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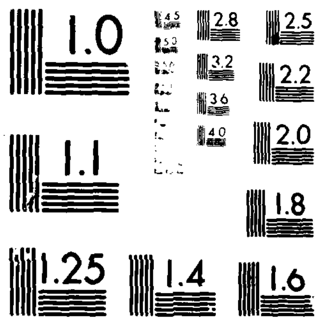

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1. REPORT NUMBER DTNSRDC-80/009	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) AN ASSESSMENT OF REPRESENTATIONS OF LONG- AND SHORT-TERM OCEAN WAVE SPECTRA AND SHIP RESPONSE PREDICTIONS,		5. TYPE OF REPORT & PERIOD COVERED (9) Final rept,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ming-Shun Chang (10)		8. CONTRACT OR GRANT NUMBER(s) (16)
9. PERFORMING ORGANIZATION NAME AND ADDRESS David W. Taylor Naval Ship Research and Development Center Bethesda, Maryland 20084		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Project R02301 Work Unit 1552-101
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Sea Systems Command Code 03412 Washington, D.C. 20360		12. REPORT DATE (11) Jun[redacted]80
		13. NUMBER OF PAGES 49
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 48		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

PE 61153

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Ship Response
Wave Spectrum
Long-Term Predictions
Extreme Values

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper presents a review of the present method for predicting ship responses in a seaway. The applicability of the existing wave spectral representations for global ship response predictions are discussed, and problems associated with the extreme response predictions are illustrated. The study indicates that the errors induced by the use of different spectral representations are of the same order of magnitude as the errors generated from the other assumptions used in the prediction scheme.

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NOTATION

$d\zeta(\omega)$	Complex random amplitudes of response
$d\xi(\omega)$	Complex random amplitudes of sea elevations
$E(X)$	Expected values of X
g	Acceleration due to gravity
$H_{1/3}$	Significant wave height
$\|H(\omega)\|^2$	Frequency response amplitude operator
$h(t)$	Response function in time
k	Wave number
$M(a,b,c)$	Kumman's confluent hypergeometric function
M_N	N^{th} moment of the spectrum
$P(X)$	Probability distribution of X (cumulative probability function)
$p(X)$	Probability density function of X
R_{max}	Absolute maximum of response r(t)
$r(t)$	Response
$S_B(\omega)$	Bretschneider spectral representation
$S_G(\omega)$	Polynomial spectral representation
$S_g(\omega)$	General Bretschneider spectral representation
$S_r(\omega)$	Spectral density function of response
$S(\omega)$	Spectral density function of ocean waves
T	Characteristic wave period
T_2	Zero-crossing wave period
t	Time

W	Wind speed
x	Space position
Γ	Gamma functions
γ_t	Theoretical value of R_{max} such that $P(R_{max} \geq \gamma_t) = 0.01$
γ_m	Theoretical value of most probable R_{max}
η	Surface elevation of the sea
η_{max}	Local maximum of η
$\theta(\omega)$	Random phase angles
ξ	Spectral bandwidth
σ^2	Variance squared
ω	Frequency
ω_0	Frequency of dominant energy

ABSTRACT

This paper presents a review of the present method for predicting ship responses in a seaway. The applicability of the existing wave spectral representations for global ship response predictions are discussed, and problems associated with the extreme response predictions are illustrated. The study indicates that the errors induced by the use of different spectral representations are of the same order of magnitude as the errors generated from the other assumptions used in the prediction scheme.

ADMINISTRATIVE INFORMATION

The study reported herein was conducted in 1974 under the General Hydromechanics Research program of the Naval Sea Systems Command. Funding was provided under Project R02301 and Work Unit 1552-101.

INTRODUCTION

The most direct approach for predicting the long-term seakeeping properties of a ship is to examine the long-term history of the ship's seakeeping behavior, for example twenty years, and to use statistical methods to predict the long-term future from this history.[1*,2] Unfortunately, this method cannot be used at the present time, since there are no long-term ship-motion histories available which have sufficiently long-term spans. However, suppose that there are ships for which every needed detail of the seakeeping history is available; there are still problems in applying this approach. The serious shortcoming of this direct approach is that the ship designer does not know which, if any, of the available seakeeping histories to use to improve a particular seakeeping property.

This paper presents an evaluation of the commonly used alternate approaches for predicting the seakeeping properties of a ship. The approach traditionally employed is an indirect one for it uses the statistical properties of the sea to predict the statistics of the response, rather than the seakeeping history. In this method, the designer first predicts the expected probability distribution of ocean waves which the ship will encounter. Then calculated are the motions of the ship as a response to

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*A complete listing of references is given on page 39.

those expected waves. The seakeeping property of interest is assumed to be a function of the ocean wave; this function, which can be determined by experiments and theory, has to be known in order to use the indirect prediction method. Determining this response function may not be easy, since the function is dependent upon not only the geometry of the ship but also the operating condition such as the speed of the ship. However, throughout this paper, the discussion will be made assuming the response function is known.

Our evaluation of the indirect method is broken into four parts. In the Ship Response Analysis section, the ship response formulation is presented. The sea spectra obtained from weather ships are discussed in the Spectral Representation section from the point of view of ship response. Short-term predictions and simulations are discussed next. Finally discussed is the long-term prediction method. For more recent information on ocean wave data, References 25 and 26 have been added to the reference list at the end of the report. Overall, it is shown that, within the present framework of long-term predictions, the use of a more sophisticated spectral representation will not lead to an improvement of the long-term response predictions but may lead to more confusion.

SHIP RESPONSE ANALYSIS

Considering the surface elevations of the sea η as a stationary normal random process, Cartwright and Longuet-Higgins[3] and Longuet-Higgins[4,5] were able to express many useful statistical qualities of the sea in terms of the amplitudes of its Fourier transforms. They represent the sea elevations $\eta(t,x)$ at a given time t and location x by

$$\eta(t,x) = \int_{-\infty}^{\infty} e^{i(\omega t-kx)} d\xi(\omega) \qquad (1)$$

where ω = wave frequency

k = wave number

g = acceleration due to gravity

The function $d\xi$ is a normally-distributed complex random variable with zero mean, which is related to the spectral density $S(\omega)$ of the sea; if $E[X]$ denotes the expected value of a random variable X, the relation between $d\xi(\omega)$ and the spectral density is given by

$$E[d\xi(\omega)\ \overline{d\xi(\omega')}] = S(\omega)d\omega;\ \text{when}\ \omega = \omega'$$

and

$$= 0 \qquad ;\ \text{when}\ \omega \neq \omega'$$

Then $\overline{d\xi(\omega)}$ denotes the complex conjugate of $d\xi(\omega)$, and $\overline{d\xi(-\omega)} = d\xi(\omega)$.

If X denotes a statistical property of the sea, it is a function of the sea spectrum $S(\omega)$ and properties of the specturm. For example, the average zero-crossing wave period T_2 is given by

$$T_2 = 2\pi \left[\frac{\int_{-\infty}^{\infty} \omega^2 S(\omega)d\omega}{\int_{-\infty}^{\infty} S(\omega)d\omega} \right]^{-1/2} \tag{2}$$

and significant wave height $H_{1/3}$ can be approximated by

$$H_{1/3} = 4\left[\int_{-\infty}^{\infty} S(\omega)d\omega\right]^{1/2} \tag{3}$$

Let M_N denote the N^{th} moment of the spectrum;

$$M_N = \int_{-\infty}^{\infty} \omega^N S(\omega)d\omega \tag{4}$$

and the spectral bandwidth parameter ξ is given by

$$\xi = \left(1- \frac{M_2^2}{M_0 M_4}\right)^{1/2} \tag{5}$$

The probability density of the local maximum in the wave elevation $p(\eta_{max})$ is then given by[3]

$$p(\eta_{max}) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{M_0}} \left[\xi \exp\left(- \frac{1}{2\xi^2} \frac{\eta_{max}^2}{M_0}\right) + \sqrt{\frac{2\pi(1-\xi^2)}{M_0}}\, \eta_{max} \exp\left(- \frac{1}{2} \frac{\eta_{max}^2}{M_0}\right) \mathcal{f}\left(\frac{\sqrt{1-\xi^2}}{\xi} \frac{\eta_{max}}{\sqrt{M_0}}\right)\right] \tag{6}$$

where
$$\mathcal{f}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\,(-y^2/2)dy \tag{7}$$

Within <u>linear</u> theory the statistical properties of a ship's response can also be expressed as functions of properties of the sea spectrum. A given ship response $r(t)$ can be expressed as a convolution of the sea elevation with response function $h(t)$, which relates the response r to the wave elevation η. The ship response is then expressed as

$$r(t) = \int_{-\infty}^{\infty} e^{i\omega t}\, d\zeta(\omega)$$

Since the sea was assumed to be a Gaussian random process with zero mean, $d\zeta(\omega)$ is a normal complex random variable with zero mean:

$$E[d\zeta(\omega)\overline{d\zeta(\omega')}] = \begin{cases} S_r(\omega)d\omega; & \omega = \omega' \\ 0 & ; \omega \neq \omega' \end{cases}$$

and

$$\overline{d\zeta(\omega)} = d\zeta(-\omega)$$

The ship response spectrum $S_r(\omega)$ is given by $S_r(\omega) = |H(\omega)|^2 S(\omega)$, where $|H(\omega)|^2$ is the response amplitude operator.

Our expression for $r(t)$ is similar to the expression for the sea elevation $\eta(t)$. The role that $d\zeta$ takes in the expression for r is equivalent to the corresponding role that $d\xi$ takes in the expression for η. Consequently, the formulas--Equations (1)-(7)--for calculating statistical properties of the sea can be applied to calculate the corresponding properties of the response with $S(\omega)$ being replaced by $S_r(\omega)$. It is these formulas which constitute the fundamental expressions for long-term indirect ship-motion prediction methods. In these methods, prediction of the distribution of a statistical property of the response is equivalent to prediction of the corresponding distribution of the sea spectra.

SPECTRAL REPRESENTATIONS

Sea spectra have to be calculated from continuous measured data for the sea elevations. The measurements required for these calculations are generally not available. Weather ships and wave buoys have been the primary sources for continuous sea elevation measurements, but their output of data is relatively small in comparison to the amount of data needed for long-term predictions. Even if today all the ships at sea were equipped to continuously measure the sea elevation, it would be some time before a sufficient amount of data were recorded for long-term predictions.

Many ships at sea record the wind speed, wave heights, and wave periods at six-hour intervals. It is generally believed that the distribution of these observed parameters can be used to predict the distribution of the sea states. For example, it is known that for fully developed ocean waves the spectrum is, ideally, a function of wind speed only.[6,7]

Bretschneider[8] concluded from measured data that wave spectra could be represented in terms of wave height and wave period. Others[9,10] have sought more complex representations of wave spectra in terms of more parameters than wind speed or the two parameters of Bretschneider.

Since the prediction of the spectral density of a ship response is equivalent to the prediction of the sea distribution, the response of a ship in a fully developed sea* is then, ideally, also a function of wind speed only. The fully developed Pierson-Moskowitz (P-M)[11] spectra could then be used to evaluate the response of a ship to a fully developed sea. If the distribution of the sea spectra is represented by the two-parameter Bretschneider distributions, then this spectral family can be used to predict the response of a ship as suggested by Cummins.[12] Other representations could also be used in the long-term indirect prediction method, if they are truly good representations of the sea-spectra density distributions.

Let us first consider the use of the P-M fully developed sea spectra in the indirect method. Suppose it is assumed, as in the past, that if a ship survives in a fully developed sea which is the result of a given wind speed, it will survive in any state of sea with the same wind speed. Because the sea is not always fully developed and because it is assumed that the worst conditions encountered are in a fully developed sea, the use of the P-M spectra would result (by assumption only) in over design at least from the probabilistic point of view. For instance, suppose one is interested in ship fatigue stresses that result from waves exceeding a given wave height; the expected lifetime-encounters of a ship with waves exceeding this wave height would be larger when predicted from the P-M spectrum than if one used the expected value calculated from the distribution of the sea conditions. On the other hand, if the conditions in a sea that is not fully developed are not subordinate to those in a fully developed sea, the application of the P-M spectra would lead to a meaningless prediction.

*A fully developed sea is defined as the limiting sea state under a given wind speed. This limiting sea state is a function of only wind speed.

Figure 1 shows a case in which the sea spectrum is not subordinated to the P-M spectrum at both high and low frequency regions. In view of uncertainty of the observed wind speed a more generous wind speed of 40 knots was used for comparison. It is seen that for responses which are most sensitive to the excitation force in these frequency regions the measured responses will certainly exceed those calculated from the P-M spectrum. Hence, in general, a one parameter fully developed sea spectrum will not give good results.

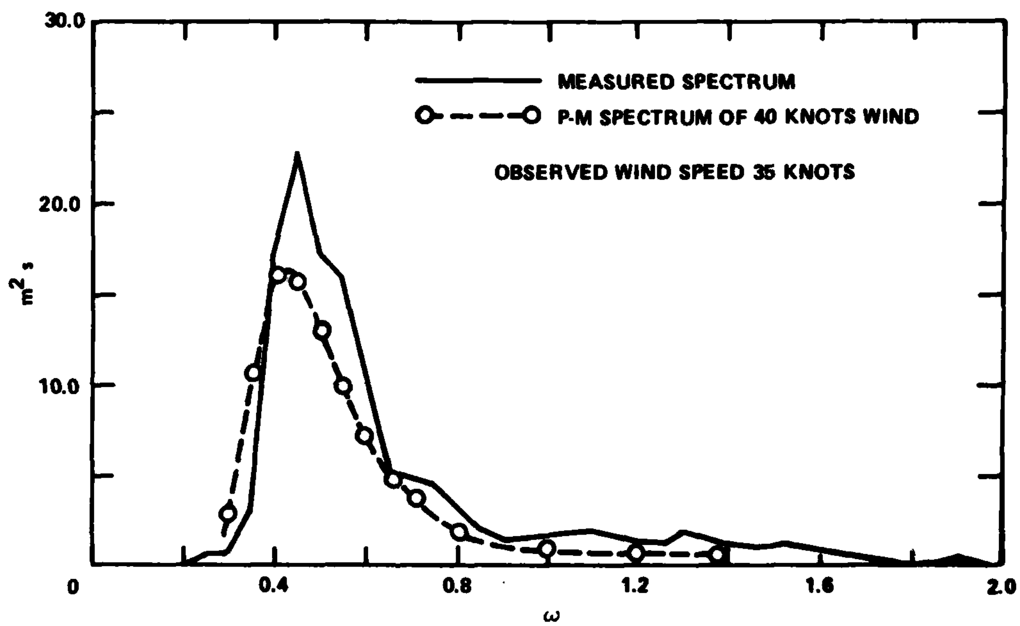

Figure 1 - A Comparison of Measured and Pierson-Moskowitz Spectra

British weather ships[13] have been continuously measuring the wave elevations at weather stations for the past twenty years. The wave elevations obtained by these weather ships are measured by Tuckermeters aboard the ship, and the measurements are recorded on strip charts. Because of the complications involved in obtaining the early charts for the purpose of digitizing the data, and because of the costs that would be involved in such a large project, few of these records have been analyzed for their spectral content. The records have primarily been analyzed[13] to determine significant wave heights, zero-crossing periods, maximum wave heights, and bandwidth parameters as these parameters have been considered the most significant ones for engineering applications.

With the increased demands for characteristics of sea spectra, Moskowitz et al.[14] used a correlation method to obtain the spectra from a large set of ocean-wave measurements. Their work resulted in the publication of 461 sea spectra. The data from which these spectra were taken covered the years 1955 through 1960. The measurements were made in the North Atlantic Ocean at the weather stations I, J, A, and K by two weather ships: WEATHER EXPLORER and WEATHER REPORTER. The strip recordings which were analyzed were originally selected to facilitate a study of the development of wind-generated seas.

Miles[15] used the fast fourier transform (FFT) method to obtain sea spectra from a second large set of weather ship wave elevation measurements. His set contained 323 spectra computed from wave elevation measurements between the years 1955 and 1967. The strip recordings were originally selected so as to obtain a population with an equal number chosen over the four seasons and over preselected ranges of wind speed. The measurements were made at weather station I by the following three weather ships: WEATHER EXPLORER, WEATHER REPORTER, and WEATHER ADVISOR.

Since Moskowitz et al. selected their spectra to study the development of wind-generated seas, it is generally believed that the statistical results from their set of spectra cannot be used to represent ocean waves in general. On the other hand, since Miles'[15] spectra have a nearly equal number of samples from the four seasons and wind speed ranges, some researchers have assumed that they should be considered more representative of the average sea spectra than the set published by Moskowitz et al.

There are differences between the spectra calculated by Miles and those calculated by Moskowitz et al., when the same wave elevation measurements were analyzed. They both analyzed the wave record of April 8, 1955. The resulting spectra, shown in Figure 2, are in good agreement for $\omega \leq 0.8$, but fail to agree at higher frequencies. Moskowitz et al. calculated

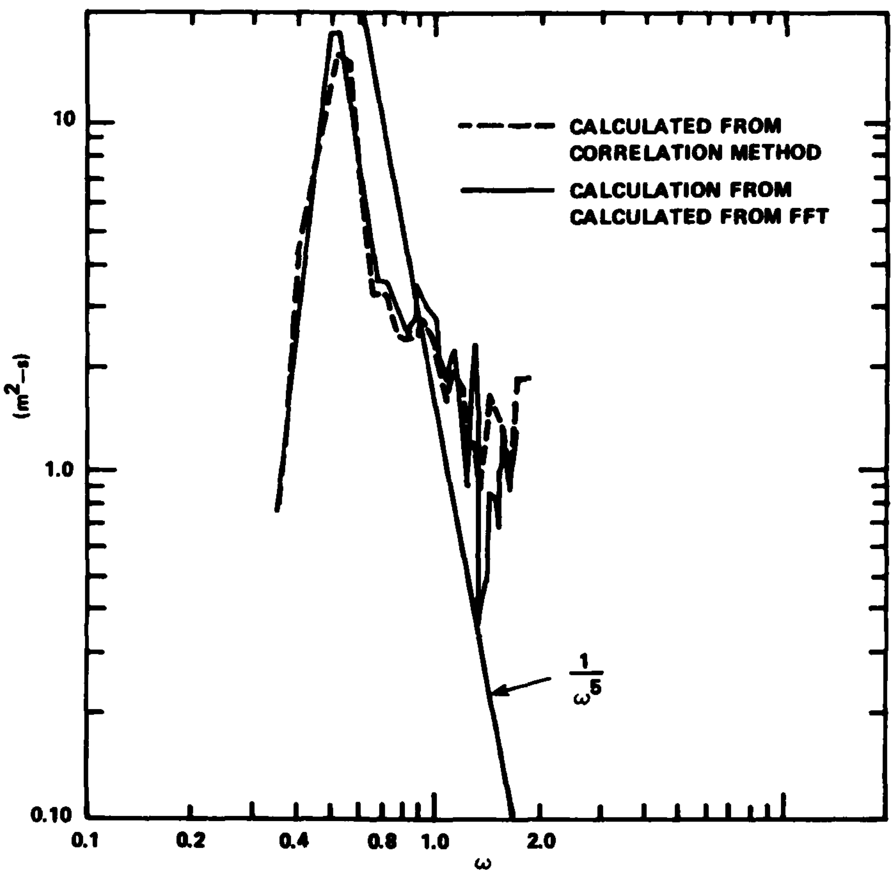

Figure 2 - Comparison of Spectra Calculated from the Fast Fourier Transform and the Correlation Methods

the significant wave height $H_{1/3}$ to be 27.4 feet (8.3 m) and the average zero-crossing wave period T_2 to be 8.1 seconds; Miles calculated $H_{1/3}$ and T_2 to be 28.4 feet (8.6 m) and 7.4 seconds, respectively. Although the agreement for the significant wave heights is good, there is approximately a 10-percent difference in the wave periods; hence, from a two-parameter point of view, there is a significant disagreement between the Miles' spectrum and the spectrum of Moskowitz et al. for the case of April 8, 1955.

The disagreement between the two spectra at the higher frequencies $\omega > 0.8$ is important. Errors in the high frequency end of the sea spectra result in errors not only in the high frequency response of the ship but also in the distributions of the wave parameters and would definitely lead to errors in the long-term prediction. The higher-order moments of the spectra are very sensitive to the accuracy of the spectral densities at the higher frequencies; hence, estimates of the statistical properties which are dependent on the higher-order must be handled carefully when there are erroes in the spectra at the higher frequencies. With the differences between the Miles' spectrum and that of Moskowitz et al. for $\omega > 0.8$, the higher-order predicted by these spectra should differ significantly so that long-term predictions based on each two-parameter spectrum of wave height and wave period would differ.

Consider for the moment the values of T_2 and $H_{1/3}$ calculated by Miles from the wave elevation measurements for station I. Diagrams of $H_{1/3} - T_2$ are plotted for the three ships WEATHER EXPLORER, WEATHER ADVISOR, and WEATHER REPORTER in Figures 3a, 3b, and 3c, respectively. There is considerable scatter in the data obtained by each ship. The mean period T_2 seems to increase with increasing significant wave height with a slope exceeding 0.5 for the data from WEATHER ADVISOR, whereas, the slope is closer to 0.25 for the data from the other two ships. The mean period for the data from WEATHER REPORTER are generally higher than that from WEATHER EXPLORER. Thus, there is an apparent significant difference between the $H_{1/3} - T_2$ diagrams for the three ships.

The large circles in Figure 3 are the averaged $H_{1/3} - T_2$ taken from Moskowitz for given $H_{1/3}$. These points tend to give higher values of T_2 than the averaged data from WEATHER EXPLORER but slightly lower values of T_2 than the median of the WEATHER REPORTER data or the WEATHER ADVISOR data. The differences between the data from the three ships are of the same order of magnitude as the differences between the Moskowitz et al. average $H_{1/3} - T_2$ and the Miles' average. (The differences between the spectra for the case of April 8, 1955 has already been noted.) Thus, the differences between the properties of the Miles' spectra and the

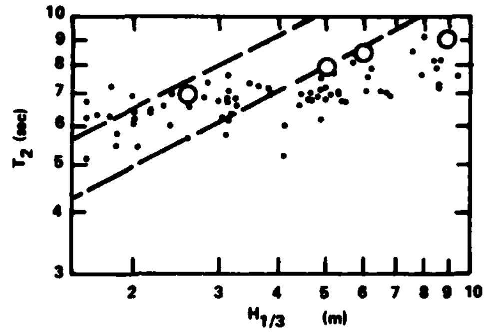

Figure 3a - Data from WEATHER EXPLORER

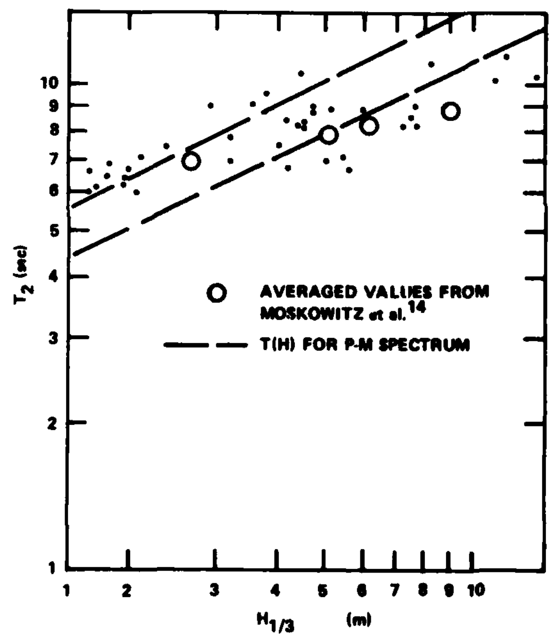

Figure 3b - Data from WEATHER ADVISOR

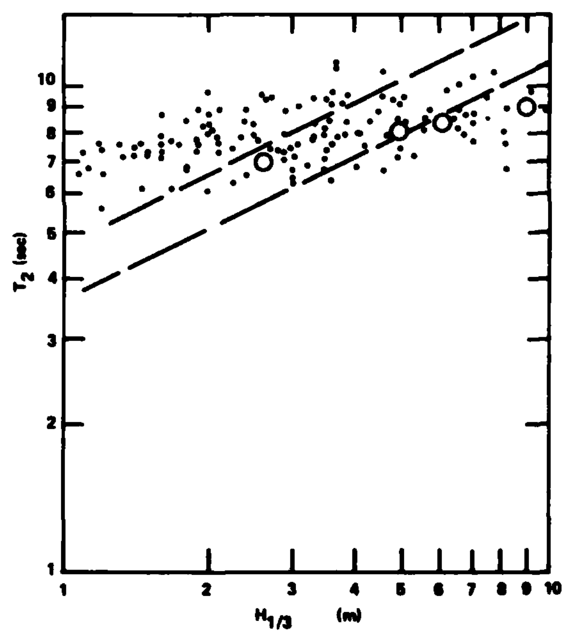

Figure 3c - Data from WEATHER REPORTER

Figure 3 - Variations of Wave Periods with Wave Heights from Weather Ship Measurements

corresponding properties of the Moskowitz et al. spectra may not be due entirely to the differences in sample population. While both sets of raw data were collected by the same agency in similar manners, seemingly minor differences in the collection and analysis procedures can lead to significant differences in statistics of the sea waves.

Hoffman[16] made use of the Miles' sea spectra to evaluate the two-parameter characterizations of wave families. The main spectral parameters of the measured wave data were shown to vary substantially from those expressed by the idealized mathematical spectrum $S_B(\omega)$ of Bretschneider; i.e., $S_B(\omega) = AH^2_{1/3}\, T_2(T_2\omega)^{-5} \exp[-B(T_2\omega)^{-4}]$, where A and B are constant. The peak distribution of the nondimensional spectra was shown to deviate significantly from the basic two-parameter formulation. Location of spectral peak as a function of the mean period T_1, presented in Reference 16, is included in the present paper as Figure 4. With regard to ship response,

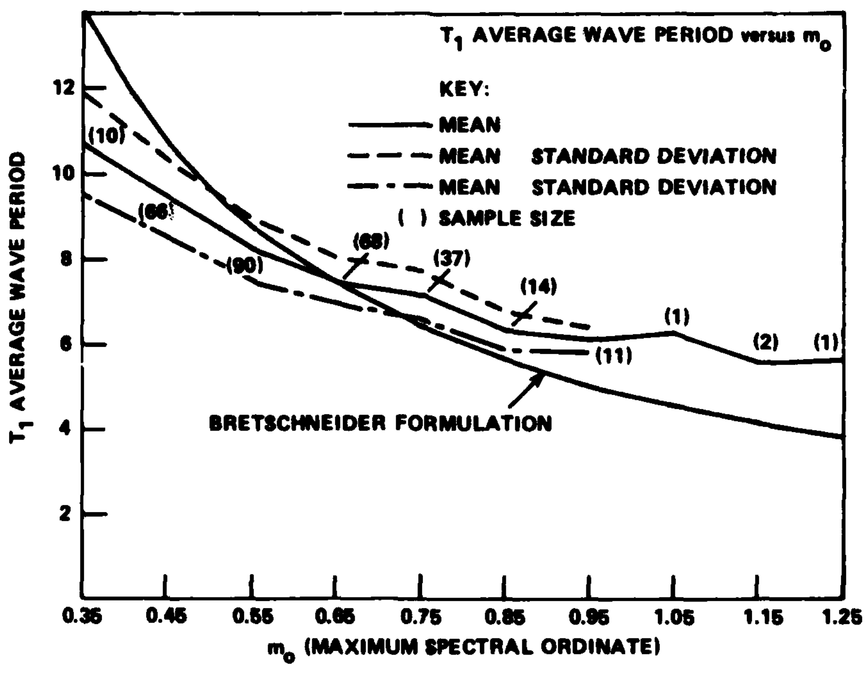

Figure 4 - Location of Spectral Peak as a Function of the Mean Period T_1 (Reference 16)

Hoffman[16] investigated the limitation of the Bretschneider spectral family as a spectral representation of the sea at station I. Hoffman concluded that the effect of the specific choice of spectral shape on the response prediction was dependent upon the size of a ship.

In view of the insufficiency of the Bretschneider spectral family for evaluating ship responses, Hoffman[16] recommended the use of statistically derived families of spectra. He selected a group of 80 spectra from Miles to represent the sea at station I. The long-term distributions of bending moment calculated from the use of this new group of spectra were compared with those predicted from the P-M spectrum and Bretschneider spectrum. The calculation of S.S. WOLVERINE STATE shows that the probability distributions for stress levels less than 8 kpsi (55160 kPa) differ little from those predicted from the new family of 80 spectra and the two-parameter family. Above stress levels of 8 kpsi (55160 kPa), the probability densities calculated from different spectral families become very different, as can be seen in Figure 5 taken from Reference 16.

Gospodnetic and Miles[17] investigated the average shape of spectra at station I by the use of a polynomial representation. They represent a spectrum by

$$S(\omega) \doteq S_G(\omega;\ T_{-1},\ H_{1/3})$$

$$= H_{1/3}^2\ T_{-1}\ (32\pi)^{-1} \sum_{L=0}^{N} \sum_{M=0}^{N-L} A_{ML}(\omega)\ (H_{1/3}-\bar{H})^M\ (T_{-1}-\bar{T})$$

where N = the order of the polynomial

$\bar{H}$ and $\bar{T}$ = the average wave height and wave period calculated from the ocean wave spectra of Miles[15]

$A_{ML}(\omega)$ = the coefficients of the polynomial

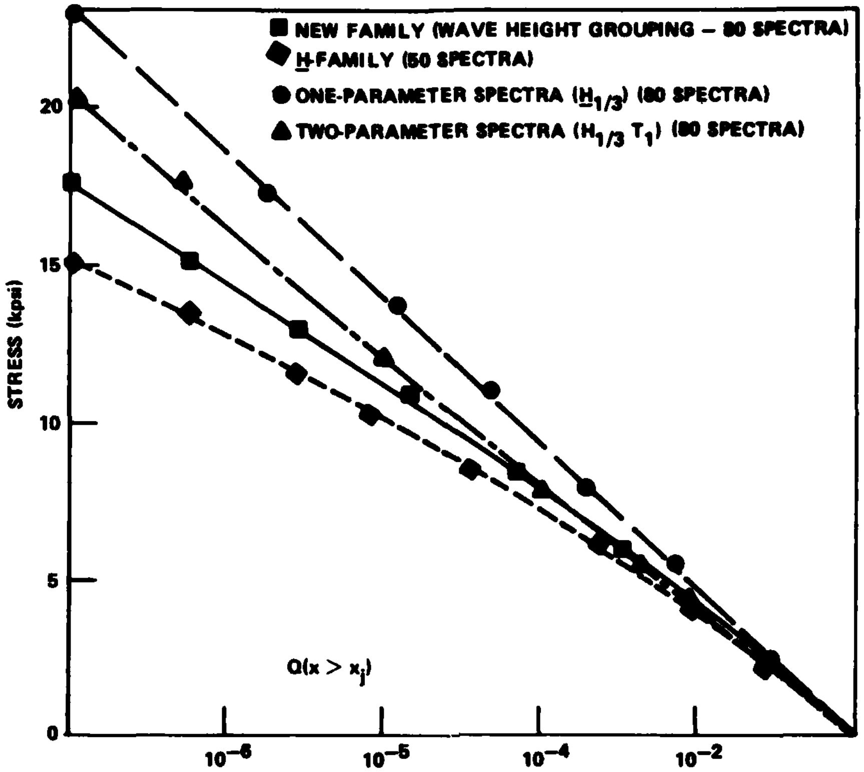

Figure 5 - Long-Term Prediction of Bending Moment Responses (S.S. WOLVERINE STATE) as Obtained from Four Alternative Spectral Sources (19.3 Foot Draft; 16 Knots) (Reference 16)

Coefficients $A_{ML}(\omega)$ were calculated for both a first and a second order polynomial representation. Figures 6 and 7 show the comparison of the measured spectra with the Bretschneider representation and with the second-order polynomial representation, respectively. The polynomial representation shows a good approximation to the measured average spectra; the improvement over the Bretschneider spectral family can be seen. Figure 8 snows the corresponding $A_{ML}(\omega)$ coefficients.

With respect to the shape of average spectra, the use of a polynomial representation is successful for station I data. However, in view of the oscillations of the $A_{ML}(\omega)$'s with ω (see Figure 8), the use of polynomial

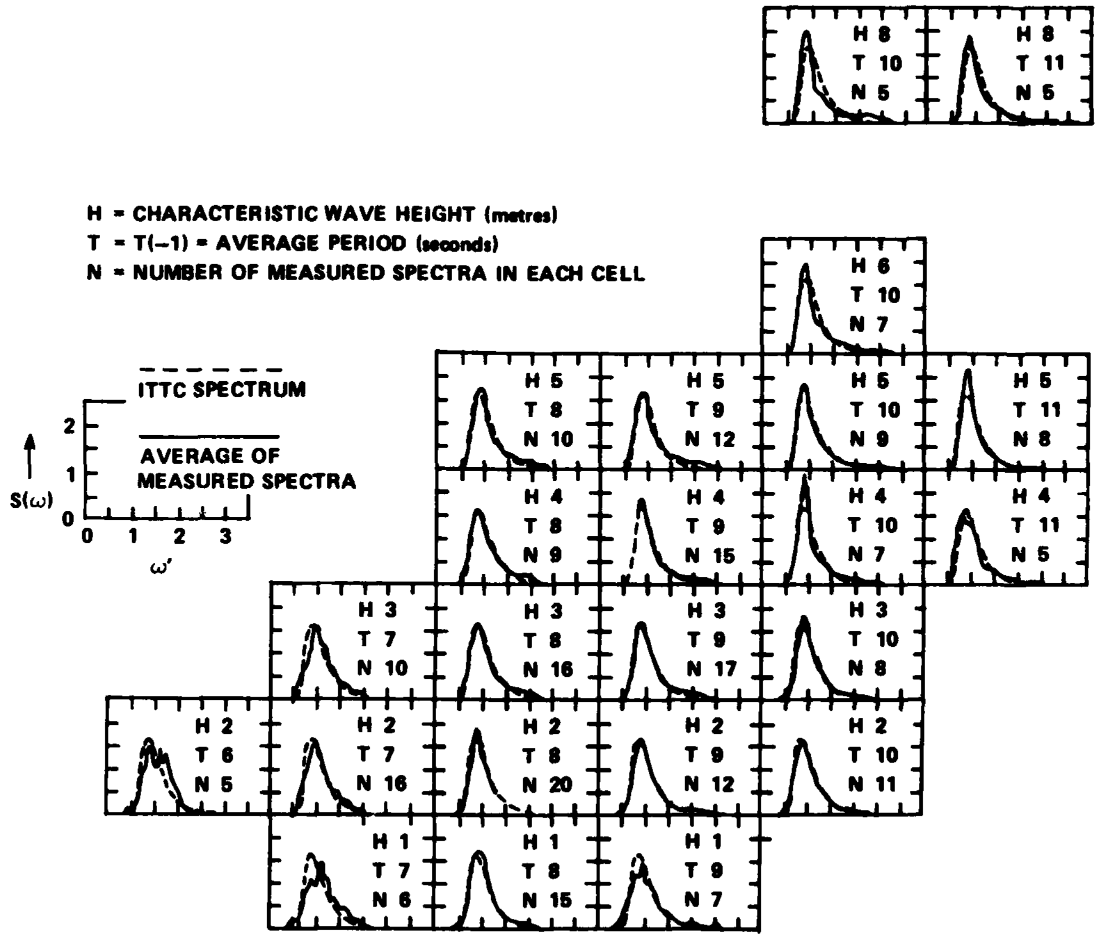

Figure 6 - Comparison of International Towing Tank Conference Spectrum and Averages of Measured Spectra in Each Cell (Reference 17)

representations for global ship-response predictions may be limited. This is because the detailed oscillations of the $A_{ML}(\omega)$ functions are probably local features which can vary from one place to another. To estimate the $A_{ML}(\omega)$'s for a global scale requires a large number of measurements, which are not available at the present time. Simple calculations show that approximations of the $A_{ML}(\omega)$'s by smooth curves, up to a 50-percent change

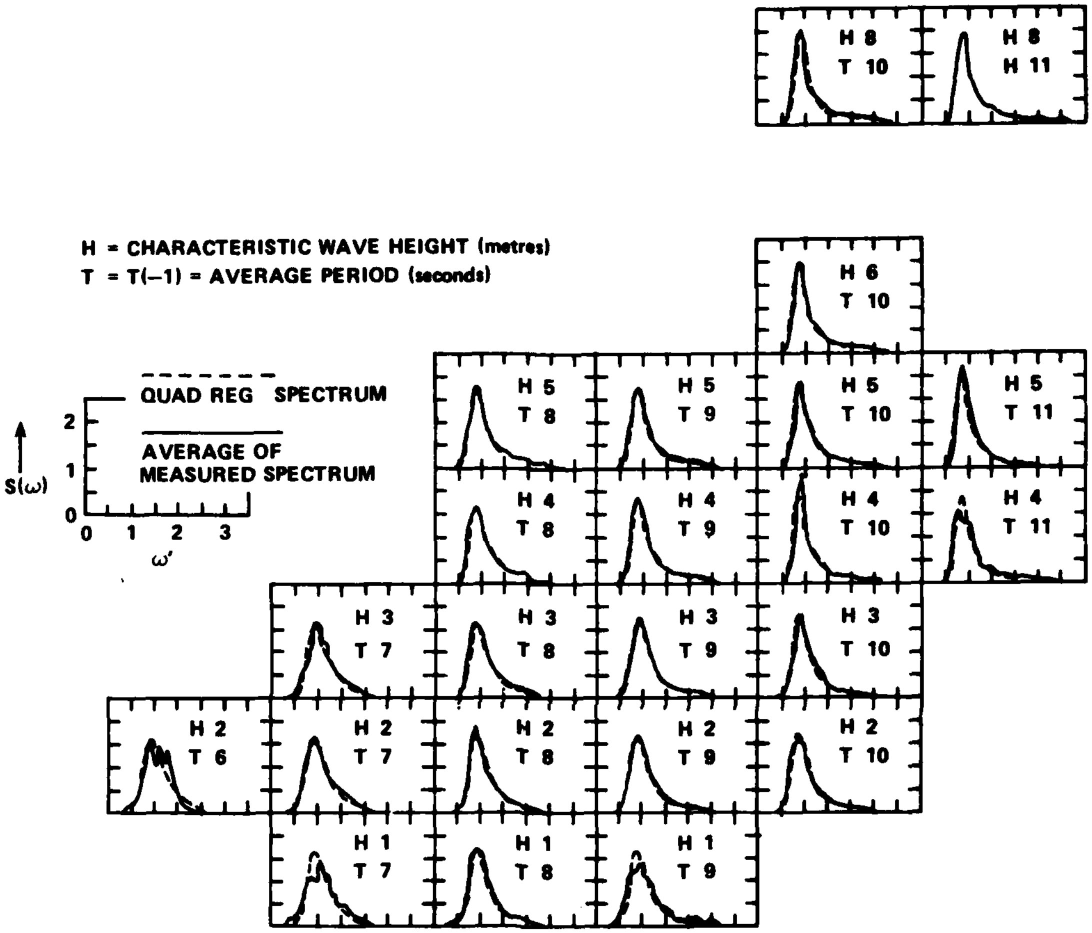

Figure 7 - Comparison of Quadratic Regression Spectrum and Averages of Measured Spectra in Each Cell (Reference 17)

were found in the spectral values for frequencies away from the main peak when wave height and wave period are significantly higher than their averaged values.

The major difficulty in the use of the generalized Bretschneider spectral family, $S_g(\omega)$; $S_g(\omega) = AH^2_{1/3}T_2(T_2\omega)^{-\ell} \exp[-B(T_2\omega)^{-n}]$, is that the measured spectra have significant energy away from the spectral peaks while

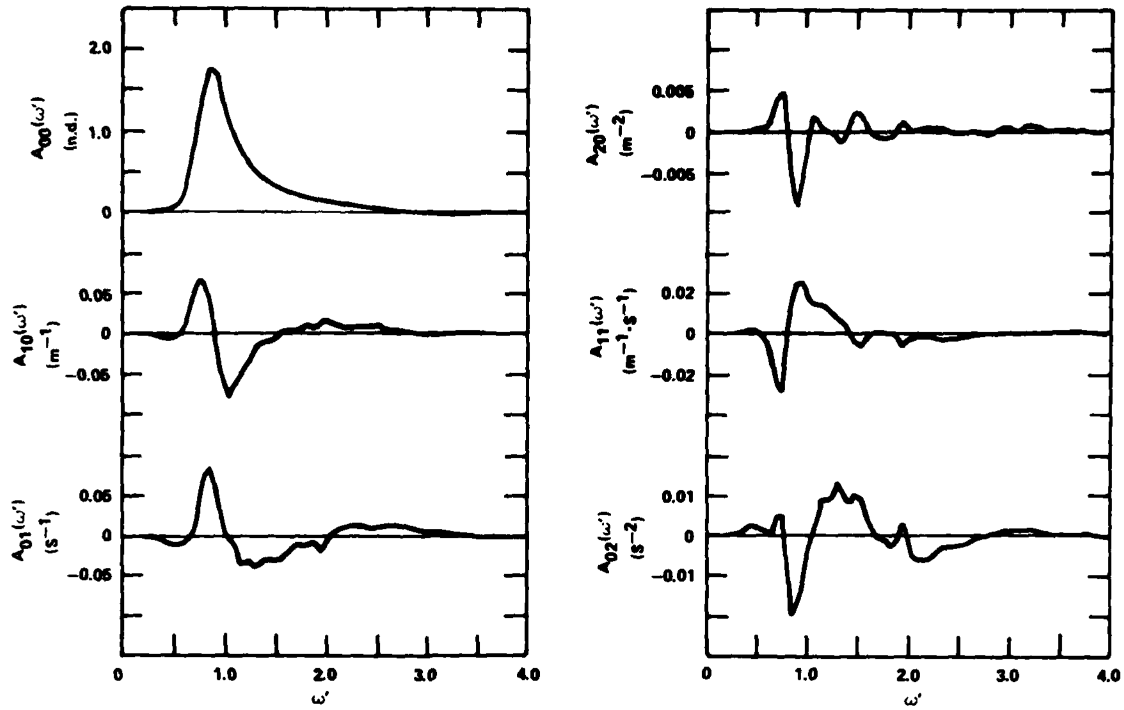

Figure 8 - Quadratic Regression Spectrum Coefficients (Reference 17)

the mathematical expression for the spectrum falls off rapidly as shown in Figure 9 taken from Bretschneider.[9] As this figure shows, the spectrum above $f/f_0 = 1.5$ fits the general form of $\ell = 9$ and $n = 4$ while major energy containing region fits $\ell = 8$ and $n = 9$ better.

In view of this difficulty, Bretschneider[18] suggested the use of a series of general Bretschneider forms. A two-term series of this form is given by:

$$S_T(\omega) = H_{1/3}^2 \; T_{-1}\{\alpha A_1 (T'\omega)^{-\ell} \exp[-B(T'\omega)^{-n}]$$

$$+ (1-\alpha) A_2 (T''\omega)^{-p} \exp[-B(T''\omega)^{-m}]$$

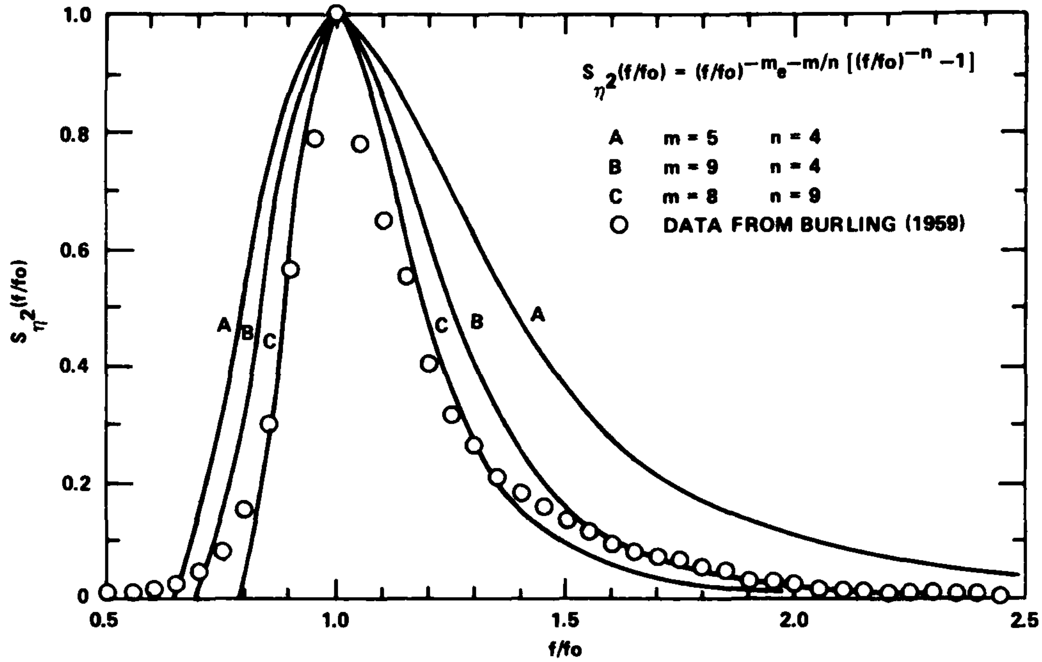

Figure 9 - Several Wave Spectra for K = 0.354 (Reference 9)

where T', T", ℓ, p, m, and n are parameters which have to be determined from observations. This two-term form has been applied to the ocean wave spectrum of Miles[15] with good results. Figures 10a, 10b, and 10c show three examples. The solid curves are the measured spectra and the curves with dots are calculated from the two-term series expressed above.

With regard to the long-term prediction, the above two-term series of the generalized Bretschneider form does not have significant advantage over the polynomial representation[17] or the statistical family of Hoffman[16] at the present time. This is because there is not sufficient information available to accurately determine the global joint-probability distribution of nine parameters; the information available now is limited to the joint probabiltiy distribution of only wave heights and wave periods. This is not enough information to construct the joint distribution of nine parameters as required for two-term series of the generalized Bretschneider

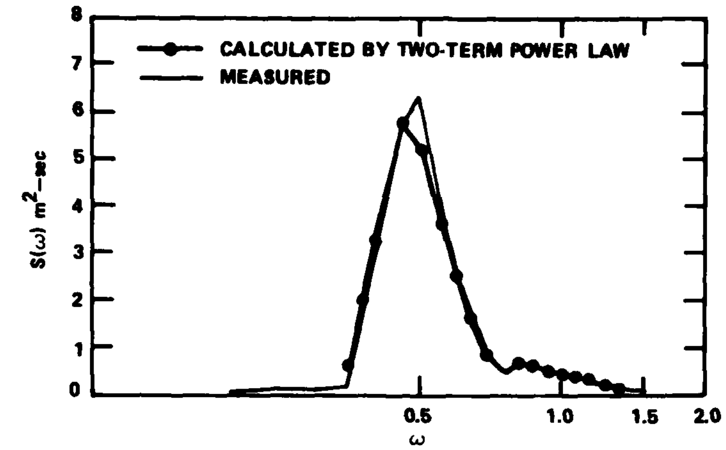

Figure 10a - Calculated for N = 6 and $\ell n\ BT^{-N}$ = -4.6

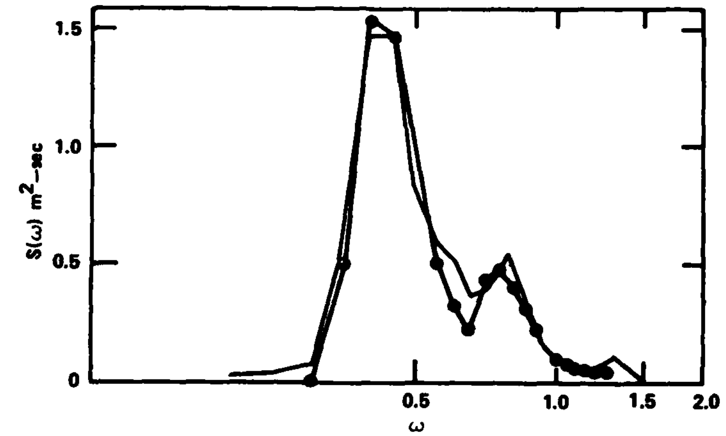

Figure 10b - Calculated for N = 6 and $\ell n\ BT^{-N}$ = -2.4

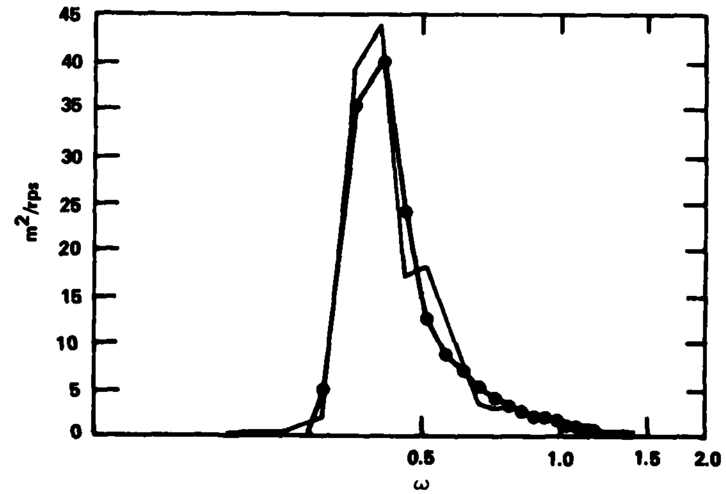

Figure 10c - Calculated for N = 7 and $\ell n\ BT^{-N}$ = -6.714

Figure 10 - Comparison of Measured Wave Spectrum and Its Corresponding Spectrum Calculated from Two-Term Power Law

family. However, because it only involves nine parameters, the determination of the joint distribution will require fewer measurement samples than those required by the previously discussed polynomial and statistical family spectral representations. The joint distribution could have an advantage because it requires fewer measurement samples.

SHORT-TERM PREDICTION AND SIMULATION

With a given spectral representation and its distribution, to predict responses of a ship over her lifetime requires the knowledge of her response statistics with respect to each sea spectrum. For linear phenomena, the probability model of linear ship responses discussed in the Ship Response Analysis section should theoretically enable one to estimate all statistical properties of the responses required for prediction, if the response operators are known. Practically, the calculations can be very involved when all of the sequential events are correlated. To simplify the calculations, short-term prediction techniques have been proposed which consider the members of a sequence of responses to be independent. For example, the magnitude of a response amplitude for the successive cycle is independent of the magnitudes of the amplitudes of the present and the past cycles. By definition, the probability density function for a positive local maximum $p_+(\eta_{max})$ is given by

$$p_+(\eta_{max}) = \frac{p(\eta_{max})}{\int_0^\infty p(\eta_{max})\, d\eta_{max}} = \left(\frac{2}{1+\sqrt{1-\xi^2}}\right) p(\eta_{max})$$

where Equation (6) has been applied.

The probability density function $p(\gamma)$ for the extreme γ of N local positive maximums is given by

$$p(\gamma) = N\left[p_+(\gamma)\left(\int_0^\gamma p_+(x)dx\right)^{N-1}\right]$$

when the local maximums are considered to be independent.

Discussions on the above distribution have been given by Ochi,[18,19] who has used this distribution to determine design load criteria for the bending moments on the MARINER hull. The mathematical criteria he used for the design are that the probability that the extreme bending moment is greater than the design value, is 0.01 when a ship is operating in the most critical sea state. The confidence factor C_T, used by Baitis et al.[20] for predicting the extreme accelerations, is equivalent to the most probable value of γ. They have concluded that the predictions on extreme waves are more influential than the choices of the probability level for predicting the acceleration.

Available measurements of the sea elevations and the motions of a ship indicate that each of these random processes can be correlated for consecutive events. Figure 11 was derived from full-scale trial data for USS BOWEN[21] to demonstrate the correlation between the roll amplitudes for two consecutive cycles. The horizontal line shown in the figure is a

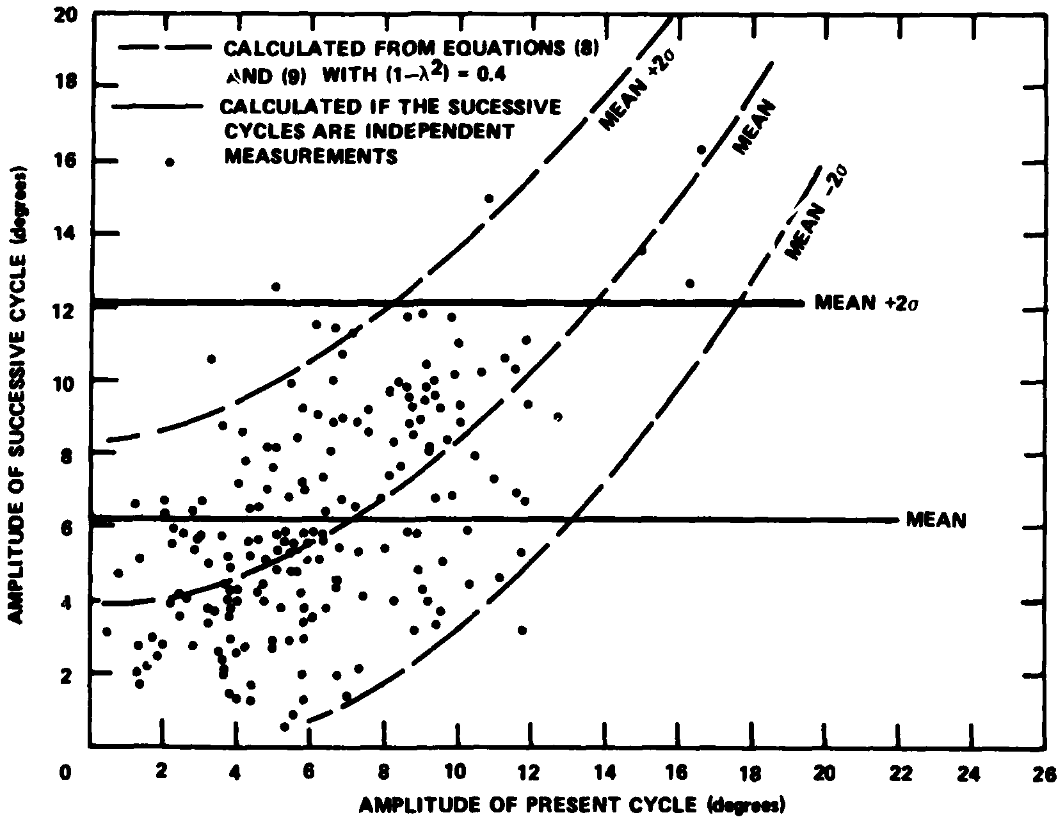

Figure 11 - Variation of Roll Amplitudes with Amplitudes of Its Preceding Cycles

theoretical expected roll amplitude for adjacent successive cycles when the consecutive cycles are assumed to be statistically independent. The measurements have a significant deviation from this theoretically predicted line. The measurements indicate that when the response at present is higher than its averaged value, the response during the next cycle will more likely be higher than that predicted from the theoretical horizontal line. The correlation between consecutive cycles is about 0.6 for the data shown in Figure 11. Now, if one applies the joint Rayleigh distribution[22] to these measurements, the expected roll amplitudes A_2 for the next cycle will be a function of the amplitude A_1 of the present cycle, given by

$$E[A_2^2;A_1] = \left[\frac{1}{\sigma^2(1-\lambda^2)}\right] \exp\left[\frac{-\lambda^2 A_1^2}{2\sigma^2(1-\lambda^2)}\right] \Gamma(3/2)\sqrt{2}$$

$$\times\ [\sigma^2(1-\lambda^2)]^{3/2}\ M\left(3/2,\ 1,\ \frac{\lambda^2 A_1^2}{2\sigma^2(1-\lambda^2)}\right) \tag{8}$$

The variance squares are given by:

$$E[A_2^2;A_1] = 2\sigma^2(1-\lambda^2)\left[\frac{1+\lambda^2 A^2}{2\sigma^2(1-\lambda^2)}\right]$$

and

$$E[(A_2-E[A_2;A_1])^2;A_1] = E[A_2^2;A_1] - (E[A_2;A_1])^2 \tag{9}$$

where A = successive events

σ = variance of A

λ = constant related to the correlation of A_1 and A_2

Γ = gamma function

M = Kumman's confluent hypergeometric function

The curves for the expected value and the variance of A_2 are plotted as dashed lines in Figure 11 with $\lambda^2 = 0.6$. These predictions show better agreement with the measurements than the theoretical predictions which assumed statistical independence.

The correlation function for the consecutive events of the response varies from one type of response to another. In general, nonadjacent successive cycles are weakly correlated, yet the correlation between the cycles can persist for many cycles. Figure 12 shows the correlation function for roll measurements used to construct Figure 11. It is seen that the correlation function oscillates between 0.2 and -0.2 even after five cycles. To predict these weakly-correlated sequential events, one has to formulate a conditional distribution. It can be very involved. In cases where one is only interested in the very immediate future of the motions of

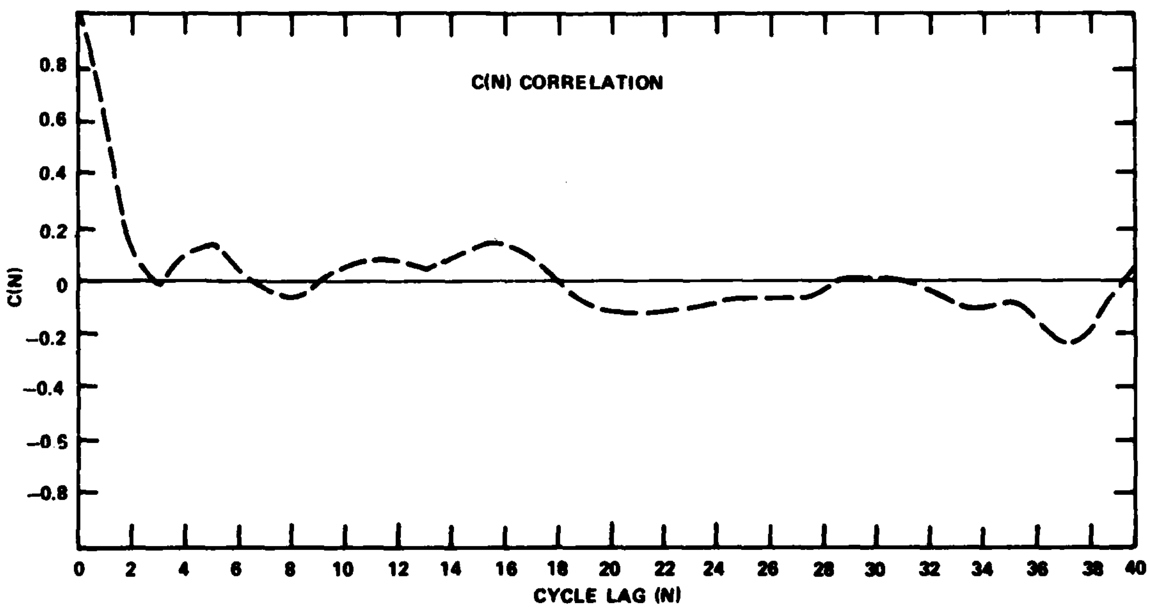

Figure 12 - Correlation of Roll Amplitudes with Respect to the Number of Cycle Lag

a ship, such as for launching a missile, a different prediction model such as an autoregression model could be more useful. The choice of parameters for the autoregression model depends on the physics of the phenomena being predicted. While techniques for predicting the immediate waves and roll motions could be developed, a regression model cannot be used to predict the long-range future with any degree of confidence.

For a nonlinear system, the response may no longer be a Gaussian random process even if the sea elevation is Gaussian. In this case, the formulation of an analytic distribution function for a given event will be quite difficult, if at all possible, using present techniques. One, therefore, needs an alternative to the probability model of linear ship response.

One such hypothetical alternative is to simulate the sea and the ship responses either experimentally in a wave tank or numerically on a computer. In this approach, one generates a series of waves in a towing tank and measures the response of the model to these waves, or one generates a series of sea surfaces on the computer and through numerical techniques determines the response of a theoretical ship. In either case one generates a series of waves and determines a series of responses to these waves. If enough trials are run, this series of responses contains all possible events permitted by the response model under consideration. The probability of occurrence of a certain event can then be computed from this series of responses. In the following, the inherent deficiencies of this approach will be discussed.

In order for the simulation to work, one must first have a good simulation of the ocean surface. The sea elevation was expressed in Equation (1) in terms of the random variable $d\xi(\omega) = d(A(\omega)e^{i\theta(\omega)})$, where A is a given function of ω and θ is the random phase angle, which is assumed to have a uniform distribution. For any given sea spectrum $S(\omega)$, the surface elevation $\eta(t,x)$ varies with the choice of $\theta(\omega)$. The responses corresponding to each choice of θ will differ from each other; however, the statistics obtained from each choice should be approximately the same if the process is simulated for a sufficiently long time.

A discrete form of Equation (1) must be used in the digital computer simulation approach. The following form is typical of the discrete forms which have been used in place of Equation (1).

$$\eta_d(t,x) = \sum_{i=1}^{M} (2S(\omega_i)\Delta\omega_i)^{1/2} \cos\ (\omega_i t - k_i x + \theta_i) \qquad (10)$$

where η_d is the sea elevation obtained from the discretized spectrum. One picks the number of intervals M into which the frequency range is divided and the length of each interval $\Delta\omega_i$, so that the statistics of η_d approximate the statistics of η. In regions where the slopes of the spectrum $S(\omega)$ are steep, the interval lengths $\Delta\omega_i$ must be small; whereas, in regions of gentle slope these lengths can be taken larger. One checks the choice by computing the spectrum $S_d(\omega)$ and comparing the resulting spectrum with $S(\omega)$. If agreement is not obtained, the intervals are refined, and $S_d(\omega)$ is recomputed; this process is repeated until agreement to within a given error is obtained. These calculations are generally done on a digital computer.

Although their spectra might agree to within an extremely small tolerance, there are major differences between the statistics of η_d and those of η. In the first place, η_d is a singular stochastic process; in other words, η_d is determined for all (t,x) once either the M phase angles θ are given or the values of η are given at M distinct points (t_N, x_N). Although the function η_d is a realization of η, for every choice of phase angle $\theta(\omega_1)$, $\theta(\omega_2)$,...., $\theta(\omega_N)$, not every realization of η can be represented in the form of Equation (10).

The extreme values of the function η_d are limited by the choice of

$$\Delta\omega = M_{in}(\Delta\omega_i)$$

$$i=1,\ldots,M$$

whereas the extreme values of η are dependent only on the physical parameters. Verification of the former statement follows from Equation (8):

$$|\eta_d(t,x)| = \left| \sum_{i=1}^{M} (2S(\omega_i)\Delta\omega_i)^{1/2} \cos [\omega_i t - k_i x + \theta(\omega_i)] \right|$$

$$\leq \sum_{i=1}^{M} [(2S(\omega_i))]^{1/2} (\Delta\omega_i)^{1/2}$$

Application of the well-known Cauchy inequality and the definition of $H_{1/3}$ yields

$$|\eta_d(t,x)| \leq \left(\frac{H_{1/3}}{2.83}\right) \left(\frac{\sum_i \Delta\omega_i}{\Delta\omega}\right)^{1/2}$$

where

$$\Delta\omega = M_{in} \, \Delta\omega_i$$

$$i=1,\ldots, M$$

This inequality represents a limitation on max $|\eta_d(t,x)|$; hence, the extreme value of wave amplitude derived from the simulation is restricted to one's choice of numerical bandwidth.

A further deficiency of Equation (10), as a simulation of the sea elevation, is that it is single valued. The sea surface of a wave which is in the process of breaking is multivalued. Hence, Equation (10) cannot be used to study any response phenomena which are related to breaking waves. Perhaps, one can use a Lagrangian model of the sea in simulating breaking waves. Pierson[23] showed that such a model can be multivalued and can account for breaking conditions. The validity of a Lagrangian simulation model should be investigated further.

The waves generated in a wave tank may not follow the same dynamics as the waves in the open sea; the dimensions of the tank, the characteristics of the wave generators and other random environmental effects can all influence the statistical behavior of the waves generated. The differences in the statistics of waves measured in the open sea and of those simulated will result in differences in statistics between responses measured in the open sea and simulated. Since it is not possible to simulate every detail of the statistics of the waves, a proper simulation conserves only the major influential parameters of the waves. At the present time, the wave spectra, the significant wave heights, and the wave periods have been considered as the most influential parameters for studying the response transfer operators. To study the extreme responses from simulations for long-term predictions, one has to include some other influential parameters such as the maximum wave elevation. To conserve this parameter is not easy, as has been discussed previously, and the extreme prediction based on the simulations can be biased.

A comparison of extreme wave elevations measured[21] in the open sea, in a seakeeping basin, and simulated by a digital computer is presented in Table 1 which gives the ratios of maximum wave elevations to $\sqrt{2\times\text{variance}}$. Since the ratios are a monotonically increasing function of the length of each record, comparisons can be made only for comparable record lengths. Table 1A presents the ratios from records with approximately 100 wave cycles and Table 1B is for 150 wave cycles. It seems that the digital computer-simulated waves have a lower maxima in comparison with the measured open sea waves while the maxima generated in the basin are higher than in the open sea. The deviation between ratios calculated for these three kinds of waves is not large. However, the differences can be critical for long-term ship predictions which will be discussed in the next section.

TABLE 1 - COMPARISON OF EXTREME WAVE ELEVATIONS MEASURED IN THE OPEN OCEAN, IN THE MODEL BASIN, AND SIMULATED BY A DIGITAL COMPUTER

TABLE 1A - RECORDS WITH APPROXIMATELY 100 WAVE CYCLES

Digital Simulation	Measured Ocean Data	Tank Simulation
1.79	1.90	2.78
1.89	2.00	
1.90	2.00	
1.98	2.05	
1.99	2.24	
2.03	2.26	
2.10	2.43	
2.19	2.44	
2.28	2.47	
-	2.57	2.78

TABLE 1B - RECORDS WITH APPROXIMATELY 150 WAVE CYCLES

Digital Simulation	Measured Ocean Data	Tank Simulation
1.89	2.11	2.08
2.00	2.13	2.17
2.12	2.17	2.38
2.14	2.20	2.38
2.26	2.20	2.40
2.28	2.23	2.51
2.44	2.25	2.56
-	2.32	2.79
-	2.33	-
-	2.39	-
-	2.48	-
-	2.65	-

LONG-TERM PREDICTION

Theoretically, the probability model for long-term ship responses can be obtained from the ship responses in the anticipated seas. First, one predicts the distribution of sea conditions in which the ship will operate. A short-term response model is formulated for each of the sea conditions; the long-term ship response distribution is calculated using the predicted sea conditions. Finally, the expected occurence of the various response events are calculated; for instance, the probability P that a response R will exceed a given level, for example a, is customarily given by

$$P(R>a) = \sum_{N=1}^{M} P_N(R>a)\ p(\text{sea} = N) \tag{11}$$

The procedures that are presently followed in carrying out these calculations are not correct. In particular, the calculation of the long-term ship response distribution from the distribution formulation, Equation (11), for predicted sea conditions is not correct. Also, the required accuracy of $P(R\leq a;N)$, the probability that the response is less than or equal to a in the N^{th} sea state, is higher than can be obtained from either the existing data or the previously discussed simulations. These two points will be considered in detail.

Let us consider a problem analogous to the problem of long-term ship response. Consider a city which has five different timing ratios on its traffic lights. Let the five ratios between the time the light is green and the time it is red be: 3/2, 1/2, 1/3, 1/4, and 1/11. Suppose on a given trip through the city a driver expects to pass through 20 different traffic lights with 10 percent of the lights in the city being a 3/2 ratio, 20 percent a 1/2 ratio, 30 percent a 1/3 ratio, 35 percent a 1/4 ratio, and 5 percent a 1/11 ratio. What is the probability p that the driver will pass through without having to stop for a light? The answer is straight forward if the lights are statistically independent, i.e.,

$$p = (3/5)^{20\times0.1} (1/3)^{20\times0.2} (1/4)^{20\times0.3} (1/5)^{20\times0.35} (1/12)^{20\times0.05}$$

$$= 1.15 \times 10^{-12}$$

However, if the customary procedure, Equation (11), for predicting the long-term ship response were used, the answer would be

$$p = (3/5\times0.1 + 1/3\times0.2 + 1/4\times0.3 + 1/5\times0.35 + 1/12\times0.05)^{20}$$

$$= 6.5 \times 10^{-12}$$

The answers differ by a factor of 5.7.

In this problem, the percentages of the traffic lights correspond to the predicted probability distribution of the sea states. The timing ratios correspond to the ratio of times when the response is significant and when the response is negligible. From the example one sees how the commonly used method for predicting long-term ship responses can over-estimate. The correct formulation should be

$$P(R_{max} \le a) = \prod_{N=1}^{M} P(R \le a;\ \text{sea} = N;\ \text{response parameters})$$

and

$$p(R_{max} = a) = \frac{dP(R_{max} \le a)}{da}$$

Applying this formula to compute the probability that $R_{max} \le a$, one has

$$P(R_{max} \le a) = \prod_{N} [1-\exp(-a^2/2\sigma_N^2)]^{M_N} \qquad (12)$$

and

$$p(R_{max}=a) = \sum_{\ell} \left\{ M_\ell \frac{a}{2\sigma_\ell} \exp\left(\frac{-a^2}{2\sigma_\ell^2}\right) \left[1-\exp\left(\frac{-a^2}{2\sigma_\ell^2}\right)\right]^{M_\ell - 1} \right.$$

$$\left. \prod_{N \neq \ell} \left[1-\exp\left(\frac{-a^2}{2\sigma_N^2}\right)^{M_N}\right] \right\} \qquad (13)$$

where the short-term uncorrelated Rayleigh distribution is applied for demonstation purposes. As discussed previously, the assumption that events are uncorrelated could introduce significant error.

For a typical long-term prediction, the order of M for low sea states is greater than 10^7. For high sea states, it is greater than 10^3. The probability distribution function for extremes, Equation (13), is then of the form $\sum_{\ell} \left[(M_\ell \times b_p) \prod_N (C_N)^{M_N}\right]$ with $b_N << 1$, $(1-C_N) << 1$ and $10^3 < M_N < 10^8$. Since $(0.999/0.999999)^{10^3}$ is equal to e^{-1} and $(0.999/0.999999)^{10^7}$ is e^{-9995} a small error in the estimated probability C_n can result in a large error in the probability for extremes. To evaluate $p(R_{max}=a)$ accurately, it is required that the probability distribution for the response at low sea state be accurate to 10^{-7}. This is much more accuracy than the accuracy that can be obtained from the present techniques. To predict the probability of the occurrence of larger responses is thus not practical and the many methods for predicting the extreme value of ship response are largely academic.

To avoid the difficulties of constructing the probabilities accurately to 10^{-7} at low sea states, Ochi[19] considers only the extremes of a response for the most severe sea conditions; the number of cycles M is on the order of 10^4 for severe seas. The probability of R_{max} being greater than a given value a can be calculated from Equation (12). It is $P(R_{max} \geq a) = 1 - [P(R \leq a)]^M$ with $M = 0(10^4)$. If one chooses a design value such that $P(R_{max} \geq a) = 0.01$, one has

$$P(R \leq a) = (1-0.01)^{10^{-4}} = (0.99)^{10^{-4}} = 1 - 1.005 \times 10^{-6}$$

or (14)

$$P(R>a) \simeq 10^{-6}$$

where $M = 10^4$ has been applied in the above calculation.

It is seen from Equation (14) that the extreme design value a has a probability level of 10^{-6}. Because of nonlinear effects and physical constraints, it is doubtful that the present response distribution function can accurately estimate probability to this level. An error factor of two or more can easily be obtained from the approximated distribution for R.

The pitch motions of USS BOWEN[21] operating at a State 2 sea have been used to study the distribution of extreme values. Table 2 presents the ratios of positive maximum to $\sqrt{2\times\text{variance}}$ for 16 different runs. The ratios γ_m, calculated from the measurements, are presented in column two. The corresponding theoretical values γ_t of $p(R_{max} \geq \gamma_t) = 0.01$, calculated from Equation (17) of Reference 19 are given in column four for comparison. Also given in the table are the theoretically calculated most probable values for R_{max}. Since the number of observations is limited, to reject or accept the theoretical distribution has to be studied from probability theory. The measurements indicate that at twelve out of sixteen runs the measured ratios are greater than their corresponding theoretically calculated most probable values. Theoretically, one would expect $(1-e^{-1}) \times 16$ runs, i.e., 11 runs. The comparison is good. However, if one compares the measured ratios to those of low probability level, one has little confidence of the theoretical distribution for R_{max}; the measured ratio for run 12 is 3.15 which is about 3 percent larger than the theoretical value γ_t of 3.05. This sample is thus falling in the region of probability level less than 0.01, theoretically. The probability that one out of sixteen runs falls in the same probability level as run 12 is less than 7 percent.

TABLE 2 - COMPARISON OF MEASURED EXTREME PITCH ANGLES AND CORRESPONDING THEORETICAL ESTIMATIONS

Run	Measured Max. Positive/$\sqrt{2}\sigma$	Theoretical Most probable Max./$\sqrt{2}\sigma$ (γ_m)	Max. with 0.01 level/$\sqrt{2}\sigma$ (σ_t)
1	2.36	2.16	3.04
2	2.46	2.21	3.08
3	2.58	2.20	3.08
4	2.64	2.27	3.13
5	2.78	2.23	3.10
6	2.30	2.43	3.24
7	$\boxed{3.04}$	2.33	3.1[illegible]
8	2.46	2.19	3.07
9	2.30	2.34	3.18
10	2.08	2.25	3.10
11	2.36	2.38	3.20
12	$\boxed{3.15}$	2.17	3.05
13	2.82	2.40	3.22
14	2.26	2.15	3.03
15	2.34	2.15	3.04
16	2.86	2.32	3.16

The measured ratio for run 7 is 3.04, which is less than the corresponding γ_t. Thus, this sample is outside of the probability level less than 0.01 region. However, it can be demonstrated from formulas[19] that this sample is in the region of probability level of 1/44. Hence, this set of samples, results in 2 out of 16 samples falling in the probability level of 1/44. The probability of more than 1 out of 16 samples falling in the 1/44 region has been estimated to be less than 5 percent. If one takes a 95 percent confidence interval to evaluate the theoretical distribution, the distribution is rejected. Since the theoretical distribution is rejected when one considers a very small probability level, one will have little confidence in design criteria based on theoretical consideration of that probability level.

Now if one returns to Figure 8, it is seen that the stress predicted by the use of different spectral families is within a factor of two. Thus, from the long-term extreme value prediction point of view, the errors caused by the insufficiency of the spectral representation are not greater than the errors introduced by other approximations in present prediction schemes.

CONCLUDING REMARKS

Since the work of St. Denis and Pierson[24] and of Cartwright and Longuet-Higgins,[3] one has been able to relate the general statistical features of the response of a ship to the sea spectrum. Comparison of the statistics of the measured response with those calculated from theoretical formulas has indicated that the theoretical calculations are quite good from an engineering point of view; e.g., the ratio of significant response to the variance of the response is close to the theoretical predicted value of four. As a result, the use of theoretical formulations has been a common practice.

In modern ship design, rough estimates of the ship response may not always be adequate; for example, differences of 10 percent in predicted stress levels could represent substantial increases in hull structure costs, and a 10 percent increase of roll angle could classify a design as acceptable or unacceptable. Thus, in some cases, the probability distribution for the ship response has to be formulated in order to optimize the design. To obtain a higher accuracy for the prediction, one needs an accurate representation of the sea, an accurate distribution function for the sea, and an accurate transfer function between the sea and the responses.

The present spectral representations are not adequate for optimal design. In general, the P-M fully developed sea spectra should only be used for fully developed seas. There are cases in which the energy of sea spectra in a nonfully developed sea are not subordinate to the P-M spectra; at other times the P-M spectra results in overdesign. There are significant differences at the higher frequencies between spectra calculated by FFT and those calculated by the correlation method. Errors at the higher frequency end of the sea spectra result in errors in the high frequency response of the ship, in the long-term response predictions, and in the statistical properties dependent on the higher order moments of the sea spectra. Hoffman[16] has already pointed out the shortcomings of the Bretschneider spectral family. However, Hoffman's proposed statistical families of spectra are only for station I. Spectra for other areas of the ocean still remain to be determined.

Because of the persistent wave energy at high frequencies, the power law of the general Bretschneider form is unable to fit the measured spectra for both the energy contain region and the high frequency region. Separate treatments for these regions are necessary if one is interested in a mathematical representation. The use of two-term series of the general Bretschneider form gives good results, but involves nine parameters which have to be determined from measurements. The determination of these nine parameters for the purpose of global ship response predictions is not economical. The Gospodnetic[17] and other polynomial representations, which reduce the dependence of spectral density at one frequency on that of the other frequencies, show improvement over the Bretschneider spectral family, yet they too cannot be applied globally because of the oscillations in the series coefficients.

To correct the high frequency region, one may also represent the spectrum by $S(\omega) = S_B(\omega) + f(W, H_{1/3}, T)$ where f is an unknown function of wind speed W and wave height $H_{1/3}$, $S_B(\omega)$ is the general Bretschneider spectrum, and T is the characteristic wave period. This representation could reduce the unknown parameters, which are nine for two-term series of Bretschneider forms, to a smaller number. However, the applicability of this form to the measured spectrum needs to be studied.

The present short-term prediction scheme considers each cycle of response to be independent of the past cycles. However, there are responses which have considerable correlation between successive cycles. A knowledge of these correlations can improve response predictions especially for predicting the occurrence of severe responses which are, in general, the result of several successive critical waves.

The lifetime of a ship is generally considered to be twenty years. The period of one cycle of a ship response is on the order of 10 seconds. Thus, over the lifetime of a ship, she undergoes 10^8 response cycles. To estimate the probability distribution of extreme values of response with such a large population requires a very accurate distribution function for each cycle of the response. For example, the expected extreme value of 10^8 samples drawn from a Gaussian distribution can be much higher than from the physical distribution which is a truncated Gaussian distribution. The present knowledge of sea spectra, the distributions of sea states, the transfer functions between the sea and ship response, and the short-term distribution of the response are not accurate enough to allow one to predict the extreme behavior of a ship within a useful confidence level.

Knowing the transfer operator and the ocean wave spectral distribution is not sufficient for predicting the extreme behavior of a ship over her lifetime. A knowledge of the physical constraints on the extreme of a response, and a better understanding of the response distribution at its extreme end are necessary for better predictions. Since, by definition, the occurrence of extreme values is rare, much data are required to emphasize the extreme end of the response distribution. To collect data economically, one needs to have a prior knowledge of when the extreme value is most likely to occur. This will require a better physical understanding of the phenomena, as well as engineer's intuition and experience. Mathematics cannot be used to solve the extreme value problem without the aid of physics.

ACKNOWLEDGMENTS

The author sincerely wishes to thank Justin McCarthy for his help and encouragement. She also expresses her thanks to Dr. Thomas Langan for reviewing the manuscript.

REFERENCES

1. Parzen, E., "Stochastic Processes," Holden Day (1962).

2. Box, G.P. and G.M. Jenkins, "Time Series Analysis," Holden Day (1970).

3. Cartwright, D.E. and M.S. Longuet-Higgins, "The Statistical Distribution of the Maxima of a Random Function," Proc. Roy. Soc. of London, Series A, Vol. 237 (1956).

4. Longuet-Higgins, M.S., "On the Statistical Distribution of the Height of Sea Waves," Journal Marine Research, Vol. 11, No. 3 (1952).

5. Longuet-Higgins, M.S., "The Distribution of Intervals Between Zeros of Stationary Random Function," Phil. Trans. Roy. Soc. of London, Series A, Vol. 254 (1962).

6. Phillips, O.M., "The Dynamics of the Upper Ocean," Cambridge University Press, London (1966).

7. Kitaigorodskii, S.A., "Application of the Theory of Similarity to the Analysis of Wind Generated Wave Motion as a Stochastic Process," Izv. Akad. Nauk SSSR, Ser. Geophys, Vol. 1 (1961).

8. Bretschneider, C.L., "Wave Variability and Wave Spectra for Wind Generated Gravity Waves," Beach Erosion Board Technical Memorandum 118 (1959).

9. "Ocean Wave Spectra," National Academy of Science (1963).

10. Barnett, T.P., "Observation of Wind Wave Generation and Dissipation in the North Sea: Implication from the Off-Shore Industry," Offshore Technology Conferences (1972).

11. Pierson, W.J. and L. Moskowitz, "A Proposed Spectral Form for Fully Developed Wind Seas Based on the Similarity Theory of S.A. Kitaigorodskii," Journal of Geophys. Res., Vol. 69, No. 24 (1964).

12. Cummins, W.E., "A Proposal on the Use of Multiparameter Standard Spectra," Proceedings of the 12th International Towing Tank Conference (1969).

13. Draper, L. and E.M. Squire, "Waves at Ocean Weather Ship Station India," Transaction, Royal Inst. Nav. Arch., Vol. 109 (1967).

14. Moskowitz, L. et al., "Wave Spectra Estimated from Wave Records Obtained by the OWS WEATHER EXPLORER and the OWS WEATEHR REPORTER, I, II, and III," School of Engineering and Science, New York University (1962, 1963).

15. Miles, M., "Wave Spectra Estimated from a Stratified Sample of 323 North Atlantic Wave Records," NRC Report LTR-SH-118 (1971).

16. Hoffman, D., "Analysis of Measured and Calculated Spectra," International Symposium on the Dynamics of Marine Vehicle and Structure in Waves, University College, London (1974).

17. Gospodnetic, D. and M.D. Miles, "Some Aspects of the Averaged Shape of Wave Spectra at Station India," International Symposium on the Dynamics of Marine Vehicle and Structure in Waves, University College, London (1974).

18. Ochi, M.K., "On Prediction of Extreme Values," J. Ship Res., Vol. 17 (1973).

19. Ochi, M.K. and L.E. Miller, "Prediction of Extreme Ship Response in Rough Seas in the North Atlantic," International Symposium on the Dynamics of Marine Vehicle and Structure in Waves, University College, London (1974).

20. Baitis, A.E. et al., "Prediction of Lifetime Extreme Accelerations for Design of LNG Cargo Tanks," Department of Transportation, United States Coast Guard Report CG-D-89-74 (1974).

21. Baitis, A.E., "Trial Results on Ship Motions and Their Influence on LAMPS Helicopter Operations for the DE 1052 Class BOWEN (DE 1079)," NSRDC Ship Performance Department Evaluation Report 556-H-01 (1974).

22. Miller, K.S. et al., "Generalized Rayleigh Process," Quart. Appl. Math., Vol. 16, No. 2 (1958).

23. Pierson, W.J., "Models of Random Seas Based on the Lagrangian Equations of Motion," Tech. Report 285(03), College of Engineering, Res. Div., New York University (1961).

24. St. Denis, M. and W.J. Pierson, "On the Motions of Ships in Confused Seas," Trans. Soc. Nav. Arch. and Marine Eng., Vol 61 (1953).

25. Bales, S.L. and W.E. Cummins, "Wave Data Requirements for Ship Design and Operation," Ocean Wave Climate, edited by Marshall D. Earle and Alexander Malahoff, Plenum Publishing Corp., pp. 93-107 (1979).

26. "Seakeeping in the Ship Design Process," Report of the Seakeeping Workshop, U.S. Naval Academy, 11-13 June 1975, prepared jointly by the Naval Ship Engineering Center and Naval Ship Research and Development Center (Jul 1975).

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