A WIDE RANGE K EXPRESSION FOR THE C-SHAPED SPECIMEN

J. KAPP
J. C. Newman
J. H. Underwood

March 1980

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED
DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The use of trade name(s) and/or manufacturer(s) does not constitute an official endorsement or approval.

DISPOSITION

Destroy this report when it is no longer needed. Do not return it to the originator.
9 June 1980

ERRATA SHEET
(Change Notice)

C1 TO: TECHNICAL REPORT ARLCB-TR-80009

A WIDE RANGE K EXPRESSION FOR THE C-SHAPED SPECIMEN

by

J. A. Kapp
J. C. Newman, Jr.
J. H. Underwood


US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER WEAPON SYSTEMS LABORATORY
BENET WEAPONS LABORATORY
WATERVLIET, N. Y. 12189
**REPORT DOCUMENTATION PAGE**

<table>
<thead>
<tr>
<th>1. REPORT NUMBER</th>
<th>2. GOVT ACCESSION NO.</th>
<th>3. RECIPIENT'S CATALOG NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARLCB-TR-80009</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. TITLE (and Subtitle)</th>
<th>5. TYPE OF REPORT &amp; PERIOD COVERED</th>
</tr>
</thead>
<tbody>
<tr>
<td>A WIDE RANGE K EXPRESSION FOR THE C-SHAPED SPECIMEN</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. PERFORMING ORG. REPORT NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7. AUTHOR(s)</th>
<th>8. CONTRACT OR GRANT NUMBER(s)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>9. PERFORMING ORGANIZATION NAME AND ADDRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, NY 12189</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10. PROGRAM ELEMENT, PROJECT, TASK AREA &amp; WORK UNIT NUMBERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMCM No. 53970M63500 PRON No. 1A9241540GGG</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>11. CONTROLLING OFFICE NAME AND ADDRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Army Armament Research &amp; Development Command, Large Caliber Weapon Systems Laboratory, Dover, New Jersey 07801</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12. REPORT DATE</th>
<th>13. NUMBER OF PAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 1980</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>14. MONITORING AGENCY NAME &amp; ADDRESS (IF different from Controlling Office)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>15. SECURITY CLASS. (OF THIS REPORT)</th>
<th>15a. DECALASSIFICATION/DOWNGRADING SCHEDULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNCLASSIFIED</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>16. DISTRIBUTION STATEMENT (OF THIS REPORT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approved for public release; distribution unlimited.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>17. DISTRIBUTION STATEMENT (OF THE ABSTRACT ENTERED IN BLOCK 20, IF DIFFERENT FROM REPORT)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>18. SUPPLEMENTARY NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presented to ASTM Taskgroup E24.01.05, 30 Oct 79, Pittsburgh, PA.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>19. KEY WORDS (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
</tr>
<tr>
<td>Fracture Mechanics</td>
</tr>
<tr>
<td>Fracture Properties</td>
</tr>
<tr>
<td>Fracture Testing</td>
</tr>
<tr>
<td>Toughness</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20. ABSTRACT (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A new expression has been developed to calculate K for the C-shaped specimen over a wider range of specimen parameters, namely a/W, X/W, and r_1/r_2, than had previously been available. The rationale used to derive the expression was to utilize known stress intensity factor solutions for short and deep cracks to develop a nondimensional form of K which approaches finite values as a/W goes to both zero and one. Numerical K solution results from prior work were then nondimensionalized to this form with the finite limiting...</td>
</tr>
</tbody>
</table>
20. **ABSTRACT** (Cont'd)

values, and the dependence of \( K \) on \( a/W \) was determined by multi-variable, linear regression. The final expression agrees with the numerical \( K \) solutions within \( \pm 1.0\% \) for \( 0.45 \leq a/W \leq 0.55 \) for all \( r_1/r_2 \) and \( X/W \) of either 0 or 0.5; within \( \pm 1.5\% \) for \( 0.2 \leq a/W \leq 1 \) for all \( r_1/r_2 \) and \( X/W \) equal to 0 or 0.5; and within about \( \pm 3\% \) for \( 0.2 \leq a/W \leq 1 \) for all \( r_1/r_2 \) and 0 \( \leq X/W \leq 1 \). The accuracy of this expression will allow expanded use of the C-shaped specimen for R-curve determination and fatigue crack growth rate testing.
TABLES

1. Comparison of \( f(a/w) \) Values from Equation. (10) with Corresponding Values, \( f_c \), from Collocation and Limit Solutions. 9

2. Comparison of \( f(a/w) \) Values from Equation. (10) with Corresponding Values, \( f_c \), from Reference 4. 10

3. Values of \( f(a/w) \) for the Range of \( a/w \) used in \( K_{IC} \) Testing. 11

4. Values of \( f(a/w) \) for Wide Range of \( a/w \). 11

ILLUSTRATION

Fig. 1. The C-Shaped Specimen, Indicating Geometric Parameters 8
INTRODUCTION

Recently the C-shaped specimen has been included as a standard specimen in ASTM Test for Plane-Strain Fracture Toughness of Metallic Materials, E-399-78. The stress intensity factor (K) expression for the C-shaped specimen in the ASTM Test was obtained by Underwood and Kendall [1] by fitting a three parameter expression to boundary collocation results. The expression applies over the following ranges of the three parameters:

\[ 0.3 \leq a/W \leq 0.7, \quad 0 \leq x/W \leq 0.7, \quad 0 \leq r_1/r_2 \leq 1.0 \]

where \( a \) is crack length, \( W \) is specimen depth, and \( r_1 \) and \( r_2 \) are respectively the inner and outer radius of the specimen as shown in Figure 1. For fatigue crack growth rate testing and R-curve measurements using the C-shaped specimen, K must be known over a wider range of \( a/W \) than that of the current expression. The purpose of this paper is to develop a wider range K expression for the C-shaped specimen.

PROCEDURE

Srawley [2] has developed wide range K expressions for two \( K_{IC} \) specimens used in ASTM Method E-399-78, the compact specimen and the bend specimen. An approach similar to that used by Srawley was used here to develop a K expression for the C-shaped specimen. Since K varies from zero to infinity as \( a/W \) goes from zero to one for the C-shaped specimen, it is very difficult to determine an expression which accurately represents the variation in K with crack depth. This problem can be circumvented by using a nondimensional form of K which has the correct limiting values as \( a/W \) approaches both zero and one. Known K solutions for a finite crack in a semi-infinite plane and
for a semi-infinite crack in a semi-infinite plane were applied as limiting cases for short and deep cracks respectively. This resulted in an expression which, for both limits of \(a/W\), converges to the correct limiting value. The \(a/W\) dependence between the limits was described by fitting a polynomial in \(a/W\) to the available \(K\) solutions [3-6] for the C-shaped specimen.

Because the C-shaped specimen is intended for use with a range of hollow cylinder geometries, the \(K\) expression must account for load eccentricity, \(X/W\), and the radius ratio, \(r_1/r_2\), in addition to \(a/W\). The \(X/W\) dependence was determined by examining the limiting \(K\) solutions, and the \(r_1/r_2\) dependence was found using the numerical \(K\) results.

First we consider the deep crack limit which is a function of the resultant normal force, \(P\), and the resultant moment, \(M\), acting on the uncracked ligament of the specimen. If \(P\) is assumed to act at the center of the uncracked ligament with dimension \((W-a)\), the stress intensity factor is given as [7]:

\[
K_I = 0.464 \frac{2P}{B\sqrt{\pi(W-a)}} .
\]

For the case of the resultant bending moment, \(M\), \(K\) is [7]:

\[
K_I = 3.975 \frac{M}{B(W-a)^{3/2}} .
\]

The total stress intensity factor as the crack depth approaches through penetration of the specimen is the superposition of equations (1) and (2). By calculating \(M\) in terms of C-shaped specimen parameters and after some algebra, the limiting value as \(a/W\) approaches 1, of the familiar nondimensional form of \(K\) is determined.
By rearranging equation (3), a nondimensional form of $K$ which approaches a finite value for all $X/W$ and $r_1/r_2$ as $a/W$ goes to 1 is obtained.

$$\lim_{a/W \to 1} \frac{KBVW}{P} = \frac{1.325(3X/W + 1.926 + 1.104a/W)}{(1-a/W)^{3/2}}$$

To determine a nondimensional form of stress intensity factor that has a finite limit for short cracks a different $K$ solution must be used, the well known Wigglesworth solution,

$$K = 1.12 \sigma \sqrt{a}$$

where $\sigma$ in this case is the total stress acting perpendicular to the plane of the crack at $r_1$ in an uncracked C-shaped specimen. This stress has two components, a normal component and a bending component, of which the bending stress is strongly dependent on the curvature ($r_1/r_2$) of the specimen. Since the deep crack limit is unaffected by $r_1/r_2$, the effects of curvature must disappear as the relative crack depth increases to 1. The $r_1/r_2$ dependence cannot be totally accounted for by analysis of the short crack limit. Therefore, the stress $\sigma$, used in equation (5) is initially calculated assuming no effects of curvature and the $r_1/r_2$ dependence will be determined separately.

Using linear bending theory and basic mechanics of materials concepts, the stress $\sigma$ is easily determined in terms of C-shaped specimen parameters. Substituting this in equation (5), the limiting value of the familiar non-dimensional form of stress intensity factor is
Upon rearranging equation (6) a nondimensional form of K is obtained that has a finite limit for all $X/W$ as $a/W$ goes to zero.

$$\lim_{a/W \to 0} \frac{KBvW(1-a/W)^2}{P\sqrt{a/W}(3X/W + 2 + a/W)} = 2.24\sqrt{\pi}$$

(7)

Although the form of equations (4) and (7) are somewhat different, some changes can be made to both equations that do not change these limits. Because $\sqrt{a/W}$ approaches 1 as $a/W$ approaches 1, this term may be included in the denominator of equation (4) without disturbing that limit. Also for $a/W$ close to zero, $(1-a/W)^2 = (1-a/W)^{3/2}$ to a first order approximation, thus $(1-a/W)^{3/2}$ can replace $(1-a/W)^2$ in the numerator of equation (7) leaving that limit unchanged. The remaining difference between the two equations does affect the limits. Since the form of equation (4) gives a better description of the $X/W$ dependence of the collocation solutions than that of equation (7), it was decided to use a simplified form of equation (4) as the form of nondimensional K used to fit the collocation data. Thus the final form of the wide range expression is:

$$\frac{KBvW(1-a/W)^{3/2}}{P(a/W)^{1/2}(3X/W + 1.9 + 1.1a/W)} = f(a/W) \cdot g(r_1/r_2, a/W)$$

(8)

The functions $f$ and $g$ were then fit to the collocation data with the conditions that for the limits $a/W \to 1$ and $a/W \to 0$ the product of $f$ and $g$ equals the values in equations (4) and (7) respectively.
The function $g$ was difficult to determine exactly. For all $r_1/r_2$ as $a/W$ approaches 1, $g$ must approach 1, and as $a/W$ approaches 0, $g$ is the ratio of the stress normal to the plane of the crack at $r_1$ using curved beam theory to that stress acting at $r_1$ using straight beam theory. This ratio was determined and found to be highly complex and has a strong dependence on $X/W$ in addition to $r_1/r_2$. Therefore, $g$ was developed by fitting a polynomial that describes the $r_1/r_2$ dependence of the available collocation results for the standard C-shaped specimens, that is for $X/W = 0$ and 0.5. For $a/W$ ranging from .2 to 1, $g$ was found to be:

$$g(r_1/r_2, a/W) = 1 + .25(1-a/W)^2(1-r_1/r_2) .$$

(9)

The function $f$ was determined from the available collocation data and the deep crack limit for the standard C-shaped specimens. Using multivariable linear regression, these data were fit to the following cubic polynomial, for $a/W$ from 0.2 to 1.

$$f(a/W) = 3.74 - 6.30 a/W + 6.32(a/W)^2 - 2.43(a/W)^3$$

(10)

COMPARISON WITH COLLOCATION DATA

To illustrate the goodness of fit of equation (10), values of $f(a/W)$ calculated from the collocation results for standard specimens are compared in Table 1 to the corresponding values determined from equation (10). For comparison we have defined an error function $E$, which is the difference between the value of $f(a/W)$ from equation (10) and the value $f_c$ from the collocation results divided by the value of $f_c$. 
Based on the comparisons in Table 1 and on the information from the limit solutions, equation (10) agrees with all these solutions within ±1.5% for \(0.2 \leq \frac{a}{W} \leq 1\) and within ±1% for \(0.45 \leq \frac{a}{W} \leq 0.55\) for all \(\frac{r_1}{r_2}\) and \(X/W\) of either 0 or 0.5.

Comparisons are made in Table 2 between equation (10) and other collocation results for nonstandard specimens. Analysis of the relative errors in these cases indicate that the derived \(K\) expression should not be used for values of specimens with \(X/W\) greater than about 1. As \(X/W\) becomes large, the bending component of the stress distribution becomes dominant. Since the bending component is significantly affected by the curvature of the specimen, the variation in \(K\) could have been explained by the function \(g\) derived above. However, \(g\) was not obtained exactly, but by fitting to the numerical solutions for the standard specimens, so good agreement can not be expected for \(X/W\) larger than 0.5.

RESULTS

To calculate \(K\) using equations (8), (9), and (10) is cumbersome. To simplify \(K\) determination, equations (8), (9), and (10) can be arranged to a more convenient form:

\[
K = \frac{P}{B \sqrt{W}} \left[ 3 \frac{X}{W} + 1.9 + 1.1 \frac{a}{W} \right] \left[ 1 + 0.25(1-\frac{a}{W})^2(1-\frac{r_1}{r_2}) \right] F(\frac{a}{W})
\]

where

\[
F(\frac{a}{W}) = \frac{\sqrt{\frac{a}{W}}}{(1-\frac{a}{W})^{3/2}} \left( 3.74 - 6.30 \frac{a}{W} + 6.32(\frac{a}{W})^2 - 2.43(\frac{a}{W})^3 \right)
\]

Numerical values of \(F(\frac{a}{W})\) have been determined for two ranges of \(\frac{a}{W}\) and are presented in Tables 3 and 4.
Equation (11) agrees with the numerical solutions available [3-6] within ± 1.0% for .45 ≤ a/W ≤ .55 for all r₁/r₂ and X/W of either 0 or .5, within ± 1.5% for .2 ≤ a/W ≤ 1 for all r₁/r₂ and X/W equal to 0 or .5; and within about ± 3% for .2 ≤ a/W ≤ 1 for all r₁/r₂ and 0 ≤ X/W ≤ 1. The K expression of equation (11) is as accurate as that which is currently given in ASTM Method E-399-78. The new expression can be used for other fracture testing over a wider range of a/W than that of the current expression.
Fig. 1. The C-Shaped Specimen, Indicating Geometric Parameters
TABLE 1. COMPARISON OF \( f(a/W) \) VALUES FROM EQ. (10) WITH CORRESPONDING VALUES, \( f_c \), FROM COLLOCATION AND LIMIT SOLUTIONS

\[
E = \frac{f(a/W) - f_c}{f_c}
\]

SPECIMENS WITH \( X/W = 0 \):

<table>
<thead>
<tr>
<th>( a/W )</th>
<th>( f(a/W) )</th>
<th>( f_c )</th>
<th>( r_1/r_2 ): Reference: #3</th>
<th>( E )</th>
<th>( f_c )</th>
<th>( E )</th>
<th>( f_c )</th>
<th>( E )</th>
<th>( f_c )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2.713</td>
<td>2.713</td>
<td>1.0</td>
<td>0.000</td>
<td>2.726</td>
<td>-0.005</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>2.353</td>
<td>2.343</td>
<td>0.4</td>
<td>0.002</td>
<td>2.319</td>
<td>0.015</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4</td>
<td>2.076</td>
<td>2.074</td>
<td>0.01</td>
<td>2.053</td>
<td>0.011</td>
<td>2.067</td>
<td>0.004</td>
<td>2.059</td>
<td>0.008</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>1.866</td>
<td>1.871</td>
<td>0.002</td>
<td>1.865</td>
<td>0.001</td>
<td>1.861</td>
<td>0.003</td>
<td>1.869</td>
<td>0.002</td>
<td>-</td>
</tr>
<tr>
<td>0.6</td>
<td>1.710</td>
<td>1.712</td>
<td>0.001</td>
<td>1.716</td>
<td>-0.004</td>
<td>1.702</td>
<td>0.005</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.7</td>
<td>1.593</td>
<td>1.585</td>
<td>0.005</td>
<td>1.593</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.8</td>
<td>1.501</td>
<td>1.483</td>
<td>0.012</td>
<td>1.492</td>
<td>0.006</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>1.330</td>
<td>1.326</td>
<td>0.003</td>
<td>1.326</td>
<td>0.003</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

SPECIMENS WITH \( X/W = 0.5 \):

<table>
<thead>
<tr>
<th>( a/W )</th>
<th>( f(a/W) )</th>
<th>( f_c )</th>
<th>( r_1/r_2 ): Reference: #3</th>
<th>( E )</th>
<th>( f_c )</th>
<th>( E )</th>
<th>( f_c )</th>
<th>( E )</th>
<th>( f_c )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2.713</td>
<td>2.698</td>
<td>1.0</td>
<td>0.006</td>
<td>2.753</td>
<td>-0.015</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>2.353</td>
<td>2.339</td>
<td>0.4</td>
<td>0.006</td>
<td>2.344</td>
<td>0.004</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4</td>
<td>2.076</td>
<td>2.075</td>
<td>0.001</td>
<td>2.072</td>
<td>0.002</td>
<td>2.073</td>
<td>0.001</td>
<td>2.078</td>
<td>-0.001</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>1.866</td>
<td>1.873</td>
<td>0.003</td>
<td>1.877</td>
<td>-0.006</td>
<td>1.871</td>
<td>-0.003</td>
<td>1.879</td>
<td>-0.007</td>
<td>-</td>
</tr>
<tr>
<td>0.6</td>
<td>1.710</td>
<td>1.713</td>
<td>0.002</td>
<td>1.722</td>
<td>-0.007</td>
<td>1.702</td>
<td>0.005</td>
<td>1.721</td>
<td>-0.006</td>
<td>-</td>
</tr>
<tr>
<td>0.7</td>
<td>1.593</td>
<td>1.586</td>
<td>0.004</td>
<td>1.596</td>
<td>-0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.8</td>
<td>1.501</td>
<td>1.483</td>
<td>0.012</td>
<td>1.493</td>
<td>0.005</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>1.330</td>
<td>1.326</td>
<td>0.003</td>
<td>1.326</td>
<td>0.003</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
TABLE 2. COMPARISON OF $f(a/W)$ VALUES FROM EQ. (10) WITH
CORRESPONDING VALUES, $f_c$, FROM REFERENCE 4

$$E = \frac{f(a/W) - f_c}{f_c}$$

<table>
<thead>
<tr>
<th>$X/W = 0.560$ ; $r_1/r_2 = 0.481$</th>
<th>$X/W = 0.712$ ; $r_1/r_2 = 0.500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a/W$</td>
<td>$f_c$</td>
</tr>
<tr>
<td>.275</td>
<td>2.492</td>
</tr>
<tr>
<td>.366</td>
<td>2.137</td>
</tr>
<tr>
<td>.458</td>
<td>1.939</td>
</tr>
<tr>
<td>.549</td>
<td>1.801</td>
</tr>
<tr>
<td>.641</td>
<td>1.679</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X/W = 0.983$ ; $r_1/r_2 = 0.584$</th>
<th>$X/W = 1.600$ ; $r_1/r_2 = 0.667$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a/W$</td>
<td>$f_c$</td>
</tr>
<tr>
<td>.281</td>
<td>2.463</td>
</tr>
<tr>
<td>.421</td>
<td>2.086</td>
</tr>
<tr>
<td>.562</td>
<td>1.831</td>
</tr>
<tr>
<td>.702</td>
<td>1.540</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 3. Values of $F(a/W)$ for the range of $a/W$ used in $K_{IC}$ testing

<table>
<thead>
<tr>
<th>$a/W$</th>
<th>$F(a/W)$</th>
<th>$a/W$</th>
<th>$F(a/W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.450</td>
<td>3.23</td>
<td>0.500</td>
<td>3.73</td>
</tr>
<tr>
<td>0.455</td>
<td>3.27</td>
<td>0.505</td>
<td>3.79</td>
</tr>
<tr>
<td>0.460</td>
<td>3.32</td>
<td>0.510</td>
<td>3.85</td>
</tr>
<tr>
<td>0.465</td>
<td>3.37</td>
<td>0.515</td>
<td>3.91</td>
</tr>
<tr>
<td>0.470</td>
<td>3.42</td>
<td>0.520</td>
<td>3.97</td>
</tr>
<tr>
<td>0.475</td>
<td>3.47</td>
<td>0.525</td>
<td>4.03</td>
</tr>
<tr>
<td>0.480</td>
<td>3.52</td>
<td>0.530</td>
<td>4.10</td>
</tr>
<tr>
<td>0.485</td>
<td>3.57</td>
<td>0.535</td>
<td>4.17</td>
</tr>
<tr>
<td>0.490</td>
<td>3.62</td>
<td>0.540</td>
<td>4.24</td>
</tr>
<tr>
<td>0.495</td>
<td>3.68</td>
<td>0.545</td>
<td>4.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.550</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4. Values of $F(a/W)$ for wide range of $a/W$

<table>
<thead>
<tr>
<th>$a/W$</th>
<th>$F(a/W)$</th>
<th>$a/W$</th>
<th>$F(a/W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>1.70</td>
<td>0.60</td>
<td>5.24</td>
</tr>
<tr>
<td>0.25</td>
<td>1.94</td>
<td>0.65</td>
<td>6.42</td>
</tr>
<tr>
<td>0.30</td>
<td>2.20</td>
<td>0.70</td>
<td>8.11</td>
</tr>
<tr>
<td>0.35</td>
<td>2.49</td>
<td>0.75</td>
<td>10.70</td>
</tr>
<tr>
<td>0.40</td>
<td>2.82</td>
<td>0.80</td>
<td>15.01</td>
</tr>
<tr>
<td>0.45</td>
<td>3.23</td>
<td>0.85</td>
<td>23.15</td>
</tr>
<tr>
<td>0.50</td>
<td>3.73</td>
<td>0.90</td>
<td>42.53</td>
</tr>
<tr>
<td>0.55</td>
<td>4.38</td>
<td>0.95</td>
<td>119.90</td>
</tr>
</tbody>
</table>
REFERENCES


<table>
<thead>
<tr>
<th>Internal Distribution List</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TECHNICAL REPORT</strong></td>
</tr>
<tr>
<td><strong>INTERNAL DISTRIBUTION LIST</strong></td>
</tr>
<tr>
<td><strong>COMMANDER</strong></td>
</tr>
<tr>
<td><strong>CHIEF, DEVELOPMENT ENGINEERING BRANCH</strong></td>
</tr>
<tr>
<td><strong>CHIEF, ENGINEERING SUPPORT BRANCH</strong></td>
</tr>
<tr>
<td><strong>CHIEF, RESEARCH BRANCH</strong></td>
</tr>
<tr>
<td><strong>CHIEF, LWC MORTAR SYS. OFC.</strong></td>
</tr>
<tr>
<td><strong>CHIEF, IMP. 81MM MORTAR OFC.</strong></td>
</tr>
<tr>
<td><strong>TECHNICAL LIBRARY</strong></td>
</tr>
<tr>
<td><strong>TECHNICAL PUBLICATIONS &amp; EDITING UNIT</strong></td>
</tr>
<tr>
<td><strong>DIRECTOR, OPERATIONS DIRECTORATE</strong></td>
</tr>
<tr>
<td><strong>DIRECTOR, PROCUREMENT DIRECTORATE</strong></td>
</tr>
<tr>
<td><strong>DIRECTOR, PRODUCE ASSURANCE DIRECTORATE</strong></td>
</tr>
</tbody>
</table>
| **NOTE:** PLEASE NOTIFY ASSOC. DIRECTOR, BENET WEAPONS LABORATORY, ATTN: DRDAR-LCB-TL, OF ANY REQUIRED CHANGES.
ASST SEC OF THE ARMY RESEARCH & DEVELOPMENT
ATTN: DEP FOR SCI & TECH THE PENTAGON
WASHINGTON, D.C. 20315

COMMANDER
US ARMY MAT DEV & READ. CMD
ATTN: DRCDE
5001 EISENHOWER AVE
ALEXANDRIA, VA 22333

COMMANDER
US ARMY ARRADCOM
ATTN: DRAR-IC
-ICA (PLASTICS TECH EVAL CEN)
-ICE
-IEM
-ICS
-ICW
-TSS(STINFO)

DOVER, NJ 07801

COMMANDER
US ARMY ARRCOM
ATTN: DRSAR-LEP-L
ROCK ISLAND ARSENAL
ROCK ISLAND, IL 61299

DIRECTOR
US Army Ballistic Research Laboratory
ATTN: DRDAR-TSB-S (STINFO)
ABERDEEN PROVING GROUND, MD 21005

COMMANDER
US ARMY ELECTRONICS COMD
ATTN: TECH LIB
FT MONMOUTH, NJ 07703

COMMANDER
US ARMY MOBILITY EQUIP R&D COMD
ATTN: TECH LIB
FT BELVOIR, VA 22060

NOTE: PLEASE NOTIFY COMMANDER, ARRADCOM, ATTN: BENET WEAPONS LABORATORY, DRAR-LCB-TL, WATERTOWN ARSENAL, WATERVIKT, N.Y. 12189, OF ANY REQUIRED CHANGES.
<table>
<thead>
<tr>
<th>COMMANDER</th>
<th>NO. OF COPIES</th>
<th>COMMANDER</th>
<th>NO. OF COPIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>US ARMY RESEARCH OFFICE</td>
<td>1</td>
<td>DEFENSE TECHNICAL INFO CENTER</td>
<td>12</td>
</tr>
<tr>
<td>P.O. BOX 12211</td>
<td></td>
<td>ATTN: DTIA-TCA</td>
<td></td>
</tr>
<tr>
<td>RESEARCH TRIANGLE PARK, NC 27709</td>
<td></td>
<td>CAMERON STATION</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>ALEXANDRIA, VA 22314</td>
<td></td>
</tr>
<tr>
<td>US ARMY HARRY DIAMOND LAB</td>
<td>1</td>
<td>METAIS &amp; CERAMICS INFO CEN</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: TECH LIB</td>
<td></td>
<td>BATTELLE COLUMBUS LAB</td>
<td></td>
</tr>
<tr>
<td>2800 POWDER MILL ROAD</td>
<td></td>
<td>505 KING AVE</td>
<td></td>
</tr>
<tr>
<td>ADELPHIA, MD 20783</td>
<td></td>
<td>COLUMBUS, OHIO 43201</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIRECTOR</td>
<td>1</td>
<td>MECHANICAL PROPERTIES DATA CTR</td>
<td>1</td>
</tr>
<tr>
<td>US ARMY INDUSTRIAL BASE ENG ACT</td>
<td></td>
<td>BATTELLE COLUMBUS LAB</td>
<td></td>
</tr>
<tr>
<td>ATTN: DRXPE-MT</td>
<td></td>
<td>505 KING AVE</td>
<td></td>
</tr>
<tr>
<td>ROCK ISLAND, IL 61201</td>
<td></td>
<td>COLUMBUS, OHIO 43201</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CHIEF, MATERIALS BRANCH</td>
<td>1</td>
<td>MATERIEL SYSTEMS ANALYSIS ACTIV</td>
<td>1</td>
</tr>
<tr>
<td>US ARMY R&amp;S GROUP, EUR</td>
<td></td>
<td>ATTN: DRXSY-MP</td>
<td></td>
</tr>
<tr>
<td>BOX 65, FPO N.Y. 09510</td>
<td></td>
<td>ABERDEEN PROVING GROUND</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MARYLAND 21005</td>
<td></td>
</tr>
<tr>
<td>NAVAL SURFACE WEAPONS CEN</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: CHIEF, MAT SCIENCE DIV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAHLGREN, VA 22448</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIRECTOR</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US NAVAL RESEARCH LAB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: DIR, MECH DIV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CODE 26-27 (DCG LIB)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, D. C. 20375</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASA SCIENTIFIC &amp; TECH INFO FAC</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P.O. BOX 8757, ATTN: ACQ BR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BALTIMORE/WASHINGTON INTL AIRPORT MARYLAND 21240</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: PLEASE NOTIFY COMMANDER, ARRADCOM, ATTN: BENET WEAPONS LABORATORY, DRDAAR-LCB-TL, WATERVLIET ARSENAL, WATERVLIET, N.Y. 12189, OF ANY REQUIRED CHANGES.