PROGRAM ANALYSIS AND EVALUATION DIRECTORATE

ACTIVITIES SUMMARY

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US ARMY ARMAMENT MATERIEL READINESS COMMAND

PROGRAM ANALYSIS AND EVALUATION DIRECTORATE

ROCK ISLAND, ILLINOIS 61299
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Program Analysis and Evaluation Directorate Activities Summary

**Note - Final**

**US Army Armament Materiel Readiness Command**
Program Analysis and Evaluation Directorate
Rock Island, IL 61299

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This report contains five Memoranda for Record (MFRs) representing some of the activities of the Program Analysis and Evaluation Directorate, US Army Armament Materiel Readiness Command, Rock Island, IL 61299. Subjects dealt with are the systems analysis of cannon damage in the M110/M110A1 systems and an approximation for the standard cumulative production cost using learning theory.
19. Key Words:

M650 projectile
M422 projectile
8-inch ammunition
Production cost
Learning theory
Wholesale logistics
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Memorandum for Record

METHODS FOR PROCESSING SIGNALS
FROM ROTARY POTentiOMETERS
WITH APPLICATION TO THE
REDUCTION OF M110A2 LOADER-RAMMER
TEST DATA

George Schlenker
Lanny Wells
MEMORANDUM FOR RECORD

SUBJECT: Methods for Processing Signals from Rotary Potentiometers with Application to the Reduction of M110A2 Loader-Rammer Test Data

1. Reference:

2. Background

The work reported in this memorandum completes one task in a project associated with a problem experienced by the M110A2 SP howitzer system. Cannon damage reported after fielding the M110A2 in Europe in 1978 was found to be associated with firing the M106 projectile from an unseated position. Some projectiles had apparently been improperly rammed and had fallen back upon the propelling charge when the gun tube was elevated. Because the loader-rammer (L/R) was implicated in this problem, a series of tests were devised to explore the limits of rammer function. Certain proposals to improve the standard loader-rammer were submitted and as a consequence, the scope of the loader-rammer tests was expanded in order to compare the performance of the standard configuration with the proposed modification. Tests have been conducted at Yuma Proving Ground (YPG), as reported in Reference a. An additional L/R test program was conducted at Jefferson Proving Ground (JPG) during August 1979 using a different M110A2 weapon. This test was considerably more modest in scope than the Yuma tests, having been intended as a corroborative test. The JPG tests measured only displacement of the rammer head using a rotary potentiometer (pot) attached to the axle of the chain sprocket. From these data it was desired to derive velocity and acceleration of the rammer as functions of time. The velocity functions for different experimental systems (or treatments) would then be compared to identify differential effects.
3. In practice, however, rotary potentiometer signals contain spurious noise and end-of-cycle non-linearity. Additionally, the displacement cycles must be smoothly pieced together, i.e., "stacked", to yield a continuous displacement function which can be differentiated. These effects (1) (and some others) pose analytic difficulties which must be addressed to obtain meaningful velocity functions.

4. **Purpose**

There are two principal purposes in writing this MFR: to set down the lessons learned from trying various methods of analysis and to indicate what differences appear in the velocity functions estimated for various experimental systems run at JPG. Relative to the first purpose, the authors would be gratified if some of the techniques used in the present analysis were applied or refined in future analyses by others confronted with similar problems.

5. **Organization**

The remainder of this memorandum is organized as follows: Data reduction problems and methods of treatment are contained under Methodology. Results of applying these methods to the present problem are presented under Results, where some of the lessons are illustrated. Finally, the methodological conclusions and specific inferences for the JPG loader-rammer data are presented under Conclusions. For the reader who is not interested in supporting evidence the final section may alone serve as an adequate summary.

6. **Methodology**

The displacement of the rammer head as measured by the rotary potentiometer is divided into cycles each of which represents one rotation of the pot. The potentiometer signal rises to a maximum voltage and then abruptly falls to its minimum value at the end of a cycle. However, experience indicates that the potentiometer has a finite resolution. This causes non-linearity and uncertainty in voltage at the beginning and end of each cycle and produces a finite fall time in passing from

---

(1) In view of all of the problems encountered in obtaining a valid velocity function from rotary pot signals, one may ask why not use an alternative type of instrument such as a tachometer? In the present case, the authors did not have a choice of instruments and had to use the available data.
max to min values. Generally also, the voltage swing corresponding to one rotation of the pot is not known precisely but must be estimated from the noisy output signal. To obtain a displacement signal one must scan the (analog) record for max and min values \( y_{\text{max}} \) and \( y_{\text{min}} \) -- or some average max and min over cycles \( \bar{y}_{\text{max}} \) and \( \bar{y}_{\text{min}} \) -- and obtain a scale factor of displacement per volt by dividing the cycle length (24 inches in the present case) by the voltage swing \( (y_{\text{max}} - y_{\text{min}}) \). This scheme is implemented in the computer program provided in Annex 2.

7. Processing of the signals will generally be performed on a digital computer, so that an analog-to-digital (A to D) conversion of the pot signals is required. Our experience indicates that this operation is also a source of noise. In fact, A to D conversion seems to yield quantization of the signal in which certain "favorite" numbers repeatedly appear. Ways to minimize this effect include recording at a maximal signal level and time sampling at a high data rate. This rate may be limited by the available high-speed data storage but should be at least 20 times the highest significant* frequency seen in the signal. For example, our analog pot signals were time-sampled at 500 hertz (2 ms intervals) since an upper frequency limit of 25 hertz was expected. Present experience indicates that filtering the analog signal with a 100 hertz pass band before digitizing creates more problems than it solves. This type of filtering exaggerates the non-linearity in the end-of-cycle pot signal and leads to stacking difficulty.

8. To obtain a stacked displacement signal which is sufficiently smooth to tolerate a differencing procedure requires some digital filtering. Two types of non-recursive filters were tried -- a symmetric moving average and a filter having exceptional discrimination and low ripple outside the passband. The latter was designed by a method devised by W. D. Hibler (Reference b). The modulus and squared modulus of the transfer function of these filters are shown in Figures 1 through 4 of Annex 1. In spite of the theoretical advantage of Hibler's filter, it did not prove to be as satisfactory in this application as a simple symmetric moving average. Consequently, the results displayed in the figures of Annex 1 were generated using a symmetric moving average filter having a maximum absolute lag of 25. With a sampling period of 2 ms, this moving-average filter has a resolution bandwidth of 10 hertz (equivalently, an averaging period of 0.1 sec). An averaging period of about 0.1 sec is required

* At the -20 db level.
SUBJECT: Methods for Processing Signals from Rotary Potentiometers with Application to the Reduction of M110A2 Loader-Rammer Test Data to achieve the proper degree of smoothing of the displacement judged necessary for this application.

9. When successive potentiometer cycles are stacked by adding the previous cumulative full-cycle value to the pot reading, small discontinuities appear at the ends of cycles. To eliminate these and other outliers from the displacement record it was desirable to use an outlier detecting and purging algorithm. Simply relying on filtering to reduce the effect of these outliers was not practical. The outlier-detecting procedure works as follows: First, the unmodified, stacked displacement signal is fitted with a (6th degree) polynomial function of time using multiple linear regression. The standard error of the estimate from this regression is then used as a measure of discrepancy of a data point from the trend. Data points which are more than about three standard errors from the fitted function were replaced by the value of the fitted function. Finally, the moving average was calculated to attenuate the pot noise. Actually, expurgation of outliers and filtering can occur in the same DO-loop of a computer program as was done in the program in Annex 2.

10. Having obtained a suitably smooth displacement function, velocity is estimated using a first-order central-difference approximation:

\[ \frac{dy_i}{dt} \approx \frac{y_{i+1} - y_{i-1}}{2h}, \]

with time step \( h \).

Although the values of \( y_i \) above may be moving-average displacements, the differencing operation produces high-frequency noise which may be distracting to the person who examines this unsmoothed velocity estimate. Consequently, we have found it convenient to smooth the above estimate by passing the unsmoothed velocity through a digital filter whose passband somewhat exceeds that of the moving-average filter which was applied to the displacement signal. In the present application, a non-recursive filter with a max lag of 6 was used for this purpose.

11. Another, completely different approach to estimating velocity and acceleration was used and found to produce valid results if properly restricted. This approach is referred to as analytic since the velocity and acceleration can be represented analytically as polynomial functions of time. In our algorithm the moving-average displacement is fitted with a sixth degree polynomial. (A fifth degree polynomial works almost as well.) This function is, of course, analytically differentiable yielding a fourth degree polynomial estimate of acceleration. Both velocity and acceleration estimates which are derived in this manner are displayed among the results in Annex 1. Clearly, analytic estimates may be rather gross if the domain of the fitted function is too large or if time derivatives beyond the sixth are numerically significant anywhere over the domain.
12. Results

The results of this study are presented in the form of plots of output from the computer program. The same types of graphs are given for each of 24 rammer tests. The sequence of the plots is as follows:

(a) Raw potentiometer signal (a function of time);

(b) Displacement signal produced by stacking the pot signal, purging and replacing outliers and scaling;

(c) Analytic estimate of velocity using the 6th degree polynomial fit to the displacement data in (b);

(d) Analytic estimate of acceleration;

(e) Unsmoothed estimate of velocity obtained by differencing the moving average displacement;

(f) Smoothed velocity estimate; from (e);

(g) Crossplot of the smoothed velocity versus the analytic estimate of displacement.

The last estimate was the most useful in comparing the results from different rams since it is independent of time zero which generally varied between rams. The sequence of graphs proceeds from the bottom to the top of each page through all ram numbers without a page break.

13. A description of the experimental systems tested is provided in Table 1. Table 1 also summarizes the average (AVG) and standard deviation (SD) of the ram speed obtained in the last full cycle of rotary pot motion. The first 10 rams were conducted using the M2A2 cannon, i.e., an M110 howitzer configuration. The remaining 14 rams were conducted in the M10A2 howitzer using an M201 cannon. Because of the difference in cannons the total travel of the rammer head is 85 inches in the M110 and about 93 inches in the M10A2. Due to this difference the same rammer produces a somewhat differently shaped velocity function in the two systems. This difference is quantified in the Conclusions. The standard loader-rammer (L/R STD) was used for the first five rams and the modified configuration (L/R MOD) for the next five. As noted in Table 1, both L/R configurations were also tested with the M201 cannon. For each of these configurations testing was done at maximum
acceptable L/R system pressure (5 rams each) and at minimum acceptable
L/R system pressure (2 rams each). Although velocity differences between
L/R configurations in the M110A2 are slight, the effect of L/R system
pressure on velocity is noticeable. A reduction in the ram speed near
end-of-travel of from 0.4 to 1.0 f/s accompanies the noted pressure reduction.

14. The data set obtained from digitizing the analog rotary pot signal for
a typical ram consists of approximately 600 points. Sets of this size per-
mit data processing of the sort described above while retaining an adequately
large value of degrees of freedom. Of course, the use of a non-recursive
filter shortens the unfiltered data set at each end by the value of the
maximum lag used in the filter. Thus, in this application 25 data points
were eliminated from each end of the displacement data to obtain the moving
average displacement.

15. The first twenty figures in Annex 1 are intended to illustrate the
effect of various errors associated with the data and with the processing
methods. These illustrations support previous claims of lessons learned
and give the reader some "feel" for the problems encountered in using data
from rotary potentiometers. Starting with Rammer Test No 1 shown in Fig
5, one can observe the kind of stacking error typically encountered in pro-
cessing the pot signal. Without identifying and purging the resulting out-
liers from the displacement signal (Fig. 5) a characteristic set of spikes
is produced in the unsmoothed velocity estimate (Fig. 7). Even after smooth-
ing the velocity, anomalous bumps (reduced spikes) are seen in the velocity-
time plot (Fig. 8) and velocity-displacement plot (Fig. 9). One can also
see the effect on velocity of potentiometer non-linearity -- the 0.1 sec
shallow waves appearing in the velocity after level off.

16. Figure 10 illustrates another approach to filtering the displacement
signal prior to differencing to obtain velocity. The Hibler filter, describ-
ed earlier, was applied to the unexpurgated displacement (Fig. 5) and differ-
enced to yield the unfiltered velocity shown in Figure 10. Because of the
sharp frequency-domain cutoff and low ripple in the stop band, one would
expect the Hibler filter to perform better than a simple moving average.
However, the smoothed velocity estimate (Fig. 11) is not subjectively better
than its moving-average counterpart (Fig. 9). Further reduction of the
cutoff frequency of the filter would improve smoothness and reduce the
effect of spurious spikes in the displacement but would lack resolution and
introduce velocity bias. A better strategy for data reduction is to prepro-
cess the unfiltered displacement to detect and purge outliers. This proce-
dure does not sacrifice resolution bandwidth as does narrow-passband filter-
ing. See Figure 19 for a better estimate using data from ram test number 1.
DRSAR-PEL
13 November 1979
SUBJECT: Methods for Processing Signals from Rotary Potentiometers with Application to the Reduction of M110A2 Loader-Rammer Test Data

Figure 12 also illustrates the effect of smoothing with Hibler's filter—in this case on the data from ram number 2. Here also, one may conclude that the velocity estimated from differencing the filtered displacement is not better than simply using a moving average on the expurgated data. See Figure 29 for comparison.

17. A surprising thing happened to the data for several rams: An A to D transcription and/or tape copying error caused intervals of data to be omitted in the final (digital) record. Figure 13 illustrates this incident in the pot signal for ram test number 5 and Figure 14 indicates the resulting error in the expurgated displacement. Evidently the outlier-detecting algorithm cannot cope with this kind of error. The consequence of this error is serious as can be seen in Figures 15 and 16. In this case some manual editing of the data was necessary before reprocessing (Figs. 43 and 44).

18. One additional type of error is noteworthy— inclusion of too many pre-ram and post-ram data points in the displacement record. If a long post-ram plateau occurs, as in Figure 23, the analytic velocity estimate (Fig. 24) will be quite poor. The sixth-degree-polynomial fit becomes inadequate if the domain is not properly bounded. By contrast, a displacement record having a truncated plateau (Fig. 25) yields an excellent velocity estimate (Fig. 26) and a reasonable acceleration function (Fig. 27). No attempt was made to rectify the analytic estimate of acceleration outside of its applicable domain.

19. One is able to observe specific differences in the ram velocity functions between different experimental systems (treatments) even when the data contain the imperfections described here. One reason for this discrimination is the consistent manner with which stacking error and filtering bias enter the velocity estimated from run to run. These velocity differences in the JPG data are discussed under Conclusions.

20. Conclusions

Conclusions are summarized here under two categories: general methodological observations and specific inferences about the M110/M110A2 L/R tests at JPG. Relative to methods for analyzing rotary potentiometer signals:

(1) It is best to stack digitized rotary potentiometer signals which have an abrupt drop from max to min values. Therefore, a broad-band analog signal is a desirable starting point since this affords the most abrupt drop.
(2) Some stacking error appears to be unavoidable. Therefore, it is essential to process the stacked displacement signal with an outlier-detecting and purging algorithm. As a minimum, such an algorithm should obtain departures from a global average, as was done here by polynomial regression.

(3) Occasional analog-to-digital encoding errors and digital data omissions occur. Therefore, it is important to display the digitized potentiometer signal graphically to assist in detecting this type of error. In this case manual editing of the data is necessary.

(4) Elimination of stacking and encoding errors solely by high-discrimination digital filters (such as Hibler's) seems to be impractical, since a sufficiently narrow filter frequency passband (or long period) to reduce the effect of the errors in the output signal would incur an objectionable degree of bias.

(5) Although somewhat cosmetic, applying a digital filter to the velocity signal to smooth it is recommended. To avoid additional bias the passband of this filter should be larger than that applied to the displacement signal used to develop the velocity estimate.

(6) If a gross-average velocity estimate is desired, one can obtain a good analytic approximation from a high-degree polynomial in time fitted to the displacement data over a restricted domain of this function. Do not expect this analytic velocity estimate to hold outside of this limited domain.

(7) Because of the generally arbitrary nature of time zero and because of temporal shifts due to filtering, it is recommended that comparisons of two distinct runs be made in phase space, i.e., via a crossplot of velocity vs displacement.

(8) In making phase-space plots it is desirable to use a highly smoothed estimate of displacement, e.g., by using regression, so as not to incur anomalous multi-point function incidents.

21. With respect to the results of the M110/M110A2 L/R tests at JPG:

(1) The form of the velocity versus displacement plots are nearly identical for repeated rams with a specific experimental system -- cannon type, L/R type, and L/R system pressure.

(2) Differences in these plots appear between experimental systems in two respects: the max velocity level achieved and the displacement at which a given level is achieved.
During the first ten ramming tests using the M2A2 cannon at max L/R pressure, rams 1 through 5 apply to the L/R STD configuration and rams 6 through 10 to the L/R MOD. The former generally achieve a slightly higher peak velocity than the latter. Another distinction in these tests is in the shape of the velocity function. The velocity of L/R STD exhibits a somewhat more convex (hump-backed) shape than that of the L/R MOD.

The shape of the velocity function which characterizes rams 1 through 5 also differs from that of rams 11 through 15. The latter set has a decidedly sway-back shape with a shallow dip in the velocity at midram. Additionally, a somewhat shorter rise to peak velocity is noted in the latter set. The difference in experimental system in these sets is simply a difference in cannon type; the M2A2 cannon was used in rams 1 through 5 whereas the M201 cannon was used in rams 11 through 15. Apparently, the standard rammer tends to increase its speed slightly at the longer travel experienced in the M201 cannon -- 93 inches versus 85 inches in the M2A2 cannon.

A reduction in the L/R system pressure from max to min acceptable under otherwise identical conditions causes a reduction in peak ramming speed. This reduction is anticipated because the level-off speed is controlled by a restricted flow of oil under a pressure differential which is nearly constant throughout a ram cycle. In the L/R STD configuration this speed reduction amounted to only about 0.4 f/s based upon the average over the last complete pot cycle and about 1 f/s based upon the difference in maxima (rams 11 through 15 vs 16 and 17) whereas in the L/R MOD configuration this reduction was about 1 f/s (rams 18 through 22 vs 23 and 24). For both configurations the shape of the velocity curve in phase space was not altered by the reduction in L/R system pressure.
ANNEX 1

RESULTS FROM M110/M110A2 LOADER-RAMMER TESTS AT JEFFERSON PROVING GROUND, AUGUST 1979
### Table 1. Average Final Ramming Speed for the M110A2 Loader-Rammer Obtained in the JPG Tests, 11 Aug 79

<table>
<thead>
<tr>
<th>Round Number</th>
<th>Speed (F/S)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.98</td>
<td>M2A2 Cannon</td>
</tr>
<tr>
<td>2</td>
<td>9.90</td>
<td>L/R STD(2)</td>
</tr>
<tr>
<td>3</td>
<td>9.80</td>
<td>System Pressure Max(3)</td>
</tr>
<tr>
<td>4</td>
<td>9.76</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.85</td>
<td></td>
</tr>
<tr>
<td>AVG/SD</td>
<td>9.858/0.086</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>9.55</td>
<td>M2A2 Cannon</td>
</tr>
<tr>
<td>7</td>
<td>9.44</td>
<td>L/R MOD(2)</td>
</tr>
<tr>
<td>8</td>
<td>9.44</td>
<td>System Pressure Max(3)</td>
</tr>
<tr>
<td>9</td>
<td>9.41</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>9.45</td>
<td></td>
</tr>
<tr>
<td>AVG/SD</td>
<td>9.458/0.054</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>9.70</td>
<td>M201 Cannon</td>
</tr>
<tr>
<td>12</td>
<td>9.59</td>
<td>L/R STD(2)</td>
</tr>
<tr>
<td>13</td>
<td>9.58</td>
<td>System Pressure Max(3)</td>
</tr>
<tr>
<td>14</td>
<td>9.66</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>9.44</td>
<td></td>
</tr>
<tr>
<td>AVG/SD</td>
<td>9.594/0.099</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>9.50</td>
<td>System Pressure Min(3)</td>
</tr>
<tr>
<td>17</td>
<td>9.28</td>
<td></td>
</tr>
<tr>
<td>AVG/SD</td>
<td>9.39/0.156</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>9.68</td>
<td>M201 Cannon</td>
</tr>
<tr>
<td>19</td>
<td>9.89</td>
<td>L/R MOD(2)</td>
</tr>
<tr>
<td>20</td>
<td>9.80</td>
<td>System Pressure Max(3)</td>
</tr>
<tr>
<td>21</td>
<td>9.79</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>9.67</td>
<td></td>
</tr>
<tr>
<td>AVG/SD</td>
<td>9.766/0.092</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>8.65</td>
<td>System Pressure Min(3)</td>
</tr>
<tr>
<td>24</td>
<td>8.84</td>
<td></td>
</tr>
<tr>
<td>AVG/SD</td>
<td>8.745/0.134</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(1) The "final" ramming speed was calculated as the average over the last complete rotary pot cycle.

(2) Two loader-rammer configurations were tested -- the current standard (L/R STD) and a proposed modification (L/R MOD).

(3) The pressure in the oil or nitrogen of the loader-rammer system was adjusted prior to each ram using the gage provided with the system.
FIGURE 1. MODULUS OF THE TRANSFER FUNCTION OF A SYMMETRIC MOVING AVERAGE DIGITAL FILTER

MAX LEAD (LAG) = 25
TIME STEP = 2 MS

FREQUENCY (HZ)
FIGURE 2. SQUARED MODULUS OF THE TRANSFER FUNCTION OF A SYMMETRIC MOVING AVERAGE DIGITAL FILTER

MAX LEAD (LAG) = 25
TIME STEP = 2 MS
FIGURE 3. MODULUS OF THE TRANSFER FUNCTION OF A SYMMETRIC NONRECURSIVE DIGITAL FILTER USING HIBLER'S METHOD

PARAMETERS:
MAX LEAD (LAG) = 50
PASS BAND = 5 HZ
N1 = 1
TIME STEP = 2 MS
FIGURE 4. SQUARED MODULUS OF THE TRANSFER FUNCTION OF A SYMMETRIC NONRECURSIVE DIGITAL FILTER USING HIBLER'S METHOD

PARAMETERS:
MAX LEAD (LAG) = 50
PASS BAND = 5 HZ
N1 = 1
TIME STEP = 2 MS
FIGURE 5. Illustration of one type of stacking error obtained for the rotary potentiometer signal given below. The outlier identification and replacement technique was not performed here.

FIGURE 6. Analog rotary potentiometer signal is digitized at a 500 hz rate. Digital signal is displayed here.
Velocity estimate obtained from the moving-average displacement record having the stacking error. Anomalous spikes appear at the ends of the moving average window.
Smoothed velocity vs displacement plot showing the effect of the original stacking error.

Smoothed velocity vs time plot showing the residual effect of the original stacking error.
Velocity estimate is the smoothed version of that given below.

Unsmoothed velocity estimate is obtained from a signal generated by using Hibler's filter (with 5 Hz passband) on the stacked potentiometer data.
Unsmoothed velocity estimate is obtained from a signal generated by using Hibler's filter (with 5 Hz passband) on the stacked potentiometer data.
Illustration of the performance of the algorithms for stacking and detecting and eliminating outliers. Note the effect of missing data at 0.55 sec.

Missing rotary potentiometer data at 0.55 sec caused by A to D plus transcription error.
Figure 15. Illustration of the effect of missing data on the velocity estimated from a moving average displacement signal.

Figure 16. Illustration of the effect of missing data on the smoothed velocity estimate.
FIGURE 18.

Illustration of the stacked rotary potentiometer signal (displacement) after automatic identification and deletion of outliers.

FIGURE 17.

Note that the decay to zero level occurs with finite slope.
Smoothed version of the velocity-time curve given below. Smoothing is done with a symmetric non-recursive filter having 25 hz cutoff.

Velocity estimate obtained from first-central-difference approximation applied to moving average displacement signal.
RAMMER TEST 02 AT J.P.G. FIGURE 22.

Note similarity of this rotary pot signal with that of rammer test 01.

Crossplot of velocity versus displacement. Displacement estimate used here is obtained from a sixth degree polynomial (in time) fit to the stacked and outlier-purged potentiometer data.
Illustration of a poor velocity estimate obtained from a polynomial fit to the displacement data when its domain includes a terminal plateau, as shown below.
Analytic estimate of the velocity obtained from a 6th degree polynomial function in time fitted to the displacement data shown below.

Displacement signal with an abbreviated terminal plateau. Compare with previous displacement signal given for test 02.
Figure 28

Velocity estimate obtained from a first-central-difference approximation applied to the moving average displacement signal.

Figure 27

Analytic estimate of the acceleration obtained from a 6th degree polynomial function in time fitted to the displacement signal.
Crossplot of velocity versus displacement. Velocity is the smoothed first-central-difference approximation shown below as a function of time.
Figure 32.

Figure 31.
RAMMER TEST 03 AT J.P.G.  FIGURE 34.

RAMMER TEST 03 AT J.P.G.  FIGURE 33.
RAMMER TEST 04 AT J.P.G. FIGURE 38.

RAMMER TEST 04 AT J.P.G. FIGURE 37.
ANNEX 2

COMPUTER PROGRAM SOURCE LISTING FOR EST. VEL. ACC

A PROGRAM FOR PROCESSING DISPLACEMENT SIGNALS FROM ROTARY POTENTIOMETERS TO ESTIMATE VELOCITY AND ACCELERATION
PROGRAM FOR PROCESSING DISPLACEMENT SIGNALS FROM ROTARY
POTENTIOMETERS TO ESTIMATE VELOCITY AND ACCELERATION

THIS PROGRAM CONTAINS EXTERNAL REFERENCES TO PRIME 400 COMPUTER
SYSTEM ROUTINES AND CALCOMP 925/936 SOFTWARE. ON THE ARRCOM S&E
SYSTEM THE FOLLOWING LIBRARIES MUST BE LOADED:
1. CWVLIB
2. VAPPLIB
3. VPLTLIB
4. VSP00$

REAL 8 DATA(1200,7), X(7), FJ, FJMJ, VEL, ACC, S, RSQ, OISPL, VELT, DELT,$RESI, XT, YEST, A(66)
DIMENSION Y(1200), YBAR(1200), S2(1200), INFO(12), N(2), TM(1200),$VMAS(1200), VMAS(1200), COFZ(13), YEST(1200), RPOT(1200)
INTEGER TN(16), TITLE(40), COMOFL(16), BUF(60)

COMMON/CPLCOM/ RXL, RYL

THE DATA(I,J) ARRAY IS USED TO STORE THE I'TH VALUE OF TIME TO THE
J'TH POWER FOR EXPONENTS UP TO AND INCLUDING NOEO. THE VECTOR
WITH J=NOEO+1 CONTAINS THE VALUES OF THE SCALED AND STACKED DEPEND-
ENT VARIABLE, DISPLACEMENT (IN). THE UNSCALED ROTARY POTENTIO-
METER DATA ARE STORED IN RPOT,

INSERT SYSCOM>RKEYS
INSERT SYSCOM>KEYS.F
INSERT SYSCOM>ERRD.F

DATA DATA/8400*0.0/, INFO/1, 2: 100000, 3= 6*0/, VMAS/1200*0.0/,$YEST/1200*0.0/
DATA COF2/1.1286702E-02, 3.0793781E-02, 5.706223E-02, 8.6293384E-02, 1.1312581E-01, 1.3202178E-01,$.3883263E-01, 1.3202178E-01, 3.1286702E-02/
DATA COF3/1.1286702E-02, 3.0793781E-02, 5.706223E-02, 8.6293384E-02, 1.1312581E-01, 1.3202178E-01,$.3883263E-01, 1.3202178E-01, 3.1286702E-02/

CALL OPENL("INPUT FILE TREENAME?...> ", 26, A$READ, TN, 32, 1)

PROCESS DATA DIRECTLY FROM TAPE

45
CALL TNOUR('TAPE UNIT=2...> ',16)
READ(1,*) NU
NU=4
CALL CM13(1.0,NU,0)  /* OPEN TAPE TO READ
CALL OPEN'A(A*READ,'PEXLOW>TAPE-DUMP',18,NU)

DO 104 NFILE=1,24
CALL CM13(6.0,NU,0)  /* SKIP TO NEXT FILE
CALL CM007(NU)
IF(NFILE.LT.6) GO TO 104  /* SKIP FIRST 4 FILES

OPEN SCRATCH FILE TO SAVE TERMINAL DIALOGUE

CALL OPNSFL(3,COMOFL)
CALL CLOS'A(3)
CALL COMOSS(:000020,COMOFL,32.0,COOE)

IF(COCE.GT.0) GO TO 103
CALL OPEN'A('OUTPUT FILE TREENAME?...> ',26,A*WRIT,TN,32,2)
ENCOD(32,12,TN) NFILE
12 FORMAT('RAMMER-TEST * ',B'***')
CALL OPEN'A(A*WRIT,TN,32,2)
CALL TRNC'SA(2)  /* TRUNCATE OUTPUT FILE

READ TITLE OF DATA SET

READ(5,2) TITLE
2 FORMAT(40A2)
CALL I*AM13(NU,TITLE,40.0)
CALL I*AO07(NU,TITLE,40.0)
WRITE(6,1) TITLE
1 FORMAT(1H ,40A2)
CALL I*AM13(NU,TITLE,40.0)
CALL I*AO07(NU,TITLE,40.0)
WRITE(6,1) TITLE
CALL CM13(5.0,NU,0)  /* SKIP 3RD REC
CALL I*AO07(NU,BUF,40.0)
CALL I*AM13(NU,BUF,60.0)  /* READ RECORD INTO BUF
CALL I*AO07(NU,BUF,60.0)
OECOC(120.11,BUF) POINTS,DELTAT
11 FORMAT(2016.8)
TIME=0.0
NLZERO=0
NDATA=POINTS-1.0
NSET=NDATA
YSCALE=10.909
YMIN=-0.01
IHIB=0   /* INDICATOR VARIABLE FOR HIBLER'S FILTER
FC=5.0   /* CUTOFF FREQUENCY (HZ) IN FILTER
YD=0.0
NCH=NLEN*(TITLE,4D)

C
READ CONTROL PARAMS
C
READ(5, *) TIMED,NZERO,NSET,NDATA,DELTAT,YSCALE,YMIN,YO
NSCALE=YSNO*Q('SELF SCALING DESIRED?...> ',26,A*NEF)
NSCALE=.FALSE.
DELT=DELTAT
ISTART=NZERO+1
ISTOP=NZERO+NDATA

C
READ THE OUTLIER LAG PARAM
C
CALL TNOUA('OUTLIER LAG PARAMETER?...> ',27)
READ(1, *) MLAQ
MLAQ=25

C
READ MIN NUMBER OF STANDARD DEVIATIONS TO DECLARE AN OUTLIER
C
CALL TNOUA('NUMBER OF SO S FOR OUTLIER?...> ',32)
READ(1, *) SDNO
SDNO=2.64

C
READ DEGREE OF POLYNOMIAL FIT
C
CALL TNOUA('DEGREE OF FITTING POLYNOMIAL?...> ',34)
READ(1, *) NDEG
NDEG=6
NDEG1=NDEG+1
MAVG=2*MLAQ+1
FMVG=FLOAT(MAVG)
ISTART=ISTART+MLAQ
ISTOP=ISTOP-MLAQ
I1=ISTART+MAVG-1

47
C REAO(5,*) (Y(I),I=ISTART,ISTOP)
NREC=NDATA/6+MINO(1,MOD(NDATA,8))
DO 13 I=1,NREC
C CALL IAM13(NU,BUF,60,15)
CALL IA007(NU,BUF,60,15)
00 TO 13
15 CALL PRERR
CALL EXIT
C IF(NFILE.NE.29) GO TO 105
C CALL GETERR(BUF,2)
C IF(BUE(1).NE.'IE') GO TO 106
C CALL IAM13(NU,BUF,60,0)
C CALL IA007(NU,BUF,60,0)
13 DECODE(120,*,BUF) (Y(8*I+ISTART-9+J),J=1,8)
C
C DETERMINE AN AVERAGE INITIAL VALUE
C
SUM=0.0
ISPI9=ISTART+19
DO 94 I=ISTART,ISP19
94 SUM=SUM+Y(I)
YO=SUM/20.0
YLIM=93.0/YSCALE+YO
C
YRANGE=24.0/YSCALE /* 24 INCHES PER POT CYCLE
YMAX=YMIN+YRANGE
TOLER=0.01*YRANGE
IF(.NOT..NSCALE) GO TO 90
C
C SCAN DATA FOR MAX AND MIN
C
NCYCLE=1
YMAX=Y(ISTART)
YMIN=Y(ISTART)
IOO=ISTART+1
DO 80 I=IOO,ISTOP
IF(Y(I).GT.YMAX) YMAX=Y(I)
IF(Y(I).LT.YMIN) YMIN=Y(I)
80 CONTINUE
YRANGE=YMAX-YMIN
YSCLAE=24.0/YRANGE
TEST=YRANGE/2.0
YIM1=Y(ISTART)
DO 82 I=ISTART,ISTOP
IF(ABS(YIM1-Y(I)) .GT. TEST) NCYCLE=NCYCLE+1
YIM1=Y(I)
82 CONTINUE
DISPF=YSCALE*(Y(ISTOP)-YO+FLOAT(NCYCLE-1)*YRANGE)
IF(DISPF .GT. 83.0 .AND. DISPF .LT. 94.0) GO TO 90

C
C    FIRST SELF-SCALING PROCEDURE PRODUCES FINAL DISPLACEMENT OUT-OF-
C    BOUNDS. TRY A SECOND.
C
WRITE(1,85) DISPF
85 FORMAT( 'FIRST SELF-SCALING FAILED! DISPF = ' , F10.6 , ' IN' )
YRANGE=(Y(ISTOP)-YO)/(92.5/24.0-FLOAT(NCYCLE-1))
YMAX=YRANGE+YMIN
YScale=24.0/YRANGE
TOLER=0.0
C
C    STACK DISPLACEMENT CYCLES AND ASSIGN THE ROTARY POT SIGNAL TO
C    THE VECTOR RPOT(I)
C
90 TEST=YRANGE/2.0
YIM1=Y(ISTART)
RPOT(ISTART)=Y(ISTART)
YI=Y(ISTART)
ADDN=0.0
ISP1=ISTART+1
Y(ISTART)=0.0
DO 5 I=ISP1,ISTOP
RPOT(I)=Y(I)
IF(Y(I) .GT. YMAX+TOLER) GO TO 101
IF(YIM1-Y(I) .GT. TEST .AND. YI.LT.YLIM) ADDN=ADDN+YRANGE
YIM1=Y(I)
Y(I)=Y(I)+ADDN-YO
5 YI=Y(I)
DO 3 I=ISTART,ISTOP
TM(I)=TIME0+FLOAT(I-ISTART)*DELTAT
II=I-ISTART+1
DATA(II,1)=TM(I)
DO 3 J=2,MDEG
DATA(II,J)=DATA(II,J-1)+DATA(II,1)
3 CONTINUE
ESTIMATE NOISE VARIANCE OF THE STACKED SIGNAL VIA POLYNOMIAL REGRESSION

\[ \text{DATA}(I) = Y(I) \]

CALL MLR(X, XT, N, ND1, 1, TRUE, S, RSQ)

\[ \text{VARERR} = S \]
\[ SD2 = \sqrt{\text{VARERR}} \]
\[ SUM = 0.0 \]
\[ SSQ = 0.0 \]

\[ \text{DO 10 I = ISTART, ISTOP} \]
\[ SDM = \text{SUM} + Y(I) \]
\[ 10 \text{ SSQ} = \text{SSQ} + Y(I)^2 \]

\[ YBAR(I) = \frac{\text{SUM}}{\text{MAXV}} \]
\[ S2(I) = \frac{\text{SSQ}}{\text{MAXV}} - YBAR(I)^2 \]

WRITE HEADINGS

WRITE(6, 14)

14 FORMAT(1H1, 'RESULTS OF TEST FOR OUTLIERS -- ORIGINAL VALUES -- REP LACEMENTS', /H0, 2X, 'INDEX', T12, 'POT VALUE (MV)', T29, 'MOVING AVG', + T46, 'STO OEV', T58, 'REPL VAL (MV)', T76, 'MOVING AVG', T89, + 'REPL STD OEV')

RECURSIVELY CALCULATE MOVING AVERAGES AND MOVING VARIANCES AND PURGE DATA OF OUTLIERS. IF THE HIBER-FILTER OPTION IS SELECTED, THE MOVING AVERAGE IS REPLACED BY A LOW-PASS FILTERED OUTPUT.

IF(IHIB .EQ. 1) GO TO 107

NOUT = 0
SERR = 0.0

\[ \text{DO 20 L = LSTART, LSTOP} \]
\[ LPRINT = L - ISTART + 1 \]
\[ IOUT = 0 \]
\[ III = L - ISTART + 1 \]
\[ YESTD = X(1) \]
\[ \text{DO 9 J = 2, ND1} \]
\[ 9 \text{ YESTD} = \text{YESTD} + X(J) \times \text{DATA}(II, J-1) \]
\[ \text{YEST}(L) = \text{YESTD} \times \text{YSIZE} \]
\[ \text{SERR} = \text{SERR} + (\text{YEST} - Y(L))^2 \]

50
S01=SQRT(SERR/FLOAT(L-LSTART+1))

IF(ABS(Y(L)-YBAR(L)) .LE. S00*S02) GO TO 25
IOUT=1
NOUT=NOUT+1
C
C Y(L) IS AN OUTLIER. REPLACE WITH Y(L-1) AND CORRECT MOVING MEAN
C
H0L01=Y(L)
Y(L)=YEST0
H0L02=YBAR(L)
YBAR(L)=YBAR(L)+(Y(L)-H0L01)/FMAVG
C WRITE(6,16) LPRINT,H0L01,H0L02,S01,Y(L),YBAR(L),S02
16 FORMAT(1H ,I5,5X,1P6G15.6)
25 YBAR(L+1)=YBAR(L)+(Y(L+1+MLAG)-Y(L-MLAG))/FMAVG
IF(IOUT) 26,26,20
26 CONTINUE
C WRITE(6,18) LPRINT,Y(L),YBAR(L),S01
18 FORMAT(1H ,I5,5X,1P3G15.6)
20 CONTINUE
WRITE(6,23) NOUT
23 FORMAT(1HO,'NUMBER OF OUTLIERS FOUND ANO REPLACED IS ',I5)
24 CONTINUE
C WRITE(6,19)
19 FORMAT(1H1,T6,'TIME (S) ',T19,' M A VEL (F/S) ')
L00=LSTART+1
LSM1=LSTOP-1
00 21 L=L00-LSM1
VMA(L)=YSCLAE/24.0/DATAT*(YBAR(L+1)-YBAR(L-1))
C WRITE(6,22) TM(L),VMA(L)
22 FORMAT(1H ,1P2G15.6)
21 CONTINUE
C
C COPY OUTLIER-PURGED DATA INTO DATA(I,NDEOP1)
C
DO 30 I=ISTART,ISTOP
Y(I)=Y(I)*YSCLAE
30 DATA(I-1START+1,NDEOP1)=Y(I)/12.0
CALL MLR(DATA,X,1200,7,NDATA,NDEOP1,1..TRUE..S.RSQA
C
C WRITE HEADINGS FOR DISPLACEMENT, VELOCITY AND ACCELERATION
C
WRITE(6,38)
DISPL=0.00 00
VELT=0.00 00
DO 42 I=ISTART,ISTOP
II=I-ISTART+1
XT=X(1)+X(2)*DATA(II,1)
VEL=X(2)
ACC=2.00 00*X(3)
DO 40 J=2,NDEO
XT=XT+X(J+1)*DATA(II,J)
FJ=FLOAT(J)
FJMJ1=FLOAT(J-1)
VEL=VEL+FJ*X(J+1)*DATA(II,J-1)
IF(J.OE.3) ACC=ACC+FJMJ1*X(J+1)*DATA(II,J-2)
40 CONTINUE
RESID=XT-DATA(II,NDE0+1)
DISPL=DISPL+6.00 00*(VELT+VEL)
VELT=VEL
XT=12.0*XT
WRITE(6,43) DATA(II,1),DISPL,VEL,ACC,XT,RESID
43 FORMAT(1H1,T6,'TIME (S)'.T19,'I OF V (IN)'.T35,'VELOCITY (F/S)'.
*T60,'ACCEL (F/S/S)'.T62,'SIGNAL (IN)'.T62,'RESIDUAL (FT)')

ASSIGN VELOCITY TO YBAR AND ACCELERATION TO S2 FOR PLOTTING

YBAR(I)=VEL
S2(I)=ACC
42 CONTINUE

PLOT ORIGINAL POT DATA

N(1)=1
N(2)=DATA
WRITE(6,41)
41 FORMAT(1H1,T20,'ORIGINAL ROTARY POTENTIOMETER SIGNAL (MV)')
CALL PLOT(TM(ISTART),RPOT(ISTART),N,7.0,4.0,0.0,'O')
WRITE(6,46)
CALL CLOT(TM(ISTART),RPOT(ISTART),N,7.0,4.0,0.0,0.1)
XPAGE=0.5
YPAGE=-0.5
CALL SYMBOL(XPAG,YPAG,0.084,'TIME (SEC)',0.0,10)
XPAG=-0.5
YPAGE=0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'DISPL SIG (MV')',90.0,14)
XPAGE=0.5
YPAGE=AYL+0.25
CALL SYMBOL(XPAGE,YPAGE,0.098,TITLE,0.0,NCH)

C PLOT UNFILTERED DATA

WRITE(6,44)
44 FORMAT(1H1,T20,'UNFILTER DISPLACEMENT SIGNAL (MV)')
46 FORMAT(1H1,T20,'TIME (SEC)')
CALL PPLOT(TM(ISTART),Y(ISTART),N,7.0,4.0,0,'O')
WRITE(6,46)
CALL CPLOT(TM(ISTART),Y(ISTART),N,7.0,4.0,0,0.1)
XPAGE=0.5
YPAGE=-0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'TIME (SEC')',0.0,10)
XPAGE=-0.5
YPAGE=0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'DISPL SIG (MV')',90.0,14)
XPAGE=0.5
YPAGE=AYL+0.25
CALL SYMBOL(XPAGE,YPAGE,0.098,TITLE,0.0,NCH)

C PLOT ESTIMATE OF VELOCITY

WRITE(6,74)
74 FORMAT(1H1,T20,'ESTIMATE OF VELOCITY (F/S)')
N(2)=NOATA
CALL PPLOT(TM(ISTART),YBAR(ISTART),N,7.0,4.0,0,'V')
WRITE(6,46)
CALL CPLOT(TM(ISTART),YBAR(ISTART),N,7.0,4.0,0,0.1)
XPAGE=0.5
YPAGE=-0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'TIME (SEC')',0.0,10)
XPAGE=-0.5
YPAGE=0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'VEL (F/S')',90.0,9)
XPAGE=0.5
YPAGE=AYL+0.25
CALL SYMBOL(XPAGE,YPAGE,0.098,TITLE,0.0,NCH)
PLOT ESTIMATE OF ACCELERATION

WRITE(6,76)
76 FORMAT(1H-,T20,"ESTIMATE OF ACCELERATION (F/S/S)'")
CALL PLOT(TM(ISTART),S2(ISTART),N,7.0,4.0,0.0,'A')
WRITE(6,46)
CALL CPlot(TM(ISTART),S2(ISTART),N,7.0,4.0,0.0,0.1)
XPAGE=0.5
YPAGE=-0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'TIME (SEC)',0.0,0.10)
XPAGE=-0.5
YPAGE=0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'ACC (F/S/S)',90.0,0.11)
XPAGE=0.5
YPAGE=AYL+0.25
CALL SYMBOL(XPAGE,YPAGE,0.098,TITLE,0.0,NCH)

PLOT THE MOVING AVERAGE VELOCITY

N(2)=NDATA-2*(MLAO+2)
WRITE(6,78)
78 FORMAT(1H1,T20,"EST. OF M. A. VELOCITY (F/S)'")
CALL PPlot(TM(LOO),VMA(LOO),N,7.0,4.0,0,'V')
WRITE(6,46)
CALL CPlot(TM(LOO),VMA(LOO),N,7.0,4.0,0.0,0.1)
XPAGE=0.5
YPAGE=-0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'TIME (SEC)',0.0,0.10)
XPAGE=-0.5
YPAGE=0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'VEL (F/S)',90.0,0.9)
XPAGE=0.5
YPAGE=AYL+0.25
CALL SYMBOL(XPAGE,YPAGE,0.098,TITLE,0.0,NCH)

SMOOTH THE MOVING AVERAGE VELOCITY

M=6
COP=40.0*DELTAT
KMAX=2*M+1
II1=LOO+M
II2=LSM1-M
C WRITE(6,29) COP
29 FORMAT(1H1,T10,'SMOOTHED MOVING-AVERAGE VELOCITY',/1H0,T10.
    *'CUT-OFF PERIOD (S) = ',1P015.6,/1H0,/1H0,T6,'TIME (S)',.T20.
    +'FILT VEL (F/S)',.T36,'DISPL (IN)')
   00 31 II=II1,II2
   SUM=0.0
   00 33 K=1,KMAX
   33 SUM=SUM+COF2(K)*VMAS(II-K-MMAX-1)
   VMAS(II)=SUM
C WRITE(6,32) TM(II),VMAS(II),Y(II)
31 CONTINUE
32 FORMAT(1H1,1P3915.6)
C PLOT THE FILTERED MOVING-AVERAGE VELOCITY
C
N(2)=N(2)-KMAX
WRITE(6,79)
79 FORMAT(1H1,T20,'EST. OF FILTERED M. A. VELOCITY (F/S)')
   CALL PPL0T(TM(II1),VMAS(II1),N,7.0,4.0,0.'V')
   WRITE(6,46)
   CALL CPLOT(TM(II1),VMAS(II1),N,7.0,4.0,0,0.1)
   XPAGE=0.5
   YPAGE=-0.5
   CALL SYMBOL(XPAGE,YPAGE,0.084,'TIME (SEC)',0.0,0.10)
   XPAGE=-0.5
   YPAGE=0.5
   CALL SYMBOL(XPAGE,YPAGE,0.084,'VEL (F/S)',90.0,9)
   XPAGE=0.5
   YPAGE=AYL+0.25
   CALL SYMBOL(XPAGE,YPAGE,0.098,TITLE,0.0,NCH)
C PLOT VELOCITY VERSUS DISPLACEMENT
C
WRITE(6,83)
83 FORMAT(1H1,T20,'CROSSPLOT OF FILT VEL (F/S) VS DISPL (IN)')
   CALL PPLOT(YEST(II1),VMAS(II1),N,7.0,4.0,0.'V')
   WRITE(6,48)
48 FORMAT(1H1,T20,'DISPL (IN)')
   CALL CPLOT(YEST(II1),VMAS(II1),N,7.0,4.0,0,0.1)
   XPAGE=0.5
   YPAGE=0.5
   CALL SYMBOL(XPAGE,YPAGE,0.084,'DISPL (IN)',0.0,0.10)
XPAGE=-0.5
YPAGE=0.5
CALL SYMBOL(XPAGE,YPAGE,0.084,'VEL (F/S)',90.0,9)
XPAGE=0.5
YPAGE=AYL+0.25
CALL SYMBOL(XPAGE,YPAGE,0.098,TITCE,0.0,NCH)

CLOSE COMO FILE AND COPY TO OUTPUT

CALL O$ADO7(2,'--- TERMINATE DIALOGUE --',12,0)
CALL COMO$$('10000B0.COMOFL,32.0,COOE')
IF(CODE.GT.0) GO TO 103
CALL OPEN$A('A*READ.COMOFL,32.3)
91 CALL I$ADO7(3,BUF,40,*92)
WRITE(6.93) BUF
93 FORMAT(1H,40A2)
GO TO 91
92 CALL CLOSE$A(3)
   CALL DECE$A(COMOFL,32)

CLOSE INPUT AND OUTPUT FILES

CALL CLOSE$A(1)
CALL CLOSE$A(2)

SPOOL OUTPUT

CALL SPOOL$(1,TN,32,INFO,DATA,7168,ICODE)
IF(ICODE.GT.0) GO TO 60
CALL TNOU(' ',1)
CALL TNOUA('YOUR SPOOL FILE IS ',19)
CALL TNOU(INFO(8),6)
104 CONTINUE

106 CALL C*M13(-4.0,NU,0)
106 CALL CLOSE$A(4)
   CALL PLOT(8.0,0.0,0.999)
   CALL EXIT
105 CALL TNOU('UNEXPECTED EOF! ',16)
   GO TO 106
107 CONTINUE

FILTER THE STACKED INPUT SIGNAL AND PLACE IN YBAR(I)
CALL HFI7TR(FC.DELTAT,MLAO,NDATA,ISTART,Y,YBAR)
GO TO 24

PRINT ERROR MESSAGE

CALL ERRPR$(K$NRTN,ICOOE.0.0.'SPool$.6)
CALL EXIT

WRITE(1,102) YMAX,YMIN,YSCALE,I,Y(I)
102 FORMAT('INCONSISTENT INPUT DATA'./'YMAX = ',1PG15.6/'YMIN = '
,$1PG15.6/'YSCALE = ',1PG15.6/'Y(I.I3) = ',1PG15.6)
CALL TNOUA('ERF ',4)
CALL EXIT

CALL ERRPR$(K$NRTN,COE.0.0.'CO.MOD ',6)
END
SUBROUTINE C*ADO7(NU)
1 CALL I*ADO7(NU,IBUF,1.0)
IF(AND(IBUF.:177400)(:400400)1.2,1
2 RETURN
END
SUBROUTINE HFI77R(FC,DELTAT,MLAO,NDATA,ISTART,Y,Z)

HIBLER'S NON-RECURSIVE, GENERAL-PURPOSE, LOW-PASS FILTER

EXTERNAL H
DIMENSION C(101),Y(1),Z(1)

DATA PI/3.1415926536/

INPUTS:
FC - CUTOFF FREQUENCY (HZ)
DELTAT - TIME STEP (SEC)
MLAO - MAX NUMBER OF LAGS USED IN THE SYMMETRIC FILTER
NDATA - NUMBER OF DATA POINTS IN THE (Y) INPUT
ISTART - POSITION OF FIRST DATA POINT IN INPUT ARRAY
Y - INPUT ARRAY

OUTPUTS:
Z - OUTPUT ARRAY

NOTE: LENGTH OF OUTPUT VECTOR HAS BEEN SHORTENED BY
MLAO POINTS AT THE START AND BY MLAO POINTS AT THE END
EMLAG=FLOAT(MLAG)
NCOEFS=2*EMLAG+1
LSTART=ISTART+MLAG
LSTOP=ISTART+NDATA-1-MLAG
IF(LSTOP.LE.LSTART) GO TO 104

N1=2.0*EMLAG-FC*DELTAT+0.001
IF(N1.LT.1) GO TO 106

C
CALCULATE FILTER COEFS
C
DO 10 I=1,NCOEFS
N=INT(I-MLAG-1)
EN=FLTAT(N)
JMAX=MLAG-1
SUM=0.0

DO 20 J=1,JMAX
FJ=FLTAT(J)
SUM=SUM+H(J,N1)*COS(PI*EN*FJ/EMLAG)
20 CONTINUE

C(I)=(SUM+H(0,N1)/2.0+H(MLAG,N1)/2.0*COS(PI*FN))/EMLAG
10 CONTINUE
C
C(I)=C(I)/2.0
C(NCOEFS)=C(NCOEFS)/2.0
C
PERFORM THE FILTERING (CONVOLUTION) OPERATION
C
DO 25 L=LSTART,LSTOP
SUM=0.0
DO 26 J=1,NCOEFS
K=L-MLAG+J-1
26 SUM=SUM+C(J)*Y(K)
26 Z(L)=SUM
RETURN

104 WRITE(1,106)
106 FORMAT('ERROR IN INPUT TO SUBROUTINE HFILTR, LENGTH OF FILTERED S

58
SERIES IS NOT POSITIVE!
CALL EXIT
106 WRITE(1,107)
107 FORMAT('ERROR IN INPUT TO SUBROUTINE HFILTER. N1 IS LESS THAN UNIT $YI$')
CALL EXIT
END

FUNCTION H(I,N1)
IF(I.GT.N1) GO TO 1
H=1.0
RETURN
1 IF(I.EQ.N1+1) H=0.77
IF(I.EQ.N1+2) H=0.23
IF(I.GT.N1+2) H=0.00
RETURN
END
Memorandum for Record

SENSITIVITY OF INTERIOR BALLISTICS IN THE M110A2 TO PROPELLING CHARGE TEMPERATURE

George Schlenker
MEMORANDUM FOR RECORD

SUBJECT: Sensitivity of Interior Ballistics in the M110A2 to Propelling Charge Temperature

1. Reference:
   a. FONECON between Brian Walters (DRSAR-HA) and George Schlenker (DRSAR-PEL), 29 Nov 79, subject as above.
   b. Firing Tables No. FT 8-Q-1, HQDA, Jan 76, title: Cannon, 8-Inch Howitzer, M201 on Howitzer, Heavy, Self-Propelled, 8-Inch, M110A1 Firing Projectile, HE, M106.

2. Background

The brief study reported in this memorandum was initiated in response to the Reference a phone conversation. Although information concerning the sensitivity of muzzle velocity of the subject system to propelling charge temperature is readily available, as in Reference b, it did not appear that comparable information on sensitivity of peak chamber pressure existed for all charge zones. Pressure information was urgently needed to satisfy the requirements of government ammunition contractors. This MFR provides such estimates in a consistent manner for all zones of the M1 and M2 propelling charges and for the M106 and M650 projectiles.

3. Methodology

Ballistic calculations were made using the ARRCOM interior ballistics simulation for the M110/M110A2 howitzer system. This computer model had been developed during CY 79 to analyse the performance of the howitzer with various projectiles fired from both well seated positions and from fallback positions, i.e., with the projectile resting upon the charge. Only well seated projectiles were simulated here. It was anticipated that configurational differences in the M106 and M650 projectiles would produce different ballistic sensitivity to charge temperature. Consequently, both projectiles were examined using the M1 (green bag) propelling charge. Additionally, the M106 projectile was examined with the M2 (white bag)
propelling charge*. Peak chamber pressure and muzzle velocity were obtained in the M201 cannon using these charges at charge temperatures of 0, 70, and 145 deg F. These results are summarized in Table 1.

4. The effect of propelling charge temperature on model parameters is treated in the following way. Heat added (or removed) from the charge by raising (or lowering) the temperature relative to ambient, 70 deg F, is considered to appropriately raise (or lower) the flame temperature by requiring conservation of energy.** Additionally, the effect of heat addition (or removal) increases (decreases) the linear burning rate of the solid propellant. This phenomenon can be treated (over a restricted range of temperature) by making the linear burning rate at 1 ksi, \( \beta \), a linear function of temperature. Thus,

\[
\beta(T) = \beta_0 + \frac{\partial \beta}{\partial T} (T - T_0),
\]

where \( \beta(T) \) is the burning rate coefficient function of temperature \( T \), and where \( \beta_0 \) is the nominal value at temperature \( T_0 \). The partial derivative, \( \partial \beta/\partial T \), is considered constant over the domain of \( \beta(T) \). However, it is noted that experimental evidence indicates that \( \partial \beta/\partial T \) decreases with decreasing temperature. Comparison of the calculated with experimental (Reference b) decrease in muzzle velocity with change in charge temperature from 70 to 0 deg F (Table 2) shows a somewhat larger calculated decrease. By inference, \( \partial \beta/\partial T \) must decrease with decreasing temperature. Unfortunately, the form of \( \partial \beta/\partial T \) is not provided from propellant data available in the SPIA manual. Therefore, the accuracy of ballistic estimates suffers for charge temperatures less than about 0 deg F.

* Thermochemical parameters, propellant grain dimensions, and charge weights specific to certain charge lots were used in the simulations. These lots were: for the M1 charge IND 69797, and for the M2 charge BAJ 67951.

** Energy conservation requires that the isochoric adiabatic flame temperature, \( T'_V \), existing after changing the charge temperature by \( \Delta T \) be related to the ambient flame temperature \( T_V \) by the relation:

\[
RT'_V/(\gamma - 1) = RT_V/(\gamma - 1) + C_{\text{solid}} \Delta T,
\]

where \( R \) is the gas constant and \( \gamma \) is the ratio of specific heats for the propellant gas. The specific heat of the solid propellant, in consistent units, is \( C_{\text{solid}} \).
5. Results

The peak chamber pressures and muzzle velocities calculated for all computer runs are given in Table 1. Average values of the pressure sensitivity coefficient \( \frac{\partial p}{\partial T} \) and of the velocity sensitivity coefficient \( \frac{\partial V_0}{\partial T} \) are given here for each charge zone. It is generally a good approximation to take these coefficients as constants for zones 1G through 5G as can be seen by the linearity of \( p_{\text{max}} \) and \( V_0 \) with \( T \) in Figures 1 and 2. However, at the higher zones using the M2 charge there is a noticeable departure from linearity. This phenomenon has been observed experimentally in other systems, and is not just an artifact of the simulation. Consequently, the calculated average values of \( \frac{\partial p}{\partial T} \) and \( \frac{\partial V_0}{\partial T} \) for zones 5W, 6W, and 7W should not be used for extrapolation.

6. There are two conspicuous aspects of the numerical results which deserve attention: (1) Ballistic sensitivity increases in absolute value with increasing zone; (2) The multi-perforated 5W charge is substantially more sensitive than the single-perforated 5G. One should also notice that the ballistic sensitivity is less with the M650 projectile than with the M106 projectile at the same zone.

7. To provide some measure of validity in the calculated sensitivities, a comparison is made between calculated and nominal values of muzzle velocity sensitivity obtained from the Firing Tables (Reference b). One possible reason for differences observed is simply due to differences between propelling charge lots. Apart from this, the assumptions regarding the linear burning rate, mentioned above, also contribute to error.

GEORGE SCHLENKER
Operations Research Analyst
TABLE 1. CALCULATED SENSITIVITIES OF CHAMBER PRESSURE AND MUZZLE VELOCITY TO PROPELLING CHARGE TEMPERATURE IN THE M201 CANNON WITH THE M106 AND M650 PROJECTILES

<table>
<thead>
<tr>
<th>Proj. Type</th>
<th>Charge Zone</th>
<th>$P_{\text{max}}$ (ksi) at Chg Temp (°F)</th>
<th>Avg* $\frac{\partial p}{\partial T}$ (psi/°F)</th>
<th>$V_o$(f/s) at Chg Temp (°F)</th>
<th>Avg $\frac{\partial V_o}{\partial T}$ (f/s/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M106</td>
<td>1G</td>
<td>8.6 9.0 9.4 5.37 820 827 835</td>
<td>0.102</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2G</td>
<td>10.1 10.6 11.1 7.15 905 913 922</td>
<td>0.114</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3G</td>
<td>12.2 12.8 13.5 9.26 1007 1016 1026</td>
<td>0.128</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4G</td>
<td>16.0 16.9 17.8 12.49 1154 1164 1176</td>
<td>0.148</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5G</td>
<td>23.1 24.5 26.0 19.68 1377 1389 1402</td>
<td>0.177</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5W</td>
<td>13.2 14.7 16.6 23.40 1450 1477 1505</td>
<td>0.377</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6W</td>
<td>18.9 21.2 24.1 35.74 1683 1716 1748</td>
<td>0.446</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7W</td>
<td>27.5 31.1 34.7 49.39 1945 1982 2019</td>
<td>0.512</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M650</td>
<td>1G</td>
<td>8.3 8.6 8.9 3.92 811 818 826</td>
<td>0.101</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2G</td>
<td>9.7 10.1 10.5 5.61 896 904 912</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>3G</td>
<td>11.6 12.2 12.8 7.72 997 1005 1014</td>
<td>0.123</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4G</td>
<td>15.1 15.8 16.6 10.48 1142 1152 1162</td>
<td>0.143</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5G</td>
<td>21.7 23.0 24.2 17.21 1361 1373 1386</td>
<td>0.172</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Average over the temperature range -- 0 to 145 deg F.
TABLE 2. COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF VELOCITY SENSITIVITY TO PROPELLING CHARGE TEMPERATURE IN THE M201 CANNON WITH THE M106 PROJECTILE

<table>
<thead>
<tr>
<th>Charge Zone</th>
<th>Temperature (deg F)</th>
<th>Calc. $\Delta V_o$ (f/s)</th>
<th>Tabulated* $\Delta V_o$ (f/s)</th>
<th>$V_o$ at 70°F (f/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1G</td>
<td>0</td>
<td>-7.0</td>
<td>-7.5</td>
<td>833</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>7.8</td>
<td>9.2</td>
<td></td>
</tr>
<tr>
<td>2G</td>
<td>0</td>
<td>-8.0</td>
<td>-8.5</td>
<td>912</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>8.6</td>
<td>9.8</td>
<td></td>
</tr>
<tr>
<td>3G</td>
<td>0</td>
<td>-8.9</td>
<td>-9.5</td>
<td>1012</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>9.7</td>
<td>11.2</td>
<td></td>
</tr>
<tr>
<td>4G</td>
<td>0</td>
<td>-10.4</td>
<td>-10.8</td>
<td>1156</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>11.1</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>5G</td>
<td>0</td>
<td>-12.5</td>
<td>-12.5</td>
<td>1388</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>13.2</td>
<td>14.4</td>
<td></td>
</tr>
<tr>
<td>5W</td>
<td>0</td>
<td>-27.4</td>
<td>-23.3</td>
<td>1464</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>27.3</td>
<td>26.4</td>
<td></td>
</tr>
<tr>
<td>6W</td>
<td>0</td>
<td>-32.4</td>
<td>-27.2</td>
<td>1708</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>32.2</td>
<td>30.3</td>
<td></td>
</tr>
<tr>
<td>7W</td>
<td>0</td>
<td>-37.2</td>
<td>-30.5</td>
<td>1995</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>37.0</td>
<td>34.1</td>
<td></td>
</tr>
</tbody>
</table>

* Firing Tables No. FT 8-Q-1, HQDA, Jan 76, title: Cannon, 8-Inch Howitzer, M201 on Howitzer, Heavy, Self-Propelled, 8-Inch, M10A1 Firing Projectile, HE, M106.
Figure 7a: Peak Chamber Pressure Versus Propelling Charge Temperature with the M106 Projectile in the M201 Cannon.
Figure 7b: Peak Chamber Pressure versus Propelling Charge Temperature with the M106 Projectile in the M291 Cannon.
Figure 2a. Muzzle Velocity Versus Propelling Charge Temperature with the M106 Projectile in the M201 Cannon.
Figure 2b. Muzzle Velocity Versus Propelling Charge Temperature with the M186 Projectile in the M201 Cannon.
Memorandum for Record

INTERIOR BALLISTICS
OF THE
M106, M650 AND M422 PROJECTILES
IN THE
M110A2 HOWITZER
WHEN FIRED FROM FALBACK POSITIONS

George Schlenker
MEMORANDUM FOR RECORD

SUBJECT: Interior Ballistics of the M106, M650, and M422 Projectiles in the M110A2 Howitzer When Fired From Fallback Positions

1. Reference:
   a. Conversations between LTC Stiehl, DRCPM-NUC-M (RIA), and George Schlenker, DRSAR-PEL, Nov 79, subject as above.
   d. Firing Record No. P-82733, TECOM, APG, 29 Nov 78, subject: Malfunction Investigation Projectile, 8-Inch, M106 in M201 and M2A1 Howitzer Tubes (Projectile Fallback).

2. Background

The study reported in this memorandum grew out of the Reference a conversations. The request to study the interior ballistics of the M422 projectile when fired from fallback is motivated by concern over the risk of catastrophic damage to the projectile (and weapon) when fired in this manner. Previous emphasis in fallback studies in the M110/M110A2 weapon system had focused on damage to the cannon. Following an in-process review (IPR) of the Cannon Damage Study in Sep 79 (Reference b), increased attention was given the risk of damaging the projectile when firing from fallback. This increased emphasis is illustrated by a proposed test program request (TPR) (Reference c), suggested by the M110A2 system manager, for investigating the frequency and type of damage incurred by each of the following types of projectiles fired from fallback: M106, M650 and M509. Although the M422 was omitted from this TPR, the present study is a first step in assessing the risk of damage to this projectile.
3. Present evidence for the kind of damage to the projectile expected when firing from fallback comes from Reference d. In firing an inert M106 projectile in hard* fallback at zone 2, smear camera coverage recorded a separated fuse and a cracked projectile ogive. From the foregoing, considerable interest has been generated in describing the collision between projectile and cannon when firing from fallback. This memorandum describes the relative velocity of projectile and tube at first major collision when the area on and near the front bourrelet first contacts the rifling. Also calculated and displayed is the peak axial force arising from this interaction. For comparative purposes results are obtained for the three projectile types: M106, M650, and M422.

4. Methodology

Ballistic calculations were made using the ARRCOM interior ballistics simulation for the M10 and M10A2 howitzer systems. This computer model had been developed during CY 79 to analyse the performance of the howitzer with the M106 projectile fired from both well seated positions and from fallback using the M1 and M2 propelling charges. To accommodate the needs of this study, changes** were made to the program to facilitate the analysis of other projectiles and other propelling charges, including the multi-grain M80 charge.

5. Phenomena simulated in this program include: (1) the burning of black powder in the base pad igniter with accompanying change in black powder grain size and unburnt mass; (2) heat transfer to and ignition of the main propelling charge; (3) heat loss to the projectile and cannon with conduction of heat within the materials; (4) mixing of the gas components -- air, igniter gas, and charge gas -- producing the thermochemistry of the mixture; (5) gas mass loss past the obturator (blowby) throughout the interior ballistic cycle; (6) burning of the propellant grain components with associated grain dimensional change; (7) pressurization of the area behind the obturator; (8) development of forward pressure resisting projectile motion due to blowby and compression of the gas in front of the projectile; (9) axial acceleration of the projectile; (10) acceleration of the unburnt charge; (11)

* When fallback occurs abruptly from a nearly seated position at a high quadrant elevation, viz, 1150 mils.

** Projectile and charge data were formerly represented in DATA statements. With the requirement to simulate several other projectile and propelling charge types, the program was restructured to transmit projectile and charge data through COMMON statements from subroutines to the main program. Additional program changes were required to accommodate the use of charges having a mixture of grains from different propellant lots (having different thermochemistry), as is the case for the M80 propelling charge.
rearward acceleration of the recoiling parts; (12) development of axial resisting forces on the projectile generated by interaction with the gun tube. The pitch and transverse dynamics of the projectile are, specifically, not simulated.

6. Due to its extraordinary nature in interior ballistics, the force of interaction between projectile and tube during collision merits delineation here. Empirical evidence regarding damage to the lands of the M201 cannon indicates that the M106 projectile, typically, plastically deforms the driving surface of the lands over a printed area, $A_{cln}$, which subsequently fails mechanically. ($A_{cln}$ is approximately 2 (in$^2$).) This collision area experiences the material yield stress, $\sigma_{yld}$, during the interaction. Thus, an empirical estimate of the peak interfacial lands loading experienced in the M201 cannon - M106 projectile interaction is just

$$ F_n = A_{cln} \sigma_{yld} . $$

The axial force associated with $F_n$ is

$$ F_{zo} = F_n (\sin \alpha + \mu \cos \alpha) $$

with $\mu$ the coefficient of friction and with $\alpha$ the helix angle of the rifling of twist $T$, given by

$$ \alpha = \tan^{-1}(\pi/T) , $$

neglecting slip with respect to the rifling. However, this force can be expected to be a function of the relative velocity, $\dot{z}$, of the colliding bodies. To account for this, the model assumes that $F_{zo}$ occurs at an axial collision speed $\dot{z}_0$ (which can be estimated) and that the maximum axial force due to an initial collision at speed $\dot{z}$ is a linear function of $\dot{z}$. Thus,

$$ F_{z \max} = F_{zo} \frac{\dot{z}}{\dot{z}_0} . $$

If the projectile were being torqued to follow the rifling, an axial resisting force would be generated $F_{zT}$:

$$ F_{zT} = \frac{2 \pi \rho T}{(\cos \alpha - \mu \sin \alpha)} \frac{\dot{z} M}{g} , $$

* $\sigma_{yld} = 1.6 \times 10^5$ (psi), approximately.

** $\dot{z}_0 = 540$ (f/s), as given in this program.
where
\[ p \] is the projectile radius of gyration in calibers,
\[ z \] the relative axial acceleration, \( M \) is the projectile weight, and \( g \) is the gravitational constant. If the projectile angular velocity exceeds that required to follow the rifling during collision, the axial force due to collision is assigned \( F_z \).

7. Data

The author's previous experience with interior ballistic simulation supports the claim that calculated ballistics will be in better agreement with experimental firings if the specific chemical composition of a propellant lot -- as opposed to nominal composition -- is used to calculate the thermochemical parameters of the combustion products and these parameters are used in the simulation. Experience also indicates that the distribution of grain size actually measured from samples of the propellant lot should be used for best results rather than using a fictitious single average grain geometry. (The computer simulation actually uses nine grain size classes obtained by treating grain dimensions as independent gaussian random variables.) Lots of M1 and M2 propelling charges have been treated in the manner described for previous studies.

8. For this study, data for the two propellant lots used in the 1976 lots of M80 charges (PA-76E001A001/2) were used to calculate* the thermochemical parameters which characterize each propellant lot. Results of these calculations are given in Table 1.

9. Data on the M80 propelling charge were, parenthetically, difficult to obtain and contained inconsistencies. For example, the data on pages 48 through 51 of Reference e indicates that the zone 1 charge contains single-perforated propellant with an 0.0375 (in) web, whereas information from Radford Arsenal, the propellant producer, indicates that the correct description should have been seven-perforated propellant with a nominal 0.0360 - 0.0365 (in) web. Additionally, the web of the MP zone 2 and zone 3 increments given in Reference e is incorrect. The value given there is 0.055 (in) whereas the propellant lot acceptance data sheets show an average web of 0.068 (in). Charge weight increments provided by the PM-NUC were also incorrect. The correct values, found in Table 1, were obtained from propellant acceptance sheets and confirmed by lot firing.

* Calculations use the Hirschfelder-Sherman method with chemical constants supplied by the Naval Ordnance Station, Indian Head.
DRSAR-PEL

11 December 1979

SUBJECT: Interior Ballistics of the M106, M650, and M422 Projectiles in the M110A2 Howitzer When Fired From Fallback Positions

records. Since the zones of the M80 charge consist of a mixture of propellant lots, values of the thermochemical parameters are adjusted for each zone by taking a mass-weighted average of the lot-specific parameters. These results are found in Table 2.

10. During fallback, the projectile compresses the somewhat loosely bagged propelling charge. Due to the great sensitivity of fallback interior ballistics to initial position of the base of the projectile, it is important to properly estimate the extent of propelling charge compression.* The degree of compression depends upon the manner in which the projectile falls back. If fallback is abrupt and occurs at a high quadrant elevation (QE), compression of the charge is much greater than if the projectile gradually slides back at a low QE. The former condition is referred to as hard fallback and is the condition simulated here.

11. In hard fallback on the M1 propelling charge, the stresses induced in the bag can rupture bag seams. However, when rupture does not occur, the resulting charge deformation appears to occur at nearly constant charge volume, with charge diameter increasing to preserve volume. Based upon experiments with compression in the M1 charge, axial compression of the M80 charge at constant charge volume is expected to be limited by the maximum diameter permitted by the chamber. The uncompressed and compressed dimensions of the M80 charge assumed for this study are given in Table 3A. Table 3B displays the empirical basis for the assumption of constant charge volume. Note that the different studies of charge compression have yielded quite different results. This indicates that a substantial inherent uncertainty exists regarding the initial position of the base of projectile when fallback occurs. In the M201 cannon the presence of standoff lugs on the breechface presents an additional complication. The one-inch deep lugs can be expected to partially penetrate the charge during fallback compression. A penetration of 0.5 inch is assumed here.

12. The parameters which characterize the projectile in the interior ballistics program are shown in Table 4. Of particular significance for fallback is the maximum diameter of the obturator and its axial position. The leakage of combustion gas is controlled by the minimum annular cross section over the projectile between the cannon internal profile and the projectile.

* The difference in ballistics associated with a highly compressed as opposed to a moderately compressed charge is marked. In the YPG fallback tests reported in Reference b, the two conditions were produced experimentally. With the M106 projectile and the M1/Z2 charge, hard fallback produced a peak pressure of 18.6 ksi, whereas a peak pressure of 15.0 ksi occurred with a nearly uncompressed charge at 200 mils QE.
SUBJECT: Interior Ballistics of the M106, M650, and M422 Projectiles in the M110A2 Howitzer When Fired From Fallback Positions

Generally, this throat occurs at the forward lip of the obturator max diameter. The projectile volume behind this position is considered in calculating the free volume for the gas. All axial dimensions pertaining to the gun projectile interface -- ZSEAT, ZCLN -- are referenced to the reference end of the gun tube on cannon drawings.

13. Results

An abbreviated set of ballistic output parameters, calculated for both fallback and well seated projectiles is displayed in Tables 5 and 6. For the fallback case a relatively limited number of applicable experimental firings exists. Peak chamber pressure and muzzle velocity have been the only comparable variables successfully measured in experimental firings. Attempts to measure collision accelerations have been unsuccessful. The simulated results which can be compared are in substantial agreement with the pressures and velocities obtained from the Reference d Fallback Tests for zones 2 through 5. Calculated velocities are generally somewhat in excess of those measured (20 to 30 f/s) but peak pressures agree within the measurement error. As expected, there is better agreement with experiment for the case of well seated projectiles.

14. The ballistics for well seated projectiles are offered here primarily for comparison with those for fallback. For the M650 and M422 projectiles there have been no fallback firings to validate the simulated results. However, these projectiles do not appear to introduce any new phenomena with respect to fallback interior ballistics. Consequently, the accuracy of results is considered comparable to that with the M106 projectile.

One should note that the collision velocity at first major collision may not be the best measure of the severity of the interaction because this model does not consider the effects of pitching and transverse motions. Using axial collision velocity as the measure of severity, one can assert that the worst case for the M106 and for the M650 occurs at zone 2. For the M422 projectile the worst case occurs at zone 1 of the M80 propelling charge. This fact is due primarily to the longer travel to first collision at zone 1 -- 13.5 (in) versus 8.1 (in) at zone 2 and to the approximately 1 (in) travel at start of ignition of the M80 propelling charge.

15. Graphical results for the worst-case zones are included in Annex 1. A glossary of the ballistic variables displayed in Annex 1 is given in Table 7. Fallback simulations for the M106, M650 and M422 projectiles are presented first, in that order followed by the standard (well seated) results for each of these projectiles for the same zones. In discussing the graphical results the following points are noted. The interval from the time at which the black powder igniter starts to burn until the main
charge is ignited is shorter for the fallback case than for the standard for all projectiles. The projectile has not traveled very far at the instant of ignition -- typically, about an inch. For all projectiles there is a shorter risetime to peak pressure for fallback than for the corresponding standard case. The pressure and acceleration curves for standard ballistics are seen to be quite smooth after shot start. However, for the fallback case two major disturbances in the smoothness of acceleration are evident following the occurrence of peak pressure. The discontinuities are due, first, to the forward bourrelet striking the rifling, which starts projectile spinup, and, second, to the rotating band engaging the rifling. There is a significantly greater peak pressure in the fallback case than in the standard case. For example, at zone 2 with the M106 projectile, in hard fallback the peak pressure is 18.8 ksi versus 10.6 ksi in the standard case. With the M650 projectile using the M1/Z2 charge, fallback peak pressure exceeds 17 ksi whereas standard peak pressure is about 10 ksi. The amplification of pressure relative to standard due to fallback is not as great with the M422 using the M80/Z1 -- 12.0 versus 9.8 ksi. For the M106 and M422 projectiles, the calculated muzzle velocity is significantly higher (about 50 f/s) in fallback than in the standard case. However, for the M650 projectile the calculated muzzle velocity is not greatly different in the two cases. The likely cause of the lower velocity for the M650 than for the M106 projectile at the same zone in fallback is the greater mass loss -- 1.66 versus 1.15 lbm -- due to the smaller max obturator diameter of the M650. In fact, without the large amount of blowby the muzzle velocity in fallback would be much greater than it is.*

Summary

16. This study examines the ballistics of several projectiles - M106, M650, and M422 fired from both fallback and well seated positions in the M201 cannon. Motivation for this study is a growing concern for the safety of these projectiles (and weapon) when firing from fallback. The ballistic simulation produces estimates of the state of the projectile at first major collision with the gun tube. The largest axial collision velocities are produced with zone 2 of the M1 propelling charge using M106 and M650 projectiles. These collision velocities are 558 and 486 f/s, respectively, the former being sufficient to cause damage to the M106. For the M422 projectile, the worst case occurs with zone 1 of the M80 propelling charge where the calculated collision velocity is 389 f/s. Graphs

* Experimental muzzle velocities with M106 fallback are 20 to 30 f/s greater than standard. Because of this relatively small incremental velocity and a higher drag, the change in range due to fallback may go unnoticed. Change in range is not a reliable indication of fallback.
SUBJECT: Interior Ballistics of the M106, M650, and M422 Projectiles in the M110A2 Howitzer When Fired From Fallback Positions

of ballistic variables are presented in Annex 1 to display the differences between fallback and standard cases for each of the projectile types.

GEORGE SCHLENKER
Operations Research Analyst
**TABLE 1**

COMPARISON OF NOMINAL THERMOCHEMISTRY OF
M6 PROPELLANT WITH THAT USED IN THE
M80 PROPELLING CHARGE

<table>
<thead>
<tr>
<th>Propellant Composition</th>
<th>Weight Fraction</th>
<th>Nominal</th>
<th>RAD 69535</th>
<th>RAD 69693</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Active Ingredients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrocellulose (13.15%N)</td>
<td>0.8689</td>
<td>0.8707</td>
<td>0.8709</td>
<td></td>
</tr>
<tr>
<td>Dinitrotoluene</td>
<td>0.0975</td>
<td>0.0976</td>
<td>0.0988</td>
<td></td>
</tr>
<tr>
<td>Dibutyl phthalate</td>
<td>0.0336</td>
<td>0.0317</td>
<td>0.0303</td>
<td></td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td><strong>Additives</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diphenylamine</td>
<td>0.0091</td>
<td>0.0100</td>
<td>0.0112</td>
<td></td>
</tr>
<tr>
<td>Potassium sulphate</td>
<td>0.0099</td>
<td>0.0109</td>
<td>0.0110</td>
<td></td>
</tr>
<tr>
<td>Water (liq)</td>
<td>0.0050</td>
<td>0.0080</td>
<td>0.0044</td>
<td></td>
</tr>
<tr>
<td>Ethyl alcohol (resid.)</td>
<td>0.0090</td>
<td>0.0021</td>
<td>0.0134</td>
<td></td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>0.0330</td>
<td>0.0310</td>
<td>0.0400</td>
<td></td>
</tr>
<tr>
<td><strong>Thermochemical Parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_v$ (deg K)</td>
<td>2549.0</td>
<td>2597.7</td>
<td>2517.8</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.2544</td>
<td>1.2524</td>
<td>1.2552</td>
<td></td>
</tr>
<tr>
<td>$n$ (gm mol/gm)</td>
<td>0.04405</td>
<td>0.04355</td>
<td>0.04426</td>
<td></td>
</tr>
<tr>
<td>$F$ (ft lbf/lbm)</td>
<td>$3122 \times 10^6$</td>
<td>$3146 \times 10^6$</td>
<td>$3099 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>$M$ (gm/gm mol)</td>
<td>22.703</td>
<td>22.963</td>
<td>22.593</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 2

VARIATION OF THERMOCHEMICAL PROPERTIES WITH ZONE IN THE M80 PROPELLING CHARGE USING PROPELLANT LOTS RAD69535 AND RAD69693

<table>
<thead>
<tr>
<th>Parameter (dimension)</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>charge mass (lbm)</td>
<td>7.456</td>
<td>15.925</td>
<td>31.206</td>
</tr>
<tr>
<td>mass fract. of zone 1 propellant</td>
<td>1.0000</td>
<td>0.4682</td>
<td>0.2389</td>
</tr>
<tr>
<td>$T_v$ (deg K)</td>
<td>2598</td>
<td>2555</td>
<td>2537</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.2524</td>
<td>1.2539</td>
<td>1.2545</td>
</tr>
<tr>
<td>$n$ (gm mol/gm)</td>
<td>0.04355</td>
<td>0.04393</td>
<td>0.04409</td>
</tr>
<tr>
<td>$\dot{F}$ (10$^6$ ft lbf lbm)</td>
<td>0.3146</td>
<td>0.3121</td>
<td>0.3110</td>
</tr>
<tr>
<td>$\eta$ (in$^3$/lbm)</td>
<td>29.881</td>
<td>30.068</td>
<td>30.148</td>
</tr>
<tr>
<td>$M$ (gm/gm mol)</td>
<td>22.963</td>
<td>22.766</td>
<td>22.681</td>
</tr>
<tr>
<td>grain OD (in)</td>
<td>0.2065</td>
<td>0.3745</td>
<td></td>
</tr>
<tr>
<td>grain PD (in)</td>
<td>0.0215</td>
<td>0.0346</td>
<td></td>
</tr>
<tr>
<td>nom. web (in)</td>
<td>0.0355</td>
<td>0.0677</td>
<td></td>
</tr>
<tr>
<td>grain length (in)</td>
<td>0.4800</td>
<td>0.8610</td>
<td></td>
</tr>
<tr>
<td>std dev OD (%)/(10$^{-3}$ in)</td>
<td>3.32/6.8</td>
<td>1.65/6.2</td>
<td></td>
</tr>
<tr>
<td>std dev. len. (%)/(10$^{-3}$ in)</td>
<td>1.65/7.9</td>
<td>1.18/10.2</td>
<td></td>
</tr>
<tr>
<td>$\alpha$, burn rate expon.</td>
<td>0.875</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$, burn rate at 1 ksi (in/s)</td>
<td>0.235</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\partial \alpha}{\partial T}$ (deg K$^{-1}$)</td>
<td>6.810$^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\partial \beta}{\partial T}$ (in/s/deg K)</td>
<td>1.010$^{-3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Inferred from strand burner data given in SPIA Manual.

TABLE 3A

ESTIMATE OF COMPRESSION IN THE M80 PROPELLING CHARGE

Dimensions of the M80 Propelling Charge Uncompressed Diameter 7.75 (in)
Uncompressed Total Length 24.25 (in)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incremental Length (in)</td>
<td>5.20</td>
<td>6.50</td>
<td>12.55</td>
</tr>
<tr>
<td>Overall Uncomp. Length (in)</td>
<td>5.20</td>
<td>11.70</td>
<td>24.25</td>
</tr>
<tr>
<td>Est. Overall Comp. Length (in)</td>
<td>4.32</td>
<td>9.73</td>
<td>20.16</td>
</tr>
<tr>
<td>Initial Position in M201 Cannon</td>
<td></td>
<td>13.0</td>
<td>23.5</td>
</tr>
<tr>
<td>(in) (ZSTART)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 3B
ESTIMATES OF COMPRESSION IN THE M1, 8-INCH PROPELLING CHARGE

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.25</td>
<td>8.25</td>
<td>7.50</td>
<td>5.25</td>
</tr>
<tr>
<td>2</td>
<td>11.00</td>
<td>10.25</td>
<td>8.80</td>
<td>6.25</td>
</tr>
<tr>
<td>3</td>
<td>13.25</td>
<td>11.50</td>
<td>10.75</td>
<td>11.50</td>
</tr>
<tr>
<td>4</td>
<td>17.00</td>
<td>14.50</td>
<td>13.75</td>
<td>15.25</td>
</tr>
<tr>
<td>5</td>
<td>22.00</td>
<td>19.50</td>
<td>19.00</td>
<td>15.00</td>
</tr>
</tbody>
</table>

(1) APG Firing Record P82733, Sep 78.

(2) Based upon compression of lot IND 69797 with a static load of 150 lb. in a 8.5 (in) cylinder. Reference: Mario Miranda, YPG, Jul 79.

(3) Based upon the ARRADCOM Charge Damage Assessment Tests using hard fallback at 1150 mils. Reference: Carl Gardner, DRDAR-LC, Aug 79.

Using the uncompressed length(1) and associated nominal max diameter of 6.5 (in) and the experimental(2) compressed length, the following are the constant-volume charge diameters after compression:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Diam. (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.2</td>
</tr>
<tr>
<td>2</td>
<td>7.3</td>
</tr>
<tr>
<td>3</td>
<td>7.2</td>
</tr>
<tr>
<td>4</td>
<td>7.2</td>
</tr>
<tr>
<td>5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

The initial position of the base of the projectile resting on the propelling charge after a hard fallback from a seated position in the M201 Cannon is estimated to be 3.3 (in) plus the compressed charge length, as measured from the reference end of the gun tube. This value assumes that the standoff lugs on the breech penetrate to 0.5 (in) depth into the propelling charge.
<table>
<thead>
<tr>
<th>Description of Parameter</th>
<th>Program Name</th>
<th>Projectile Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M106</td>
</tr>
<tr>
<td>Projectile mass (lbm)</td>
<td>EMP</td>
<td>200</td>
</tr>
<tr>
<td>Radius of gyration (cal)</td>
<td>RGRYN</td>
<td>0.3842</td>
</tr>
<tr>
<td>Dist. from base of proj. to max diam. of obturator (in)</td>
<td>ZBOPRB</td>
<td>6.49</td>
</tr>
<tr>
<td>Volume aft of obturator (in³)</td>
<td>VBOP</td>
<td>294.4</td>
</tr>
<tr>
<td>Max diam. of obturator (in)</td>
<td>DIAOBT</td>
<td>8.28</td>
</tr>
<tr>
<td>Width of band (in)</td>
<td>WDBAND</td>
<td>1.94</td>
</tr>
<tr>
<td>Dist. from obturator to seated position (in)</td>
<td>ZRBSET</td>
<td>0.00</td>
</tr>
<tr>
<td>Diameter of seat (in)</td>
<td>DIASET</td>
<td>8.28</td>
</tr>
<tr>
<td>Dist. from reference end of tube to base of seated proj. (in) in M201 cannon</td>
<td>ZSEAT</td>
<td>36.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28.26</td>
</tr>
<tr>
<td>Dist. from reference end of tube to base of proj. at collision (in) in M201 cannon</td>
<td>ZCLN</td>
<td>28.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20.7</td>
</tr>
</tbody>
</table>
### TABLE 5

**SUMMARY OF CALCULATED INTERIOR BALLISTICS IN THE M201 CANNON FOR SEVERAL PROJECTILES FIRED FROM HARD* FALLOUT**

Simulated conditions are given below.

<table>
<thead>
<tr>
<th>Proj. Type</th>
<th>Chg. Type</th>
<th>Zone</th>
<th>Peak Press (ksi)</th>
<th>Muzzle Velocity (f/s)</th>
<th>Collision Velocity Axial** Col Force (k Ibf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M106</td>
<td>M1</td>
<td>1</td>
<td>16.1</td>
<td>865</td>
<td>539</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>18.8</td>
<td>959</td>
<td>558</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>21.6</td>
<td>1066</td>
<td>552</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>26.3</td>
<td>1218</td>
<td>523</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>31.1</td>
<td>1428</td>
<td>275</td>
</tr>
<tr>
<td>M650</td>
<td>M80</td>
<td>1</td>
<td>14.5</td>
<td>815</td>
<td>473</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>17.2</td>
<td>910</td>
<td>486</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>20.0</td>
<td>1019</td>
<td>467</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>24.6</td>
<td>1174</td>
<td>411</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>29.8</td>
<td>1395</td>
<td>93</td>
</tr>
<tr>
<td>M422</td>
<td>M80</td>
<td>1</td>
<td>12.0</td>
<td>936</td>
<td>389</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>19.7</td>
<td>1392</td>
<td>329</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>36.7</td>
<td>1995</td>
<td>0</td>
</tr>
</tbody>
</table>

* Abrupt drop from the seated position at 1150 mils QE.

** The ratio of axial to cross-axial collision force in the M201 cannon is 0.4237 under the assumption that the projectile is following the rifling.

**Conditions:**

- Propelling charge temperature -- 70 deg F.
- Cannon condition -- new.
- M1 charge lot no IND 69797.
- M80/Z1 propellant lot no RAD 69535.
- M80/Z2/Z3 propellant lot no RAD 69693.
## TABLE 6

**SUMMARY OF CALCULATED INTERIOR BALLISTICS IN THE M201 CANNON FOR SEVERAL PROJECTILES FIRED FROM A SEATED* POSITION**

Simulated conditions are given below.

<table>
<thead>
<tr>
<th>Proj. Type</th>
<th>Charge Type</th>
<th>Zone</th>
<th>Peak Press (ksi)</th>
<th>Muzzle Velocity (f/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M106</td>
<td>M1</td>
<td>1G</td>
<td>9.0</td>
<td>827</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2G</td>
<td>10.6</td>
<td>913</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3G</td>
<td>12.8</td>
<td>1016</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4G</td>
<td>16.9</td>
<td>1164</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5G</td>
<td>24.5</td>
<td>1389</td>
</tr>
<tr>
<td>M106</td>
<td>M2</td>
<td>5W</td>
<td>14.7</td>
<td>1477</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6W</td>
<td>21.2</td>
<td>1716</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7W</td>
<td>31.1</td>
<td>1982</td>
</tr>
<tr>
<td>M650</td>
<td>M1</td>
<td>1G</td>
<td>8.6</td>
<td>818</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2G</td>
<td>10.1</td>
<td>904</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3G</td>
<td>12.2</td>
<td>1005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4G</td>
<td>15.8</td>
<td>1152</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5G</td>
<td>23.0</td>
<td>1373</td>
</tr>
<tr>
<td>M422</td>
<td>M80</td>
<td>1</td>
<td>9.8</td>
<td>874</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>14.5</td>
<td>1279</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>29.3</td>
<td>1880</td>
</tr>
</tbody>
</table>

* Projectile is assumed to have been rammed 0.120 (in) beyond the initial contact position.

**Conditions:**

Propelling charge temperature -- 70° F.
Cannon condition -- new.
M1 charge lot no IND 69797.
M2 charge lot no BAJ 67951.
M80/Z1 propellant lot RAD 69535.
M80/Z2/Z3 propellant lot RAD 69693.
### TABLE 7

ABBREVIATIONS USED IN PLOTS OF BALLISTIC SIMULATION OUTPUT

<table>
<thead>
<tr>
<th>Label Used on Graphs</th>
<th>Detailed Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE</td>
<td>Space-mean or chamber pressure (psia)</td>
</tr>
<tr>
<td>DPDT</td>
<td>Pressure time derivative (psi/s)</td>
</tr>
<tr>
<td>ACCELERATION</td>
<td>Projectile axial acceleration (g)</td>
</tr>
<tr>
<td>GRAIN ID (ENDS)</td>
<td>Propellant grain inside diam. at ends of single perf. grain (in)</td>
</tr>
<tr>
<td>GRAIN OD</td>
<td>Propellant grain outside diam. for both SP and MP grain (in)</td>
</tr>
<tr>
<td>GRAIN ID (CNTR)</td>
<td>Propellant grain inside diam. at the center of single perf. grain (in)</td>
</tr>
<tr>
<td>CHG VOLUME</td>
<td>Volume of unburnt propellant in propelling charge (in³)</td>
</tr>
<tr>
<td>BURNING AREA</td>
<td>Area of burning surface of propellant (in²)</td>
</tr>
<tr>
<td>MASS LOSS</td>
<td>Mass of gas lost by escape past obturator (1bm)</td>
</tr>
<tr>
<td>MASS OF GAS</td>
<td>Mass of gas remaining behind projectile (1bm)</td>
</tr>
<tr>
<td>VELOCITY</td>
<td>Projectile velocity (f/s)</td>
</tr>
<tr>
<td>INTERNAL ENERGY</td>
<td>Internal energy of remaining gas (ft-lbf)</td>
</tr>
<tr>
<td>DISPLACEMENT</td>
<td>Displacement of projectile relative to initial position (in)</td>
</tr>
<tr>
<td>POS RE REF</td>
<td>Position of the base of the projectile relative to the reference end of the tube (in)</td>
</tr>
<tr>
<td>WALL TEMPERATURE</td>
<td>Temperature at surface of chamber wall (deg K)</td>
</tr>
<tr>
<td>TEMPERATURE</td>
<td>Space-mean temperature of gas (deg K)</td>
</tr>
<tr>
<td>HEAT LOSS</td>
<td>Thermal loss to cannon and projectile (ft lbf)</td>
</tr>
<tr>
<td>SPIN</td>
<td>Projectile rotational frequency (Hz)</td>
</tr>
</tbody>
</table>
ANNEX 1

GRAPHICAL RESULTS OF INTERIOR BALLISTICS FOR M106, M650, AND M422 PROJECTILES FIRED FROM FALBACK AND WELL SEATED POSITIONS IN THE M201 CANNON
INTERIOR BALLISTICS WITH FAL BACK

Cannon: M201
Projectile: M106
Charge: M1/Z2

Chamber Pressure Versus Position of Base of Projectile Relative to Reference End of Tube.
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M106
Charge: M1/22

Comparison of Chamber Pressure and Rate of Change of Pressure as Functions of Time.

TIME (MILLISECONDS)
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M106
Charge: M1/22

Time Derivative of Chamber Pressure Versus Burning Area.

Time Derivative of Chamber Pressure Versus Pressure.
INTERIOR BALLISTICS WITH FALIBACK
Cannon: M201
Projectile: M106
Charge: M1/22

Comparison of Chamber Pressure and Projectile Acceleration as Functions of Time.
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M106
Charge: M1/Z2

Propellant Grain Inside Diameter at Grain Ends Versus Time.

Propellant Grain Outside Diameter Versus Time.
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M106
Charge: M1/22

Propellant Grain Inside Diameter at Grain Center Versus Time.

Comparison of Propellant Grain Dimensions as Functions of Time.
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M106
Charge: M1/22

Volume of Unburnt Propellant in Charge Versus Time.

Burning Area of Charge Versus Time.
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M106
Charge: M1/Z2

Mass of Gas Lost Past Obturator Versus Time.

Mass of Gas Aft of Projectile Versus Time.
INTERIOR BALLISTICS WITH FALBACK

Cannon: M201
Projectile: M106
Charge: M1/Z2

Projectile Velocity Versus Time.

Internal Energy in Propellant Gas Versus Time.
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M106
Charge: M1/22

Projectile Velocity Versus Position of Base of Projectile Relative to Reference.

Projectile Velocity Versus Projectile Displacement.
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M106
Charge: M1/22

Position of Projectile Relative to Reference Versus Time.

Displacement (in) Projectile Displacement Versus Time.
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M106
Charge: M1/Z2

Chamber Wall Surface Temperature Versus Time.

WALL TEMPERATURE (DEG K)

Space-Mean Temperature of Gas Versus Time.

TEMPERATURE (DEG K)
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M106
Charge: M1/Z2

Projected Spin Rate Versus Time.

Heat Loss (FT-LBF)

Thermal Loss to Cannon and Projectile Versus Time.
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M106
Charge: M1/Z2

Projectile Spin Rate Versus Position of Base of Projectile Relative to Reference.
PRESSURE (PSI)

INTERIOR BALLISTICS WITH FALBACK

Cannon: M201
Projectile: M650
Charge: M1/Z2
Memorandum for Record

INTERIOR BALLISTICS
OF THE
M509E1 PROJECTILE
IN THE
M110A2 HOWITZER
WHEN FIRED FROM FALBACK POSITIONS

George Schlenker
MEMORANDUM FOR RECORD

SUBJECT: Interior Ballistics of the M509E1 Projectile in the M10A2 Howitzer When Fired From Fallback Positions

1. Reference:
   b. DF, DRSAR-PE, HQ ARRCOM, 20 Nov 79, subject: Proposed Test Program Request (TPR) to Investigate Projectile Damage in Firing from Fallback in the M10A2 SP Howitzer, with inclosures.

2. Background

   This memorandum is a sequel to Reference a. Concern about the consequences of firing projectiles from a fallback position in the M10A2 howitzer (M201 cannon) extends to all projectiles which can be fired in that system. Although the original incentive for studying this problem was to understand the cause and likelihood of cannon damage -- specifically, stripped lands -- a more recent emphasis of concern is to assess the damage to the projectile fired from a "hard" fallback position. Hard fallback occurs when the nearly seated projectile abruptly falls back on the propelling charge during tube elevation. In this condition the charge is considerably compressed.

3. As indicated in Reference b, the M509 projectile is one of the types of projectiles whose vulnerability to firing from fallback in the M201 cannon was to have been examined experimentally. The present analytic study is intended to support a more extensive study of cannon and projectile damage and to complement experimental work which may proceed from the Reference b proposal.

4. Data

   Data relative to the dimensional and inertial characteristics of the M509E1 were received from DRDAR-LCU-SS, 3 Jan 80, after the Reference a study had been completed. These data reflect the most recent changes to the aft main body and boattail. Inferences regarding the parameters used in a computer simulation of the M509E1 were made from the primitive data. The simulation
parameters characterizing the M509E1 are found in Table 1.

5. Methodology

The ballistic simulation described in Reference a was used to calculate collision velocity and peak collision force occurring at first major collision between the projectile and gun tube during a fallback trajectory. This event occurs when the forward bourrelet encounters the rifling at the reference position ZCLN. (See Table 1.) Another major collision occurs when the rotating band encounters the rifling.

6. The dynamic behavior of the projectile is not described in detail in the ballistics simulation. Rather an ad hoc semiempirical model is used to characterize the effect of the collision on projectile (Z-) axial and spin motions. See Reference a for details. The computer program reports a variety of measurable variables such as chamber pressure and muzzle velocity as well as many endogenous variables which would be at best difficult to measure. The simulation outputs are displayed graphically for two sample runs in Annex 1. Ballistics are calculated for two conditions: when the projectile starts from a hard fallback position and, second, when the projectile is fully seated. The latter runs are made for the purpose of comparison with the former.

7. Results

A brief summary of results is provided in Table 2 for the fallback case and in Table 3 for the seated case. It is noted that the largest collision velocity using the M509E1 projectile occurs at zone 1 of the M1 propelling charge. This contrasts with the situation using the M106 and M650 projectiles (Reference a) in which the worst-case collision velocity occurs at zone 2. Further, differences are noted in the magnitude of the collision velocity. For the M106 the largest velocity is 558 f/s; for the M650 a comparable value is 486 f/s; and for the M509E1 the largest collision velocity is 421 f/s. Due to its greater inertia and shorter travel to first collision, the M509E1 suffers a less severe impact. Whether even this type of impact would damage the M509E1 is not addressed here. It is also noted that the peak initial collision force for the M509E1 is nearly the same for the first four zones. This fact appears to be due to the combined effects of projectile velocity and acceleration during the collision, notwithstanding the fact that collision velocity declines with increasing zone.
8. One should note from Tables 2 and 3 that the muzzle velocities achieved with the M509E1 are not greatly different between the fallback and seated cases. This is so in spite of the fact that the peak chamber pressure is much larger with fallback, being almost twice as great in fallback as standard at zones 1 and 2. Although combustion occurs at a higher pressure (and with greater thermodynamic efficiency) in the case of fallback, the losses, due principally to blowby, compensate for the gains in this system.

9. **Summary**

This study examines the interior ballistics of the M509E1 projectile fired in the M201 cannon from both fallback and well seated positions. Motivation for the study is a growing concern for the safety of this projectile if it were fired from fallback. The largest axial collision velocity of the M509E1 projectile is 421 f/s, which occurs at zone 1 of the M1 propelling charge. Although this velocity is less than the maximum of 558 f/s produced with the M106 projectile, a velocity of this magnitude may still suffice to damage the projectile. Additional studies are required to evaluate this possibility. Graphs of ballistic variables are presented in Annex 1 to display differences between fallback and standard cases for a typical firing zone.

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1 Incl

Annex 1

GEORGE SCHLENKER

Operations Research Analyst
# TABLE 1

PROJECTILE PARAMETERS USED IN THE INTERIOR BALLISTICS OF FALLBACK PROGRAM

<table>
<thead>
<tr>
<th>Description of Parameter</th>
<th>Program Name</th>
<th>Value for M509E1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile mass (lbm)</td>
<td>EMP</td>
<td>207.7</td>
</tr>
<tr>
<td>Polar radius of gyration</td>
<td>RGRyn</td>
<td>0.3819</td>
</tr>
<tr>
<td>Distance from base of proj. to max diam. of obturator (in)</td>
<td>Zboprb</td>
<td>4.239</td>
</tr>
<tr>
<td>Proj. volume aft of obturator (in)</td>
<td>Vbop</td>
<td>209.7</td>
</tr>
<tr>
<td>Max diam. of obturator (in)</td>
<td>Diaobt</td>
<td>8.21</td>
</tr>
<tr>
<td>Width of band (in)</td>
<td>Wdband</td>
<td>2.29</td>
</tr>
<tr>
<td>Dist. from obturator to seated position (in)</td>
<td>Zrbset</td>
<td>1.708</td>
</tr>
<tr>
<td>Diameter of seat (in)</td>
<td>DiaSet</td>
<td>8.186</td>
</tr>
<tr>
<td>Dist. from reference end of tube to base of seated proj. (in)</td>
<td>Zseat</td>
<td>37.74</td>
</tr>
<tr>
<td>in M201 cannon</td>
<td></td>
<td>29.74</td>
</tr>
<tr>
<td>Dist. from reference end of tube to base of proj. at collision (in)</td>
<td>zcln</td>
<td>in M201 cannon</td>
</tr>
<tr>
<td>in M2A2 cannon</td>
<td></td>
<td>13.52</td>
</tr>
<tr>
<td>Plateau pressure (psia) relative to base of proj. in M201 cannon</td>
<td>pplat</td>
<td>450</td>
</tr>
<tr>
<td>in M2A2 cannon</td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>Peak engraving pressure (psia)</td>
<td>Ppeak</td>
<td>3000</td>
</tr>
</tbody>
</table>
TABLE 2

SUMMARY OF CALCULATED INTERIOR BALLISTICS IN THE M201 CANNON FOR THE M509E1 PROJECTILE FIRED FROM HARD* FALlBACK

Simulated conditions are given below

<table>
<thead>
<tr>
<th>Charge Type</th>
<th>Zone</th>
<th>Peak Press (ksi)</th>
<th>Muzzle Velocity (f/s)</th>
<th>Collision Velocity (f/s)</th>
<th>Axial** Col Force (k 1bf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1</td>
<td>16.7</td>
<td>806</td>
<td>421</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>19.6</td>
<td>900</td>
<td>412</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>22.4</td>
<td>1009</td>
<td>357</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>27.5</td>
<td>1164</td>
<td>232</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>32.5</td>
<td>1380</td>
<td>0</td>
<td>24</td>
</tr>
</tbody>
</table>

* Abrupt drop from the seated position at 1150 mils QE.
** The ratio of axial to cross-axial collision force in the M201 cannon is about 0.4237 under the assumption that the projectile follows the rifling. Values of peak axial force are given.

Conditions:

Propelling charge temperature -- 70 deg F.
Cannon condition -- new.
M1 charge lot no IND 69797.
Position of base of projectile relative to ref. end of tube after fallback (in) at zone:

1: 10.8 (in)
2: 12.2
3: 14.3
4: 17.0
5: 22.3
### TABLE 3

**SUMMARY OF CALCULATED INTERIOR BALLISTICS IN THE M201 CANNON FOR THE M509E1 PROJECTILE FIRED FROM A SEATED* POSITION**

Simulated conditions are given below.

<table>
<thead>
<tr>
<th>Charge Type</th>
<th>Zone</th>
<th>Peak Press (ksi)</th>
<th>Muzzle Vel (f/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1G</td>
<td>8.7</td>
<td>808</td>
</tr>
<tr>
<td></td>
<td>2G</td>
<td>10.3</td>
<td>892</td>
</tr>
<tr>
<td></td>
<td>3G</td>
<td>12.5</td>
<td>993</td>
</tr>
<tr>
<td></td>
<td>4G</td>
<td>16.4</td>
<td>1138</td>
</tr>
<tr>
<td></td>
<td>5G</td>
<td>23.9</td>
<td>1358</td>
</tr>
<tr>
<td>M2</td>
<td>5W</td>
<td>14.5</td>
<td>1442</td>
</tr>
<tr>
<td></td>
<td>6W</td>
<td>20.9</td>
<td>1676</td>
</tr>
<tr>
<td></td>
<td>7W</td>
<td>30.9</td>
<td>1938</td>
</tr>
</tbody>
</table>

* Projectile is assumed to have been rammed 0.120 (in) beyond the initial contact position.

**Conditions:**

- Propelling charge temperature - 70 deg F.
- Cannon condition - new.
- M1 charge lot no IND 69797.
- M2 charge lot no BAJ 67951.
ANNEX 1

GRAPHICAL RESULTS OF INTERIOR BALLISTICS FOR THE M509E1 PROJECTILE FIRED FROM FALBACK AND WELL SEATED POSITIONS IN THE M201 CANNON
<table>
<thead>
<tr>
<th>Label on Graph</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE</td>
<td>Space-mean or &quot;chamber&quot; pressure (psia)</td>
</tr>
<tr>
<td>DPDT</td>
<td>Pressure time derivative (psi/s)</td>
</tr>
<tr>
<td>ACCELERATION</td>
<td>Projectile axial acceleration (g)</td>
</tr>
<tr>
<td>GRAIN ID (ENDS)</td>
<td>Propellant grain inside diam. at the ends of single perf. grain (in)</td>
</tr>
<tr>
<td>GRAIN ID (CNTR)</td>
<td>Propellant grain inside diam. at the center of single perf. grain (in)</td>
</tr>
<tr>
<td>GRAIN OD</td>
<td>Propellant grain outside diam. (in)</td>
</tr>
<tr>
<td>CHG VOLUME</td>
<td>Volume of unburnt propellant in propelling charge (in³)</td>
</tr>
<tr>
<td>BURNING AREA</td>
<td>Area of burning surface of propellant (in²)</td>
</tr>
<tr>
<td>MASS LOSS</td>
<td>Mass of gas lost by escape past obturator (Ibm)</td>
</tr>
<tr>
<td>MASS OF GAS</td>
<td>Mass of gas remaining behind projectile (Ibm)</td>
</tr>
<tr>
<td>INTERNAL ENERGY</td>
<td>Internal energy of remaining gas (ft lbf)</td>
</tr>
<tr>
<td>VELOCITY</td>
<td>Projectile axial velocity (f/s)</td>
</tr>
<tr>
<td>DISPLACEMENT</td>
<td>Axial displacement of projectile relative to initial position (in)</td>
</tr>
<tr>
<td>POS RE REF</td>
<td>Position of the base of the projectile relative to the reference end of the tube (in)</td>
</tr>
<tr>
<td>WALL TEMPERATURE</td>
<td>Temperature at surface of the chamber wall (deg K)</td>
</tr>
<tr>
<td>TEMPERATURE</td>
<td>Space-mean temperature of the gas (deg K)</td>
</tr>
<tr>
<td>TOT FWD RESISTANCE</td>
<td>Total resistance to projectile forward motion (lbf)</td>
</tr>
<tr>
<td>ENGR RESISTANCE</td>
<td>Resistance to projectile forward motion due to band engraving and friction (lbf)</td>
</tr>
<tr>
<td>HEAT LOSS</td>
<td>Thermal loss to cannon and projectile (ft lbf)</td>
</tr>
<tr>
<td>FILM COEFFICIENT</td>
<td>Film coefficient of heat transfer (ft lbf/ in²/s/deg F)</td>
</tr>
<tr>
<td>SPIN</td>
<td>Projectile rotational frequency (hz)</td>
</tr>
</tbody>
</table>
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Chamber Pressure Versus Position of Base of Projectile Relative to Reference End of Tube

Chamber Pressure Versus Time
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201

Projectile: M509E1

Charge: M1/Z2

Comparison of Chamber Pressure and Rate of Change of Pressure as Functions of Time.
INTERIOR BALLISTICS WITH FALBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z-2

Time Derivative of Chamber Pressure Versus Burning Area.

Time Derivative of Chamber Pressure Versus Pressure
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Comparison of Chamber Pressure and Projectile Acceleration as Functions of Time

ACCELERATION (G)
PRESSURE (PSI)

ACCELERATION (G)

Projectile Acceleration Versus Time
INTERIOR BALLISTICS W/FALLBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Propellant Grain Inside Diameter at Grain Ends Versus Time

Propellant Grain Outside Diameter Versus Time
INTERIOR BALLISTICS W/FALLBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

GRAIN I.D. (AT CNTR) (IN)

Propellant Grain Inside Diameter at Grain Center Versus Time

Comparison of Propellant Grain Dimensions as Functions of Time
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Volume of Unburnt Propellant in Charge Versus Time

Burning Area of Charge Versus Time
INTERIOR BALLISTICS WITH FALLBACK
Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Mass of Gas Lost Past Obturator Versus Time

Mass of Gas Aft of Projectile Versus Time

TIME (MILLISECONDS)

MASS LOSS (LB)

MASS OF GAS (LB)
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

VELOCITY (FT/SEC)

Projectile Velocity Versus Time

INTERNAL ENERGY (FT-1.8F)

Internal Energy in Propellant Gas Versus Time
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Projectile Velocity Versus Position of Base of Projectile Relative to Reference

Projectile Velocity Versus Projectile Displacement
INTERIOR BALLISTICS WITH FALLOUT

- Cannon: M201
- Projectile: M509E1
- Charge: M1/Z2

**Position of Projectile Relative to Reference Versus Time**

**Displacement (in)**

**Projectile Displacement Versus Time**
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Chamber Wall Surface Temperature Versus Time

WALL TEMPERATURE (DEG K)

Space-Mean Temperature of Gas Versus Time

TEMPERATURE (DEG K)
INTERIOR BALLISTICS WITH FALLBACK

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

WALL TEMPERATURE (DEG K)
Chamber Wall Surface Temperature Versus Displacement

TOTAL FORWARD RESISTANCE (LBF)
Total Forward Resistance Versus Time
INTERIOR BALLISTICS WITH FALLBACK
Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Total Forward Resistance to Proj.
Motion Versus Displacement

Engraving and Frictional Resistance
Versus Displacement
INTERIOR BALLISTICS WITH FALLBACK
Cannon: M201
Projectile: M509E1
Charge: M1/Z2

HEAT LOSS (FT-LBF)

Thermal Loss to Cannon and Projectile Versus Time

TIME (MILLISECONDS)

FILM COEFFICIENT

Film Coefficient of Heat Transfer Versus Time

TIME (MILLISECONDS)
INTERIOR BALLISTICS WITH FALLOUT

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Projectile Spin Rate Versus Position of Base of Projectile Relative to Reference

Projectile Spin Rate Versus Time
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Chamber Pressure Versus Position of Base of Projectile Relative to Referenced End of Tube

Chamber Pressure Versus Time
INTERIOR BALLISTICS WITH SEATED PROJ.

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Comparison of Chamber Pressure and Rate of Change of Pressure as Functions of Time

Time Derivative of Chamber Pressure Versus Time
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Time Derivative of Chamber Pressure Versus Burning Area

Time Derivative of Chamber Pressure Versus Pressure
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Comparison of Chamber Pressure and Projectile Acceleration as Functions of Time

Projectile Acceleration Versus Time
INTERIOR BALLISTICS WITH SEATED PROJ.

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Propellant Grain Inside Diameter at Grain Ends Versus Time

Propellant Grain Outside Diameter Versus Time
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Propellant Grain Inside Diameter at Grain Center Versus Time

Comparison of Propellant Grain Dimensions as Functions of Time
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon:        M201
Projectile:     M509E1
Charge:        M1/Z2

CHG VOLUME (IN**3)

Volume of Unburnt Propellant in Charge Versus Time

TIME (MILLISECONDS)

BURNING AREA (IN**2)

Burning Area of Charge Versus Time

TIME (MILLISECONDS)
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Mass of Gas Lost Past Obturator Versus Time

Mass of Gas Aft of Projectile Versus Time
INTERIOR BALLISTICS WITH SEATED PROJ.

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

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**Projectile Velocity Versus Time**

**Internal Energy in Propellant Gas Versus Time**
INTERIOR BALLISTICS W/SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Projectile Velocity Versus Position of Base of Projectile Relative to Reference

Projectile Velocity Versus Projectile Displacement

VELOCITY (FT/SEC)

0.00 10.00 20.00 30.00 40.00

0.00 25.00 50.00

0.00 25.00 50.00

DISPLACEMENT (IN)

0.00 10.00 20.00 30.00

10.00 20.00 30.00 40.00

X 10^1

X 10^1
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

Position of Projectile Relative to Reference Versus Time

Projectile Displacement Versus Time

DISPLACEMENT (IN)

TIME (MILLISECONDS)
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

WALL TEMPERATURE (DEG K)
Chamber Wall Surface Temperature Versus Time.

TEMPERATURE (DEG K)
Space-Mean Temperature of Gas Versus Time.
INTERIOR BALLISTICS WITH SEATED PROJECTILE
Cannon: M201
Projectile: M509E1
Charge: M1/Z2

WALL TEMPERATURE (DEG K)
Chamber Wall Surface Temperature Versus Displacement

Total Forward Resistance Versus Time
INTERIOR BALLISTICS WITH SEATED PROJECTILE

HEAT LOSS (FT-LBF)

Thermal Loss to Cannon and Projectile Versus Time

Cannon: M201
Projectile: M509E1
Charge: M1/Z2

FilM COEFFICIENT

Film Coefficient of Heat Transfer Versus Time
INTERIOR BALLISTICS WITH SEATED PROJECTILE

Cannon: M201
Projectile: M509E1
Charge: MI/ZZ

Projectile Spin Rate Versus Position of Base
of Projectile Relative to Reference

Projectile Spin Rate Versus Time

TIME (MILLISECONDS) 150
Memorandum for Record

APPROXIMATION
FOR THE
STANDARDIZED
CUMULATIVE PRODUCTION COST
USING
LEARNING THEORY

George Schlenker
MEMORANDUM FOR RECORD

SUBJECT: Approximation for the Standardized Cumulative Production Cost Using Learning Theory

1. Reference:

2. Background

   I recently had the opportunity to attend a course offered by ALMC which was concerned with cost estimation. During the course, materials were presented concerning the theory of learning or experience related to product manufacturing cost. This MFR was stimulated by that course.

3. Where applicable, learning theory is useful in predicting the relation of manufacturing unit cost (or man hours) of a product to the number of units produced. Manufacturing experience with missiles and with tanks and automotive vehicles has shown that the specific reduction of unit cost with units produced is commodity dependent. However, for a variety of products and over a considerable range of production quantity, the expected direct cost of manufacturing the 2n th unit is proportional to the unit cost for the n th unit for n an integer up to some limit, where the constant of proportionality, s, is called the "slope" of the learning curve and is frequently expressed as a percentage.* Thus, the unit cost for the n th unit, u(n), is given by

   \[ u(2n) = s \, u(n), \quad 0 < s < 1 \]  

   This result implies

   \[ u(n) = A n^B, \]  

*Production experience with missiles indicates a 79 to 84% slope and with automotive and tank vehicles a 92 to 96% slope.
with \( A \) the first unit cost and
\[
B = \frac{\log(s)}{\log(2)}. \tag{3}
\]
From (2) the expected total cost of manufacturing a lot of products starting with the \( n_1 \) th and proceeding through the \( n_2 \) th is given by
\[
\text{total lot cost} = \sum_{n_1}^{n_2} c(n) \tag{4}
\]
or
\[
\text{total lot cost} = A \left[ \sum_{n=1}^{n_2} n^B - \sum_{n=1}^{n_1} n^B \right]. \tag{5}
\]
The sum terms on the right in equation (5) have been named cumulative total factors for the \( n_2 \) th and \( n_1 \) th quantities. Because the term
\[
Q(n) = \sum_{n=1}^{N} n^B \tag{6}
\]
represents the cumulative production cost from the first through the \( N \) th unit standardized by division by the first unit cost, it is here called the standardized cumulative cost. With this notation equation (5) becomes
\[
\text{total lot cost}/A = Q(n_2) - Q(n_1-1). \tag{7}
\]
4. Applications

A variety of applications of (7) are possible. For example, one may have prior estimates of \( A \) and \( B \) and wish to calculate the total lot cost or average lot cost --
\[
\text{total lot cost}/(n_2 - n_1 + 1). \tag{8}
\]
Alternatively, one may have a total lot cost and an estimate of \( B \) from prior experience and wish to estimate \( A \). Finally, one may wish to estimate the learning slope \( B \) from values of total lot cost and first unit cost. Additionally, the "algebraic lot midpoint", i.e., the value of \( n \) (=K) such that
\[
A^KB
\]
is the average lot cost, is obtained from (7) via the equation
\[
k^B = \frac{Q(n_2) - Q(n_1-1)}{n_2 - n_1 + 1} = \frac{\Delta Q}{n_2 - n_1 + 1}. \tag{9}
\]
SUBJECT: Approximation for the Standardized Cumulative Production Cost Using Learning Theory

5. Because of the significance of the standardized cumulative cost and the effort to calculate it directly from (6), this quantity has been tabulated for a large set of the parameters B (or s) and n. Some examples of tables are given in Reference a. Without recourse to tables or a computer, one can calculate Q(n) using a numerical approximation.

6. Approximations for Q(n)

One approximation for Q(n) is obtained directly from the approximation for the algebraic lot midpoint, \( K_1 \), proposed by ALMC in Reference b:

\[
K_1 = \frac{F + L + 2\sqrt{FL}}{4} ,
\]

with

F = first unit in the lot
L = last unit in the lot.

With \( F = 1 \) and \( L = n \) and using the definition of algebraic lot midpoint, the ALMC approximation for Q(n) is

\[
\hat{Q}_1(n) = nK_1^B = n\left[\frac{(1 + n + 2\sqrt{n})/4}{\sqrt{B}}\right]^B .
\]

7. Another approximation, proposed by the author, is derived by treating n as a continuous variable and by solving the integral analog of equation (6). This approximation, designated \( \hat{Q}_2(n) \), has the form

\[
\hat{Q}_2(n) = (B + 1)^{-1}\left[\frac{(n + 0.5)^{B+1} - 0.5^{B+1}}{\sqrt{B}}\right] .
\]

The presence of the term 0.5 as a correction to n is motivated by the need for a symmetric integral approximation to a discrete variable in the same spirit as the gaussian distribution function approximates a binomial distribution.

8. Using the approximation \( \hat{Q}_2(n) \), the standardized cumulative cost of manufacturing a product from the \( n_1 \) th through the \( n_2 \) th units is given approximately by

\[
\Delta Q \approx (B + 1)^{-1}\left[\frac{(n_2 + 0.5)^{B+1} - (n_1 - 0.5)^{B+1}}{\sqrt{B}}\right] .
\]
In the limit as \( B \) approaches zero, i.e., as no learning occurs,
\[ \Delta Q = n_2 - n_1 + 1, \]
which is exactly what would be expected. Further, the standardized average unit cost of manufacturing \( n \) units from the \( n_1 \) th through the \( n_2 \) th:
\[ n_2 = n_1 + n - 1 \]
is given by
\[
\text{lot std avg unit cost} = \frac{(n_2 + 0.5)^{B+1} - (n_1 - 0.5)^{B+1}}{(B + 1)(n_2 - n_1 + 1)}
\]

9. Numerical Comparisons of Accuracy

With the availability of scientific calculators the two approximations given in (10) and (11) can be easily carried out. Some numerical examples are provided in Table 1. The range of slope values shown there is representative of a variety of products. Although lot sizes, \( n \), may exceed 1000 units, Table 1 does not display \( Q(n) \) for larger values since the theoretical or ideal learning may have leveled off by that point. Note that for values of \( n \) less than about 20, \( Q_1 \) and \( Q_2 \) are equally good approximations. However, for \( n > 100 \), \( Q_2 \) is distinctly superior to the ALMC approximation, \( Q_1 \). Note also that the error in \( Q_1(100) \) is about 2.4% whereas that of \( Q_2(100) \) is only 0.02% for a 90% slope. The approximation \( Q_2(n) \) improves as \( n \) increases, while \( Q_1(n) \) becomes a progressively poorer approximation with increasing \( n \). The foregoing results are generally valid for learning slopes above 80%.

10. Summary and Recommendations

It has been shown that application of learning theory requires the use of the standardized cumulative cost (sometimes called the cumulative total). To retain tables of this quantity is unnecessary if one has use of a presently commonplace, scientific calculator. The use of an approximation for \( Q(n) \) seems justified since the error of the approximation for the best approximation \( Q_2(n) \) is quite small relative to other errors. Since \( Q_2 \) is just as
SUBJECT: Approximation for the Standardized Cumulative Production Cost Using Learning Theory

easy to calculate as \( \hat{Q}_1 \), and since it is generally more accurate, \( \hat{Q}_2 \) is recommended. Further, \( \hat{Q}_1 \) is to be avoided for \( n \) in excess of 200 because of a sizeable error of approximation.

1 Incl

Table 1

GEORGE SCHLENKER
Operations Research Analyst
TABLE 1. COMPARISON OF APPROXIMATIONS FOR THE STANDARDIZED CUMULATIVE MFG COST WITH THE EXACT VALUE FOR SEVERAL LEARNING CURVES

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<th>Percent Learning Slope</th>
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Notes:

*B = \log_{10}(\text{slope})/\log_{10}(2)

(1) Cumulative using the ALMC algebraic lot midpoint is \(n\left[\frac{1 + n + 2\sqrt{n}}{4}\right]^B\)

(2) \(Q_2 = (B + 1)^{-1}\)\([(n + 0.5)^{B+1} - 0.5^{B+1}]\)

(3) \(Q = \sum_{i=1}^{n} i^B\)
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