one may have to take into consideration time, available resources of all kinds, or consider secondary damage areas that might be effected, etc.

The problem of weapon-target allocation is somewhat different from that of force structure analyses. In the force-mix problem, one is interested in determining just what weapon mix should be developed for limits on funds and constraints on time to "maximize" the damage to a projected or hypothesized enemy force. On the other hand, for the weapon-target allocation problem one starts with the premise that given a weapon family of certain characteristics against certain targets appearing at different ranges, then just how should the weapons of the family be assigned for actual firing?

A review of the literature on the weapon-target matching or "missile allocation" problem to about 1970 is given by Matlin (Ref. 1). In Ref. 1, Matlin covers the model characterization, including the weapon system, the target complex, the engagement, the damage submodel, and the algorithm description. He also lists in his Table I (Ref. 1) many of the parameters involved for the attacker, the defender, the scenario, objectives, and intelligence problems. Matlin's Fig. 1 gives a flow chart as guidance for users, and his Fig. 2 summarizes the characteristics of the missile allocation submodel.

The types of models described by Matlin include the allocation model for a single weapon, the allocation models for multiple weapon types, game models, and some special feature models. Finally, Matlin gives abstracts of some 41 papers or reports concerning the weapon-target allocation problem.

We will first illustrate the weapon-target assignment problem by starting with a simple evaluation strategy.

32-2 A TYPICAL WEAPON-TARGET ALLOCATION MODEL

Let us start with a very typical weapon-target assignment problem. Suppose there are B Blue infantrymen with rifles in position and under attack by R Red riflemen approaching them. Then, let us use the following, rather general, notation to analyze this limited, homogeneous weapon situation.

Let:

f_{ij} = relative frequency with which, or the chance that, the ith Blue weapon (type) takes on or is assigned to defend against the jth Red weapon (type). These relative frequencies may be zero in some cases or several Blue weapons may attack one Red target. $i = 1, 2, \ldots, B$ and $j = 1, 2, \ldots, R$.

p_{ij} = conditional or average probability, for the engagement, that the ith Blue weapon (type) will destroy the jth Red target (type), given that the jth Red target survives all other Blue weapons. The p_{ij} may be determined or estimated by means of the methods in Chapters 14, 17, and 20.

w_j = worth or value of the jth Red target. This may be estimated on a cost basis, measure of threat, risk, or by any other means of interest.

Hence, the chance that the ith Blue weapon takes on the jth Red target and then defeats it is $f_{ij}p_{ij}$, and the chance that this does not occur, which means that the jth Red target survives, is

$$(1 - f_{ij})(1) + f_{ij}(1 - p_{ij}) = 1 - f_{ij}p_{ij}. \qquad (32\text{-}1)$$

Furthermore, an overall measure of the threat Blue should consider for the engagement is then easily seen to be the sum of worths for each Red target multiplied by the chance that no Blue weapons kill that Red target, i.e., the threat T is

$$T = \sum_{j=1}^{R} w_j \prod_{i=1}^{B} (1 - f_{ij}p_{ij}) \qquad (32\text{-}2)$$

and Blue will want to minimize this overall threat value subject to the conditions for assigning the ith Blue weapon, or

$$\sum_{j=1}^{R} f_{ij} = 1 \qquad (32\text{-}3)$$

and

$$f_{ij} \geq 0. \qquad (32\text{-}4)$$

Now having established a somewhat general formulation of the problem, let us return to the simple situation of Blue riflemen versus Red riflemen. In this case, we may just as well assume that each Blue rifleman has an equal chance of taking on a Red rifleman, and each Blue has the same chance of killing a Red, and all Red targets have equal value or threat to Blue. Thus, for such a homogeneous case of combat, the various factors simplify to

$$\left.\begin{aligned} w_j &= w \\ f_{ij} &= f \\ p_{ij} &= p \end{aligned}\right\} \text{for all } i, j. \qquad (32\text{-}5)$$

Further, Eq. 32-2 then collapses to the total threat value

$$T = wR(1 - fp)^B. \qquad (32\text{-}6)$$

To be very specific, recall that f in Eq. 32-6 is the chance or proportional frequency with which the ith Blue rifleman engages a Red rifleman, and p is the conditional probability that given a detection and an engagement that the ith Blue rifleman kills a Red rifleman. Thus, given p and the threat value w for each Red rifleman, then what should the relative frequency f be? Since the chances of engaging all Red targets by the ith Blue weapon sum to unity as in Eq. 32-3, and with the assumption that all f_{ij} are equal, then $Rf_i = Rf = 1$, or f has to be equal to $1/R$. That is to say, each Red rifleman will be taken on by a Blue rifleman with the equal relative frequency of $1/R$. This may be proven more rigorously by setting $f_{ij} = f_j$, noting that $\sum_{j=1}^{R} f_j$ must equal unity, and using the Lagrange multiplier technique to find under "optimum" conditions that $f_j = 1/R$. Thus, each Red rifleman is engaged by a Blue rifleman with a frequency equal to the reciprocal of the number of Reds.

The appropriate value of T we seek under the assumptions is thus attained at $f = 1/R$, and is given by

$$T = wR(1 - p/R)^B. \tag{32-7}$$

An example will be instructive.

EXAMPLE 32-1:

Given that 20 Red riflemen attack 15 Blue riflemen in position and that for close ranges each Blue rifleman has a 90% chance of killing a Red rifleman as he approaches Blue in the open. How should Blue allocate his fire and just what is the initial overall "value" of this simple engagement?

We might just as well take $w = 1$, for equal threats, to obtain relative numerical answers, and we know that each Blue should select any one of the Reds with equal frequency. Thus, the chance that each Blue selects a Red, or allocates fire to that Red, is 1/20—and this incidentally has nothing to do with the total number of Blue infantrymen, being quite independent of that. Thus, each Blue will pick a Red at random and the threat computation is found using Eq. 32-7 to be

$$T = (1)(20)(1 - 0.9/20)^{15} = 10.02.$$

One notes, incidentally, that the highest value to Red occurs when no Reds are taken under fire, i.e., $f = 0$, and the value T is then equal to 20. Had each Blue taken on a Red more frequently than $f = 1/R = 1/20$, say $f = 1/15$, then T would be less than 10.02, i.e., $T = 7.91$. Had Blue engaged a Red less frequently, say, $f = 1/30$, then $T = 12.67$, a higher "value" to Red. Had the Blue force consisted of 20 riflemen, then the value to Red would have been less, of course; i.e., we would then have

$$T = 20(1 - 0.9/20)^{20} = 7.96$$

for in this case Blue, with five more riflemen, would have been able to bring on more effective fire.

Continuing a bit, suppose now that Blue's single-shot kill probability were reduced to 0.5, which might occur for somewhat longer engagement ranges. Then the threat T for 15 Blue riflemen becomes

$$T = 20(1 - 0.5/20)^{15} = 13.68$$

and, as expected, the value to Red is much increased.

Blue may always, therefore, reduce the overall value by throwing in more troops or using more potent weapons. On the other hand, one may easily see that this example is rather superficial, for Red will try to sneak up on Blue, or choose paths toward Blue, which will keep his chance of being detected as low as possible—i.e., so that he may gain an advantage, namely, that of surprise. Hence, Blue will not then see all Red riflemen at once, but only one or a few occasionally, and the selection problem will boil down to taking each Red target as it suddenly appears. Then again, when fewer Red targets appear on the scene, we see that Blue can allocate more fire per Red and possibly obtain some advantage since the kill probability per Red may increase. Moreover, after each shot the numbers of Blues and Reds, or B and R, will change. Consequently, the threat value of the conflict will vary accordingly, and perhaps on a stochastic basis, unless Blue or Red can allocate their firing systematically. Obviously, we are merely trying to illustrate the allocation problem of weapons to targets in an elementary manner and for the battle at a given stage, so to speak. Uniform randomness here does not seem to be unreasonable.

Now to return to the more general weapon-target allocation problem as formulated in Eq. 32-2. One sees that the i's and j's may refer to different types of weapons, which take on different types of targets at very different ranges of engagement, and the probability p_{ij} that the ith weapon type kills the jth target type may vary considerably; accordingly, the general allocation problem becomes much more complex. In fact, this general formulation is actually one such type of allocation problem set up in the literature by Manne (Ref. 2), and apparently one originally formulated by Dr. Merrill Flood at a Princeton University Conference on Linear Programming in March 1957. Furthermore, as Manne (Ref. 2) points out, this particular formulation of the weapon-target allocation problem is a highly simplified one because the typical military problem in the field will obviously involve target assignments which will have to be made also on a priority, sequential, and/or surprise basis, as we have indicated, and not at all on a simultaneous allocation.

Generally speaking, the minimization of Eq. 32-2 subject to conditions (Eqs. 32-3 and 32-4) is rather difficult—as pointed out by Manne (Ref. 2)—and some modifications of the general formulation have to be made, or more restrictive assumptions taken into account. In this vein, a very useful and important reformulation of the problem due to Ash (Ref. 3) would seem to have rather wide ranges of application. We will illustrate the rather general usefulness of Ash's variation of the original problem of Flood by outlining an important practical example.

EXAMPLE 32-2:

Blue intelligence indicates that Red infantry numbering about 100 riflemen and a separate group of Red tanks numbering 10 are preparing for an attack on Blue's position, although the Red forces are still some 8000 m away. Knowing his weapon capabilities from systems analysis studies, blue decides that he can stop Red's probable attack with a Battery of six 105-mm howitzers, which has 1000 HE rounds available, and a Battery of six 8-in. howitzers equipped with 500 HE rounds, with the latter also having 1250 of a new type of projectile which has some capability to kill tanks at very long ranges. Blue estimates, using the methods of Chapter 15, that each 105-mm HE round has about 0.05 chance of killing a Red infantryman, but only 0.01 of causing an enemy tank mobility kill. For the 8-in. howitzer battery the new projectile has a single-shot kill probability of about 0.04 against an enemy rifleman and 0.08 against an enemy tank, whereas corresponding values for the 8-in. HE round are 0.1 and 0.02, respectively. How then should Blue allocate his combat load of ammunition to minimize the Red threat?

Clearly, a problem of this nature is of much practical importance since it represents an application of the use of a new weapon and just how it may fit into the family. For Ash's formulation (Ref. 3) which assumes relatively small single-shot kill probabilities, some additional definitions are needed as defined below. In particu' ,r, he assumes that there are several composite targets, or target complexes, each consisting of severa or many target elements.

The additional definit..ons needed are:

B_i = number of Blue rounds available to be fired from the ith type weapon

x_{ij} = number of rounds allocated for firing at the jth type Red target from the ith type Blue weapon

t_j = number of target elements in the jth type Red target complex

c_{ij} = x_{ij}/t_j = commitment ratio of the ith type Blue weapon rounds to the jth Red target complex

w_j = threat or value potential of the jth Red type target complex (as before for simply the jth Red target).

Note that for our reformulated problem the commitment ratio c_{ij} corresponds roughly to the relative frequency or allocation ratio f_{ij} already defined, it being redefined somewhat here. Also since we are interested in the number of rounds of each type fired by Blue, and have defined that number as B_i, we see that total of Blue rounds of all types is B_0, or

$$B_0 = \sum_{i=1}^{B} B_i. \tag{32-8}$$

Thus, we may as well let B be the total number of different types of Blue weapons, and R the total number of different types of Red target complexes.

With these considerations, then the total value of the Red threat can be expressed as

$$T = \sum_{j=1}^{R} w_j t_j \prod_{i=1}^{B} (1 - c_{ij} p_{ij}) \tag{32-9}$$

$$= \sum_{j=1}^{R} w_j t_j \prod_{i=1}^{B} (1 - x_{ij} p_{ij}/t_j)$$

and the desired assignment is the determination of the c_{ij}, or hence the x_{ij}, which makes T a minimum subject to

$$\sum_{j=1}^{R} c_{ij} t_j = \sum_{j=1}^{R} x_{ij} = B_i \tag{32-10}$$

for $i = 1, 2, \ldots, B$ and

$$c_{ij} \geq 0. \tag{32-11}$$

By assuming, as will often be the case, that the product of factors $c_{ij} p_{ij}$ is suitable small, i.e., perhaps no greater than about 0.1, then Ash (Ref. 3) shows that T is very approximately given by

$$T \approx \exp(B/2) \sum_{j=1}^{R} w_j t_j \exp\left[-\sum_{i=1}^{B}(1 + c_{ij}p_{ij})^2\right] \quad (32\text{-}12)$$

through second order terms in p_{ij}, which therefore should be of sufficient accuracy. The approximate threat value T of Eq. 32-12 has geometric characteristics and is of the form that Ash (Ref. 3) was able to use Lagrange multiplier methods to prove that the allocations that minimize Red's approximate total threat (Eq. 32-12) are given by

$$c_{ij} = 1/p_{ij} - (\eta_i - n_i)/[(w_j \prod_{i=1}^{B} p_{ij})^{1/(B-1)} f(R,B)] \quad (32\text{-}13)$$

where

$$\eta_i = \sum_{j=1}^{R} t_j/p_{ij} \quad (32\text{-}14)$$

and

$$f(R,B) = \sum_{j=1}^{R} t_j(w_j \prod_{i=1}^{B} p_{ij})^{-1/(B-1)}. \quad (32\text{-}15)$$

Hence, the allocation computations are straightforward, although a bit tedious perhaps, and for the very complex problems the use of a high-speed calculator may be very desirable.

We are now ready to return to and give a solution of Example 32-2.

EXAMPLE 32-3:

Given the data of Example 32-2 and assuming that a Red tank is equivalent to a threat equal to 19 Red infantrymen, determine the best allocation of 105-mm HE, 8-in. HE, and the new long-range antitank rounds for Blue's combat load.

With the given data, the subscript i represents Blue, and i = 1, 2, and 3 (B = 3) will be taken to mean 105-mm HE rounds, 8-in. HE rounds, and new projectiles, respectively. Similarly, j means Red, and j = 1 and 2(R = 2) will denote Red infantrymen and Red tanks, respectively. Moreover, B_1 = 1000, B_2 = 500, and B_3 = 1250 ($B_i = n_i$ in Eq. 13-13); t_1 = 100 Red riflemen and t_2 = 10 Red tanks, and w_1 = 0.05 and w_2 = 0.95. Finally, for the kill probabilities, we have

$$p_{11} = 0.05, \quad p_{12} = 0.01, \quad p_{21} = 0.1, \quad p_{22} = 0.02, \quad p_{31} = 0.04 \text{ and } p_{32} = 0.08.$$

Then from Eq. 32-15, we find that

$$\begin{aligned} f(R,B) = f(2,3) &= \sum_{j=1}^{2} t_j(w_j \prod_{i=1}^{3} p_{ij})^{-1/2} \\ &= 100[(0.05)(0.05)(0.1)(0.04)]^{-1/2} + 10[(0.95)(0.01)(0.02)(0.08)]^{-1/2} \\ &= 31{,}622.8 + 2564.9 = 34{,}187.7. \end{aligned}$$

For the η_i, we have from Eq. 32-14

$$\eta_i = \sum_{j=1}^{2} t_j/p_{ij}$$

$$\eta_1 = 100/0.05 + 10/0.01 = 3000$$

$$\eta_2 = 100/0.1 + 10/0.02 = 1500$$

$$\eta_3 = 100/0.04 + 10/0.08 = 2625.$$

Finally, the c_{ij} are determined from Eq. 32-13 and are

$$c_{11} = 1/0.05 - (3000 - 1000)/\{[(0.05)(0.05)(0.1)(0.04)]^{1/2}(34,187.7)\}$$

$$= 1.50$$

and since $x_{ij} = c_{ij}t_j$

$$x_{11} = 1.50(100) = 150.$$

$$c_{12} = 1/0.01 - (3000 - 1000)/\{[(6.95)(0.01)(0.02)(0.08)]^{1/2}(34,187.7)\}$$

$$= 85$$

and

$$x_{12} = 85(10) = 850.$$

Similarly, we calculate $c_{21} = 0.75$ or $x_{21} = 75$; $c_{22} = 42.5$ or $x_{22} = 425$; $c_{31} = 12.28$ or $x_{31} = 1228$; and $c_{32} = 2.18$ or $x_{32} = 22$.

In order to avoid classification, this problem has been formulated in terms of "artillery" weapons with very low kill chances to meet the assumptions and for illustrative purposes only. The reader may easily see that there is an inordinately high level of "wasted" rounds for the total allocation which nevertheless checks out well. Obviously, one has to be very careful that Ash's assumptions are fully met; negative allocations for the c_{ij} are *prima facie* evidence of failure to meet the assumptions, and zero allocations would have to be assigned upon recheck.

32-3 A MODIFICATION OF FLOOD'S ALLOCATION MODEL

As we have pointed out, the minimization of Eq. 32-2—subject to the conditions of Eqs. 32-3 and 32-4 under all conditions—is apparently very difficult. For this reason, Manne (Ref. 2) indicates that Dr. George Dantzig has suggested replacing the factor $(1 - f_{ij}p_{ij})$ by the factor $(1 - p_{ij})^{f_{ij}}$. In fact, since

$$(1 - f_{ij}p_{ij}) \geq (1 - p_{ij})^{f_{ij}}, \quad 0 \leq f_{ij}, p_{ij} \leq 1 \tag{32-16}$$

one might consider minimizing

$$T = \sum_{j=1}^{R} w_j \prod_{i=1}^{B} (1 - p_{ij})^{f_{ij}}. \tag{32-17}$$

It is noted in this connection that Eqs. 32-2 and 32-17 take on identical values when the f_{ij} are unity, and should be very similar for fractional f_{ij}. Moreover, the minimum of Eq. 32-17 would represent a lower bound to that of Eq. 32-2.

The forms of Eqs. 32-2 and 32-17 are nonlinear, but Eq. 32-17 may be converted into a form, which may use a linear-programming procedure by defining a new variable y_j given by

$$y_j = \sum_{i=1}^{B} f_{ij} \ln(1 - p_{ij}) \tag{32-18}$$

and then Eq. 32-17 becomes

$$T = \sum_{j=1}^{R} w_j \exp(-y_j). \tag{32-19}$$

Hence, the original problem reduces to minimizing Eq. 32-19 subject to the conditions of Eqs. 32-3 and 32-4, for which Manne (Ref. 2) indicates a method of solution. However, dynamic programming techniques may be applied to obtain the values of f_{ij} which minimize Eq. 32-17. We discuss this in par. 32-5, rather than proceeding with Manne's approximate and somewhat restrictive solution. In fact, any solutions which give fractional values of the f_{ij} may be undesirable, so techniques which give integer values have more meaning physically.

Some points of interest concerning peripheral matters of the optimal target assignment problem are worthy of mention here.

32-4 OTHER WEAPON-TARGET ALLOCATION MODELS

den Broeder, Ellison, and Emerling (Ref. 4) consider the problem of allocating and firing B missiles of the same type simultaneously against R Red targets. They define the following:

p_j = chance that a Blue missile kills the jth Red target

$q_j = 1 - p_j$ = chance the jth Red target survives a Blue missile

B_j = number of Blue missiles assigned to the jth Red target

a_k = chance Blue destroys exactly k Red targets

P_k = chance Blue destroys at least k Red targets

$= \sum_{j=k}^{R} a_j$

V_k = value, or reduction in threat, to Blue for destroying exactly k Red targets. The Red targets are assumed to be more or less similar, although they could be of somewhat different types if the V_k and a_k take care of that possibility. (Note that V_1 = value of destroying one target, V_2 = value of destroying two targets, etc., so that the assumed targets are just about restricted to being of equal value, or that is $V_k = k\omega$, where ω is the value of a single target.)

V = total value to Blue, which is to be maximized, and which is given by

$$V = \sum_{k=1}^{R} V_k a_k. \tag{32-20}$$

den Broeder, Ellison, and Emerling (Ref. 4) do not solve the general problem for different p_j, but assume initially that they are all equal, i.e.,

$$p_1 = p_2 = \cdots = p_j = \cdots = p_R = p \tag{32-21}$$

and prove the following two Theorems:

Theorem I: *If the V_k are nondecreasing functions of k, then the maximum value of V is attained when the B_j's differ by at most one.*

Theorem II: *The probability $P_k = \sum_{j=k}^{R} a_j$ of destroying k or more targets is, for each k, a maximum when the B_j's differ by at most one.*

The reader should note that we have used T for the threat in Eq. 32-2 when Red's survival chances are considered, and in Eq. 32-20 we use V for value when kill probabilities of Red are involved. Also, the w_j of Eq. 32-2 are not equal to the V_k of Eq. 32-20, the latter being cumulative for the number of targets k.

In Ref. 4, the authors also consider the Flood model of Eq. 32-2, which in their notation to be minimized is W the total worth of the surviving Red forces

$$W = \sum_{j=1}^{R} w_j (q_j)^{B_j} \tag{32-22}$$

subject to

$$\sum_{j=1}^{R} B_j = B \tag{32-23}$$

and

$$B_j \geq 0. \tag{32-24}$$

In connection with minimizing Eq. 32-22, one could instead maximize

$$\sum_{j=1}^{R} w_j [1 - (q_j)^{B_j}]. \tag{32-25}$$

Based on Eqs. 32-22, 32-23, and 32-24, den Broeder, Ellison, and Emerling prove the following theorem:

Theorem III. Given that the B_j *minimizes*

$$W(B) = \sum_{j=1}^{R} w_j(q_j)^{B_j} \tag{32-26}$$

then $\hat{B}_j$ *minimizes*

$$W(B+1) = \sum_{j=1}^{R} w_j(q_j)^{B_j} \tag{32-27}$$

for $\hat{B}_j \geq 0$ *and* $\sum_{j=1}^{R} \hat{B}_j = B + 1$, *if*

$$\hat{B}_j = B_j \text{ for } j \neq k, \text{ and } \hat{B}_k = B_k + 1$$

where k satisfies

$$w_k(q_k)^{B_k} p_k = \max_{1 \leq j \leq R} [w_j(q_j)^{B_j} p_j]. \tag{32-28}$$

Note that Theorem III merely states that if one interprets

$$w_k(q_k)^{B_k}$$

as a revised estimate of the value of the kth target based upon an optimum assignment of B missiles, then an added missile should be assigned to the target for which the expectation of the revised value destroyed is largest. Hence, Theorem III affords a very nice algorithm for computation.

One may note that the value model of Eq. 32-20 and the worth or value form given by Eq. 32-25, i.e.,

$$W_V = \sum_{j=1}^{R} w_j[1 - (q_j)^{B_j}] \tag{32-29}$$

are identical if

$$w_j = v, \quad V_k = kv, \quad \text{and} \quad q_j = q. \tag{32-30}$$

For example, we illustrate for the case of two targets with $q_1 = q_2 = q$, and B_1 missiles assigned to target 1, and B_2 to target 2.

Then,

$$a_0 = \text{chance that both targets survive}$$

$$= 1 - (1 - q^{B_1} \cdot q^{B_2}) = q^{B_1 + B_2}$$

a_1 = chance that exactly one target is killed

$$= (1 - q^{B_1})q^{B_2} + (1 - q^{B_2})q^{B_1} = q^{B_1} + q^{B_2} - 2q^{B_1+B_2}$$

a_2 = chance that both targets are killed

$$= (1 - q^{B_1})(1 - q^{B_2}) = 1 - q^{B_1} - q^{B_2} + q^{B_1+B_2}$$

From Eqs. 32-20 and 32-30

$$V = \sum_{j=1}^{2} V_k a_k = va_1 + 2va_2 = (2 - q^{B_1} - q^{B_2})v.$$

Also, from Eqs. 32-26 and 32-30

$$W(B) = \sum_{j=1}^{2} w_j(1 - q^{B_j}) = v[(1 - q^{B_1}) + (1 - q^{B_2})]$$

$$= (2 - q^{B_1} - q^{B_2})v \text{ also.}$$

As pointed out in Ref. 4, for equal survival probabilities $q_j = q$, the procedure of the authors' Theorem III may be interpreted as "an attempt to make all $w_j(q)^{B_j}$ equal and, if the B_j were not required to be integral, this interpretation would be quite precise". To quote den Broeder, Ellison, and Emerling further:

"Proceeding formally from $w_j(q)^{B_j} = w_k(q)^{B_k}$, one obtains

$$B_j - B_k = |\ln q|^{-1}\ln(w_k/w_j) = b_{jk}, \text{ say.} \tag{32-31}$$

"Thus, it appears that for $q_j = q$, one should attempt to assign so that the differences between the B_j are equal to specified constants (whereas most analysts interrogated have guessed that their ratios would be significant). It may be recalled that in Theorem I we attempt to satisfy $B_j - B_k = 0$. Indeed, if $w_j \equiv \omega$ and $q_j \equiv q$, the b_{jk} are all zero and we have a heuristic proof of Theorem I for the special case $V_k = k\omega$."

This discussion suggests the following.

Theorem IV: The function

$$V = \sum_{k=1}^{R} V_k a_k$$

is, for arbitrary $q_j = 1 - p_j$ and $V_k = k\omega$, maximized according to Theorem III with $w_j = \omega$.

We will illustrate these results with an example.

EXAMPLE 32-4:

Blue is planning an attack on Red, and has intelligence information to the effect that the Red force he is assigned to neutralize consists of an infantry company, an artillery battery, and a tank company.

Blue has 10 missiles with large warheads, each consisting of a payload of 1000 shaped-charge, fragmentation submissiles, and the payload of each missile has a kill probability of 0.2 against the Red infantry company, 0.05 against the tanks, and 0.1 against the Red artillery. If Blue considers that the relative worths or values of Red infantry, Red tanks, and Red artillery are about equal, then how should he allocate the firing of his 10 missiles so as to maximize the value of neutralizing the Red forces?

Let $j = 1$ denote infantry, $j = 2$ denote tanks, and $j = 3$ denote artillery. Then

$$p_1 = 0.2 \qquad p_2 = 0.05 \qquad p_3 = 0.1$$

$$q_1 = 0.8 \qquad q_2 = 0.95 \qquad q_3 = 0.9$$

$$w_1 = \omega \qquad w_2 = \omega \qquad w_3 = \omega.$$

We note that $V_k = k\omega$, for $k = 1, 2, 3$, and for computational purposes take $\omega = 1$, so that the requirements of Theorem IV are satisfied. Thus, we may use Theorem III with $w_j = \omega = 1$, and find the B_j which minimizes the worth of Red surviving, or

$$W(B) = \sum_{j=1}^{R} w_j(q_j)^{B_j} = \sum_{j=1}^{3} (q_j)^{B_j}$$

Suppose we were to fire only one missile, then clearly that single missile should be fired at Red infantry, since its kill worth is then $w_1p_1 = 0.2$, whereas $w_2p_2 = 0.05$ and $w_3p_3 = 0.1$. Note that the expected value of the Red surviving the threat is then 0.8, and if Blue shot the single round at Red artillery or tanks, it would be 0.9 and 0.95, respectively. Since we have minimized the threat to Blue for one round, we may now apply Theorem III to obtain allocations for 2, 3, etc., rounds. (Actually, for the next round or so, it is easily seen that Blue should continue to shoot at Red infantry.) The computations are carried forward in Table 32-1.

The algorithm of Theorem III therefore starts with $B_1 = 1$, and keeps increasing B_1 by one until

$$(q_1)^{B_1}p_1 < q_2^0 p_2 = p_2 \text{ or } q_3^0 p_3 = p_3$$

TABLE 32-1. APPLICATION OF THEOREM III TO EXAMPLE 32-4

Line	B_1	B_2	B_3	$(q_1)^{B_1}p_1$	$(q_2)^{B_2}p_2$	$(q_3)^{B_3}p_3$
1	1	0	0	0.1600	0.0500	0.1000
2	2	0	0	0.1280	0.0500	0.1000
3	3	0	0	0.1024	0.0500	0.1000
4	3	0	1	0.1024	0.0500	0.0900
5	3	0	2	0.1024	0.0500	0.0810
6	4	0	2	0.0819	0.0500	0.0810
7	4	0	3	0.0819	0.0500	0.0729
8	4	0	4	0.0819	0.0500	0.0656
9	4	0	5	0.0819	0.0500	0.0590
10	5	0	5	0.0655	0.0500	0.0590

and this occurs on line 4 of Table 32-1, at which point a missile is allocated to target $j = 3$ or Red artillery. Following line 4, Red artillery is allocated 2 missiles through line 6. After line 6, Red artillery gets the additional allocation through line 9 since $(q_3)^{B_3}p_3$ is greater. The final allocation is to shoot 5 missiles at Red infantry, 5 at Red artillery, and none of the 10 rounds at Red tanks. In fact, no missiles would be allocated for firing at Red tanks until Blue had at least 15 missiles to fire. (The final value from Eq. 32-29 is 1.08.)

Calculations such as these should make the analyst stop and think. For example, are the basic input data sound or should they be altered? Perhaps if the data can be refined, then a more exact calculation could be made and a more reasonable (?) allocation of missiles to targets scheduled. Then again, which type of Red unit would really be the greatest threat to Blue? Would it be Red's infantry, tanks, or artillery? Offhand, it would seem to be the Red artillery, and hence our allocation may not therefore be too poor. But aren't the Red tanks more of a threat than the Red infantry? To settle such questions, one would have to consider the entire Blue forces in the battle area, and based on this, then perhaps he might place different values on the Red units rather than the assumption of equality we have made here. In such cases, the analyst may desire to change models as needed to fit the battle situation.

Lemus and David (Ref. 5) also developed some rules for optimum allocation of different weapons to a target complex. They consider that the threat is composed of groups of attacking units (e.g., Red weapons) to an assemblage of targets, each of which has a certain worth or value to the attacker, and for each of which the chance of hitting and killing is known. They assume also, as is often necessary in analyses of this kind, that the engagement time is short so that there is no time to evaluate the outcome of individual engagements, and the attacker must therefore assign all of his weapons before the effects of individual shots are assessed. Furthermore, the targets are assumed to be fixed in dispersed positions so that the attacker cannot possibly knock out more than one of them with a single shot.

We now adapt their analysis to our basic problem of Blue allocating his weapons and rounds against Red forces, and state Lemus and David's formulation (Ref. 5) as follows:

1. There are $j = 1, 2, \ldots, R$ Red targets, each of which has a certain worth or threat value w_j to Blue, and $w_j \geq 0$.

2. There are $i = 1, 2, \ldots, B$ different types of Blue weapons, and the number of weapons (or rounds) of the ith type is B_i. Also, $\sum_{i=1}^{B} B_i = B_0$, the total of weapons (rounds).

3. The probability that the ith Blue weapon destroys the jth Red target is p_{ij}, and the chance that the jth Red target survives the ith Blue weapon is then $q_{ij} = 1 - p_{ij}$.

4. The number of Blue weapons (or rounds) of the ith type which are assigned to the jth Red target is x_{ij}.

5. All targets are within the range of all weapons considered.

Then consider the problem of finding the x_{ij} by maximizing Blue's kill value against Red:

$$V = \sum_{j=1}^{R} w_j \left[1 - \prod_{i=1}^{B} (1 - p_{ij})^{x_{ij}}\right] \tag{32-32}$$

subject to

$$\sum_{j=1}^{R} x_{ij} = B_i \quad , \text{ for } i = 1, 2, \ldots, B \tag{32-33}$$

and

$$0 < p_{ij} < 1, \quad x_{ij} \geq 0. \tag{32-34}$$

Lemus and David (Ref. 5) point out that there are essentially two different types of solution in the literature for this kind of problem. One is the "digital" solution which yields integral values of the x_{ij}, and the other is the "analytical" type of solution which is based on the use of Lagrange multipliers, and treats the number of weapons (or rounds) as a continuous variable, usually producing fractional numbers. If the number of weapons is of the order of magnitude of the number of targets, then the digital solutions are most desired; whereas if the number of weapons is large compared with the number of targets then analytical methodology may be quite acceptable since it will require less computational time and at the same time yield sufficiently accurate results. In Ref. 5, Lemus and David develop an analytical solution based on Lagrange multipliers and compare it through an example with that obtained from digital methods. A very useful account of Lagrange multiplier methods for the systems analyst is given by Dorn in Ref. 6.

The analysis of Lemus and David (Ref. 5) is based on the analytical treatment that follows. The q_{ij} are first expressed as

$$q_{ij} = (A_j)^{\rho_i} \tag{32-35}$$

where A_j is a "common" part of the q_{ij} for the jth target, and it may as well be taken in terms of one of the probabilities q_{ij}, for example $A_j = (q_{1j})^{1/\rho_1}$; i.e., selected as a "base" here. Hence, the ρ_i turn out to be different exponents and depend on the ith Blue weapon. Then, some factors L_i, with ρ_1 of Eq. 32-35 referring to the 1st Blue weapon type, are defined in terms of the ρ_i as

$$L_i = \rho_i/\rho_1 \tag{32-36}$$

and finally quantities S_j, giving the total allocation of weapons (rounds) to the jth target, are

$$S_j = \sum_{i=1}^{B} L_i x_{ij} \tag{32-37}$$

i.e., in terms of the allocation factors x_{ij}. We then have that

$$\sum_{j=1}^{R} S_j = \sum_{i=1}^{B} L_i B_i = B_e \tag{32-38}$$

where B_e is the total number of Blue weapons (or often rounds) available for firing.

One may note using these definitions that this simply means

$$\prod_{i=1}^{B} (q_{ij})^{x_{ij}} = \prod_{i=1}^{B} [(A_j)^{\rho_i}]^{x_{ij}} = \prod_{i=1}^{B} [(q_{1j})^{\rho_i/\rho_1}]^{x_{ij}} = \prod_{i=1}^{B} [(q_{1j})^{L_i}]^{x_{ij}} = (q_{1j})^{S_j} \tag{32-39}$$

where

q_{1j} = chance of survival of jth Red target when fired on by the 1st Blue weapon type.

Moreover the value (Eq. 32-32) then becomes

$$V = \sum_{j=1}^{R} w_j[1 - (q_{1j})^{S_j}] \quad (32\text{-}40)$$

Next, to obtain the optimum allocations, one calculates the quantities

$$\beta = \sum_{j=1}^{R} \{\ln[w_j \ln(1/q_{1j})]/\ln(1/q_{1j})\} \quad (32\text{-}41)$$

$$\gamma = \sum_{j=1}^{R} [1/\ln(1/q_{1j})] \quad (32\text{-}42)$$

$$\ln\lambda = (\beta - B_0)/\gamma \quad (32\text{-}43)$$

$$S_j^* = \{\ln w_j - \ln\lambda + \ln[\ln(1/q_{1j})]\}/[\ln(1/q_{1j})] \quad (32\text{-}44)$$

where S_j^* is the optimum value of S_j. In passing, and as it turns out, we have that

$$\sum_{j=1}^{R} S_j^* = B_0 \qquad S_j^* \geq 0. \quad (32\text{-}45)$$

Finally, the optimal allocations x_{ij} of Blue weapons (rounds) against Red targets are given by

$$x_{ij} = B_i S_j^* / B_0 \quad (32\text{-}46)$$

and the final value V for the allocation values may be calculated easily from

$$V = \sum_{j=1}^{R} w_j - \lambda\gamma. \quad (32\text{-}47)$$

If some of the allocations turn out to be negative, those targets are dropped, and the procedure repeated for those targets for which $S_j^* \geq 0$.

EXAMPLE 32-5:

Use the data of Example 32-4. Find the optimal allocation of Blue's missiles to Red's three targets of infantry, tanks, and artillery, assuming (1) equal target values, and (2) the relative worths for Red infantry, Red tanks, and Red artillery are $w_1 = 1$, $w_2 = 2$, and $w_3 = 3$, respectively.

Although Blue's 10 missiles are all alike, we may treat them as being "distinct", with $B_i = 1$ and $\Sigma B_i = B_0 = B = 10$. Also, we see that

$$p_{11} = 0.2 \qquad p_{12} = 0.05 \quad \text{and} \quad p_{13} = 0.1$$

$$q_{i1} = 0.8 \qquad q_{i2} = 0.95 \qquad \text{and} \qquad q_{i3} = 0.9$$

$$u_1 = 1\,, \qquad u_2 = 1\,, \qquad \text{and} \qquad u_3 = 1 \text{ (for Assumption No. 1).}$$

We now compute β from Eq. 32-41:

$$\beta = \ln[\ln(1/0.8)]/\ln(1/0.8) + \ln[1/0.95)]/\ln(1/0.95)$$

$$+ \ln[\ln(\ln(1/0.9)]/\ln(1/0.9) = -85.987.$$

Then compute γ from Eq. 32-42:

$$\gamma = 1/\ln(1/0.8) + 1/\ln(1/0.95) + 1/\ln(1/0.9) = 33.468.$$

Then compute $\ln\lambda$ from Eq. 37-43

$$\ln\lambda = (-85.987 - 10)/33.468 = -2.868.$$

Finally, from Eq. 32-44, we obtain the total allocation of Blue missiles to Red infantry as

$$S_1^* = \{\ln 1 - (-2.868) + \ln[\ln(1/0.8)]\}/\ln(1/0.8) = 6.13.$$

In a like manner,

$$S_2^* = -1.99 \quad \text{and} \quad S_3^* = 5.86.$$

We note that S_2^* is negative (or minus two Blue missiles), hence the second Red target (tanks) should be omitted, and a recalculation made.

By way of passing, $S_1^* + S_2^* + S_3^* = 10.0$, nevertheless.

A recalculation, using only the Red infantry with $q_{11} = 0.8$, and the Red artillery with $q_{13} = 0.9$, gives the new values

$$\beta = -28.081, \quad \gamma = 13.973, \quad \ln\lambda = -2.725$$

and

$$S_1^* = 5.49 \quad \text{and} \quad S_3^* = 4.51 \quad (\text{and we take } S_2^* = 0).$$

Since there is only one type of Blue missiles or rounds, the individual type Blue allocations of Eq. 32-47, which become $x_{ij} = S_j^*/B_w$ are of no particular interest in this example.

Hence, rounding would give the same answers,

$$S_1^* = 5, \quad S_2^* = 0, \quad \text{and} \quad S_3^* = 5$$

as the digital or integer values in Example 32-4, using the methods of den Broeder, Ellison, and Emerling (Ref. 4).

For Assumption No. 2 of Example 32-5, using the unequal values—i.e., $w_1 = 1$, $w_2 = 2$, and $w_3 = 3$ —we calculate

$$\beta = -62.046 \ , \quad \gamma = 33.468 \ , \quad \ln\lambda = -2.153$$

$$S_1^* = 2.93 \ , \quad S_2^* = -2.43 \ , \quad S_3^* = 9.50.$$

Again, S_2^* is negative, and hence the allocation of -2.43 Blue missiles against Red tanks should be ignored, i.e., made zero. Recalculation using data for only the Red infantry and Red artillery yields

$$\beta = -17.653 \ , \quad \gamma = 13.973 \ , \quad \ln\lambda = -1.979$$

$$S_1^* = 2.15 \ , \quad S_2^* = 0 \ , \quad S_3^* = 7.85$$

or rounding

$$S_1^* = 2 \ , \quad S_2^* = 0 \ , \quad S_3^* = 8.$$

Thus, the final allocation is to fire 2 of the 10 Blue missiles at Red infantry and 8 against Red artillery. Without a change in single-shot kill probabilities, Red artillery now gets 3 more Blue missiles against it, since its value relative to infantry was tripled. On the other hand, Red tanks are still not assigned any Blue missiles, for the increased value to $w_2 = 2$ is apparently not enough to overcome the lower single-shot kill probability of 0.05. The Red tanks would be fired upon, of course, with increases in the Blue missiles above 10 to appropriate numbers.

Lemus and David (Ref. 5) give a numerical example of three different types of weapons, which are fired at 20 targets of different values, with the numbers of types of each of the three weapons being 50, 80, and 45, respectively. Their calculations for the allocations are carried through using both the methodology for the "analytical" approximation previously given, and also for the application of Theorem III of den Broeder, Ellison, and Emerling (Ref. 4) for the "digital" or integer value allocations. In this connection, their results show a very close correspondence between the two methods of allocation, i.e., the analytical or continuous versus the digital or interger values as did our Examples 32-4 and 32-5.

32-5 THE DYNAMIC PROGRAMMING APPROACH TO WEAPON-TARGET ALLOCATION PROBLEMS

Perhaps the most powerful method in recent years of solving many applied problems, including the weapon-target allocation problem described in this chapter, is that developed by Richard Bellman (Ref. 7) through use of his "principle of optimality." It is especially adaptable to nonlinear problems, and we state it here.

The Principle of Optimality. An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. (Ref. 7 and Ref. 9, p. 16).

We will describe Bellman's dynamic programming technique here by returning to the threat of Eq. 32-17, since it represents an important practical problem and also gives a lower bound to Eq. 32-2 first discussed. We note that the threat T is a function of the allocation factors or relative frequencies f_{ij}, which we will now change to or replace by allocation integers x_{ij}—since they may now exceed unity—the chances p_{ij} that the ith Blue weapon (or type) kills the jth Red target (type), and some measure of worth w_j of the jth Red target. Thus, the total value of the threat which Blue must minimize is, as similarly in Eq. 32-17,

$$T = \sum_{j=1}^{R} w_j \prod_{i=1}^{B} (1 - p_{ij})^{x_{ij}} = \sum_{j=1}^{R} w_j \prod_{i=1}^{B} (q_{ij})^{x_{ij}} \tag{32-48}$$

where Blue has B_i weapons (rounds) of type i, and a total load of

$$\sum_{i=1}^{B} B_i = B_0. \tag{32-49}$$

In order to approach and describe the dynamic programming approach to weapon-target allocation problems, we follow Bellman (Ref. 7), Bellman and Dreyfus (Ref. 8), and especially the treatment of Sacco and Shear (Ref. 9) In Eq. 32-48, there are R terms and each term, depending on final allocations, may consist of as many as $(B + 1)$ factors. The B allocation numbers x_{ij}, giving the number of Blue i type weapons assigned to the jth Red (type) target, are to be integers. It is helpful here to write out just what we are seeking from the dynamic programming approach, i.e., we want the

$$\min_{\text{all } x_{ij}} \sum_{j=1}^{R} w_j \prod_{i=1}^{B} (q_{ij})^{x_{ij}} = \min_{\{x_{i1}\}} w_1 \prod_{i=1}^{B} (q_{i1})^{x_{i1}}$$

$$+ \min_{\{x_{i2}\}} w_2 \prod_{i=1}^{B} (q_{i2})^{x_{i2}} + \cdots + \min_{\{x_{ij}\}} w_j \prod_{i=1}^{B} (q_{ij})^{x_{ij}}$$

$$+ \cdots + \min_{\{x_{iR}\}} w_R \prod_{i=1}^{B} (q_{iR})^{x_{iR}} \tag{32-50}$$

for $i = 1, 2, \ldots, B$, subject to

$$0 \le q_{ij} = 1 - p_{ij} \le 1 \,, \quad \text{for all } i \text{ and } j \tag{32-51}$$

$$x_{ij} \ge 0 \quad , \text{ (all nonnegative integers)} \tag{32-52}$$

and

$$\sum_{i=1}^{B} B_i = B_0 \tag{32-53}$$

the total number of weapons (or rounds) allocated.

We note that the jth term in the expanded summation (Eq. 32-50) represents a minimization of the threat defined for the jth Red target. Hence, it represents a minimization over the set of values, i.e., the number of rounds of each Blue weapon system against the jth Red target,

$$\{x_{1j}, x_{2j}, x_{3j}, \ldots, x_{Bj}\}$$

for the threat of the jth Red target, which we now concentrate on, or

$$w_j \prod_{i=1}^{B} (q_{ij})^{x_{ij}}$$

subject to

$$\sum_{i=1}^{B} x_{ij} = x_j \leq B_0. \tag{32-54}$$

We have used x_j in Eq. 32-54 to denote the total number of Blue weapons (or rounds) which are fired at only the jth Red target, and it certainly has an upper bound of B_0 in the (most unusual) case that all rounds may be fired a. the jth Red target. Now it will usually turn out that all B different types of Blue weapons (or rounds) will not be fired at only the jth Red target for the optimum allocation of rounds. Thus, we need another letter, say k, to describe this particular allocation and, therefore,

$$k \leq B. \tag{32-55}$$

Hence, for k different actions or allocations against the jth Red target, we will denote the minimum value of the jth term of Eq. 32-50 by $h_k^j(x_j)$, or

$$h_k^j(x_j) = \min_{\{x_{1j}, \ldots, x_{kj}\}} [w_j(q_{1j})^{x_{1j}}(q_{2j})^{x_{2j}} \cdots (q_{kj})^{x_{kj}}] \tag{32-56}$$

which is subject to the constraints of Eqs. 32-54 and 32-55, with x_j replaced by x_k.

We note that the minimization indicated by Eq. 32-56 is really a multidimensional problem, although it can be accomplished in k one-dimensional minimization processes by using Bellman's Principle of Optimality (Ref. 7). This is accomplished by an imbedding process of including all previous functions or factors in the current one being minimized. In fact, the Bellman principle of optimality will be applied first to minimize the threat of the jth Red target alone, i.e., Eq. 32-56, and then ultimately to the entire threat (Eq. 32-48) of all Red targets. Thus, Bellman's powerful principle of optimality is to find the minimum of Eq. 32-56 by employing the functional arrangement

$$h_k^j(x_j) = \min_{x_{kj}} [(q_{kj})^{x_{kj}} h_{k-1}^j(x_j - x_{kj})] \tag{32-57}$$

for $k = 2, 3, \ldots, B$, where x_{kj} can assume the values of $0, 1, 2, \ldots, x_j$. When $k = 1$, then we find

$$h_1^j(x_j) = \min_{x_{1j}} [w_j(q_{1j})^{x_{1j}}] \tag{32-58}$$

for $x_{1j} = 0, 1, 2, \ldots$, or x_j, or thus it is seen that if x_{1j} is as large as x_j, then

$$h_1^j(x_j) = w_j(q_{1j})^{x_j} \quad \text{for all } j = 1, 2, \ldots, R. \tag{32-59}$$

At this stage x_j is not known, and all we really know concerning it is that

$$0 \le x_1 + x_2 + \cdots + x_R \le B_0, \text{ and } 0 \le x_j \le B_0.$$

In other words, the total allocation of Blue's weapons and rounds against all Red targets, as well as that against the jth target, cannot exceed the total number of rounds B_0 available. Hence, it is seen that we will have to tabulate the quantities $h_k^j(x_j)$ over a "sufficiently appropriate" grid of values for x_j and obtain the sequence of values for Eq. 32-56 or Eq. 32-57.

So far, we have discussed allocations against only the jth Red target, so that now we must face the problem of handling all of the other Red targets also. This is accomplished also by the use of Bellman's principle of optimality. In fact, we notice that there are R different Red targets, although of course we must recognize that all of them may not be shot at for an optimal allocation of weapons and/or rounds. Hence, suppose that $r \le R$ of the Red targets will actually be attacked. Then, we define a new function $g_r(x,B)$ representing a minimization over the total Red threat, which is given by

$$g_r(x,B) = \min_{\{x_1,\ldots,x_r\}} [h_B^1(x_1) + h_B^2(x_2) + \cdots + h_B^r(x_r)] \tag{32-60}$$

where $r = 1, 2, \ldots, R$ and

$$\sum_{j=1}^{r} x_j = x \le B_0. \tag{32-61}$$

Note that when $r = R$, the final optimal allocations of rounds to the Red targets are obtained, and that also these allocations by target designation numbers are the quantities

$$x_1, x_2, \ldots, \text{ and } x_R$$

some of which may be equal to zero.

In order to obtain this final optimal allocation, we again use Bellman's principle of optimality for Eq. 32-60, which now becomes, or is expressed by,

$$g_r(x,B) = \min_{x_r} [h_B^r(x_r) + g_{r-1}(x - x_r, B)] \tag{32-62}$$

for $r = 2, 3, \ldots, R$, and where also x_r may take on the values $0 \le x_r \le x$.

When $r = 1$, then the function $g_1(x,B)$ is

$$g_1(x,B) = h_B^1(x). \tag{32-63}$$

Now Eqs. 32-62 and 32-63 hold for all B, R, and x, and, using Bellman's theory, we can assess the effect on the final optimal policy or allocations which may result from some variation in any of the parameters. Moreover, in carrying out Bellman's principle of optimality, as just described, many such problems are solved, including that for $R = 1, 2, 3, \ldots,$ different Red targets. Hence, Bellman's dynamic programming technique reduces what would ordinarily be the need for hundreds, thousands, or millions of enumerations to a systematic and powerful procedure for determining the optimal allocations or arrangements.

We will illustrate the dynamic programming approach by applying it to the data of Examples 32-4 and 32-5, but for the relative worths of Red infantry, Red tanks, and Red artillery equal to $w_1 = 1$, $w_2 = 2$, and $w_3 = 3$, respectively—Assumption No. 2 of Example 32-5. Although the reader will no doubt notice that for this particular application dynamic programming seems more complex than the solutions given for Examples 32-4 and 32-5, he should, nevertheless, notice its general use for otherwise formidable or "impossible" problems.

EXAMPLE 32-6:

Find the optimal allocation of Blue's 10 missiles against the Red infantry, Red tanks, and Red artillery for relative target worths of $w_1 = 1$, and $w_2 = 2$, and $w_3 = 3$, respectively, given the data of Example 32-4.

Although in Example 32-5, it was convenient using the method of Lemus and David (Ref. 5) to treat Blue's 10 missiles as being distinct or different i type weapons, $i = 1, 2, \ldots, 10$, such is not the case for Bellman's dynamic programming approach here. In fact, there is only one type of Blue weapon, i.e., missiles, and $i = 1$ only; however, there is a constraint of 10 missiles to be fired, or

$$B_i = B_1 = B_0 = 10.$$

Thus, there is $B = 1$ or only one type of Blue weapon, but $j = 1, 2, 3$, or $R = 3$, different types of Red targets: infantry, tanks, and artillery. Furthermore, for a single Blue weapon, the k of Eq. 32-57 is limited to, or simplified to $k = 1$, and the B of Eq. 32-60 is also restricted to $B = 1$. Thus only the computations

$$g_r(x,B) = g_r(x,1)$$

of Eqs. 32-60 and 32-62 need be carried out to determine the optimum target assignments.

We proceed by first constructing Table 32-2 for the individual allocations

$$x_{1j}(x_k) = x_{1j}(x_1) \ , \ j = 1, 2, 3$$

and the functions to be minimized (Eq. 32-59)

$$h_1^j(x_k) = h_1^j(x_1) = w_j(q_{1j})^{x_{1j}} \ , \ j = 1, 2, 3.$$

Since there are a total of 10 missiles to be allocated against 3 targets, then we will make computations through $x_k = 10$, although the allocations to individual Red targets, or $x_{1j}(x_k)$, are stopped at 5, which turn out to be sufficient for the problem considered here. One may often have to make such initial guesses. One should note in Table 32-2 that the computations are strictly as indicated in the example at the bottom of the table, and hence no minimizing takes place at this stage of initial calculations.

Since there is only one type of Blue weapon, then only the $h_1^j(x_k)$ have to be computed. That is to say, what would ordinarily be the next table—i.e., based on $h_2^j(x_k)$, for example, or Eqs. 32-56 and 32-57 for $k = 2, 3$, etc.—become altogether unnecessary, or do not really exist. Instead, one may proceed to the minimization of Eq. 32-60 or that is Eq. 32-62.

TABLE 32-2. TABLE OF INITIAL FUNCTIONAL VALUES, $h_1^j(x_1) = w_j(q_{1j})^{x_{1j}}$ FOR EXAMPLE 32-6

	Values of Function $h_1^j(x_1)$			Allocation Values to Individual Red Targets $x_{1j}(x_1)$		
	$h_1^1(x_1)$	$h_1^2(x_1)$	$h_1^3(x_1)$	$x_{11}(x_1)$	$x_{12}(x_1)$	$x_{13}(x_1)$
x_j \ j	**1**	**2**	**3**	**1**	**2**	**3**
0	1	2	3	0	0	0
1	1	2	3	0	0	0
2	0.8	1.90	2.7	1	1	1
3	0.8	1.90	2.7	1	1	1
4	0.64	1.805	2.43	2	2	2
5	0.64	1.805	2.43	2	2	2
6	0.512	1.7148	2.187	3	3	3
7	0.512	1.7148	2.187	3	3	3
8	0.4096	1.6290	1.9683	4	4	4
9	0.4906	1.6290	1.9683	4	4	4
10	0.3277	1.5476	1.7715	5	5	5

$q_{11} = 0.80$, $q_{12} = 0.95$, $q_{13} = 0.90$

$w_1 = 1$, $w_2 = 2$, $w_3 = 3$

Example: $h_1^2(6) = w_2(q_{12})^{x_{12}} = 2(0.95)^3 = 1.7148$

(The allocation values in this table are estimated as the most likely ones needed for the minimization process of Table 32-3).

The next table to construct is Table 32-3 which does involve finding minima. Table 32-3 introduces the second target $j = 2$ for the Red tanks, in addition to the values $h_1^1(x_1)$ for Red infantry, which we also use just now to determine the $g_r(x,B)$ of Eq. 32-62. In fact, we will calculate all entries for Table 32-3, for illustrative purposes, even though only the minimum values need be recorded ordinarily. In the calculation of entries for Table 32-3, one:

a. Selects a total number of rounds x_k to be fired.

b. Selects an x_2 (rounds fired at tanks) = 0, or 1, or ..., to x_k (less than or equal to x_k).

c. Locates the entry $h_1^2(x_1)$ for $x_k = x_2$ in Table 32-2.

d. Adds the entry $h_1^1(x - x_2)$ for the difference $x_k = x - x_2$ from Table 32-2. (Here $x \equiv$ initial x_k selected in step a.)

To illustrate the procedure, the Table 32-3 entry for $x_k = 5$ and $x_2 = x_{12}(x_1) = 2$ is

$$h_1^2(2) + h_1^1(5 - 2) = 1.9 + 0.8 = 2.7$$

using the appropriate entries from Table 32-2. However, the minimum occurs not for $x_2 = x_{12}(x_1) = 2$, but for $x_2 = x_{12}(x_1) = 0$, or $x_2 = x_{12}(x_1) = 1$, and is 2.64. Hence, for 5 rounds against the first two targets, Red infantry and tanks, the optimal allocation is either 5 Blue missiles against Red infantry only, or (5 − 1 = 4) 4 missiles against Red infantry and 1 against Red tanks. Note in Table 32-3 that the minimum values are underlined, and only these will be used in the next and final calculation of the

TABLE 32-3
TABLE OF $g_2(x,1)$ BASED ON FIRST AND SECOND TARGETS (RED INFANTRY AND RED TANKS)

Total Allocation — Different values of x_2 used in calculating $h_1^2(x_2) + h_1^1(x - x_2)$

x_k	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$x_2 = 5$	$x_2 = 6$	$x_2 = 7$	$x_2 = 8$	$x_2 = 9$	$x_2 = 10$
0	3										
1	3	3									
2	2.8	3	2.9								
3	2.8	2.8	2.9	2.9							
4	2.64	2.8	2.7	2.9	2.805						
5	2.64	2.64	2.7	2.7	2.805	2.805					
6	2.512	2.64	2.54	2.7	2.605	2.805	2.715				
7	2.512	2.512	2.54	2.54	2.605	2.605	2.715	2.715			
8	2.4906	2.512	2.414	2.54	2.445	2.605	2.515	2.715	2.63		
9	2.4096	2.4096	2.412	2.412	2.445	2.445	2.515	2.515	2.63	2.63	
10	2.328	2.4906	2.3096	2.412	2.317	2.445	2.355	2.515	2.43	2.63	2.548

x_2 = compact notation for $x_{12}(x_1)$ = 0, 1, 2, ..., x_k.
Optimal $x_{12}(x_1)$'s underlined.
x_k = total allocation, or rounds.

$g_2(x,1) = \min_{x_2} [h_1^2(x_2) + g_1(x - x_2, 1)] = \min_{x_2} [h_1^2(x_2) + h_1^1(x - x_2)]$ with $h_1^j(x_2)$ from Table 32-2.

minima $g_2(x,1)$. Note from Table 32-3, for example, that if only 2 Blue missiles are fired against only 2 of the Red targets, infantry and tanks, then the minimum for $x_k = 2$ is 2.8 at the value of x_2 j 0, so that 2 Blue missiles would be fired at Red infantry and none at Red tanks. Other optimal allocations against infantry and tanks are clear.

For the final calculations, we could construct a table of

$$g_3(x,1) = \min_{x_3} [h_1^3(x_1) + g_2(x - x_3, 1)]$$

where the $h_1^3(x_1)$ are listed in the column $j = 3$ of Table 32-2, i.e., for the added Red artillery, and the $g_2(x - x_3, 1)$ are the *minimum* values for the allocation entries of Table 32-3 already calculated. However, there is no point really in constructing such a complete table for our problem here since we know that exactly 10 Blue missiles will be fired. Therefore, we see that only the values of $g_3(10,1)$ need be calculated, and we select that x_3 which gives the least or minimum value. In fact, for $x_3 = 0, 1, 2, \ldots, 10$, the values of $g_3(10,1)$ are, respectively:

5.3096, 5.4096, 5.1096, 5.212, 4.942, 5.07,
4.827, 4.987, 4.7683, 4.4683, and 4.7715.

(Example calculations for generating these $g_3(10,1)$ values follow:

$$x_3 = 4: \; h_1^3(4) + \min x_k = 10 - 4 = 6$$

$$2.43 \text{ (from Table 32-2)} + 2.512 \text{ (from Table 32-3)} = 4.942$$

$$x_3 = 7: \; h_1^3(7) + \min x_k = 10 - 7 = 3$$

$$2.187 \text{ (from Table 32-2)} + 2.8 \text{ (from Table 32-3)} = 4.987.)$$

We see that the least value, 4.7683, is for $x_3 = x_{13}(x_1) = 8$, and hence 8 Blue missiles would be fired against Red artillery, leaving 2 Blue missiles, which from Table 32-3 would be fired against Red infantry only since the minimum occurs for $x_2 = 0$.

Therefore, and in summary, Blue would fire 2 missiles against Red infantry, zero missiles against Red tanks, and 8 missiles against Red artillery, which is precisely the answer obtained before by the (much simpler) method of Lemus and David (Ref. 5) in Example 32-5, involving the "analytical" calculation, as compared to integer values for the dynamic programming approach.

For this particular example, therefore, Bellman's dynamic programming solution is much more involved than the rather straightforward "analytical" approach of Lemus and David; however, the Bellman principle of optimality is much, much more general, and can be used to solve a very wide range of allocation problems no other technique can even approach. We illustrate this now with an important type of problem not readily handled by other methods.

Instead of placing a limit on the total number of Blue weapons (or rounds) to be fired, suppose we were interested in a limit on total weight of the rounds, or the cost, or even other measures of effectiveness. Sacco and Shear (Ref. 9) cover a very appropriate application of Bellman's dynamic programming procedure for a limitation on the total weight of rounds to be fired. (This might, for example, be a measure of logistic effort.) Hence, we consider the analysis that follows.

Before, we defined the w_j as the worth or value of the jth Red target. To distinguish, we will now define W_i as the weight, for example in pounds of one round fired from the Blue ith type weapon. Hence, if x_{ij} rounds fired from the Blue ith type weapon are allocated against the jth Red type target and W_i is the weight of one Blue i type round, then the total weight of all Blue i type rounds to be fired against all $j = 1, 2, \ldots, R$ Red targets is

$$W_i \sum_{j=1}^{R} x_{ij}. \tag{32-64}$$

Moreover, if the total weight of all of Blue's rounds fired from all Blue weapons is not to exceed, say, W, then we must have the restriction

$$\sum_{i=1}^{B} W_i \left(\sum_{j=1}^{R} x_{ij} \right) \leq W \tag{32-65}$$

as a constraint in our new type analysis here. Hence, instead of a limitation on simply the total number of rounds fired (for example, $B_0 = 10$ missiles in Example 32-6), the allocations must now be made in terms of an upper limit on the total weight of rounds to be fired. This is easily handled, however, by replacing Eqs. 32-54 and 32-55 by

$$0 \leq \sum_{i=1}^{k} x_{ij} W_i = z_j \leq W \qquad (32\text{-}66)$$

and Eq. 32-57 by

$$h_k^j(z_j) = \min_{x_{kj}} [(q_{kj})^{x_{kj}} h_{k-1}^j(z_j - W_k x_{kj})] \qquad (32\text{-}67)$$

Note also that we have changed the former number of rounds x_j to a new designation z_j which now represents *weights* of warheads as in Eq. 32-66. The only other problem to handle in this analysis is to note that for the x_{kj} to be integers, then the values it can assume are

$$0, 1, 2, \ldots, [z_j/W_k], \qquad [z_j/W_k] = [y]$$

where $[y]$ means "greatest integer not exceeding y".

The $h_1^j(z_j)$ for Eq. 32-67 is found from an equation similar to Eq. 32-58, or

$$h_1^j(z_j) = w_j(q_{1j})^{x_{1j}}$$

and then $k = 2, 3$, etc., is used in Eq. 32-67 as required by the particular problem or application.

With these modifications, we are now ready to give an example of dynamic progamming for this new type of application. In order to illustrate the principles rather fully for more complex problems, we should consider at least two different Blue weapon types, and also we will use three different types of Red targets.

EXAMPLE 32-7:

In a very hypothetical planning situation, Red has a truck, an armored personnel carrier, and a light tank, all widely spaced and staggered, approaching Blue's position over a hill. Blue has an antitank weapon which fires an armor piercing projectile weighing 2 lb, and which has single-shot kill probabilities of 0.9, 0.6, and 0.4, respectively, against the Red truck, armored personnel carrier, and light tank. Blue also has a recoilless rifle, located somewhat off to the side, but rather advantageously, which may be aimed rather accurately, which fires an HE projectile weighing 3 lb and may produce single-shot mobility kills of 0.5, 0.7, and 0.8, respectively, against the given Red targets. If, for logistic planning purposes, only 10 lb of warheads should be fired, representing a day-of-supply limitation, then just how should Blue allocate the firing of his weapons, assuming that the relative worths of the Red truck, armored personnel carrier, and light tank, on a scale of unity, are 0.2, 0.3, and 0.5, respectively?

Again, our problem is to find the allocations of now two Blue weapons to three Red targets, designated by $\{x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}\}$, and which will minimize the threat (Eq. 32-48). Unlike Example 32-6, Blue now has two weapon types ($B = 2$), so that Eq. 32-57 for $k = 2$ must be calculated also. The data for this problem have been thoroughly analyzed by Sacco and Shear (Ref. 9), and we record their detailed analysis here for the benefit of the reader.

We begin the calculation with Eq. 32-58 and compute the value of $h_1^j(z_k)$, for $j = 1, 2, 3$, and for values of $z_k = 0, 1, 2, \ldots, 10$ lb. For each j and z_k we record the values of $h_1^j(z_k)$ and the value of x_{1j}

which gives this value of $h_1^j(z_k)$, i.e., we construct Table 32-4 similar to Table 32-2. Note that x_{1j} range from 0 to no higher than $[z_k/W_1] = [10/2] = 5$ rounds allocated.

The next step is to compute $h^j(z_k)$, $j = 1, 2, 3$, since Blue now has added a second type of weapon. Again, we record the value of x_{2j} (in the adjacent Table 32-6) which gives h_2^j, these being listed in Table 32-5. Since $h_2^j(z_k)$ can be interpreted as the "return" from the direction of two Blue weapon types against the jth Red target, we must obtain and record the x_{1j} in Table 32-6 for the pairs of (x_{2j},x_{1j}) allocations. Each x_{1j} is obtained from Table 32-4 and is found in the row corresponding to $(z_k - 3x_{2j})$ and the appropriate column for j. Thus, all results are exhibited in the side-by-side Tables 32-5 and 32-6.

Having computed the h_1^j and h_2^j, the calculations of $h_B^j(z)$ are now complete, and we may compute the g functions in Eq. 32-62. In fact, since $g_1(z,2) = h_2^1(z)$, we can proceed to the calculation of $g_2(z,2)$, which is given by

$$g_2(z_k,B) = \min_{z_2} \{h_2^2(z_2) + g_1(z_k - z_2,B)\} \tag{32-68}$$

TABLE 32-4
TABLE OF FUNCTIONAL VALUES $h_1^j(z_k) = w_j(q_{1j})^{x_{1j}}$
FOR EXAMPLE 32-7

	Values of Functions $h_1^j(z_k)$			Values to Individual Red Targets Allocation $x_{1j}(z_k)$*		
	$h_1^1(z_k)$	$h_1^2(z_k)$	$h_1^3(z_k)$	$x_{11}(z_k)$	$x_{12}(z_k)$	$x_{13}(z_k)$
z_k \ j	1	2	3	1	2	3
1	0.20	0.30	0.50	0	0	0
2	0.20	0.12	0.30	1	1	1
3	0.02	0.12	0.30	1	1	1
4	0.002	0.048	0.18	2	2	2
5	0.002	0.048	0.18	2	2	2
6	0.0002	0.0192	0.108	3	3	3
7	0.0002	0.0192	0.108	3	3	3
8	0.00002	0.00768	0.0648	4	4	4
9	0.00002	0.00768	0.0648	4	4	4
10	0.000002	0.003072	0.03888	5	5	5

z_k = total projectile allocation in pounds

***$x_{1j}(z_k)$ represents the number of warheads (projectiles) allocated to each of the three targets: $j = 1$ (Red truck), $j = 2$ (Red APC), and $j = 3$ (Red tank). The x_{1j} of Table 32-4 are chosen hopefully to cover all particular allocations needed for the dynamic programming solution limited to $z_k = 10$ lb of warheads.**

$$q_{11} = 0.1, \quad q_2 = 0.4, \quad q_3 = 0.6$$

$$w_1 = 0.2, \quad w_2 = 0.3, \quad w_3 = 0.5$$

EXAMPLE. $h_1^2(10) = w_2(q_{12})^{x_{12}(10)} = 0.3(0.4)^5 = 0.003072$

TABLE 32-5. TABLE OF FUNCTIONAL VALUES $h_2^j(z_k)$

z_k \ j	1	2	3
0	0.2	0.3	0.5
1	0.2	0.3	0.5
2	0.02	0.12	0.3
3	0.02	0.09	0.1
4	0.002	0.048	0.1
5	0.002	0.036	0.06
6	0.0002	0.0192	0.020
7	0.0002	0.0144	0.020
8	0.00002	0.00768	0.012
9	0.00002	0.00576	0.004
10	0.000002	0.003072	0.004

TABLE 32-6. TABLE OF PAIRS $[x_{2j}(z_k),\ x_{1j}(z_k - 3x_{2j})]$

z_k \ j	1	2	3
0	(0,0)	(0,0)	(0,0)
1	(0,0)	(0,0)	(0,0)
2	(0,1)	(0,1)	(0,1)
3	(0,1)	(1,0)	(1,0)
4	(0,2)	(0,2)	(1,0)
5	(0,2)	(1,1)	(1,1)
6	(0,3)	(0,3)	(2,0)
7	(0,3)	(1,2)	(2,0)
8	(0,4)	(0,4)	(2,1)
9	(0,4)	(1,3)	(3,0)
10	(0,5)	(0,5)	(3,0)

$$h_2^j(z_k) = \min_{x_{2j}} [(q_{2j})^{x_{2j}} h_1^j(z_k - 3x_{2j})\ ,\ \text{obtain } h_1^j(z_k - 3x_{2j}) \text{ from Table 32-4}$$

$$x_{2j} = 0, 1, \ldots [y]\ ;\ [y] = [z_k / W_2]$$

$$q_{21} = 0.5 \quad , \quad q_{22} = 0.3 \quad , \quad q_{23} = 0.2$$

EXAMPLE. Check $h_2^2(8) = 0.00768$

$$h_2^2(8) = \min_{x_{22}} [(q_{22})^{x_{22}} h_1^2(z_k - 3x_{22})]$$

$$[y] = 8/3 = 2$$

$$x_{22} = 0, 1, 2$$

$$(0.3)^0 h_1^2(8) = (1)(0.00768) = 0.00768 \text{ (minimum)}$$

$$(0.3)^1 h_1^2[8 - 3 \times 1] = (0.3) h_1^2(5) = (0.3)(0.048) = 0.0144$$

$$(0.3)^2 h_1^2[8 - 3 \times 2] = (0.3)^2 h_1^2(2) = (0.09)(0.12) = 0.0108$$

The minimum occurs when $x_{22} = 0$. Therefore no rounds from Blue weapon 2 (recoilless rifle) are expended against $j = 2$ (armored personnel carrier). Since $z_k = 8$ lb and a Blue weapon 1 (antitank weapon) round weighs 2 lb, 4 can be expended against $j = 2$. In Table 32-6, one may find this information at the intersection of $z_k = 8$ and $j = 2$ given as (0,4).

Our objective is to find the value of z_2 which minimizes the expression on the right side of Eq. 32-68 for the given $z_2 \leq z_k$. The values, z_2 and $(z_k - z_2)$, are used to enter the Table 32-6 and obtain the policy $\{x_{22}(z_k), x_{12}(z_k - 3x_{22})],\ [x_{21}(z_k - z_2), x_{11}(z_k - z_2 - 3x_{21})]\}$. For example, when $z_k = 6$ lb, $g_2(6,2) = 0.068$ by use of Eq. 32-68 and the value of z_2 which gives this value of $g_2(6,2)$ is $z_2 = 4$ lb. Thus, entering Table 32-6 at row $z_k = 4$ lb, we find in the $j = 2$ column that $x_{22}(4) = 0$, $x_{12}(4) = 2$. In the same table in row $z_k - z_2 = 6 - 4 = 2$ lb, we find in the $j = 1$ column that $x_{21}(2) = 0$ and $x_{11}(2) = 1$. Thus, the optimal policy for only two targets, the two Blue weapon types, and a total warhead allocation of $z_k = 6$ lb is $\{x_{11}, x_{12}, x_{21}, x_{22}\} = \{1,2,0,0\}$.

Proceeding in this manner, we construction Table 32-7.

To sum up a bit at this point, we have so far the optimum allocations of two weapons versus the first two of the targets, i.e., the Red truck and the APC, for all total weights of warheads up to 10 lb as shown in Table 32-7. Now we must add the third target, i.e., the Red tank to complete our problem.

TABLE 32-7. CALCULATIONS OF THE MINIMA $g_2(z_k,2)$

z_k	$g_2(z_k,2)$	$\{x_{11}, x_{12}, x_{21}, x_{22}\}$
0	0.5	0, 0, 0, 0
1	0.5	0, 0, 0, 0
2	0.32	1,0,0,0 or 0,1,0,0
3	0.29	0, 0, 0, 1
4	0.14	1, 1, 0, 0
5	0.11	1, 0, 0, 1
6	0.068	1, 2, 0, 0
7	0.056	1, 1, 0, 1
8	0.0392	1, 3, 0, 0
9	0.0344	1, 2, 0, 1
10	0.0212	2, 3, 0, 0

Now let us stop momentarily and examine the status of our solution at this stage, i.e., for adding the third target, remembering especially that the weight of the warhead for the first weapon—the Blue antitank weapon—is 2 lb and that for the Blue recoilless rifle is 3 lb. Now looking at Table 32-7, let us select the solution at the stage for the allocation {1,2,0,0} for 6 lb. We note here that the first weapon fires 1 round at the Red truck and 2 rounds at the Red APC, making a total of 6 lb. Thus, we cannot fire a warhead weighing 3 lb from the second Blue weapon and come out with exactly 10 lb, so we would be locked into firing only with the first Blue weapon—the antitank one. Thus, if we are to fire the second Blue weapon at all, i.e., the recoilless rifle, the total allocation at this stage of Table 32-7 must probably be something like z_k = 5 lb, i.e., the allocation {1,0,0,1}; or perhaps that of z_k = 7 lb, i.e., the allocation {1,1,0,1}; and indeed only one 3-lb warhead can be fired to reach 10 lb total, and that is the second Blue weapon. In this connection, we begin to see that we "are getting locked in" on the overall problem solution merely by looking ahead a bit. In any event, we will demonstrate that beyond the first two targets the recoilless rifle will fire a shot at the Red tank because of the constraint of 10 lb total weight and its effectiveness. In this connection, however, we do not need to calculate a complete table such as Table 32-7 for $g_3(z_k,2)$, for there is no fourth target to consider. Furthermore, we will be interested only in calculations involving the exact, specified total weight of warheads, i.e., 10 lb. Thus, we need to calculate $g_3(10,2)$ only by using Eq. 32-62 which is now expressed for our purposes here as

$$g_3(10,2) = \min_{z_3} \{h_3^2 + g_2[(10 - z_3), 2]\}.$$

Hence, to proceed we run z_3 over the values of 0, 1, 2, . . . , 10 and look for the minimum. It occurs at the value of z_3 = 3 lb, and we find

$$h_3^2(3) + g_2(7,2) = 0.1 + 0.056 = 0.156.$$

Therefore, we see that a 3-lb warhead is fired at the third target and the remaining 7 lb at the first two targets, making up 10 lb. This means that the final optimal allocation is

$$(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}) = (1,1,0,0,1,1)$$

or the Blue antitank weapon fires a 2-lb projectile at the Red truck, a 2-lb projectile at the Red APC, and the Blue recoilless rifle fires two 3-lb projectiles—one at the Red APC and one at the Red tank.

Examining the structure of the calculations in Table 32-7, however, it is readily seen that we have actually solved many simpler types of problems in the process. For example, we have solved the problem of 2 targets (index 1 and 2), two Blue weapon types, and a total weight constraint of 7 lb. The optimal policy for this problem is given in Table 32-7. Furthermore, if, for example, one is interested in the same targets and two weapon types, but with a total weight constraint of, say, 9 lb, then $g_3(9,2)$ must be computed. For this example, $g_3(9,2) = 0.168$, and the optimal policy is {1,2,0,0,0,1}.

32-6 A LINEAR PROGRAMMING FORMULATION OF A WEAPON-TARGET ALLOCATION PROBLEM

As we have seen, practically all problems in allocating weapons to targets in some optimal manner involve the application of mathematical programming principles since we are searching for maxima or minima subject to certain constraints. In connection with cost-effectiveness analyses conducted some years ago at the USA Ballistic Research Laboratories, Sacco and Masaitis (Ref. 10), and O'Neill, Sacco, and Westerman (Ref. 11) developed a weapon-target allocation model of some practical importance. The typical problem they covered was expressible in terms of standard linear programming principles and was formulated as follows. Let:

T_j = jth Red target class, referring to type of target, size, and location

f_j = number of targets in the jth Red target class

B_i = number of rounds (or sorties) available for the ith type of Blue weapon. (This may be a combat day of expenditure.)

r_{ij} = number of rounds (sorties) of ammunition required to defeat the jth Red target with the ith Blue weapon

x_{ij} = number of targets in the jth Red class of a single threat assigned to the ith Blue weapon

i = 1, 2, ..., B as before

j = 1, 2, ..., R as before.

The problem developed by Sacco and Masaitis (Ref. 10), and O'Neill, Sacco, and Westerman (Ref. 11) is that of finding the maximum number of times K (not necessarily an integer) that the Red target threat can be defeated for the available supply of ammunition. It becomes clear that this problem can be stated mathematically as that of finding the allocations x_{ij}, which maximize the quantity k, subject to:

$$\left.\begin{aligned}
&K(r_{11}x_{11} + r_{12}x_{12} + \cdots + r_{1R}x_{1R}) \leq B_1\\
&K(r_{21}x_{21} + r_{22}x_{22} + \cdots + r_{2R}x_{2R}) \leq B_2\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&K(r_{B1}x_{B1} + r_{B2}x_{B2} + \cdots + r_{BR}x_{BR}) \leq B_B\\
&x_{11} + x_{12} + \cdots + x_{1R} = f_1\\
&x_{21} + x_{22} + \cdots + x_{2R} = f_2\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
&x_{B1} + x_{B2} + \cdots + x_{BR} = f_R\\
&x_{ij} \geq 0.
\end{aligned}\right\} \qquad (32\text{-}69)$$

The authors (Refs. 10 and 11) suggest the use of "slack" variables L_i to replace the top half of Eq. 32-69, or the inequalities, by equalities expressed as:

$$\left.\begin{aligned}
&K(r_{11}x_{11} + r_{12}x_{12} + \cdots + r_{1R}x_{1R} + L_1) = B_1\\
&K(r_{21}x_{21} + r_{22}x_{22} + \cdots + r_{2R}x_{2R} + L_2) = B_2\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&K(r_{B1}x_{B1} + r_{B2}x_{B2} + \cdots + r_{BR}x_{BR} + L_B) = B_B\,.
\end{aligned}\right\} \tag{32-70}$$

Instead of maximizing the quantity K, it is clear that we may minimize $1/K$ or, using the first equation of Eq. 32-70, this means finding the minimum

$$\min_{x_{ij}} (r_{11}x_{11} + r_{12}x_{12} + \cdots + r_{1R}x_{1R} + L_1)/B_1 \tag{32-71}$$

subject to

$$\left.\begin{aligned}
&B_1(r_{21}x_{21} + r_{22}x_{22} + \cdots + r_{2R}x_{2R} + L_2) = B_2(r_{11}x_{11} + r_{12}x_{12} + \cdots + r_{1R}x_{1R} + L_1)\\
&B_1(r_{31}x_{31} + r_{32}x_{32} + \cdots + r_{3R}x_{3R} + L_3) = B_3(r_{11}x_{11} + r_{12}x_{12} + \cdots + r_{1R}x_{1R} + L_1)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&B_1(r_{B1}x_{B1} + r_{B2}x_{B2} + \cdots + r_{BR}x_{BR} + L_B) = B_B(r_{11}x_{11} + r_{12}x_{12} + \cdots + r_{1R}x_{1R} + L_1)
\end{aligned}\right\} \tag{32-72}$$

and the R equalities of Eq. 32-69, where we used the common K of Eq. 32-70 to obtain the first $(B - 1)$ equations (Eq. 32-72). In summary, and using matrix notation, this means simply that we are to find the x_{ij} such that Eq. 32-71 holds, or is a minimum, subject to the linear equations

$$[A][x] = [b], \quad x_{ij} \leq 0 \tag{32-73}$$

where

$$[x]^T = [x_{11}, x_{12}, \ldots, x_{1R}, L_1, \ldots, x_{B1}, x_{B2}, \ldots, x_{BR}, L_B] \tag{32-74}$$

is the row vector of unknowns to be solved and

$$[b]^T = [0, 0, \ldots, 0, f_1, f_2, \ldots, f_R] \tag{32-75}$$

is the row vector of constants, there being $(B - 1)$ zeros and R of the f_j's. The coefficient matrix $[A]$ is given by

$$[A] = [r_{ij}]. \tag{32-76}$$

The values of K and those of the x_{ij} are not restricted to integers. The elements r_{ij} of the coefficient matrix, which give the number of rounds from the ith Blue type weapon required to defeat the jth Red target, may be found for many targets and weapons as listed in the *Joint Munitions Effectiveness Manual* (Ref. 12).

Hence, and by way of summary, this type of weapon-target allocation problem involving Eq. 32-71, subject to Eq. 32-73, reduces to a standard linear programming problem, the algorithms of which may be found, for example, in the book of Gass (Ref. 13).

32-7 RANDOM TARGET ALLOCATION AND THE HOMOGENEOUS POINT FIRE MODEL

In this chapter our primary discussion was on the "optimum" allocation of weapons of specific capability to targets they are most capable of attacking. Except for Eqs. 32-6 and 32-7, this did not cover random selection or allocation of weapons to targets, as will often be the case. Therefore, we believe it desirable to close out this chapter by discussing briefly something more about the random allocation of weapons to targets. This will be done for uniform random allocation—or, that is, equally likely assignment—of weapons against targets for the case of homogeneous forces. The case of assignment of weapons to targets otherwise discussed in this chapter should be considered, of course, to involve heterogeneous forces anyway. Thus, it does not make sense to fire rifles at tanks, for example, nor is it necessary to shoot a tank or artillery projectile at a single infantryman. In fact, it can be seen that between homogeneous forces random firing may indeed make much sense, except especially for those situations where, for example, particular targets at the closer combat ranges necessarily deserve and are given the highest priorities for firing. In what follows, we will discuss briefly the homogeneous point fire model of Karr (Ref. 14) which has to do with a class of binomial attrition processes, and, in fact, one may obtain simple expressions for the expected number of enemy casualties or kills and also chances that the number of kills will be equal to selected or prespecified numbers.

In the formulation of Karr (Ref. 14), he considered a "one-sided" combat between two opposing homogeneous forces, e.g., riflemen or tanks, where a force of B Blue indistinguishable "searchers" seek out and fire upon a force of R Red indistinguishable "targets". The assumptions he makes concerning the combat situation are:

1. By some fixed time, all R Red targets become vulnerable to detection and attack by the B Blue searchers.
2. The probability that any Blue searcher detects any Red target is p_d, and each particular Blue searcher detects a different Red target independent of one another.
3. A Blue searcher who makes no detection makes no attack, and a Blue searcher who makes one or more detections chooses one target to attack randomly according to a uniform distribution over the set of targets he has detected, independent of his detection process.
4. The conditional probability that a Blue searcher kills a Red target, given detection and attack, is p_{ssk} for all Blue searchers and Red targets.
5. No Blue searcher may attack more than one Red target.
6. The target detection and attack process of different Blue searchers are mutually independent.

With these assumptions and the uniform random allocation of Blue firers against Red targets, Karr (Ref. 14) shows that the expected number $E(R_k)$ of Red targets killed is given by the relatively simple expression

$$E(R_k) = R\left\langle 1 - \left\{1 - \frac{p_{ssk}}{R}\left[1 - (1 - p_d)^R\right]\right\}^B\right\rangle^{*} \qquad (32\text{-}77)$$

*Apparently, this relation was originally suggested or worked out by LTG Glenn A Kent, USAF and later proven rigorously by Karr (Ref. 14).

When the chance of detection p_d is zero, then Eq. 32-77 reduces to

$$E(R_k) = 0 \tag{32-78}$$

as one would expect, and when $p_d = 1$, then Eq. 32-77 reduces to

$$E(R_k) = R\{1 - [1 - p_{ssk}/R]^B\} \tag{32-79}$$

$$\approx R[1 - \exp(-Bp_{ssk}/R)] \tag{32-80}$$

when p_{ssk}/R is suitably small.

Karr (Ref. 14) also shows that the chance that the number of Red kills is exactly equal to, say, α is

$$Pr(R_k = \alpha) = \binom{R}{\alpha} \sum_{i=1}^{\alpha} (-1)^{\alpha-i} \binom{\alpha}{i} [(1 - Q) + iQ/R]^B \tag{32-81}$$

where

$$Q = p_{ssk}[1 - (1 - p_d)^R]. \tag{32-82}$$

Karr (Ref. 14) also extends this type of model to the case of applications to heterogeneous forces, or the "heterogeneous point fire model", and to an area fire model. We will not go into such matters any further here, however.

32-8 SUMMARY

We have introduced the problem of allocating weapons to targets in order to improve the effectiveness of weapons on the basis of worth, potential threat, logistic effort, or other considerations. The treatment of weapon-target allocation problems involves operations research techniques such as mathematical programming (particulary the dynamic programming principles of Bellman), linear programming techniques, Lagrange multipliers, and other useful analytical tools. Weapon-target allocation models are very valuable and should be applied to problems of the weapon systems analyst—especially in planning for the fielding of optimum families of weapons to be used in combat.

It is realized that weapon-target allocation problems cannot often be handled in a combat environment, although the methodology may be of considerable importance for logistic planning purposes. minimization of costs of weapons to perform intended tasks, the study of families of weapons and how they fit together, the prudent use of weapons in the field, and a host of other considerations.

A sufficient number of examples are given to illustrate some typical applications.

The analyst may extend his knowledge in various or desirable directions by studying thoroughly the References and the Bibliography.

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CHAPTER 33
INTRODUCTION TO HUMAN FACTORS AND WEAPON SYSTEMS ANALYSIS INTERFACE PROBLEMS

Since humans operate, perform with, and/or maintain weapons or weapon systems, human factors and human engineering considerations become an important part of the overall system to be evaluated—especially from the standpoint of maximizing effectiveness and establishing training requirements. Although there is some overlap, it seems desirable, nevertheless, to define and discuss human engineering, human performance, and human reliability somewhat separately insofar as weapon systems analysis is concerned. This leads to the discussion of typical problems and examples the analyst might possibly face in his evaluations. This introduction to the human factors role in the field of weapon systems analysis hopefully should serve to give the young analyst a start in the direction of appreciating the importance of human factors and human engineering interface problems.

33-0 LIST OF SYMBOLS

C = high explosive charge weight
E = effectiveness of weapon system
m = measure of the soldier's proficiency
m = mean value
P = percent targets detected, or probability of detecting a target
t = parameter describing tactical employment capability
t = time variable or parameter
v = variance
W = weight of metal casing
w = inherent capability of the weapon
θ = parameter of the exponential distribution
$\sigma = \sqrt{\sigma_\mu^2 + \sigma_\nu^2}$
σ_D = standard deviation in deflection for grenades
σ_R = standard deviation in range for grenades
σ_x = round-to-round standard deviation in range
σ_y = round-to-round standard deviation in deflection
σ_μ = standard deviation of C of I (aiming error) in range direction
σ_ν = standard deviation of C of I (aiming error) in deflection direction

33-1 INTRODUCTION

Considerations of the man and his interface with a weapon system have to be very important factors in analyzing the potential effectiveness of the weapon system. Every new successor weapon system should be better than its predecessor, and hopefully we should be able to predict the performance of each weapon in the field or its potential under combat conditions. In order to evaluate weapon systems properly, we must know accurately what human performance contributes to system effectiveness and bring about any needed training of personnel in the fielding of any weapon or weapon system.

Weisz (Ref. 1) gives a most appropriate description of the man-weapon interface problem in weapon systems analyses, and we quote him here:

"No weapon system analysis can be considered valid unless it includes the contribution of a very important component, namely, "man", as part of the system. Man's contribution to the *performance* of the weapon can either enhance or degrade its performance, depending on whether or not his capabilities and/or limitations were appropriately considered in the first place as part of the initial, total system concept. A growing body of literature is slowly being assembled and methodologies developed which, if properly utilized, will materially assist system analysts in conducting their analyses throughout the life cycle of a weapons system.

"Most of the research supporting this human factors field is being conducted in the behavioral sciences and human factors engineering fields utilizing experimental methodology and multi-variable statistical techniques drawn from experimental psychology. Operation research techniques have also been added to the list of tools which human factors researchers are using in performing their portion of a given weapon system analysis. There is at present no single comprehensive reference source which attempts to describe and pull together all the techniques and methods which have been utilized in this field. . .

"There are at least four areas to be considered in an analysis of a system or systems to which the field of human factors can make contributions:

"*Manpower Requirements*

Manpower is a critical commodity and thus needs to be considered on all proposed system concepts in terms of the number of personnel to be used and the skill levels of the personnel required to operate and maintain the system. These inputs can be expressed in quantitative and qualitative terms and in the monetary cost for each competing system concept being analyzed. If special skills are required, whether there are a sufficient number of such personnel available to the Army becomes an extremely important question which must be considered in the analysis.

"*Training Requirements*

How difficult will it be and how much will it cost to train operating and maintenance personnel for a proposed system? Will it be easier and thus less costly to train personnel on one competing system compared to another?

"*Performance Requirements*

Can personnel perform, and how effectively can they perform, the duties required to operate a system? Are the capabilities (auditory, visual, physical strength, judgment, etc.) of the operating personnel used appropriately in the system? What is the error potential of the man component of the system as differentiated from the equipment error potential? The error potential of man can usually be determined and expressed quantitatively.

"The contribution of man to system performance must always be considered in the particular environment in which the system will be utilized. Thus such environmental conditions as dust, heat, cold, fog, mud, noise, blast, smoke, etc., must be considered insofar as they affect the man doing his tasks as part of total system operation. Obviously, total system functioning or its effectiveness will be influenced directly or indirectly by the degree that these conditions affect man's performance. Providing quantitative data with regard to how seriously these environmental aspects affect man's performance comprises part of the contribution which human factors thinking can make to a specific system analysis.

"*System Design*

A large bulk of human factors data is available in the area of human factors engineering where the main objective is to provide assistance to designers of displays, controls, layouts of crew work areas and operator compartments, etc. Analyses of how various design approaches in these areas will affect system effectiveness can usually be made throughout the development process of a particular system.

"Although human factors contributions have not been treated here in their entirety, it is clear that the area of human factors should definitely be included but not treated separately from other areas presently embraced in the system analytic thinking process. Since man is an integral part of the total system, his contributions must be included each and every time that such areas as *system performance*, *system effectiveness*, *system dependability*, *system reliability*, *system capability*, and *cost effectiveness* are considered. The US Army Human Engineering Laboratories (HEL) have shown that man's contributions can be determined and expressed quantitatively as accurately as most of these other factors.

"Human factors personnel are unanimously in agreement that, if the man-component variable is introduced early into pre-development system analytic thinking and then appropriately considered in the actual design process, tremendous savings in time and money will usually result. In addition, the Army R & D manager will have more complete and more valid information available to him for decision-making purposes."

Man has often been looked upon as one of the primary components of a complete weapon system, and his interface with the weapon or weapon system he employs may likely strike the balance between success and defeat in a battle. It is a very natural approach for the analyst to identify each anticipated source of variation in the expected performance of the weapon system, to estimate the relative sizes of the components of variation, and to find their effect on predicted overall system performance. Should it turn out that the man is contributing too large a component of variation toward expected system performance, then further training of the soldier, or operator, may be indicated, or perhaps the improved design of weapons will be made mandatory. Otherwise, it becomes important to estimate the size of natural human variations which may be involved in operation of a weapon system, and to take such amounts of variability into account in effectiveness studies. Personal equipment—such as the helmet, body armor, clothing, shoes, and communication equipment—must be compatible with the military personnel who use it. In addition, for example, pilots of helicopters and aircraft have special interface problems to meet in connection with the complex equipment they operate. Thus, such considerations as we have enumerated bring forward three important areas of interest concerning human factors which the weapon systems analyst must take into account, namely:

1. Human engineering
2. Human performance
3. Human reliability.

In this chapter, we will discuss some evaluations of these in turn and thereby introduce the analyst to some of the aspects of each type of human factors problem.

33-2 GENERAL DEFINITIONS AND SOME GUIDELINES

33-2.1 HUMAN ENGINEERING

Historically human engineering has been defined as the study of interface problems between man and machine. Engineering can be said to be the process of planning, designing, fabricating, producing, and testing things—such as a rifle, a vehicle, a tank, or a weapon. The human engineer represents the user of equipment with regard to ease of operation, maintenance, safety, comfort, etc., and in his role evaluates man as a system component and his contribution or relation to the whole system. The human engineer plays an important role in product and system design. Accordingly, it is necessary that he contribute to the selection among alternatives to system design so that the most appropriate piece of equipment for use in the field will become available. For many new systems, the engineering process of study is rather highly conceptual, and the experienced human engineer is perhaps needed here very critically.

A fairly complete and comprehensive treatment of human engineering guidelines to equipment design is that of Ref. 2. This book was prepared under the sponsorship of a Joint Army-Navy-Air Force Steering Committee, and the various chapters include topics on:

1. System and Human Engineering Analyses
2. Man as a System Component
3. Visual Presentation of Information
4. Auditory and Other Sensory Forms of Information Presentation
5. Speech Communication
6. Man-Machine Dynamics
7. Data Entry Devices and Procedures
8. Design of Controls
9. Design of Individual Workplaces
10. Design of Multi-Man-Machine Work Areas
11. Engineering Anthropology
12. Design for Maintainability
13. Training System Design
14. Training Device Design
15. Human Engineering Tests and Evaluation.

The various chapters were written by selected authorities in the fields of interest.

There exists also a military standard (Ref. 3) on human engineering design criteria for military systems, equipment, and facilities. The purpose of this military standard is to present human engineering design criteria, principles, and practices to achieve mission success through integration of the human into the system, subsystem, equipment, and facility, and achieve effectiveness, simplicity, efficiency, reliability, and safety of system operation, training, and maintenance. The standard is applicable to the design of all systems, subsystems, equipment, and facilities and is apparently intended to cover humans who are between the 5th and 95th percentile body dimensions. Applicable body dimensions are those dimensions which are design-critical to the operation, manipulation, removal, or replacement tasks involved.

33-2.2 HUMAN PERFORMANCE

Human performance usually is taken to mean a quantitative measure of the degree to which an individual is capable of operating with the equipment. It should be recognized that individuals vary in capability and that the assessment of such variability may be of importance insofar as weapon system operation is concerned. As examples, we may desire to know just how well a soldier can sight or aim a rifle, how accurately he can throw a hand grenade, or how well can he aim and fire a machine gun, an artillery piece, etc. Also, when we consider that for many pieces of equipment or weapon systems the human operator may in effect be a component of the whole system, then his performance in terms of variability has to be considered and estimated along with other variations or errors that affect system performance.

33-2.3 HUMAN RELIABILITY

Human reliability might be considered by many to be perhaps a part of or includable in human performance, and indeed it could be so argued. However, there is the tendency in recent years to break out separately the concept of human reliability, and indeed this can often amount to a very pertinent treatment in view of the recent marked advances for the field of reliability analyses in general. A simple concept of reliability might be the proportion of errors a soldier would make in handling or using a piece of equipment or a weapon. Or it might be the "mean time in battle" until fatigue occurs for the soldier. Also humans come into the picture in the maintenance of systems so that they will be ready once a demand is placed upon them to operate, fire, etc. Thus, the fraction of the time the system is "up" and ready to start a mission may become very important (it being a measure of availability), and the degree of maintainability by individuals would become a study of human reliability, so to speak. Thus, in many problems it may be advisable to study human reliability separately from the concept of performance in order to give proper emphasis to the new field. Moreover, the principles developed in Chapter 21, *Part One*, of this handbook would be applicable to the analysis of many problems of human reliability.

We will illustrate each of these areas in turn, although first we consider an early example, which borders on both human engineering and human performance. It concerns a 1952 Ballistic Research Laboratories (BRL) study of hand grenades and an analysis of the ability of soldiers to throw them, in order to select an optimum grenade.

33-3 AN EARLY EXAMPLE

Throughout World War II, the Army retained the old Mk II hand grenade, an egg-shaped, serrated, and heavy grenade, weighing 22 oz. The serrated fragments were large, but small in number, whereas terminal ballistic research during World War II indicated that lethality could be improved with a design calling for much smaller, although higher velocity fragments. Also, it was realized that the throwing accuracy of the soldier should be determined and taken into account in any effort toward developing a new hand grenade. Thus, it is easily seen that in the early 1950's the Army was faced with a human engineering and a human performance problem, and indeed both should be taken into account in efforts to develop any new hand grenade. In fact, some thought that a spherical and lighter grenade could be developed which could be thrown farther than the Mk II and more accurately. Others thought that the surface finish, whether smooth or rough, might have an important effect. Another thoughtful person argued that the grenade should be much like a baseball, because of the popular American sport! Obviously, this presented a good opportunity to design and carry out an appropriate experiment for studying human performance in throwing the grenade and to clear up the human engineering problem of hand grenade size, weight, and shape. The BRL decided consequently to approach the problem from an overall or system concept, the man being an important system "component", and to conduct a good weapon system analysis to recommend the optimum hand grenade.

A preliminary analysis indicated that the optimum shape for lethality effects would be spherical and that some six different types of grenades, covering three different weights and four different diameters, should be considered in the study. The physical features of the six types of grenades are given in Table 33-1.

In view of the fact that accuracy of throwing is a very important characteristic of hand grenades, it became necessary to conduct throwing tests involving the proposed grenades in order to evaluate this factor rather extensively. For the throwing tests, it was decided to conduct a program which would involve the given six types, but would also include some smooth surface grenades and some rough surface grenades of the same type, the old standard Mk II grenade, and in addition the "Beano" grenade experimented with during World War II. The (inert) grenades selected for the throwing tests were identified by code letter and had the various characteristics outlined in Table 33-2.

TABLE 33-1. PHYSICAL DESCRIPTION OF TEST GRENADES

Wt of Grenade, oz	C/W*	% Explosives	Fragment velocity, ft/s	Average density, g/cm³	Volume, cm³	Diam, in.
8	0.327	25	4250	3.99	56.9	1.88
8	1.00	50	6530	2.66	85.4	2.15
12	0.327	25	4250	3.99	85.4	2.15
12	1.00	50	6530	2.66	128.0	2.47
18	0.327	25	4250	3.99	128.0	2.47
18	1.00	50	6530	2.66	192.0	2.82

*C/W = charge to metal weight ratio

TABLE 33-2. CHARACTERISTICS OF ELEVEN TEST GRENADES

Code Mark or Assigned Letter*	Surface Finish	Lot No. PA-E-	Ave. Wt, oz	Std. Dev. of Wt, oz	Ave. Diam, in.	Std. Dev. of Ave. Diam, in.
C	Rough	7037	8.01	0.07	1.98	0.00
D	Rough	7038	8.02	0.09	2.23	0.01
H	Smooth	7039	8.04	0.06	2.23	0.01
G	Rough	7040	12.04	0.08	2.27	0.01
A	Rough	7041	11.94	0.16	2.57	0.02
B	Smooth	7042	12.19	0.10	2.59	0.00
I	Rough	7043	18.24	0.22	2.61	0.01
E	Rough	7044	18.15	0.20	2.86	0.01
F	Smooth	6247	17.95	0.10	2.85	0.01
J (Beano)	Smooth	—	12.00	Unk.	2.95	Unk.
K (Mk II)	Serrated	Unk.	22.00	Unk.	---	Unk.

*Letters assigned randomly to grenades for the test design.

The nine experimental types of hand grenades (first nine in Table 33-2) had a "pip" on their spherical surface so that they could be oriented in the hand of the thrower to simulate the fuze which would probably be developed, but did not have fuze handles since the exact type of fuzing was not known at the time and was not considered to be of great importance for the throwing tests. The standard Mk II (inert) fragmentation hand grenade was loaded with a filler equivalent in density to that of TNT, and the fuze handle was used on this grenade since the combat soldier was accustomed to this conventional type. It was necessary to throw the "Beano" grenades without simulated fuzes, i.e., the fuze wells were covered with tape.

It was decided that the (inert) grenades would be thrown from three positions—i.e., prone, kneeling, and standing—and for four different ranges consisting of 20 yd, 30 yd, 40 yd, and maximum range. Thus, combinations of throwing positions and ranges consisted of four in number and were as follows:

1. Prone position—aiming stake at 20 yd range
2. Kneeling position—aiming stake at 30 yd range
3. Standing position—aiming stake at 40 yd range
4. Standing position—maximum range.

The grenade thrower was allowed to throw the grenades in any manner he considered best for himself; therefore, the method or manner of throwing was not restricted to being "standard". For the kneeling position, it was specified that the grenade thrower should keep at least one knee on the ground. For the standing position-maximum range, a stake was placed at 60 yd from the throwing position and the grenade thrower was instructed to throw the grenade toward the stake, but at the same time to throw as far as he could (i.e., the purpose of the stake was only to establish a well-defined direction of throwing). During practice periods of the throwing tests, it was noticed that the grenade throwers exhibited considerable difference in technique for the prone position, some arising almost to the kneeling position as the grenade left their hand. Because of this, it was specified that in throwing from the prone position a grenade thrower must not in any way come to a position approximating the kneeling position.

Before throwing each of the different types of grenades for record, the grenade throwers were allowed a preliminary practice period in order to get the "feel" of the particular type of grenade they were about to throw. When throwing the grenades for record, all throws counted unless the thrower indicated that he had a legitimate alibi and that the deficiency of a particular throw was his own and not a fault of the grenade. Actually, no "alibis" were made or recorded during the entire throwing test.

In the throwing tests, stakes driven in the ground were used as targets. These stakes were placed in the direction of throw at distances of 20 yd, 30 yd, 40 yd, and 60 yd. After each throw, two observers noted the position at which the grenade first impacted on the ground and measured the deviation of the impact from the stake in both the range and deflection directions. (The point of first impact with the ground was selected because it was understood that the fuzing system for the new grenade would involve an impact element and the grenade would detonate on first coming into contact with the ground.) Also, the grenade throwers were asked to comment on each of the types of grenades tested, giving, for example, their opinion on ease of handling, size, weight, or any other observations which might be considered important insofar as the selection of an optimum grenade was concerned.

The grenade throwers for this program were selected from combat soldiers who recently had returned from Korea and who were stationed at Aberdeen Proving Ground. In order that the throwing tests could be accomplished quickly, it was decided to construct a layout of eleven "throwing ranges", side by side, in order that the eleven different types of grenades could be thrown or tested during the same time period by the eleven grenade throwers. However, a single soldier tested only one type at a time and finished with this particular type before proceeding to a new type. A suitably flat and sandy area of ground was constructed on which the eleven throwing ranges were laid out, and directions for the throwing tests were given to all grenade throwers by the proof officer over a loud-speaker system.

The primary purposes of the throwing tests were to compare the different types of grenades insofar as inherent accuracy of throwing was concerned and to obtain information on the accuracy with which the "average" combat soldier could aim and throw the grenades. The important parameters which were considered to introduce probable or significant variations into the experiment were, therefore, the different types of grenades, the different individuals throwing the grenades, the fact that an individual's arm may become sore as he proceeded with the throwing test (i.e., a time parameter) and, finally, a possible source of bias which might be introduced by the different "throwing ranges". It was decided, therefore, to carry out the throwing tests using an experimental design known as the Graeco-Latin square, with four grenades of the same type thrown per cell, in order that variation from the various sources could be stripped out and analyzed separately. In this connection, the eleven different types of grenades were identified and assigned the letters A, B, C, D, E, F, G, H, I, J, K at random, as in Table

33-2, and eleven grenade throwers were used at a time, these being assigned the numbers 1-11 at random. The entire throwing experiment, with detailed analyses, has been reported by Grubbs and Shank (Ref. 4), some of the results being of sufficient interest to record here. For the entire experiment, there were 3872 throws of the hand grenades, half the experiment being with gloves and half without. A rather complete account of the statistical analysis is given in Ref. 4.

Results of the analysis indicate that there existed no significant differences in round-to-round dispersion (standard deviation or variance) in range for the different types of grenades, or for dispersion in deflection for the different types of grenades. (Throwing accuracy could be different nevertheless.) In some cases there were significant differences in round-to-round dispersion in range which were attributed to the throwers but not to the types of hand grenades tested. Table 33-3 gives an indication of the sizes of the round-to-round standard deviations in range and deflection for the average of all types of grenades tested. It can be seen from Table 33-3 that the standard deviation in range is about 1.3 times that in deflection and that both values appear to be of acceptable magnitude.

On average range and with reference to the aiming stake thrown at, the analyses revealed the smaller diameter (2 in. and 2.5 in.), and the small and medium weight grenades (8 oz and 12 oz), to be superior in accuracy. The larger diameter and greater weight grenades (2.47 and 2.82 in. diameter and 12 and 18 oz) appear to be the most inaccurate as did also the old standard MkII grenade. The "Beano" grenade fell more or less between the group of best accuracy in range and the group of poor accuracy in range. If one were to rank the grenades in decreasing order of accuracy in range, the result would be D, C, H, A, B, G, J, I, K, E, F (correlate with Table 33-2). There were highly significant differences among the *throwers* in so far as accuracy in range was concerned.

On average deflection, there were generally no significant differences among the grenade types, although there was some evidence to indicate that the heavier grenades tended to land to the right of the stakes and the lighter grenades to the left of the stakes—perhaps because practically all of the throwers were right-handed.

Although the standard deviations in range and deflection are different in magnitude, the accuracy of throwing may be described in terms of a very meaningful measure: namely, the chance that a thrown grenade will come within a given distance of the stake aimed at. This is illustrated in Fig. 33-1 for the aiming stake placed at 40 yd and the thrower in the standing position. Note in particular (1) the superiority of the smaller weight and hence smaller diameter grenades, and (2) that the old Mk II hand grenade was the least accurate one. Similar curves for the prone position-20 yd range and the kneeling position-30 yd range are given in Ref. 4.

The effectiveness of a hand grenade as an antipersonnel weapon depends on the distance from the target the grenade functions, and on the fragment size and velocity. From this standpoint, therefore, a characteristic of primary importance in analyzing the throwing test data would be the radial deviations from the stakes and the dispersion in radial distances. Hence, we see the importance of Fig. 33-1. Actually, significant differences for the different types of grenades were indicated for the average radial distances, as one may suspect, due to the results on average range previously mentioned. If one were to

TABLE 33-3. SUMMARY OF ROUND-TO-ROUND DISPERSIONS

Position	Range, yd	σ_R, ft	σ_D, ft
Prone	20	4.44	3.81
Kneeling	30	4.98	3.58
Standing	40	7.47	5.54

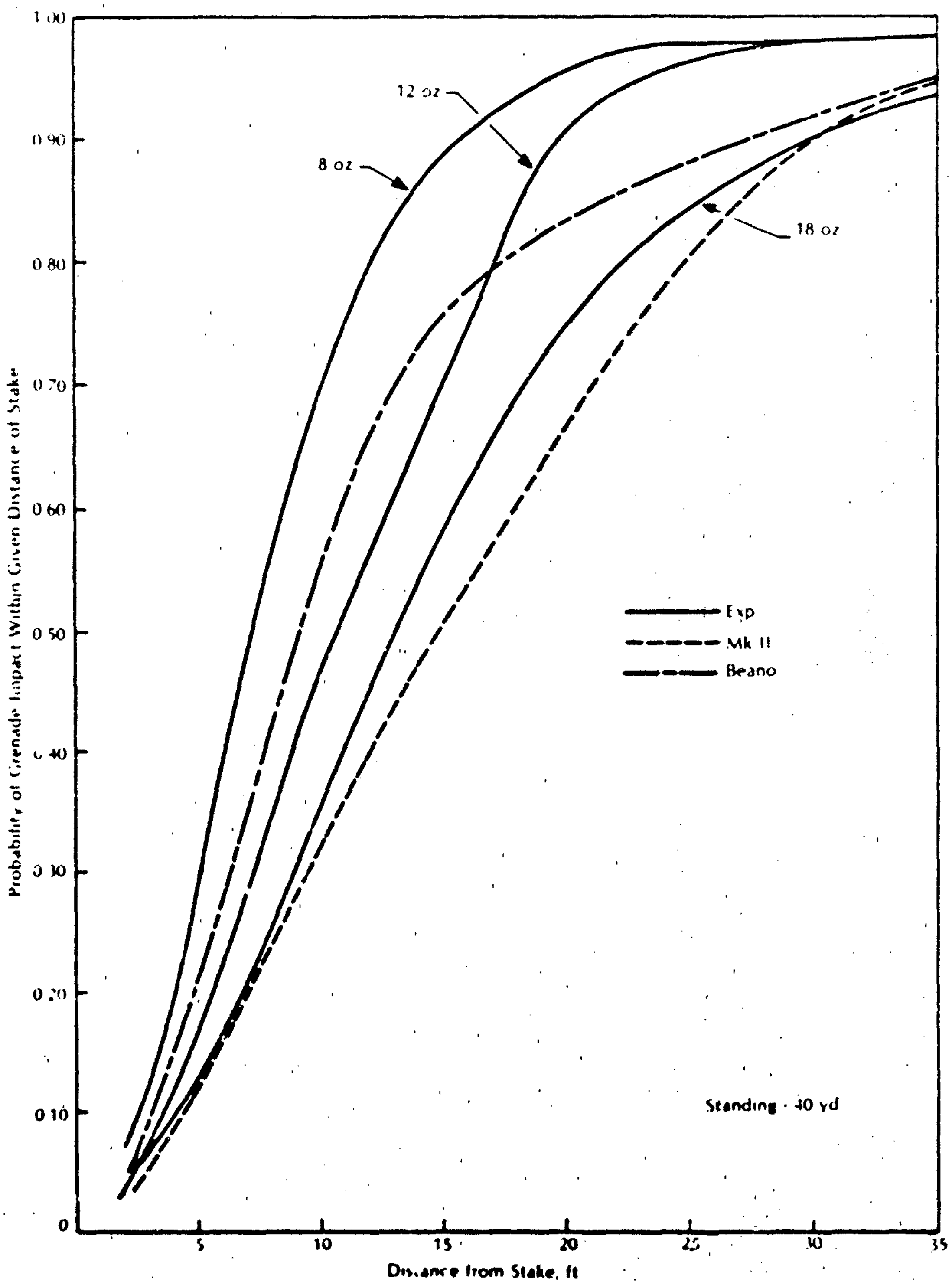

Figure 33-1. Cumulative Probability Curves for Hand Grenade Throwing Test (Standing Position, Range 40 yd)

rank the grenades in accuracy using sums of average radial deviations (omitting the throwing test for maximum range), the order of decreasing accuracy for the designated grenades would be D, C, H, B, G, A, J, I, E, K, F.

Thus, it appears that irrespective of whether one analyzes the average range and deflection deviations independently, or the important characteristic of radial deviations, the smaller diameter and lighter weight or medium weight grenades turn out to be somewhat superior in accuracy. The larger diameter and heavier grenades were the more inaccurate, whereas the "Beano" grenade held the middle ground, and the old Mk II grenades were the most inaccurate.

The smaller diameter, lighter weight grenades could also be thrown farther, as indicated in Table 33-4.

In evaluation studies of hand grenades, the probability of a casualty, i.e., the single-shot kill probability, may be used as a measure of effectiveness. The chance of a casualty depends on the product of the chance that a thrown grenade will land within a given distance of the target and the conditional chance that it will cause an incapacitation at that distance. Thus, a very important reason for conducting the throwing experiment was actually to match terminal ballistic properties with the throwing accuracy ability of the average soldier. In this connection, one notes in Table 33-3 that the maximum standard deviation occurs in range and is about 7.5 ft. This value, combined with the bias or deviation from the stake thrown at, could be used to help in designing the lethality parameters of the grenade. Suppose, for example, that the radial standard deviation of the impact points from the aiming stake amounts to 10 ft. Then, the charge to metal weight ratio for the grenade could be determined such that fragments emanating from the grenade upon detonation would be lethal out to about $3\sigma = 30$ ft for the fragment weights and initial velocities decided upon. Thus, the further importance of the type of experiment carried out here which involved both human engineering and human performance. In fact, this account, though for a simple weapon, should nevertheless give a clear picture of the type of interface problems between human factors, engineering design, and weapon systems analysis. Indeed, the design of the hand grenade, including its size and shape, and the fragment weights and velocities, should be such that the throwing accuracy of the soldier is fully considered.

TABLE 33-4. AVERAGE MAXIMUM RANGE DATA FOR GRENADE TYPES AVERAGED OVER THROWERS

Grenade Type	Grenade Description	Actual Ave. Range (all throwers), yd
C	2 in.-8 oz	48.75
D	2.25 in.-8 oz (Rough)	46.93
H	2.25 in.-8 oz (Smooth)	44.75
B	2.6 in.-12 oz (Smooth)	42.41
A	2.6 in.-12 oz (Rough)	41.71
G	2.25 in.-12 oz (Rough)	41.02
J	Beano	40.06
I	2.6 in.-18 oz (Rough)	37.25
K	Mk II	37.10
E	2.9 in.-18 oz (Rough)	36.94
F	2.9 in.-18 oz (Smooth)	36.38

33-4 HUMAN ENGINEERING

In par. 33-2.1 we defined human engineering as a study of the interface problems between man and machine and indicated that human capabilities to perform the various operations should be taken into account in designing the machine. This is especially true insofar as weapon systems are concerned, for the soldier is often an important component, or he may indeed amount to the major element affecting system performance or vice versa. A simple example is use of the rifle. Thus, those charged with the development of a weapon system should consider a method of analysis which integrates the human elements of the system with the overall performance of the system. The advantage of doing this is that one can better determine just how well the human elements affect overall system performance and also how the system affects the human capabilities or limits them in some way. The best designers of a weapon system will try to develop designs which will not only facilitate their use or operation by an individual but will also aim for designs which tend to reduce human error rates, the time to operate a system, or to maintain it, etc Thus, careful planning, analysis, synthesis, and management control of a weapon system development project is necessary to assure proper integration of the man and the weapon. Shapero and Bates (Ref. 5) have suggested a Systems Analysis and Integration Model (SAIM), or matrix method, to establish the relations among elements of a system, trace their effects on performance, and hence fully establish the important aspects of system operation. They view the weapon system as being "composed of subsystems consisting of mechanisms, men, and facilities having inputs, outputs, physical characteristics, and environmental characteristics". This implies the need for a way of treating the human and nonhuman contributions to the system compatibly. Accordingly, in their Systems Analysis and Integration Model the human components of a system are treated as "human operational components" that are the "black-boxes" of human operations, but these components are not equated to man. Rather, man is visualized as participating in the functioning of the weapon system through combinations of operational components that can vary in size from those that represent operations for less than an hour to those that represent many men for more than a day. To quote Shapero and Bates (Ref. 5) further, we note that they indicate the following:

"More specifically, a human operational component is a combination of operations in which rigid arrangements or constraints compel events to take a certain course. The constraints of the component are intended to exclude all possibilities of action which would not be in line with the intended course; and typically the constraints cannot be altered by the action forces.

"In this context, those responsible for the human factors in a man-mechanism system are responsible for designing sets of operations to be performed by men and for assuring that these operations have fixed characteristics. Steps usually taken to ensure that these operations are reproduced within given tolerances and with given reliability are:

1. Design and allocation of systems functions to be performed by man.
2. Design of work area, work place, and work environment in order to minimize variability.
3. Establishment of fixed operational procedures.
4. Training of personnel in these procedures.
5. Performance of research to provide data and methods for the better accomplishment of 1, 2, 3, and 4 above.

"As with mechanisms, the human operational components defined in the model are typical of the system level being discussed, and may be a mission segment, a function, a task, or a job element. At one level, for example, it may be the operation of a tracking console. At another level, it may be a monitoring operation of a simple job of assembly. At every level, as with mechanisms, the characteristics of the human operational components are described in terms of their inputs, outputs, physical characteristics, and environmental characteristics."

The Systems Analysis and Integration Model approach to include human factors in weapon systems analysis studies of Shapero and Bates (Ref. 5) is suggested as good background reading for the young systems analyst, and Appendix A of Ref. 5 might well be an aid to the analyst because it covers some suggested classifications of human operations components and mechanism terms for use in the SAIM methodology.

Sometimes the analyst will have to be familiar with the anthropometric studies of humans, and how they may affect human engineering applications. Anthropometry is the technology of measuring various human physical traits—primarily such factors as size, mobility, and strength—i.e., characteristics that will have decided effects on weapon system performance. Engineering anthropology is the field of endeavor which aims to apply anthropometric data to equipment, workplace, and clothing design, in order to enhance the efficiency, safety, and comfort of the operator. An authoritative treatment of engineering anthropology is given by Hertzberg in Chapter 11 of Ref. 2. The weapon systems analyst must realize that the concept of "the average man" is either a myth or a fallacy, and hence that variations in human characteristics and traits do indeed exist and may be large in magnitude. Hence, such variations must be properly considered in designing and fielding worthwhile or useful weapon systems.

Man-machine dynamics are covered by Frost in Chapter 6 of Ref. 2, and the design of controls is discussed in Chapter 8 of Ref. 2. These references represent appropriate reading for the systems analyst who will be dealing with human engineering studies in connection with weapons.

In recent years, there has existed a considerable amount of interest in determining the design constraints for shoulder fired weapons, and the Human Engineering Laboratory constructed a test facility to collect human performance data which could be used to determine limits for length, weight, and other physical properties of shoulder fired weapons. Parallel to this effort, the Ballistic Research Laboratories collected physical measurements of some one hundred modern shoulder fired weapons, both U.S. and foreign. In fact, the weapons considered included rifles, rocket launchers, recoilless rifles, grenade launchers, carbines, submachine guns, machine guns, shotguns, and various miscellaneous shoulder fired weapons. Moreover, the opinion of experts in the use and maintenance of weapons were sought and analyzed in order to estimate the percent of US Army men who could easily handle, transport, and fire US weapons. This study is reported by Moore and Strickland (Ref. 6), and it should be very useful to designers concerning the practical length and weight of any new shoulder fired weapon. Moore and Strickland (Ref. 6) also established a regression equation to estimate the percent of men who could easily use weapons of various lengths and weights. Such studies should be very useful in future analyses of human engineering problems concerning shoulder fired weapons.

Some particular examples of human engineering applications in connection with weapon systems might be helpful toward orienting the systems analyst a bit and will be mentioned here. For example, in assemblying the 40-mm XM172 grenade launcher, a very large number of operator errors were found and reported in a study by Miles, Kramer, and Ellis (Ref. 7) on what appeared to be a simple task of inserting a locking bar into a hole in the upper rotor of the grenade launcher. It was found that the locking bar was required to pass through four separate pieces; the misalignment of any one of which would prevent the locking bar from being seated properly. In this connection, a human engineering redesign recommendation was made to bevel the leading edges of the locking bar so that fine alignment would occur automatically. Thus, the importance of human engineering studies for even a rather simple operation of this kind becomes evident.

During the late 1950's and early 1960's, the Army conducted a series of studies on the Special Purpose Individual Weapon (SPIW) as a possible replacement or supplementary shoulder fired weapon to the standard rifle. For a series of seven "systems" tests conducted in support of the SPIW, involving rifles and men, Torre (Ref. 8) presents the findings of human factors or human engineering studies pertinent to design of future rifles. A problem of much concern at the time was that limited hits were obtained for ranges of 200 yd or more in an earlier experiment involving pop-up man-silhouette targets. In fact, the average aiming error for an infantryman amounted to about 3.5 mils linear standard deviation, and the need for firing several rounds was thus indicated. Also, other possible causes of inaccuracy needed study. In particular, Torre (Ref. 8) was interested in determining the effects of rifle design parameters such as impulse, stock configuration, and cyclic rate on semiautomatic and automatic fire accuracy, especially for multiple projectiles (usually three) per trigger pull. Generally, impulse and stock configuration both had effects on delivery accuracy. In fact, if was found, for example, that the reduction in the moment arm of the rifle would lead to reduced round-to-round dispersion. There seemed to be little difference in dispersion between firing from the arm as opposed to the shoulder. The effect of weapon weight seemed to have little effect for serially fired bullets, and the effect of a pistol grip at two positions along the weapon produced negligible results. The effect of firing from a support condition with the "sport" type stock was quite pronounced at least for low-impulse values of the rifle. Although dispersion did not increase between slow and rapid fire for quick turns up to about 90 deg, dispersion did increase when the riflemen turned as much as 180 deg and in a rapid manner. For the rapid firing of three rounds per trigger pull, the second and third rounds departed markedly from the first, the largest displacements being vertical rather than horizontal. Hence, such an effect and its size was found to be of importance in developing any policy of firing such weapons, especially, since the mean radial distance of the second projectile from the first was between one-third and one-half that of the distance between the first and third projectiles. These and other considerations developed in Ref. 8 bring out the importance of human engineering studies—which were very helpful in efforts to make recommendations for a dual-purpose type weapon—which might consist both of a point target system and an area fire system.

A fairly typical human factors evaluation of an experimental item is that reported by Miles, Ellis, and Kramer (Ref. 9) in a study of the XM174 Automatic Grenade Launcher. This human factors study included a design and operation type of evaluation and a field evaluation employing nine enlisted subjects with Infantry MOS's. The study was aimed toward answering three questions: (1) can randomly-selected infantrymen be taught to operate and maintain the XM174 Automatic Grenade Launcher, (2) just how fast and accurate is their fire, and (3) what features of the weapon design inhibit optimum man-system performance?

An experiment designed to answer these questions was conducted (Ref. 9) and led to several suggested design modifications which would meet current human factors standards. The other conclusions reached in Ref. 9 are:

"The XM174 weapon can reasonably be fired in a tripod-mounted mode with or without the T&E (training and elevation) mechanism engaged. However, significantly greater hit probability against vertical targets (e.g., apertures in buildings or fortifications) and targets beyond 200 meters is achieved when the tripod and T&E are both used.

"The weapon is suitable for hand-held (assault) firing, although right-handed persons are more likely to favor this mode and to achieve greater accuracy.

"Persons experience carrying a loaded XM174 (without tripod and T&E) as heavy and awkward, but an individual's loss of speed and maneuverability are no greater than if he were carrying a loaded M60 Machinegun."

The quantitative values of parameters answering the three questions are given in Ref. 9 for future reference and for aids in further human engineering studies of similar launchers.

In summarizing the problem of human engineering factors in the area of weapon systems analysis, one might say that the effectiveness of the weapon or system might well be expressed as a function of three variables or parameters, thus involving the relationship

$$E = f(m,w,t) \tag{33-1}$$

where

E = overall effectiveness of the weapon system

m = proficiency of the soldier or soldiers manning the system

w = inherent capability of the weapon as designed

t = tactical technique of employment of the weapon in combat.

Many other human engineering studies or examples are to be found in the Bibliography of this chapter.

33-5 HUMAN PERFORMANCE

Whereas human engineering is concerned primarily with the development of appropriate interfaces between man and equipment or weapon, human performance has more to do with (1) the assessment of just how well man performs with the weapon or system, and (2) the evaluation, usually in a quantitative way, of just how well he actually operates the pieces of equipment. If, for example, the man can operate one design of a weapon or system much better than some competing designs, then the best system to field will usually be the former one. Whereas it is not always easy to distinguish very sharply between human engineering and human performance, and indeed some studies actually involve both areas of interest (par. 33-3); it is often worthwhile nevertheless to treat the two subjects somewhat separately.

A useful example of a human performance study is the hand grenade throwing test of par. 33-3. A more recent and illustrative study, however, would be that of studying target detection capability, such as that reported by Horley, Eckies, and Dax (Ref. 10). Their study covered a comparison of several vision systems mounted in stationary and moving tanks in the target detection role. It was considered that the Army's requirement for future tanks to be able to penetrate nuclear battlefields, and survive there, would really change the basic tank design since the crew would have to perform many of its tasks in a sealed compartment isolated from the environment. Hence, target surveillance and detection in the traditional manner from the open hatch may not be possible at all, and other detection methods needed more extensive evaluation. These considerations led to the study and evaluation of four tank vision systems for the vital surveillance tasks of tanks, as follows:

1. Open hatch with 7 × 50 binoculars
2. Closed hatch (greenhouse vision cupola) with 6 × 30 binoculars
3. Gunner's vision block and periscope
4. Closed-circuit vidicon TV with zoom lens.

A field experiment was conducted with these four detection systems which were tested simultaneously against the same target arrays, and a factorial type of experimental design was used to help in stripping out such effects as learning and the transfer of crews and systems. The results of the study are summarized in terms of informative graphs which give the percent detection for each surveillance system as a function of time to detect targets. Thus, the probability of detection of the targets sought within any given time for the four surveillance systems, including human operation, is available as is also a comparison of the four systems.

Fig. 33-2 is a reproduction of Fig. 2 from the report of Horley, Eckles, and Dax (Ref. 10) and illustrates the type of summary information gathered from the experiment on crews. One sees that the television system turned out to be the poorest, and that although the open hatch with binoculars gives the best target detection probabilities, it may be that the gunner's periscope for the closed-hatch tank would indeed be acceptable, perhaps especially in a nuclear environment. Ref. 10 contains some 15 graphs giving detection chances with respect to time for the other conditions of test. Hence, studies of this character often produce valuable and useful information on human performance as well as for the systems under study.

Although not mentioned in Ref. 10, we have noted that many of the graphs on the times-to-detect targets may be summarized by use of an exponential type law. For example, the curve for the gunner's periscope of Fig. 33-2 may be fitted approximately by the equation

$$P = 1 - \exp(-t/\theta) \tag{33-2}$$

where

P = percent targets detected
t = time (to detect), min
θ = exponential parameter = about 1.8 or 2.0 min.

In any event, it would certainly seem that the two-parameter Weibull type law (see Eq. 21-147, for example) would fit many of the time-to-detect curves of Ref. 10. Such a fitted law would give the weapon systems analyst a rather general analytical function he could often use in his evaluation studies.

An example of a rather huge and somewhat more complex human performance study is HELBAT I, or the Human Engineering Laboratories Battalion Artillery Test, reported by Horley and Giordano (Ref. 11). As indicated in Ref. 11, the Army needed additional information concerning the operational accuracy of conventional artillery systems. In fact, the continuing assessment of various delivery errors for conventional artillery systems represents information of importance to the weapon systems analyst, weapon design and development engineers, military tacticians, and artillery training agencies. HELBAT I involved three artillery battalions of M109 self-propelled 155-mm howitzers from the First Armored Division, Ft. Hood, TX. Each of the battalions consisted of three batteries of six howitzers each, for a total of 18 weapons. The purpose of HELBAT I was to determine the capability of a battalion of M109 self-propelled 155-mm howitzers to deliver surprise mass fire accurately in the shortest possible time without adjustment. We quote further from the study:

"The mission selected was surprise fire using the predicted fire technique, Meteorological + Velocity Error Transfer (MET + VE). There are basically three types of artillery techniques for firing indirect missions: (a) adjustment onto target, (b) transfer fire using registration corrections, and (c) MET + VE transfer. The MET + VE transfer is the technique most likely to produce human errors at the Fire Direction Center (FDC), and since speed of engagement is critical to massing surprise fire, many human errors will also be committed by forward observers and gun crews. Human error, therefore, will be greatest in this type of mission, although the resulting inaccuracy does not preclude the mission from being one of the most important. Its importance is, in fact, stated by Army Artillery Doctrine: 'In order to inflict a maximum number of casualties, and to achieve the greatest demoralizing effect on the enemy, the immediate objective is to deliver a mass of accurate and timely fire from many pieces in the shortest possible time without adjustment.' "

The planned objectives of HELBAT I were listed as:

"1. Determine the total system accuracy of a battalion of self-propelled 155-mm howitzers using surprise-fire techniques.

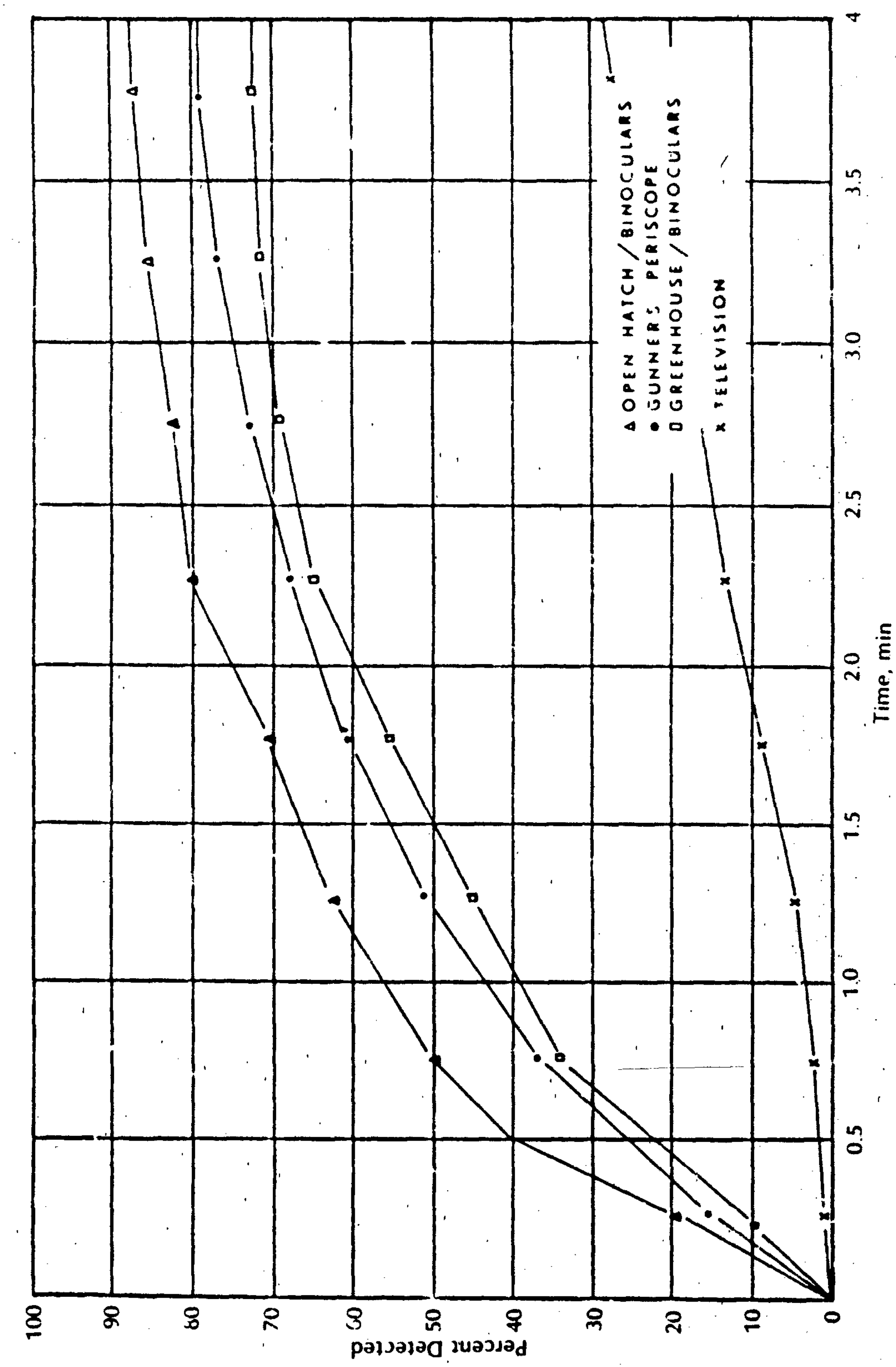

Figure 33-2. Probabilities of Target Detection for Stationary Observers as a Function of Time Using Combined Attacking and Defending Targets

2. Determine the total time (from survey to time-on-target) required by a battalion to deliver fire on the target.

3. Measure the time distributions of rounds striking the target area.

4. Measure the warning sound of incoming volley fire at the target area.

5. Determine what portion of the total system error is human error and isolate this error in each of the sections of an artillery unit."

Concerning the procedure, each battalion conducted surprise-fire missions against stationary targets. The battalions would move rapidly into a firing position, engage a target with massed time on target (TOT) fire, then change to new firing positions and repeat the operation against a new target.

"For experimental purposes, the targets were considered point-type, and they were engaged by mass volley fire (TOT), parallel sheaf, center range.

"Two firing techniques were used to achieve the element of surprise: (a) MET + VE mission, where the firing data is computed from current MET data and existing VE data on each battery, and (b) single-piece registration, where one piece from the center battery registers as far away from the target as possible but within K transfer limits, with the registration data then applied to all three batteries. Both of these techniques rate high in terms of expected errors at the FDC, because of the complexity in computing the firing data and the many approximations associated with applying corrections from both meteorological messages and registrations."

For the HELBAT I experiment, there were 24 firing missions for the MET + VE method of firing and 18 firing missions involving the registration procedure. Although very desirable, it was not possible in HELBAT I to carry to completion the objective listed as planned objective 5, i.e., being able to strip out the effect of humans as a component of variance in assessing overall system error or variation. Thus, although we speak of HELBAT I as being an example of "human performance", it turns out—as will so often be true for many other such experiments—that one is really studying overall system delivery accuracy, i.e., with the human elements incorporated into and being a part of the test. Thus, although it is certainly quite proper to include and fully account for the human engineering or human factors part of a weapon system, it is also proper and indeed necessary as well to be sure that total system accuracy is assessed since this is really what is needed in the first place. Fortunately, modern techniques of analysis apply to both problems. Of course, it is highly desirable to assess all of the possible components of variability in the entire system as operated in order that those parts causing major variations or errors will be known promptly and corrected as need be; this often would include some special emphasis for human factors problems.

Before presenting some results of HELBAT I for system delivery accuracy, including errors introduced by personnel operating and manning the battalion equipment, it might be well to point out that system accuracy will be based primarily on the capability of artillery batteries to place the center of impact (C of I), or mean point of impact (MPI), as it is often called, on the target center or aim point. For surprise fire, it is desirable to do this as quickly as possible, and then fire a sufficient number of rounds to neutralize the enemy threat. Thus, one desires to determine whether the C of I for a battalion can be placed within 50, 100, or 200 m of the intended aim point; knowing full well that there is a sizeable round-to-round normal variation in the fall of shot, or "precision of fire", as it is often referred to in terms of the standard deviation.

Results of HELBAT I indicate that for the combined firings of all three battalions the round-to-round standard deviation in range, or precision fire, was about $\sigma_x = 70$ m and the round-to-round standard deviation in deflection amounted to $\sigma_y = 45$ m, thus exhibiting a ratio of the range sigma to deflection sigma of 1.56. These range and deflection sigmas are for individual or single rounds fired.

Now, with regard to the major point of interest concerning just how well surprise fire for a battalion may be brought to bear on enemy targets employing the MET + VE method of fire adjustment, results indicate that the standard deviation in range for a C of I, or MPI, will be about $\sigma_\mu = 136$ m, and the standard deviation in deflection will amount to about $\sigma_\nu = 88$ m. Thus, the inability to place battalion C of I on the target aim point involves delivery standard errors of just about double that for range and deflection sigmas, respectively, for individual rounds fired from the howitzers. This is obviously a very important piece of information concerning expected aiming errors since such data on the values of the parameters involved are needed to estimate the percentage of casualties that might be anticipated, for example, by using the methods of Chapter 20 to make such an assessment. The errors in placing battery or battalion C of I's on target are also of interest to the weapon designer, or the logistician in planning for ammunition supply based on the weapon systems analyst's studies and others as well.

The delivery accuracy standard errors in range and deflection of the MPI placement may also be used to determine relative frequencies, such as for example the 50% point or the circular probable error (CEP), which indicates a circle within which half of the C of I's can be expected to lie.

In spite of there being very different values for the standard deviation in range ($\sigma_\mu = 136$ m) and standard deviation in deflection ($\sigma_\nu = 88$ m), Grubbs (Ref. 12, p. 32), nevertheless, gives a very accurate approximation for calculation of the CEP. It consists of determining or using the following parameters:

$$\sigma^2 = \sigma_\mu^2 + \sigma_\nu^2 = (136)^2 + (88)^2 = 26240, \text{ or } \sigma = 162$$

$$m = \text{mean value} = 1, \text{ in this case}$$

$$v = \text{variance} = 2(\sigma_\mu^4 + \sigma_\nu^4)/\sigma^4 = 1.168.$$

Then, the CEP of the delivered C of I's is found from p. 32, Ref. 12

$$\text{CEP} \approx \sigma\sqrt{m}[1 - v/(9m^2)]^{3/2} = 131.5 \text{ m}. \tag{33-3}$$

Hence, the CEP for expected battalion MPI's or C of I's is about 132 m. This value compares with and in fact is smaller than the value of CEP = 150 m determined in Ref. 11 by a graphical plotting procedure.

The MET + VE method turned out to be somewhat superior to the registration procedure—see Table 7 of Ref. 11.

Table 9 of Ref. 11 gives pertinent data on errors for the forward observers' functions carried out in the HELBAT I experiment. Finally, as pointed out in Ref. 11, the HELBAT I results agree quite well with the analysis of data from Korea by the former Army Operations Research Office and that of the British.

33-6 HUMAN RELIABILITY

Human reliability, as discussed in par. 33-2, may be considered to be the relative frequency of success with which an individual soldier, crew, etc. performs the various assigned tasks. Human reliability is a rather new field, relatively speaking, and more and more effort apparently is being devoted to it currently because of its importance as a special subject. One writer, Miles (Ref. 13), seems to argue that the proper term should be "human performance reliability"; although we have already indicated that human performance and human reliability are perhaps closely allied, it simply being desirable to give some special emphasis to characterizations of reliability due to the current importance of that now widely known and accepted field of special interest. Then again, there seems to be some emphasis to the effect that reliability studies have not properly taken account of the human element, and hence may be "misleading" (Ref. 13, p. 2). In any event, the reader should and no doubt will realize that studies of system reliability will be understood to include all of the elements or component parts of the system which may have an effect on overall weapon reliability, and this, of course, includes humans. Indeed, it may be that in some applications the human initiated malfunctions are the most frequent, or the most important, and hence they have a major effect or impact on system reliability. Moreover, the basic principles and theories given in Chapter 21 will still apply to human and system reliability problems in a general manner.

As a simple example of human reliability, we might consider the effectiveness of infantrymen in properly cleaning and assemblying an infantry weapon, such as a rifle. In this connection, a dirty rifle or one not perfectly assembled may produce a failure of some kind before a properly cleaned rifle fails. Thus, for a group of "rookie" riflemen, who have just cleaned and assembled their rifles, one might start a firing program and count the number of shots until a malfunction occurs for each rifleman and his rifle. It might be, for example, that the number of rounds to first malfunction might be distributed in an exponential fashion (Chapter 21), so that the reliability is defined as the chance that a rifleman will be able to fire some mission number of rounds, such as 50, before his first malfunction. The better the cleaning and assemblying job, then the greater the reliability, or, that is, the chance of successful operation beyond the mission number of rounds. This is a measurement of reliability on a more or less continuous scale, the principles of which are rather fully covered in Chapter 21.

A somewhat allied form of system reliability, including human operational ability, might be seen by referring to Fig. 33-2. For a time of 2 min, the chance that a target will be detected with open hatch and binoculars is about 0.75, whereas that for the gunners periscope the corresponding chance is only about 0.64, or that is to say, the human operation involving open hatch and binoculars is the more reliable one. This is another example of what could be called human reliability on a continuous scale.

A different type of reliability for humans would be the relative frequency of errors encountered in performance tasks of individuals. For example, McCalpin and Miles (Ref. 14) conducted a study to measure the extent and consequence of human errors in the operation and maintenance of Stoner weapons. Twenty-four subjects were tested six times on each of several maintenance actions resulting in 144 trials, with a human error potential involving some 11,520 possibilities for error. The error categories, the frequency of error, and the observed human error rates are given in Table 33-5. The

TABLE 33-5. FREQUENCY OF ERROR AND OBSERVED PROBABILITY BY ERROR CATEGORY

ERROR CATEGORY	FREQUENCY	OBSERVED HUMAN ERROR RATE
1. Catastrophic	2	0.00018
2. Inadvertent Activation	7	0.00061
3. Weapon Stoppage	157	0.014
4. Procedural Errors	252	0.022
Total Errors	418	0.036
(Total Number of Error Possibilities = 11,520)		

authors go ahead to point out that the highest incidence of operator error leading to Stoner rifle failure was experienced in the weapon cleaning process. This finding indicated that the weapon was dirt- and carbon-sensitive and, further, that the operator cannot easily ascertain when the weapon is really clean. One characteristic in particular appeared to be the source of the problem, that being the excessive friction in the bolt carrier group which caused binding in the receiver slides; this resulted in weapon action being slower in a dirty weapon than in a clean one. Thus, in the firing tests there resulted increased failures for both the rifle and machine gun configurations when proper cleaning and maintenance were not carried out. McCalpin and Miles (Ref. 14) concluded that reliable operation of the Stoner rifle and machine gun is rather heavily dependent upon reliable human performance and maintenance tasks.

McCalpin (Ref. 15) suggests setting up a data bank to collect, process, and analyze human reliability data of all kinds. Thus, and in summary, there seems to be a rather fertile area of future interest in gathering human reliability data and taking pains to apply it to overall system reliability problems encountered in the various weapon systems analyses.

33-7 SUMMARY

We have introduced and discussed some of the interface type problems between human engineering and weapon systems analysis studies. It was thought somewhat desirable to discuss three perhaps different areas for the analyst—i.e., the human engineering problem, the human performance problem, and the human reliability problem. It is seen that the weapon systems analyst may face studies which could involve all three facets of human factors, and consequently some typical example application possibilities have been covered. An extensive Bibliography of some Human Engineering Laboratories publications which might be of some interest to the weapon systems analyst is included.

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CHAPTER 34
ANALYSIS OF COSTS AND OTHER RESOURCE MEASURES — INTRODUCTION AND GENERAL GUIDELINES

The analyses of weapon systems must necessarily take proper account of associated costs for any equipment that is eventually fielded since resources are most usually of a finite character. Therefore, the weapon systems analyst must possess a proper background knowledge of the analysis of costs and other resource measures, and apply cost analyses in his evaluations. The material presented in this chapter should serve as a broad introduction to the problem of cost analyses of weapon systems, and it should give the systems analyst an appreciation of the scope and importance of the Army's cost analysis activities. In particular, the weapon systems analyst will always strive to determine the overall effectiveness of the weapons he evaluates in terms of cost, and hence effect economies in overall effort.

34-1 INTRODUCTION

The purpose of this chapter is to provide a general introduction to weapon system costing methodology and some rationale for the major concepts in military cost estimating. Quantitative techniques for cost analyses and their applications in support of the Army's weapon system development program will be addressed in greater detail in Chapters 35 and 36.

In addition to models of evaluation discussed so far, the function of weapon systems analysis has been described also as that of assisting the decision process by providing comparative measures of cost-benefit/cost-effectiveness for the prudent selection of alternatives which can attain specified objectives. Conceptually the emphasis is on comparison, both in the physical context (technical effectiveness) and the economic context (cost-benefit). Within the framework of weapon systems analysis, physical and economic assessments should be made as an integrated activity.

As a weapon system progresses through successive life cycle phases, technical estimates become more definitive, allowing further refinement of the cost estimates. Conversely, the more definitive the cost estimate, the greater the capability to delineate system design alternatives which will optimize effectiveness per unit of effort or expenditure. The overall objective is to determine the weapon system or weapon system alternatives which achieve the best balance between cost and technical effectiveness in combat.

Cost analysis is a disciplined process founded in consistent and uniform application of cost estimating techniques and methodology. Therefore, in order to provide cost comparisons which are meaningful and appropriate to the decision at hand, it is necessary that a visible cost framework be developed with which to:

1. Relate the costs of weapon system design alternatives, in a uniform manner, to system objectives
2. Provide a structure to relate costs to the decision process and budget programs
3. Provide a traceable path to assess the validity and accuracy of the cost estimates themselves.

Subsequent paragraphs of this chapter will address briefly this framework of cost analysis within the Army and some considerations in applying cost analysis techniques and methodology to weapon assessment problems.

34-2 THE FRAMEWORK OF COST ANALYSIS

The concepts of cost comparison and cost estimating are fundamental to the cost analysis approach. In the early conceptual stage of a weapon systems analysis, requirements are characterized by numerous uncertainties. The objective in developing cost estimates during this preliminary stage is to provide gross estimates of the comparative life cycle costs of competing alternatives, identifying the most significant cost relationships wherever practical. During successive stages of the weapon system life cycle, when system parameters and technical requirements become definitive, a more refined cost estimate can be developed.

Throughout the costing process, the focus is on cost estimating rather than detailed and precise cost projections in the normal accounting sense. Such an approach is necessary as well as practical since the objective is to present primarily cost information which is relevant to the decision to be made. For certain types of cost estimates, extraneous cost detail which does not contribute materially to differentiating the cost impact of economic alternatives contributes little to the final technical decision and, in fact, may waste some project time. For example, if the same mechanized component support item is common to two design alternatives and its fuel consumption rates historically have been evaluated at 10% of operating hours, detailed cost comparisons based on fuel consumption rates may be relatively meaningless. (Major system decisions should be made on full 100% life cycle costs, including sunk costs, non-add items, "wash costs"*, etc. Traditionally, cost and operational effectiveness analyses (COEA) do not consider "wash costs" at all.)

If one assumes the usage rate of some component is different for each alternative, i.e., differences in quantity required or weapon system operating characteristics, the relevant costs may be those related to operating hours. Further, such differential may be expressed adequately as a percent increase in total operating costs for one alternative over the other based on usage rates. In short, the cost estimating process seeks to compare the relevant differences in cost per unit of effectiveness sought in determining the economic merits of competing alternatives.

34-2.1 CONSISTENCY AND UNIFORMITY OF COSTING PROCEDURES

A valid cost comparison requires that cost parameters receive standard treatment and that costs be expressed in compatible units. Uniformity and consistency are the cornerstones of cost analysis and they serve to:

1. Facilitate the development of cost relationships which accurately reflect the comparative economic merit of competing alternatives
2. Promote compatible cost categories and cost information systems within the Army and Department of Defense (DOD) to support costing efforts
3. Provide a means of relating resources to budgets in order that funds may be programmed for system development.

Uniform treatment of cost elements is also necessary to assure the relevancy of economic comparisons among alternatives. For example, evaluation of supporting data for two alternative items may reveal quite similar costs. Further investigation, however, may show that the basis of the cost estimate supporting one alternative was expressed in constant dollars while the basis of the other was current dollars — a fact not necessarily documented in the original source data. In this case, the preliminary estimate therefore did not reflect a valid cost comparison.

*"Wash costs" has been used to describe identical costs — ignored in evaluation — for two or more competing systems. "Wash costs" is not an official term.

Another reason for uniform and consistent costing procedures is to promote standardization in cost documentation and data bases. Cost estimates are generally developed using one of two basic approaches or methodologies. First, they may be developed using historical cost records and past cost analysis studies which provide either directly relatable cost experience on similar systems, or cost information on dissimilar systems having certain characteristics analogous to the system to be costed. This approach is the parametric or "top-down" approach. Unfortunately, expeditious and accurate development of cost relationships is handicapped by the incompatability of much of the available historical data — i.e., differences in cost categories, cost elements, level of aggregation, units of measure, constant dollars vs current dollars, and other disparities. Therefore, much time and effort must be devoted to manual data search, data validation, and appropriate adjustment before cost estimating relationships can be developed. Second, cost estimates may be developed using the engineering or "bottoms-up" approach where each part is estimated or priced and then summed to derive the relevant total cost. Recognition of this problem has resulted in directive efforts by DOD and the Department of the Army (DA) to promote consistency and uniformity in cost analysis procedures. Army Regulation (AR) 11-18 (Ref. 1) and Department of the Army Pamphlets (DA PAM) 11-2 through 11-5 (Refs. 2-5) outline the policy of the Army Cost Analysis Program. These documents further identify the responsibilities of the Army Staff and Commands in supervising the development, operation, and flow of an Army-wide uniform system of cost analysis within the DOD Acquisition System (Ref. 6). Army cost analysis offices are required to maintain close coordination with research and development activities, procurement and production activities, and project managers to obtain their input to cost analysis programs.

Consistent and uniform costing procedures also assist in equitably relating resource cost to budget programs. In the final analysis, limited budgets control the total commitment of resources, and weapon system costs must be related to budget accounts so that funds can be programmed to accomplish Army force posture objectives. The major Army budget program accounts to which weapon system development, investment, and operating costs are charged are:

1. Research, Development, Test, and Evaluation (RDTE)
2. Procurement Appropriation (aircraft missiles, weapons, ammunition, other) (PA)
3. Military Personnel Army (MPA)
4. Military Construction Army (MCA)
5. Operation and Maintenance Army (OMA).

34-2.1.1 Cost Categories

Consistency in developing cost estimates requires uniformity in the manner in which costs are developed. The major military groupings of costs throughout the life of a weapon system correspond to the program phases in which costs are incurred. These are (Refs. 2-4):

1. Research and Development. Those costs resulting from applied research, engineering design, analysis, development, test, evaluation, and management of development efforts related to a specific materiel system. Examples of major cost categories are:

   a. Development engineering
   b. Producibility engineering and planning (PEP)
   c. Prototype manufacturing
   d. System test and evaluation
   e. Tooling
   f. System/project management

g. Research and Development (R&D) facility construction
h. Training services and equipment.

2. Investment Nonrecurring. Those cost elements which generally occur only once in the production cycle of a weapon/support system. Examples of major cost categories are:
a. Initial production facilities
b. Industrial facilities/production base support.

3. Investment Recurring. Those cost elements which occur repeatedly in the production of a weapon/support system or its component, including the costs of delivery to the user. Examples of major cost categories are:
a. Manufacturing
b. Engineering changes
c. System test and evaluation
d. Initial spares and repair parts
e. Transportation
f. Training services and equipment
g. System/project management.

4. Operating and Support Costs. Those direct costs resulting from the operation, maintenance, and consumption of materials and supplies for a weapon/support system after acceptance into the Army inventory. Examples of major categories are:
a. Military personnel costs
b. Depot maintenance
c. Replacement training
d. Replenishment spares and repair parts
e. Unit training, ammunition and missiles
f. Petroleum, oils, and lubricants (POL).

Fig. 34-1 shows a typical distribution of system dollar expenditures over time (cost streams) for R&D, investment, and operating and support (O&S) costs over the life of a weapon system.

34-2.1.2 Work Breakdown Structures

Department of Defense Directive 5010.20 (Ref. 7) establishes the policy governing the preparation and application of a work breakdown structure (WBS) for use during the acquisition of systems,

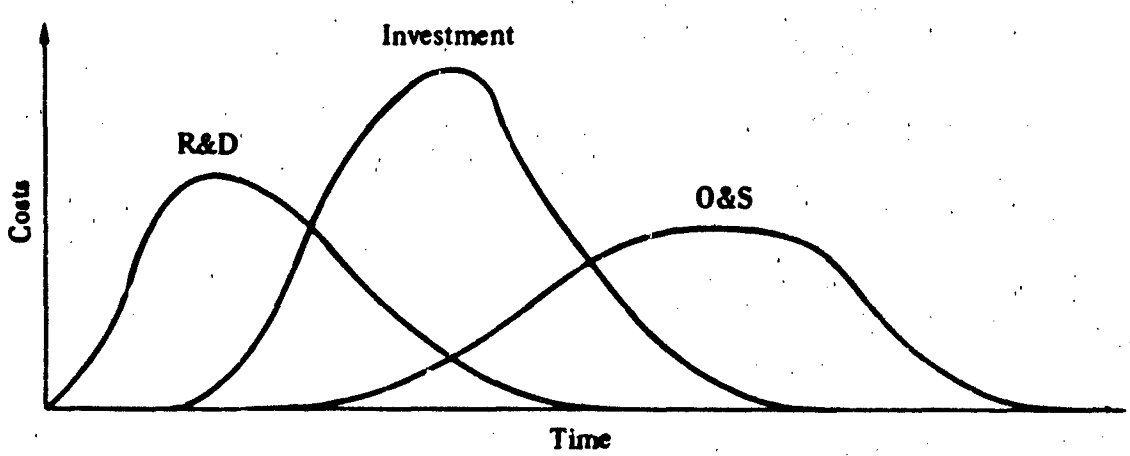


Figure 34-1. Typical Cost Stream Pattern Over a Weapon System Lifetime

equipment, and materiel items. The program objective of the WBS as given in Ref. 7 is to provide a consistent, visible cost framework that facilitates:

"A. Planning and assigning management and technical responsibility; and,

"B. Controlling and reporting the progress and status of engineering efforts, resource allocations, cost estimates, expenditures, and procurement actions throughout the development and production of a defense materiel item."

The criteria for the preparation of military WBS's are prescribed in Military Standard 881 (Ref. 8). Refs. 1 and 2-4 establish weapon/support life cycle cost structures and definitions for cost categories and elements in support of the WBS defined in Ref. 8. The WBS is a product oriented family tree of hardware, software, services, and other work tasks which completely defines the project program. Fig. 34-2 illustrates in a conceptual matrix the relationship between the cost categories of Refs. 2-4 and elements of a typical WBS for a major materiel item.

The work breakdown structure was developed to assure that the uniformity objectives of Ref. 7 are met and that the approach to the development of the first three levels of cost breakdown are consistent as a minimum. The three levels are defined in Ref. 8 as:

1. Level 1. An entire defense materiel item, e.g., the XM138 Self-Propelled Howitzer System (hardware, software, services). This level usually is identified in the DOD programming/budget system either as an integral program element or as a project within an aggregate program element.

2. Level 2. A major element of the defense materiel item, e.g., an aircraft, a missile, a track vehicle such as the XM1 main battle tank; or an aggregation of services or activities, e.g., system test and evaluation. Within Army system development programs, this level will normally be a major end item.

3. Level 3. Elements subordinate to Level 2 major elements — e.g., an airframe, launch and guidance control equipment, or the power package drive train of the XM1 tank.

By the use of the WBS categories as guidelines, costs can be subdivided or aggregated into cost elements which relate directly to design and performance variables. A further breakdown of a WBS Level 3 item into essential components is provided in Ref. 9.

34-2.1.3 Major Cost Terms

Cost categories and WBS elements provide an input framework for cost analysis. That is, they assist in structuring the resource categories (equipment, facilities, manpower, materiel items, etc.) and functional categories (maintenance, logistics, training, etc.) necessary to support successfully the development, acquisition, maintenance, other support and operation of a proposed weapon system design specifications (physical, operational, and performance characteristics) developed to satisfy the military need generating the system requirement.

The presentation of the results of cost analysis involves the development of an *output* structure which presents system life cycle costs in a form useful for assessing the economic implications of the weapon system decision under consideration. However, specific output formats are highly context dependent. That is, they depend upon the scope and level of the costing effort, e.g., individual weapon system costs, force-mix costs, total force structure costs; the evaluation criteria of the decision maker; the objectives which the weapon system is being designed to achieve; etc.

At a more general level, comparative cost estimates must be related to the time-phased application of resources over the lifetime of a weapon system and the impact of those resource commitments on Army budget appropriations. The latter consideration is an important part of the weapon system decision since year-to-year system funding requirements must compete with demands for funds from other

SPECIFIC / GENERIC	AIRCRAFT SYSTEM		MISSILE SYSTEM		SURFACE VEHICLE SYSTEM		ELECTRONIC SYSTEM		ORDNANCE SYSTEM	
	LIFE CYCLE COST MATRIX	MIL STD 881	LIFE CYCLE COST MATRIX	MIL STD 881	LIFE CYCLE COST MATRIX	MIL STD 881	LIFE CYCLE COST MATRIX	MIL STD 881	LIFE CYCLE COST MATRIX	MIL STD 881
1 Frame	Airframe	Airframe	Airframe	Missile Integration & Assembly	Hull, Turret & Suspension	Body/Cab Hull/Frame Turret Assembly Suspension/Steering Integration & Assembly	Automatic Data Processing Subsystem	Automatic Data Processing Equipment	Carriage	Carriage Launch System Integration & Assembly
2 Propulsion	Power Plant	Power Plant	Propulsion	Propulsion	Power Package	Power Package/ Drive Train	Data Display Subsystem	Data Displays		
3 Guidance and Control Communication Equipment	Communications, Navigational Guidance	Communications, Navigational Guidance Auto/Flight Control Penetration Aids-Recon Equip	Missile Guidance & Control	Missile Guidance & Control Equipment	Communications Equipment	Communications & Navigation Equipment	Communications Subsystem	Communications	Communications Equipment	
4 Fire Control	Fire Control	Fire Control	Fire Control & Other Communication Equipment	Surveillance, Ident & Tracking Sensors Launch & Guidance Control Communications Data Processing	Fire Control	Fire Control	Sensor Subsystem	Sensors	Fire Control	Fire Control Equipment
5 Armament	Armament	Armament Weapons Delivery Equipment Aux Arm/Wpn Del Equipment	Launcher	Launcher Equipment Command & Launch Equipment Integration & Assembly	Armament	Armament	Auxiliary Equipment Subsystem	Auxiliary Equipment	Launcher	Launcher
6 Payload Ammunition	Ammunition		Missile Payload	Launched Payload Payload Shroud	Ammunition				Complete Round	Complete Round (WBS Level II)
7 To be Specified	Specified		Specified		Specified		Specified		Specified	
8 Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment	Peculiar Support Equipment
9 Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment	Common Support Equipment
10 Other	Other	As Required for 100% Coverage	Other	As Required for 100% Coverage	Other	As Required for 100% Coverage	Other	As Required for 100% Coverage	Other	As Required for 100% Coverage

Figure 34-2. Conceptual Matrix Relating Cost Categories and Elements to a Typical WBS

existing and proposed Army systems. In this context, general output cost terms have been standardized to permit a common frame of reference in relating input cost structures and budget categories over the life of the weapon system. The definitions of these key cost terms (from Ref. 10) follow:

1. Development Cost. Development cost includes:

a. RDTE funded costs — i.e., conceptual, validation, and full-scale development phases from the point the program/system is designated by title as a program or system

b. All costs — both contract and in-house — of the research and development cost category to include the cost of specialized equipment, instrumentation, and facilities required to support the RDTE contractor and/or Goverment installation.

2. Flyaway (rollaway) Cost. Flyaway is used as a generic term related to the creation of a usable end item of hardware/software. Flyaway cost includes:

a. WBS elements of major system equipment such as structure, propulsion, electronics, and Goverment furnished equipment (GFE)

b. System/project management, and system test and evaluation if any of the system text and evaluation effort is funded by procurement funds

c. Procurement funded costs

d. All costs — both contract and in-house — of the production nonrecurring and recurring cost categories to include allowances for engineering changes, warranties, and first destination transportation unless first designation transportation is a separate budget line item.

3. Weapon System Cost. Weapon system cost includes:

a. The same WBS elements as in Flyaway Cost — i.e., major system equipment, system/project management, system test and evaluation (if any of the system test and evaluation effort is funded by procurement funds) plus WBS elements of training, peculiar support equipment, data, operational/site activation, and industrial facilities unless industrial facilities are funded as a separate budget line item or by RDTE funds

b. Procurement funded costs

c. All costs — both contract and in-house — of the production nonrecurring and recurring cost categories to include allowances for engineering changes, warranties, and first destination transportation unless first destination is a separate budget line item.

4. Procurement Cost. Procurement cost includes:

a. The same WBS elements as in weapon system cost — i.e., major system equipment system/project management, system test and evaluation (if any of the system test and evaluation effort is funded by procurement funds), training, peculiar support equipment, data, operational site activation, and industrial facilities (unless industrial facilities are funded as a separate budget line item or by RDTE funds), plus the WBS element of initial spares and initial repair parts

b. Procurement funded costs

c. All costs — both contract and in-house — of the production nonrecurring and recurring cost categories to include allowances for engineering changes, warranties, and first destination transportation unless first destination transportation is a separate budget line item.

5. Program Acquisition Cost. Program acquisition cost consists of development costs, procurement costs, and any construction costs which are in direct support of a system or project. Program cost and program acquisition cost are synonymous terms. Program acquisition cost includes:

a. The WBS elements of major system equipment, system/project management, system test and evaluation (except operational test and evaluation funded from military personnel funds or operation and maintenance funds), training, peculiar support equipment, data, operational/site activation, industrial facilities (unless industrial facilities are funded from procurement funds as a separate budget line item), initial spares, and initial repair parts

b. RDTE, procurement, and military construction funded costs

c. All costs — both contract and in-house — of the research and development, and production (nonrecurring and recurring) cost categories to include allowances for engineering changes, warranties, and first destination transportation except when first destination transportation is a separate budget line item.

6. Ownership. Ownership cost encompasses the cost elements within the operating and support cost category exclusively. O&S costs include those costs associated with operating, modifying, maintaining, supplying, and supporting a weapon/support system in the inventory.

7. Life Cycle Cost. Life cycle cost includes *all* WBS elements, *all* related appropriations, and encompasses the costs — both contract and in-house — for *all* cost categories. It is the *total* cost to the Government for a system over the full life of it; and it includes the cost of development, procurement, operation, support, and, where applicable, disposal.

Fig. 34-3 graphically displays the relationships between the seven cost terms, cost categories, appropriations, and WBS.

34-2.2 EMPHASIS ON ECONOMIC DECISION ALTERNATIVES

A decision to acquire a proposed weapon system involves a commitment of future economic resources to attain a specific capability. Since resources are finite, their cost to satisfy one objective is the foregone value of their best alternative use in satisfying other objectives. In economic theory, this is referred to as "opportunity costs", and it implies that any resource allocation decision either directly or indirectly involves the consideration of alternatives.

However, the dollar cost of a resource allocation decision is but one measure of value. The other side of value is the return expected or the benefits to be accrued for the resources consumed. That is, the greater economic return to be realized from the alternative use of resources is a function of the ratio of benefits to costs, e.g., the return is greatest when "opportunity costs" are minimized. In this context, the economic analysis objectives of AR 11-28 (Ref. 11) includes guidance to the effect that every effort should be made to examine two or more alternative means of providing the same type and level of benefits so that the alternative can be identified whose total discounted cost is lowest. *Costs can be appropriately measured in dollars, but not necessarily in benefits.* Therefore, various effectiveness measures should be calculated and correlated to the amount of resources required.

It is important to recognize that costs have two dimensions:

1. The value of resources and their application in a particular system configuration

2. The socioeconomic value of these resources as a function of the return expected (benefits to be derived). While the former may appropriately be measured in dollars, the latter often cannot be. In evaluating alternatives, a prudent decision maker can be expected to maximize his return (benefits) for costs (resources). Therefore, a weapon system decision cannot unequivocably be based solely on dollar costs alone, and hence the socioeconomic benefit may prove to be the margin of acceptability. At the very least, to justify the incremental cost outlays to attain the new capability, a comparison would be required to demonstrate the gain in military capabilities provided by the new system over existing systems.

Cost-benefit and cost-effectiveness are often used interchangeably in current literature, leaving the impression that the two terms are synonymous. However, they are different and the distinction between the two is addressed in the paragraphs that follow.

TERM	COST CATEGORIES					APPROPRIATIONS					WORK BREAKDOWN STRUCTURE			
	Research & Development		Production		Operating & Support	RDTE	Procurement 3/	MILCON	O & M	OTHER	Major System Equipment; System Project Management; System Test and Evaluation	Training; Peculiar Support Equipment; Data; Oper Site Activation	Initial Spares & Initial Repair Parts	Industrial Facilities (PBS); Common Support Equip
	Non-recurring	Recurring	Non-recurring	Recurring										
Development Cost	$	$				$					$	$		5/
Flyaway 1/ Cost			$	$			$				$			
Weapon System Cost			$	$			$				$	$ 6/		7/
Procurement Cost			$	$			$				$	$	$	7/
Program 2/ Acquisition Cost	$	$	$	$		$	$	$			$	$	$	
Ownership Cost					$		$	$	$	$	8/	8/	8/	8/
Life Cycle Cost	$	$	$	$	$	$	$	$	$	$	$	$	$	$

Shaded Areas Are Excluded from Definitions

Note: The sum of program acquisition, ownership, and other costs (e.g., military and civilian management pay) equals life cycle cost.

Notes:
1. Cancer term depending upon commodity can also be called rollaway or sailaway
2. Also known as Acquisition Cost or Program Cost
3. Includes shipbuilding and conversion, Navy appropriation
4. Other appropriations (e.g., Military Personnel) and funds are included as appropriate
5. When industrial facilities are funded by RDTE it will be included as appropriate
6. Generally other program peculiar WBS elements (i.e., flight support operations and services for spacecraft) apply when procurement supported
7. Excludes industrial facilities when funded as a separate budget line item
8. The MIL-STD-881 work breakdown structure does not apply

Figure 34-3. Discrete Cost Definitions

34-2.2.1 Cost-Effectiveness

Each system design, employing resources in a specific configuration, has associated with it certain attributes related to standards of technical efficiency or operational effectiveness which can be quantified. Specifically, the physical characteristics of a piece of equipment — such as its size, shape, weight, type of construction materials, and operating design characteristics — can be associated with various standards of durability, maintainability, ease of operation and use, etc., to develop quantifiable measures of effectiveness. Similarly, the effectiveness of a proposed operational procedure can be measured using comparative standards of man-hours saved, number of procedural actions consolidated, reductions in setup time or assembly time, etc. These and other examples of system benefits can be quantified in terms of returns accrued for the resources, or the cost of resources, consumed. These quantifiable system attributes are normally the basis for what is termed "cost-effectiveness" in support of comparisons among system alternatives. Comparisons are based on quantifiable, tangible, physical, operational, and performance merits of each alternative.

34-2.2.2 Cost Benefit

Each alternative also has "benefit" attributes which are intangible and related to the various levels of abstraction which are not subject to explicit quantification. These might be termed the "extra dimension" value of alternatives relative to sociopolitical considerations, external and internal environmental influences, and other qualitative factors. The importance of such intangible benefits should not be underestimated because they can be the overriding consideration in making a weapon system decision. Some examples of nonquantifiable benefit considerations might include:

1. Political acceptability. Does the proposed weapon system alternative present such a significant arms escalation as to cause a reaction on the part of a major foreign power?
2. Horizon technology. Are there indications that a technological breakthrough is imminent which will make the proposed system obsolete before its useful lifetime is realized? Therefore, would a cheaper and less durable system be more beneficial in the long run than the preferred alternative for which the cost-effectiveness ratio is highest? Would such a trade-off compromise result in a higher loss of human life?
3. External organizational influences. Is another alternative more compatible with overall Army mission objectives, although less cost-effective in terms of specified mission objectives? Would proposed new concepts in logistics, operations, and training have a significant impact on any of the proposed alternatives?
4. Human factors. How much radiation shielding (increase in weight) is required relative to the specified mission environment? How might this reduce maneuverability and hence raise the vulnerability of other mission essential combat elements? Does the design configuration present a significant safety hazard under present refueling procedures? Are proposed operating procedures compatible with organizational manning levels? Will they cause an increase in labor burden resulting in fatigue (higher accident rates) or lower morale?

These and other intangibles are considered in making rational resource allocation decisions. Not only are they often difficult to quantify, but assigning a cost value to them may often be meaningless. For example, how can a cost be associated with the value of human life? However, these intangibles should be an integral part of a thorough economic analysis, and their impact on the weapon system decision must be assessed subjectively and/or qualitatively, as by consensus of expert opinion, priority ranking, etc. The utility of a weapon system is a function of the political and social system it serves.

Therefore, the benefits of a particular system cannot be assessed solely on the basis of technical effectiveness per unit of resource (cost) expended. Often, a less effective and/or more costly alternative may be acceptable. From a technical/scientific point of view, the difference between use of a nuclear weapon and an equivalent amount of conventional bombs is the efficiency in delivery of the former. From a political/social viewpoint, the use of nuclear weapons would have serious international significance.

The broader issues of "benefit" analysis are beyond the scope and purpose of this handbook, which is oriented toward the quantitative attributes of benefits or "cost-effectiveness" considerations. The subject of cost benefit has been introduced here only to promote an awareness of the fact that (1) any resource allocation decision inherently involves consideration of alternatives, and (2) the value, or merits, or a particular alternative cannot always be based solely on quantitative system characteristics versus costs.

34-2.2.3 Cost Comparisions

Estimates of cost-effectiveness are based on two fundamental approaches (Ref. 12):

"1. Fixed effectiveness approach. For a specified level of effectiveness to be attained in the accomplishment of a given objective, the analysis attempts to determine which alternative (or feasible combination of alternatives) is likely to achieve the specified level of effectiveness at lowest economic costs.

"2. Fixed budget approach. For a specified cost level to be used in attainment of a given objective, the analysis attempts to determine that alternative (or feasible combination of alternatives) which is likely to produce highest effectiveness.".

While the initial approach taken in a cost-effectiveness analysis might be directed toward optimizing one or the other of these objectives, in practice (as additional considerations and constraints are uncovered) the overall objective usually becomes one of maximizing effectiveness per unit of cost or minimizing costs per unit of effectiveness. The final presentation of results may be a comparison of cost-effectiveness curves such as displayed in Fig. 34-4 with appropriate supporting information indicating the sensitivity of alternative choices to effectiveness level. In this case, alternative A is most cost-effective at higher cost levels while alternative C is more cost-effective at lower cost levels.

In making cost-effectiveness comparisons, it is also important to recognize that costs must be comparable over time with respect to both:

1. The life cycle phase (R&D, investment, and O&S) within which costs will be incurred
2. The total life cycle costs of each alternative.

Fig. 34-5 illustrates the fact that weapon system design alternatives often have diverse cost stream patterns over their respective life cycles. We might assume, for example, that each alternative has system characteristics as shown in Table 34-1. While the characteristics of the three hypothetical design alternatives in the example are exaggerated to illustrate a concept, in practice there are many cases where radically different proposals have been made to satisfy a weapon system objective.

34-2.2.4 Weapon System Cost Characteristics

In the example given, there are many important trade-offs which must be evaluated with respect to how much and when resource costs will be incurred, the returns expected from each alternative relative to internal and external system influences, year-to-year budget limitations in each of the Army appropriation categories, and competing demands on resources by existing and other proposed weapon system development programs, etc. Some of the more important considerations in reaching weapon system cost-effectiveness comparisons are discussed in the next paragraphs.

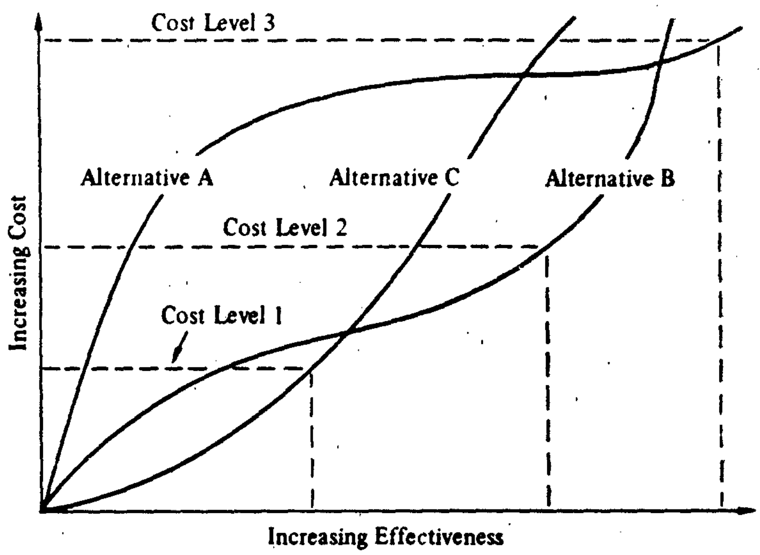


Figure 34-4. Conceptual Cost-Effectiveness Comparison

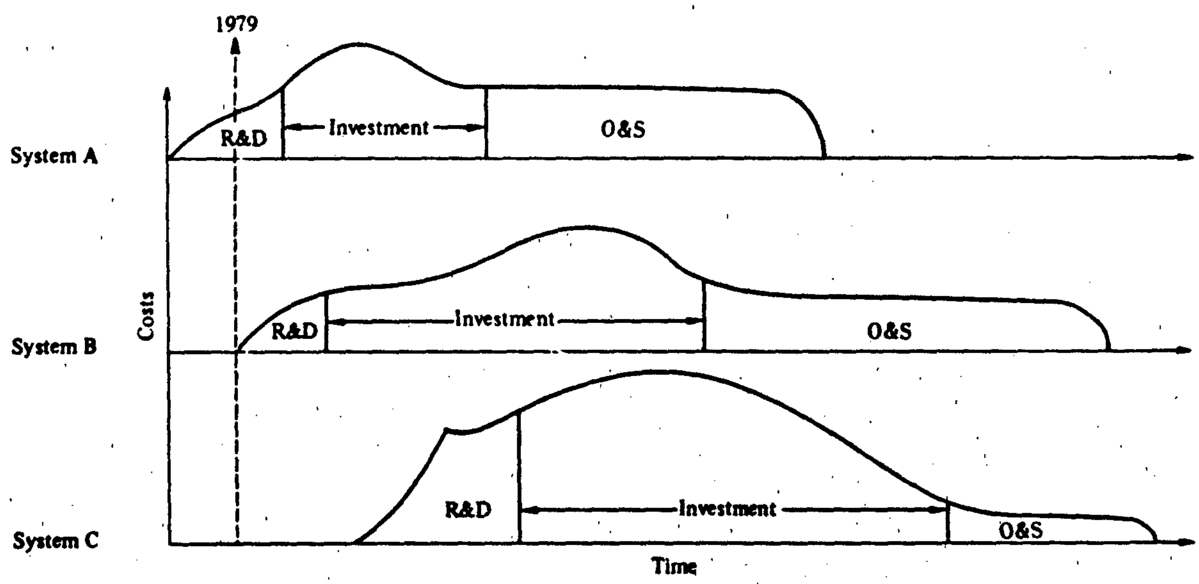


Figure 34-5. Cost Streams for Three Hypothetical Weapon System Alternatives

34-2.3 RELEVANCY OF COSTS

Development of the input cost structures (WBS elements and cost categories) described in previous paragraphs, although essential to the cost analysis approach, serve only as a point of departure in making cost-effectiveness comparisons. That is, they provide a viable, consistent, and comparable framework for describing the cost associated with each weapon system alternative.

TABLE 34-1. HYPOTHETICAL ALTERNATIVE WEAPON SYSTEM COST CHARACTERISTICS

Life Cycle Costs	Alternative System A	Alternative System B	Alternative System C
R&D	Extensive modification to existing weapon system	New weapon system	New weapon system
	Low to moderate technology	Moderate technology	High technology
	Moderate multiple mission adaptability	Moderate multiple mission adaptability	High multiple mission adaptability
Investment	Existing assets; real estate plant and equipment	Existing assets; real estate, same plant	New assets required
	Moderate support system retrofit costs	Moderate support (hardware) system development	Extensive support (hardware) system development
	High development "learning curve"	Moderate development "learning curve"	Minimum development "learning curve"
	Minimum training requirements	Moderate training required	High training requirements
	More units required	Less number of units required	Minimum number of units required
O&S	High operating costs	Moderate operating costs	Low operating costs
	High maintenance costs	Moderate to high maintenance costs	Low maintenance costs
	High replacement costs	Low replacement costs	High replacement costs

A subsequent step in cost analysis is to determine which costs are relevant in making a valid comparison of the merits of each alternative. Which costs are relevant depends, in turn, on the scope and level of the cost estimate required and the criteria specified for evaluating alternatives. Normally, for major weapon/support systems, all life cycle costs, including sunk and "wash costs", are included and documented for decision purposes and historic cost track visibility.

Cost relevancy may also be related to the difficulty in distinguishing variable cost elements from those that are fixed, and what share of fixed costs are to be included as valid determinants of total costs. US Army Materiel Command supplement to AR 37-13 cautions that:

"Substantial distortion can be introduced (into the cost estimate) by applying to direct costs an overhead rate based on both variable and fixed cost overhead elements. The overhead cost accounts should be examined to identify any costs that would be the same for all alternatives. Such fixed costs may be applied in the same dollar amount (rather than rate) to all alternatives; or particularly (when making cost comparisons to identify the least costly of several project alternatives) they might be excluded from all alternatives as 'wash items'."

Implicit in this guidance is the admonition that only the portion of fixed costs which change as a result of differences in investment between alternatives should be included in the cost comparison. Fixed costs which are common to each alternative or which do not change significantly with changes in variable costs do not materially affect the decision to be made; hence they should not be included in the cost comparison for historic cost tracking purposes and cost visibility.

34-2.3.1 External and Internal Costs

Deciding which costs to include in the cost estimate must begin with establishing bounds to the costing problem. Differentiating internal from external costs defines the scope of the costing effort. It involves a determination of how much weight and value should be given to cost influences which bear on the weapon system program for which alternatives have been identified. External costs have been defined by G. H. Fisher (Ref. 12) as those ". . . that fall beyond the boundaries of the decision maker's organization or beyond the scope of interest of the cost analyst's customer.". More specifically, which costs are external depends upon the type of cost estimate required — e.g., individual weapon system cost comparisons, force-mix costing, total force structure costing, and the level at which the final weapon system decision will be made. Obviously, if the cost analysis is directed toward developing comparative cost estimates for a single weapon system, e.g., a tank, concurrent developments in other Army weapon systems most likely will have limited impact on the decision to be made. On the other hand, if the cost analysis is directed toward the determination of the total force structure impact of a weapon system, past and future weapon system development programs become internal to the analysis.

As a practical matter, the time available to develop the cost estimate will dictate to a large extent the detail to be considered in the costing effort, thereby forcing concentration on those major costs which directly affect the decision to be made. While this may result in some degree of suboptimization, this is partially offset by the diseconomies or marginal returns from a more extensive (and therefore more costly) cost development effort.

34-2.3.2 Direct and Indirect Costs

The particular charges to include in a weapon system cost estimate have been the subject of continuing debate and revision. In general, all costs which can be traced to hardware, direct consumption of material, or personnel who directly operate or maintain the system are direct charges to the system.

The difficulty often arises in determining what prorated share of indirect costs — e.g., maintenance, shared assets, line item logistic management — should be included in the cost estimate. The general guidelines in Ref. 11 and the definition of costs to be included in various cost elements within cost categories established by Refs. 1-4 should be consulted for an indication of the costs to be charged to a weapon system development program. Questionable cost items, when encountered, should be referred to higher authority for evaluation.

However, in consonance with the objective of developing comparative estimates, a preferable approach is to include in the final cost estimate those costs, direct and indirect, which would be affected by one of the alternatives. Costs which will be incurred regardless of the alternative chosen, or which are equally applicable to each alternative, are not always relevant for making comparisons to identify the least costly system and hence may be of little value in supporting the decision to be made. However, total life cycle costs are required for budget purposes; the "delta costs" or cost variance among alternatives will not suffice.

34-2.3.3 Recurring and Nonrecurring Costs

The classification of cost elements into recurring and nonrecurring costs is essential to the time-phased economic comparison of life cycle costs for competing system alternatives. In general, nonrecurring costs are those associated with research and development and those that occur only once in the investment phase (and usually independent of the size of the buy). Recurring costs are those that are incurred repeatedly in the production cycle (and are dependent upon the size of the buy) and the cost associated with wear-out replacement items, operations, and maintenance. Ref. 3 provides some guidelines for classification of recurring and nonrecurring costs.

The purpose of separating recurring from nonrecurring cost elements is to facilitate determination of variable costs as a function of the buy-quantity changes and variations in operational requirements. Each system design alternative will incur certain fixed costs regardless of the number of units of the system to be procured or the length of time they will see service. Therefore, to determine the cost of each alternative for a specified effectiveness level, or conversely, to determine differences in effectiveness levels for a given cost, trade-offs are generally made in variable cost elements. Further, weapon system development programs are often subject to design changes, engineering changes, performance requirement changes, schedule changes, procurement level changes, etc. Separation of costs into recurring and nonrecurring elements provides greater flexibility in assessing the cost impact of such changes.

34-2.3.4 Emphasis on Incremental Costs

Incremental costing usually refers to the additional costs associated with some increase in capability or effectiveness. Since each new weapon system evolves from some base of military resources (advanced R&D studies, expansion of maintenance and training facilities, etc.), costs are viewed from the perspective of both the total absolute life cycle costs and the additional resources necessary to develop, field, and operate the new system. In essence, these are the costs over which the decision maker has control. Some of the more significant considerations in determining the incremental costs of a new system are identified and briefly described in the paragraphs that follow.

34-2.3.4.1 Sunk Costs

Costs which have been incurred as a result of past decisions may not be relevant to many current weapon system decisions under consideration. Also they may not be part of the incremental costs relevant to the current weapon systems decision either. Such "sunk" costs, therefore, may not represent meaningful alternatives because funds or resources have already been expended, whether used in support of the new system or not. Consider the costing problem for three competing systems — A, B, and C — in Fig. 34-5, and assume system A represents a weapon system design alternative for which $3 million in R&D funds have been expended as part of a previously cancelled development program, whereas no R&D funds have been expended for systems B and C. The $3 million already spent represents a sunk cost no matter which alternative — A, B, or C — is chosen and therefore should be included in the cost comparison, but explicitly identified as sunk, i.e., not relevant to the decision at that point in time*.

34-2.3.4.2 Inherited Assets

Inherited assets are similar to sunk costs in that they may not be meaningful to a cost comparison. For example, system design alternative A in Fig. 34-5 may eventually be described as a modification of an existing weapon system. In this case, the costs associated with assets of the older system — e.g., real estate, plant, and equipment — have already been incurred and may be essentially "free" to the proposed system if the decision does not require replacement of assets used. Only those costs which are necessary to effect modification requirements and the additional costs to operate and maintain the new system would, in this case, be included in the cost comparison.

*Major system sunk costs are generally used and so specified.

34-2.3.4.3 Shared Costs

Shared costs are those which are shared by two or more projects and often present a difficult estimating problem. An example might be a repair facility for a component common to two or more weapon systems. The facility is available to each weapon system whether used or not. However, how much of the pro rata share of costs should be charged to each system may often be difficult to estimate. Should the costs be shared equally, or on a cost percentage based on expected usage rates, e.g., scheduled maintenance vis-a-vis reliability or maintainability parameters plus an unscheduled maintenance factor? The rationale for including these costs and their relevance must be explicitly determined beforehand to avoid bias in the comparison of cost estimates between alternatives.

34-2.3.4.4 Salvage Value

Not all inherited assets can be treated as "cost free". Many existing systems/support systems may have intrinsic value beyond their useful lifetime to the project or mission under study. They may have residual market value, salvage value, or even a substantial value to some other weapon system program. In such cases, where residual/salvage value is significant, an attempt should be made to estimate a value (in terms of opportunity costs) to provide a more equitable basis for comparison among alternatives. The method to be used will depend upon the alternative uses of the system/subsystem and could be based on commercial scrap value, future market value, or depreciated value using a reasonable depreciation formula.

34-2.3.4.5 Wartime Costs

Estimating the additional cost of wartime operations — i.e., additional reserve stocks, ammunition, spares, repair parts, other consumable items, personnel casualties, wartime attrition factors, etc. — often presents a difficult problem, particularly in long range weapon system development programs initiated during peacetime. The difficulty arises in attempting to predict the scope and duration of a potential war in which the weapon system will see service. Because of this, the common practice is to assume that all alternatives have equal wartime capability or to compare only the wartime costs necessary to sustain operations for a fixed duration, e.g., 30 to 120 days, after hostilities begin until reinforcement/replenishment operations can be initiated.

While in most cases this is a practical approach (particularly for most individual weapon system costing problems), there are some costing problems (e.g., weapon systems in a force mix) in which omission or limited treatment of the costs of wartime operations may result in a substantial bias in the real differential costs among competing alternatives. This differential may be illustrated conceptually in Fig. 34-6. Assume two equally effective alternatives, A and B, for which cost-effectiveness curves have been plotted as shown in Sections I and II of Fig. 34-6 (peacetime cost plus the costs of 60 days sustained combat support and consumption stocks). In this example, alternative A appears to provide the greater return in effectiveness per unit of cost. However, extending the cost comparison into Section III (sustained combat operations for 1 to 2 yr), alternative B begins to show greater effectiveness returns per unit of expenditure and a crossover point occurs at 60 + n number of days after the initiation of hostilities. This change might have been as a result of higher replacement costs (combat losses) for alternative A over replacement costs of alternative B.

34-2.4 COSTS ASSOCIATED WITH TIME

A complete determination of the cost differences among weapon system alternatives often require consideration of the time value of resources, i.e., the amount and rate at which resources (funds) a

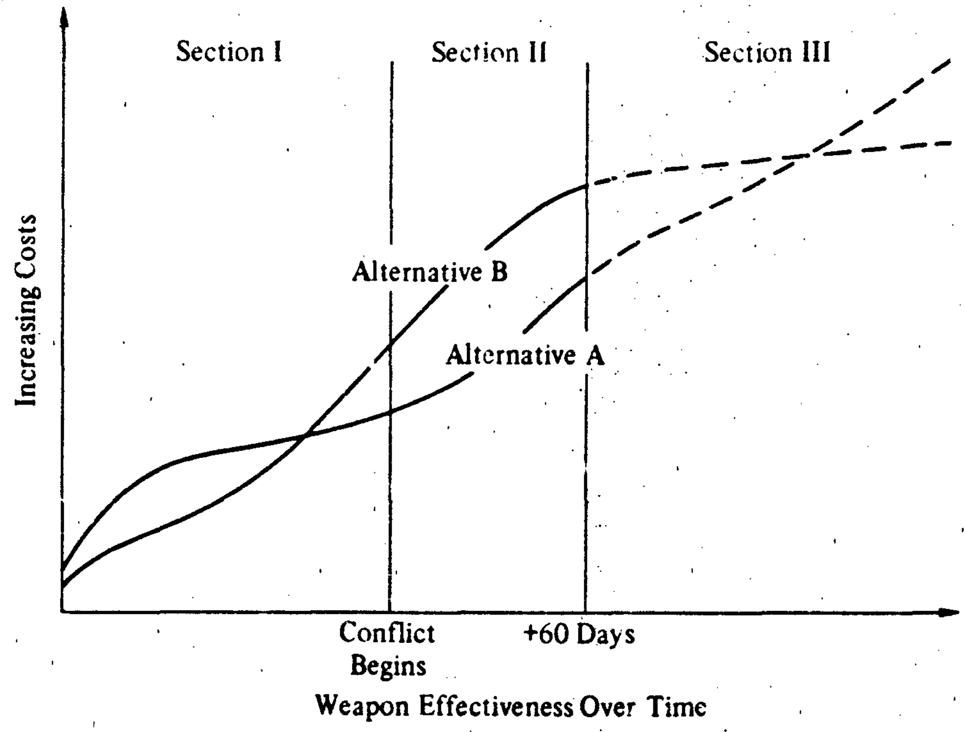


Figure 34-6. Conceptual Cost Crossover Point Under Wartime Conditions

consumed over the life cycle of each alternative. The concept involves the principles of "present value" of resources (funds) and "discounting" theory. The objective is to assure that weapon system costs are commensurable in both the static context (the absolute value in dollars of resources consumed) and the dynamic context (the alternative value of resources over time).

The life cycle cost streams generated by each proposed alternative may differ significantly (see Fig. 34-5). If it is anticipated that the time value of money in any given case is sufficiently great to change the ranking of choices, a method must be employed to reduce the cost comparison to a common frame of reference, i.e., the value in present dollars of dollar expenditures projected for future years. Fundamentally, if the prevailing interest rate is 5% compounded annually, the consumption of a dollar now is equivalent to investing the dollar for deferred consumption of $1.05 one year from now. Conversely, if the period of investment is ten years, the discounted value of $1.0 million for consumption ten years from today, at 5% interest, is equivalent to spending $644,000 now. The interest rate, therefore, is one measure of the opportunity cost (see par. 34-2.2) of alternative investments over time.

The opportunity cost of a weapon system investment decision is the alternative use of funds to support other weapon system development programs. In this context, the alternative which defers the use of resources furthest into the future releases resources for their next best *immediate* use. This partially offsets the present value of the system costs to be incurred in the future. The present value of future system cost is therefore equal to the undiscounted cost minus the benefits accrued by the immediate alternative use of resources until required by the system at some later date.

Both the discount rate and the differences in expenditure patterns over the investment period may be significant in comparing alternatives. Consider the hypothetical weapon system comparison at zero discount rate presented in Fig. 34-7. The time period of expenditure is 15 yr; alternative A incurs higher start-up costs (R&D and investment), but is an efficient operating system. Alternative B, on the

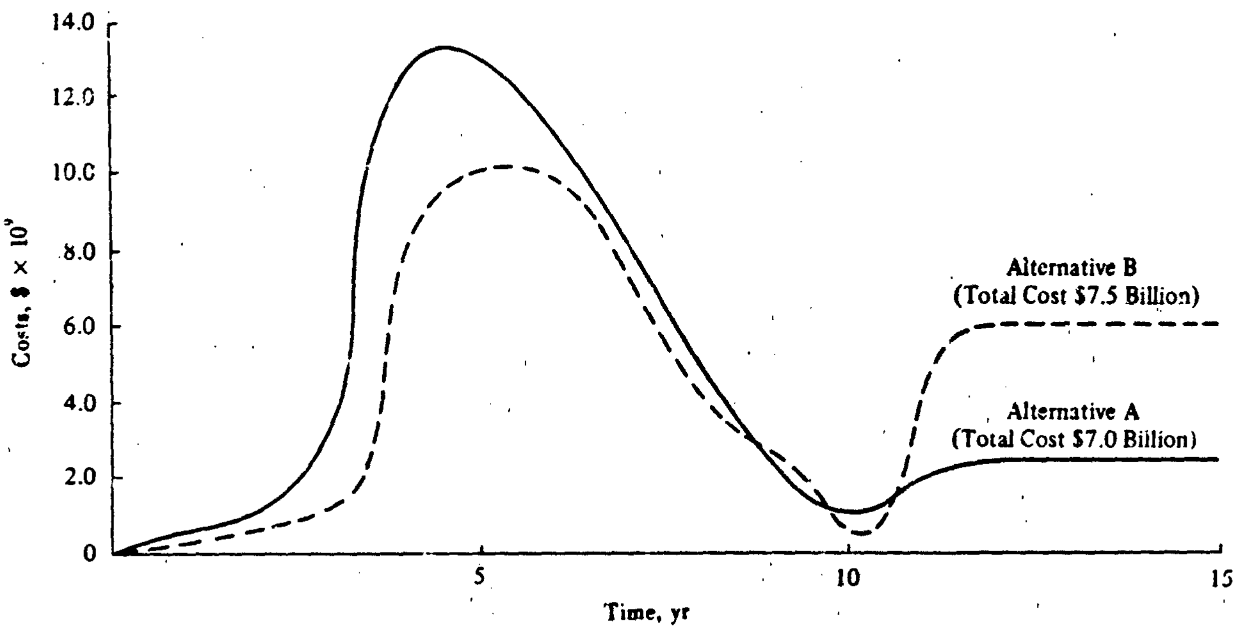


Figure 34-7. Time-Phased Life Cycle Expenditure Patterns, Undiscounted, for Two Alternatives Over a 15-yr Period

other hand, is a less efficient operating system (high operating and maintenance costs), but requires less R&D and investment costs. The undiscounted total cost for alternative A is $7.0 billion, while the undiscounted total cost for alternative B is $7.5 billion. The expenditure patterns of the life cycle costs (assuming complete replacement by the 16th year and no salvage value) for each alternative is shown in Table 34-2. Table 34-3 shows (column 2) the 15-yr discounted present costs for each alternative at various discount rates (column 1) Notice that the absolute cost difference (column 3) is $500 million in favor of system A at zero discount rate, but only $70 million in favor of system A at 5% discount rate. A crossover point takes place somewhere between 5% and 10%; at 10% an absolute cost difference of $320 million occurs in favor of B (indicated by minus sign). The relative difference (column 4) shows an even greater increase with higher discount rates. Which of the two systems would be the preferred alternative depends upon many considerations other than their discounted present value costs. However, the discounted costs of each alternative are an important input to the investment decision, particularly during the peak periods of competing demands for funds in a particular budget appropriation category. The problem is not in calculating present value, but in determining the appropriate discount rate to use. As stated in AR 11-28 (Ref. 11), the discount rate — currently 10% — is specified by the Office of Secretary of Defense.

A practical approach (analysis time permitting) is to calculate present value for a range of discount rates for each alternative in order to determine the relative "break even" rate. The decision maker can then decide whether this discount rate (or rate range) is above or below what he considers the appropriate discount rate to be.

34-2.5 APPROACHES TO COST ESTIMATING

A cost estimate may often be referred to as a judgment or opinion, developed formally or informally, regarding the anticipated cost of a piece of hardware, a service, a commodity, or an integrated system,

TABLE 34-2. EXPENDITURE PATTERNS FOR ALTERNATIVES IN FIG. 34-7

		R&D and Investment									O&S						Total
	Future Years	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
Costs, \$ $\times 10^6$	Alternative A	50	100	300	1300	1300	1100	800	500	200	100	250	250	250	250	250	7.0 billion
	Alternative B	25	75	100	900	1000	1000	700	400	250	50	600	600	600	600	600	7.5 billion

TABLE 34-3. DISCOUNTED PRESENT VALUE OF TWO ALTERNATIVE EXPENDITURE PATTERNS (TABLE 34-2) AT THREE DISCOUNT RATES (5%, 10%, AND 15%)

Discount Rate (15 yr)	Present Value (Billions of Dollars)		Absolute Difference, \$ $\times 10^9$	Relative Difference, %
	Alternative A	Alternative B		
Base	7.00	7.50	+0.50	—
5%	5.25	5.32	+0.07	1.3
10%	4.18	3.86	−0.32	7.6
15%	3.37	2.95	−0.42	12.4

using past experience as a guide. The greater the gap between a proposed capability reflecting innovations in technical design, materials, operational or organizational concepts over past capabilities, the less the confidence that experience will provide a reliable guide to the future. Since most new military capabilities involve advances over preceding systems, cost estimating often requires projections from the known to the unknown. This inevitably results in uncertainty regarding how much a cost prediction could differ from actual costs. (See par. 34-2.6 for a discussion of cost uncertainty.) The magnitude of this uncertainty is a function of how far the cost analyst must project from the known to the unknown and how adequately he can identify and resolve the impact of unknowns on cost.

There are very few approaches currently available with which cost predictions involving elements of uncertainty can be quantified with any great mathematical precision. Rather, cost estimating must be viewed as a process of hypothesis testing which requires caution and good judgment in the use of available techniques (both statistical and nonstatistical). The cost analyst must recognize the limitations of these techniques and use them appropriately to support, test, and validate assumptions and informed judgments as to the relationship between costs and cost influencing factors. The techniques used in cost estimating may vary from a sophisticated mathematical expression to an engineer's considered opinion of expected costs. There are few standard guidelines regarding techniques to use and when to use them; selection depends to a large extent on the:

1. Scope of the analysis
2. Purpose of the cost estimate
3. Time available to develop costs
4. State in the weapon system life cycle, i.e., definitive level of system requirements and specifications
5. The volume, completeness, and accuracy of available cost data.

The different methods used and their approach to developing cost estimates are briefly described as follows:

1. Parametric Cost Estimating. An approach which treats the relationships between costs and cost influencing factors as a function of key cost explanatory variables having a range of values as opposed to a single value. Its functional form is the cost estimating relationship (CER) which expresses the link between a physical or performance characteristic, a resource, or an activity with a particular cost associated with it; or the link between independent but related costs. (Commonly called a cost factor.) A CER of the first type might be a complex mathematical function relating the investment cost of a piece of hardware (dependent variable) to its weight, range, and accuracy (independent or explanatory variables). A CER of the second type might be a simple functional expression of the annual hardware maintenance cost as a fixed percentage of the original investment costs. The parametric approach to cost estimating normally is most useful in the early conceptual stage of a weapon system acquisition program when system parameters are only grossly defined. As a weapon system matures, the parametric approach is frequently used to "validate" or make "about right" conclusions of other costing methodologies.

It is to be recognized, however, that this approach deals with a relatively high level of aggregation, and cost estimating relationships use cost explanatory variables (weight, speed, power, frequency, etc.) to predict costs when limited detailed knowledge about system configuration characteristics and requirements are available. As such, the adequacy with which a cost projection closely approximates actual system costs depends to a large extent upon: (a) the reliability of historical source data as valid predictors of future costs, and (b) the ability of the cost analyst to establish comprehensively the proper relationships between system costs and cost influencing factors.

2. Engineering Estimate. An approach to cost estimating involving a detailed examination of separate segments of work and system components at a relatively low functional level, e.g., level 4 or lower in the work breakdown structure, and synthesizing these individual estimates into a total cost. Its application requires a detailed knowledge of the system, the production process, work standards, organizational procedures, manufacturing methods, and operations. Hence, engineering estimates are most applicable during the production stage of a weapon system acquisition cycle when requirements have been fairly well defined.

The cost estimator works from sketches, blueprints, engineering drawings, word descriptions of items not completely designed, work statements, organizational descriptions, manufacturing process descriptions, etc. From these, he attempts to specify, in detail, each engineering and production task to be performed and the man-hours and materials required; tooling requirements; sequence of production operations to include fabrication, assembly and checkout, etc. — developing detailed cost estimates for each process and aggregating costs at each level of the work breakdown structure. The engineering approach is a laborious and tedious task typically involving thousands of calculations to estimate the cost of a major end item of equipment.

3. Estimates by Analogy. An approach to cost estimating which depends upon a direct comparison of a proposed item, piece of equipment, system function, or operation with comparable or analogous capabilities of some prior system. Although it has been the most widely used method in the past, it is a judgment process and requires expertise and experience on the part of the cost analyst. The term "judgment" is applied because the validity of the estimate depends upon how precisely the chosen analogous system mirrors the system to be costed in terms of size, weight, performance, complexity, etc., and the accuracy with which the cost analyst can identify and adjust for specific differences.

Ref. 13 defines the analogue techniques as based on the construction of relationships using cost data from logically similar systems. There are two types of analogues:

a. Direct comparison with similar systems having the same operational/performance characteristics, e.g., using cost data on commercial vehicles to develop estimates for combat vehicles

b. Direct comparison with dissimilar systems having many of the same cost characteristics as the system to be costed, e.g., estimating missile costs based on aircraft experience.

4. Expert Judgment Estimate. This is little more than an educated guess by those who have comprehensive knowledge of a system or system components Its use is often necessitated by gaps in empirical data or when a sufficient statistical sample is not available. It is recommended primarily for cross-checking the validity of estimates developed by other methods to determine their reasonableness. Alternatively, an expert judgment may be used to assist in identifying the key cost variable in a particularly elusive cost relationship.

34-2.6 COST UNCERTAINTY

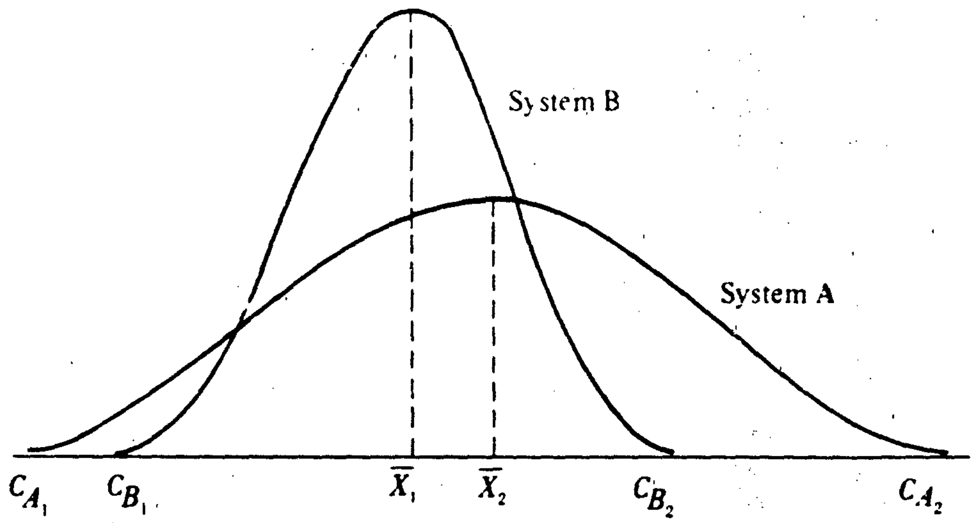
Uncertainty is an inherent characteristic of system and cost analyses, particularly in support of major weapon system programs which require long development lead times (eight to ten years). The changing threat, advancing technology, fluctuating economic conditions (wages and prices), development time-horizon uncertainty, necessary design modifications to accommodate new requirements or to overcome unforeseen technical barriers, etc., all contribute to the difficulty in accurately predicting eventual system costs. Additionally, there is the problem of incompleteness and inconsistencies in much of the available historical cost data which is fundamental to the development of cost projections for new weapon systems. As pointed out by Hitch and McKeen (Ref. 14): "But the actual costs of developing, producing, and operating complete weapon systems have frequently exceeded cost estimates made prior to development by factors of ten or more, largely because of technological uncertainty that existed when the costs were estimated."

However, it is essential to the weapon system decision process to be explicit in identifying cost uncertainties and determining the range of error they are likely to introduce into the cost estimate. Developing a point estimate of alternative system cost provides no indication of the possible cost variability associated with each alternative. This could be misleading and even bias the weapon system decision to be made. For example, consider the cost distributions for two hypothetical systems shown in Fig. 34-8. Although the actual or final cost of System A may turn out to be less than B, the expected distribution of costs for B has a much narrower range and therefore could encounter less risk that the probable cost will be seriously in error. Depending upon the decision maker's preference for risk, his upper cost threshold, etc., his choice of System A or B can be made with better insight into the consequences of his decision if such information is available to him.

Although there are many individual factors contributing to cost uncertainty, they can generally be classified into two primary categories:

1. Requirements uncertainty
2. Cost estimating uncertainty.

Further, it has often been observed that cost estimates are often as different as the cost analysts developing them. Inevitably, there will be some bias in the estimate reflecting subjective judgments on how uncertainty should be handled and its impact on costs. However, it is important to recognize that uncertainty in the cost data does not necessarily invalidate the cost estimate, as long as cost uncertainty is treated uniformly in the comparison of alternatives.



$\overline{X}_1$ = expected or mean value of cost curve for System B

$\overline{X}_2$ = expected or mean value of cost curve for System A

C_{A_1} = lower bound of cost for System A

C_{A_2} = upper bound of cost for System A

C_{B_1} = lower bound of cost for System B

C_{B_2} = upper bound of cost for System B

Figure 34-8. Hypothetical Cost Distributions for Two Systems

34-2.6.1 Requirements Uncertainty

Requirements uncertainty has the greatest impact on the accuracy of cost projections. Historical evidence suggests that much of the difference between initial weapon system cost estimates and actual costs can be attributed to system configuration changes resulting in cost growth. Some examples, given by G. H. Fisher (Chapter VI, Ref. 15) are:

1. The original hardware design may fail to meet the desired performance characteristics, and hence hardware configuration must be changed.
2. Performance characteristics may be changed in response to the changing threat, causing hardware specification changes.
3. A decision might be made that the system is required sooner than originally planned, requiring substitution of resources for time.
4. A change in system specifications may be induced by errors or omissions in initial requirements for some part of the system.
5. Indirect effects of specification changes may impact on other parts of the system, e.g., personnel requirements may change.
6. The strategic situation may change, leading to a change in operational performance characteristics, or methods of employing or deploying the system, or changes in force size or number of years of system operation.

It is not possible to anticipate all contingencies which could arise and result in unscheduled changes in a weapon acquisition program, particularly when the development cycle spans an 8-10 yr period However, requirements uncertainty can be treated explicitly if grouped into two categories: (1) uncertainty which is subject to probabilistic quantification over a relevant range of values, and (2) uncertainties which are not subject to prediction within known limits of confidence.

Requirement uncertainties in the first category can be dealt with using parametric costing methods. That is, key cost explanatory variables, e.g., production quantity, operating hours, etc., can be examined through a range of values (discrete or continuous) to determine the sensitivity of cost to changes in these variables. This approach permits an indication of both the probable magnitude of the cost uncertainty and the rate of change in costs with changes in requirements.

Requirements uncertainty in the second category is characterized by unanticipated variations from circumstances such as budget changes, safety or logistic policy changes, changes in threat or technology, administrative delays, and deferments. While in the past these factors have contributed significantly to cost growth (particularly weapon systems with long lead times), it is almost impossible to predict with reasonable confidence the probable magnitude of such costs. These uncertainties can usually be dealt with only in a general manner through the use of cost adjustment factors derived from experience with predecessor systems.

34-2.6.2 Cost Estimating Uncertainty

Cost estimating uncertainty refers to variations in the cost estimates themselves. These can be attributed to:

1. The variability or errors in the available cost data used to develop cost estimating relationships
2. Errors or variations in cost estimating relationships due to weaknesses in the values chosen as valid predictors of costs, or variability in the data samples of explanatory variables
3. Extrapolation errors which occur when it is necessary to develop estimates beyond the sample range of historical data
4. Errors resulting from aggregation of costs at higher levels as a result of overlooking important intersystem dependencies
5. Price changes which induce errors in the cost estimate, particularly if the cost estimate is required in terms of prices expected in future years.

Cost estimating uncertainty is inherent in any prediction of future weapon system costs based on projections or extrapolations from past experience with analogous or similar systems, even if the requirements of the system are clear and certain. This uncertainty is inherent because projections of future weapon system costs normally are based on a statistical sample (or population) of historical data on similar or analogous systems. While generalized statistical estimating techniques provide measures of data variability, e.g., standard error of estimate and the use of confidence intervals, which can be used to make range error estimates of the costs within the data sample, extrapolating beyond the range of the sample introduces uncertainty as to whether the characteristics of the sample still hold. (See Ref. 15.)

34-2.7 COST SENSITIVITY ANALYSIS

Cost sensitivity analysis is a systematic approach to examining the impact on total system costs resulting from variations in the values of key cost generating variables or changes in assumptions about major system requirements. It is one of the primary tools available to the cost analyst for testing hypotheses relative to:

1. Requirements uncertainty
2. Cost estimating uncertainty
3. Dominance in the preferred ranking of system alternatives in cost-effectiveness comparisons.

Cost sensitivity analysis attempts to determine how much significance uncertainty about key system cost parameters will have on total system cost. Specifically, if a cost generating variable — e.g., weight, speed, or range — in a cost estimating relationship is allowed to vary through a relevant range of values, sensitivity analysis can determine what impact this will have on the total cost outcome. As an example, assume that the effect of a key cost generating variable, value X, is uncertain on the total cost C_T in an estimating equation. Assume also that informed judgment indicates that the relevant range of values of X varies from X_0 to X_2 with a "most likely" value of X_1. Suppose various values of X through this range are tested in the equation to determine what the impact will be on the total cost C_T. Fig. 34-9 illustrates three possible cost curves which could result from such a sensitivity analysis. Cost curve AB indicates that C_T is relatively insensitive to changes in X; cost curve AC indicates that C_T is relatively insensitive up to X_1, but it changes dramatically for higher values of X; and cost curve AD has an almost linear sensitivity with respect to changes in X.

The results of sensitivity analysis, therefore, may assist in identifying and isolating critical cost parameters which could introduce significant variation in total system cost. This information can then serve as the basis to (1) identify ways to hedge against uncertainty; (2) indicate areas where empirical testing may be necessary to reduce uncertainty; (3) point out areas where more information or further research is required; or (4) test for dominance in cost-effectiveness comparisons of weapon system alternatives.

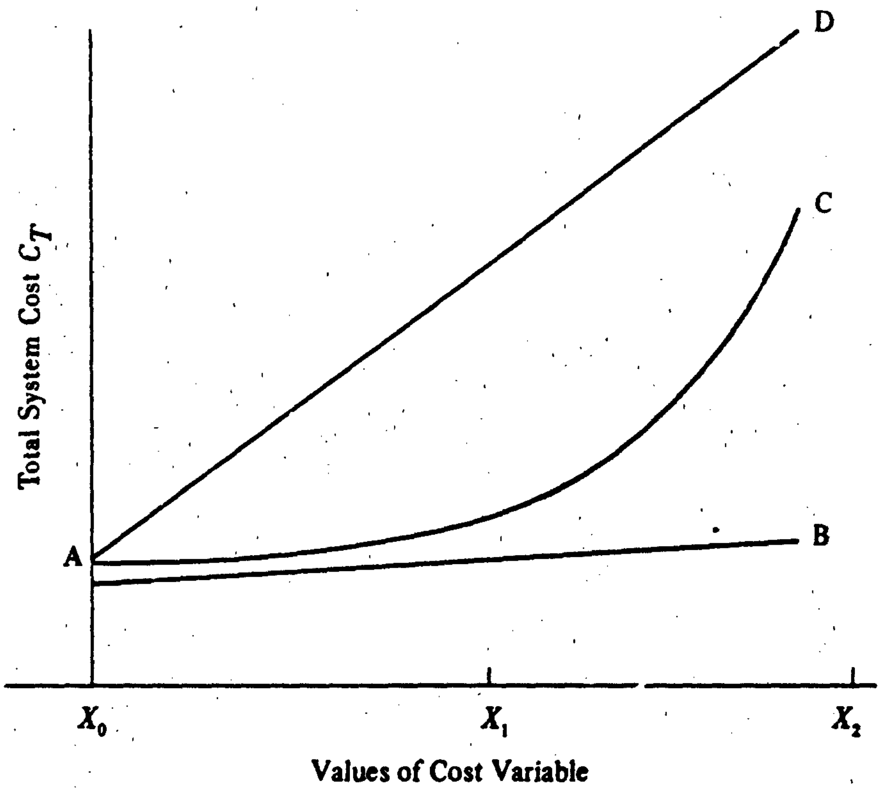


Figure 34-9. Hypothetical Cost Sensitivity Curves

Cost sensitivity analysis takes place after cost estimating relationships have been clearly specified, and it attempts to assess the magnitude of uncertainty surrounding the key system cost parameters that can be tolerated and still yield a reasonable cost estimate.

34-3 COLLECTING COST DATA

While cost estimating techniques and cost structures provide the tools and methodology in support of cost analysis, the validity of the estimate depends to a large extent on the adequacy and accuracy of available cost data. Since many cost estimates of future systems are based on a comparison with previous cost experience, the major task facing the cost analyst is identifying appropriate cost data and deciding just which techniques and methods should be applied.

The identification and selection of data sources should begin after a clear description of the system to be costed has been obtained and after the ground rules specifying the structure and detail of the cost estimate are established. Normally, substantial time and effort are required to obtain a precise description of the system and to determine the desired content of the output of the cost estimate. If the cost estimate is prepared early in the concept formulation phase of the system life cycle, the imprecise description of the system may limit the estimating techniques that can be used and may influence the selection of data sources. System descriptions required by the cost analyst may differ considerably from those required by the weapon systems analyst. For example, the weapon systems analyst may not be concerned with activity rates or operating hours which are important to the cost analyst. An example of both equipment specifications and operational assumptions which could be required in a system description is given in Table 34-4. Additionally, the costing ground rules underlying the study should be firmly established, including, for example:

1. Kind of cost index to be used (example: ten-year system cost)
2. Date when all prior cost will be considered sunk cost (example: FY 1978)
3. Rules regarding amortization or discounting
4. Rules regarding costs of other agencies
5. Special rules regarding base operating support personnel, attrition rates, etc.

In collecting cost data in support of a particular costing problem, the cost analyst will invariably face difficulties in obtaining useful data. Typical problems experienced in the past include the following:

1. Data are not system related.
2. Data are not standardized within reporting systems.
3. Data are obsolete.
4. Data are biased.
5. Access to data is difficult.
6. Data are program and budget oriented instead of system oriented.
7. The credibility of the data is often unknown.

The cost analyst should recognize that he must adapt data to fit his purpose. As data sources are uncovered and evaluated, techniques must be devised to adapt or normalize the data that are available to the needs of the analysis. Data collection is likely to be the most time consuming part of any cost analysis; therefore, careful planning of the data collection and adaptation effort is necessary.

34-3.1 COST INFORMATION STANDARDIZATION PROGRAMS

DOD Directive 7000.1 (Ref. 16) provides the overall policy guidance with regard to resource management systems within the Department of Defense. This directive states that the resource

TABLE 34-4. SYSTEM SPECIFICATIONS AND ASSUMPTIONS (EXAMPLES)

I. Primary equipment specifications (if possible, by major components, e.g., airframe or structure, propulsion, guidance).

A. Performance specifications:

1. Examples for airframes:

a. Speed
b. Combat radius
c. Climb
d. Ceiling
e. Range
f. Load

2. Examples for electronics:

a. Frequency
b. Continuous vs spasmodic operation
c. Functions to be performed and speed of computation
d. Accuracy (e.g., in terms of deviation over time and/or drift rate, discrimination capability)
e. Jammability

3. Examples for engines:

a. Rating
b. Specific fuel consumption
c. Operating temperature

B. Weight data

C. Other physical data:

1. Examples for airframes:

a. Size data (e.g., fuselage length, wing area, wing span)
b. Construction characteristics:

(1) Sheet and stringer
(2) Sandwich, waffle, etc.
(3) Foamed metal
(4) Welded vs riveted
(5) Castings, forgings, extrusions, weldments, etc.

c. Basic metal types (with respect to items in b, above)
d. Tolerances (with respect to items in b, above)

2. Examples for electronics:

a. Volume
b. Type of construction technique (tube, transistor, modular)
c. Number of tubes or transistors
d. Number of stages

(cont'd on next page)

TABLE 34-4. (cont'd)

e. Power requirement
f. Antenna diameter (for radars)

D. Who the manufacturer is or is likely to be.

II. Ground support equipment specifications analogous to those listed under I.

III. Operational concept specifications or assumptions and related matters. Examples are:

A. Force size
B. Geographical deployment (especially overseas vs ZI)
C. Dispersal scheme
D. Activity rates
E. Fixed or mobile system and description thereof
F. "Hard" or "soft" system, and psi specification, if hard
G. Organizational concept: wing, group, etc., and number of squadrons per wing or group
H. Alert capability and related manning concept
I. Degree of system automation, stated by function if possible, in relation to manning and GSE requirements
J. Number of years the system is to be in the operational inventory
K. Training concepts; and in the case of missile systems: (a) number of missiles to be used in initial training, (b) number of live firings for "proficiency" training purposes per year
L. Logistic support concepts, especially regarding depot maintenance (DARCOM depot, or contractor?). Is there to be a "central support" area?
M. Permanent or temporary facilities
N. Tenant or nontenant operation
O. Main aspects of the development program, especially number of vehicles in the test inventory.

management systems will be oriented to the needs of all levels of DOD management, and also they will provide the information required by the Congress, Bureau of the Budget, Treasury Department, and other Government agencies. Another requirement of the directive is that the resource management systems be standardized and controlled to the extent practical so as to minimize the data gathering and reporting work load imposed on contractors and in-house activities. The Assistant Secretary of Defense (Comptroller) is charged with the overall supervision of all DOD resource management activities, and some of his responsibilities include: (1) review and approve proposed significant changes in resource management systems or proposed new systems, (2) insure compatibility and uniformity among resource management systems, (3) provide policy guidance for the characteristics and general criteria governing resource management systems, and (4) insure standardization of data elements and data codes. As noted, the Assistant Secretary of Defense (Comptroller) has overall or general, but not working level, supervision; this level of supervision is delegated to the individual military departments.

The Department of the Army Cost Analysis Program, outlined in Ref. 1, establishes policy and assigns responsibilities for improving the analysis of all major costs related to organizing, equipping, maintaining, and developing Army forces. Within the US Army Materiel Development and Readiness

Command. AMCR 37-4 (Ref. 17) prescribes program and policy guidance for establishing Cost Estimate Control Data Centers (CECDC's) in each subordinate command. These CECDC's are intended to serve as the central cost analysis activity at each commodity command for registration, control, review, and validation of cost information. Each CECDC is required to maintain cost tracks on selected weapon systems in a consistent and traceable format. Although progress in the past has been somewhat slow, this systematic accumulation of cost data should materially improve the availability and validity of the data required by Army cost analysts.

34-3.2 COST INFORMATION SOURCES

Contractor Cost Data Reporting (CCDR), discussed in DOD Instruction 7000.11 (Ref. 18), provides cost information in a format compatible with the standard/weapon/support system cost categories of AR 11-18. The CCDR consists of the following four reports of potential value to the weapon systems cost analyst:

1. Contract Cost Data Summary
2. Functional Cost-Hour Report
3. Progress Curve Summary
4. Plant-Wide Data Report.

Another source of standard cost data is the Selected Acquisition Reports (SAR's) defined in DOD Instruction 7000.3 (Ref. 19). This is a summary report designed for submission to Congress and other Government agencies and provides pertinent information regarding weapon system program changes that influence costs to include:

1. Engineering changes
2. Procurement quantity changes
3. Support requirements changes
4. Schedule changes
5. Unpredictable changes
6. Economic changes
7. Estimate changes
8. Contract performance incentives
9. Contract cost overrun/underrun.

DOD Instruction 7000.10 (Ref. 20) provides information and instructions for preparation of three summary reports that are used to collect summary level cost and schedule performance data and funding data from contractors for use by program management personnel. These reports may provide cost information of value to the cost analyst. Discussion of these three reports follows:

1. The Cost Performance Report (CPR) is intended to provide early identification of problems having significant cost impact, effects of management actions taken to resolve the problems, and program status information for use in making and validating management decisions. This report, prepared by contractors, consists of five formats containing costs and related data for measuring contractor's cost and schedule performance. The formats are:

a. Format 1 provides data to measure costs and schedule performance by summary level WBS elements.

b. Format 2 provides a similar measure for organization or functional cost categories.

c. Format 3 provides the budget baseline plan against which performance is measured.

d. Format 4 provides manpower loading forecast for correlation with budget plan and cost estimating predictions.

e. Format 5 is a narrative report used to explain significant costs and schedule variances and other identified contract problems.

2. The Contract Funds Status Report (CFSR) is designed to supply funding data about defense contracts to program managers for:

a. Updating and forecasting contract fund requirements

b. Planning and decision making on fund changes

c. Developing fund requirements and budget estimates in support of approved programs

d. Determining funds in excess of contract needs and availability for deobligation.

3. The Cost/Schedule Status Report (C/SSR) is prepared by contractors and provides summary costs and schedule performance information on programs where application of the CPR is not appropriate.

AMCR 715-22 (Ref. 21) provides for estimates of all negotiated procurements in excess of $1 million to be made according to a specified functional cost structure compatible with Ref. 1.

While these reports (SAR and CCDR) are summary in nature, their consistency of format provides a good point of departure in searching for essential characteristics of a weapon system to be costed.

34-3.3 COST DATA HANDBOOKS

The *Army Force Planning Cost Handbook* (Ref. 22) has been prepared by the Comptroller of the Army to provide cost and other factors for typical building block organizations to permit calculation of the estimated cost of alternative force structures. It provides detailed investment, personnel, and operating costs for selected TOE's or mixes of TOE's using unit identification numbers. Since equipment authorizations vary somewhat in different theaters, separate costs are shown for each theater where this difference may be significant. Sufficient data are available to enable the analyst to identify the cost impact of changes in:

1. Force structure
2. Force deployments
3. Activity rate, e.g., flying hour program
4. Training requirements
5. Airlift requirements
6. Equipment modernization
7. Logistic guidance
8. Manpower.

The data factors are intended to enable the analyst to develop a first approximation of the equipment and training costs of alternative force structures, the added costs that result when they are deployed, and the operational costs that apply in different theaters.

The explanatory notes included in the Handbook are of particular interest and value. They outline the techniques used to accumulate various costs and the dollar value estimating factors that are used. Such estimating factors (training costs per man-year of training and central supply costs per year per tables of organization and equipment strength) can be used in other costing exercises, such as weapon system life cycle costs. This detailed explanation of the procedures used to derive unit costs illustrates the point that cost estimates should clearly present methodology and assumptions so that users of derived cost data may judge the suitability of the estimates for their analytical purposes.

Ref. 22 is revised periodically to reflect price changes, authorization changes, and improved cost factors. As the cost data base expands, the Handbook will incorporate new data and improved methodology. Automated cost calculation assists in revising data to encompass the continuous changes that take place.

The engineering handbooks contained in most libraries provide more technical descriptive material than cost information. They are useful to the cost analyst in providing an understanding of the technical process of subsystems that must be costed. Catalogs prepared by manufacturers that specify the various hardware subsystems in more detail can then be reviewed for detailed cost information. Care must be exercised in using such prices however. Most frequently, new developments are based on extending the state of the art of the components of the system. Catalog prices reflect on-the-shelf prices that may substantially understate the acquisition costs of one-of-a-type systems not available in the market. Catalog prices should be used only when it is clear that the items are directly applicable to the system.

It is unlikely that the cost analyst will find all the cost data he might desire in handbooks. In fact, prior studies on similar systems are more likely to provide the data he requires or to provide data references that are pertinent to his needs.

34-3.4 PREVIOUS STUDIES

The analyst frequently estimates an unknown future cost by relating it to some known similar system that has been accurately costed. Detailed cost data derived from prior studies are essential for accurate cost analysis of new systems. Such data are used to develop valid estimating relationships. Estimating relationships are of five basic types:

1. Per-unit catalog price or planning factor, e.g., helicopter engines, by type and current production costs; helicopter maintenance in terms of dollars per flight hour
2. Cost-to-cost estimating relationships, e.g., initial repair parts, as a percent of initial equipment costs
3. Non-cost-to-cost relationships, e.g., communication equipment with frequency and range as cost influencing variables
4. Specific analogy, e.g., speed and range requirement by cost of similar specific systems
5. Expert opinion, e.g., new laser sensor by production state of the art.

Previous costing studies will have used some or all of these estimating techniques, no doubt. Ideally, the system to be costed will be comparable in many details with a system that has recently been developed. In such cases, much of the available cost data will be useful with minimum adjustment. More likely, however, the new system will be unlike older systems for which extensive cost data exist. In these cases, the similarities in subsystems and in their development, production, or maintenance procedures should be identified and specific analogy techniques should be used whenever practical. If specific analogy techniques cannot be used because of basic differences between the systems, it may be practical to adapt the data that are available for use in performance estimating relationships. In such cases, the data from many studies are used to derive statistical non-cost-to-cost estimating relationships.

Prior studies contain much data that have been derived from cost and engineering handbooks. The reference documentation for such data provides an excellent lead to similar data that are more directly applicable to the new system to be costed. Prior studies also contain references to other cost studies that were used in compiling estimating data; these references enable the analyst to explore such studies more fully for data that may be applicable to his analysis.

34-4 DOCUMENTATION

Proper documentation of a cost study is essential to both the presentation of the estimate and providing a traceable record of how costs were derived. The latter is of fundamental importance in accommodating timely revisions to the estimate as a result of program modifications and for recording detailed results for subsequent weapon system studies.

The documentation that the analyst prepares must provide sufficient detail to assure that the data, procedures, rationale, and CER's that are derived can be validated step by step by review authorities. All data sources, data used, assumptions, discussions, and calculations must be summarized in a logical fashion that makes clear each action taken in the analysis process Various types of formats have been suggested to assure that the cost analyst identifies and records each step in the analysis. If several groups are participating in an analysis, explicit instructions must be prepared to avoid duplications and omissions. Interim reports should follow the format of the final report. Table 34-5 illustrates a sample format that identifies the various components of a cost estimate report. Further, DA documentation policy is provided by DA PAM 11-5 (Ref. 5). Circumstances may modify the reporting requirements. Usually, the detail in the full-length report is summarized to present the study results more clearly. However, the full report is the supporting base that provides the essential backup for the summary submission.

34-5 SUMMARY

Army cost analysis requires a thorough understanding of the systems analysis structure, the use of appropriate cost categories, knowledge of cost element structures, consideration of the effect of uncertainties, the use of cost sensitivity analysis, and a clear definition of the actual hardware to be costed.

Some major considerations that should be stressed in cost analysis are:

1. Emphasis on total program cost, to include the entire spectrum of cost implications over the life of the program
2. Use of cost categories that highlight the major phases in the life cycle
3. Considerations of the timing of cost
4. Explicit handling of sunk costs
5. Identifying the net resource requirements, thereby allowing for inheritance from existing systems and recoverable value at retirement of the system.

Inaccurate or imprecise estimation of costs of competing weapon or weapon systems may result in failure of an overall evaluation to discriminate properly among alternatives.

TABLE 34-5. COST ESTIMATE WORKING REPORT OUTLINE

Cover Page:
- Title
- Prepared for ____________
- Prepared by ____________
- Participating Organizations
- Approving Authority (name, rank, and/or title and signature)
- Date of Approval
- Key Analysis

Cost Task Summary Sheet
Cost Estimate
Cost Estimate Change Summary
Cost Estimate Quality Control Checklist
Contents
Introduction:
- Purpose of Estimate
- History of Estimate
- Brief Description of Items or Systems (including developmental status)

Study Ground Rules
Constraints
Assumptions
Work Breakdown Structure
Organizational Responsibilities
Information Summaries
Product Characteristics:
- Tabular
- Graphic (comparisons of data base with products being estimated)
- Schedule (tabular or graphic)
- Resource Expenditure

Reference Unit Costs
Data Base Items (brief description)
Cost Estimating Relationships Used (with constants)
Cost Uncertainty
Cost Sensitivity Analysis
Validation Report
Cost Escalation Implications
Results (cost estimate details)
Conclusions:
- In Response to Task Assignment
- Relative to Cost Analysis and/or Cost Estimating

Appendixes:
- A — Study Directives, Instructions, and Correspondence (a bibliography list by date of receipt or date of origin for outgoing documents followed by copies of study)
- B — Cost Estimate Schedule
- C — Bibliographic List of Cost Information Survey
- D — Bibliographic List of Interim Reports, Working Reports, and Ancillary Reports
- E — Cost Estimate Change Detail
- F — Summary Report (annotated as to Working Report source for each item of cost information.)

REFERENCES

1. AR 11-18, *The Cost Analysis Program.*
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4. DA PAM No. 11-4, *Operating and Support Cost Guide for Army Materiel Systems.*
5. DA PAM No. 11-5, *Standards for Presentation and Documentation of Life Cycle Cost Estimates for Army Materiel Systems.*
6. DOD Directive 5000.1, *Major System Acquisitions.*
7. DOD Directive 5010.20, *Work Breakdown Structures for Defense Materiel Items.*
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9. J. O'Flaherty, *Work Breakdown Structure*, Report No. RAC-CR6, Research Analysis Corporation, McLean, VA, February 1971.
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17. AMCR 37-4, *Cost Estimate Control Data Center.*
18. DOD Instruction 7000.11, *Contractor Cost Data Reporting (CCDR).*
19. DOD Instruction 7000.3, *Select Acquisition Reports (SAR).*
20. DOD Instruction 7000.10, *Contract Cost Performance, Funds Status and Cost/Schedule Status.*
21. AMCR 715-22, *Independent Government Cost Estimates (IGCE's).*
22. *Army Force Planning Cost Handbook*, Office of the Comptroller of the Army.

CHAPTER 35

WEAPON SYSTEM LIFE CYCLE COST ESTIMATION (LCCE)

The problem of determining the life cycle costs of a weapon system is an involved process. Outlined herein are various cost guides for consideration of the weapon systems analyst and some of the more general and widely practiced cost estimation procedures. The Army divides the weapon system life cycle into three phases—the research and development (R&D) phase, the investment or procurement phase, and the operating and support (O&S) phase. Cost considerations for each of these three phases are covered in some explanatory detail for the analyst, and the practice of using either the bottoms-up or top-down approach to cost estimation problems is discussed. These two different approaches may be considered to involve either the analogy method, the analytical or cost estimation relation (CER) method, or the well-known and widely practiced engineering methods of cost determination. A very important parameter in costing the life cycle of a weapon system relates to the estimation of the useful life of the system, and this must be included in the cost analysis process.

An example outlining the current approach to cost estimation for the life cycle of a system is given and illustrated for the Utility Tactical Transport Aircraft System (UTTAS).

35-0 LIST OF SYMBOLS

A = constant to be determined
A_i = amount of expected cost in the ith year, dollars
AEW = aircraft empty weight
$AWGT$ = $AMPR$ weight, lb
a = $\ln A$
b = improvement learning curve slope
C = general symbol used for cost
C_i = cost of ith item or element
C_T = $C(n)$ = cost of producing the first n items
$\overline{C}$ = average cost per cannon for the first 1000 cannons
d = exponent to be determined
F = cost of producing the first unit
n = number of items produced
n = period of the life cycle, yr
n = number of elements costed over life cycle
Op = number of actual operating personnel, in thousands
P = present value, dollars
P = peak monthly production rate
r_i = (variable, if applicable) discount rate used for the ith year, expressed as a decimal
S = "shift factor" to account for inflation
S = number of support personnel (Eq. 35-19), in thousands
W = gun weight, lb
X_1 = shaft horsepower, HP
X_2 = dry weight, lb
x = $\ln W$
Y_1 = prototype $AMPR$ weight, lb

Y_2 = adjustment in cost to account for use of composite materials
y = $\ln \bar{C}$
σ^2 = average variance of costs for all elements costed
σ_C = standard error of total cost
$\hat{\sigma}_{\bar{C}W}$ = estimate of standard error of residuals for $\bar{C}$ on W
$\sigma_C(Inv)$ = standard error of cost for Investment phase
$\sigma_C(O\&S)$ = standard error of cost for the Operating and Support phase
$\sigma_C(R\&D)$ = standard error of cost for the R&D phase
σ_i = standard error of ith element cost
$\hat{\sigma}_{yx}$ = estimate of standard error of residuals for y on x

35-1 INTRODUCTION

During the 1960's, major weapon system programs experienced uninhibited cost growth over original estimates of expected life cycle costs. This has been variously attributed to overoptimism in cost estimates, design changes, modifications to accommodate "desired" characteristics, overemphasis on pushing the limits of technology (high risk solutions), schedule slippages, rapid inflation, etc. For these and other reasons, escalation in weapon systems program costs has been the subject of continuing criticism and debate by many, including the Congress of the United States. As a result, the Deputy Secretary of Defense issued a memorandum to all military departments (Ref. 1) emphasizing the need to improve weapon system acquisition practices, specifically the need for cost realism and improvements in cost estimating and validating capabilities. This memorandum was followed by a revision to DOD Directive 5000.1, *Major System Acquisitions* (Ref. 2), which stressed the importance of careful study of the need for weapon systems and adequate assessment of the risks involved. Primary emphasis was placed on trade-offs among capabilities, cost of acquisition and ownership, along with development schedules. Key among the policy objectives was a "fly-before-buy" concept stressing prototype and hardware testing to validate performance and cost prior to full-scale development.

In response to this guidance, the Department of the Army has made some major revisions to improve weapon system acquisition and management practices as reflected in AR 1000-1, *Basic Policies for Systems Acquisition* (Ref. 3). This regulation outlines basic procedures required to implement Army system acquisition policy objectives.

35-2 THE WEAPON SYSTEM LIFE CYCLE

The preparation of a weapon system life cycle cost estimate must be responsive to the requirements of the formal process to obtain funds to proceed through the acquisition cycle. The key management-decision milestones, requiring cost estimates, correspond to the following major phases of the weapon system life cycle:

1. Conceptual
2. Validation
3. Full-Scale Development
4. Production and Deployment
5. Operating and Support (formerly ownership phase).

A determination that cost estimates are complete, realistic, and acceptable is very important in order to proceed to a subsequent life cycle phase. The following guidelines for preparation, assessment, and revision of life cycle cost were specified in the 1972 edition of AR 1000-1, *Basic Policies for Systems Acquisition by the Department of the Army*.

"1. Prior to Approval to Initiate Conceptual Phase. The materiel developer will perform an initial broad based 'cost assessment' of acquisition cost mainly using summary level parametric estimating techniques. This initial assessment will be of major cost categories and to the extent feasible will estimate cost of major items or components below the system level.

"2. Prior to Entering Validation. The materiel developer will prepare a baseline cost estimate as detailed as possible in conformance with the single work breakdown structure (WBS) established. This estimate will also establish all discrete cost elements for the program, cost parameters, and the initial 'design to' unit cost goal. The estimate for program acquisition cost, a part of the baseline cost estimate, will serve as a basis for submission to DSARC/ASARC* review and establishment of the Army Planning Estimate. Beginning with the planning estimate, cost estimates and parameters must be sufficiently realistic so that significant changes to cost parameters will not occur subsequently. An Independent Parametric Cost Estimate (IPCE) will be prepared and submitted for DSARC/ASARC I review.

"3. Prior to Entering Full-Scale Development. The materiel developer will perform a deliberate re-estimate of cost for DSARC/ASARC II and will recommend establishment of the development estimate baseline for program acquisition cost. An Independent Parametric Cost Estimate (IPCE) will be prepared and submitted concurrently. The cost parameters previously established will be reassessed and resubmitted to provide realistic 'design to' unit cost goals in RFP's (Request for Proposals) and contracts. The impact of the costs of ownership (operating costs) will be updated only when necessary for specific decision purposes.

"4. Prior to Entering Low-Rate Initial Production. The developing agency will reassess the development estimate for program acquisition cost and will reassess the impacts of 'design to' requirements on cost parameters for DSARC/ASARC IIa review.

"5. Prior to Entering Full-Scale Production. The developing agency will again perform a deliberate re-estimate of the investment (procurement) cost portion of the development estimate. At this time an industrial engineering type estimate will consider detail levels of the work breakdown structure (WBS). The objective is to obtain the most comprehensive assessment possible of the expected production cost. The contractor's experience during low rate initial production will be reviewed and analyzed to determine validity of the 'design to' unit cost and the cost for production/procurement reported. This cost estimate supports the DSARC/ASARC III review."

Throughout the life cycle, the baseline cost estimate established during the conceptual phase serves as the point of departure for all subsequent cost reassessments. This provides for the traceability of the cost estimate for evaluating program cost variance in accordance with Ref. 4. The assignment of responsibility for the preparation of an Independent Parametric Cost Estimate (IPCE) is done by the Comptroller of the Army, and this estimate is developed independently of the developer estimates for submission through command channels for each DSARC/ASARC review.

Through the materiel acquisition process, life cycle cost estimates—to including funding profiles—are necessary to support funding decisions to continue with program development. Further guidance on cost-estimating requirements in support of management decisions can be obtained from Refs. 5, 6, and 7.

It will be very helpful at this point to discuss briefly the life cycle of a materiel system which is shown schematically in Fig. 35-1 (Fig. 2-1 of Refs. 8, 9, and 10). Note in particular that the figure is oriented toward indicating program costs throughout the life cycle. On a time scale basis, one notes the in-house planning phase, the conceptual phase, the full-scale development phase, the production and

*DSARC/ASARC = Defense Systems Acquisition Review Council/Army Systems Acquisition Review Council

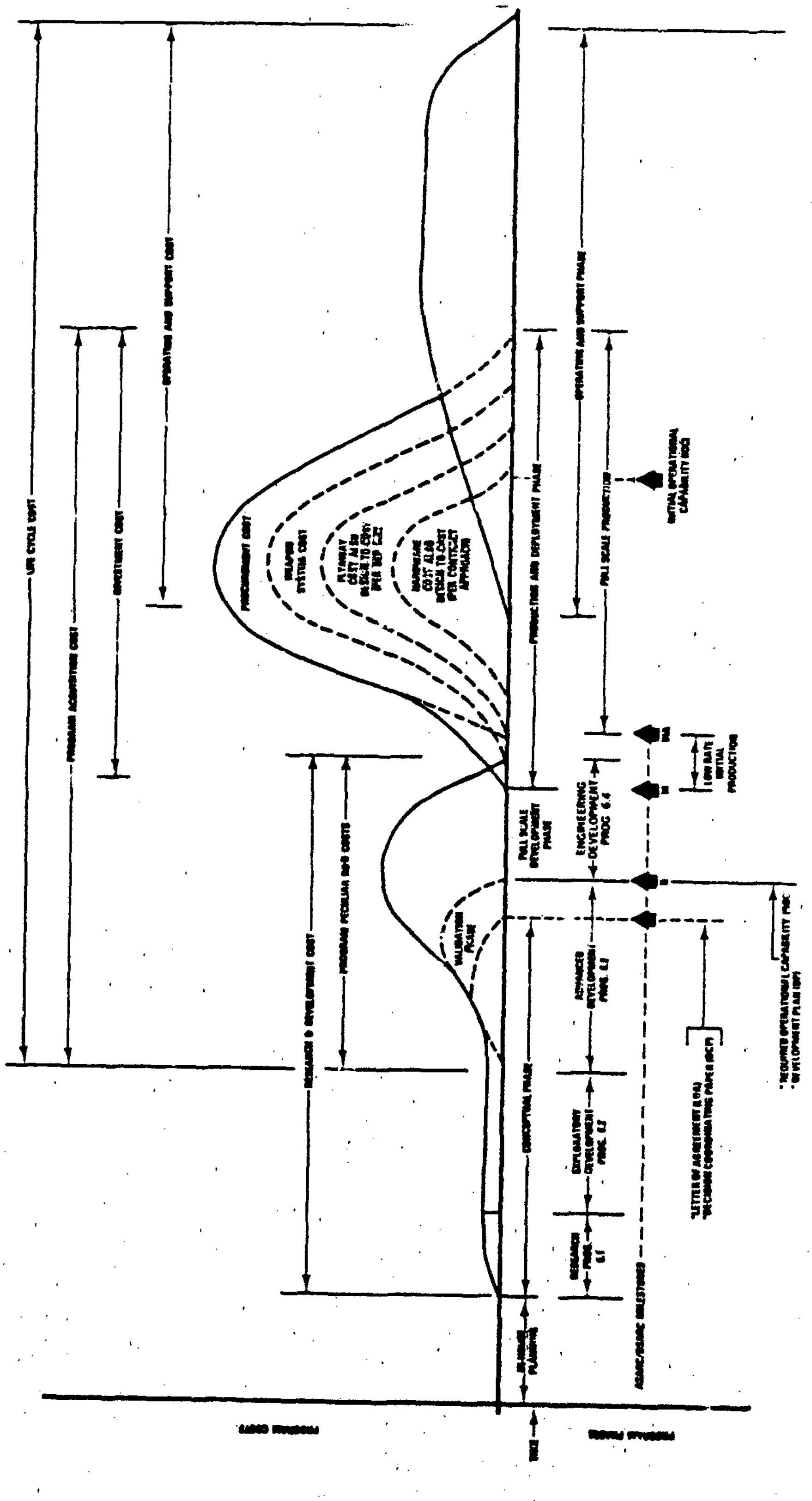

Figure 35-1. Life Cycle of a Materiel System (Ref. 8)

deployment phase, and the operating and support phase which overlaps the production and deployment phase. Note also the division into research, exploratory development, advanced development, engineering development, and full-scale development. Generally, the more significant milestones—in terms of the ASARC/DSARC—are indicated and include the letter of agreement (LOA), the decision coordination paper (DCP), the required operational capability (ROC), the development plan (DP), the low-rate initial production, and initial operational capability (IOC). The more important cost phases in the whole life cycle cost analysis include the research and development (R&D) costs, the investment costs, and the operating and support (O&S) costs. The height of the curves indicates the relative costs for each of the significant cost phases. Indeed, if the program costs (ordinates) could be estimated accurately, and especially if the shown costs were costs per unit of time, the integration of these curves over time, i.e., over the life cycle, would give an acceptable account of total costs. Obviously, the costs over each of the individual phases will vary from one weapon system to another, and it is the job of the cost analyst to notice the differences and to determine projected costs as precisely as possible. Hopefully, experience in estimating the different cost phases might apply to several weapon systems.

Fig. 35-1—applying to Army "materiel systems"—is rather general in character.

A "materiel system" consists of all tangible items—including ships, tanks, self-propelled weapons, aircraft, etc., and related spares, repair parts, and support equipment; but excluding real property, installations, and utilities—necessary to equip, operate, maintain, and support military activities without distinction as to its application for administrative or combat purposes. A "system", for our purposes, is a combination of components which function together as an entity to accomplish a given objective.

The boundaries of different materiel systems may seem to vary, but the key words in the given definition, "function together as an entity", serve to define the system. For example, a large missile such as the PERSHING may function within a force unit of a battalion; while a rifle, together with the man who carries it and some maintenance men, would constitute a functional entity. As a practical matter, it is often possible to define many systems at battalion level or lower. In some cases, however, there will still be difficulty in deciding what costs should be attributed to the materiel system. The applicable principle is "if a given component would not exist if the system did not exist, then that component must be included in the definition of the materiel system".

Divisional and nondivisional support cost estimates are major topics of interest for the cost analysts, nevertheless.

35-3 THE COST GUIDES AND GENERAL COSTING METHODS

As we have indicated, there are three major areas or categories which have been developed for estimating the life cycle costs of Army materiel systems; these are the research and development phase, the investment phase, and the operating and support phase. In this connection, a common framework for general cost communication among interested personnel is given on Fig. 35-2, the Army life cycle cost matrix. Note the division into rows and columns such that (1) the rows represent the cost elements for R&D, investment, and O&S costs; and (2) the columns refer to system elements, such as system frame, propulsion, guidance and control, and fire control. The framework must be compatible with cost analysis policy and convention; it must be capable of capturing 100% of the costs; and it must be manageable in size. Simplification in level of cost analysis detail is very essential also.

The US Army has published three very useful and informative major cost guides for the cost analyst or the weapon systems analyst, namely: the *Research and Development Cost Guide for Army Materiel Systems*

ROW	FUNDS APPRO	DEFN REF	COST ELEMENT / SYSTEM STRUCTURE*	(1) FRAME	(2) PROPULSION	(3) GUIDANCE CONTROL/ COMMUNICATIONS	(4) FIRE CONTROL	(5) ARMAMENT	(6) PAYLOAD/ AMMUNITION	(7)** (TO BE SPECIFIED)	(8) PECULIAR SUPPORT EQUIPMENT	(9) COMMON SUPPORT EQUIPMENT	(10) OTHER	(11) TOTAL	(12) PERCENT
1		1.0	RESEARCH AND DEVELOPMENT												
2	RDTE	1.01	DEVELOPMENT ENGINEERING												
3	RDTE	1.02	PRODUCIBILITY ENGINEERING AND PLANNING (PEP)												
4	RDTE	1.03	TOOLING												
5	RDTE	1.04	PROTOTYPE MANUFACTURING												
6	RDTE	1.05	DATA												
7	RDTE	1.06	SYSTEM TEST AND EVALUATION												
8	RD/OM	1.07	SYSTEM/PROJECT MANAGEMENT												
9	RD/OM	1.08	TRAINING												
10	RD/MC	1.09	FACILITIES												
11	RDTE	1.10	OTHER												
12		2.0	INVESTMENT												
13	PR/MC	2.01	NON RECURRING INVESTMENT												
14	PROC	2.02	PRODUCTION												
15	PROC	2.03	ENGINEERING CHANGES												
16	PR/OM	2.04	SYSTEM TEST AND EVALUATION												
17	PR/OM	2.05	DATA												
18	PR/OM	2.06	SYSTEM/PROJECT MANAGEMENT												
19	PR/MC	2.07	OPERATIONAL/SITE ACTIVATION												
20	PR/OM	2.08	TRAINING												
21	PR/OM	2.09	INITIAL SPARES AND REPAIR PARTS												
22	PR/OM	2.10	TRANSPORTATION												
23	PR/OM	2.11	OTHER												
24		3.0	OPERATING AND SUPPORT COST												
25		3.01	MILITARY PERSONNEL												
26	MPA	3.011	CREW PAY AND ALLOWANCES												
27	MPA	3.012	MAINTENANCE PAY AND ALLOWANCES												
28	MPA	3.013	INDIRECT PAY AND ALLOWANCES												
29	MPA	3.014	PERMANENT CHANGE OF STATION												
30		3.02	CONSUMPTION												
31	PR/OM	3.021	REPLENISHMENT SPARES												
32	OMA	3.022	PETROLEUM, OIL AND LUBRICANTS												
33	PROC	3.023	UNIT TRAINING, AMMUNITION AND MISSILES												
34		3.03	DEPOT MAINTENANCE												
35	OMA	3.031	LABOR												
36	PA/OM	3.032	MATERIEL												
37	OMA	3.033	TRANSPORTATION												
38	PROC	3.04	MODIFICATIONS, MATERIEL												
39		3.05	OTHER DIRECT SUPPORT OPERATIONS												
40	OMA	3.051	MAINTENANCE, CIVILIAN LABOR												
41	OMA	3.052	OTHER DIRECT												
42		3.06	INDIRECT SUPPORT OPERATIONS												
43	MP/OM	3.061	PERSONNEL REPLACEMENT												
44	MPA	3.062	TRANSIENTS, PATIENTS AND PRISONERS												
45	OMA	3.063	QUARTERS, MAINTENANCE AND UTILITIES												
46	MP/OM	3.064	MEDICAL SUPPORT												
47	OMA	3.065	OTHER INDIRECT												
48			TOTAL SYSTEM COST (LESS ERDA)												
49	ERDA	4.0	ERDA COST												
50			TOTAL SYSTEM COST (WITH ERDA)												

100%

*THE COLUMN HEADINGS SHOWN ARE GENERIC, USED FOR DISCUSSION OF SYSTEMS IN GENERAL. FOR SPECIFIC STUDY, SPECIFIC COLUMN HEADINGS ARE THOSE PROVIDED IN APPENDIX A.

**USED FOR A SPECIFIC HARDWARE COMPONENT/SUBSYSTEM OF HIGH MANAGEMENT INTEREST, NOT CAPTURED ELSEWHERE

Figure 35-2. Army Life Cycle Cost Matrix (Ref. 8)

(Ref. 8), the *Investment Cost Guide for Army Materiel Systems* (Ref. 9), and the *Operating and Support Cost Guide for Army Materiel Systems* (Ref. 10).

Ref. 8 indicates that there are two major cost estimating approaches, and we quote:

"Estimates for materiel system acquisition costs are either derived from detailed, grassroot calculations (the industrial engineering approach) or based on the relationships between more aggregate components of system cost and the physical and/or performance characteristics of the system. These relationships should be derived from cost histories on prior programs. The latter method is often called the parametric approach. Two additional descriptors, which will be used in this guide, have come into common usage because of the clarity with which they capture the essential difference: *Bottoms-up* for the detailed industrial engineering approach and *Top-down* for the parametric approach.

"Historically, defense contractors have employed the bottoms-up approach in their proposal pricing and planning purpose estimates for the Government. Because of Government Program Managers (PM's) responsibilities in connection with defense contractors, it has evolved that PM estimates of program costs mirror the detailed work breakdown structure (WBS) associated with contractor cost estimates. Thus the PM estimate, described as the Baseline Cost Estimate (BCE), usually reflects bottoms-up cost estimating methodology.

"The advent of top-down cost estimating methodology brought the opportunity for a genuine cross-check of detailed bottoms-up cost estimates. The descriptor, Independent Parametric Cost Estimate (IPCE), has been given to those estimates employing the top-down cost estimating methodology.

"For the cross-checking or validation process to be productive it is necessary that a common ground be created whereupon differently derived estimates may be compared, analyzed, and judged. A crucial criterion in the selection of such a common ground is that the WBS selected should not preclude (by its inherent composition) the choice of either top-down or bottoms-up methods. While it is possible to aggregate detailed cost (i.e., to pyramid) it is *not* possible to disaggregate composite costs (to un-pyramid) any lower than the level of costs used in creating the cost estimating relationship. (Example: If you cost the construction of a new 1800-square-foot home on the basis of $22.00/square foot [a factor derived by averaging the square foot costs of several new homes in the area] which yields a composite cost of $39,600, you cannot say how much of the $39,600 is the plumber's or electrician's cost.) Thus, the common ground, as would be reflected in a WBS, must take into consideration perceptions of the general level of detail upon which cost estimating relationships are based. This is a function of the overall quality and general structure of the historical data base and the levels of cost aggregation at which cost analysts conventionally work.

"During the early phases of the acquisition process only limited requirements information is available. The top-down approach is particularly suited to making estimates based on limited physical and performance information. The descriptor, 'Cost and Operational Effectiveness Analyses' (COEA), has been given to those cost/effectiveness studies performed, principally, in the early acquisition phases. Research and Development phase costs of system alternatives evaluated in COEA are derived using, principally, the top-down cost estimating approach.".

Trainor (Ref. 11) adds a third basic method of costing materiel systems—the "analogy" method. Estimation by analogy involves establishing a cost-oriented relationship between a project whose cost is known and the project for which costs are to be estimated. As an example, it is sometimes possible to make a simple cost per pound analogy. During much of World War II, for example, it was not too inappropriate to cost some weapon systems at about $1 per lb for the nonelectronic components, while the cost for VT fuzes, for example, was about $35 per lb. Trainor (Ref. 11) points out that if a combat vehicle costs $2.70 per lb, then, as a first approximation, it may be possible to conclude that its replacement, even though 20% heavier, will also cost about $2.70 per lb. Thus the chief advantages of the analogy technique are its ease of application and the fact that it may even avoid much detailed costing of the system items or components.

By way of summary, we see that the bottoms-up method is the usual engineering approach which has long been used, while the top-down approach attempts to identify some of the key or more important parameters and develops a cost estimation relation (CER) in terms of the finally adopted parameters. Naturally, it would be expected that the top-down or parametric approach could involve the principles of statistical regression theory, and in fact it does.

The engineering approach is one of the most common and certainly the most detailed method of cost estimation. Indeed, it involves a complete description of the system, a detailed list of activities and all the operations that are required for completion of the tasks, etc. Thus the engineering estimate, although very complex, has been widely used in industry and is often important in project justification because it may present solid evidence that the engineers will have a sufficiently detailed understanding of the system they are to develop. When properly used, the engineering estimate can provide a detailed and accurate cost estimate. The technique is not commonly used by DOD, however. The engineering type estimate is applicable to nearly all kinds of major projects and weapon systems and is also used for the production phase of a system, especially along with the so-called "learning curve" type of projections for many units produced. The degree of learning has been shown to be quite predictable within acceptable limits for repetitive operations involving weapon system components, electronics, and automotive equipment. It has not been shown to have been entirely successful in many field or environmental conditions where things are not so well controlled as in the factory. The engineering approach, it can be seen, is very time-consuming and somewhat complex; therefore, quicker, less costly techniques are desired.

The top-down or parametric cost estimating procedure, which establishes the relation between cost and the characteristics or major parameters of weapon systems, often is very economical and entirely satisfactory. Some typical cost-related parameters or variables, for example, might include system or component weight, speed, and horsepower. In fact, it might be said that the parametric approach is an extension of the analogy technique to include more parameters and a statistically supportable basis. The parametric technique has become increasingly popular perhaps due to work at the RAND Corporation through the 1960's. In fact, the parametric technique often was used for estimating procurement costs of airframes, engines, and various categories of electronic equipment or power supplies. It has not been used often to estimate development costs. Statistical regression techniques have been found to be very valuable, and increased sample sizes in such regression analyses would be expected to reduce risks of estimation. Thus the analogy technique may often be a "back of the envelope" type calculation, often involving a single "parameter"; whereas the parametric approach uses available data to fit a curve relating costs to values of the parameters and hence may become quite sophisticated indeed. In the parametric approach, the analyst must be careful to develop a good law or the proper relationship between expected cost and sensible functions of the parameters used. Parametric techniques are nearly always preferred to analogy techniques for estimating costs when a sufficient data base is available, and a keen cost analyst develops proper regression relations. There is often a limitation in the use of parametric techniques—one must be careful not to extrapolate for inferences beyond standard errors of prediction. In fact, cost equations may be very nonlinear, and hence linear regression methods may not actually apply.

All three estimating procedures—the engineering or bottoms-up, the analogy, and the parametric or top-down methods—are most commonly applied to the estimation of production costs, although they may be applied to the estimation of development costs also. However, development cost estimation may be expected to be more unreliable than production cost estimation. Nevertheless, once a good parametric or regression equation is available, it can be used quickly to get answers. On the other hand, the ordinary engineering approach would be too detailed, costly, and time-consuming.

In the paragraphs that follow we discuss research and development costs, which actually are related primarily to development rather than research; investment costs; and operating and support costs.

35-3.1 RESEARCH AND DEVELOPMENT COST GUIDE

The basic principles and guidelines for treatment of research and development cost estimation problems are covered in Ref. 8. R&D cost estimates are needed to permit labor savings, to permit trade-offs between life cycle phases, to achieve better balance between equipment purchase and repair, to perform better comparisons between new materiel systems, and to provide management visibility of critical resource requirements. The R&D cost guide (Ref. 8) is expected to apply as a minimum to:

"(1) All programs for which there is an expectation of a Cost Analysis Improvement Group (CAIG) review.

(2) All programs expected to enter the Army Systems Acquisition Review Council/Defense Systems Acquisitions Review Council (ASARC/DSARC) process.

(3) All alternatives expected to be listed in the final Decision Coordination Paper (DCP).

(4) All other programs as may be directed by the Comptroller of the Army.

"The presentation and documentation requirements do not apply to the myriad alternatives, options or scenarios conceived in the course of the iterative process of defining and redefining systems requirements. They apply once it can be concluded that the process has yielded alternatives that are candidates for senior decision makers (DA Staff) attention."

"The term, 'R&D Cost', is defined, in general, to be the sum of all costs resulting from applied research, engineering design, analysis, development, test, evaluation, and managing development efforts related to a specific materiel system. The term R&D cost includes:

(1) All costs to the Government, defined as contractor costs plus in-house costs, of products and services necessary to bring a specific materiel system from concept to serial production.

(2) All costs to the Government of developing the specific capability, irrespective of how such costs are funded, i.e., irrespective of which appropriations (RDTE, MPA, MCA, or OMA) are cited, and irrespective of which organization within the Army has responsibility."

"Specifically excluded from R&D cost estimates are:

a. Costs incurred during the Investment and Operating and Support phases of a program.

b. Research and Development costs which cannot be directly related to the system itself or which cannot be reasonably allocated to the system. Research (6.1) and Exploratory Development (6.2) categories are not considered to be program peculiar R&D costs.".

A major fraction of the total R&D costs are for services performed, as compared to products produced. In fact, the cost of R&D services may be some 2/3 of total R&D costs. For civilian services, some useful algorithms have been developed; an example is Fig. 35-3. The cost analyst is expected to make explicit formulations such as in Fig. 35-3 which underpin the methodology employed in his estimate.

For military personnel, the cost per man year should include the following (Ref. 8):

Concept:

Use a parametric expression based on aggregations of contract pricing type elements. Thus:

Cost Per Man Year =	(Number Man-hours Per Yr)	(Avg Cost Per Manhour)	(Ratio of Total Direct to Direct Labor)	(1.0 + Avg Overhead Rate Expected)	(Out-of-House to 1.0 + In-House Ratio)	(1.0 + Fee)
=	1960	5.65	1.25	2.25	1.50	1.07
	2080 hrs per yr reduced 3 weeks for vacation, sick lv includable in overhead.	Avg cost per direct labor hr in FY 71 $ for Investment	This ratio picks up other direct charges beyond direct labor.	This is an aggregate overhead multiplier capturing all burden/G&A expense.	Factors captures materials and subcontracts awarded during Investment Program. Says for each $1.00 in-house you have 50¢ out-of-house.	This factor captures fee or estimated profit.

Cost = $50,000 (FY 71 $)

NOTE: The overall formulation has the purpose of capturing the essential variables underpinning an aggregate cost factor, cost per Investment manyear. Considerable care and understanding must be exercised in changing one or more of the variables as such variables are highly interrelated and in the case above reflect a mean expectation of cost of a manyear of Investment in any major defense plant. Given the condition of a contractor also performing the role of system manager/integrator, costs of the role of system manager, primarily expressed through the out-of-house ratio, must be eliminated and replaced by the more normal expectation of a small out-of-house contract effort.

Figure 35-3. Cost Per Man Year Methodology (Ref. 8)

"(1) Actual mean pay based on grade and MOS assuming a weighted average length of service for each grade.

(2) Basic Allowance for Subsistance and Basic Allowance for Quarters (BAS and BAQ) based on grade structure and percent by grade receiving BAQ.

(3) Special pay, if applicable, received by individuals based on the personnel requirements specified in the table of distribution and allowances (TDA) or requesting authority. These pays should be added to the mean cost per person based on the percentage required to possess the particular skill.".

Such methods of estimating costs of services may be expected to apply as well to the investment phase and in some instances to the operating and support phase also.

There are two types of estimates of costs; one is a point estimate, and the other a range estimate. The point estimate does not reflect any uncertainty associated with it, and too often, therefore, it implies a very precise estimate. For this reason, a range of costs—based on the inherent cost estimating uncertainty—should be provided. In fact, costs estimated from statistical considerations are often very valuable because the statistician may be able to place confidence bounds about many cost estimates.

As we have mentioned, the term "research and development cost estimation" is somewhat of a misnomer because cost estimation problems related to this phase are predominantly for development. In other words, costs for research may be mostly for the use of brains and some special (but often costly) equipment, and it is to be realized that many results from any research effort may apply to a variety of development projects. In fact, even programmed research may turn out to be applicable mostly to a very different field of interest than that for which the effort has been expended.

Development costs are not easy to estimate, and the process often turns out to be a much more involved or complex problem than that of estimating costs for the investment (procurement) or for the O&S phase. A very interesting account of the development cost estimating problem has been given by Trainor (Ref. 12), who made a study of some seven techniques of estimating costs of weapon systems—"all wrong", as he puts it. Trainor (Ref. 12) discusses the seven techniques—the component buildup method (an engineering type approach), the component buildup method plus a correction factor, a risk analysis-network theory type of approach, analogy techniques, a "remote" analogy technique (as compared to the direct analogy process), a complexity factor analysis type of approach, and the parametric cost analysis approach. He points out the advantages and disadvantages of all of these cost analysis approaches in some rather convincing detail. Clearly then, there is always the need for much cross-checking in cost analysis methodology, and different approaches have to be pursued. For development costs in general, Trainor (Ref. 12) indicates that variations between the estimates and actual development costs for some major systems can often vary by as much as 200-300%.

35-3.2 INVESTMENT COST GUIDE

Ref 9 gives guidelines for costs relative to the investment or procurement phase. The investment costs are the one-time expenditures that must be made to introduce a new weapon into the operational force. Investment costs are needed for the same reasons as those listed for R&D in par. 35-3.1, and the investment cost guide (Ref. 9) applies as a minimum to the four conditions at the beginning of par. 35-3.1 also. The investment cost elements are listed on Fig. 35-2, although the element No. 2.01, Nonrecurring Investment, is actually broken down further into:

1. Initial Production Facilities (IPF)
2. Industrial Facilities/Production Base Support (PBS)
3. Other Nonrecurring

Furthermore, the element No. 2.02, Production, is broken down into:

1. Manufacturing
2. Recurring Engineering
3. Sustaining Tooling
4. Quality Control
5. Other.

Of course, costs for each of the various elements must be estimated, as well as additional significant elements not listed if they occur.

"The term 'investment cost' is defined, in general, to be the sum of all costs resulting from the production and introduction of the materiel system into the Army's operational inventory. The term 'investment cost' includes:

(1) All costs to the Government, defined as contractor costs plus in-house costs, of products and services necessary to transform the results of R&D into a fully operational system consisting of the hardware, training, and support activities necessary to initiate operations.

(2) Costs of both a nonrecurring [nature], i.e., costs which are required to establish a production capability, and recurring nature, i.e., costs which occur repeatedly during production and delivery to user organizations.

(3) Costs of all production products and related services, irrespective of how such costs are funded, i.e., irrespective of which appropriations (PROC, MPA, OMA, MCA), are cited, and irrespective of which organization within the Army has responsibility.

(4) All costs resulting from production and introduction into operational inventory irrespective of how allocated among Unit Equipment (UE), Maintenance Float (MF), and Training Usage classifications." (Ref. 9)

Production cost estimating is also an activity involving uncertainty or error, but it involves much less error than that of estimating costs for development. Estimates often are within 20% of actual costs, and they seldom vary by as much as 200%. The practice of using parametric cost estimating techniques for production has worked well indeed and has been especially useful in reducing errors of estimation. Much of the variation in estimated unit production costs is actually the result of inadequate resource allocation, which in turn results in reductions in the planned quantities or planned production rates themselves.

The investment phase includes production, and it is for this item of the investment cost matrix that we may illustrate an example of the top-down cost estimating approach and the use of cost estimation relations (CER's). This particular application is for the so-called "learning curve", which is an equation expressing the cumulative cost of producing any number of items in terms of the cost for the first item produced and the "learning curve slope", as it is called.* To illustrate, let

$C_T = C(n)$ = cost of producing the first n items
n = number of items produced
F = cost of producing the first unit
b = improvement learning curve slope.

Then, the equation usually employed to estimate the total cost of producing n items is

$$C_T = Fn^{1+b} \qquad (35\text{-}1)$$

* See also Chapter 36.

It is easy to linearize Eq. 35-1. Taking natural logarithms of both sides, we have

$$\ln C_T = \ln F + (1 + b)\ln n. \tag{35-2}$$

Thus with cost data for the first item produced and the total cumulative cost for any number n of the items, the "learning slope" b may be determined and used for future, similar production. Alternatively, once the exponent b is known for some typical item, and the cost F of the first item is also known, Eq. 35-1 may be used to determine the total cost of any number n of items produced. Eq. 35-2 is especially useful for linear regression studies and the determination of errors of prediction.

Due to inflation or rising costs for many military items, it is often necessary to modify Eq. 35-1 to account for changing dollar values. This is done in terms of a "shift factor" S that represents the value of the dollar for any given year to that for a reference or "standard" year. Thus Eq. 35-1, modified to account for inflation, would become

$$C_T = Fn^{1+b}S. \tag{35-3}$$

Ref. 9 covers a very good and informative example of the top-down or parametric approach for costing artillery type weapons, or cannons. The example is in terms of the "cumulative average cost" for various calibers of cannon as a function of the gun weight only. The cumulative average cost is defined as the total cost of producing some number n of cannons divided by that particular number of cannons. In the example, it was found that an acceptable estimate of the cumulative average cost could be found in terms of only one parameter, i.e., the gun weight. The initial data studied by the cost analyst are given in Table 35-1, which is Fig. B-6 of Ref. 9. Fig. B-5 of Ref. 9, reproduced here as Fig. 35-4, indicates the original cost information for different calibers of cannon plotted against gun weight in pounds from the appropriate columns of Table 35-1. The cost analyst has fitted a curve of the form

$$\bar{C} = AW^d \tag{35-4}$$

where

$\bar{C}$ = average cost per cannon for the first 1000 cannons
A = constant to be determined
d = exponent to be determined
W = gun weight, lb.

To illustrate the cost estimation relation (CER) approach for determining predicted cost of any weight of cannon, one may easily apply linear least squares by working with logarithms for both sides of Eq. 35-4. Thus this transformation gives

$$\ln\bar{C} = \ln A + d\ln W \tag{35-5}$$

or

$$y = a + dx \tag{35-6}$$

TABLE 35-1. MANUFACTURING CANNON DATA

PRE/ POST AUTO FRETTAGE	CANNON DESCRIPTION & CALIBER		END ITEM APPLICATION	LENGTH, in.	GUN WEIGHT a, lb	TUBE WEIGHT, lb	APPLIED MOMENTUM, lb-s	LENGTH TO WEIGHT RATIO	AVE. COST C PER CANNON BASED ON FIRST 1000 CANNONS, FY 72$	MANUFACTURE
PRE	M18A1	— 57mm	Recoilless	61	40	40	N/A	1.53	686	FIRESTONE
PRE	T83E7	— 75mm	M51 Towed AA	177	1590	1030	N/A	11	7951	WHELAND
PRE	M32	— 76mm	M41/M41A1	187	1352	952	N/A	.14	6442	AM TYPE FOUND
PRE	M41	— 90mm,	M48/M48A1	193	2370	1582	N/A	08	9449	WHELAND
PRE	M41	— 90mm,	M48/M48A1	193	2370	1582	N/A	08	6465	FIRESTONE
PRE	M36	— 90mm,	M47 Tank	20[illegible]	2650	1750	N/A	08	11829	FIRESTONE
PRE	M27	— 105mm,	Recoilless	134	365	365	N/A	37	2648	OLIVER CORP
POST	M68	— 105mm,	M60/M60A1	210	2485	1660	N/A	08	8639	WATERVLIET
POST	M2A2	— 105mm,	M101 LTH	102	1064	706	1950	09	3800	WATERVLIET
PRE	M58	— 120mm	M103 Tank	298	6280	4600	1950	04	25839	WATERVLIET
PRE	M45	— 155mm,	M44 SPH	157	2970	2140	12775	05	8581	MINN MOLINE
PRE	M126A1	— 155mm,	M109 SPH	152	2840	N/A	7175	05	18000	WATERVLIET
POST	M113A1	— 175mm	M107 SPH	426	13800	12050	22500	.03	27288	WATERVLIET
POST	M2A2	— 203mm	M110 SPH	215	10350	4777	16800	02	20545	WATERVLIET

Ave C = $11297

SOURCES

a. End-Item Production Report-March 1956
b. CER for Manufacturing Hardware Costs of Gun/Howitzers. ARMCOM
c. TM 9-500 Hardware

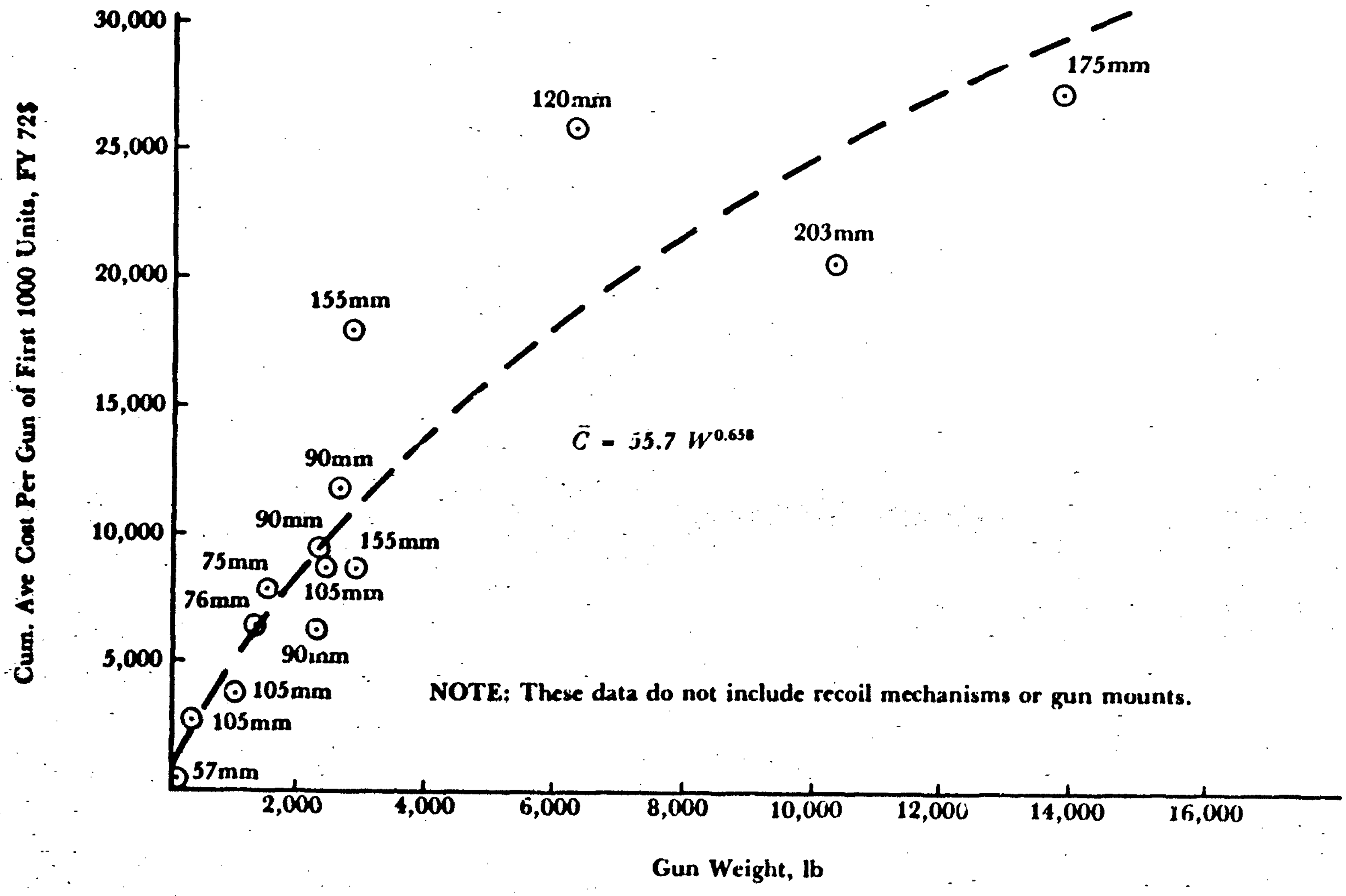

Figure 35-4. Cannon Manufacturing Cost vs Gun Weight

where

$$y = \ln \bar{C} \tag{35-7}$$

$$a = \ln A \tag{35-8}$$

$$x = \ln W. \tag{35-9}$$

The calculations to fit the line of Eq. 35-6 are indicated on Table 35-2; the notation is that of Ref. 13. Thus, the line is fitted using logarithms of the costs in 1972 dollars and the weights of cannon in pounds. The line fitted is

$$\ln \bar{C} = 4.02 + 0.658 \ln W \tag{35-10}$$

which may be returned to the original scale by taking antilogs, i.e.,

$$\bar{C} = e^{4.02}\, W^{0.658}$$

or

$$\bar{C} = 55.7\, W^{0.658}. \tag{35-11}$$

Eq. 35-11 then gives a CER from which the average cost per cannon in 1972 dollars may be estimated from any cannon weight W in pounds. Suppose, for example, one were interested in the cost of a new

TABLE 35-2
CALCULATION OF CER

$\bar{C}$, Dollars	W, lb	$y = \ln \bar{C}$	$x = \ln W$
686	40	6.53	3.69
7951	1590	8.98	7.37
6442	1352	8.77	7.21
9449	2370	9.15	7.77
6465	2370	8.77	7.77
11829	2650	9.38	7.88
2648	365	7.88	5.90
8639	2485	9.06	7.82
3800	1064	8.24	6.97
25839	6280	10.16	8.75
8581	2970	9.06	8.00
18000	2840	9.80	7.95
27288	13800	10.21	9.53
20545	10350	9.93	9.24

$n = 14$
$\Sigma x = 105.85,\ \Sigma y = 125.92$
$\bar{x} = 7.56,\ \bar{y} = 8.99$
$\Sigma x^2 = 827.26,\ \Sigma y^2 = 1145.09$
$\Sigma xy = 969.78$

CER: $\bar{C} = AW^{d}$

Fitted equation: $y = a + dx$
with $y = \ln \bar{C}$
$a = \ln A$
$x = \ln W$

$A_{xx} = n\Sigma x^2 - (\Sigma x)^2 = 377.42$
$A_{yy} = n\Sigma y^2 - (\Sigma y)^2 = 175.38$
$A_{xy} = n\Sigma xy - (\Sigma x)(\Sigma y) = 248.29$
Slope $d = A_{xy}/A_{xx} = 248.29/377.42 = 0.658$
Intercept $a = \ln A = \bar{y} - d\bar{x} = 4.02$
Variance of residuals $= (A_{yy} - A_{xy}^2/A_{xx})/[n(n-2)] = 0.0718$
Std error of residuals $= \hat{\sigma}_{y_x} = \sqrt{0.0668} = 0.268$
Transformation back to original scale gives: $\bar{C} = 55.7\, W^{0.658}$ and $\hat{\sigma}_{\bar{C}_W} \approx \$3028$

or proposed cannon that weighed 10,000 lb. Then, we calculate the average cost per cannon for the first 1000 such cannons to be

$$\bar{C} = 55.7(10000)^{0.658} = \$23,870$$

from the fitted CER.

The standard deviation of residuals about the line on the logarithmic scale (Eq. 35-6) is

$$\hat{\sigma}_{y_x} = 0.268$$

which converts to about

$$\hat{\sigma}_{\bar{C}_W} \approx (11297)(0.268) = \$3028$$

for the original cost in dollars and the cannon weight scale. Hence the standard error of prediction from the fitted curve at the mean value $\bar{x} = 7.56$ is $0.268/\sqrt{14} = 0.07$, which means that on the original scale at $W = e^{\bar{x}} = e^{7.56} = 1920$ lb the standard error of prediction would be about

$$(11297)(0.268)/\sqrt{14} = \$809.$$

Thus at $\bar{x} = 7.56$ or $W = 1920$ lb, the standard error of residuals or prediction from the fitted curve would be at the minimum value of

$$\hat{\sigma}_{\bar{C}_W} = \$809.$$

However, the standard error of prediction for a weight of cannon equal to 10,000 lb substituted into the fitted curve (Eq. 35-4) would amount to

$$\hat{\sigma}_{\bar{C}_W} = (\text{Ave } \bar{C})(\hat{\sigma}_{y_x})\sqrt{(1/n) + n(7.56 - \ln 10000)^2/A_{xx}} \tag{35-12}$$

$$= (11297)(0.268)\sqrt{(1/14) + 14(7.56 - \ln 10000)^2/377.42)}$$

$$= \$1257.$$

(See any standard statistical textbook on regression, or see Ref. 13, for example). Thus the \$1257 (and such similarly computed values) represent a rather significant increase in the error of prediction at cannon weights departing from the mean value of about 1920 lb.

The predicted average cost for 1000 cannons weighing 10,000 lb would be found from Eq. 35-11 and is

$$\bar{C} = \$23,870$$

so that 95% confidence bounds on the true cost at this weight would be about

$$\bar{C} \pm 1.96(1257) \text{ or}$$

$$\$21{,}406 \text{ to } \$26{,}334$$

or a spread of \$4928.

We note from Eq. 35-1 that the average cost of the first n items produced is C_T divided by n or

$$C_T/n = Fn^b \tag{35-13}$$

where as before F is the cost of the first item produced and b the logarithm of the learning curve slope. Hence from the data just calculated we may estimate F and then determine the general cost equation for any number of cannons produced. In fact, by definition, the exponent b is known to be

$$b = \ln 0.95/\ln 2 \tag{35-14}$$

$$= -0.0740$$

for a 95% learning rate. Therefore, suppose we are interested in a cannon weight of, say, 2495 lb. Then the average cost per cannon of the first 1000 cannons from Eq. 35-11 would be

$$\bar{C}_{1000} = AW^a = 55.7(2495)^{0.658} = \$9575$$

so that from Eq. 35-1 we may easily find the cost of the first unit, or

$$F = \frac{(C_T/n)}{n^b} \tag{35-15}$$

which for the initial 1000 cannons produced of weight 2495 lb would give

$$F = \bar{C}_{1000}/(1000)^b = \$9575/1000^{-0.074} = \$15{,}964.$$

Finally then, the total cost for any number n of cannons at any weight W is given by the equation

$$C_T = Fn^{1+b} = 15{,}964\, n^{0.926} \tag{35-16}$$

We therefore have a CER also for calculating the total cost of any number of cannons of any given weight and have demonstrated the extreme usefulness of the top-down or cost estimating selection approach.

We emphasize that so far in this series of computations we have estimated only one of the investment phase costs and that all of the other costs for investment or production on Fig. 35-2 must also be estimated and added together. Nevertheless, this example illustrates the value of regression techniques in cost analysis problems, particularly those involving the top-down approach.

An illustrative example for the bottoms-up or engineering type approach to cost estimation during the investment phase is given with all the necessary detail in Appendix C of Ref. 9. In this example, some 50 cost elements are listed for which costs have to be estimated and cost data sheets may have to be prepared. This should give some idea of the necessary detail and the many numerous calculations which have to be carried out for the bottoms-up approach.

Chapter 6 of Ref. 9 lists many general cost estimating equations which may be used to determine investment costs generally. As an isolated example, the cost of transportation may be estimated from the relation

$$\text{Transportation } 2.10 = \left(\begin{array}{c} \text{1st + 2nd Des-} \\ \text{tination Trans-} \\ \text{portation Cost} \\ \text{Per Unit} \end{array} + \begin{array}{c} \text{1st + 2nd Des-} \\ \text{tination Trans-} \\ \text{portation Cost} \\ \text{For Spares Per} \\ \text{Unit} \end{array} \right) \times \left(\begin{array}{c} \text{Total} \\ \text{Quantity} \\ \text{To Be} \\ \text{Manufactured} \end{array} \right) \qquad (35\text{-}17)$$

35-3.3 OPERATING AND SUPPORT COST GUIDE

The guide which provides a framework for the presentation, documentation, and reporting of cost estimates of the O&S phase of a materiel system life cycle is Department of the Army Pamphlet No. 11-4 (Ref. 10). The major objectives of this particular guide are:

"a. To achieve consistent preparation and documentation of Army materiel system Operating and Support phase cost estimates. This objective is accomplished through establishing and maintaining uniform cost structures and formats employing standardized cost elements and definitions.

"b. To improve the perception of Operating and Support cost estimating. This objective is accomplished by illustrating, in a highly simplified manner, the types of generic expressions employed to estimate Operating and Support phase costs.".

It can be expected that the operating and support costs incurred during the useful life of a weapon system may exceed the production cost of the system. Therefore, the high O&S costs warrant a very comprehensive review during the acquisition process of the weapon system. Some specific reasons why O&S cost estimates are needed are given in Ref. 10 and are:

"a. To permit personnel savings. Personnel costs, not only of the crew, but also of the maintenance and the indirect personnel, make up a major share of O&S costs. Only if these costs are made visible can Army planners realize the opportunity for reducing these costs, for example, by considering other ways to integrate the hardware into the force units.

"b. Under the current design-to-cost concept, potential cost performance trade-offs and engineering changes must be evaluated in terms of their impact upon the overall cost of ownership of the system, and appropriate weight should be given to this factor during source selection evaluation. High acquisition costs are acceptable provided that the additional investment will be amortized in a reasonable period of time through lower operating costs. By preparing full O&S cost estimates the proper trade-offs can be made.

"c. To achieve better balance between equipment purchase and repair. Before making its annual budget request the Army must determine the best balance between the purchase of new modern equipment and the repair of its existing equipment. This involves a trade-off between new capability and present readiness. Visibility of O&S costs is required in such determinations.

"d. To perform better comparisons between materiel systems. Credible estimates of the Operating and Support cost impacts of new systems will permit the Army to discriminate better between competing systems. In a scenario in which two or more competing systems have comparable performance and production costs, it is conceivable that the systems could have significantly different operating and support costs.

"e. To provide management visibility of critical resource requirements.".

One should note that annual operating costs are the recurring outlays that are required to maintain and operate a system after it becomes a part of the military force. The estimates of weapon system operating costs are sometimes quite accurately predictable. This is perhaps especially true for systems that are not too complex, such as trucks or even armored personnel carriers. For such systems, the operating costs will likely be dominated by the costs of the operating crews and the cost of fuel and maintenance. Moreover, maintenance costs usually may be treated on a cost per hour basis or a cost per unit basis based on similar predecessor vehicles, for example.

Returning to Fig. 35-1, we note a very significant omission is time in years on the abscissa. Thus one needs to know the "useful life" of a weapon system to determine O&S costs, and hence the life cycle cost. In fact, perhaps one of the most difficult facets of estimating the life cycle cost is the determination of the useful life of a weapon system. Indeed, by actual observation, the useful life of materiel items depends on type of weapon, design, usage, environment, etc., so that useful life itself may be expected to vary randomly. Some attempts are usually made by the Army to specify a standard useful life for a selected class of equipment. For example, in 1968, the accepted life of US Army helicopters was 9 yr, whereas the accepted useful life is now some 15 yr. Then again, a recent study of the Utility Tactical Transportation System (UTTAS) indicates that a value of 20 yr should be considered acceptable. Useful life is an important and critical factor in determining O&S costs, or even in planning production.

As an aid to the cost analyst, the general cost estimating equations for the operation and support phase of the system life cycle are given in Chapter 6 of Ref. 10. In fact, there is a cost estimation equation for each of the elements listed on Fig. 35-2 for the operating and support phase cycle. As a simple example, the cost estimation equation for petroleum, oils, and lubricants (POL) is listed in Fig. 6-6 of Ref. 10 as:

$$\begin{array}{c}\text{POL}\\ \text{Cost}\end{array} = \begin{pmatrix}\text{Total Quantity}\\ \text{Operational}\\ \text{Equipment}\end{pmatrix} \times \begin{pmatrix}\text{Annual Activity}\\ \text{Rate Per Equip-}\\ \text{ment (Miles, e.g.)}\end{pmatrix} \times \begin{pmatrix}\text{POL Cost Per}\\ \text{Mile, e.g.}\end{pmatrix} \times \begin{pmatrix}\text{Number of}\\ \text{Operating}\\ \text{Years}\end{pmatrix} \quad (35\text{-}18)$$

Appendix B of Ref. 10 documents an example of an O&S cost element. It is for the crew pay and allowances. The cost model itself is very simple, and a cost data sheet (CDS) is completed for this element. The cost model simply states that the total estimated cost for crew pay and allowances may be found by multiplying together the total quantity of operational equipment, the number of crewmen per item of equipment, the sum of average annual base pay and theater cost and flight pay per crewman, the number of operating years, and the constant dollar shift factor for military pay and allowances (see Fig. B-3 of Ref. 10).

Appendix C covers a very good illustrative example of the operating and support costs for typical new transport helicopters and is thoroughly documented.

Thus as with the general problem of accurate cost estimation, one needs to identify all significant elements throughout the life cycle of a military system and properly plan for all of the detailed costs, summing them up to the total for a given organization, etc.

There may be quite an advantage gained by trying to establish some general top-down cost estimation procedures for the operating and support phase also. For example, there may exist a useful relation between the number of support personnel and the number of actual operating personnel for the equipment. Therefore, one might be able to establish a regression equation of the kind:

$$S = 1500 + 0.32(Op) \tag{35-19}$$

where

S = number of support personnel in thousands
Op = number of actual operating personnel in thousands.

Thus it is easily seen that with such relationships cost could be more readily estimated with sufficient accuracy

35-3.4 STANDARDS FOR PRESENTATION OF LCCE

Standards for the presentation and documentation of Life Cycle Cost Estimates of Army materiel systems are given in Ref. 14. This guide has been prepared primarily for those who prepare cost estimates and those who review cost estimates prepared by others. Ref. 14 has been prepared primarily to indicate the need for standardized formats, while at the same time it attempts to recognize that substantial reliance must be placed on the freedom of expression inherent in the professionalism of individual cost analysts throughout the Army even though the two aims may appear to be somewhat conflicting. Therefore, the guide really aims to address the minimum standards that are required. The guide takes into cognizance and is, in fact, based on the wide-spread use of Refs. 8, 9, and 10.

There are three broad classes of standard life cycle presentations that are needed, namely: (1) static cost presentation which displays cost estimates on a nontime-phased basis and is the most frequently encountered in cost analyses, (2) time-phased cost presentation which reflects static costs that have been displayed over time usually by fiscal year in relation to the program schedule, and (3) cost sensitivity presentation which portrays the sensitivity of a point estimate to changes in the cost driving variables. There is also a need in the cost analysis community for standardization of the required degree of cost calculation precision and accuracy; accordingly Ref. 14 covers significant figures and the rounding of numbers. Appendix B of Ref. 14 suggests a life cycle cost structure that might well be used, and a good summary of the needs are outlined in Fig. 35-5, which is Fig. B-2 of Ref. 14.

35-4 ADDITIONAL FACTORS FOR LCCE PROCEDURES

So far in this chapter we have indicated some of the more ordinary or standard procedures associated with the problem of life cycle cost estimation for weapon or materiel systems. However, there are some rather significant additional factors that must be taken into account in the cost analysis process. The general aim is for an accurate cost estimation procedure. In Chapter 34 we discussed the matter of sunk costs and showed that they may not apply to the system under study, especially if they have already been expended and hence are not proratable to a particular weapon or system of current interest. However, such detailed considerations may become of considerable importance in charging costs to any current system undergoing cost analysis.

There are three additional significant factors affecting cost analysis which are worthy of mention, namely: (1) the matter of useful life versus combat life, (2) residual or scrap value of a materiel item,

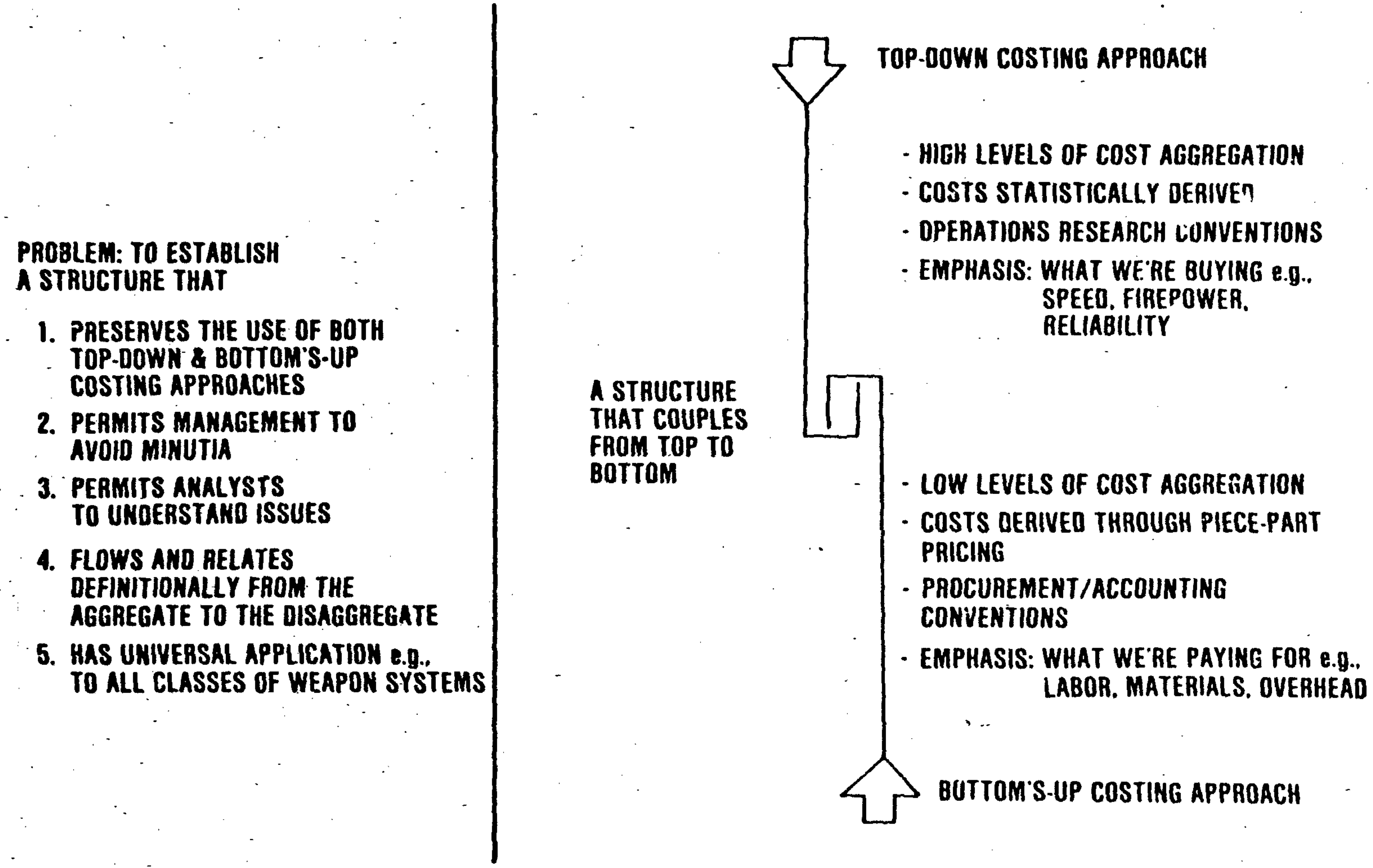

Figure 35-5. A Basic Need in Life Cycle Cost Analysis (Ref. 14)

and (3) discounting future expected costs. We now consider these factors, but treat (1) and (2) together.

35-4.1 RESIDUAL OR SCRAP VALUE, USEFUL LIFE, AND COMBAT LIFE

As everyone knows, most commercial items have a scrap value or a trade-in value toward the purchase of a new item. Moreover, it can be seen that when one considers the residual or scrap value of an item, that particular point in time may also determine the useful life. Therefore, it can be expected that the concepts of residual value and useful life are closely related. Normally, it has been US Army practice to make product improvements on weapons or materiel systems and to overhaul or rebuild many items from time to time in which case the estimation of useful life after modification would become of importance. On the other hand, at least for some Army materiel, it has been customary to consider that the residual value of an item is zero, or in other words simply ignore the problem of residual value of a materiel system. Nevertheless, there is always the problem of timely replacement of items. Also a factor or concept not previously mentioned—and one which would affect all of these considerations—is that of the *expected combat life.* Clearly, the determination or estimation of combat life is a much more difficult problem than estimation of noncombat useful life since combat life depends on the number, types, and intensity of combat engagements over time. The reader may easily see that the expected combat life of materiel items or weapons may have a most dominant effect on system "useful" life. We cannot treat this difficult problem of estimation in any detail here even though it will have a very marked effect on cost determination. Nevertheless, we can point out that the various methods of analysis presented in this handbook might well be applied to the estimation of combat life for many military items, and hence this might give a useful approach toward estimation of the combat lives of some military systems.

35-4.2 DISCOUNTING OF COSTS

The amount of funds appropriated for weapon systems are not allocated all at one time, but rather the custom has been that of appropriating a fraction each year. This is true for R&D, investment or procurement, and for O&S costs as well—all phased in over time. Consequently, the matter of discounting to determine costs, which are comparable for alternative weapon system buys, has to be considered and should be developed aside from the concept of useful life, although useful life will determine the total discounted cost of any system. Some new weapon development may have a higher investment cost, and this may lead to expected lower operating costs for the same overall effectiveness as a predecessor military system which is replaced. Inherent in the estimation of discount values is the discount rate and the time period over the life cycle of a weapon system considered to be of importance. For example, as many as 25 or 30 yr may be required from the time of initial development of a weapon system to final phaseout of it. Discount rates themselves vary over time although it is also desirable for the Army to specify or set standard rates in many cases. It is understood that for the present a discount rate of 10% has been specified; however, discounting in general is not always put into practice in the Department of Defense (Ref. 11) for life cycle cost estimation problems. Nevertheless, the appropriate equation for discounting future costs to the present value is

$$P = \sum_{i=1}^{n} \frac{A_i}{(1 + r_i)^i} \qquad (35\text{-}20)$$

where

P = present value, dollars

n = period of the life cycle, yr

A_t = amount of expected cost in the tth year, dollars

r_t = (variable, if applicable) discount rate used for the tth year, expressed as a decimal.

Should a constant discount rate $r_t = r$ be considered valid over the period, Eq. 35-20 then becomes

$$P = \sum_{t=1}^{n} \frac{A_t}{(1 + r)^t} \tag{35-21}$$

We emphasize that Eqs. 35-20 and 35-21 are for constant dollars. Since inflation rates may be quite pertinent for any discounting analysis, discounted costs have to be accordingly adjusted. An excellent treatment of discounting, and as a matter of fact all of the subjects related to life cycle cost estimation, is that of Fisher (Ref. 15).

To illustrate the importance of discounting in cost analyses, Trainor (Ref. 16) has prepared a convincing example which we include in Table 35-3. Trainor (Ref. 16) develops a comparison of the military adaptation of a commercial helicopter with a helicopter specifically designed for military missions to illustrate the use and importance of discounting.

In his example Trainor uses the undiscounted cumulative costs (columns 3 and 7) for which it can be seen that after 25 yr (5 yr development, 5 yr production, and 15 yr of operation) the new design has cost \$3,551,000, and the military adaptation has cost \$3,802,000. The lower operating cost of the new design (\$160,000/yr vs \$190,000/yr for the military adaptation) has caused this design to break even in the 17th year, i.e., after 7 yr in operation. By the end of the 25th year, however, the new design is 7% less expensive and, assuming equal effectiveness, would be a clear winner. However, using discounting costs, a totally different picture emerges. The military adaptation is less expensive beginning in the third year (\$67,200 vs \$74,800) and never relinquishes its lead. By 17th year (the break-even year in undiscounted costs) the military adaptation is still 6% less expensive. By the 25th year, the gap is nearly closed. Hence, Trainor points out that the "prudent" manager might tend to choose the military adaptation in preference to the new design. Certainly he would be tempted if he is concerned that both designs might become obsolete in less than 15 yr of operation. He then indicates

"The illustration shows the power of discounting. Yet discounting is seldom practiced in life cycle costing. Two possible reasons are offered. First, many analysts and managers simply do not understand the concept of discounting, its importance in the selection of alternatives and the ease with which it can be applied. The second reason for the lack of use of discounting, especially in the past three-four years is the confusion between the use of inflation factors and discounting. The casual observer may question why one should inflate costs at the rate of 10 percent per annum only to turn around and discount these same costs by 10 percent. Actually, discounting and inflation are different concepts for different purposes. Discounting is appropriately used to distinguish between alternate investment opportunities while inflation factors are used to assure that programmed funds are adequate during periods of rising prices. In other words, discounting is used to select an alternative and inflation factors are used to be sure the selected alternative can be purchased. These two reasons may offer some rationale for the lack of acceptance of discounting. Whatever the reasons, the failure to discount seriously weakens the credibility of much of today's life cycle costing."

TABLE 35-3
THE USE OF DISCOUNTING TO COMPARE ALTERNATIVES
(DISCOUNTING AT 10%)

	NEW DESIGN, $1000's				MILITARY ADAPTATION, $1000's			
	UNDISCOUNTED		DISCOUNTED		UNDISCOUNTED		DISCOUNTED	
YR	COST/ YEAR	CUM. COST	COST/ YEAR	CUM. COST	COST/ YEAR	CUM. COST	COST/ YEAR	CUM. COST
				DEVELOPMENT				
1	10	10	9.1	9.1	10	10	9.1	9.1
2	25	35	20.6	29.7	25	35	20.6	29.7
3	60	95	45.1	74.8	50	85	37.5	67.2
4	75	170	51.2	126.0	40	125	27.3	94.5
5	26	196	16.1	142.1	15	140	9.3	103.8
				PRODUCTION				
6	62	258	35.0	177.1	50	190	28.2	132.0
7	220	478	112.9	290.0	200	390	102.6	234.6
8	300	778	139.8	429.8	220	610	102.5	337.1
9	250	1028	106.0	535.8	220	830	93.3	430.4
10	123	1151	47.4	583.2	122	952	47.0	477.0
				OPERATION				
11	160	1311	56.0	639.2	190	1142	66.7	544.1
12	160	1471	50.9	690.1	190	1332	60.4	604.5
13	160	1631	46.2	736.3	190	1522	55.0	659.5
14	160	1791	42.1	778.4	190	1712	50.0	709.5
15	160	1951	38.2	816.6	190	1902	45.4	754.9
16	160	2111	34.7	851.3	190	2092	41.2	796.1
17	160	2271	31.7	883.0	190	2282	37.6	833.7
18	160	2431	28.8	911.8	190	2472	34.2	867.9
19	160	2591	26.1	937.9	190	2662	31.1	899.0
20	160	2751	23.7	961.6	190	2852	28.3	927.3
21	160	2911	21.6	983.2	190	3042	25.6	952.9
22	160	3071	19.7	1002.9	190	3232	23.3	976.2
23	160	3231	17.9	1020.8	190	3422	21.2	997.4
24	160	3391	16.2	1037.0	190	3612	19.2	1016.6
25	160	3551	14.7	1051.7	190	3802	17.5	1034.1

35-5 SOME CONSIDERATIONS IN DEVELOPING COST RELATIONSHIPS

Some further points in cost analyses of which the weapon systems analyst should be aware are discussed now. Once the data have been collected, normalized, and evaluated, the next task is to develop an estimate for each category of cost. The estimating method best suited to a particular application may depend upon many factors, including, for example:

1. Position of the weapon system in the life cycle at the time the estimate is required
2. The availability of historical cost information
3. The level of detail of the cost information available
4. The level of detail available on system specifications (design, performance, and operational characteristics)
5. The time available to prepare the cost estimate.

Early in the concept formulation phase—when limited design information is available and considerable uncertainty exists regarding system specifications, development, and production requirements—only order of magnitude cost estimates may be possible. Parametric cost estimating and estimating by analogy are particularly applicable during this phase. While DOD guidelines may suggest that the use of statistical parametric methods is the preferred approach to the development of early cost estimates, their use may often be invalidated by an inadequate sample of historical cost data. If only one or two predecessor systems or items have been developed, estimating by analogy with the most comparable system may have to be used.

The parametric method involves the development of CER's between cost and one or more cost generating variables. This method has the advantage of objectivity and flexibility in that the general form of the influence on cost of the explanatory variables is determined; however some "correlations" may be "spurious" and should be guarded against. Specific numerical values can be derived through standard statistical regression techniques. Another advantage of CER's is that they provide some indication of the quality of the cost relationships—whether by coefficient of "determination", standard error of estimate, confidence, or prediction intervals—as valid predictors of cost. We have covered one such example in Table 35-2. Again, the primary limitations to the successful use of parametric methods are the quality and quantity of historical data reflecting the essential characteristics of the item or system to be costed, and the fitted "law".

The analogy estimating approach may involve the identification of a single predecessor item or system and adjusting the cost of that system for changes in price levels, production quantity and schedules, etc., to develop a baseline cost. This baseline cost is then adjusted to account for differences in physical, operational, and performance characteristics between the predecessor system and the system to be costed. Obviously, the validity of the cost prediction is a function of:

1. The ability to make a good analogy
2. The accuracy with which the essential differences between a predecessor system and a new system have been determined, both of which may involve considerable expertise and good judgment.

If a good analogy is available, the advantage of this over the parametric approach is the time saved in data search, adjustment, evaluation, and curve fitting. The disadvantages of estimating by analogy are that costs are related to a single explanatory variable and costs are assumed to be proportional to the magnitude of the explanatory variable, e.g., "a 25% increase in weight will result in a 25% increase in cost".

Both the analogy and parametric approaches can be used to estimate costs in all of the cost categories of Refs. 8, 9, and 10. Historically, estimating by analogy has perhaps predominated (until recently) due to the data availability problem. However, with the current emphasis on parametric estimation, the DOD and the Army are currently engaged in measures to correct these deficiencies by:

1. Development of uniform costing procedures both from within the DOD and from contractors engaged by the DOD
2. Development of CER libraries, the cost estimate control data centers within the DARCOM commodity commands and automated cost information systems where feasible.

When technical data packages and engineering drawings become available during the later stages of the development phase of the life cycle, the engineering estimate is often used to determine investment and recurring production cost. This is a time-consuming and expensive approach to cost estimating because numerous calculations are required in the buildup of costs from basic work tasks, materiel fabrication and assembly processes, item physical dimensions, etc. While this method has the potential for greater accuracy in cost estimates, the time consuming aspect suggests that this method be used primarily in those circumstances in which:

1. There is considerable uncertainty regarding estimates developed parametrically or by analogy

2. There is no useful historical counterpart for the item or system being costed.

Many of the cost relationships developed in the early stages of the weapon system life cycle, however, may be made by using cost ratios and standard cost factors. Examples are construction cost factors (cost/square foot), facilities maintenance cost as a percent of facilities investment costs, and standard labor rates for specific work processes. These may be available from current indexes and tables developed through historical observations of similar cost elements.

In the conceptual stage, Research, Development, Test, and Evaluation (RDTE) costs are of primary importance. Accurate estimates of investment and operating costs cannot, at that time, be developed because some of the basic cost parameters are not yet established. The total requirement or the initial tables of organization and equipment (TOE) allowances, plus additional allowances such as maintenance float and combat consumption, will vary as decisions are made on how many and which organizations are to be equipped with the materiel items. Operating costs will be a factor of total authorizations, and operational system maintenance and consumable demands. In this early phase, much of the R&D cost may be based on comparable research efforts for similar items of equipment and related components. As the detailed design specifications emerge in the development phase and prototype hardware is produced and tested, parametric or analogy estimates of costs are gradually replaced by actual costs—e.g., R&D costs will have been expended, and more accurate projections for production costs based on updated CER's and engineering estimates will be available. Operating costs may still be developed using cost factors, but these can be updated through experience with prototype testing.

Since cost relationships are based on experience with "similar" systems, there is always the element of uncertainty concerning whether past cost experience will hold in predictions about future costs. This is particularly applicable to advanced weapon systems which are normally characterized by predecessors of lesser technological sophistication. Therefore, each cost estimate should also reflect a judgment regarding the possible variability about the most likely costs based on the quantity or quality of information used to derive cost relationships. An assessment of uncertainty is essential to both a realistic prediction of component costs and an indication of potential areas for system cost growth.

The results of the uncertainty analysis also support trade-off analyses among weapon system alternatives during the concept formulation phase.

35-6 CONSTRUCTION OF TABULAR COST MODELS

Cost estimating relationships, cost factors, etc., may be considered to be the transformation devices for relating the cost of functional, resource, and program elements of the system to cost generating or explanatory variables. These parameters must be combined in a systematic, logical, and valid manner to provide some insight into the total resource impact of the weapon system decision to be made. The cost model serves the role of an "integrating" device which relates the cost categories, cost estimating relationships, cost factors, etc., to the cost summaries required for a particular study. The cost model is an analytical tool for examining the behavior of system costs relative to assumptions about the weapon system, e.g., number of items to be procured, number of years of operation, and utilization rates. In this context, therefore, the cost model represents an efficient summary of the essential system cost elements in a format which lends itself readily to manipulation and prediction.

An example of the cost elements and factors constituting a cost model for a surveillance aircraft system is shown in tabular form in Table 35-4 with the identification of the variables or symbols given

TABLE 35-4. MATHEMATICAL STATEMENTS FOR THE SURVEILLANCE AIRCRAFT SYSTEM COST MODEL

Cost Category and Element	Symbol
1.0 Research and development	= (1.1) + (1.2)
1.1 Aircraft	= RDAC
1.2 Surveillance	= RDS
2.0 Initial investment	= (2.1) + (2.2) + (2.3) + (2.4)
2.1 Personnel	= (2.1.1) + (2.1.2)
2.1.1 Training	= (2.1.1.1) + (2.1.1.2) + (2.1.1.3) + (2.1.1.4)
2.1.1.1 New pilots	= PPN X TPN
2.1.1.2 Transitional pilots	= PPT X TPT
2.1.1.3 Officer, other	= POO X TOO
2.1.1.4 Enlisted	= PE X TE
2.1.2 Travel	= (TVO X PO) + (TVE X PE)
2.2 Installations	= FACI
2.3 Equipment	= (2.3.1) + (2.3.2) + (2.3.3) + (2.3.4)
2.3.1 Aircraft	= ACT X CAC
2.3.2 Ground support	= SG X CSG
2.3.3 Other specified	$= \sum_{i=1}^{N} IA_i (1 + MF_i) C_i$
2.3.4 Organizational	= PM X COE
2.4 Initial stocks	= (2.4.1) + (2.4.2) + (2.4.3)
2.4.1 Aircraft spares	= SP X ACT X CAC
2.4.2 Aircraft POL	= SM/12.0 X POL X FH X ACI
2.4.3 Other specified	$= \sum_{i=1}^{N} IA_i \times CC_i \times LOG \times C_i$
3.0 Annual operating	= (3.1) + (3.2) + (3.3) + (3.4) + (3.5)
3.1 Personnel	= (3.1.1) + (3.1.2) + (3.1.3)
3.1.1 Training	= (3.1.1.1) + (3.1.1.2) + (3.1.1.3) + (3.1.1.4)
3.1.1.1 Pilots	= YRS X TPN X PP X TORP
3.1.1.2 Officer, other	= YRS X TOO X POO X TORO
3.1.1.3 Enlisted	= YRS X TE X PE X TORE
3.1.2 Pay and allowances	= (3.1.2.1) + (3.1.2.2) + (3.1.2.3) + (3.1.2.4)
3.1.2.1 Officer, rated	= YRS X PR X PAR
3.1.2.2 Officer, nonrated	= YRS X PNR X PANR
3.1.2.3 Enlisted	= YRS X PE X PAE
3.1.2.4 Civilian	= YRS X PC X PAC
3.1.3 Travel	= YRS [TVO X (PP X TORP + POO X TORO) + TVE X PE X TORE]
3.2 Equipment	= (3.2.1) + (3.2.2)
3.2.1 Aircraft attrition	$= YRS \left(ACI \times FH \times \frac{RCAC}{100{,}000} \ \text{round integer} \right) CAC$
3.2.2 Other replacement/consumption	$= \sum_{i=1}^{N} YRS\ [(IA_i \times RC_i) C_i]$
3.3 Maintenance	= (3.3.1) + (3.3.2) + (3.3.3)
3.3.1 Facilities	= YRS X FACM
3.3.2 Aircraft	= YRS X ACI X FH X CMFH
3.3.3 Other equipment	= YRS X PM X CMU
3.4 POL	= YRS X ACI X FH X POL
3.5 Services and other	= YRS X PM X CO

in Table 35-5. The total system model is developed from subsystem models which in turn are composed of CER's, cost factors, and engineering estimates. The layers of submodels will depend upon the level of WBS detail and cost estimation relationships which can be developed from historical cost data. The total system cost model should identify costs by major cost categories and elements, as shown. Within each cost category, submodels and estimating relationships should include only those mathematical statements necessary to determine uniquely the link between cost and cost generating variables. A simplified example of mathematical expressions within a nonrecurring investment submodel is shown in Fig. 35-6.

Tables 35-4 and 35-5, and Fig. 35-6, illustrating submodels, should give the weapon systems analyst a good idea and some appreciation of the overall approach for life cycle cost estimation of a system.

35-7 DEVELOPING A BASELINE COST ESTIMATE

To provide a point of departure in estimating system costs, an initial "baseline" estimate should be developed early in the life cycle under a set of logical assumptions regarding the number of items to be procured, base case production schedules, utilization rates, and other pertinent factors. Since limited information is available to project accurately the ultimate system configuration at this point, variations in the values of these key variables can be explored in the cost model through sensitivity analyses. For example, the base case may be costed for N configuration end-items and the sensitivity of total system costs determined for variations in production schedules and rates (experience curves). Similarly, the number of years of operation of the system can be varied over 5 to 20 yr, for example, to determine the sensitivity of system costs to utilization rates and operating assumptions. Other critical assumptions about key variables can be tested in the cost model, using the base case as a point of departure. The purpose of such an exercise is to determine those key variables for which total system cost is most sensitive in the relative comparison among weapon system design alternatives. The ultimate objective is to attain the best overall balance between system design, operational effectiveness, and cost over the lifetime of the system.

As the system progresses through successive life cycle stages, the baseline model is retained and updated to reflect more definitive information available from development, design, and testing efforts, and, ultimately, actual production schedules developed during the initial production stage.

The importance of the baseline estimate cannot be overemphasized because all subsequent cost analysis effort should perhaps be an extension of it; i.e., as more definitive system design information becomes available to improve the life cycle cost estimate, any updates or revisions in costs should be made relative to the baseline estimate. This procedure is essential to the systematic recording of a cost trail which:

1. Facilitates development of consistent data bases for subsequent cost analysis studies and, more importantly,
2. Provides the framework for tracing system cost growth in support of a "variance analysis" which indicates the reasons for cost differences between current and previous life cycle cost estimates.

Life cycle costs must also be time-phased to provide an indication of comparative year-to-year expenditure patterns (total obligational authority) for system configuration alternatives. Funding implications on fiscal year budget levels in major Army appropriation accounts are important input to the weapon system decision. Time phasing should further require that future cost streams be discounted to determine the financial value (time/money preference) of various investment options. Year-to-year cost information is also an important input to formal budget requests, e.g., program change requests (PCR's), to provide funds to proceed into the next phase of the weapon system life cycle.

TABLE 35-5. VARIABLE DATA NAMES FOR THE SURVEILLANCE AIRCRAFT SYSTEM COST MODEL

Data Name	Identification
	Major Names
ACI	= Initial allowances – aircraft
ACT	= Initial allowances plus maintenance float-aircraft
CAC	= Average unit cost – aircraft
CMU	= Maintenance – cost per military personnel
CMFH	= Maintenance – cost per flying hour
CO	= Services and other
COE	= Cost – organizational equipment cost per military personnel
CSG	= Average unit cost – sensor
FACI	= Initial installations
FACM	= Maintenance of facilities
FH	= Peacetime annual flying hour per aircraft
LOG	= Number of months of logistic guidance
PAC	= Pay and allowances – civilian
PAE	= Pay and allowances – enlisted
PAN	= Pay and allowances – nonrated officers
PAR	= Pay and allowances – rated officers
PC	= Total number of civilian personnel
PE	= Total number of enlisted personnel
PM	= Total number of military personnel
RNR	= Number of nonrated officers
PO	= Total number of officers
POO	= Number of other officers
POL	= POL – cost per flying hour
PP	= Total number of pilots
PPN	= Number of new pilots
PPT	= Number of transitional pilots
PR	= Number of rated officers
RCAC	= Peacetime aircraft attrition per 100,000 flying hours
RDAC	= Research and development – aircraft
RDS	= Research and development – surveillance
SG	= Initial allowances plus maintenance float – sensor
SM	= Months of stockage
SP	= Initial spares factor – aircraft
TE	= Initial training cost – enlisted
TOO	= Initial training cost – other officers
TORE	= Turnover ratio – enlisted
TORO	= Turnover ratio – other officers
TORP	= Turnover ratio – pilots
TPN	= Initial training cost – new pilots
TPT	= Initial training cost – transitional pilots
TVE	= Travel cost – enlisted
TVO	= Travel cost – officers
	Other Specified Equipment Names
CC_i	= Combat consumption factor for the i^{th} equipment item
IA_i	= Initial allowance for the i^{th} equipment item
MF_i	= Maintenance float factor for the i^{th} equipment item
RC_i	= Replacement/consumption factor for the i^{th} equipment item

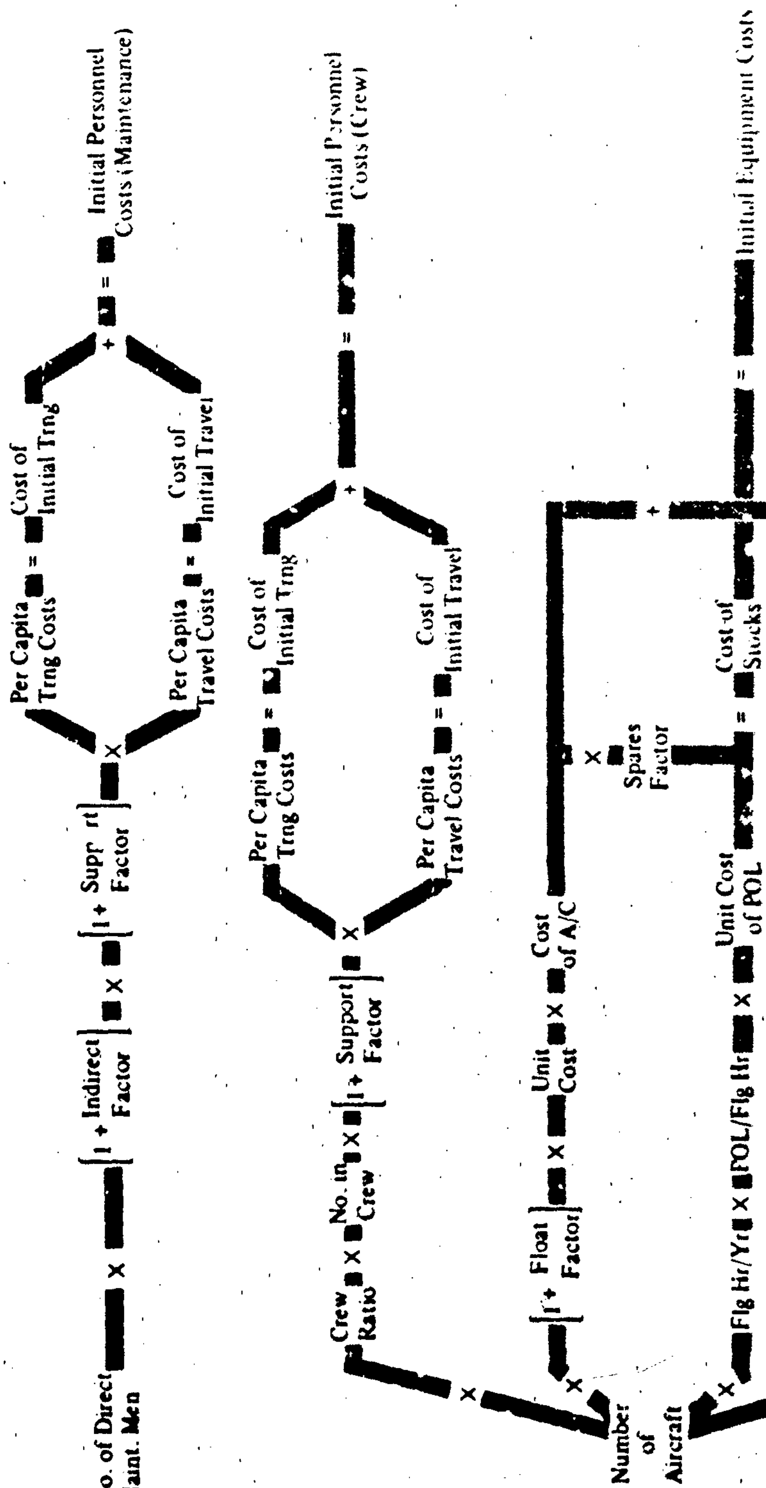

Figure 35-6. Example of Nonrecurring Investment Cost Submodels

In the initial system proposal, time-phasing considerations will be a planning estimate only based on specified target dates with appropriate lead and lag factors derived from experience. As the system proceeds through the development phase and more definitive information on a system specification becomes available—e.g., number and types of organizations to be equipped with the major end-item (basis of issue phases), and tentative production and delivery schedules established—then year-to-year estimates of funding requirements can be improved.

With such background details, we are now in a position to present an excellent example of the problem of LCCE; namely, that for the Utility Tactical Transport Aircraft System (UTTAS).

35-8 A LCCE EXAMPLE FOR UTTAS

A current and highly illustrative example of the LCCE process is that related to the UTTAS. This particular cost analysis study is covered and fully documented in Army Cost Analysis Paper, ACAP-8, of the Directorate of Cost Analysis, Office of the Comptroller of the Army—Refs. 17, 18, 19, 20, 21, and 22—October 1976. As would be expected, the study is quite voluminous and contains the six stated references and some 385 pages of detail and justification. Ref. 17 is the Executive Summary and Comparative Analysis; this broad description should give the weapon systems analyst a good appreciation of the probable extent and expected scope, size, and required detail of a suitable life cycle cost analysis.*

"The UTTAS is a new twin engine helicopter to replace the UH-1 in the air assault, air cavalry and medical evacuation missions. This aircraft has been designed as the Army's first true squad assault helicopter, since the UH-1 was originally designed to perform the medical evacuation mission, which only later was expanded to include the air assault role also. The result was an increase in mission gross weight with related performance reductions. The UTTAS will perform the missions of transporting troops and equipment into combat, resupplying the troops while in combat and performing associated functions of aeromedical evacuation, repositioning of reserves, and command control. UTTAS development program considers overall cost-effectiveness with particular emphasis on reliability, maintainability, and survivability in combat/field operations.

"The UTTAS was approved (DCP signed) for full scale development on 22 June 1971. The General Electric Company was selected (from three responding offerors) to develop engines for UTTAS and a contract to that effect awarded on 6 March 1972. On 5 January 1972, a Request for Proposal (RFP) was issued to the aircraft industry for proposals to develop the UTTAS airframe. Two contractors (Boeing Vertol and Sikorsky Aircraft Division) were selected from three responding manufacturers to proceed into full scale engineering development. Airframe development contracts were awarded on 30 August 1972. The initial Army plans called for six flying prototypes, one ground test vehicle (GTV) and one static test article (STA) from each contractor. The House Appropriations Committee in the FY-73 Research and Development budget request indicated that the number of UTTAS prototypes from each contractor should be reduced to three flying prototypes, one ground test vehicle, and one static test article. The Army has therefore proceeded with a three flying prototype UTTAS development program. Following a competitive prototype fly-off program, a single contractor will be selected to enter into low rate initial production program.

"The October 1976 BCE is an update to a previous (15 May 72) study and has been prepared by the Office of the Project Manager, UTTAS, US Army Materiel Development and Readiness Command

* We are indebted to Col William Clough, formerly assigned to the Office of the Comptroller of the Army, for the material of this example and other suggestions on this chapter.

(USA DARCOM). While partial updates have been prepared to the May 72 BCE (R&D phase estimates were updated in July 1973 and May 1974 and Investment phase was updated in December 1974), the present BCE is the first comprehensive update of the entire UTTAS life cycle cost estimate. The principal difference between the current estimate and previous PMO estimates lies in the areas of methodology. The present estimate places greater emphasis on contractor cost data and engineering buildup approach than did the previous studies.

"On the IPCE side, the current study is the second estimate of the UTTAS program. The initial IPCE, prepared by a Joint DA/USA DARCOM cost team, is dated 16 December 1974." (Ref. 17)

The system addressed is a generic UTTAS of 8500 lb aircraft manufacturer's production report (*AMPR*) weight and 10,500 lb empty weight, powered by two GE-T-700 engines. Both the Baseline Cost Estimate and the Independent Parametric Cost Estimate follow identical program scenarios in which the competitive R&D phase produced three flying prototypes, with a single contractor starting with either an 85 aircraft low rate initial production (LRIP) option or a 200 aircraft LRIP option. Subsequent full-scale production plans call for buying out the remainder of the 1107 aircraft of the Army's UTTAS program. For the O&S phase, estimates are based on a fleet of 914 deployed aircraft, with the remainder going to training, and as float and attrition. Each aircraft is to be operated 324 flying h/yr for a 20-yr life span. Costing, was performed in accordance with AR 11-18 and DA Pamphlets 11-2 through 11-5, i.e., Refs. 8, 9, 10, and 14.

For our purposes here, we will give illustrative examples for isolated, but perhaps typical, costing of elements for each of the R&D, Investment, and O&S phases—the complete study is covered by Refs. 17, 18, 19, 20, 21, and 22.

35-8.1 EXAMPLES OF R&D PHASE ELEMENT COSTS

In the R&D phase of costing approaches for UTTAS, the parametric approach or top-down method proved to be of considerable advantage, and CER's were developed for many of the elements.

A CER for airframe development (Element 1.01,1) costs was fitted to previous data for the OH-6A, UH-1A, SH-3A, AH-56A, CH-46A, CH-47A, and CH-53A airframe costs (see Fig. 1.01, 1-1 of Ref. 19). The cost for airframe development, or the CER, is given by

$$C = \frac{AWGT}{0.0080665(AWGT) + 48.819}, \text{ FY 74\$} \tag{35-22}$$

where

C = cost of airframe, FY 74\$
$AWGT$ = *AMPR* weight of airframe, lb.

Engine R&D costs were based on a CER which expresses cost as being dependent on only the shaft horsepower X_1 and the dry weight X_2. Thus the engine cost is estimated from the equation

$$C = 41.3 + 0.208X_1^{0.802} - 5.4X_2^{0.412}, \text{ FY 72\$} \tag{35-23}$$

where

C = cost of engine, FY 72\$
X_1 = shaft horsepower, HP
X_2 = dry weight of engine, lb.

The CER for the prototypes airframe first unit cost may be estimated from a regression relation, determined from data on the HO-4, HO-6, US-1A, AH-56A, CH-47A, CH-53A, and SH-3A airframes. The first prototype airframe cost equation fitted is given by

$$C = 28.227 Y_1^{0.613} + Y_2, \text{ thousands FY 72\$} \tag{35-24}$$

where

C = cost, thousands FY 72$
Y_1 = prototype *AMPR* weight, lb
Y_2 = adjustment to account for use of composite materials in prototypes.

35-8.2 EXAMPLES OF UTTAS INVESTMENT PHASE COST METHODOLOGY

Ref. 20 covers costing methodology for the Investment phase of the UTTAS program. The Investment Cost Matrix includes such elements as

1. Nonrecurring Investment (2.01)
2. Production (2.02)
3. Engineering Changes (2.03)
4. System Test and Evaluation (2.04)
5. Data (2.05)
6. System/Project Management (2.06)
7. Operational/Site Activation (2.07)
8. Training (2.08)
9. Initial Spares and Repair Parts (2.09)
10. Transportation (2.10)
11. Other (2.11)

for system structure items of airframe, engines, guidance and control, armament, fire control, ammunition payload, etc.

As an example, the estimated airframe nonrecurring cost for the investment phase is found from the CER given by

$$C = 5.5047 + 0.0001088(AWGT)P, \text{ millions FY 72\$} \tag{35-25}$$

where

C = nonr urring airframe tooling cost, millions FY 72$
$AWGT$ = *AM* ? weight of airframe, lb
P = peak monthly production rate of airframes.

For a 9180-lb airframe with composite material and a peak monthly airframe production rate of 15, the nonrecurring investment cost is readily calculated from Eq. 35-25 as

$$C = 5.5047 + 0.0001088(9180)(15) = \$20.5 \times 10^6 \text{ FY 72\$}.$$

The FY 77 inflated dollar amount is determined by using the multiplier 1.5268, or the cost is thus $1.5268 \times 20.5 \times 10^6 = 31.3 \times 10^6$ FY 77$.

Data for the airframe production cost basis are displayed graphically on Fig. 35-7 (same as Fig. 2.02, 1-1 of Ref. 20). This figure gives the cumulative average cost for the airframe recurring investment at the 1000th production unit in FY 72$ versus the average *AMPR* airframe weight in pounds. A linear fit was decided upon and yielded the equation

$$C = 63{,}210 + 68.646\,(AWGT),\ \text{FY 72\$} \qquad (35\text{-}26)$$

where

C = recurring investment cost for the airframe at 1000 production units, FY 72$

$AWGT$ = ave *AMPR* weight of airframe, lb.

The first unit cost for manufacturing of the airframe—including recurring engineering, sustaining tooling, and even quality control—is readily found by dividing the cost in Eq. 35-26 by 1000 raised to the power for the relative learning curve slope, i.e.,

$$F = [63{,}210 + 68.646(AWGT)]/1000^{(\ln 0.86/\ln 2)}] \qquad (35\text{-}27)$$

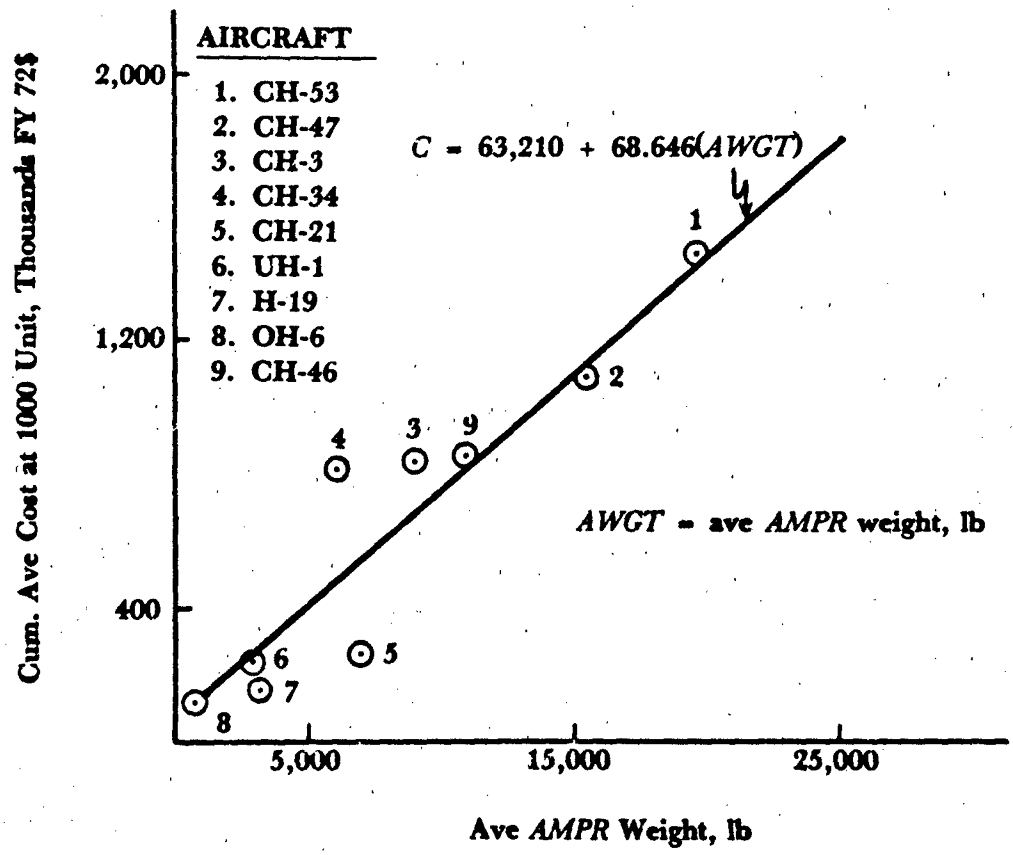

Figure 35-7. Airframe Recurring Investment Cost CER

where

F = first unit cost

0.86 = learning curve slope or rate of 86% assumed.

A point of some interest for any item or element costed would be the actual or observed cost versus that predicted or the development of a "CER" for such a relationship. The observed production cost study and "CER" developed for the larger size engines versus predicted cost are depicted on Fig. 35-8 (i.e., Fig. 2.02, 2-1 of Ref. 20). Note in this case that the fitted CER is linear in terms of the dry weight X_2 and the *ratio* of shaft horsepower X_1 to dry weight X_2 of the engine. The CER fitted is

$$C = 91522 + 209.78X_2 + 36943.2(X_1/X_2),\ \text{FY 72\$} \tag{35-28}$$

where

C = average cost per engine for first 100 engines, FY 72$

X_1 = shaft horsepower *SHP*, HP

X_2 = engine dry weight *DWGT*, lb.

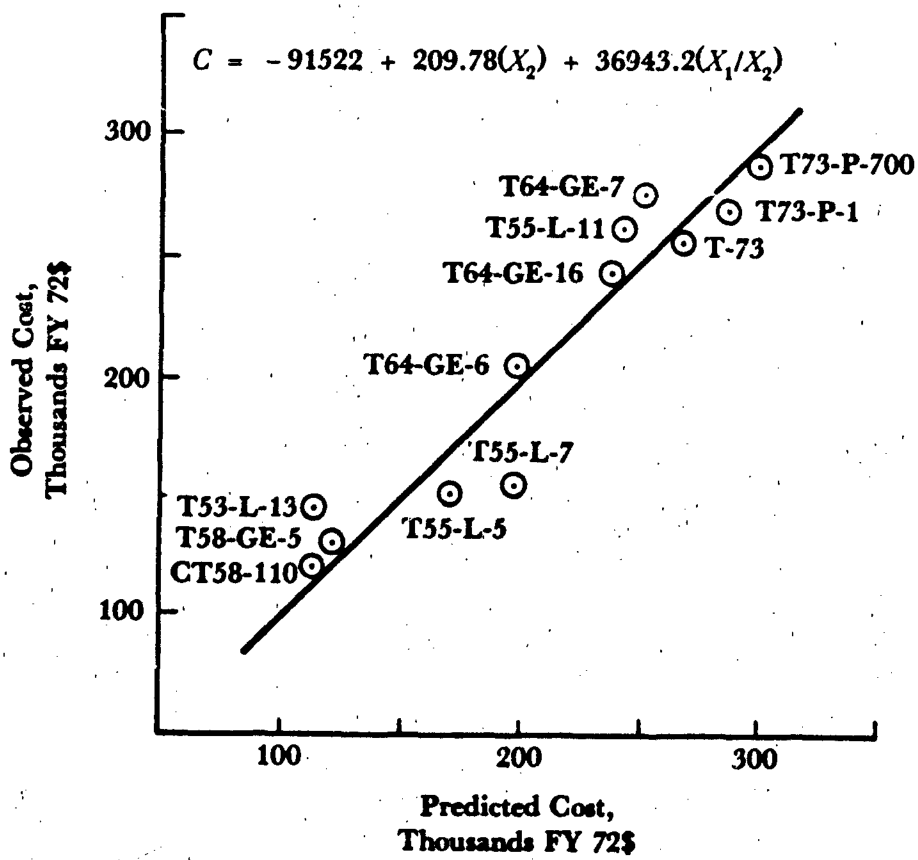

Figure 35-8. Large Engine Production Cost CER (Plot of Residuals Ave Cost for First 100 Engines)

For this CER, the mean cost is \$207,730, and the standard error of the estimate is \$22,649 or about 9%.

Also included in the production costs are the communications/navigation equipment, the armament system, and ammunition, the ground support equipment, subsystem integration features, engineering changes, test and evaluation programs, initial spares and repair parts, and other needs. Again we see that there are many items of cost which are to be identified, estimated as accurately as possible, and summed to obtain all of the investment costs of the UTTAS system.

35-8.3 ESTIMATING COSTS FOR THE OPERATING AND SUPPORT PHASE

Ref. 21 contains the details of the methodology used to derive the costs for the operating and support phase for the UTTAS system. As would be expected, one must determine the organizations that will be required to operate and support the UTTAS system. He must also consider maintenance, any materiel modifications, direct and indirect support operations, and other relevant costs. For the military personnel involved, one must determine total costs—i.e., costs based on crew pay and allowances, maintenance personnel pay and allowances, indirect pay and allowances, and charges for permanent change of station (PCS). The number of expected operating years is, of course, a very important parameter in all of the calculations. Furthermore, the various costs have to be estimated for each grade of military personnel involved, the different theaters of operation, the tables of organization and equipment, and summed over all such categories.

For the UTTAS system, the young weapon systems analyst may get some idea of the enormous amount of detail involved in costing the operating and support phase by scanning Ref. 21 of some 163 pages. Many of the costs are very direct calculations, building up, it might be said, from a bottoms-up type of approach. Therefore, there would seem to be plenty of room for some analytical studies to develop regression relations or other models which might give satisfactory tools for cost estimation in the operating and support phase. Of course, CER's may be used in the operating and support phase, as well as for the R&D and the investment phases. As an example for the UTTAS system, this was done for costing the replenishment spares for the airframe and engines. In fact, the operation and maintenance cost per flying hour in 1977 dollars may be estimated from the CER

$$C = -33.9 + 0.040(AEW)\,,\ \text{FY } 77\$ \qquad (35\text{-}29)$$

where

C = cost, FY 77\$

AEW = aircraft empty weight, lb.

35-8.4 COMMENT ON THE CER's AND THE ACCUMULATION OF ERRORS IN COSTING

Having covered some of the methodology for estimating R&D costs, investment costs, and O&S costs it is of some pertinence here to make some remarks about the accumulation of errors in estimating the overall or projected cost of a weapon system over the life cycle.

One may note in particular that most of the CER's are linear, or they are often converted to linear form by using logarithms. Of course, there is a very basic problem in identifying the more important parameters and just how they should fit together in any cost model. A useful feature of the regression type CER's is that standard errors of prediction may be calculated, as for example for the learning curve of par. 35-3.2. Hence when all such costs are added, one finds the standard error of prediction for the system or sum of all costs to be very large indeed. For example, even for a single item or element, the mean of the observed costs on Fig. 35-7 is about $638,970, and the standard error of the estimate is a whopping $201,000—nearly a third! Therefore, it can be expected that in many cases the standard errors of prediction may be intolerably large for the items or elements costed. This could invalidate the cost model. Furthermore, it can easily be seen that systems as complex as the UTTAS, or even much simpler systems, may have literally hundreds of items or elements for which to estimate costs. Moreover, irrespective of the use of CER's or engineering approaches to determine estimates, there will be errors in estimating the true or actual costs. This leads us to reflect just briefly on the total error of prediction for the overall costing of a system life cycle. In fact, the problem may be examined statistically as indicated in the paragraphs that follow.

Consider adding up the costs of $i = 1, 2, \ldots, n$ items or elements of a weapon system, where C_i is the estimated cost for the ith element but has a standard error of estimate equal to σ_i. Then clearly, the total of the estimated costs, or the overall system life cycle cost, is

$$C = \sum_{i=1}^{n} C_i \tag{35-30}$$

and the variance of the total estimated cost C is

$$\sigma_C^2 = \sum_{i=1}^{n} \sigma_i^2 \tag{35-31}$$

which mounts rather quickly with increasing n. Thus if the average variance per element of the system is taken to be σ^2, then

$$\sum_{i=1}^{n} \sigma_i^2 = n\sigma^2 \tag{35-32}$$

and we see that the standard error of the estimated system life cycle cost is

$$\sigma_C = \sqrt{n}\sigma. \tag{35-33}$$

As an example, suppose we have under consideration a weapon system, the life cycle costing of which requires the summing of 100 cost elements. Suppose further that we would like for the standard error of estimate for the system life cycle cost to not exceed 10%. Then from Eq. 35-33, it is easily seen that the "average" standard error of estimate for each element costed cannot exceed 1% of the system life cycle cost, which may be difficult indeed.

Finally, we see the need to develop and have available some kind of confidence statements for system life cycle costs. As a hypothetical example and to avoid classified cost data for UTTAS, let us consider LCCE data for "UTAH", a Utility Tactical Assault Helicopter. By use of the independent parametric cost estimate (IPCE) approach to place upper and lower bounds on estimated costs, it was found in this connection that the BCE's were within the ranges of limits determined by the IPCE's, as shown in Table 35-6.

The expected total cost of the UTAH system over the life cycle may be taken to be (in FY 77$)

$$581 + 2146 + 6423 = \$9,150 \times 10^6.$$

TABLE 35-6
SUMMARY OF LCCE OF UTAH
(MILLIONS FY 77$)

Phase	Lower IPCE Cost Estimate	Most Likely BCE Cost Estimate	Expected IPCE Cost	Most Likely IPCE Cost	Upper IPCE Cost Estimate
R&D	488	540	581	601	713
Investment	1768	2086	2146	2380	2986
O&S	4633	5143	6423	6642	7527

The standard error of this estimate could be determined with the aid of Eq. 35-31 if the variances of the individual element costs were known, but such information is not available. To get an approximate standard error, we might perhaps assume safely that the "lower" and "upper" IPCE cost estimates for the R&D, investment, and O&S phases are "in the tails" of the distributions, and consider for illustration here that they are at about the 0.025 and 0.975 points of a normal distribution. Then the standard errors of the three phases are:

$$\sigma_C(R\&D) \approx (713 - 488)/3.92 = 57.4$$

$$\sigma_C(Inv) \approx (2986 - 1768)/3.92 = 310.7$$

$$\sigma_C(O\&S) \approx (7527 - 4633)/3.92 = 738.3.$$

The standard error of the total cost is then estimated as

$$\sigma_C = \sqrt{(57.4)^2 + (310.7)^2 + (738.3)^2} = \$803.1 \times 10^6.$$

Or in other words, we might "state with 95% confidence" that the true life cycle cost of UTAH will be given by

$$\$9150 \times 10^6 \pm 1.96(\$803.1 \times 10^6)$$

or

$$\$7576 \times 10^6 \text{ to } \$10724 \times 10^6$$

indicating a difference of about $\$3,148 \times 10^6$, or a very wide "confidence" interval!

35-9 HIGH RELIABILITY AND SYSTEM COST

Finally, perhaps we should make a remark about system reliability since it certainly is one of the parameters which drives up the cost of weapon systems. As the saying goes, the higher the required reliability of the system, the higher and higher the total cost, and extreme reliability will mean very high costs. No doubt, the problem of guaranteeing high reliability is one of the most difficult engineering and quality assurance areas of current interest for many weapon systems; therefore, this deserves some special consideration in cost analysis studies of many weapons. An informative study of such a problem is covered in Ref. 23. This reference makes a study of industrial practices and procedures and makes some recommendations for improvements in the general area of guaranteeing reliability of electronic systems. Without doubt, much more emphasis will be placed on the problem since "reliability growth" of military systems is now rather widely analyzed and practiced. (See also Chapter 36, par. 36-5).

35-10 SUMMARY

We have introduced the problem of estimating weapon system life cycle costing and have indicated the very involved nature of the process. It is clear that life cycle costing of systems must fit within the framework of the required documentation, coordination, review, and budget approval procedures of the DOD and Army. In fact, the year-to-year financial impact of weapon system costs on Army appropriations must be estimated as accurately as possible to insure that adequate funds will be available to complete all of the overall weapon system programs. This clearly requires a thorough knowledge of the DOD and Army program planning and budget programs.

It appears very convenient to determine life cycle cost of systems by dividing the costing effort into three phases: (1) the R&D phase, (2) the investment or procurement phase, and (3) the operating and support phase. Inherent in the preparation of LCCE is the determination of the useful life of a system.

We have covered many of the current techniques for estimating costs over the life cycle of a weapon system, and we have illustrated the process by using an example involving UTTAS and a hypothetical system called UTAH, Utility Tactical Assault Helicopter.

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CHAPTER 36
SOME SPECIAL COST ESTIMATION TECHNIQUES

In view of its importance, the concept of the learning curve is discussed more fully, and the usual model for it is derived in an elementary manner. Because it is often necessary to provide rather rough estimates of costs for Army systems, the application of the Program Evaluation and Review Technique (PERT) is covered in sufficient detail for the weapon systems analyst. Since it is seen that design changes at various periods during late development and production will have an important impact on costs, the concept of altering the learning curve when such changes occur represents an area of some particular interest to the analyst. Reliability growth costs are also discussed.

36-0 LIST OF SYMBOLS

A = absolute minimum value for the beta distribution
A = constant to be determined (see Eq. 36-46)
a = coefficient in Eq. 36-14
a = estimate used in PERT (see par. 36-3)
a = estimate of A, the minimum
B = absolute maximum value for the beta distribution
B = constant which represents the slope of the fitted cost line in Eq. 36-46
b = $\ln L/\ln 2$ = exponent of the learning curve
b = estimate used in PERT (see par. 36-3)
b = estimate of B, the maximum
C = $C(R)$ = cost to achieve reliability R
$\bar{C}$ = average cost per item for n produced items
$C(T)$ = $N(T)/T$ = cumulative failure rate to time T
C_T = total cost of all n items
C_1 = cost of first item produced
C_2 = cost of second item produced
C_4 = cost of fourth item produced
C_i = cost of ith item produced
C_n = cost of nth or last item produced
$C_0(n)$ = cost of nth produced item before the first design change
$C_0(n)$ = C_n where there was no design change
$C_0(n - n_1)$ = new cost of items produced after $n = n_1 = K_0(n - n_1)^b$
$C_0(n - n_i)$ = new cost at the n_ith design change
c = cost variable for PERT
d = one-time tooling cost
$E(c)$ = expected value of cost c
$f(c)$ = probability density function of costs
$h(t)$ = failure rate, hazard rate, or intensity function
K_0 = cost of 1st produced item before first design change
K_0 = C_1 for no change in design
K_i = $K_0 \prod_{j=0}^{i} [(1 - \lambda_i) + \gamma_i]$ = constant representing the changed cost

k = $1/[(B-A)^{p+q+1}\beta(p+1, q+1)]$ = constant to make the area under the beta distribution curve equal to unity

k = 1, 2, ..., n = stages in system development

L = learning rate = 80%, 90%, etc.

M = true mean value, or the population mean

M_0 = true modal value, or the population mode

$M(T_o)$ = mean time between failures (MTBF) at $T = T_o$

$\hat{M}(T_n)$ = maximum likelihood (ML) estimate of MTBF at time T_n

m = PERT estimate of $M = (a + 4m_0 + b)/6$

m_0 = estimate of M_0, the mode

$N(T)$ = total number of failures to time T

n = item number (even for changes in design during production)

n_1 = item number at the first change in design

n_2 = item number at the second change in design

n_i = item number at the ith change in design

$n - n_1$ = number of items beyond the first design change where the old learning curve is corrected or changed to start the learning process over

p, q = shape parameters for the beta distribution

R = reliability level

$\hat{R}(T_n, t)$ = ML estimate of reliability at T_n for new random failure time t

r = last or rth design change

$r(T)$ = instantaneous failure rate

T = test time-to-fail for an item at some stage of development

T_i = cumulative test times to any failure for the ith stage of development

T_o = specified development test time

t = random failure time of item on test

t_m = mission time

t_m = unit or system specified mission time for service use after the last development test

u_α = lower α probability level of u (see Table 36-2)

$u_{1-\alpha}$ = upper α probability level of u (see Table 36-2)

v = PERT estimate of variance = $(b - a)^2/36$

x = natural logarithm of the number of the item

y = natural logarithm of the cost of an item

2^u = general geometrically numbered item for the learning curve (ratio of 2)

$\ln\lambda$ = ordinate intercept for $\ln T = 0$

$1 - \alpha$ = upper probability level (> 0.5)

α = lower probability level (< 0.5)

β = Weibull shape parameter

$\hat{\beta}$ = ML estimate of β

$\beta(p,q) = \dfrac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ = complete beta function

γ = slope of a fitted (logarithmic) line

γ_1 = fractional increase in cost based on the first design change as it affects new production at the $(n_1 + 1)$st item

δ = characteristic life = $\lambda^{-1/\beta}$
λ = Weibull scale parameter
λ_1 = fractional reduction in production cost due to first design change
λ_i, γ_i = similar fractional quantities for the ith design change
$\hat{\lambda}$ = ML estimate of λ
σ^2 = true variance
$\hat{\sigma}$ = estimate of standard deviation of σ

36-1 INTRODUCTION

In Chapter 35, we presented a rather detailed formulation and discussion of the problem of life cycle costing of Army weapon systems, and we indicated the usual methods or techniques of estimating costs. The prominent techniques for estimating costs consisted of the analogy approach, the bottoms-up or itemized engineering approach, and the top-down or usually statistical regression approach. In the process of estimating life cycle costs, we found it necessary to illustrate by actual example the use of the so-called learning curve during production. In view of its importance, we derive the law or equation for the learning curve in this chapter and introduce some special techniques the analyst might need in his costing problems. One of these is PERT which may often be found to be of some particular use when it becomes necessary to develop rough cost data.

Since it is very likely there will be some design changes in Army systems during late development, prototype production, and production in quantity, the analyst should be aware of the fact that learning curves will correspondingly change in a significant or even drastic manner. For this reason, the analyst should be aware of models which can alter properly the learning curve because of design changes. Finally, reliability growth may have to be costed.

36-2 THE LEARNING CURVE AND ITS CHARACTERISTICS

The original work and idea for the so-called learning curve theory has been credited to Leslie McDill, who was commanding officer of Cook Field, OH (now well known as Wright-Patterson Air Force Base). McDill's work on learning curves was performed about 1925. Later in 1936, T. P. Wright published a paper on learning curve theory in the *Journal of the Aeronautical Sciences*, the title of his paper being, "Factors Affecting the Cost of Airplanes". Wright (Ref. 1) showed among other things that as the number of production aircraft increased, the cumulative average unit cost to produce the aircraft decreased at a constant rate. This finding has been referred to as the "cumulative average theory". Wright also expressed costs in dollars per pound. While working for the Lockheed Corporation during World War II, J. R. Crawford advanced the proposition that as the number of aircraft produced continued to increase, the unit cost to produce an airplane would decrease at a constant rate. Crawford's propostion became known as the "unit theory". Naturally, as experience with the production of any item increases, efficiency improves and the cost of the items exiting the production line is bound to go down in some describable fashion. Thus, there must be some kind of law which should describe this trend, and the law developed has been labeled the "learning theory" or "learning curve" law.

The learning curve perhaps has a somewhat strange formulation, involving the number two in a very prominent way, for it indicates that production costs are lowered by the multiplication of the learning rate factor every time the number of manufactured articles or items is doubled. We illustrate the process and its formulation in Table 36-1 for a learning rate L of 90%.

TABLE 36-1. THE LEARNING CURVE PROCESS

ITEM NUMBER	EQUATION OF PRODUCTION COST OF ITEM	PRODUCTION COST OF ITEM
1	$C_1 = C_1$	\$1,000
2	$C_2 = C_{2^1} = LC_1$	900
4	$C_4 = C_{2^2} = L^2C_1$	810
8	$C_8 = C_{2^3} = L^3C_1$	729
16	$C_{16} = C_{2^4} = L^4C_1$	656.10
32	$C_{32} = C_{2^5} = L^5C_1$	590.49
...	...	...
2^u	$C_{2^u} = L^uC_1$	$(0.90)^u(1000)$

We let the cost of the first, second, etc., items produced be designated respectively by $C_1, C_2, C_3, \ldots, C_i, \ldots, C_n$, where C_i = cost of ith item produced, and C_n = cost of nth or last item produced. Thus, in the original formulation of the learning curve, the first item costs C_1, or \$1,000 as shown in the last column of Table 36-1. Then, when production is doubled to two articles, the cost of the second item produced is the learning rate $L = 0.90$ times the cost of the first produced article. Then, when production is doubled again, from 2 to 4 items, the cost of the 4th item is the learning rate L times the cost of the second article, or $(L)(L)C_1 = L^2C_1$, or for the example, we get $(0.90)^2$ (1000) = \$810. The process continues in this progression, so the cost of the 8th item is L^3C_1, or $(0.90)^3$ (1000) = \$729, for the example; the cost of the 16th item is L^4C_1, or \$656.10, etc. Note that the item numbers proceed in a geometric progression with the ratio of adjacent terms equal to two, and the costs proceed also in geometric progression with ratio equal to the learning rate L. Thus, for any general power of 2, say u, we have that the 2^uth item will cost L^uC_1, or $(0.90)^u(1000)$ dollars.

To find the costs for any (and all) numbered production item(s), i.e., 1, 2, 3, 4, 5, 6, 7, etc., we see that the cost law is geometric at the geometrically numbered articles, and we may therefore linearize the process by taking logarithms and fit a line passing through the end points, or for the first item and its cost, and the last or nth item and its cost. Letting y = the logarithms of the costs, and x = the logarithms of the item numbers, the fitted line or equation on the logarithmic scale is seen to be

$$\frac{y - \ln C_1}{x - \ln 1} = \frac{\ln(L^uC_1) - \ln C_1}{\ln 2^u - \ln 1} = \frac{u \ln L}{u \ln 2} \tag{36-1}$$

or

$$y = \ln C_1 + (\ln L/\ln 2)x. \tag{36-2}$$

However, since $x = \ln n = \ln 2^u$ for the nth or last produced article, and $y = \ln C_n$, the cost of that article, then from Eq. 36-2

$$\left.\begin{aligned} \ln C_n &= \ln C_1 + (\ln L/\ln 2)(\ln n) \\ \text{or} \quad C_n &= C_1(n^{\ln L/\ln 2}) \\ &= C_1 n^b \end{aligned}\right\} \tag{36-3}$$

where

$$b = \ln L/\ln 2. \tag{36-4}$$

Eq. 36-3 states that the cost of the nth produced item is equal to the cost C_1 of the first item multiplied by the number n of the item designated raised to the power equal to the logarithm of the learning rate L divided by the ln 2. Thus, using the geometric points of the learning curve, we have found an equation for the cost of any numbered item produced. (Eq. 36-3 is not a geometric series, however.)

Since the learning rate L is always less than unity, the exponent b is always negative. For example, if $L = 0.90$, then $b = \ln 0.90/\ln 2 = -0.15$.

The total cost C_T of all n items from the production line is also of much interest and is

$$C_T = \sum_{i=1}^{n} C_i \tag{36-5}$$

But since

$$C_i = C_1(i)^b \tag{36-6}$$

for all i as determined in Eq. 36-3, then

$$C_T = C_1 \sum_{i=1}^{n} i^b$$
$$\approx C_1[n^{b+1}/(b+1) + n^b/2 + bn^{b-1}/12 - \cdots]. \tag{36-7}$$

The series expansion (Eq. 36-7) may be determined from equations for summing series by the methods of the Calculus of Finite Differences. Thus, when the number of items n produced is sufficiently large, then the second, third, etc., terms of Eq. 36-7 nearly vanish, for b is negative. Then the total cost of n produced items becomes

$$C_T = C_1 n^{b+1}/(b+1) \tag{36-8}$$

giving an equation that is not often used. Instead, many investigators recommend use of the approximation

$$C_T \approx C_1 n^{b+1} \tag{36-9}$$

apparently assuming that b is suitably small when the learning rate is sufficiently high. Eq. 36-9 is easy to fit by least squares by taking logarithms of both sides, whereas the more exact Eq. 36-8 becomes a bit more awkward. Eq. 36-8 is well worth using, however, since b may be found by iteration, if necessary, for example. Eq. 36-9 is widely suggested for its simplicity, however, but gives a smaller b than the true one.

One notes that given appropriate cost data on the items produced then the exponent b may be determined by the method of least squares (usually working on the logarithmic scale) by the aid of Eq. 36-3, 36-8, or 36-9. Moreover, when the exponent b is found, then the learning rate L may be determined from Eq. 36-4

$$L = 2^b. \tag{36-10}$$

For example, if b is found by linear least squares (from logarithmic data) to be -0.3, then the value of L is

$$L = 2^{-0.3} = 0.81 \text{ or } 81\%.$$

The cumulative average cost per item $\bar{C}$ may be found by dividing the total cost for all items produced by the total number of items. That is, from Eq. 36-8, we see that

$$\bar{C} = C_T/n = C_1 n^b/(b+1) \tag{36-11}$$

or from Eq. 36-9

$$\bar{C} = C_T/n \approx C_1 n^b. \tag{36-12}$$

If the cumulative average cost and the unit cost are plotted together on a graph versus the number of produced units, then as perhaps expected, the unit cost curve will drop and continue below the cumulative average cost curve.

Andress (Ref. 2) gives an account of the learning curve as a production tool, including background information; some of the hazards that may be involved in using learning curve theory; and recommended applications such as in electronics, home appliances, residential home construction, shipbuilding, machine shop operations, product innovation, proportion of assembly time, and advanced planning. Thus, Andress seemed to think that the uses of learning curves should be widespread indeed. In our account here so far, we have applied the theory only to cost estimation, although it may as well be applied to man-hours to perform production operations, other measures of effort, and other areas as well, no doubt.

An example on fitting the learning curve given by Eq. 36-9 is fully illustrated in par. 35-3.2, based on data displayed in Table 35-1, and the computations in Table 35-2. Moreover, some of the more pertinent information on the original scale of costs is given after transformation of the least square parameters for the equation fitted on the logarithmic scale.

The previous account might be referred to as the "conventional" learning curve theory. Vardeman and Laney (Ref. 3), based on a study of flyaway costs for some Navy aircraft, have suggested some modifications of the conventional learning curve models to improve accuracy of cost predictions. The first change is to include one-time tooling costs, so that Eq. 36-9 is modified to become

$$C_T = C_1 n^{b+1} + d \tag{36-13}$$

where d is the one-time tooling cost.

Another adjustment attempts to take into account the combined effects of the introduction of engineering changes and the diminishing rate of learning, both of which require the log-linear curve to be adjusted. Vardeman and Laney (Ref. 3) point out that this modification may be of particular concern when one is trying to project future costs based on currently available data. The idea here is to make a continuous adjustment to the slope of the conventional learning curve, and they suggest fitting the total cost curve given by

$$C_T = C_1 n^{b+1+a\ln n} + d \tag{36-14}$$

where the new coefficient or parameter a depends on the type of aircraft.

The learning curve approach represents a particular method of estimating cost, labor, or effort applied to a project; namely, that to the repetitive operations of production. One might consider also a "rough" but somewhat more general approach to many cost estimation problems, and this is through the use of methodology for PERT which we next discuss.

36-3 COST ESTIMATION BY USE OF THE PERT TECHNIQUE

We have seen in Chapter 35 that even though some otherwise refined methods for estimation of costs were applied, the cost estimates themselves were often quite variable or uncertain—to say the least. Moreover, the total costs for the life cycle of a weapon system were found to be subject to rather wide variations or statistical uncertainty. For this reason, and for the sake of making quick and cheap estimates wherever possible, then occasionally and for certain applications at least, the use of approximate or rough techniques of estimation may be permissible. In this connection, and for certain phases of life cycle cost estimation, one might well consider the possible use of subjective cost estimates based on PERT. PERT—originally referred to as the Program Evaluation Research Technique—was developed for the Special Projects Office of the Navy Bureau of Ordnance in connection with the

POLARIS Fleet Ballistic Missile Program and is described by Malcolm, Rosebloom, Clark, and Fazar (Ref. 4). Their paper describes the development and application of the PERT technique for measuring and controlling development progress for the POLARIS Fleet Ballistic Missile Program. The PERT methodology the authors initiated was for the purpose of developing, testing, and implementing a systematic methodology for providing management with integrated and quantitative evaluation of progress to date and the outlook for accomplishing the objectives of the fleet ballistic missile program. It also involved studying the validity of established plans and schedules for accomplishing the program objectives and determining the effect of any changes proposed in established plans. In fact, the research and development program was characterized as a network of interrelated events to be achieved in a properly ordered sequence. The basic data for the PERT analysis consisted of estimates of elapsed time for the various activities which connect the dependent events of the network. The time estimates were obtained from "responsible, technically qualified" individuals and were subsequently expressed in "probability" terms or, that is, by a method of quantifying uncertainty. Thus, although the original development of PERT was associated with times to complete various phases of a project, we nevertheless will deal with costs here instead. Moreover, it can be argued that time, cost, and often other measures of effort will be equivalent.

The PERT team for the Navy project felt that the most important requirement for the evaluation was the provision of detailed, well-considered estimates of the time constraints on future activities, and hence that only carefully considered time estimates should be obtained. In fact, the person making any such estimates must have a very thorough understanding of the work to be done. PERT recognized that the time estimates for some activities, such as research and development, are highly uncertain, and that such uncertainty should be quantified and made known. Thus, it is seen that one is really involved in estimating the significant parameters of distributions in times to perform certain activities or work. Another requirement is knowledge of the precise sequencing of the various activities to be performed with the realization that any specific step in an isolated area of work cannot be completed until a specific step in another activity has been accomplished. In a like vein, we will regard these considerations as being important also for the estimation of costs. In fact, we might consider that the problem of fielding a weapon system, or the pursuit of a development program, etc., may be considered to be a function of several key variables. These variables are essentially of three kinds: resources, usually in the form of dollars, or what dollars represent—manpower, materials, and methods of production; technical performance of the weapon system and its components; and finally time. Here, our problem is to estimate costs in a meaningful way.

Of course, we recommend that costs always be estimated in the most accurate way, if possible and if time permits. However, there often will be the requirement or the existence of a deadline to preclude the most accurate estimation of project costs. It is then that the rather subjective PERT approach that follows may be used, perhaps with some success.

Once it had been decided to employ the PERT technique to determine likely costs, it becomes necessary to study thoroughly the element or characteristic involved and for such a task to estimate the most optimistic (least) cost, the most pessimistic (greatest) cost, and the most likely cost. Thus, the minimum, the maximum, and the modal values or estimates represent the key parameters of study and the very basis for the PERT. Hence, further, one might postulate or adopt the concept of a distribution of costs for any item or activity in a manner that the "distribution" has a maximum value B, a minimum value A, and a most likely or modal value M_0. The situation is described graphically in Fig. 36-1.

One notes that there is very little chance that either the most optimistic cost A or the most pessimistic cost B would be realized—the true cost would likely be somewhere in between—so that the

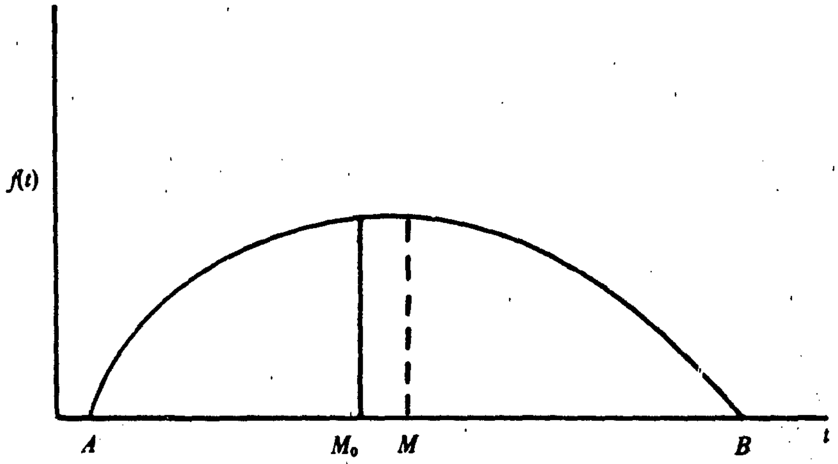

Figure 36-1. Typical Beta Distribution

probable distribution of costs would reach a maximum somewhere in the middle. No assumption is really made about the position of the point M_0, the most likely value of the cost, since this would depend on the estimator's judgement. Nevertheless, these concepts fit in rather well with the idea of a generalized beta distribution which is widely referred to in the statistical literature. The PERT idea is not to get so involved, however, since the beta distribution is characterized by four parameters—the mean, the standard deviation (or such measure of dispersion), the skewness, and the amount of kurtosis—with full realization of the abrupt endpoints, instead of long tails going off to infinity. An adequate statistical model of the beta density to represent probable costs is

$$f(c) = k(c - A)^p(B - c)^q, A <<< B \quad (36\text{-}15)$$

where

$f(c)$ = probability density function of costs
c = cost
k = constant to make the area under the distribution curve equal to unity
$= 1/[(B - A)^{p+q+1}\beta(p + 1, q + 1)]$
$\beta(\cdot,\cdot)$ = complete beta function
A = minimum cost
B = maximum cost
p, q = parameters determining the shape of the beta distribution.

Whereas, the mode, or most likely value, of the distribution is taken to be M_0, then we may designate the average value, or true mean, of the distribution to be $E(c) = M$.

For the PERT estimation problem, or the sampling problem, the recommendation and practice, according to Ref. 4, is to get qualified experts to estimate the least cost A, the maximum cost B, and the most likely cost M_0. Then, the estimates of these are denoted by

a = estimate of A
b = estimate of B
m_0 = estimate of M_0.

With these expert estimates, the true mean M and true variance σ^2 of the (general) beta distribution are determined from

$$m = \text{estimate of } M = (a + 4m_0 + b)/6 \quad (36\text{-}16)$$
$$v = \text{estimate of } \sigma^2 = (b - a)^2/36. \quad (36\text{-}17)$$

Thus, with these principles, one has a simple, straightforward approach to estimate the mean cost of an item, a project, etc., and the variance or standard error of the distribution of costs.

To illustrate, let us return to the data of Table 35-6 for various life cycle costs of the Utility Tactical Assault Helicopter (UTAH). Suppose, for example, that experts estimated the least, the greatest, and the most likely independent parametric cost estimate (IPCE) costs as

$$a = \text{least IPCE cost of R\&D phase} = \$483 \times 10^6$$
$$b = \text{greatest IPCE cost of R\&D phase} = \$713 \times 10^6$$
$$M_0 = \text{modal IPCE cost of R\&D phase} = \$601 \times 10^6.$$

Then, with these data one could use Eq. 36-16 to find the expected or mean IPCE cost, which would be

$$M = [488 + 4(601) + 713]/6 = \$601 \times 10^6.$$

Note that the expected IPCE cost, calculated by more refined methods, turned out to be $\$581 \times 10^6$. The reader realizes, of course, that the least, greatest, and most likely IPCE cost estimates of Chapter 35 were actually calculated by more refined methods than that of PERT. Our interest here is for illustrative purposes only.

The estimate of the standard deviation of costs is calculated from Eq. 36-17 as

$$\hat{\sigma} = (713 - 488)/6 = \$37.5 \times 10^6.$$

By contrast, in par. 35-8, we considered the least and greatest costs of Ref. 17, Chapter 35, to be at the 2.5% and 97.5% points of a "normal" distribution, and estimated the standard deviation to be

$$\hat{\sigma} = (b - a)/3.92 = (713 - 488)/3.92 = \$57.4 \times 10^6.$$

Thus, one may see that the assumption of the exact distribution to use for cost determination, along with the method of estimation of the parameters, could indeed be somewhat critical—at least for accurate estimates of dispersion.

In any event, this illustrates PERT and just how it might be applied to the problem of estimating costs, or times to perform certain tasks, amount of effort to be expended, etc. We emphasize, nevertheless, that PERT is, or may be, very subjective, and the user should proceed with such caution. PERT does, however, provide a very quick method of estimating means and dispersions of probable costs.

What are the limitations of PERT? On a statistical basis, this has been studied by Grubbs (Ref. 5). In this connection, he shows by equating moments that the possible number of different beta distributions which are implied is very limited. In fact, the shape parameters are limited to three cases:

$$p = 2 + \sqrt{2} = 3.41 \qquad q = 2 - \sqrt{2} = 0.59$$
$$p = 2 - \sqrt{2} = 0.59 \qquad q = 2 + \sqrt{2} = 3.41$$

and

$$p = 3 \qquad q = 3$$

That is to say, the number of possible beta distributions is limited to three "fat" and "flat" shapes, aside from the endpoints. Of course, weights other than the values involving the factor 6 in the denominators of Eqs. 36-16 and 36-17 could be used. Such changes would alter the possible shapes a bit, but this problem may well require some further detailed study if additional advances are to be made in the PERT methodology. To illustrate, for example, we note that the estimate of the mean actually weights the most likely value four times as much as the endpoints, i.e., the least and greatest values. No doubt, one may argue this to be somewhat proper, although it is well known that in much of the sampling theory in statistics the individual observations making up the sample mean are weighted equally. (In this case, it is well to mention that least and greatest values are hardly ever attained.) Perhaps the greater source of some error would be in the variance or standard deviation. As we have seen, the PERT standard deviation is $(b - a)/6$, whereas we used 3.92 instead of the value 6 for truncation at the 2.5% and 97.5% points of the distribution in par. 35-8. In contrast, if we were dealing with a uniform distribution on the interval (a,b), then we know that the standard deviation must be $(b - a)/\sqrt{12} = (b - a)/3.46$, and is nearer our estimate in par. 35-8. The assumption of the distribution, perhaps especially for the variance, is seen to be important.

It would be beyond the scope of this handbook to give an account of the general analysis of PERT-type networks. However, it should perhaps be mentioned that at least some of the phases of the life cycle cost analyses of weapon systems could in fact be characterized by PERT networks or system flow plans, and cost analyses correspondingly made in a systematic manner. We should mention also that the Delphi Method (Refs. 6 and 7) also might be considered in connection with cost estimation problems.

36-4 UNIT COSTS FOR DESIGN CHANGES

In Chapter 35, we indicated the possibility of design changes during development of a weapon system. Such design changes will alter the costs, and indeed the learning curve must be modified correspondingly. James (Ref. 8) has carried out an analytical study to provide predictive expressions or models taking account of a perturbed environment due to design changes. He proceeds as follows, assuming initially that the cost of the nth item produced before a design change occurs is, as in Eq. 36-3, given by

$$C_0(n) = K_0 n^b \tag{36-18}$$

where in view of subsequent notation for design changes during production we define:

$C_0(n)$ = cost of nth produced item before first design change = C_n of Eq. 36-3
K_0 = cost of 1st produced item before first design change = C_1 of Eq. 36-3
n = number of item—and will be used here for any general number of the produced articles, including design changes
b = $\ln L/\ln 2$, as before, where L is the learning rate.

Thus, James (Ref. 8) assumes that production proceeds until the first design change occurs, and assumes this occurs at item $n = n_1$. The next design change occurs at item number $n = n_2$, etc., so that the ith design change is at $n = n_i$, with the subscript $i = 1, 2, 3, ..., r$ for r design changes.

Now if the first design change is made at item $n = n_1$, the cost of the $(n_1 + 1)$st produced item will not follow the usual learning curve, but will be changed, perhaps abruptly. In fact, at this stage, it may be possible to reduce or remove some of the original effort, or part of the production line, etc. Suppose that the original effort is reduced by the fractional amount λ_1 at the first design change, then the new cost at this stage, or discontinuity, is changed to

$$(1 - \lambda_1)C_0(n) = (1 - \lambda_1)K_0 n^b \tag{36-19}$$

where λ_1 = fractional reduction of cost at the first design change.

However, to this reduced cost there must be added a new cost due to the first design change. This new cost may be expressed as a fraction or percentage of the cost or the first item produced $(n - n_1)$ or

K_0. Hence, taking account of both the reduced and added costs, we see that the new cost for producing the item numbered $(n_1 + 1)$ and beyond follows the equation

$$\left.\begin{aligned} C_1(n - n_1) &= (1 - \lambda_1)C_0(n) + \gamma_1 C_0(n - n_1) \\ &= (1 - \lambda_1)K_0 n^b + \gamma_1 K_0 (n - n_1)^b \end{aligned}\right\} \quad (36\text{-}20)$$

where we define or use

$n - n_1$ = number of items beyond the first design change where the old learning curve is corrected or changed to start the learning process over

γ_1 = fractional increase in cost based on the first design change as it affects subsequent production

$C_0(n - n_1)$ = new cost for items produced after $n = n_1 = K_0(n - n_1)^b$.

It is assumed that learning curve rate or slope does not change as evidenced by the continued use of the same parameter b. The asymptote to the cost function (Eq. 36-20) is

$$C_1(n - n_1) = K_0[(1 - \lambda_1) + \gamma_1]n^b. \quad (36\text{-}21)$$

The new cost, just after the first design change, continues until the second design change is made at item number $n = n_2$. (After the first design change, the number n of the item ranges over $n_1 < n \leq n_2$.)

For the second design change, we proceed in a manner analogous to that for the first design change (Ref. 8) but use subscript notation to designate the establishment of a new (third) learning curve, as compared to the second one of Eq. 36-20. Also, we use new percentage (fractional) cost changes, λ_2 and γ_2. Thus, for items produced over the interval given by $n_2 < n \leq n_3$, we establish the new cost learning curve for item numbers exceeding n_2 to be

$$C_2(n - n_2) = (1 - \lambda_2)C_1(n - n_1) + \gamma_2 C_1(n - n_2) \quad (36\text{-}22)$$

where $C_1(n - n_1)$ is given by Eq. 36-20, and $C_2(n - n_2)$ is calculated from Eq. 36-20 substituting new λ_2 and γ_2. Expansion of Eq. 36-22 gives actually, in terms of the initial quantities, the cost curve at the second design change as

$$\begin{aligned} C_2(n - n_2) = {} & (1 - \lambda_1)(1 - \lambda_2)K_0 n^b + \gamma_1(1 - \lambda_2)K_0(n - n_1)^b \\ & + \gamma_2[(1 - \lambda_1) + \gamma_1]K_0(n - n_2)^b \end{aligned} \quad (36\text{-}23)$$

and hence becomes a bit involved. In any event, it is always more convenient to program cost calculations for any design change stage i by using the recursion relation (Ref. 8):

$$\left.\begin{aligned} C_i(n - n_i) &= (1 - \lambda_i)C_{i-1}(n - n_{i-1}) + \gamma_i C_0(n - n_i) \\ &= (1 - \lambda_i)C_{i-1}(n - n_{i-1}) + \gamma_i K_{i-1}(n - n_i)^b \end{aligned}\right. \quad (36\text{-}24)$$

where

$$\lambda_0 = \gamma_0 = n_0 = 0 \text{ for zero subscripts}$$

$$K_i = K_0 \prod_{j=0}^{i} [(1 - \lambda_i) + \gamma_i]. \quad (36\text{-}25)$$

and

$$C_0(n - n_i) = K_{i-1}(n - n_i)^b. \quad (36\text{-}26)$$

Otherwise, the general formula for direct evaluation at the beginning of the n_ith unit cost learning curve is

$$C_i(n - n_i) = K_0 n^b \prod_{j=1}^{i} (1 - \lambda_j) + \gamma_i K_0 (n - n_i)^b + \sum_{j=2}^{i} \gamma_{j-1} K_0 (n - n_{j-1})^b \prod_{k=j}^{i} (1 - \lambda_k). \quad (36\text{-}27)$$

The asymptote to Eq. 37-22 for the second design change incidentally is

$$C_2(n - n_2) = K_0[(1 - \lambda_1) + \gamma_1][(1 - \lambda_2) + \gamma_2]n^b. \quad (36\text{-}28)$$

Thus, James was able to establish the overall cost learning curve for the important, and perhaps very usual, case where design changes are made from time to time in the production process. Every time a design change is made, there occurs an abrupt or discontinuous change in item cost at that particular stage of the production process. In order to illustrate the computational process, James (Ref. 8) gives an example for two design changes during the production process, the data in terms of the parameter being:

$\lambda_1 = 0.113$	$\gamma_1 = 0.207$	$n_1 = 30$
$\lambda_2 = 0.137$	$\gamma_2 = 0.333$	$n_2 = 70.$

He assumes an 80% learning curve, for which

$$b = \ln 0.8/\ln 2 = -0.322.$$

The reader may verify by calculation with the aid of the given equations that

1. For production up through the 30 items, the cost learning curve is (Eq. 36-18)

$$C_0(n) = K_0 n^{-0.322}, \quad 0 < n \leq 30.$$

2. For production from the 31st through the 70th item, the cost learning curve is (Eq. 36-20)

$$C_1(n - 30) = 0.887K_0 n^{-0.322} + 0.207K_0(n - 30)^{-0.322}, \quad 30 < n \leq 70.$$

3. Beyond production of the 70th item, where the second design change is made, the cost learning curve becomes (Eq. 36-23)

$$C_2(n - 70) = 0.765K_0 n^{-0.322} + 0.179K_0(n - 30)^{-0.322} + 0.364K_0(n - 70)^{-0.322}, \quad n > 70.$$

In his Figs. 1 and 2, James (Ref. 8) plots the item costs and learning curves in terms of the percent of the initial item cost K_0. His Figs. 1 and 2 are plotted on linear and logarithmic scales, respectively, and are reproduced as Figs. 36-2 and 36-3. One may note the abrupt item cost changes at the design changes, which occur at the 30th and 70th items, for both the linear and logarithmic plots.

The cost analyst must determine the initial item cost, the learning curve rate, and values of the λ_i and γ_i. Then, knowing at what points in production there are design changes, he will be able to calculate costs accurately. The methodology given herein may be particularly suitable for automatic data processing.

36-5 RELIABILITY GROWTH AND COST IMPLICATIONS

In *Part One*, par. 21-8, we discussed the Weibull failure time or reliability model, and pointed out in Eq. 21-146 that the (variable, time-dependent) conditional failure rate, hazard rate, or intensity function is given by

$$h(t) = \beta\lambda t^{\beta-1} \quad (36\text{-}29)$$

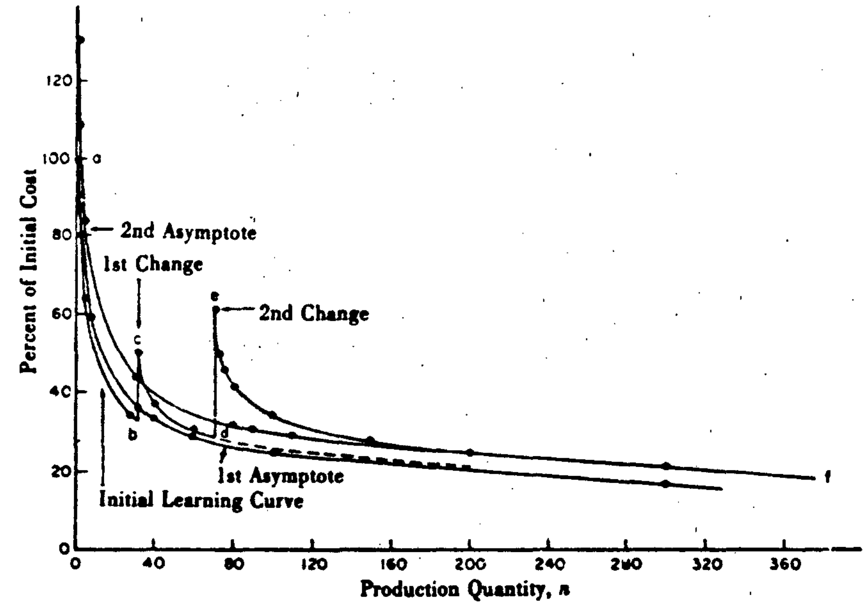

Figure 36-2. Unit Cost Expressions, With Perturbations Caused by Design Changes, Plotted on Linear Scales*

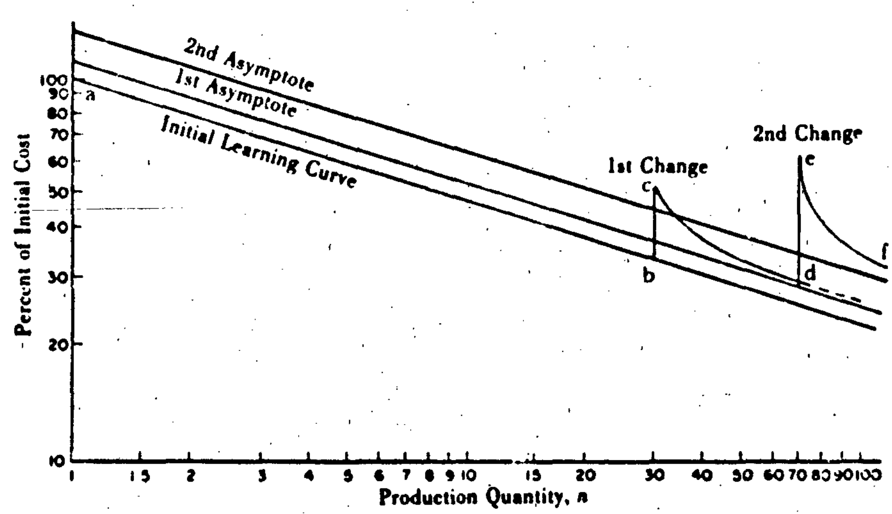

Figure 36-3. Unit Cost Expressions, With Perturbations Caused by Design Changes, Plotted on Logarithmic Scales*

* Reproduced from James' paper, Ref. 8

where

$h(t)$ = intensity or failure rate
t = random failure time of item on test
β = Weibull shape parameter
λ = Weibull scale parameter.

Here, we emphasize that under the Weibull model specified, t is a random life time or random time-to-fail for a general item, component, or system, and the reliability or $R(t_m)$ at the mission time t_m, or the chance that any general item will survive to at least the mission time t_m is given by

$$R(t_m) = \exp(-\lambda t_m^{\beta}). \tag{36-30}$$

With this brief introduction or background, we now turn to the analysis problem of increasing or improving reliability of the system through design changes, development, or quality control procedures, i.e., the concept of "reliability growth".

Now let us suppose a system is still undergoing development, and either design changes are made, or quality assurance procedures are being used to improve the reliability of the system as development progresses. Then, it has been noticed (e.g., Duane, Ref. 9) that the failure rate of the item, component, or system will usually decrease with time. In particular, if the total of the development test times is T, and $N(T)$ is the total number of observed failures of the equipment put on test up to time T, then it has often been shown on a log-log plotting scale that a linear relation is experienced, i.e.,

$$\ln[N(T)/T] = \ln\lambda + \gamma \ln T \tag{36-31}$$

where

$\ln\lambda$ = ordinate intercept for $\ln T = 0$
γ = slope of fitted line.

Furthermore, we see that

$$N(T) = \gamma T^{\gamma+1} \tag{36-32}$$

from the logarithmically fitted line of Eq. 36-31. Thus, when there are efforts to improve system reliability, then as the development test time T of the system increases, it is found that the cumulative failure rate or

$$C(T) = N(T)/T \tag{36-33}$$

decreases and can be expressed analytically by Eq. 36-31. Note, in this connection, that the slope γ of the fitted line on the log-log plot would then be negative. Furthermore, once data on $\ln[N(T)/T]$ are plotted against $\ln T$, the intercept $\ln\lambda$ and the slope γ may be estimated, for example, by least squares (Ref. 10), and Eq. 36-31 firmly established.

Continuing, we next see that the item, component, or system instantaneous failure rate $r(T)$ during development for a test time T in Eqs. 36-31 or 36-32 may be found by taking the derivative of Eq. 36-32, and in fact is

$$r(T) = \frac{d}{dt}[N(T)] = \lambda(\gamma + 1)T^{\gamma}. \tag{36-34}$$

We note that since the system configuration is changing, or quality assurance measures are being applied to improve system reliability, the data are not "homogeneous" in the sense of sampling a single or fixed population. Thus, in reliability growth studies one is not dealing with, for example, a single Weibull failure-time distribution, and the usual Weibull theory of Chapter 21 will not apply. In fact, Crow (Ref. 11) points out that when the configuration of the system is changing during a development program, and failures are governed by the failure rate of Eq. 36-29, then the system failure times follow a nonhomogeneous Poisson process with Weibull type intensity function $r(T)$ equivalent to Eq. 36-34. Hence, for that particular stage of development, Crow, due to the nonhomogeneous Poisson process model, suggests equating the $h(t)$ of Eq. 36-29 to the $r(T)$ of Eq. 36-34, i.e.;

$$h(t) = r(T)$$

or

$$\beta\lambda T^{\beta-1} = \lambda(\gamma + 1)T^{\gamma} \tag{36-35}$$

showing that this simply means

$$\beta = \gamma + 1. \tag{36-36}$$

Hence, at a development test time T_o the Weibull failure rate is

$$r(T_o) = \beta\lambda T_o^{\beta-1} \tag{36-37}$$

and if no further system improvements in reliability are made after time T_o, then it is reasonable to assume that the failure rate of the system would remain constant at the value $r(T_o)$ of Eq. 36-37 if testing were continued with that latest design. In particular, if the system were put into production with the configuration fixed as it was at time T_o, then it would be expected that the life distribution of the produced systems would be exponential with mean time between failures (MTBF) of

$$\text{MTBF} = M(T_o) = [r(T_o)]^{-1} = T_o^{1-\beta}/(\lambda\beta). \tag{36-38}$$

Furthermore, since system reliability is improving as the development goes on, the MTBF is increasing and in view of Eq. 36-38 is proportional to $T_o^{1-\beta}$. We can, therefore, say that β may be referred to as a "growth parameter" reflecting the rate at which the system reliability is improving, or equivalently as the MTBF is increasing with development. We note that since the MTBF is increasing, and the scale and shape parameters λ and β are positive, then necessarily β has to be less than unity for the nonhomogeneous Poisson type model considered here.

Although the scale parameter λ and the shape parameter β for the Weibull failure rate (Eq. 36-37) may be estimated by establishing the (linear least-squares) line of Eq. 36-29, we will use the more efficient method of maximum likelihood (ML) estimation by Crow (Ref. 11). We consider a system development program that proceeds in stages $k = 1, 2, ..., n$ and where we test the equipment at these stages to determine the failure times. It is expected that since the equipment is undergoing "reliability growth", the (total) cumulative test times to any failure, $T_1, T_2, ..., T_i, ..., T_n$ are increasing, as are the mean times between failures, or $T_i - T_{i-1}$, for all $i = 2, 3, ...$. These "development" test times at the stages of testing are used by Crow (Refs. 11 and 12) to determine maximum likelihood estimates of the parameters β and λ which are

$$\hat{\beta} = n/[\sum_{i=1}^{n-1} \ln(T_n/T_i)] \tag{36-39}$$

and

$$\hat{\lambda} = n/T_n^{\hat{\beta}}. \tag{36-40}$$

The reader may note that the estimates—Eq. 36-39 for the shape parameter and Eq. 36-40 for the scale parameter—are equivalent to those of Eqs. 21-185 and 21-186, where $\lambda = 1/\delta^{\beta}$, and δ is the so-called characteristic life. The estimates of the shape and scale parameters of Eqs. 36-39 and 36-40 are based on a "fixed number of failures" truncation, or Type II censoring. For time truncation estimates and multiple tests of items, see Crow (Ref. 12).

The ML estimate of the MTBF at the last test time T_n is from Eq. 36-38 given by

$$\hat{M}(T_n) = T_n^{1-\hat{\beta}}/(\hat{\lambda}\hat{\beta}) = T_n/(n\hat{\beta}). \quad (36\text{-}41)$$

Moreover for the item, component, or system, the ML reliability estimate for any random time t is then

$$R(T_n, t) = \exp(-\hat{\lambda}\hat{\beta}t/T_n^{1-\hat{\beta}}) = \exp(-n\hat{\beta}t/T_n). \quad (36\text{-}42)$$

Crow (Ref. 12) gives confidence bounds on the true unknown MTBF for the system configuration at the last development test time T_n, or that is for the parameter

$$M(T_n) = T_n^{1-\beta}/(\lambda\beta). \quad (36\text{-}43)$$

These confidence bounds are based on the estimate of MTBF or $\hat{M}(T_n)$ given in Eq. 36-41. Thus, if α and $(1 - \alpha)$ (for $0 < \alpha < 0.5$) are respectively the lower and upper probability levels, then one can make the following confidence statement:

$$Pr\{u_\alpha \hat{M}(T_n) \le M(T_n) \le u_{1-\alpha}\hat{M}(T_n)\} = 1 - 2\alpha$$

or

$$Pr\{T_n u_\alpha/(n\hat{\beta}) \le M(T_n) \le T_n u_{1-\alpha}/(n\hat{\beta})\} = 1 - 2\alpha \quad (36\text{-}44)$$

where probability levels u_α and $u_{1-\alpha}$ are found in Table 36-2. For example, suppose we want 95% confidence limits on the true MTBF and there were $n = 10$ test times, the last one being T_{10}, and we determine $\hat{\beta}$ from Eq. 36-39. Then we can make the confidence statement (at T_n) that

$$Pr\{0.5171T_n/(10\hat{\beta}) \le \text{MTBF} \le 3.286T_n/(10\hat{\beta})\} = 0.95.$$

Furthermore, it follows that if at that stage the item or system exhibits an exponential failure-time distribution, or constant failure rate, then the confidence bounds on the true unknown reliability at T_n are found from

$$Pr\{\exp[-n\hat{\beta}t_m/(T_n u_\alpha)] \le \exp[-t_m \cdot r(T_n)] \le \exp[-n\hat{\beta}t_m/(T_n u_{1-\alpha})]\} = 1 - 2\alpha \quad (36\text{-}45)$$

where t_m = unit or system specified mission time for service use after the last development test.

This account of the current state of the art for reliability growth modeling gives the needed theory for estimating system reliability at various stages of the development process. However, just what can be said about the cost of improving reliability insofar as the problem of the life cycle cost estimation of a system is concerned? Apparently, so far very little has been accomplished on establishing cost models to predict this particular part of the overall system cost, and hence this area will deserve some research in the future. Perhaps, nevertheless, there are some guesses that can be made in order to round out this part of the subject insofar as the total analyses of weapon system costs are involved. In fact, we can rather easily see that military systems will have to achieve suitably high reliability, for otherwise they will risk casualities or result in defeat of our forces, cause inefficiency, and bring about loss of confidence in the equipment. In this connection, the reliabilities we are interested in will most surely be at least 90% or higher. Moreover, the cost of producing systems which have, say, 95% reliability may be much higher than the cost for a design of about 90% reliability. When there exists the need to produce and field systems with reliabilities of 99%, 99.9%, or even higher, it is easy to see that costs will go up at perhaps an exponential rate, or be "astronomical" indeed. As a guess therefore, we might consider the possible use of a cost model of the type that follows insofar as increasing system reliability is concerned.

TABLE 36-2 PERCENTAGE POINTS u_α SUCH THAT $\Pr[M(T_n)/\hat{M}(T_n) < u_\alpha)] = \alpha$

n \ α	0.005	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
2	0.2378	0.2944	0.4099	0.5552	0.8065	33.76	72.67	151.5	389.9	788.6
3	0.2627	0.3119	0.4054	0.5137	0.6840	8.927	14.24	21.96	37.60	55.52
4	0.2902	0.3368	0.4225	0.5174	0.6601	5.328	7.651	10.65	15.96	21.31
5	0.3151	0.3603	0.4415	0.5290	0.6568	4.000	5.424	7.147	9.995	12.68
6	0.3372	0.3815	0.4595	0.5421	0.6600	3.321	4.339	5.521	7.388	9.076
7	0.3569	0.4003	0.4760	0.5548	0.6656	2.910	3.702	4.595	5.963	7.162
8	0.3746	0.4173	0.4910	0.5668	0.6720	2.634	3.284	4.002	5.074	5.993
9	0.3906	0.4327	0.5046	0.5780	0.6787	2.436	2.989	3.589	4.469	5.211
10	0.4052	0.4467	0.5171	0.5883	0.6852	2.287	2.770	3.286	4.032	4.652
11	0.4185	0.4595	0.5285	0.5979	0.6915	2.170	2.600	3.054	3.702	4.233
12	0.4308	0.4712	0.5391	0.6067	0.6975	2.076	2.464	2.870	3.443	3.909
13	0.4422	0.4821	0.5488	0.6150	0.7033	1.998	2.353	2.721	3.235	3.650
14	0.4528	0.4923	0.5579	0.6227	0.7087	1.933	2.260	2.597	3.064	3.438
15	0.4627	0.5017	0.5664	0.6299	0.7139	1.877	2.182	2.493	2.921	3.262
16	0.4719	0.5106	0.5743	0.6367	0.7188	1.829	2.114	2.404	2.800	3.113
17	0.4807	0.5189	0.5818	0.6431	0.7234	1.788	2.056	2.327	2.695	2.985
18	0.4889	0.5267	0.5888	0.6491	0.7278	1.751	2.004	2.259	2.604	2.874
19	0.4967	0.5341	0.5954	0.6547	0.7320	1.718	1.959	2.200	2.524	2.777
20	0.5040	0.5411	0.6016	0.6601	0.7360	1.688	1.918	2.147	2.453	2.691
21	0.5110	0.5478	0.6075	0.6652	0.7398	1.662	1.881	2.099	2.390	2.614
22	0.5177	0.5541	0.6132	0.6701	0.7434	1.638	1.848	2.056	2.333	2.546
23	0.5240	0.5601	0.6186	0.6747	0.7469	1.616	1.818	2.017	2.281	2.484
24	0.5301	0.5659	0.6237	0.6791	0.7502	1.596	1.790	1.982	2.235	2.428
25	0.5359	0.5714	0.6286	0.6833	0.7534	1.578	1.765	1.949	2.192	2.377
26	0.5415	0.5766	0.6333	0.6873	0.7565	1.561	1.742	1.919	2.153	2.330
27	0.5468	0.5817	0.6378	0.6912	0.7594	1.545	1.720	1.892	2.116	2.287
28	0.5519	0.5865	0.6421	0.6949	0.7622	1.530	1.700	1.866	2.083	2.247
29	0.5568	0.5912	0.6462	0.6985	0.7649	1.516	1.682	1.842	2.052	2.211
30	0.5616	0.5957	0.6502	0.7019	0.7676	1.504	1.664	1.820	2.023	2.176
35	0.5829	0.6158	0.6681	0.7173	0.7794	1.450	1.592	1.729	1.905	2.036
40	0.6010	0.6328	0.6832	0.7305	0.7894	1.410	1.538	1.660	1.816	1.932
45	0.6168	0.6476	0.6962	0.7415	0.7981	1.378	1.495	1.606	1.747	1.852
50	0.6305	0.6605	0.7076	0.7513	0.8057	1.352	1.460	1.562	1.692	1.787
60	0.6538	0.6823	0.7267	0.7678	0.8184	1.312	1.407	1.496	1.607	1.689
70	0.6728	0.7000	0.7423	0.7811	0.8288	1.282	1.367	1.447	1.546	1.618
80	0.6887	0.7148	0.7553	0.7922	0.8375	1.259	1.337	1.409	1.499	1.564
100	0.7142	0.7384	0.7759	0.8100	0.8514	1.225	1.293	1.355	1.431	1.486

Let

$C = C(R)$ = cost of system for the level of reliability R
A = constant to be determined
B = constant which represents the slope of the fitted cost line
R = value of system reliability which must be achieved for producing and placing the system in the field.

The reliability R at some development stage may be estimated from Eq. 36-42, for example, or from lower confidence bound on R based on Eq. 36-45. Then, as a suggestion or guess, we hint that one might consider predicting the linear cost relation from

$$\ln C = \ln A + B \ln[R/(1 - R)]. \tag{36-46}$$

Thus, for example, as the necessary reliabilities of a system increase from 0.90 to 0.95 or from 0.95 to 0.99, or to 0.999, etc., the ratio $R/(1 - R)$ from Eq. 36-46 equals 9, 19, 99, etc., and the natural logarithm of the ratio becomes 2.20, 2.94, 4.60, 6.91, etc., representing a fairly rapid increase. Moreover, we also have the slope B to work with, which may be determined or appropriately adjusted over a wide range of suitable values for the fitted cost line. From Eq. 36-46, we see that the actual cost of high reliability at a stage of development may perhaps be estimated therefore by the fairly robust model

$$C = A[R/(1 - R)]^{B}. \quad (36\text{-}47)$$

These matters are recorded here because they may represent costs over and above normal development.

36-6 OTHER CONSIDERATIONS FOR COST ESTIMATION AND RELATED PROBLEMS

As we have seen in Chapter 35, and aside from estimating costs by means of the analogy technique or the bottoms-up engineering approach, many of the methods involve the statistical regression approach. The problem for the regression approach is first to choose the proper model; the model selected is most often a linear form. In any event, the analyst should endeavor to select a meaningful model to fit to any existing cost data, and also a model which hopefully will lead to as precise a cost prediction as possible. If the linear regression model does not result in a good fit, then a more meaningful model should be found—the analyst always striving to select a final fit which exhibits small variance of residuals. Some of the background for military cost analysis and the use of regression techniques to estimate costs are given in Refs. 13 through 16. For a complete treatment of linear and multiple regression procedures, and generalized least squares, the analyst should refer to appropriate literature on statistical methods. Also, a BRL Report (Ref. 10) covers most of the ramifications that the cost analyst may be in need of, as well as the various tests of statistical significance needed in regression studies, whether to estimate costs or for other applications. We believe it to be beyond the scope of this handbook to cover such analytical techniques in the space permitted here, especially since such methodology is readily available in the military libraries.

Needless to say, there are many different types of military operations research problems that require the study of costs in one way or the other. For example, the whole field of cost effectiveness studies may be mentioned, and the newer interests of the military in the so-called cost and operational effectiveness analyses as well. Thus, proper costing of all types of systems studied represents a major activity of the Army systems analysis community, and the weapon systems analyst must have sufficient background in cost analysis methodology.

The paper of Wright (Ref. 1) seems especially noteworthy for its day or time (1936), because it covers the subject of costs of aircraft in some detail for that period. Wright indicates that the unit cost of aircraft structures would vary from about $5.25 to $6.25 per lb, whereas for the whole airplane (lightweight metal) the cost would likely be between $5.75 and $6.75 per lb in terms of 1936 dollars. For that period (1936), he even compares the cost of automobiles with that of airplanes. Wright indicates that for airplanes and automobiles carrying 4-5 passengers, in terms of 1936 dollars the airplane at a production quantity of 25 would cost about $8.18 per lb and at a production quantity of 1000 the airplane unit cost per pound would go down to about $3.20. At 10,000 produced airplanes, the average cost per pound was estimated by Wright to be $1.90 per lb, whereas at 1,000,000 copies the price per pound would have gone below one dollar, i.e., $0.95! For the 4-5 passenger automobile, the cost per pound at 1000 production line autos would have been only $0.784, and at a million autos the cost in 1936 dollars would have been an amazing $0.224 per lb! On the average for the airplane, which would have to fly and weigh much less, with its more expensive construction. Wright (Ref. 1) points out that the ratio of airplane to automobile cost per pound was then about 4.75 to 1. Thus, Wright advantageously used the concept of dollars per pound back then and as well he promoted the use of cost estimation by the method of analogy! His investigations, it would seem, show the value of keeping a close tab on the art of costing various production items; accordingly, the Army would do well to continue an appropriate cost data bank on all of its items undergoing R&D, production, and logistic studies.

There are many special applications also where costs come into considerable prominence, and again something we cannot go heavily into here. To mention an example, however, one might refer to the problem of repair parts for military systems. In this connection, it is highly desirable to have maximum assurance of continued operation of systems in combat zones, especially for any fixed expenditure limits, and one might be interested in the proper allocation of repair parts for the highest value of reliability that can be achieved for a given cost. Black and Proschan (Ref. 17) have made a study of this type of problem and related ones.

36-7 SUMMARY

As special techniques in the costing of Army items, we have covered the learning curve in a somewhat thorough manner and extended it to encompass the probability that there will be design changes during the production of Army systems. Accordingly, the analyst must expect changes to occur in learning curves as development and production proceed. Also, we have introduced the concepts of PERT since there may be some possible application of it in various Army costing problems. Finally, we pointed out that reliability growth may cause additional costs.

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CHAPTER 37
INTRODUCTION TO COST-EFFECTIVENESS STUDIES

An introduction is given to the analytical field of cost-effectiveness studies for weapon systems. There are two primary ways of conducting cost-effectiveness studies: (1) The first sets a level of effectiveness which is necessary or desirable to achieve and then determines the relative costs of two or more competing systems or alternatives. (2) The other allocates some given, affordable cost or amount of effort for competing alternatives and the mission under study, and then determines which alternative attains the highest level of effectiveness. Often, this type of approach to the analysis of weapon systems gives answers which are rather clear cut and hence of direct interest and value to the decision maker. It can be said that cost-effectiveness studies are more general than simply effectiveness studies, or the use of measures of effectiveness (MOE's), since they treat and analyze both the costs and the effectiveness factors.

Two simple examples are given to illustrate applications of cost-effectiveness type studies, and this should give the analyst a beginning toward more inclusive investigations.

37-0 LIST OF SYMBOLS

A = area of illumination of at least one footcandle
BA = burst altitude of round to initiate flare, ft
BT = burning time of flare, s
c = constant
E = level of illumination, footcandles
$\bar{f}(n)$ = expected fraction of the enemy force that becomes casualties for n rounds fired
H = altitude of flare, ft
I = intensity of flare, candles
IR = required intensity of flare light, candles
n = number of rounds, or in some cases battery volleys
R_T = target radius
UA = unit area of interest, ft^2
σ_{kx} = parameter of a kill function for the range direction, similar to a standard deviation (see Chapter 20 also)
σ_{ky} = parameter of a kill function for the deflection direction, similar to a standard deviation (see Chapter 20 also)
σ_x = round-to-round standard deviation in the range direction (used in this chapter also to involve increased dispersion due to artificial aiming of rounds)
σ_y = round-to-round standard deviation in the deflection direction (used in this chapter also to involve increased dispersion due to artificial aiming of rounds)
σ_μ = standard error of the aim error of C of I in the range direction
σ_ν = standard error of the C and I or aim error in the deflection direction

37-1 INTRODUCTION

The various evaluation models and many different measures of effectiveness of weapon systems were covered in previous chapters, and the treatment of costs was discussed in Chapters 34, 35, and 36. Logically then, this brings us to the idea of cost-effectiveness studies of weapon systems. Such studies relate to the relative effectiveness of competing weapons or weapon systems for some given cost or allowable amount of funds available, or the cost-effectiveness studies may otherwise revolve around the determination of the relative costs of two or more competing systems for some specified level of effectiveness against targets. If we were to study a family of rockets and guided missiles as a possible replacement for an existing family of artillery weapons, for example, then the cost and effectiveness study should be made from either of these two viewpoints. Moreover, in the study of costs, it seems clear that life cycle costs for both systems must be compared for equal values of effectiveness, or the effectiveness determined for equal life cycle costs. (The latter may often be the most difficult to do.) One sees easily that for the newer rockets and guided missiles, there might be somewhat more of a problem of determining accurate life cycle costs. In fact the problem may be somewhat like, or at least as difficult as, that of the UTTAS system referred to in Chapter 35. The new inventory item one proposes to introduce into any existing weapon family may involve some special or unique effort in developing accurate costs. The reader should realize that many high-level systems analyses for current development programs could be very voluminous, take many months of preparation on the part of the analysts performing the study, and ordinarily would involve a rather high level of security classification. Therefore, we will go into a somewhat simplified version of a cost-effectiveness study here in order to give the analyst an idea of the problem of performing effectiveness studies along with simple cost estimation techniques. In fact, since the basic principles for costing the life cycle of army weapon systems were discussed in Chapter 35, we may indicate in this chapter the principle more or less, but for a very simple, yet illustrative, case. Based on this, the analyst should be able to proceed to the analysis of more complex cost-effective cases.

It might well be said that a cost-effectivenes study should define and attack the overall problem in terms of an objective or a number of objectives. Moreover, there must be alternative means, or alternative weapons or systems, by which the objective may be accomplished or at least there could be different strategies that one is interested in comparing. Then the costs of the alternatives should be determined for each of the competing warheads, weapons, strategies, etc. These costs, or estimated value or uses of resources, form the basis of one of the prime tools in the cost-effectiveness evaluation. Continuing, one sees also that it is necessary to develop a model—i.e., a mathematical model or a computerized model, for example—which describes and sets the relationships among the objectives, the alternative means of possibly achieving them, the environment of conflict with its probable effects on mission accomplishment, and finally the costs or resources for the alternatives. In order to make the selection of the best alternative, it then becomes necessary for the analyst to choose the criterion—and hopefully the best one—which leads to the superior alternative. This criterion should relate the objectives and costs in an effective and useful way, for example, by setting the stage for maximizing the achievement of the objective or objectives for some fixed budget determined in a relevant manner for the problem at hand.

As an introduction to the general ideas of a cost-effectiveness study, we will first set up a problem, the aim of which is to select the better of two possible alternative projectiles for an artillery weapon. Next we will consider a rather special and often isolated type of cost-effectiveness study relating to the choice of flares to illuminate targets.

37-2 COST-EFFECTIVENESS COMPARISON OF TWO TYPES OF 155-mm PROJECTILES

37-2.1 BACKGROUND DATA

In order to illustrate the concept of a simple cost-effectiveness type study, let us recall that the "standard 155 mm" howitzer projectile is a high explosive projectile primarily for antipersonnel fire.

Now consider that a proposal to increase the lethality of artillery projectiles might consist of loading a special type of projectile with submissiles or grenades, which will be dispersed on the downward slope of the trajectory to spread over a fairly wide area on the ground, and hence attack more enemy ground troops should they be present. For an assumption on the range of engagement of 155-mm fire, let us postulate that it is expected enemy personnel targets will most likely be attacked at about 11,000 m; and that charge 7 ordinarily will be used for such firing.

For the standard HE projectile, the round-to-round standard deviation in range for 11,000 m is about 45 m, and the round-to-round standard deviation in deflection is only about 6 m.* The expected error in placing the center of impact (C of I) on the target aim point is about 120 m standard deviation in range and 45 m in deflection.* For these delivery conditions, the lethal area of each projectile against enemy troops—30% of which are standing, 40% of which are kneeling, and 30% are prone or are under cover—is about 1800 m², although for the simultaneous delivery of 6 rounds fired in volley by an artillery battery, the total lethal area is about 9,000 m². These data give the basic effectiveness parameters of the standard 155 mm HE round, and the known cost of such a round, properly fused, is determined [illegible] the production learning curve to be $170.

For the other competing 155-mm round, consisting of multiple grenades—which are ejected at proper altitude and detonated by submissile fuze action on striking the ground—the lethal area for a volley from a battery has been calculated to be about 25,000 m² for the same target, although the round-to-round standard deviation in range is about 172 m and the round-to-round standard deviation in deflection is about 25 m. For the new submissiled projectiles, the delivery standard errors for the C of I are expected to be the same as for the 155 mm HE projectile although the cost per projectile for the newer grenade-filled projectiles is expected from cost estimation relation (CER) studies to be $300. Normally, such enemy personnel targets have been found to occupy an area of about 140,000 m².

37-2.2 THE PROBLEM AND OBJECTIVE

The given detailed characteristics, involving both performance data and cost data, are sufficient to carry out a limited cost-effectiveness study. The question we seek to answer is whether the new projectile consisting of submissiles is "cost-effective". In other words, for its cost, is the new or proposed round as effective against personnel targets as the present standard round? To answer such a question, our objective is first to establish a good or proper model of evaluation. We must determine a suitable measure of effectiveness, and then we can substitute the performance and cost data into the model to determine whether we should change over to production of the new type of 155-mm projectile loaded with submissiles.

In order to answer the question posed, it would certainly appear that a very suitable method of comparison would involve determining the expected fraction of a unit that becomes casualties for each of the two type of projectiles, and hence establish the superiority of one over the other. Now, shall this be done by fixing the cost level at some value and finding the effectiveness for both cases of interest, or shall we determine relative costs for a given level of effectiveness of a unit, i.e., say the 30% level of expected fraction that becomes casualties? Actually, this is a "low-level" type of cost-effectiveness (CE) study, so it seems more natural to determine the number of rounds for each type of projectile to give 30% casualties, and then cost each type for comparison. We note also in this connection that the analyst does not have to go to the trouble of estimating life cycle costs for two major systems. Rather,

*See Ref. 1, for example.

both projectiles may be fired from existing howitzers, and hence we may deal simply with a cost-effectiveness comparison of the two different projectiles, and nothing more. This is especially true if logistic costs or effort are about the same—which we expect in this case.

Obviously, there is such a jumble of numbers for the characteristics of the two projectiles that we need a single measure of effectiveness! Hence, we proceed as follows.

37-2.3 AN APPROPRIATE SOLUTION

As a pertinent solution to this "component" CE study, we will find the number of standard rounds and the number of new rounds necessary to attain 30% enemy casualties. Then, these required numbers will be multipled by the appropriate cost per round to determine which round results in the lesser cost.

For the standard HE round, the lethality pattern of the battery volley (6 rounds) is about twice as wide in the deflection direction as in the range direction. Therefore, using the notation of Chapter 20, Eq. 20-57, with $c = 1$, we have

$$2\pi c\sigma_{kx}\sigma_{ky} = 2\pi\sigma_{kx}(2\sigma_{kx}) = 9000 \text{ m}^2$$

or

$$\sigma_{kx} = 26.8 \text{ m and } \sigma_{ky} = 53.6 \text{ m}.$$

For the new round with submissiles, it was found from firings that the pattern was almost circular. Thus, for this round we see from Eq 20-57

$$2\pi\sigma_{kx}\sigma_{ky} = 2\pi\sigma_{kx}^2 = 25{,}000 \text{ m}^2$$

or

$$\sigma_{kx} = \sigma_{ky} = 63.1 \text{ m}.$$

Since the enemy personnel normally occupy an area of about 140,000 m^2, we will take the target to be approximately circular and determine the target radius R_T from

$$\pi R_T^2 = 140{,}000 \text{ m}^2$$

or

$$R_T = 211.1 \text{ m}.$$

The expected fraction $\bar{f}(n)$ of the enemy force that becomes casualties will be calculated by Eq. 20-77, for which we recall that the round-to-round ballistic dispersion is assumed to follow the bivariate normal distribution, and the error in placing the C of I of the rounds on the target aim point also follows a bivariate normal delivery distribution. The enemy target elements or personnel are assumed to be distributed uniformly over the entire target area of 140,000 m^2. We note that Eq. 20-77 depends critically on the number of volleys in this case (rounds ordinarily) that will be fired. Thus, to find the number of volleys required, we try some number of volleys in Eq. 20-77 and change the n until we get the desired 0.30 fraction of casualties, i.e.,

$$\bar{f}(n) \approx 0.30.$$

Then that n so determined gives the required number of battery volleys.

Case I (Standard Round)

Since R_T is large (211.1 m), the analyst decides for the comparison to improve target coverage by increasing the round-to-round dispersion in range and deflection to 110 m and 15 m, respectively.

$$\sigma_x = 110/\sqrt{6} = 44.9 \text{ m} \qquad \sigma_y = 15/\sqrt{6} = 6.1 \text{ m}$$
(for volley movement)*

$$\sigma_\mu = 120 \text{ m (given)} \qquad \sigma_\nu = 45 \text{ m (given)}$$

$$\sigma_{kx} = 26.8 \text{ m} \qquad \sigma_{ky} = 53.6 \text{ m}$$

Using Eq. 20-77, we find

$$\bar{f}(n) = \bar{f}(20) \approx 0.302.$$

Case II (New or Proposed Round)

$$\sigma_x = 172/\sqrt{6} = 70.2 \text{ m} \qquad \sigma_y = 25\sqrt{6} = 10.2 \text{ m (for volley movement)}$$

$$\sigma_\mu = 120 \text{ m (given)} \qquad \sigma_\mu = 45 \text{ m (given)}$$

$$\sigma_{kx} = 63.1 \text{ m} \qquad \sigma_{ky} = 63.1 \text{ m}$$

$$R_T = 211.1 \text{ m}$$

By using Eq. 20-77, we calculate

$$\bar{f}(3) = 0.290 \text{ and } \bar{f}(4) = 0.336.$$

Hence, only three volleys of the submissile rounds are needed to obtain about 30% enemy casualties, whereas for the standard HE round about 20 would be necessary, and using the original round-to-round standard deviations of $\sigma_x = 45$ m and $\sigma_y = 6$ m does not help! (Perhaps it might appear to the reader that three is a surprisingly low number of rounds for the new projectile, but its lethal area is nearly three times larger than that of the standard projectile, and the increase in round-to-round dispersion for the new projectile is still lost in the relatively large aiming error. (For three volleys, there are only four terms in Eq. 20-77, so that the results are easily checked.)

Now for the relative costs to achieve 30% casualties:

1. Total standard round cost is (6)(20)($170) = $20,400
2. Total new proposed round cost is (6)(3)($300) = $5,400.

Hence, it might be said that the proposed submissile round is nearly four (3.78) times as effective as the standard 155-mm projectile. Moreover, the saving in dollars for each such enemy personnel target threat destroyed is $15,000. Therefore, it would be concluded that the new round would be very "cost-effective" indeed and is recommended.

We emphasize here that, indeed, we have illustrated a very simple example of a cost-effectiveness type of study, although the reader might well visualize its generalization to a more complex study. For example, if we return momentarily to the concept of possibly replacing a family of artillery weapons by a family of rockets and missiles, then we might start with a "division slice" or a "corps slice" and the

*The battery volley of 6 rounds gives a lethal area of 9000 m^2 which moves randomly as the C of I of 6 rounds.

amount of effort in terms of dollars that might be allocated for performance of the artillery mission in expected engagements. Then we would proceed to determine the relative effectiveness of the cannon type artillery versus the family of rockets and missiles for the given allocated cost. In such a CE study, one would lay out the entire complex of targets that would be attacked in combat, evaluate weapons against it, and perhaps even conduct a computerized war game of some kind to determine relative effectiveness of the two types of competing weapon families.

We now turn to a problem involving a very different type of munition, i.e., an illuminating projectile. Nevertheless, the cost-effectiveness study can be treated in much the same way as for the fragmenting projectile analysis previously covered, except that we will illustrate by fixing costs as a starting point.

37-3 ILLUSTRATION OF EFFECTIVENESS FOR A FIXED COST

Illuminating projectiles are used to light up target areas at night, and hence are needed to determine the type of target, size, location, etc., so that appropriate fire can be brought to bear upon any threatening enemy units. In 1975, Sheldon (Ref. 2) performed a "logistic/cost-effectiveness" study of flares. His primary purpose in developing a model of cost-effectiveness was to give the user a simple tool which would translate the characteristics of pyrotechnic illumination into numbers of flares and procurement costs, so that a technique would be readily available to determine costs to accomplish a given mission. In his study, Sheldon (Ref. 2) attempted to determine how much light is necessary to see and engage enemy targets, especially enemy tanks at night, and to find out what size area on the ground would be representative of a typical illumination mission. The study of Sheldon involved the following parameters:

I = intensity of flares, candles
H = altitude of flare
BA = burst altitude of round to initiate flare, ft
BT = burning time of flare, s
E = level of illumination, footcandles
IR = required intensity of flare light, candles
UA = unit area of interest, ft^2.

He developed the relationship in Eq. 37-1 which gives the required intensity IR in terms of the unit area UA to be illuminated, the flare altitude H, and the needed level of illumination E to see a target of interest:

$$IR = (E/H)[(UA/\pi) + H^2]^{3/2}, \text{ candles.} \quad (37\text{-}1)$$

The particular target of interest considered in Ref. 2 was taken to be a tank, and the viewable ranges were determined for both front and back lighting of the target. Fig. 2 of Ref. 2, reproduced here as Fig. 37-1 shows the viewable range as a function of the level of illumination E in footcandles. It is noted from Fig. 37-1 that 1 footcandle will allow an observer to see and engage tanks at ranges beyond two km, for either front or back lighting. Thus, it would seem that 1 footcandle would certainly be sufficient for our illustrative purposes of evaluation here, i.e., the illumination of a troop concentration.

The study, Ref. 2, showed that dispersion in height of burst to release the flare had very little effect on viewable range, although the target area it was desired to illuminate, i.e., 5 km^2, was so large that the delivery errors of mortars or artillery would have no effect on the evaluation and were therefore of inconsequential interest. In order to formulate for illustration a relative effectiveness evaluation for a given fixed cost, we will approach the problem somewhat differently here than in Ref. 2.

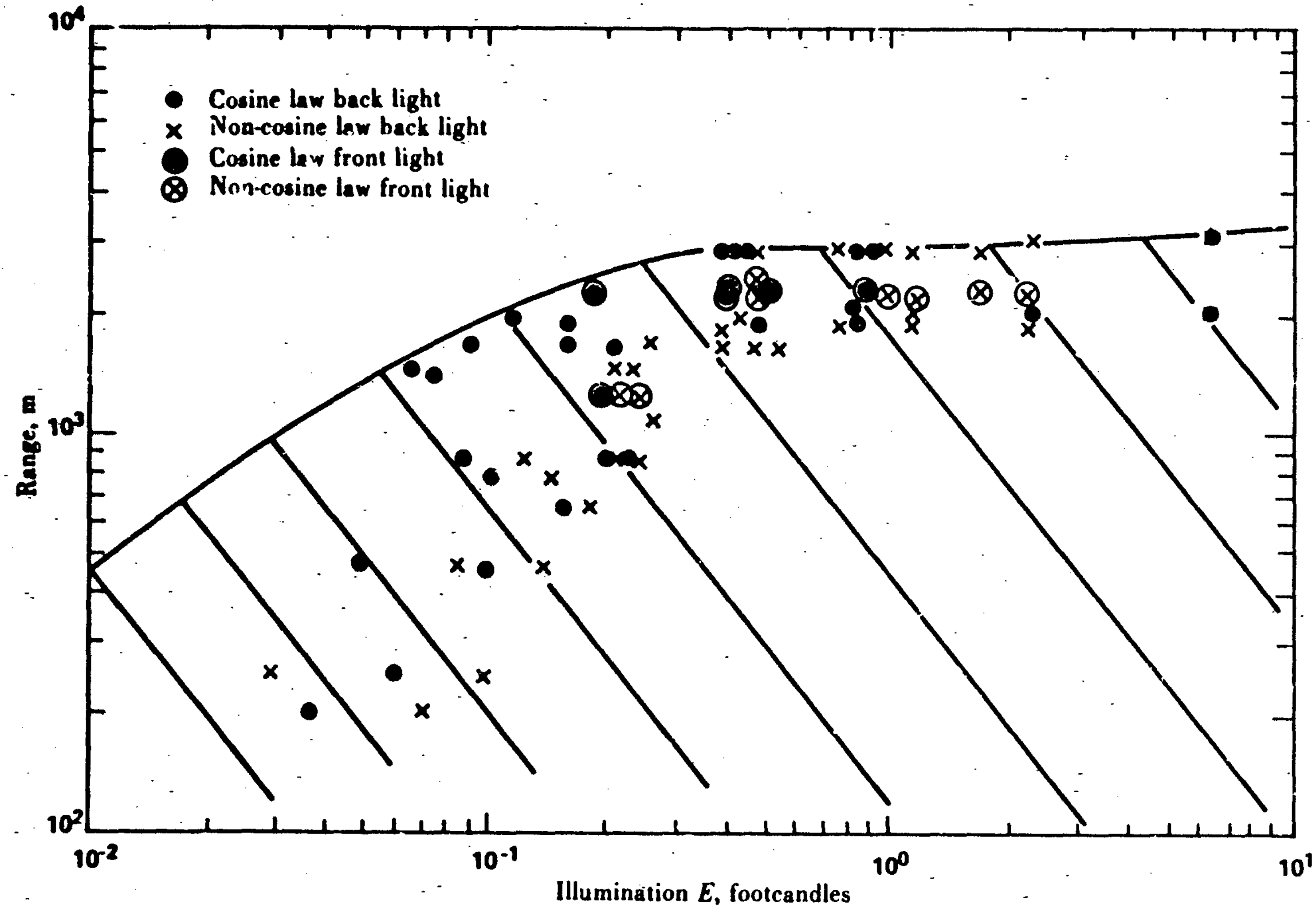

Figure 37-1. Viewable Range for a Tank (Side-on to the Observer) vs Available Illumination

The current 155 mm illuminating projectiles—M485, M485E1, and M485A2—have an intensity of about 700,000 candles, whereas an improved 155 mm illuminating projectile NR109 possesses an intensity of 2,200,000 candles. The area A in square meters on the ground for proper fuzing altitude and at least 1 footcandle of illumination may be determined from Eq. 37-2 (Ref. 2):

$$A = 1.2I/E, \text{ ft}^2 = 0.11148\ I/E, \text{ m}^2. \qquad (37\text{-}2)$$

Hence, the standard round will illuminate an area on the ground, to at least 1 footcandle, equal to

$$A = 0.11148(700,000) = 78,039 \text{ m}^2.$$

Likewise, the area illuminated to at least one footcandle by the NR 109 155-mm round will be

$$A = 0.11148(2,200,000) = 245,264 \text{ m}^2.$$

Since Sheldon (Ref. 2) has shown that the 155 mm NR109 round is cost-effective for an area of rather large size, i.e., about 2240 m on a side, then let us rephrase the problem for a "target" radius somewhat larger than the 211.1 m used in the previous fragmenting projectile example. Thus, a new target of radius 300 m which we would consider illuminating, to at least 1 footcandle, would include the one of 211.1 m, and the total area of the new "target" is 282,743 m². Certainly, the actual target of radius 211.1 m we desire to neutralize with fragmenting projectiles would be found if the larger area were illuminated to at least 1 footcandle everywhere.

For round-to-round delivery errors, we might expect that once the fuze functions and the flare and parachute are ejected from the projectile, then the flares would disperse over a wider area than projectiles hitting the ground. Thus, we might take $\sigma_x = 60$ m and $\sigma_y = 20$ m.

For the aiming errors, we may as well use the same values as for the high explosive projectiles in par. 37-2.

Finally, concerning costs of rounds, Sheldon (Ref. 2) puts the cost of the 155 mm M485 series illuminating round at \$106, and that of the NR109 at \$115. In this connection, let us suppose that the cost of the NR109 round were even 1/3 higher than the stated \$115 cost, i.e., \$153.33 and that the budget to find the target is \$1060. In other words, we could fire 1060/106 = 10 rounds for the M485 series illuminating projectile and 1060/153.33 = 6.91 or 7 of the NR109 illuminating projectile. With the fixed cost of \$1060 and the increased cost of the NR109 illuminating round, we want to determine whether it is still "cost-effective".

The NR109 round burns for 69 s and the M485 burns for 153 s. For this illustration of relative effectiveness for equivalent costs, we are assuming that 69 s burning time is sufficient to locate the tank target.

Our procedure will be that of determining the expected fraction of a "target" of radius 300 m which will be illuminated to at least 1 footcandle by 10 of the M485 type rounds, as compared to that of only 7 of the NR109 rounds. Clearly, moreover, for this case involving delivery errors, we may think of the problem also as a "target coverage" study and use Eq. 20-77, i.e., the same as for the first example (par. 37-2) of this chapter. To summarize, we have the following basic data:

M485 Type Round

$n = 10$

$\sigma_x = 60$ m $\qquad \sigma_y = 20$ m

$\sigma_u = 120$ m $\qquad \sigma_v = 45$ m

$\sigma_{hx} = \sigma_{hy} = \sqrt{78,039/(2\pi)} = 115.5$ m

$R_T = 300$ m

NR109 Round

$n = 7$

$\sigma_x = 60$ m $\qquad \sigma_y = 20$ m

$\sigma_\mu = 120$ m $\qquad \sigma_\mu = 45$ m

$\sigma_{kx} = \sigma_{ky} = \sqrt{245{,}264/(2\pi)} = 197.6$ m

$R_T = 300$ m.

Then using these data and Eq. 20-77, we find for the M485 type round that

$$\bar{f}(10) = 0.669$$

and for the NR109 illuminating projectile

$$\bar{f}(7) = 0.938.$$

Hence, to summarize, the 10 standard 155 mm illuminating rounds would provide sufficient light to cover 66.9% of the extended "target" of radius 300 m. whereas only 7 of the newer NR109 rounds, provided at an equivalent cost of $1060, would still give sufficient light to expose 93 8% of the extended target. Thus, in addition to agreeing with Sheldon (Ref. 2), we find using a different approach-—namely, that of treating the problem as a target coverage one—that the newer NR109 illuminating projectile is still very "cost-effective" indeed.

In summary, pars. 37-2 and 37-3 give two different approaches to the problem of integrating costs and effectiveness in an operations research type study. Both provide rather clearcut answers to the questions that were raised, and hence provide the analyst with very useful tools in his analyses. Especially do they seem to give the manager the concrete type of information he is interested in or is actually seeking.

37-4 SOME FURTHER CONSIDERATIONS ON COST-EFFECTIVENESS

As an elementary introduction to cost-effectiveness in this chapter, we have aimed to outline by example the approach to determining costs of competing alternatives for a fixed level of effectiveness, and the relative effectiveness for a given fixed cost of alternative or competing systems. Our "systems" were merely components of artillery cannon or gun delivery means to illustrate the general idea. In the first illustration, we considered the problem of whether it might be desirable to "break up the payload" by dividing it into smaller "packages" in the way of submissile or grenades and scatter them over an area of the battlefield for improvement in effectiveness, especially as compared to the single warhead or fragmenting projectile of the artillery weapon. Moreover, the analyst, noting that the target was possibly a bit large, decided to improve target coverage by scattering the rounds from a battery so as to increase the round-to-round dispersion somewhat. However, this did not appear to payoff. Also a very concrete judgement on a particular engineering principle was evaluated through means of a cost-effectiveness approach.

In the second illustration, we concentrated on hypothesizing a fixed cost to be allocated for detecting targets at night, and even increased the cost of one of the alternatives to see if the improved illuminating projectile was still cost-effective when delivery errors and smaller target areas were involved in the analysis. In other words, we did somewhat of a "sensitivity analysis" which backed up another study approach. Again, a particular and important engineering type of question was answered through the use of a cost-effectiveness analysis. Thus, the advantages of employing the cost-effectiveness approach over and above the regular effectiveness type of analysis are evident because it can be seen that costs of weapon systems are clearly of major importance.

The reader will see rather easily that we have only introduced the idea of applying the general principles of cost and effectiveness analyses. Nevertheless, the Army literature on the subject of cost-effectiveness analyses is very extensive and represents an area of much continuing interest to the analyst. Like many other topics, it might be said that cost-effectiveness studies were always of much value and importance, especially since part of the weapon systems planning game involves costing. Our President and the Congress use cost analyses to assist in reaching decisions whether or not to proceed with major weapon systems. Because of the probability [illegible]ed budgets, practically all systems have to be "cost-effective" or they otherwise cannot be dev[illegible]d and produced.

In recent years, the approach has been somewhat away from the use [illegible] cost-effectiveness analysis, perhaps to some extent because some individuals have criticized the activity as being too much oriented in the engineering direction! Also, there has been more and more emphasis on the "fly before buy" principle, so that "operational" type testing has appealed to a wide segment of the Army operations analysis community. In fact, we hear more and more of the importance of performing "cost and operational effectiveness analyses", implying a more pertinent and general type of useful analytical "pastime". In many cases, it becomes very difficult to distinguish a cost and operational effectiveness analysis (COEA) from a cost-effectiveness analysis, and in any event a good overall pertinent analysis that answers important questions is still a *good* and useful analytical "pastime" COEA's are covered in Chapters 45 and 46.

Due to the nature of the cost-effectiveness analyses, however, we should bring out here a few additional comments on references of possible interest to the reader. The Research Analysis Corporation published a primer on cost-effectiveness in 1967, apparently thinking of its ubiquitous applications and to inform the layman, so to speak, this being in Ref. 3 by Sutherland. Visco (Ref. 4) discusses the beginning of cost-effectiveness analysis. At one of the early Army Operations Research Symposia (1965), a Deputy Assistant Secretary of Defense, Dr. Alain Enthoven, (Ref. 5) gave a discussion of cost-effectiveness analyses of Army divisions in the keynote speech. He pointed out, among other important considerations, that the analysts cannot make a useful calculation for all of the factors that may come into prominence in a cost-effectiveness analysis, and that judgements will have to be made on some of the key factors. Hopefully, the decision maker and the study team can work out and resolve possible disagreements, so that the best solution to cost-effectiveness type problems may be reached.

A theory of cost-effectiveness for military systems analysis has been published by Fox (Ref. 6). Schlenker (Ref. 7) has studied the foundations of a cost-effectiveness methodology for weapons, and the idea of using dynamic programming approaches to cost-effectiveness studies is discussed by Sacco and Schlegel (Ref. 8). Barfoot (Ref. 9) initiated work on a preliminary cost-effectiveness handbook in 1963, and Quade (Ref. 10) had indicated some limitations of the cost-effectiveness approach to the problem of military decision-making. Ref. 11 is a fairly extensive bibliography on cost-effectiveness type studies in the Department of Defense.

A partial bibliography of references which the Army weapons systems analyst might have some interest in is included in this chapter for possible further study.

37-5 SUMMARY

We have introduced the idea of cost-effectiveness studies to evaluate problems in the analysis of weapon systems. An advantage of such studies is that they take into consideration simultaneously both the effectiveness and the cost issues in an evaluation. Moreover, cost-effectiveness studies appear to give the decision maker the very type of information he usually seeks, and the answers are likely to be rather clear-cut. The two primary ways of conducting cost-effectivenss studies involve either the analysis of costs for competing alternatives when the level of effectiveness has been set, or the determination of relative effectiveness for alternatives once the allocated or affordable cost has specified for the particular mission.

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CHAPTER 38

SURVIVABILITY

Survivability considerations for personnel, weapon systems, and materiel must be taken into account in evaluating the effectiveness of Army capabilities in the field. The term "survivability" currently is taking on a somewhat newer connotation and is quite distinct from the well-established fields of vulnerability and vulnerability reduction. After defining survivability in a workable and useful manner, we proceed to discuss its realm of possible applications to an informative extent and find that the weapon systems analyst should regard the concept as an element of the systems analysis process. In particular, we foresee that quantification and modeling of survivability parameters will be required in order to make the more meaningful and useful trade-offs of survivability characteristics with other measures of effectiveness of weapon systems.

38-1 INTRODUCTION

The vulnerability of targets to attack has been under study for many years; in reality, from the very beginning of the problem originally of determining the relative effectiveness of weapons. In fact, it could be argued that the initial evaluations of weapons involved the calculation of hit probabilities and the estimation of the conditional chance that a hit is a kill, i.e., the latter taking into account target vulnerability considerations. Thus, it was absolutely natural and necessary that the vulnerability of all kinds of targets to all types of weapons be studied from the very start. Moreover, the necessity of vulnerability programs, mostly of an experimental character, emerged as a needed activity to make valid comparisons of weapons for different roles and missions. It was seen very often over the years that reduced vulnerability led to improved survivability for personnel, aircraft, tanks, and other combat systems in general.

It can be said additionally that survivability of personnel has long been an area of prime concern to the Armed Forces, and that, for example, the planning of medical facilities for the field and the medical workload always needed somewhat precise estimates to cope with all probable occurrences. Indeed, at one of the early Army Operations Research Symposia, Laughlin, Scoles, and Eyler (Ref. 1) — working in the interests of the US Army Medical Service Combat Developments Agency, Fort Sam Houston, TX — gave a paper establishing survival curves for possible casualties in a war, along with the various levels of medical type treatments. Much less attention, however, has been paid to the survivability of materiel until recently, and now such considerations are being regarded as a "must" in evaluations of all kinds.

The general term "survivability" is a relatively recent concept, coming to the forefront in the past few years, and there seems little doubt that survivability will receive more and more attention in the weapons planning process from now on. Obviously, survivability involves more than improvements (decreases) in the vulnerability of weapons or targets as too much protection will clearly result in some loss of mobility for personnel and weapon systems. Thus, it becomes of much necessity and interest to explore the concept of survivability on an overall or operations research basis. This exploration is the purpose of this chapter in connection with the normal role of the weapon systems analyst.

The new field of "survivability" is of much interest to the Army because: (1) large numbers of personnel and large quantities of materiel have been lost in past conflicts; (2) the costs of developing, acquiring, and maintaining weapon systems have escalated enormously; (3) reliability and maintainability problems in a combat environment are continually plaguing the Armed Forces; (4) many new relatively cheap and unsophisticated weapons have the capability of destroying expensive combat systems; (5) with the growing complexity of weapon systems there is a need for highly trained personnel, and hence more costly operation of equipment; and (6) it is expected that our potential enemies may have a numerical superiority in the numbers of personnel and perhaps in materiel (see Ref. 2).

Survivability might well be considered to be a concept which must be taken fully into account in the weapon systems analysis or evaluation process. For this reason, survivability should be regarded as an element of systems analysis. Moreover, it can be seen that studies of survivability will be necessary to assure the accomplishment of many Army missions in future combat. Indeed, the term "survivability" currently encompasses a rather general field of activity, and it may involve the much older disciplines such as vulnerability, vulnerability reduction, reliability, maintainability, and (loss of timely) repairability. To sum up, the weapon systems analyst will need to study and apply the principles of survivability analysis in his process of evaluating weapon systems and materiel.

In order to lead up to a definition of survivability, we will first review the definitions of vulnerability and vulnerability reduction, as there has been some confusion among the three terms, and all three concepts need to be somewhat distinct for future reference and applications. *Vulnerability* is a quantitative measure of the susceptibility to damage of a target structure or item of materiel to a given attack mechanism. Vulnerability often is expressed as a "vulnerable area". *Vulnerability reduction* consists of the application of design techniques to materiel items in order to reduce or eliminate the effects of combat damage mechanisms. Vulnerability reduction may often be expressed as a percent decrease in vulnerable area. With these two definitions as a background, and the lack of any definition of survivability in the current issue of the Army dictionary, the Army Materiel Systems Analysis Activity which set up a Survivability Office, proposed the following definition:

"Survivability is that characteristic of personnel and materiel which enables them to withstand (or avoid) adverse military action or the effects of natural phenomena which ordinarily and otherwise would result in the loss of capability to continue effective performance of the prescribed mission."

Put simply, therefore, survivability represents a new area of interest and effort which aims to increase as much as possible the probability that personnel and materiel of all kinds will survive in a battlefield environment.

Ref. 2 is a "Survivability Primer", which covers some of the current basic approaches regarded as important in improving survivability, and points out that any improvements in the chance of survival must be achieved without sacrificing the ability of the system to perform its mission within realistic and resource constraints. For example, some realistic trade-offs may be necessary between survivability and other aspects of effectiveness such as reliability, mobility, and lethality. We might well add here that the Army mission also is to win in battle, and survivability should not detract from that.

Ref. 2 discusses four areas considered of prime interest in the study of weapon systems to enhance survivability, namely: (1) detectability, (2) hitability, (3) vulnerability reduction, and (4) repairability. Thus, Ref. 2 recommends that targets should be designed or used on the battlefield so that they cannot be detected, if at all possible. And if they are detected, then system characteristics or cover should be such that they are not easily hit. If and when targets are hit, then they should be as invulnerable as possible; and if friendly systems are damaged, they should be designed so that they are easily repaired. In summary, it can be seen that much preliminary thought should go into the problem of designing and producing weapon systems for future conflict — once committed to a battlefied environment, it is often too late to enhance survivability. Thus, the best way to assure survivability is to "build it into" systems in the planning stages, and then make the necessary trade-offs through proper analyses.

To the list of the given four areas of effort, we should add another important area of effort—one which has more or less been concentrated on in the past—namely, that of continuing to enhance the effectiveness of weapon systems wherever possible, especially since winning the battle may often be the best way of assuring survivability.

In the paragraphs that follow we will discuss each of the five areas of interest for survivability in order that the analyst will have available an introduction to the general subject and will always remember the role of system survivability in his effectiveness studies.

38-2 DETECTABILITY

Perhaps one of the more important problems of warfare centers around the timely detection and location of targets, so that they can be brought under fire before the enemy can employ his weapons effectively against friendly forces. Enemy targets—especially those that will be threats to friendly advance—should be detected promptly; and recognized and identified as to type, characteristics, and location. In fact, these elements or phases constitute the target acquisition process. It can be said that targets are detected by their "signature", whether the detection is by visual means, sound, radio wave propagation, infrared (ir) radiation, ultraviolet radiation, or other propagation means. Thus, to minimize the chance of being detected by enemy forces, Army equipment and personnel should have reduced signatures wherever possible. Indeed, the techniques of camouflaging systems and personnel, reducing electromagnetic radiation, and the use of acoustic mufflers and ir type suppressors should be considered in equipment design for lowering the chances that the enemy will detect Army systems and personnel on the battlefield. (See Chapter 27 also.)

Ref. 2 gives a good, somewhat detailed discussion of serious enemy surveillance threats—including visual, noise (acoustic and seismic vibrations), infrared/thermal, and electronic means of identification—which we must counter to preclude detection. In this connection, Table 2 of Ref. 2, which is reproduced here as Table 38-1, gives some examples of the various enemy threats to detecting our targets, along with some possible countermeasures we might take to lower the chances of being detected. Also, Table 4 of Ref. 2, reproduced here as Table 38-2, lists some survivability enhancement

TABLE 38-1. HIGH THREAT AREAS—ENEMY CAPABILITIES AND OUR COUNTERMEASURES

Surveillance Threat	Threat	Enemy Capability	Our Countermeasures
Visual	Naked eye Conventional photographs Image intensifiers	Yes	Pattern painting Lightweight screening system (LSS) Smoke
Noise (acoustic and seismic vibrations)	Directional microphones Seismic sensors	Yes	More efficient mufflers Quiet operations
Infrared/thermal	Infrared photographs Infrared searchlight	Yes	Improved paint LSS Smoke
Electromagnetic	Radar Monitoring of radio transmission Detection of spurious radiation Direction finding units	Yes	LSS Jamming Remote antennas

TABLE 38-2. SURVIVABILITY ENHANCEMENT MEASURES FOR CONSIDERATION—DETECTABILITY

Item	Measures for Consideration	Expected Benefit
Aircraft	Smoke-screen warhead for 2.75 in. rocket	Protect aircraft from observation by enemy air defenses and troops on the ground
Vehicles	Noise-reduction — tracks, engines	Reduce distance at which vehicle may be detected by its noise
Vehicles	Signature reduction — ir, acoustic, optical, magnetic, seismic, and gases	Reduce detection probabilities
Personnel	Lightweight camouflaged clothing and equipment	Reduce chances for detection of individual
Vehicles and aircraft	Engine advanced technology — regeneration aircraft engine; reduced noise, reduced ir signature	Reduce signature and maintenance
Various equipment	Camouflage through reflectance of natural environment by mirrors	Reduce detectability
Various equipment	Camouflage through use of urethanes colored at local site	Reduce detectability
Aircraft and ships	Reduced radar cross section	Reduce detectability
Materials	Coatings to increase radar absorption	Reduce detectability

measures for consideration in current activities to reduce the chances of detection of our battlefield systems.

Hopefully, these few brief accounts will serve to acquaint the analyst with a useful overview of the target detectability part of the problem to improve survivability. We emphasize in particular that systems with low signatures or systems which cannot be seen have less chances of being detected, and hence improved chances of survival in a combat environment.

38-3 HITABILITY

Detected targets will usually be shot at and, in spite of all kinds of efforts to avoid detection, some of our weapons and personnel will be identified and located by the enemy during a battle. Once detected, then it becomes of increasing importance to avoid being hit, if at all possible. The coined term "hitability" has therefore been defined as the susceptibility of a target to being hit (Ref. 2).

Small targets are harder to hit than larger ones; consequently, proper attention must be paid to size. For example, low silhouettes are very desirable, and it has long been observed that some Russian tanks have lower silhouettes than their American counterpart. The design of materiel in a manner to preclude bulkiness or large exposed areas should be kept in mind by the engineer.

The shape of a target may be of some importance as far as hitability is concerned, although a more important rule is to present as small an area as possible for any probable detection by the enemy. This would include also the exposure of as small a part of the target as possible by taking full advantage of terrain and vegetation to shield or hide the remaining parts of the target.

The use of suppressive fire against the enemy may be of much importance, because if the enemy cannot freely use his weapons to bring fire on detected or exposed friendly targets, then decided advantages in survivability will be gained for friendly forces. Artillery strikes are valuable for this.

The use of electronic countermeasures to avoid detection or to jam enemy weapons depending on electronics also may clearly improve survivability.

Some targets may avoid being hit by virtue of their speed, acceleration, or by changing direction suddenly. Thus, the maneuverability and the agility of weapon systems may often decrease the chances of being hit, and hence improve the chances of survivability. On the other hand, it must be remembered that moving targets are often more easily detected. For some weapon systems, such as tanks, the reader should recall from Chapter 22 "Mobility, Maneuverability, and Agility", for example, that one of the continuing fundamental problems for overall tank design is to strike the right balance between aggressive firepower, armor protection, and the use of mobility to avoid being hit (in addition to shock action).

Harrassment fire by friendly artillery will also keep the enemy off guard and reduce his chances of bringing effective fire on our systems. Thus, there is much to be gained by judicious use of friendly artillery as a continuing deterrent to enemy efficiency.

Probabilities of hitting depend on the aiming errors of weapons, the round-to-round ballistic dispersion, the target size and the weapon-target range. Thus, any means of provoking the enemy to open fire at as long a range as possible will often enhance survivability through guaranteeing lower hit probabilities. Note in particular that our target size (except for direction) and the enemy's round-to-round ballistic dispersion cannot usually be altered, although some consideration might well be advanced toward trying to increase the opponent's aiming error. This may be accomplished, for example, by mobility, presentation of false targets at different locations, enticing the enemy to fight during adverse weather conditions especially when our forces are on the defense, and any other countermeasures to bring on wild shooting from the enemy.

38-4 VULNERABILITY REDUCTION

The classical approach to improving the survivability of materiel has been through efforts to reduce the vulnerability of systems (targets) to enemy weapon fire. This includes protection from most of the typical damage mechanisms such as fragments; kinetic energy projectiles; shaped charge rounds; blast; thermal and nuclear radiation; and chemical and biological attack mechanisms.

The Ballistic Research Laboratories have always been one of the prime agencies studying the vulnerability of targets to attack and the consequent reduction in vulnerability that might be achieved through certain basic principles. For example, some of the ways for reducing target vulnerability include the use of armor plate to protect personnel and critical components of a system; designing redundancy into the system where applicable; locating critical components behind noncritical components; and using temporary ballistic protection such as "ballistic" or nylon blankets, or sand bags, logs, etc., in the field.

In general, special experiments have to be designed and conducted to study target vulnerability; otherwise the collection of data from analyses of target damage in combat too often leads to uncontrolled and unexplained variations.

Naturally, the armoring of vehicles or systems increases weight and reduces mobility although there might be some optimum trade-off, for example, for tanks. In fact, the design of tanks should be such that the crew is protected by armor, and the ammunition and fuel supplies should not be inside the crew compartment if at all possible. In addition, the use of spall suppression liners on the interior walls and floor aids in crew survivability.

Spaced armor mounted on tanks or other combat vehicles provides protection against shaped charge ammunition, and may also tip monobloc type armor penetrating projectiles, thereby reducing their effectiveness in penetration.

For Army aircraft, one of the prime sources of vulnerability is that of fire hazards. For this reason, self-sealing fuel cells or tanks and various self-closing fittings for fuel systems on Army aircraft have proven to be most successful. Also, plastic foam material has been inserted inside aircraft fuel cells to counter the occurrence of fires from incendiary bullets. Ref. 2, which provides a current overall summary of survivability efforts, gives the following guidelines for reducing fires aboard aircraft:

"1. Place flammable fluid containers or tankage within the airframe to avoid leakage into potential ignition areas.

"2. Provide scuppers or drains to dump leakage overboard into areas where such leakage is not likely to be ignited downstream.

"3. Where structural compartments or voids adjacent to such containers cannot be avoided, provide fire/explosion suppressant materials or extinguishing systems to prevent the ignition or propagation of fires or explosions.

"4. Isolate oxygen systems from flammables. Where such isolation is not practical, provide structural containment or fire barriers.

"5. Provide fire suppression methods in those areas, such as an engine accessory bay, where a sustained fire would cause loss of the aircraft."

Vulnerability may also be reduced by eliminating unnecessarily complex or sophisticated components of systems and by using the principle of miniaturization wherever possible. Also the use of modular construction provides for quick replacement of damaged components (Ref. 2).

Ref. 2 discusses techniques for reducing the vulnerability of stacked ammunition, the use of ballistic blankets and flexible armor around equipment, the use of shrouds for artillery, the protection of communication and electronic equipment, the reduction of vulnerability to helicopter rotor blades, eye protection for armored vehicle crewmen, and other possible areas for decreasing target vulnerability. To show somewhat in general the type of current thinking relative to survivability enhancement measures for further consideration, we present Table 38-3, which is Table 12 of Ref. 2.

Needless to say, the most extensive efforts of past investigations in connection with survivability have been that of studying target vulnerability and the possible implementation of measures to reduce target vulnerability. However, it can be easily seen that armor and passive defense types of protection can only go so far in improving survivability. Hence, in future studies the problem of survivability has to be approached more from an overall point of view, or a more complete operations research type of investigation, for the system in the combat operational environment. What we are saying is that the newer concept of survivability has to make more headway by trade-offs between the use of vulnerability reduction measures and other means of survival such as preventing target detection by the enemy, reducing chances of being hit, promoting ease of repairability of equipment, and enhancing the effectiveness of friendly systems to damage enemy targets as quickly as possible. This also means or calls for better quantification, perhaps best on a probabilistic study basis, of the competing survivability measures which might possibly be implemented.

38-5 REPAIRABILITY

The fourth area of interest covered by the "Survivability Primer" (Ref. 2) is that of repairability. Ref. 2 defines repairability as the "characteristic of military equipment which determines how readily and easily that equipment is repaired or replaced when it sustains combat damage". Most characteristics of repairability for systems or items must be considered and formed during the design stages.

TABLE 38-3. SURVIVABILITY ENHANCEMENT MEASURES FOR CONSIDERATION—VULNERABILITY REDUCTION

Item	Measures Under Consideration	Expected Benefit
Aircraft	Advanced structure design concepts and composite materials, fiberglass rotor blade, composite material rotor hub, composite fuselage, composite tubular rotor, composite transmission housing, boron rotor blade	Reductions in maintenance, in vulnerability, in fatigue of components
Aircraft	Fire-safe fuels	Fewer fires in flight
Aircraft	Ballistically tolerant flight controls such as the bellcrank on the AH-1G	Reduced vulnerability of controls
Aircraft	Improved crash survivability—crashworthy fuel system	Fewer injuries to personnel, reduced damage to aircraft
Aircraft	Improved cargo restraints	Fewer crew injuries under crash conditions
Aircraft	Crashworthy armored seat for helicopter crews	Fewer crew injuries
Aircraft	Ducted replacement for tail rotor	Reduced vulnerability to foreign objects and fewer injuries to personnel
Aircraft	Oil-mist lubrication as emergency back-up in turbines	Provides additional running time for engines with damaged lubrication systems
Helicopters	Sensors of serious blade damage would initiate shaped charges to sever damaged blade and opposite blade	Restores rotor balance after severe ballistic damage to rotor system
Aircraft	Armor blanket	Protect parked aircraft from low velocity fragments
Aircraft (UTTAS)	Suction boost fuel system	Fewer fires in event fuel lines are broken or engine stops running
Aircraft	Integrated actuator package	Reduced vulnerability of hydraulic systems through reduction of vulnerable areas
Aircraft	Lateral axis redundancy for actuators	Reduced vulnerability by providing redundant paths and redundant controls
Aircraft	Oil-starvation-tolerant transmission systems	More aircraft returning to base after sustaining ballistic damage to transmission system
Aircraft	Improved structural adhesives (polyamide adhesive)	Greater load bearing under conditions of metal fatigue
Aircraft (UTTAS)	Minimize hydraulic ram effect	Reduced damage to fuel tanks from penetrators
Aircraft (UTTAS)	Fail-safe lubrication	Reduced damage to moving parts from failure of lubrication system
Aircraft	Transparent plastics for high speed flight vehicles with exceptional mechanical properties at high temperatures	Reduced vulnerability for plastic windshields
Track vehicles	Split track	Reduced damage from mines
Vehicles	Armor kit for fuel tanks	Protect fuel tanks, fewer fires and leaks

(cont'd)

TABLE 38-3 (cont'd)

Item	Measures Under Consideration	Expected Benefit
Personnel	Vest to accept survival components and also serve as body armor	Improved survivability of personnel
Personnel	Lightweight clothing using new fiber types of graded protection from chemical agents, camouflage for the individual, body armor, eye protective devices	Reduction in injuries from the effects of various weapons
Personnel	Transparent armor	Reduced vulnerability of personnel
Personnel	Prophylaxis against lethal chemicals	Fewer personnel casualties from poisoning
Personnel	Fire-resistant clothing for naval personnel	Fewer injuries from fire
Personnel	Buoyant cold weather clothing	Reduce probability of drowning or injury from exposure for personnel forced into water
Personnel	Lightweight armor for trucks	Protect personnel from small arms fire
Personnel	Armor by the yard	Protect personnel in boats, trucks, and emplacements
Personnel	Sealed and actively pressurized flight suit	Personnel protection in event cabin pressure collapses at high altitude
Personnel	Miniature oxygen regulator	Increased reliability for oxygen flow
Artillery	Confining effects of burning ammunition	Reduced casualties and damage due to fires
Ammunition	Decrease sensitivity of propellants to impact by projectiles	Fewer fires and explosions
Cables	Fireproof cable sheath	Fewer fires in cables
Fuel systems	Harden fuel systems to withstand effects of high energy lasers	Reduced vulnerability of fuel system
Materials	Coatings to harden materials against laser damage and increase radar absorption	Reduced damage and fewer fires
Communications	Fault-tolerant digital communication by integrating and time sharing use of circuits	More reliable communications through less vulnerable area and resistance to electromagnetic interference and lighting
Fuel systems	Fuel solidification upon projectile impact	Fuel gelling will result in less leakage following penetration and fewer fires
Transmissions	Integral cooling/lubrication system for transmission	Less vulnerable transmission by virtue of less vulnerable area
Controls	Integrally-armored servo-actuators	Reduced vulnerability to penetrators
Hydraulic system	Silicon-based hydraulic fluids	Reduced flammability
Communications	Propagation of low frequency radio waves	Provides backup communication means in event other systems are useless because of severe ionospheric disturbances
Airbase	Nondestructive pavement	Reduced damage to air strips

Thus, the designer must include ease of repairability and replacement of parts or modules in his overall study of any proposed military system. Preventive maintenance and servicing should be kept firmly in mind, so that the extent of any possible combat damage and its probable effects may be reduced. Nevertheless, the repair of much of the newer Army hardware is made more difficult by the increase in complexity of military systems. Concerning the ease of repairability, however, the following design objectives are listed by Ref. 2 as principles or standards to follow:

"1. Provisions for greater accessibility to equipment and components that may require repair or replacement.

"2. Reduction of mean time to repair/replace a given component to assure combat and operational readiness of the equipment.

"3. Provisions for interchangeability of like components wherever feasible.

"4. Provisions for modular construction as appropriate and design-for-repair wherever feasible."

Finally, we remark that cannibalization, or the use of parts from damaged pieces of equipment to repair another damaged item in order to make it immediately serviceable, must continue to be explored for prompt repairability of systems in combat areas. This procedure is not a recommended solution, however, under normal conditions.

38-6 EXAMPLE OF A SURVIVABILITY STUDY

Since we have so far discussed only some of the more general types of guidelines and rules to improve survivability, it might be helpful to indicate an application of the principles of survivability. There seems little doubt that survivability analysis and applications, even though they are still new, will flourish as an important activity and also as an area of prominence for the systems analyst in his future evaluations. As a case in point, Redwinski and Smith (Ref. 3) have made a study of the HAWK air defense system site and have reported on possible survivability measures to reduce vulnerability or improve survivability in their "Improved HAWK Survivability Primer". The HAWK Battery was chosen since the Improved HAWK system is a very effective air defense system for the Army, and any probable enemy will very likely expend considerable effort and resources to neutralize or destroy such air defense capability.

The approach in Ref. 3 to improve HAWK system survivability consists of the following four essential elements of survivability:

1. Make the HAWK air defense system hard to detect from the air.
2. Make the HAWK system hard to hit if it is detected.
3. Make HAWK hard to kill if hit.
4. Make HAWK easy to repair if damaged.

The commander usually has some control over the first two elements, whereas elements three and four usually must be addressed during the design and production of the HAWK system.

Fig. 38-1 gives a baseline layout for the improved HAWK battery, with the key distances in meters. Also, the direction of the possible main threat is indicated at the top of Fig. 38-1.

The keys to the acronyms in the text and especially Figs. 38-1 and 38-3 of this chapter are:

ICWAR = Improved Continuous Wave Acquisition Radar
IROR = Improved Range Only Radar
ICC = Improved Control Center
IHPIR = Improved Highpower Illuminator Radar
ILCHR = Improved Launcher
ILSCB = Improved Launcher Section Control Box

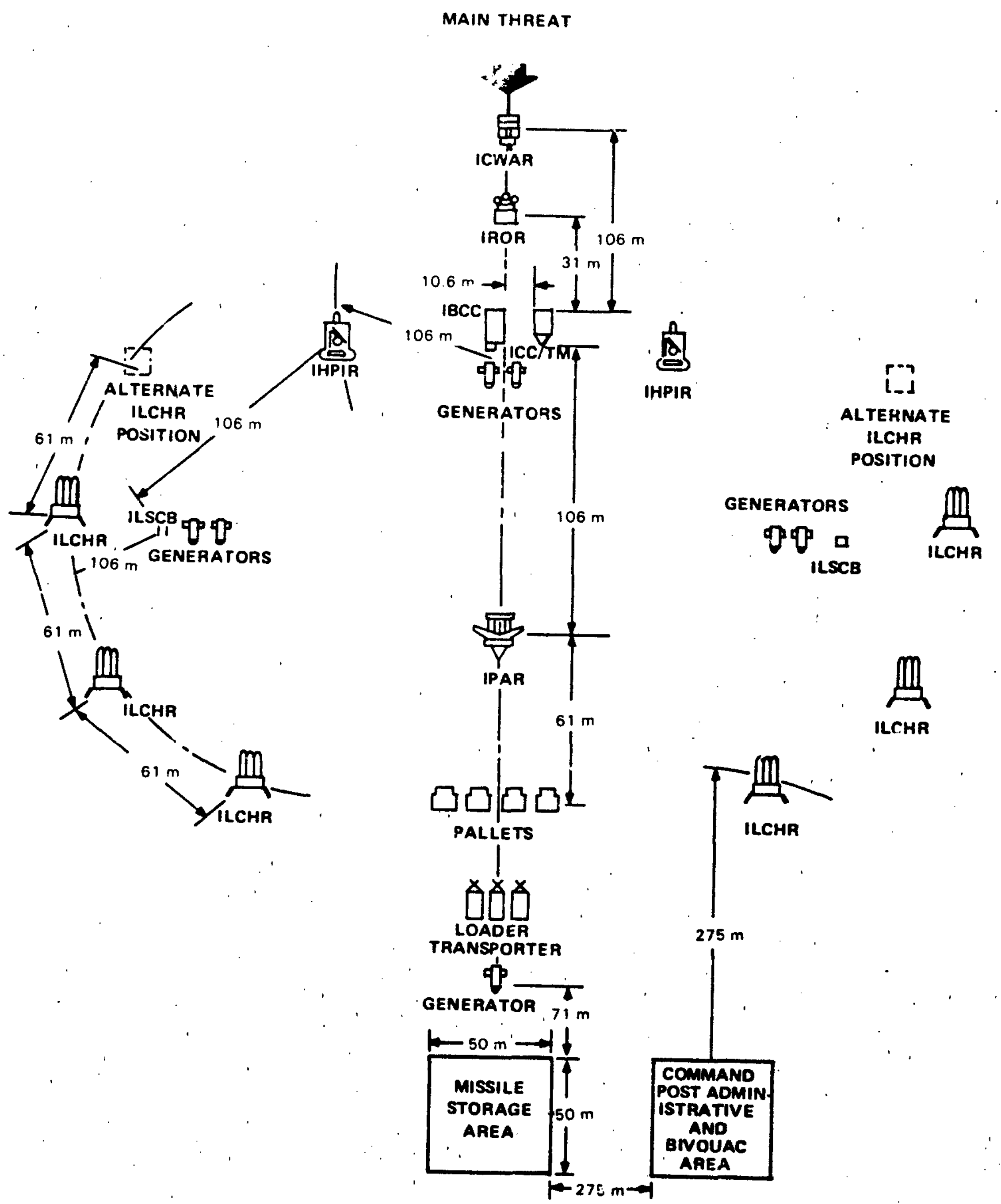

Figure 38-1. Baseline Layout for an Improved HAWK Battery

IPAR = Improved Pulse Acquisition Radar
IBCC = Improved Battery Control Center
IHPI = Improved High Power Illuminator
CW = Continuous Wave
ARM = Antiradiation missile
EMCM = Emission Control
FEBA = Forward Edge of the Battle Area
FROG = Free Rocket Over Ground
SHORADS = Short Range Air Defense System

Actually, it is to be expected that enemy aircraft may attack from more than one direction. The HAWK battery has reasonably good radar coverage against lower altitude aircraft attack. However, the enemy probably would decide to deny one HAWK battery any support from the adjacent one if feasible and be willing to accept the possibility of some exposure and subsequent loss for low altitude attack because of better survival chances. The probable procedure would be that enemy aircraft would begin the attack by employing electronic countermeasures and search visually for the HAWK site. There seems to be little doubt that approaching enemy aircraft will probably select the Improved Pulse Acquisition Radar (IPAR) and the Improved Continuous Wave Acquisition Radar (ICWAR) as the aim points, because any rotation of the antennas "keys in" the target area and would provide a good cue visually. Moreover, it can be expected that 500-lb general purpose bombs would ordinarily be used by enemy aircraft.

Ref. 3 indicates that the attack profile would be that shown in Fig. 38-2 and points out that the enemy attack aircraft have only a short period of time to locate the HAWK site visually, for the HAWK battery will be engaging as many attackers as possible anyway. Each attacker will probably make only a single pass because they know that REDEYE teams and Short Range Air Defense Systems (SHORADS) will be deployed in the vicinity of the HAWK site. This background leads us to consider some pertinent survivability measures for the HAWK battery in its mission.

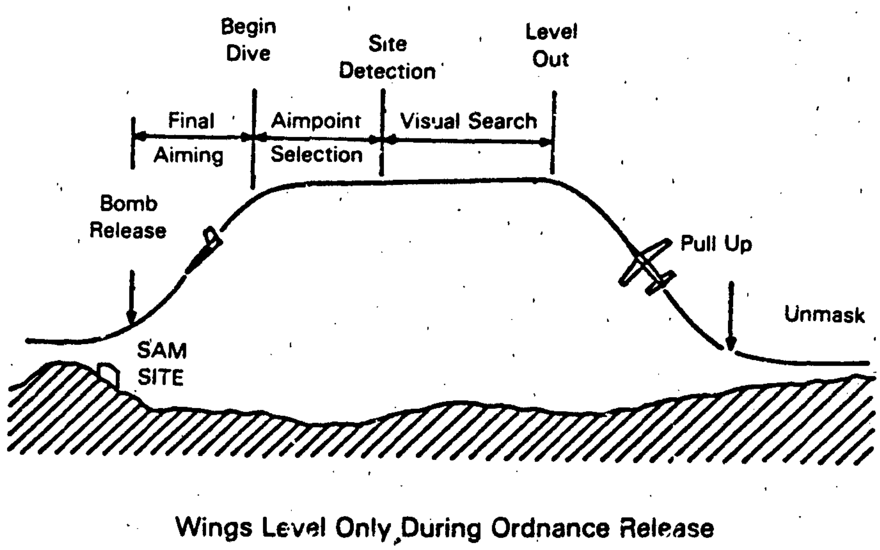

Figure 38-2. Maneuvering Aircraft Attack Profile

The measures which were considered in Ref. 3 for increasing the survivability of HAWK batteries to direct aircraft attack included:

1. The influence of dispersion
2. The importance of site hardening
3. The importance of camouflage
4. The probable impact of decoy sites
5. The importance of HAWK site location
6. The effect of combinations of survivability measures.

The baseline layout in Fig. 38-1 for the improved HAWK battery is based on TM 9-1425-525, and is so recommended. However, a study of European and Korean sites indicates the IPAR and ICWAR radars were deployed only some 40 m and 20 m apart, respectively, whereas the baseline configuration recommends 212 m, perhaps affecting survivability.

Site hardening is especially important and the use of revetments as shown in Fig. 38-3 is especially recommended.

Camouflage will be very effective since it makes the HAWK site much more difficult to detect and hence aim bombs during the attack phase.

Decoys force the enemy to weaken his attack when he mistakenly shoots at them, and hence they are highly recommended.

Ref. 3 recommends placing the HAWK battery along a tree line or slope of a hill since using such a configuration may have the effect of decreasing the engagement envelope by 180 deg.

Naturally, combinations of the various survivability measures, if put into effect, will usually add to more than individual improvements in survivability.

Table 38-4 gives a brief summary of the suggested priorities of Ref. 3 for implementing the various survivability measures for a HAWK battery site.

Ref. 3 warns that survivability defense of the HAWK site should give consideration to the likelihood that the enemy inventory will include two standoff attack weapons, namely, the Antiradiation Missile (ARM) and the Free Rocket Over Ground (FROG) weapon. Survivability against the ARM may be increased by prematurely activating the ARM proximity fuze, interfering with the ARM guidance, shooting at the ARM in flight, and reducing the vulnerability of the HAWK site to blast and fragments discussed earlier, in particular by the use of revetments. ARM guidance degradation can be achieved by shutdown of the radars. The FROG rocket has such a large dispersion in range and deflection that a large number of FROG's would have to be fired at HAWK sites to achieve considerable damage. The FROG threat can likely be ignored, therefore.

It can be seen from this discussion that it will pay to keep the concept of survivability thoroughly in mind throughout the weapon acquisition process, and that the implementation of survivability measures based on systems analysis procedures will not only increase the chances of being able to live on the battlefield, but will also clearly improve the overall capability and effectiveness of our weapon systems.

We should remark, nevertheless, that at the current state of development much of the survivability effort is at the "guidelines" or "general rules" stage, so to speak, and therefore extensive quantification and modeling of survivability parameters are called for in the future to bring about the most reasonable and effective trade-offs. Indeed, it might be said that survivability considerations and recommendations require overall systems analysis procedures to determine their real worth or advisability, especially as compared to the general effectiveness of weapon systems otherwise. With this thought, we turn to some analytical considerations which have a bearing on improving the effectiveness of weapons and hence may have an important effect on survivability also.

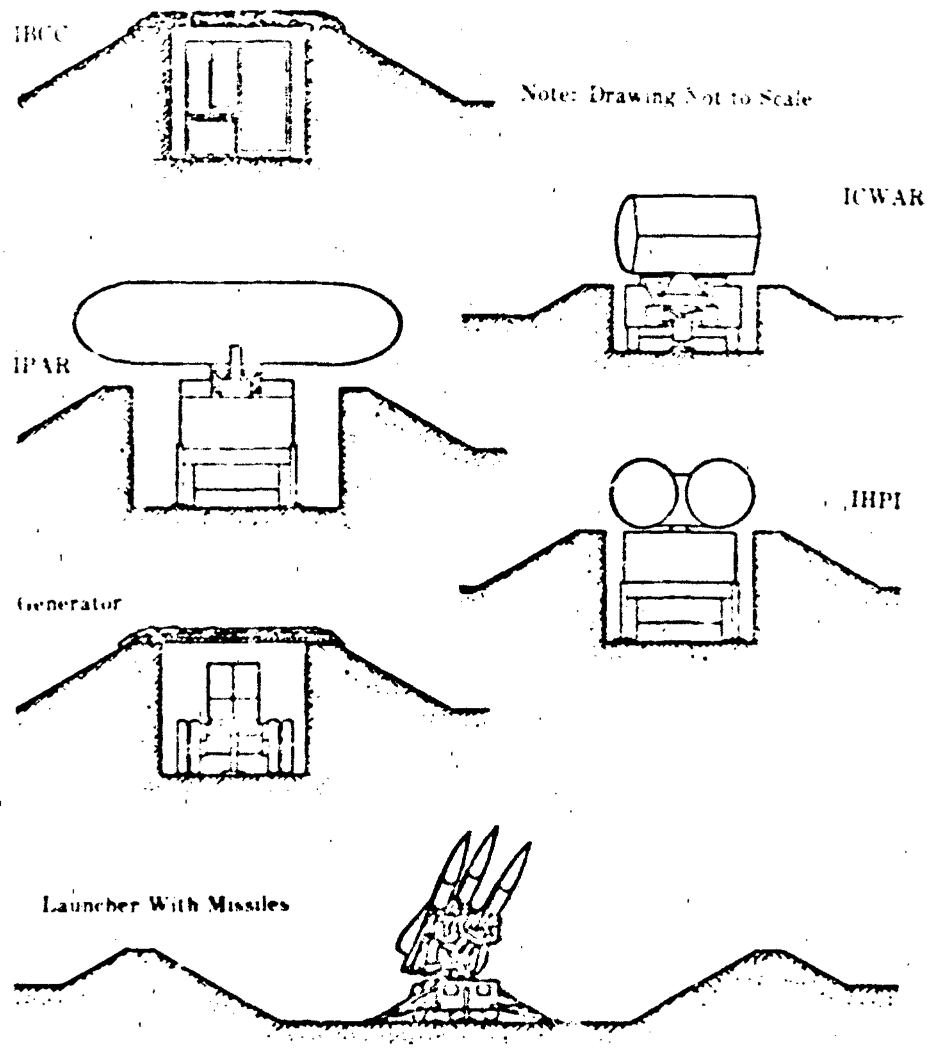

Figure 38-3 Typical Revetments for Improved HAWK Components

38-7 THE ROLE OF SOME ANALYTICAL EFFECTIVENESS MEASURES IN SURVIVABILITY

The role of the weapon systems analyst, as we are well aware, revolves around the problem of evaluating properly all of the so-called measures of effectiveness related to overall system performance. The aim of troops in the field must involve more than implementation of survivability measures for personnel and equipment, fo, otherwise there may be "no contest", and the problem of the systems analyst would be reduced to a most improper role! Thus, the concept of survivability, as we have said, must be evaluated in competition with the other desirable or pertinent characteristics of weapon systems. By this, we simply mean that the key problem is the evaluation of overall performance, and hence we cannot assign any one factor too much prominence or weight. To illustrate a bit, suppose we raise the question, "Just how important are survivability measures of the type discussed so far in this

TABLE 38-4. SURVIVABILITY MEASURES IMPLEMENTATION PRIORITY FOR HAWK BATTERY SITE

Survivability Measures	Poor Visibility Conditions		Good Visibility Conditions	
	Light Attack	Heavy Attack	Light Attack	Heavy Attack
1 Disperse site components to the maximum extent feasible	X	X	X	X
2 Protect cables if spares are not available	X	X	X	X
3 Revet IPAR, ICWAR, IBCC, ICC, both IHPIs		X	X	X
4 Protect cables and use limited camouflage to at least cover earth scarring caused by revetment construction and cable protection measures			X	X
5 Employ one or more decoy sites.				X
6 Employ revetments around rest of components not revetted in Measure No. 3 and employ full camouflage measures				X

chapter as compared to another very desirable measure of effectiveness (MOE) such as the kill rate of a weapon?" We recall that the kill rate of a weapon system is basically the rate of fire multiplied by the probability of hitting, multiplied by the conditional chance that a hit is a target kill. Moreover, kill rate turns out to be the fundamental parameter describing the effectiveness of a weapon for stochastic duels (Chapter 17), target coverage and target damage studies (Chapter 20), combat theory (Chapters 28 and 29), weapon equivalence studies (Chapter 30), optimal firing policies (Chapter 31), weapon-target allocation problems (Chapter 32), cost-effectiveness studies (Chapter 37); and potential kill rates will also reduce the logistic burden (Chapter 23). With such an impressive list indicating the importance of kill rates, it nevertheless is easy to demonstrate that kill rate also means improving the chance of survival. Thus, it is reasonable to assume that combat might well consist of a series of individual stochastic or random duels, or a "bunching-up" of weapons on one side against single or a smaller number of targets on the other side — in which case the kill rates add up quickly! Now, define the following:

$KR(BR)$ = kill rate of a Blue weapon against a Red target (weapon)

$KR(RB)$ = kill rate of a Red weapon against a Blue target (weapon).

Then, for the stochastic duels that are likely to occur in a battle, we know from Chapter 17 that the chance $P(B)$ of Blue winning an individual engagement is practically

$$P(B) = KR(BR)/[KR(BR) + KR(RB)] \tag{38-1}$$

and thus the higher the kill rate for Blue, the higher his chance of winning, or i.e., the greater the chance of survival in an engagement. This obviously leads us to raise the question just asked! In other words, just how should the kill rate of a weapon be traded off with any of the other survivability type measures covered in the chapter so far? We contend, of course, as is quite proper, that survivability is and will continue to be an element of systems analysis procedures which requires appropriate evaluation also.

Let us continue this general line of thought a bit further. System response time may be of much importance in combat as far as the chances of being hit are concerned, for once the two sides detect each other, then that side with the shorter response time will often avoid being hit—to say nothing of obtaining a decided advantage in firing first.

The use of multiple rounds in optimal firing doctrines may also make the difference in superiority, for the expected number of rounds to produce a hit for many targets is greater than one, along with the fact also that multiple rounds, or especially rapidly fired rounds, will increase kill rates of our weapons. The result is that increased kill rates against an enemy means improving our chance of survival in combat. Thus, overwhelming the enemy with firepower will increase the chances of hitting and destroying his forces, while increasing our chances of winning an engagement. This may well mean the difference between survival and defeat, or being killed on the battlefield.

As an important area of endeavor for survivability studies in the future we point out that there exists a huge volume of unanswered questions which center around the incorporation of survivability parameters or considerations in determining survival probabilities for weapons, individuals, and organizations. For example, Robertson (Ref. 4) has developed a model for calculating the survival probabilities of several targets against several weapons. Some survivability models have also been studied by Schoderbeck (Refs. 5 and 6). Dubins and Morgenthaler (Ref. 7) studied the inclusion of detection in survivability models. Thus, there is a need to generalize the concept of survivability as now practiced to effect its proper role in the weapon systems analysis process.

Other evaluations of probable interest to the systems analyst concerning the general subject of survivability include a survivability evaluation of selected communication and electronic equipment by Groves et al. (Ref. 8), helicopter survivability in a REDEYE type threat environment by Hagis (Ref. 9), survival enhancement provided by remote antennas by Sohn (Ref. 10), a comparison of selected methods to improve the survivability of stored nuclear weapons by Westerman (Ref. 11), an aircraft survivability analysis by Paris (Ref. 12), a vulnerability model for weapon sites with interdependent elements by Firstman (Ref. 13), and the minimization of fatalities in a nuclear attack by Owen (Ref. 14).

There is included herewith also a bibliography of some other relevant studies, models, or evaluations.

38-8 SUMMARY

There has been some increasing concern in recent years about the survivability of our personnel, weapons, and materiel systems on the battlefield. The current activity of survivability, therefore, means that much increased emphasis will be placed on the design, production, analysis, and protection of systems and personnel in future conflicts. Survivability investigations are now necessary in the evaluations of all new systems, and the term survivability is to be regarded as an element of systems analysis. The various parameters representing the survivability role of systems undergoing evaluation must be traded off in some optimal way with other competing parameters of effectiveness, or otherwise survivability will be achieved only at some additional cost and may even impair mobility, for example, or other key system characteristics. It is seen that the analyst has an increasing problem in quantifying survivability elements into his evaluations.

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CHAPTER 39

COUNTERMEASURES AND THEIR ANALYTICAL TREATMENT

The importance of considering countermeasures in the evaluations of weapon systems is discussed, and some introductory analytical techniques to evaluate counter and counter-counter measures are covered.

39-0 LIST OF SYMBOLS

A_T = target cross section area, m^2

e, E = efforts expended on Tactics 1 and 2, respectively (see Eq. 39-12)

e_j = the jth action of Red

G = gain of the transmitter, dimensionless

H_0 = null statistical hypothesis of no effect — to be tested

H_1 = alternative statistical hypothesis

I_{a_1} = Blue uses ir jamming

I_{a_2} = Blue does not use ir jamming

I_{e_1} = Red uses ir command guidance

I_{e_2} = Red does not use ir command guidance

I_R = intensity of incident radiation, W•m^{-2}

$\ell(a,e)$ = loss function depending on Blue's action a and Red's action e

m, M = successes associated with Tactics 1 and 2, respectively, when efforts e and E are expended, respectively (Eq. 39-12)

m = number (of tanks, etc.) committed to battle

m_0 = number (of tanks, etc.) lost in m committed to battle

n, N = number of tanks committed under the two different conditions (see Eq. 39-10)

n = number (of tanks, etc.) committed to battle

n_0 = number (of tanks, etc.) lost in n committed to battle)

p, P = probabilities of tank destruction under two different conditions or hypotheses

P_J = jammer power, W

P_T = power source, W

$p(e_j)$ = probability that Red uses action e_j

$p(x_k)$ = probability x_k occurs

$p(e_j|x_k)$ = conditional probability of e_j given x_k

$p(x_k|e_j)$ = conditional probability of x_k given e_j

$p(e_j x_k)$ = chance that e_j and x_k both occur = $p(x_k|e_j) \cdot p(e_j)$

p_1 = true unknown rate of loss for one period of time

p_2 = true unknown rate of loss for another period of time

R = range, m

$R(a_i)$ = Blue's risk for action a_i

R_{ss} = crossover or self-screening range, m^2

s, S = number of tanks destroyed under two different conditions (see Eq. 39-10)

t = variable of integration, dimensionless

u = unit standard normal deviate

x = upper limit of cumulative normal integral

x = a variable

x_k = outcome of observation or experiment under a series of $k = 1, 2, \ldots$ conditions

$Pr(p > P)$ = chance that one unknown probability p exceeds another unknown probability P

$$\phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^{x} \exp(-t^2/2)dt$$ = cumulative standard normal probability intergral

χ^2 = chi-square random variable.

39-1 INTRODUCTION

The practicing weapon systems analyst must be rather well acquainted with the more general principles of countermeasures, for his task is often that of evaluating the potentials of proposed countermeasures. Also, countermeasures are often likely to improve survivability (Chapter 38), and hence they have some interest in their own right. Indeed, war is often based on "see-sawing" advantages anyway.

This chapter provides the analyst with:

1. Basic definitions of concepts
2. An analytic framework
3. Some specific examples of measures, countermeasures, and counter-countermeasures, and their employment.

While it is usually impossible to anticipate enemy reactions to an initiative, there is a rather satisfactory or useful mode of approach to this problem.

The history of warfare can almost be written in terms of measures and countermeasures. The shield was the countermeasure to the arrow and sword, while the battle-ax was the counter-countermeasure to the shield. The tank was the countermeasure to the machine gun, while the antitank mine and shaped charge warheads are counters to the tank threat. While such concepts are simple, there are facets of the subject which bear some close study; therefore, we will discuss them here.

A measure, in the present context, is a weapon, weapon system, or mode of warfare which is capable of neutralizing or destroying an enemy, and against which he has no adequate defense other than flight, evasion, or battle, using a new weapon, weapon system, or mode of warfare designed specifically to counter the measure. A distinction must be drawn between measures and countermeasures on the one hand, and tactics and counter-tactics on the other. Since, however, a new tactical combination or array might be considered a measure and a purely tactical maneuver—such as evasive maneuvering by aircraft under surface-to-air missile attack which may be considered properly to be a countermeasure—the distinction is sometimes a fine one.

Similarly, measure and countermeasure are distinct from attack and counterattack even though they may be associated by tactical considerations. An attack may be made in order to exploit a new measure. The German attack on England with the V-1 pulse or "Buzz" Bomb is a classic association of measure and attack. The British counterattacked in several ways, one of which was through conventional air attack against the launch sites on the continent. This form of counterattack would not be considered a countermeasure. However, the missiles were also attacked in flight by the effective combination of the VT-fuzed projectiles, the M-9 gun director, and the SCR-584 radar, which combination was classified by Herman Kahn (Ref. 1) as a countermeasure.

Any analysis of countermeasures must account for the dynamic element. Between application of a measure and the deployment of a countermeasure, there inevitably will be a time lag which may be of critical importance. During this lag, the possessor of the measure will enjoy a degree of freedom and an advantage which may prove decisive. Such dynamic considerations must be accounted for also in the use of counter-countermeasures. For example, radar has done perhaps as much to revolutionize warfare as any other single technical development. It greatly extended the range and accuracy of target detection, and detection is not only the first, but also an essential requirement of any military action (Chapter 27). Radar detection devices found early and important uses in World War II in the air war over Britain in the summer and fall of 1940. Moreover, the strategic importance of radar was demonstrated repeatedly at sea as well. It is not widely appreciated how close Germany came to victory with her submarine campaign; fortunately, radar played a crucial part in preventing this victory.

In the summer of 1942, the German U-boat Command learned that British patrol aircraft were using radar in the one-meter (old L-band) wavelength band to detect and attack their submarines as they cruised on the surface at night to recharge their batteries. The Germans set to work developing a receiver to monitor these radar signals so that upon being illuminated by the British radar the U-boat could submerge before a detection and position fix could be obtained. The successful development of this receiver, a superheterodyne type having considerable radiant energy of its own, reduced British success. In response to this, the British developed an S-band radar which they used with renewed success. A German counter to this was delayed by their failure to attribute their again increasing losses to the correct source. Two coincidences intervened. First, they had been concerned that infrared detection was being used by the British, and secondly, they thought that the radiation of their superheterodyne L-band receiver was being homed on. The value of such information is clearly seen here The development of an S-band transmitter depended entirely upon the perfection of the magnetron. To the Germans, this technical possibility was unknown. Thus, the upward extension of the frequency spectrum just did not seem, at first, possible. In the world of measure-countermeasure, the watchword or slogan is "Expect the unexpected".

A bizarre note on this period is pointed out by Morse and Kimball in Ref. 2, page 96. The German Air Force captured intact, in Rotterdam, one of these British 10-cm radars in March 1943, yet knowledge of this event and technological characteristics of the device were not made known to the German Navy until six months later. Those six months were crucial to the U-boat effort. The reason for this lapse remains an intriguing mystery. An S-band receiver was eventually developed, i.e., a counter-counter-countermeasure. Later the British developed a counter-counter-counter-countermeasure, an X-band radar, which the Germans quickly again countered.

The measure, countermeasure, counter-countermeasure chain is an endless one. By the nature of warfare, one side does not tell the other what it is doing. The answer to the question, "Why have our losses increased so rapidly?" may be extremely difficult to find. It can be considered in two parts. First, "Is the opposition using some new device or measure?"; if so, "What is it?" The second question is extremely difficult when an entirely new technology breakthrough is possible. If answers are found to these questions, the next question is "What is to be done about it?" This is a problem in part technical and in part tactical, i.e., what and how?

39-2 EMPLOYMENT

Secrecy is vital in military operations and is essential in slowing down the use of countermeasures. Essential to the successful use of the British airborne radar was ignorance on the part of the Germans that it was being used—ignorance even that it was, or could be, in existence. This ignorance arose from the fact that the higher frequencies required totally new and different technologies. The S-band transmitter, as pointed out, depended on the development of the magnetron. Thus, often it will not be known what measure an enemy is employing, or even whether or not he has a new weapon. Clearly, such knowledge is of vital importance.

Such determinations must be made in the light of combat experience, of course. To reach the conclusion that a new weapon or technology is being used by an enemy, the tactical weapon systems analyst needs to have a data base in which time records of appropriate operational measures of effectiveness are maintained. For example, along a stabilized front or in some theater of operations it may be known that tank losses are X tanks per tank engagement per month. If this quantity is viewed as a random variable, then a sudden jump upward in the losses, say by 2 standard deviations, would indicate the possibility at least of some new factor having been added to the tactical picture. A new warhead, fuzing system, fire control system, or similar factor, might have been introduced secretly by the other side. This sort of analysis is the same as that encountered in statistical tests of hypotheses.

In viewing his problem, the analyst or the commander will take the position that it must be decided whether or not the enemy has introduced, in this example, a new antitank weapon. The null hypothesis that could be formed in this case is:

H_0: Nothing has changed, no new measures are being taken by the enemy. The variation noted is due to chance alone.

This hypothesis would have as its alternative the hypothesis H_1, i.e.,

H_1: A new, more effective, antitank weapon is available to the enemy.

Dependent on the degree of seriousness to be associated with a continuation of an unopposed use of a new weapon, a decision could be made that would result in the rejection of a true null hypothesis with a low probability.

Here is an illustration for tank warfare. At some point in time, let us suppose that for a given number of engagements there were m_0 tanks lost in a total of m committed to battle, and for a like period following that point in time there were n_0 tanks lost in n committed to battle. The true rates of loss before and after the point of time selected might be called p_1 and p_2, respectively, but they are unknown. We want to know, however, whether our sample results would give credence to establishing that

$$p_2 > p_1 \tag{39-1}$$

for, in that case, we would conclude that enemy technology or tactics could be such that he is gaining much superiority, and we had better do something about it quickly.

Our null hypothesis H_0 is

$$H_0: \quad p_1 = p_2$$

and our alternate hypothesis H_1 is

$$H_1: \quad p_1 < p_2.$$

For an appropriate statistical test of these hypotheses, we calculate

$$\chi^2 = \frac{(mn_0 - m_0n)^2(m + n)}{mn(m_0 + n_0)(m + n - m_0 - n_0)}$$

which is distributed as the chi-square distribution with one degree of freedom (df). Equivalently, take $u = \sqrt{\chi^2}$, or

$$u = \frac{(mn_0 - m_0n)\sqrt{m + n}}{[mn(m_0 + n_0)(m + n - m_0 - n_0)]^{1/2}} \tag{39-2}$$

where u is a unit standard normal deviate, and make a one-sided test by referring the calculated u to an upper significance level of the standard normal distribution.

Thus, if we select the 95% point, then if the calculated u is less than $u_{0.95} = 1.645$, we conclude that $p_1 = p_2$ and also conclude there is no significant increase in our tank loss rate. But if the calculated u is 1.645 or greater, we conclude that the enemy is gaining an advantage over us, and we had better investigate the cause—which might involve looking for countermeasures.

We might well add here that, while such calculations often might require some special talents not necessarily in the field of interest, it does not pose a great problem. Assistance is readily found due to the availability of statistical or operations research type personnel almost anywhere.

Once a decision has been made—by statistical inference or direct knowledge—that an enemy possesses a new weapon, a combatant will seek a counter to it. In general, the availability of a countermeasure, and its employment, produces a benefit to the employer, but may be subject itself to a counter-countermeasure which could prove disadvantageous. As a result, it might not be very clear to a combatant whether or not he should use the countermeasure.

Here is a concrete example. One side, Red, has a new ir guidance system for an antitank missile. This materially enhances the accuracy of the missile. Blue, to counter, has developed an ir jammer. In response to this, Red uses a home-on-jam (HOJ) missile which can seek out and destroy Blue when he is jamming. The spectrum of tactical possibilities is shown in Fig. 39-1.

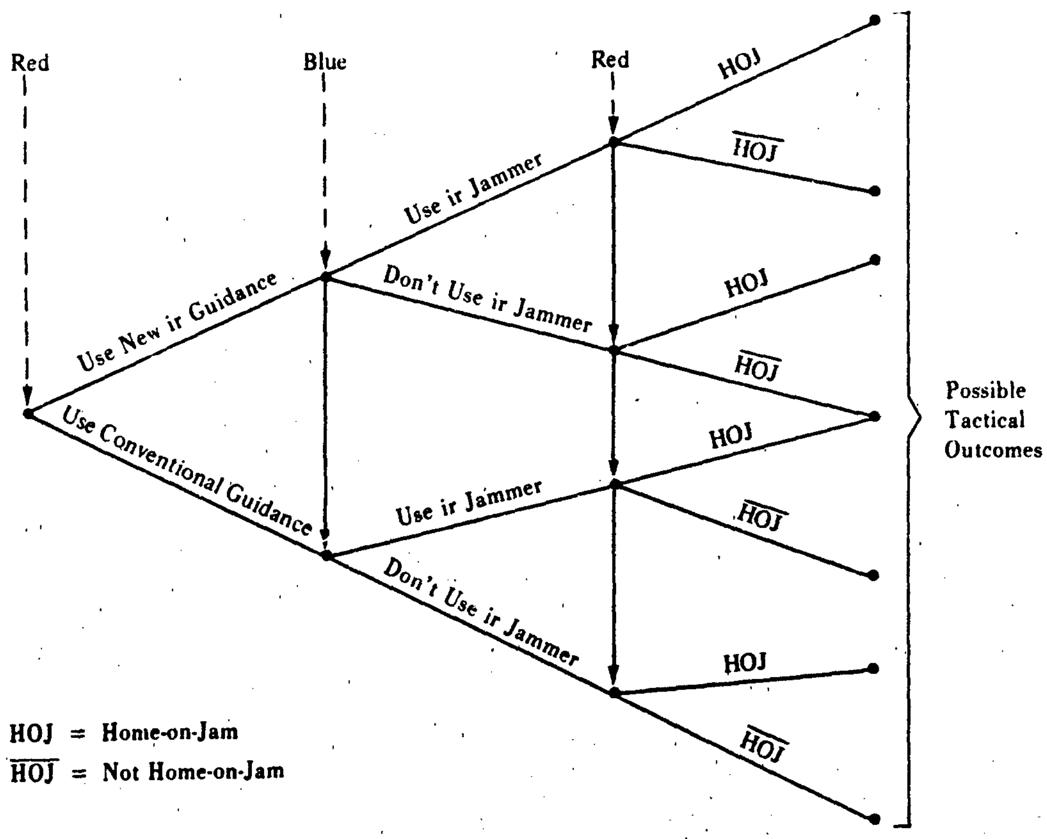


Figure 39-1. Spectrum of Tactical Possibilities

Several choices are open to each side. For the time being, assume that a choice of tactical posture must be made by both sides without knowledge of the decision of the other. In such a case, a payoff matrix can be constructed as shown in Table 39-1.

The ℓ_{ij} are numerical values of some measure of effectiveness (MOE) and, by convention, may be thought of as being the values received by Red. If, in addition, they flow from Blue to Red and Blue's sole objective is to minimize the payoff, while Red's is to maximize the payoff, the problem is called a zero-sum two-person game.

A complete discussion of the solution of such games is not really necessary or appropriate here. However, some conclusions can be drawn by inspection once the matrix of MOE's has been given numerical values. If there were no penalty to Red associated with his use of the home-on-jam missile, then there is no reason why he would fail to use it. The payoff matrix would make this clear by the fact that all elements in the second and fourth columns of the ℓ_{ij} would be as small or smaller than their corresponding elements in the first and third columns, respectively. In this case, it would be said that Red's second and fourth alternatives (strategies) were dominated. By increasing the values of the elements in the first and third columns, Red could always cause a loss as great, or greater, to Blue by using his home-on-jam missile, regardless of Blue's action. Hence, Red would not rationally choose either the second or the fourth course of action.

It is frequently possible to reduce the size of the matrix by examining the columns and rows for dominance. Row dominance in a loss matrix would occur whenever every element in one row is as small or smaller than its corresponding element in any one other row. The latter row is then said to dominate the former row. This discussion is perhaps somewhat informative, but far from complete. See Ref. 3 for more details.

In the game theory context, the decisions to employ the measures, countermeasures, etc., are made in advance and a tactical commitment is then made. An alternative view is possible, however, and is a "dynamic" view based on Bayesian analysis. Assume that the question facing Blue is whether or not to use his jammer. The decision is to be made in the face of uncertainty regarding Red's use of his ir command guidance.

TABLE 39-1. PAYOFF MATRIX

		Red			
		Use ir Guidance		Don't Use ir Guidance	
		Home-On-Jam Missile		Home-On-Jam Missile	
		Yes	No	Yes	No
Blue	Jam	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}
	Don't Jam	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}

Define Blue's loss function as $\ell(a,e)$. In this function, a stands for Blue's action and e stands for Red's action. Define the following terms:

I_{a_1} = Blue uses ir jamming
I_{a_2} = Blue does not use ir jamming
I_{e_1} = Red uses ir command guidance
I_{e_2} = Red does not use ir command guidance.

The list of possible values of e could be enlarged to include Red's use not only of ir command guidance, but also his home-on-jam missiles. To simplify exposition, however, the problem will be limited to the two alternatives.

If it were known, for example, that e_2 (Red does not use ir command guidance) were in effect, all that would be required would be for Blue to choose a_i such that ℓ_{a_i,e_2} would be minimized, i.e., knowing what the enemy is doing makes it possible to choose optimal action in the face of that knowledge. In the absence of effective espionage, however, this will not generally be known. There are several ways to deal with this lack of knowledge, aside from the Game Theory approach. One may be able to say that Red chooses his various strategies without regard for Blue's possible actions by some random process.

The risk for Blue is defined as the expected value of the loss funtion, where the expectation is with respect to the probabilities, or probability distribution, associated with Red's action e_1 and e_2, and is denoted $R(a_i)$,

$$R(a_i) = \sum_{j=1}^{2} \ell(a_i, e_j)p(e_j), \text{ for } i = 1, 2 \tag{39-3}$$

where

$p(e_j)$ = probability that Red is using action e_j and

$$p(e_1) + p(e_2) = 1. \tag{39-4}$$

If Red is using the ir command system, it can reasonably be expected that there will be some evidence of this. The evidence may be direct, such as direct reception of ir radiation of known frequency and modulation; or it may be indirect, such as an unusually high Blue tank casualty rate.

In order to quantify or model the discussion, let x be a number representing an observation, or experiment, designed to tell Blue whether or not the enemy is using ir command guidance. There is no reason, of course, why x cannot be a continuous variable, such as a reading on a continuous scale denoting ir signal intensity; or x may be discrete, such as the number of tank kills per month, etc. For expositional simplicity, let x take only three values: x_1 implies that Red is using ir guidance, x_2 implies that he is not, and x_3 implies a toss-up between Red using ir and not using ir guidance.

Finally, account for possible error in the experimental outcome since the indication could be that Red is using ir guidance when he is not, etc. To do this, let

$$\left.\begin{aligned}
p(x_1|e_1) &= \text{probability of observing } x_1 \text{ when } e_1 \text{ is true}\\
p(x_1|e_2) &= \text{probability of observing } x_1 \text{ when } e_2 \text{ is true}\\
p(x_2|e_1) &= \text{probability of observing } x_2 \text{ when } e_1 \text{ is true}\\
p(x_2|e_2) &= \text{probability of observing } x_2 \text{ when } e_2 \text{ is true}\\
p(x_3|e_1) &= \text{probability of observing } x_3 \text{ when } e_1 \text{ is true}\\
p(x_3|e_2) &= \text{probability of observing } x_3 \text{ when } e_2 \text{ is true}
\end{aligned}\right\} \tag{39-5}$$

where

$$\sum_{k=1}^{3} p(x_k|e_1) = 1 \tag{39-6}$$

and

$$\sum_{k=1}^{3} p(x_k|e_2) = 1. \tag{39-7}$$

The conditional probabilities can be obtained from field testing or deduced from engineering considerations, or from theory. The problem then has been reduced to one of making an observation in the field, noting the outcome x_k designed to reveal enemy measure employment.

If the $p(e_j)$ are referred to as *a priori* probabilities—i.e., probabilities that prevail, on an equally likely basis or preset subjectively on the basis of "experience" or otherwise, and prior to actual observations—what one seeks is the chance of e_j given x_k, i.e.,

$$p(e_j|x_k)$$

i.e., the probability distribution revised or "inverted" on the basis of the observations x_k. This is called the *a posteriori* distribution, and from Bayes' Theorem we obtain

$$p(e_j|x_k) = \frac{p(e_j x_k)}{p(x_k)} = \frac{p(x_k|e_j)\cdot p(e_j)}{\sum_{j=1}^{2} p(x_k|e_j)\cdot p(e_j)} \tag{39-8}$$

where

$$\left. \begin{aligned} p(e_j x_k) &= \text{chance of } e_j \text{ and } x_k \text{ both occurring} \\ &= p(x_k|e_j)\cdot p(e_j). \end{aligned} \right\} \tag{39-9}$$

and

$$p(x_k) = \sum_{j=1}^{2} p(x_k|e_j)\cdot p(e_j).$$

Several aspects of this mode of analysis should be noted.

The first of these is the loss function itself. No rational decision is possible without some idea of the possible conquences. In the worst case, it should at least be possible to assign an ordinal ranking for the consequences, in this case from best to worst.

Secondly, the $p(x_k|e_j)$, the conditional probabilities of the experimental outcomes when the various actions available to the enemy are assumed to be actually taken, can usually be calculated or estimated. Such calculations are often a routine part of the designer's and operations researcher's art.

Finally, and most controversially, there is the *a priori* distribution $p(e_j)$ on the array of possible Red actions. The controversy arises over the question of what is meant by probability, and specifically to the point, what is meant by such a statement as "The probability that Red is using his new ir command guidance system is 0.3.". For a fuller discussion of this very significant question, see Refs. 3, 4, and 5. Under appropriate circumstances, these numbers may be merely the frequency of occurrence of the enemy's past employment of his tactical alternatives. Such an interpretation is not at variance with the classical concepts of probability. However, this distribution has a much different possible interpretation in the "subjectivist" school. Here these probabilities are the degree of belief decision maker or analyst attaches to each of the enemy's possible actions. The justification for this point of view is twofold. First, no universally accepted definition of "probability" exists and, since probability assignments in every case are implied recognition of uncertainty, the degree of belief point of view is as valid as any other. Second, the decision maker, especially the military decision maker who is responsible for success or failure associated with his decisions, must have, in the absence of certain knowledge, a way of making his store of subjective experience bear on his decision. The Bayesian structure thus combines the objective elements, i.e., the $p(x_k|e_j)$ with the subjective elements $\ell(a_i,e_j)$ and $p(e_j)$.

Another variation to this analysis is possible. This is presented as an adaptation from Ref. 2 to the hypothetical tank warfare situation under discussion. Blue is employing his tanks against Red. Red is known to have deployed his new ir guidance system and the question to be answered by Blue is whether or not his jammer is effective. Questions relating to the effectiveness of Red's home-on-jam missile can be easily incorporated as follows. Let

p = probability that Red's missiles destroy a tank when Blue's jammer is not used (Tactic 1) and

P = probability when Blue's jammer is used (Tactic 2).

Blue's question is: "Is $p > P$?", which means, "Is the jammer effective?"

To answer this question, Blue observes the number of tanks s that Red's missiles destroy out of the n attacking tanks when the jammer is not in use, and the number of tanks destroyed S out of the N attacking tanks when his jammer is used—when such information is available. The question is answered approximately on a probabilistic basis by the computation

$$Pr(p > P) \approx \phi \left\{ \left[\frac{s}{n} - \frac{S}{N} \right] / [s(n-s)/n^3 + S(N-S)/N^3]^{1/2} \right\} \tag{39-10}$$

where

$$\phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^{x} \exp(-t^2/2)dt \tag{39-11}$$

$$= \text{cumulative standard normal probability integral.}$$

This result is displayed graphically in Fig. 39-2. For s successes out of n discrete trials of Tactic 1, S successes out of N Trials of Tactic 2, a point is determined on the plot, giving the rule (tactic) to employ.

In the example just given, the effort expended was discrete, since n and N are integers. The operational situation may be more appropriately characterized by continuous endeavor. Ref. 2 (Morse and Kimball) again gives a useful result. In this case, e is a continuous measure of effort using one tactic and E the same measure using the alternative, e.g., time. Under Tactic 1, m successes are achieved; under Tactic 2, M successes. Let p be the effectiveness of Tactic 1, and P of Tactic 2, then

$$Pr(p > P) \approx \phi \left\{ \left(\frac{m}{e} - \frac{M}{E} \right) / [(m/e^2) + (M/E^2)]^{1/2} \right\} \tag{39-12}$$

This result is incorporated graphically in Fig. 39-3 in terms of continuous efforts e and E expended on Tactics 1 and 2, resulting in m and M successes.

39-3 COUNTERING

39-3.1 METHOD

The weapon systems analyst does not ordinarily invent new weapons, nor does he ordinarily invent their countermeasures (although he may). He does need to be able to anticipate more or less successfully the general nature of a counter or counter-counter, however. This kind of anticipation is in part scientific and in part simply art. There are very few advances in science which are unique.

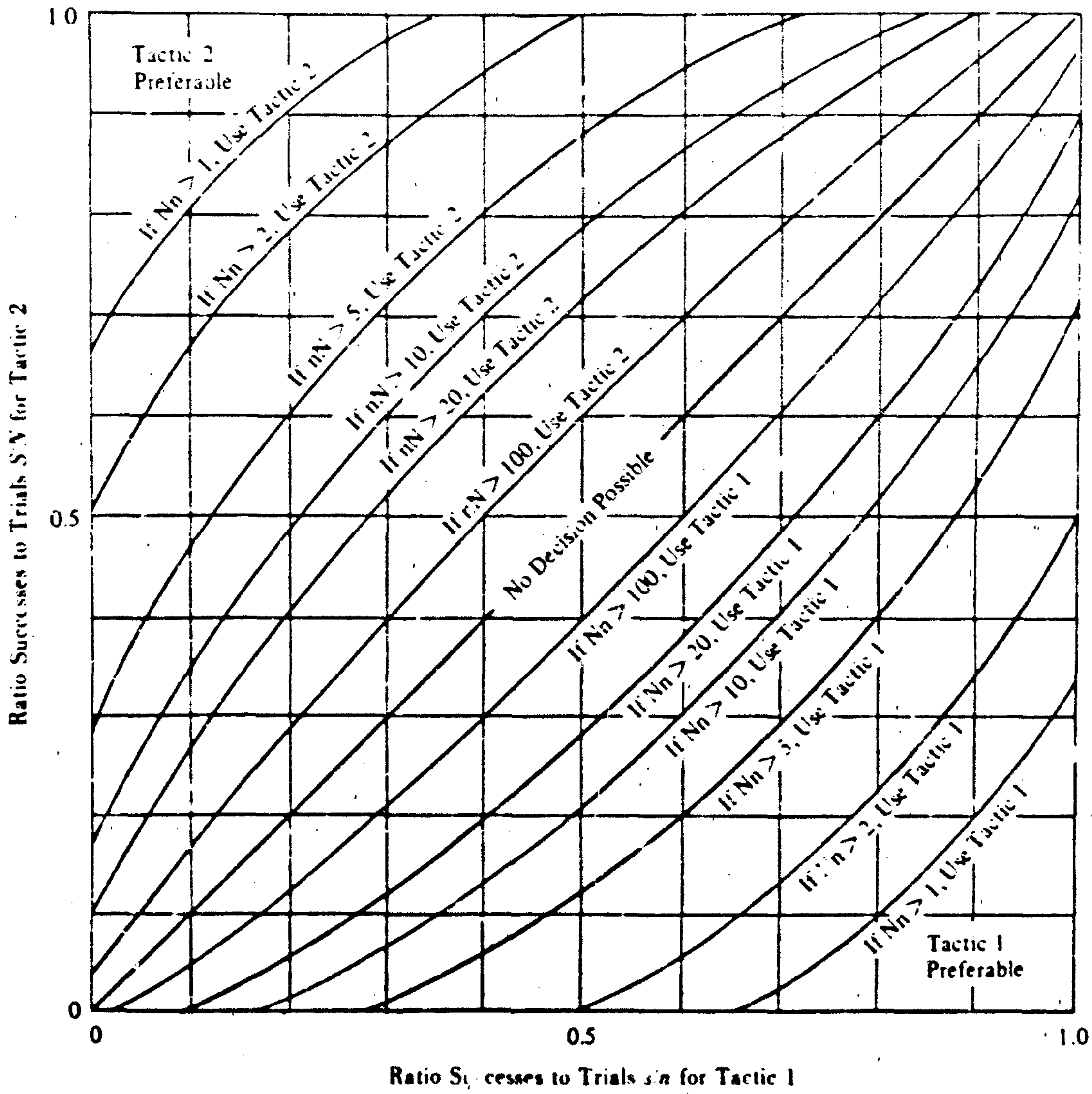
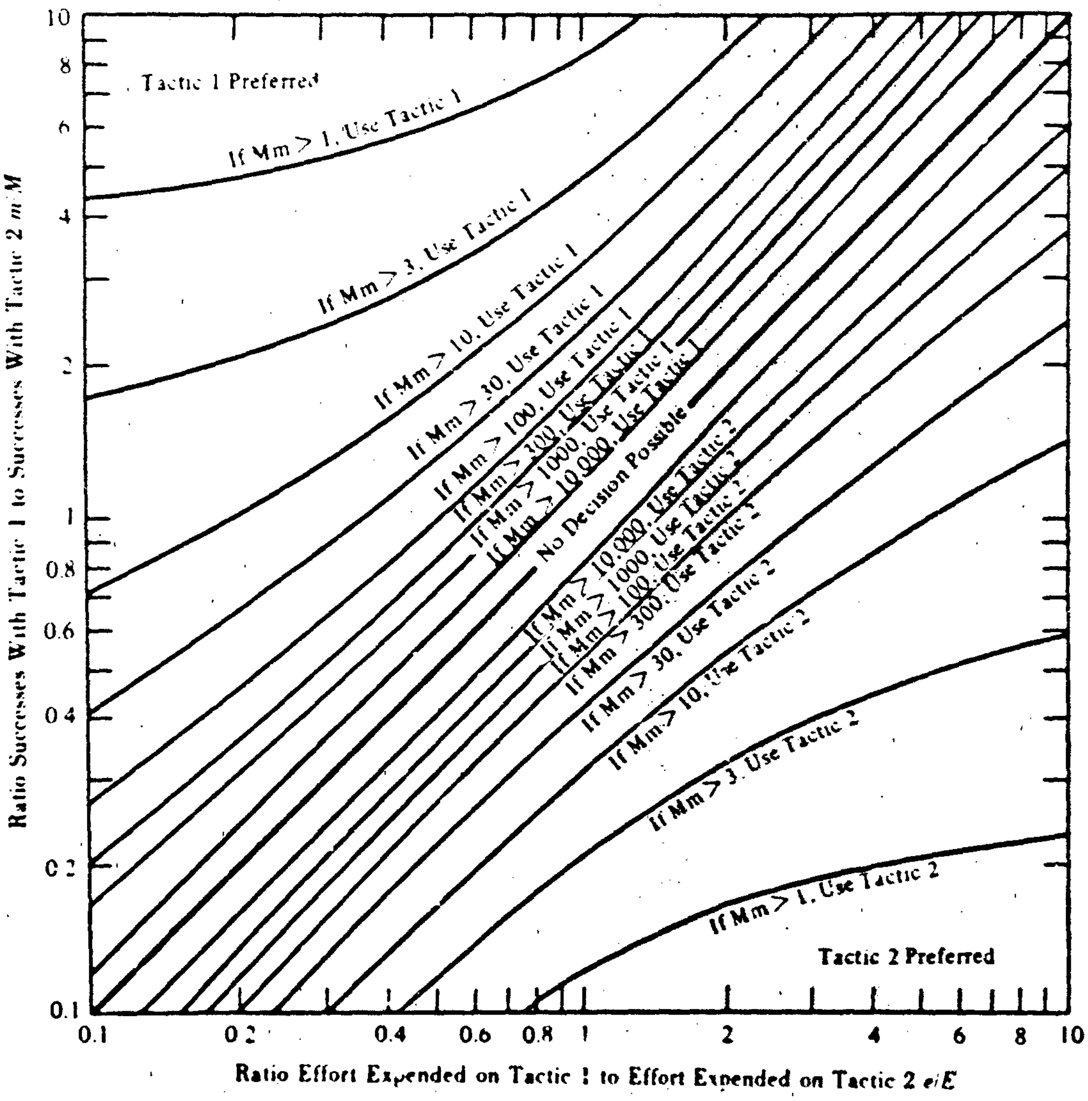


Figure 39-2. For s Successes Out of n Discrete Trials of Tactic 1 and S Successes Out of N Trials of Tactic 2, a Point is Determined on the Plot

Generally, the state of knowledge in any particular field is such that a breakthrough could be accomplished by any of a number of experimenters, even with wartime security measures. This diffusion of knowledge, during peacetime at least, is more or less intentional and in keeping with the scientific tradition, taking place largely through professional literature, symposia, etc. During wartime, the scientists of one nation have the peacetime baseline of their enemy plus their own experiments and



From *Methods of Operations Research*, First Edition, Revised, by P. M. Morse and G. M. Kimball. Copyright © 1950 by The MIT Press. Reprinted by permission of The MIT Press.

Figure 39-3. Continuous Effort e and E Expended on Tactics 1 and 2, Respectively, Resulting in m and M Successes, Respectively

developments to formulate an estimate of enemy capability. This estimate can be enhanced through overt and covert observation of enemy progress. Overt observation can be in the form of a systematic scanning of the several natural phenomena associated with potential weaponry—e.g., the electromagnetic spectrum, radiological background condition, or seismography. Covert observation—i.e., espionage and related activities—while important, is not, however, the subject of this discussion.

A weapon system or subsystem, by its nature, uses energetic processes. Previous chapters have pointed out to some extent how these processes work and how they interact with the enemy. If a causal network is constructed from a measure to its impact on its intended victim—and if this chain is considered in relationship to the natural, physical environment—then a large number of the countering possibilities can be enumerated. Consider, in this light, the German U-boat experience. The link in the causal chain that concerned the Germans was detection. They rightly considered the possibility of detection means other than radar, i.e., ir emission and radiation from their superheterodyne receiver. Fig. 39-4 conveys the idea.

The analysis suggested by Fig. 39-4 is incomplete in that it lacks a dimension, namely, frequency. This is suggested by revising Fig. 39-4 into that shown in Fig. 39-5. The vertical axis may be thought of as "means of detection", the axis perpendicular to the page, labeled "frequency", is the missing dimension.

Identifying the dimensions is where "art", or subjective thinking, is involved. To make this point in extreme fashion, the analyst should note that science is far from certain that all the natural field phenomena—such as electrostatic, magnetic, or gravitational—have been identified. Were some such phenomenon to be discovered in secret and turned to military purposes, an enemy, in seeking to construct a causal network similar to that depicted in Fig. 39-5, would fail because the one essential

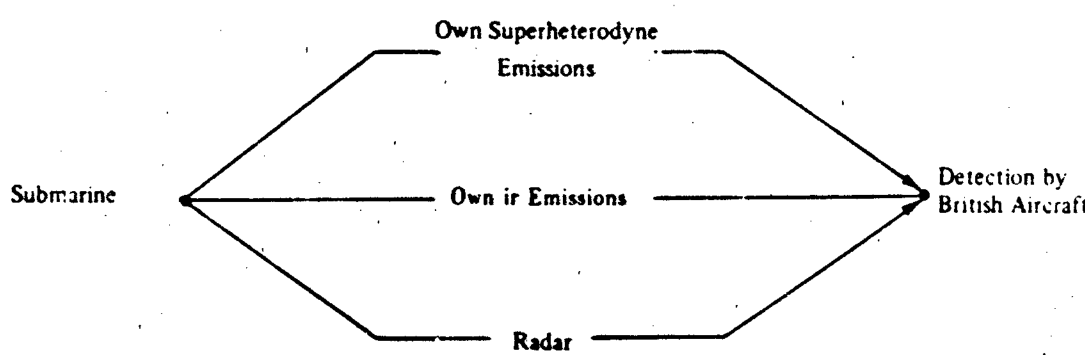


Figure 39-4. Causal Network

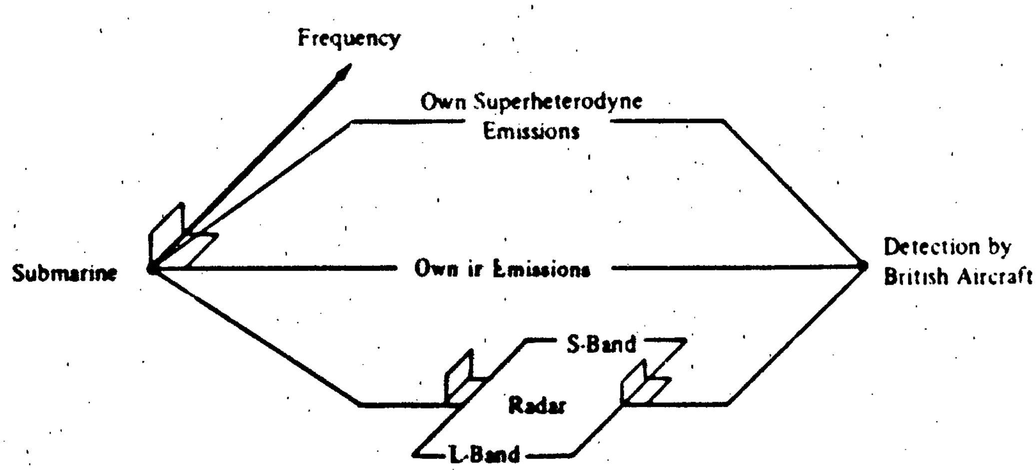


Figure 39-5. Two-Dimensional Causal Network

dimension of his analysis would be missing. Descriptions of light sources, prior to the discovery of the laser, would have been limited to those characteristics which could be defined by the spectral density, i.e., amplitude and frequency. The characteristics now known as "coherence" would most likely not have been mentioned.

39-3.2 MODES OF CLASSIFICATION

Measures, countermeasures, and counter-countermeasures may be classified by whether or not their employment is directly observable by the enemy. This amounts to classification by the energy source which enables them to produce their effect.

39-3.2.1 Passive Mode

The passive mode is characterized by the system being an absorber of energy or information in some channel of interest, even though it may be a net emitter also. A radio direction finder is such a device. It listens in a frequency band, even though it radiates heat energy and perhaps radio energy at some other frequency.

The passive mode characterizes most countermeasures designed to protect. As exemplified by the radio direction finder, many radiation receiving devices are passive devices used for search, detection, and tracking. However, this classification can also be extended to include many other objects and systems designed for defense—such as armor, shielding, and evasive maneuver.

39-3.2.2 Semipassive and Semiactive Modes

These modes are descriptive of information gathering systems. Information must be conveyed by a transfer of energy. If the source of energy is some part of the natural environment, such as the sun, and this energy falls upon some object, is then modulated in some unique way, and then is received and processed by a third object or device, this third device is said to be semipassive. If the source of energy is an object other than the receiver-processor and the subject of observation, and if this energy source is under the control of the observer, then such system is said to be semiactive. A man reading by the light of the sun constitutes a semipassive system, while one reading by candlelight is a semiactive system.

39-3.2.3 Active Mode

Active systems are those that contain the energy necessary for them to produce their designed and desired effect. In the field of sensors, especially those employing echo ranging, e.g., radar and sonar, the energetic source is located within the device. This notion can be extended to include those subsytems which actually fire projectiles; hence, the machine gun is considered an active device. For other obvious reasons, so is the nuclear bomb.

39-4 ELECTRONIC WARFARE

Most of the terminology and concepts used in this chapter were developed to meet the needs of electronic warfare. Electronic warfare is defined as that division of the military use of electronics involving actions taken to prevent or reduce an enemy's effective use of radiated electromagnetic energy and actions taken to ensure our own effective use of radiated electromagnetic energy. Electronic warfare, including the visible portion of the electromagnetic spectrum, embraces both communication systems and missile guidance systems. Hitherto, these activities have been more or less concentrated on gathering and transmitting military information in the search, detection, tracking, or guidance phases of the combat encounter. Recent developments, however, indicate that the laser may be used as a destructive weapon as well as a metering or guidance device.

Measures taken to counter electronic warfare activities are called electronic countermeasures (ECM), and measures taken to counter ECM activities are called electronic counter-countermeasures (ECCM).

The treatment here of electronic warfare—vulnerability, ECM, and ECCM—is elementary, just to introduce these concepts as they relate to countermeasures and counter-countermeasures. Refs. 6-12 present a detailed treatment of the subject.

39-4.1 PASSIVE SYSTEMS

39-4.1.1 Radio and Radar Receivers

Whether they are used as measures or countermeasures, receivers are subject to countering. The simplest and most effective way to counter a passive receiver is simply to shut down transmission.

An alternative to closing down transmission entirely is to transmit at random times for random time durations. If this is coupled with a random selection of frequencies, over some band, then the passive receiver operator is confronted with a search problem that will lessen the flow rate of received information.

It may be that the combatant believes other tactical considerations to be overriding and will elect to radiate rather than shut down. In some instances, shielding may be used, either natural or manmade.

An example of the use of natural shielding is the placement by the US Air Force of its extensive ECM training facilities in the western desert. The purpose was to shield the signals generated there from foreign electronic intelligence (ELINT) trawlers operating off the California coast. The shielding effect is provided by the sharply rising Sierra Nevada Mountain Range. See Ref. 13, page 101.

High gain transmitting antennas, in effect a form of shielding, are often effective counters. By suppressing radiation in all but the preferred direction, the combatant is able to deny his signal to receivers not located along this axis.

Decoys and other deceptions may also be employed. A decoy generates a radiant signal similar in frequency, modulation, and power to that of the real target, but follows a separate path; thus presenting the receiver with the problem of selecting which of possibly many signals belongs to the true target. A totally spurious signal could be generated to confuse an ELINT operator, causing him to attempt to analyze or make inferences concerning a nonexistent electronic weapon.

As has been mentioned, some purely passive receivers radiate in channels, i.e., frequencies, other than those being monitored. Those that do are subject to passive countermeasures such as homing or direction finding.

39-4.1.2 Electro-Optical Receivers

Guidance systems based on television and ir receivers have been shown to be subject to countermeasures. Camouflage, smoke, and other shielding arrangements could effectively counter such devices. If guidance data are transmitted to a warhead, such as a bomb or missile, from a remote command or processing station, then such transmissions could be jammed.

39-4.2 SEMIPASSIVE AND SEMIACTIVE SYSTEMS

These systems employ a longer energy path than do passive systems, since the object of interest must be irradiated. More countering opportunities exist, therefore, along this longer path. Semiactive systems are subject to attack on the illumination source, be it radar or light. For example, a warhead which is guided to its target by a reflected laser-orginated light could be rendered impotent by a successful attack on the source laser. The same principle applies to systems using ir illumination.

Evasive maneuvering can be an effective counter. Such maneuvering can be considered to be either radial, along the line of sight (LOS); or transverse, across the LOS. If the source of illumination is man-made, the effectiveness of the two forms of evasive maneuver will depend on the range. If the illumination is natural, e.g., the sun or moon, then—because of the pervasiveness of the illuminant—maneuvering is not as effective as hiding or using camouflage. Consider man-made illumination, either electromagnetic or acoustic. All such energy transmissions are subject to the inverse square law. Let I_R be the intensity of incident radiation (including acoustic radiation) on a target. Then

$$I_R = P_T G/(4\pi R^2),\ \text{W}\cdot\text{m}^{-2} \tag{39-13}$$

where

P_T = power source, W
G = gain of the transmitter, dimensionless
R = range, m.

Increasing the range, i.e, radial flight, by an amount ΔR will change the incident intensity by

$$\Delta I_R = -P_T G \Delta R/(2\pi R^3). \tag{39-14}$$

Eq. 39-14 shows that at great ranges, i.e., large R, the incremental effect of flight is smaller than when the target is close in.

On the other hand, if the target can evade in a direction perpendicular to the LOS and contact is broken, the radiant source would have to enter a search mode. In general, this requires an increase in beam width, resulting in a reduction in gain if the power is constant. This will, from Eq. 39-13, reduce incident intensity. Since detection probability depends on the signal-to-noise ratio, this reduction could make redetection very difficult, if not impossible. Thus, there will generally be a range beyond which tracking is possible, but redetection is not. At such ranges tangential maneuvering is likely to be effective. At closer ranges, direct radial flight might be preferred.

39-4.3 ACTIVE SYSTEMS

Active systems may require an even longer information path. All of the previously mentioned countering techniques pertain to active systems, plus a few more.

Jamming is perhaps the most effective counter. If conducted at sufficiently high power levels, it can completely negate the effect of the radar. Jamming can itself be countered by frequency shifting or through the use of home-on-jam missiles. If the jammer is the target itself, and if the target is closing on the radar, as an incoming air target would, then there is a range short of which the jammer signal will be less than that of the target itself. This range is called the crossover range or self-screening range R_{ss} and is given by

$$R_{ss} = [P_T G A_T/(4\pi P_j)]^{1/2},\ \text{m}^2 \tag{39-15}$$

where

P_T = transmitter power, W
G = antenna gain, dimensionless
A_T = target cross section area, m^2
P_j = jammer power, W.

Another counter to radar is chaff. Airborne targets, such as attacking aircraft, may be preceded by other aircraft which dispense the chaff particles or ribbons. Chaff is composed of lightweight conducting materials cut to lengths designed to produce maximum echo intensity in the radar. Thus, an approach corridor of more or less solid radar retu... is established within which raiding aircraft can approach. The radar operator may know that attacking aircraft are approaching, but he may not be able to estimate their number or altitude, nor will he be able to conduct intercepts or provide a missile battery with an acquisition assignment. Several tactical variations can be built around a basic chaff attack, one being the creation of diversionary corridors against which the radar operator and his defense will have to respond, but within which there will be no attacking aircraft. See Ref. 13 for details.

An attacking aircraft may dispense its own chaff. Thus it will be continually moving out of its chaff cover, but some protection is provided.

Decoys are also used to counter radar. A decoy is simply a device having the same radar echo as the target which it protects. It will move at like speeds and maneuver in similar fashion to the real target. Decoys may also be used to protect stationary targets.

Coating materials are available which reduce the radar reflectivity of a target.

Several forms of deception are possible for targets having sufficiently sophisticated equipment. The radar target could carry a receiving device with the capability to measure the pulse repetition rate, sweep rate, frequency, pulse length, etc., and then generate spurious signals at times before and after its own illumination by the radar. If these signals are emitted at the correct power and are coordinated properly, the radar operator is presented with a multitude of false targets at various ranges and bearings moving on various paths. A variant of this, somewhat akin to jamming, is to amplify the received radar pulse and transmit it back with a near zero delay time and thus saturate the radar receiver.

A counter to this form of deception, a counter-countermeasure, is to vary the pulse repetition rate randomly. This is called jittering. By doing this, the time of the next target illumination is unknown to the target. A counter to this is possible if the jitter rate is not truly random but follows a discernible pattern.

Finally, the anticipated use of high powered coherent light beams, generated by continuous wave lasers, is becoming a reality. Ref. 14, page 17, cites the achievement of continuous power outputs of 8.8 kW, which is more than enough to cause physical damage to various kinds of targets. Anticipated uses of such devices range from antiair warfare and missile defense to direct battlefield application. Since the destructive energy travels at 186,300 $mi \cdot s^{-1}$ (the speed of light), the fire control problem is simplified, and the waiting time to evaluate results is reduced. Potential countermeasures are shielding and, due to the extremely narrow beamwidth, evasive maneuver.

39-5 COUNTERING DAMAGE MECHANISMS

Throughout this handbook, we have discussed the means and mechanisms by which a target may be damaged or killed, often evaluating the specific ways in which the mechanisms produce their destructive effect. By applying appropriate methodology, the analyst can develop an exhaustive list of possible countermeasures. This technique amounts to an algorithm for producing such lists, and for this reason it is unnecessary to produce such an exhaustive list here. An example of the method may be provided, however.

Example: Nuclear warheads produce the following effects:

1. Blast
2. Heat
3. Radiation:
   a. Initial:
      (1) Neutrons
      (2) Alpha particles

(3) Beta particles
(4) Gamma rays

b. Residual, radioactive isotopes.

Thus, to construct a list of potential countermeasures to a nuclear warhead, examine each effect and consider a countermeasure. Consider the radiant heat accompanying the explosion and fireball, for example. Water particles, dust, and other airborne particles between the source and the target greatly attenuate and scatter infrared radiation. Thus, a blanket of fog or smoke might protect a target from a large proportion of the radiant heat. Although naturally occurring fog or smoke cannot be relied upon to be present at the time of attack, generating fog or smoke cheaply and in quantity may be listed as a counter. This has, in fact, been considered. Ref. 15 shows that 90% of incident energy can be shielded at costs as low as $33 per person shielded. This reference also discusses further refinements and approaches to this kind of shielding. Perhaps this type of approach will be of some help to the analyst.

39-6 SUMMARY

Measures of military importance arise in two ways:

1. In response to perceived military needs
2. To exploit a new scientific development or capability.

A perceived military need may not necessarily find a means of satisfaction however, and a new technological advance need not have a military application. There are thus elements of uncertainty and chance permeating the entire area of weapon systems analysis. The analyst must, therefore, allow his imagination some reign, but at the same time he must possess scientific knowledge, at least to the extent that excessive time and funds are not expended in pursuit of objectives which require violation of the known physical laws, e.g., the second law of thermodynamics. On this last point, even here the analyst's door must always remain slightly ajar!

The analyst's ability to forecast the overall effect of a system depends in large part on his ability to anticipate countermeasures which the enemy might take. The effectiveness of a system, or its "measure", is a conditional thing. Its potential worth is inversely related to the ease with which an enemy can counter it, and this point must be strongly emphasized.

There are two ways that counters can be anticipated:

1. Search for effective counters in the space of the "measure".
2. Search for the counter in an enlarged space, i.e., search for ways in which the space could be enlarged.

The first method is easier. To counter a heavier arrow, where the space is the dimension termed kinetic energy, one could build a thicker shield, which seeks only to diminish the effect in the original dimension. However, an order of magnitude improvement, an increasing return, may be gained if the arrow is countered in a new and unexpected way, e.g., by gunfire. Here the new dimension can be viewed as "counter with chemical energy".

In order to search for a counter in an enlarged space, the analyst breaks down the process in question into its most essential characteristics, supplying as he can dimensions which are not really in the state of the art, but which are feasible or conceivable. These define the regions in which the least expected, and therefore most potentially effective, counters are possibly to be found.

There remains much room for novel analyses or play of games.

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CHAPTER 40

INTRODUCTION TO WAR GAMES AND COMPUTERIZED SIMULATIONS OF COMBAT

A historical sketch is given of war games, their development, and uses or play over the many, many years of military experience with them. Computerized simulations of combat are also covered in some illustrative detail, and many of the curr t combat analysis models are summarized or highlighted for the new or young weapon systems analyst. War games and other simulations of combat have been found to be of enormous benefit in the training of military strategists and tacticians. Also combat analysis models represent the best currently known attainments in military operations research to study the effectiveness of tactics, weapons, and the environment in two-sided, complex conflicts between opposing forces. However, some cautionary measures must be taken to apply and develop further many of the combat analysis models — especially in connection with guaranteeing realism, accounting properly for interactive processes, and establishing significant results through analytical means. The chapter is concluded with the idea of a "near real time casualty assessment" type of field experiment which might be used as an aid in the validation process of ome combat analysis models.

40-0 LIST OF SYMBOLS

A = remaining number of rounds
a = standard deviation in dispersion at zero range
b = standard deviation in dispersion at 0.707 maximum range
c = standard deviation in dispersion at maximum range
DGF = radar degradation factor
d = constant
F = number of weapons fired
H = total number of hits = $\sum_{i=1}^{z} h(i)$
$h(i)$ = designation which is 1 if a hit occurs and zero if not
k = designation which is 1 if a target kill is obtained and is zero otherwise
k = firing symbol for CARMONETTE (or a "kill")
M = square of the maximum range of the weapon employment in CARMONETTE
N = number of rounds per trigger pull
$P(k|h)$ = conditional chance that a hit is a kill
$P(r)$ = single-shot hit probability at range r
P_D = chance of target detection
p = priority
R = equivalent radius of ("circular") target
RR_{adj} = $r/[(DGF)(r_{max})]$ = adjusted range ratio
r = observer-target range
r = rate
r = range
r_{max} = maximum effective range
$s_{i,t-\Delta t}$ = speed of unit i at time $(t-\Delta t)$

t = time
Δt = small time increment
tt = given time in a simulation
x, y = coordinates for two dimensions
xx, yy = representation for a square or grid in a simulation
$x(i)$ = random number generated from a uniform distribution
$x(j)$ = random number generated from a uniform distribution
x_{it}, y_{it} = coordinates at time t for unit i
$x_{i,t-\Delta t}, y_{i,t-\Delta t}$ = coordinates at time $(t - \Delta t)$ for unit i
z = minimum of A and $NF = \min(A, NF)$
$\theta_{i,t-\Delta t}$ = azimuth on which unit i is moving at the instant $(t - \Delta t)$
$\sigma(r)$ = standard deviation of weapon dispersion at range r

40-1 INTRODUCTION

Because of the complexities that may be involved, it is almost impossible or at least very improbable to find suitably accurate analytical models for describing all types of combat, especially the more complex ground battles among heterogeneous forces. The better military decisions, therefore, must depend on expert or suitably capable fighting experience, or special training and study. Moreover, when a major nation is involved, as is always true, in the development of new weapons, the task of finding the best ways to fight a potential enemy becomes more critical. Then again, there is also the need to train new troops and new commanders for the possibility of military conflict with all probable enemies. This latter problem often has been solved by the use of war games, or simulations of combat conditions of interest. With the advent of the high-speed electronic computer, there exists a huge capability to simulate battles and conduct appropriate analyses of all kinds to study various hypotheses concerning weapons, tactics, the effectiveness of different individuals as commanders, or other military problems of interest.

The experience in military operations research in the last decade has established the need for increasing the emphasis on operational gaming as a research and development tool, as well as for training purposes. Since the young analyst will very likely be required to program and conduct combat simulations of different kinds in his various applications of weapon systems analysis techniques, it is important to give an informative account of war games and combat simulations.

40-2 WAR GAMES

40-2.1 SOME HISTORICAL HIGHLIGHTS UP TO WORLD WAR I

A study of pertinent literature on the subject will show that the concept of war games is very old. In fact, war games may be traced to very early times when they involved mostly a modification of the game of chess, used principally for pleasure, and advanced all the way through to a period of great importance in the military profession as an aid to training officers. A good historical account of war games is given by Young in Ref. 1. Young points out that as the value of the war game grew over the years, so did the complexity of the scheduled play to represent reality.

For our purposes, war games may be defined as an imaginary military operation—usually conducted on a map or terrain board—employing various movable devices which are intended to represent the opposing (Blue) and (Red) forces, and which are moved about to reflect the conditions of actual warfare. Thus it is seen that many different types of possible military operations may be played—including offensive or defensive type operations, infantry battles, tank battles, small engagements, large engagements, or even beachhead type activities. Obviously, the potential of military games is very great, especially if the players can represent realism accurately. Thus the war game offers an invaluable tool for training and other purposes. In fact, the amount of intelligence may be controlled on both sides so that very informative results may be derived from appropriately played war games.

We quote from Young (Ref. 1) and summarize much of his contributions which are needed for the inexperienced analyst:

"Most war games are played on a map, although sand tables are often used, and small blocks or special pins are used to represent troops and their equipment. When blocks are used, they are often made to scale and occupy a proportionate area on the map. The blocks are moved under the supervision of an umpire and according to the desires of the opposing players at rates determined by the various arms of service represented. When the blocks have attained positions so that there exists a good possibility that hostile troops are within sight or range of each other, it is assumed that a battle will occur. It then becomes the umpire's duty to decide the result. This decision, depending on the type of war game being played, may be based solely on the judgment and experience of the umpire, or it may be based on prepared data, charts, tables, calculations, and other detailed aids. Data are frequently obtained from similar circumstances in actual battles. Occasionally, dice or Monte Carlo methods are used to decide the outcome of those situations involving chance elements.

"Many of the modern map maneuvers utilize celluloid sheets, which are placed over the maps, and on which are penciled the positions of the opposing troops and their formations. After each phase of a battle, such sheets may be stored for future reference, or the penciled marks may be erased and redrawn according to the changing conditions.

[One of the very early war games was "Kriegsspiel", or war-play, which represented a detailed combat simulation that originated in the early Prussian Army. The primary use of Kriegsspiel was to study tactics and strategy, and for this purpose it was played by high-ranking officers of the Prussian Army.]

"The apparatus required for playing Kriegsspiel included maps that were carefully drawn to scales of 6 to 8 inches to the mile; blocks that were proportioned for use with the maps and were intended to represent the various branches of the service involved; and strings of beads that were laid on the maps and were used to represent frontages of formations and movements of cavalry. The game was directed by an umpire, who had several assistants available and who determined the course of the game after evaluating the movements and decisions made by players assigned the command of the opposing armies Neither of the commanders was permitted to see in detail the dispositions of the opposing forces. Limited information was given to each commander regarding strength and disposition of the enemy, state of the roads, season of the year, and the supply situation. Tables and charts were usually prepared in advance from which losses were calculated by the umpire, and those situations which could not be resolved by the use of such data were frequently decided by dice. The action in the game was developed until a point was reached whereby victory could be declared for one side or the other.

"According to Farrow (Ref. 2):

The principal utility of the [Kriegsspiel] game appears to be in the arrangements previous to and during the early conduct of the action. When the troops get to close quarters, the element of chance enters so largely into the game that it destroys to a very great extent the dependence that may be placed on the issue of the battle. The game, however, affords great practice in the drawing up of the order of march of columns previous to any action, and the development of the columns of march into formation for attack. In the hands of men having some military experience, this game becomes a certain means of acquiring and perfecting a science which in time of peace cannot be easily acquired. It raises questions which are strategical problems of great interest."

Thus we see that even though war games often are played according to some very definite rules and may be controlled to various desired extents as deemed necessary, there ultimately may come a stage at which randomness sets in and could more or less take over, so to speak, as often occurs in combat between forces. Games that are controlled in one way or the other are often referred to as "deterministic" or "rigid", whereas if chance elements are somewhat predominant—as is often the case in actual combat—such war games are termed "stochastic". Obviously, the personnel conducting the war games have a very decided effect on the battle simulated and hence on the usefulness of results or lessons learned—regardless of whether the war games are for training purposes or for studying the effectiveness of tactics, weapons, etc., or the effect of terrain or intelligence. Hence war games are about as good and useful in drawing proper inferences as is the very wisdom of the personnel planning or conducting the games.

Points in favor of conducting war games are that they are relatively inexpensive, they may be played over and over again with variations likely to be expected under real combat situations, and analyses of the outcomes may be examined for possible inferences to future conflicts.

Well-known games such as checkers, chess, bridge, etc., are played according to very strict rules, and the competition between players must proceed according to such boundary conditions. On the other hand, games simulating combat between two military forces permit much more leeway and involve the experienced judgment of field commanders so that the better ways of fighting under various sets of conditions in the field can be studied with some realism. Moreover, a voluminous and accurate amount of bookkeeping usually is required in the conduct of war games, even to make valid comparisons or improve the training of personnel.

The Germans were not the only ones to employ war games extensively. Farrow (Ref. 2) points out that the Americans developed a series of war games called "Strategos"—Strategos comes from the Greek word for general—based on military principles which were designed to provide training and to assist both beginners and advanced students in war gaming procedures applied to studies of tactics, strategy, military history, and other aspects of war. Whereas Kriegsspiel was apparently used primarily for part-time play and to train the upper ranks of the military, Strategos was divided into two parts—the Battle Game and the Advanced Game—which it was argued was more detailed and comprehensive; this enabled both the lower and the upper ranks to be trained and to benefit from such games.

According to Lt. Totten (Ref. 3):

"The *battle game* was played on a game board using various blocks to represent armies of any size and organization. The different components of the armies were assigned special moves and powers, and very close attention was paid to distances and orders of battle. The progress of the game was governed by carefully compiled rules. However, the game was generally kept simple, although rather artificial, and was designed primarily to educate novices to the point where the more advanced game could be played. The battle game was intended to permit military men of any rank to practice the organization of forces, their dispositions, the formation of battle lines, and orders of battle.

"The inclusion of the battle game in Strategos was an application that was claimed to have been . . . completely overlooked in the haste to present an advanced and necessarily very complicated game to those few special students whose interest and professional studies may lead them to it. The great mass of [American] military aspirants . . . will thus always fail to find interest in Kriegsspiel because of its complexity at the very outset. This too, in small degree accounts for the extreme slowness with which this game [Kriegsspiel], though so long in existence abroad, becomes known in America even among officers of the regular army, who alone, perhaps, as a class, can afford to be constant players of it. But

Strategos . . . is not open to these objections, nor does it by any means neglect the special wants of those few whose extensive knowledge of the military art, science, and history, and whose more than passing object in studying these matters would demand the highest and most scientific application of the outfit. With such an application, embodying all that is valuable in the German and English games, and introducing many new and noticeable improvements in the matter of methods, men and tables . . . the game of Strategos does naturally and appropriately terminate. Its advanced game thus affords to the professional military men every opportunity that could be desired for pursuing studies, commenced in more elementary fields to their legitimate termination. In this . . . all arbitrary assignments of values and moves are of course entirely out of the question and improper. The whole game is required to base itself upon actualities, upon the results of careful investigations, and upon the tabulated statistics of experience, of actual practice and of former battles and campaigns.

"Considerable effort was made in the *Advanced Game* portion of Strategos to provide a more complex type of game suitable for students with more experience and military knowledge. An attempt was made to conform to the best military knowledge available, and to make the game, through its direction, rules, orders of procedure, tables and charts, approximate as closely as possible all the features that would be present in actual battle."

Young (Ref. 1) gives an account of "ancient" war games to about 1824; he discusses the uses of mathematics and scientific principles for developing models of combat as the "vogue of military mathematics" from about 1700 to 1800, and then covers developments in the early nineteenth century. It is an interesting historical note that Napoleon did not become very fascinated with the "vogue of military mathematics" applied to war games at the time. In fact, Napoleon's tactics showed very little respect for such then-prevailing concepts of fighting wars—yet he swept victoriously across Europe! It is said (Ref. 1) that Napoleon probably planned his campaigns well in advance by maneuvering pins with colored heads over detailed maps of the actual scenes of the expected military operation. Such actions also may have aided the Germans very much in both World War I and World War II because they apparently developed such techniques of planning and practice for fighting battles to a marked degree.

The credit for the war game as we know it today should probably go to Lt. von Reisswitz, Jr., who was a Lieutenant in the Prussian Guard Artillery and a member of the Artillery Examining Committee. Lt. von Reisswitz acquired his interest in the war game from his father, who was a civilian and also the Prussian War Counselor at Breslau (Ref. 1). The elder von Reisswitz made a significant advance in the war game by transferring it in 1811 from the chessboard to the sand table which used a scale of 1:2373. Troops were represented by squares of wood on which symbols for the branches of the service were pasted, and more realistic maneuvering and marching of columns were made possible because they were not restricted to the squares of the chessboard.

Lt. von Reisswitz conceived the general idea of adapting the war game to actual military operations or campaigns, and in 1824 he transferred the game to realistic map-like charts with a scale of 1:8000. Young (Ref. 1) gives the following account of the apparatus and method of play for Lt. von Reisswitz's war game.

"The game maps, although very realistic were "idealized"; they did not represent *actual* terrain. They were usually drawn to a scale of 1:8000, and showed approximately 4 square miles of ground. Terrain details were made as complete as possible, considering the state of the art of mapmaking during this period. Blocks or pawns, which represented the troops, were made of little squares of lead, and were painted red for one side and blue for the other. They were made to the same scale as the map used, and conventional symbols were marked on the blocks in order to indicate the arm of the service and the numerical strength of the organic subdivisions. The game outfit included dice, dividers, and scales of distances and ranges.

"The number of players varied. Since subordinates were permitted each side, but the minimum needed to play the game was three: a commander for each side and an umpire. The decisions of the umpire were final; they were not questioned during the course of the game, and no discussion was permitted until after the conclusion of the game. The umpire saw to it that the troops were moved in accordance with the spirit of the orders and applicable drill regulations. This was done in order to familiarize the players with field service regulations and to create an appreciation of the time required for the movements and maneuvers.

"The umpire was kept informed of all plans, marches, movements, ambuscades, etc., so that he could control their execution and avoid improbable situations. He required the players to maintain silence; and all orders, reports, and intelligence were passed through him and were transmitted at the appropriate time to the parties addressed. He divulged to each player only such information as, in his judgment, could reasonably come from patrols or other recognized sources.

"It appears that the important factor in every action in the game was the estimation of the time required for the execution of its different phases. In order to provide a means for controlling the duration of troop movements, and indirectly to estimate the losses sustained on both sides during a battle, the time during which an action was developed was divided into intervals of 2 minutes each. These were called rounds (giri), and only such movements were made as were realistically possible in this time. The game could be stopped, as desired, and a particular round could be studied in detail.

"To begin the game, the umpire gave each commander a 'general hypothesis' and a 'special theme'. The general hypothesis was the same for both sides; the special theme was not, and depended on whether the side was attacking or defending and on various aspects of the situation. The general hypothesis contained all the necessary information on the projected operation and the general situation, and was designed to promote contact by the opposing troops on the terrain included on the map. The special themes were derived from the general hypothesis and comprised such information as each commander would normally possess concerning the strength and composition of his own forces, the mission, and the enemy.

"The players were required to be familiar with the function of each arm of service, and the dispositions they made of the forces were often a test of their knowledge. The movements on the map were the same as would have been executed on actual terrain under similar conditions, but very complicated or unusual maneuvers were excluded from the game.

"Having received his general hypothesis and his special theme, each commander prepared written orders and delivered them to the umpire. From this information the umpire was able to establish the initial situation, and only the troops actually visible to the enemy at this time were represented on the map. Detachments, patrols, etc., were represented only as they became visible to the enemy or his reconnaissance elements. Account was taken of the time required for information gained by patrols to be transmitted to the leaders, and pawns representing troops thus discovered were not put on the map until the necessary time had elapsed. The same rule applied to the issuing of orders; the time required to transmit the orders was taken into account. A commander also was not permitted to communicate directly with subordinates who were more than 1000 paces from him.

"After the original dispositions were established, the umpire could direct that several movements be made simultaneously if the opposing forces were so widely separated as to prevent an engagement. However, once the troops were engaged, the game proceeded in an orderly fashion, round by round.

Each side made such movements as were possible during the period of 2 minutes. During an engagement, the umpire determined the losses by means of dice. These losses were reflected by withdrawing pawns, and by substituting others of lesser value, according to established rules.

"The game proceeded until some predetermined result was arrived at or until the umpire saw fit to stop the match. However, it was contended that the object of the game was not a question of winning or losing, as in cards or chess.

'. . . the well grounded decisions of the umpire and approbation of one's comrades are the only possible rewards. Whoever best follows up his movement, adopts the simplest and most natural means to the end, and departs least from the general idea of the operation, will have won the match, even though he may have lost a few more pawns than his adversary. . . . The advantages they will derive from it will be to acquire skill in reading maps, in the selection of movements best suited to the different arms of the service, in the choice of positions, etc. The interesting discussions which are sure to follow a match will be of incontestable value in the study of the military art.' "

As pointed out in Ref. 1, Young notes the following about Lt. von Reisswitz's war game:

"Several notable improvements in the war game are now evident. The chessboard type of chart was finally abandoned by Lt. von Reisswitz, Jr., and increased attention was given to the use of charts which more faithfully represented actual terrain. Chess-like pawns, with their conventional and limited movements, were discarded in favor of blocks that represented as nearly as possible the parts of a regiment. The rules governing movements were those which actually applied to troops in the field. Finally, and perhaps most important, the concepts of limited intelligence play were introduced, together with more realistic control of troop movements and time requirements by an umpire.

"The game contained a modification of an old feature; the use of dice in order to decide the results of actions and the losses sustained by the opposing troops. In each action it was supposed that one of the sides retained an advantage over the other due to superior strength, position, or some other factor. The combinations of various factors were assigned odds, corresponding to the faces of the dice. In this manner, an unlucky throw of the dice might cause the superior side to withdraw from the action with heavy losses. Seven dice were usually used, separately marked for indicating the success or failure of an attack, whether an assailant merely retreats or is routed, number of losses suffered, etc."

Young (Ref. 1) gives very extensive coverage of the development of war games in America during the period 1872-1918. In particular, Major W. R Livermore of the Corps of Engineers developed the American Kriegsspiel based on the previous work of German investigators. Livermore developed much detail for the conduct of the American war games, with many instructions for movement of troops on the battlefield and times to accomplish certain meneuvers, et al. The data used by Livermore were obtained from the US Civil War and from Prussian Wars of 1866 and 1870. The American Kriegsspiel, at the time, was considered by many to be an extremely flexible war game and perhaps a much closer approximation to the actual conditions of war than many contemporary games. Livermore succeeded in eliminating much of the detailed bookkeeping and extensive and voluminous records in earlier Kriegsspiel type games and introduced many devices, charts, and tables to speed up the conduct of play and reduce the labor formerly required to make many computations during the operation of the war game.

There were also other American efforts and contributions toward war game development. Lt. C. A. L. Totten published his book on the "Strategos" war game in 1895. Totten claimed that his Strategos war game was much better than any German game or even the American Kriegsspiel since Strategos was divided into the Battle Game and the Advanced Game referred to earlier. Totten felt that time was the most important element of tactics, and his book contains many tables of estimated times to accomplish the many movements and maneuvers of the different types of service elements of the various Army organizations. Although in some ways Totten's Strategos was more flexible and slightly easier to play and manipulate than Livermore's American Kriegsspiel, it was only marginally different from the forms of the "Rigid" Kriegsspiel for that period. Neither Livermore's American Kriegsspiel nor Totten's Strategos received as much enthusiasm in America as the German type Kriegsspiel war games in Germany because they both were considered too complex.

The early twentieth century saw a growing use of a type of war game referred to as "Free Kriegsspiel", which (1) tended to depart from rigid moves and rules, and place an increasing amount of responsibility on the directors of the war games, as well as the various calculations when made; and (2) attempted to make use of the more realistic data gathered from studies of recent wars. There was also a very definite trend toward the use of maps of actual terrain, drawn to a suitable scale, which broke away from the "ideal" maps of von Reisswitz and others. In fact, map maneuvers underwent very rapid development during the next twenty or so years, and a two-sided conflict was taken into account in the use of actual terrain maps in many geographical areas of interest.

Although present day analysts can see rather easily that losses, caused by being fired upon, involve chances of hitting and the conditional chance that a hit is a kill, it was true, nevertheless, for the early war games that the players had to use "play" tables constructed for the purpose since the more exact scientific methods were not then really developed. Such aids involved tables of multipliers, and an example is given in Table 40-1. Note that there are multipliers, and the corresponding "points", for both field artillery firing and rifle firing. Moreover, these are dependent on range of engagement, posture, motion, formation, and position as indicated. Thus the reader may begin to appreciate the growing importance of the need for automatic handling of such details for the players.

Prior to about 1910, it was fairly customary for the players to use three maps in a war game—one for each side and one for the referee or controller. By 1910 the use of the three-map game in tactical exercises had all but disappeared, and in its place was introduced the use of a large, single map of large scale which all participants used. However, each participant was given a small-scale map of the local area similar to those they might have in actual battles.

The development of the map maneuver during this period prior to World War I was marked chiefly by the great advance in the maps used and their improved accuracy. Thus war games had a very pronounced effect on the techniques of mapmaking and the terrain details needed for simulating battles.

40-2.2 BRIEF ACCOUNT FROM WWI THROUGH WWII

From just after World War I until about 1940, the Germans continued to make the greatest use of war game procedures. In fact, just after World War I, the war game received more emphasis by the Germans because the victorious allies precluded Germany's building up any sizeable armies or even carrying out any large-scale field maneuvers. Consequently, the Germans seemed to have no other choice since funds were lacking anyway. The war game was hence an important outlet for the Germans under the circumstances and fitted in quite well for any military training programs the Germans were allowed to conduct.

As a point of interest concerning the confidence that might be placed in war games, Ref. 4 points out that, near the end of World War I, the German Army High Command issued orders requiring a rehearsal of the spring offensive in 1918. This practice was performed by means of a strategic war game which was played at the headquarters of the Army Group Crown Prince Ruppert, with the very

TABLE 40.1. TABLE OF MULTIPLIERS FOR USE IN COMPUTING FIRE LOSSES

		Multiplier / Points	0.95 / -24	0.10 / -20	0.12 / -18	0.16 / -16	0.20 / -14	0.25 / -12	0.30 / -10	0.40 / -8	0.50 / -6	0.60 / -4	0.80 / -2	1.00 / 0	1.20 / +2	1.60 / +4	2.00 / +6	2.40 / +8	3.00 / +10	4.00 / +12	6.00 / +16	8.00 / +18	10.00 / +20	16.00 / +24	20.00 / +26
Troops Firing	Range yd	Field Artillery						4000			3500	3300	3000	2500	2000	1500	1000								
		Rifle			1800	1700	1600	1500	1400	1300	1200	1100	1000	900	800	700	600	500	400	300	200		100		
		Rate of Fire												6/min		Rapid									
		Position											Standing	Lying	Rest										
		Instruction									Recruits			Average			Sharp-shooters								
		Physical Condition										Fatigued		Fresh											
		Morale									Surprised		Shaken	Unshaken											
		Under Fire									4 Times	Double		Equal		None									
Troops Fired on		Angle of Front												90°	60°	45°	30°								
		Motion						Gallop oblique			Trot or D oblique	Gallop to front	Trot or D to front	0											
		Formation						5-yd interval				2-yd interval		2 pace interval			1 man/yd		1 Rank		2 Ranks	Cavalry skirmishers	*Artillery in action	Cavalry in line	Artillery limbering
		Position	Loopholes					Trench		Crest	Wood			Lying			Kneeling				Standing				

*Unsheltered

Additional Multipliers . . . 0.06 0.11 0.14 0.18 0.22 0.27 0.35 0.45 0.55 0.70 0.90 1.1 1.4 1.8 2.2 2.7 3.5 4.5 5.5 5.5 7 9 11 12 13 18 24 30 40 100

Corresponding Points . . . −24 −19 −17 −15 −13 −11 −9 −7 −5 −3 −1 +1 +3 +5 +7 +9 +11 +13 +14 +15 +17 +19 +21 +22 +23 +25 +26 +30 +32 +40 significant result that the High Command could predict quite well just what a slim chance existed of any decisive success!

After World War I, the Germans continued their extensive application of war gaming techniques and developed much literature on the general subject. Their war games were conducted on maps of from 1:5000 to 1:8000, or approximately 12-18 in. to the mile, and the games played on these maps involved many minor tactics and operations for detachments composed of all the arms up to the strength of a brigade. A war game called the "Great Kriegsspiel" was also practiced by the older officers of a regiment and the staffs of corps and divisions on maps with a scale of about 1:10,000 or about 6 in. to a mile. Such games normally involved military operations of brigades, divisions, and larger forces. The German General Staff officers were trained and instructed by means of a "Strategic War Game" which used maps with a scale of 1:100,000.

In the U.S., a rather extensive amount of war games was conducted at the Army Command and General Staff College at Fort Leavenworth, the Naval War College, and elsewhere, although budgetary restraints practically precluded any serious undertakings of war game studies, and nothing beyond the advanced stages of war games by the Germans was really developed as indicated in the quote from Ref. 1:

"Two new works may be mentioned here because of their relatively immature nature in contrast to the sophisticated techniques of the Germans. The first of these was *Little Wars*, published a short time prior to 1938 by H. G. Wells. Although it was intended originally as a child's game, it forms an exercise that can readily be identified with the earlier types of war games. The game was worked out in collaboration with the distinguished Orientalist, Colonel Mark Sykes, and was designed for use on a drill-hall type of floor or some similarly large space. The terrain was built up in three dimensions through the use of 1-inch boards cut into chords and angles. Houses were similarly cut of wood and twigs were painted to represent trees and shrubbery. Lead soldiers were used, about 1 inch in height. Toy guns, which actually emitted projectiles, were used to simulate artillery and infantry fire. Although it was intended that these devices would eliminate the need for tables and calculations of losses, it is evident that the accuracy of fire is questionable in terms of aiming errors, ballistic similarities, etc. The umpire in this game was relegated to the background, and was used only to administer simple rules involving bayonet charges and the like. Although this game is said to have had the color and dash that Kriegsspiel lacked, it never attained any real popularity due mainly to its inaccuracies and the expensive and cumbersome apparatus required. However, many of the features of the game were desirable, such as the miniature houses, soldiers, guns, etc., and these have been extensively adapted to the sand table (Ref. 5).

"The second piece of work was published in 1948 [1938] by E. A. Raymond, a lieutenant in the Field Artillery Reserve (Ref. 5). He proposed the transfer of the game from the map or sketch to a board that had previously been specially prepared. This board, usually about 4 by 10 feet, would have contours built up on it in cork and would be covered with graph paper of from 1- to 1/2-centimeter scales. Railroads, rivers, roads, etc., would be painted on the graph paper. Pins were to be used, with heads of various colors and shapes, to represent the armies. Infantry, artillery, tanks, and aircraft were to be designated by additional pins of various sizes, shapes, and colors. Tanks, ships, and trains were to be modeled mostly in rubber and affixed with pins. The moves were to be made in proportion to the arm of the service and according to the scale of the map used. Fire effectiveness was to be determined by range, and by chance. As an aid, three dice were to be used, with all the spots blocked out except the one-spot; for each which spot thrown an enemy unit was to be considered to have been wiped out.

"The game was intended to be played on the battalion or regimental levels. Three men and a commanding officer were to be assigned to each side. Fifteen-minute moves were to be used during which all the pieces on a side had to be moved. This was intended to simulate the usual haste and inattention present on the battlefield, and to require the commanders to come to a swift decision each time. However, the element of surprise was removed from the game since each side would have full view of the game board at all times. In addition, no provision was made for taking any captives, or for their exchange. This game, like 'Little Wars', did not attain any high degree of popularity or usage, although many of its features were also borrowed for use in regular map maneuvers and sand table exercises (Ref. 5)."

Prior to and during World War II, the Germans developed some of the more significant and widespread applications of war games by planning many of their intended military operations. Examples involved the initial campaigns against France during the early stages of the war, and a map exercise which was intended to check whether the possibility and time allowances for traversing the Ardennes with armored units could be accomplished with any degree of success, as estimated by the German Chief of Staff. It was reported that the German war games at the time were largely responsible for the rapidity of success and the smoothness of the German movements, and no doubt saved many lives and much labor and material that would have been expended otherwise. Moreover, Hitler's decision to invade Poland in 1939 resulted in failure as is well-known, and no such planning of campaigns for that operation was undertaken because of time and the uncertainty of the situation.

Perhaps the reader might think we have gone into too much detail to introduce the subject of war games although he may also understand that the concept of playing war games is not only very old indeed, but literally hundreds of years have been devoted to developing war games for various purposes, and huge amounts of effort have been expended to make such games as realistic as possible. The thrust in war game development has been to provide meaningful results which can be depended on to give sound inferences and train military personnel. Even at present, war games have not been "perfected", and therefore they still have a long way to go. Nevertheless, there exists enough experience in the documented literature to enable the serious gamer studying the more complex problems of military actions to make a judicious selection of appropriate techniques to play a meaningful game and thus develop some insight into many complex problems that could not be handled analytically. Moreover, the analyst can use the results from war games and try to identify the more important parameters so that he may develop an analytical model which hopefully can be used in applications and also be economical in both dollar costs and time.

At this stage we should point out and have the reader appreciate that an enormous amount of instructions, details, rules of play, and bookkeeping is required in war game operations, especially if they are to represent the realism of probable battles. Moreover, if we are to use war games for more than the training of military personnel, then the games must be rather "fine-tuned" or made sensitive enough, for example, if they are to be employed advantageously for comparing weapon systems or for force structure analyses. Thus it is seen that modern day computers not only can be used to great advantage for storage and timely extraction of needed information, but also can be an enormous aid in the overall problem of bookkeeping, analysis of results, etc. This indicates, as the reader no doubt expects, that we will ultimately lead up to the considerations of computerized combat simulations.

The war game was the forerunner to all kinds of simulations and hence became a very handy tool to obtain some insight into rather complex types of military operations. The so-called map exercises could be played as a one-sided game, whereas the war game is a two-sided conflict type operation for appropriate treatment and analysis of the effects of the more important interactions between the opposing forces.

40-2.3 SOME PRELIMINARY COMMENTS ON DEVELOPING THE PLAY AND ANALYSIS OF WAR GAMES

When setting up the procedure and play of war games, one should consider the amount and validity of all available background information. Based on this information, the sponsor and the players may develop some of their own ideas as to possible outcomes of the play or, at least, they might conjecture just what the war game might prove. The next important point is the development of a good scenario for play of the war game, along with the data and other information which will be available to the controller and the Blue and Red players.

The assignment of missions to the Blue and Red forces are given, along with the "controlled" information supplied to the players, and then the sequence of decisions, actions, and control of these is decided upon It will be important to study and be well aware of possible consequences which result from the decisions made and actions taken. One of the key problems and areas of interest is that of record keeping, for the analysis of war game results and the lessons learned may well depend on proficient bookkeeping. Good bookkeeping naturally will involve the status of Blue and Red forces (including location, survivors or losses, etc.) versus time. The characteristics of the environment and terrain; the amount of intelligence assumed available; relative information used in making decisions to resupply, to commit any reserves and replacements; the controller decisions, and measures of effectiveness—all must be developed and recorded.

Finally, sensible game termination criteria must be developed to bring the game to a halt, and it is important to critique the war game played to provide information for future improvements.

For our purposes here, we state that the play of modern war games usually involves the use of a Blue Room, a Red Room, and a Control Room. This arrangement has been found to be very satisfactory in the play of the war game; the programming of event steps in proper order; the handling of decisions made at time points during play; and the control of instructions, information, intelligence, etc., to Blue and Red forces. Thus the amount of detailed information available to the Blue and the Red forces can be controlled to any extent desired, and the controller or umpire becomes so very much involved in the management and conduct of the war game, that his decisions during play must be considered final. In fact, the control or umpire function is designed, among other things, to monitor the exchange of information and all the information gathering procedures of the various combat elements to limit these processes according to the rules of the battle and the restrictions imposed by the performance characteristics of the weapons, type of units, and the cover and concealment associated with the terrain.

Unlike completely computerized simulations, the war game or map exercises are such that play may be varied during the course of "engagement", should this become desirable or necessary. In fact, some special situations (e.g., an atomic attack) might be encountered during war game play, and personal, but experienced, command judgment might well be in order. Thus it can be said that war games are not as "rigid" in this respect as are completely computerized simulations of combat, which must be usually programmed for "once and all". Perhaps the more a war game is automated on a computer, the less the training value of it.

Very briefly, once the Blue and Red forces are "in position" on the terrain, the play of the game involves very detailed programming of the required steps or events according to a "logical battle". There must be provisions during play of the game for the movement of the troops, i.e., the various units and the weapons on the "terrain" according to realistic time intervals, and the relative location of Blue and

Red units then established from time to time for the record and analysis. Reconnaissance activities aid in the initial detection of enemy elements. The troops on each side will then move forward as ordered, and eventually forces on one side may come into contact with elements or units on the opposite side. Of course, line-of-sight calculations, based on terrain analyses, may be used to determine detection and acquisition of enemy units. Additional information on subjects such as terrain, weather, and obstacles in the area may also be involved in the play. Eventually, sufficient contact will be effected, and the "battle" begun. Orders are given for the firing of weapons as they become capable of engaging the enemy units. After some period of play there must be an assessment of conditions on each side—such as losses, new locations of elements, any required movement of personnel and weapons—and information on replacement and supply, if applicable. Thus there is a considerable amount of detailed and important information to be handled, programmed, stored, and later analyzed. Play of the engagement may be recycled, new assessments made, etc., and the game eventually terminated, perhaps according to predetermined criteria. The results are then analyzed, and the training proficiency assessed.

A remark is in order here about time intervals of play in war games. In many games, the play may proceed in equal time intervals, such as one minute, for example, and assessments made at the end of each equal time interval of play. Other war games frequently may use "critical event" timing, as do some simulations, in which the actions will proceed from one event to another, i.e. the next event in a time sequence does not necessarily involve equal time intervals. There are advantages to both approaches although current practice favors the equal time interval approach for convenience. However, something may be said for stopping action at critical events, such as the loss of an important piece of equipment, e.g , a tank, because the more exact time-to-kill data may be analyzed in terms of reliability type distributions as illustrated in par. 28-12.4.

Perhaps one of the most important considerations we should bring out at this stage is the design, development, and play of war games—and computerized simulations of combat as well—so that the analysis of final results may be more meaningful, sound, and useful for inferences. Because of the (often unidentified) interactions that invariably creep into the more complex battles involving diverse weapons, it is very desirable to design war games in accordance with statistical principles and play them so that the significance of results, if attained and true, may be established. Thus we point out that usually it would make little sense to control all other sources of possible variation in a war game so that only a single comparison is made since the controls may not turn out to be meaningful under all possible conditions of combat variability. For example, it might be far better, depending on the particular problem under study, to make fewer overall runs but to program a series of runs to statistically establish superiority of tactics, types of weapons, etc. To illustrate our point, a statistical design of experiment involving a Greco-Latin Square may be used, in which the Latin letters would be used to designate various mixes of Blue weapons, and the Greek letters various choices of the probable Red weapons capabilities. Or, as another example, factorial experimentation in connection with war games or computer simulations could often prove advantageous and profitable, or perhaps applicable standard designs of statistical experimentation might be exploited. War games and computer simulations are indeed "experiments"; and therefore it makes considerable sense to conduct them so that the more important problem areas are covered and the significance of the results, if true, is established. Otherwise, it is well-known that many ordinary war games and computer simulations simply produce confusing or useless results. We will illustrate these very points in Chapter 41.

We will discuss by example some other details of actual play of the war game type of simulation in connection with a discussion of computerized combat simulations in par. 40-3; however, it is important to list some of the modern uses and applications of war games for the reader at this point (see par. 40-2.4).

40-2.4 SOME MODERN USES OF WAR GAMES

Some of the more important uses of modern war games include applications such as:

1. Training, especially of combat officers
2. Assessing the potential performance of subordinate officers
3. Historical battle study purposes. Thus, given a famous battle in the past, the war game can incude many of the key inputs and can be run to study possible outcomes. In this way, war games can be very educational and informative for future military operations.
4. Planning for campaigns. As we have seen, the Germans did indeed profit from practicing war exercises just before engaging in actual conflicts with an enemy.
5. Logistical training. There is a need to train supply, transportation, communication, and other such personnel in their respective fields. War games can be valuable in this regard.
6. Weapon family studies. If some standard or desired tactics are assumed in a battle, various weapon mixes can be run in a simulation of combat and the most desirable mix of weapons, or the best "family", selected.

In connection with war games support of the TRADOC Systems Analysis Activity (TRASANA) to the Allied Forces South (AFSOUTH), Naples, Italy, Willis (Ref. 6) advances the following alternative objectives for war games:

1. Facilitate communication between members of a staff and between organizations on plans, procedures, policies, and planning factors relative to war games.
2. Evaluate the impact of constraints on force effectiveness.
3. Identify deficiencies in a force (in the areas of procedures, plans, tactics, systems, and training) and evaluate the effectiveness of any current force.
4. Provide training, experience, and practice in the control and coordination of combined arms (or joint service) operations and in preparing plans and orders.
5. Determine possible and/or likely enemy courses of action and identify exploitable enemy weaknesses.
6. Exercise and test various plans and procedures.
7. Provide a context or scenario for more detailed studies, including estimates of the relative frequencies of various situations.
8. Identify subject areas in which tests or experiments are required.
9. Determine requirements for forces, new systems, reserves, or replacements.
10. Evaluate alternatives for overcoming deficiencies, including changes in:
    a. Procedures
    b. Tactics
    c. Organization
    d. Systems
    e. Force mix.
11. Determine the most effective way to incorporate new systems.
12. Evaluate and compare:
    a. Alternative strategies
    b. Alternative tactics, countermeasures, or procedures

c. Alternative force deployments
d. Alternative organizations
e. Alternative systems.

Thus there are wide uses and important applications of modern war game techniques.

40-3 COMPUTER SIMULATIONS OF COMBAT

40-3.1 DEFINITION AND DESCRIPTION

So far in this chapter, we have reserved the term "war game" to mean usually a manual simulation of combat between opposing forces; however, we also could speak of "computerized" war games, a commonly used term, so that the big differences between a war game as an entity now and a computer simulation of combat may disappear to some degree. There seems to be no progressive satisfaction in arguing or sticking to the strict use of such terms since they will likely be used in a more or less loose fashion anyway. In any event, it becomes very natural indeed to employ a computer in simulating combat because the computer is a natural aid for the enormous amount of bookkeeping needed to conduct war games or combat simulations. In fact, the computer can result in great savings in the amount of human effort. It can make calculations quickly and efficiently; it can be instructed to make "decisions" if so programmed; it can easily generate random numbers for stochastic type study; and also, no doubt, it will decrease overall cost. Thus it becomes very desirable to have the computer perform as much of the war game simulations as it possibly can; the ultimate is feeding the computer war game input data, pushing the button, and waiting for the results!

In addition to bookkeeping, it seems desirable to illustrate by example a few of the problems the computer can easily handle in the "electronic" play of a war game, map exercise, or combat simulation. For example, the given terrain characteristics may be stored in the memory of a computer; then, for known weapon and target coordinates, calculations of the chances of a line of sight—and hence target detection with suitable sensors—can be determined. Given a fire order, the computer—knowing target size, its vulnerability, and the weapon characteristics involved—can calculate the chance of hitting and killing targets at all ranges. In this connection, the computer may make calculations based on the weapon delivery errors as a basic input, or it may query a hit probability distribution versus range. (The statistical description of this for CARMONETTE is given in par. 40-3.2.2.)

For the new or young systems analyst, a rather detailed description of several possible analytical roles of computers in the computer-assisted war game is given by Murray, Sewell, Chandler, and Winslow (Ref. 7). They illustrate some, but not all, of the typical mathematical techniques which may be used to interface the computer properly with the play of the war game or combat simulation. For example, these authors, who consulted with the US Army Strategy and Tactics Analysis Group (now the US Army Concepts Analysis Agency), illustrate some details through the use of their "Influence Diagram" which we display as Fig. 40-1. Their calculation, indicated by [1] in Fig. 40-1, determines the new location of a unit at time t from the direction and speed if movement begins at time $(t - \Delta t)$. Hence the x- and y-coordinates at time t for unit i, which are represented by x_{it} and y_{it}, are easily calculated by the computer as

$$\left.\begin{aligned} x_{it} &= x_{i,t-\Delta t} + (\Delta t)(s_{i,t-\Delta t})\cos\theta_{i,t-\Delta t} \\ \text{and} \quad y_{it} &= y_{i,t-\Delta t} + (\Delta t)(s_{i,t-\Delta t})\sin\theta_{i,t-\Delta t} \end{aligned}\right\} \qquad (40\text{-}1)$$

where

Δt = time increment

$s_{i,t-\Delta t}$ = speed of unit i at time $(t - \Delta t)$

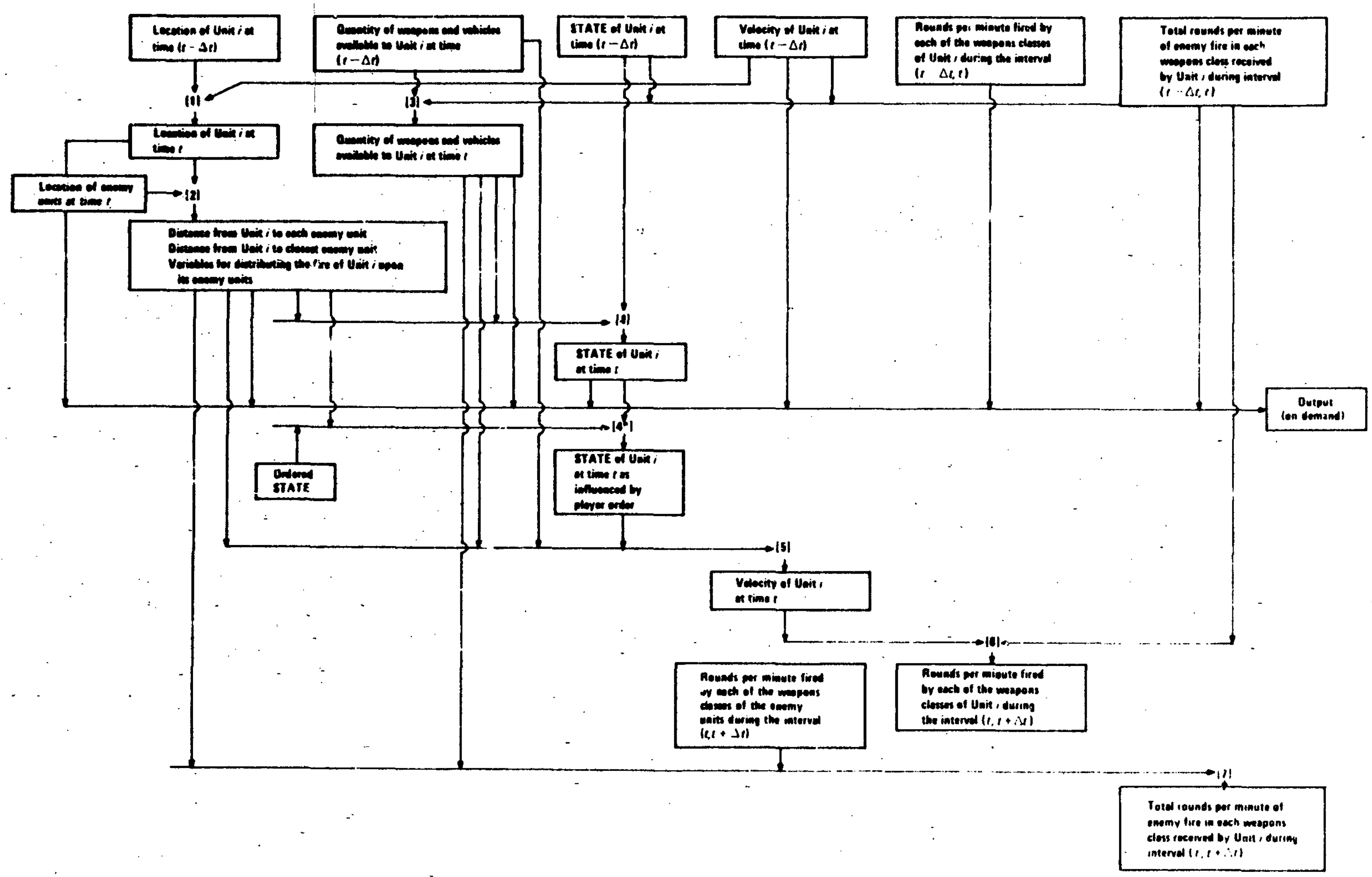

Fig. 40-1. Influence Diagram

$\theta_{i,t-\Delta t}$ = the azimuth on which unit i is moving at instant $(t - \Delta t)$

$x_{i,t-\Delta t}$ and $y_{i,t-\Delta t}$ = "old" coordinates of unit i, i.e., at time $(t - \Delta t)$.

Refer again to Fig. 40-1; calculation [2] determines the distance between any two elements on the terrain. After firing orders and assessments, then calculation [3] is used to determine the losses suffered by unit i. And so the process proceeds in the use of the computer, and calculation [7] determines the total rounds per minute received by unit i from each class of enemy weapons during the interval $[t, (t + \Delta t)]$.

There is an important element missing here, however, which we describe briefly, and that relates to the use of a "clock" for scheduling the events in proper order. For battlefield time, each unit has associated with it an "alarm clock" or set of numbers representing the time at which it will next be able to act—i.e., open fire, move, etc.—and at the beginning of the battle, or time zero, these "clocks" will be set at a few seconds unless some of the units have been prohibited from firing or moving, etc. A computer, or "clock", routine is programmed to examine the unit clocks for the purpose of finding that one with the smallest time. Thus, this will correspond to the first event that is scheduled to happen, and this particular time interval becomes the new battlefield time. The computer then performs the various calculations which are required to simulate the next scheduled event. If, for example, the event were the firing of an antitank weapon, then once the weapon is fired, its clock would be reset for the time it is able to fire again. Also all other "clocks" of the elements or units would be reset, with the "clocks" routine determining the very next (minimum time) event so that the battlefield time is again adjusted, and the calculations continue in play of the game. This process often is referred to as "event sequencing" and is used in the play of many high resolution computer combat simulations.

40-3.2 BRIEF DISCUSSION OF CARMONETTE

An example of one of the key and very successful concepts of a tactical war game, which has been computerized and widely used, is the battle simulation called "CARMONETTE" — described by Zimmerman (Ref. 8), and Refs. 9 and 10. CARMONETTE is a fully computerized Monte Carlo type of simulation which covers a typical but isolated battle in a simple step-by-step procedure outlining the actions that might occur. CARMONETTE was designed so that all calculations could be performed on a standard general purpose digital computer, except that a limited number of high level decisions or orders could be injected into the battle during the calculations. The CARMONETTE simulation detailed the manner in which combat activities of the individual participants fought the battle and integrated these separate combat activities into larger and more meaningful conflicts between forces. Initially, the concept and play of CARMONETTE centered around "small units", or ones of about company size, involving "some hundreds of men and dozens of heavy weapons like tanks". CARMONETTE is a critical event, computerized war game with time recorded to 1/10,000 of a minute. Originally CARMONETTE had the capability of playing up to 36 independent combat elements of each side; a typical organization might include, for example, 18 tanks, 2 platoons of infantry, some mortars, and perhaps a few special purpose weapons, such as a guided missile. CARMONETTE has been developed through some six major iterations. Currently, CARMONETTE (Ref. 10) is a computerized Monte Carlo simulation of battalion size or lower units in ground combat. The primary activities of the simulation consist of the movement of units, the detection of targets, and the firing of weapons at the targets. Movement is either under preplanned orders or may be conditional. Target acquisition may involve different chances of detection in order to pinpoint targets on each side. The firing of weapons and the assessment of effects use computer calculated probability of hit and kill calculations. Unit resolution can be an individual soldier, a tank, or a platoon; and the

Blue and Red forces can have up to a total of 48 units each. The game can play up to 56 weapon types including indirect fire by artillery and mortars (12 types), and direct fire weapons with both a fragmenting capability (22) and direct fire bullets or armor-piercing projectiles (22). There is a limitation of no more than four weapons per unit. Although the map grid size may be selected, usually a 100-m grid on a 6 km by 6.3 km total map area is played. Table 1 of Ref. 10 gives the relation between grid size, unit size, force size, and zone of action (our Table 40-2). As summarized in Refs. 9 and 10, CARMONETTE input requirements include detailed descriptions of the terrain of play, appropriate target detection probabilities, and a set of orders for each unit based on tactical doctrine in accordance with a predetermined scenario. The terrain inputs for each grid square include average elevation, height of vegetation, indices of cover and concealment, and road and cross-country trafficability. Basic outputs of CARMONETTE involve a computer listing of every event assessed during the battle and include elements killed, beginning and final positions of units, beginning and final strengths, rounds fired and rounds received, time-to-death of unit, ammunition expended, number of engagements, and number of rounds fired and targets killed by specific weapon types and related range brackets.

In CARMONETTE the forces to be gamed are organized into no more than 48 units for each side, and each of these units may have no more than 63 "killable" elements. The CARMONETTE units are individual weapon systems such as a tank, antitank guided missile, or they may be groupings of elements having the same degrees of mobility and vulnerability, the same sensors, and which are located within the same referenced grid. The entire unit is considered detected when a single element of the unit is detected. When a unit is fired upon, however, all of its elements are considered equally vulnerable although the probability of a hit, or a kill, is calculated for each element individually. A unit can have up to four groups of weapons. For example, a tank may have a main gun, an air defense machine gun, and a coaxial machine gun; or the rifle squad may have two antitank weapons, one machine gun, one grenade launcher, and five rifles; etc.

There are sixteen target classes in the CARMONETTE game, and the target classes are given priorities. For example, an armored personnel carrier mounting an antitank guided missile would be a higher priority target than an armored personnel carrier without the missile. The two factors associated with the assignment of a friendly unit to target class are the vulnerability to the various weapons and the firepower possessed by the unit. Targets are also classified according to size, mobility, fire response, and sensor class.

Detailed orders that will be followed throughout play are given to units, and if a unit is killed it simply stops following orders. A typical order might be, for example, "move at rate *r* without stopping to square

TABLE 40-2
RELATION BETWEEN GRID SIZE, UNIT SIZE, FORCE SIZE, AND ZONE OF ACTION

Grid size, m	Approximate Unit Size				Maximum Force Size		Maximum Zone of Action	
	Infantry	Mechanized Infantry	Artillery	Aviation, aircraft	Infantry	Mechanized Infantry	Width, m	Depth, m
10	1 Man	n/a	1 tube	n/a	2 Plts	n/a	600	630
25	2 Men	1 Veh	2 tubes	1	1 Co	1 Co	1500	1575
50	½ Sqd	2 Vehs	4 tubes	1	1 Bn	1 Bn	3000	3150
100	1 Sqd	3 Vehs	6 tubes	2	2 Bns	2 Bns	6000	6300
250	1 Plt	7 Vehs	12 tubes	4	4 Bns	4 Bns	15000	15750

xx,yy with kind of fire k and priority p", "stay until time $tt.tt$, or fire s shots with kind of fire k and priority p", etc.

Two kinds of moving commands may be provided. One is to move without stopping to a given square, and the other provides a doctrine that permits the unit to stop occasionally and fire as it moves along.

It is through CARMONETTE's simulation of time and space that the interaction of the forces and their environment can be accounted for. In an actual battle, the various activities of firing, moving etc., may take place simultaneously, although for a computer simulation no two events can take place simultaneously. The computer requires the discrete and sequential handling of the events that occur in combat, and this is all that is really necessary. For example, it is not necessary for the computer to trace the path of a projectile in flight to its target, but the time of its impact is a very significant event in play of the battle, and it can be handled easily by a computer. The movement of a unit from one square to another is also an event for which only the end time points are of interest.

The periodic assessment for any target acquisitions and for neutralization of targets is also handled sequentially. The probability of detecting a target is determined for the most part by the dwell time of the sensor, the range from sensor to target, and the characteristics of the target in relation to its environmental cover. For neutralization, the number of rounds impacting in the vicinity of a unit is calculated although only the more recent rounds, or those in the last interval of time, are considered effective in establishing unit integrity.

The activities simulated in CARMONETTE which are common to more than one arm are target acquisition, target selection, firing of rounds and their impact, neutralization, movement, selection of units, and communications. Perhaps of particular interest to the analyst would be some examples which illustrate the processes of target detection, the calculation of hit probabilities, and the calculations of casualties since these might give some concrete insight into the operation of CARMONETTE in a computer. Many of the available models for such calculations do indeed require some very notable simplifications "to get on with the game", so to speak!

40-3.2.1 Target Detection

The CARMONETTE detection model is very much a simplification of the theory of target detection because of the lack of realistic field experiments to validate detection under most combat conditions. We know (Chapter 27) that the reflectivity characteristics of the target or its "contrast" is important in detectability, as well as atmospheric attenuation of signals, camouflage of the target, experience of the observer, range to target, motion of target, and many other factors. The CARMONETTE target detection model, however, uses the existence of a line of sight, the response state of the observer, whether or not the target or sensor is moving, the target solid angle subtended at the observer, and the type of sensor employed to look up the chance of target detection in programmed tables. Some six sensor types are permitted. The visual detection routine and the image intensifier routine are rather complex and are given in Appendix A of Ref. 10 for the interested reader. Here, we will illustrate only the radar detection routines which are for the plan and position scopes, PPS 4 and PPS 5. Up to six such radars can be played, but the present program routine does not consider a threshold target speed or the direction of movement of a unit—only whether or not the target is classed as moving. The radar model input calculations depend on the radar degradation factor DGF, the maximum effective range r_{max}, and the observer-target range r as follows:

DGF = radar degradation factor $\approx$ 1.23 (typical value)

r_{max} = maximum effective range of radar, i.e.,

$$= \begin{cases} 3500 \text{ m for personnel for PPS 5} \\ 10{,}000 \text{ m for vehicles for PPS 5} \\ 1700 \text{ m for personnel for PPS 4} \\ 2500 \text{ m for vehicles for PPS 4, where} \end{cases}$$

r = observer-target range, m

RR_{adj} = adjusted range ratio

$= r/[(DGF)(r_{max})]$.

The chances of target detection P_D are calculated from the following equations:

$$P_D = 0.9 - (RR_{adj})/8, \text{ if } RR_{adj} < 0.8 \tag{40-2}$$

or

$$P_D = 2.0 - 1.5(RR_{adj}), \text{ if } 0.8 \le RR_{adj} < 1. \tag{40-3}$$

(The reader may contrast these "approximate" CARMONETTE target detection chance models with those in Chapter 27 of this handbook.)

40-3.2.2 Firing and Assessment of Casualties

The firing and impact computerized simulation includes possible position disclosure for the firing weapon, ammunition expenditure, and the assessment of casualties. When a weapon fires, that weapon produces a signature which the opposing side may detect and hence locate the weapon; this is important for the battle procedure. Moreover, every time a weapon fires such rounds are removed from the unit's supply of ammunition. This applies also to more than a single round per trigger pull.

The probability of hitting for CARMONETTE is determined by knowing the range to the target, the exposed area of the target, the "total tactical dispersion" of the weapon at the given range, and whether the particular round fired is the first round or a subsequent one. The equation currently used for single-shot hit probability $P(r)$ depends on the equivalent radius R of the target area and the total tactical dispersion, or standard deviation $\sigma(r)$ of the weapon in one direction and is

$$P(r) = 1 - \exp\{-R^2/[2\sigma^2(r)]\} \tag{40-4}$$

where

$$\sigma(r) = a + [(b-a)/(0.5M)]r^2 + [(c-2b)/(0.5M)](r^2/M)(r-0.5M) \tag{40-5}$$

a = standard deviation in dispersion at zero range

b = standard deviation in dispersion at 0.707 maximum range

c = standard deviation in dispersion at maximum range

M = square of the maximum range of the weapon employed in CARMONETTE

r = range to the target.

If the range r is in meters, then the target radius R is also to be taken in meters, i.e., r and R must be expressed in the same units. For CARMONETTE, one notes that Eq. 40-4 is quite similar to Eq. 14-2 of Chapter 14, where σ of Eq. 14-2 equals $\sigma(r)$ here. Thus Eq. 40-5 is an estimate of weapon delivery dispersion, for convenience in CARMONETTE, which depends primarily on and is a strong function of the range r to the target. By contrast, the reader may have noted that elsewhere in this handbook sometimes we have used a hit probability calculation based on

$$P(r) \approx 1 - \exp(-dr) \tag{40-6}$$

where d is a constant, and r the range.

For CARMONETTE, values of the "total tactical dispersion" are stored for each of some 12 conditions of volley history, firer activity, and target activity and the two types of ammunition permitted each weapon as three coordinates of a parabolic curve. As indicated in Eq. 40-5, the three values are taken at zero range, 0.707 maximum range, and maximum range of the weapon. The parabolic approximation is used as shown in Eq. 40-5 to obtain total tactical dispersion for the intermediate ranges. Even though the minimum range of a weapon may be greater than zero, the value of total tactical dispersion sometimes must be extrapolated back to zero range; "Negative values are not permitted." (Ref. 10).

Since a hit is not necessarily a "kill" of a target, the conditional chance that a hit is a kill must be multiplied by the hit probability, or otherwise as in CARMONETTE, "For the calculation of the probability of a hit of killable elements of multiple element targets, the target area is [taken as] the area of one killable element of the unit." (Ref. 10).

The probability of hit calculations results in decimal values between zero and unity; however, in the play of the game the computer must decide whether a "hit" or a "kill" is obtained for each target fired upon. In order to do this, a uniform random number $x(i)$ is generated for each shot fired at a target and is compared to the calculated $P(r)$ in Eq. 40-4. A hit occurs if the $h(i)$ in Eq. 40-7 is unity, i.e.,

$$h(i) = \begin{cases} 1, \text{ if } P(r) \geq x(i) \\ 0, \text{ if } P(r) < x(i) \end{cases} \text{ for } i = 1,2,\ldots,z \tag{40-7}$$

and z is the number of rounds given by

$$z = \min(A, NF) \tag{40-8}$$

where

A = remaining number of rounds

N = number of rounds per trigger pull

F = number of weapons fired.

Thus, the total number of hits H scored is then summed from

$$H = \sum_{i=1}^{z} h(i). \tag{40-9}$$

The number of kills cannot exceed the number of hits, as we have said, and is often much less than the number of hits. Also the casualty computation for vehicles and personnel is very different. It is easy to determine whether a kill is or is not obtained against a vehicle in CARMONETTE; this is done if k is unity from the calculation

$$k = \begin{cases} 1, \text{ if } P(k|h) \geq x(j) \\ 0, \text{ if } P(k|h) < x(j) \end{cases} \text{ for all } j \leq H \tag{40-10}$$

where

$P(k|h)$ = conditional chance that a hit on the vehicle is a kill

H = number of hits from Eq. 40-9

$x(j)$ = uniform random number drawn for the case of a hit.

For the case of using fragmenting weapons, such as artillery, against personnel, all troops in the impact area of indirect-fire weapons must be considered; an approximate, but very detailed, algorithm is covered in Ref. 10 for the interested reader. Random number generation is also covered in Ref. 10.

40-3.2.3 Command, Control, and Communications

The command, control, and communications (C^3) functions of CARMONETTE are described in Ref. 10 as being somewhat complex, i.e., in terms of the true situation being simulated. For units and task forces of the CARMONETTE size, the mission-type order is employed as the appropriate way to cause the various desired actions to take place. Other elements of C^3 are the identification of friend or foe, the transfer of information among units, and the formation of various elements of a unit. Firings cannot take place against units on the same side; therefore once a "target" is pinpointed, it may be fired upon immediately. CARMONETTE uses a "Communications Routine" to transfer information among units on the same side, and thus during such communication cycles each weapon unit can report the nearest square that contains enemy units to its immediate headquarters. Commanders at all levels relay such information to their superior, subordinate, and adjacent headquarters. Any headquarters can be given the capability of calling for attack by helicopter and/or artillery fire. In certain circumstances even fire units may call for artillery fire which will be provided if the support unit is not already committed to some other mission. In CARMONETTE the actual combat formation of the elements of a unit is not simulated. As a result of this simplification, when an element of an enemy unit is detected, the other side will have full knowledge of all of the elements of the target unit. There are, of course, area weapons whose effects depend on the formation of elements in a target unit although in CARMONETTE this distinction is not made.

40-3.2.4 Examples of CARMONETTE Records

The record of events in CARMONETTE is referred to as the history tape which contains all move selections, target selections, boundary crossings, firings, impacts, and status information, such as "out-of-ammunition", response to fire, line of sight, intelligence level, and recognition of target death-time for each live unit. If, for example, some unit does not select a target, then no message is transmitted or recorded, i.e., nonevents are not recorded. Examples of the types of reports that CARMONETTE produces are:

1. Table 40-3, a report of chronological cumulative casualties
2. Table 40-4, a report of target kills by weapon type
3. Table 40-5, an operational-status report giving information on initial and final unit locations, numbers of moves, number of rounds fired or received, initial number and final number of troops and vehicles, and time of unit death.

For the information of the reader, some of the management aspects of CARMONETTE are reported in Table 40-6.

Many other details and coverage of CARMONETTE are given in Ref. 10, and we remark that the CARMONETTE simulation is part of a hierarchy of combat analysis models. Thus the output from CARMONETTE may be used as input to higher organizational level games such as COMANEX (Combat Analysis Extended), DBM (Division Battle Model), ATLAS (A Tactical, Logistical, and Air Simulation), or others described in par. 40-4.

Hopefully, this somewhat sketchy and exemplary detail of the completely computerized CARMONETTE combat simulation will give sufficient insight into the model of programming a two-sided conflict on a computer. The new analyst perhaps sees rather easily that many compromises or simplifications have to be made and that the incorporation of some of the more exact theory summarized in this handbook will take years to incorporate, even if it is necessary input to more refined combat simulations.

As a final comment, we may note that computerized combat simulations generally will involve very detailed programming of the events of two-sided conflict which cannot yet be modeled or described analytically. On the other hand, it would not make much sense to conduct one-on-one stochastic duels, which may be modeled analytically as in Chapter 17, as a simulation on a computer. Such analytical calculations, however, may well be inputs to a computerized battle.

TABLE 40-3. CHRONOLOGICAL CUMULATIVE CASUALTIES REPORT

TREATMENT9901 SUMMARY OF REPLICATION 1 07/19/74

CHRONOLOGICAL CUMULATIVE CASUALTIES

RED	27	2.0035	32,50,	0	CASUALTY WPN NO.	36	FIRER NO.	34	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	3
RED	1	5.3366	26,55,	0	CASUALTY WPN NO.	13	FIRER NO.	31	VEH BEFORE	4	AFTER	3	MEN BEFORE	40	AFTER	38	CUMULATIVE	5
RED	1	5.6284	26,55,	0	CASUALTY WPN NO.	13	FIRER NO.	31	VEH BEFORE	3	AFTER	2	MEN BEFORE	38	AFTER	36	CUMULATIVE	7
RED	1	5.7039	26,55,	0	CASUALTY WPN NO.	35	FIRER NO.	25	VEH BEFORE	2	AFTER	1	MEN BEFORE	4	AFTER	2	CUMULATIVE	9
RED	7	6.1694	29,49,	0	CASUALTY WPN NO.	36	FIRER NO.	35	VEH BEFORE	4	AFTER	3	MEN BEFORE	40	AFTER	36	CUMULATIVE	13
RED	2	6.8474	25,55,	0	CASUALTY WPN NO.	6	FIRER NO.	37	VEH BEFORE	0	AFTER	0	MEN BEFORE	32	AFTER	31	CUMULATIVE	14
RED	1	7.2383	26,55,	3	CASUALTY WPN NO.	13	FIRER NO.	30	VEH BEFORE	1	AFTER	0	MEN BEFORE	2	AFTER	0	CUMULATIVE	16
BLUE	39	7.9632	6,56,	0	CASUALTY WPN NO.	16	FIRER NO.	20	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	3
RED	11	8.5942	27,45,	0	CASUALTY WPN NO.	13	FIRER NO.	31	VEH BEFORE	3	AFTER	2	MEN BEFORE	30	AFTER	28	CUMULATIVE	18
RED	2	8.6692	24,56,	3	CASUALTY WPN NO.	6	FIRER NO.	37	VEH BEFORE	0	AFTER	0	MEN BEFORE	31	AFTER	29	CUMULATIVE	20
BLUE	31	8.7966	15,39,	3	CASUALTY WPN NO.	14	FIRER NO.	26	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	6
RED	38	8.8974	26,49,	0	CASUALTY WPN NO.	35	FIRER NO.	26	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	23
RED	11	9.2883	27,45,	0	CASUALTY WPN NO.	45	FIRER NO.	30	VEH BEFORE	2	AFTER	1	MEN BEFORE	28	AFTER	26	CUMULATIVE	25
RED	11	9.9043	27,45,	0	CASUALTY WPN NO.	44	FIRER NO.	17	VEH BEFORE	1	AFTER	0	MEN BEFORE	2	AFTER	0	CUMULATIVE	27
RED	19	9.9858	28,54,	0	CASUALTY WPN NO.	41	FIRER NO.	8	VEH BEFORE	3	AFTER	2	MEN BEFORE	9	AFTER	6	CUMULATIVE	30
RED	22	10.8276	26,52,	0	CASUALTY WPN NO.	35	FIRER NO.	26	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	33
RED	8	10.7114	24,44,	0	CASUALTY WPN NO.	6	FIRER NO.	37	VEH BEFORE	0	AFTER	0	MEN BEFORE	30	AFTER	29	CUMULATIVE	34
BLUE	38	11.5342	15,40,	3	CASUALTY WPN NO.	14	FIRER NO.	21	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	9
RED	10	11.6448	24,43,	0	CASUALTY WPN NO.	1	FIRER NO.	40	VEH BEFORE	0	AFTER	0	MEN BEFORE	24	AFTER	23	CUMULATIVE	35
BLUE	34	12.7189	6,56,	0	CASUALTY WPN NO.	16	FIRER NO.	20	VEH BEFORE	1	AFTER	0	MEN BEFORE	3	AFTER	0	CUMULATIVE	12
BLUE	1	14.5132	14,46,	0	CASUALTY WPN NO.	46	FIRER NO.	5	VEH BEFORE	1	AFTER	0	MEN BEFORE	1	AFTER	0	CUMULATIVE	13

TABLE 40-4. EXAMPLE OF THE TARGET-KILL REPORT

TARGET KILLS BY WEAPON TYPE

RED WEAPON NUMBERS	BLUE TARGET CLASSES											
	CLASS 1		CLASS 2		CLASS 4		CLASS 7		CLASS 8		CLASS 10	
	MEN	VEH	MEN	VEH	MEN	VEH	MEN	VEH	MEN	VEH	MEN	VEH
3	0	0	0	0	0	0	0	0	2	0	0	0
4	0	0	0	0	0	0	1	0	0	0	0	0
14	6	2	0	0	4	1	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	6	2
46	0	0	1	1	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	1	0	0	0
52	0	0	0	0	0	0	0	0	5	0	0	0
TOTALS	6	2	1	1	4	1	1	0	8	0	6	2

RED WEAPON NUMBERS	TOTAL KILLS	
	MEN	VEH
3	2	0
4	1	0
14	10	3
16	6	2
46	1	1
50	1	0
52	5	0
TOTALS	26	6

TABLE 40-5. EXAMPLE OF THE OPERATIONAL-STATISTICS REPORT

BLUE UNITS	LOCATION INITIAL	LOCATION FINAL	NUMBER OF MOVES	NUMBER OF ROUNDS FIRED	NUMBER OF ROUNDS RECEIVED	TROOPS INITIAL	TROOPS FINAL	VEHICLES INITIAL	VEHICLES FINAL	UNIT DEATH TIME
1	10,46, 0	10,46, 0	0	6	24	11	0	1	0	14.5132
2	10,46, 0	10,46, 0	3	0	36	0	10	0	0	0.0000
3	10,44, 0	10,44, 0	0	22	18	11	1	1	1	0.0000
4	10,44, 0	10,44, 0	0	0	0	0	10	0	0	0.0000
5	13,48, 0	9,46, 0	4	3	0	11	1	1	1	0.0000
6	13,48, 0	9,46, 0	1	0	81	0	4	0	0	0.0000
7	13,45, 0	10,45, 0	3	5	266	3	2	1	1	0.0000
8	13,48, 0	13,43, 0	0	5	163	3	3	1	1	0.0000
9	7,41, 0	7,41, 0	0	0	0	11	1	1	1	0.0000
10	7,41, 0	7,41, 0	0	0	0	0	10	0	0	0.0000
11	7,41, 0	7,41, 0	0	4	0	11	1	1	1	0.0000
12	7,41, 0	7,41, 0	0	0	0	0	10	0	0	0.0000
13	15,41, 0	15,41, 0	0	34	55	11	1	1	1	0.0000
14	15,41, 0	15,41, 0	0	0	0	0	0	0	0	0.0000
15	7,41, 0	7,41, 0	0	0	0	3	3	1	1	0.0000
16	15,41, 0	15,41, 0	0	5	32	3	3	1	1	0.0000
17	16,34, 0	16,34, 0	0	32	0	11	1	1	1	0.0000
18	16,34, 0	16,34, 0	0	0	0	0	10	0	0	0.0000
19	15,32, 0	15,32, 0	0	0	0	11	1	1	1	0.0000
20	15,32, 0	15,32, 0	0	0	0	0	10	0	0	0.0000
21	15,31, 0	15,31, 0	0	0	0	11	1	1	1	0.0000
22	15,31, 0	15,31, 0	0	6	0	0	10	0	0	0.0000
23	16,33, 0	16,33, 0	0	0	0	3	3	1	1	0.0000
24	14,31, 0	14,31, 0	0	0	0	3	3	1	1	0.0000
25	10,44, 0	10,44, 0	0	20	38	4	0	1	0	17.7961
26	10,45, 0	10,45, 0	0	29	267	4	4	1	1	0.0000
27	15,33, 0	15,33, 0	0	0	0	4	4	1	1	0.0000
28	15,32, 0	15,32, 0	0	0	0	4	4	1	1	0.0000
29	14,41, 0	14,41, 0	0	198	55	3	3	1	1	0.0000
30	15,40, 0	15,40, 0	0	47	27	3	0	1	0	11.5342
31	15,39, 0	15,39, 0	0	21	22	3	0	1	0	8.7944
32	8,41, 0	8,41, 0	0	0	0	3	3	1	1	0.0000
33	7,41, 0	7,41, 0	0	0	0	3	3	1	1	0.0000
34	2,58, 0	6,56, 0	98	4	14	3	0	1	0	12.7109
35	2,58, 0	6,56, 0	51	3	14	3	0	1	0	7.9602
36	1,21, 0	1,21, 0	0	64	0	40	40	5	5	0.0000
37	1,34, 0	1,34, 0	0	96	0	40	40	7	7	0.0000
38	1,18, 0	1,18, 0	0	96	0	40	40	7	7	0.0000
39	2,21, 0	2,21, 0	0	24	0	33	33	5	5	0.0000
40	4,39, 0	4,39, 0	0	36	0	24	24	4	4	0.0000
TOTALS				778	1118	331	385	54	48	

TABLE 40-6
MANAGEMENT ASPECTS OF CARMONETTE

Task	Technical Effort	Calendar Time	Computer Cost
Terrain Inputs	0 - 2 TMM*	0 - 1 mon	0 - $1000
Program Revisions	0 - 4 TMM	0 - 4 mon	0 - $4000
Scenario and Inputs	1 - 6 TMM	1 - 3 mon	$1000-$2000
Production Runs	2 - 6 TMM	1 - 3 mon	$10,000-$20,000 (40-200 replications)
Analysis	2 - 6 TMM	1 - 3 mon	$1000-$3000
Report Preparation	2 - 6 TMM	1 - 3 mon	
TOTALS	7 - 20 TMM	4 - 17 mon	$12,000-$60,000

*Technical man months

40-4 DESCRIPTION OF SELECTED COMBAT ANALYSIS MODELS

Over the years, many different types of combat simulations have been developed for various purposes of analysis or for training. These range from war games, which are mostly manual in character, to computer-assisted combat simulations, and all the way through to the completely computerized combat simulations such as CARMONETTE. Generally, such simulations are now referred to as "combat analysis models", irrespective of the amount of computer assistance. Moreover, there has been a tendency to develop combat analysis models into a hierarchy of simulations leading up to large-scale operations. Thus the small unit, high-resolution models may be used to generate performance measures of military units which will feed into battalion and division level games, and on to corps, army, or theater level studies.

Of course, there are also analytical types of combat analysis models—such as Lanchester's linear and square laws, Deitchman's guerrilla warfare models, and others—which were presented in Chapters 28 and 29. The analytical models are readily used in many calculations of interest although they depend critically on estimation of the most proper attrition coefficients for particular applications. These models should be used whenever they describe two-sided combat adequately since they may save considerable effort, time, and/or cost as compared to war games or computerized combat simulations. Also in some cases it will be convenient and useful to employ some of the analytical models in connection with either war game exercises or computerized combat simulations described in this chapter. As an example, the kill times for units on each side may be analyzed and fitted to a model of combat, such as covered in par. 28-12.2, or the data on losses for each side at various stages in the battle may be used to predict attrition as in the Bonder IUA model described later.

The selection of the best models for different applications often comes within the realm of responsibility of the weapon systems analyst, and his decision will depend on the nature of the problem and just how well an existing simulation might fit. Otherwise, the analyst may find it necessary to develop a new model or simulation to obtain the most appropriate answers. He also will have to decide on the need for and the particular points in a simulation, or "combat analysis model", for which stochastic routines are appropriate and on the number of runs which must be made to judge something about convergence of results for the simulation process to suitably stable predictions.

Once we depart from the purely analytical type of model, we can categorize the other two types of combat simulations as either "Delphic" on one hand or a computerized combat simulation on the other. The Delphic term is used as a description which involves much player and controller involvement; the war game is such an example here. Thus although the Delphic type war game or simulation may require much time and the use of rather expensive resources, it does provide for direct involvement of the military players or sponsors and gives more visible battle detail for training purposes, for example. Because computer simulation type models are much faster, many combat situations may be developed, many runs made, if desirable, and the sensitivity of key variables and interactions determined.

In the paragraphs that follow, we will summarize some of the current combat analysis models (other than the analytical ones of Chapters 28 and 29, for example) to acquaint the analyst with them and provide a quick reference in connection with his daily job. Some of the combat models are completely manual in nature, such as the Theater Battle Model (TBM); some are computer assisted; some are completely computerized such as CARMONETTE which is summarized (again but in brief form); and one model is even a one-sided deterministic simulation (Legal Mix IV). Some models are completely "deterministic"; some may be considered to be stochastic; and others involve various combinations or mixtures of the more desirable methods of play which enable the analyst to pick the particular combat analysis model he can use to advantage.

Although we will discuss the models more or less in the order from the high-resolution type—involving the quantification of measures and trade-offs, such as weapon delivery accuracy or probability of hitting, lethality, and rate of fire (i.e., kill rates)—to the large-scale types of simulations, such order may not mean much to the reader since he may be more interested in the particular content of the battle played.

Many of these descriptions are available from a summary by Braddock, Dunn, and McDonald provided the Army under contract or are rephrased from Refs. 9 and 11.

40-4.1 DETERMINISTIC END GAME ASSESSMENT SIMULATION (DEGAS)

DEGAS is a computerized, deterministic simulation developed by the General Research Corporation to produce estimates of the results of hostile encounters essentially isolated from the general context of battle and involving relatively few participants on each side. In fact, the DEGAS model was developed primarily for the investigation of the effectiveness of Army helicopters, and it may be thought of as an "end-game" simulation. The DEGAS model simulates engagements involving a maximum of six helicopters attacking a ground target complex having a maximum of 16 elements, and the time duration of the simulated engagement is limited to about 10 min. In its present form, the airborne weapons represented in the DEGAS model include the TOW missile, the 30 mm aircraft gun, HELLFIRE, and the 2.75-in. rocket. Ground weapons attacked include air defense weapons and missiles, tank main armament, and small arms. The inputs include the attack helicopter weapon characteristics, vulnerable areas for each helicopter and each armored ground target, and lethal areas against personnel. Movement paths, movement rates, and line-of-sight conditions are listed as inputs for each participating element for the total duration of the simulated engagement, and the tactics, target priority, and selection rules are input by a combination of the model logic and input variables. The model logic involves target selection based on target priorities, which are influenced by range to the target and line-of-sight conditions as they evolve during the simulation or engagement. After target selection, whether new or old, the lines of sight are checked, and the attackers and defenders fire at each other. Then damage to the attackers and defenders is assessed, followed by movement of the attackers and defenders and a reassessment of priorities, which leads to selection of new targets for further engagement, etc. The outputs from DEGAS include attacker losses, defender losses, ammunition expenditures by type, and elapsed times. The DEGAS model is quite sensitive to relatively small variations of weapon characteristics or tactics employed. This feature permits the evaluation of the consequences of input variations not practicable with a Monte Carlo type of simulation because a stochastic model might require many, many replications before stabilization of outcomes. Thus the advantage of DEGAS is apparent—it is practicable to simulate the engagement repeatedly, varying the engagement range with each repetition, and thereby determining the particular range at which the exchange rate is optimum. This type of information could be of great importance to the larger scale computerized battles. The General Research Corporation is the point of contact.

40-4.2 ASARS IIX (ARMY SMALL ARMS REQUIREMENTS STUDY) BATTLE MODEL

The ASARS Battle Model is a two-sided, high-resolution, dynamic, Monte Carlo simulation of dismounted combat between less-than-company sized units. ASARS represents, with a high degree of realistic detail, a substantial portion of the factors involved in or impacting on small infantry unit combat. The model was designed to serve as an operations research tool for evaluating the comparative effectiveness and utility of small arms (pistols, rifles, automatic rifles, machine guns, grenades, and grenade launchers), operational concepts of various organizations, and tactics of weapon employment in an operational context. Movement paths of the units are generated dynamically within the model to reflect leaders' perceptions of current battle conditions. The dismounted forces can be supported with artillery and mortar fires represented in detail, and firing from aircraft can be approximated. Antipersonnel minefields are represented, with options to breach, traverse, or bypass. Intelligence representation focuses on line-of-sight acquisition of opposing personnel and small arms but also includes unattended ground sensors. The model represents decision processes and events in great detail and affords much flexibility for the user to specify situations and tactical decision rules. Although vehicles and direct fire weapons larger than grenade launchers are not represented, model design permits modifications or expansions into many areas. Terrain elevations are specified at 12.5-m intervals from map-based digitized tapes of the Topographic Command. Each of up to 150 soldiers is individually represented in up to 30 separate maneuver units. Each exposed man is individually assessed for weapon effects from individual bullets or flechettes and from fragments from each exploding munition. Hits are recorded for five areas of the body. Probability of incapacitation is computed for each body part hit. These probabilities are translated into the man's inability to observe, move, fire, or fire and move. Suppressive effects of hits and misses are also represented for small arms rounds, grenade fragments, and artillery and mortar fire. ASARS IIX was developed by US Army Combat Developments Command, Systems Analysis Group and documented in May 1973. The model is maintained by the US Army Infantry School, Fort Benning, GA. The model is programmed in FORTRAN IV for the CDC 6400/6500 computer and requires 84k (decimal) words of core, plus tape and disc, for a 31-element scenario, which requires on the order of 25 min of central processor unit (CPU) time. A 60-element scenario requires 3-5 h of CPU time and 95k (decimal) words of core.

40-4.3 CAC INTERACTIVE WAR GAME (JIFFY)

The "Jiffy" Game is a computer-assisted, manual, two-sided war game. Players manually manipulate forces, using maps and performance indicators developed in previous nonmanual studies to simulate ground combat. The game can accommodate from battalion through theater level forces. It was developed or evolved as a highly flexible, simple, and rapid procedure for preliminary investigation of the relative value or effectiveness of different force designs. Units are identified and placed in their position on the map. Firepower scores are aggregated for each side, force ratios are calculated and modified in accordance with the situation, and rates of advance are determined. Previous nonmanual studies and Field Manuals provide the factors used to quantify the performance of weapon systems and to calculate attrition resulting from combat. Personnel and equipment losses, and use are determined, and requirements for replacements are derived. Use of and requirements for artillery and engineer support are determined. Elements whose status is specifically addressed include field artillery; ADA; TACAIR; trains elements; dismounted infantry; antitank weapons; armed helicopters; command posts; tanks, APC's, and ICV's; mortars; and minefields. Resolution is to the level required, but normally it addresses the battalion. The battle is assessed periodically for time intervals during which committed combat power remains constant, termed "critical incidents" (significant events). It requires approximately two to five days to run a critical incident; however, this depends on the evaluation objectives assigned. The output of the exercise is a narrative, photographic, and statistical display of the progress of the battle to include the listing of personnel losses, major supplies consumed, and equipment losses. The value judgments of the gaming team dictate the relationships of study inputs and their specific adaptation to the gaming process. The point of contact is the Directorate of Scenarios and War Gaming, US Combat Arms Combat Developments Activity (CACDA), Fort Leavenworth, KS.

40-4.4 CARMONETTE VI

(See also par. 40-3.1). CARMONETTE VI is a two-sided, high-resolution, Monte Carlo simulation of small unit combined arms combat involving ground units ranging in size from platoon to reinforced battalion. Activities simulated include movement, target acquisition, communication, and employment of a variety of weapons, including missiles, fired by infantrymen, tanks, armored personnel carriers, helicopters, and air defense units. Resolution can be set from platoon level to the individual vehicle or dismounted soldier. CARMONETTE plays a battle area of 60 × 63 terrain cells, with cell size variable from 10 m to 250 m on a side (total battle area from 600 m × 630 m to 15 km × 15.8 km) (100-m cell size normally is used). For each cell the average value is input for terrain height, cover, concealment, height of vegetation, and trafficability (road and cross-country). Up to 63 units on each side can be represented; 48 of which can be weapon units, and 15 can be command, control, and surveillance units. A predetermined scenario explicitly controls the action of all units, with the exception of certain orders whose execution is dependent on knowledge of and action by enemy or other friendly units. Battles as long as 90 min can be simulated. Stable results often can be achieved with 5 to 20 replications. CARMONETTE was essentially the first high-resolution computer simulation of this type. It was programmed in 1959 and since has been under modification and use by Research Analysis Corporation (now General Research Corporation (GRC)). Version III was used in the Small Arms Weapons Study (1967); version IV was used for a study of night vision devices (1969); version V was used in the equal Cost Firepower Study (1971); and version VI has been employed in the SCAT-II helicopter study. The model can be run by GRC and by the US Army Concepts Analysis Agency. CARMONETTE is programmed in FORTRAN IV and requires 65k (decimal) on the UNIVAC 1108, and 175k (octal) on the CDC 6600 for core storage.

40-4.5 COMANEX (COMBAT ANALYSIS EXTENDED)

COMANEX is a Monte Carlo ground combat model designed to extrapolate rapidly the results of the high-resolution CARMONETTE model for a given force mix to other force mixes. Results so extrapolated include losses of dismounted infantry, tanks, APC's, and helicopters. The short running time of COMANEX also lends itself for use in division level games/simulations for assessing small unit engagements. Detailed battle history results from a high-resolution model are preprocessed by COMANEX to form a set of Lanchester-type parameters which represent, essentially, the kill rates for each weapon-target combination in the engagement. These parameters are then used to predict battle results when varying input-specified numbers of these weapons are involved. COMANEX, running about 100 times faster than CARMONETTE, can provide 30 replications of a 30-min battle in less than one minute. The model is a revision by GRC of the COMAN model developed by Dr. Gordon Clark of Ohio State University in 1970. COMANEX is programmed in FORTRAN IV for the CDC 6400 computer and can be operated by GRC. The point of contact is GRC, McLean, VA.

40-4.6 IUA (INDIVIDUAL UNIT ACTION)

IUA is a two-sided, high-resolution, large-scale Monte Carlo simulation of mounted ground combat. IUA can represent up to a battalion task force in offense, defense, and delay at engagement ranges up to 3000 m. The model was developed for evaluating the combat effectiveness of equal-cost mixes of armor and antiarmor weapons. A strong capability of IUA is the ability of it to simulate in detail direct fire weapon effectiveness to include weapons such as tanks, APC's, recoilless rifles, rocket launchers, and guided missiles. IUA has a limited capability to portray minefields, artillery, helicopter-borne weapons, and TACAIR. Dismounted infantry is not played. Movement is on predetermined routes. The defender does not maneuver but can withdraw to a secondary position. The simulation of mobility and line of sight are done deterministically by mobility and terrain preprocessor computer programs. The defenders are always considered in hull defilade. Terrain is represented by up to 999 triangles, with map elevation to the nearest meter specified for each vertex. Generally, a battle area of 5 km × 5 km is represented. Input can include five soil types, 13 obstacle types, three concealment heights, six terrain roughness types, and three cover heights. The attacking force has one to three prespecified axes of advance. A total of up to 12 routes are prespecified with two force sections. IUA was developed by Lockheed in the mid-sixties to support TATAWS (Tank Antitank Weapon Study); the model was improved by Booz Allen in 1970. IUA has been used in nine studies, such as TATAWS III, ATMIX, and CONFADS. The model requires approximately 162k (octal) of core storage for execution and approximately 10 min of CPU time for 30 replications of one case on the TRADOC CDC 6500 computer at Fort Leavenworth. Input data preparation time—for a new terrain, scenario, and weapon data set—requires on the order of 10-12 man-weeks. One full-time analyst, plus programmer support, is required to operate the model. The point of contact is CACDA, Fort Leavenworth, KS.

40-4.7 BONDER/IUA

Bonder/IUA is a differential model based on the IUA (Individual Unit Action) model. As such, Bonder/IUA is a two-sided, high-resolution, large-scale, analytical model of tank-antitank combat which can represent up to a battalion task force in offense and defense (the delay role cannot be played) at engagement ranges up to 3000 m. Bonder/IUA uses the same terrain, route, and mobility data as IUA, and therefore it depends upon the same deterministic mobility and terrain preprocessor programs as IUA to simulate mobility and line of sight. The principal difference between Bonder/IUA and IUA is in the attrition assessment portion of the program. While IUA uses Monte Carlo techniques in this area, Bonder/IUA uses analytic techniques involving modified Lanchester equations. Bonder/IUA requires no replication, however. Bonder/IUA uses approximately five minutes of CPU time to execute on the TRADOC CDC 6500 computer, as compared to approximately 10 min for 30 replications by IUA. The short running time of Bonder/IUA and the relative ease of changing tactics and weapon mixes (provided no changes are required in the prespecified routes) enable users to review a number of weapon mixes and tactics with relatively small cost. Bonder/IUA requires about 150k (octal) of core storage to execute. With respect to the assumptions made as to the combat process and the limitations therein. Bonder/IUA and IUA are identical. Bonder/IUA was developed in 1970 by Vector Research, Incorporated, Ann Arbor, MI, and has been used by the Studies, Analysis, and Gaming Agency of JCS for several studies; by ACSFORS for the DRAGON Cost Effectiveness Analysis; by Rock Island Arsenal and the Weapon System Analysis Directorate, OAVCSA, in the MBT Study and the M60A1 Improvement Study; and by USACDC in the ATMIX Study. Although the model must not be considered validated against actual test data, the model has been validated against IUA, and the validation effort underway for IUA applies also to Bonder/IUA. The model is maintained at CACDA.

40-4.8 AMSWAG (AMSAA*) WAR GAME

The AMSWAG model is a time-sequenced, deterministic, battalion level, force-on-force computer model that simulates a classical attack and defense. Up to 64 defenders are deployed in fixed positions in hull defilade. The attacking force has already deployed in fixed positions and moves along predetermined routes of advance toward the defender. The attacking force is allowed a maximum of 12 routes of approach, and these routes are administratively broken into one to three groupings of up to four routes each. Such groupings are called "axes", and each axis nominally contains a company sized force. Thus each route nominally contains one platoon, and the platoon may be further split into two homogeneous sections (of two to four vehicles each) which maneuver together down the route. Normal movement techniques for these sections are either alternate bounds or successive bounds. The AMSWAG model conducts the battle in uniform time steps of 10 s each.

The history of the AMSWAG model, it might be said, started with the Individual Unit Action (IUA) simulation, a time-sequenced, Monte Carlo, battalion level, force-on-force computer model developed by the Ground Vehicle Systems Analysis organization of the Lockheed Missiles and Space Company for the US Army Combat Developments Command's study on the Tank Antitank Assault Weapon System (TATAWS III) study. The next evolutionary step was the development of the Bonder/IUA model, which uses the same data base input card formats as those required for the Lockheed IUA model. Beginning in about 1973, AMSAA effected modifications and improvements to the Lockheed IUA and Bonder/IUA models for their evaluations to include individual line of sight, revised target priorities and round choices, multiple kill criteria, dismounted infantry, revised data bases, ease of computer running cases, improved acquisition, expected time-to-kill tables, basic load constraints/ammunition summaries, and improved output (Ref. 11).

For AMSWAG, the majority of the input is preprocessed and stored on magnetic disks due to the relatively large amount of individual inputs required by the model (approximately 250,000 numbers per case) and because there are so many similar cases. These are stored in a packed form, which reduces significantly the computer time and memory required to read and store such data. The preprocessed information includes mobility data, intervisibility data, vulnerability data, probability of kill given a hit data, expected time-to-kill data, and weapon and round type data.

AMSWAG repeats the same sequence of events during each 10-s interval of a case, and, for the most part, AMSWAG does not maintain knowledge of just what happened during the preceding time intervals. The decisions about what to do during an interval depend on the conditions that exist during that time interval and not on what happened in the previous intervals. The basic sequence of events for each interval includes target acquisition, weapon-target allocations, firing, assessment, a suppression routine, movement of units, and output of results for each interval. At the end of each 10-s interval, the computer prints out the following:

*AMSAA = US Army Materiel Systems Analysis Activity

1. Total vehicle and personnel losses for each side
2. Nominal ranges between forces
3. Current vehicle exchange ratio (total Red vehicle losses divided by total Blue vehicle losses)
4. Current vehicle force ratio (total surviving attacker vehicles divided by total surviving defender vehicles)
5. Game time.

At the end of each 60 s of game time and at the end of the game, a more complete output summary occurs. This summary contains the previous printouts plus:

1. List of units which have been killed
2. Status of surviving units (ammunition remaining, location, unit movement speed, survivors)
3. Victim-killer scoreboards (the amount of kills as a function of weapon type versus weapon type and the number of survivors of each type).

The point of contact for the AMSWAG simulation is AMSAA, Aberdeen Proving Ground, MD.

40-4.9 LEGAL MIX IV

Legal Mix IV is a one-sided, high-resolution, deterministic simulation developed to evaluate artillery mixes at field army and lower levels. Artillery weapons are employed against a time-phased set of acquired targets. Primary uses of the model are to provide data on artillery support requirements and to provide comparative analyses on the effectiveness of alternative mixes of artillery weapons. The model computes percentage of missions lost, personnel casualties inflicted, armored vehicles damaged, missions accomplished, targets defeated, accrued units of military worth for missions accomplished, and cost and weight of ammunition expended to achieve effects. Military worth is an average value assigned to each target processed in the model and was derived from questionnaires in which military officers assigned priorities to the existing Legal III target list. Weapon system rates of fire, ammunition basic loads and resupply rates, predicted and precision weapon circular probable errors, weapon-range capabilities, ammunition lethality data, and ammunition costs are used as inputs. Legal Mix IV is written in FORTRAN and is operational on the TRADOC CDC 6500 computer. Required core space is 110k (octal) for the largest of four basic computer programs. Computer run time can take from 8-25 min for the effectiveness program. Preparation time is substantial. The point of contact is the US Army Field Artillery School, Combat Training and Developments Activity, Fort Sill, OK.

40-4.10 DYNTACS X (DYNAMIC TACTICAL SIMULATION EXTENDED)

The DYNTACS X model is a two-sided, small-unit, high-resolution, dynamic, Monte Carlo, event-sequenced, highly interactive, land combat simulation. The model is capable of representing battalion or smaller size armor and mechanized units. The basic elements are vehicles and crew-served weapons; dismounted infantry is not played. Casualties for vehicular mounted crews of direct fire, crew-served weapons and helicopters are represented and accounted for by the model. Systems which can be represented are vehicles (tracked and wheeled), antiarmor ground weapons (large direct fire ballistic weapons, rapid fire ballistic weapons, and guided missiles), indirect fire (cannon, missile, and mortars), air defense weapons (air defense guns, passive homing missiles, and semiactive homing missiles), terminal homing systems for direct and indirect weapons, minefields, helicopters (reconnaissance, gun, and utility), and the artillery fire control system. The principal types of operational variables which can be included are terrain type, roughness, trafficability, obstacles, and day/night conditions. Other variables are engagement type and size, and the type, size, organization, doctrine, and tactics of both Blue and Red forces. The operational area addressed tends to be limited by computer considerations (primarily core storage) to 5 km × 10 km with resolution of 100 m × 100 m (a potential exists to reduce grids to 6 m × 6 m).

The core storage of the computer being used regulates the number of battle elements represented (along with area and terrain resolution) and time required for a replication. Examples of the type of computer in relation to core storage, number of battle elements, and CPU time for one replication are CDC 6600 (MERDC)/172k words (OCTAL)/47 elements/7.6 min; CDC 6500 (CACDA)/105k (OCTAL)/24 elements/10 min; IBM 360-65/670k bytes/47 elements/20 min; and IBM 390-91 (Johns Hopkins)/670k bytes/47 elements/4 min. A typical run involves 20-30 min of battle time. Manpower expenditures to run the model run from approximately 2 man-months for a routine exercise, 2 to 3 man-months to convert from one computer to a similar computer, 6 to 8 man-months for introduction of a new or different system into the model, several months for force structure and number of elements for a newly located scenario, to man-years of effort for an entirely new or different type system not currently represented by the model. Points of contact are US Army Missile Research and Development Command, Redstone Arsenal, AL, CACDA, Fort Leavenworth, KS; and Systems Research Group, Department of Industrial Engineering, Ohio State University, Columbus, OH.

40-4.11 DBM (DIVISION BATTLE MODEL)

DBM is a division-level, computer-assisted manual war game designed for study of the combat impact of varying weapon mixes, organizations, tactics, and support levels. It can address a Blue division opposed by a Red combined arms or tank army with supporting artillery and airpower. Resolution is generally to company on the Blue side and to battalion on the Red side. Up to 350 units per side can be accommodated by the computer program. The game is played on 1:50,000 or 1:25,000 maps which are reposted each 15 min in an open, semiclosed, or closed mode. In the closed mode, a control team is necessary to process gamer orders (according to game rules), translate to computer inputs, and distribute information to gamers. Gamers perform all battle decision-making functions. The computer performs assessment and bookkeeping. In the closed mode, a team of 11 can process 2 to 4 h of combat in a working day. In the open mode, speed can be doubled with a smaller team. Normally, about 4 h of battle (up to some critical event) is laid out by gamers before the computer is called upon to assess losses and replacements, and to update the status of units. Assessment employs the COMANEX model to determine unit losses based on inputs from the high-resolution, small unit engagement CARMONETTE model. Air-to-air, air-to-ground, ground-to-air engagements, and air-mobile operations can be played in DBM. Conventional and nuclear munitions can be delivered by air or artillery. Computer portions of DBM are programmed in FORTRAN IV for a CDC 6400 computer. DBM was developed by Research Analysis Corporation (now GRC) in 1970-1971. Point of contact is GRC, McLean, VA.

DBM is one of the important models of the hierarchy of combat analysis models developed by GRC. Company or battalion organizations, weapons and performance data, terrain descriptors, and tactics are input to the CARMONETTE model, and the forces played cover a range of forces expected to be fought in DBM. COMANEX may be used to obtain results of battles not played explicitly in CARMONETTE, so that they will become input for the DBM model. This is an illustrative linkage for the hierarchy of models developed by GRC. Also, a division computer simulation based on the proven DBM games gives an output which may be used to assess the close combat engagements or actions in the theater model ATLAS discussed in par. 40-4.19. Thus it might be said that the DBM model simulation may be used to "drive" theater models.

40-4.12 DIVWAG (DIVISION WAR GAME)

The DIVWAG model is predominantly a deterministic, two-sided division level, player-assisted computer simulation. It was designed for use in force composition and doctrine studies in mid- and high-intensity environments. It simulates combat between one Blue division level force and a Red force composed of up to four divisions. The model addresses all the functions of land combat. It achieves this comprehensiveness of functional coverage only through some sacrifice in the resolution with which specific activities are treated. Therefore, the model should be considered as a medium-resolution model. The user retains general control of the battle by issuing orders to individual units. These orders may be given in a manner that the unit will execute them sequentially, or execution may be made dependent upon the condition of some dynamic element in the battle (e.g., time, the location of a unit, or the number of personnel remaining in a unit.) Functions and activities simulated by the DIVWAG model include intelligence, ground combat, area fire, air-ground engagement, mobility, engineer, combat service support, air mobile operations, and the effects of nuclear weapons. The DIVWAG model is capable of simulation of up to 14 d of continuous combat. Although designed for force composition and doctrine studies, the basic design of it allows a great deal of flexibility in use. In addition to employment in a war game of successive periods of play, it can be employed as a pure simulation without gamer intervention. For example, a single period of engagement, using a scenario from a larger game, can be played to examine the performance of specific systems.

Following extensive testing of DIVWAG in 1972, further refinement and testing have been conducted by the War Games Division, Scenarios and War Gaming Directorate, (CACDA). The DIVWAG model develops casualties from direct and indirect fires of all weapons except that small arms and other short-range weapons of dismounted infantry are not fully represented, principally because of spatial aggregation in the model.

Some of the more recent studies supported by the DIVWAG model include: Family of Scatterable Mines (FASCAM) studies, September 1973 - August 1974, and January - August 1977; Integrated Intelligence From All Sources (IIFAS) Study, May - September 1975; Antiarmor Systems Program Review (ASPR) Study, November 1975 - April 1976; Legal Mix V Studies Phase I, May - August 1976, Phase II, January - March 1977, Phase III, August - December 1977.

The DIVWAG model is operational on the CDC 6500 computer at Fort Leavenworth. The program, with overlays, requires approximately 57,000 (decimal) words of central memory storage to execute. Approximately 3 million (decimal) words of disk storage (used to store the data files), and three tape drives are also required. The time required to perform the simulation is dependent upon the number of units being played and the complexity of the activities in which they are ordered to engage, but approximately 3 s of CPU time are required to simulate 1 s of game time. The point of contact is CACDA, Fort Leavenworth, KS.

40-4.13 FAST-VAL

The FAST-VAL Model is a two-sided, deterministic computer model which simulates the ground engagement between two infantry forces with and without varying amounts of fire support (air, artillery, and mortar). The model was developed to assist the Air Force in selecting weapons, vehicles, and operational techniques for their close air support role. The simulation can represent an infantry force of up to five companies on the defense in prepared and unprepared positions and up to five companies attacking the defender's forces. Infantry units are identified down to company size, and the artillery and mortars are played as batteries. Those weapons represented in the infantry force are rifles and machine guns. The supporting fires represented are air-delivered, artillery, and mortar weapons.

Two degrees of protection can be given to the infantry and support personnel in dismounted positions. Protection factors can be assigned on a permanent or a temporary basis to bunkers for defenders and for the attacking force mounted in APC's moving to the line of departure. The battle area is broken into 100-m grid squares with personnel and weapons played being identified with each grid. Weapons effects (personnel losses and material losses) are calculated for artillery, mortar rounds, volleys, and for concentrations and air-delivered sticks or patterns. The Full Spray Lethal Area Program is used to evaluate the effects of fragmentation of the individual rounds. Round-to-round ballistic and volley aim dispersion errors are used to transform full spray damage functions into volley pattern damage functions. These pattern damage functions are used to calculate casualties at targets in both the vicinity of and at the aim point. Rifle and machine gun weapon effects are expressed as expected casualties, as a function of range and posture of targets for both single round fire and burst of rounds fire. Provision for reduced efficiency in delivery of firepower and speed of movement due to suppressive fire also has been incorporated into the model. The input requirements for the model include definition of the attacking unit-characteristics of riflemen and support personnel, posture/time tables, weapons/vehicle characteristics, rifle company characteristics, delivery schedule, range limits and firing rates; definition of defending units—same as those for the attacking unit; definition of the infantry action—engagement table; advance characteristics—engagement ranges, troop carrier characteristics, influence of suppression, influence of cumulative fraction of casualties upon advance rates; and small arms characteristics for attacker and defender. A summary of the status of all units and a summary of the status of the several engagements are printed for each simulation cycle. Additionally, the user may request print-outs describing the status of all units at the end of each cycle and the aim points selected for mortars, artillery, and air-delivered munitions. As can be seen by the description, the model is of high resolution. It produces detailed output and is free running once started. However, limitations are that only rifles, mortars, and machine guns of the infantry force are represented while, in reality, today's mechanized infantry units have many more supporting weapons—grenade launchers, LAWS, recoilless rifles, tanks, etc.—organic to the organization or attached during battle. Additionally, FAST-VAL does not discriminate between killed in action (KIA) and wounded in action (WIA) although such a capability could be added, as in most of the other models. FAST-VAL does address the small unit engagement area in considerable depth, and significant efforts have been made to compare its results to those of a series of actual small unit engagements in Vietnam. In number of Blue casualties incurred, these comparisons are surprisingly close. However, the outcome or winner of the fight does not so much prove the rectitude of FAST-VAL as a predictor of small unit infantry casualties and fight outcomes, as it confirms what has been shown elsewhere: given operational inputs that are correct in essentially all respects, a reasonably designed model can recreate accurately historical results. FAST-VAL attempts to predict the outcome (winner and casualties) of a fire fight as a function of weapons and tactics employed rather than predicting, for example, what engagement will occur in a battle or how much of what munition will be employed in a fight. The program is written in FORTRAN IV for an IBM 360/65 computer and requires 190k bytes of core storage memory. The model was developed by RAND for the Air Force during 1970-1971 to support Air Force requirements for close air support. The point of contact for this model is the Deputy Chief of Staff, Research and Development, ATTN: RAND Project Office, Headquarters, US Air Force, Washington, DC.

40-4.14 CEM III [CONCEPTS (FORMERLY CONAF) EVALUATION MODEL III]

CEM III is a two-sided, deterministic, theater-level warfare simulation (fully computerized). It was designed to encompass all combat aspects of theater warfare, in a dynamic way, covering an entire campaign and permitting evaluation of a force alternative in about one week, while remaining sensitive to important force characteristics. CEM represents ground combat engagements on given terrain. CEM resolution is at the level of Blue brigade and Red division. Much input of a judgmental or historical data nature is used. CEM concentrates on representing the sequential decision making at theater, army, corps, and especially division levels, to determine the allocation of resources and the missions to be undertaken by the various units as the battle progresses. Periodically, estimates of the situation are represented—on the basis of which decisions are reached and implemented at each of those four echelons. At division level, this process is repeated every 12 h, at corps level every 24 h, at army levels every 48 h, and at theater levels every 96 h. Decisions are determined by input alternatives and criteria which are compared with the status of units, estimated unit force ratios, missions, postures, and anticipated engagement outcomes down to the brigade level, by minisector. Unit status reflects losses and replenishments. Losses are a function of engagement type and outcome. Replenishments include personnel and materiel. Estimated force ratios and anticipated outcomes reflect imperfect knowledge. Up to 1000 minisectors can be represented; each designates the front of a resolution unit which may be opposed by one or more adjacent resolution units whose minisector boundaries need not be coincident with those of the opposer. Minisector traces must be specified as pregame input, conforming to map terrain features. Firepower potential is modified to reflect the circumstances of each engagement, in which only the firepower is counted for which there are targets present. To simplify firepower calculations, ground targets are classed as "hard" (e.g., tank weapons), "medium", or "soft". Similarly, ground missions are classed in three categories: attack, defend, and delay. Four types of terrain are defined: roadway passage only, cross-country possible with difficulty, no impedance to movement, and barrier. Decisions made include distribution of replenishments, commitment or retention of reserves, assignment of newly arriving (input scheduled) reinforcing units, allocation of close air support and artillery, and the unit mission and posture to be adopted during the next period. Air resources are similarly allocated to air defense, counterair, armed reconnaissance/interdiction, and close air support. Assessment of employment of air resources includes losses to aircraft inventories, aircraft ground facilities, and air defenses. This assessment also determines whether the air environment is friendly for ground forces whose delays between allocation and availability are affected accordingly. Fire support is allocated to strong units in attack and to weak units in defense. Engagement outcomes are win, lose, or draw. Rate of the forward edge of the battle area (FEBA) movement is based on these outcomes plus input data. Although deterministic, the model may yield substantially different results from similar forces because of the complex dependent sequence of threshold-type decisions made during the course of a lengthy battle. Thus it is difficult to relate cause and effect. CEM outputs WIA and KIA based on input adapted from FM 101-10-1. Other elements of theater personnel replacement are similarly treated. CEM III is an improved version of CEM which was developed in 1971 by the Research Analysis Corporation (now GRC) for use in the CONAF (Conceptual Design of the Army in the Field) methodology and study. CEM can be run by GRC and by US Army Concepts Analysis Agency. The model is programmed in FORTRAN IV and requires 100k on the UNIVAC 1108 and CDC 6600 series computer for core storage. Two days of combat require about one minute of CPU time.

40-4.15 TARTARUS IV

TARTARUS is a player-assisted, two-sided, differential model of theater level combat, with resolution to the brigade or division. The model is designed to study the effect of weapon systems and their mixes and can simulate the attack, defense, delay, and counterattack. Firepower scores modified by the interactions are used in differential equations to assess movement and casualties. The model represents the effects of tank, infantry, and tank/infantry forces supported or not supported by artillery or like forces. Target acquisition, engagement, and movement are played. Four to 300 brigade/division size units can be played with 94 items of equipment identified (10 weapons classes and 3 firing classes can be programmed). Every battle hour, or as inputted opposing units are acquired, the target list is updated. The firing interval may be as small as one minute. Weapons are assumed to distribute their fires among available targets within range according to a formula based upon unit mission, range to the targets, surveillance factors, and maximum range-firing fraction (a factor given to weapons based upon their capability to fire at maximum range in a particular type mission—i.e., hasty defense, attack, etc.). The computer-developed assessment is highly sensitive to the weapons-class versus weapons-class effectiveness factors which combined with unit "hardness indicators" and "breakpoints" will determine the outcome of any simulated engagement. The outputs of the model are a unit status report which gives the general status of the unit; detailed strength and loss report (strengths and losses of each unit by weapon class); ammunition and fuel expenditure report; summary of losses by weapon class and side; number of weapons lost by unit and weapon type; and displays (off-line Calcomp Plotter) showing unit location, frontages, unit movement routes, and terrain data set. The model was developed by the US Army Strategy and Tactics Analysis Group (STAG)—now the Concepts Analysis Activity (CAA)—and is written in FORTRAN V for the UNIVAC 1108 computer. The effort required to run the model is based on the number of units and size of the area played. The point of contact is the USA Concepts Analysis Agency, 8120 Woodmont Avenue, Bethesda, MD.

40-4.16 TBM (THE THEATER BATTLE MODEL)

TBM is a comprehensive manual war game of tactical combat operations involving all types of theater forces (land, sea, and air) under a conventional or nuclear environment. The level of resolution for land forces is the division; for air elements, the flight for conventional weapons and the sortie for nuclear weapons; and, for sea forces, the task force. The Research Analysis Corporation (RAC) was directed in 1968 to develop a family of compatible models adding capabilities to simulate CBR, air mobile operations, and counterguerrilla warfare operations to the 1963 version of TBM. The models were Theater War Game, Theater Quick Game, Division Operations, Amphibious Warfare, and Counterguerrilla Warfare Model. While elements of this TBM appear in several different war games which carry TBM in their name, the version referred to here is a tactical war game which has been reported to acquire 30 gamers and to proceed at a 1:1 ratio of combat to real time. The point of contact is the National Military Command System Support Center (NMCSSC), the Pentagon, Washington, DC.

40-4.17 THEATER AMMORATES (THEATER NONNUCLEAR AMMUNITION COMBAT RATES MODEL)

THEATER AMMORATES is more properly called a methodology than a model since a group of nine models are employed separately—with the results of one being a partial input to another—in a series of off-line steps culminating in a final processing and aggregating run by the THEATER AMMORATES model. THEATER AMMORATES was designed to predict Army expenditures of nonnuclear ammunition in hypothetical theater campaigns of 90 or 180 days in Europe and the Pacific. Such predictions are to serve as the basis for Department of the Army plans and decisions on ammunition stockage and procurement, as a part of the annual DOD budgetary process. The THEATER

AMMORATES model which generates the final output, is a two-sided, deterministic model of theater level ground warfare, including artillery and helicopters. The model uses specially developed scenarios, and input data from the various submodels, to simulate a theater campaign, including intense initial periods of conflict and subsequent sustaining periods. The eight submodels are of various types. The Tank-Antitank submodel and the Helicopter Antiarmor model are both two-sided, small unit, high-resolution, deterministic models. The Infantry submodel is a two-sided, small unit, high-resolution, Monte Carlo simulation. The Helicopter Antipersonnel submodel is a one-sided, small unit, high-resolution, Monte Carlo simulation. The Artillery Casualty Assessment submodel is a one-sided, high-resolution, Monte Carlo, munition delivery and target effects simulation. It is supported by a one-sided, Monte Carlo target acquisition simulation and separate deterministic models for Red and Blue artillery, representing tactical rules and weapon allocation processes of the fire direction center and the availability of weapons to respond to the time-phased fire missions. The Air Defense submodel is essentially a one-sided manipulator of judgmentally-derived input data. As a whole, THEATER AMMORATES represents most of the major types of weapon system-versus-unit interaction that result in personnel casualties. Close air support by fixed-wing aircraft, however, is not represented, except by Air Force input data. THEATER AMMORATES is unusual, moreover, in being intended to generate, with limited resources, numbers having a reasonable degree of *absolute* validity, rather than simply the *relative* validity which is often sufficient for comparative evaluation of forces or weapon systems. Thus the makeup and development of this overall model reflects some concern with the matter of "representativeness" of the subnumbers used and generated and with the matter of creating realistic rates of battle activity as far as ammunition expenditures, and to some degree casualties, are concerned. Typically, in operation of the model, military experience is used to define—based on a detailed scenario—the small unit engagements likely to occur in each of a series of consecutive six-hour periods for a typical division slice. As many as 40-50 such engagements may be identified in one such six-hour period. Based on those defined engagements, about 100 representative engagements are simulated with the relevant high-resolution submodels. Results of those simulations are used to fill 80 main cells in a limited-situation matrix, reflecting four operation types or "postures" (attack, defense light, defense heavy, and delay), five types of engagement (infantry, tank-antitank, helicopter antitank, helicopter antipersonnel, and indirect fire support), and four six-hour portions of the day and night.

THEATER AMMORATES was initially developed in 1967-68 by Eyler Associates, Frederick, MD, for what is now known as US Army Concepts Analysis Agency (CAA). Modifications and improvements have been made by CAA. Also CAA has maintained the model with a staff of five analysts, and it has been exercised annually since 1968. It is programmed in FORTRAN IV for the UNIVAC 1108 computer, on which the various submodels each require from 20k to 50k of core and consume from about one minute to three hours of CPU time per "case" run. The point of contact is CAA.

40-4.18 TXM (TANK EXCHANGE MODEL)

TXM is a two-sided medium-high resolution stochastic simulation model developed to assess tank lethality and vulnerability. A total of 10 elements can be used as inputs and can be either tanks or antitank weapons. Any combination of the 10 elements may be examined, a constraint being that attacking tanks must be all of the same type. Attacking tanks are allowed movement along straight line predetermined paths, whereas defending tanks and antitank weapons remain stationary. Line of sight is prescribed by the user, and scoring is on a one-to-one basis. Into each such cell data is loaded on Blue and Red ammunition expenditures, personnel casualties, armor losses, and helicopter losses. The

THEATER AMMORATES model then accesses these data and aggregates and extrapolates to the theater (US Army Sector). In performing this process, the model updates the Index of Comparative Firepower (ICF) scores of the opposing forces to account for losses, reinforcements, replacements, and returns to duty, and uses the ICF—together with scenario data, criteria, and doctrine—to define frontal activity on a period-by-period basis. A cumulative total of expenditures, casualties, and losses is recorded for each period and at the end of the campaign, for the theater.

40-4.19 ATLAS (A THEATER LEVEL COMBAT SIMULATION OR A TACTICAL, LOGISTICAL, AND AIR SIMULATION)

ATLAS is a computerized theater level combat simulation consisting primarily of four models: the Ground Combat Model, the Tactical Air Model, the Logistics Model, and the Tactical Decision Model. The following quote from Ref. 9 describes ATLAS and the models.

"A game scenario which states the specific objectives, the constraining policies to be followed, and the combat forces available, essentially guides the simulation from the start. From this scenario and the developing tactical situation comes information which triggers the tactical decision model into sending troops, supplies, and equipment to the other models. These models then interact in a tactical sense and thus develop the combat situation.

"The simulation regards the tactical battlefield as being divided into non-interacting battle areas called sectors.

"In general, the smallest discrete combat unit simulated in any given sector is a combat division. Since the ground combat model is designed to determine unit advance, each sector is further divided into *segments* so that trafficability within each segment may be considered constant. Terrain and natural or man-made barriers affect military movement, so six types of terrain-barrier combinations are simulated in the model.

"Each battle sector also has a logistics system, a one-dimensional supply system extending from theater ports or central staging areas to the most forward supply area near the combat zone. Intermediate supply centers or supply nodes are simulated to service airbases and other rearward elements.

"To complete the view of the ATLAS battlefield, it shows tactical aircraft activities: interdiction, air-to-air engagements, and close air support.

"Tying these three models together so that the simulation may proceed day-by-day with no interruptions is the function of the tactical decision model."

40-4.19.1 Ground Combat Model (Ref. 9)

"The functions of the ground combat model are to compute rates of advance of forces, casualties per day and unit effectiveness. In attempting to calculate the daily advance of the attacking force, this model examines the forces assigned to combat on each side and determines their present level of combat effectiveness in accordance with a loss of personnel or supplies and equipment.

"The measure of combat effectiveness used in the model is called ICE (Index of Combat Effectiveness), based on the relative firepower of units.

"In determining how effective a combat unit is on a given day, it is assumed that the unit's effectiveness can be measured as a function of the percent casualties to the unit, the level of the unit's supplies and equipment, and the particular activity of the unit—attacking or defending. To determine at what point a combat unit becomes ineffective is a difficult procedure. However, the effects of casualties must be taken into account and this is accomplished by the effectiveness curves. These curves indicate the percent degradation of unit effectiveness as a function of casualties received when attacking or defending. The effect of a given casualty level is greater on an attacking unit than on a defending unit. This is because an attack normally requires rapid movement, good coordination, and higher organizational integrity.

"Other curves of the ATLAS Model are available for degrading effectiveness as a function of supply level. The effectiveness value finally used is the minimum of the value due to casualties or the value due to lack of supplies."

40-4.19.2 Logistics Model (Ref. 9)

"In order that ATLAS be capable of realistically assessing the outcome of deploying forces rapidly to meet a given threat, a model of the theater logistics capability is required. A model of this nature should simulate such things as the movement of supplies to the deployed combat units, the interdiction of supplies being forwarded, the movement of new units through the theater to the combat zone, and the stockpiling of supplies within the theater, if desired. A basic premise of the model is that the resupply of deployed units takes priority over the deployment of new units, with the building of stockpiles taking third priority.

"Within each battle sector, the network of LOCs [lines of communication], both rail and road, are represented by a single series of supply nodes. These supply nodes are located for each side approximately 1 day's overland journey apart. Each node is described by certain characteristics that indicate the maximum daily output by ground means, light helicopters, or fixed wing transport aircraft. If a specific node is required to stock a certain level of supplies, this is also indicated. Nodes which simulate ports or large airbases generally are the receiving points for direct delivery of troops, supplies, and equipment into the theater. The operational capabilities of these nodes are scenario dependent and are specified in the input data.

"For each combat sector the node immediately behind the FEBA is designated the forward supply point, and is responsible for resupplying all the combat troops in that sector. For the first day of combat, the supply node which is to be the forward node is specified in the input data. Thereafter, the movement of the FEBA is examined to discover whether the previous day's forward supply node has been overrun, or if it can now be moved to a more forward node.

"The logic that simulates the flow of supplies is the same for each sector and deals first with the forward node. A demand from the ground combat units, which varies with the number and type of demanding unit as well as its combat posture, is created and sent to the forward supply node. If this node cannot meet the demand, the next most rearward node attempts to meet it. If this node also fails, supplies may be forwarded by air from a more rearward node if the capability is available.

"When the daily movement of supplies has been completed all remaining ground and airlift capacities are used to move fresh troops and equipment to the combat zone.

"The logic of this model was designed so that for stable combat conditions and adequate logistic support, supplies should flow smoothly into the forward supply node and hence to the consuming units. However, if the movement capabilities are low, or the enemy interdiction effort is heavy, the combat effectiveness of active units may be degraded and the total number of combat missions that could be flown from any one airbase may be restricted.

"The ground model handles the daily combat actions (and the subsequent FEBA advance) and the logistics model attempts to keep supplies moving forward to insure maximum combat effectiveness."

40-4.19.3 Tactical Air Model (Ref. 9)

"The tactical air model accounts for mission assessment relating to SAM suppression, airbase interdiction, air defense, close air support, and supply interdiction. Daily operation of the air model depends on a tactical air controller, simulated within the decision model, to assign combat aircraft to each sector. Aircraft are assigned to sectors on the basis of the tactical environment, the airbase capability and the aircraft availability.

"Once aircraft are assigned to sectors, the air model makes assignments to specific airbases within the sector for a home base location and logistical support. The home base location is necessary as a basis for evaluating the combat radius of the aircraft. A combat radius determines the maximum depth to which missile sites, airbases, and supply nodes may be interdicted. All distances are calculated from the node that supplies the home base to the nodes associated with the target elements.

"Target elements such as missile sites and air bases are specific missions in the general air superiority role. The logic of the air model assumes that all active airbases and missile sites within range of the combat aircraft are vulnerable to attack." (Ref. 9 gives further details.)

To have the air model operate on a 24-h cycle from day to day without additional mission type orders requires a routine to assign aircraft to tactical missions each day. This is done by a set of mission assignment curves. The number of aircraft assigned to each mission are determined by the relative strength of air power per sector. As one side achieves air superiority, more and more aircraft are assigned to close air support and interdiction missions. The curves are entirely arbitrary. Any set of such (appropriate) curves may be used.

40-4.19.4 Tactical Decision Model (Ref. 9)

"The function of the fourth and last model, the tactical decision model, is to allow the simulation to proceed through an entire war without interruption. One application of the simulation is to assist in rapid deployment studies, hence in situations where troops, supplies, and equipment will be scheduled to arrive at ports and air bases at various times during a war. This model is specifically designed to determine the sectors to which newly arrived combat units might best be deployed, to determine the distribution of supplies and missile units as they enter the theater, and to allocate tactical aircraft on a daily basis to each sector for both sides.

"The decision model assigns a new unit to a particular sector in the following way: after viewing the type combat actions in all battle sectors, the model determines in which sector the attacking force could reach some predesignated defensive position in minimum time. This position may be a strategic phase line or the enemy's final objective itself. If there is no movement on the front when this assessment is made, minimum distance becomes the criterion instead of minimum time.

"Instances may well occur, however, where an additional unit assigned to a sector will overburden the logistics capability of the sector. Therefore, before the new unit is assigned to the sector, the ability of that sector to resupply existing combat units, to transport replacement items and supplies, and to move the new unit through the system is carefully evaluated. If the sector in its present condition is not able to handle the new unit, other sectors are then evaluated as to their capability, always keeping the tactical need foremost in mind.

"The allocation of combat aircraft to battle sectors is a function of the tactical situation existing in the sector. Three tactical situations are possible: (1) the aggressor forces advancing, (2) the aggressor forces retreating, and (3) the forces stalemated (i.e., no FEBA movement).

"These situations are assumed to be assignment priorities in the order 1, 2, 3 for the aggressor force. Thus each day the aircraft are assigned to the highest priority available. If the same situation exists in more than one sector, aircraft are assigned in proportion to the ICE of the opposing force in the sectors involved. Hence any desired change in the logic or priority assignment of aircraft may be made by reordering the above situations.

"As supplies enter the theater they may be either earmarked for a specific combat unit or distributed to the various sectors where need is the greatest."

40-5 SOME CRITICISMS OF COMBAT ANALYSIS MODELS

Perhaps the reader will easily see that although many of the combat analysis models are very valuable for training purposes, for studying weapon mix problems, force structure analyses for identifying problem areas of combat evaluations, etc., they are still under development and evolving somewhat slowly into completely acceptable methodology. Although better alternatives to war game studies and computerized combat simulations do not now exist, it should not be said that "all is well" with the combat analysis models generally. This state of affairs is rather well recognized and continues to be under discussion, for example as indicated in Refs. 12-17 and the Bibliography. Davis, Magee, and Pfortmiller (Ref. 12) point out that, "Increasingly, we are faced with choosing between some closed combat model and a manual or computer-assisted war game, where the choices present long lists of generally disjoint advantages.". These authors think that the Scenario Oriented Recurring Evaluation System (SCORES) ". . . is maturing and is beginning to provide a solid framework for identifying the important independent variables and environmental conditions that must be investigated" in connection with combat analysis models. Further, they state, "With or without adequate data, and whether the model logic is gross or detailed, all combat simulations attempt to represent five generic elements in some way, i.e., the tactical situation, the tactical interactive processes, the weapons and supporting systems, the physical interactive processes, and the physical environment. In general, closed combat models treat the weapon and supporting systems and the physical interactive processes fairly well; and progress is being made currently toward improving the representation of the physical environment in these models. It is the tactical situation and the interactive processes that are represented poorly, with but a few *minor* exceptions, in our combat models." . . . "To summarize, our concept for interactive gaming is to optimize the interaction of the gamers with the models, and the models with each other at an appropriate level of resolution. Computer graphs and other recent computer hardware developments, such as storage/refresh graphics, will be major considerations in the design and development of input/output processors."

Bross (Ref. 13) states that the use of "complex modeling techniques is no guarantee of realistically portraying combat interactions, even with the sophisticated computer models now available. Indeed, we have seen that complexity and realism are not synonymous, and to pretend that they are perpetuates a very dangerous myth."

Thorp (Ref. 14) discusses a modification to CARMONETTE to reflect observations gathered in a series of tests conducted at the US Army Combat Developments Experimentation Command for evaluating antitank missile systems effectiveness. He indicates a number of discrepancies between model runs and field experiments, including excessive line-of-sight occurrences at long ranges caused by inadequate coding of vegetation, improper quantification of the activities of the attackers, and a much larger percentage of false targets engaged by the attackers in the field experiment than in the CARMONETTE model—just to mention a few.

In a paper entitled, "DYNTACS-X: Is It Worth the Price?", Burnham (Ref. 15) states:

"Since its conception the DYNTACS-X (Dynamic Tactical Simulator-Extended) has served as a basis for numerous Masters and Ph.D theses; it has been praised and cursed; its results have been believed and doubted. Replicating the model for a single set of conditions has cost over $10,000 for computer time alone! Countless hours have been spent poring over its output. The data base required for DYNTACS is the most extensive of all combat models attempting to simulate up to battalion level armored combat engagements. With the theme for the Army Operations Research Symposium XV being "The Complexity Crisis and How to Avoid It," a cursory inspection of the detailed data, the potential volume of output or the magnitude of the FORTRAN logic would surely cause one to question whether or not DYNTACS is worth the price."

After further appropriate discussion of the DYNTACS-X simulation, Burnham, however, summarizes:

"Although all combat models may appear to be the same on the surface, only DYNTACS provides the necessary details of an engagement which are important to interpret the gross MOE properly. However, DYNTACS by itself is a luxury which can scarcely be justified for use in trivial parametric variations. An approach . . . whereby DYNTACS is mated to one of the lower resolution models (TXM, CARMONETTE, or BDM), would seem to offer the greatest flexibility and most efficient use of personnel and computer resources in support of an in-depth systems analysis effort."

Bechtlofft and Wiley (Ref. 16) point out the need for visual data representation as an aid in analyzing DYNTACS-X results. They say "a reason for visual data representation of DYNTACS-X results, which anyone who has used DYNTACS-X should be familiar with, is the large volume of very detailed information which is normally printed from a typical run. A typical run will produce from 500 to 1000 pages of output, with some runs going as high as 2500 pages."! Thus this should give the analyst some appreciation of the data presentation and analysis problem in DYNTACS.

In spite of many, many years of effort, therefore, there seems to be much room for improving the general areas of war games and combat simulations on a computer, and the interested analyst might well attempt to make some contributions in this area. In addition, we might state the need for continuing effort to inject realism into the combat models; improving the analytical approximations now used in the computer runs or simulations; conducting much research on the physical and tactical interactions that seem to be of much importance; and last, but not least, the use of statistical design considerations of combat simulations so that improved analysis of final results and outcomes will be guaranteed. We will illustrate this latter point in an example on infantry type evaluations in Chapter 41. We also stress that many of the combat analysis models are costly in terms of man-months of effort and even in computer time for many simulations.

There is also a problem in assuring the decision maker of the value of combat analysis models and that they meet his particular requirements in the critical decisions that must be made occasionally. In addition, it is important that the full range of analysis requirements at multiple levels of combat fits into an appropriate hierarchy of combat analysis models, so that consistency is guaranteed for the weapon acquisition process. Managers also desire better documentation of the models used by the analyst in his studies—especially perhaps in the area of interactions among tactics, systems, and the physical environment. Finally, there seems to be plenty of room for standardization of the inputs and the methods of analysis of results. Chapter 30 addressed a method of analysis termed weapon equivalence studies; this technique should be considered.

40-6 NEAR REAL TIME CASUALTY ASSESSMENT

In closing this chapter, we should mention an investigation under way at the US Army Combat Developments Experimentation Command to improve realism in experiments covering combat simulations. Hollis (Ref. 18) indicates that field experimentation has the potential of being a closer simulation to reality than do many of the proposed techniques otherwise recommended for combat analyses. Also he points out that practicing "real time casualty extraction" can add the threat of "kill or be killed", which heightens realism and generates combat-like player reactions. Until recently, such freedom of action could not be permitted in two-sided combat type experiments, although now the hardware/software system—having been developed to a satisfactory state—is available for such field experiments. The instrumentation system developed and now employed to collect the required data in field experiments, process the data, and extract the "casualties" consists of a range-measuring system, a range-timing system, a direct fire simulator, and a medium scale computer. The direct fire simulator is a cooperative laser transmitter/detector system that, when installed on player elements, permits the positive identification of firer-target pairings during a field trial or combat simulation. The output energy of the laser transmitter is such that no eye hazard exists. Hence, in the "battle" when a target is taken under fire by an attacker system employing its organic sighting system to which the laser transmitter of the direct fire simulator system has been bore sighted, the actual "firing" or "trigger pull" of the laser transmitter initiates a coded signal which is sent to the computer, and the range measuring system identifies the laser which has been fired. The laser beam may or may not illuminate the laser detector mounted on a target; this depends on whether a "hit" occurs. However, in the event of a "hit", the laser detector on the target is illuminated and complete identity of the firer-target combination is recorded in the computer, including times of actions, ranges, whether a kill occurs, etc. Thus the laser transmitter/detector system may be used in connection with a very "realistic" battle or field experiment along with a computer which records all of the important events and information needed for an improved "combat" analysis. There is a Handbook (Ref. 19) on the whole process of real time casualty assessment.

Hopefully, some cross-checking with other types of combat simulations might lead to improved model validations.

40-7 SUMMARY

We have highlighted some of the historical developments in war games over the years and have introduced some of the techniques for playing simulations of combat in connection with computers—whether the latter are used for bookkeeping purposes or for actual running of the combat analysis model. Also we have outlined and recorded in brief form some of the more well-known or widely used combat analysis models for possible study and use by the practicing analyst. Finally, in case the practicing analyst would have some interest in such areas of endeavor, we have indicated some of the problems connected with trying to make combat simulations realistic and responsive. It is hoped that the chapter will provide the new or unfamiliar weapon systems analyst with sufficient material and understanding to extend somewhat his knowledge of combat simulations or his expertise on the subject.

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CHAPTER 41

AN EVALUATION OF SOME MIXES OF INFANTRY SMALL ARMS WEAPONS

A battle situation for Blue versus Red infantry, especially for hand-held small arms types of weapons, is outlined, and the possible use of several different mixes of Blue infantry weapons to produce as many Red casualties as possible discussed. A statistical design of experiment, the Latin Square, is employed to carry out runs using the ASARS II X type combat simulation for the purpose of making a realistic evaluation of the different small arms weapon mixes. An example is given to illustrate just how superiority of one infantry weapon mix over another can be rather easily established with the suggested statistical analysis procedure.

41-0 LIST OF SYMBOLS

$S_{\bar{x}}$ = standard error of an average

T = David's Studentized extreme deviate

$\bar{\bar{x}}$ = grand average

$\bar{x}_n$ = largest of n averages

ν = number of degrees of freedom (df)

41-1 INTRODUCTION

With our introduction to and coverage of war games and computerized combat simulations, we are now ready to illustrate the utility of simulations as an aid in predicting the relative performance of some weapons under approved or recommended standard tactics of close combat. In particular, our interest here will center around the employment of different mixes of hand-held weapons in close combat between Blue and Red infantrymen. Our purpose—in addition to illustrating uses of simulations—will be that of presenting a combat simulation procedure with an appropriate method of analysis, which will lead to establishing superiority of one mix of infantry hand-held weapons over that of other candidate mixes of interest.

Concerning the battle situation, we will focus attention on an isolated area of expected conflict where (1) only hand-held weapons are feasible for employment, and (2) contact between Blue and Red infantry is so closely involved that supporting artillery and heavy mortar fire by each side would result in risking the loss of both Blue and Red infantry. Thus, only small infantry units with hand-held weapons in close combat should be involved in this particular battle situation. The weapons which can be used are more or less limited to rifles, such as the M14 or M16, automatic rifles, pistols, hand grenades, grenade launchers, and light machine guns. Hand-held antitank weapons are not considered by Blue in this particular study since Red tanks cannot be used in an assault on Blue's position, at least not initially, due to terrain features.

As developed in the discussion that follows, the Blue systems analysts who were assigned the evaluation study decided that the analytical models available were either so complex or inadequately developed that they would not apply very well, especially for the possible interactions among the weapons employed. Therefore, after a search of combat analysis models, it was decided that the ASARS II X (Army Small Arms Requirements Model, par. 40-4.2) simulation would be very suitable for the purposes at hand. In fact, ASARS II X would also give a reference point for previous, typical evaluations of small arms type weapons since it had shown realism and promise for extended studies of this type.

41-2 THE GENERAL SITUATION

Blue's 21st Infantry Division was assigned the mission of preparing a defensive position to prevent a frontal assault approach up a narrow valley toward a key town by two Red divisions as reported by Blue intelligence sources. Both sides or boundaries of the valley were protected by mountain ranges with very intensive canopy, the eastern range being of critical importance to the Blue. Blue's A Company of infantry was ordered to dig in and protect the eastern mountain range, especially the southern slope which sloped rather gradually toward the main line of resistance (MLR) between Blue and Red forces. Red's C Company was ordered to attack Blue A Company's position, annihilate as many Blue infantrymen as possible, and secure the entire slope for Red so that Red engineers might build a suitable trail or limited road up the southern slope. Red Tanks with mounted infantry could then proceed up a narrow trail to the mountaintop for a mopping-up action of Blue's position atop the mountain. The direct capture and occupation of the mountaintop would provide Red forces a considerable advantage in any further attacks against the general areas held by Blue forces. Generally speaking, Red C Company was organized according to and would attack in accordance with the principles of FM 30-40, *Handbook on Soviet Ground Forces*, Ref. 1, although Red infantrymen were dismounted for this particular engagement.

Before Red's infantry attack on the southern slope of Blue's dug-in position, Red artillery fire was brought to bear on the slope, although it was realized that trees and canopy might render such fire somewhat ineffective. Immediately after, nevertheless, Red infantry assaulted the Blue infantry positions on the slope. In such a close-combat position, Blue's primary interest centered around the particular mix of hand-held weapons that would inflict the most casualties on the assaulting Reds since this would no doubt stop his attack. The hand-held weapons primarily under consideration consisted of pistols, rifles, automatic rifles, grenades, and light machine guns. In particular, and due especially to the canopy, it was realized by Blue that "quick fire" in any direction would be required against suddenly appearing Red infantrymen. Since high single-shot kill probability against Red in a man vs man close-combat situation seemed essential, multiple flechette cartridges could prove very advantageous. Also, multiple flechette rounds might be very appropriate for machine guns, which jump around and give large aiming errors. Thus, key questions might be that of the use of triple-flechette cartridge rounds and that of the relative percentage of them to use for rifles and machine guns. The Blue analysts selected the ASARS III X combat analysis model to make the evaluation and ran a series of simulations based on the five different weapon mixes as indicated in par. 41-3.

41-3 INFANTRY WEAPON MIXES OF INTEREST

As a result of several conferences, Blue weapon systems analysts and experienced infantry combat officers decide that five particular mixes of possible hand-held (except for the light machine guns) infantry weapons might well be considered in the overall evaluation and that each of these mixes would be distributed among about 90-100 infantrymen who might actually be involved in the close-combat situation outlined. We will refer to the five different mixes as A, B, C, D, and E for simplicity and define them as follows:

1. Mix A: 50 M14 rifles, 20 M16 rifles, 15 light machine guns, and the standard loads of pistols, hand grenades, and grenade launchers. All ammunition consists of standard ball type rounds and no flechette cartridges at all.

2. Mix B: 20 M14 rifles, 50 M16 rifles, 15 light machine guns, and the standard loads of pistols, hand grenades, and grenade launchers. This mix was used to help settle the question as to whether

the lighter M16 rifle would possibly be more effective in the close-combat situation studied, although no flechette rounds would be involved in this mix either.

3. Mix C. 50 M14 rifles, 20 M16 rifles, 15 light machine guns, with standard loads of pistols, hand grenades, and grenade launchers. In this mix only the M14 rifles would fire flechette cartridges; standard ball ammunition would be fired from the M16 rifles and the light machine guns. Note that the quantities of rifles and machine guns are the same as in Mix A.

4. Mix D. The same quantities of rifles as in Mix A and Mix C, but only the M16 rifles fire the triple-flechette cartridges. There are no other changes.

5. Mix E. For a somewhat very significant departure, the conferees decided to get some idea of the importance of the triple-flechette rounds, especially in "swallowing up" the aiming errors perhaps. Accordingly, the same quantities of rifles and machine guns as in Mix B would be played in ASARS II X; however, only flechette rounds would be fired from all the M14 rifles, M16 rifles, and the machine guns. It was considered that this Mix E very likely would settle the question concerning the effectiveness of the proposed flechette cartridges.

Blue could see no problem in providing A Company with Class V supplies, and hence there were no restrictions on the availability of ammunition for the combat simulations outlined. Moreover, it was deemed that cost considerations should not enter this particular study, although one might ultimately consider the cost per kill of Red infantrymen, or some other more appropriate measure of effectiveness (MOE).

41-4 THE EXPERIMENTAL DESIGN

Available information on previous runs of ASARS II X seemed to indicate that good stability of output concerning results would be guaranteed if there were about five runs on each type of weapon mix. This particular decision, therefore, was made, and it also was based somewhat on the determination to settle finally the ever-plaguing flechette cartridge proposal. Indeed, it seemed further that certainly a single run would not be enough, and eight or ten such runs would be too costly perhaps in dollars, time, and personnel.

A rather good experimental design for the combat simulation appeared to be that of the 5 × 5 Latin Square as indicated in the discussion that follows. The order of the runs for the different mixes of the first row were determined by a random drawing of the five letters, A, B, C, D, and E. Thus, the order of all 25 runs was as indicated in Table 41-1.

Note that each infantry weapon mix, i.e., Latin letter, appears once and only once in each column and row for the Latin Square arrangement. In this arrangement each of the five weapon mixes would be fired in each column and each row of Table 41-1. This further means that any possible variations due to time or order of the runs could be stripped out of the overall experiment, so that the effects of only the different weapon mixes could be accurately compared. The 25 ASARS II X runs were

TABLE 41-1

LATIN SQUARE ARRANGEMENT FOR ASARS II X RUNS OF WEAPON MIXES

D	C	B	E	A
E	A	D	C	B
A	B	C	D	E
B	D	E	A	C
C	E	A	B	D

programmed so that each row of Table 41-1 would be carried out on a single day. It might be well argued that such particular care was not necessary at all; however, some precautions against extraneous variations possibly creeping into the experiment should be guarded against. Moreover, should it turn out that the possible variations creeping into the experiment due to rows or columns be trivial indeed, then the results of all the runs for each of the Mixes A, B, C, D, and E could be grouped any way for analysis. One never knows in advance just what might be a limitation of such a proposed experiment; hence the precautions taken.

41-5 RESULTS OF THE ASARS II X SIMULATIONS

The 25 runs for the weapon mixes studied were made. The results, given in terms of the numbers of Red infantry casualties, are shown in Table 41-2. For each weapon mix—A, B, etc.—in Table 41-2, the number of Red casualties for that particular run is given in parentheses beside the weapon mix letter.

The total numbers of casualties for rows and columns of the Latin Square design are also listed in Table 41-2, and the total number of Red casualties for the whole experiment is 704. Note that there are several variations in the numbers of casualties for columns and rows, so that this characteristic may be worth analyzing. The average number of casualties per run is 704/25 = 28.16, which represents the average per combat battle for the situation studied. It was thought in this connection that perhaps attaining 30-33% casualties would stop any such Red infantry attack; also the higher the percentage of Red casualties, the better were Blue's chances of winning.

If in Table 41-2 we strip out the Red casualties due to the five different weapon mixes, then the total Red casualties for each weapon mix, the average number per an expected battle, and the percentage of Red losses—assuming about 100 Red infantrymen in C Company assault Blue's position—are listed in Table 41-3.

It is seen from Table 41-3 that only Blue's weapon Mix E with flechette rounds attains the desired level of casualties. However, it becomes very worthwhile to establish whether our results for Mix E are statistically significant from the others—since the variations could be accidental—and also let us take a look at other possible inferences from this particular experimental combat simulation.

41-6 LATIN SQUARE ANALYSIS

The details of statistical analysis for Latin Square experiments may be found in most textbooks on the statistical design of experiments or in many textbooks on statistical methods. Nevertheless, we will give a suitably brief indication of the calculations to establish the analysis of variance table. In this connection, we proceed stepwise as follows, where the analysis will be carried out on the basis of the observed individual number of casualties for one run or battle:

TABLE 41-2
NUMBERS OF RED CASUALTIES FOR WEAPON MIXES A, B, C, D, AND E

						Row Total
	D-(33)*	C-(29)	B-(27)	E-(34)	A-(21)	144
	E-(37)	-(26)	D-(29)	C-(27)	B-(21)	140
	A-([illegible])	B-(22)	C-(26)	D-(31)	E-(33)	132
	B-(25)	D-(34)	E-(38)	A-(19)	C-(30)	146
	C-(28)	E-(35)	A-(25)	B-(24)	D-(30)	142
Column total	143	146	145	135	135	704

*D-(33) means 33 Red casualties for weapon mix D on first run.

TABLE 41-3

RED TOTAL CASUALTIES, AVERAGE CASUALTIES, AND EXPECTED PERCENTAGES OF CASUALTIES DUE TO BLUE'S WEAPON MIXES

	Blue Weapon Mix				
	A	B	C	D	E
Total Red Casualties for the 5 Runs	111	119	140	157	177
Average Red Casualties per Run	22.2	23.8	28.0	31.4	35.4
Expected Percentage Losses for Red, %	20.2	21.6	25.5	28.5	32.2

1. Correction term from Table 41-2:

$$(\text{Table total})^2/25 = (704)^2/25 = 19824.64$$

2. Sum of squares (S.S.) due to rows (i.e., days) from Table 41-2:

$$(144^2 + 140^2 + \cdots + 142^2)/5 - 19824.64 = 19848.00 - 19824.64 = 23.36.$$

3. Sum of squares (S.S.) due to columns (i.e., times of day) from Table 41-2:

$$(143^2 + 146^2 + \cdots + 135^2)/5 - 19824.64 = 19848.00 - 19824.64 = 23.36.$$

(The result happens to be the same S.S. as for rows.)

4. Sum of squares (S.S.) attributable to different weapon mixes, using totals from Table 41-3, for weapon mixes A, B, C, D, and E:

$$(111^2 + 119^2 + \cdots + 177^2)/5 - 19824.64 = 20412.00 - 19824.64 = 587.36.$$

5. The total sum of squares (S.S.), using number of casualties in parentheses in Table 41-2, is found:

$$(33^2 + 29^2 + \cdots + 30^2)/1 - 19824.64 = 20518.00 - 19824.64 = 693.36.$$

6. Degrees of freedom (df):

a. The total number of degrees of freedom (df) for the 5×5 Latin Square is $(5)(5) - 1 = 24$.

b. The numbers of degrees of freedom for rows, columns, and weapon mixes are each 4, leaving $24 - 4 - 4 - 4 = 12$ df for the residual or experimental error term.

The results of this analysis are appropriately brought together in the Analysis of Variance (ANOVA) of Table 41-4.

An examination of Table 41-4 shows a very well controlled experiment, since the F-ratios for days and times of a day for the runs are quite insignificant. On the other hand, the observed $F = 29.72$ for the five different weapon mixes on the basis of 12 df for residual variance is very large indeed. Thus, and perhaps as expected, the source of variance due to the different weapon mixes is very highly significant, since the probability of a chance occurrence for equivalent weapon mixes would be much less than 0.005, as seen from a table of the percentage points of the Snedecor-Fisher "F" statistic for 4 and 12 df. Hence, we conclude that the choice concerning the mix of infantry weapons for the battle situation outlined may be of much critical importance to guarantee a high effectiveness level. In particular, the flechette cartridge round turns out to be highly desirable in the battle simulated, and we see that Mix E is the more desirable one, which may be justified statistically as follows.

TABLE 41-4

ANOVA TABLE FOR ANALYSIS OF CASUALTIES DUE TO WEAPON MIXES

Source of Variation	df	S.S.	Variance	F-Ratio	
Rows (Days)	4	23.36	5.84	1.18	
Columns (Time of Day)	4	23.36	5.84	1.18	
Different Weapon Mixes	4	587.36	146.84	29.72	(Highly significant)
Residual or Error	12	59.28	4.94		
Total	24	693.36			

In Table 41-2, there are 25 numbers of casualties, one for each run. The residual variance for one of these is from Table 41-4 equal to 4.94, and the variance for an average of 5 runs for each of the 5 different mixes is $4.94/5 = 0.988$, or, that is, the standard error is $\sqrt{0.988} = 0.994$. Now the average number of Red casualties per run or battle for Mix E is 35.4 from Table 41-3, and the grand average of all 25 runs is found from Table 41-2 as $704/25 = 28.16$. Hence, we may use David's test (Ref. 2 or Ref. 3) to judge whether the highest average number of casualties for Mix E, i.e., 35.4, is significantly greater than that expected from random sampling of 5 such averages. David's test for this case is

$$T = (\bar{x}_n - \bar{\bar{x}})/S_{\bar{x}} \qquad (41\text{-}1)$$

where

T = David's Studentized extreme deviate

$\bar{x}_n = \bar{x}_5$ = largest of n averages (Mix E)

$\bar{\bar{x}}$ = grand average

$S_{\bar{x}}$ = residual standard error of an average ($n = 5$ in this case).

Hence, the observed value of T is

$$T = 35.4 - 28.16/\sqrt{0.99} = 7.28$$

and is very highly significant, since from Ref. 2 the upper 1% point or probability level for David's T for $n = 5$ and $\nu = 12$ df is only 3.17.

Actually, perhaps we may improve on the precision of the test by increasing the error number of degrees of freedom from 12 to 20, especially since we can—due to insignificance—pool the sums of the squares for rows and columns with the residual sum of squares. Doing this, we get a new residual variance, which is

$$(23.36 + 23.36 + 59.28)/(4 + 4 + 12) = 5.30$$

for an individual number of casualties based on 20 df.

Moreover, the new observed value of T for 20 df then becomes

$$T = (35.4 - 28.16)/\sqrt{5.30/5} = 7.03$$

which is still very highly significant indeed since the new 1% point for $n = 5$ and $\nu = 20$ df. is 2.91. We conclude with little doubt, therefore, that Mix E is substantially superior to the other mixes in causing Red casualties and that it should be highly recommended.

As a matter of some interest, we might look at only the four mixes, A, B, C, and D, to make a judgment concerning whether weapon Mix D—with only the M16 rifles firing flechettes (and not the M14's)— should be superior to Mixes A, B, or C. In this case the observed value of David's T is given by

$$[31.4 - (22.2 + 23.8 + 28.0 + 31.4)/4]/\sqrt{5.30/5} = 4.90.$$

Now the upper 1% probability level for David's T with $n = 4$ and $\nu = 20$ df is 2.73. Hence, we conclude the Mix D is significantly better in causing Red infantry casualties than Mixes A, B, or C. This would seem to substantiate the use of flechette-type cartridges for small arms. In fact, the aiming errors in such a typical battle situation may be sufficiently large that multiple flechettes per round would improve the chance of at least one hit per cartridge.

Finally, and as a somewhat fine point concerning statistical analyses of the kind covered here, we might ask whether the assumptions of the analysis of variance model (ANOVA) are really met with sufficient accuracy here. To this question, we could well reply that we might examine the previous analysis with the square roots of the numbers of casualties instead of the observed number directly. Or, we might use a transformation of the data to some other scale which might be more nearly normally distributed, as is often done by statisticians. However, the assumption of normality becomes of significant importance only when the F-ratios are compared to critical values in F-distribution tables. Otherwise, the F-ratios more or less speak for themselves, implying an observed value so large for the different mixes that "there must be some physical difference of interest". Then again, the numbers of casualties for about 100-110 combatants are not so small as to suggest a Poisson type analysis, but rather the approximate percentage of casualties averages about 28.2%. In this connection, we know from appropriate theory that the normal approximation to the binomial distribution is satisfactory for such percentages, and we therefore conclude that our analysis of variance is sufficiently accurate to make the judgments we have shown.

41-7 SUMMARY

We have outlined a rather useful or typical engagement of Blue infantry versus Red infantry for the case where only small arms can be used, and in particular the hand-held type of weapons. Our purpose was to study the relative effectiveness of a hypothetical flechette-type cartridge as compared to standard ball ammunition in producing Red casualties. We were able to make comparisons of five different weapon mixes, employing the Latin Square design for an efficient statistical analysis procedure to guarantee any possible existence of superiority. Our conclusion is that the multiple flechette type of rounds or cartridges offers considerable promise as being highly effective.

Finally, we do not mean to imply, of course, that only a Latin Square should be used in this kind of an analysis or combat simulation. Indeed, other statistical designs of experiments could well be used—such as the factorial design, completely randomized blocks, lattice squares, and others. In fact, one might well aim to use any design which would result in establishing the regression of the numbers of casualties on key weapon parameters. In this way, additional valid predictions could be made, perhaps.

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CHAPTER 42

AN EXAMPLE OF TANK WARFARE IN THE AGE OF THE GUIDED MISSILE

An example of possible tank warfare in the age of antitank guided missiles (ATGM's) is developed in this chapter to illustrate a highly useful method of analyzing the overall combat capability and potential of antitank weapons in quantitative terms. The technique employed is that outlined in Chapter 30, and it is seen that perhaps the analysis of more complex combined arms forces on both sides could also be studied in this same way. In fact, with the promising method of analysis described, the accurate determination of relative worths of the different types of weapons in a pertinent interactive combat simulation could well lead to weapon equivalence values, or "building blocks", to help establish improved combined arms teams for future wars.

42-0 LIST OF SYMBOLS

APDS = armor-piercing, discarding sabot (projectile)
ATGM = antitank guided missile
BCBT = Blue chief battle tank
HEAT = high explosive, antitank (projectile)
MLR = main line of resistance
RBMP = Red armored infantry combat vehicle
RBRDM = Red scout vehicle which mounts ZWATTER ATGM's
R63 = main Red battle tank
WOW = Blue's ATGM (wire command link)
ZAGGER = Red ATGM (wire command link)
ZWATTER = another Red ATGM (radio controlled)
k = constant of proportionality
r_{ij} = kill rate of weapon i against weapon j
r_{13} = kill rate of a Blue WOW against a Red ZAGGER
r_{14} = kill rate of a Blue WOW against a Red ZWATTER
r_{15} = kill rate of a Blue WOW against a Red R63 tank
W_1 = relative worth or combat power of a Blue WOW ATGM
W_2 = relative worth or combat power of a Blue CBT
W_3 = relative worth or combat power of a Red ZAGGER ATGM
W_4 = relative worth or combat power of a Red ZWATTER ATGM
W_5 = relative worth or combat power of a Red R63 tank
$[\beta]$ = Red's kill rate matrix against Blue targets
$[\rho]$ = Blue's kill rate matrix against Red targets

42-1 INTRODUCTION

All of the great armies of the world rely heavily on the premise that the tank should be the major combat system for ground fighting. In fact, it appears currently that all of the modern armies of NATO, the Warsaw Pact, the Arab nations, and the Israelis generally agree that the main offensive weapon of the ground forces is the tank (Ref. 1). The tank is a weapon system that possesses good mobility on the road; it has very acceptable cross-country mobility for many battle situations; it has the best available armor protection for the crew and stowage; and its firepower is quite suitable for neutralizing or destroying many, many ground targets that could be encountered. Such targets include, for example, enemy infantry and enemy tanks which our tank main armament can engage at relatively long ranges with good delivery accuracy. The tank secondary armament can also neutralize enemy personnel; the tank may quickly defeat lightly armored personnel carriers; it can attack buildings and fortifications; it can mop up or hold key ground areas; and it can especially aid in withdrawals as required. In addition, an important characteristic of the tank is its shock action effect in advancing swiftly to critical points of battle and providing the element of surprise for almost any phase of battle, no matter what the battle plans of the enemy may be. With its armor protection, its heavy armament or great firepower, and cross-country mobility, only the tank can break through an enemy force and defeat it decisively.

As we learned in Chapter 39, the art of war consists of measures and countermeasures, or the modern battlefield is a contest of measures and countermeasures, and the side that is fortunate enough or smart enough in fielding effective countermeasures to any new enemy measure will often have the advantage or capability to stop any threat and reduce enemy capability to a marked degree. Ref. 1 indicates that " 'The Yom Kippur War,' that is the Arab-Israeli War of 1973, very much reaffirmed the offensive potential of the tank, but it also dramatized the lethality of modern antitank weapons, particularly the high velocity tank cannon and the long range antitank guided missile. The effect of these modern antitank weapons in this war was devastating. Not since the Battle of Kursk between the Germans and Russians in World War II had there been a comparable loss of tanks in such a short period of time. If the rate of loss which occurred in the Yom Kippur War during the short 20 days of battle were extrapolated to the European battlefields over a period of 60-90 days, the resulting losses would reach levels for which the United States Army is totally unprepared. While it is impossible to say precisely how many losses were attributable to a certain weapon system, we can say, particularly in view of the vast numbers of ATGM's employed, that *the antitank guided missile was responsible for a high percentage of the Israeli tank losses at the beginning of that war*. In the Arab/Israeli War of 1967, the Israelis were able to dominate the battlefield principally with tanks and fighter aircraft. Extensive Arab air defenses in 1973, however, seriously degraded effective close air support. Thus in the first several days of the 1973 war, Israeli armor units, advancing without close air, infantry or artillery support, attacked in the face of large numbers of Soviet-made ATGM's and suffered wholesale destruction. This same situation is, of course, possible on European battlefields. Thus, we should conclude that:

'On today's battlefield, unsupported tank attacks face mass destruction from accurate and lethal antitank guided missiles.' "

42-2 PURPOSE OF THIS ANTITANK EVALUATION

Insofar as the tank is concerned, therefore, it becomes of great importance to determine whether our tanks can survive the advances in technology attained in connection with light, highly mobile antitank weapons of all kinds. Thus, we must examine very closely and evaluate the potential capabilities of Red antitank guided missiles, or any short-range hand-held weapons having a shaped charge warhead or high explosive antitank capability, in addition to the major armament for any new enemy tanks. Moreover, close combat turns out to be too costly in terms of personnel and materiel losses, so that consideration should be given to attacking enemy tanks at longer ranges with artillery type weapons if at all possible. The advent of the guided missile and its widely applicable technology has resulted in gun-launched projectiles which may be guided very accurately along artillery type trajectories to hit enemy tanks at the longer ranges. Hence, if our artillery can have this capability in addition to that of its normal antipersonnel effectiveness, then such a method of attacking tanks could well result in a considerable advantage indeed. There is a need, therefore, to simulate future battles involving all of the probable antitank weapons which may be used, and play them in a suitable context so that their effectiveness on the battlefield can be assessed properly in a quantitative way. To include the antitank capability of artillery in this particular study, however, would go beyond our purpose of illustration, and hence we will not address this larger problem of evaluation here. Instead, the present study is more or less directed toward the quantitative evaluation of some typical current or proposed prototypes of antitank guided missiles which are relatively inexpensive and potent. It is with this in mind that we proceed to set up an armor and antiarmor type engagement for a future hypothetical battle and develop a useful systems analysis study of tank and antitank capabilities, especially in a close combat situation involving tanks and antitank guided missiles (ATGM). In fact, we wonder about the real combat capability of some of the ATGM's to stop Red tanks in their tracks, so to speak. Clearly, inexpensive weapons could be proliferated easily on the battlefield if they possessed the capability to prevent a Red striking force from overrunning Blue's position and then moving forward to control larger and larger ground areas in Blue's territory. To particularize a bit, we might say that tank weapon systems are, for example, eight or more times as expensive as ATGM's so that we might be able to afford many ATGM's in the hands of infantrymen on the battlefield simply to stop the enemy tank threat. Thus, our interest centers around developing a good quantitative measure of ATGM's just for the purpose of stopping the huge number of Red tanks that will very likely be brought to bear in any European conflict. Moreover, since tanks are often considered to be the main killers of other tanks, then we should search for the relative worth of ATGM's in killing Red tanks, especially as compared to the combat value of modern tanks. Can we answer the question, "If x ATGM's are available at the same cost as that of the Main Battle Tank (MBT) and there are sufficient military personnel available to man the ATGM's, what is the military worth of the ATGM relative to our main MBT weapon for the purpose of only killing Red tanks?" We aim to develop a method of answering this type of question in the sequel, and perhaps the reader can see that it falls into the area of weapon equivalence studies in a particular role only, and hence the analysis here is not to be considered a multipurpose or a total system evaluation of modern tanks.

We will proceed by setting up a rather hypothetical war in Western Europe and then develop the weapon list of prime interest, along with the scenario of a typical battle engagement. Then, from the results of a combat simulation, we will be able to illustrate our suggested methodology for the measurement of relative worths of ATGM's relative to that of tanks as tank-killers.

42-3 THE GENERAL BATTLE SITUATION

Early on the morning of 21 April 1983, Red forces, without warning, broke into West Berlin with their tanks and mounted infantry, and had almost all of West Berlin under control before the day was over, for Blue was surprised, stunned, and not ready for battle under the circumstances. Within another two days, the Red Sixth Army had advanced through Potsdam, Brandenburg, Magdeburg, and Dessau without any heavy losses, and so they proceeded to push on and take over the area of Germany involving the towns of Hannover, Brunswick, and Haberstadt. It seemed quite evident to Blue at this point that Red forces would drive toward the Ruhr Basin with their quick and successful tank and mounted infantry attacks, and hence that some quick and decisive means would be necessary to stop and annihilate Red tanks over a wide area of probable conflict. In this connection, Blue visualized that two or three Red armies would likely attack with their armored divisions all along the line running approximately from Bremen to Nuremberg, and that the primary problem, at least initially, was to counter the tank threat immediately. Blue then decided to make a stand along the west banks of the Weser and Werra rivers. In fact, Blue believed they could bring up forces and be quite well prepared to stop any such tank threat, for they had played pertinent war games and conducted many combat simulations for this very possibility a number of years earlier. Also, Blue's completely computerized combat simulation called CARMONETTE had been developed to a very advanced degree of realism and accuracy of assessment, and much of the input details and standard decisions had been validated through field exercises and many experiments conducted at Blue's Combat Development Experimentation Center. In the paragraphs that follow, we will outline the type of combat simulations that led to a rather accurate establishment of the relative values of tanks and ATGM's in killing enemy tanks, and it was on the basis of such studies that Blue could determine just how to organize his divisions in terms of the relative balance of tanks and ATGM's needed to counter the Red tank threat.

42-4 BRIEF DESCRIPTION OF KEY BLUE AND RED ANTITANK WEAPONS

For a tank battle in Western Europe, such as outlined in par. 42-3, Blue expected that each of his mechanized companies would have to repulse or stop Red forces up to the size of a Red tank battalion. In a Red tank battalion, it was expected that there would be about 31 R63 tanks mounting 120 mm smooth-bore guns firing an armor-piercing discarding sabot (APDS) projectile. In addition, Red would have 10 armored infantry combat vehicles (RBMP's) which featured a 73 mm smooth-bore gun capable of firing HEAT ammunition, the usual 7.62 mm machine gun, and a ZAGGER ATGM for each RBMP. The Red ZAGGER ATGM weighs about 25 lb, has a 5-in. diameter HEAT warhead, a wire command link, and a range of about 3,000 m.

In addition to the R63 tanks and the RBMP's with ZAGGER missiles, Blue intelligence indicated that Red may have attached to its tank battalion some five RBRDM's (scout vehicles) which mount ZWATTER ATGM's. The Red ZWATTER ATGM's have a range capability of about 3,300 m, and mount a HEAT warhead of about 5.3 in. diameter. However, the Red ZWATTER ATGM guidance systems incorporates a radio command link which could be jammed.

Hit probability for the Red ZAGGER ATGM increases from 0.3 to 0.4 at 200 m to about 0.6 at ranges beyond 1000 m, while the Red ZWATTER would attain hit probabilities of about 0.7 or slightly better at all ranges within its range capability. Red ZAGGER and ZWATTER ATGM's with their HEAT warheads could be effective also against Blue personnel and Blue crews firing ATGM's, either dismounted or protected in armored personnel carriers.

In summary, Red possessed three different types of weapons with antiarmor capability against Blue's tanks, along with capability also to defeat mounted or unmounted infantrymen firing ATGM's.

Ordinarily, a Blue mechanized company would have five Blue chief battle tanks (BCBT's), armed with highly accurate 115-mm guns firing an XM 731 fin-stabilized, slowly-rotating projectile with a newly developed and highly potent HEAT type warhead which could easily defeat any Red tank or vehicle on the battlefield. Moreover, Blue tank fire control equipment had been developed which had amazingly small aiming errors. The Blue CBT's have the capability to attack personnel targets of any kind, including any Red ATGM crews in their armored personnel carriers.

Blue mechanized companies also have the capability of some six ATGM's known as WOW's (Weapon-launched, optically tracked, wire command link missiles) with HEAT type warheads, and some shorter-range "Dragon" antitank weapons. Blue's WOW missile appeared to be one of the best for antitank capability to be fielded in recent years, and it had been very successful in extensive field trials. Blue's WOW missile could be fired very accurately to a range of perhaps some 3500 m from weapon mounts on its armored personnel carriers, or from relatively simple ground mounts operated by infantrymen, or from a helicopter as well.

Thus, Blue some years earlier had reached the decision that its new BCBT's and the highly developed WOW missile could handle the Red tank threat expected in Western Europe. Therefore, only these two Blue weapons were played in Blue's combat simulation called CARMONETTE to determine their relative worth in tank combat.

For the type of Red tank threat described in par. 42-3, Blue considered that on the equivalent of a division front the total cost of Blue WOW ATGM's fielded could be made equal approximately to the cost of the Blue CBT's in the division, if need be. In this way, Blue might then be able to judge directly the effectiveness of WOW ATGM's with that of the main antitank gun on an equal cost or effort basis. Furthermore this meant that Blue could have at least eight WOW's for each BCBT on the average at the division level. However, Blue decided the eight WOW's per BCBT seemed a bit "too far out", and really wanted to know if he could get by with as few as only two WOW's for each BCBT. Also, Blue desired as many BCBT's as possible for use in a counterattack. Blue thus proceeded as outlined in par. 42-5.

42-5 PLAN OF THE SIMULATION AND THE RESULTS

In conducting the CARMONETTE simulations, Blue realized that the Red tank threat would no doubt be over many miles along the main line of resistance (MLR), and Blue also knew from past experience with CARMONETTE simulations that some 5 to 20 runs or replications would be required to obtain dependable and stable results. In addition, Blue realized that for any wide-scale Red attack through the heart of West Germany he could not depend entirely on "going by the book", especially insofar as current doctrinal use of ATGM's was concerned. In fact, he had the following rather isolated, but important, information from Ref. 1 concerning an experience during the Yom Kippur War:

> "We were advancing and in the distance I saw specks dotted on the sand dunes. I couldn't make out what they were. As we got closer, I thought they looked like tree stumps. They were motionless and scattered across the terrain ahead of us. I got on the intercom and asked the tanks ahead what they made of it. One of my commanders radioed back: 'My God, they're not tree stumps. They're men!' For a moment I couldn't understand. What were men doing standing out there—quite still—when we were advancing in our tanks towards them? Suddenly all hell broke loose. A barrage of missiles was being fired at us. Many of our tanks were hit. We had never come up against anything like this before . . ."
>
> ISRAELI TANK COMMANDER IN THE SINAI, October 1973

Blue also realized that his defense along the Weser and Werra rivers might well represent his best available opportunity to counter the Red threat. Thus, Blue reached the decision that he would do well to bring up special reserves for the occasion of the generally described battle, even though the Blue policy had been that of expecting a Red to Blue force ratio of about three to one. Based on as careful planning as he could perform at the time and with his available resources, Blue decided that on the average he should pit about 10 CBT's and 20 WOW's against Red's 31 R63's, 10 ZAGGER's, and 5 ZWATTER's. This meant a restructuring of Blue's basic mechanized and armored divisions to fight the expected battles, although Blue knew that this would represent only a slight problem, since his armored company teams might have as many as 17 CBT's and two WOW's on one hand, or on the other could accommodate 8 CBT's and some 24 WOW's. Blue's choice of weapon ratios made it evident that

the simulation would be run in order to safely give the most useful results for future planning. Blue therefore would base his findings on ten CBT's and 20 WOW's against Red for the CARMONETTE comparisons desired.

With reference to the number of CARMONETTE simulations to run, Blue decided on 15 replications with somewhat varying scenarios, representing especially the different terrain conditions along the MLR and the probable protection thereby provided for Blue defenses. Blue also had to depend on the hopeful possibility that Red tanks would advance far beyond their infantry and hence might be "easy pickings" for Blue's generally protected CBT's and his hidden WOW's in the defense.

The CARMONETTE simulations based on the restructured organizations were run at the battalion level and the average numbers of kills per weapon based on 15 replications for the various 90-min battles along the MLR, allowing some resupply and system replacements, were found to be the following:

1. Each Blue WOW averaged 0.15 ZAGGER kill, 0.30 ZWATTER kill, and 0.15 R63 kill.
2. Each CBT averaged 0.45 ZAGGER kill, 0.60 ZWATTER kill, and 0.45 R63 kill.
3. Each Red ZAGGER averaged 0.30 WOW kill and 0.15 CBT kill.
4. Each Red ZWATTER averaged 0.15 WOW kill and 0.30 CBT kill.
5. Each Red R63 tank averaged 0.45 WOW kill and 0.15 CBT kill.

Blue further calculated that the average number of Red weapon system kills would be about 0.35 per Blue weapon and the standard deviation of the number of Red kills was about 0.05. Thus, the standard deviation of an average of 15 such kill numbers would amount to only about 0.013, which Blue decided would be acceptable for his analytical purposes of relating tank kills from guided missiles as compared to that of the tank main armament.

42-6 ANALYSIS TO DETERMINE WEAPON EQUIVALENCE VALUES OR WORTHS

With the kill data enumerated in par. 42-5, we may now set up the killer-victim scoreboard for the Blue-Red CARMONETTE tank battle simulations as indicated in Table 42-1. (Note how the Blue and Red weapons are numbered consecutively, W_1, W_2, W_3, W_4, and W_5.)

Next, we assume that the worth of any weapon depends on its killing power, i.e., kill rate, against weapons on the other side in a battle. That is to say, the worth W of a weapon is defined as being proportional to the number of opposing enemy weapons it can kill per unit time, where each opposing weapon is weighted according to its own worth. As an example in Table 42-1, we may say that the worth W_1 of the Blue WOW may be estimated from the following worth equation determined from the first row of Table 42-1 in terms of the worth W_3 of the Red ZAGGER, the worth W_4 of the Red ZWATTER, and the worth W_5 of each Red R63:

$$kW_1 = r_{13}W_3 + r_{14}W_4 + r_{15}W_5 \tag{42-1}$$

where

k = constant of proportionality

r_{ij} = kill rate of weapon i against weapon j = number kills/battle time.

Hence, with the battle time of 90 min used in the CARMONETTE runs, we may easily calculate the relative kill rates in kills per hour and set up the killer-victim worth matrix of Johnsrud (Ref. 2 or Chapter 30), as in Eq. 42-2.

TABLE 42-1

KILLER-VICTIM SCOREBOARD (AVERAGE NUMBER KILLS AGAINST EACH SIDE)

	W_1 Blue WOW	W_2 Blue CBT	W_3 Red ZAGGER	W_4 Red ZWATTER	W_5 Red R63
W_1—Blue WOW	0	0	0.15	0.30	0.15
W_2—Blue CBT	0	0	0.45	0.60	0.45
W_3—Red ZAGGER	0.30	0.15	0	0	0
W_4—Red ZWATTER	0.15	0.30	0	0	0
W_5—Red R63	0.45	0.15	0	0	0

$$\begin{matrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \end{matrix} \begin{bmatrix} -k & 0 & 0.1 & 0.2 & 0.1 \\ 0 & -k & 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & -k & 0 & 0 \\ 0.1 & 0.2 & 0 & -k & 0 \\ 0.3 & 0.1 & 0 & 0 & -k \end{bmatrix} \tag{42-2}$$

Thus, following the analysis of par 30-4 in Chapter 30, we have that Blue's kill rate matrix against Red weapons $[\rho]$ is

$$[\rho] = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \tag{42-3}$$

and Red's kill rate matrix against Blue weapons $[\beta]$ is given by

$$[\beta] = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix} \tag{42-4}$$

The product of these two matrices which will be needed in the killer-victim analysis is

$$[\beta][\rho] = \begin{bmatrix} 0.05 & 0.08 & 0.05 \\ 0.07 & 0.10 & 0.07 \\ 0.06 & 0.10 & 0.06 \end{bmatrix} \tag{42-5}$$

Now, as in Johnsrud's procedure of par. 30-4, we seek the largest eigenvalue for k from the following determinantal equation:

$$|k^2[I] - [\beta][\rho]| = \begin{vmatrix} k^2 - 0.05 & -0.08 & -0.05 \\ -0.07 & k^2 - 0.10 & -0.07 \\ -0.06 & -0.10 & k^2 - 0.06 \end{vmatrix} = 0. \tag{42-6}$$

This determinantal equation, however, simplifies for our use to

$$k^4 - 0.21k^2 - 0.0016 = 0 \qquad (42\text{-}7)$$

and the largest real positive root of this equation is found to be

$$k = 0.46622. \qquad (42\text{-}8)$$

The worth matrix to be evaluated, therefore, is that of Eq. 42-2 with k replaced by the numerically largest eigenvalue $k = 0.46622$.

One can easily triangularize the resulting matrix of Eq. 42-2 by using successively the first row, then the resulting second row, etc., to produce zeros below the principal diagonal. The final triangularized matrix is found to be

$$\begin{bmatrix} -0.46622 & 0 & 0.1 & 0.2 & 0.1 \\ 0 & -0.46622 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & -0.35897 & 0.17159 & 0.10725 \\ 0 & 0 & 0 & -0.17996 & 0.19500 \\ 0 & 0 & 0 & 0 & 0.0000 \end{bmatrix} \qquad (42\text{-}9)$$

A good check on the calculations is obtained since the last element is only 0.0000 (to four decimal places).

Hence, we see that

$$0.17996 W_4 = 0.1950 W_5$$

or

$$W_4 = 1.084 W_5. \qquad (42\text{-}10)$$

Further,

$$0.35897 W_3 = 0.17159 W_4 + 0.10725 W_5 \qquad (42\text{-}11)$$

or

$$W_3 = 0.817 W_5. \qquad (42\text{-}12)$$

Similarly,

$$W_2 = 2.099 W_5 \qquad (42\text{-}13)$$

and

$$W_1 = 0.855 W_5. \qquad (42\text{-}14)$$

Finally, the relative worths, "killing powers" or effectiveness values for the Blue and Red weapon systems are from Eqs. 42-10 through 42-14

One Blue CBT = 2.099 Red R63's
One Blue WOW = 0.855 Red R63
One Red ZAGGER = 0.817 Red R63
One Red ZWATTER = 1.084 Red R63's.

Also,

One Blue CBT = 2.099/0.855 Blue WOW's
= 2.455 Blue WOW's

$$\text{One Red R63} = 1.224 \text{ Red ZAGGER's}$$
$$\text{One Red ZAGGER} = 0.754 \text{ Red ZWATTER.}$$

Next, let us look at the relative overall values of Blue antitank weapon systems versus those of Red tank forces we have programmed in CARMONETTE. For Blue, we have 10 CBT's and 20 WOW's, the total value of which may be described as

$$\text{Blue's Total Value} = 10(2.099) + 20(0.855) = 38.1 \text{ R63's.}$$

Correspondingly, we get for Red:

$$\text{Red's Total Value} = 30(1) + 10(0.817) + 5(1.084) = 43.6 \text{ R63's.}$$

We therefore conclude from the relative numerical worths of Blue and Red forces that although Red possesses some superiority over Blue in weapon potential, Blue has nevertheless done quite well in his weapon employment tactics indeed. In fact, in an hour Blue has been able to kill four Red ZAGGER's, as many as six Red ZWATTER's with resupply or if Red had them, and four Red R63's, the worth of which is

$$4(0.817) + 6(1.084) + 4(1) = 13.8 \text{ R63's}$$

which is 13.8/43.6 = 0.32 or 32% of Red's total combat worth. Hopefully, Blue's planned defense might go a long way toward stopping Red's thrust through West Germany, and moreover Blue has a quantitative method of comparing overall forces on each side and "building up" his own effective force as needed.

We should emphasize again that the measurement of worth in this example involves only the capability of Blue to kill Red tanks and antitank weapons, and vice versa. It does not measure the overall worth of either a Blue or a Red tank, for they possess much more versatility in mobility, crew protection, shock action capability, etc. It does seem possible nevertheless that if a suitable combat simulation could be developed in depth to account for the key parameters in a battle—including maneuver, armor protection, weapon effectiveness, tactical mobility, etc.—then the battle outcome might be sufficiently scenario dependent to generate rather general conclusions. To our knowledge, such accurate description of a real combat simulation has not yet been attained. This leads us to some additional comments concerning actual battles and combat simulations.

42-7 SOME PERTINENT COMMENTS ON TANK WARFARE

Concerning the advent of the ATGM, Ref. 1 indicates that the ATGM must be considered a potent weapon on the battlefield. Some of the strengths of probable enemy ATGM systems include their long range accuracy, high degree of lethality, ease of employment, high reliability for the wire command link, invulnerability of the wire-guided weapon to countermeasures, and remote firing capability. On the other hand, some of the ATGM weaknesses include the need for highly trained gunners, the minimum range capability, the slow speed of the missile, the susceptibility of electronic systems to countermeasures, the requirement for good visual contact with both target and missile during flight, and the lack of good responsiveness in tracking erratically moving targets. It is expected that second generation missiles will show considerable improvements, however.

Now a word or two about Soviet Ground Forces. FM 30-4 (Ref. 3) is a fairly current account of Soviet ground force operations on the modern battlefield. It points out that the historic emphasis of Russia on defense has changed dramatically to that of offensive in current Soviet military doctrine and that armor will probably be massed in multiple formations, except perhaps for the possibility of nuclear warfare. Moreover, Soviet infantry is a mechanized force, and the armored personnel carrier is both a carrier and a fighting vehicle, with soldiers trained to ride directly into combat firing their weapons through side ports. However, if heavy antitank fire is encountered, then Soviet doctrine is for the men to dismount and attack from behind supporting tanks, followed by their APC's giving support with heavy machine guns. In fact, to quote Ref. 3, we must realize that

"Tanks provide the offensive punch that is so important in all Soviet tactical concepts, with riflemen being carried in armored personnel carriers or infantry combat vehicles that can keep pace with the tanks and deliver the infantry directly into combat. High-speed armored strike forces are designed both to attack enemy concentrations directly and also to penetrate as far as possible into the rear of enemy concentrations."

Thus, it may surely be expected that dismounted Soviet infantry and sometimes those in armored personnel carriers will be fully integrated with their tanks in any future conflict.

With reference to the Mideast War results, we quote from Ref. 1:

"Initial news media reports from the October War heralded the demise of the tank and the ascendancy of the antitank guided missile. The Israeli tank losses in the war tended to support the view that the tank was dominated by SAGGER's and RPG-7's. However, subsequent reports and analysis indicated that, in fact, the tank was the principal tank-killer. The effect of the ATGM was significantly degraded by the use of proper tactics and techniques.

"During the first few days of the war, Israeli armor units attacked without adequate artillery or infantry support. Few artillery units had been mobilized, and what few mechanized infantry units were available were mostly mounted in halftracks and could not keep up with the tanks. The result was devastating destruction of Israeli tanks by Arab ATGM's and RPG-7's.

"The Israelis, however, soon modified their tactics to employ the combined arms team—infantry, armor and artillery. By firing artillery on likely or suspected locations for SAGGER's and employing infantry with the tanks to add suppressive fire to SAGGER and RPG-7 positions, the effectiveness of the antitank guided missile was significantly reduced. The infantry was employed with the tanks in the three ways:

(1) APC's together with tanks
(2) APC's leading tanks
(3) Dismounted infantry leading tanks.

"The role of the infantry in the attack was primarily to add more suppressive fire. Infantry fought mounted, except only when heavy antitank fire prevented forward movement.

"These simple tactics were not, of course, invented nor developed independently by the Israelis. They are US tactics. They represent the application of the combined arms team concept which has long been taught in US Army schools. The lesson to be learned is that the October War has once again proved the validity of the combined arms doctrine."

Thus, it would seem from this account that the combined arms doctrine, its development and exploitation, represents the proper direction for improving combat effectiveness.

In this handbook, we have often remarked that perhaps one of the most important, current problems is that of developing suitable methodology for analyzing and quantifying the relative effectiveness of combined arms teams or heterogeneous forces of all kinds. In fact, the particular example covered here would seem to add some weight to such a belief, and we would therefore encourage weapon systems analysts to proceed with research in this general area. We believe that there is much to be accomplished concerning the development of better methodology to analyze more accurately the battle potential of combined arms through the medium of the combat simulations, as this procedure would appear to be relatively inexpensive and productive. In order to bring such thoughts into sharper focus, we note that Howes and Thrall (Ref. 4) give an eigenvalue type analysis for Red versus Blue combat involving small arms, armored personnel carriers, tanks, armed reconnaissance vehicles, antitank weapons, mortars, and artillery on each side. Thus, seven different types of weapons were involved in their study of a "battle" where Red forces were in the attack and Blue in the defense. The authors point out that no claims are warranted concerning the general representativeness of their simulation results since this would depend very much on the scenario used. Also, the random statistical variations inherent in the game model would have some effect in achieving consistent results. Nevertheless, the principles of their analysis, i.e., the techniques outlined in this chapter and that of Chapter 30, would seem to show considerable promise as perhaps the best available method of studying combined arms in a systematic and quantifiable way. That is to say, we need study procedures which will measure the relative worths of the different weapons employed in combined arms simulations for these establish the relative combat values of each weapon type or various grouping that might be considered as well. The Army weapon systems analysts could well accumulate, catalog, and have available for ready reference the weapon equivalence values extractable from the large number of combat simulations now being conducted at Army agencies. In one simulation, for example, a BCBT might be worth 2 R63's, while from another combat simulation an R63 might be worth 100 infantrymen or two howitzers, etc., so that all relative worths could be related. Once such values are obtainable, then they may be used as building blocks of tables or organization and equipment, which would indicate perhaps the best structures of future army divisions. It is recommended, therefore, that attempts to accomplish results in this direction would well be worthwhile.

42-8 SUMMARY

We have developed an example of Blue versus Red tank combat which might be fought in a future war and have indicated a useful method of analysis which quantifies the relative "combat worths" of the different types of antitank weapons involved. An advantage of the approach is that the weapon system worths developed could possibly represent a powerful technique of developing building blocks to field optimum combat forces. The analysis again shows that kill rates are the primary MOE's.

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3. FM 30-40, *Handbook on Soviet Ground Forces*.
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CHAPTER 43
AN EVALUATION METHOD FOR ARTILLERY OR SUPPORT TYPE WEAPON FAMILIES

We outline and discuss in some detail the current techniques of evaluating artillery weapons, especially those calibers which are combined in various ways to give a "mix" or "family". The aim is to present a method of comparing the different competing weapon mixes in order to establish superiority of one mix over the other, if possible. It is found that a very good measure of effectiveness is that of determining which artillery weapon mix can engage the most targets at fairly reasonable cost, and this analysis is accomplished through the use of the Legal Mix simulation. An example in some informative detail is given which approaches the question of determining the best mix of 155 mm, 175 mm, and 203 mm (8 in.) artillery weapons for an expected conflict in Europe.

43-1 INTRODUCTION

In contrast to Chapter 41 on the evaluation of infantry hand-held weapon mixes to produce as many casualties as possible and stop Red infantry, and the analysis of Chapter 42 on combat between tanks and guided missiles on both sides, this chapter involves an example of the evaluation of artillery or support type weapons. Note that, for the infantry and tank types of battles, there are invariably some exchanges or losses of weapons on both sides due to the direct firing between forces or the close-combat situation. In contrast, artillery and supporting weapon systems in general must often be evaluated in a rather different manner in the absence of direct exchanges. For combat situations such as that depicted for tanks in Chapter 42, it might be said that we are quite capable of measuring the relative worth of weapons on one side against those on the other in a very direct manner, and particularly in terms of their capability to destroy opposing close-combat forces. In the analysis of support weapons such as artillery, there may often be little or no direct exchange of fire. Hence it becomes necessary to measure the worth of, or that is to say, evaluate artillery in terms of its capability to destroy many deep targets, usually of much larger sizes, in areas adjacent to or beyond the direct battle between infantry and tank elements, or to attack rear areas that may become threats in due course of the overall battle action. This would include also counterbattery fire as needed.

As indicated in Chapter 8, artillery caused more than half the casualties in World War II. In Table 43-1, we give the percent deaths and percent wounds for US troops for World War II, Korea, and Vietnam. Note the effectiveness of artillery (fragmenting projectiles) relative to small arms, except for

TABLE 43-1
PERCENT DEATHS AND PERCENT WOUNDS BY CAUSATIVE AGENT FOR US ARMY TROOPS IN WORLD WAR II, THE KOREAN WAR, AND VIETNAM

	Deaths			Wounds		
Cause	WWII	Korea	Vietnam	WWII	Korea	Vietnam
Small Arms	32	33	51	20	27	16
Fragments	53	59	36	62	61	65
Other	15	8	13	18	12	19

(The above percentages are for US casualties caused by opposing forces) the Vietnam war where rifle firing dominated. We should also make particular note of the fact that artillery weapons are becoming more and more versatile, as evidenced by the current capability of some of them to engage and destroy tanks at long ranges, and no doubt that artillery will continue to be one of the main arms of the future. Field artillery can destroy or suppress enemy infantry at short ranges; it can destroy antitank guided missiles and crews at the short to medium ranges, and our support weapons can attack enemy artillery or enemy air defense systems at the longer ranges. Suppression is also very important since if enemy crews fail to take cover or evasive actions during an artillery attack, a high chance of destruction invariably follows.

It can be said that field artillery is the land combat arm with a mission to provide continuous and timely fire support to various field commanders by destroying or neutralizing in proper order of priority those ground targets that may have the effect of jeopardizing the accomplishment of the field commander's mission. In order to accomplish its mission objectives, field artillery must:

1. Support the maneuver forces with continuous, timely, accurate, and close fire support as required.
2. Deliver counterbattery fires throughout the range of the weapon systems capability.
3. Extend depth to combat by delivering fires on logistical installations, reserves, command posts, communication facilities, and other enemy targets throughout the area of influence of the supported force.

As contrasted to infantry and tank units in direct combat, artillery may be placed or hidden in rear areas not under direct fire, yet deliver devastating fire on enemy targets of all kinds located at various distances from the main line of resistance (MLR). For example, the 155 mm M109A1 Howitzer can deliver fire on the enemy from a few thousand meters to a range of about 18.1 km, or with the newer extended range projectile this field artillery piece can attack targets out to a range of about 24.5 km.

Artillery represents one of the main combat support arms of the division in combat; the others are aviation, air defense, engineer, signal, and military police. Additional combat support elements and reinforcements in the combat support areas may be attached from higher headquarters when the tactical situation demands such support. Thus, we begin to see some of the inherent versatility in employment of artillery in the combat zone to annihilate the enemy over the wide areas where he may be found.

Artillery also represents one of the key weapon systems for combined arms effectiveness studies. We need to establish the relative performance and combat interrelationships among aviation, artillery, other gun fire, infantry, and armor. In addition, we need to quantify the artillery firepower effectiveness improvements that can be made through enhanced intelligence activities or improved targeting; logistic operations of support; mobility and maneuverability with mechanized infantry; and the command and control functions as well.

Surprise fire is the most effective fire that can be delivered on enemy personnel, as is well known. FM 6-141-2 (Ref. 1) gives a good and informative account of field artillery effectiveness analysis and artillery weapon employment for nonnuclear combat. In particular, Ref. 1 covers the following topics of interest for artillery:

1. Concepts of employment
2. Cannon weapon systems characteristics and effectiveness, including lethal areas
3. Employment of standard, high explosive, and selected ammunition
4. Employment of toxic chemical ammunition
5. Rocket and missile weapon systems
6. Comparison of high explosive and selected ammunition.

A very useful compendium of field artillery facts, organizations, tactics, operations, weapon systems and terminology of relative current usage is that of Reichard and Downs (Ref. 2). Also, an extension of this account to include classified field artillery facts is given by Reichard and Downs in Ref. 3. In particular, Ref. 3 covers the history of weapon-range studies, range-extension alternatives for artillery, artillery weapon systems lethality figures, and the Soviet military organization and equipment.

An informative account of artillery suppression analysis techniques is given by Kinney in Ref. 4. This reference also gives a good coverage of this important application of field artillery type weapons. An overview of some techniques of fire support analysis has been provided by Crane (Ref. 5), and Girard (Ref. 6) discusses the structural approaches to fire support system mix analyses.

Field artillery cannon gunnery principles are thoroughly covered in FM 6-40 (Ref. 7) which is the standard and universal text for artillery usage in the US Army. The artillery weapon systems analyst should be familiar with much of the contents of FM 6-40.

In this chapter, our main interest centers around some of the current and rather typical techniques of evaluating artillery weapon mixes for the fire support role in combat. Many of the principles and details of the present methodologies for studying the overall effectiveness of artillery are covered in a series of studies called "Legal Mix" analyses. The Legal Mix methods of evaluation have been developed over a period of some ten years now and their results have been of considerable use to artillery weapon planners. The serious reader should study Refs. 8-15 to develop some expertise in the Legal Mix type methodology for evaluations and the Task Force Battleking study of Hardison et al. (Ref. 16) because field artillery weapon planning purposes should be of continuing interest. Earlier principles and studies concerning the development of methods for evaluating artillery in its combat role of fire support are given in Refs. 17-19 which contain pertinent background information for the new analyst.

In the paragraphs that follow, we will use much of and amplify somewhat the study of DeArmon (Ref. 20) in order to present a rather typical and useful procedure for comparing one artillery force mix of weapons against that of another gun mix. In what follows, we will approach the evaluation of support weapons by first indicating an expected complex of enemy targets, with some of their general characteristics (personnel in various degrees of cover, armor, trucks, etc.), criteria to defeat such targets, some brief characteristics of the artillery weapons which might be employed to attack these targets, the alternative mixes of artillery weapon families which are compared, along with weapon delivery errors, lethality, and cost information.

In evaluating artillery weapons, the reader should understand that the targets are not often point targets, that they may be large in size (up to several hundred meters in diameter), and that many rounds must be fired to neutralize many of the targets. In fact, the principles covering multiple round hit and kill probabilities, especially those of target damage of Chapter 20, are seen to be relevant and useful in this connection. Also, it will be observed that computers are indispensible for most of the current support weapon evaluations.

Before proceeding with the analytical details, we will hypothesize briefly a general battle situation.

43-2 GENERAL BATTLE SITUATION

For an illustration of some evaluation principles of artillery in support of our Blue combat organizations, we have hypothesized a probable battle situation in the European Theater, where Blue and Red forces oppose each other in a major conflict or full-scale war. The Red target complex that Blue artillery is assigned the mission to attack consists of some 16 different types of targets (Ref. 20) found in a Red Motorized Rifle Combined Arms Army for a combat intensity level called "Intense Defense" for Blue. The problem of this illustrative example is to determine the comparative effectiveness of three different mixes of possible Blue artillery weapons against the subject target complex and, in particular, select the best one for further study and planning of the weapon acquisition process.

The artillery weapons to be considered in neutralizing the enemy target complex consist of the M109A1 155 mm Howitzer, the M110 Self-Propelled 8-in. (203 mm) Howitzer, and the M107 175 mm Self-Propelled Gun. The different artillery weapon mixes or "families" of interest consist of various combinations and numbers of these weapons as indicated in the sequel, and these three different mixes will be compared by evaluating them against the same target complex. The total artillery force is to consist of nine battalions (Ref. 20) which will contain the three different combinations of the 155 mm, the 175 mm, and the 203 mm artillery weapons considered here. The structure of the Blue artillery force represented is known as a "Division Force Equivalent" (DFE) for which the direct support and the general support artillery units are treated in their normal role in rather general combat operations expected for a future conflict in Europe.

The methodology for evaluation of these typical and current artillery weapons will now be outlined and developed in some logical detail and order for the young weapon systems analyst.

43-3 TARGETS, WEAPON CHARACTERISTICS, AND PRELIMINARY BACKGROUND DATA FOR THE EVALUATION

We see the desirability of employing support type weapons for extending depth to the combat zone in view of the importance of artillery in causing casualties, e.g., as indicated in Table 43-1. Thus, and in principle, we might say that we use artillery to neutralize the enemy insofar as possible at the longer ranges so that he will not be able to accomplish his mission of overrunning our Blue position through close-combat action. We intend to use our artillery to hit the enemy and destroy his will to fight long before he is able to engage our forces in costly, close combat.

In Table 43-2 we have listed the various target types our artillery family is to attack, the primary target elements to be considered, and the attack criterion — or level of damage desired on a single firing engagement — and the defeat criterion for Blue's intense defense in Europe. In case the attack criterion can be met by more than one available firing unit, then the unit that can meet the attack criterion at the lowest cost will be selected for the engagement. As will be seen in the sequel, firing decisions also are made on the basis of the attack criterion assigned to the target. The attack and defeat criteria are listed in terms of the percent of targets (personnel, armor, trucks, etc.) that must be engaged and defeated insofar as our artillery is concerned. Also given in Table 43-2 are the number of target acquisitions our friendly target acquisition and surveillance equipment is able to acquire in this particular study of artillery effectiveness. Note that there is a total of 1510 targets acquired during the hypothesized battle for some 16 different enemy target types of artillery interest. The number of target acquisitions for each type of target indicates the expected frequencies of occurrence for each of the enemy threats and hence also gives some idea of the possible threat value and perhaps the priority we must consider in allocating our artillery weapons to the problem of neutralizing the total enemy threat. For example, the number of enemy troops in the final assault phase and those prior to the assault add to 367 targets, or nearly one-fourth of the total number of targets to be attacked. Also, for example, we see that about a fifth of the targets acquired is armor in the attack, which may pose a serious threat.

Some of the prominent or more important characteristics of the 155 mm, and the 175 mm, and the 203 mm artillery weapons the analyst will need to consider in his evaluation are listed in Table 43-3. Obviously the range of the artillery piece is important, for it might be necessary in deciding whether a given weapon can be considered as having the capability of engaging a target. Speed of the weapon,

TABLE 43-2
TARGET TYPES, NUMBER OF TARGET ACQUISITIONS, AND DEFEAT AND ATTACK CRITERIA REPRESENTED IN THE COMBAT SIMULATION OF 24 h OF BLUE INTENSE DEFENSE IN EUROPE

			Defeat Criterion	Attack Criterion
Target Type	Primary Target Element	Number of Target Acquisitions	Percent of Target Elements	Percent of Target Elements
1 Dismounted troops prior to final assault	Personnel	239	30	30/10[a]
2 Dismounted troops in final assault	Personnel	128	30	30/10
3 Dismounted troops in assembly area (hasty position)	Personnel	26	30	30/10
4 Dismounted troops in assembly area (prepared position)	Personnel	28	30	30/10
5 Infantry in prepared defense	Personnel	18	30	30/10
6 Service units (hasty positions)	Personnel	113	30	30/10
7 Service units (prepared positions)	Personnel	18	30	30/10
8 Command posts and observation posts	Personnel	85	30	30/10
9 Dismounted troops in approach march	Personnel	14	30	30/10
10 Artillery units (hasty positions)	Personnel	183	30	30/10
11 Artillery units (prepared positions)	Personnel	14	30	30/10
12 Armor in assembly area (hasty positions)	Armor	155	20	20/7[b]
13 Armor in assembly area (prepared positions)	Armor	64	20	20/7
14 Armor in column	Armor	106	20	20/7
15 Armor in attack	Armor	296	20	20/7
16 Truck convoy (stopped)	Trucks	21	30	30/10
TOTAL		1510		

[a]30/10 indicates that 30% decreases to 10% at ranges beyond 10 km.
[b]20/7 indicates that 20% decreases to 7% at ranges beyond 10 km.

cruising range, water-crossing ability, and time required to emplace an artillery piece are of interest in determining the availability of the weapon and its general usage in a combat maneuver situation. Note the relative basic loads for the three different gun tubes and the amount of resupply of ammunition by caliber. To be available for a mission, a weapon system must have sufficient ammunition for the attack. The round-to-round probable errors of the weapons and especially the aiming errors due to uncontrollable variations in trying to place the center of impact of the guns on the desired target point will be very important indeed to the weapon systems analyst since these errors will determine the number of rounds needed to defeat the various targets. Note, in particular, that the aiming errors for both range and deflection are listed generally as being at least twice the round-to-round delivery errors. The aiming errors, or mean point of impact (MPI) errors, demonstrate on the basis of their relatively large size that the main problem — at least initially in firing — is the inability of gunners to place the (unknown) center of impact (C of I) on or at the desired target aim point whether it is the

TABLE 43-3
ARTILLERY WEAPON SYSTEM CHARACTERISTICS

	155-mm, M109A1			175-mm, M107	8-in. (203-mm), M110	
Projectiles	M107	M483	M549	M437	M106	M509
Max Range, km	18.0	18.0	24.5	32.7	16.5	16.3
Battery Basic Load, rounds		1734		456		630
Resupply per Battery per Howitzer, round		84		32		32
Man/Crew		10		13		13
Weapons/Battery		6		4		4
Weapons/Bn		18		12		12
Weight Combat Loaded, tonne		24.1		28.2		26.5
Maximum Speed, km/h		56		55		55
Cruising Range, km		349		725		725
Water-Crossing Capability		amphibious		fordable		fordable
Time to Emplace, min		1		3		2
Probable Error (PE) in Range, %		0.33		0.28		0.30
PE in Deflection, mil		0.60		0.57		0.20
MPI or Aiming Error PE Range, %		0.74		0.71		0.72
Deflection, mil		3.37		3.62		3.33
Mean Rounds Between Failures (MRBF)		775		100		100
Mean Miles Between Failures (MMBF)		245		580		580
Cost of weapon, $		195,000		165,683		154,324

center of the target or some other point of more interest. Moreover, the aiming error is usually so large that the effectiveness of the smaller round-to-round dispersion cannot "take over" so to speak and bring its effectiveness to bear in terms of "precision" fire. Thus, and in summary, we see that an important problem relates to that of placing the C of I of the rounds at the desired aim point, or at least effecting an adjustment procedure which will guarantee convergence of the C of I onto the desired aim point. FM 6-40, *Field Artillery Cannon Gunnery* (Ref. 7) describes in some detail the recommended standard procedure for adjusting the C of I of the rounds to the intended aim point for the case of sensing shots as "over" or "short", and "right" or "left". This, however, is a "go-no go" type of analysis and adjustment, with some loss of efficiency. In those cases where the fall of shot can be sensed on a quantitative basis as being a deviation in so many meters beyond or short of the target aim point — e.g., 35 m over or short, etc. — then optimum procedures to be followed which will guarantee the convergence of the true unknown C of I onto the desired aim point are given in Refs. 21 and 22. Both of these studies point out that a full correction should be made for the observed deviation of the first round sensed, whereas only half the deviation for the second round should be corrected for, and one-third of the deviation sensed for the third round, one-fourth for the fourth round, etc. This optimum policy to guarantee convergence was also referred to in par. 14-1, *Part One*, and hence may be the best way to conserve on rounds fired. Also, the laser range finder has resulted in improved accuracy of fire. The system delivery errors listed in Table 43-3 are in broad or general contextual terms or values, whereas more refined values for the DeArmon study (Ref. 20) and the particular projectiles used are given in terms of the CEP in Table 43-5 in the sequel.

The mean rounds between failures (MRBF) and the mean miles between failures (MMBF) for a weapon will have a direct bearing on weapon availability or readiness. Finally, the costs of the competing weapon systems are very relevant in any cost effectiveness study or cost and operational effectiveness study and hence are given at the end of Table 43-3.

The alternative artillery forces or mixes of weapons (A, B, and C) considered in this evaluation are listed in Table 43-4. There we give the number of artillery battalions of each caliber considered for the three mixes we are to evaluate, while the number of weapons per battery (6 or 4) and number of weapons per battalion (18 or 12) are listed in Table 43-3. In this study of the effectiveness of the three different mixes, the problem of availability of weapons to engage targets must also be considered where availability is defined as the chance that a particular weapon type will be ready for a mission when needed and called upon to fire. In Table 43-4, the availability levels covered are for 100%, 75%, 50%, 25%, and 0% of the 8-in. and 175 mm force; also the actual numbers of 155 mm, 175 mm, and 203 mm artillery pieces correspondingly in the three mixes are shown. Note that wherever 155 mm artillery pieces are employed, they are used in both direct and general support functions, whereas both the 175-mm and 203-mm guns are used only in general support of the combat divisions.

Projectile-gun combinations will result in different delivery errors due to the different physical and range characteristics of the projectiles. The study of Ref. 20 takes this into account and the "relative"

TABLE 43-4
ALTERNATIVE ARTILLERY FORCES CONSIDERED IN STUDY

Artillery Force[a]	Percentage of 8-in., 175-mm, or Both Available[b]	Number of Weapons Initially Available		
		M109A1 155-mm	M110 8-in.	M107 175-mm
A	100	90	48	0
5 Bn's of M109A1 155-mm	75	90	36	0
and	50	90	24	0
4 Bn's of M110 8-in.	25	90	12	0
	0	90	0	0
B	100	90	24	24
5 Bn's of M109A1 155-mm	75	90	18	18
2 Bn's of M110 8-in.	50	90	12	12
and	25	90	6	6
2 Bn's of M107 175-mm	0	90	0	0
C	100	90	0	48
5 Bn's of M109A1 155-mm	75	90	0	36
and	50	90	0	24
4 Bn's of M107 175-mm	25	90	0	12
	0	90	0	0

[a]In all cases three battalions of 155-mm are in direct support, and the remainder of the force is in general support.
[b]Considers an equal number of weapons are available to each 8-in. or 175-mm battery.

CEP's for the three-mix evaluation are given in Table 43-5. We note in this connection that the M110 artillery weapon, which delivers a rather heavy projectile out to ranges not exceeding about 16.5 km (especially the M509 projectile), shows some superiority in delivery accuracy over the other projectiles and weapons. Thus, on this basis alone we might expect that the M110 weapon system has some potential in overall effectiveness over the other weapons. Moreover, when one considers that the M110 weapon system fires a 203-mm projectile, i.e., the largest projectile of all three weapons, its lethality would ordinarily be much higher — at least on the basis of the relative cube of the calibers. Thus, there is also some indication of superiority here in terms of potential lethality, but only as far as the caliber is concerned. For ranges of 18 km and beyond, we note in particular from Table 43-5 that the delivery errors of both the 155-mm and the 175-mm weapons deteriorate considerably, and the analyst might well expect large decreases in effectiveness potential here. (The artilleryman has long favored the 8-in. howitzer for its delivery accuracy, but ballisticians also have long realized that perhaps the potential of this artillery piece has not been exploited fully due to its relatively short range capabilities.)

In addition to the delivery accuracy capabilities of the three artillery pieces, the next rather key and important parameter in establishing effectiveness against the targets listed in Table 43-2 is the lethality or lethal areas of the various projectiles considered in the three force-mix comparisons. Relative lethality values for the different projectiles against personnel and materiel targets are given in Table 43-6. Actual numerical values are classified Confidential, and hence relative lethality values are

TABLE 43-5
RELATIVE SYSTEM DELIVERY ERRORS (CEP's) USED IN STUDY

Weapon and Caliber	Projectile	Type Error	Range, km					
			8	14	16.5	18	24.5	32.7
			Relative CEP Ratios[a]					
M109A1 155-mm	M483	Precision[b]	1.00[e]	1.32	1.74	2.00	—	—[d]
		Total[c]	3.53	6.21	7.79	8.74	—	—
	M107	Precision	1.37	1.95	2.47	2.84	—	—
		Total	3.83	6.37	8.47	9.79	—	—
	M549	Precision	1.58	2.05	2.37	2.53	4.00	—
		Total	3.84	6.32	7.84	8.79	13.84	—
M107 175-mm	M437	Precision	1.84	2.37	2.74	2.95	3.74	6.58
		Total	5.00	6.84	7.68	8.16	11.42	20.25
M110 8-in. (203-mm)	M106	Precision	1.53	2.00	2.32	—	—	—
		Total	3.79	6.32	7.84	—	—	—
	M509	Precision	0.89	1.37	1.63	—	—	—
		Total	2.68	5.26	6.74	—	—	—

[a]CEP = Circular Probable Error, or radius of circle which includes 50% of the shots. The relative CEP ratio is the ratio of the system and projectile CEP to that of the precision CEP of the M483, 155-mm projectile at 8 km. Actual CEP's are Confidential and are given in Table 3 of Ref. 20.
[b]Precision — round-to-round ballistic dispersion
[c]Total = total system CEP including both round-to-round ballistic dispersion and variation of the MPI or C of I (aiming error)
[d]Beyond maximum range

TABLE 43-6
RELATIVE (RATIOS OF) PROJECTILE LETHAL AREAS AT A RANGE OF 14 km

Weapon and Caliber	Projectile	Relative (Ratio of) Lethal Areas[a] m² (Open/Woods[b])					
		Personnel[c]			Materiel[d]		
		Standing	Prone	Foxhole	Tank	APC	Truck
M109A1 155-mm	M107	103/42.5[b]	70.8/22.3	6.67/2.0	1.50/1.0[a]	[illegible]	[illegible]
	M483	981/580	598/308	14.2/9.5	19.8/9.5	24.7/12.2	67.0/32.0
	M549	100/44.8	82.2/26.0	1.50/0.83	5.83/3.5	[illegible]	[illegible]
M110 8-in (203-mm)	M106	198/96.2	124/40.0	12.0/7.3	2.17/1.5[illegible]	11.5/6.8	[illegible]
	M509	2012/1234	1086/667	31.3/20.8	43.8/20.8	54.5/27	[illegible]
M107 175-mm	M437	181/71.7	139/48.3	21.0/7.3	2.83/2.0	14.8/8.8	[illegible]

[a]Figures in the table list the ratio of the lethal area for the weapon, projectile, and condition to the lethal area against a tank in the woods for the M109A1 weapon

[b]The first figure gives the ratio of lethal areas for the target in the open, and then the ratio of lethal areas for the same target in woods

[c]Assault 5-min criterion

[d]M or F (mobility or firepower) kill for tanks and APC's, 40-min interdiction kill for trucks

listed for personnel for standing, prone, and foxhole conditions of protection. Also, the major materiel target relative lethality values are given for tanks, trucks, and armored personnel carriers. (Lethal areas instead of vulnerable areas are appropriate here for vehicles since projectile explosions nearby may cause mobility or firepower damage.) Table 43-6 provides some insight into the relative toughness of the different types of targets to artillery attack — as may be noted. Note the relative protection provided by tanks, APC's, and foxholes as compared to that of trucks or standing and prone personnel. We might also say that overall the largest caliber, the 203-mm, appears to be superior in lethality against all of the various targets, although cost should also be a consideration.

Relative costs of the different projectiles are listed in Table 43-7 and will be relevant in terms of ammunition expended or total cost to achieve the levels of defeat criteria given in Table 43-2. Note the higher costs of the VT fuzed rounds and, especially, the much higher cost of the M509 projectile for the 8-in. howitzer since this will be significant in a cost-effectiveness analysis. Costs will also be a driving factor in the determination of which weapon should attack targets, given that either two or all three of the different artillery pieces could do the job.

Having discussed the artillery weapons, projectiles, expected delivery errors, lethality, and costs, we are now ready to present the model for evaluation which will be used in comparing the three competing mixes of support weapons. Since there are a very large number of characteristics to be considered in the very extensive computations needed to evaluate artillery support weapon families, it should be clear to the reader that many detailed computer programs are needed to handle this rather formidable problem. In fact, this is precisely what has been developed for the Legal Mix analysis models (Refs. 8-15).

43-4 DISCUSSION OF THE EVALUATION MODEL

As indicated in par. 40-4.9, the Legal Mix IV model is a one-sided, high resolution, deterministic simulation which has been developed in recent years to evaluate the effectiveness of artillery type mixes of weapons at the Army and lower organization levels. The artillery weapons are employed against a set of time-phased targets which have been "acquired" by available target detection devices.

TABLE 43-7
RELATIVE COSTS OF COMPLETE ROUNDS, COMPONENTS, AND TRANSPORTATION AS COMPARED TO COST OF M107, 155-mm PROJECTILE W/PD FUZE[a]

Caliber	Projectile/Fuze	Relative Costs: Projectile	Fuze	Prop Charge[b]	Transportation	Total
155-mm	M107/PD	1.00[a]	0.15	0.63	0.10	1.88
	M107/VT	1.00	0.81	0.63	0.10	2.54
	M549/PD	2.98	0.15	1.20	0.10	4.43
	M549/VT	2.98	0.81	1.20	0.10	5.09
	M483/VT	4.91	0.58	0.63	0.10	6.22
8-in.	M106/PD	1.71	0.15	1.03	0.21	3.10
	M106/VT	1.71	0.81	1.03	0.21	3.76
	M509/MT	8.73	0.58	1.03	0.21	10.55
175-mm	M437/PD	1.54	0.10	1.50	0.17	3.31
	M437/VT	1.54	0.92	1.50	0.17	4.13

[a]Relative costs in the table are the ratios of item costs to that of the 155 mm, M107 projectile. Actual costs are given in Table 5 of Ref. 20.

[b]Weighted to account for range.

The Legal Mix model is used to compute the percentage of missions lost, the number of personnel casualties inflicted, number of targets defeated, armored vehicles damaged, missions accomplished, accrued units of military worth for the missions accomplished, and the cost and weight of ammunition expended to achieve the levels of effectiveness desired. The DeArmon study, (Ref. 20) primarily covered here to illustrate a method of artillery evaluation, extended the Legal Mix type of analysis to some extent in order to determine the relative operational effectiveness capabilities of artillery forces — composed in part of the larger caliber 175-mm and 203-mm guns — which were subject to changes in weapon availability due to dynamic effects of reliability, availability, and maintainability that might actually occur in a combat operation. Thus, weapon availability was to be somewhat of a primary input parameter for the 175 mm and 203 mm artillery pieces since they were to be employed in the general support role of the fighting division.

For a preliminary bird's-eye view of the analysis, perhaps the reader may see that for each of the three competing families it is necessary to determine whether each weapon is available and can undertake fire at the various targets which have been detected, the employment of the best weapon-target allocation criterion, the number of rounds and costs of ammunition to achieve the level of defeat desired, the number of targets actually engaged, and casualties inflicted for the 24-h Blue intense defense of Europe. It might be said that "operational effectiveness" here is measured in terms of the number of targets effectively engaged and the target elements killed. Moreover, comparisons made between artillery force levels are based on measures of effectiveness attributed to the total artillery family.

In the study of Ref. 20, the indirect fire systems model which was used is an updated version of the Legal Mix IV model in the references, and the primary purpose was to simulate a Blue artillery force versus the Red target threats during the 24-h time period, considering 8-in. and 175-mm availability.

The model simulates the firing decision process; selects the weapon system to engage targets; fires the number of rounds necessary to meet the attack criterion; assesses damage to each target engaged; and accumulates ammunition expenditures by type of weapon, type of ammunition, and engagement range. The Red target list is an input. In this list, targets are identified by type, size, location, environment, arrival time, departure time, and Blue echelon (direct or general support) that acquires the target. The type, size, location, and environment of predicted targets may be somewhat different from actual targets. Decisions on whether or not to fire, what to fire, how much to fire, etc., are based on predicted targets; damage assessments are based on the examination of actual targets. The fire mission is assigned to the same echelon (direct or general support) as that of the acquiring unit. If, in that echelon, fire units are overloaded, the fire mission can be passed on to another echelon.

As previously indicated, firing decisions are made on the basis of the attack criterion assigned to the target. If the attack criterion can be met by more than one available fire unit, the unit that can meet the attack criterion at lowest cost is always selected to make the engagement. However, before fire units can be used against a target, the firing time and supply of ammunition needed to complete the mission must be available. The mission cannot be fired if the required ammunition is not available or if the expected firing time exceeds the time that the target will be in range. In the case of direct support units, a fire unit will be selected if its tube-to-target range is less than that of other direct support units and the target is a direct support acquisition. In the case of general support units, a fire unit will be selected if it can meet the attack criterion at a lower cost than other general support fire units regardless of the tube-to-target range. Within the fire unit, the most cost-effective round is selected for use. Mass fire (engaging a single target with more than one fire unit, simultaneously) occurs if available independent fire units cannot meet the attack criterion.

The defeat criterion, which usually exceeds the attack criterion, is assigned to each target. If, as a result of one or more engagements, the target damage equals or exceeds the defeat criterion, the target obviously is withdrawn and cannot be reacquired.

The model output provides the following:

1. Number of targets engaged; number of targets defeated; number of personnel, tanks, APC's, and trucks killed; ammunition expenditures (by rounds); and ammunition expenditure costs.

2. The characteristics of each fire mission. This includes the acquisition time, the range, the weapons fired, the kinds and amounts of ammunition fired, the number of target elements killed, and the number of elements in the target area.

The main effectiveness measures for the Blue artillery force were taken to be:

1. The number of targets engaged
2. The number of personnel and materiel target elements killed.

The cost measures for the Blue artillery force were:

1. The number of rounds of ammunition expended
2. Ammunition dollar cost.

43-5 RESULTS OF THE LEGAL MIX IV SIMULATION

As a result of the Legal Mix IV simulation for comparing the three different artillery weapon systems or families, Ref. 20 lists the key findings in its Tables 6 and 7, which are classified Confidential. Table 6 of Ref. 20 gives, for each of the three mixes, the number of targets actually engaged, the number of rounds by type expended, and the ammunition expenditure cost by alternative Blue artillery forces during the simulated combat day of 24 h of Blue's intense defense of Europe. Table 7 of Ref. 20 gives the relative effectiveness figures and relative costs by each Blue artillery force for the same combat day. The key results on an unclassified basis are brought together for our purposes in Table 43-8. The relative number of rounds expended for each caliber may be used in conjunction with the ammunition costs to indicate relative total costs for the firing missions. For Table 43-8, we found

TABLE 43-8
RELATIVE NUMBER OF TARGETS ENGAGED AND RELATIVE NUMBER OF ROUNDS EXPENDED BY PERCENTAGE OF 8-in. AND/OR 175-mm WEAPONS AVAILABLE TO THE ALTERNATIVE BLUE ARTILLERY FORCE[a]

Artillery Force	Percentage of 8-in., 175-mm or Both Available[d]	Relative Number of Targets Engaged									Relative Number of Rounds Expended[b]		
		Personnel				Materiel							
		Range, km			Relative Subtotal	Range, km			Relative Subtotal	Relative Total			
		0-10	10-20	>20		0-10	10-20	>20			155-mm	8-in.	175-mm
A													
5 Bn's of	100	1.08	1.22	1.24	1.16	1.70	1.77	1.27	1.69	1.30	0.80	0.21	—[c]
155-mm, and	75	1.08	1.22	1.26	1.16	1.60	1.81	1.45	1.66	1.29	0.83	0.20	—
4 Bn's of	50	1.09	1.23	1.21	1.16	1.45	1.85	1.45	1.59	1.28	0.89	0.17	—
8-in.	25	1.06	1.19	1.17	1.13	1.29	1.69	1.55	1.45	1.22	0.96	0.13	—
	0	1.00[a]	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	—
B													
5 Bn's of	100	1.09	1.23	1.24	1.17	1.46	1.85	1.45	1.60	1.28	0.80	0.13	0.11
155-mm, 2	75	1.08	1.21	1.24	1.15	1.46	1.87	1.36	1.60	1.27	0.87	0.13	0.10
Bn's of 8-	50	1.08	1.23	1.26	1.16	1.33	1.83	2.00	1.56	1.27	0.91	0.12	0.09
in., and 2	25	1.05	1.22	1.13	1.13	1.19	1.60	1.55	1.36	1.19	0.96	0.07	0.07
Bn's of 175-mm	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00
C													
5 Bn's of	100	1.09	1.25	1.29	1.18	1.24	1.77	1.82	1.48	1.25	0.91	—[c]	0.25
155-mm, and	75	1.09	1.25	1.19	1.16	1.25	1.77	1.55	1.46	1.24	0.93	—	0.24
4 Bn's of	50	1.04	1.22	1.20	1.14	1.14	1.58	1.64	1.34	1.19	0.93	—	0.20
175-mm	25	1.02	1.13	1.17	1.09	1.08	1.35	1.45	1.20	1.12	0.96	—	0.15
	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—	0.00

[a] Within each mix, all numbers are relative to the case for zero percent 8-in. and 175-mm weapon availability for each condition.

[b] Relative rounds expended are with reference to 155-mm for zero availability of 8-in. and 175-mm weapons.

[c] Indicates that weapons of indicated caliber were unavailable to the artillery force.

[d] Considers an equal number of weapons were available to each 8-in. and 175-mm battery.

it more informative to determine the relative number of targets engaged in terms of the base which is the actual number of targets engaged for the case of zero availability of the 8-in. and 175-mm weapons since such numbers stay constant regardless of the three different mixes. In addition, for the relative number of rounds fired, these have been calculated with respect to the 155-mm weapon as a base, where the availability of the 8-in. and 175-mm weapons was zero. Again, when none of the larger caliber weapons are involved or available, each of the three families has exactly the same 155-mm weapons. This has been done because we then have a constant and unified base of comparison, which would not be the case for 100% availability of the 8-in. and 175-mm weapons as was done in Ref. 20.

Based on Tables 6 and 7 of Ref. 20, and Table 43-8, DeArmon — on the basis of the calculations of the measures of effectiveness — arrived at the following findings:

"On the basis of the *total* number of targets engaged, the 155-mm/8-in. mix is best, the 155-mm/8-in./175-mm is second best, and the 155-mm/175-mm mix is worst. However, the maximum difference among alternative forces is only about 4%. The number of *personnel* targets engaged by the three alternative forces are essentially equal. The numbers of materiel targets engaged by the 155-mm/8-in. mix exceed those engaged by the 155-mm/8-in./175-mm mix by 5%, and those engaged by the 155-mm/175-mm mix by 13%. At short ranges (0-10 km), the 155-mm/8-in. mix engages 14% more materiel targets than the 155-mm/175-mm mix. At long range (>20 km), the 155-mm/8-in./175-mm and 155-mm/175-mm mixes engage more materiel targets than the 155-mm/8-in. mix, by 14% and 43%, respectively.

"The 155-mm/8-in. mix is the most costly, and the 155-mm/175-mm mix is the least costly. The maximum difference in costs among the three alternatives, however, is only about 4%."

As shown by the data in Table 43-8, as the availability of large caliber 8-in. and/or 175-mm weapons is reduced, there is considerable variability in number of targets engaged from range to range. As the weapon availability decreases, the decrease in number of personnel targets engaged is less than the corresponding decrease in the number of engagements of materiel targets.

For all alternative mixes, the relative proportional decrease in effectiveness is small compared to the proportional decrease in availability of the larger caliber weapons.

The data in Table 43-8 show that for each alternative mix, as the availability of large caliber weapons is reduced, the ammunition expenditure of the 155 mm is increased.

Concerning the cost information involved, DeArmon indicates the data show that for the reduction of each 8-in. weapon in the 155-mm/8-in. mix, the 8-in. ammunition expenditure is decreased (on the average) by 110 rounds and the 155-mm ammunition is increased by 190 rounds. For the reduction of each large caliber weapon in the 155-mm/8-in./175-mm mix, the 8-in. expenditure and 175-mm expenditure are reduced by 160 rounds and 111 rounds, respectively, while the 155-mm expenditure is increased by 190 rounds. For the reduction in force of each 175-mm weapon in the 155-mm/175-mm mix, the 175-mm expenditure is reduced by 130 rounds and the 155-mm expenditure is increased by about 60 rounds.

43-6 CONCLUSIONS

On the basis of this operational effectiveness involving larger caliber weapon availability, DeArmon indicates that the 155-mm/8-in. mix is regarded best, the 155-mm/8-in./175-mm mix is second best, and the 155-mm/175-mm is the poorest. However, the numbers of personnel targets engaged by alternative forces were essentially equal. The 155-mm/8-in. mix engaged more materiel targets at short range (0-10 km) than the 155-mm/175-mm mix; but at long range (>20 km), the 155-mm/175-mm mix engaged more materiel targets than the 155-mm/8-in. mix, due to its extended range.

On the basis of ammunition expenditure cost, however, the 155-mm/8-in. mix is most costly and the 155-mm/175-mm mix is least costly.

In all alternative force mixes, the effectiveness decreased as the number of available weapons decreased, but the proportional decrease in relative effectiveness is small in comparison to the corresponding proportional decrease in the number of larger caliber weapons.

43-7 COMMENTS ON THE ARTILLERY EVALUATION STUDY

In this particular support weapon evaluation study, we have outlined one of the prime current methods for comparing the worth of artillery type families of weapons. It is seen in this connection that the most effective family is not the one of least cost, and this is not always to be expected anyway. We note here that the difference in costs, however, is only about 4%, so that the 155-mm/8-in. mix seems well worth the extra cost. A point of some interest is that even though the 8-in. (203-mm) weapon has only about one-half the maximum range of the 175-mm gun, the 8-in. weapon still showed up in the most effective family. Some artillerymen might well question this occurrence, but it may be that the 175-mm gun has inordinately high delivery errors at the extended ranges and the greater lethality of the 8-in. weapon also came into play insofar as the chosen measures of effectiveness were concerned.

We see that operational effectiveness may be significantly dependent on weapon availability, and indeed this places a much increased burden on the firing of the 155-mm weapons.

Finally, it should be noted that the evaluation of artillery support weapons, at least in a way, is not as direct as that of either the infantry or tank type combat evaluations. Indeed, we see that in the case of comparing families of artillery weapons there are several (competing) measures of effectiveness, and really not one stands above the others in all respects. For example, it is very important to have a family of support weapons which will be able to engage as many targets as possible, for that means more casualties and more effectiveness. On the other hand, it must be conceded that cost is also a primary consideration, but perhaps relatively small cost differences should not be the governing factor. In any event, the game here is to select the family which is superior in overall effectiveness, while at the same time ensuring that its cost is not inordinately high.

43-8 SUMMARY

We have presented the current method of evaluation for artillery or support type weapon families. The suggested procedure is necessarily different from that of evaluating infantry and tank weapons in close combat since the artillery weapons can attack a large variety of enemy targets of rather large size at almost any range at which such threats might appear on the battlefield. In addition, but not evaluated here, is the future capability of artillery even to attack enemy tanks at very remote ranges before they become threats in close combat.

There are several important and different types of measures of effectiveness for evaluating support weapons, and the decision maker must necessarily consider these on a joint basis — at least to some degree.

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CHAPTER 44
THE MODERN GUN EFFECTIVENESS MODEL (MGEM) FOR EVALUATING AIR DEFENSE GUNS

The problem of evaluating the effectiveness of air defense guns for the field army is discussed in this chapter. The technique used is known as the Modern Gun Effectiveness Model (MGEM) which is a computerized simulation of the terminal engagement between gun projectiles and the aerial targets. An important and very useful consideration concerning the application of the MGEM is that extensive and successful simulation efforts have been made to validate the model, which gives much confidence in the accuracy and applicability of it.

An example is given to illustrate some of the typical computations of hit and kill probabilities, as well as the expected number of rounds fired in an engagement.

44-0 LIST OF SYMBOLS

"A" = a kill of the aircraft which results in loss of manned control in 5 min
A_p = presented area
A_v = vulnerable area
az_t = azimuth at time t
az_{t+1} = azimuth at time $t + 1$
ϵ_{az} = random deviate from $N(0,\sigma_{az})$
ϵ_{el} = random deviate for $N(0,\sigma_{el})$
el_t = elevation at time t
el_{t+1} = elevation at time $t + 1$
ϵ_r = random deviate from a normal distribution $N(0,\sigma_r)$ with zero mean and standard deviation σ_r
"K" = a quick, 30-s kill
$N(0,\sigma)$ = normal distribution with zero mean and indicated σ
P = total kill chance
$p(k|h)$ = probability of a kill, given a hit
p_{ssk} = single-shot kill probability
R = ballistic range
r = range
r_t = range at time t
r_{t+1} = range at time $t + 1$
t = time
t_f = time-of-flight
v_m = muzzle velocity
β = drag-related constant for a gun projectile
σ = delivery standard error
σ = ballistic dispersion
σ_{az} = standard error in azimuth
σ_{el} = standard error in elevation
σ_r = standard error in range

σ_x = round-to-round delivery standard error (one direction)
σ_μ = one-directional aiming error sigma

44-1 INTRODUCTION

Like the evaluation of close combat for infantry (Chapter 41) and tanks (Chapter 42) and the evaluation of the role of artillery (Chapter 43), the field of Army air defense represents a rather large area of application for the weapon systems analyst. It can be said that the problem of modeling ground combat is indeed very complex, and that many of the important parameters or boundary conditions, for example that of the terrain, contribute greatly to the complexity of any competent analysis. In contrast, it would seem that the analysis of air defense weapons against aerial targets might be somewhat simpler, at least in some respects. Thus, and in principle, we might expect that one merely needs to deal with the chances of at least one hit against typical aerial targets, and include the vulnerability of the target or the conditional chance that a hit is a kill in the analysis. Nevertheless, we must appreciate that somewhat different or special evaluation models are required in air defense studies, as might be expected. Indeed, for air defense evaluation problems, we are dealing with relatively high-speed targets; accordingly, the terminal engagement geometry between the warhead or projectile and the aircraft or aerial target becomes of critical importance. In addition, aerial targets can fly paths to avoid being seen, detected, or acquired; also they may adopt evasive tactics to reduce, as much as possible, the chance of being hit once under acquisition and attack. It is also true that the problem of directing guns along the predicted flight paths of enemy aerial targets will very likely involve autocorrelation analyses of one kind or another. (See Chapter 20 for an introduction to such methodology.)

In view of the analytical complexity concerning the terminal conditions of a warhead or projectile approaching a high-speed aerial target, it would seem that Monte Carlo type simulations would be very valuable in estimating kill probabilities of air defense weapons against aircraft or other aerial targets, and indeed this approach often turns out to be a necessary mode of analysis. One does have to to be rather confident that he can simulate accurately the warhead delivery errors and the fuzing errors of the air defense system under evaluation, as well as any possible evasive tactics the aerial target might take during the encounter. Furthermore, the conditional chance that a hit is a kill against the aerial target must be determined accurately from aerial target vulnerability analyses for the warheads involved.

In the case of guided missiles with rather large warheads, for example the HAWK missile system, Monte Carlo simulations of engagements to produce effectiveness data have worked out very well indeed. Also, once the guidance and fuzing errors are determined or estimated with sufficient accuracy, an analytical procedure for evaluating blast-fragmentation warheads in air defense can be used to estimate kill chances. (See for example, the work of Banash, Ref. 1.)

We cannot expect to cover the many ramifications of the vast field of air defense evaluation techniques for the Army here, but our intention is to give some of the present methodology for evaluating the effectiveness of air defense guns of calibers from about 30 mm to 40 mm to satisfy the Army need for its relatively short-range weapons against aerial targets. In fact, a quote from a *Wall Street Journal* article of Wednesday, 30 November 1977, is of interest and provides pertinent background at this point:

"WASHINGTON—The Army selected General Dynamics Corp. and Ford Aerospace & Communications Corp., a unit of Ford Motor Co., to develop a new short-range air defense gun for use in the 1980s.

"Selection of the two companies means they also will compete for a production contract that's expected to exceed $1 billion. It was as much a victory for Pentagon planners who want U.S. services to build weapons that are compatible with those used by North Atlantic Treaty Organization allies, as it was for the two companies.

"General Dynamics plans to build its system around a 35 mm Swiss Oerlikon gun, built here under license, which is widely used by both the West Germans and the Dutch. Ford will use a Swedish 40 mm Bofors gun, also widely deployed in Europe.

"The two companies were selected to begin full-scale engineering development of the new radar-directed gun called DIVAD, for division air defense gun, over General Electric Co., Sperry Rand Corp. and Raytheon Co. One of the factors in the selection was the winners' incorporation of European guns into their proposals, according to Percy A. Pierre, Assistant Secretary of the Army for research, development and acquisition.

"NATO members frequently develop and build weapons that aren't compatible with similar weapons built by other allies. To increase the effectiveness of NATO forces, the Pentagon is telling contractors that they can increase their chances of winning business if they design weapons that are compatible with European systems.

"The Army plans to purchase about 430 of the new guns from U.S. manufacturers. It also plans to purchase 171 West German air defense guns, called the Gepard, for about $900 million for quick deployment to U.S. forces in Europe. 'The advantage of the Gepard is you can get them right off the production line,' Mr. Pierre said, while it will be five or six years before the more advanced—and cheaper—U.S. system can be deployed. The Gepard system uses the same Swiss gun included in the General Dynamics proposal. The purchase of the Gepard is still subject to approval by Defense Secretary Harold Brown.

"Both the Gepard and the new DIVAD gun are designed to improve the Army's ability to shoot down enemy aircraft at ranges up to about three miles. The DIVAD will be an all-weather, radar-directed gun system in an armored turret mounted on an M48 tank chassis. The guns will be used to protect combat forces based near battle lines.

"The Army hasn't awarded contracts to General Dynamics or Ford Aerospace yet. It plans to begin contract negotiations soon, leading to agreements with the companies to develop and build two prototypes of their guns over the next 29 months. After testing the models and comparing their cost, the Army will select one of the companies to produce the gun system."

44-2 BACKGROUND FOR THE MODERN GUN EFFECTIVENESS MODEL (MGEM)

The main technique for evaluating air defense guns to be presented here is known as Modern Gun Effectiveness Model, or "MGEM", and is based primarily on a digital simulation to compare the various candidate or competing weapons of calibers from about 30 to 40 mm. To give some limited background information, the current Army air defense gun system is the VULCAN Gun, which is a 20 mm six-barrel Gatling type weapon, which has been procured as an interim system during the 1960's. However, the VULCAN system evaluation studies led to the conclusion that some improvement in range capability and weapon delivery accuracy was highly desirable, and this led to a product improvement program. In fact, a study called the GADES, or Gun Air Defense Effectiveness Study, was initiated to develop the appropriate methodologies to compare future candidate air defense guns through simulation techniques. The GADES study indicated that effectiveness seemed to depend rather critically on the accuracy of the tracking and fire-control loop, and indeed this very finding was arrived at independently by the Johns Hopkins University Applied Physics Laboratory. Meredith and Scheder (Ref. 2) give a more informative account of the background which we have paraphrased somewhat here:

"The GADES study brought to light the fact that the modern digital control processor could and should be used in air defense gun systems. This concept was tested by the Gun Low Altitude Air Defense (GLAAD) study, when a twin-barrel 25 mm test bed was built for the Army with a digital computer fire control. The computer was programmed using modern optimum control techniques (Kalman filter) for target state estimation and second-order target prediction. During the GLAAD project, digital simulations were built which emulated both the gun system hardware and its fire control concepts software. The results of the GLAAD testing demonstrated that modern fire control could significantly enhance gun air defense systems, especially against maneuvering targets, and that digital simulation techniques could accurately predict gun system performance.

"A current objective of the ARGADS (Army Gun Air Defense System) Project Manager is to field a new air defense gun which utilizes modern technology. Other studies indicate a need for an air defense gun in the 30 to 40 mm caliber range. A number of companies expressed an interest in supplying the Army with a gun system and have offered preliminary designs and potential capabilities. To help the Project Manager sort out and evaluate such proposals, the Army Materiel Systems Analysis Activity (AMSAA) was requested to conduct a preliminary study using simulation techniques developed and validated by the GADES and GLAAD studies. Related studies were concurrently conducted by Frankford Arsenal, the Ballistic Research Laboratory, Harry Diamond Laboratories, along with studies within the Project Manager's Office.

"The method of analysis used in this study was to examine the various gun candidates through a digital simulation of a one-on-one engagement. The simulation used was the Modern Gun Effectiveness Model (MGEM) developed precisely for this type of study. MGEM is a digital Monte Carlo simulation which contains submodules for the target flight path, the gun system's sensor/tracker, fire control and doctrine, predictor and gun servo, the projectile's ballistics and lethality, and the target's vulnerability. A single engagement assumed no terrain effects and long range detection and identification. It consists of burst fire at the target, commencing at a prespecified projectile-target intercept range and continuing intermittently until either the target is "killed" or recedes out of range.

"For this study, two aspects of realism not usually considered in one-on-one investigations have been interjected. One is the inclusion of gun barrel cool times in the fire doctrine. Each gun system had a cooling period associated with a burst size which should not be exceeded for normal barrel wear. Two firing doctrines were used in the study—a shoot-shoot when only one-second intervals were programmed between bursts and a shoot-look-shoot doctrine when a projectile time of flight plus a one-second wait was required between bursts. In both cases, however, the cool time criterion overrides the preprogrammed time between burst intervals.

"The second aspect of simulating realism was the use of several maneuvering flight paths and many gun locations against each flight path. All of today's fielded air defense gun systems have linear prediction fire control which are effective only against nonmaneuvering targets, and presumably should be studied and tested against nonmaneuvering targets. This lends itself to neat tables such as effectiveness versus range versus offset. The new gun should be able to achieve a respectable effectiveness against maneuvering targets. Some method had to be devised to compare gun candidates in an unstructured array or situation without showing favoritism to a particular characteristic. We decided to use five fixed-wing flight paths (straight, jinking, pop-up and dive, constant 'G' turn, turning flyby) and one single rotary-wing path. Each path contains regions where guns do well and regions where they do poorly. Past studies have encountered situations when one gun appears superior primarily because it happened to shoot at the right time—a matter of luck and geometry. To alleviate this phenomenon in this study, the gun site was randomly situated for each Monte Carlo trial against each flight path. Thus, the effectiveness of a given gun system candidate is the engagement hit or kill probability against a target flying a particular type of flight path and integrated over many gun locations relative to the flight path. The effect of this procedure is to force the guns to fire at many points along the flight path, resulting in a value representative of expected overall effectiveness.

"The characteristics of the gun systems that are considered in this study are the caliber, ballistic dispersion, muzzle velocity variations, ballistic trajectory, rate of fire, projectile lethality and cool times. Each system was assumed to possess an identical, state-of-the-art sensor/tracker, digital computer fire control (target state estimator and predictor), and gun servomechanism, since the characteristics of these modules cannot be ascertained until industrial proposals have been submitted. For each gun system considered, the latest available information was used to describe the system. This information was obtained from both industrial and government sources, compiled and selected by the Project Manager's Office, and submitted to AMSAA for use in this study. These data were used, but parameters had to be determined and frozen before the study could begin. Guns and bullets which have yet to be designed are inherently ill-defined, so the only recourse is to obtain the best possible estimates at the time of need.

"Since the object of this study is to compare gun systems, a comparison of each in its own best light is desirable. But the optimum burst sizes and firing doctrine for the gun candidates are unknown. One criticism of performing this study at the time was that the firing doctrine is a key gun system characteristic, and until such doctrine is recommended by the system's manufacturer, no impartial comparison can be made. In an attempt to overcome this very valid objection, a matrix of strategies was conducted—two firing doctrines (shoot-shoot and shoot-look-shoot and three burst lengths—for a total of six situations for each gun against each flight path. Obviously, we were quickly overwhelmed by an avalanche of output. So, to select the optimum strategy for each gun system, the hit or kill probabilities from the five flight paths were averaged with equal weight. The justification for this is the assumption that a gun system in the field is equally likely to encounter each type of flight path. Of course, this assumption may be false, but there are little data to the contrary, and it would probably be unwise to optimize a gun system against only a particular type of flight path.

"Two types of optimization were achieved. One was based upon maximum effectiveness while the other was for least cost per unit of effectiveness. We were able to perform the latter because we were furnished gun and ammunition costs by the Project Manager's Office. One other figure of merit for the gun system . . . is the number of kills per stowed load using optimum strategy."

44-3 GUN SYSTEM AND AMMUNITION CHARACTERISTICS, AND TARGET VULNERABILITY DATA

Certain performance characteristics of the typical gun systems which may be considered for the role of low altitude air defense for the US Army are given in Table 44-1. The gun systems listed may be the proposed candidates for the DIVAD requirement. Note in particular that the round-to-round ballistic dispersion of the different weapons is rather unpredictable as a function of caliber and that the Gatling Gun 30G has a relatively high (one-directional) ballistic standard deviation in mils. The system rate of fire, also listed in Table 44-1, decreases to some extent with increasing caliber, as would be expected. Finally, the stowed load of rounds does indeed depend markedly on caliber and

TABLE 44-1. AIR DEFENSE GUN SYSTEM CHARACTERISTICS

Gun Designation	Caliber, mm	No. of Barrels	Ballistic Dispersion σ, mils	System Rate of Fire, rd/min	Stowed Load, rd
30G[a]	30	7	1.40	3000[*]	1000
30M[b]	30	3	0.31	2400	1000
35F[c]	35	2	0.56	1100	640
35S[d]	35	6	0.85	2400	288
40	40	2	0.56	600	450

*Rate of fire is 4200 rd/min after 1 s

σ = linear (one-directional) standard deviation

[a]GE Gatling Gun
[b]Mauser Model Gun
[c]Oerlikon Gun
[d]Sperry Gatling Gun

decreases rather systematically with increasing caliber, except for the Sperry Gatling Gun which seems low.

The aiming errors, or movement of the C of I's of salvos, are not listed in Table 44-1 since they depend on the target tracking capability of the radar or optical sensor, the filtering or "smoothing" of such data to predict future target position, and the servo or gun-pointing mechanism. The aiming errors are in fact produced and properly accounted for in the MGEM simulation.

Table 44-2 gives the maximum salvo or rapid fire burst sizes for gun-cooling times of 1 s, 5 s, and 60 s in order to preclude overheating of the gun tubes. Such schedules would be followed in evaluations allowing for gun-cooling fire doctrines.

Pertinent ammunition characteristics of the projectiles by caliber are given in Table 44-3. As indicated in par. 44-2, final design data were not necessarily available, although for study purposes somewhat homologous projectiles may be evaluated, at least in a preliminary way, to determine the importance of caliber. Note in particular that the 40-mm projectile has three times the weight of high explosive filler as does the 30-mm round. Muzzle velocities of the rounds in meters per second, flight times to 3 km, and ranges at which the remaining velocity of each round reaches the sonic condition also are listed in Table 44-3.

The operating and supply costs of the ammunition for the competing air defense gun systems are listed in Table 44-4. These basic data would be used in a cost-effectiveness type study.

Obviously, terminal ballistic performance of the projectile against aerial targets is of fundamental importance in the evaluation of air defense gun systems. In this connection, Table 44-5 gives target vulnerable areas in square meters for a typical fixed-wing aircraft and a rotary-wing aircraft for uniformly distributed random impacts on the front, side, rear, bottom, and top of the aerial targets, and as a function of the 30 mm, 35 mm, and 40 mm projectile calibers. Point-detonating fuzes with appropriate delay are to be used for the projectiles since a hit is required to produce aircraft damage. Vulnerable areas are listed for "K" (quick, 30 s) kills and "A" kills or loss of manned control of the aircraft in 5 min. Any vulnerable area divided by the presented area under consideration gives $p(k|h)$, the conditional chance that a hit is a kill. For example, for a 35-mm projectile random hit against the side of a fixed-wing aircraft, the conditional chance of an "A" kill is

$$p(k|h) = A_v/A_p \qquad (44\text{-}1)$$
$$= 11.3/28.2 = 0.40$$

TABLE 44-2. GUN SYSTEM COOL TIME SCHEDULE

Gun System	Burst Size for 1-s Cool Time	Burst Size for 5-s Cool Time	Burst Size for 60-s Cool Time
30G	10-25	50	140
30M	20	40	150
35F	8	20	60
35S	25	50	75
40	5	20	50*

*30-s cool time

TABLE 44-3. AIR DEFENSE AMMUNITION ROUND CHARACTERISTICS

Caliber, mm	HE wt, gm	Muzzle Velocity, m/s	Time-of-Flight to 3 km, s	Fire Range,* m
30	54	1112	4.5	3305
35	112	1175	3.8	4067
40	160	1040	4.2	4039

*Range at which remaining velocity is 1.1 Mach

TABLE 44-4. OPERATING AND SUPPLY COSTS

Gun System	Ammunition Cost[a]	Total Costs[b]
30G	$ 7.40	$ 8.80
30M	7.40	10.46
35F	12.59	16.70
35S	12.59	16.70
40	17.73	22.00

[a]Per round
[b]Total costs include ammunition cost plus operating and support costs.

TABLE 44-5
TABLE OF FIXED-WING AND ROTARY-WING AIRCRAFT VULNERABLE AREAS VERSUS PROJECTILE CALIBER

Caliber, mm	Vulnerable areas A_v, m²									
	Front	Side	Rear	Bottom	Top	Front	Side	Rear	Bottom	Top
	Fixed-Wing "K" Kill					Rotary-Wing "K" Kill				
30	0.4	5.0	0.4	1.8	3.9	2.9	12.6	2.7	12.3	15.9
35	0.8	8.2	0.7	3.1	9.2	5.2	22.1	3.0	18.6	21.5
40	1.2	9.6	0.9	4.5	9.7	6.8	27.0	4.0	21.8	23.4
	Fixed-Wing "A" Kill					Rotary-Wing "A" Kill				
30	0.5	9.5	0.8	7.5	9.1	4.2	18.3	2.7	20.1	18.7
35	0.8	11.3	1.2	8.2	10.3	6.0	24.9	3.0	22.0	21.6
40	1.3	12.4	1.6	9.1	11.5	7.3	28.9	4.2	23.4	23.9
Presented Areas A_p	4.7	28.2	4.7	39.6	39.6	14.6	40.7	14.6	51.3	51.3

where

A_v = vulnerable area, m^2
A_v = vulnerable area, m^2

44-4 PRELIMINARY CALCULATIONS OF SOME KILL PROBABILITIES

In order to get some idea of the magnitudes of kill probabilities, we will calculate a few optimistic kill chances or boundary values for the reader. For example, if there were no aiming errors and only ballistic dispersion (see Table 44-1 for values of ballistic dispersion σ), the approximate single-shot kill probability p_{ssk} of the 30 mm Mauser projectile for an "A" kill against the bottom of the fixed-wing aircraft at 1500 m is given by

$$p_{ssk} = A_v/(A_v + 2\pi\sigma^2) \quad (44\text{-}2)$$
$$= 7.5/\{7.5 + 2\pi[(0.31)(1.5)]^2\}^* = 0.85$$

which is to be regarded as a very optimistic value, especially since the aiming error would be expected to be several or many times the ballistic error.

An approximate single-shot kill probability considering the existence of an aiming error per round may be calculated by replacing the term $2\pi\sigma^2$ in Eq. 44-2 by the equivalent or total ballistic and aiming dispersion (an expression similar to Eq. 20-95), i.e.,

$$p_{ssk} \approx A_v/(A_v + 2\pi\sigma_x^2 + 2\pi\sigma_\mu^2) \quad (44\text{-}3)$$

where

σ_x = one-directional ballistic sigma
σ_μ = one-directional aiming error sigma.

For the example previously given for which the ballistic sigma is 0.31 mil, suppose the aiming error σ_μ is, say, 3 mils as it may well be for an air defense gun. Then

$$p_{ssk} = 7.5/\{7.5 + 2\pi[(0.31)(1.5)]^2 + 2\pi[(3)(1.5)]^2\}$$
$$= 0.055$$

which represents a crucial change indeed.

Consider a salvo or burst size of 40 rounds and a cooling time of 5 s between bursts (Table 44-2). Then, for complete independence among adjacent rounds, the most optimistic total kill chance P is about

$$P = 1 - (1 - p_{ssk})^n \quad (44\text{-}4)$$
$$= 1 - (1 - 0.055)^{40} = 0.896.$$

As will be observed in the sequel, such calculations may not be realistic since they do not account for the tracking sensor, filtering, prediction, and gun-aiming problems—especially for which the MGEM simulation has been "validated" (Ref. 3).

If the aim error were accurately known, another method of calculating kill probability would be estimation, by the models of Chapter 20, of the chance of at least one hit on the vulnerable area of the target. Indeed, appropriate modeling of the aim error to place the C of I on the target is of critical importance in air defense weapon systems effectiveness. Also the air defense guns considered here have such a high rate of fire that the group of rounds fired in a burst may be considered to have the same C of I. We now turn to the most promising known procedure for evaluating air defense guns.

*To convert the standard deviation from mils to meters, the standard deviation in mils is multiplied by the range in kilometers.

44-5 SUMMARY OF THE MGEM SIMULATION AND VALIDATION

The proposed model (MGEM) for evaluating low altitude, relatively short-range air defense guns consists essentially of describing the gun projectile-aerial target engagement simulation encounter schematically as in Fig. 44-1 (Ref. 3).

A sensor (radar, optical, etc.) detects and acquires the target, and gives "noisy" flight path data. A filter then processes the raw tracking data to determine target position, velocity, and acceleration estimates. The filter function will involve smoothing of current and past target position data on the basis of some criteria in order to filter out noise or extraneous data. Next, the filtered or smoothed flight data are weighted in some "optimum" way to extrapolate for future target position at the projectile time-of-flight in the future. This prediction, along with the known trajectory of the projectile, determines where to aim the gun to achieve a projectile-target intercept one time-of-flight in the future. The servomechanism directs or aims the gun tube in a continuous fashion so that a burst of rounds may be fired from the gun at the correct instant.

MGEM involves a complete air defense gun-target engagement simulation which has been fully programmed on a computer. Five representative flight paths for enemy aircraft are used in the MGEM simulation (Ref. 2), namely:

1. Straight Flyby (Fig. 44-2)
2. Constant 2-G Turn (Fig. 44-3)
3. General Maneuver (Fig. 44-4)
4. Pop-Up and Dive (Fig. 44-5)
5. Jinking Flyby (Fig. 44-6).

The reader may comprehend that such flight paths may be made "noisy" with appropriately added random numbers, and then the filter, predictor, and servo subsystems simulated so as to estimate accurately gun pointing data. Projectile-target intercept conditions are determined for one time-of-flight for the burst, and aircraft damage is then assessed.

The fire control includes a forward looking infrared (flir) or visual optic target sensor and a digital computer which uses Kalman statistical estimation techniques. Thus, conventional fire control design for air defense guns has been updated by the use of modern control theory techniques, and the superiority of this approach has been verified and validated by Meredith, Scheder, and Lufkin (Ref. 3).

Perhaps a brief sketch of some of the salient points of the validation process would be of interest here since it might give the reader an appreciation and some confidence that the best known model for evaluating air defense guns has been found.

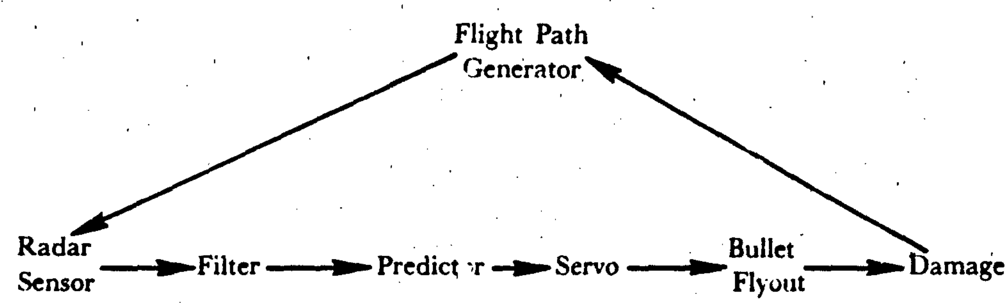


Figure 44-1. MGEM Simulation

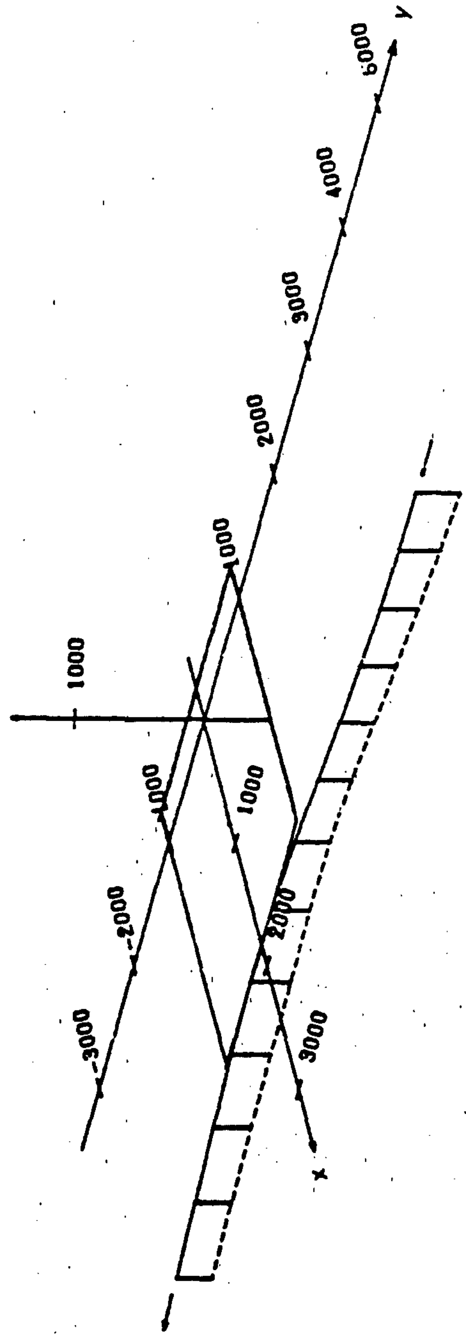


Figure 44-2. Straight Flyby

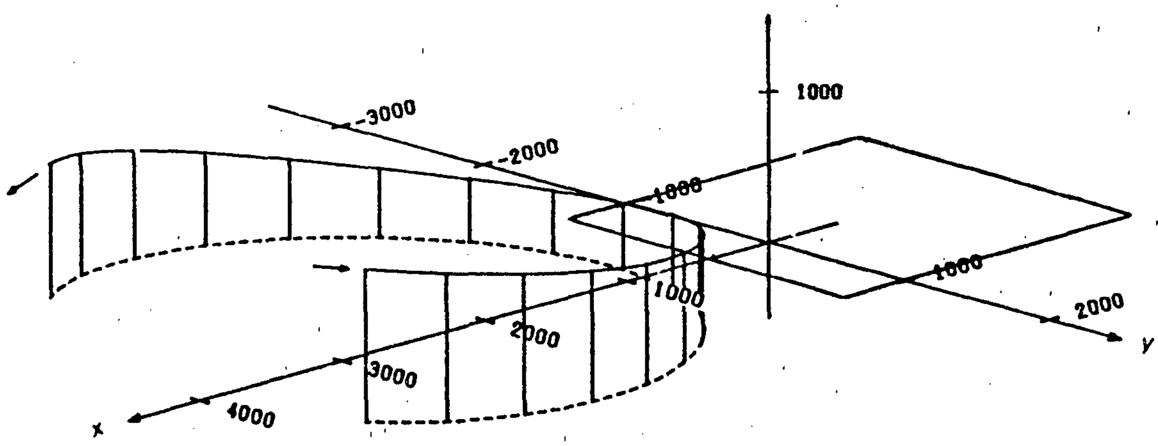


Figure 44-3. Constant 2-G Turn

44-5.1 THE SENSOR OR TARGET TRACKING RADAR

As described in Ref. 3, the manually controlled flir tracking sensor is simulated by adding an appropriate error to the actual target position at each time interval. The "errors" added are, of course, modeled as an appropriate random process in the sensor line-of-sight coordinate frame. For the GLAAD, typical errors used were:

1. The range standard deviation error σ_r was estimated to be

$$\sigma_r = 2 \text{ m} \tag{44-5}$$

2. The azimuth and elevation standard errors were determined from

$$\sigma_{az} = [(4/r^2) + 1]^{1/2}, \text{ mrad} \tag{44-6}$$

$$\sigma_{e\ell} = [(1/r^2) + 1]^{1/2}, \text{ mrad} \tag{44-7}$$

where

σ_{az} = standard error in azimuth
$\sigma_{e\ell}$ = standard error in elevation
r = range to target, m.

To produce such values, a normal distribution $\mathcal{N}(0,\sigma)$ with zero mean and the indicated σ was sampled to obtain "noisy" path data.

In processes of the kind modeled here, we are dealing with a time series, or autoregressive process, and a prediction from one time point to the next may be generated by a Markov process (since the new additional value depends on immediately preceding value). Thus, the range, azimuth, and elevation values for time $t + 1$ may be predicted from data at time t as follows:

$$r_{t+1} = 0.6r_t + 0.8e_r, \text{ m} \tag{44-8}$$

$$az_{t+1} = 0.9az_t + 0.44e_{az}, \text{ mil} \tag{44-9}$$

$$e\ell_{t+1} = 0.9e\ell_t + 0.44e_{e\ell}, \text{ mil} \tag{44-10}$$

where

r_{t+1} = range at $t + 1$, m
r_t = range at time t, m

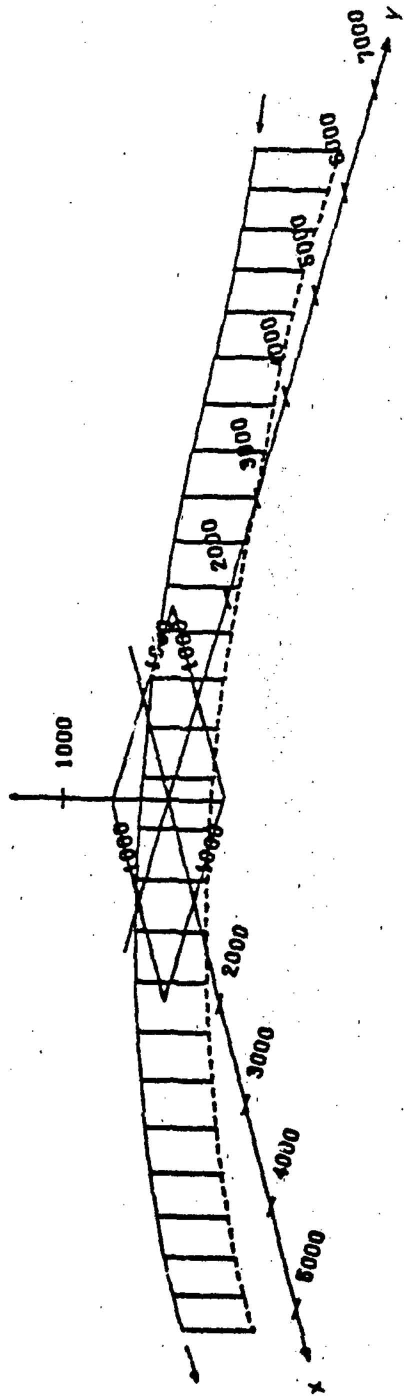


Figure 44-4. General Maneuver

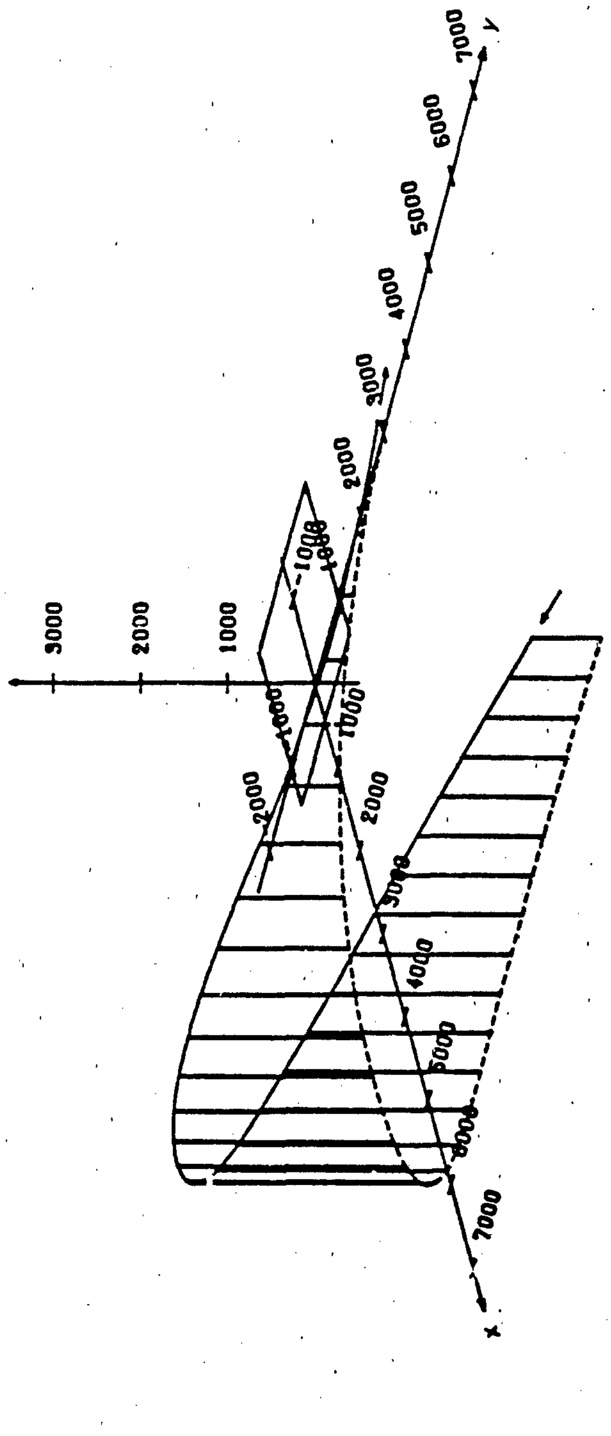


Figure 44-5. Pop-Up and Dive

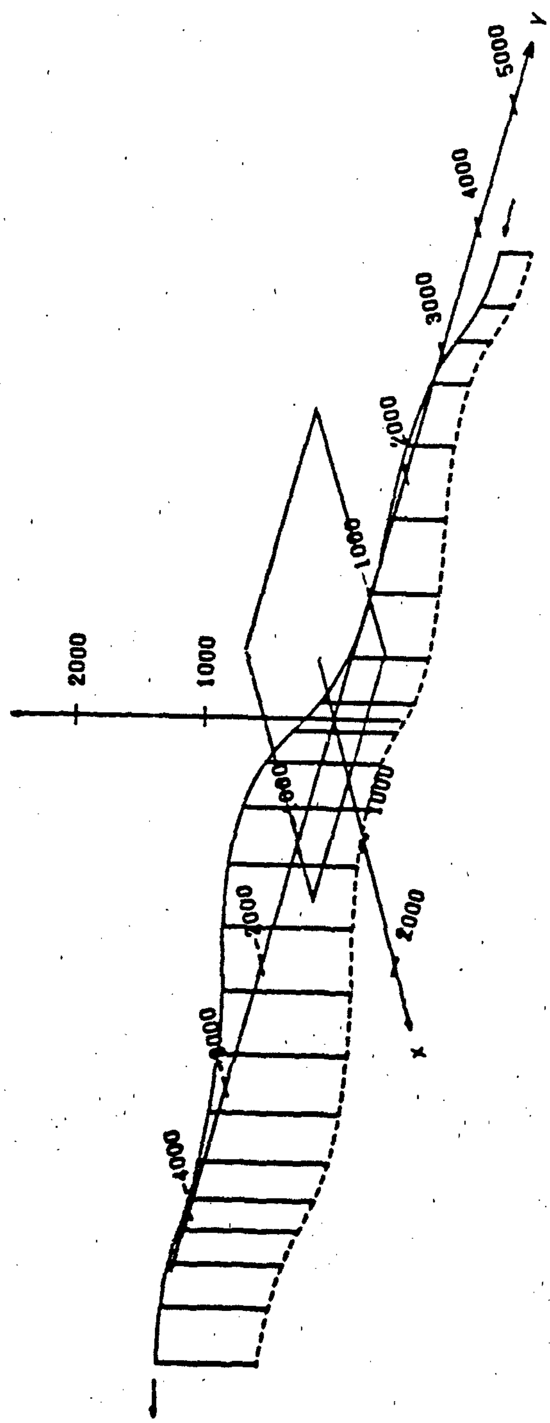


Figure 44-6. Jinking Flyby

e_r = random deviate from $N(0,\sigma_r)$, m
az_{t+1} = azimuth at time $t + 1$, mil
az_t = azimuth at time t, mil
e_{az} = random deviate from $N(0,\sigma_{az})$, mil
$e\ell_{t+1}$ = elevation at time $t + 1$, mil
$e\ell_t$ = elevation at time t, mil
e_e = random deviate from $N(0,\sigma_{e\ell})$, mil.

Suitable numerical coefficients were determined from an analysis of gun tracking data.

Target position data, determined thereby with "error", may be determined in cartesian coordinates, and the resulting values fed into the Kalman filter as input data.

44-5.2 THE FILTER

The heart of the MGEM simulation and perhaps the real advance in technology and evaluation of air defense guns center around the so-called Kalman filter. The purpose of the filter is to process the raw target tracking data, and supply present and future (extrapolated) target position, velocity, and acceleration estimates. For a brief background, and by way of comparison, one could take the raw position data acquired by the sensor in tracking an aerial target and employ a least-squares fit and prediction procedure for future target coordinates without the aid of any other assumptions or considerations. On the other hand, the Kalman filter or estimator is built around an algorithm that uses sensor data, the statistical properties of the sensor errors, the equations of expected target motion, and the statistical properties of present versus past errors to produce minimum variance estimates of target position, velocity, and acceleration data. The reader may study Refs. 3 through 7 to the extent desired for the mathematical background—considered to be beyond the scope of this chapter—and, furthermore, an extensive field of interest in its own right. Our point of emphasis is that the use of the Kalman filter has been found to give the best predictors of target flight data insofar as the proper modeling of the air defense aim error is concerned.

44-5.3 THE PREDICTOR

The purpose of the predictor is to determine just where to aim the gun to achieve projectile-target intercept one time-of-flight t_f in the future. In this connection, the ballistics of the projectile must be modeled while the drag coefficient of the projectile, the effects of winds, and gravitational decelerations—just to mention a few requirements—are being considered. For supersonic velocities of projectiles, which apply to the air defense gun projectiles considered here, the accurate time-of-flight information of the projectile may be determined from the equation

$$R = v_m t_f/(1 + \beta t_f) \qquad (44\text{-}11)$$

where

R = ballistic range of the projectile, m
v_m = muzzle velocity, m/s
t_f = projectile time-of-flight, s
β = a drag-related constant for the projectile considered, s^{-1}.

Since at intercept the projectile and the target are at equal range from the gun, aiming may be determined by comparing the predicted target range with that of the projectile in flight. Then, it can be seen that the time-of-flight for intercept conditions may be determined with the aid of target travel prediction equations and Eq. 44-11. Hence, intercept conditions are established; and once these are found, wind corrections and gravity drop are fed into the ballistic theory equations: and the aiming commands for the pointing of the gun are determined. These aiming commands are used to drive the gun servomechanism.

44-5.4 THE SERVOMECHANISM

Operation of the servomechanism is simulated by adding errors to the aiming commands; the process is very similar to that of treating the sensor operation. In fact, the errors for the servomechanism are modeled as Markovian noise also. For interested readers, the noise levels and the autocorrelation coefficients are given in Table 2.3 of Ref. 3. Included also in the servomechanism simulation are typical boresight errors.

44-5.5 PROJECTILE TRAJECTORY

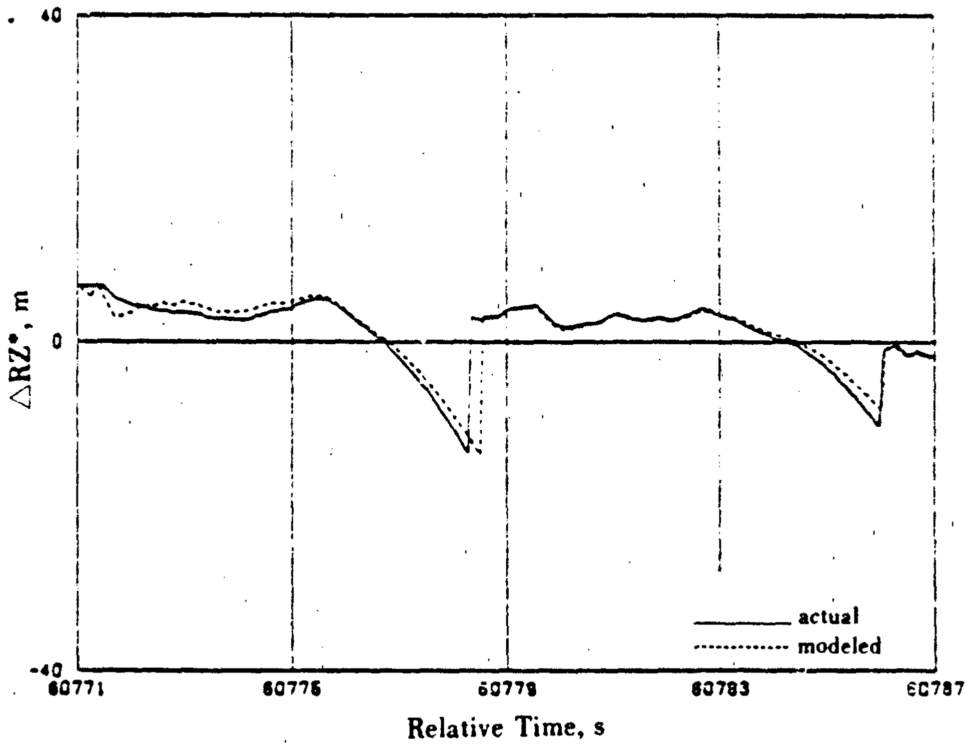
The difference between the computed trajectory used for obtaining the intercept solution and the actual projectile path is modeled by adding trajectory differential effects to the nominal straight line path. Thus muzzle velocity and drag variations are simulated, as are dispersion in azimuth and elevation deflections, wind measurement errors, and gravity drop.

44-5.6 TARGET DAMAGE

Finally, we come to the target damage problem. Computer simulations and calculations locate the point of closest approach of the projectile in flight to the target, and this is done by a search routine which compares miss distance at 0.05-s intervals along the target flight path. The relative position of the projectile and the target at closest approach is checked against the projected area of the target to determine whether a hit on the aircraft has been obtained. The projected target area is the presented area corrected for any roll, and thus the aircraft vulnerable area may be seen as being projected in a plane normal to the relative velocity vector of the projectile. For convenience, the projected area is assumed to be circular, although it has been determined that actual shape will not adversely affect hit probability chances as seen to some extent, for example, in Chapters 14 and 20, *Army Weapon Systems Analysis, Part One*, Handbook. Once a "hit" on a presented area is achieved, the conditional chance that a hit is a kill, or $p(k|h)$ as determined from Table 44-5, is used and matched against a randomly drawn number from the appropriate uniform probability distribution to either score or not score an aircraft "kill".

44-5.7 VALIDATION OF THE MGEM SIMULATION

Procedures for verifying and validating the GLAAD or MGEM simulations of air defense guns are discussed in much detail by Meredith, Scheder, and Lufkin (Ref. 3). In brief, field data were either gathered or were available to compare the results from MGEM simulations for the sensor, filter, predictor, and gun servomechanism. As some specific examples, Fig. 4.9 of Ref. 3 is reproduced here as Fig. 44-7 and illustrates a comparison of the actual (solid curve) and the modeled (MGEM) estimator of position output for the target range in meters. The dotted curve gives the GLAAD or MGEM prediction. Notice the rather close agreement. Fig. 44-8, reproduced from Fig. 4.12 of Ref. 3, gives a comparison of the actual and modeled estimator velocity output or target speed for the y-component, and Fig. 44-9 gives a comparison of the actual and modeled acceleration output. Similar comparisons are graphed in Ref. 3 for the predictor, gun aiming commands, and actual aiming points; and in some comparisons "confidence bands" from the Monte Carlo simulations are displayed to determine whether the miss distance, etc., lies within predictions. In summary, a sufficient number



*ΔRZ ≡ Difference between the actual and modeled values of the Z-component of range.

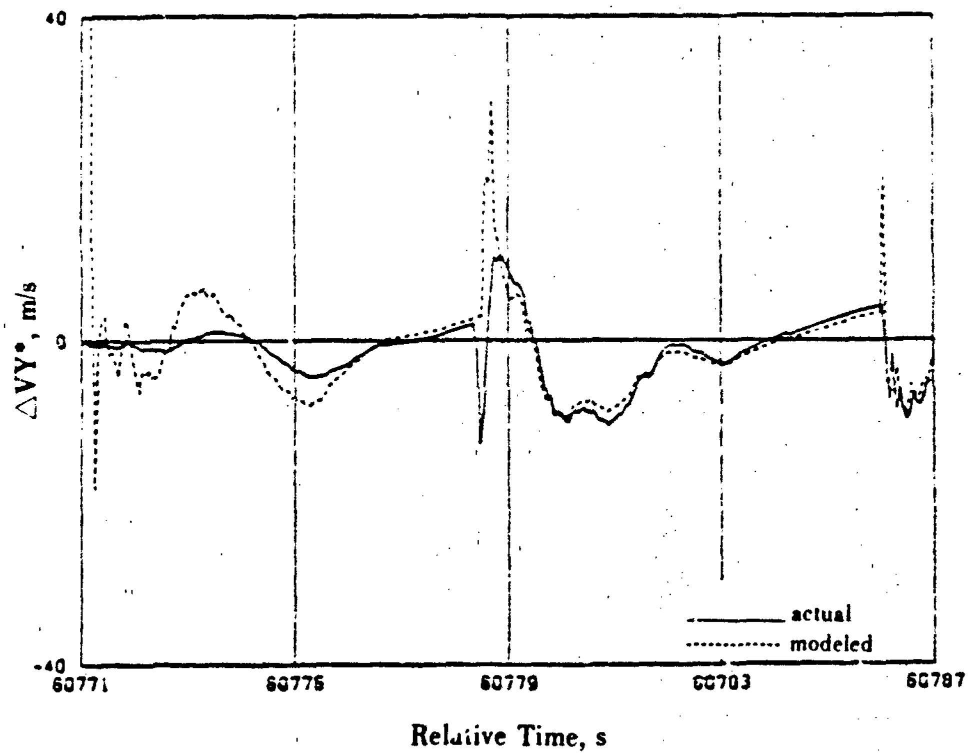
Figure 44-7. Comparison of Actual and Modeled Estimator Position Output

of comparisons have been made similar to Ref. 3, which give considerable confidence that the MGEM simulation represents not only a suitable, but also the best available computerized model to date for evaluating the overall effectiveness of air defense gun state of the art. With the establishment of this confidence, therefore, we now proceed to some example data on air defense gun effectiveness, or MOE's, which may be used to compare some different systems.

44-6 A SAMPLE MGEM COMPUTER RUN OF ENGAGEMENT KILL PROBABILITIES

In order to indicate a somewhat typical comparison of possible air defense gun systems having calibers from 30 to 40 mm, we have used the data of Tables 44-1 through 44-5, along with three flight paths, to determine hit and kill chances as an illustration. The three flight paths were for the straight flyby (Fig. 44-2) and the pop-up and dive (Fig. 44-5) courses for a typical enemy jet aircraft, and in addition, a target helicopter was considered at 2500 m. The MGEM computerized simulation was programmed and run under the direction of Mr. John Meredith of the Air Warfare Division, AMSAA, with the results given by caliber (30 mm, 35 mm, and 40 mm) in Table 44-6.

The reader should understand that the figures of Table 44-6 are for engagement hit and kill probabilities for the various numbers of rounds that can be fired during a target flyby from the three different calibers of air defense guns. In particular, many more rounds can be fired from the 30 mm



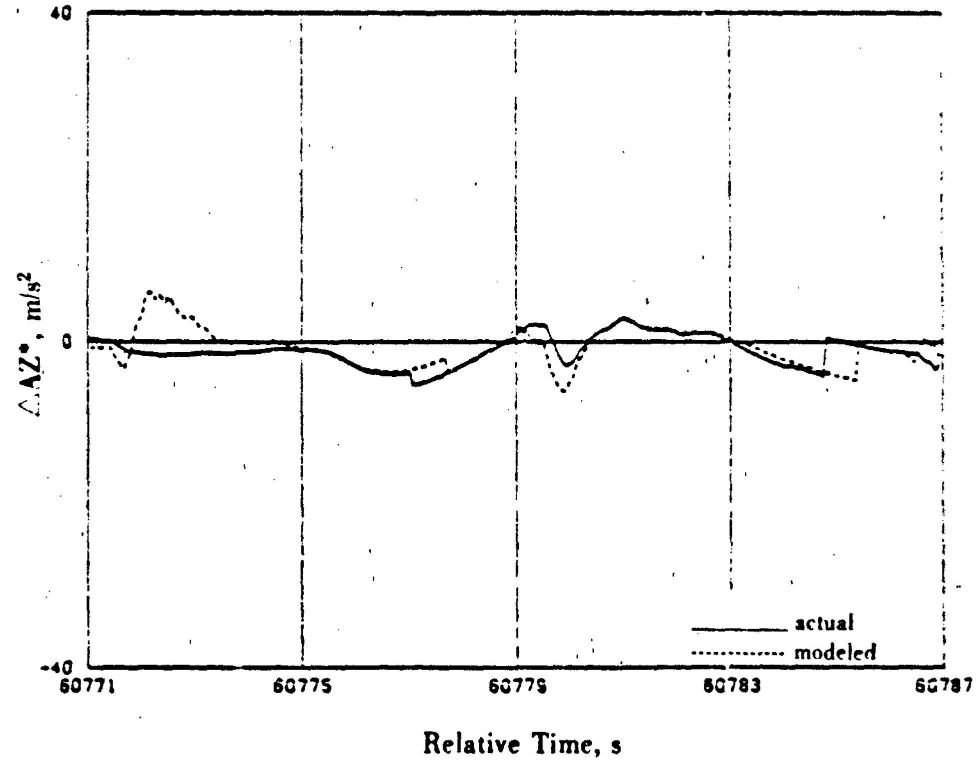
*. ΔVY = Difference between the actual and modeled values of the Y-component of velocity.

Figure 44-8. Comparison of Actual and Modeled Estimator Velocity Output

air defense gun than from the 35 mm, and the least from the 40-mm weapon. Note that the chances of at least one hit, and the chances of an "A" kill or a "K" kill decrease strikingly and uniformly with increasing caliber of weapon. Thus, the 30 mm has sufficient terminal effectiveness—or conditional chance that a hit is a kill—along with its higher rate of fire to possess higher kill chances than either the 35- or 40-mm weapons. By contrast, the number of rounds expended during the typical engagements by the 30-mm, 35-mm, and 40-mm weapons are given in Table 44-7. One notes in particular that the 30 mm air defense gun may get off as many as five times the number of rounds as the 40-mm weapon, and the 35-mm weapon about 1.5 times the number of rounds as the 40-mm weapon.

In connection with Table 44-6, we remark that during an engagement the aspect angle of the aerial target changes over the whole flight path, so that the approaching projectiles attack the enemy aircraft under changing impact conditions, and hence the MGEM simulation takes the proper vulnerable areas of Table 44-5 into account, which change with engagement time.

On the basis of simple hit and kill probabilities, we may easily see from Table 44-6 that the smaller caliber 30-mm weapon is the superior air defense gun, and it would be recommended on this basis since we would certainly desire to have in the field those weapons which possess the greatest kill chances against enemy aircraft. On a cost basis, however, using the individual round costs of Table 44-4, it is seen that the 30 mm may be more expensive. Thus to go further into real or expected costs, or a cost-effectiveness comparison, one would have to come to grips with the most desirable tables of



ΔAZ = Difference between the actual and modeled values of the Z-component of acceleration.

Figure 44-9. Comparison of Actual and Modeled Estimator Acceleration Output

TABLE 44-6. ENGAGEMENT HIT AND KILL PROBABILITIES

Flight Path \ Caliber, mm	Probability of at Least One Hit			Probability of "A" Kill			Probability of "K" Kill		
	30	35	40	30	35	40	30	35	40
Straight Flyby	0.87	0.80	0.62	0.68	0.55	0.48	0.57	0.44	0.38
Pop-Up and Dive	0.92	0.78	0.76	0.71	0.45	0.39	0.52	0.33	0.31
Helicopter at 2500 m	0.81	0.38	0.20	0.52	0.14	0.11	0.36	0.11	0.10

TABLE 44-7. AVERAGE NUMBER OF ROUNDS EXPENDED PER ENGAGEMENT

Flight Path \ Caliber, mm	Number of Rounds Expended		
	30	35	40
Straight	279	77	52
Pop-Up	280	92	62
Helicopter	218	50	38

organization and equipment for the air defense organization, as for example in a fielded division, and consider overall costs for a given level of effectiveness on one hand, or the costs to attain some desired or minimum level of effectiveness or protection on the other. We believe that this further direction of study is more or less expressly for the practicing analyst; consequently, we will not delve more into such details, having covered the matter of an air defense gun effectiveness simulation here.

As a final comment of some interest, we might note as an example that for an "A" kill the engagement kill probability against the pop-up and dive target course is 0.71, and the expected number of 30-mm rounds expended is 280. Hence the average kill probability per round is only 0.71/280 = 0.0025, i.e., a rather low figure and one indicating the difficulty of placing the C of I of the rounds on the target. Moreover, the expected number of engagements to achieve a kill is 1/0.71 = 1.41, and this converts to about 1.41 × 280 = 394 rounds, or a cost of about 8.80 × 394 = $3467. We see, therefore, that the expected cost to achieve a kill by the 30 mm air defense gun is perhaps unexpectedly high indeed. The expected cost for the 40-mm weapon at the lower "A"-kill probability of 0.39 is about $3497. This simple cost-effective analysis brings out the danger of making a determination without considering all the information—such a determination is made earlier in this paragraph.

44-7 ADDITIONAL COMMENTS AND CAUTIONARY NOTES

With reference to the air defense problem generally and the use of guns as defense weapons against enemy low-altitude type attacks, some might question the need for an "A" kill. This is because the air defense mission is to prevent the delivery of any "ordnance" on the target, and consequently the need primarily for a "K" kill or a mission abort.

The problem of "validation" of a model brings forth many, many questions. The MGEM model was "validated" primarily through the use of simulations on a computer; however, many may well argue that actual firing experiments or field trials are required to validate any military operations research model. It must be remembered, however, that actual firings to validate a model are not only very expensive or prohibitive, especially to cover all probable missions, flight paths, etc., but also are perhaps too dangerous to personnel conducting such tests. Nevertheless, it is agreed that certain firings, if they could be conducted properly in a field experiment, would indeed add much confidence to any model validation attempt.

An obvious criticism of the MGEM model is that it may not actually handle the terrain features problem very well, especially for target detection, identification, acquisition, and tracking capability. Realistic terrain quantification is thus an important problem to continue to study. Also the attacking aircraft may not be as successful if their target acquisition ranges are less than about 1.5 km. More study may be very desirable.

Some authorities might question the need or appropriateness of all or the particular flight paths covered in the MGEM type evaluation. For example, it would be very difficult to deliver ordnance from the 2-G constant turn or the jinking flyby maneuvers, so that some further consideration or study in detail is in order concerning such flight paths in combat situations.

Also it should be clear that the somewhat "rigid" weapon cooling-time requirements in Table 44-2 cannot be strictly enforced, and, for example, that cooling times more than three or four seconds would not be very feasible. Again, this type of problem may require more experience.

The problem of properly matching the caliber of the projectile, its fuzing, its HE content, and its fragmentation effect against future targets will require continuing research in the area of vulnerability effectiveness studies. Also, the ballistics of the round, including especially trajectory dispersion and loss of velocity, will play a crucial role. Moreover, this likely will lead to various revisions of the cost-estimation studies or will require continual updating for any valid inferences on cost-effectiveness.

Finally, in our rather limited, unclassified evaluation—which has been highlighted for illustrative purposes only—we found that the caliber 30 mm looked very good or the best, probably because of the numerous rounds that could be fired during an engagement. Nevertheless, the previously enumerated points of further consideration might lead to the requirement for large calibers, or even perhaps the 40-mm projectile sometime in the future.

Therefore, we see for the purposes of this chapter that a very extensive and completely satisfactory evaluation of the low altitude air defense problem cannot actually be covered, although the discussion presented should be adequate for an initial orientation of the systems analyst.

44-8 SUMMARY

We have described one of the more recent methodologies for evaluating the general effectiveness of air defense guns for the field army. The procedure covered is that of the MGEM which is a computerized simulation of the encounter between gun projectiles and typical or expected enemy aerial targets. The usefulness of the MGEM would seem to be of much interest and wide application since its accuracy has been validated through the study of many simulations.

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CHAPTER 45
ARMY COST AND OPERATIONAL EFFECTIVENESS ANALYSES (COEA's)*

This chapter gives clear, concise, comprehensive, and otherwise detailed accounts and procedures for performing a cost and operational effectiveness analysis (COEA). Indeed, the guidelines presented here should be followed by all individuals who prepare a COEA for staffing through Army activities and echelons.

45-1 INTRODUCTION

45-1.1 PURPOSE

Army cost and operational effectiveness analysis (COEA) efforts need to be more timely, less expensive, more pertinent, less ponderous, more comprehensible, less redundant, and more illuminating of relevant issues. This chapter is intended to help improve COEA's by explaining the fundamental ideas of a good COEA. The discussion that follows indicates the standards which should be used in preparation of COEA's and which will be used in judging them.

45-1.2 OBJECTIVES AND SCOPE

This discussion is for anyone who becomes involved with an Army COEA; for example, an action officer responsible for preparation of a study directive, a Study Advisory Group (SAG) member, a study director, a member of study team, or a user of the final product. It is not intended as a primer of technical methods or as a source of data for a COEA. It does not discuss matters such as agency responsibilities and procedures. These are covered in regulations such as AR 1000-1, AR 71-9, AR 5-5, TRADOC Reg 11-6, and DARCOM publications.

The chapter is also not a handbook of data sources useful in COEA's, and it does not aim to prescribe a format with which every COEA must comply in minute detail. Rather, it seeks to set the tone of thought that should be present in COEA's. It tries to articulate the spirit and philosophy of the COEA effort and tries to avoid the "by-the-numbers" approach.

The chapter aims to explain, in simple terms, each of the following:

1. What an Army COEA is.
2. Why an Army COEA is done — especially to include its use in the materiel acquisition process.
3. When an Army materiel COEA should be done, and how the emphasis should change with development phase.
4. What a COEA of an Army materiel system should contain.
5. How to do a COEA.
6. How to get a COEA done.

The exposition in this chapter is from the point of view of a COEA done in connection with hardware decisions. However, by changing a few lines, the ideas apply as directly to other major decisions

*This chapter has been prepared by Mr. David C. Hardison, Deputy Under Secretary of the Army for Operations Research. Generally, only the format of Mr. Hardison's original paper has been changed in spots to make it consistent with the writing of this *Army Weapon Systems Analysis* Handbook. We are indebted to Mr. Hardison for use of his material as a pertinent chapter of this Handbook.

which necessitate choices among alternatives that differ in resources required or potential to accomplish objectives. Examples are training programs, weapons mix, and force decisions.

The chapter is divided into several major paragraphs for special emphasis. Following this introduction, par. 45-2 answers the what, why, and when of COEA; and par. 45-3 tells what a COEA should contain and how to do it. The matter of how to get a COEA done is discussed in par. 45-4. Two appendices are attached. Appendix A gives the purpose, scope, and characteristics of analysis during the various phases of major system acquisition. Appendix B is a discussion of the types of models used in COEA's.

45-2 THE WHAT, WHY, AND WHEN OF ARMY COEA

45-2.1 WHAT AN ARMY COEA IS

An Army COEA is an analysis of the costs and operational effectiveness of each of a set of alternative courses of action to meet selected Army needs. In the materiel acquisition process, the COEA produces information regarding the estimated costs and operational effectiveness of alternative materiel systems and alternative programs for acquiring the materiel systems.

The basic framework of the Army COEA is that of a problem of choice. A COEA is thus a comparative or relative analysis. The problem takes a form wherein the decision maker must judge the merits of several alternative courses of action to meet perceived objectives and, ultimately, choose one from among them. Each of the alternatives is an entity, or system, which is comprised of people, materiel, and procedures. Each system has a set of attributes which enables it to achieve some relevant performance when used according to a concept of operations to accomplish the tasks pertinent to a given set of objectives. Each system also entails use of scarce resources or, put differently, has some costs. There are limits, or constraints, on the acceptability of alternatives, and there are threat factors which oppose attainment of objectives. There is an environmental context within which attainment of objectives must be sought. Evaluation of each alternative is made by identification, generation, and assessment of resource implications, i.e., measures of cost; and a set of indicators of the extent to which objectives are anticipated to be attainable, i.e., measures of effectiveness (MOE). A sense of importance is associated, at least implicitly, with each measure of cost and effectiveness. Criteria of choice are developed and applied to the measures of costs and effectiveness in order to determine whether the costs and effectiveness of one alternative are, on balance, more preferred than the costs and effectiveness of each of the other alternatives.

An Army COEA, therefore, should include several subanalyses: mission needs, deficiencies, and opportunities; enemy threats and other operational environments; constraints; operational concepts; specific functional objectives; system alternatives; system characteristics, performance, and effectiveness; costs; uncertainties; and the preferred alternative. It must provide measures of costs and measures of effectiveness. It should suggest useful criteria of choice. Good COEA's treat each of these elements explicitly and distinctly.

45-2.2 WHY COEA ARE DONE

The Army has many needs, of course. The resources — personnel, facilities, and funds — available to the Army are limited. It is important that each need be met and that each need be met efficiently. There usually are several alternative ways to meet a need. A COEA is done in order to produce understandable information that adds to the appreciation of the relative merits of the several alternatives by estimating their costs and operational effectiveness.

45-2.3. WHEN COEA ARE DONE

Overall policies for acquisition of Army materiel are provided mainly in DOD Directive 5000.1 and AR 1000-1. These policies establish thresholds and milestones at which management must make program decisions. These decisions require information about the projected costs and effectiveness of the system/program at issue both in absolute terms and in comparison with other ways to meet the objectives. As a consequence, a COEA is needed at each major milestone during the life of a program, i.e., each time the system/program enters a new "phase" of its life cycle.

The initial COEA of a system is a most important one. It should be done well, and when it is done well, subsequent COEA's should be limited to updating. Updating means to take account of changes in objectives, concepts, constraints, threats, environments, alternatives, costs, effectiveness, or criteria of choice that occurred since the previous COEA was completed. If significant change has not occurred, no new COEA is required. However, that determination should nevertheless be documented. It follows that before beginning a subsequent COEA, there should be an assessment of change since the last COEA. This would help to determine the scope of the update, and it might show that an update is not needed.

In the past, the initial COEA has not always been done well. There has been a tendency to do each successive COEA as if it were an initial one. This practice is neither efficient nor even effective. One important aim of this chapter is to encourage that the initial COEA be structured so that it can be easily updated. This is not only a worthy goal, but a necessary one so that the demands for analysis can be met within available personnel resources.

45-3 WHAT A COEA SHOULD CONTAIN AND HOW TO DO IT

45-3.1 PARTS OF THE ANALYSIS

The elements of a COEA are listed in par. 45-2. These elements, in slightly different form, now are discussed in more detail. The structure of a good COEA is emphasized in this paragraph.

45-3.2 ANALYSIS OF MISSION NEEDS, DEFICIENCIES, AND OPPORTUNITIES

The objectives for any system are derived from its mission. The mission of a system is to fulfill certain of the needs of the next higher level system. Therefore, a good COEA usually is begun by determination of the specific level of the system — e.g., tank gun ammunition, tank armament, tank, tank battalion, armored division — which is at issue. The next higher level system is then examined to determine the implications for the system at issue. It is sometimes appropriate to go beyond the next higher system. For example, one might wish to examine the needs of a theater force when comparing the merits of alternate concepts for an important materiel item. Usually however, an extension beyond the next higher system is not helpful.

There are several ways that mission needs can be examined. It often is helpful to do so in the context of a set of future wartime situations (scenarios). When this is done, the scenarios should:

1. Be derived from higher echelon scenarios.
2. Represent the spectrum of relevant situations encountered over the whole of the conflict.
3. Represent the spectrum of expected environmental conditions -- terrain, visibility, and weather.
4. Represent the qualities of enemy materiel and numbers of enemy forces and their tactical use.
5. Define the organizational and temporal context.

Other techniques, such as survey and historical analysis, are also useful. Whatever the technique of analysis, the aims are the same: identification of mission needs, definition of deficiencies of current . . .

systems in meeting those needs, and discovery of opportunity areas where efficiency may be improved.

45-3.3 ANALYSIS OF THREATS AND OTHER ENVIRONMENTS

The threat analysis determines the elements that our systems would be used against and the employment forces that would be used against our systems. It includes broad matters such as opposing forces and detailed matters such as the strength of ballistic attack or level of ECM/IRCM.* The threat is analyzed in detail in order to understand the set of conditions that might exist at the time of employment of the systems at issue. When analyzing the threat, one should:

1. Take full advantage of available intelligence and guidance, and get the intelligence community involved early in the study.
2. Examine opponent objectives as carefully as our own. The objectives of the two forces are different, and they may not be symmetrically opposite.
3. Explore implications of constraints on the threat. The laws of nature apply. The enemy has resource and other constraints that may limit his freedom of action, as do we.
4. Develop a realistic range of plausible threats since the opposition to future systems is always to some extent unknown. Postulate reasonable countermeasures or threat responses to our system(s) at issue. What would we do if we were in the enemy's situations? What might he do given his past performance, philosophy, operational concepts, etc.?
5. Be aware that gross overestimation of the qualities or quantities of opposing forces can lead to preference of inferior alternatives.

Hardly less important than the enemy threats are factors of the natural environment — terrain, weather, temperature, altitude, and visibility conditions — within which the systems would operate.

45-3.4 ANALYSIS OF CONSTRAINTS

Constraints are factors that limit the set of admissible alternatives. They should be studied carefully and stated explicitly. Progress sometimes comes from finding that a presumed constraint — personnel, funding, or technical — does not exist. Understanding the consequences of constraints is appropriate since some of them sometimes change or can be changed.

45-3.5 ANALYSIS OF THE OPERATIONAL CONCEPTS

As used herein, the term operational concept includes the full set of notions regarding the ways in which people and things would be arranged and employed. The concept includes operational doctrine and tactics. It is concerned with the matter of how the system is to be used to accomplish its objectives. It may include organizational issues. It forms a thought framework within which the systems undergoing evaluation are envisaged to operate capably and efficiently. In the absence of a sound and elaborated operational concept, a COEA usually flounders.

Inasmuch as a system is evaluated properly in the context of an operational concept, it is not surprising that one can find examples of poor quality COEA's that have done without clear statements of operational concept. One even can find cases where different views on the merits of a system can be shown to be more the result of differing assumptions regarding the purpose of the system and how it would be used, than the result of disagreements regarding costs and performance of alternative systems. A good COEA includes a good statement and analysis of the operational concepts.

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*ECM/IRCM = electronic countermeasures/infrared countermeasures

Since operational concepts can be changed, the overall set of alternatives considered often should include cases where employment concepts, rather than hardware systems or acquisition programs, are varied.

45-3.6 ANALYSIS OF SPECIFIC FUNCTIONAL OBJECTIVES

The analyses of mission needs and deficiencies, threats, constraints, and operational concepts produce information that enables one to understand the context of the system at issue. The next step is to express this understanding in terms of specific functional objectives for the system. Unfortunately, these specific functional objectives are referred to variously depending upon the type of system at issue. When analyzing a transportation system, one might describe movement requirements; when analyzing a firepower system, one might describe the acquired target list; when analyzing a communication system, one might describe the traffic demand schedule; etc. Each of these, and others, is used as a definite surrogate for a less definite and more complex set of operational objectives. Each is an expression of the tasks that must be done by the system at issue. In application, the functional objectives are treated as specific goals or standards, and the effectiveness of alternative systems is measured in terms of the extent to which they would be achieved.

Failure to establish specific functional objectives is a fault common to many poor quality COEA's. One reason (for such failures) is the variety of circumstances of potential use of the systems and the resulting difficulty of stating directly the essence of the wide spectrum of objectives. One consequence of failure to state functional objectives is the adoption of measures of effectiveness which are not clearly relevant. It is important to understand how the meeting of basic operational needs depends upon the level of performance of the systems at issue. In the end, the value of differences in system performance must be assessed to determine whether they are worth the differences in system costs. A key part of the logic therefore is an analysis to develop a clear and defendable statement of specific functional objectives. Without the results of such an analysis, there could be differing views on how effective a system would be in war and little understanding of why the views differ.

45-3.7 ANALYSIS OF SYSTEM ALTERNATIVES

Alternatives are the candidate courses of action or system solutions that offer the prospect of meeting the mission objectives. Discovery of the key alternatives is one of the most important tasks in doing a COEA since the remainder of the work at best can only identify which of the recognized alternatives is best.

Concerns of all major participants in the materiel systems acquisition process should be recognized in identification of alternatives. This should include concerns of persons associated with OSD, GAO, OMB (Office of Management and Budget), and Congress. Alternatives that might not be of interest within the Army might be of central concern to influential people in other organizations.

When identifying the set of alternatives, each of the following should be considered:

1. The current system
2. The current system product improved in one or more ways
3. Systems in development
4. Conceptual systems
5. Systems of other services (current, product-improved, developmental)
6. Foreign systems (current, product-improved, developmental).

An alternative should not be eliminated solely because it is in the concept stage and in competition with a "real" system. Rather, recognize that there are varying degrees of uncertainty associated with all systems depending upon their stage of development. The differing dates of operational availability can be one of the factors considered in the decision process. When generating the set of alternatives, check that:

1. Where possible, a reference alternative (or base case) is included; e.g., the current equipment in the current organization is used according to current tactical doctrine.
2. A range of alternatives is included (as opposed to numerous small variations on a single theme) covering variations in tactics, materiel, and organization as appropriate.
3. Each alternative is fully defined; including specification of materiel, organization, and tactics. Describe the operational concepts for using the system, and units within which it is embedded, to accomplish the defined objectives. Describe how the system/unit operates in conjunction with the other systems/units in accomplishing the objectives.
4. The set of alternatives is structured as an orderly sample which is systematically representative of the total set of feasible alternatives.

If there is a question as to whether an alternative should or should not be included, resolve in favor of including it. If it is a "bad" alternative, the subsequent analysis will show this to be the case, and the alternative can be dropped from further consideration at that time. On the other hand, if it has merit that was not immediately apparent, subsequent analysis will also demonstrate that fact.

Allow for development and consideration of new alternatives as the study proceeds. Frequently, new alternatives emerge as a result of the cost and effectiveness analyses and from direction provided by OSD and Congress.

45-3.8 ANALYSIS OF SYSTEM CHARACTERISTICS, PERFORMANCE, AND EFFECTIVENESS

An effective system is one which accomplishes its functional objectives. Measures of effectiveness are used to indicate the extent to which a system would meet its objectives. In practice, begin by defining what the system would be, i.e., its characteristics. The work to define system characteristics, such as weight, size, shape, color, and materiel, is useful in two ways: it forces one to describe unambiguously the specific system that is being evaluated, and it is the first step towards inference of system performance and cost. Unless what the system being evaluated actually would be is rather well known, it is impossible to estimate with high confidence what it would cost and what it would be capable of doing.

With a clear definition of the characteristics of a system in hand, it is proper next to determine system performance, i.e., to determine what the system would be capable of doing. Rate of fire, cross-country speed, range, number of channels, service time, lethal area, payload, detection range, and armor penetration capability are typical of the many useful measures of performance that apply to Army system 1 each tells something of importance about what a system would be capable of doing. A measure of performance does not indicate whether a system would be adequate; it merely indicates what it would be capable of doing.

Inference of performance must be based on data from a variety of sources: empirical relationships derived from tests of similar systems, data from tests of the systems at issue, and theoretical calculations, to name but three. Most COEA's must deal with situations where system performance data are not known precisely. Good operations research practice demands that the sources of the performance data be documented in a form conducive to review and assessment of credibility.

When system performance and functional objectives are understood, one can proceed to estimate the effectiveness of the alternative systems on the battlefield. As stated before, measures of effectiveness are used to indicate the extent to which the performance of a system enables it to meet the objectives of the requirement. There usually are several important objectives and several important measures of performance. Similarly, it rarely is possible to find a single measure of effectiveness which captures the richness of the information that should be considered in the process of choosing one of the alternative systems. Instead, it usually proves best to use several measures of effectiveness that are selected so as to be not overlapping but, taken altogether, reasonably inclusive.

The central challenge in any good effectiveness analysis is accurate estimation of the extent to which each system would meet the objectives. The variety of procedures that can be used to estimate effectiveness can be divided into two broad classes depending upon whether the procedure is mostly subjective or mostly objective; it is recognized that all approaches are in fact combinations of both objective and subjective parts. Procedures that are mostly objective, i e., those which attempt to express a reality apart from personal feelings, generally are preferred in COEA to procedures that are mostly subjective, i.e., those which express reality as perceived and conditioned by personal experiences.

The desire to treat reality objectively leads most analysts to the use of explicit models. Each model is a representation of selected relevant parts of the perceived real world. The models try to reflect the logical framework and functional interdependencies of the elements of the system at issue within the environment of the system. In support of COEA's, models are built to facilitate examination of the way the represented systems would behave. Models cannot represent all aspects of reality. They are designed to represent the inputs, processes, and outputs that are judged to be most essential to the purpose for which the model is developed. Like systems, models rarely are intrinsically "good" or "poor"; they are "good for . . ." or "poor for . . .". (An argument can, of course, be made that models which are founded on violations of physical laws are good for nothing and bad absolutely because they are dangerously misleading. Apart from this qualification, the linking of quality to use seems appropriate.)

An analyst can use an available model or build and use a new model to estimate system effectiveness. But this is possible only after mission needs, operational concepts, system functional objectives, and system performance are established.

It is good operations research practice to exercise each model and produce value estimates for the various measures of effectiveness. The model-checking results should be clearly understood both in regard to *what* they are and *why* they occur. The logic must track from system characteristics to system performance to attainment of functional objectives to accomplishment of mission needs. This model-checking process involves detailed examination of data well beyond the measures of effectiveness. It is tedious work, but the effort is well spent and, minimally, tends to:

1. Check the operation and credibility of the model.
2. Develop an understanding necessary to explain and interpret the results.
3. Provide insights into the utility of the proposed measures of effectiveness, and possibly suggest new and better ones.
4. Lead to ideas and methods of displaying the results in condensed, comprehensible, and meaningful form.

There is no point in going beyond the reference system alternative until one is comfortable that the model accurately depicts the relevant behavior of it. As a rule, systems other than the reference alternative are less well understood than it is. It is rarely possible to be confident that other systems are properly assessed when the reference system seems not to be. When the model(s) is (are) accepted as operating properly, it (they) can be used as a tool to aid in the assessment of other system alternatives.

The measures of effectiveness for all alternatives and the results of sensitivity analyses should be collected, condensed, displayed graphically, and studied to discover patterns that permit generalizations about the class of systems and environments studied. This can permit extrapolation and interpolation to other alternatives and situations and may allow creation of simple analytic expressions relating characteristics, performance, environment, and measures of effectiveness. It also is a thought process which assists the drawing of conclusions from the effectiveness analysis.

It would be wrong to think that a COEA should use just one model or that every model should be a large-scale computer-played force-on-force battle simulation. These large computer-played battle simulation models certainly are powerful tools that have a place in the total set of tools available to the analyst. But many COEA issues can be better approached by using a more straightforward mathematical model or other techniques. Similarly, some issues of importance are quite difficult to analyze in the large-scale simulations. The analyst must concentrate on system missions, objectives, characteristics, and performance when determining what kinds of modeling tools are most appropriate to infer system effectiveness. Sometimes a simulation model is called for, but often it is not.

45-3.9 ANALYSIS OF COSTS

The objective of the cost analysis is to determine the resource implications of choice of each of the alternative systems/programs. Army cost analyses usually seek to determine the costs of acquiring, operating, and maintaining a quantity of each system during a presumed period of peace. The quantity of systems presumed to be needed usually reflects projections of peacetime structure and wartime usage. At least part of the cost analysis effort must be done after the effectiveness analysis results are in hand. For example, the wartime attrition and expenditure rates estimated in the effectiveness analysis often are used to develop the quantities of the system to be acquired and maintained during the period of peace that is presumed to precede use of the system during war.

Typically, there are several kinds of resource implications that should be analyzed in a COEA. Some think that resources should be thought of basically in monetary terms. In this view, the nonmonetary aspects of other scarce resources — such as numbers of personnel, electromagnetic bandwidth, nuclear materials, strategic lift assets, and energy resources — should also be considered and treated as constraints. Others think that cost, like effectiveness, is treated best as a matter having several important partially interdependent dimensions. Neither view is universally better. A key task in the cost analysis is the determination of the kinds of costs that are most relevant to the issues for decision.

Monetary costs normally are shown according to the categories of R&D, investment, and operations and support. Subcategories provide additional details. All costs that are predicted to occur during the life of a system are sometimes summed to obtain an overall estimate called "life cycle cost". This practice has some value, but it should not be followed to the exclusion of exhibition of the component cost. The objections to the provision of only a single roll-up life cycle cost are several: operating costs often are poorly known but large compared to acquisition costs, operating life often is poorly known and selected somewhat arbitrarily, and future costs are treated as summable with current costs. Current DA regulations indicate that future costs should be discounted. Most Army COEA's have not discounted future costs. Arguments continue over whether discounting is appropriate and what discount rates should be applied. An often miscoupled, but basically unrelated, matter of projected future inflation rates adds confusion to the matter. The study leader and cost analyst should reach explicit agreement with the SAG on this somewhat messy matter.

One further matter regarding monetary costs is mentioned. It is somewhat difficult to determine the extent to which an indirect cost should be charged to a system. The term "systems costs" has been used almost as a synonym for direct cost, and the term "systems slice" or "force cost" almost as a synonym for the total direct and indirect costs of having the system in the force. Which approach is preferred depends on how dominant a part of the force's effectiveness is provided by the system in question. In any case, cost and effectiveness should be of the same system or slice of force. For example, one should use the costs of a fuzed projectile when he has done an effectiveness analysis of fuzed projectiles even though fuzes are the only system at issue. Similarly, he should consider the cost of armed helicopters rather than just missiles, even when only the missile is at issue if the effectiveness model analyzes armed helicopters rather than missiles.

Analysis of personnel costs is quite important in most Army COEA's. Key factors that influence personnel costs are the logistic support policy and estimates of system failure rates and required availability rates. Such analyses require much more than a simple counting of military structure spaces.

Estimates of the costs of materiel systems are obtained using approaches that fall into two broad categories. The industrial engineering cost estimating approach ("bottoms up") is based on a detailed work breakdown structure and, in effect, rolls together a large number of fine-grain estimates of the cost of materiel, labor, and capital to acquire and operate the system. The parametric cost estimating approach makes use of less detailed cost estimating relationships that have been developed empirically as a result of experience with earlier systems. Neither approach is always better. The parametric approach poorly reflects changes in technology, and the industrial engineering approach fails to include the cost of work not recognized but ultimately required. It is reasonable to presume that parametric approaches are useful primarily in the early stages of the life cycle of a system and that the industrial engineering approaches become increasingly more accurate as information is gained during the later stages. In any case, Army policies require the use of both cost estimating approaches. The technical methods for application of the techniques are provided in many publications; the most recent and authoritative ones are DA Pamphlets 11-2, 11-3, 11-4, and 11-5 prepared by the Office, Comptroller of the Army.

Some COEA's have been of poor quality because of shortcomings in the cost analysis. Problems have included cases where the systems being costed were not well defined, cases where the costing logic was flawed, and cases where the prices were poorly estimated. The cost analyst should exercise the same discipline of checks for accuracy of inputs, consistency of logic, and appropriateness of information that the effectiveness analyst should use. The cost estimates should be validated by the cost estimating/cost data centers at the DARCOM commands. The cost estimates should be examined carefully to understand the relationship of both system and force structure costs to system design, system performance, and system effectiveness. The sensitivity of the cost estimates to system design, performance, and mission effectiveness must be comprehended so that we can have field systems that are effective and efficient.

45-3.10 ANALYSIS OF UNCERTAINTIES

COEA's deal with important decisions relative to future courses of action and are replete with uncertainty. Uncertainty is associated with each of the factors discussed in this chapter: operational concepts, environments, mission needs, functional objectives, threats (including countermeasures to the alternatives), system characteristics, system performance, and costs. Additionally, both the effectiveness models and the cost models are only partial representations of reality.

The analyst should identify the main areas of uncertainty and estimate the extent of uncertainty. The implications of the uncertainties should be examined by sensitivity testing using the cost model(s) and effectiveness model(s). Cost uncertainty should be examined from the standpoints of uncertainty in cost estimating methods, and uncertainty in system performance and deployment requirements. A most important part of a sensitivity analysis is to establish the range within which a system can perform and still be an "attractive" solution. The uncertainties which most affect the analysis should be highlighted.

The extent to which sensitivity analyses can be done depends mainly upon the availability of analysis techniques, including cost models, that can be exercised rapidly and repeatedly. This argues strongly against highly elaborate models that are so cumbersome that, in practice, cases of interest cannot be examined. Results of the sensitivity analyses should be displayed along with the results of the main effectiveness analysis and cost analysis.

45-3.11 ANALYSIS OF THE PREFERRED ALTERNATIVE

The COEA is not intended predominately to decide which alternative is preferred. It is supposed to generate information that will assist decision authorities in making their decisions.

Each COEA addresses complex issues. Each attempts to illuminate the issues by showing several kinds of costs and several effectiveness indicators for each of the alternative systems/programs. Even when each alternative has been given the most careful and rigorous study to establish costs and effectiveness, evaluation of the merits of the alternatives can rarely be accomplished mechanistically.

There is no magic or universal formula which can be used to combine the several cost and effectiveness measures to identify the most preferred alternative. The dimension of value, or worth, must be added. This judgment of value is found in human attitudes and perceptions of the relative importance of competing needs at higher levels. Therefore, the COEA should not try to make the decision, but it should present the information in such a way as to permit easy comprehension. In this regard, experience indicates it usually helps to do the following:

1. Depict the absolute values of all of the measures as compactly as possible. Make all of the facts available and visible. Show all the measures of cost and all the measures of effectiveness for each alternative.
2. Avoid unrealistic schemes in which several measures are weighted and combined into an overall score. Decision makers are able to consider several measures in their judgmental assessments of overall worth, but they cannot decompose an aggregate score. Sometimes weighting schemes are valid and helpful, but even then it is best that they be explicitly portrayed so that their implications are fully recognized.
3. Be cautious in constructing ratios. They tend to address only some of the measures, ignore questions of sufficiency, and they can hide important differences in absolute results. Relative worth, defined as the ratio of relative effectiveness to relative cost, is an example that is easily misused.
4. Point out any dominance relations.
5. Identify any alternatives which have proved to be false alternatives due to lack of technical feasibility, economic feasibility, or violation of policy.
6. Identify any alternatives that do not meet any well established criterion of sufficiency or adequacy.
7. Identify the more effective alternatives which are indifferent in costs, and the less costly alternatives that are indifferent in effectiveness.

8. For alternatives which have no readily apparent differences, identify those which are weaker at the more important and/or more frequent objectives and alternatives which, relative to others, have risks without compensation.

9. Highlight the factors which tend to rank-order the remaining alternatives. Sensitivity to key variables, robustness to counteraction, preservation of the flexibility for resolution by later decision, contribution to longer term goals, and deferral of resource requirements are examples of factors that may be of interest in discerning which alternative is best.

10. Reexamine the status quo alternative in the light of the new insights. It may well be better than was realized, or such a bad choice as to make otherwise unattractive alternatives quite appealing. When no other alternative is clearly superior, the status quo alternative probably should be chosen. In other words, the current system probably should win the "ties" because of less uncertainty as to what it is, can do, and costs.

If the conclusions and recommendations include identification of a preferred alternative, the criterion of choice must be explicit. Good operations research practice suggests several criteria of choice and association with each of the alternatives showing most promise.

45-3.12 THE DRY RUN

At the outset of a COEA, one should work through all of the previous steps using such data as are available. This dry run exercise should be done, if possible, when preparing the study plan. In any event, it should be done during the first half of the study. This practice insures that the logic tracks from beginning to end and that the work that must be done during the study is reasonably well understood. The subsequent study probably won't work out exactly as planned, but the analyst will be better prepared to cope with the unexpected things that occur.

45-3.13 SIMPLIFY THE MODELS

Experience indicates that the time spent making models more elaborate doubly subtracts from the time available to do the substantive work of the COEA. Conversely, the time spent simplifying existing models — paradoxically — adds to the time available for thought. The analyst who becomes fascinated by the shovel and digs not the ditch deserves to be inundated by the flood which surely will follow.

45-3.14 THE COEA REPORT

COEA reports should be as brief as possible and written as plainly as possible. They are not a proper vehicle for publication of technical methods of interest mainly to the analyst. The style of exposition can vary widely and still be effective. Whatever the style, the report normally should include the ten subanalyses discussed in this chapter. And in the absence of good analysis, the report will be useless regardless of editorial appeal.

45-4 HOW TO GET A COEA DONE

45-4.1 PLANNING FOR A COEA

The responsible action officer should start thinking about the COEA update in sufficient time to support the next decision milestone. He should see that the following activities are accomplished:

1. Identification of criticial issues, particularly issues that were raised and unresolved in earlier parts of the decision-making process.
2. Identification of alternatives raised subsequent to the most recent program decisions.

3. Roughing out what will have to be done in the next COEA.

4. Working out when to begin formal planning and execution in order to finish the COEA so it will be responsive to the next decision milestone.

5. Identification of data needed, especially from testing, and initiate actions to obtain the data. The testers should be involved early so that they know test requirements.

The action officer should continuously monitor critical variables to which the previous studies and decisions were sensitive. A significant change (i.e., one that might bring into serious question the previous decisions) should trigger an immediate reaction to update the COEA.

45-4.2 THE STUDY DIRECTIVE

The study directive should be initiated at a time dictated by the schedule discussed in par. 45-4.1. The study sponsor (in coordination with the study agency) should prepare and distribute an initial draft directive to interested parties and ask, "What do you consider to be the issues regarding system X?". Based on the responses, the study sponsor should issue a draft directive to the study agency. The study agency should prepare an outline study plan and a proposed directive. The outline study plan should indicate how the study agency proposes to do the work, and it should show the extent to which the issues and alternatives are planned to be treated in the COEA. A prestudy planning session of major HQ DA/MACOM* participants (general officers and civilian equivalents) should then be held to finalize issues and alternatives, and to agree on the outline plan of study. The study sponsor should then issue a study directive and work should proceed. This procedure involves the study doer in the process of formulating the directive and provides the best chance that the study will be responsive to the needs as perceived by higher headquarters.

The approach described in the previous paragraph is not intended as prescriptive of the procedure which must be followed but, instead, as descriptive of a procedure which when followed well is successful. Simpler procedures for preparation and issuance of the directive are appropriate to many cases. In any event, the prime focus should be on the content of the directive rather than the paperwork flow pattern.

45-4.3 THE STUDY PLAN

The study plan should parallel steps discussed in par. 45-2 - - tell what will be done in each step and how and when it will be done. Involve testers early so that they will know test requirements and will be able to generate data in the form needed.

Check out the plan by conducting a dry run analysis as described in par. 45-3.

45-4.4 THE SCHEDULE

COEA's must be scheduled so that their results will be useful in the decision made as part of the ASARC/DSARC* process and the planning, programming, and budgeting cycle. Initial planning usually provides for this, but program revisions often create problems of asynchronism. For example, COEA analyses are sometimes desired to be completed in step with, or even before, provision of data acquired during DT/OT† tests. In such cases, "work arounds" are necessary. Analyses can be done based on forecast performance and validated when test data are available. Interim test data can be

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*MACOM = major command
ASARC = Army Systems Acquisition Review Council
DSARC = Defense Systems Acquisition Review Council

†DT = Development Testing
OT = Operational Testing

provided even while tests are in progress. Partial analyses can be provided pending availability of completed work. There is a clear need for a continuing effort to provide COEA results when they are useful. There is no pat solution that will apply to all situations but in most cases a way can be found.

It is good practice to plan to complete all of the work in no more than about half of the remaining time. Whatever the constraints of the time available to do the COEA, it is also good practice to:

1. Plan for thoughtful interpretation of the results of the effectiveness analysis and cost analysis. An arrangement that contemplates continued computer runs up to very near the study deadline invites disaster. The results will not be understood, effectively explained, or accepted.
2. Plan for introduction of new alternatives that emerge from the work done in the early portions of the study.
3. Plan for adequate analysis of the alternatives, issues, and "What if's?" that almost inevitably will arise during staffing of the draft report.

The demanding and sometimes conflicting schedule requirements described in this paragraph can be accommodated mainly to the extent that "coarse grain" methods of analysis are used.

45-4.5 GUIDING A COEA

A most important act in guiding a COEA is the preparation of a study directive that will lead to a sensible study plan. A second is the provision of adequate resources and access to information to do the study. A third is refraining from the provision of excess and contradictory guidance to the study team. Each act sounds easy but does not occur always.

Most COEA studies are provided a SAG. The SAG is intended to do the following:

1. Provide advice to the study team.
2. Provide assistance to the study team primarily with regard to threat information, technical methods, input data, and costing. Subcommittees are helpful in this regard.
3. Keep the Army participants in the acquisition process informed on study progress and results. Observers from outside the Army, e.g., OSD, should be invited when the issue is of direct interest to them.

A SAG is successful to the extent that:

1. The SAG Chairman is a person of high ability who can stay on the job throughout the study and who runs orderly meetings.
2. The SAG members are knowledgeable about COEA and their areas of interest, and stay on the job throughout the study.
3. The needed advice and assistance are provided early in the study when they are most useful. (The SAG should have finished well over two-thirds of its work when the study is one-third done.)

When these conditions for success are present, the SAG helps more than it hurts. When these conditions are not mostly met, the SAG fails and must be changed so that favorable conditions are present.

45-4.6 REVIEWING A COEA

The following questions can serve as a checklist when reviewing the quality of a COEA:

1. Is the problem stated the real problem?
2. Is the context (environment, scenario, and threat) representative?
3. Are assumptions stated and reasonable?
4. Are the constraints unduly restrictive? Are they really constraints?
5. Are there threats rather than a threat?

6. Are any feasible and significant alternatives omitted?
7. Are the stated "facts" correct? Are the sources given?
8. Are there measures of effectiveness rather than a measure of effectiveness?
9. Are the measures of effectiveness appropriate to the mission objectives?
10. Are there costs rather than a cost?
11. Are all the relevant costs considered?
12. Are the models (costs and effectiveness) adequately identified and explained?
13. Are the models (costs and effectiveness) appropriate for estimating values of the measures?
14. Are there sensitivity analyses?
15. Have the critical variables been identified?
16. Is there a presentation of all the costs and effectiveness measures for all the alternatives?
17. Are the criteria for suggesting the order of preference of the alternatives meaningful? Are they consistent with higher echelon objectives?
18. Has the COEA taught anything not already known?
19. Are the conclusions and recommendations intuitively satisfying? If not, are they convincingly substantiated?
20. Is the study adequately documented to include key input data?

If the answers are yes, you have just reviewed a COEA that probably is good. If some are no, you probably have just reviewed a COEA which was prepared by persons who have not adopted the practices encouraged in the several paragraphs of this chapter.

CHAPTER 46
COST AND OPERATIONAL EFFECTIVENESS ANALYSIS OF THE WICV-WOW ARMORED INFANTRY FIGHTING VEHICLE

A cost and operational effectiveness analysis (COEA) of an armored infantry fighting vehicle, the WICV-WOW, is discussed in this chapter. The evaluation is carried out on the basis that such a vehicle should have the capability not only to carry mounted infantry, but it should also possess the capability to kill enemy tanks occasionally, conduct some suppressive fire, permit riflemen to fire from ports while riding in it, and perhaps even change the tide of some close combat engagements. Indeed, the WICV-WOW type vehicle should be able to operate effectively in both the mechanized infantry role and also the armored cavalry role, often conducting scout type activities as well. Although parts of the operational effectiveness issues have to be judged on a subjective basis covered herein, many of the important combat capabilities may be evaluated on a quantifiable basis, and the costs and operational effectiveness parameters may be integrated into an overall measure of effectiveness (MOE).

Our example centers around a COEA study of parameters for the WICV-WOW armored infantry fighting vehicle, i.e., a hypothesized member of a combined arms team. The material presented here is more or less an executive summary.

46-0 LIST OF SYMBOLS

FER = fractional exchange ratio (See Eq. 46-2)
= (fraction of Red equipment losses)/(fraction of Blue equipment losses)
LER = loss exchange ratio (See Eq. 46-1)
= (number of Red equipment losses)/(number of Blue equipment losses)
RC = relative cost normalized on the base case
RE = relative effectiveness normalized on the base case
RW = RE/RC = relative worth

46-1 INTRODUCTION

Whereas the major portion of this handbook covers the principles of effectiveness analyses, or the evaluations of weapons or weapon systems, Chapters 34, 35, and 36 discuss the estimation and analysis of costs of systems, and Chapter 37 introduces the reader to cost-effectiveness studies. The present chapter is devoted to the discussion of a cost and operational effectiveness analysis (COEA) type of study. Our prime objective in a COEA is to try to integrate the system costs, its "operational" capabilities, and its effectiveness on the battlefield into a single or overall type of study. Thus, the systems analyst in a COEA must bring together in a single package all of the important analytical aspects of any system being evaluated and present these in a very systematic manner for the decision maker. Chapter 45 gives a discussion and the recommended procedure of how COEA's should be performed and how they fit into the Army weapon programs. Therefore, in this chapter we will attempt to illustrate a COEA. Unfortunately, at the time of preparation of this handbook, there did not seem to exist a good, unclassified example of a typical COEA or one that was fully staffed and approved by the Department of the Army. Therefore, we had to take the liberty to bring together those remnants of some of the aspects of COEA's which would at least illustrate the procedure for a very hypothetical case. In addition, however, we also take the time to criticize our example insofar as the state of the art is concerned. Hopefully, therefore, the material presented in this chapter wi·· at least give the young analyst a suitable introduction to the problem of COEA's since it currently appears that the more or less "standard" evaluations of future weapon systems will center around the general principles of cost and operational effectiveness analyses. We are a long way from attaining solid analytical descriptions of all of the "operational" characteristics of weapon systems in the field under combat conditions, although we can, for example, estimate system reliability and effectiveness measures such as kill probabilities fairly well indeed. In addition, the problem of integrating system costs, effectiveness, and operational capabilities may not be straightforward.

With this preliminary introduction, we now turn to a brief, broad historical description of just what might typically lead up to the present type of COEA. We draw upon Ref. 1.

46-2 PERTINENT HISTORICAL BACKGROUND

Prior to about 1964, the Army required a protected—but not an *armored fighting*—vehicle to transport infantry personnel on the battlefield. The M113 series of armored personnel carriers (APC) was developed over the years to meet this requirement of mechanized infantry doctrine, and studies conducted between about 1964 and 1972 concluded that a "weaponized" infantry combat vehicle (WICV) with a stabilized automatic cannon of about 27 mm would be required to provide the "lightly armored" fighting vehicle desired for the 1982 time frame. A concurrent, but separate, operations research type study conducted during 1968 resulted in a qualitative materiel requirement (QMR) for an armored reconnaissance scout vehicle to replace the M113 as the Army's scout vehicle. In fact, a study subsequent to 1968 and some prototype testing of possible candidate vehicles led to the conclusion that the current scout platoon organization lacked both sufficient antitank and long-range suppressive fire capabilities, even when equipped with existing prototype vehicles. This led to Department of the Army approval of a conceptual armored cavalry platoon consisting of two "WICV-WOW" (weaponized infantry combat vehicles with weapon-launched-optically tracked-wire command link antitank guided missile) vehicles, three scout vehicles with cannon, and four chief battle tanks (CBT). WOW antitank capability was covered in Chapter 42.

Finally, a task force review at the Department of the Army level of the entire infantry combat vehicle and scout vehicle programs directed that:

1. Former materiel need/engineering development (MN/ED) documents be revised to provide for a two-man weapons station mounting both the 27-mm and the WOW weapons in all WICV's.

2. The WICV, with appropriate interior modifications if necessary, be capable of operating in both the mechanized infantry and the armored cavalry roles. In this connection, a single study effort—COEA—was decided upon to evaluate both the WICV and armored scout vehicle concepts. In fact, this COEA was conducted to select the preferred alternative for future WICV-type systems program reviews and is the one covered in this chapter.

The primary objectives of this COEA are:

1. To quantify all probable candidate alternative vehicles in terms of their operational effectiveness, cost, and overall integration of cost and operational effectiveness

2. Recommend if possible a preferred alternative vehicle based on the integrated COEA rankings, which may be coupled with and tempered by expert military judgment

3. Especially demonstrate effective antitank capability.

46-3 SKETCH OF THE ORGANIZATIONAL ROLES AND FUNCTIONS AS THEY AFFECT THE SCOPE OF THE WICV COEA

In performing the WICV-WOW COEA, we must keep in mind the two rather distinct combat roles the Army has in mind, i.e., the mechanized infantry role and the armored cavalry role.

Concerning the mechanized infantry role, the WICV-WOW would engage in very heavy fighting and would be expected to perform as an antitank weapon system when needed. Thus, the WICV-WOW would be expected to perform well in both defensive and offensive operations in Europe, and its prime role in other areas of the world might be that of proving its value in the offensive role.

In the armored cavalry role, the WICV-WOW would not engage in heavy fighting, but rather it would be depended upon to conduct reconnaissance operations, screen our forces, scout for any information of value, and hence protect our primary forces, especially perhaps by precluding surprise attacks by the enemy. Thus, in the armored cavalry role, the WICV-WOW would aid greatly in covering force operations, be effective in screening, and provide very valuable service in movement-to-contact operations. (As of the present time, it is rather widely recognized that the armored cavalry role would be difficult to evaluate analytically; accordingly, a map exercise or manual type war game might well be necessary).

Finally, in both roles, the suppressive fire of the WICV-WOW might be very effective since automatic cannon can be used efficiently here. Moreover, it should be kept in mind that the WICV-WOW type weapon system would no doubt be quite helpful in increasing somewhat the survivability of our CBT.

It is of some interest to discuss briefly approximate organizational aspects and tentative concepts of organization and equipment brought on by deployment of the WICV-WOW. These concepts and ideas, as could best be judged at the time, are discussed in the paragraphs that follow.

In the mechanized infantry role, the concept arrived at was one WICV per rifle squad, but for the platoon there would be an extra WICV in addition to one each for the three rifle squads. At the rifle company level, the three platoons would then have a total of 16 WICV's including four additional WICV's equipped with 81-mm mortars. At the battalion level there would be 48 plus 18, or 66 WICV's, including five with mortars. The mechanized infantry battalion would consist of about 850 men.

For the armored division (which is not evaluated in this COEA), it was projected that a tank platoon would be equipped with five tanks, and the tank company would be equipped with $3 \times 5 + 2$, or 17 tanks. At the tank battalion level, the concept was to have $17 \times 3 + 3$, or 54 tanks with, in addition, 6 WICV's including 4 with mortars. There would be about 550 men per tank battalion.

For the present cost and operational effectiveness analysis, it was decided to evaluate these projected organizational elements in the simulations.

In keeping with the scope of the present COEA, it was decided to conduct the evaluation based on the approved European I and Middle East II scenarios, and the approved threat and doctrine which were postulated for the 1982 time frame.

For the mechanized infantry mission directed by the Department of the Army level task force, both defensive and offensive operations in Europe would be played in a realistic computerized combat simulation, whereas only offensive operations for the Middle East conflict would be played.

For the armored cavalry type mission, on the other hand, covering force operations in Europe and movement to contact for the Middle East simulation would be played and evaluated.

It was decided that alternatives would be examined primarily at the company-team/troop level with these units employed as members of a combined arms team. A division level excursion was conducted for mechanized infantry in the defense.

For evaluating the scout role, the armored cavalry platoon, Table of Organization and Equipment (TOE) 17-307H, would serve as the basic organization, whereas the battalion scout platoon would be evaluated separately in a manual war game.

Any night operations were to be addressed by means of an analysis of data available from the Fort Knox night-fighting test results.

As some final comments of interest concerning the intended scope and intent of the WICV-WOW COEA, it is noted that current Army doctrine centers around the desired use of a WICV type vehicle and the CBT to be integrated into a combined arms team, so that for the first time our combined arms forces can fight in a mounted posture. In order to fulfill the intended combined arms role, it will be necessary for the WICV to be able to keep up with our unique CBT which is capable of cross-country speeds up to about 43 km/h. There was much concern that an infantry fighting vehicle might be fielded that was too slow, too tall, less protected—making it a "sitting duck"—and much more conspicuous due to smoke and noise from it. If such were the case, then the WICV concept would not be a gain, and indeed it would result in a combat incapability or inefficiency. In this connection, it is seen that expert military judgment may well be required to evaluate these characteristics, for any available analytical models would fall far short of covering such details with any great accuracy.

46-4 THE COMPETING CANDIDATE SYSTEMS FOR THE INFANTRY FIGHTING VEHICLE (IFV) TYPE ROLE

In connection with the cost and operational effectiveness analysis of the combined arms role for the infantry fighting vehicle, the three main systems to be considered here consist of the M113A1 Armored Personnel Carrier, the armored infantry fighting vehicle, and our newly conceived WICV-WOW vehicle which was designed to meet the requirements of the combined arms team role. The vehicles are now described briefly:

1. M113A1. The M113A1 is a tracked, diesel-powered vehicle, the primary armament of which is a pintle-mounted (one-man weapon station) cal .50 machine gun. The M113A1 was designed for a crew of two with payload space for the additional nine infantry squad members. The combat weight is 24,590 lb. (The scout vehicle has only a five-man capacity, including the crew.)
2. AIFV (ARMORED INFANTRY FIGHTING VEHICLE). The AIFV consists of an M113A1 chassis and hull but has the following modifications: turbocharged diesel engine; improved, heavy-duty transmission; and tube-over-bar suspension. The primary armament of AIFV is a turret-mounted (one-man station) Oerlikon KBA 25 mm dual feed, unstabilized, automatic cannon with a coaxial Belgian MAG 7.62 mm secondary weapon (machine gun). The vehicle is designed for a crew of two with space for only eight additional infantry squad members (The scout vehicle has only a five-man capacity, including the crew.). The combat weight of the AIFV is 30,135 lb.
3. WICV-WOW 27 mm (XM 769). The WICV with a WOW antitank guided missile system is a tracked, diesel-powered type of infantry combat vehicle. It has improved cross-country mobility which is achieved through a combination of tube-over-bar suspension, dual road wheels, and greatly improved shock absorbers. The primary armament of the WICV consists of the WOW missile system in addition to the turret-mounted (one-man weapon station) 27 mm XM 769 dual feed, stabilized automatic cannon, with also a coaxial 7.62 mm secondary weapon (machine gun). The WICV normally has a crew of two, with space for nine additional infantry squad members. (The scout vehicle has a total capacity of five men). Combat weight is about 44,322 lb.

Table 46-1 gives a condensed tabulation of candidate system features for the COEA.

Table 46-2 gives the quantity of the candidate vehicles which this COEA will consider as required.

46-5 ASSUMPTIONS AND LIMITATIONS OF THE INFANTRY FIGHTING VEHICLE COEA

The assumptions and limitations that follow were considered to be very desirable, and they therefore were specified for this COEA.

46-5.1 ASSUMPTIONS

Assumptions used in this analysis follow:

1. All performance characteristics for each candidate vehicle in its production configuration will be those properly staffed and approved by the US Army Materiel Development and Readiness Command (DARCOM), and they will be verified by applicable development tests/operational tests (DT/OT) wherever possible.

2. The final cost figures for each alternative are those determined or estimated by DARCOM and approved by the Comptroller of the Army (COA).

3. All Blue equipment, force structure, and doctrine employed in the COEA are to be representative of the 1982 time frame as also is the postulated Red threat which will be simulated in the evaluations.

4. The RAM-D* and logistic concepts, applicable to this COEA, are those specified by the MN/EP documents, the maintenance support plan (MSP), and applicable logistic system analyses prescribed by current Army doctrine.

*RAM-D = reliability, availability, maintainability, and durability

TABLE 46-1. COEA ALTERNATIVE INFANTRY FIGHTING VEHICLES

Alternative	Brief Description	Combat Weight, lb	Squad* Size
M113A1	Current mechanized infantry carrier, with pintle-mounted (one-man weapon station) cal .50 machine gun.	24,590	11
AIFV	Armored infantry fighting vehicle (AIFV), with turret-mounted (one-man weapon station) 25 mm KBA B02 dual feed, unstabilized, automatic cannon.	30,135	10
WICV-WOW	A new prototype weaponized infantry combat vehicle (WICV), which is a tracked, diesel-powered infantry combat vehicle. Has improved cross-country mobility. Armed with WOW antitank guided missiles and 27 mm XM 769 automatic cannon.	44,322	11

*Includes two-man crew.

TABLE 46-2. PROBABLE VEHICLE QUANTITIES TO BE PROCURED

WICV Alternative	Mechanized Infantry	Armored Cavalry	Totals
M113A1	3222	1746	4968
AIFV	3222	1746	4968
WICV-WOW	3222	2582	5804

5. Finally, it is assumed that the performance of each competitive vehicle is based on manning it with an adequately trained crew—as proper training and familiarization are considered essential.

46-5.2 LIMITATIONS

Only a mid-intensity (nonnuclear) type of conflict is portrayed in this COEA, which certainly seems appropriate under the circumstances of this study. Moreover, night operations, attack helicopters, tactical aircraft, the family of scatterable mines (FASCAM), and the cannon-launched guided projectiles (CLGP) are not considered or played in this particular COEA. Rather, a more general battle model problem would have to be investigated elsewhere.

In connection with assumptions, limitations, and scope of the COEA study, a graphical indication of the general methodology is indicated in Fig. 46-1. In fact, Fig. 46-1 gives a more or less graphical description of the remainder of this chapter, and it is believed to be a rather desirable procedure in general. Note that the operational effectiveness analysis (or the analyses) is (are) carried out, usually in the normal way, and then the cost analyses are made along the lines of Chapters 34, 35, and 36. Then, these are combined in a cost-effectiveness type of integration, and once any additional important considerations are determined and brought into the COEA study, the preferred alternative is seen and recommended.

46-6 OPERATIONAL EFFECTIVENESS ANALYSES*

For convenience, the operational effectiveness analyses were divided into two parts. Some of the evaluations could be studied accurately through the application of existing models or methodologies, while other phases of the overall analysis were concerned with rather intangible or subjective measures which had to be determined on the basis of expert military judgment. To approach these problems, it was decided to consider the following five different scenarios:

1. Blue defense (Europe)
2. Blue counterattack—long-range, short-range, and dismounted in Europe

*It is not intended that any of the MOE's or evaluations of this paragraph reflect accurately on the capability or performance of the M113A1 or the AIFV. Therefore, accurate (and hence classified) evaluation data must be obtained from appropriate Army sources.

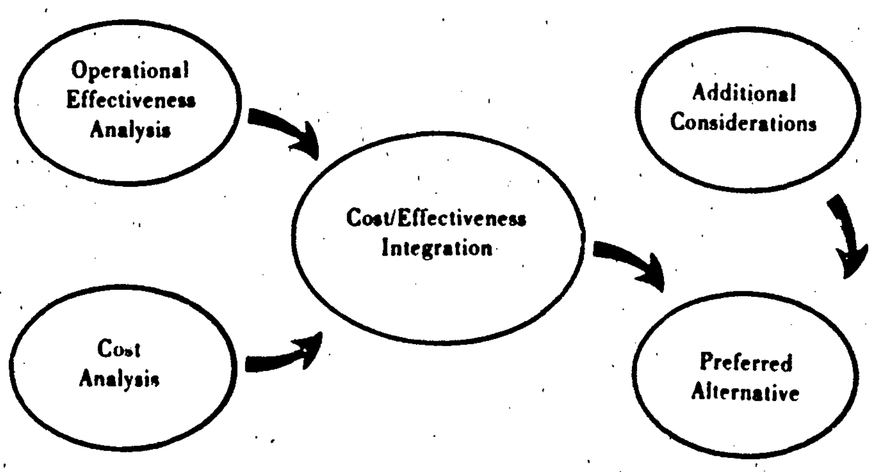

FIGURE 46-1. Methodology

3. Blue covering force—light and heavy force and tanks forward
4. Blue attack (Middle East)
5. Blue movement to contact (Middle East).

Also considered was the concept of a maneuver battalion and scout platoon type of operation, and the results were incorporated into the final judgments, wherever possible.

Experience in the methods of Army systems analysis, such as topics covered in this handbook, would indicate that the relative effectiveness of the candidate vehicles could be assessed rather accurately through use of war games or computerized combat simulations—for example, the division battle model (DBM) of par. 40-4.11. Moreover, exchange ratios or any other type of measures of effectiveness could be used to assess relative worths of the three vehicles. On the other hand, scout operations, relative mobility of the vehicles, parts of movement to contact, covering force type activities, and the best judgments relative to the combat worths of the three vehicles as efficient members of combat combined arms teams seemed a bit too difficult to quantify or model. In view of this, it was decided to convene a panel of military experts with combat experience, and conduct either map exercises, map maneuvers, or "table top" simulations as required to gain confidence in final judgments. For this purpose, a panel of fifteen experienced officers of various grade levels was selected, given any needed or relevant training, and the expertise of the officers sought in the design and play of the exercises.

For our purposes here, and considering the effect on the overall COEA, it is necessary only to summarize some of the more pertinent findings of the panel of military experts.

46-6.1 MILITARY EXPERT PANEL SUBJECTIVE EVALUATIONS

The panel of military experts was briefed thoroughly on the overall problem of the cost and operational effectiveness analysis, and the experts participated in the map exercises, war games, and other combat simulations required. Indeed, the judgment of the panel on important events and critical decisions, which arose during the COEA, was considered very valuable in many respects. The panel members were in continuous contact with the players of the simulations who acted as vehicle crews when needed. There is, of course, much available experience with the M113A1 APC, only brief experience with the AIFV, and practically no development test or engineering test information on the WICV-WOW. As is often the case with many new or proposed systems, many subjective assessments had to be made relative to the WICV-WOW, and these were quantified as accurately as possible.

The military experts seemed quite satisfied with the firepower capabilities of the WICV-WOW especially as it compared to that of the M113A1 and the AIFV. Also, they fully realized that this part of the evaluation would be accurately accounted for in war games or computerized combat simulations. They were concerned about the ability to fire on the move since loss of accuracy could develop here, especially at longer ranges.

The problem of mobility turned out to be one of the major problems for the panel to consider because it is important that infantry fighting vehicles keep pace with the CBT's. The average cross-country speed of the CBT's was projected to be about 42 km/h and, based on rather solid past information, neither the M113A1 nor the AIFV seemed able to achieve any speed approaching that capability. Therefore tests were run at the Army Combat Development Experimentation Center at Fort Ord, CA, involving the M113A1 and prototypes of the CBT, the AIFV, and the WICV-WOW to aid in proper assessment of this problem. It was found that both the M113A1 and the intended design of the AIFV would not meet the desirable requirements in this connection, but that the WICV-WOW could average somewhat over 32 km/h for cross-country runs and keep pace with the CBT's about 92% of the time. At the higher speeds, i.e., 39 km/h or more, the WICV-WOW turned out to be very uncomfortable—over half the crew members complained of discomfort. The M113A1 and the AIFV had the same problem. The WICV-WOW failed to negotiate some 20% of the normal terrain and water obstacles expected in typical combat situations, whereas these percentages for the M113A1 and the AIFV were much higher. Thus, there seemed to be no insurmountable problems for the WICV-WOW and, in fact, it was superior to the other candidates.

Concerning carrier capability, all three vehicles appeared to satisfy all requirements, either for the scout vehicle type of requirement or for heavy fighting when needed.

From the command and control standpoint, the WICV-WOW turned out to be at least as good as either the M113A1 or the AIFV. In fact, crews felt much more secure and could conduct their functions better and easier in the WICV-WOW.

On the matters of vulnerability and survivability on the battlefield, it is known that none of the candidate infantry fighting vehicles could survive in direct battles with enemy tanks or many antitank weapons. Moreover, vulnerability calculations by the US Army Ballistic Research Laboratory estimated the WICV-WOW to be perhaps somewhat more vulnerable than either the M113A1 or the AIFV because it is somewhat larger than either. Hence, it became necessary to settle such a problem in the actual combat simulations described in the sequel.

Panel officers felt both the M113A1 and the AIFV, due to their armament, should rarely engage in heavy fighting. Rather, they should hide and ambush wherever possible.

The panel of military experts judged the WICV-WOW to be at least as acceptable as the M113A1 or the AIFV in scout or reconnaissance operations, night operations, and dismounted operations. Moreover, expected logistic burdens seemed to cause no formidable problems for any of the competitors.

Finally, the panel was very much concerned about the noise, smoke, and dust of all three candidate systems—especially the WICV-WOW, because they could be more easily detected on the battlefield than the CBT and, therefore, much enemy fire could be brought to bear upon them. Hence, the panel of experts recommended that a special engineering study be initiated to correct this problem. The cost of the WICV-WOW seemed quite high indeed to panel members (see par. 46-7); therefore this fighting vehicle must "prove its worth" and desirability in a cost and effectiveness type study.

Thus it is readily seen by the reader that some parts of a COEA have to be judged or determined by nonquantifiable methods.

46-6.2 AVAILABILITY AND RELIABILITY DURING MISSION SCENARIOS

An appropriate and important evaluation to be made in COEA's concerns that of system availability and system reliability. The procedures and analysis techniques of Chapter 21 were employed, and the system characteristics exhibited in Table 46-3 developed. The point estimates are given and are self-explanatory.

TABLE 46-3. SYSTEM AVAILABILITY AND RELIABILITY

System Candidate	Availability	Reliability During Mission	Overall Performance Reliability (Product)
M113A1	0.93	0.95	0.88
AIFV	0.91	0.94	0.86
WICV-WOW	0.90	0.93	0.84

On road tests and cross-country runs, evaluations indicated that the combinations of engine, transmission, road wheels, and tracks would have just about the same mean-miles-to-failure for all three competing vehicles. Thus, there did not seem to be any special problem for the WICV-WOW in this area.

Finally, maintenance problems for each of the three candidate systems were judged to be about equivalent as determined by Army logistic personnel and vehicle mechanics.

46-6.3 COMBAT TYPE OPERATIONAL AND EFFECTIVENESS ANALYSES

With reference to the combat type operational and effectiveness analyses, battle simulations were conducted based especially on the TRADOC Standard Scenarios Europe I, Sequences 2 and 2a; and the Middle East II, Sequence 3. As explained in the sequel, the loss exchange ratio (*LER*), i.e., the ratio of the number of Red equipment losses to the number of Blue equipment losses, was considered adequate for the primary measure of effectiveness (MOF) in this evaluation. Moreover, no attempt was made to determine the weapon equivalence values of the systems employed in the simulation as discussed in Chapter 30 because the military experts considered the breakpoints of the battle would surely be reached when about 50% of the equipment on either side was defeated or put out of action regardless of the types of systems defeated. For the simulations, two rather high resolution models, TRACOM and CARMONETTE, were used to simulate combat operations at the company-team/troop level of action. Also, a division level excursion was conducted for mechanized infantry in the defense role in Europe using the division battle model (DBM) of par. 40-4.11.

Operational effectiveness analyses were conducted for each of the alternatives in the five scenarios (par. 46-6), representing three mechanized infantry and two armored cavalry scenarios for the 1982 time frame. As very brief descriptions of the models, we might say that the TRASANA* combat model (TRACOM) is a deterministic combat simulation or model which has been modified for a COEA of the type presented here to incorporate defender moves, smoke operations, and concealed defender routines over the battlefield. TRACOM was used in simulations of the mechanized infantry operations. CARMONETTE was employed in the simulation of the armored cavalry scenarios and is the Monte Carlo, critical-event-sequenced combat model as explained in par. 40-4.4.

The major Blue and Red weapon systems used in the combat simulations for Europe and the Middle East are listed in Table 46-4; the "alternative" is the M113A1, the AIFV, or the WICV-WOW

*TRASANA = TRADOC Systems Analysis Activity

TABLE 46-4
MAJOR BLUE AND RED WEAPON SYSTEMS PLAYED IN THE COMBAT SIMULATIONS

	Mechanized Infantry Role		Armored Cavalry Role	
	Blue Systems	Red Systems	Blue Systems	Red Systems
EUROPE	CBT DRAGON ALTERNATIVE WOW	R10 BMP BRDM ZAGGER ZWATTER	CBT MAW WOW ALTERNATIVE	R10 BMP BRDM ZAGGER
MIDDLE EAST	CBT DRAGON ALTERNATIVE	T62 BMP BRDM SPG-9	CBT MAW ALTERNATIVE	T62 BMP T54

weapon systems. These combined arms combinations were selected by the panel of military experts as being the most promising ones for the expected or typical conflicts for the immediate future.

As indicated previously, the *LER* is defined as

$$LER = (\text{number of Red equipment losses})/(\text{number of Blue equipment losses}). \quad (46\text{-}1)$$

There was much discussion that the *LER* is the best MOE for all combat simulations, but several of the panel members pointed out the weakness of the *LER*. They indicated the *LER* may not be very descriptive of systems such as the M113A1 or even the AIFV when equipped with smaller caliber weapons such as the cal .50 or the 20 mm automatic weapons and, therefore, should not necessarily be depended upon. Indeed, it was not really expected that such systems would participate in any predominant way in the direct fire battles; rather, if at all possible, they would best hide and only occasionally ambush enemy weapon systems. It was decided, therefore for these cases, to use what is referred to as the fractional exchange ratio (*FER*), which is defined as

$$FER = \frac{\dfrac{\text{number of Red equipment losses}}{\text{initial Red equipment strength}}}{\dfrac{\text{number of Blue equipment losses}}{\text{initial Blue equipment strength}}} \quad (46\text{-}2)$$

The *LER* is certainly easy enough to comprehend, and *FER* = *LER* when initial Red and Blue strengths are equal. It is seen that if the *FER* exceeds unity, Red's fractional losses from the start of battle increase faster than Blue's. Hence Blue is gaining an advantage, so to speak.

It is realized that neither the *LER* nor the *FER* could be the most accurate descriptors of battle potential although they were considered to be adequate and useful in this COEA. Also, as many experienced analysts have commented, such ratios often leave much to be desired as overall descriptors.

Generally speaking, the Scenario Oriented Recurring Evaluations System (SCORES) was used to develop the various operational scenarios needed for the battle plays and evaluations. For the mechanized infantry combat situations, the Red to Blue force ratio was taken as 3.5 to 1, and battle simulations run in the West Germany region. A summary of these results is given in Table 46-5. It is seen that the M113A1 and the AIFV would not stem the tide of the advancing Red forces, but the WICV-WOW could at least bring on a draw. The relative effectiveness of the WICV-WOW normalized on the M113A1 turns out to be about 1.5 to 1.

For the Blue counterattack scenario in Western Germany, long-range mounted, short-range mounted, and short-range dismounted battle simulations were run, and the Red to Blue force ratio was

TABLE 46-5. BLUE DEFENSE RELATIVE EFFECTIVENESS SUMMARY

Alternative in Force	Red System Losses	Blue System Losses	Battle Trend	*LER*	Normalized on M113A1
M113A1	93.6 (43.7%)	24.8 (46.6%)	Red Favored	3.8	1.00
AIFV	92.7 (43.3%)	25.0 (46.9%)	Red Favored	3.7	0.98
WICV-WOW	94.5 (44.1%)	17.2 (32.3%)	Draw	5.5	1.46

about 3 to 1. We see from the summary of battle results depicted in Table 46-6 that the battle trend favored Blue no matter which of the alternative fighting vehicles was used although the WICV-WOW turns out to be more effective.

With reference to the Blue Middle East attack scenario, the Blue task force—comprised of a mechanized infantry battalion plus a tank company—attacked a Red motorized rifle battalion with an attached Red tank platoon; the results are portrayed in Table 46-7. Blue is favored here in a force ratio of Blue to Red equal to about 3.2 to 1. In this battle-simulated case, any of the candidate systems for Blue would develop a favorable battle trend, although again the WICV-WOW seems significantly more effective as shown by the normalized figures with respect to the M113A1 in Table 46-7.

Finally, we give results for the armored cavalry roles in Germany and the Middle East. First, for the Blue covering force comprised of an armored cavalry troop in Western Germany opposing the advanced guard of Red forces outnumbering the Blue about 2.7 to 1, we exhibit the simulated battle results in Table 46-8. In this situation, the WICV-WOW outperforms both the M113A1 and the AIFV. Hence, it would be recommended on the system effectiveness basis comparison.

The last battle simulation covered in our COEA concerns the Blue force movement to contact in the Middle East. Here Red and Blue force strengths were equal, and a Blue armored cavalry troop was engaged in conducting a zonal reconnaissance mission against a Red reinforced tank battalion. We see from the battle results of Table 46-9 that none of the candidate systems for Blue would stem the tide in

TABLE 46-6. BLUE COUNTERATTACK RELATIVE EFFECTIVENESS SUMMARY

Alternative in Force	Red System Losses	Blue System Losses	Battle Trend Favored	*LER*	Normalized on M113A1
		(A) LONG-RANGE MOUNTED			
M113A1	16.4 (68.3%)	14.7 (26.4%)	Blue	1.12	1.00
AIFV	15.7 (67.4%)	14.9 (26.7%)	Blue	1.05	0.94
WICV-WOW	17.1 (69.2%)	11.2 (20.1%)	Blue	1.53	1.36
		(B) SHORT-RANGE MOUNTED			
M113A1	11.3 (82.1%)	12.3 (21.3%)	Blue	0.92	1.00
AIFV	11.4 (83.2%)	11.7 (18.2%)	Blue	0.97	1.05
WICV-WOW	12.1 (85.6%)	10.0 (17.1%)	Blue	1.21	1.32
		(C) DISMOUNTED			
M113A1	22.1 (85.7%)	47.2 (72.2%)	Blue	0.47	1.00
AIFV	23.2 (90.8%)	33.3 (51.0%)	Blue	0.70	1.49
WICV-WOW	24.3 (91.2%)	29.8 (45.6%)	Blue	0.82	1.7[illegible]

TABLE 46-7. BLUE ATTACK RELATIVE EFFECTIVENESS SUMMARY

Alternative in Force	Red System Losses	Blue System Losses	Battle Trend Favored	*LER*	Normalized on M113A1
M113A1	19.8 (63.1%)	23.2 (32.5%)	Blue	0.85	1.00
AIFV	18.6 (60.0%)	22.3 (31.7%)	Blue	0.83	0.98
WICV-WOW	19.0 (61.3%)	12.4 (15.3%)	Blue	1.53	1.80

TABLE 46-8. BLUE COVERING FORCE RELATIVE EFFECTIVENESS SUMMARY

Alternative in Force	Red System Losses	Blue System Losses	Battle Trend Favored	*FER*	Normalized on M113A1
M113A1	25.2 (56%)	3.7 (26%)	Blue	2.0	1.00
AIFV	26.7 (58%)	4.8 (27%)	Blue	1.9	0.95
WICV-WOW	24.9 (54%)	5.3 (30%)	Blue	2.3	1.15

TABLE 46-9. BLUE MOVEMENT TO CONTACT RELATIVE EFFECTIVENESS SUMMARY

Alternative in Force	Red System Losses	Blue System Losses	Battle Trend Favored	*LER*	Normalized on M113A1
M113A1	7.6 (28%)	9.8 (32%)	Red	0.78	1.00
AIFV	7.1 (27%)	9.2 (31%)	Red	0.77	0.99
WICV-WOW	7.0 (26%)	8.9 (29%)	Red	0.79	1.01

favor of Blue. Rather, Red would win, and the WICV-WOW—showing no very significant superiority at all here—would be barely more effective than the M113A1 and slightly better than the AIFV.

Summarizing the effectiveness studies, we see that the WICV-WOW outperforms the M113A1 and AIFV on an overall basis, but we obviously need to examine the cost picture rather closely. This is done next.

46-7 COST ANALYSES

In this paragraph, we summarize the dollar costs relative to including each of the alternative vehicles in the US Army force structure and the fielding of Blue forces played in each of the five scenarios indicated in par. 46-6.

46-7.1 COST METHODOLOGY

The two types of costs described in this subparagraph were calculated for a complete examination of the cost implications of each alternative candidate or competing vehicle. These costs are as follows:

1. Life Cycle Costs (LCC's). Separate LCC's were computed for each alternative vehicle over its 15-yr operational time frame. These costs represent the total costs to the Army of acquisition and ownership if each alternative were procured in sufficient quantity to satisfy the IFV program requirements. The LCC's are aggregates of DARCOM estimates of those peacetime costs incurred during vehicle research and development (R&D), all associated recurring and nonrecurring investment costs, and the operating and support costs for the vehicle operational time frame. (See Chapters 34, 35, and 36 for an account of cost analysis procedures.)

2. Force Costs. Separate force costs were computed for each Blue force exercised during the five scenarios played. They represent the cost of fielding the specific force in the scenarios and in no way reflect the cost of ownership over any period of years. These force costs were developed by means of the force cost information system (FCIS), using the appropriate TOE for the type units included in the forces.

Additional analyses, called sensitivity analyses, were conducted to identify the major cost-driving parameters and cost magnitudes of several particular parameters of interest—such as a rebuilt vs a new M113A1 chassis, ammunition rates, and logistic support, to mention some.

All estimates are expressed in constant FY 77 dollars, rounded to the nearest million, and include 15-yr peacetime operating and support (O&S) costs.

46-7.2 COST DISCUSSION

The 15-yr LCC's estimates for the total buy to satisfy both the mechanized infantry and the armored cavalry requirements are listed in Table 46-10 Separate LCC's for a mechanized infantry buy only, or for an armored cavalry buy only, are not included in our analyses here.

The contributions of certain parameters to LCC's are presented as percentage ranges in terms of minimum and maximum percent of LCC's among alternatives. The parameters shown are those determined to be major contributors to the cost or those of particular interest to the cost analysts. They are:

1. Investment cost of vehicle with weapon station — 7-26 %
2. "War reserve" ammunition of 180 d — 0-4 %
3. O&S cost of vehicle with weapon station — 15-22 %
4. O&S cost of dedicated crew — 22-44 %
5. Training ammunition cost — 3-12 %
6. Total O&S cost — 50-70 %.

The force costs for each Blue force employed in the five scenarios played are summarized in Table 46-11. Also presented are these same costs normalized on the M113A1 force costs for each scenario. Shown next are the parameters identified from the analytical effort as major cost contributors or identified by the cost subcommittee as being of particular force cost interest. They are expressed in terms of minimum and maximum percent of total force costs among alternatives:

1. Total personnel cost — 41-54 %
2. CBT tank cost — 18-38 %
3. Logistic support cost — 0-3 %
4. Candidate vehicle cost — 3-21 %.

The sensitivity analysis resulted in the following observations:

1. Cost analysis results are not affected by the difference between new and rebuilt M113A1 chassis.

2. Personnel costs constitute about half the total cost as might have been expected.

3. Annual recurring costs represent 50% or more of the total cost for an expected life of 15 yr or longer.

TABLE 46-10. LIFE CYCLE COST — $ × 10^6

Alternatives	R&D	Investment	15-yr O&S	Total	Total Cost Normalized on M113A1
M113A1	$ 0	$ 612	$2,830	$3,442	1.00
AIFV	$28	$1,573	$3,180	$4,781	1.39
WICV-WOW	$88	$3,000	$3,000	$6,800	1.98

TABLE 46-11. FORCE COST SUMMARY — $ × 10⁶

Scenario	Alternatives	Nonrecurring	15-yr Recurring	Total	Total Cost Normalized on M113A1
Blue Defense (Europe)	M113A1	$16	$142	$158	1.00
	AIFV	$19	$146	$165	1.04
	WICV-WOW	$26	$148	$174	1.10
Blue Counterattack (Europe)	M113A1	$43	$240	$283	1.00
	AIFV	$45	$243	$288	1.02
	WICV-WOW	$50	$244	$294	1.04
Blue Attack (Middle East)	M113A1	$24	$266	$290	1.00
	AIFV	$31	$275	$306	1.06
	WICV-WOW	$46	$278	$324	1.12
Blue Covering Force (Europe)	M113A1	$12	$ 54	$ 66	1.00
	AIFV	$13	$ 56	$ 69	1.05
	WICV-WOW	$19	$ 59	$ 78	1.18
Blue Movement to Contact (Middle East)	M113A1	$18	$ 76	$ 94	1.00
	AIFV	$19	$ 79	$ 98	1.04
	WICV-WOW	$26	$ 83	$109	1.16

4. The LCC's rank ordering appears to be insensitive to either a 10- or 20-yr operating life.

5. Cost impacts of training, additional logistic support, and variance in ammunition stockage/usage rates are not significant for comparative purposes.

46-7.3 COST SUMMARY

We might summarize cost findings as follows:

1. Life Cycle Cost. Based on normalized LCC's presented in the preceding paragraphs, the cost of alternatives when compared to the base case are presented in Table 46-12. Primary differences between alternatives are attributed to successively increasing investment costs associated with introducing progressively more complex new hardware into the inventory, e.g., the WICV-WOW.

2. Force Unit Cost. All alternative forces were considered to have equal rank order in the Blue defense and Blue counterattack scenarios since alternative vehicle costs were overshadowed by costs for personnel and the CBT. In the Blue attack scenario and both armored cavalry scenarios, however, the WICV-WOW force cost was significantly higher than that of the base case force cost. Additionally, in both cavalry scenarios the WICV-WOW force cost was significantly higher than those for the M113A1 or the AIFV. The differences in both cases are due to larger ratios of alternative vehicles within each force which prevent personnel and CBT costs from dominating vehicle costs within the force structure.

In summary we see that, although the WICV-WOW has greater potential effectiveness, it is also much more costly than either the M113A1 or the AIFV. In view of this, we must now integrate the cost and effectiveness values in some way.

46-8 INTEGRATION OF COST ESTIMATES AND OPERATIONAL EFFECTIVENESS VALUES

The WICV-WOW is an effective armored infantry type of fighting vehicle and it could, as a matter of fact, possess potential value as an important member of a combined arms team; however, it does

TABLE 46-12. RELATIVE LIFE CYCLE COSTS RANK ORDER

Alternative in Force	Life Cycle Cost
M113A1	1
AIFV	2
WICV-WOW	3

seem to be expensive. Therefore, it becomes incumbent upon us to integrate costs and operational effectiveness in some way to make a final judgment for our analysis. In our outline of the WICV-WOW COEA in this chapter, we did not fix organizational costs and then find the relative effectiveness of the WICV-WOW; nor did we fix the desired level of effectiveness needed and then calculate costs of alternative combined arms forces as indicated or implied in Chapter 37. Rather, we started from the premise that there should be certain numbers of armored infantry fighting vehicles in our proposed new combat organizations, and then we war-gamed several battle situations of critical interest to see just how the candidate or competing systems would function—operationally speaking. Of course, available expert military judgment might argue that a new weapon for the combined arms team generally of the characteristics of the WICV-WOW must be procured for the inventory anyway, no matter what, and that costs should be of little concern. In particular, there must be more tank killers on the battlefield—especially infantry fighting vehicles that can defeat tanks also. Nevertheless, we should perhaps attempt to quantify our COEA a bit better at this stage, and this may now be done rather easily.

A very simple and perhaps adequate method of combining costs and operational effectiveness is that of the concept of relative RW. RW may be defined as follows:

$$RW = RE/RC \tag{46-3}$$

where

RW = relative worth (an overall descriptor)
RE = relative effectiveness normalized on the base case
RC = relative cost normalized on the base case.

In view of the fact, as indicated in par. 46-7, that personnel costs are often the most dominant cost of all, the relative worth may often depend primarily on the relative effectiveness RE which represents no disadvantage. We have calculated the RW values for the five different scenarios, and they are displayed in Table 46-13.

Scrutiny of Table 46-13 indicates that the WICV-WOW vehicle performs in quite a valuable way as a member of the combined arms team in terms of its operational effectiveness and force costs, even though it is more expensive. In fact, it is only in the role of Blue movement to contact that the WICV-WOW is somewhat deficient, and this is not expected to represent the most important battlefield condition for the armored infantry fighting vehicle anyway. Our conclusion then on the basis of all of the subjective evaluations and the quantifiable measures of effectiveness and costs is that we believe the WICV-WOW is a highly desirable addition to the combined arms teams as compared with the M113A1 or the AIFV. WICV-WOW should, therefore, be recommended for procurement.

46-9 ADDITIONAL COMMENTS

The reader can appreciate that a COEA attempts to go beyond the intended scope of an effectiveness analysis or even a cost-effectiveness analysis. In fact, the term "operational" has been added

TABLE 46-13. SUMMARY OF RELATIVE WORTHS *RW*

Alternative in Combat Force	Blue Defense	Blue Counterattack			Blue Attack	Blue Covering Force	Blue Movement to Contact
		Long-Range	Short-Range	Dismounted			
M113A1	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AIFV	0.97	1.05	1.11	1.69	1.08	0.90	0.98
WICV-WOW	1.38	1.38	1.24	1.73	1.94	0.97	0.89

to this type of systems analysis, and presumably it really requires in some way the quantification of operational factors not previously dealt with very extensively. It might be argued that a whole new era, or method of analysis, has therefore been opened up for the practicing systems analyst. Although at the present stages of development, it also may be argued that cost and operational effectiveness analyses have not really advanced significantly beyond cost-effectiveness studies. Indeed, command, control, and communication characteristics are not easily integrated into battle scenarios through the presently available quantitative methods; nor is mobility, night operations, the use of smoke, logistics, suppressive fire, or enemy air attack, for example. About the best that can be done for the present time is to try to program scenarios for battle simulations—whether manual, war game, or computerized—and work the more intangible factors into such simulations insofar as possible. The present COEA did, as a matter of fact, try to cover some of the operational factors by using battle scenarios in two different probable combat zones as well as different types of battle conditions, which probably should be expected in campaigns of the projected future. We do realize, nevertheless, that there exist many additional areas of interest for the systems analyst to quantify in any future work on COEA's.

46-10 SUMMARY

We have attempted to display some of the methodology which might be used in COEA-type analyses or studies. Our example covers the projected combat capabilities and analyses of armored infantry fighting vehicles, particularly the WICV-WOW, and this phase of combined arms studies seems very important indeed. In fact, it is easily seen that armored infantry fighting vehicles very likely will have to be armed because occasionally they will be required to participate in the direct fire battles and to have a good capability of killing enemy tanks, armored personnel carriers, and other Red weapon systems. Also, infantry riflemen should be able to fire from them with some protection. The WICV-WOW was found to be capable of turning around otherwise favorable Red trends in the critical battle situation of Blue defense. Also, for the first time we have found the concept of an armored infantry fighting vehicle which allows Blue forces to fight mounted in accordance with current and projected Army doctrine.

REFERENCE

1. US Army Training and Doctrine Command, *Mechanized Infantry Combat Vehicle and Mechanized Infantry Combat Vehicle Scout Cost and Operational Effectiveness Analysis* (Short Title: MICV and MICV/SCOUT COEA), Vol. I, Executive Summary (U), US Army Training and Doctrine Command, Fort Monroe VA, 1 June 1977 (CONFIDENTIAL).

APPENDIX A
RELATIONSHIP OF COEA TO THE MAJOR SYSTEMS ACQUISITION PROCESS

System Acquisition Phase	Questions Addressed by Analysis During Phase	Scope and Characteristics of Analysis Performed During Phase	Supporting Documentation and Decision Milestone at Conclusion of Phase
Early Investigation	• What will be the missions that must be performed?	*Family of Systems Study* (FOSS) • Address major mission areas (e.g., field artillery, tank/antitank).	• Mission Element Need • Milestone 0 — Decision on entering Program Initiation Phase
	• What will be the threats and what is the existing capability to perform the missions?	• Study current and projected threats, capabilities, and deficiencies. • Study trade-offs among systems within a family and between families.	
	• What would be the deficiencies in DOD capability for mission accomplishment especially in light of improvement opportunities?	• Establish the need for generic systems (e.g., tanks). • Update periodically to reflect major changes in threat, doctrine, forces, and technology.	
	• What would be the impact of not rectifying the deficiencies?		
Program Initiation	• What will be the missions that must be performed?	*Cost and Operational Effectiveness Analysis*	• Letter of Agreement (LOA)
	• What will be the threats and what is the existing capability to perform the missions?	• Solicit issues and alternatives from major interested parties (e.g., HQ, DA, OSD). • Reassess mission deficiency. • Develop operational concepts.	• Concept Formulation Package • Initial Development Concept Paper (DCP)
	• What would be the mission deficiencies? • What could be done to remedy the deficiencies; i.e., what are the alternatives?	• Consider wide range of alternatives; e.g., product improvements, and systems of other branches, services, and nations. Use rapid analysis techniques-analytic effectiveness models, parametric costing models.	• Milestone I — Decision on entering Demonstration and Validation Phase (ASARC/DSARC I)
	• To what extent would the alternatives remedy the mission deficiencies? • What would be the costs associated with each alternative?	• Conduct extensive sensitivity analyses to: •• Identify key elements of cost, performance, and effectiveness. •• Establish bounds of cost, system characteristics, performance, and effectiveness.	

(cont'd on next page)

APPENDIX A (cont'd)

System Acquisition Phase	Questions Addressed by Analysis During Phase	Scope and Characteristics of Analysis Performed During Phase	Supporting Documentation and Decision Milestone at Conclusion of Phase
Program Initiation (cont'd)		• Identify uncertainties. • Identify key issues for testing.	
Demonstration and Validation	• What will be the missions that must be performed? • What will be the threats and what is the existing capability to perform the missions? • What would be the mission deficiencies? • What could be done to remedy the deficiencies; i.e., what are the alternatives? • To what extent would the alternatives remedy the mission deficiencies? • What would be the costs associated with each alternative?	*Cost and Operational Effectiveness Analysis* • Solicit issues and alternatives from major interested parties (e.g., HQ DA/OSD) • Update and analysis of mission deficiencies done in the Program Initiation Phase. • Focus on a more limited set of system alternatives. • Use more detailed analysis techniques as appropriate to the issues and alternatives, combat simulations, engineering cost estimates. • Use DT/OT I test data and FDTE data as appropriate.	• Required Operational Capability (ROC) • Updated DCP • Milestone II — Decision on entering Full-Scale Engineering Development (ASARC/DSARC II)
Full-Scale Engineering Development	• What will be the missions that must be performed? • What will be the threats and what is the existing capability to perform the missions? • What would be the mission deficiencies? • What could be done to remedy the deficiencies; i.e., what are the alternatives? • To what extent would the alternatives remedy the mission deficiencies? • What would be the costs associated with each alternative?	*Cost and Operational Effectiveness Analysis* • Solicit issues and alternatives from major interested parties (e.g., HQ DA/OSD). • Update analysis of mission deficiencies done in Demonstration and Validation Phase — may be abbreviated depending on circumstances (e.g., little change in the threat force, etc., since previous analysis). • Focus on a limited set of system alternatives. • Update cost and effectiveness estimate using DT/OT II data and detailed costing.	• Updated DCP • Milestone III — Decision on entering Production and Deployment

(cont'd on next page)

APPENDIX A (cont'd)

System Acquisition Phase	Questions Addressed by Analysis During Phase	Scope and Characteristics of Analysis Performed During Phase	Supporting Documentation and Decision Milestone at Conclusion of Phase
Production and Deployment	• Does production hardware meet specifications? • Does ILS work? • Can system meet readiness objectives?	*Cost and Operational Effectiveness Analysis* • To be determined on a case-by-case basis.	N/A

APPENDIX B
MODELS

Models represent systems in a mathematical or physical form which is suitable for examining the way the represented systems perform. Models do not depict reality in all aspects; instead, they are designed to represent the inputs, internal characteristics, and outputs essential to the purpose for which the model is developed. Models are not "good" or "poor"; they are "good for . . ." or "poor for . . .".

Model(s) to be used in a COEA should be selected based on examination of all objectives, alternatives, issues, the range of environmental/threat conditions to be considered, and the time and resources available to the study. There are three major sources of models, any one of which, or combination thereof, may be appropriate:

1. Existing models can be used and have the advantage of familiarity, comparability of results with earlier studies, and ease of start-up and operation. They have the disadvantage that there is danger of tailoring the problem to the model rather than the reverse.

2. Modifications of existing models can be made, and this approach has the same advantage as the use of existing models except ease of start-up. The disadvantages are the same except that modification can be a time- and resource-consuming activity requiring a lengthy checkout period.

3. Development of a new model can be undertaken. This has the advantage that the model can be tailored to the problem under study. The disadvantage is that it can be very time-consuming, usually much more so than recognized at the time the approach is contemplated.

Models can be grouped according to various attributes of classification: representational form, treatment of chance elements, methods of use, and extent of human integration are but a few.

Viewed according to representational form, models are of three basic types, with a fourth type being a compound of the basic types:

1. Iconic models "appear similar" to the systems they represent. They result from metric scaling. They represent static systems or a time snapshot of a dynamic system. A model of a missile built for test in a wind tunnel is an example. A globe is another example.

2. Analogic models do not usually look like the systems they represent but — in relevant part — they act like them. A laboratory lash-up of tubes, valves, pumps, and fluids may be an analogic model of the human circulatory system. The electrical circuits in an analog computer can serve as a model of the control surfaces of a missile system.

3. Symbolic models represent the elements of a system in its environment in symbolic — usually mathematical — form that can be manipulated according to rules of logic. Symbolic models are abstract, and they do not in appearance resemble the system that they represent. Most COEA's use symbolic models.

4. Compound, or hybrid, models represent systems by using a combination of submodels of different representational forms. Many war game models are compound, using submodels that are iconic, analogic, and/or symbolic.

Viewed according to treatment-of-chance, models are of two types:

1. Exact, or deterministic, models do not represent chance elements, except insofar as input values may be probabilistically derived. In a sense, many of the mathematical "laws" of physics, such as $s = at^2/2$ are exact models.

2. Stochastic, or probabilistic, models represent the chance elements in a system and/or its environment. The performance of many systems is highly variable, with predictions of performance being reliable only in a statistical sense. Ballistic dispersion of gunfire, time to detect a target, and time to failure of a hardware component are examples.

APPENDIX B (cont'd)

Viewed according to the methods of integration of humans into them, models are of two types:

1. Systemic models do not include human beings in the representation of the system at issue except that inputs may be a function of human performance, e.g., detection probabilities. Rather, the elements, relations, and rules are specified — to include all contingencies — and the model is run autonomously to examine the behavior of the represented system. Many computer-played battle simulations — such as CARMONETTE and DYNTACS — are systemic models.

2. Role-playing models include human beings in the representation of the system at issue. Many manual war games, business games, military-political games, etc., elect to integrate human decision making directly into the model. (This obviates the need to develop specific decision rules, but it also means that they remain implicit and largely not understood or necessarily agreed.)

There are three methods that are used widely with symbolic models to examine system behavior:

1. Analytic solution methods involve the straightforward application of the techniques of the branch of classical mathematics known as analysis. Analytic methods are extremely powerful and efficient, but not all systems are readily represented by models that are solved analytically.

2. Numerical solution methods are often used to solve systems of equations that result from symbolic models of systems. Numerical methods are generally less elegant than analytic methods but they — subsequent to the introduction of high speed digital computers — have become very powerful tools when dealing with models that defy analytic solution. Most numerical methods involve selection of a trial solution and then proceed through rapid successive iterations to converge to an acceptably good solution.

5. Monte Carlo solution methods often are used in connection with symbolic models which contain numerous stochastic elements. In the Monte Carlo method, a "suitably random" sample number is produced each time a random chance variable is encountered in representation of the system. Available or not available, detect or not detect, retention time, hit or miss, kill or not kill, time to repair, etc., are examples of variables that often are approached using Monte Carlo techniques. Since the model results are influenced by the samples that are drawn at random, Monte Carlo methods repeat the process, drawing other random samples, to produce a distribution of outputs that must then be analyzed statistically. It is often found that the results of the exercise of a Monte Carlo model can be nearly duplicated by analytic models. This can lead to a substantial reduction of cost and time if additional results are desired.

From the foregoing, it is plain that analysts build and use models of systems in order to learn about the systems. There are many different types of models, and they are exercised in many different ways to learn of system behavior. Model building is an art form of high fascination to analysts and sometimes — sadly — the model replaces the system as the focus of prime interest.

(DRCMT)

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INDEX

INDEX (cont'd)

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