COMPRESSION FATIGUE LIFE PREDICTION METHODOLOGY FOR COMPOSITE STRUCTURES - LITERATURE SURVEY

JUNE 1980

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<thead>
<tr>
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<tbody>
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<td>Technical Director, Naval Air Development Center</td>
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<td>Comptroller</td>
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<td>Directorate Command Projects</td>
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<td>Software Computer Directorate</td>
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<td>Aircraft &amp; Crew Systems Technology Directorate</td>
</tr>
<tr>
<td>70</td>
<td>Planning Assessment Resources</td>
</tr>
<tr>
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<td>Engineering Support Group</td>
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J. STURM
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This report summarizes literature concerning static strength and fatigue analysis of bolted jointed in composite materials. The literature survey was a portion of the first Task of the Navy Contract N62269-79-C-0214, "Compression Fatigue Life Prediction Methodology for Composite Structures". The objective of this program is to develop a fatigue life prediction methodology that will improve design efficiency, facilitate structural certification and provide guidelines for service life management. This survey provides a basis for formulation of the experimental and analytical programs to be performed in later tasks.
20. Abstract (Cont.)

This report supplements and updates a previous literature survey on bolted composite joints, AFFDL-TR-78-137. The previous survey emphasizes stress and static strength analyses of bolted composite joints and this survey includes discussion of recent research in strength analysis to update that survey.

The primary emphasis of this report is to discuss current understanding of fatigue behavior of composite materials and to examine several analytical methods for describing that behavior. Recent research programs concerning fatigue of composite materials have allowed identification of a general degradation sequence in fatigue of composite materials. Fatigue life prediction based on this sequence is in its infancy, and programs such as this provide valuable progress toward development of a rational methodology for fatigue life prediction for advanced composite structures.
This report was prepared by McDonnell Aircraft Company (MCAIR), St. Louis, Missouri, for the Naval Air Development Center (NADC), Warminster, Pennsylvania, under Contract No. N62269-79-C-0214. Mr. E. F. Kautz is the NADC Project Engineer.

The Structural Research Department of McDonnell Aircraft Company is responsible for performance of this program. Principal author of this report is C. R. Saff, Mr. H. D. Dill is Program Manager and Dr. R. Badaliance is Principal Investigator for MCAIR.

This report covers work accomplished during the period October 1979 to January 1980.
**TABLE OF CONTENTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>IV</td>
<td>15</td>
</tr>
<tr>
<td>V</td>
<td>25</td>
</tr>
<tr>
<td>VI</td>
<td>50</td>
</tr>
<tr>
<td>VII</td>
<td>51</td>
</tr>
<tr>
<td>VIII</td>
<td>58</td>
</tr>
</tbody>
</table>

**I** INTRODUCTION.

**II** SUMMARY

**III** STRESS AND STRENGTH ANALYSIS OF BOLTED JOINTS

**IV** FATIGUE LIFE BEHAVIOR OF COMPOSITE MATERIALS.

**V** FATIGUE ANALYSES OF COMPOSITE MATERIALS

1. EMPIRICAL METHODS
2. SIMPLE DEGRADATION MODELS
3. FATIGUE DAMAGE MODELS
4. COMPARISON OF FATIGUE ANALYSIS MODELS

**VI** CONCLUSIONS

**VII** REFERENCES.

**VIII** BIBLIOGRAPHY.
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comparison of Analysis Using BJSFM with Bolted Joint Test Data of Hyer and Lightfoot (Series A)</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Comparison of Analysis Using BJSFM with Bolted Joint Test Data of Hyer and Lightfoot (Series B)</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Comparison of Analysis Using BJSFM with Open Hole Test Data of Hyer and Lightfoot.</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Test Data Correlation with Predicted Bearing-vs-Bypass Strength Envelope (50/40/10 Lay-up)</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Test Data Correlation with Predicted Bearing-vs-Bypass Strength Envelope (30/60/10 Lay-up)</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Predicted Ply Strengths Under Pure Bearing Loads.</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>Correlation of Predictions with Off-Axis Bearing Test Data.</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>Comparison of Measured and Predicted Fracture Stresses of Angle-Plied Laminates AS/3501-5 with 1.8 Percent Moisture (Chamis).</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>Effect of Temperature on Strength of Coupons with an Unloaded Hole.</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>X-Ray Photographs of Fiber Dominated Lay-Up at Quarter and Half Life.</td>
<td>16</td>
</tr>
<tr>
<td>11</td>
<td>X-Ray Photographs of Matrix Dominated Lay-Up For Two Specimens</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>Fracture Obtained from Matrix Dominated Graphite/Epoxy Laminate with a Hole.</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>Effect of Load Frequency on Fatigue Life of Graphite/Epoxy Laminates.</td>
<td>21</td>
</tr>
<tr>
<td>14</td>
<td>Effect of Load Frequency on Life of Boron/Epoxy Laminates.</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>Effect of Load Frequency on Life of Boron/Aluminum Laminates (Batch 1).</td>
<td>23</td>
</tr>
</tbody>
</table>
### LIST OF ILLUSTRATIONS (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Effect of Load Frequency on Life of Boron/Aluminum Laminates (Batch 2)</td>
<td>24</td>
</tr>
<tr>
<td>17</td>
<td>Miner's Rule Applied to Composites</td>
<td>27</td>
</tr>
<tr>
<td>18</td>
<td>Linear Fatigue Damage Models</td>
<td>28</td>
</tr>
<tr>
<td>19</td>
<td>Comparison of Linear Strength Reduction and Miner's Rule Predictions</td>
<td>29</td>
</tr>
<tr>
<td>20</td>
<td>Wear-Out Model Analysis</td>
<td>30</td>
</tr>
<tr>
<td>21</td>
<td>Material Degradation Model Predicts Residual Strength Increases</td>
<td>33</td>
</tr>
<tr>
<td>22</td>
<td>Local Heterogeneous Zone Concept of Kanninen et. al.</td>
<td>35</td>
</tr>
<tr>
<td>23</td>
<td>Correlation of Strain Energy Density With Fatigue Life</td>
<td>36</td>
</tr>
<tr>
<td>24</td>
<td>Results of Spectrum Fatigue Tests of Composite Laminates</td>
<td>37</td>
</tr>
<tr>
<td>25</td>
<td>Interlaminar Stress Distribution in the (+45/902/+45/902)_s Laminate Between the Outer +45-Degree Plies</td>
<td>38</td>
</tr>
<tr>
<td>26</td>
<td>Observed C-Scans in the (+45/902/+45/902)_s Laminate (Ratwani and Kan)</td>
<td>39</td>
</tr>
<tr>
<td>27</td>
<td>Delamination Crack Geometry, Coordinate System, and Hybrid Stress Finite Element Mesh Configuration in Composites (Wang)</td>
<td>40</td>
</tr>
<tr>
<td>28</td>
<td>Comparison of Experimental and Computed Delamination Crack Growth (Wang)</td>
<td>41</td>
</tr>
<tr>
<td>29</td>
<td>Comparison of Analysis and Test Results for Load Frequency/Temperature Effects (Sun and Chan)</td>
<td>43</td>
</tr>
<tr>
<td>30</td>
<td>Effect of Load Frequency on Temperature at the Edge of a Hole in Graphite/Epoxy</td>
<td>44</td>
</tr>
<tr>
<td>31</td>
<td>Correlation of the Effect of Stress Level and Frequency on Temperature at the Hole</td>
<td>45</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>Effect of Ambient Temperature on Shifting Factors for S-N Data in Graphite/Epoxy Lamina</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>Fatigue Analysis Model Evaluation Summary</td>
<td>48</td>
<td></td>
</tr>
</tbody>
</table>
SECTION I
INTRODUCTION

This report summarizes literature concerning static strength and fatigue analysis of bolted joints in composite materials. The literature survey was a portion of the first Task of the Navy Contract N62269-79-C-0214, "Compression Fatigue Life Prediction Methodology for Composite Structures". This survey provides a basis for formulation of the experimental and analytical programs to be performed in later tasks.

This report supplements and updates a previous literature survey on bolted composite joints prepared by Garbo and Ogonowski (Reference 1). Their survey emphasizes stress and static strength analyses of bolted composite joints and this survey includes discussion of recent research in strength analysis to update that survey.

The primary emphasis of this report is to discuss current understanding of fatigue behavior of composite materials and to examine several analytical methods for describing that behavior. Recent research programs concerning fatigue of composite materials have allowed identification of a general degradation sequence in fatigue of composite materials. Fatigue life prediction based on this sequence is in its infancy, and programs such as this provide valuable progress toward development of a rational methodology for fatigue life prediction for advanced composite structures.
Inherent in design of composite aircraft structure is selection of joining techniques. Currently, bolted joints are used in joining primary structures because of the requirements for inspection, assembly and equipment access, and replacement of damaged structure. For these reasons, while research and development of adhesive bonding, co-curing, stitching, and other joining techniques continues, bolted joints will remain a primary joining technique for aircraft developed in the near future.

Static strength is the primary design criterion for composite structures. Strength reductions caused by holes and other stress concentrations generally require stress levels low enough that fatigue lives in tests are long. Strength analyses for bolted composite joints are improving as a result of recent research activities. Several of these improvements are examined herein.

Fatigue life has long been a consideration in design of bolted joints in metal structures because fatigue failures often originate at bolt holes. For many years research has been conducted to produce reliable techniques for determining fatigue life of bolted joints in metals. Research continues to explore methods to improve joint life and life prediction methods.

Fatigue life of bolted joints in composite materials has not been considered as great a problem as in metal joints because joint loads are limited by static strength to a level which produces long lives in fatigue tests. However, more recent research has shown that compression and reversed stress cycling loadings, and low frequency loadings, can significantly reduce fatigue life of notched composite panels.

Results of recent research programs have been used to identify a general degradation sequence for fatigue of composite materials. This sequence consists of initial local intralaminar cracking, eventual delamination, and final rupture. Identification of this degradation sequence has led investigators away from statistical schemes for prediction of fatigue life and residual strength and toward analytical modeling of postulated failure mechanisms. Current investigations are concerned with modeling and analysis of particular failure mechanisms.
Recently a thorough assessment of literature concerning mechanically fastened joints was prepared by Garbo and Ogonowski (Reference 1). Their study summarized the state-of-the-art with respect to commonly used types of joints, common design practices, load distribution analysis, joint failure criteria, and fatigue life methodology. Some conclusions from their report are repeated as background for discussion of more recent research.

Aircraft ranging from lightweight fighter aircraft to the space shuttle use composite structural components which include mechanically fastened composite-to-composite or composite-to-metal joints. With the exception of the modifications to the composite constituent plies (e.g. inserts or softening strips), mechanical joints are configured much like those in conventional metal structure.

Bolted composite joint design practices with respect to edge distance and fastener spacing are not clearly defined among the aircraft companies. Minimum allowed edge distances and fastener spacings ranged respectively from 2-3 and 3-4 times fastener diameters. Generally, only tension head fasteners are used with composites to avoid bearing damage and pull-through problems presented by shear head fasteners. Interference fit fasteners, hole filling fasteners, or vibration driving are not recommended. Torque values are consistent with conventional metal structure guidelines.

Analysis of bolted composite joints in aircraft structural components is similar throughout the industry. Analysis proceeds from overall structural analysis, to localized joint idealization and bolt-load distribution analysis, to assessment of strength through utilization of joint failure analysis performed at individual fastener holes. Detailed stress analysis at individual fastener holes and associated application of failure criteria is the primary area of research activity.

Theoretical and empirical methods are currently used to determine detailed stress distributions in the immediate vicinity of the fastener hole. Theoretical approaches include analytic, finite element, and strength of materials approximate methods. Analytic methods are preferred because of their potential generality, economy, and exactness. These analytic methods are principally formulated from two-dimensional anisotropic elasticity theory.
Current joint failure analyses can be grouped into five classifications: (1) empirical, (2) elastic failure analysis, (3) inelastic failure analysis, (4) phenomenological failure analysis, and (5) fracture mechanics models. Physical variables considered relevant for accurate solutions were generally agreed upon throughout the industry. However, the degree to which variables were accounted for was different in particular methods. No single methodology accounted for all of the important variables (e.g. orthotropy, finite geometry, non-linear or inelastic material behavior).

In each type of joint failure analysis, after detailed stress distributions are determined, strength is assessed by using material failure criteria. No single material failure criterion is uniformly endorsed, studies of the accuracy of various material failure criteria for joint failure analysis are very limited. Failure criteria which account for biaxial stress fields appear to have greatest capability for prediction of both failure load and failure modes for bolted composite joints.

Fatigue of bolted composite joints is currently accounted for through an iterative design-test procedure. Bolted composite joints are generally designed for static strength and tested to assure adequate fatigue lives. Fatigue analysis methodologies include: (1) empirical methods, (2) cumulative damage models and (3) fracture mechanics models. Few methods were found to have extensive experimental verification. Research on composite fatigue is continuing throughout the industry.

The MCAIR developed Bolted Joint Stress Field Model (RJSFM) was selected in the Reference 1 program as a base for development of a validated joint strength analysis routine. This closed-form stress analysis methodology has the capability to handle material strength and stiffness anisotropy, general in-plane loadings, hybrid laminates, and arbitrary hole sizes. It computes ply-by-ply failure mode information for any of five failure criteria (maximum strain, maximum stress, Tsai-Hill, Modified Tsai-Wu, and Hoffman, See Reference 1). Inelastic or non-linear behavior in the area immediately surrounding the fastener hole is accounted for by using a "characteristic dimension" failure hypothesis (Reference 2). Only elemental testing of the basic lamina (unidirectional ply) of a material system and one laminate configuration is required.

Since publication of Garbo and Ogonoowski's survey, Aagarwal has presented a method (Reference 3) for predicting the strength of bolted composite joints based on comparison of average stresses, determined over a characteristic dimension from the hole, with laminate strengths predicted by maximum strain theory. A finite element analysis is used to determine stress distributions near a hole loaded by a rigid frictionless bolt. Bolted joint tests of Verette and Labor, Reference 4, were used to determine characteristic dimensions for tensile, bearing and shearout.
modes of failure. Data from Van Siclen, Reference 5, was used to verify the analysis. While preliminary results are encouraging, Agarwal concludes that further investigations are required to determine the applicability of the approach to different material systems, structural geometries and loadings.

Hyer and Lightfoot (Reference 6) have recently published results of an experimental investigation of the effects of finite width, laminate thickness and bolt diameter on strength of bolted composite joints. In this test series, joint failure generally occurred as net section tensile failures. The test specimens were fabricated from a T300/5208 fiber resin system in a quasi-isotropic lay-up.

This author has analyzed Hyer and Lightfoot's test data using BJSFM to compute first-ply failure using the maximum stress criterion and a characteristic dimension, 0.036 inch from the hole. Lamina properties for analysis of this material system were obtained from Hart-Smith (Reference 7). Comparisons of analysis and test are shown for two different test series in Figures 1 and 2. Strengths in the thicker laminate fall below predicted value, perhaps due to bolt bending effects which were not accounted for in this analysis. In general, correlation between test and analysis is good. Comparison with results of tests of open hole specimens is shown in Figure 3. Here the characteristic dimension was reduced to 0.018 to correlate with the data. While noting that the characteristic dimension changed, correlation of predicted and measured effects of hole diameter and finite width on laminate strength is good.

In recent MCAIR research programs, ten orthotropic layups were tested, having percentages of $0^\circ$ plies ranging from 10% to 70% in the family of $0^\circ$, $\pm 45^\circ$, and $90^\circ$ ply orientations. All layups were analyzed using BJSFM to predict bearing-versus-bypass strength envelopes. Predicted strengths were correlated with test data obtained from specimens having load conditions ranging from unloaded fastener holes, through intermediate bearing-versus-bypass load ratios, to pure bearing. Strength envelopes were predicted on a first-ply failure criterion using the maximum strain failure criterion in conjunction with the "characteristic dimension" failure hypothesis. In all cases, the characteristic dimension was a constant .02 inch for tensile loadings and .025 inch for compression loadings. Load deflection data was used to determine first ply failures.

Representative correlation of test data with predicted strength envelopes for the 50/40/10 (Reference 8) and 30/60/10 (MCAIR Data) is shown in Figures 4 and 5. Test data presented as open circles represent first-ply fiber failures as indicated by abrupt deflection offsets and/or substantial stiffness changes
Figure 1. Comparison of Analysis using BJSFM with Bolted Joint Test Data of Hyer and Lightfoot (Series A)
Figure 2. Comparison and Analysis using BJSFM with Bolted Joint Test Data of Hyer and Lightfoot (Series B)
Figure 3. Comparison of Analysis using BJSFM with Open Hole Test Data of Hyer and Lightfoot
Figure 4. Test Data Correlation with Predicted Bearing-vs-Bypass Strength Envelope (50/40/10 Lay-Up)
occurring in load-versus-deflection data. Test data presented as filled circles represent the maximum load obtained in each static test. For low or zero bolt bearing stress, first-ply fiber failure was coincident with the ultimate load, and net-section failure modes occurred. At higher bolt bearing loads considerable local damage at the hole boundary occurred prior to ultimate load, with failure modes transitioning to local compression failures (bearing modes). The difference between ultimate load and load at first-ply failure reflects additional load-carrying capability of remaining undamaged plies. This effect may or may not be present depending on laminate orthotropy (percentage of 0° plies relative to ±45° plies), loading directions, and ratio of bearing stress to bypass stress.
To further verify strength predictions using BJSFM, Garbo, in MCAIR research programs (Reference 8), analyzed a series of pure bearing specimens loaded to failure at various angles off the principal material axis (parallel to 0° plies). Effects of off-axis angles were evaluated at 0°, 10°, 22.5°, 45°, 67.5°, 80° and 90°. The 50/40/10 laminate was selected for testing. The single-fastener specimens were loaded in double shear. A ply-by-ply evaluation of strength for the 50/40/10 lay-up is presented in Figure 6. Analysis results shown in this figure indicate that initial failure should occur in 90° and +45° plies. More interesting is that as off-axis angles increase, laminate strength limited by the least strength of these plies is relatively constant. This would imply that while unloaded holes show a pronounced sensitivity to off-axis by-pass loading, bearing strengths remain relatively constant. Correlation of test data with theory is illustrated in Figure 7 and predicted trends were confirmed. The dashed line represents an empirically determined 35% increase in bearing capability attributed to bolt torque-up.

![Figure 6. Predicted Ply Strengths Under Pure Bearing Loads](image)

11
While many investigators have performed experimental studies concerning temperature and moisture effects on lamina and laminate strengths, very few have attempted to use measured variations in lamina properties due to moisture and/or temperature effects to predict laminate response under similar conditions. Chamis, et.al., (Reference 9) predicted laminate strengths for several lay-ups using room temperature-dry strength properties and room temperature-wet elastic properties for the plies. Correlation of these predictions with measured properties for three laminates is shown in Figure 8.
Figure 8. Comparison of Measured and Predicted Fracture Stresses of Angle-Plied Laminates AS/3501-5 with 1.8 Percent Moisture. (Chamis)

Garbo (Reference 8) has shown that coupling temperature dependent ply properties with the stress and failure analyses of BJSFM can be used to accurately predict the effects of temperature on compression strength of laminates containing unloaded holes (Figure 9). Lamina mechanical properties based on unnotched element tests at 250°F were used with BJSFM to predict compression strengths of open hole coupons at 250°F. A single characteristic dimension (0.025 inch) was used for both 250°F and room temperature predictions.
Figure 9. Effect of Temperature on Strength of Coupons with an Unloaded Hole
Recent research into the fatigue behavior of composite materials has concentrated on experimental determination of fatigue damage initiation, progression, and failure modes. There has been little modeling of such behavior. Knowledge of the nature of fatigue damage progression in composites is a first step toward understanding and modeling this behavior. Considerable knowledge of damage progression has been gained in these programs, allowing a general fatigue failure sequence for composite materials to be recognized.

This sequence begins with intralaminar matrix cracking which initiates as debonding of the fiber-matrix interface at fiber discontinuities after very few load cycles. Intralaminar cracking progresses along fibers in a manner which reduces in-plane stress concentrations (often resulting in increased residual strength in some laminates) but increases interlaminar stresses, eventually promoting delamination. In properly designed laminates, delaminations generally initiate in areas which have seen extensive previous matrix cracking. Delamination and intralaminar cracking interact to rapidly degrade the matrix. Reduced fiber support leads eventually to fiber ruptures or microbuckling and overall failure.

While intralaminar cracking occurs very early in life regardless of lay-up or stacking sequence, the time required to initiate delaminations is heavily dependent upon lay-up and stacking sequence, since these control interlaminar stresses. For the same lay-up, a stacking sequence which promotes early delamination will produce a shorter fatigue life than a stacking sequence which delays delamination. However, there is no direct evidence which proves that delamination life is directly proportional to fatigue life. Evidently, intralaminar cracking and delamination synergistically combine to produce eventual fatigue failure.

This progression of failure has been observed by many investigators for both notched (References 10-13) and unnotched coupons. For unnotched coupons matrix cracking and delamination initiate at the edge of the specimen (References 14-15). Unidirectional laminates under uniform loadings generally do not show the delamination stage of damage progression (References 16 and 17), perhaps due to the absence of interlaminar stresses.

Lay-up distribution has considerable impact on the fatigue damage progression in composite materials. Tests of notched coupons (References 10, 11 and 12) have shown that fiber dominated lay-ups initially crack within 0° plies and may delaminate between 0° fibers bounding the notch before significant matrix cracking occurs outside this region, Figure 10. This intralaminar cracking
can reduce a notched specimen effectively to two unnotched laminates separated by damaged material between 0° fibers bounding the notch. Thus intralaminar cracking can reduce the notch stress concentration significantly and, if the unnotched strength of the remaining material is greater than the notch strength of the original laminate, produce residual strengths higher than static ultimate in some cases (References 10 and 18).

Figure 10. X-Ray Photographs of Fiber Dominated Lay-Up at Quarter and Half Life

As cycling of a fiber dominated laminate is continued, the cracked and delaminated material begins to induce further cracking and delaminations in the two columns of undamaged material. Ultimately the matrix deteriorates via cracking and delaminations to the point when individual fibers, having a little or no matrix support, begin to fail either by breaking under tensile loading or buckling under compressive loading (References 19-21).

Matrix dominated laminates show fatigue damage progression somewhat similar to that occurring in fiber dominated laminates but with significant differences caused by the dominance of ±45° plies. In these laminates damage appears to initiate as intralaminar cracks near fiber discontinuities, e.g., a hole, in the ±45° plies. Cracks along fibers tangent to the hole appear to be the most severe and bound a region perpendicular to the loading direction near the hole as shown in Figure 11. Within this region intralaminar cracks appear to occur in all ply orientations and cracks tangent to the hole in the 0° plies often extend beyond this region.
While damage in this bounded area is primarily intralaminar cracking, the area can serve as a "softening strip" to reduce the stress concentration and increase residual strength above the initial static strength. Matrix dominated laminates delaminate very rapidly in the damaged zone, and cracking and delamination progress perpendicular to the load.

Ultimately, fiber breakage or buckling starting near the hole initiates failure. Roderick and Whitcomb (Reference 11) state that initial fiber breakage under tensile loads occurs in the $\pm 45^\circ$ fibers. However, instances are reported in References 20, 22 and 23 wherein 0$^\circ$ fibers break, allowing the $\pm 45^\circ$ plies to separate along intralaminar cracks without fiber breakage, as shown in Figure 12.

These assessments of fatigue damage progression in composite materials are based on the results of recent investigations of NDI techniques for use in such materials. Various techniques including x-ray radiography (Reference 20), scanning electron microscopy (Reference 11), ultrasonic scan (Reference 12), thermography (Reference 25), and acoustic emission monitoring (Reference 17), have been used to track fatigue damage initiation and propagation during fatigue tests. TBE enhanced dye penetrant has recently been selected by several investigators (References 10, 20, 24, 26, and 27) for x-ray radiographic inspections of test coupons. There
Figure 12. Fracture Obtained from Matrix Dominated Graphite/Epoxy Laminate with a Hole
has been some concern over the effect of TBE exposure on fatigue life of composite material coupons. Pettit, et.al., (Reference 26) have compared fatigue life test results from TBE exposed specimens with those from baseline specimens tested in laboratory air. Their comparison shows no change in life for a 32 ply quasi-isotropic laminate but some reduction in life for a 24 ply fiber dominated laminate.

Several investigators have correlated matrix damage during fatigue loading with increased specimen compliance (References 11, 18 and 28) but the relationship of compliance to fatigue failure is not completely understood. Reifsnider, et.al., (Reference 18) used compliance measurements to determine failure during fatigue tests. They considered an 18 percent increase in compliance to represent failure in boron/epoxy. However, Roderick and Whitcomb (Reference 11) have shown that compliance increases can vary significantly for different lay-ups having the same fatigue lives.

Stacking sequence has been shown (References 12, 20, 24 and 29) to have considerable impact on the life and failure appearance of laminates. For the same lay-up distribution, delamination of the outer plies occurs earlier when the outer plies are 0° than when they are +45° and lives are shorter for stacking sequences having 0° outer plies.

Environmental factors known to have significant impact on static strength of composite materials are moisture and temperature.

The effects of moisture on fatigue life have not been documented as extensively as the effects of moisture on static strength. Watamabe (Reference 30) has shown that fatigue life for preconditioned uniaxial fiber glass/epoxy specimens tested in air is slightly less than that for similar dry specimens tested in air. Similar specimens tested while immersed in water showed shorter lives than preconditioned specimens. Whiteside, et.al., are performing a comprehensive test program to study the effects of moisture on fatigue life and residual strength of composite materials (Reference 31). Their results have shown very little impact of moisture on residual strength after fatigue cycling as compared to static strength. Their fatigue life results are too few to offer conclusive evidence of moisture effects on life.

Sendeckyj, et.al. (Reference 27) found that elevated temperatures promote delamination in fatigue of epoxy resin composites. Delaminations were not found to effect residual strength. Delamination causes axial and shear stresses within the laminate which aggravate matrix cracking and thus should hasten fatigue failure, however, because of the reliance on residual strength testing, direct evidence of this effect was not found.
Temperature, moisture, and mechanical loading interact to change the mechanical properties of composite materials. McKague has published several papers (References 32-34) on the effect of temperature spikes to permanently increase moisture diffusion rates and equilibrium moisture levels. Since absorbed moisture reduces the glass transition temperature of epoxy, thermal spikes can significantly reduce allowable service temperatures which are usually limited to the glass transition value. Browning (Reference 35) has shown evidence that increased diffusion rates and increased equilibrium moisture levels result from microcracks formed by stress gradients introduced by thermal shock. It is expected that intralaminar cracking under mechanical loadings would cause the same irreversible change in diffusion properties in the epoxy. No publications were found that show impact of high moisture equilibrium levels on fatigue life.

In fatigue, cyclic frequency is coupled with both moisture and temperature effects since high frequencies produce significant heat in graphite/epoxy laminates and can also drive off moisture. Sendeckyj, et.al. (Reference 15) determined that fatigue lives of unnotched laminates tested under trapezoidal wave loadings with frequencies of 10 cpm and 1 cpm are directly proportional to frequency. Their specimen laminate was [(0/+45/90)s]2 tested at gross stress levels exceeding 77 percent of static ultimate and producing lives less than 6000 cycles.

Sun and Chan (Reference 28) have recently published results of fatigue tests at frequencies from 1 cps to 30 cps on (+45)2S laminate specimens with holes. At gross stress levels varying from 53.3 to 66.6 percent of static ultimate, they show that fatigue life is proportional to frequency at low frequencies wherein specimen heating is small. However, at high frequencies and stress levels, temperatures at the stress concentration rise significantly during the test and fatigue lives are no longer proportional to frequency. Figure 13 summarizes the data of Sendeckyj, et.al., and Sun and Chan. Even such disparate tests indicate that at low frequencies fatigue life is proportional to frequency.

The effect of frequency on fatigue life has been attributed to viscoelastic properties of the matrix by Sun and Chan. However, Reifsnider, et.al., (Reference 18) show that, for both boron/epoxy and boron/aluminum coupons, low frequency cycling produces failure in fewer cycles than high frequency cycling. For open hole coupons of [0, +45, 0]g boron/epoxy, increasing the cyclic frequency from 5Hz to 30Hz increased fatigue life 40 percent as shown in Figure 14. For similar boron/aluminum coupons, Figures 15 and 16, increasing the cyclic frequency from 5Hz to 30Hz increased life 70 percent, yet the aluminum matrix does not have the viscoelastic sensitivity of epoxy. It appears that temperature and sustained load (frequency) effects must be considered in determining fatigue life of composite materials regardless of matrix material.
Figure 13. Effect of Load Frequency on Fatigue Life of Graphite/Epoxy Laminates
Figure 14. Effect of Load Frequency on Life of Boron/Epoxy Laminates
Figure 15. Effect of Load Frequency on Life of Boron/Aluminum Laminates (Batch 1)
Figure 16. Effect of Load Frequency on Life of Boron/Aluminum Laminates (Batch 2)

Biaxial loadings have not yet received much attention in the literature primarily because of the difficulties encountered in applying biaxial loads to simple element specimens. Daniel (Reference 36) has performed an extensive test program on static biaxial loading of plates having open holes. Francis, et.al. (Reference 37) have shown, from tests of notched tubes of [±45]s lay-up, that, as the ratio of tension-to-torsion stress increases, the stress concentration factor decreases and the fatigue life increases.
Modeling of fatigue damage in composites is in its infancy. Knowledge of the nature of fatigue damage propagation in composites, gained in recent research programs, is a valuable first step toward understanding and modeling fatigue behavior. Much work needs to be done to determine the mechanisms of damage growth and the significance of matrix cracking and delamination on residual strength and fatigue life. Fatigue life prediction techniques being used or developed currently fall into three levels of analysis depth: empirical techniques which assume little or no analysis capability, simple degradation assumptions (such as cumulative damage or wear-out approaches) which involve varying degrees of sophistication in developing damage parameters, and methods which attempt to model the damage modes of composites, to define damage mechanisms, and to provide a deterministic approach to predicting the impact of damage on residual strength and fatigue life.

1. **EMPirical METHODS** - State-of-the-art life prediction approaches rely on spectrum fatigue tests using specimens representative of specific design details. These empirical techniques are still widely used due to lack of confidence in and verification of analytical fatigue life prediction techniques. In meeting various military durability specifications (MIL-A-8866, MIL-A-83444, etc.), current and near term military aircraft development programs (F-15, F-16, F-18, AV-8B) are using empirical methods to assess the effects of cyclic loading on composite life (References 1, 38 and 39). These investigations have demonstrated that sufficient fatigue life is generally achieved by composite structure designed to satisfy static strength requirements.

Most research and development programs on composite fatigue have emphasized experimental investigations. Conclusions and recommendations reached in these studies have been based on test data. Fatigue failure modes are typically not the same as static modes of failure (References 14 and 20). Extrapolation of conclusions is difficult due to the lack of physical understanding of the failure mechanisms involved.

Inherent with empirical methods is the large data base needed to generate confidence. Scatter in fatigue life data has required either large numbers of replicates or considerable conservatism to produce allowables with confidence levels typical of metals. Most published fatigue data have been on unnotched laminates or laminates with an unloaded hole. Relatively little fatigue data exists on bolted composite joints. Of the existing data, results are often for specialized specimen design, lay-up, or test conditions.
2. SIMPLE DEGRADATION MODELS - These methods are based on assumptions of the degradation of life or residual strength under cyclic loads, rather than attempting to model the physical damage process. Accuracy of these methods depends on definition of damage parameters which account for the degradation found in tests. These models include cumulative damage and wear-out approaches.

a. Cumulative Damage Models - Cumulative damage models are used to correlate and predict fatigue life in structure when no measurable damage occurs before failure or when relationships between damage growth and applied loading have not been determined. These models are based on the assumption that a predictable portion of the structural life is depleted with each application of load. The rate at which life is computed to be depleted depends on the assumed damage parameter and the constant amplitude fatigue (S-N) data used. Spectrum fatigue life predictions are commonly accomplished by analyzing the loading as blocks of constant amplitude load levels and stress ratios.

Miner's linear cumulative damage rule has been the most-explored cumulative method for analyzing composites. Miner's rule is simple to use. It requires only constant amplitude fatigue data (S-N curves) for the applied stress ratios in the spectrum (Figure 17). While there is currently no generally accepted technique for predicting (correlating) the effects of stress ratio on composite fatigue life, several investigators are beginning to formulate such techniques. These techniques are discussed in detail under Fatigue Damage Models. Such techniques are required to permit prediction of spectrum life, without resorting to extensive stress ratio testing. Miner's rule is also geometry-dependent. The S-N data required must be generated from the same specimen geometry as used in spectrum tests.

Other cumulative damage rules are applied in a fashion similar to Miner's Rule. Non-linear damage rate assumptions can be accommodated by use of damage parameters which are not linearly related to stress level.

Attempts to use Miner's rule have met with varying degrees of success. In some cases, it has been reported that Miner's rule is grossly unconservative in predicting life of composite materials (References 40 and 41). Others have found it to be an adequate technique for preliminary design studies (References 42 and 43).

b. Linear Residual Strength Degradation - Linear residual strength degradation (Reference 44) is a model based on the assumption that a portion of residual strength (rather than life as in Miner's Rule) is depleted with each application of the load. This results in a slightly different interpretation of damage rates as determined from S-N data (Figure 18). A recent investigation by MCAIR showed predicted life using the linear strength reduction model to be in better agreement with spectrum fatigue life results than Miner's Rule (Figure 19).
Figure 17. Miner's Rule Applied to Composites
Miner's Rule: 
\[ \sum \frac{n_i}{N_i} \leq 1 \]

Linear Residual Strength Prediction:
- \[ f_{\text{Residual}} = f_{\text{cu}} - \sum \frac{n_i}{N_i} (f_{\text{cu}} - f_i) \]
- \( f_{\text{Residual}} > f_{\text{max}} \)

where: \( f_{\text{max}} \) = Maximum Spectrum Stress

**Figure 18. Linear Fatigue Damage Models**
c. Wear-Out Models - Various analytical procedures based on the assumptions of flaw growth under cyclic loading have been developed for composite laminates. Of documented composite fatigue life methodologies, the "wear-out model" (Reference 45) has had considerable evaluation. This model assumes the damage growth rate can be determined by residual strength testing after cyclic loading. A growth rate equation relates both life and residual strength distributions to the initial static strength distribution.

Using the wear-out model requires experimental determination of parameters needed for formulation of residual strength predictions. These include the static shape parameter, static scale parameter and the fatigue shape parameters. "Maximum likelihood estimates" (Weibull estimates) of the static parameters are determined using static test results. To determine the fatigue shape parameter, it is necessary to have fatigue failure data (life-time data) for each fatigue variable, e.g., stress level, stress ratio. The wear-out model generates probability-of-survival curves for a given spectrum. Estimates of specimen lifetimes for the spectrum loading are made based on wear-out model predictions as shown in Figure 20.
The wear-out model, as an analytic tool, has limitations. It is primarily an empirical procedure which applies to a restricted set of experimental results. Each new configuration must be fatigue tested to determine parameters needed in the wear-out equation. Predicted residual strength is a monotonically decreasing function of time. The model is incapable of predicting increases in residual strength which have been found to occur in composite structure (Reference 46). The accuracy of the model in prediction of effects of stress level shifts is dependent on failure modes remaining constant. Spectrum modifications may cause failure modes which are not the same as for a baseline spectrum. Ryder (Reference 23) has done an extensive investigation of the wear-out model. He states that "... the real value of any formulation to describe the wear-out or strength degradation rate is to accurately predict the time to first failure of coupons or components undergoing fatigue loading at different stress levels than those previously used to obtain life data. The data from this report indicates that any such prediction based on the wear-out model as formulated above would have a low accuracy and thus little utility. ...The model is inadequate beyond satisfaction of the boundary conditions".
In studying the residual strength of composite materials under fatigue loading, Halpin et.al. (47) proposed a degradation equation that is based on the crack propagation of homogeneous materials. Realizing that fatigue failure of composites is not dictated by the initiation and growth of a dominant crack, Hahn and Kim (Reference 48) introduced the concept of rate of change of residual strength. Without referring to any crack, they assumed the time rate of decrease of residual strength is inversely proportional to residual strength, to a power. From this deterministic residual strength equation, and the static strength distribution, they derived a fatigue life distribution.

Following the same approach as Hahn and Kim, Yang and Liu (Reference 49) further derived the residual strength distribution and compared the results with several groups of experimental data. In recent papers Yang has shown that the same wear-out model can be derived from either Halpin's assumption of crack growth or Hahn and Kim's assumption of residual strength degradation. Yang has developed analysis procedures to predict constant amplitude fatigue life under tension-tension loadings, tension-compression loadings, and shear loadings. Recent publications demonstrate the application of the model to initial or periodic high loads and to simple blocked loadings (References 50 and 54).

Like the wear-out model, Yang's residual strength degradation model is a statistical approach. A large data base is needed to accurately determine values of parameters used in the model. Due to sample size limitations, these values are computed by minimizing the mean square difference between various statistical equations. Slight variations in test data results in the value of one of these parameters varying by orders of magnitude. This indicates the sensitivity of the residual strength degradation model to data scatter.

The assumption of a continuously decreasing residual strength is questionable since there is evidence showing that initial residual strength increases in many configurations. Yang has recognized this fact and stated (Reference 50) that "Under such a situation, it is recommended that a different theoretical model should be developed".

McLaughlin et.al. (Reference 55) derived an analytical model which theoretically can predict residual strength increases. The analysis predicts failure of a laminate having a stress concentration (unloaded hole) from uni-directional material fatigue properties. Residual strength is treated as a static failure utilizing fatigue degraded lamina properties. A fatigue failure occurs when the residual strength of the laminate falls to the level of applied maximum cyclic stress. The fatigue analysis methods consist of the following:
(1) Calculating laminate stresses for a given tensile loading.

(2) Calculating lamina stresses.

(3) Computing fatigue-induced material property changes in each ply.

(4) Predicting new laminate properties.

(5) Calculating the changed residual strength properties of the laminate.

The steps are repeated until failure is predicted. Failure modes treated in the analysis include:

(1) Axial cracking.

(2) Transverse cracking.

(3) Cracking at an angle to the load axis along a fiber direction.

The failure loads are predicted by computing the laminate stress causing each mode of failure, using a strength model. Failure stresses are compared, the lowest being the predicted laminate failure stress and indicating the dominant mode of failure.

An example of the capabilities of this model is illustrated in Figure 21. Residual strength curves for transverse crack propagation mode and axial crack propagation mode are plotted from calculations using the static fracture model. The laminate's residual strength is determined by the lower of the two curves (solid line). This example shows the residual strength initially increasing, then decreasing until the strength equals the maximum cyclic stress when failure is predicted to occur.

This model is still under development. Additional capabilities have recently been added to the model by Ramkumar, et.al. (Reference 56). They have performed three-dimensional finite element analyses to determine the influence of interlaminar stresses on delamination near the hole. Due to lack of statistically significant base of residual strength data for uniaxial laminates measured after axial and off-axis constant amplitude fatigue loadings, correlation with more general lay-up distributions has been inaccurate.

The major drawback to the residual strength models is that residual strength does not usually show significant degradation during most of the fatigue life. Scatter in static strength is large compared with the small reductions in residual strength caused by fatigue. A considerable number of tests have to be performed to provide confidence in any conclusion drawn from such tests.
Figure 21. Material Degradation Model Predicts Residual Strength Increases

Recognizing that residual strength does not fall in many cases, Chou and Croman (Reference 57) have proposed a sudden-death model which assumes that the damage produced by each cycle does not necessarily effect residual strength. This uses the same damage rate assumption as that used by Yang (Reference 50) but restricts the residual strength data used to that from specimens which survive fatigue testing. For the sudden-death model, it is assumed that there is a unique relation between static strength and fatigue, the stronger specimens last longer. This relation is only implied in Yang's degradation model. A comparison of predictions of residual strength using wear-out and sudden-death models was made for graphite/epoxy laminates having six different lay-up distributions. Residual strength data for each laminate after constant amplitude fatigue loading were obtained from literature. Both models predicted mean residual strength well for each lay-up distribution. The sudden-death model predicted the distribution of residual strengths for unidirectional laminates slightly better than the wear-out model since unidirectional laminates do not show the gradual degradation of more general lay-up distributions. Regardless of direct applicability, the sudden-death model is useful as a limiting case in residual strength studies.

3. FATIGUE DAMAGE MODELS – With the knowledge of damage progression provided by advanced NDI techniques and recent research programs, several investigators have initiated attempts to model the measurable damage found to occur in fatigue tests of composite coupons. The objective of these studies is threefold: (1) deter-
mine mechanisms which drive the damage growth in composites and thus control fatigue life, (2) characterize damage so that remaining strength and fatigue life can be assessed, and (3) combine damage characterization and growth mechanism parameters to develop a deterministic fatigue life prediction methodology. So far analyses have been restricted either to intralaminar cracking or to delamination.

Kanninen, et al. (Reference 58) have described their preliminary development of a mathematical model for the strength of fiber reinforced composites containing specific flaws. While their studies have been limited to static loading cases in uniaxial laminates, their approach toward modeling the flaw is noteworthy. The approach is to embed a local heterogeneous region (LHR) surrounding a crack tip into an anisotropic elastic medium. The LHR is modeled by spring-like elements which permit orthotropic properties of the fiber, matrix and interface to be individually accommodated. The basic element and its use in the LHR are shown in Figure 22. It is assumed (1) that the LHR is large enough relative to the microstructural dimensions of the composite so that the boundary displacements are given closely by continuum theory and (2) small enough relative to the flaw length and dimensions of the body that the singular behavior of the continuum solution at the crack tips dominates the LHR. By comparison of energy release rate values computed within the components of the LHR and critical values for each of the constituents (fiber, matrix or interface) it is possible to trace progression of fracture of each constituent.

Computations for arbitrary flaw size and orientation have been performed for unidirectional composites with linear elastic-brittle constituent behavior. The mechanical properties were nominally those of graphite/epoxy. With the rupture properties arbitrarily varied to test the capability of the model to predict real fracture modes in fiberous composites, it was shown that fiber breakage, matrix crazing, crack bridging, matrix-fiber debonding, and axial splitting all can occur during a period of increasing load prior to catastrophic fracture. Qualitative comparisons with experimental results on edge notched unidirectional graphite/epoxy specimens have also been made.

A simple model of intralaminar cracking has been used by Badaliance (Reference 59) to develop a correlation parameter, $S$, based on strain energy density to account for lay-up and stress ratio effects on constant amplitude fatigue life. This parameter was used in conjunction with linear fatigue damage models to predict spectrum fatigue life.

Since tests have shown the matrix to be the weak link of the composite system, the matrix in each ply of the laminate was modeled as an isotropic layer containing a through-the-thickness crack sandwiched between two semi-infinite orthotropic plates. Far field stresses are obtained through laminated plate theory, which assumes a uniform strain distribution through-the-thickness.
These far field stresses are used to determine stress intensities and strain energy densities at crack tips. Strain energy density can be associated with the concept of stress intensity by substitution of stress intensity factors for stresses in the standard equations for strain energy density. Thus the strain energy density factor (the coefficient of the singular term in the strain energy density) can be expressed as:

\[ S = a_{11} K_1^2 + a_{22} K_2^2 + 2 a_{12} K_1 K_2, \]  

where \( a_{11}, a_{22} \) and \( a_{12} \) are elastic constants and \( K_1 \) and \( K_2 \) are Mode I and Mode II stress intensities.
This strain energy density factor was simplified and normalized with respect to laminate ultimate strengths (compressive and tensile) to obtain $\bar{S}$. The parameter $\bar{S}$ was then used to correlate constant amplitude fatigue data for three laminates. Results shown in Figure 23, indicate that the current formulation of $\bar{S}$ does not correlate the effects of R-ratio, so an empirical relationship was developed. This empirical relationship was then used in conjunction with the linear residual strength reduction model to predict spectrum fatigue life. The linear strength reduction model (Reference 44) is a modification of Miner's Rule based on the assumption that residual compressive strength due to constant amplitude cyclic loading is linearly related to the number of applied cycles. Results in Figure 24 show good correlation between spectrum test results for three different lay-ups and predictions of the linear strength reduction model coupled with the strain energy density parameter, $\bar{S}$.

![Figure 23. Correlation of Strain Energy Density with Fatigue Life](GP03-0299-36)
The concept of crack tip strain energy density appears to be a useful tool for explaining the growth of matrix cracks. Both Mode I and Mode II crack loadings result from the stresses within each ply are treated in a far simpler and less expensive manner than in Kanninen's method (Reference 59). Another advantage of Badalliance's technique is that Mode III crack loadings, which cause delaminations, can be accommodated by simply adding the additional term $a_{33} K_3^2$ to the formulation $S$. Current work is being directed toward developing reliable estimates of interlaminar stresses. Methods for determining interlaminar shear stress have been developed by several authors (References 60–63). That of Puppo and Evenson (Reference 63) is currently being examined as a basis for formulating the mode III term for $S$. 
An interesting analysis of interlaminar stresses and delaminations near holes in composite materials is presented by Ratwani and Kan (Reference 64). In this work three dimensional finite element analyses of several 16 ply symmetric laminates having holes were performed using the NASTRAN computer routine. Using orthotropic solid elements, each ply is modeled by one layer of elements. From these analyses, plots of interlaminar shear and tensile stress distributions were made as shown in Figure 25. The stresses shown (in Figure 25) are those between the outer \(+45\) degree plies of a \((\pm 45/90_2/\pm 45/90_2)_S\) laminate. Although matrix crack and delamination singularities are not modeled, Ratwani and Kan have compared the stress analysis results to delamination propagation found in test specimens. C-scans of four specimens of this laminate are shown in Figure 26. The delamination is shown to propagate perpendicular to the loading direction as predicted by the interlaminar stress analyses. The stress analyses and test results for \((\pm 45/0_4/90/0)_S\) laminate show similar agreement.

Figure 25. Interlaminar Stress Distribution in the \((\pm 45/90_2/\pm 45/90_2)_S\) Laminate Between the Outer \(\pm 45\)-Degree Plies. (Ratwani and Kan)
Ratwani and Kan have implemented their interlaminar stress analyses into a wear-out model to predict fatigue life at different constant amplitude stress ratios for a (+45/902/+45/902)6 laminate. The constant amplitude fatigue life results published in Reference 64 for stress ratios of -- and -1 agree well with prediction. These results show promise for analytical prediction of the effect of measured delaminations on fatigue life and residual strength of composite materials.

A more fundamental approach toward delamination analysis is being pursued by Wang (Reference 65). In this work Wang is coupling analytical studies with an experimental investigation of delamination crack growth in unidirectional laminates under static and cyclic loadings. Experiments using simple coupons with notches perpendicular to the loading direction and extending completely across the section (Figure 27) allowed monitoring the delamination growth optically at the edge of the specimen. In static loading tests, delaminations initiated at low load levels and grew in a stable, well-defined manner as the nominal stress increased monotonically. Similar behavior was found in constant amplitude loading at 5Hz. Delaminations initiated and grew in the resin between plies. Rapid propagation occurred at a critical stress level in static tests and at a critical number of cycles, depending on load level, in cyclic tests.
Wang's analytical approach is based on introducing a singular element into a hybrid-stress finite element model. The crack-tip supplement includes singular and higher-order terms derived from the complex variable formulation of Muskhelishvili stress functions (Reference 66). The composite laminate is modeled as an assembly of anisotropic plies bonded by thin isotropic resin interply layers. The delamination crack is modeled as a crack completely embedded in a resin-rich interlaminar region.

Analysis of a uniaxial laminate having a delamination crack shows: (a) an intensified stress field just ahead of the crack tip within the resin-rich interlaminar region, (b) a large stress gradient through the laminate with high concentration in the ply adjacent to the delamination, and (c) a local longitudinal compressive stress developed in the delaminated ply adjacent to the crack tip. The local, high stress concentrations in continuous plies adjacent to delaminations could cause intralaminar cracking in that ply, particularly if those plies are angled with respect to the loading direction. The compressive stresses found in discontinuous adjacent layers may cause surface layer buckling during delamination failure.

Wang has shown that delamination crack growth under monotonically increasing loads can be predicted using his stress analysis and a mixed-mode fracture criterion. The criterion states that delamination crack extension will occur when the total strain energy release rate in the laminate reaches a critical value, \( G_C \). Wang's stress analyses show Mode I and Mode II stress intensity factors to be nearly the same magnitude, requiring consideration of both modes in the determination of a crack growth criterion. Wang decouples the contributions of each mode to state his crack growth criterion as,

\[
\left( \frac{K_I}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IIc}} \right)^2 = 1
\]  

(2)
This criterion is subject to question since Sih (Reference 67) has shown that crack extension under biaxial loadings involves a cross product of $K_I$ and $K_{II}$ (see Equation 1). However, using his criterion, Wang has obtained the correlation between predicted and measured crack growth under monotonic loadings shown in Figure 28.

![Figure 28. Comparison of Experimental and Computed Delamination Crack Growth in Glass/Epoxy (Wang)](image)

Under cyclic loadings Wang has shown that delamination crack growth rate in a uniaxial laminate can be related to the Mode I and Mode II stress intensity factor ranges by a power law relationship. These are important first steps toward developing a validated analytical approach for predicting delamination growth in composite materials.
The literature has shown a significant effect of cyclic frequency on fatigue life of composite materials. Sun and Chan, Reference 28, have developed a method of correlating fatigue lives for various frequencies based on an extension of Schapery's theory of crack growth in viscoelastic media, Reference 68. Schapery's theory predicts a fatigue life that is proportional to the frequency of loading. This theory explains frequency dependent fatigue behavior of composites very well except at higher frequencies where life decreases with increasing frequency, as shown by the data presented in Figure 13. At high frequencies the temperature of the composite material near a notch or hole increases noticeably and fatigue life tends to decrease. Sun and Chan introduced an exponential term to account for this temperature effect. They found that fatigue lives could be correlated for several frequencies and at a given stress level by the following expression.

\[ N(\omega) = N(\omega_1) \frac{\omega_1}{\omega} e^{\eta \left( \frac{\Delta T_1 - \Delta T}{T_0} \right)} \]  

where \( N(\omega) \) is the predicted fatigue life at frequency, \( \omega \), \( N(\omega_1) \) is the measured life at frequency, \( \omega_1 \), \( \Delta T_1 \) is the measured temperature rise near the hole at frequency, \( \omega_1 \), \( \Delta T \) is the measured temperature rise near the hole at frequency, \( \omega \), and \( T_0 \) is ambient temperature. The parameter, \( \eta \), is an experimentally determined constant. Sun and Chan state that \( \eta = 9 \) gives good correlation with their test results and this is confirmed by the plot shown in Figure 29. This procedure appears to provide a good method for accounting for frequency and temperature effects in fatigue life behavior.

Both MCAIR test data, Figure 30, and Sun and Chan's data indicate that temperature increases near the hole for uncooled specimens are proportional to the test frequency. Also thermodynamic considerations imply that the temperature rise should be a function of the energy input by mechanical loading and energy lost by conduction. The energy input by mechanical loading is proportional to the frequency and to the square of the applied stress level while conduction is dependent on geometry and temperature differential. Empirical analysis of Sun and Chan's data by the author indicates that the temperature rise due to applied loading near the hole for a laminate of given geometry can be expressed as:

\[ \Delta T_w = \frac{\omega}{\alpha} \left[ \left( \frac{\Delta \sigma}{\sigma_{ult}} \right)^2 - \beta \right] \]  

where \( \alpha \) and \( \beta \) are empirical parameters determined from tests of particular laminates and geometries. By letting \( \alpha = 0.1984 \) and \( \beta = 0.1615 \), temperature changes measured by Sun and Chan can be readily interpolated as shown in Figure 31.
Figure 29. Comparison of Analysis and Test Results for Load Frequency/Temperature Effects (Sun and Chan)
Figure 30. Effect of Load Frequency on Temperature at the Edge of a Hole in Graphite/Epoxy
Rotem and Nelson (Reference 69) have analyzed temperature dependence of the fatigue behavior of graphite epoxy. They have formulated functions which describe the degradation of principal strengths with a number of cycles. These functions have the following form,

$$\sigma = \sigma_S \frac{T_0}{T} a(T) [C_{T_0} - b_{T_0} \log N] \quad (5)$$

where $\sigma$ is the maximum applied stress (constant amplitude testing assumed) at temperature, $T$, which produces failure in $N$ cycles. $T_0$ is a reference ambient temperature and $\sigma_S$ is the static strength at $T_0$. The function, $a(T)$ modifies the $S$–$N$ curve defined by $C_{T_0}$ and $b_{T_0}$ which are determined experimentally.

Rotem and Nelson found, from experiments with many unnotched laminate configurations, that use of laminated plate theory together with their fatigue behavior theory gave good correlation with test data. It was found that inplane shear is influenced most by temperature, transverse tension is influenced less, and axial tension is hardly influenced at all, Figure 32.
Figure 32. Effect of Ambient Temperature on Shifting Factors for S-N Data in Graphite/Epoxy Lamina

It is interesting to note that Rotem and Nelson's results correspond to those of Sun and Chan for cases where only ambient temperatures are changed. Rewriting Equation (5) in terms of $N_{T_0}$, the life at the given stress level $\sigma$ and the reference temperature, $T_0$, gives

$$N_T = N_{T_0} e^{\frac{\sigma}{\sigma_0} \frac{1}{b_{T_0}} \left[ \frac{1}{a(T_0)} - \frac{1}{a(T)} \right] \frac{T}{T_0}}$$

(6)
As shown in Figure 32, from the test data of Rotem and Nelson, \( a(T) \) is close to unity for both longitudinal and transverse stresses over a wide range of temperatures, thus

\[
N_T = N_{T_0} \frac{1}{bT_0} [1 - \frac{T}{T_0}] 
\]  

(7)

Which is the same as Equation (3) when \( \omega = \omega_0, \Delta T = 0, \) and \( \Delta T = T - T_0, \) that is, when the only temperature change is due to a change in ambient temperature.

The implication of these results is that the changes in constant amplitude fatigue life at a given load level due to load frequency and ambient temperature can be determined from the life at some reference frequency and ambient temperature as follows:

\[
N(\omega, T) = N(\omega_0, T_0) \frac{\omega}{\omega_0} e^{\eta [T_{H_0} - T_H]/T_0}
\]

(8)

where \( T_{H_0} \) is the half life temperature measured at the hole at frequency \( \omega_0, \) and ambient temperature \( T_0, \) and \( T_H \) is the half life temperature measured at the hole at frequency \( \omega, \) and ambient temperature, \( T. \) \( \eta \) is an empirical constant. Temperature at the hole is the sum of the temperature rise due to loading and the ambient temperature. Using Equation 4 to describe the temperature rise due to loading, then half life temperatures can be expressed as

\[
T_{H_0} = \frac{\omega_0}{\alpha} \left[ \left( \frac{\Delta \sigma}{\sigma_{ult}} \right)^2 - \beta \right] + T_0
\]

(9)

\[
T_H = \frac{\omega}{\alpha} \left[ \left( \frac{\Delta \sigma}{\sigma_{ult}} \right)^2 - \beta \right] + T
\]

(10)

where \( \alpha \) and \( \beta \) currently must be experimentally determined as was done for Sun and Chan's data. This relationship has not yet been verified by test.

4. COMPARISON OF FATIGUE ANALYSIS MODELS - A comparison of the fatigue damage analysis models is presented in Figure 33. One conclusion which can be drawn from this summary is that not many features of any model are duplicated in any of the others. Not all of the models are based on analysis of failure modes, such as matrix cracking. Rotem's and Nelson's method is used to interpolate empirical results. Sun and Chan's technique uses Schapery's model of crack growth in viscoelastic medium to correlate frequency and temperature effects. None of the models examined fiber buckling analytically.
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<th>Analysis Features</th>
<th>Fatigue Analysis Capabilities</th>
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<td>Stress Analysis</td>
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\[\triangledown \text{Badaliace (Reference 59)}\]
\[\triangledown \text{Kannenin, et. al. (Reference 58)}\]
\[\triangledown \text{Ratwani and Kan (Reference 64)}\]
\[\triangledown \text{Wang (Reference 65)}\]
\[\triangledown \text{Sun and Chan (Reference 28)}\]
\[\triangledown \text{Rotem and Nelson (Reference 69)}\]
\[\triangledown \text{Based on Referenced article this Analytic Feature(s) is present}\]

Figure 33. Fatigue Analysis Model Evaluation Summary

Ratwani and Kan are the only analysts found to be directly using finite element results to develop a damage parameter. Kannenin et. al. and Wang use finite element analyses coupled with fracture mechanics methods to estimate the strength of crack tip singularities. Modeling stresses near discontinuities with finite elements requires considerable care, coupled with some prior knowledge of expected behavior, to insure accuracy. Badaliace, Sun and Chan, and Rotem and Nelson use orthotropic plate theory coupled with appropriate stress intensity factor solutions to model matrix cracking.
Only Kanninen, et.al., actually model matrix crack growth and only Wang models delamination growth. All other techniques assume some damage accumulation rule or empirically determine life under specific conditions.

Badaliance's method allows use of either Miner's Rule or a linear residual strength degradation scheme to predict fatigue life. He found that the linear residual strength reduction scheme was more accurate. Kanninen et.al. and Ratwani and Kan have used results of finite element analyses to predict residual strength with success, and Wang has used an energy criterion to predict residual strength. The semi-empirical techniques of Sun and Chan, and Rotem and Nelson, are limited to prediction of the effects of frequency and ambient temperature on fatigue life.

None of the methods have been shown to account for moisture effects on the laminate, although, if Chamis's theory that lamina property degradation is sufficient to describe laminate behavior is correct, all methods could probably accommodate moisture effects. Sun and Chan's work was performed to determine the interaction of frequency and temperature on life. Rotem and Nelson restricted their analysis to ambient temperature effects alone. As shown herein these authors have derived (apparently independently) very similar relationships for the effect of temperature on life.

There is no single technique which offers a complete treatment of the problem of fatigue life prediction for composite materials. Each model appears to treat a single aspect of the problem. Kanninen et.al. have formulated the most detailed analytical scheme so far, but it has not been directly applied to fatigue analysis and it may be prohibitively expensive to perform such analyses. Ratwani and Kan have used finite element results to examine stress states and failure modes, particularly delamination, but they ignore the singularity strengths which define crack growth and degradation of the matrix and laminate. The remaining models have been shown to adequately model particular phases of the damage sequence.

At present, it appears that a promising procedure which would provide a complete treatment of the composite fatigue problem within the current state-of-the-art is to use Badaliance's characterization of matrix cracking to model matrix crack growth and couple this with Wang's characterization of the delamination singularity behavior. This would not be an easy task. Some criterion for initiation of delamination must be defined. The interaction of matrix cracking and delamination growth must be determined. And some criterion for fatigue failure must be defined. The strain energy density characterization used by Badaliance is attractive as a starting point since it has been shown to provide accurate predictions of failure based on analysis of matrix cracking alone. Further, in its generality for describing the severity of flaws loaded in several modes, it can accommodate flaw interaction and provide a rational failure criterion for fatigue failure.
SECTION VI

CONCLUSIONS

This report summarizes the current state of knowledge of static strength and fatigue behavior of bolted joints in composite materials. Not a great deal has been published concerning strength analysis of bolted joints in composite materials since publication of the literature survey of Garbo and Ogonowski (Reference 1). Predictions based on characteristic dimensions, coupled with various failure criteria, have been compared with recently published test data and show good correlation. BJSFM is representative of the best of these techniques and has been shown to provide excellent correlation with test results encompassing many joint parameters.

Recent research programs have provided insight into fatigue damage initiation and propagation in composite materials. Generally, damage initiates immediately as debonds between fibers and matrix at geometric discontinuities, debonds rapidly progress to matrix cracks which progress slowly until delaminations occur. Delaminations and matrix cracking interact to rapidly degrade the matrix until fiber rupture or buckling causes final failure. The sequence of damage progression appears to be laminate dependent, with 45 degree plies observed to play a key role in the process. Damage propagation from notches and holes in tension-tension fatigue initiates in the longitudinal directions and appears to be well contained between axial splits at hole edges or between the specimen edge and notch root for edge notches. Delaminations appear to be most severe at interfaces between 0 and angle plies nearest the free surface. Frequency is shown to have a pronounced effect on the fatigue behavior of composite material.

Research on the mechanisms of composite fatigue behavior is beginning to lead toward development of analytical prediction procedures. These procedures include detailed modeling of the mechanical behaviors and failure modes through finite element analyses, such as those of Kanninen, et al. and Ratwani and Kan, as well as more fundamental examination of fatigue failure modes, such as methods of Badaliance and Wang. Fundamental approaches based on rigorous analyses of simplified models is a preferred starting point for development of a fatigue analyses methodology. These approaches are more easily applied to the wide variety of laminates and geometries found in aircraft structures and their limitations are more easily identified than are those approaches based on finite element analyses. Finally, the effect of load frequency on fatigue life of composite structures requires considerably more investigation. Care must be exercised in preparing test programs so that the effects of cyclic loading frequency will not influence the trends of the data.
REFERENCE


22. 't Hart, W.G.J., "Residual Strength of (0, +45)\textsubscript{s} and (+45, 0)\textsubscript{s} Carbon/Epoxy Laminates", National Aerospace Laboratory NLR, Report NLR-TR-77115U, 1977.


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