

AFWAL-TR-80-3018

LEVEL II

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RESPONSE OF NONLINEAR STRUCTURAL PANELS SUBJECTED TO HIGH INTENSITY NOISE

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March 1980

TECHNICAL REPORT AFWAL-TR-80-3018

Final Report for Period June 1979 through August 1979

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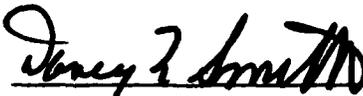
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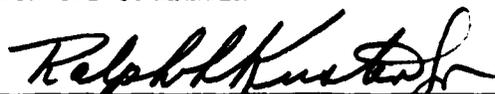


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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19 REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

18

1. REPORT NUMBER
AFWAL TR-80-3018 1

2. GOVT ACCESSION NO.
AD-A085638

9

3. RECIPIENT'S CATALOG NUMBER

6

4. TITLE (and Subtitle)
RESPONSE OF NONLINEAR STRUCTURAL PANELS SUBJECTED TO HIGH INTENSITY NOISE.

5. TYPE OF REPORT & PERIOD COVERED
Final Report, ~~Final Report~~
August 18 Jun-20 Aug 79

10

6. AUTHOR
Chuh Mei

8. CONTRACT OR GRANT NUMBER(s)

17 01

7. PERFORMING ORGANIZATION NAME AND ADDRESS
Flight Dynamics Laboratory
Structural Integrity Branch
Structures & Dynamics Division
Wright-Patterson AFB, Ohio 45433

16

10. PROGRAM ELEMENT, SUBJECT AREA & WORK UNIT NUMBERS
2401046

11

13. REPORT DATE
March 1980

12. NUMBER OF PAGES
56

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)

UNCLASSIFIED

12 56

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.

15 F49620-79-C-0169

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DRIC
JUN 15 1980

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Nonlinear Response
Sonic Fatigue
Large Amplitude Response

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
Lightweight aircraft structures exposed to a high intensity noise environment can fatigue fail prematurely if adequate consideration is not given to the problem. Design methods and design criteria for sonic fatigue prevention have been developed based on analytical and experimental techniques. Most of the analytical work was based upon small deflection or linear structural theory which did not agree with the experimental results. A large deflection geometrical nonlinearity was incorporated into the analysis methods for determining

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20. Abstract (Continued)

the structural response to high intensity noise. Using the Karman-Herrmann large deflection equations for rectangular plates, a single mode Galerkin approximation, the nonlinear differential equations of motion were obtained. The method of equivalent linearization was used to solve the nonlinear equations for mean-square displacement, mean square stresses and nonlinear frequencies at various acoustic loadings for rectangular panels. Comparisons with experimental results are presented. The results obtained agreed with the experimental results; however, additional test data are needed for an adequate quantitative comparison.

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FOREWORD

This report contains the research results on large amplitude response of aircraft structural panels subjected to high intensity broadband random acoustic excitation. The work was performed at the Department of Engineering Mechanics, University of Missouri-Rolla during the period 18 June 1979 to 20 August 1979. The research was sponsored by the Air Force Office of Scientific Research (AFSC), United States Air Force, under contract F49620-79-C-0169. The work was monitored under the supervision of Lt. Colonel Joseph D. Morgan III, Office of Aerospace Sciences, and Howard F. Wolfe, Structural Integrity Branch, Structures and Dynamics Division, Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio 45433. The work was performed under work unit number 24010146, "Sonic Fatigue Test of Advanced Materials and Structural Configurations."

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TABLE OF CONTENTS

SECTION	Page
I INTRODUCTION	1
II FORMULATION AND SOLUTION PROCEDURE	3
1. Governing Equations	3
a. Simply Supported Panels	3
b. Clamped Panels	7
2. Method of Equivalent Linearization	9
3. Solution Procedure	12
4. Stress and Strain Response	13
a. Simply Supported Panels	14
b. Clamped Panels	15
III RESULTS AND DISCUSSION	18
1. Simply Supported Panels	19
2. Clamped Panels	20
3. Comparison with Experimental Results	20
IV CONCLUSIONS	25
REFERENCES	26
APPENDIX - RANDOM RESPONSE SOLUTION PROGRAMS	39

LIST OF ILLUSTRATIONS

<u>FIGURE NO.</u>	<u>TITLE</u>	<u>PAGE</u>
1.	Geometry and Coordinates	4
2.	Mean-square deflection versus spectral density parameter of excitation for simply supported panels, $\zeta = 0.02$.	27
3.	Maximum mean-square bending stress versus spectral density parameter of excitation for a simply supported square panel, $\zeta = 0.02$	28
4.	Mean-square membrane stress versus spectral density parameter of excitation for a simply supported square panel, $\zeta = 0.02$.	29
5.	Maximum mean-square stress versus spectral density parameter of excitation for simply supported panels, $\zeta = 0.02$	30
6.	Effects of damping on mean-square deflection for a simply supported square panel	31
7.	Effects of damping on maximum mean-square stress for a simply supported square panel	32
8.	Frequency parameter versus mean-square deflection for simply supported panels	33
9.	Mean-square deflection versus spectral density parameter of excitation for clamped panels, $\zeta = 0.02$.	34
10.	Maximum mean-square stress versus spectral density parameter of excitation for clamped panels, $\zeta = 0.02$	35
11.	Effects on damping on mean-square deflection for a clamped square panel	36
12.	Frequency parameter versus mean-square deflection for clamped panels	37
13.	Skin-stringer panel	38

LIST OF TABLES

<u>TABLE NO.</u>	<u>TITLE</u>	<u>PAGE</u>
1	Frequency Comparison	22
2	Stress Comparison	23
3	Deflection Comparison	24

NOMENCLATURE

a, b	Panel length and width
A, B	Panel dimension parameters, $2\pi/a$ and $2\pi/b$
C_1, C_2	Constants
D	Bending rigidity
err	Error of linearization
E	Young's modulus
f	Equivalent linear frequency in Hz
F	Stress function
h	Panel thickness
$H(\omega)$	Frequency response function
L	Spectrum level
m	Mass coefficient
N	Membrane stress resultant
\bar{N}	Constant
p	Pressure loading
q	Generalized or nodal displacement
r	Aspect ratio, a/b
$S(\omega)$	Spectral density function of excitation pressure $p(t)$
t	Time
u, v	Displacement of midplane
w	Transverse deflection
x, y, z	Coordinates
β	Nonlinearity coefficient
β^*	Nondimensional nonlinearity coefficient
ϵ	Strain
ζ	Damping ratio, c/c_c

λ	Nondimensional frequency parameter
ν	Poisson's ratio
ρ	Panel mass density
σ, τ	Normal and shear stresses
ω	Radian frequency
Ω	Equivalent linear or nonlinear radian frequency
b	Bending
c	Complementary solution or critical
m	Membrane
max	Maximum
o	Linear
p	Particular solution

SECTION I

INTRODUCTION

Vibrations caused by acoustic pressure can frequently disturb the operating conditions of various instruments and systems, and sonic fatigue failures which occurred in aircraft structural components cause large maintenance and inspection burdens for the Air Force. The development of sonic fatigue data and design techniques were initiated to prevent sonic fatigue failures. Design methods and design criteria for many types of aircraft structures have been developed under Air Force sponsorship and by the industry in the past twenty years. Reference 1 has a complete list of the reports describing these efforts. This research led to sonic fatigue design criteria and design charts which are widely used during the design of an aircraft. Although current analytical sonic fatigue design methods are essentially based on small deflection or linear structural theory (Reference 1, page 209), many documented tests on various aircraft panels have indicated that high noise levels produce nonlinear behavior with large amplitudes in such structural panels. For example, Fitch et al, (Reference 2), van der Heyde and Smith (Reference 3), Jacobs and Lagerquist (Reference 4), and Jacobson (References 5 and 6) have repeatedly reported that a poor comparison exists between the measured and computed root mean square (RMS) displacements and/or RMS stresses. They all observed that the test panels responded with large deflections at high sound pressure levels, whereas the analytical responses were based on linear small deflection theory. The neglect of such large deflection geometrical nonlinearity in analysis and design formulations has been identified as one of the major causes for disagreement between experimental data and analytical results. The evidence of those researchers was summarized in Reference 7,

where a comprehensive review of existing analytical methods on random excitations of nonlinear systems was also given.

Because there are no reliable analysis methods available for predicting the nonlinear stress-sound pressure relation, costly and time-consuming full-scale fatigue tests of aircraft structures and components are frequently conducted. The objectives of the present work are: 1) to gain a better understanding of the random response of nonlinear panels, 2) to incorporate large deflection geometrical nonlinearity into analysis methods for determining structural response to high intensity noise, and 3) to provide analytical background material for formulation of improved sonic fatigue design procedures that would result in better and less costly designs without sacrificing safety.

The Karman-Herrmann large deflection equations for rectangular plates (Reference 8) are employed in this development. Using a single-mode Galerkin's approximation, the dynamic equations reduce to a nonlinear differential equation with time as the independent variable. The method of equivalent linearization is then applied to reduce the nonlinear equation to an equivalent linear one (References 12-14). Mean-square displacements, mean-square stresses, and nonlinear frequencies at various acoustic loadings are obtained for rectangular panels of different aspect ratios and damping factors. Both simply supported and clamped boundary conditions with immovable and movable inplane edges are considered. The results are presented in graphical form. Comparisons with experimental results are also presented.

SECTION II

FORMULATION AND SOLUTION PROCEDURE

1. Governing Equations

Assuming that the effect of both the in plane and rotatory inertia forces can be neglected, the dynamic equations of a rectangular isotropic plate undergoing moderately large deflections are (References 8 and 9):

$$L(w,F) = D\nabla^4 w + \rho h w_{,tt} - h (F_{,yy} w_{,xx} + F_{,xx} w_{,yy} - 2F_{,xy} w_{,xy}) - p(t) = 0 \quad (1)$$

$$\nabla^4 F = E (w_{,xy}^2 - w_{,xx} w_{,yy}) \quad (2)$$

where w is the transverse deflection of the plate, h is the panel thickness, ρ is the mass density of the panel material, $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity, E is Young's modulus, ν is Poisson's ratio, $p(t)$ is the exciting pressure, and a comma preceding a subscript(s) indicates partial differentiation (s). The stress function F is defined by

$$\begin{aligned} \sigma_x &= F_{,yy} \\ \sigma_y &= F_{,xx} \\ \tau_{xy} &= -F_{,xy} \end{aligned} \quad (3)$$

where σ_x , σ_y , and τ_{xy} are membrane stresses.

a. Simply Supported Panels

For a rectangular plate simply supported along all four edges as shown in Figure 1, Chu and Herrmann (Reference 8), and Lin (Reference 10) have considered that, if the fundamental mode is predominant, the motion of the panel can

be represented adequately as

$$w = q(t) h \cos (\pi x/a) \cos (\pi y/b) \quad (4)$$

where $q(t)$ is a function of time only. The maximum value of $q(t)$ coincides with the maximum deflection w_{\max} divided by panel thickness h . The expression w satisfies the boundary conditions for simple supports.

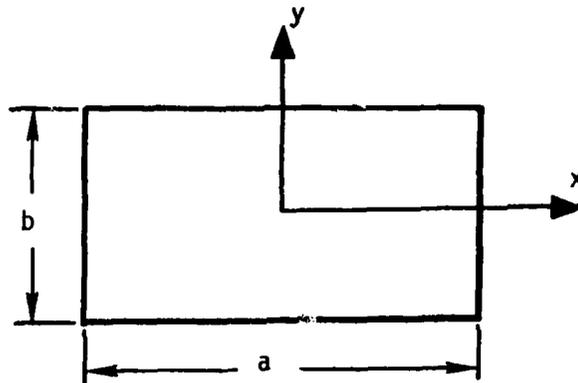


Figure 1.

Geometry and Coordinates

$$\begin{aligned} w = w_{,xx} + \nu w_{,yy} &= 0, \text{ on } x = \pm a/2 \\ w = w_{,yy} + \nu w_{,xx} &= 0, \text{ on } y = \pm b/2 \end{aligned} \quad (5)$$

Substituting the expression for w in Eq. (2) and solving for a particular solution F_p yields

$$F_p = -\left(\frac{1}{32}\right) q^2 h^2 E r^2 \left(\cos \frac{2\pi x}{a} + \frac{1}{r^4} \cos \frac{2\pi y}{b} \right) \quad (6)$$

where $r = a/b$. The complementary solution to Eq. (2) is taken in the form

$$F_c = \bar{N}_x \frac{y^2}{2} + \bar{N}_y \frac{x^2}{2} - \bar{N}_{xy} xy \quad (7)$$

where the constants \bar{N}_x , \bar{N}_y and \bar{N}_{xy} contribute to the membrane stresses

σ_x , σ_y and τ_{xy} and are to be determined from the inplane boundary, immovable or movable, conditions.

For the immovable edges case, the conditions of zero inplane normal displacement at all four edges are satisfied in an averaged manner as

$$\left. \begin{aligned} F_{,xy} &= 0 \\ \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \frac{\partial u}{\partial x} dx dy &= \iint \left[\frac{1}{E} (F_{,yy} - \nu F_{,xx}) - \frac{1}{2} w^2_{,x} \right] dx dy \end{aligned} \right\}, \text{ on } x = \pm a/2$$

$$\left. \begin{aligned} F_{,xy} &= 0 \\ \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \frac{\partial v}{\partial x} dy dx &= \iint \left[\frac{1}{E} (F_{,xx} - \nu F_{,yy}) - \frac{1}{2} w^2_{,y} \right] dy dx \end{aligned} \right\}, \text{ on } y = \pm b/2 \quad (8)$$

where u and v are inplane displacements.

For the movable edges case, the edges are free to move as a rigid body with the average inplane stress equal to zero. The inplane boundary conditions are

$$\left. \begin{aligned} F_{,xy} &= 0 \\ N_x &= h \int_{-b/2}^{b/2} F_{,yy} dy = 0 \end{aligned} \right\}, \text{ on } x = \pm a/2$$

$u = \text{constant}$

(9)

$$\left. \begin{aligned} F_{,xy} &= 0 \\ N_y &= h \int_{-a/2}^{a/2} F_{,yy} dx = 0 \end{aligned} \right\}, \text{ on } y = \pm b/2$$

$v = \text{constant}$

where N_x and N_y are membrane stress resultants per unit length in plate.

By making use of these inplane edge boundary conditions, Eqs. (8) and

(9), it easily can be shown that for the immovable edges

$$\begin{aligned}\bar{N}_x &= \frac{q^2 h^2 E \pi^2}{8a^2 (1-\nu^2)} (1 + \nu r^2) \\ \bar{N}_y &= \frac{q^2 h^2 E \pi^2}{8a^2 (1-\nu^2)} (r^2 + \nu) \\ \bar{N}_{xy} &= 0\end{aligned}\tag{10}$$

and for the movable edges

$$\bar{N}_x = \bar{N}_y = \bar{N}_{xy} = 0\tag{11}$$

the complete stress function is then given by $F = F_p + F_c$.

With the assumed w given by Eq. (4) and stress function given by Eqs. (6) and (7), Eq. (1) is satisfied by applying Galerkin's method

$$\int_{-a/2}^{a/2} \int_{-b/2}^{b/2} L(w, F) w \, dx dy = 0\tag{12}$$

from which yields the modal equation of the form

$$\ddot{q} + \omega_0^2 q + \beta q^3 = \frac{p(t)}{m}\tag{13}$$

and

$$\begin{aligned}\omega_0^2 &= \lambda_0^2 \frac{D}{\rho h b^4}, \quad \lambda_0^2 = \pi^4 \left(1 + \frac{1}{r^2}\right)^2 \\ m &= \pi^2 \rho h^2 / 16 \\ \beta &= \beta_p + \beta_c\end{aligned}\tag{14}$$

with

$$\beta_p = \beta_p^* \frac{D}{\rho h b^4}, \quad \beta_p^* = \frac{3\pi^4}{4r^4} (1 + r^4) (1 - \nu^2)$$

$$\beta_c = \beta_c^* \frac{D}{\rho h b^4}, \quad \beta_c^* = \frac{3\pi^4}{2r^4} [1 + \nu r^2 + r^2(r^2 + \nu)]$$

where ω_0 is linear radian frequency, m is mass coefficient, and β is non-linearity coefficient. The linear frequency λ_0 , nonlinearity coefficients β_p^* and β_c^* , and aspect ratio r are all nondimensional parameters.

b. Clamped Panels

Yamaki (Reference 11) considered the predominant mode

$$w = \frac{q(t)h}{4} (1 + \cos \frac{2\pi x}{a}) (1 + \cos \frac{2\pi y}{b}) \quad (15)$$

which satisfies the clamped support conditions

$$\begin{aligned} w = w_{,x} = 0, & \quad \text{on } x = \pm a/2 \\ w = w_{,y} = 0, & \quad \text{on } y = \pm b/2 \end{aligned} \quad (16)$$

By introducing Eq. (15) in Eq. (2) and solving it, the particular stress function is

$$\begin{aligned} F_p = & -\frac{1}{32} q^2 h^2 E r^2 \left[\cos \frac{2\pi x}{a} + \frac{1}{r^4} \cos \frac{2\pi y}{b} + \frac{1}{16} \cos \frac{4\pi x}{a} \right. \\ & + \frac{2}{(1+r^2)^2} \cos \frac{2\pi x}{a} \cos \frac{2\pi y}{b} + \frac{1}{16r^4} \cos \frac{4\pi y}{b} \\ & \left. + \frac{1}{(4+r^2)^2} \cos \frac{4\pi x}{a} \cos \frac{2\pi y}{b} + \frac{1}{(1+4r^2)^2} \cos \frac{2\pi x}{a} \cos \frac{4\pi y}{b} \right] \quad (17) \end{aligned}$$

The complementary stress function is assumed as the form appearing in Eq. (7). Upon enforcing the inplane edge conditions, Eqs. (8) and (9), it can be shown that for the immovable edges

$$\begin{aligned}\bar{N}_x &= \frac{3q^2 h^2 E\pi^2}{32 a^2 (1-\nu^2)} (1 + \nu r^2) \\ \bar{N}_y &= \frac{3q^2 h^2 E\pi^2}{32 a^2 (1-\nu^2)} (r^2 + \nu) \\ \bar{N}_{xy} &= 0\end{aligned}\tag{18}$$

for the movable edges

$$\bar{N}_x = \bar{N}_y = \bar{N}_{xy} = 0\tag{19}$$

the complete stress function is given by $F = F_p + F_c$. Introducing these expressions for w and F in Eq. (1) and applying Galerkin's procedure yields the modal equation

$$\ddot{q} + \omega_0^2 q + \beta q^3 = \frac{p(t)}{m}\tag{13}$$

where

$$\begin{aligned}\omega_0^2 &= \lambda_0^2 \frac{D}{\rho h b^4}, & \lambda_0^2 &= \frac{16\pi^4}{9r^4} (3 + 2r^2 + 3r^4) \\ m &= 9 \rho h^2 / 16 \\ \beta &= \beta_p + \beta_c = (\beta_p^* + \beta_c^*) \frac{D}{\rho h b^4}\end{aligned}\tag{20}$$

and

$$\begin{aligned} \beta_p^* &= \frac{4}{3} \pi^4 (1-\nu^2) \left[1 + \frac{1}{r^4} + \frac{1}{16} + \frac{2}{(1+r^2)^2} + \frac{1}{16r^4} \right. \\ &+ \left. \frac{1}{2(4+r^2)^2} + \frac{1}{2(1+4r^2)^2} \right] \\ \beta_c^* &= \frac{3\pi^4}{2r^4} [1 + \nu r^2 + r^2 (r^2 + \nu)] \end{aligned} \quad (21)$$

Equation (13) represents the undamped, large amplitude vibration of a rectangular panel with simply-supported and clamped edges.

The methods commonly used for determining the damping coefficient are the bandwidth method in which half-power widths are measured at modal resonances and the decay rate method in which the logarithmic decrement of decaying modal response traces is measured. The values of damping ratio ζ ($= c/c_c$) range from 0.005 to 0.05 for the common type of panel construction used in aircraft structures. Once the damping coefficient is determined from experiments or from existing data of similar construction, the modal equation, Eq. (13), now reads

$$\ddot{q} + 2\zeta\omega_0 \dot{q} + \omega_0^2 q + \beta q^3 = \frac{p(t)}{m} \quad (22)$$

The method of equivalent linearization is employed to determine an approximate RMS displacement from Eq. (22).

2. Method of Equivalent Linearization

The basic idea of the equivalent linearization (Reference 12-14) is to replace the original nonlinear equation, Eq. (22), with an equation of the form

$$\ddot{q} + 2 \zeta \omega_0 \dot{q} + \Omega^2 q + \text{err}(q) = \frac{p(t)}{m} \quad (23)$$

where Ω is an equivalent linear or nonlinear frequency, and err is the error of linearization. An equivalent linear equation is obtained by omitting this error term, then Eq. (23) is linear and it can be readily solved. The error of linearization is

$$\text{err} = (\omega_0^2 - \Omega^2) q + \beta q^3 \quad (24)$$

which is the difference between Eq. (22) and Eq. (23). The equivalent linear frequency Ω is chosen in such a way as to make the error of linearization term, $\text{err}(q)$, as small as possible. To this end the mean-square error

$$\overline{\text{err}^2} \text{ is minimized, that is } \frac{\partial(\overline{\text{err}^2})}{\partial(\Omega^2)} = 0 \quad (25)$$

If the acoustic pressure excitation $p(t)$ is stationary Gaussian and ergodic, then the response q computed from the linearized equation, Eq. (23), must also be Gaussian. Substituting Eq. (24) into Eq. (25) yields (References 10 and 12)

$$\Omega^2 = \omega_0^2 + 3 \beta \overline{q^2} \quad (26)$$

where $\overline{q^2}$ is the maximum mean-square deflection of the panel. Dividing both sides of Eq. (26) by $D/\rho h b^4$ yields

$$\lambda^2 = \lambda_0^2 + 3\beta^* \overline{q^2} \quad (27)$$

where λ^2 is a nondimensional equivalent linear or nonlinear frequency parameter.

An approximate solution of Eq. (23) is obtained by dropping the error term, the mean-square response of modal amplitude is

$$\overline{q^2} = \int_0^{\infty} S(\omega) |H(\omega)|^2 d\omega \quad (28)$$

where $S(\omega)$ is the spectral density function of the excitation pressure $p(t)$, and the frequency response function $H(\omega)$ is given by

$$H(\omega) = \frac{1}{m(\Omega^2 - \omega^2 + 2i\zeta\omega_0\omega)} \quad (29)$$

For lightly damped ($\zeta \leq 0.05$) structures, the response curves will be highly peaked at Ω . The integration of Eq. (28) can be greatly simplified if the forcing spectral density function $S(\omega)$ can be considered to be constant in the frequency band surrounding the nonlinear resonance peak Ω , so that

$$\overline{q^2} \approx \frac{\pi S(\Omega)}{4m^2 \zeta \omega_0 \Omega^2} \quad (30)$$

In practice, the spectral density function is generally given in terms of the frequency f in Hertz. To convert the previous result one must substitute

$$\Omega = 2\pi f$$

and
$$S(\Omega) = S(f)/2\pi \quad (31)$$

into Eq. (30), the mean-square peak deflection is simply

$$\overline{q^2} = \begin{cases} \frac{32 S_f}{\pi^4 \zeta \lambda_0 \lambda^2}, & \text{for simply supported panels} \\ \frac{32 S_f}{81 \zeta \lambda_0 \lambda^2}, & \text{for clamped panels} \end{cases} \quad (32)$$

The pressure spectral density function $S(f)/2\pi$ has the units $(\text{Pa})^2/\text{Hz}$ or $(\text{psi})^2/\text{Hz}$, and S_f is a nondimensional forcing excitation spectral density parameter defined as

$$S_f = \frac{S(f)}{\rho^2 h^4 (D/\rho h b^4)^{3/2}} \quad (33)$$

The linear frequency parameters λ_0 in Eqs. (32) are given in Eq. (14) and Eq. (20) for simply supported and clamped panels, respectively, and the equivalent linear frequency parameters λ^2 can be determined through Eq. (27).

3. Solution Procedure

The mean-square response $\overline{q^2}$ in Eq. (30) (or Eq. (32)) is determined at the equivalent linear frequency Ω (or λ) which is in turn related to $\overline{q^2}$ through Eq. (26) (or Eq. (27)). To determine the mean-square deflection, an iterative procedure is introduced. One can estimate the initial mean-square deflection $\overline{q_0^2}$ using the linear frequency ω_0 through Eq. (30) as

$$\overline{q_0^2} = \frac{\pi S(\omega_0)}{4m^2 \zeta \omega_c^3} \quad (34)$$

This initial estimate of $\overline{q_0^2}$ is simply the mean-square response based on linear theory. This initial estimate of $\overline{q_0^2}$ can now be used to obtain refined estimate of Ω_1 through Eq. (26), $\Omega_1^2 = \omega_0^2 + 3B \overline{q_0^2}$, then $\overline{q_1^2}$ is obtained through Eq. (30) as

$$\overline{q_1^2} = \frac{\pi S(\Omega_1)}{4m^2 \zeta \omega_0 \Omega_1^2} \quad (35)$$

As the iterative process converges, the relation

$$\overline{q_n^2} = \frac{\pi S(\Omega_n)}{4m^2 \zeta \omega_0 \Omega_n^2} \approx \overline{q_{n-1}^2} \quad (36)$$

becomes satisfied. In the numerical results presented in the following section, convergence is considered achieved whenever the difference of the RMS displacements satisfied the relation

$$\left| \frac{\sqrt{\overline{q_n^2}} - \sqrt{\overline{q_{n-1}^2}}}{\sqrt{\overline{q_n^2}}} \right| \leq 10^{-3} \quad (37)$$

4. Stress and Strain Response

Once the RMS nodal displacement is determined, the bending stresses on the surface of the panel can be determined from

$$\sigma_{xb} = -\frac{6D}{h^2} (w_{,xx} + \nu w_{,yy}) \quad (38)$$

$$\sigma_{yb} = -\frac{6D}{h^2} (w_{,yy} + \nu w_{,xx})$$

The corresponding strains are given by

$$\begin{aligned} \epsilon_{xb} &= -\frac{h}{2} w_{,xx} \\ \epsilon_{yb} &= -\frac{h}{2} w_{,yy} \end{aligned} \quad (39)$$

The membrane stresses in the panel are obtained from Eqs. (3) and the corresponding stress function F . Membrane strains are given by

$$\begin{aligned} \epsilon_{xm} &= \frac{1}{E} (F_{,yy} - \nu F_{,xx}) \\ \epsilon_{ym} &= \frac{1}{E} (F_{,xx} - \nu F_{,yy}) \end{aligned} \quad (40)$$

a. Simply Supported Panels

From Eqs. (3) and (38), and using Eqs. (4), (6), (7) and (10), the expressions for the nondimensional stresses on the surface of the panel with immovable edges are given by

$$\begin{aligned}
 \frac{\sigma_x b^2}{Eh^2} &= (\sigma_{xb} + \sigma_{xm}) \frac{b^2}{Eh^2} \\
 &= \left[\frac{\pi^2}{2(1-\nu^2)} \left(\frac{1}{r^2} + \nu \right) \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right] q \\
 &\quad + \left(\frac{\pi^2}{8r^2} \cos \frac{2\pi y}{b} \right) q^2 + \left[\frac{\pi^2(1 + \nu r^2)}{8r^2(1-\nu^2)} \right] q^2 \\
 \frac{\sigma_y b^2}{Eh^2} &= (\sigma_{yb} + \sigma_{ym}) \frac{b^2}{Eh^2} \tag{41} \\
 &= \left[\frac{\pi^2}{2(1-\nu^2)} \left(1 + \frac{\nu}{r^2} \right) \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right] q \\
 &\quad + \left(\frac{\pi^2}{8} \cos \frac{2\pi x}{a} \right) q^2 + \left[\frac{\pi^2(r^2 + \nu)}{8r^2(1-\nu^2)} \right] q^2
 \end{aligned}$$

The tensile strains, Eqs. (39) and (40), on the surface of the panel are then given by

$$\begin{aligned}
 \epsilon_x &= \left[\frac{\pi^2}{2r^2} \left(\frac{h}{b} \right)^2 \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right] q \\
 &\quad + \frac{\pi^2}{8} \left(\frac{h}{b} \right)^2 \left(\frac{1}{r^2} \cos \frac{2\pi y}{b} - \nu \cos \frac{2\pi x}{a} \right) q^2 \\
 &\quad + \left[\frac{\pi^2}{8} \left(\frac{h}{b} \right)^2 \frac{1 + \nu r^2 - \nu(r^2 + \nu)}{r^2(1-\nu^2)} \right] q^2 \tag{42}
 \end{aligned}$$

$$\begin{aligned}
\epsilon_y = & \left[\frac{\pi^2 (h/b)^2}{2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \right] q \\
& + \frac{\pi^2 (h/b)^2}{8} \left(\cos \frac{2\pi x}{a} - \frac{\nu}{r^2} \cos \frac{2\pi y}{b} \right) q^2 \\
& + \left[\frac{\pi^2 (h/b)^2}{8} \frac{r^2 + \nu - \nu(1 + \nu r^2)}{r^2(1 - \nu^2)} \right] q^2
\end{aligned} \tag{42}$$

For movable inplane edges, the last term in Eqs. (41) and (42) vanishes.

b. Clamped Panels

Similarly, from Eqs. (3) and (38) and using Eqs. (7), (15), (17), and (18), the expressions for the nondimensional tensile stresses on the surface of a clamped panel with immovable edges are

$$\begin{aligned}
\frac{\sigma_x b^2}{Eh^2} = & \frac{\pi^2}{2(1 - \nu^2)} \left[\frac{1}{r^2} \cos A x (1 + \cos B y) + \nu (1 + \cos A x) \cos B y \right] q \\
& + \frac{\pi^2 r^2}{8} \left[\frac{1}{r^4} \cos B y + \frac{2}{(1 + r^2)^2} \cos A x \cos B y + \frac{1}{4r^4} \cos 2 B y \right. \\
& + \frac{1}{(4 + r^2)^2} \cos 2 A x \cos B y + \frac{4}{(1 + 4r^2)^2} \cos A x \cos 2 B y \left. \right] q^2 \\
& + \left[\frac{3\pi^2 (1 + \nu r^2)}{32r^2 (1 - \nu^2)} \right] q^2
\end{aligned} \tag{43}$$

$$\begin{aligned}
\frac{\sigma_y b^2}{Eh^2} = & \frac{\pi^2}{2(1 - \nu^2)} \left[(1 + \cos A x) \cos B y + \frac{\nu}{r^2} \cos A x (1 + \cos B y) \right] q \\
& + \frac{\pi^2}{8} \left[\cos A x + \frac{1}{2} \cos 2 A x + \frac{2}{(1 + r^2)^2} \cos A x \cos B y \right. \\
& + \frac{4}{(4 + r^2)^2} \cos 2 A x \cos B y + \frac{1}{(1 + 4r^2)^2} \cos A x \cos 2 B y \left. \right] q^2 \\
& + \left[\frac{3\pi^2 (r^2 + \nu)}{32r^2 (1 - \nu^2)} \right] q^2
\end{aligned}$$

where $A = 2\pi/a$ and $B = 2\pi/b$. The corresponding tensile strains are given by

$$\begin{aligned}
 \epsilon_x = & \left[\frac{\pi^2}{2r^2} \left(\frac{h}{b}\right)^2 \cos Ax (1 + \cos Bx) \right] q \\
 & + \frac{\pi^2 r^2}{8} \left(\frac{h}{b}\right)^2 \left[\frac{1}{r^4} \cos By + \frac{2}{(1+r^2)^2} \cos Ax \cos By \right. \\
 & + \frac{1}{4r^4} \cos 2By + \frac{1}{(4+r^2)^2} \cos 2Ax \cos By + \frac{4}{(1+4r^2)^2} \cos Ax \cos 2By \left. \right] \\
 & - \frac{\nu}{r^2} \left[\cos Ax + \frac{1}{4} \cos 2Ax + \frac{2}{(1+r^2)^2} \cos Ax \cos By \right. \\
 & + \frac{4}{(4+r^2)^2} \cos 2Ax \cos By + \frac{1}{(1+4r^2)^2} \cos Ax \cos 2By \left. \right] q^2 \\
 & + \left[\frac{3\pi^2}{32} \left(\frac{h}{d}\right)^2 \frac{1 + \nu r^2 - \nu(r^2 + \nu)}{r^2(1-\nu^2)} \right] q^2 \quad (44)
 \end{aligned}$$

$$\begin{aligned}
 \epsilon_y = & \left[\frac{\pi^2}{2} \left(\frac{h}{b}\right)^2 (1 + \cos Ax) \cos By \right] q \\
 & + \frac{\pi^2}{8} \left(\frac{h}{b}\right)^2 \left\{ \cos Ax + \frac{1}{4} \cos 2Ax + \frac{2}{(1+r^2)^2} \cos Ax \cos By \right. \\
 & + \frac{4}{(r+r^2)^2} \cos 2Ax \cos By + \frac{1}{(1+4r^2)^2} \cos Ax \cos 2By \left. \right\} \\
 & - \nu \left[\frac{1}{r^4} \cos By + \frac{2}{(1+r^2)^2} \cos Ax \cos By + \frac{1}{4r^4} \cos 2By \right. \\
 & + \frac{1}{(4+r^2)^2} \cos 2Ax \cos By + \frac{4}{(1+4r^2)^2} \cos Ax \cos 2By \left. \right\} q^2 \\
 & + \left[\frac{3\pi^2}{32} \left(\frac{h}{b}\right)^2 \frac{r^2 + \nu - \nu(1 + \nu r^2)}{r^2(1-\nu^2)} \right] q^2
 \end{aligned}$$

For movable edges, the last term in Eqs. (43) and (44) vanishes.

Examining Eqs. (41) - (44), a general expression is obtained for the stress (or strain) at any point in the structure as

$$\sigma = C_1 q + C_2 q^2 \quad (45)$$

where C_1 and C_2 are constants. The expressions for C_1 and C_2 can be found from Eqs. (41) to (44). The constants can be determined from material properties, dimensions of the panel, and the location and direction at which the stress is to be measured. The mean-square stress (or strain) is then related to the mean-square modal amplitude in a general expression as

$$\overline{\sigma^2} = C_1^2 \overline{q^2} + 3 C_2^2 (\overline{q^2})^2 \quad (46)$$

Once the mean-square deflection $\overline{q^2}$ is determined, Eqs. (36) and (37), the mean-square stress (or strain) can then be obtained from Eq. (46).

SECTION III

RESULTS AND DISCUSSION

Because of the complications in analysis of the many coupled modes, only one-mode approximation is used in the formulation. The assumption for fundamental mode predominacy is admittedly overly simplified; the conditions under which this is a valid approximation remain to be investigated. However, a simple model sometimes helps to give basic understanding of the problem.

Using the present formulation, response of nonlinear rectangular panels with all edges simply supported and all edges clamped subjected to broadband random acoustic excitation are studied. Both immovable and movable inplane edges are considered. In the results presented, the spectral density function of the excitation pressure $S(f)$ is considered flat within a certain region near the equivalent linear frequency f and a value of Poisson's ratio of 0.3 is used in all computations, unless otherwise mentioned. Mean-square modal amplitudes and mean-square nondimensional stresses for panels of various aspect ratios and damping ratios are determined and presented in graphical form. These graphs can be used as guides for preliminary design of aircraft panels. The maximum mean-square deflection can be reasonably obtained from these figures; however, multiple-modes had to be considered for accurate determination of mean-square stresses. This has been demonstrated by Seide in Reference 15 for a simple beam subjected to uniform pressure excitation and in Reference 16 for large deflections of prestressed simply supported rectangular plates under static uniform pressure.

Comparison with experiment is also given. It demonstrated that the present formulation given remarkable improvement in predicating RMS responses as compared with using the linear theory.

1. Simply Supported Panels

Figure 2 shows the maximum mean-square nondimensional deflection versus nondimensional spectral density parameter of acoustic pressure excitation for rectangular panels of aspect ratios $r = 1, 2,$ and $4,$ and a damping ratio $0.02.$ It is clear from the figure that an increase of r will cause "closing" of the curve. This occurs because as r increases the panel becomes less stiff, and the mean-square deflection has to be finite. It can also be seen from the figure that the mean-square deflection of the movable inplane edges case is approximately twice as that of the immovable edges.

Figures 3 and 4 show the maximum mean-square bending stress and the mean-square membrane stress, respectively, for a simply supported square panel with $\zeta = 0.02.$ The maximum mean-square bending stress of the movable edges case is approximately twice as much as that of the immovable edges, whereas the mean-square membrane stress of the movable edges case is found to be somewhat less than that of the immovable edges. In Figure 5, the maximum mean-square nondimensional stress (bending plus membrane stress, at the center of the panel and in the y -direction) is given as a function of excitation spectral density parameter for simply supported rectangular panels of various aspect ratios and a damping factor $0.02.$ Results showed that the difference of maximum mean-square stresses between immovable and movable edges is small as compared with the difference of mean square deflections between the two edge conditions.

Figure 6 shows the mean-square deflection versus forcing spectral density parameter for simply supported square panels of different damping ratios. The corresponding maximum mean-square stress (bending plus membrane stress, at the center of panel). is shown in Figure 7. As it can be seen from the figures that the precise determination of damping ratio from

experiment is important, e.g. stress increases by 25-30% as ζ is decreased from 0.015 to 0.01 (for S_f between 5000 to 20000). Again, the difference of maximum mean-square stresses between immovable and movable edges is small as compared with the difference of mean-square deflections.

Plots of the equivalent linear or nonlinear frequency parameter λ^2 versus mean-square modal amplitude for simply supported rectangular panels of aspect ratios $r = 1, 2,$ and 4 are shown in Figure 8. The lowest value of λ^2 corresponds to the linear case.

2. Clamped Panels

In Figure 9, the mean-square deflection is given as a function of excitation spectral density parameter for rectangular panels of aspect ratios $r = 1, 2,$ and 4 and a damping ratio 0.02. The maximum mean-square deflection of the clamped panels is somewhat much less than that of the simply supported. The corresponding maximum mean-square nondimensional stress (bending plus membrane stress, in the y -direction and at the center of the long edge) versus spectral density parameter is shown in Figure 10.

Figure 11 shows the mean-square modal amplitude versus spectral density parameter of excitation for a square panel of different damping ratios. In Figure 12, the equivalent linear frequency parameter is given as a function of mean-square deflection for clamped rectangular panels of aspect ratios $r = 1, 2,$ and 4 .

3. Comparison with Experimental Results

The experimental measurements on skin-stringer panels exposed to random pressure loads reported in References 3 and 4 are used to demon-

strate the improvement in predicting panel responses by using the present formulation. The structure was a skin-stringer, 3-bay panel as shown in Figure 13. The panels were constructed of 7075-T6 aluminum alloy. Details of the test facility, noise sources, test fixture, and test results are given in Reference 3. The important properties of the panel are:

Length	$a = 27$ inches
Width between the rivet lines	$b = 6.63$ inches
Thickness	$h = 0.032$ inches
Damping ratio	$\zeta = 0.0227$
Poisson's ratio	$\nu = 0.33$
Young's modulus	$E = 9.6 \times 10^6$ psi
Weight density	$\rho = 0.1$ lb/in ³

The tests were conducted with an overall sound pressure level (SPL) of 157 decibels (dB) with a range of ± 1.5 dB which corresponds to an average spectrum level of 125.26 dB (see Table IV of Reference 3 or Table 8 of Reference 17).

The central bay of the 3-bay test panels is simulated by a flat rectangular plate. The linear frequencies for both simply supported (Eq. (14)) and clamped (Eq. (20)) support conditions are calculated and shown in Table 1.

Test measurements and finite element solution are also given for comparison. Table 1 also shows the equivalent linear or nonlinear

Table 1 Frequency Comparison

	Natural frequency f_0	Equivalent linear frequency f_{157}
Simply supported - Immovable edges	71	321
- Movable edges	71	240
Clamped - Immovable edges	159	311
- Movable edges	159	264
Finite element (Ref. 4)	155	-
Experiment (Ref. 3)	126,	-

frequencies at overall SPL 157 dB. Frequency at high intensity noise level was not reported in Reference 3. From the results shown in Table 1, it is clear that the central bay of the test panels did not respond to the acoustic excitation as though it were fully clamped on all four edges. This was also demonstrated in Figures 12 and 17 of Reference 3 in the sense that the highest measured RMS strains did not occur at the center of the long edges. The central bay of the test panels actually behaved somewhere between fully simply supported and fully clamped support conditions.

The acoustic pressure spectral density $S(f)$ is related to the spectrum level L as

$$S(f) = \begin{cases} 8.41 \times 10^{(L/10 - 18)} & (\text{psi})^2/\text{Hz} \\ 4 \times 10^{(L/10 - 8)} & (\text{dynes/cm}^2)^2/\text{Hz} \end{cases} \quad (47)$$

A white noise pressure loading with spectral density of $S(f) = 2.824 \times 10^{-5}$ (psi)²/Hz (or nondimensional spectral density parameter $S_p = 5100$), which corresponds to an average spectrum level $L = 125.26$ dB, is used in the computations. The RMS stresses (Eq. (46)) at the center of the long edges for simply supported (Eq. (41)) and clamped (Eq. (43)) boundary conditions are calculated and given in Table 2.

Table 2 Stress Comparison

	RMS Stress (kpsi)			
	$\sqrt{\sigma_x^2}$		$\sqrt{\sigma_y^2}$	
	Linear theory	Nonlinear theory	Linear theory	Nonlinear theory
Simply Supported	0.0	0.58 (Immovable) 0.17 (Movable)	0.0	3.28 (Immovable) 2.74 (Movable)
Clamped	2.17	1.12 (Immovable) 1.32 (Movable)	6.57	3.83 (Immovable) 4.24 (Movable)
Finite element (Ref. 4)	2.4	-	7.7	-
Measured (Refs. 3,4) on panel				
A		0.63		2.2
B		0.94		2.9
C		0.78		2.5
D		1.1		-
E		0.84		2.2
Measured average		0.87		*2.5

Table 3 shows the RMS deflections using the present formulation. The measured and finite element RMS stresses and RMS deflections in Reference 4 are also given in the tables for comparison. It demonstrates that a better correlation between theory and experiment can be achieved when large deflection geometrical nonlinearity effect is included in the formulation.

Table 3. Deflection Comparison

	$\sqrt{(w_{\max}/h)^2}$	
	<u>Linear theory</u>	<u>Nonlinear theory</u>
Simply Supported	8.0	1.8 (Immovable) 2.4 (Movable)
Clamped	2.7	1.4 (Immovable) 1.6 (Movable)
Finite Element (Ref. 4)	3.1	
Measured (Ref. 4)		2.0

SECTION IV

CONCLUSIONS

An analytical method for predicating response of rectangular nonlinear structural panels subjected to broadband random acoustic excitation is presented. The formulation is based on the Karman-Herrmann large deflection plate equations, a single-mode Galerkin approximation, the equivalent linearization method, and an iterative procedure. Both simply supported and clamped support conditions with immovable or movable inplane edges are considered. Panel mean-square deflection, maximum mean-square stress, and equivalent linear frequency at given excitation pressure spectral density can be determined, and they are presented in graphical form. These graphs can be used as guides for preliminary design of aircraft panels under high noise environment. Results obtained agree well with the experiment. It is suggested that further research be carried out with special attention to employ multiple-modes in the formulation, and additional test data on simple panels are needed for an adequate quantitative comparison.

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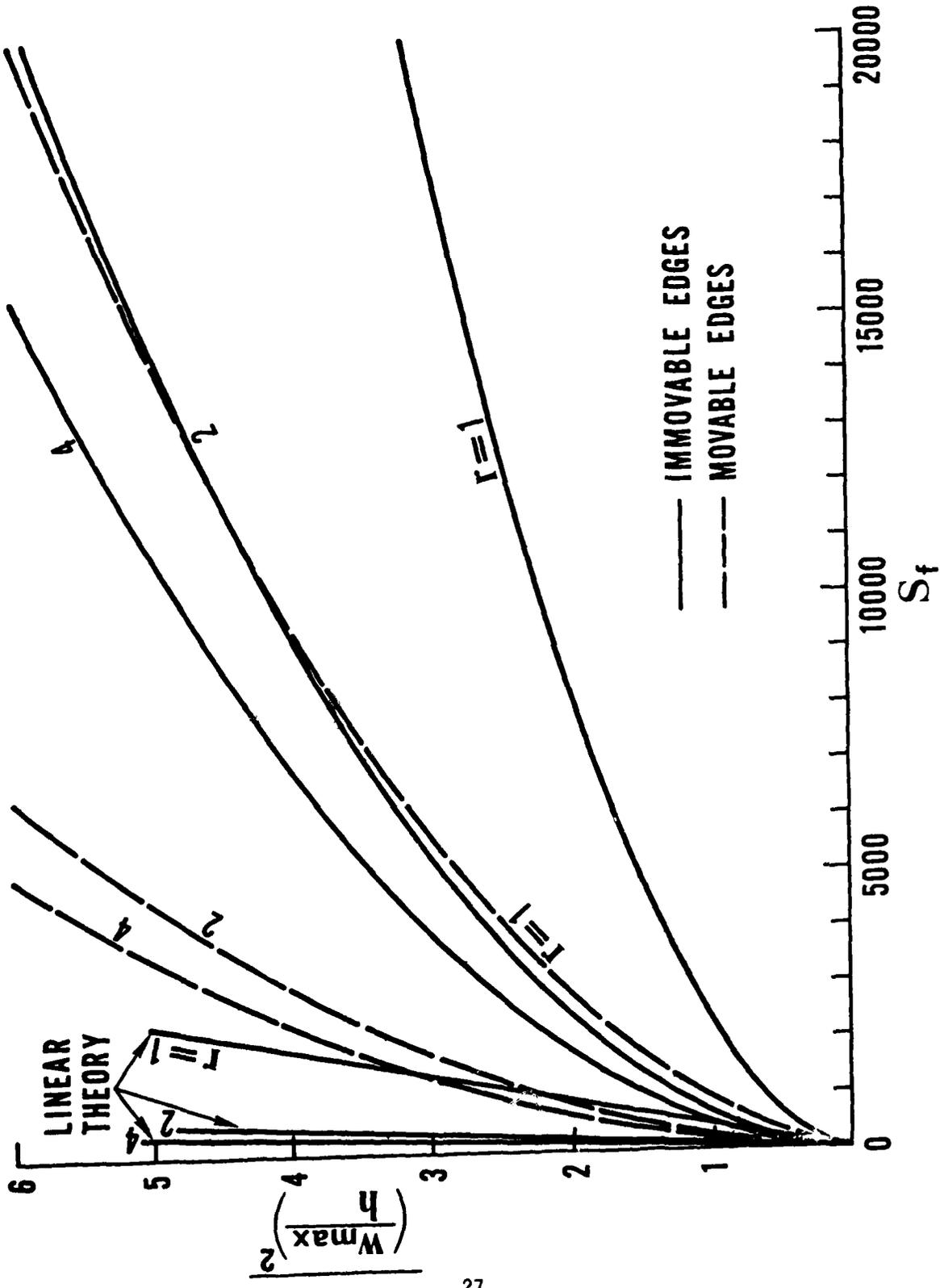


Figure 2. Mean-square deflection versus spectral density parameter of excitation for simply supported panels, $\zeta = 0.02$.

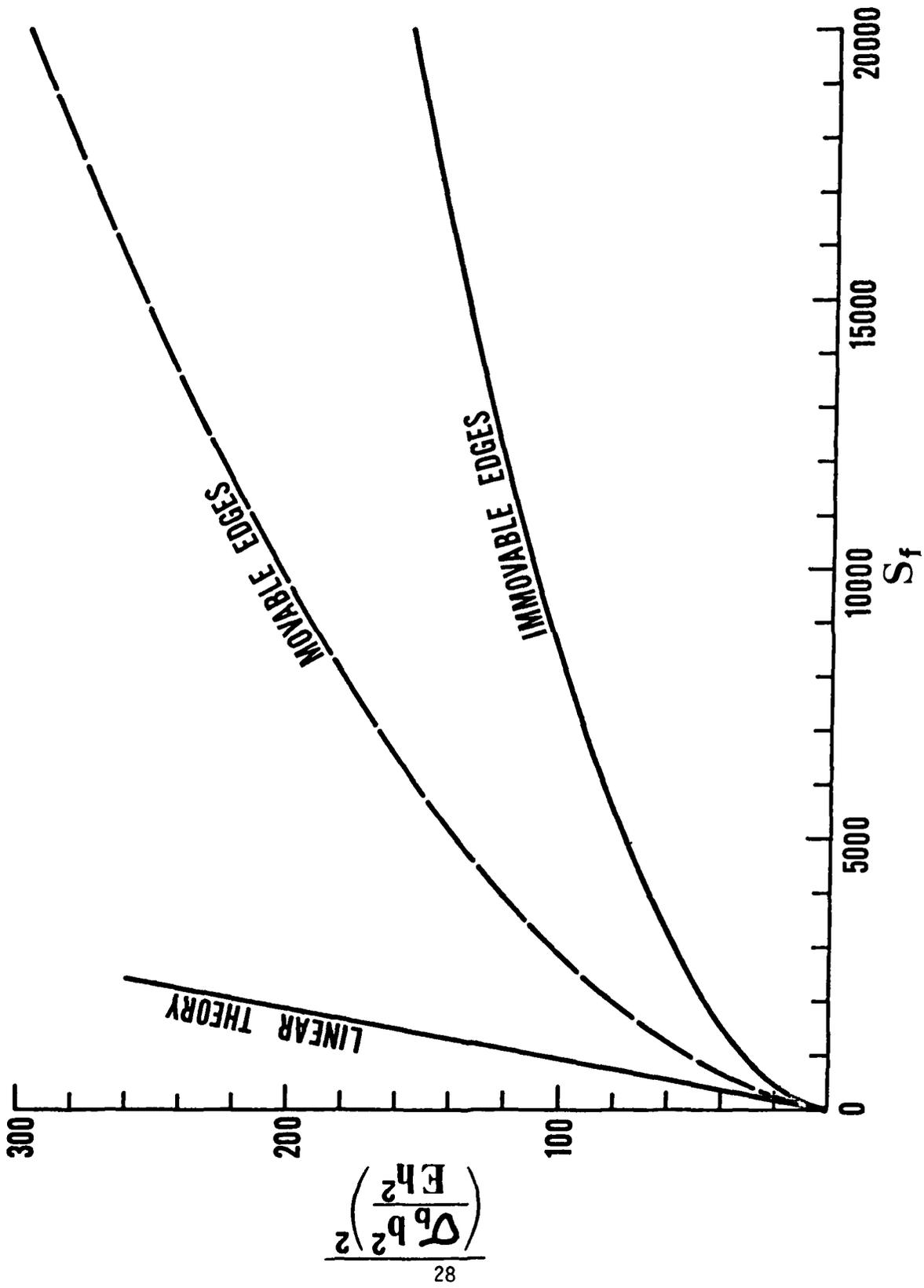


Figure 3. Maximum mean-square bending stress versus spectral density parameter of excitation for a simply supported square panel, $\zeta = 0.02$

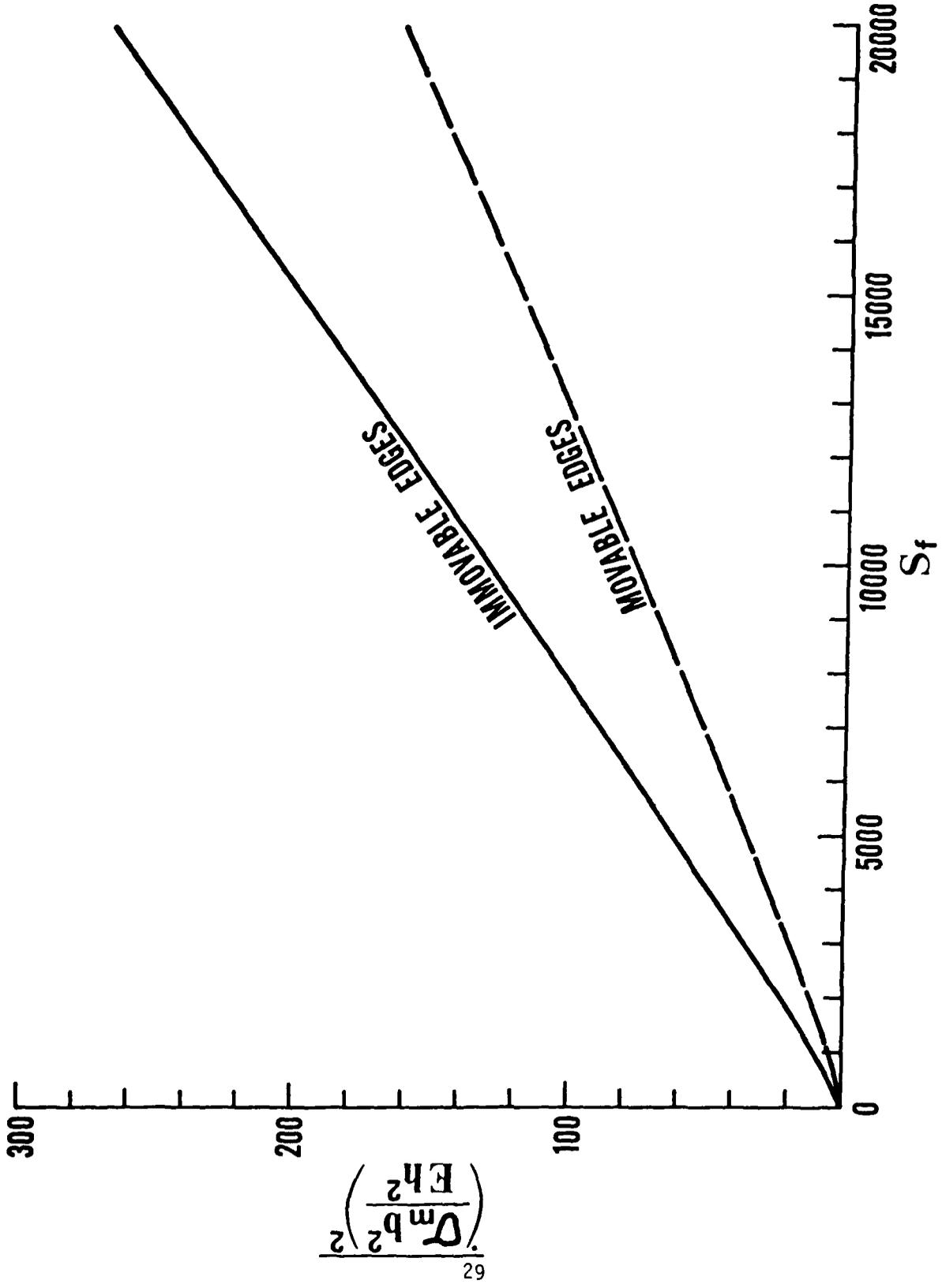


Figure 4. Mean-square membrane stress versus spectral density parameter of excitation for a simply supported square panel, $\zeta = 0.02$.

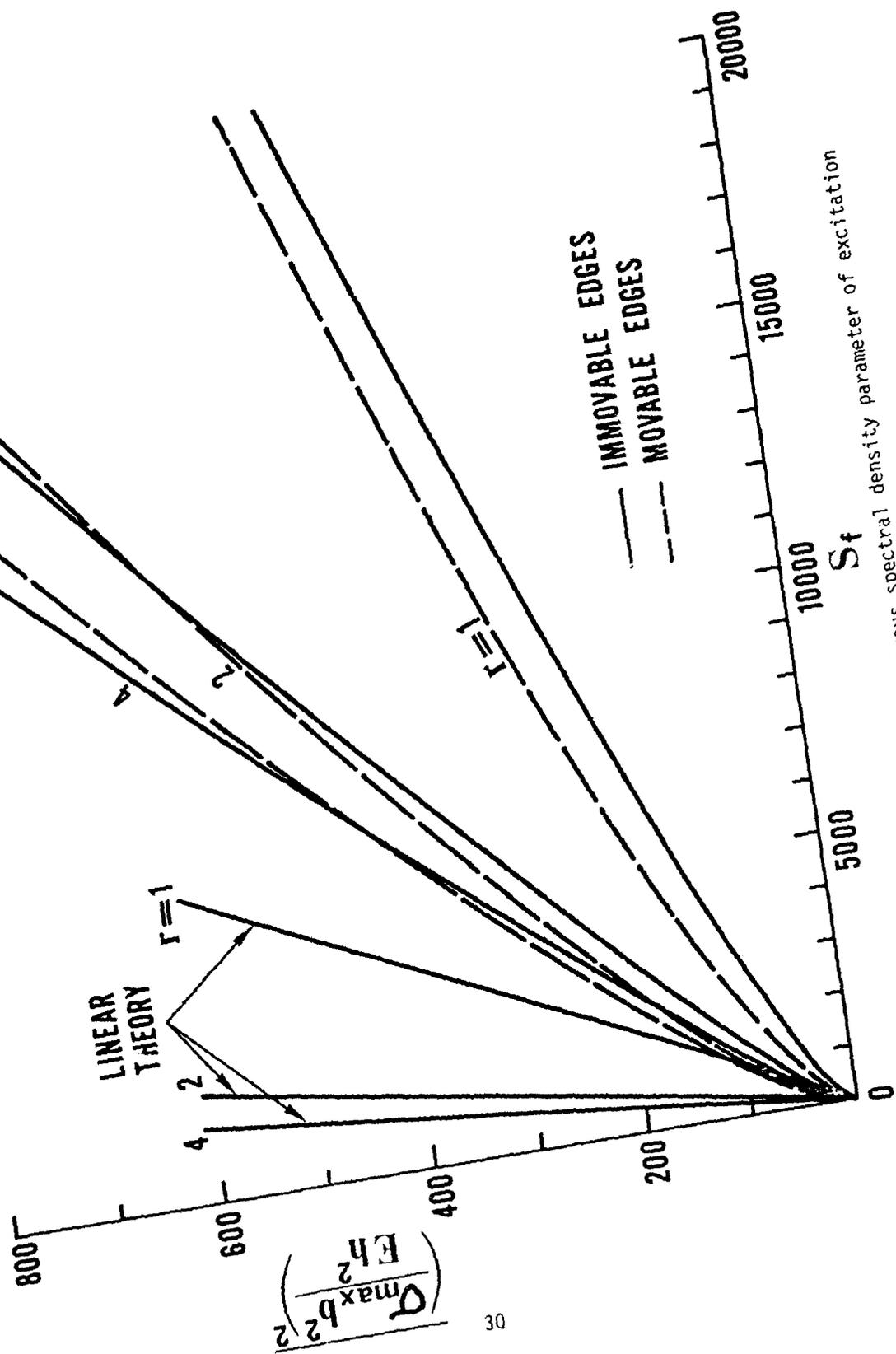


Figure 5. Maximum mean-square stress versus spectral density parameter of excitation for simply supported panels, $\zeta = 0.02$

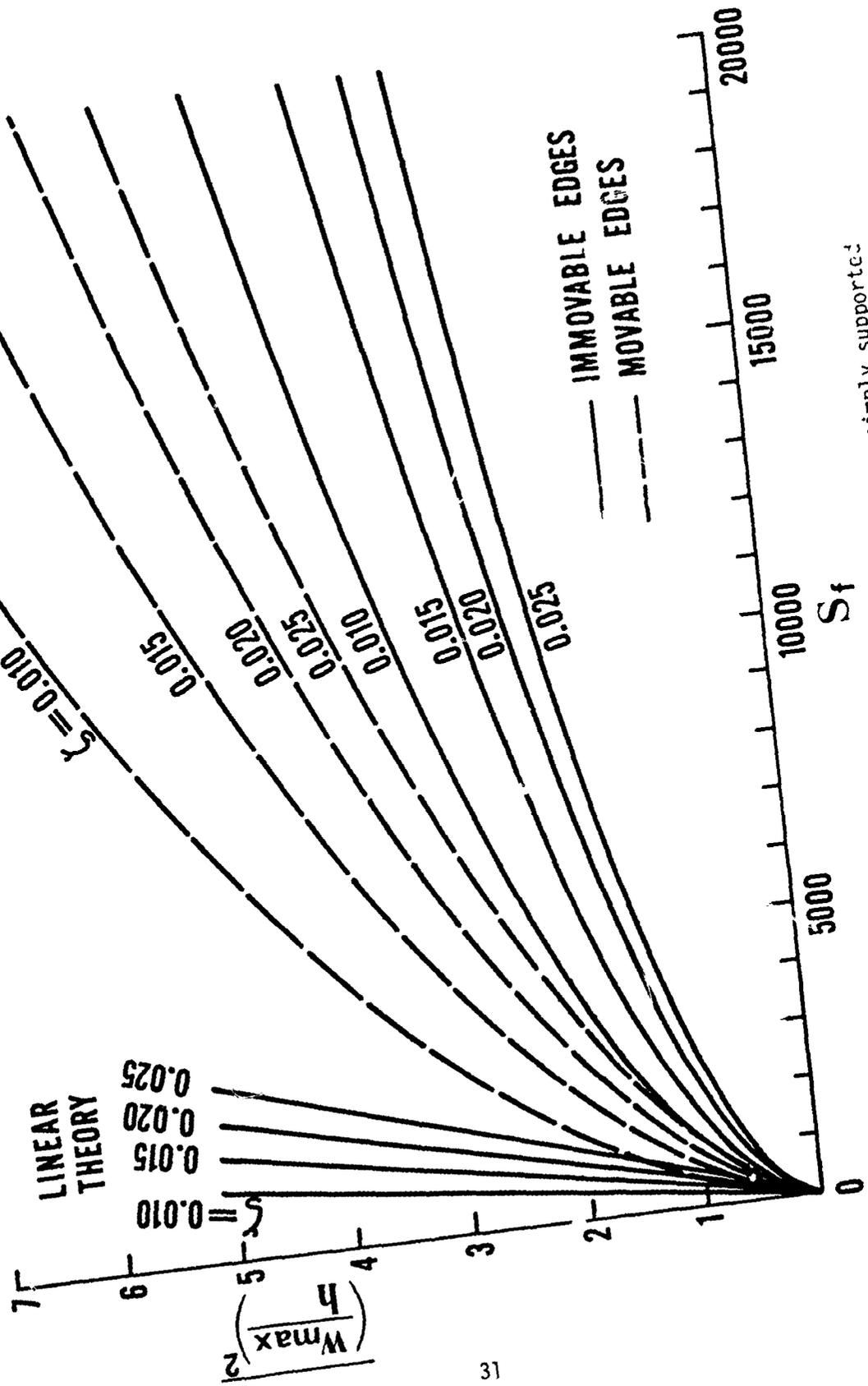


Figure 6. Effects of damping on mean-square deflection for a simply supported square panel

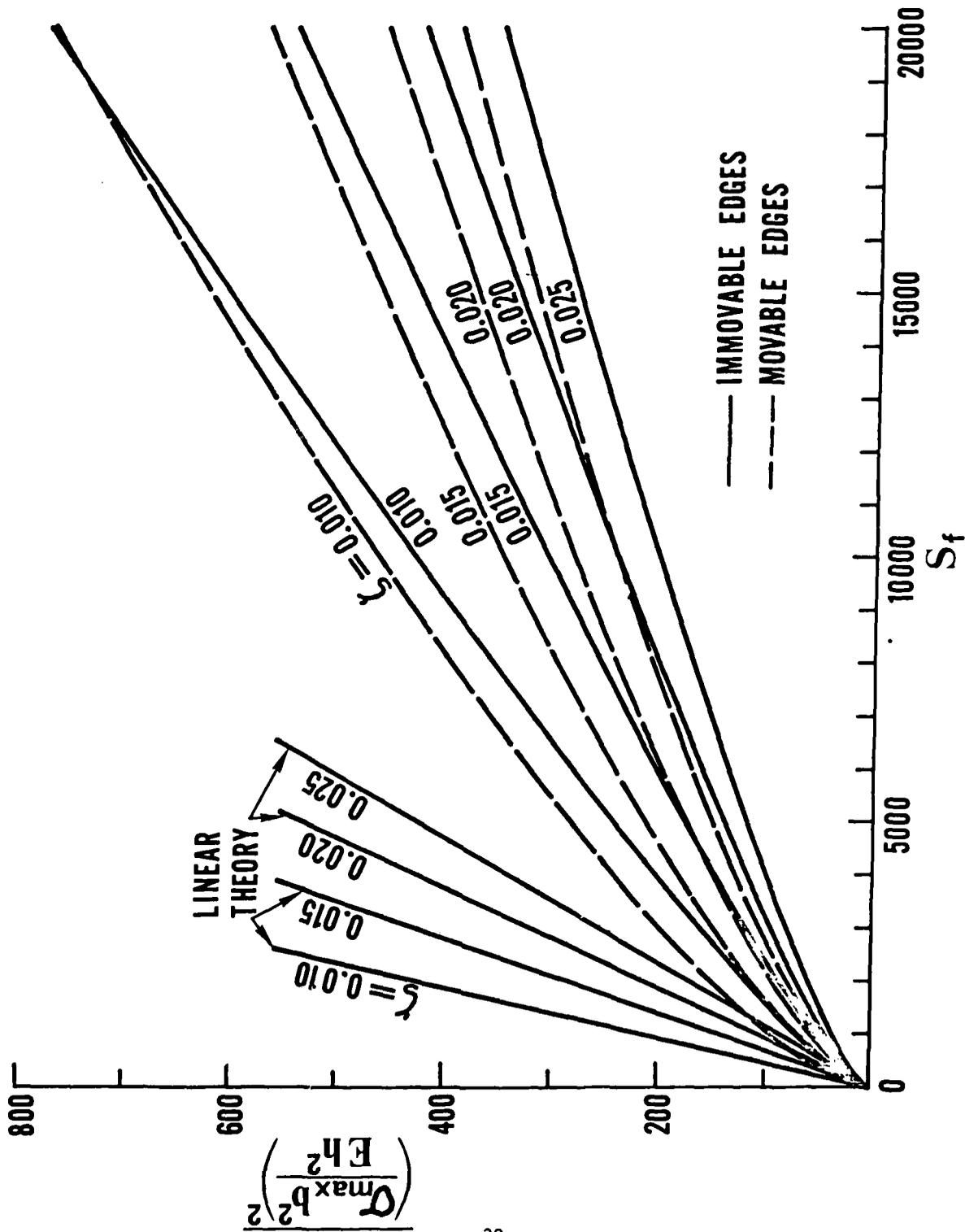


Figure 7. Effects of damping on maximum mean-square stress for a simply supported square panel

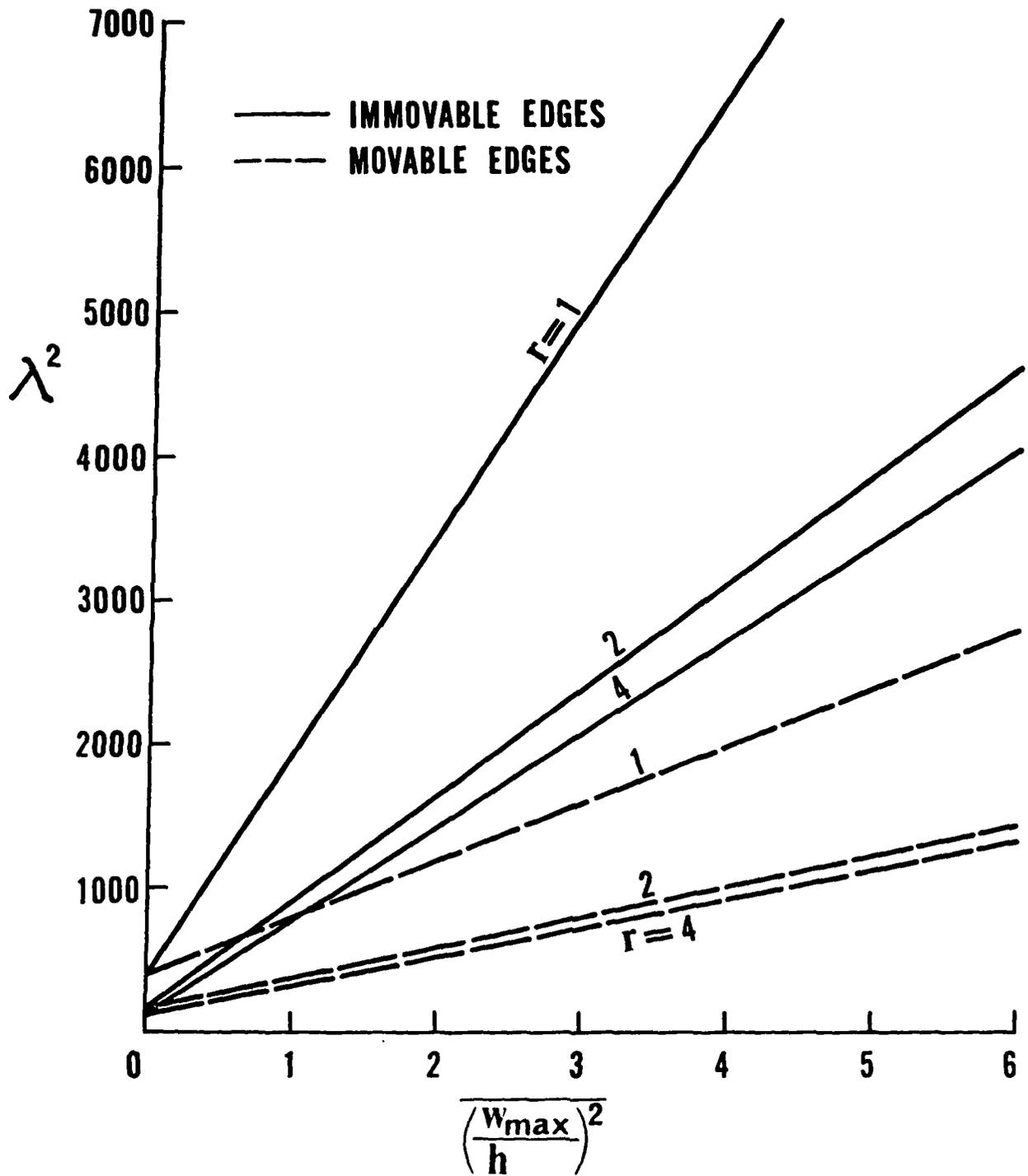


Figure 8. Frequency parameter versus mean-square deflection for simply supported panels

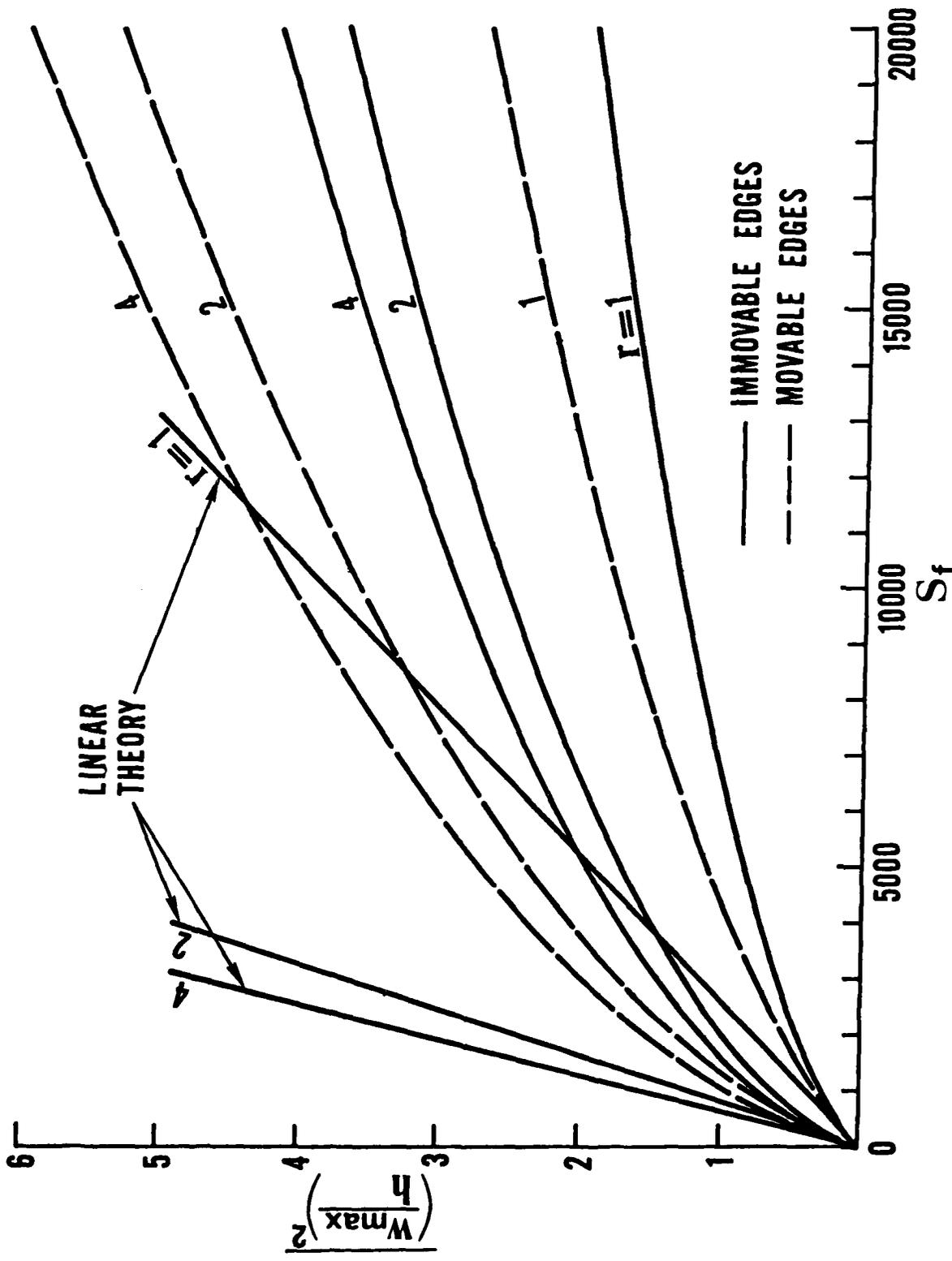


Figure 9. Mean-square deflection versus spectral density parameter of excitation for clamped panels, $\zeta = 0.02$.

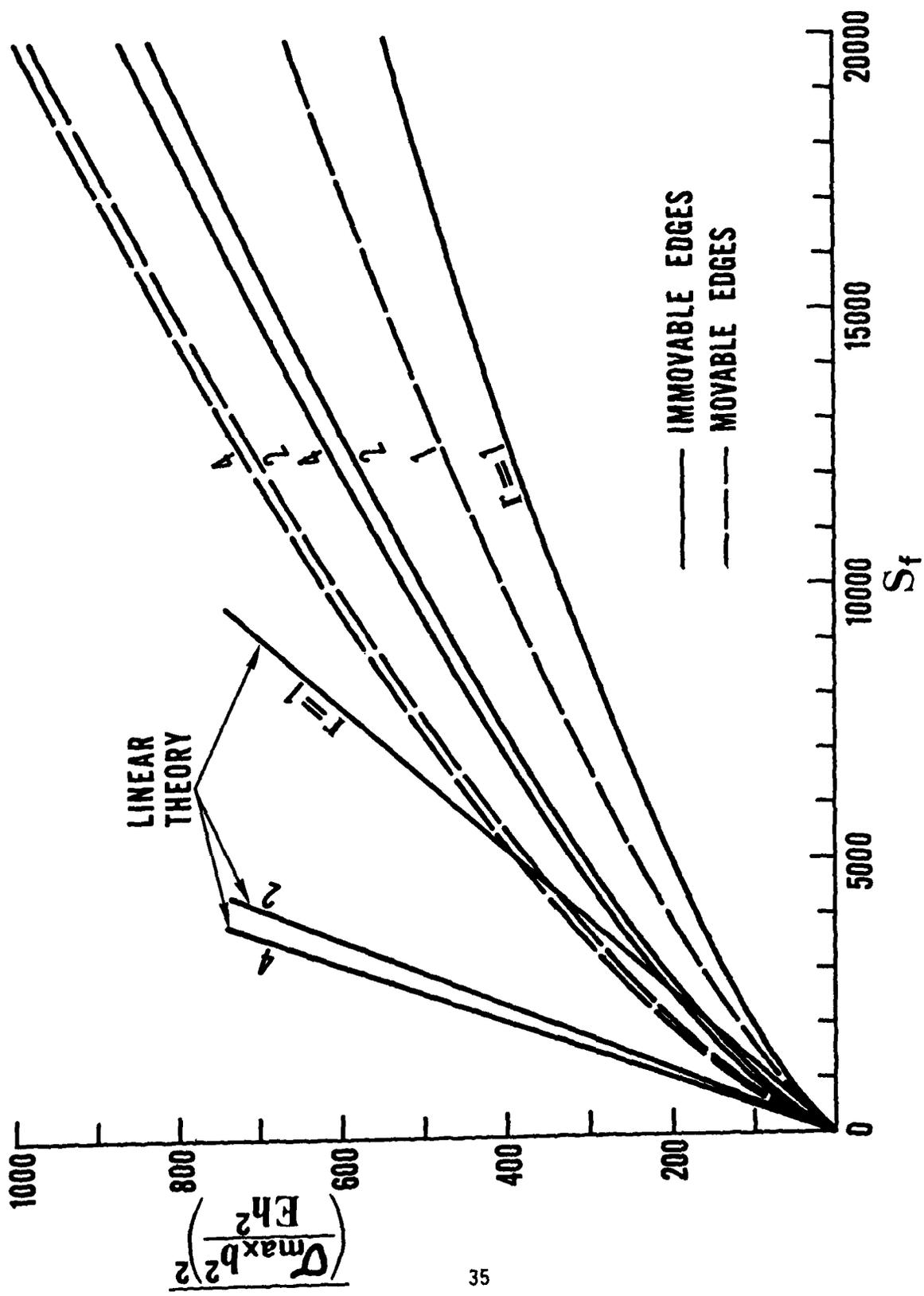


Figure 10. Maximum mean-square stress versus spectral density parameter of excitation for clamped panels, $\tau = 0.02$

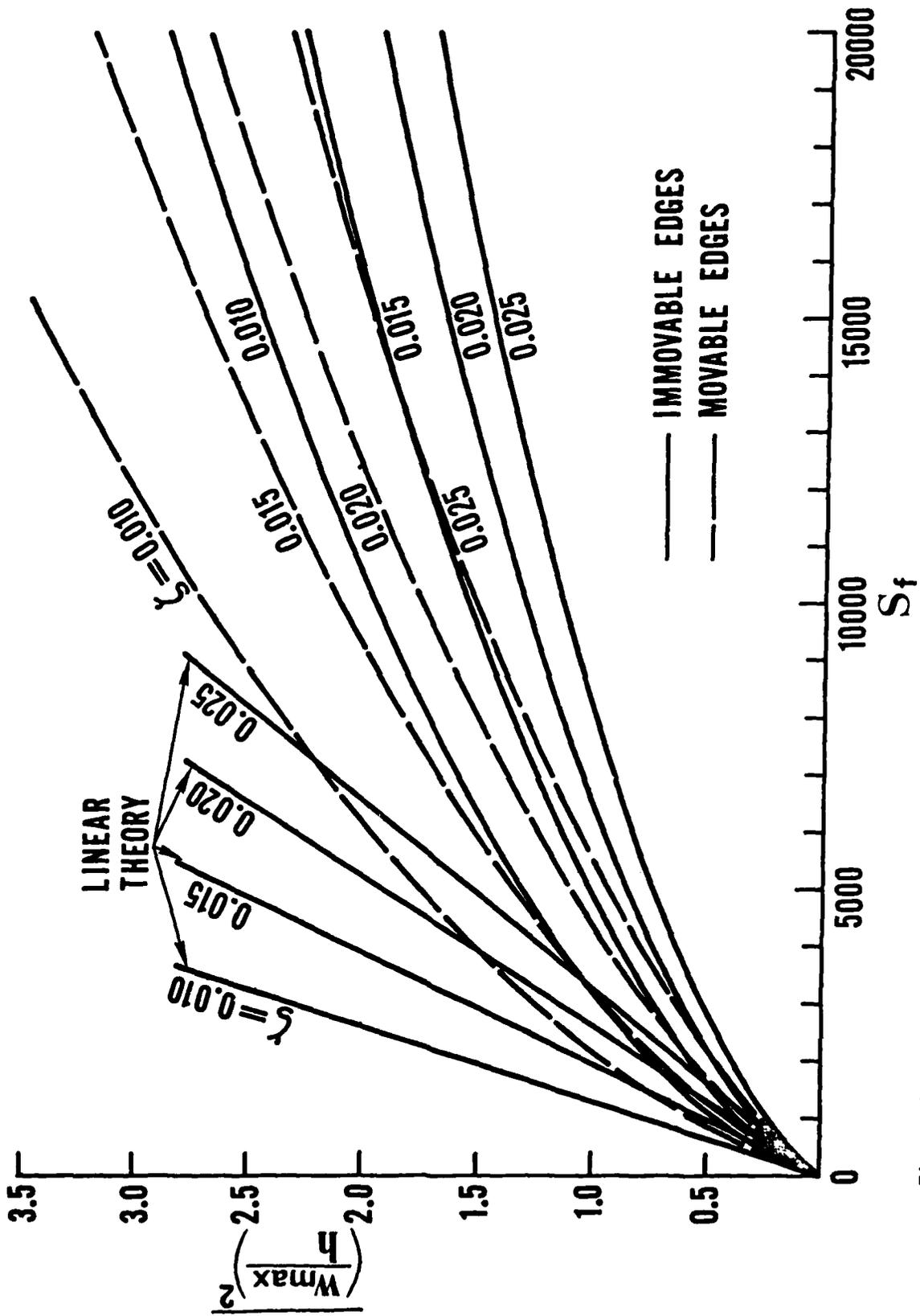


Figure 11. Effects of damping on mean-square deflection for a clamped square panel

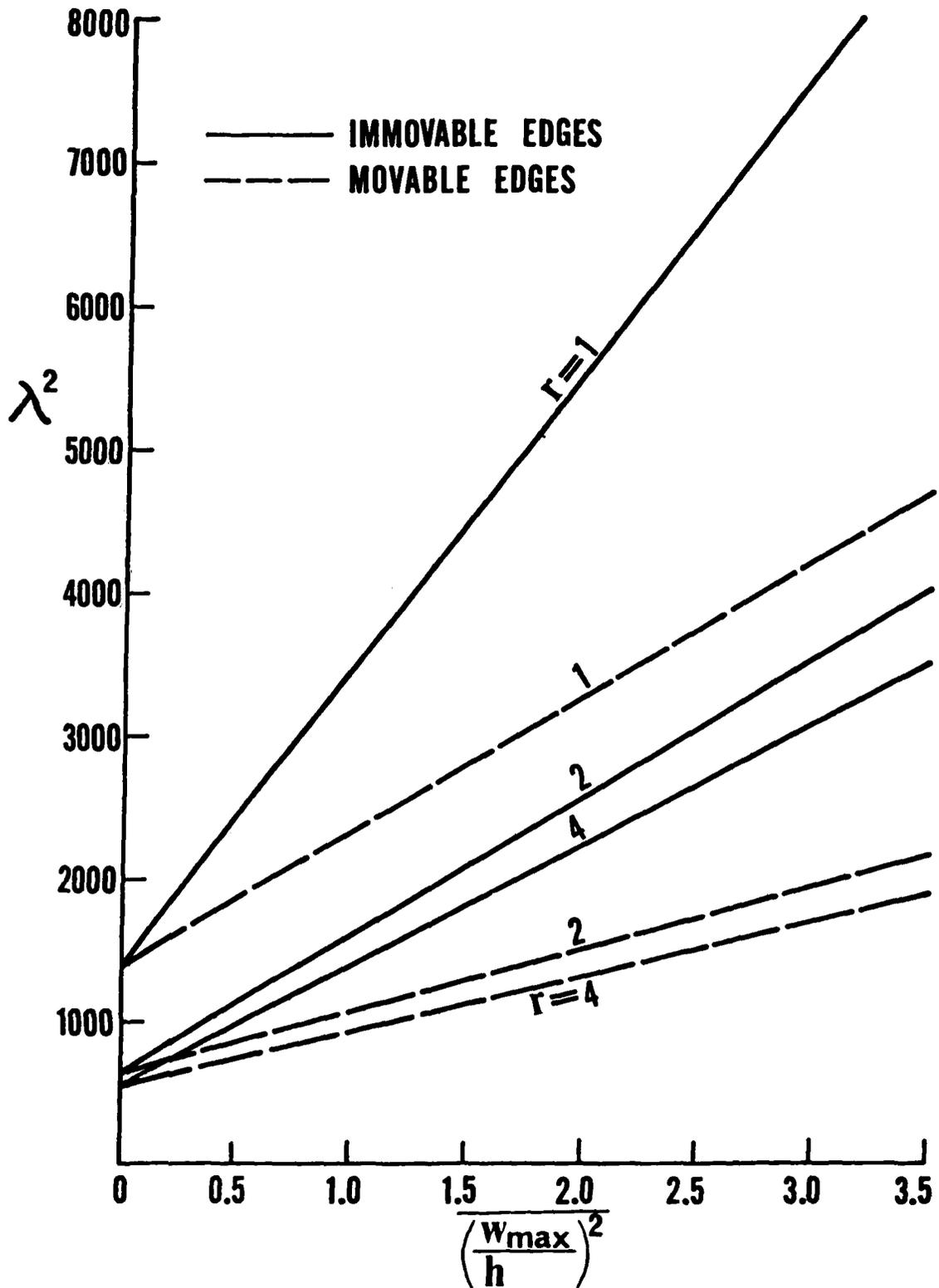
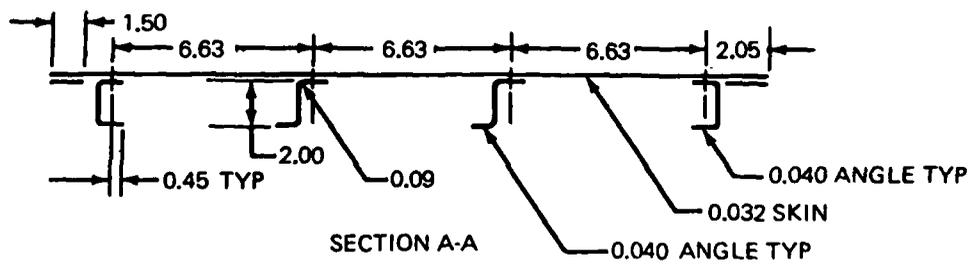
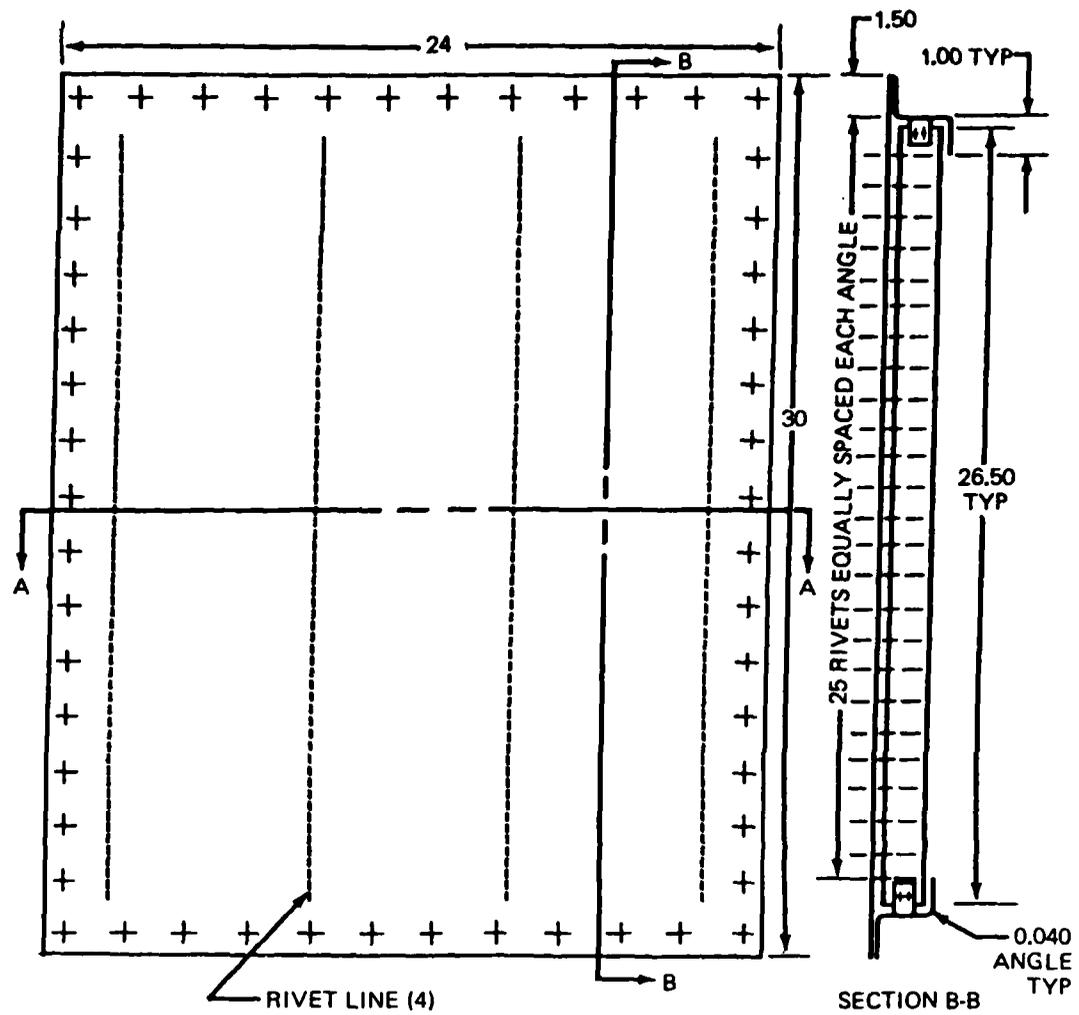


Figure 12. Frequency parameter versus mean-square deflection for clamped panels



NOTE: DIMENSIONS IN INCHES

Figure 13.

Skin-stringer panel

APPENDIX

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C LARGE AMPLITUDE RESPONSE OF AIRCRAFT PANELS SUBJECTED TO BROADBAND
C   RANDOM ACOUSTIC EXCITATIONS
C   SIMPLY SUPPORTED ALONG ALL FOUR EDGES
C RECTANGULAR PANEL - SIMPLY SUPPORTED ALONG ALL FOUR EDGES
C   SIMPLY SUPPORTED ALONG ALL FOUR EDGES
C   COMMON AR,AR2,PI,PI2,UN,ENU
C   DIMENSION SPSD(99)
C MOVABL=FLAG FOR INPLANE EDGE CONDITIONS
C   SET MOVABL= 0, FOR IMMOVABLE EDGES
C   SET MOVABL=+1, FOR MOVABLE EDGES
C MOVABL=0 OR +1 ?
C   MOVABL=1
C ASGNQ=AN ASSIGNED SMALL CONSTANT FOR CONVERGENCE TEST OF RMS
C   MAXIMUM DEFLECTION
C   ASGNQ=1.0E-3
C
C   PI=3.14159265
C   PI2=PI*PI
C   PI4=PI2*PI2
C ZETA=DAMPING FACTOR OF PANEL=C/(C CRITICAL)
C   ZETA=0.020
C UN=POISSON'S RATIO
C   UN=0.3
C   ENU=1.0 - UN*UN
9000  FORMAT(I2)
9050  FORMAT(F10.3)
C NUMSF=NUMBER OF EXCITATION SPECTRAL DENSITY
8000  FORMAT(48H HOW MANY EXCITATION SPECTRAL DENSITY ARE THERE?,I4)
      READ (5,9000) NUMSF
      WRITE (6,8000) NUMSF
      WRITE (5,9100)
      DO 100 NSF=1,NUMSF
8100  FORMAT(23H WHAT ARE THEIR VALUES? )
      READ (5,9050) SPSD(NSF)
      WRITE (6,9150) SPSD(NSF)
8150  FORMAT(5X,E15.0)
C SPSD=NONDIMENSIONAL FORCING EXCITATION SPECTRAL DENSITY
C   PARAMETER=SF(F)/((RH0**2) * (H**4) * (DFACTOR**1.5))
100  CONTINUE
C NAR=NUMBER OF ASPECT RATIOS
8200  FORMAT(24H HOW MANY ASPECT RATIOS?,I4 )
      READ (5,9000) NAR
      WRITE (6,8200) NAR
      DO 900 NTIME=1,NAR
C AR=ASPECT RATIO OF PANEL=AL/BL
C AL=LENGTH
C BL=WIDTH
      READ (5,9050) AR
      AR2=AR*AR
      AR4=AR2*AR2
C FLMD02=LAMBDA0 SQUARE - NONDIMENSIONAL LINEAR FREQUENCY SQUARE
      FLMD02=PI4*(1.0 + AR2)*(1.0 + AR2)/AR4
      FLMD0= SQRT(FLMD02)
C BETAP=NONLINEARITY COEFFICIENT
C BETAC=NONLINEARITY COEFFICIENT
      BETAP=0.75*PI4*ENU*(1.0 + 1.0/AR4)
      BETAC=1.5*PI4*(1.0 + UN*AR2 + AR2*(AR2 + UN))/AR4
C FOR MOVABLE INPLANE EDGES, BETA=BETAP
C FOR IMMOVABLE INPLANE EDGES, BETA=BETAP + BETAC
      BETA=BETAP + BETAC

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IF (MOVABL .EQ. 1) BETA=BETAP
C
DO 500 NSF=1,NUMSF
SF=SPSD(NSF)
*WRITE (6,9100) AR,SF,BETA
9100  FORMAT(/17H1ASPECT RATIO=AR=,F6.2/,
1  47H NONDIMENSIONAL EXCITATION SPECTRAL DENSITY=SF=,
2  E13.5/31H NONLINEARITY COEFFICIENT=BETA=,E15.6)
IF (MOVABL .EQ. 0) WRITE (6,8300) ZETA
8300  FORMAT(46H SIMPLY-SUPPORTED WITH IMMOVABLE INPLANE EDGES,
1  /34H DAMPING FACTOR=C/C CRITICAL=ZETA=,F7.4)
IF (MOVABL .EQ. 1) WRITE (6,8310) ZETA
8310  FORMAT(44H SIMPLY-SUPPORTED WITH MOVABLE INPLANE EDGES,
1  /34H DAMPING FACTOR=C/C CRITICAL=ZETA=,F7.4)
C ITER=0 CORRESPONDS TO THE LINEAR SOLUTION
ITER=0
*WRITE (6,9150)
9150  FORMAT(/41H ***SMALL-DEFLECTION (LINEAR) SOLUTION***)
C Q02=MEAN SQUARE MAXIMUM DEFLECTION - LINEAR STRUCTURAL THEORY
Q02=32.0*SF/(PI4*ZETA*FLMDO*FLMDO2)
C Q0=ROOT MEAN SQUARE MAXIMUM DEFLECTION
Q0= SQRT(Q02)
WRITE (6,9200) FLMDO2,FLMDO,Q02,Q0
9200  FORMAT(4X,31H FREQ. PARAMETER SQUARE=FLMDO2=,E15.7,
1  5X,23H FREQ. PARAMETER=FLMDO=,E15.7/,
2  8X,27H MEAN-SQUARE AMPLITUDE=Q02=,E15.7,
3  10X,18H RMS AMPLITUDE=Q0=,F7.3)
C COMPUTE RMS STRESSES OR STRAINS BASED ON LINEAR STRUCTURAL THEORY
*WRITE (6,9250)
9250  FORMAT(/3X,9H LOCATION,20X,19H MEAN-SQUARE STRESS,19X,
1  11H RMS STRESS/29X,9H X-COMPS.,6X,9H Y-COMPS.,14X,
2  9H X-COMPS.,6X,9H Y-COMPS.)
CALL STRESS(0.0,0.0,Q02,MOVABL,ITER)
CALL STRESS(0.5,0.0,Q02,MOVABL,ITER)
CALL STRESS(0.0,0.5,Q02,MOVABL,ITER)
CALL STRESS(0.5,0.5,Q02,MOVABL,ITER)
C STORE Q02, Q0 AND FLMDO FOR CONVERGENCE TEST LATTER
Q2PV=Q02
QPV=Q0
FLMPV=FLMDO
C
C START ON ITERATION - ITER=ITERATION COUNTER
C
ITER=1
10  CONTINUE
FLMD2=FLMDO2 + 3.0*BETA*Q2PV
Q2=32.0*SF/(PI4*ZETA*FLMD*FLMD2)
FLMD= SQRT(FLMD2)
Q= SQRT(Q2)
C CHECK FOR CONVERGENCE
DEF=(FLMD-FLMPV)/FLMD
DEQ=(Q-QPV)/Q
DEQ= ABS(DEQ)
C CONVERGENCE TEST, IF SATISFIED GO TO 12 FOR STRESS COMPUTATION
IF (DEQ .LE. ASGNJ) GO TO 12
C STORE FLMD, Q2 AND Q FOR CONVERGENCE TEST
FLMPV=FLMD
Q2PV=Q2
C SAVE COMPUTATION TIME AFTER 50 AND 100 ITERATIONS
IF (ITER .EQ. 50) Q2PV=0.5*(Q*Q + QPV*QPV)

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      IF (ITER .EQ. 100) Q2FV=0.5*(Q*Q + QPV*QPV)
      QPV=Q
      ITER=ITER + 1
      GO TO 10
12     CONTINUE
      WRITE (6,9400) ITER
9400   FORMAT(/,44H ***LARGE-DEFLECTION (NONLINEAR) SOLUTION**/,
1      4X,13H CONVERGENCE AT,14,13H-TH ITERATION)
      WRITE (6,9450) FLMD2,FLMD,Q2,Q
9450   FORMAT(4X,30H FREQ. PARAMETER SQUARE=FLMD2=,E15.7,
1      5X,22H FREQ. PARAMETER=FLMD=,E15.7/,
2      8X,26H MEAN-SQUARE AMPLITUDE=Q2=,E14.6,
3      11X,17H RMS AMPLITUDE=Q=,F7.3)
      WRITE (6,9500) DEQ,DEQ
9500   FORMAT(20X,4HDEQ=,E15.6,5X,4HDEQ=,E15.6)
C COMPUTE STRESSES AT LARGE AMPLITUDE
      WRITE (6,9250)
      CALL STRESS(0.0,0.0,Q2,MOVABL,ITER)
      CALL STRESS(0.5,0.0,Q2,MOVABL,ITER)
      CALL STRESS(0.0,0.5,Q2,MOVABL,ITER)
      CALL STRESS(0.5,0.5,Q2,MOVABL,ITER)
      IF (.NOT. 5.0) GO TO 700
500    CONTINUE
700    CONTINUE
900    CONTINUE
      STOP
      END
      SUBROUTINE STRESS(XA,YA,Q2,MOVABL,ITER)
      COMMON AR,AR2,PI,PI2,UN,ENU
C THIS SUBROUTINE COMPUTES THE RMS STRESSES IN THE X (SX) AND Y
C (SY) DIRECTIONS - THESE STRESSES ARE NONDIMENSIONAL
C XA=X/A
C YA=Y/A
C Q2=MEAN SQUARE OF Q
      CX=COS(PI*XA)
      CY=COS(PI*YA)
      C2X=COS(2.0*PI*XA)
      C2Y=COS(2.0*PI*YA)
      DX=0.5*PI2*(UN + 1.0/AR2)*CX*CY/ENU
      D1X=0.125*PI2*C2Y/AR2
      D2X=0.125*PI2*(UN + 1.0/AR2)/ENU
      IF (MOVABL .EQ. 1) D2X=0.0
      DY=0.5*PI2*(1.0 + UN/AR2)*CX*CY/ENU
      D1Y=0.125*PI2*C2X
      D2Y=0.125*PI2*(UN + AR2)/(AR2*ENU)
      IF (MOVABL .EQ. 1) D2Y=0.0
      DXX=D1X + D2X
C ITER=0 CORRESPONDS TO THE SMALL DEFLECTION LINEAR THEORY
      IF (ITER .EQ. 0) DXY=0.0
      SYB2=DX*DX*Q2
      SXB2=3.0*DX*DX*Q2*Q2
      SX2=SXB2 + SYB2
      DYY=D1Y + D2Y
      IF (ITER .EQ. 0) DYY=0.0
      SYB2=DY*DY*Q2
      SYB2=3.0*DYY*DY*Q2*Q2
      SY2=SYB2 + SX2
C SX, SY=NONDIMENSIONAL ROOT MEAN SQUARED STRESSES (STRESS**2)
C / (E**H**2)
C P=L=WIDTH OF PANEL

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C E=MODULUS OF ELASTICITY
C H=THICKNESS OF PANEL
      SX= SQRT(SX2)
      SY= SQRT(SY2)
      WRITE (6,9000) XA,SX2,SY2,SX,SY
9000  FORMAT(3x,5h X/A=,F5.2,5X,6H TOTAL,3x,2E15.6,8X,2E15.6)
9200  FORMAT(3x,5h Y/A=,F5.2,5X,8H BENDING,1x,2E15.6,8X,2E15.6)
      SXB= SQRT(SXB2)
      SXM= SQRT(SXM2)
      SYB= SQRT(SYB2)
      SYM= SQRT(SYM2)
      WRITE (6,9200) YB,SXB2,SYB2,SXB,SYB
      WRITE (6,9400) SXM2,SYM2,SXM,SYM
9400  FORMAT(18x,9H MEMBRANE,2E15.6,8X,2E15.6)
      RETURN
      END

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C LARGE AMPLITUDE RESPONSE OF AIRCRAFT PANELS SUBJECTED TO BROADBAND
C   PANDUM ACOUSTIC EXCITATIONS
C           CLAMPED ALONG ALL FOUR EDGES
C RECTANGULAR PANEL - CLAMPED ALONG ALL FOUR EDGES
C           CLAMPED ALONG ALL FOUR EDGES
C           COMMON AR2,PI,PI2,UN,ENU,PH10,PH01,PH20,PH11,PH02,PH21,PH12
           DIMENSION SPSD(99)
C MOVABL=FLAG FOR INPLANE EDGE CONDITIONS
C   SET MOVABL= 0, FOR IMMOVABLE EDGES
C   SET MOVABL=+1, FOR MOVABLE EDGES
C MOVABL=0 OR +1 ?
           MOVABL=0
C ASGNQ=AN ASSIGNED SMALL CONSTANT FOR CONVERGENCE TEST OF RMS
C   MAXIMUM DEFLECTION
           ASGNQ=1.0E-3
C
           PI=3.14159265
           PI2=PI*PI
           PI4=PI2*PI2
C ZETA=DAMPING FACTOR OF PANEL=C/(C CRITICAL)
           ZETA=0.020
C UN=POISSON'S RATIO
           UN=0.3
           ENU=1.0 - UN*UN
9000   FORMAT(I2)
9050   FORMAT(F10.3)
C NUMSF=NUMBER OF EXCITATION SPECTRAL DENSITY
8000   FORMAT(49H HOW MANY EXCITATION SPECTRAL DENSITY ARE THERE?,I4)
           READ (5,9000) NUMSF
           WRITE (6,8000) NUMSF
           WRITE (6,8100)
           DO 100 NSF=1,NUMSF
8100   FORMAT(23H WHAT ARE THEIR VALUES? )
           READ (5,9050) SPSD(NSF)
           WRITE (6,8150) SPSD(NSF)
8150   FORMAT(5X,E15.6)
C SPSD=NONDIMENSIONAL FORCING EXCITATION SPECTRAL DENSITY
C   PARAMETER=SF(F)/((RHO**2) * (H**4) * (DFACTOR**1.5))
100   CONTINUE
C NAR=NUMBER OF ASPECT RATIOS
8200   FORMAT(24H HOW MANY ASPECT RATIOS?,I4 )
           READ (5,9000) NAR
           WRITE (6,8200) NAR
           DO 900 NTIME=1,NAR
C AR=ASPECT RATIO OF PANEL=AL/BL
C AL=LENGTH
C BL=WIDTH
           READ (5,9050) AR
           AR2=AR*AR
           AR4=AR2*AR2
C FLMD02=LAMBDA0 SQUARE - NONDIMENSIONAL LINEAR FREQUENCY SQUARE
           FLMD02=16.0*PI4*(3.0 + 2.0*AR2 + 3.0*AR4)/(9.0*AR4)
           FLMD0= SQRT(FLMD02)
C BETAP=NONLINEARITY COEFFICIENT
C BETAC=NONLINEARITY COEFFICIENT
           PH10=1.0
           PH01=1.0/AR4
           PH20=0.0625
           PH11=2.0/(1.0 + 2.0*AR2 +AR4)
           PH02=0.0625/AR4

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PH21=1.0/(16.0 + 8.0*AR2 + AR4)
PH12=1.0/(1.0 + 3.0*AR2 + 16.0*AR4)
PHSUM=PH10 + PH01 + PH20 + PH11 + PH02 + 0.5*(PH21 + PH12)
BETAP=4.0*PI4*ENU*PHSUM/3.0
BETAC=1.5*PI4*(1.0 + UR*AR2 + AR2*(AR2 + UR))/AR4
C FOR MOVABLE INPLANE EDGES, BETA=BETAP
C FOR IMMOVABLE INPLANE EDGES, BETA=BETAP + BETAC
BETA=BETAP + BETAC
IF (MOVABL.EQ. 1) BETA=BETAP
C
DO 500 NSP=1,NUMSF
SF=SPSD(NSP)
WRITE (5,9100) AR,SF,BETA
9100 FORMAT(/17H1ASPECT RATIO=AR=,F6.2/,
1 47H NONDIMENSIONAL EXCITATION SPECTRAL DENSITY=SF=,
2 13.5/31H NONLINEARITY COEFFICIENT=BETA=,F15.6)
IF (MOVABL.EQ. 0) WRITE (6,8300) ZETA
9300 FORMAT(47H CLAMPED-SUPPORTED WITH IMMOVABLE INPLANE EDGES,
1 /34H DAMPING FACTOR=C/C CRITICAL=ZETA=,F7.4)
IF (MOVABL.EQ. 1) WRITE (6,8310)
9310 FORMAT(47H CLAMPED-SUPPORTED WITH MOVABLE INPLANE EDGES,
1 /34H DAMPING FACTOR=C/C CRITICAL=ZETA=,F7.4)
C ITER=0 CORRESPONDS TO THE LINEAR SOLUTION
ITER=0
WRITE (5,9150)
9150 FORMAT(/14H ***SMALL-DEFLECTION (LINEAR) SOLUTION***)
C Q02=MEAN SQUARE MAXIMUM DEFLECTION - LINEAR STRUCTURAL THEORY
Q02=32.0*SF/(31.0*ZETA*FLM00*FLM002)
C Q0=ROOT MEAN SQUARE MAXIMUM DEFLECTION
Q0=SQRT(Q02)
WRITE (6,9200) FLM002,FLM00,Q02,Q0
9200 FORMAT(4X,31H FREQ. PARAMETER SQUARE=FLM002=,E15.7,
1 5X,23H FREQ. PARAMETER=FLM00=,E15.7/,
2 8X,27H MEAN-SQUARE AMPLITUDE=Q02=,E15.7,
3 10X,18H RMS AMPLITUDE=Q0=,F7.3)
C COMPUTE RMS STRESSES OR STRAINS BASED ON LINEAR STRUCTURAL THEORY
WRITE (6,9250)
9250 FORMAT(/3X,9H LOCATION,20X,19H MEAN-SQUARE STRESS,19X,
1 11H RMS STRESS/29X,9H X-COMPS.,6X,9H Y-COMPS.,14X,
2 9H X-COMPS.,5X,9H Y-COMPS.)
CALL STRESS(0.0,0.0,Q02,MOVABL,ITER)
CALL STRESS(0.5,0.0,Q02,MOVABL,ITER)
CALL STRESS(0.0,0.5,Q02,MOVABL,ITER)
CALL STRESS(0.5,0.5,Q02,MOVABL,ITER)
C STORE Q02, Q0 AND FLM00 FOR CONVERGENCE TEST LATTER
Q2PV=Q02
QPV=Q0
FLMDPV=FLM00
C
C START ON ITERATION - ITER=ITERATION COUNTER
C
ITER=1
10 CONTINUE
FLMD2=FLM002 + 3.0*BETA*Q2PV
Q2=32.0*SF/(31.0*ZETA*FLM00*FLMD2)
FLMD=SQRT(FLMD2)
Q=SQRT(Q2)
C CHECK FOR CONVERGENCE
DEF=(FLMD-FLMDPV)/FLMD
DEQ=(Q-QPV)/Q

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      DEQ= ABS(DEQ)
C CONVERGENCE TEST, IF SATISFIED GO TO 12 FOR STRESS COMPUTATION
  IF (DEQ .LE. ASGNQ) GO TO 12
C STORE FLMD, Q2 AND Q FOR CONVERGENCE TEST
  FLMDPV=FLMD
  Q2PV=Q2
C SAVE COMPUTATION TIME AFTER 50 AND 100 ITERATIONS
  IF (ITER .EQ. 50) Q2PV=0.5*(Q*Q + QPV*QPV)
  IF (ITER .EQ.100) Q2PV=0.5*(Q*Q + QPV*QPV)
  QPV=Q
  ITER=ITER + 1
  GO TO 10
12  CONTINUE
     WRITE (6,9400) ITER
9400  FORMAT(/44H ***LARGE-DEFLECTION (NONLINEAR) SOLUTION***/,
1     4X,15H CONVERGENCE AT,14,13H-TH ITERATION)
     WRITE (6,9450) FLMD2,FLMD,Q2,Q
9450  FORMAT(4X,30H FREQ. PARAMETER SQUARE=FLMD2=,E15.7,
1     5X,22H FREQ. PARAMETER=FLMD=,E15.7/,
2     8X,25H MEAN-SQUARE AMPLITUDE=Q2=,E14.6,
3     11X,17H RMS AMPLITUDE=Q=,F7.3)
     WRITE (6,9500) DEF,DEQ
9500  FORMAT(20X,4HDEF=,E15.6,5X,4HDEQ=,E15.6)
C COMPUTE STRESSES AT LARGE AMPLITUDE
  WRITE (6,9250)
  CALL STRESS(0.0,0.0,Q2,MOVABL,ITER)
  CALL STRESS(0.5,0.0,Q2,MOVABL,ITER)
  CALL STRESS(0.0,0.5,Q2,MOVABL,ITER)
  CALL STRESS(0.5,0.5,Q2,MOVABL,ITER)
  IF (Q .GE. 5.0) GO TO 700
500  CONTINUE
700  CONTINUE
900  CONTINUE
     STOP
     END
     SUBROUTINE STRESS(XA,YB,Q2,MOVABL,ITER)
     COMMON AR2,PI,PI2,UN,ENU,PH10,PH01,PH20,PH11,PH02,PH21,PH12
C THIS SUBROUTINE COMPUTES THE RMS STRESSES IN THE X (SX) AND Y
C (SY) DIRECTIONS - THESE STRESSES ARE NONDIMENSIONAL
C XA=X/A
C YB=Y/B
C Q2=MEAN SQUARE OF Q
     CX=COS(2.0*PI*XA)
     CY=COS(2.0*PI*YB)
     C2X=COS(4.0*PI*XA)
     C2Y=COS(4.0*PI*YB)
     UX=0.5*PI2*( CX*(1.0+UN)/AR2 + UN*(1.0+CX)*CY )/ENU
     FAC1=0.125*PI2*AR2
     D1X=FAC1*(PH01*CY + PH11*CX*CY + 4.0*PH02*C2Y +
1     PH21*C2X*CY + 4.0*PH12*CX*C2Y)
     D2X=3.0*PI2*(1.0 + UN*AR2)/(32.0*AR2*ENU)
     IF (MOVABL .EQ. 1) D2X=0.0
     DY=0.5*PI2*( (1.0+CX)*CY + UN*CX*(1.0+CY)/AR2 )/ENU
     FAC2=0.125*PI2
     D1Y=FAC2*(PH10*CX + 4.0*PH20*C2X + PH11*CX*CY +
1     4.0*PH21*C2X*CY + PH12*CX*C2Y)
     D2Y=3.0*PI2*(AR2 + UN)/(32.0*AR2*ENU)
     IF (MOVABL .EQ. 1) D2Y=0.0
     DXX=D1X + D2X
C ITER=0 CORRESPONDS TO THE SMALL DEFLECTION LINEAR THEORY

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      IF (ITER .EQ. 0) DXX=0.0
      SXB2=DX*DX*Q2
      SXM2=3.0*DXX*DXX*Q2*Q2
      SX2=SXB2 + SXM2
      DYY=D1Y + D2Y
      IF (ITER .EQ. 0) DYY=0.0
      SYB2=DY*DY*Q2
      SYM2=3.0*DYY*DYY*Q2*Q2
      SY2=SYB2 + SYM2
C  SX, SY=DIMENSIONLESS ROOT MEAN SQUARED STRESSES (STRESS*B**2)
C  /(E*H**2)
C  B=BL=WIDTH OF PANEL
C  E=MODULUS OF ELASTICITY
C  H=THICKNESS OF PANEL
      SX= SQRT(SX2)
      SY= SQRT(SY2)
      WRITE (6,9000) XA,SX2,SY2,SX,SY
9000  FORMAT(3x,5h X/A=,F5.2,5X,6H TOTAL,3x,2E15.6,8X,2E15.6)
9200  FORMAT(3x,5h Y/A=,F5.2,5X,8H BENDING,1x,2E15.6,8X,2E15.6)
      SXB= SQRT(SXB2)
      SXM= SQRT(SXM2)
      SYB= SQRT(SYB2)
      SYM= SQRT(SYM2)
      WRITE (6,9200) YB,SXB2,SYB2,SXB,SYB
      WRITE (6,9400) SXM2,SYM2,SXM,SYM
9400  FORMAT(18x,9H MEMBRANE,2E15.6,8X,2E15.6)
      RETURN
      END

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