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A LANCHESTER-TYPE ANALYSIS
OF THE
COVERING FORCE COMMITMENT

by

Edwin Thomas Carlson

March 1980

Thesis Advisor: J. G. Taylor

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The key elements of combat dynamics are identified and modelled.
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A Lanchester-Type Analysis
of
The Covering Force Commitment

by

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Submitted in partial fulfillment of
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I. A BASIS FOR ANALYSIS

A. INTRODUCTION

The current U.S. Army doctrine for defense centers around the word ACTIVE. In an active defense, forces engage the enemy from battle positions that make maximum use of advantageous terrain and allow forces to attrite the enemy at the maximum effective range of U.S. weapon systems. When appropriate, units will move from battle position to battle position, so they can continually exploit the advantage of the defender on prepared terrain, make use of obstacles (natural and manmade), and engage the enemy in a series of ambushes at long range (Ref. 1).

The defensive area is organized as depicted in Figure 1 (Ref. 2). It consists of the Covering Force Area (CFA), the Main Battle Area (MBA), and the Rear Area. (See Appendix A for definitions of terms used in this thesis.)

The purpose of this thesis is to analyze the covering force; its commitment and contribution to the total defensive effort. By building some simple Lanchester-type differential combat models, important questions such as the identification of key combat dynamics, optimal force commitment, and concentration will be addressed.
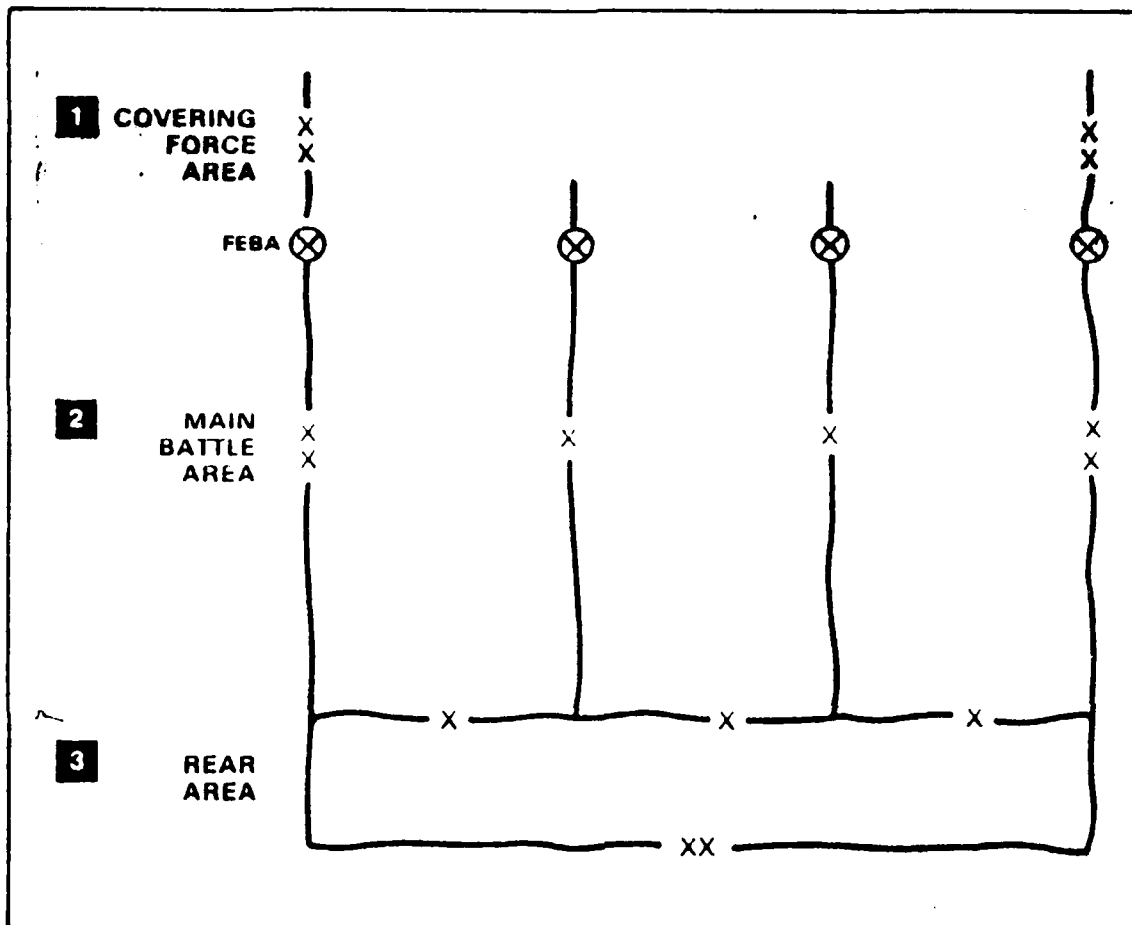
FIGURE 1

THE DEFENSIVE AREA (Ref. 2)

B. CONCENTRATION AND LANCHESTER'S ORIGINAL WORK

In 1914 F. W. Lanchester, in an attempt to quantitatively justify the principle of concentration of forces under modern conditions, formulated the classic mathematical equations that describe combat between two opposing homogeneous forces. His hypothesis was that under "modern conditions" firepower can be concentrated on opposing forces so that each side's casualty rate is proportional to the number of enemy firers. Under these conditions, it may be shown that a victorious side can significantly reduce its casualties by initially committing as many forces as possible. Conversely, a commander dilutes his combat power by committing his forces "piecemeal".

"Modern conditions" are those that stem from the technology that has produced weapons that can accurately deliver a projectile on a long-range target, thus allowing the concentration of firepower by weapons widely separated on the battlefield. Modern, in this context, is opposed to "ancient conditions" of combat whereby forces engaged one another in hand to hand combat using edged weapons. Under ancient conditions the capability to concentrate weapon effect on a single target from widely located positions on the battlefield does not exist.

Thus, Lanchester hypothesized that under "modern conditions" a side's casualty rate would be proportional to the number of enemy combatants. Mathematically, this may be expressed as:

$$\frac{dB}{dt} = -r\,R$$

$$\frac{dR}{dt} = -b\,B$$
(1.1)

where t denotes battle time (t=0, is the start of the battle) and r and b are constants, called Lanchester attrition-rate coefficients, that represent the effectiveness of each side's fire.

The instantaneous casualty ratio is obtained from equations (1.1)

$$\frac{dB}{dR} = \frac{rR}{bB} \ , $$
(1.2)

whence integration yields Lanchester's familiar Square Law:

$$b(B_o^2 - B^2) = r(R_o^2 - R^2)$$
(1.3)

The advantages of concentration are apparent from this state equation, since the effective strength of one side is proportional to the first power of its efficiency (its attrition coefficient) but proportional to the square of the number of combatants entering the engagement. Two opposing forces are then equally matched in a fight to the finish when the exchange rate, $\frac{r}{b}$, is equal to the square of the ratio of the number of combatants.

$$\frac{r}{b} = \frac{B_o^2}{R_o^2}$$

Consequently, it is more profitable for a victorious side to increase the number of combatants in an engagement than it

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is to increase (by the same amount) the exchange rate (by
increasing the effectiveness of the individual weapons).
Said in another way; a tactical or strategic use of concen-
tration may adequately counterbalance any moderate advantage
in weapon efficiency.

C. BATTLE TERMINATION

Without further specification, combat attrition will
follow the above schedule in an engagement until one side or
the other is annihilated to terminate the engagement. But
such engagements that continue until one side is wiped out
are rare. Historically, battles end when one side or the
other reaches a force level below which it is no longer able
to carry out its mission. At that point in the battle the
losing side either withdraws from the field of battle or
surrenders.

The possible battle outcomes are discussed by R. L.
Helmbold (Ref. 3). They are:

1) One side has been annihilated, with its opponent
in undisputed control of the battlefield.

2) One side surrenders and submits to the will of
its opponent, who thereby gains control of the battlefield.

3) Neither side surrenders or is annihilated, but one
of them has disengaged and either has withdrawn, or is in
the process of doing so, leaving its opponent in control
of the battlefield.

4) Neither side has surrendered or been annihilated, but both sides have disengaged and are in the process of withdrawing from the battlefield. Mutual withdrawal leaves control of the battlefield uncertain, with no certain victor.

For simplicity Taylor (Ref. 4) and Helmbold (Ref. 5), consider control of the battlefield as the criterion for victory. As implied above, the opponent who controls the battlefield is the side that retains its combat effectiveness while the other reaches a force level that renders it combat ineffective.

In modelling the battle termination phenomenon one considers a combat force to have reached its "breakpoint" when it reaches that force level at which it loses its combat effectiveness. At some percentage of its original strength a unit will lose its ability to influence the action and will abandon its mission, forcing it to "break off" the engagement, leaving its opponent in possession of the field of battle (Ref. 6). The important question here (addressed by Taylor, Helmbold, etc.) is the determination of the significant factors upon which battle termination depends. Many can be considered, but few can actually be quantified. Those factors that have been widely accepted as major contributors to a unit's breakpoint are:

1) Type of unit

2) Size of unit

3) Mission of the unit (attack or defend)

These factors have been incorporated and expressed as the Breakpoint Hypothesis (Ref. 7):

"Breakpoint Hypothesis": A unit will cease to be an effective fighting force in combat when a given force level is reached. When this event happens, the unit loses its ability to perform its mission and will "break-off" the engagement. This force level breakpoint depends on the type of unit, its size and mission."

Then, the force level at which a unit ceases to be combat effective is that unit's breakpoint force level (or simply breakpoint). The major assumption here is that the first unit that reaches its breakpoint loses the engagement. Using common notation (Ref. 8); if two homogeneous forces enter the field of combat against one another, Red (R) versus Blue (B), then Red's breakpoint is denoted as R_{Bp} and Blue's breakpoint is similarly defined as B_{Bp}.

As an example, a Blue victory may be expressed mathematically as:

$$R_f = R_{Bp}$$

B wins when $\quad B_f > B_{Bp}$ $\hfill (1.4)$

$$B(t) > B_{Bp} \text{ and } R(t) > R_{Bp} \text{ for } 0 < t \leq t_f$$

where the f subscript denotes the final values of the Red force, the Blue force and time at the end of the battle, and B(t) and R(t) denote the Red and Blue force levels at some time, t , during the action. It is also convenient to express a unit's breakpoint in terms of its initial strength, i.e.:

$$R_{Bp} = f_{Bp}^{R} \cdot R_{o}$$

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where Red's breakpoint force level is equal to some fraction $(0 < f_{Bp}^{R} < 1.0)$ of Red's initial force level (R_o). If the Red commander chose to fight until the last man, the battle termination criteria would then revert to the simple Lanchester outcome where Red fought to annihilation. Red's breakpoint force level would then be $R_{Bp} = f_{Bp}^{R} \cdot R_o$ where $f_{Bp}^{R} = 0$ and $R_{Bp} = 0$.

The above breakpoint hypotheses implies that f_{Bp}^{R} and f_{Bp}^{B} depend on the unit's type, size and mission. Common values attributed to these breakpoint fractions for company-sized units are:

$$f_{Bp}^{B} = 0.5 \text{ for a defending force and}$$

$$f_{Bp}^{R} = 0.7 \text{ for an attacking force.}$$

An interesting result of this model of battle termination is that, depending on a unit's breakpoint, the victor could be that force that has a smaller number of forces remaining on the battlefield. Reaching one's breakpoint means that one's mission must be abandoned. Thus, an attacker may give up his attack because he is not strong enough to prosecute it, although he may be numerically stronger than the defender.

For example, imagine Red attacking Blue with a force of 200 $(R_o = 200)$, and Blue is defending and numbers 100 $(B_o = 100)$. Then if $f_{Bp}^{R} = 0.7$ and $f_{Bp}^{B} = 0.5$, the battle could end with Blue victorious with $B_f = 75$ and Red the loser with $R_f = R_{Bp} = 140$. Red has reached its breakpoint $R_{Bp} \cdot R_o = (.5)(100) = 50$, since $B_f = 75 > B_{Bp} = 50$, Blue has retained its combat

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effectiveness and has achieved victory with fewer forces remaining at the end of the battle. This result can only be achieved, however, when the smaller Blue force has a combat advantage over the larger Red force. This combat advantage exists in the model when Blue's attrition coefficient exceeds Red's attrition coefficient, i.e., $b > r$ to such a degree that Blue attrites Red at a faster rate than Red attrites Blue.

D. THE USE OF A SIMPLE METHOD

Current models of combat vary in level of detail from simple, deterministic, force-on-force Lanchester-type analytical models that represent all the complexity of combat in a few differential equations, to high-resolution Monte-Carlo computer-based models that simulate in minute detail the interaction of such combat variables as terrain, weapon characteristics and individual movement. All models are abstractions from reality. For complex man-machine systems (such as military organization) there is always a trade-off between the levels of detail, convenience and accessibility, and operational realism. Analysis in this paper choses simplicity and ease of computation over complexity and high resolution.

A simple model (here considered to be an analytical model) can be constructed on the basis of several simple assumptions. Assumptions, such as "modern conditions" of warfare, may remove the model from reality, but allow the modeller to construct a logical structure that can be enriched, and from

which important insights can be drawn. An example can be easily found in Lanchester's aimed fire law. The logical structure was that modern weapons could be brought to bear on enemy targets from widely located positions. So that a force's attrition rate over time was directly proportional to the number of opposing forces and the combat effectiveness of each member of the opposing force. From this logical structure, a mathematical model was formed that, when manipulated, showed that concentration of forces in the battle achieved better results than when forces were not concentrated (Ref. 9).

Probably the most important use of the simple model is obtaining insights into the dynamics of combat (Ref. 10). The simple model allows the analyst to better understand the basic nature of combat dynamics, and to "...hopefully perceive some significant interrelationships that are difficult to discern in more complicated models." (Ref. 11).

The goal of the simple combat model is then one of exploring the dynamics, discovering the significant variables, and evaluating the sensitivity of the parameters and assumptions within a logical framework.

In the following analysis of the covering force, such a simple model is used to gain the insights, interrelationships, and sensitivity previously mentioned. This analysis cannot, nor is meant to, give definitive answers to specific problems of covering force commitment.

II. THE DYNAMICS OF THE DEFENSIVE BATTLE

A. INTRODUCTION

The intent of this chapter is to present some of the
important dynamics of the defensive battle, the relationship
between the MBA and the CFA, the approach in examining the
contribution of the covering force, and the measures of
effectiveness that will be used in later chapters.

B. THE DEFENSIVE BATTLE

The covering force area (denoted as CFA) was defined in
Chapter I, and is the area forward of the FEBA in which the
covering force is committed. For the sake of the analysis
in this thesis, the defensive force (Blue) has an initial
force size denoted as B_o. If some percentage (p • 100%, where
$0 < p < 1$) of the initial force, B_o, is committed to the CFA,
this covering force has an initial force size of B_o • p.

The mission of the covering force, as expressed in FM 100-5,
Operations is:

> "...to fight in a specified area for a specified
> period of time...to find ways not only to deceive the
> enemy as to the MBA dispositions, but also to trade
> space for time - time for the MBA force to get set to
> defend. Therefore, the covering force mission may be
> a delay, which could be terrain - specific, time-specific,
> or both."

Dynamically, the covering force would fight from key
terrain to key terrain (battle position to battle position)
in keeping with the concept of the active defense. It would
attempt to maintain continuous contact with the enemy, slow

his rate of advance, force him to reveal his main attack, inflict maximum enemy attrition, and provide continuous enemy intelligence to the MBA commander. The covering force in the delay will trade space for time, usually at the least risk to its survival. However, it may also be required to "trade risk for time" (Ref. 12). In order to gain enough time, the covering force may have to fight harder and longer on each battle position, thus risking greater losses. Sufficient resistance must be presented to the enemy to force him to commit his main attack, but the covering force commander must not allow himself to become so decisively engaged as to risk the destruction of his force.

On order, the covering force will conduct a rearward passage of lines through the FEBA and hand the battle over to the MBA forces. The covering force can them assume several missions, the most probable being to participate in the MBA battle after resupply, consolidation and reorganization.

The organization of the covering force will be predominantly armored cavalry and air cavalry, reinforced with sufficient tank, antitank, mechanized infantry, field artillery, air defense artillery, engineer and attack helicopter forces to accomplish the mission (Ref. 13).

It is submitted that the most critical tasks of the covering force in support of the defensive effort are the acquisition of enemy intelligence and the gaining of time. But these are not independent contributions, since one without the other

is of little use to the forces in the MBA. To illustrate this point, suppose that the covering force was able to delay the enemy attack for the required amount of time to enable the MBA forces to concentrate at the critical areas, but the covering force was unable to force the enemy into revealing his main attack. In this scenario, the MBA forces could react to enemy intelligence but none was available. Or, for another example, suppose that the covering force was able to correctly identify the avenue of advance of the main enemy attack, but was unable to slow the enemy advance sufficiently to gain the required amount of time needed to maneuver MBA forces to counter the identified thrust. Again, with only one element of the crucial two, the MBA force could not attain the "combat advantage" required to defeat the enemy.

The term "combat advantage" implies that the commitment of some percentage of the defensive forces to the CFA must provide an advantage to the forces in the MBA. The nature of this advantage should be explored.

During the course of the action in the CFA, the MBA forces will continue to improve their defensive posture by upgrading firing positions, implacing obstacles, and by moving logistic support forward. These improvements take time; time provided by the tenacious action of the covering force. However, if time is supplemented by accurate enemy intelligence, then not only can the MBA forces improve their combat effectiveness, but they can also redistribute themselves in order to

concentrate their forces at the critical areas and use econ-
omy of force tactics in areas of secondary importance.

The covering force can provide the crucial elements of
enemy intelligence and time by exploiting the terrain, en-
gaging the enemy in ambush fashion at the maximum range of
its weapons, and maintaining a mobility advantage over the
attacker through the use of obstacles and superior knowledge
and use of terrain.

The question of how long the covering force battle will
last varies from scenario to scenario. Col. T. N. Dupuy
(U.S. Army, Retired) in his book Numbers, Predictions and
War does a campaign analysis of the defensive battle in
Central Europe, in which he addresses this question (Ref. 14).
He postulates that the covering force battle will last about
24 hours depending on whether or not the enemy main attack is
in the sector being analyzed in combination with whether or
not the enemy has achieved surprise in this sector. (In
later chapters, the battle duration time of 24 hours will be
used as a means of modelling the intensity of combat in the
CFA).

C. EVALUATING THE COVERING FORCE

For several important reasons, the basic assumption will
be made that the CFA battle and the MBA battle are two distinct
battles. It must be acknowledged, however, that this concep-
tualization may not always agree with real-world combat opera-
tions for several reasons. Firstly, it is very possible that

the covering force may be able to defeat the enemy prior to his reaching the FEBA. For another, the fluidity of the battlefield suggests the possibility of the enemy reaching the FEBA by penetrating the covering force and being engaged by both the covering force and MBA forces simultaneously. However, as will be seen in subsequent chapters, the model building process will depend greatly on the assumption of two distinct battles. The relaxation of this assumption would put the model building process into such a high level of resolution that certain amounts of transparency would be lost.

Before proceeding to the modelling efforts, a discussion of the assessment of covering force effectiveness seems in order. Initially, it was considered that focusing on the CFA battle and its outcome would offer sufficient insights into the effectiveness of the covering force in any particular scenario. It was thought that the winner of the CFA battle, the duration of the CFA battle, the number of enemy and friendly casualties would give a sufficient basis upon which to draw conclusions concerning optimum covering force commitment and tactics. However, one must give pause, and consider that even though the defensive effort has been partitioned into two separate battles, the final outcome of the final battle is the only true indicator of whether or not any strategy, tactic, or force commitment in the CFA was effective. The criteria for assessing the combat

effectiveness of any component must rest on the overall outcome of the total defensive battle. It is submitted that the only appropriate way to analyze the contributions of the covering force is to analyze how any change in covering force dynamics affects the final outcome and not merely the outcome of the CFA battle.

A good example of how a CFA battle analysis that focuses on the wrong system-effectiveness criterion leads to erroneous conclusions can be seen when the analyst focuses only upon the win or lose criteria in the outcome of the CFA battle. If the analyst believes that a Blue win in the covering force battle provides adequate proof that the covering force was effective, he may overlook the fact that the MBA forces were deprived of forces to make the covering force of sufficient size to win the CFA battle. He may miss the fact that the depleted MBA forces could not concentrate sufficiently to defeat an enemy penetration. The possibility exists that the enemy, by a second echelon attack could penetrate to the flimsy forces of the MBA and continue relatively unopposed to his objective in the rear area. In this case the enemy could achieve victory in the final outcome of the defensive battle, regardless of suffering defeat at the hands of the covering force.

In contrast, if the covering force loses the CFA battle, but can sustain its action long enough to contribute time and enemy information, the enemy may be defeated in the MBA and

thus fail to reach his objective. These two simple examples
illustrate the fundamental point that the covering force's
contribution to the total defensive battle can only be eval-
uated in the final outcome. Further analysis will depend
upon the acceptance of this key premise.

The measures of effectiveness that are to be used in the
analysis presented in future chapters must reflect the con-
tribution of the covering force to the total force effort.
These measures of effectiveness are:

1) Defender (Blue) casualties,

2) Attacker (Red) casualties,

3) The loss ratio, Blue over Red casualties,

4) The loss difference, Blue minus Red casualties,

5) Winner of the MBA battle.

The above measures of effectiveness are preferred at
their minimum value, except for Red casualties. The winner
of the MBA battle is strictly nominal data and is the sort
of measure of effectiveness associated with a Yes-No or 0-1
outcome. This measure of effectiveness is optimum where Blue
wins the MBA battle, thus attaining the final victory in the
total defensive battle.

III. AN APPLICATION OF THE GAMOW-ZIMMERMAN MODEL

A. INTRODUCTION

In recent times the U.S. Army analysis community has
viewed the covering force and its contribution to the defen-
sive battle, from the perspective of defense in Central Europe.

This perspective cannot be reproached since the political realities and consequences of defeat in that area provide severe repercussions to national security and world peace. However, Army defensive doctrine is not limited in applicability to that particular scenario. In this chapter a more general scenario is presented so that an initial analysis of the covering force and its dynamics can be made with a more transparent analytical model than the models presented in later chapters.

B. BASIC ASSUMPTIONS (Ref. 15)

In order to model the general defensive formation consisting of a CFA and MBA, the following assumptions are made.

Consider that the troops in the CFA are distributed along a line, and that the MBA forces are distributed along a second line, that is designated as the FEBA. Imagine that all the Red forces face all the Blue forces along an infinite homogeneous front (see fig. 2). Further suppose that the relative strength of Red and Blue are so balanced that, were the Red forces to attack within any sector of width, K meters, without further concentration, then the resulting battle would be a draw, and that at the end of the assault both sides would have been destroyed within this attack sector. This situation is known as parity, and is represented as $\frac{R^2}{B^2} = \frac{b}{r}$ (Ref. 16). This must mean that Red forces outnumber the Blue forces by some factor since the defenders are in a stronger position than the attackers.
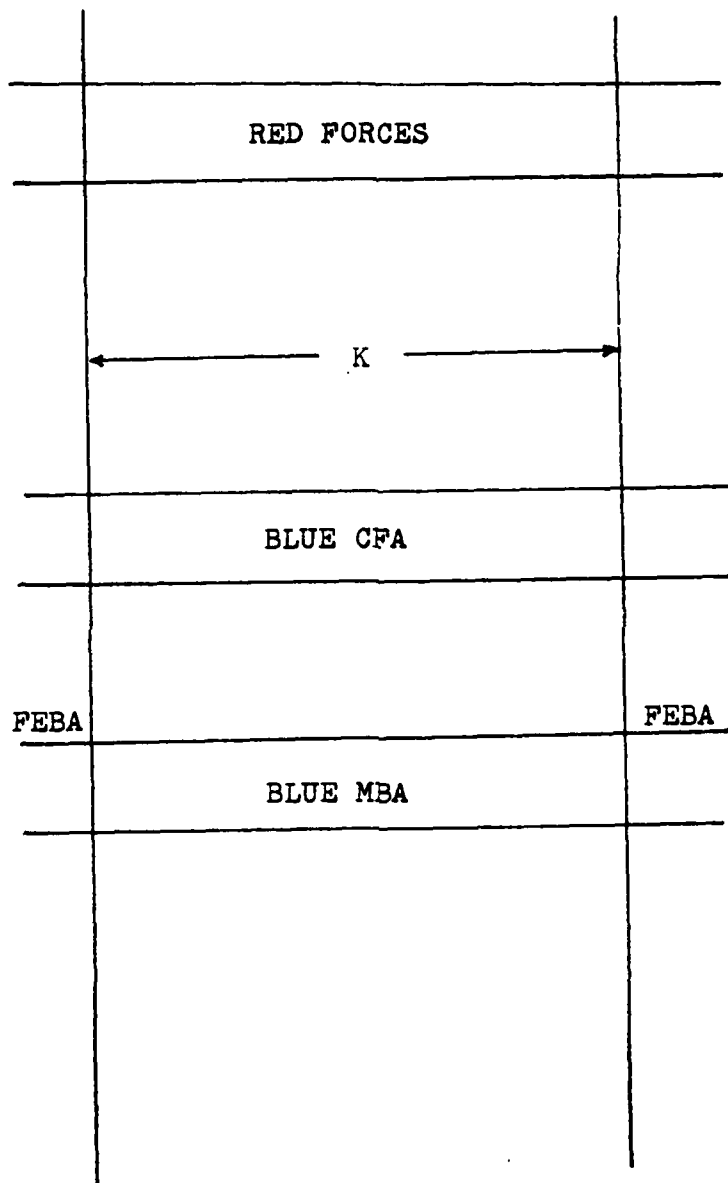
FIGURE 2

A HYPOTHETICAL DEFENSIVE SCENARIO

The problem of the CFA is defined by inquiring whether any advantage could be attained by the defenders if they were to split their forces into two lines, one behind the other, in some definite ratio of strengths, i.e., if Blue had a finite initial strength, B_o, then some fraction, p, $(0 < p < 1.0)$, would be committed to the first line (the covering force initial strength would then be B_op), and the remainder, 1 - p, would occupy the second line. (The MBA initial strength would then be $B_o \cdot (1 - p)$.

Within this defensive formation the Blues could take advantage of the time delay in the advance of the Red forces due to the combat of the CFA forces, to reinforce the FEBA positions directly behind the threatened sector by drawing in troops from adjacent positions along the FEBA (see fig. 3).

Several additional assumptions are required:

1) The assumed equality of Red and Blue strength will be expressed, for the purpose of making the mathematical calculation more transparent, as asserting that each side has an equal number of troops, each with the same killing power within any width of sector, i.e., Red and Blue forces are equally distributed homogeneously over the infinite width of the battlefield.

2) The CFA battle is not over until <u>all</u> the Blue forces in the CFA are annihilated.

3) The time for the Red forces to move through the CFA to attack the FEBA is ignored.
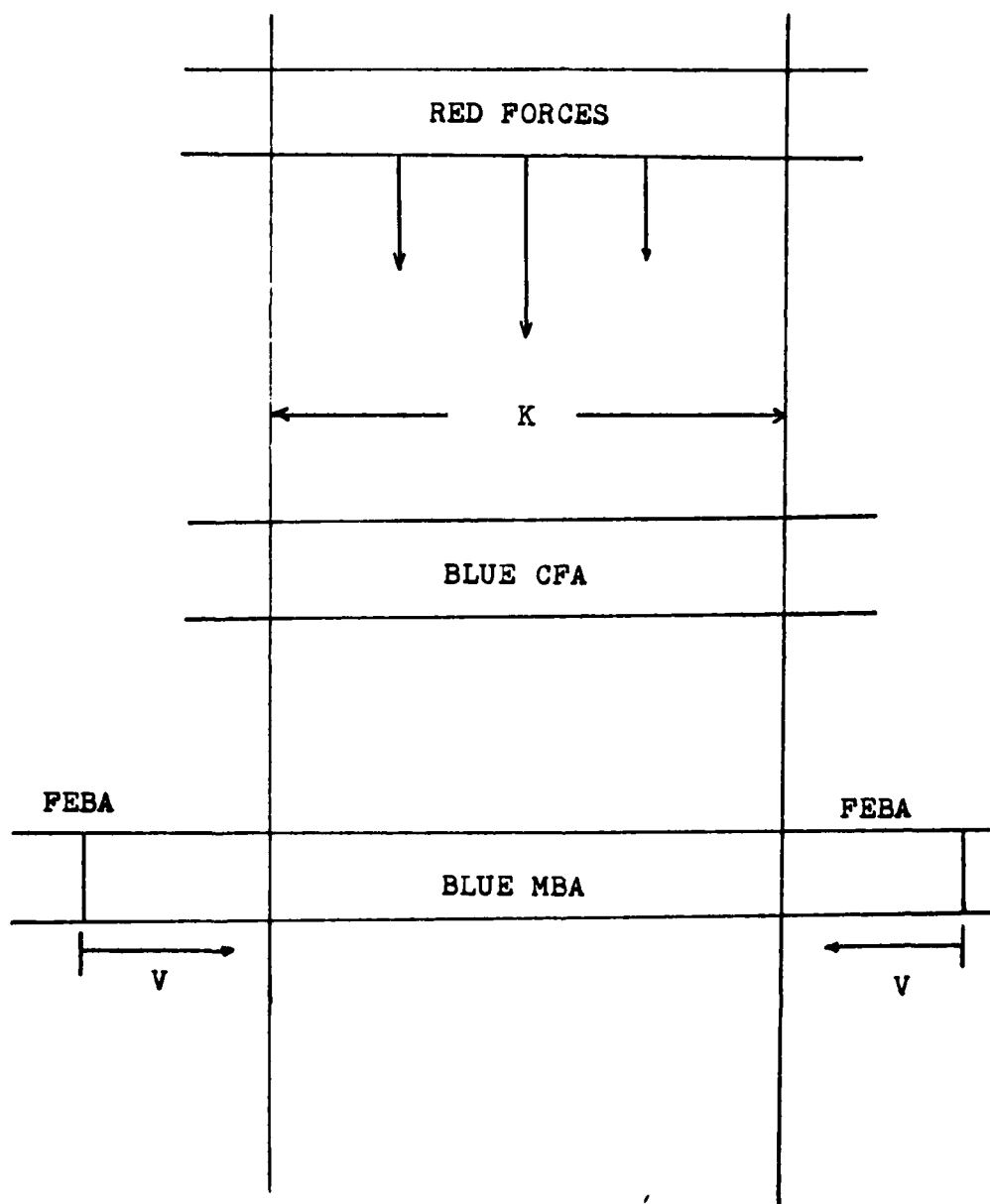
FIGURE 3

RED FORCES ATTACK IN A SECTOR OF WIDTH, K.

4) The movement of Blue adjacent units to thicken the MBA stops when the MBA battle begins.

C. DERIVATION OF LANCHESTER EQUATIONS

The model conceptualized above is now translated into a Lanchester-type model.

Let: R = number of Red forces

B = number of Blue forces

r = Red attrition coefficient

b = Blue attrition coefficient

Then: $\dfrac{dR}{dt} = -bB$

$$\dfrac{dB}{dt} = -rR \tag{1.1}$$

Let: R_1 = initial number of Red forces attacking in a sector of width, K.

B^{CFA} = initial number of Blue forces committed to CFA in attacked sector.

Then: $B^{CFA} = B_o p$

B^{MBA} = initial number of Blue forces deployed along the FEBA in attacked sector.

the state equation becomes:

$$(B^{CFA})^2 = B^2 = \dfrac{r}{b} (R_1^2 - R_2^2) \tag{3.2}$$

We assume that the individual elements of both Red and Blue forces are equally effective, then $r = b$ and $\dfrac{r}{b} = 1$.

Also, we assume that the covering force fights to annihilation, $B = 0$. Then the number of Red forces, R_2, that

survive the CFA battle and will continue the attack upon the FEBA, can be found in the expression:

$$R_2^2 = R_1^2 - (B^{CFA})^2 \qquad (3.3)$$

To determine the time of the duration of the CFA battle it is necessary to first form an expression of the Blue force level as a function of time, denoted at B(t). Solving for B in equation (3.2) and substituting into the second differential equation in (3.1), and integrating, it is found that:

$$B(t) = \tfrac{1}{2} \cdot \left((B^{CFA} - R_1) \, e^t + (B^{CFA} + R_1) \right) \, e^{-t} \qquad (3.4)$$

(Ref. 17)

If the Blue covering force fights to annihilation, as previously assumed, and t_f is the time required for annihilation to occur, (CFA battle duration), then the value of t_f can be found when $B(t_f) = 0$. Substituting yields:

$$t_f = \tfrac{1}{2} \ln \left\{ \frac{R_1 + B^{CFA}}{R_1 - B^{CFA}} \right\} \qquad (3.5)$$

(Ref. 18)

Let $u = \dfrac{R_1}{B^{CFA}}$, where u is the ratio of initial Red forces to initial Blue forces in the CFA, or, equivalently, the percent by which the covering force is outnumbered.

Then (3.5) can also be written as:

$$t_f = \tfrac{1}{2} \left(\ln \frac{u + 1}{u - 1} \right) \qquad (3.6)$$

Let y be the number of reinforcing Blue troops that will move from adjacent positions to thicken the threatened MBA sector during the time of the CFA battle, t_f. If we assume

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that these adjacent troops can move with some fixed speed, V, then y can be expressed as follows:

$$y = 2x(\text{number of troops in adjacent areas})$$

$$y = 2x(\text{length of area}) \times (\frac{\text{troops}}{\text{unit length}})$$

$$y = 2x(V \cdot t_f) \cdot (\frac{B^{MBA}}{K}) \tag{3.7}$$

Consequently, the total number of Blue troops that can defend the threatened MBA sector is equal to the number originally present plus the number that can arrive from adjacent positions in time, t_f.

Let: B_t = the number of Blue to occupy the
              threatened MBA sector

Then: $B_t = B^{MBA} + y = B_o (1 - p) + y \tag{3.8}$

For ease of calculation the above equations must be put into "dimensionless form" (Ref. 19). This is accomplished by choosing units of troop strength, time, and length as follows:

1) A unit of troop strength will be the total number of Blue forces initially present in the sector of width, K.

2) A unit of time will be the length of time required for one unit of either side to kill one unit of the enemy.

3) A unit of width will be the width of the front, K, (this choice gives, V, the units of width of front per unit kill time).

Applying the above units of measurement, equations (3.3), (3.5), and (3.7) reduce to a simpler form.

33

The number of Red troops left after annihilating the Blue covering force becomes:

$$R_2^2 = R_1^2 - (B_0 p)^2$$

and,

$$R_2^2 = 1 - p^2$$
$$R_2 = \sqrt{1 - p^2} \qquad (3.9)$$

The time of CFA battle is then determined from:

$$t_f = \tfrac{1}{2} \ln \left\{ \frac{R_1 + B^{CFA}}{R_1 - B^{CFA}} \right\}$$

$$t_f = \tfrac{1}{2} \ln \left\{ \frac{\dfrac{R_1}{B^{CFA}} + 1}{\dfrac{R_1}{B^{CFA}} - 1} \right\}$$

$$t_f = \tfrac{1}{2} \ln \left\{ \frac{\dfrac{1}{p} + 1}{\dfrac{1}{p} - 1} \right\}$$

$$t_f = \tfrac{1}{2} \ln \left(\frac{1 + p}{1 - p} \right) \qquad (3.10)$$

The number of arriving adjacent troops, y, expressed in equation (3.7) now becomes:

$$y = 2V t_f \cdot (1 - p) \qquad (3.11)$$

So, the total number of Blue defensive forces in the threatened MBA sector is now:

$$B_t = B_0 (1 - p) + y \qquad (3.9)$$

$$B_t = 1 - p + y \qquad (3.12)$$

34

In the MBA battle, the aimed fire state equation (1.3) is again applied (assuming this time that Blue can annihilate Red) to determine the number of Blue survivors, B_s.

$$B_s = \sqrt{B_t^2 - R_2^2} \qquad (3.13)$$

where: $\qquad R_2 = \sqrt{1 - p^2} \qquad (3.9)$

and: $\qquad B_t = 1 - p + y \qquad (3.12)$

To determine the total number of Blue casualties, C, the number of Blue survivors must be subtracted from the total number committed:

$$C = \text{(total committed)} - \text{total survivors}$$

$$C = (1 + y) - B_s \qquad (3.14)$$

where C is expressed in dimensionless units.

D. RESULTS

For various percentages of the Blue force committed to the CFA (p varies between .1 and .9) the Blue losses can be plotted along a series of curves for each hypothesized value of V (fig. 4).

While it is possible to express these results in the dimensionless form used to simplify the expressions, the curves may be more meaningful by assuming arbitrarily that;

1) the assault sector is 1 kilometer in width,

2) $B_o = 1000$ and $R_1 = 1000$,

3) $r = b = 1$, that is, each Red and Blue combat element has a killing power of one enemy element per hour,

35

FIGURE 4

BLUE LOSSES AS A FUNCTION OF SPEED
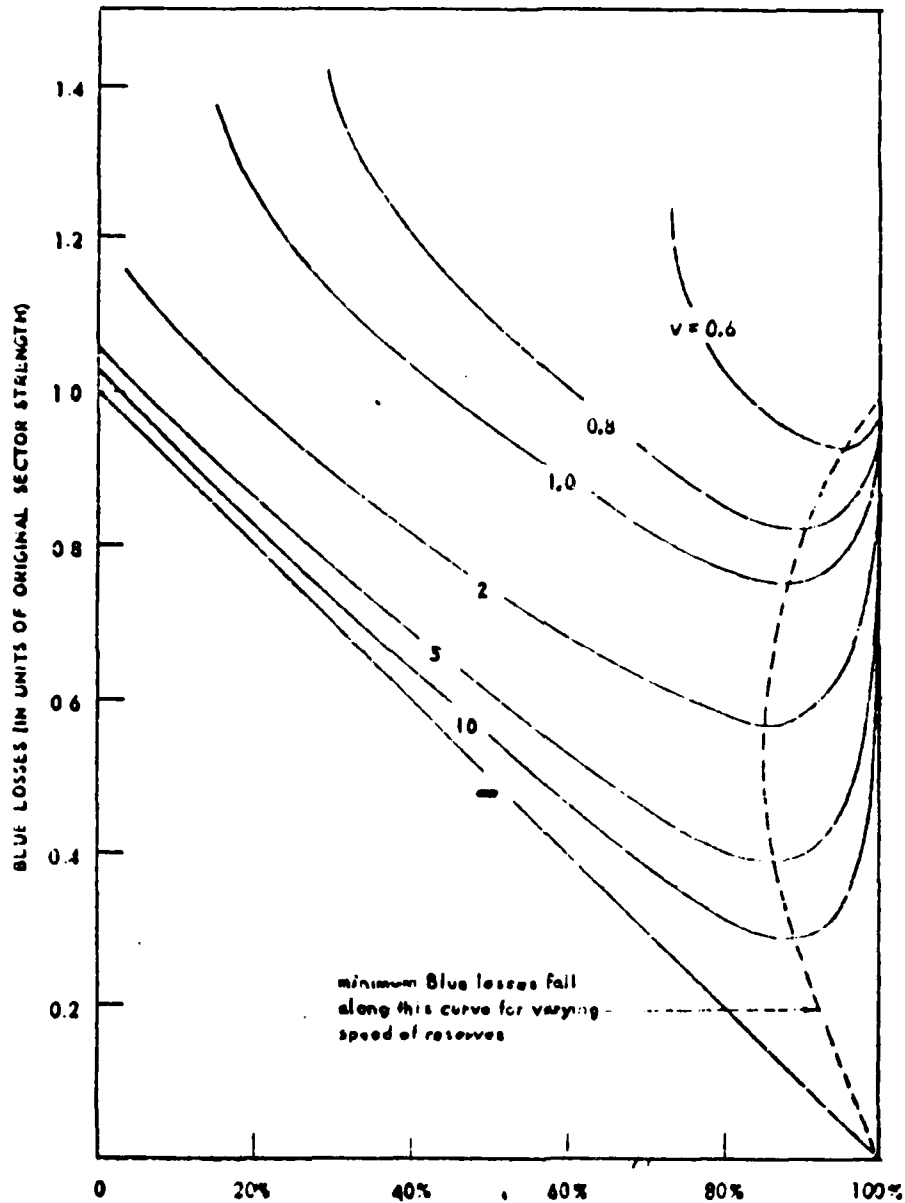
% BLUE FORCES IN MBA

Blue losses as a function of % initial strength
retained in the MBA and for various assumed speeds
of movement of reinforcing units. Red is annihilated.

4) V is measured in kilometers per hour.

It should be noted that, though in every case Blue loses the CFA battle, the scenario has been contrained so that Blue will win in the decisive MBA battle.

E. CONCLUSIONS FOR EQUAL FORCE RATIO

The results show that the optimum Blue strategy seems, in every case, to favor retaining the bulk of its defensive forces ($\cong$ 85%) in the MBA regardless of the speed of the reinforcing adjacent units.

Obviously, with such a simple model, the drawing of conclusions is dangerous, but offers a means of developing insights into the dynamics of the covering force and its contribution to the overall defensive effort.

F. SUPERIOR ODDS

An enrichment of this model can easily be achieved by relaxing the assumption requiring Red and Blue force level equality in any sector width. Equation (3.6) can now be used to calculate the CFA battle duration time, t_f, for any force level ratio:

$$t_f = \tfrac{1}{2} \ln \left(\frac{u + 1}{u - 1} \right) \qquad (3.6)$$

where u = force level ratio, $\dfrac{R_1}{B^{CFA}}$

$$R_2^2 = R_1^2 - (B^{CFA})^2 \qquad (3.3)$$

can be used to determine the Red survivors of the CFA battle.

37

Then equation (3.7) will give the number of Blue reinforcements:

$$y = 2x(V \cdot t_f) \cdot (\frac{B^{MBA}}{K})$$ \hfill (3.7)

where: $\qquad\qquad B^{MBA} = B_o \cdot (1 - p)$

and: $\qquad\qquad B_t = B_o \cdot (1 - p) + y$ \hfill (3.9)

gives the number of Blue forces that will defend the threatened MBA sector.

Solution of the aimed fire state equation for the MBA battle will provide the number of Blue survivors:

$$B_s = \sqrt{B_t^2 - R_2^2}$$

The number of Blue casualties can then be found for each hypothesized value of V by:

$$C = (B_o + y) - B_s$$ \hfill (3.15)

For a 3:1 superiority in total Red forces over total Blue forces in any width of sector, but retaining the assumption of $r = b = 1$, figure 5 shows the minimum speed with which the reinforcing adjacent units must move in order to achieve parity at the end of the MBA battle. Then, for Blue to win with any survivors, the speed, V, of adjacent units must be greater than the plotted curve. Notice that the minimum velocity required to achieve parity occurs when 46% ($p = .46$) of total Blue is committed to CFA.

Figure 6 gives the total Blue losses as a function of p for V = 20 kmph where:

MINIMUM SPEED OF REINFORCEMENTS
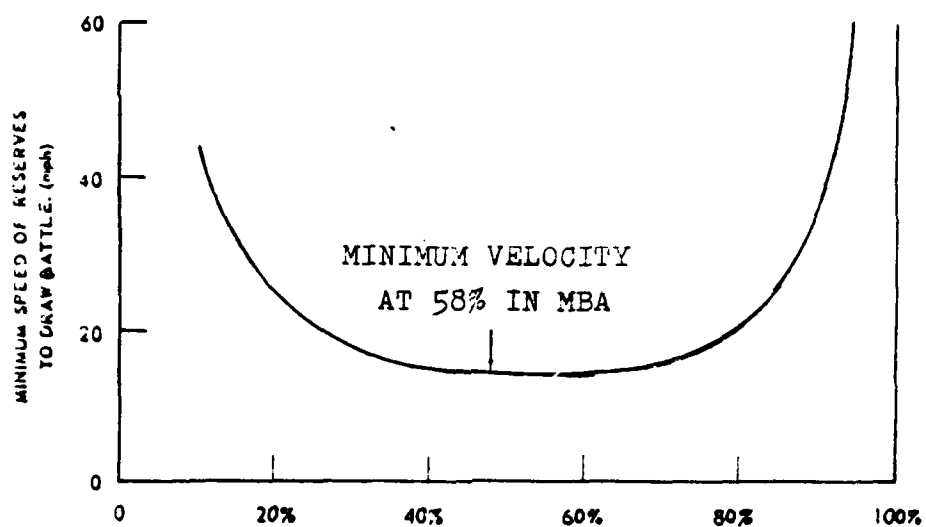
FIGURE 5

% BLUE FORCES RETAINED IN THE MBA

Minimum speed with which reinforcements must
have to annihilate Red forces (i.e., Blues are
themselves annihilated).

BLUE LOSSES
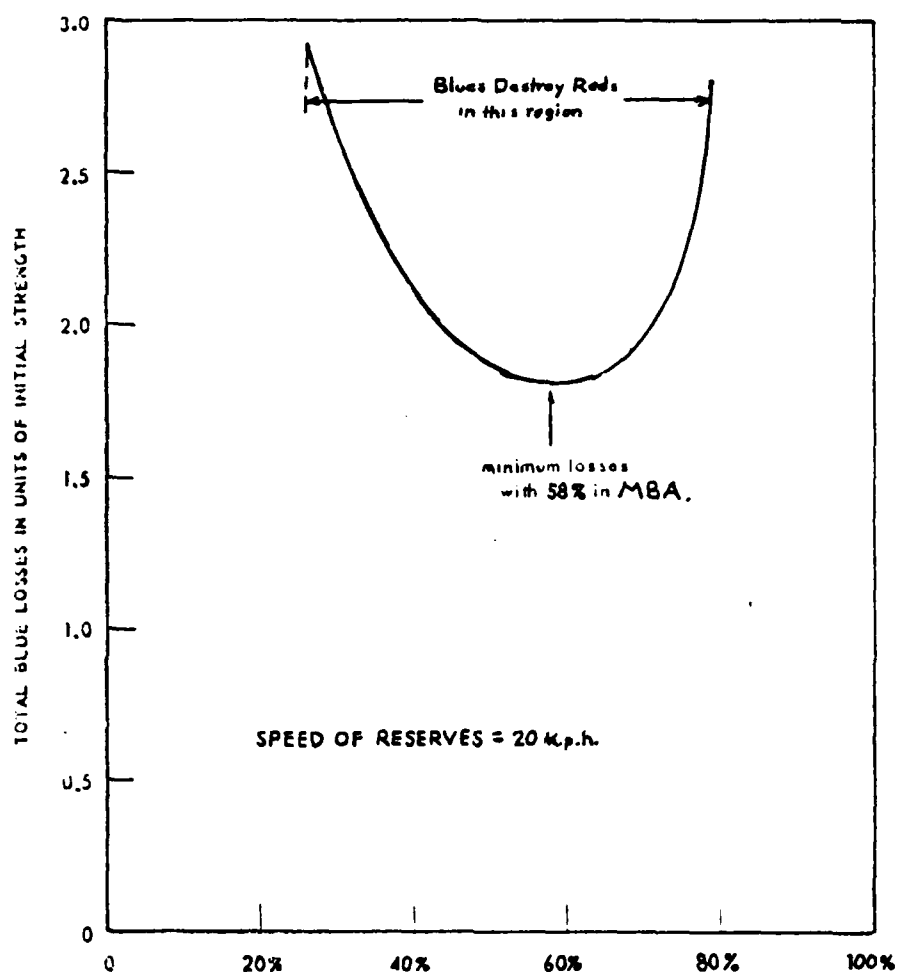
FIGURE 6

% BLUE FORCES RETAINED IN THE MBA

Blue losses versus the percentage, (1-p) of Blues originally deployed in the MBA when the Red have a 3:1 numerical superiority over the Blues.

$$B_o = 600$$

$$R_1 = 1300$$

$$r = b = 1$$

$$K = 1 \text{ Km.}$$

G. CONCLUSIONS

Figure 5 shows that an optimum Blue strategy would commit about 46% of its forces to the CFA. In this case, the optimem deployment strategy will be defined as the p value that achieves parity using the minimum speed of adjacent units as a parameter. In this case, figure 5 reveals that minimum speed is about 15 kph.

This example poses a limitation on the general application of the results found in the first battle where a 1:1 force ratio was assumed. In that case the optimum strategy was to retain about 85% of forces in the MBA regardless of the hypothesized V.

Figure 6 shows the optimum Blue strategy in the face of 3:1 odds with a fixed speed of V = 20 kph. The optimum tactic now is to retain about 60% in the MBA (p = .40).

Thus, the optimum Blue tactic, from this model, seems to require that large fractions be retained in the MBA when total opposing sides are more nearly equal and that more forces be committed to the CFA as the enemy superiority increases.

IV. AN APPLICATION OF SEVERAL LANCHESTER-TYPE MODELS TO THE DEFENSIVE BATTLE

A. INTRODUCTION

In this chapter a series of cases will be presented that will offer an opportunity for analysis of the crucial advantages that must be gained in order to quantitatively justify the tactics of organizing the defensive battlefield into two areas, the CFA and the MBA.

From the discussion of Lanchester's original work in Chapter I, it will be remembered that under the aimed fire assumptions of modern warfare, the victor can significantly reduce his casualties by concentration. A unit that could concentrate all available forces on the battlefield was found to be capable of overcoming a particular advantage in combat effectiveness that an opponent might have, due to superior firepower or organization.

The question now arises that if concentration is so advantageous under modern conditions why is it that conventional, U.S. defensive doctrine splits a defensive force into two component forces? Currently, a defensive force of finite size must be divided into two smaller forces to occupy the CFA and the MBA. Since the terrain and distances are such that neither the force in the CFA nor the force in the MBA can assist each other with direct fire, then analysis by Lanchester-type methods demands that two separate battles be modelled, each with its own set of differential equations

and initial force levels. This partition of the defensive battle into two areas with a subsequent partition of defensive forces, hardly seems to be in consonance with the principle of concentration. Yet, this tactic in reality must offer a tangible defensive advantage to be accepted doctrine. It appears that there is a contradiction between classic Lanchester theory and current defensive tactical doctrine. Many military men may react adversely to the application of Lanchester-type models to contemporary combat. This controversy generates sufficient doubt in the conclusions found in Lanchester's aimed fire conditions to wonder whether modern warfare has evolved beyond the state where attrition could be modelled by a simple analytical model.

To explore this question of concentrating all forces in the MBA versus having a covering force forward of the MBA a sequence of simple cases will be presented. Each case will be a simple application of Lanchester's analytical models to different defensive scenarios. It may be found that some modelling assumptions will be difficult to accept initially, but the reader is encouraged to continue since these aberrations will be resolved.

B. CASE I

Consider a battle in which two homogeneous forces clash. The Blue force is in a defensive posture and concentrates all its available forces on a piece of terrain such that all Blue elements can bring effective fire on the attackers. The Red

force outnumbers the Blue force by two to one, yet attacks the Blue force in two echelons, one behind the other. Assume that the defensive battle becomes two distinct battles wherein the total Blue force must defend against the attack of the Red first echelon and then the attack of the second echelon. Blue forces receive no reinforcements during the entire conflict. Battle termination is determined using breakpoint force levels in the fashion discussed in Chapter I. When either Red or Blue reaches its own breakpoint force level the battle ends, with victory going to the remaining force. Also assume that, regardless of the victor in the first echelon attack, Red first echelon survivors are assimilated into the second echelon and thus increase the initial Red force size for the second battle.

Because of the defensive posture of the Blue force, assume that it has a combat advantage over the Red force. This advantage will be reflected in its constant attrition coefficient, b, which will remain constant over the entire conflict. Likewise, the attrition coefficient, r, of the Red forces will be constant over the entire conflict and will be the same for both the first and second echelon.

A simple diagram of the progress of the battle is portrayed in figure 7. If we assign values to the constants, then the aimed fire state equations can be solved for each battle in the defensive conflict.

FIRST BATTLE

BLUE DEFENDS   b   ATTACKING 1st ECHELON   ADVANCING 2ND ECHELON

B R_1 R_2

r

SECOND BATTLE

b

$B^{(-)}$ $R_2^{(+)}$

BLUE SURVIVORS DEFEND   2ND ECHELON PLUS 1ST ECHELON SURVIVORS ATTACK
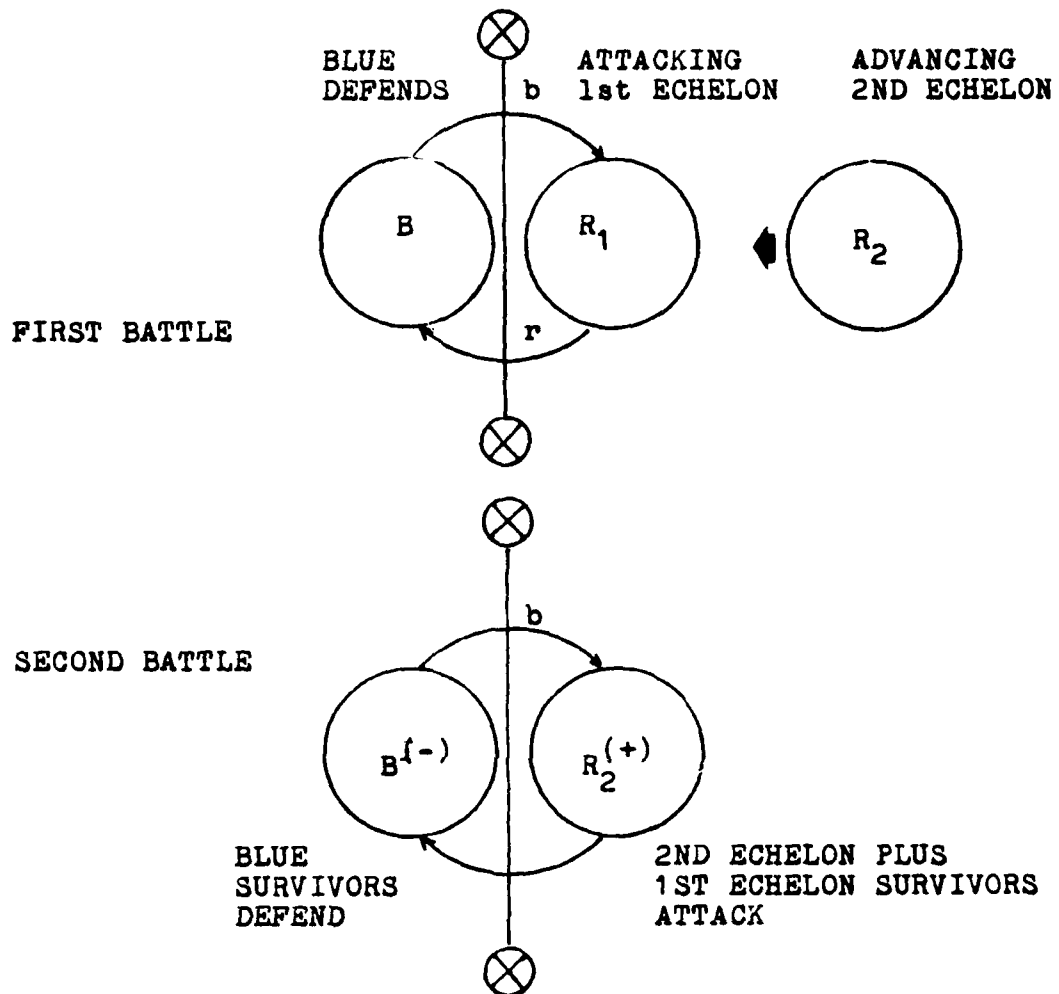
CASE I: Blue defenders concentrate all forces against a two echelon attack.

FIGURE 7

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Let: B_o = Blue force size = 1500 troops

$b = 4$

R_1 = first Red echelon force size = 2000 troops

R_2 = second Red echelon force size = 1000 troops

$R_{total} = R_1 + R_2$ = total Red strength = 3000 troops

$r = 1$

f_{BP}^R = Red's breakpoint fraction = .4

f_{BP}^B = Blue's breakpoint fraction = .5

It should be noted that the model is very sensitive to
battle termination criteria, i.e., the selection of break-
point fractions. In this chapter and in subsequent chapters
the breakpoint fraction of the Red force is lower than the
hypothesized value for an attacker presented in Chapter I.
The reason for this low breakpoint is that the Red force is
being portrayed in each model as a force that can suffer
high casualties and still continue the attack. Additionally,
it is modelled as a force that is willing to accept high
casualties in order to achieve its objectives.

The aimed fire state equation for the first echelon attack
is:

$$b(B_o^2 - B_f^2) = r(R_1^2 - (R_1 \cdot f_{BP}^R)^2)$$

Substitution of the above values reveals that Blue wins
the first battle with the number of Blue survivors being
B_f = 1187 troops. The Red survivors are found as R_{1f} =
$R_1 \cdot f_{BP}^R$ = 800 troops. Since the Red first echelon reached

its breakpoint force level first, Blue can claim victory in the first echelon battle. However, the Blue survivors must face the second echelon attack with no reinforcements. The Blue initial force size is simply the number of survivors, $B_f = 1187$ troops; and the Red second echelon survivors is $R_2 + R_1 \cdot f_{BP}^R = 1800$ troops. The attrition coefficients and breakpoint fractions remain the same, and the aimed fire state equation for the second echelon attack is:

$$b(B_f^2 - B_S^2) = r\left((R_2 + R_1 \cdot f_{BP}^R)^2 - ((R_2 + R_1 \cdot f_{BP}^R) \cdot f_{BP}^R)^2\right)$$

Substitution reveals that once again the Red force reached its breakpoint force level first and Blue can claim total victory with survivors, $B_S = 854$ troops.

The measures of effectiveness for the models in this paper, as mentioned in Chapter II, will be Blue losses, L_B, loss ratio, R, loss difference, D, Red losses, L_R, and Blue victory or defeat. These are calculated for this example and are as follows:

Blue losses, $L_B = 646$

Loss ratio, R = .283

Loss difference, D = -1634

Red losses, $L_R = 2280$

Blue victory

The values of the measures of effectiveness for each case in this chapter are listed in Table I for comparison.

C. CASE II

In the previous defensive scenario a more effective Blue force concentrated in one area, and defeated a larger Red force that attacked in two exhelons. In Case II the Blue force will partition the battlefield into two areas (CFA/MBA) and will commit a fraction of its total force into each area. $B_o p$ forces will be committed to the CFA as the covering force and $B_o \cdot (1 - p)$ forces will remain in the MBA ($o < p < 1.0$). Once again the Blue forces will face a two echeloned attack by a superior Red force, but will retain its combat advantage over Red. Figure 8 diagrams the progress of this fight.

Assume that the covering force fights the first echelon until the battle termination criteria has been reached. Then the Red and Blue survivors of the CFA battle are incorporated into the second echelon and MBA forces respectively, and the second echelon attack commences against the MBA. Also assume that the Blue covering force will not fight to annihilation, but rather to a low breakpoint force level that will reflect a tenacious CFA battle, but will not permit such an extreme defeat as annihilation. Let the fraction of Blue forces committed to the CFA be $p = .45$, with a breakpoint fraction of $f_{BP}^{CF} = .2$. Then 45% of the total Blue forces will be committed to the CFA and will abandon the CFA when 20% of its original force level is reached.

The aimed fire state equation for the CFA battle is:

$$b \left((B_o p)^2 - (B_o \cdot p \cdot f_{BP}^{CF})^2 \right) = r \left(R^2 - R_{1f}^2 \right)$$

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CFA BATTLE
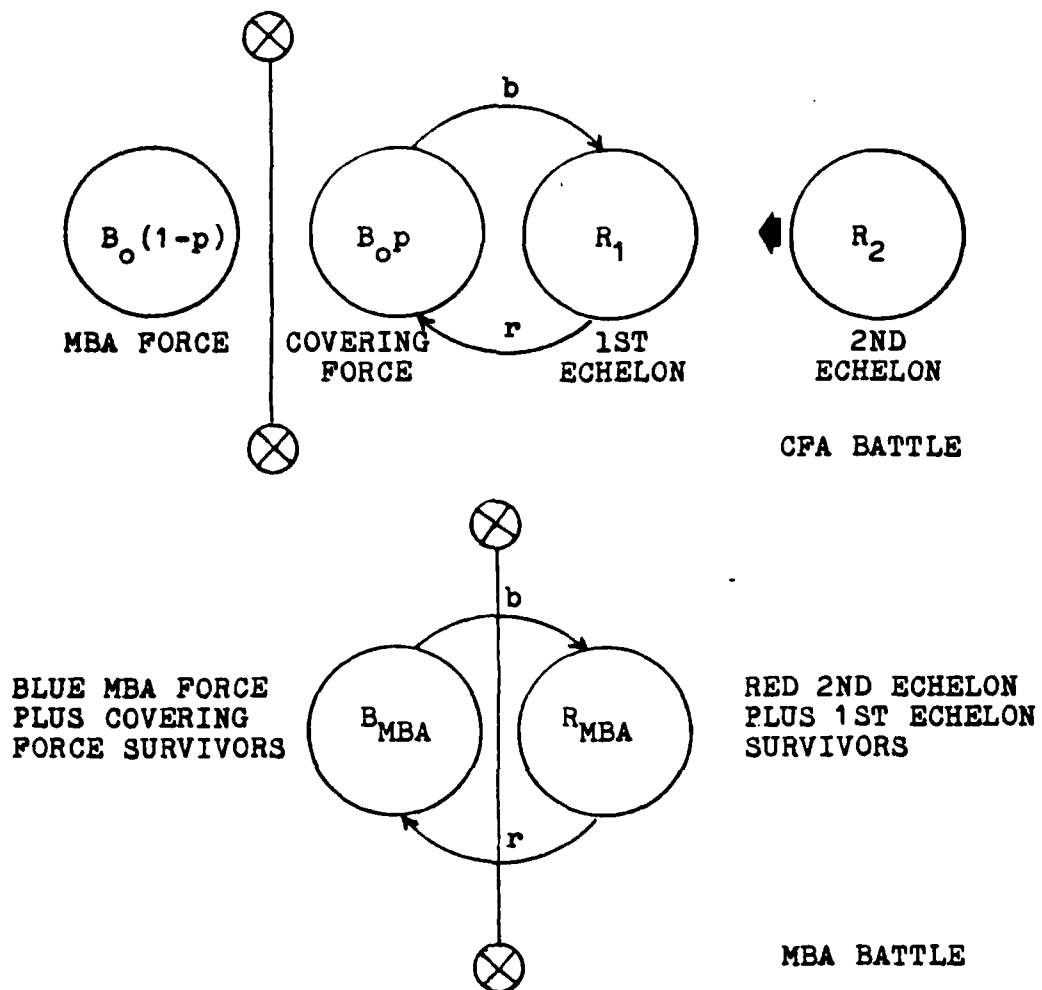
MBA BATTLE

CASE II: Blue commits a certain fraction, p , of total forces
          forward as a covering force. Two battles compose
          the defensive battle (CFA and MBA battles). No
          increase in combat effectiveness is attained due
          to the actions of the covering force.

FIGURE 8

Substitution of the numerical values for this example reveals that Red is victorious, with Red survivors, R_{1f} = 1500 troops. The Blue covering force survivors are $B_o p \cdot f_{BP}^{CF}$ = 135 troops.

After incorporation of Blue and Red survivors the initial force levels for the MBA battle can be determined where:

$$B_{MBA} = B_o (1 - p) + B_o p \cdot f_{BP}^{CF} = 825 + 135 = 960 \text{ troops}$$

and

$$R_{MBA} = R_2 + R_{1f} = 1000 + 1500 = 2500 \text{ troops}.$$

Prior to the start of the MBA battle a breakpoint fraction for the Blue forces must be hypothesized. For simplicity let us adopt the value for the breakpoint fraction of a defender discussed in Chapter I. Then f_{BP}^{MBA} = .5.

Since the Red and Blue attrition coefficients for the MBA battle do not change from those hypothesized in the CFA battle, the aimed fire state equation can be written as:

$$b \left(B_{MBA}^2 - (B_{MBA} \cdot f_{BP}^{MBA})^2 \right) = r \left(R_{MBA}^2 - R_S^2 \right)$$

Substitution shows that Red wins the MBA battle with total Red survivors for the entire defensive battle as R_S = 1867 troops. Blue survivors are B_S = 480 troops.

The computed values for the measures of effectiveness for this scenario are:

Blue losses, L_B = 1020

Loss ratio, R = .9

Loss difference, D = -113

50

Red Losses, $L_R = 1133$

Red victory

Comparison of the results of Case I and Case II indicate that Blue concentration can mean the difference not only between victory and defeat, but also in the severity of defeat. Regardless of a 4:1 relative combat effectiveness ratio, Blue casualties were almost equal to Red casualties, as seen in the loss ratio, $R = .9$, and the loss difference, $D = -113$.

D. CASE III

Once again one must question the need of some percentage of a defensive force being committed forward as a covering force if the results in Case II adequately reflect the dynamics of the defensive battle, within the limitations of an analytical model. Considering the tasks of the covering force as discussed in Chapter II, a possible resolution to this dilemma can be proposed. During the conduct of the CFA battle the MBA forces should be able to take advantage of the time and enemy intelligence provided to increase its own combat effectiveness. This can be accomplished realistically by movement of forces within the MBA to meet identified Red thrusts and the improvement of defensive positions, as well as certain logistic improvements.

This idea has a great deal of intuitive appeal since one would easily accept the assumption that the MBA forces could make profitable use of accurate enemy intelligence and the

extra time to react in such a fashion as to increase their combat effectiveness.

In this scenario we differ from Case II only in attributing to the MBA force an eight-fold increase in combat effectiveness such that the MBA attrition coefficient, b_2, has a value of $b_2 = 8b_1$ where b_1 is now the attrition coefficient of the covering force. b_1 does not differ in value from the original b used in Cases I and II. Assume that the Red has no increase in combat effectiveness between the CFA and MBA battle. Figure 9 diagrams the progress of the Case III defensive battle.

The aimed fire state equation for the CFA battle is:

$$b_1 \left((B_o p^2) - (B_o p \cdot f_{BP}^{CF})^2 \right) = r \left(R_1^2 - R_{1f}^2 \right)$$

Substitution reveals that Red wins the CFA battle with survivors, $R_{1f} = 1500$ troops, and the Blue covering force survivors are $B_o p \cdot f_{BP}^{CF} = 135$ troops.

After incorporation of Blue and Red survivors the initial force levels for the MBA battle can be determined (see Case II) as:

$$B_{MBA} = 960 \text{ troops}$$

$$R_{MBA} = 2500 \text{ troops}$$

In addition, the Blue MBA forces have increased their combat effectiveness to $b_2 = 8b_1 = 32$.

The aimed fire state equation for the MBA battle is:

$$b_2 \left(B_{MBA}^2 - B_S^2 \right) = r \left(R_{MBA}^2 - (R_{MBA} \cdot f_{BP}^R)^2 \right)$$

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CFA BATTLE
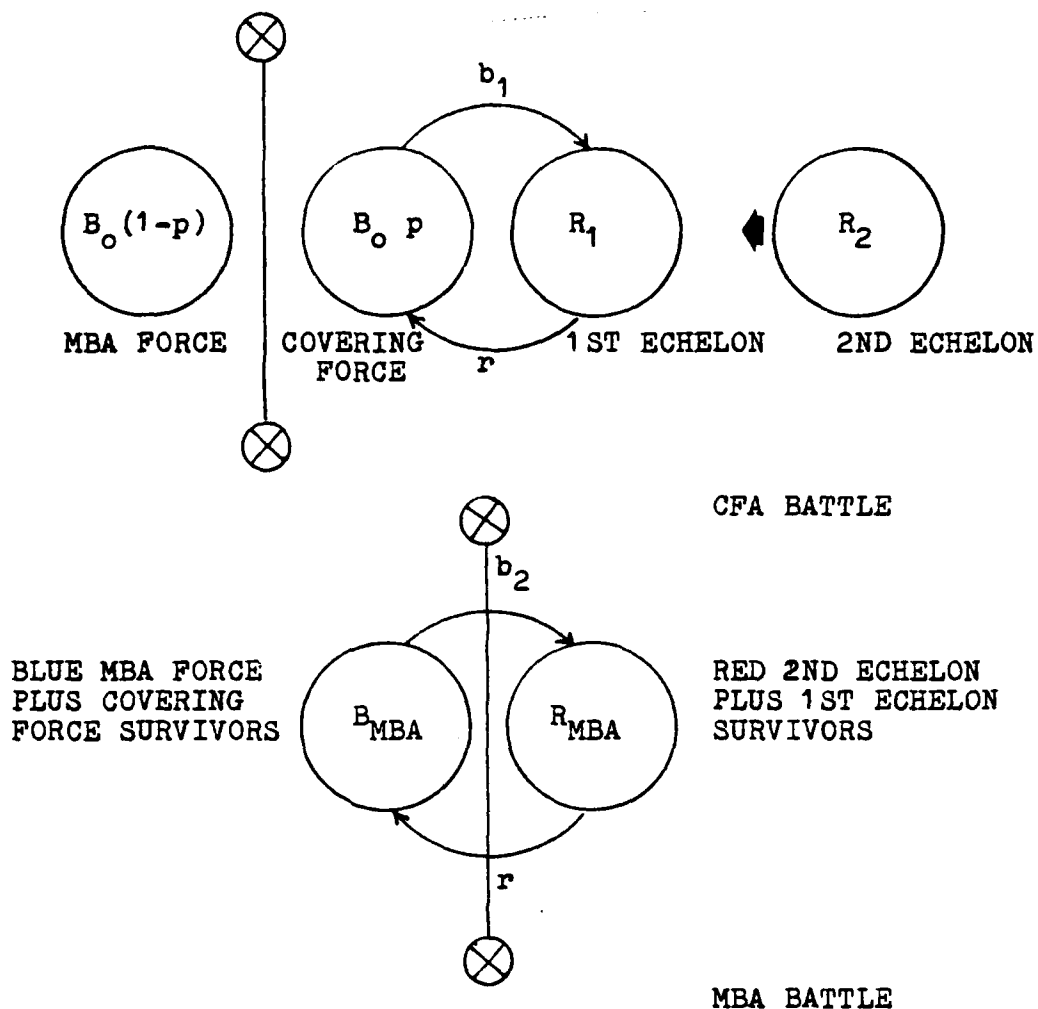
MBA BATTLE

CASE III: Blue commits a certain fraction, p, of total forces forward as a covering force. Two battles compose the defensive battle (CFA and MBA battles). An increase in combat effectiveness is attained due to the actions of the covering force ($b_2 = 8b_1$)

FIGURE 9

In this case Blue wins the MBA battle with the number of survivors being found by substitution and solving for B_S, B_S = 970 troops.

The Red survivors, R_S, are found in the battle termination expression, $R_S = R_{MBA} \cdot f_{BP}^R$ = 1000 troops.

In this case the measures of effectiveness are determined to be:

> Blue losses, L_B = 630
>
> Loss ratio, R = .31
>
> Loss difference, D = -1388
>
> Red losses, L_R = 2000
>
> Blue victory

Notice that these values for the measures of effectiveness compare favorably with values found in Case I. At these particular force levels, breakpoints and attrition coefficients for Red and Blue, it must be pointed out that for Blue to win the final (MBA) battle as successfully as in Case I, the covering force must provide the MBA force with sufficient time and information to allow it to improve its combat effectiveness by a factor of eight. This factor is not the victory transition point, but merely the increase in the attrition coefficient that compares favorably with Case I. With our knowledge of the interpretation of the Lanchester attrition coefficients this means that each Blue combatant in the MBA must be able to improve his average kill rate eight-fold. Whether this achievement is possible depends upon the optimism of the

analyst. For example, if Red and Blue are homogeneous tank forces, and if each Blue tank kills, on the average, four Red tanks an hour; and each Red tank kills, on the average, one Blue tank an hour, then, $b_1 = 4$ and $r = 1$. An increase in combat effectiveness by a factor of eight in Blue's attrition coefficient requires that each Blue tank must now kill, on the average, thirty-two Red tanks an hour ($b_2 = 8b_1 = 32$).

The observant reader may be questioning the inherent assumption that exists in this particular scenario concerning the combat effectiveness of the Blue covering force survivors that participate in the MBA battle. In this case, the assumption of homogeneous forces requires that all the Blue MBA forces have the same constant attrition coefficient, b_2. Since the CFA survivors are incorporated into one homogeneous Blue force in the MBA that fights with combat effectiveness, $b_2 = 8b_1$, this suggests that the covering force survivors magically increase their combat effectiveness eightfold when they cross the FEBA. This is hardly realistic and is acknowledged as being difficult to accept. However, in order to keep the model simple and therefore transparent, homogeneity of forces is assumed.

If two heterogeneous Blue forces exist in the MBA, problems such as Red distribution of fire between the two Blue forces arise that serve only to complicate the analysis. The reader can imagine, however, that if the covering force survivors fought in the MBA battle with their original combat effectiveness, b_1, or less, then the original MBA force

$(B_o \cdot (1 - p))$, would have to be able to increase its combat
effectiveness by an even greater amount to compensate for the
lower combat ability of the CFA survivors. This situation of
having two heterogeneous forces in the MBA cannot be ignored
and is incorporated in the computer model described in
Chapter V.

The intent of this example should not be lost. The cover-
ing force must provide the MBA forces some large advantage in
combat effectiveness for Blue to win and attain values of the
measures of effectiveness that compare favorably with those
values attained in Case I, where total concentration was
modelled. In order for comparable forces to produce compar-
able battle outcomes for Cases I and III it was necessary to
hypothesize rather unrealistic behavior for the covering
force. One must question the validity of this model since a
tremendous combat advantage had to be built into the MBA
battle.

E. CASE IV

The increase in combat effectiveness suggested by Case
III has an intuitive appeal, but one must balk at accepting
the magnitude of improvement required by the numerical example.
Some order of magnitude less than that needed in Case III
seems possible and probable. However, any degradation in
combat effectiveness would present a situation somewhere
between Case II and Case III, which would create an unfavorable
comparison between the covering force tactic and concentration

presented in Case I. Obviously, there must exist additional advantages that the commitment of the covering force can provide to the MBA forces that can be modelled to make the assumptions and results more realistic.

Recalling the discussion of the combat dynamics of the current defensive doctrine in Chapter II and the analysis in Chapter III, the possibility of modelling the movement of adjacent units into the threatened MBA sector presents itself as another enrichment that may more accurately capture the dynamics of the defensive battle. If the covering force can provide not only an increase in combat effectiveness, but also the time to respond to information, the time to alert adjacent units, and the time to move them into critical areas, then the MBA forces will be able to fight with increased force size as well as increased combat effectiveness. This idea will allow one to present in Case IV a scenario that does not require such a large increase in combat effectiveness to compare favorably with Case I.

Let us consider that there are adjacent units on the flanks of the MBA force that are not required in their particular sectors and are available to move to reinforce the MBA during the duration of the CFA battle. Also assume that the duration of the CFA battle is sufficient for the reinforcing units to arrive in the MBA and to achieve equal combat effectiveness with the original MBA forces. In keeping with the need for homogeneity within the MBA, we will also include

the CFA survivors with the same stipulations and acknow-
ledgements mentioned in Case III. Figure 10 diagrams the
progress of this battle.

The covering force battle aimed fire state equation is
the same as in Case III with the same results. Red wins,
with survivors R_{1f} = 1500 troops. The Blue CFA survivors
are $B_o \cdot p \cdot f_{BP}^{CF}$ = 135 troops. After incorporation of the
CFA survivors and the adjacent Blue units, the initial Blue
force level in the MBA is:

$$B_{MBA} = B_o \cdot (1 - p) + B_o p f_{BP}^{CF} + B_L + B_R$$

where B_L and B_R are force levels of the left and right rein-
forcing units.

The initial Red force level in the second echelon attack
is the same as in Case III.

$$R_{MBA} = R_2 + R_{1f}$$

Let the Blue reinforcing strength of B_L and B_R be,
$B_L + B_R$ = 600 troops. Then the initial Red and Blue force
levels are:

$$B_{MBA} = 825 + 135 + 600 = 1555 \text{ troops}$$

$$R_{MBA} = 2500 \text{ troops}$$

Let us also assume a more realistic increase in Blue
combat effectiveness over the duration of the CFA battle, so
that $b_2 = 3b_1$ = 12. Blue forces have more believably increased
their combat effectiveness three-fold. The aimed fire state
equation for the MBA battle is:

B_L and B_R are the left and right
adjacent Blue forces available to
thicken the MBA sector during the
CFA battle.

COVERING   1ST ECHELON   2ND ECHELON
FORCE

CFA BATTLE

BLUE MBA FORCE   RED 2ND ECHELON
PLUS COVERING   PLUS 1ST ECHELON
FORCE SURVIVORS   SURVIVORS
PLUS LEFT AND
RIGHT REINFORCE-
MENTS
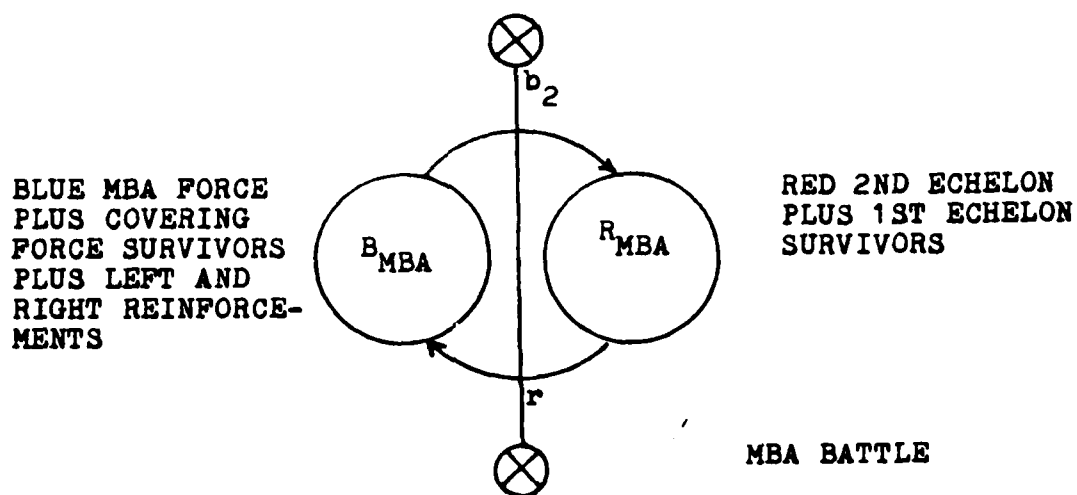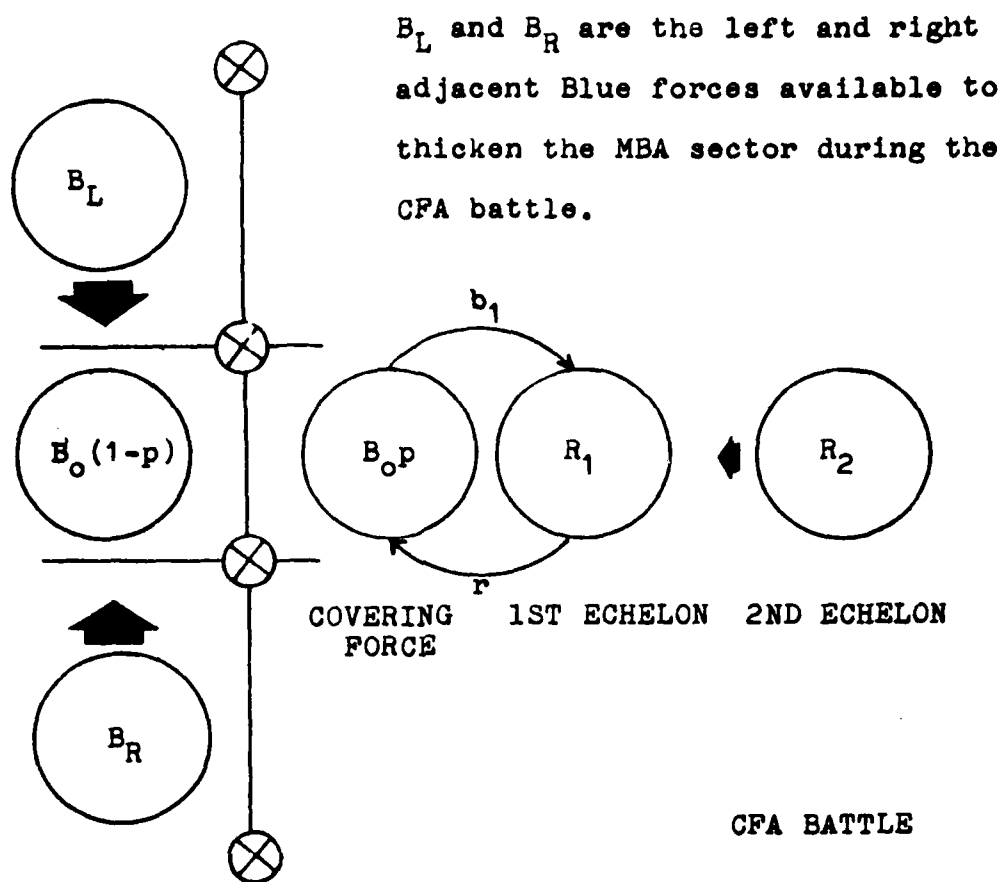
MBA BATTLE

CASE IV: Blue force move adjacent units to thicken the MBA
          and attain an increase in combat effectiveness due
          to covering force action (i.e., $b_2 = 3b_1$).

FIGURE 10

59

$$b_2 \left(B_{MBA}^2 - B_S^2 \right) = r \left(R_{MBA}^2 - (R_{MBA} \cdot f_{BP}^R)^2 \right)$$

Substitution reveals that Blue wins the MBA battle with B_S = 1407 troops, and Red once again being fought to its breakpoint force level of $R_S + R_{MBA} \cdot f_{BP}^R$ = 1000 troops.

The measures of effectiveness are found as:

Blue losses, L_B = total Blue committed - Blue survivors

$$L_B = 1500 + 600 - 1407 = 693$$

Loss ratio, R = .347

Loss difference, D = -1307

Red losses, L_R = 2000

Blue victory

F. RESULTS

Table I lists all four cases presented in this chapter and the values of the five measures of effectiveness established as a means of evaluating the covering force's contribution to the total defensive battle. All the measures of effectiveness except Red losses, L_R, are preferred at a minimum. Obviously, Red losses are preferred by Blue at their maximum value. Victory or defeat in the CFA battle can be a misleading measure of effectiveness in evaluating the contribution of the covering force, and should be avoided. As seen in Cases II, III and IV, the covering force loses the battle in the CFA. However, because of time and intelligence gained by its action the Blue MBA can profit by thickening the battlefield and attaining an increase in combat effectiveness.

	MOE's				WINNER	
	Blue Losses, L_B	Loss Ratio, R_B	Loss Difference, D	Red Losses, L_R	CFA	MBA
CASE I A concentrated force opposing a 2 echelon attack	646	.283	-1634	2280	N/A	BLUE
CASE II A CFA/MBA formation (p=.45) with no combat advantage due to CFA battle	1020	.9	-113	1133	RED	RED
CASE III A CFA/MBA formation (p=.45) with an increase in combat effectiveness (8-factor) due to CFA battle	630	.31	-1388	2000	RED	BLUE
CASE IV A CFA/MBA formation (p=.45) with a 3-fold increase in combat effectiveness and reinforcement	693	.347	-1307	2000	RED	BLUE

TABLE I. THE VALUES OF THE MEASURES OF EFFECTIVENESS

A commitment of a percentage of the total defensive force
that would be of sufficient size to insure victory in the
CFA would, in the numerical examples presented in this chap-
ter, reduce the force size in the MBA to such a level that,
regardless of the advantages attained, insufficient strength
would exist to win the final and decisive MBA battle. It
must be emphasized that the only correct way to determine
the contribution to the total defensive battle by the cover-
ing force is to analyze the outcome of the final battle, not
the outcome of the CFA battle.

G. CONCLUSIONS

The numerical examples presented in this chapter are only
a means by which the discrepancy between the principle of
concentration and the current defensive organization can be
explored, within the framework of the Lanchester differential
equations of modern warfare. The models presented are merely
excursions along paths suggested by the combat dynamics that
are easily quantified after making some simplifying assumptions.
They are also presented as a sequential enrichment process
that adapt Lanchester's original work to the unique combat
dynamics of the CFA/MBA battles.

The original problem at the start of this chapter was
whether the principle of concentration, as discussed in Chap-
ter I, could be reconciled with the tactic of partitioning a
defensive force into a CFA and an MBA. To partition a finite
defensive force into two areas in this manner seems to defy

the principle of concentration, wherein Lanchester's aimed fire model suggested the best advantage. Yet this tactic is adopted as defensive doctrine by the U.S. Army. Acknowledging the limitations of the simple model, this controversy leads one to question whether or not Lanchester's original work is still applicable or has failed to capture contemporary combat.

Case I illustrated that an outnumbered force can win against a two echeloned attack when it can concentrate and fight with a combat advantage. Case II portrayed the same force, partitioned, against the same echeloned attack. In this example the defending force was soundly defeated. Case III suggested that, if the covering force could provide the MBA force with an increase in combat effectiveness, then the defender could win favorably in the final outcome. However, the increase in combat effectiveness required was of such an order of magnitude to make it unrealistic. Case IV elaborated upon Case III and offered (as in Chapter III) the defender the ability to thicken the threatened MBA sector during the duration of the CFA battle with adjacent forces that were available to move.

By this sequential method of elaborating upon the initial aimed fire assumptions and equations of Lanchester, the discrepancy between the principle of concentration and the commitment of the covering force is reconciled. The covering force must provide to the MBA at least two distinct advantages, enemy intelligence and the time to react to that intelligence. Chapter II has gone into detail concerning how

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these advantages are to be actually obtained in combat. The
examples in this chapter serve to illustrate how these advan-
tages can be quantified and used to enrich the model. With
the original model manipulated in this fashion it can once
again grasp (albeit in a limited fashion) the dynamics of
the modern battlefield as organized by U.S. tactical defensive
doctrine, and serves to justify why such a tactic has been
adopted.

One particular factor that exists on the battlefield,
that has not been addressed or injected into any analysis
yet seen in this paper, is the element of surprise. Surprise
exists when one force presents itself on the battlefield in
such a manner as to negate its opponent's advantages in force
size or combat effectiveness. Certainly the covering force
serves as a means by which the defensive unit seeks to elimi-
nate the element of surprise in the MBA battle. If one would
attempt to model the situation wherein the covering force does
not eliminate the element of surprise in the MBA battle, it
is suggested that Case II would present a reasonable model of
this phenomenon. Any increase in combat effectiveness gained
by the MBA forces is negated and they must fight at their
original combat effectiveness. In Case II the Blue force
suffered severe defeat, and can serve as an illustration of
the importance of this particular task of the covering force.
The element of surprise and its effects has, in the past,
been difficult at best to quantify but must be recognized as

a parameter that needs to be included in any combat model. The enrichments that follow in subsequent chapters will attempt to introduce this element as well as resolve some of the uncomfortable assumptions that have been adhered to up to this point.

V. THE COMPUTER MODEL

A. INTRODUCTION

Chapters III and IV have presented some simple models from which closed-form analytical solutions were easily obtained and numerically evaluated with the aid of a hand-held calculator. Unfortunately, some important aspects of the combat dynamics had to be over simplified in order to keep the model transparent and the calculations tractable. For example, the assumption in Chapter IV that the covering force survivors and reinforcements were incorporated into the original MBA forces at the same level of combat effectiveness, was needed to retain the concept of homogeneity of forces. Obviously, this is not an acceptable assumption, and a more complex model must be developed to allow for a more realistic approach. In this chapter a model will be presented that will take advantage of the modern, high-speed computer's ability to make recursive and complicated calculations easy.

A certain reconciliation has been achieved between the need for concentration and the need for a covering force commitment in defensive combat as modelled by Lanchester-type

differential equations. There is a need now to attempt to answer the question of how many forces should be committed to the covering force. For any combat situation, given Red and Blue force levels, and their relative combat effectiveness, it is submitted that there is some optimal percentage of defensive forces that should be deployed in the covering force area. The value of this fraction of p_o will be such that the measures of effectiveness being used in this analysis will be at their preferred levels. The intent of this chapter and the model herein constructed is to evaluate by an iterative approach on the digital computer the optimal percentage, p_o, of the defensive forces that should be committed to the CFA.

B. THE SCENARIO

The general scenario used to construct the computer-based model is essentially the same situation as presented in Case IV, Chapter IV. The defensive battlefield is organized by the Blue force into two areas, the CFA and the MBA. The Blue force is attacked by a two echelon Red force, superior in strength. The defensive battle reduces to two separate battles, the CFA battle and the MBA battle. The CFA battle is between the Blue covering force and the Red first echelon. The MBA battle is between the Blue forces in the MBA (original force plus covering force survivors and reinforcements that have moved from adjacent positions to thicken the threatened MBA sector) and Red second echelon attack (consisting of first

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echelon survivors and the original second echelon). During
the covering force battle the MBA forces are capable of in-
creasing their combat effectiveness and have time to move
a certain number of adjacent units to increase the MBA force
size. In contrast, the Red forces do not achieve an advan-
tage in combat effectiveness from the CFA to MBA battles
because of their attack posture. Again, separate differen-
tial equations are used to capture these essential dynamics
in the CFA and MBA in the same fashion as in Chapter IV.

C. USE OF FINITE-DIFFERENCE APPROXIMATIONS

The Lanchester differential equations are modified in
this model due to a more complex approach to modelling the
defensive battle. The specific modelling assumptions will
be addressed in section D of this chapter. Before the actual
modelling process is described, a brief discussion of the use
of finite-difference approximations and their applicability
to this model is in order.

It is accepted that it is impossible for all practical
purposes to solve analytically the differential equations for
any but the most simple Lanchester-type combat models. In
order to obtain approximate solutions to the more complex
equations numerical methods must be used.

One such method is to use finite-difference approxima-
tions. These can be conveniently, numerically solved by
automated, iterative procedures on the digital computer (i.e.,
to do loop in Fortran). The method enables one to generate

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approximate values for the force levels at discrete points in time (as opposed to continuous time) during the course of the battle. Time is discretized by incrementing the passing of time by small uniform time steps, Δt. At each time step a finite-difference equation of the form

$$B_{n+1} = B_n - G_n (B_n, R_n) \cdot \Delta t \qquad (5.1)$$

is solved for the B force level after each time step. By applying such approximations to the continuous time combat model (equation 1.1) a discrete time model is obtained, for which the values of the approximate force levels B_n and R_n may be generated recursively at a finite number of time steps for any interval of battle. The finite-difference equations are not any easier to solve than the original differential equations, but their structure makes recursive procedures easy on a digital computer. The finite-difference equations will be the basis for the modelling effort in this chapter, which moves the battle results ahead in time with the approximate Blue and Red force levels known at the old time step, n, allowing the computer to approximate values for force levels B_{n+1} and R_{n+1}. This recursive procedure continues until a battle termination criteria has been met for each battle.

D. INITIAL MODELLING ASSUMPTIONS

The following assumptions are made:

1) The defensive battle consists of two separate battles, the CFA and MBA battles (see section B, this chapter).

2) The covering force survivors from the CFA battle
make a rearward passage of lines and are incorporated into
the MBA in such a fashion that they are uniformly distributed
throughout but retain the same combat effectiveness they
enjoyed in the CFA (i.e., the attrition coefficient of the
CFA survivors remains the same after incorporation into the
MBA). This will lead to a heterogeneous force in the MBA
and the relaxation of the assumption of homogeneity of
forces. The uniform distribution of these elements will
allow us to model their attrition by Red in proportion to
their numbers present on the battlefield.

3) The Red first echelon survivors of the CFA battle
are incorporated into the second echelon attack. This may
not accurately depict actual Red doctrine, but since no
accurate breakpoint fraction can be hypothesized, this assump-
tion makes this model more conservative.

4) The Blue and Red forces are of finite size where Red
outnumbers Blue at least 2:1 and Red attacks in two echelons
(i.e., Blue = 1500, Red = 3000).

5) The Blue forces enjoy a combat advantage over Red
forces which is reflected in the Blue and Red attrition coef-
ficients, i.e., $b = 1.2$ and $r = .2$. This gives a combat
effectiveness ratio of $\frac{b}{r} = \frac{1.2}{.2} = 6$. Blue forces are on the
average six times more effective than Red forces.

6) The intensity of combat, $\sqrt{b \cdot r}$, is of such a magni-
tude that the covering force battle lasts approximately one
time interval (one day).

7) The original MBA forces are capable of increasing their combat effectiveness due to the time and enemy intelligence gained by the action of the covering force in the CFA. It would be more accurate to be able to model this phenomonenon as being of the form $b_2 = g(t_f,$ enemy intelligence) where b_2 is the increased Blue MBA attrition coefficient, but the functional relationship between enemy intelligence and combat effectiveness is difficult to quantify. No attempt is made in this paper to undertake this task. The increase in combat effectiveness is merely portrayed as a linearly increasing function over time, until some maximum value of b_2 is reached (see figure 11). It is assumed that the MBA forces are capable of attaining a maximum increase in combat effectiveness if the CFA battle can last one day. It is not reasonable to expect this increase to continue beyond some maximum level that can be achieved in finite time.

As mentioned at the end of Chapter IV, surprise may be represented by a negation of any combat advantage that the MBA forces may achieve due to CFA action. It is reasonable to expect that if the Red forces were able to penetrate the CFA and reach the MBA prior to some preparation time, t_p, that the MBA forces would have to fight at a diminished capability represented by a diminished attrition coefficient. Graphically, this is represented in Figure 12.

8) The beginning of the CFA battle sees the covering force engaging the Red first echelon from battle positions that make maximum use of terrain and the extended ranges of

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BLUE MBA ATTRITION COEFFICIENT
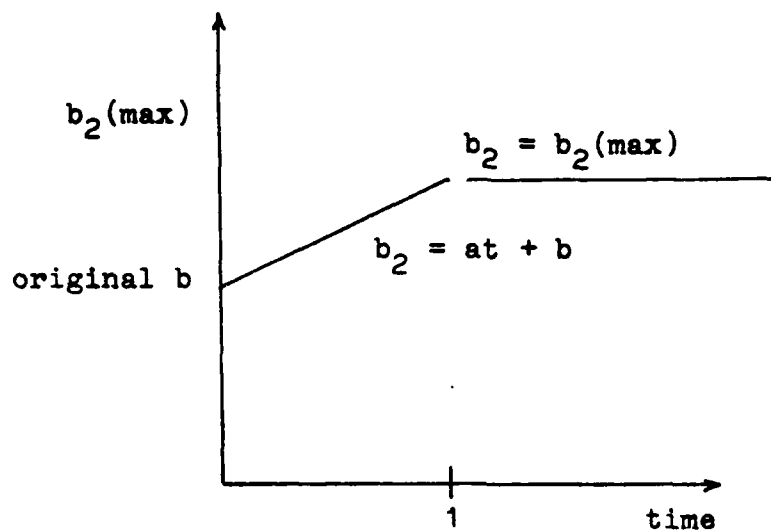
Linear increase in combat effectiveness of MBA forces over time, where $b_2(max)$ is the maximum value possible to attain in finite time.

FIGURE 11

BLUE MBA ATTRITION COEFFICIENT
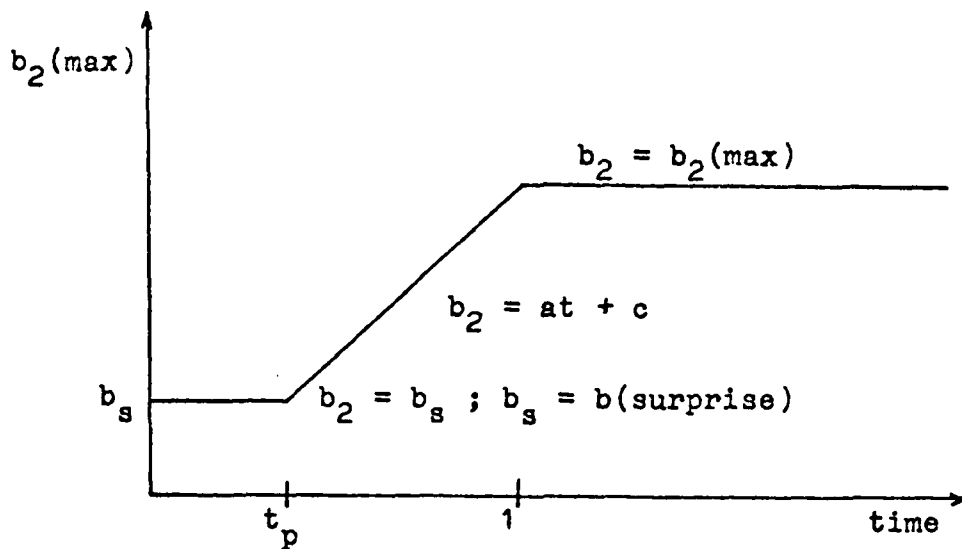
Linear increase in combat effectiveness of MBA forces over time. $t < t_p$, MBA forces are surprised and must fight at a diminished combat effectiveness. $t_p < t < 1$, MBA forces are capable of gaining a linear increase in combat effectiveness. $t \geq 1$ maximum combat effectiveness is attained.

FIGURE 12

organic weapons. These ambush techniques make effective Red aimed fire against the Blue covering force elements difficult. As the battle continues over the expanse of the covering force area Red forces become increasingly more capable of pinpointing individual covering force elements and can engage them with accurate aimed fire. Thus, the initial CFA battle can be modelled as in the Dietchman ambush model where the Blue covering force engages the Red first echelon with aimed fire, but the Red forces can only engage suspected covering force positions with area fire.

As the battle progresses a transition is made from the pure ambush model to the pure aimed fire model. This transition is expressed as:

$$B_{n+1} = B_n - (\frac{B_n}{.1 \cdot B_o} e^{-gt} + (1 - r \cdot e^{-gt})) \cdot R_n \cdot \Delta t$$

As t gets larger the equation approaches the aimed fire finite difference equation.

9) The time between the end of the CFA battle and the beginning of the MBA battle cannot realistically be considered instantaneous. Additionally, the outcome of the CFA battle will have some effect on this time lapse. For example, if the covering force is successful in defeating the Red first echelon one can readily expect that the time required for the second echelon to prepare its assault against the MBA would be longer than if the first echelon could defeat the covering force. More time would be required to launch the second echelon attack if the Red force had to recover from the defeat

of its first echelon. However, when the first echelon attains victory over the covering force, the time required to attack the MBA should be less. This idea is modelled, depending on the CFA winner, by increasing the time of the start of the MBA battle from the end of the CFA battle by two different time increments; one smaller than the other, i.e., if the first echelon wins the CFA battle, the MBA battle starts 2.4 hours later (.1 days). If the covering force wins the CFA battle, the MBA battle starts 6.0 hours (.25 days) after the end of the CFA battle. This procedure gives an added advantage to the MBA force in the computation of its new attrition coefficient, b_2, when Blue wins in the CFA. When Blue does not win the CFA battle the time lapse is not of sufficient length to allow much more of an increase in combat effectiveness than attained over the actual duration of the CFA battle.

10) The movement of adjacent units to thicken the threatened MBA sector is modelled by first determining the maximum number of forces, left and right, that are available to move. If these forces can move at a constant rate then the number of reinforcements is a linear function with respect to time, until such a time that the maximum number can thicken the MBA. However, one must accept that the movement of reinforcing units cannot happen instantaneously, but only after the CFA battle has progressed beyond a certain time in which enemy intelligence can be processed, orders disseminated and

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movement begun. A Red penetration to the MBA (surprise) would find the MBA forces at their minimum force level $(B_o \cdot (1 - p)$ with no increase due to arriving lateral reinforcements. The maximum number of reinforcements available can be a variable input into the model. Graphically, this simple subroutine is depicted in figure 13.

11) The reinforcing units arrive in the threatened MBA sector in a continuous manner and not in discrete quantities as moving company-sized units would. They also arrive at a level of combat effectiveness that is lower than the original MBA forces since the reinforcing units have not had the same amount of time to prepare for the MBA battle. For ease in modelling the reinforcing units are incorporated into a homogeneous group with the covering force survivors. All elements of this group when in the MBA have the same attrition coefficient and distribution over the battlefield.

12) Breakpoints are included for both forces in both battles. The Red breakpoint fraction is constant over the entire defensive battle and has a value of $f_{BP}^R = .4$ for reasons stated in Chapter IV. The Blue breakpoint fractions are modelled differently between the CFA and the MBA. To reflect the tenacious battle that the covering force must fight and the possibility of its annihilation a low breakpoint fraction of $f_{BP}^{CFA} = .2$ is modelled. The MBA forces, on the other hand, are modelled with the standard defensive breakpoint fraction of $f_{BP}^{MBA} = .5$.

BLUE REINFORCEMENTS
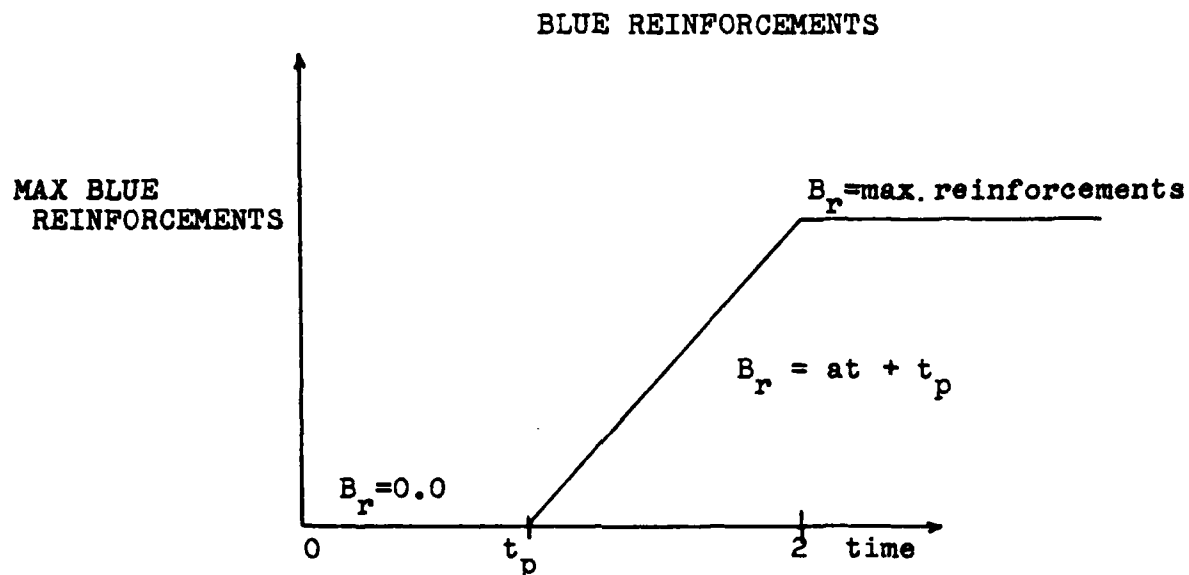
INCREASE IN BLUE REINFORCEMENTS OVER TIME

$t < t_p$ BLUE REINFORCEMENTS = 0.0

$t_p \le t \le 2.0$ BLUE REINFORCEMENTS INCREASE LINEARLY

$t \ge 2.0$ BLUE REINFORCEMENTS REACH MAXIMUM #

FIGURE 13

E. THE PROGRAM

The computer program is listed in Appendix C. The output
consists of 91 (for each value of $.1 \leq p \leq .9$) in increments
of .01) different force level attrition curves that show the
force decays over time of the total Red and Blue forces. The
jump in force levels shows the beginning of the MBA battle
when the Red force is increased by its second echelon and
the Blue force is increased by the MBA forces and reinforcing
units. Accompanying each force level attrition graph is a
list of initial force levels for each battle, CFA battle
duration time, the winner of each battle and number of sur-
vivors. If Red wins the battle a zero is printed. If Blue
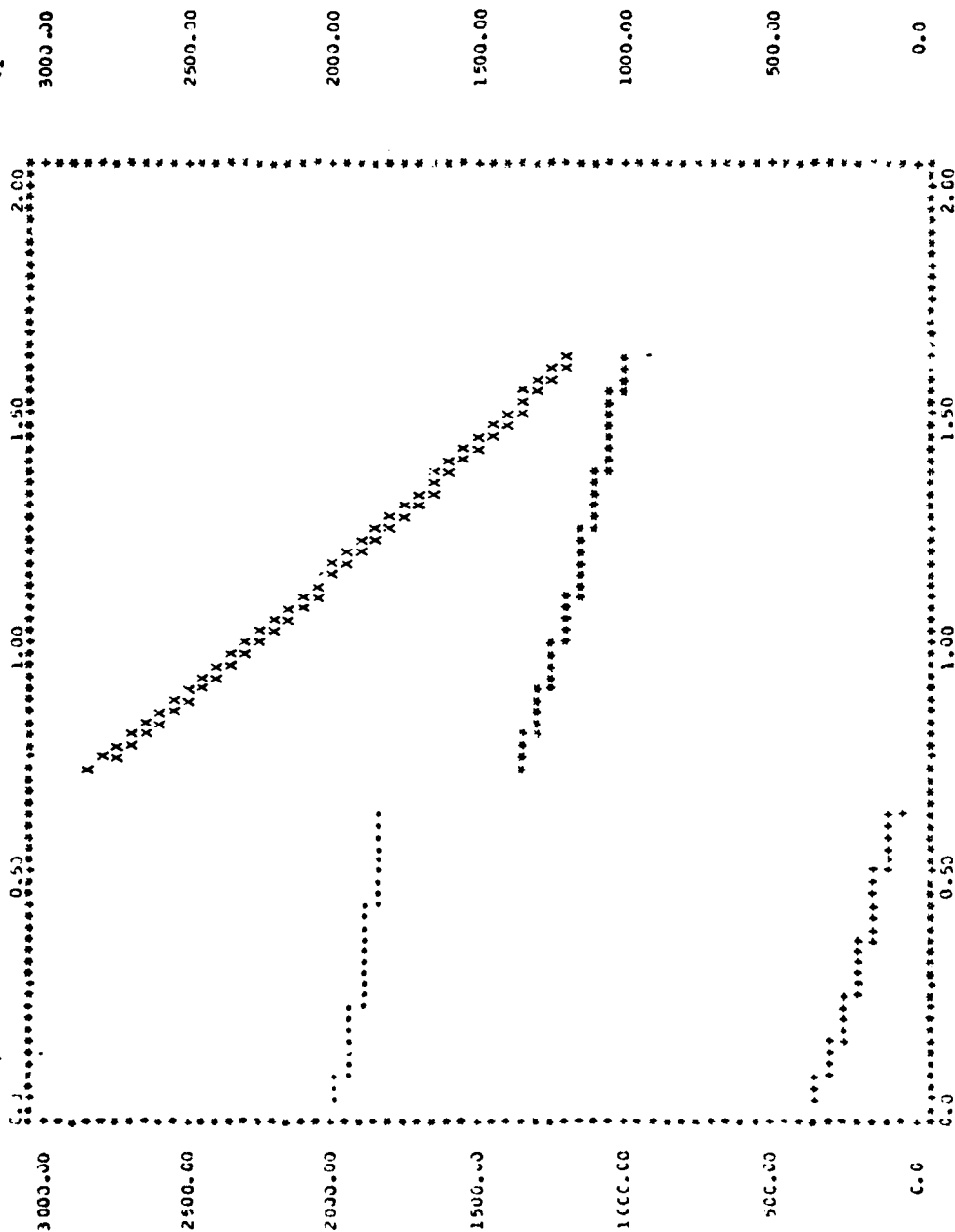wins the battle a one is printed (see figures 14 through 16).

At the end of the 91 sequential battles the values of
the measures of effectiveness are graphed versus the corres-
ponding value of p. A visual means is then provided for
determining the optimum range for the percentage of forces
to be committed forward as a covering force. The effects and
sensitivity of particular values of force levels, attrition
coefficients, and breakpoint fractions may be evaluated using
this program.

F. RESULTS

The graphs for the measures of effectiveness versus p for
the specific values of the input variables presented in Appen-
dix C (i.e., Red to Blue force level ratio is 2:1; 3000:1500)
are shown in figures 17 through 20.

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FIGURE 14 - FORCE DECAY GRAPH OF RED AND BLUE FORCES   (p = .25)

NOTE: BLUE LOSES BOTH BATTLES

FIGURE 15 - FORCE DECAY GRAPH OF RED AND BLUE FORCES (p = .30)
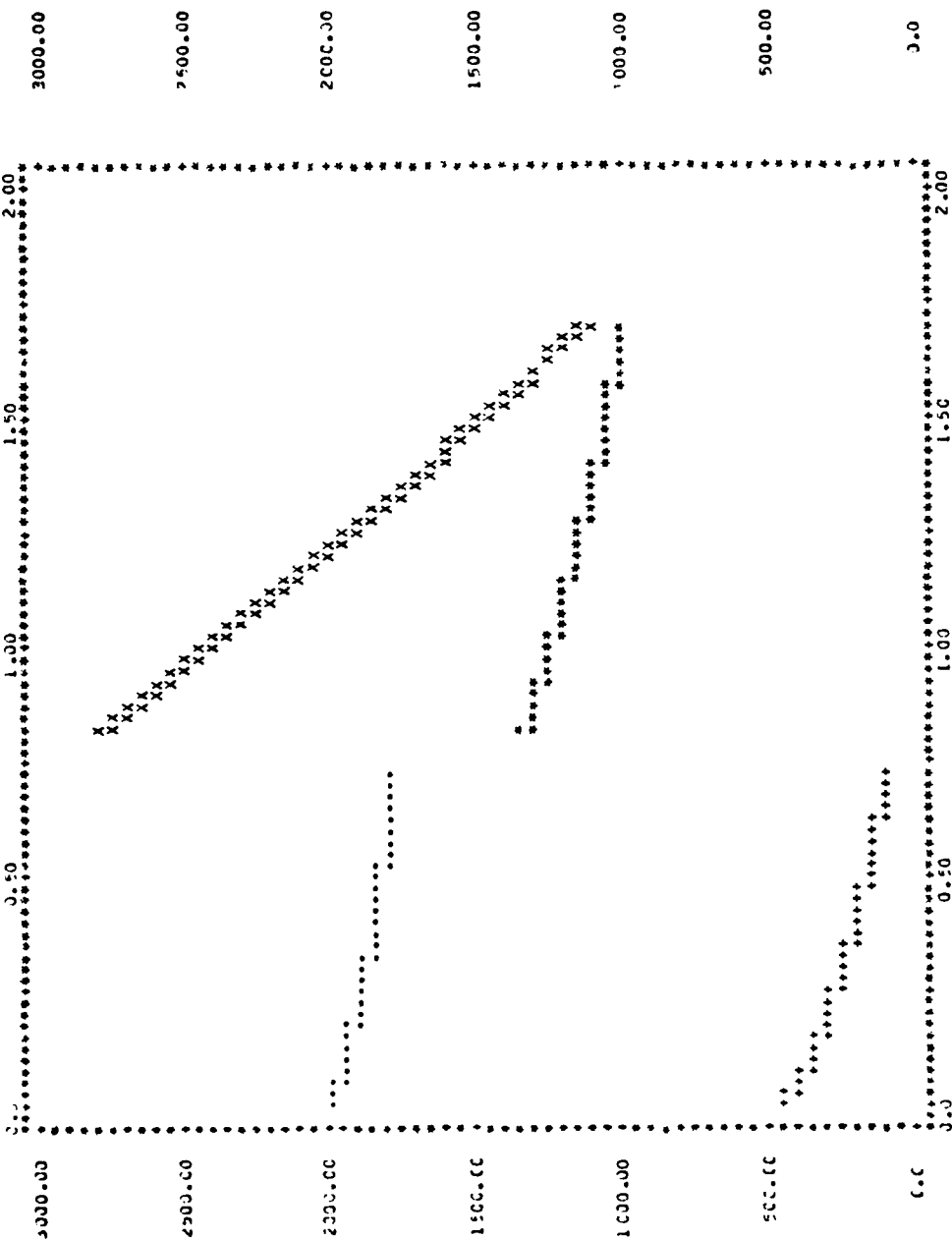
NOTE: BLUE LOSES THE CFA BATTLE BUT WINS THE MBA BATTLE

FIGURE 16 - FORCE DECAY GRAPH OF RED AND BLUE FORCES (p = .54)
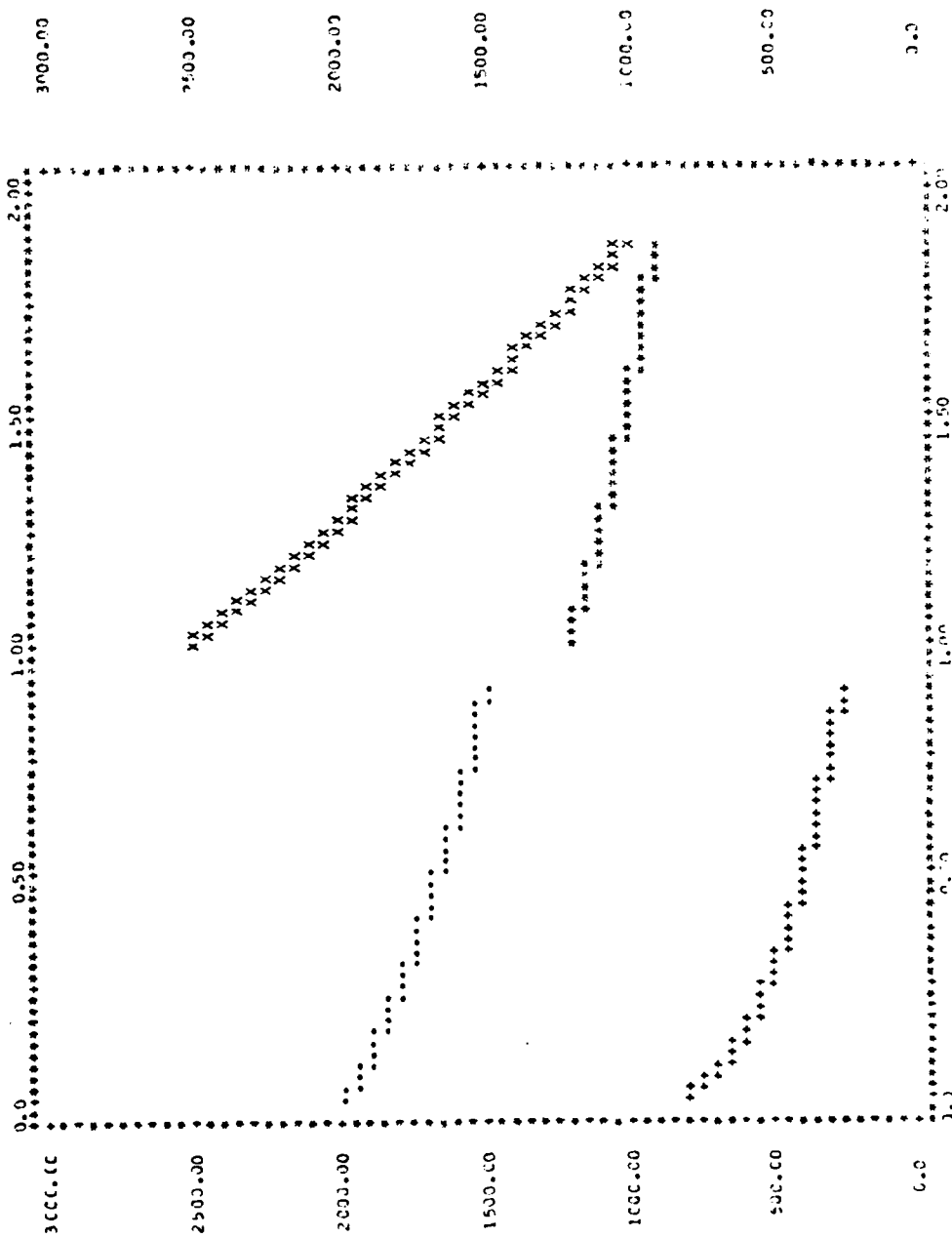
NOTE: BLUE LOSES CFA BATTLE BUT WINS THE MBA BATTLE

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FIGURE 17 - THE GRAPH OF BLUE LOSSES VERSUS p
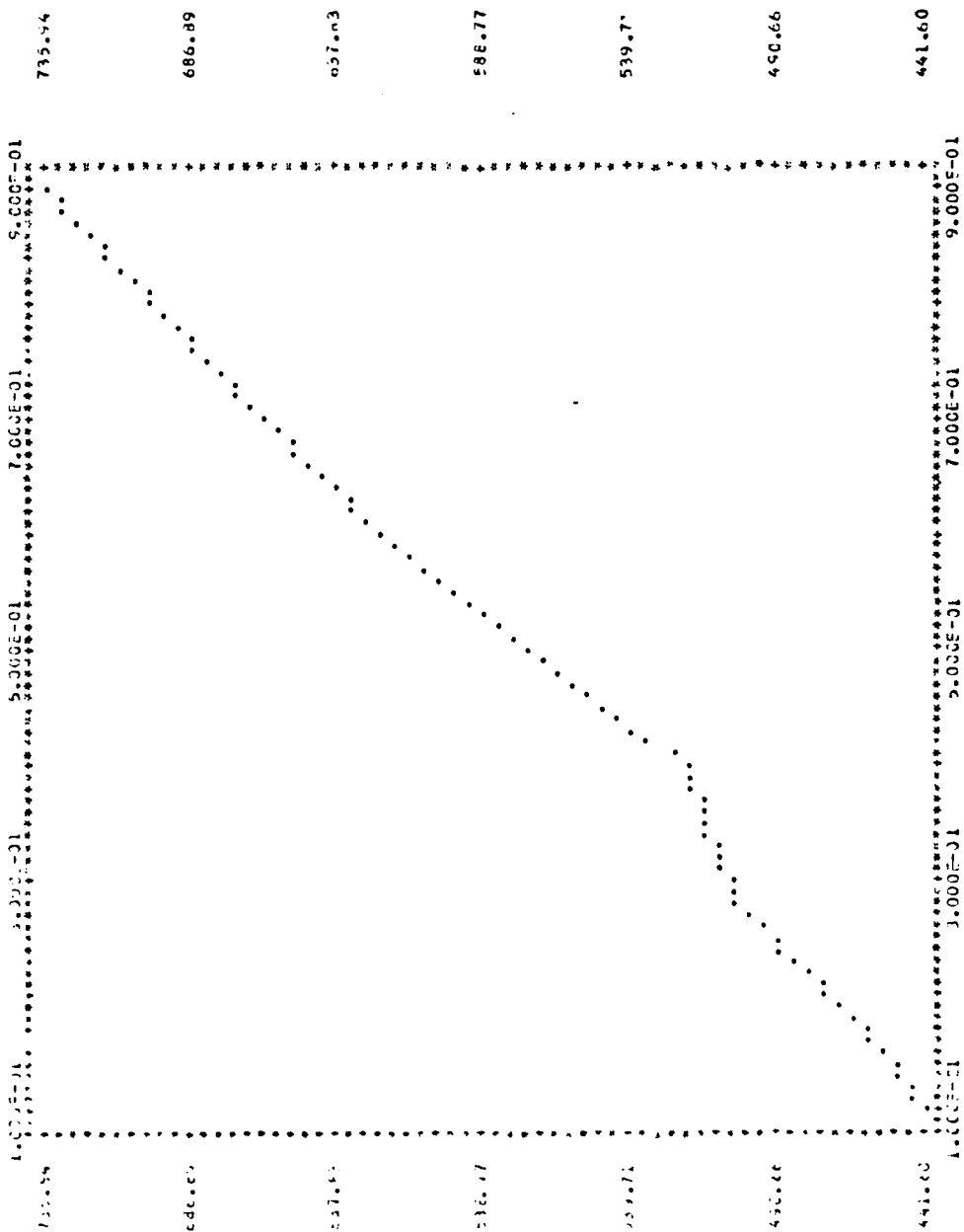
FIGURE 18 - THE GRAPH OF RED LOSSES VERSUS p
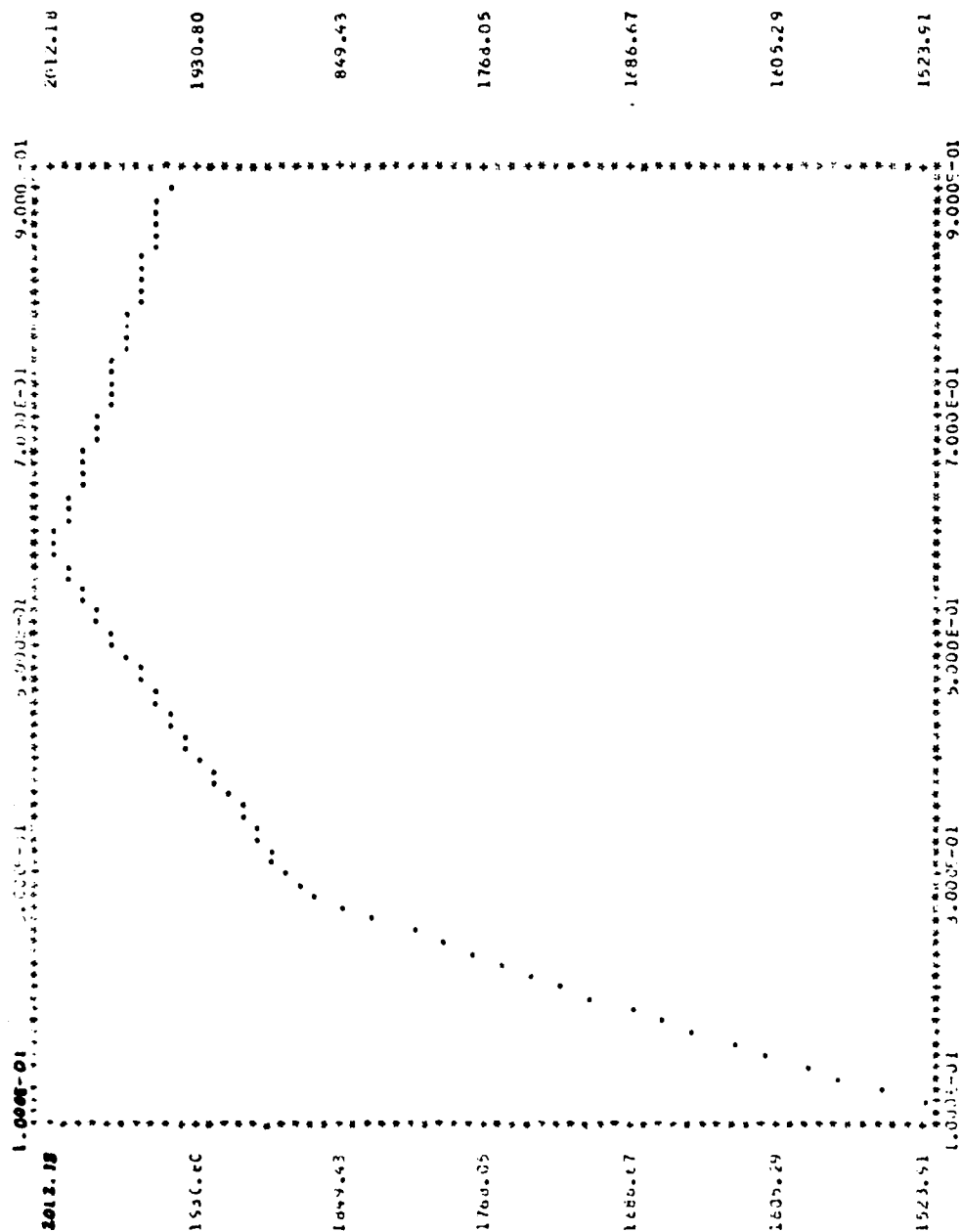
FIGURE 19 - THE GRAPH OF THE LOSS DIFFERENCE VERSUS p
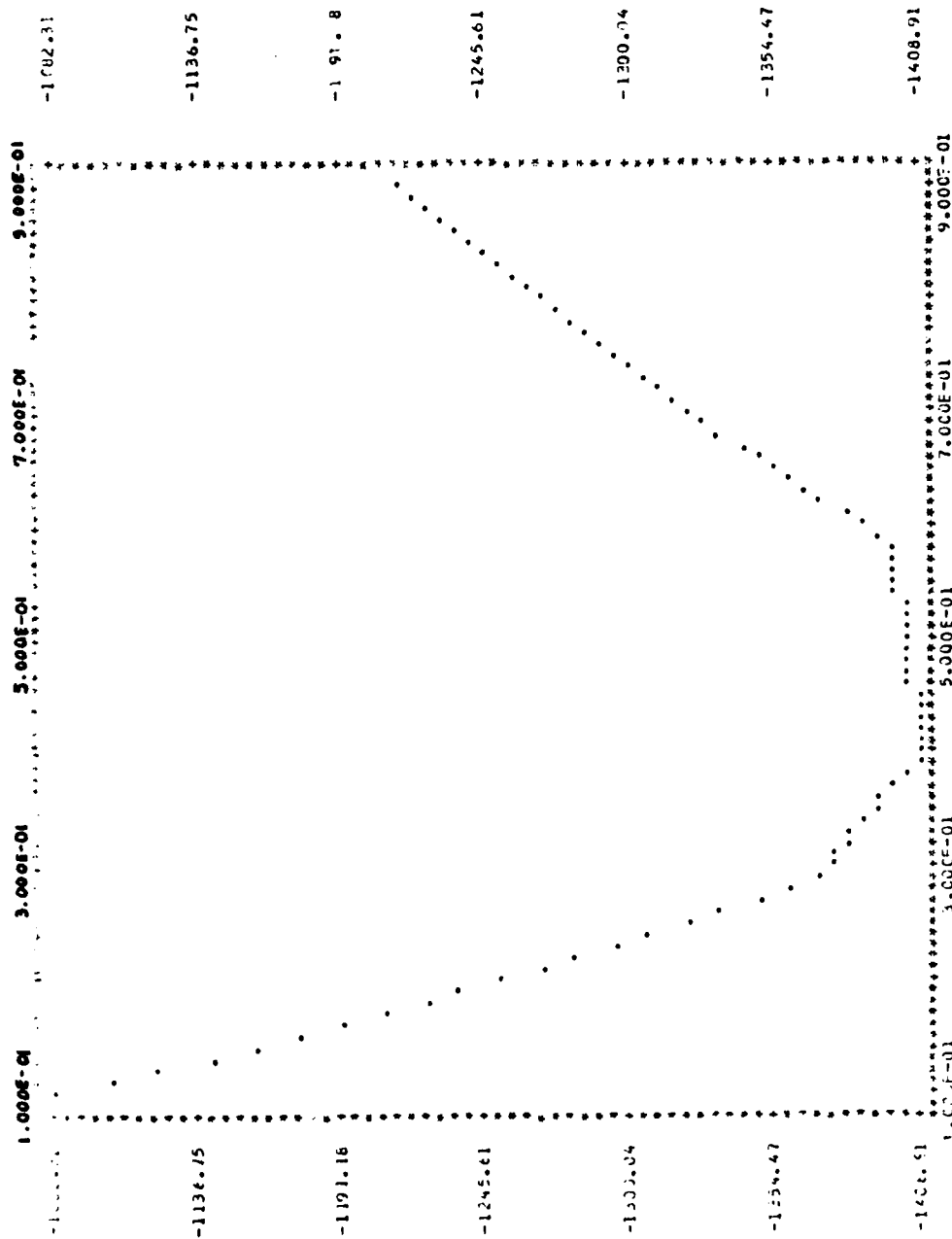
(MINIMUM VALUE IS IN THE RANGE .39 p .45)

FIGURE 20 - THE GRAPH OF THE LOSS RATIO VERSUS p
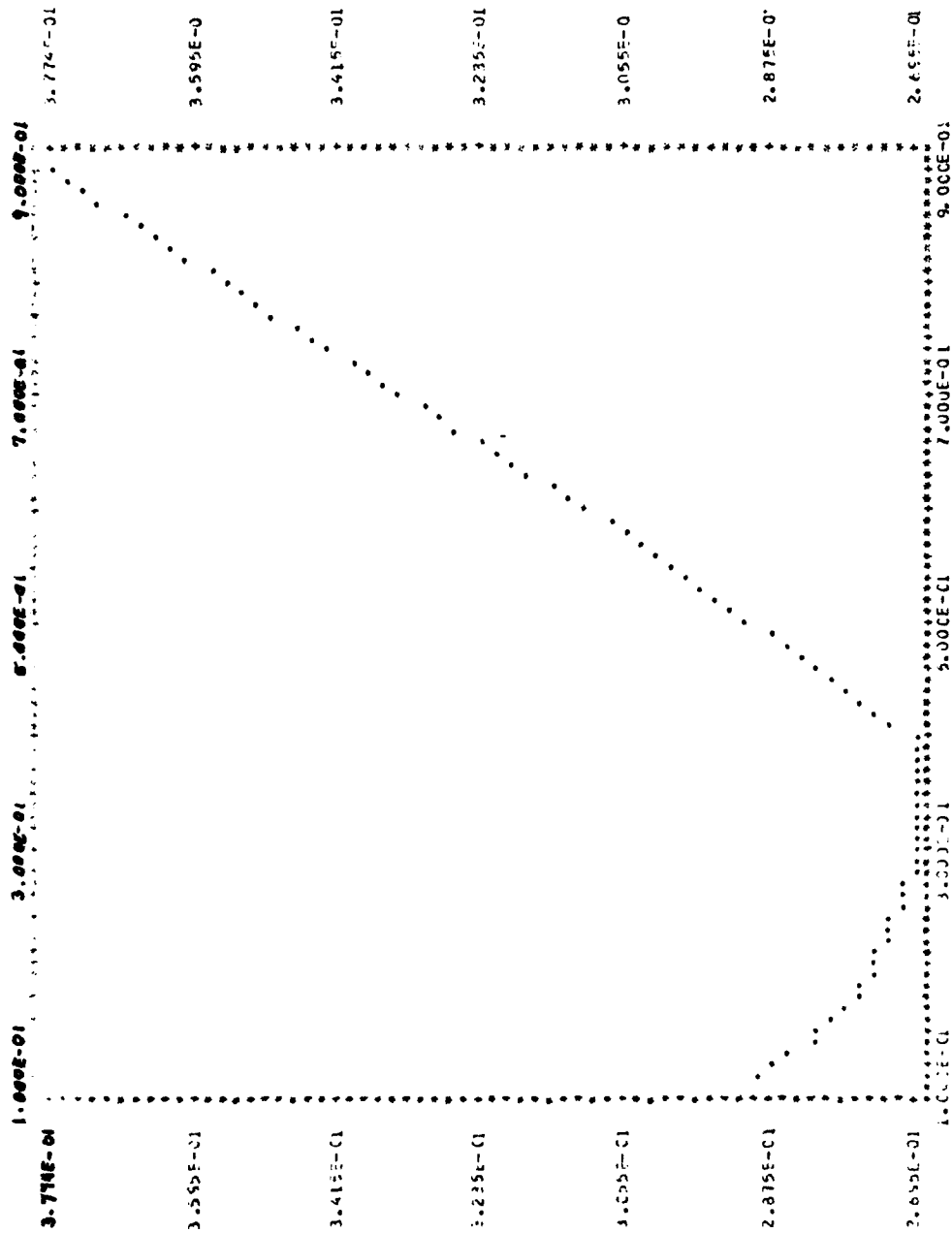
(MINIMUM VALUE IS IN THE RANGE .38 ≤ p ≤ .45)

As can be seen, the preferred Blue tactic is to commit forward as a covering force about 40% of its total force. The MBA battle was won by Blue in the p range, $.29 \leq p \leq .51$.

It must be emphasized that this optimum value of p is merely that specific value that is preferred given the specific values of the controlled variables. It is interesting that in this case and the subsequent cases used for sensitivity analysis the covering force never won the CFA battle.

G. SENSITIVITY ANALYSIS

Many additional runs of this model were made to determine its sensitivity to adjustments in the controlled variables (force levels, attrition coefficients, breakpoint fractions, and the maximum number of Blue reinforcements).

It was found that if the covering force is made to fight to annihilation the optimum value of p is about .30. This simply means that a smaller force fighting to annihilation is as effective as a larger force that breaks the battle prior to annihilation (see figure 21).

If the covering force is modelled to break off the CFA battle at a higher breakpoint fraction (i.e., $f_{BP}^{CFA} = .5$) it was found that the preferred tactic was to concentrate the majority of forces in the MBA, but at no time was the Blue side victorious in the MBA battle.

When the breakpoint fraction of the Red forces in increased (i.e., $f_{BP}^{R} = .6$) it was found that the preferred Blue tactic was to concentrate the majority of its forces in the MBA since

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FIGURE 21 - THE GRAPH OF THE LOSS RATIO VERSUS p
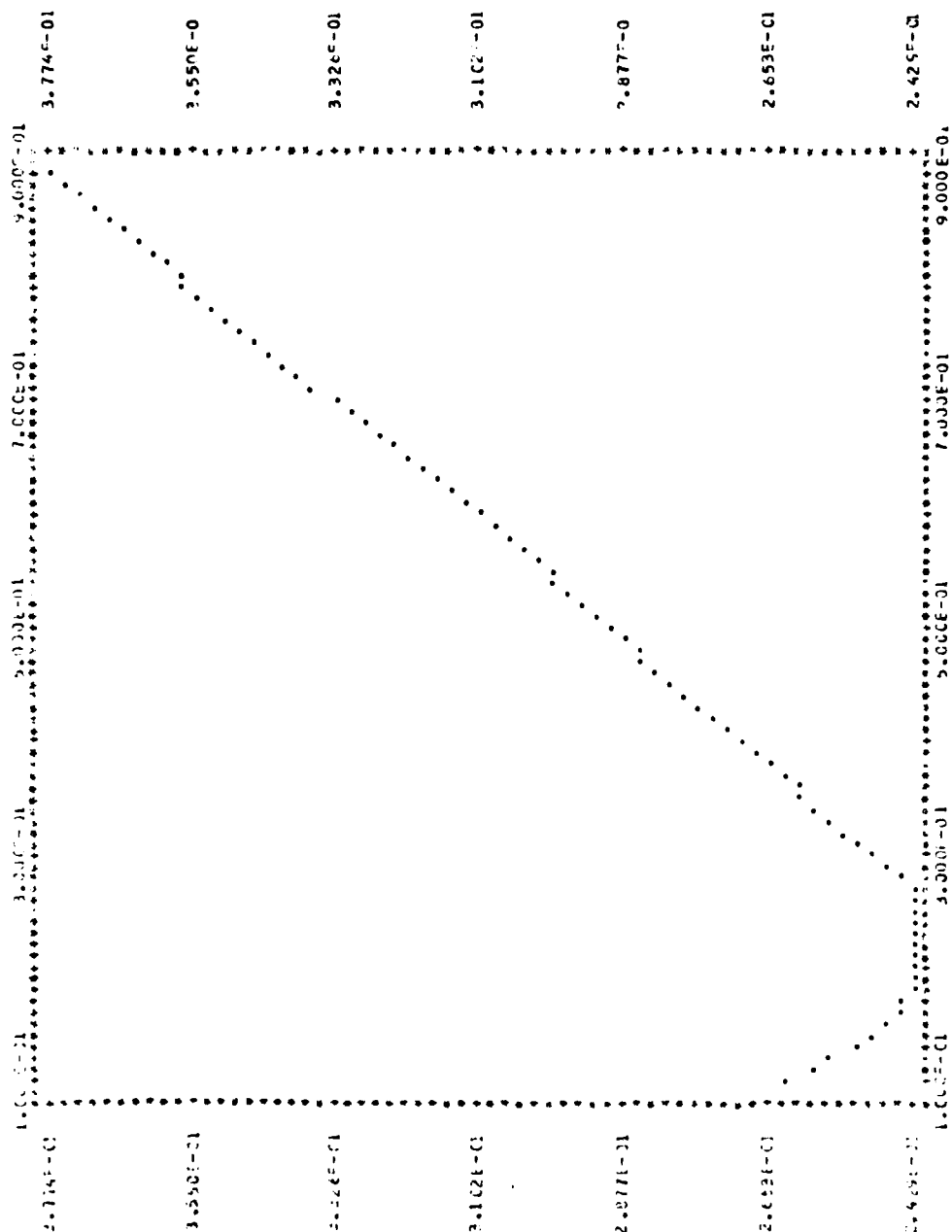
(SENSITIVITY ANALYSIS WITH FCF = 0.0)

the concentrated Blue forces were more effective at attriting the Red rapidly to its high breakpoint force level. No interior value of p was found to be optimum.

When the maximum number of available Blue reinforcements is increased a more precise value of p is attained (see figure 22).

When enemy superiority becomes too great (i.e., 3:1) in comparison with the Blue defensive forces and the maximum number of Blue reinforcing units, the preferred Blue tactic is to concentrate all forces in the MBA. However, sheer enemy numerical superiority defeats the Blue force in both battles for all values of p.

H. CONCLUSIONS

The more complex computer-based model presented in this chapter allowed for a more realistic portrayal of the defensive battle. The preferred Blue tactic is to commit 30% to 40% of all available forces forward as a covering force. If the Red forces could be routed easily or if the covering force were to fight to annihilation, it was found that fewer forces need be committed to the CFA. As enemy numerical superiority increased this model indicated that concentration in the MBA is the preferred tactic. In other words, using the language of mathematical programming, we may say that no "interior-point" solution was found for the optimal value of p (denoted here as p^*).

FIGURE 22 - THE GRAPH OF THE LOSS RATIO VERSUS p
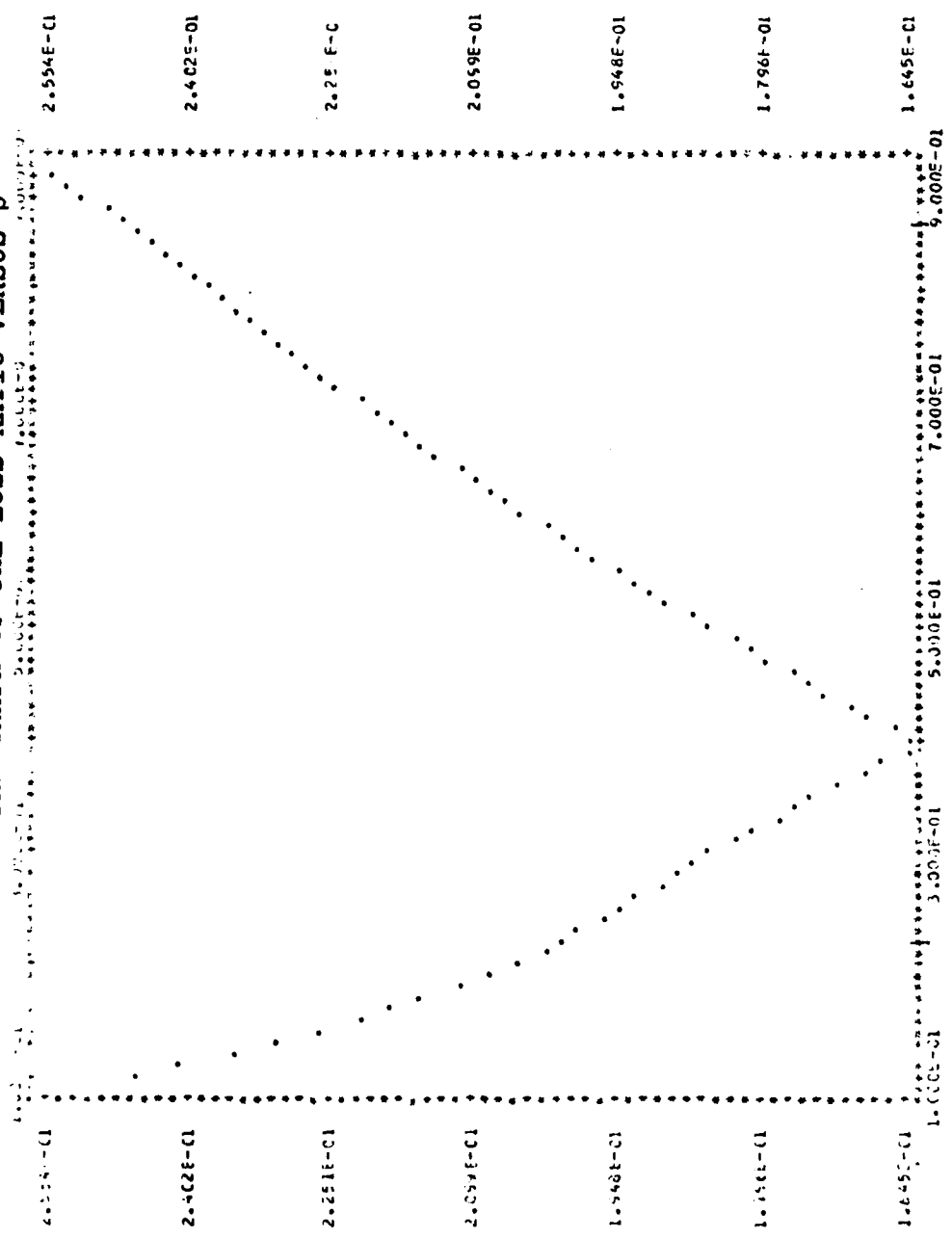
(SENSITIVITY ANALYSIS WITH BRMAX = 1000)

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VI. FINAL REMARKS

A. METHODS OF ANALYSIS

1. Basis of Analysis

The method of analysis used in this thesis depended upon using Lanchester-type differential equations to represent attrition in combat governed by the defensive doctrine of organizing the battlefield in two areas, the covering force area (CFA) and the main battle area (MBA). Initially a contradiction was apparent between the conclusions of Lanchester theory with regard to the benefits of concentration and the tactic of splitting a defensive force in order to deploy a covering force forward. This contradiction was sufficiently resolved to show that a Lanchester approach to the defensive battle could capture the advantages of such a tactic when key combat dynamics were incorporated into the model (i.e., time, increased combat effectiveness and thickening of the MBA in threatened areas).

2. Gamow-Zimmerman Model

Chapter III presented a simplistic and general approach to the concept of the two force defensive battle by viewing the covering force battle as an opportunity to move as many adjacent forces into the threatened MBA sector as possible with the time provided by the delaying actions of the covering force. The possible average speeds of these adjacent units was found to be a critical factor since the quicker the movement of these forces the more the force level could be

increased in the MBA over the duration of the CFA battle.
Within the limitations of this model, it was found that when
opposing sides were more nearly equal only a small fraction
need be deployed forward to provide the time required to in-
crease the MBA force level sufficiently to win the MBA battle.
However, when the defending force faced increased odds a
greater fraction of forces had to be committed to the CFA in
order to provide the time for a greater number of adjacent
forces to move into the MBA sector. As enemy numerical
superiority increased a greater percentage had to be committed
to the CFA to gain the needed movement time.

In order to achieve victory in the MBA, the minimum speed
of the reinforcing units and the percentage, p, of forces
committed to the CFA had to increase as the enemy's numerical
superiority increased. Simply stated, this result says that
the ability to concentrate in the MBA depends on time and
speed. The longer the duration of the CFA battle and the
quicker the adjacent forces can move, the greater the increase
in the force level in the critical MBA sector.

Unfortunately, the Lanchester-type equations are unable
to portray a gain in time by the covering force as being
achieved by any other means than an increase in the percen-
tage of forces committed to the CFA. If the attrition coef-
ficients are constant over time the only means of increasing
battle duration is by increasing the force levels. This is
why, when more time is required to reach a winning force level
in the MBA (at a fixed movement speed of adjacent units) a

greater percentage must be committed to the CFA. The key
point to consider from this insight is that when the cover-
ing force can gain time by means other than force on force
attrition, more forces can be retained in the MBA and the
need for the rapid relocation of forces to thicken threatened
MBA sectors is reduced. A natural means of achieving this
increase in CFA battle duration is by the maximum use of
obstacles and constricting terrain by the CFA forces. In
this fashion, a smaller covering force can gain time by
slowing the enemy advance to the FEBA using other means
than just continuous force on force combat.

3. The Computer Model

The speed and ease with which recursive complex
calculations can be accomplished on the digital computer per-
mitted the formulation of a model that could portray a more
detailed and realistic defensive battle. The transition
from area to aimed fire attrition in the CFA, the action of a
heterogeneous MBA force and the functional increase in combat
effectiveness and reinforcements over time was easily modelled
on the computer by using finite-difference approximation
equations. In addition, the nonlinear problem of determining
the optimum covering force commitment was solved by succes-
sively increasing the percentage of forces committed to the
CFA from 10% to 90%.

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B. KEY ASSUMPTIONS

1. A Two Force Defensive Battle

Probably the most critical assumption underlying
every model presented has been that the defensive battle
is composed of two separate and distinct battles; the CFA
battle and the MBA battle. This assumption has allowed the
separate application of Lanchester-type differential equations
to the CFA battle and then the MBA battle.

2. A Combat Advantage

The only means of manipulating the Lanchester-type
models in order to capture the key dynamics of the covering
force battle is to provide the MBA force a functional combat
advantage over the duration of the CFA battle. This combat
advantage has been reflected in two ways. These are the
increase in combat effectiveness reflected in an increased
attrition coefficient, and the increase in the force level
in the MBA due to the movement of available adjacent units
over the duration of the CFA battle. If these key dynamics
are omitted from the application of Lanchester theory it no
longer becomes an accurate tool for the evaluation of cover-
ing force effectiveness.

3. Homogeneous Forces

With the exception of the computer model, where ease
of calculation was not a problem, the need for simplicity and
transparency demanded that each phase of the defensive battle
be modelled as conflicts between homogeneous forces. Even in

the computer model the number of heterogeneous forces was kept at a minimum (i.e., two).

4. Enemy Numerical Superiority

Each model has assumed, at a minimum, an enemy numerical superiority of 2:1, where the enemy has attacked in two echelons.

5. Superior Friendly Combat Effectiveness

Each model has assumed that the Blue forces fight the battle with a superior combat effectiveness that is reflected in their larger constant attrition coefficient.

C. CONCLUSIONS

The conclusions of the analysis are as follows:

1) A Lanchester-type analysis is applicable when the basic theory is manipulated to include the key dynamics of the total defensive battle. There is no contradiction between accepted tactical defensive doctrine and the principle of concentration when a complete Lanchester model is constructed.

2) Time and enemy intelligence are the key contributions of the covering force to the total defensive battle.

3) The effectiveness of any covering force commitment can only be determined by the final outcome of the MBA battle.

4) The optimum percentage of defensive forces to be committed forward as a covering force is in the range of 30% to 40%. This depends greatly on the force levels, breakpoints, and the relative combat effectiveness of the opposing forces.

5) The covering force should prolong the duration of the CFA battle not only by force on force attrition, but also by slowing the enemy rate of advance through the maximum use of obstacles in the CFA. This additional time will allow the defensive forces to concentrate more forces in the threatened MBA sectors.

6) The greater the speed of reinforcing units the greater the force level can be increased in the threatened MBA sectors.

D. ENRICHMENTS AND FURTHER STUDY

This paper has presented only simple models to conduct its analysis. Possible enrichments and areas of further study are as follows:

1) Formulate a functional increase in the combat effectiveness that is more representative of reality than in increasing linear function.

2) Hypothesize battle termination criteria that is more accurate than the breakpoint fractions used in this paper (see Appendix B).

3) Portray the forces in the MBA as consisting of at least three different heterogeneous forces with different combat capabilities (i.e., the original MBA forces, the arriving reinforcements, and the covering force survivors).

4) The search for an optimum value of p, the fraction committed to the CFA, is essentially a nonlinear program that has been solved iteratively on the computer. The formulation of a nonlinear program that can be solved deterministically is an area for future research.

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5) A movement model may be incorporated into the computer model that will determine not only attrition but also movement of forces.

6) More extensive sensitivity analysis may be conducted to determine more exactly the affects of variation of such control variables as force levels, breakpoint fractions and attrition coefficients.
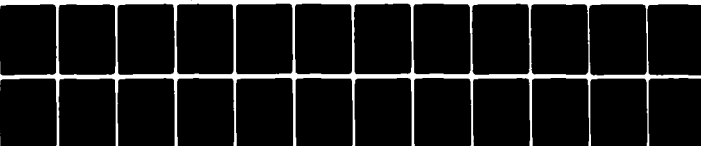
APPENDIX A

DEFINITION OF TERMS

1. <u>Covering Force Area (CFA)</u>: In defensive operations, the covering force area starts at the line of contact and ends at the forward edge of the Main Battle Area. Forces in the CFA are deployed to find the enemy and fight him with sufficient force to cause him to reveal the location of his main thrust. (Ref. 20).

2. <u>FEBA</u>: The forward edge of the (main) battle area (Ref. 21).

3. <u>Main Battle Area (MBA)</u>: That area extending back from the FEBA to the rear boundaries of the unit's subordinate elements. It is in this area that the decisive defensive battle will be fought (Ref. 22).

4. <u>Covering Force</u>:

a. A covering force provides the main body early warning, reaction time, maneuver space, and information about the enemy. It is a tactically self-contained security force which operates at considerable distance to the front of a moving or stationary force. Its mission is to develop the situation early and to defeat the enemy if possible. If the latter is not possible, then the covering force deceives, delays, and disorganizes the enemy until the main body can effectively react.

b. In defensive operations, a covering force operating apart from the main body has four basic tasks:

(1) Force the enemy into revealing the strength, location, and general direction of his main attack.

(2) Deceive the enemy or prevent him from determining the strength, dispositions, and location of friendly forces, especially in the MBA.

(3) Strip the enemy of his air defense umbrella, or force him to displace his air defense before attacking the MBA.

(4) Gain time for the main body, enabling it to deploy, move, or prepare defenses in the MBA (Ref. 22).

5. <u>Battle Position</u>: A location selected as a result of terrain and weapon analysis from which units can defend, block or attack. They can be selected for occupation by units as large as a task force or as small as a platoon (Ref. 23).

6. <u>Delay</u>: A defensive operation to fight an enemy force - usually in a specified area or a given sector and often for a specified period of time - in order to gain time for friendly forces to concentrate or deploy elsewhere (Ref. 24).

7. <u>Passage of Lines</u>: An operation in which one unit moves either forward or rearward through positions held by another friendly unit (Ref. 25).

APPENDIX B

SOME INDEPENDENT THOUGHTS ON BATTLE TERMINATION

As mentioned in Chapter I and in several of the refer-
ences, the battle termination criteria that adheres to the
breakpoint hypothesis depends upon attaining at some time
in the battle a certain force level below which a given unit
is no longer capable of completing its mission. It should
be noted that there is no consideration given to how quickly
the unit reaches this level. The rate of attrition, the
number of casualties sustained per unit time, may be rapid
or slow as the force level approaches its breakpoint. If a
battle is intense, with many casualties sustained in a small
interval of time, it is possible and realistic that a unit
would abandon the battlefield earlier (at a higher force level)
than if the battle were not very intense.

It is remembered from the brief discussion in Chapter I
that the attrition rate is expressed as $-\frac{dB}{dt}$, and is graphi-
cally the slope of the force level versus time curve at any
point (see figure 23). The more intense the battle, the
steeper the slope of the force level curve. If one compares
the force level graph of a unit in an intense battle with
the force level graph of the same unit in a less intense
battle, the breakpoint force level, B_{BP}, may be the same
but the slope of the curves will be different (see figure 24).
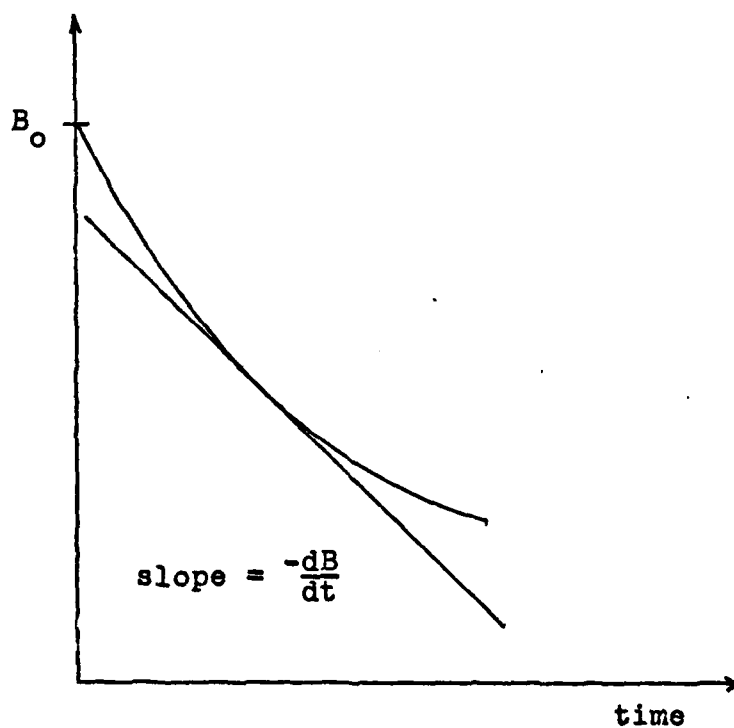
FIGURE 23

A GRAPHICAL REPRESENTATION OF THE ATTRITION RATE

INTENSE
(steep slope)

LESS INTENSE
(moderate slope)
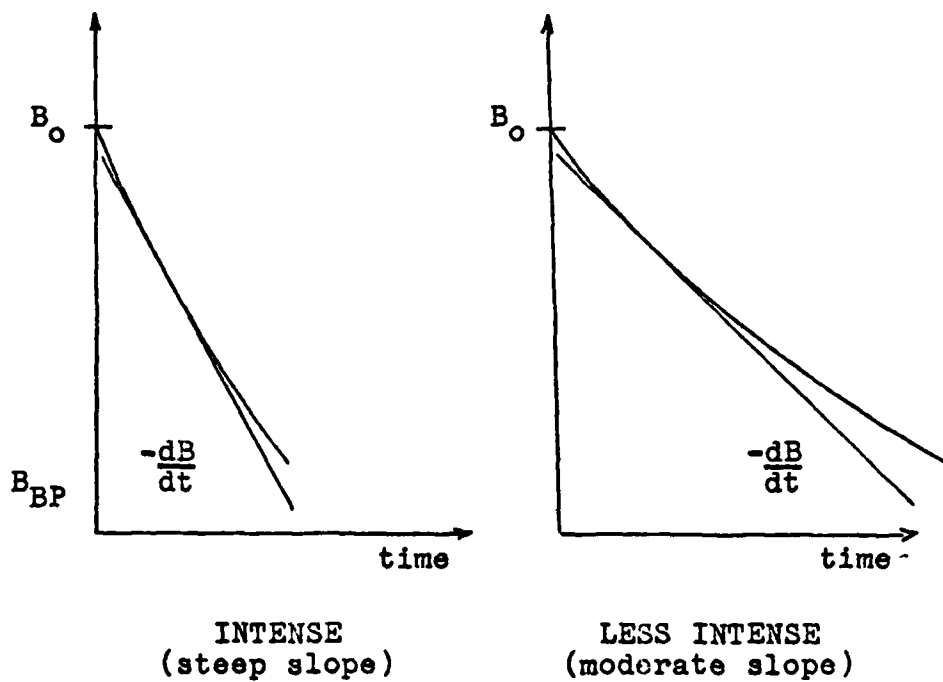
FIGURE 24

A COMPARISON OF TWO RATES OF ATTRITION
FOR DIFFERENT INTENSITIES OF BATTLE

Let us now consider a unit of initial force size, X_o, that is sufficiently prepared mentally and physically for combat. Let us believe that this is also a well disciplined unit that is capable of sustaining action in an intense combat environment for short periods without suffering any anciety at experiencing rapid attrition. Let us also assume that this unit has a breakpoint force level, X_{BP}, below which it is no longer combat effective.

It is submitted that a unit that is well disciplined and experienced can initially suffer a large number of casualties in a short period of time (a high attrition rate) and sustain no degradation in its fighting spirit. But, after a longer period of time of continuous high attrition a certain force level is reached below which a certain paranoia or anxiety sets in. At this force level the unit realizes that if casualties are to be continuously sustained at the current rate or at a more severe rate, then the decimation of its force in finite time becomes increasingly likely. If the attrition of this unit continues at a high rate then it is probable that the unit will abandon the field of battle (battle termination) at a force level higher than its hypothesized breakpoint, due solely to the attrition rate. However, if the intensity of the battle decreases and the attrition slows, it is more likely that the unit will retain its composure and continue to fight to its original breakpoint force level.

Graphically this situation is depicted in figure 25.
Irregardless of the attrition rate above the "anxiety level"
X_A, the unit will continue to fight. Below the anxiety
level, if the attrition rate is not too severe the unit will
not lose its composure, and will continue to fight until its
original hypothesized breakpoint, X_{BP}, is reached. But if
the unit sustains casualties at a fast rate (a high attrition
rate = a steep slope) the unit will lose its composure and
abandon the battlefield at a higher force level, X_H.

This relationship can be expressed mathematically as
Termination = $F(X)$ for $X_A < X < X_o$, where $X_A = f_A^X \cdot X_o$
($0 < f_A^X < 1$), f_A^X being the fraction of force size at which
anxiety sets in). The attrition rate makes no difference
above the amxiety level, X_A. But for $X \leqslant X_A$, any force
level below X_A, if the attrition rate $- \frac{dx}{dt}$ is greater than
some critical attrition rate $(- \frac{dx}{dt}_{crit})$ the unit, X, will
stop fighting.

Now the battle termination is a function of not only the
force level, but also the rate of attrition.

$$\text{Termination} = F \left(X, \frac{dx}{dt} \right)$$

It is easy to see that as the battle continues each
casualty sustained reduces the force level by one element.
If one considers that at each decrement of the force level
a new anxiety level is reached then a new, but less severe,
critical attrition rate could determine battle termination.

Thus, as the unit's force level decreases, the critical
attrition rate that can be sustained without breaking also
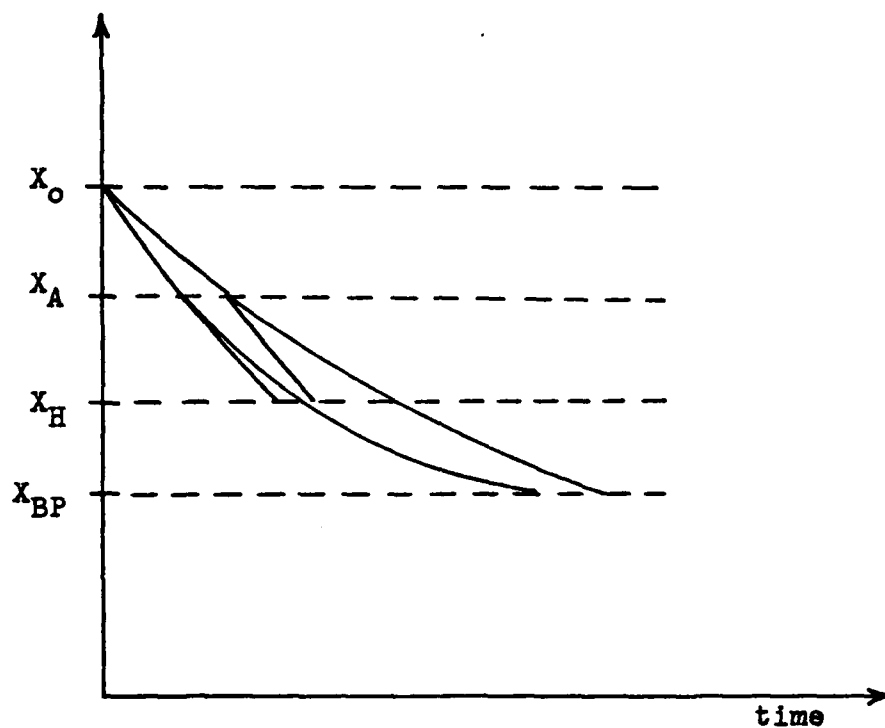
FIGURE 25

A unit with initial force size, X_O, may fight to its
breakpoint force level, X_{BP}. Or if its attrition rate
exceeds some critical value below the anxiety level, X_A,
it will break early at X_H.

BREAKPOINT VARIABILITY DUE TO ATTRITION RATE

decreases. Naturally, if the intensity of combat never
presents an attrition rate that exceeds this critical rate,
then the unit retains its composure and continues to fight
to its original breakpoint force level.

Graphically this decreasing value of the critical attri-
tion rate versus the force level is seen in Figure 26. As
the force level decreases below the anxiety level, the mag-
nitude of the critical attrition rate decreases. In real
terms this equates to the situation explained above, where a
unit in continuous combat is unable to face a certain attri-
tion rate as its force size decreases. The more its force
size decreases, the smaller becomes the rate of attrition it
can successfully sustain, without breaking.

No attempt will be made to present likely numerical values
for the critical attrition rates for any force level, or even
to postulate at what percentage the anxiety level should be
placed. However, a numerical example is in order.

Suppose that a tank company (17 tanks) occupies a defen-
sive position with the mission to defend. Assume that this
company has experienced combat and is disciplined and confi-
dent in its ability to sustain heavy combat. Further assume
that in modelling this company's fight an hypothesized break-
point of 30% of its original strength is proposed. The company
position is attacked by a superior force and the company begins
to take casualties in men and tanks. After 30 minutes of
fighting the company has lost four tanks and crews. The attri-
tion rate is 4 tanks/.5hr = 8 tanks/hr. At the end of this
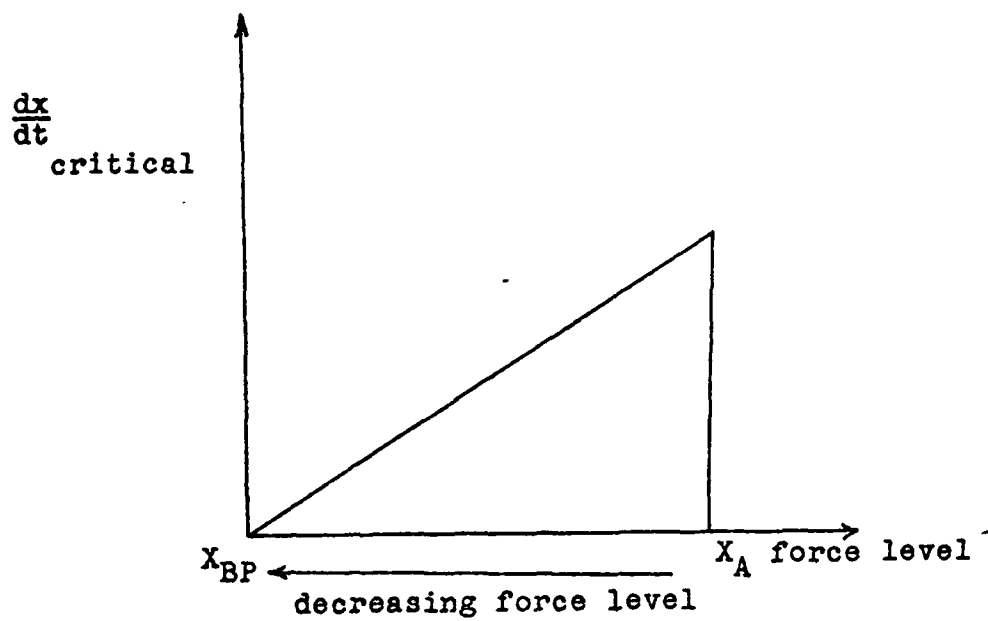
FIGURE 26

FUNCTIONAL RELATIONSHIP BETWEEN THE FORCE LEVEL
AND THE CRITICAL ATTRITION RATE

30 minute time period the company is at about 80% of its original strength (13 tanks). If the intensity of the battle continues at its present level, after 45 minutes the company has lost (3/4 hr) · (8 tanks/hr) = 6 tanks and is at about 65% of its original strength.

If the current battle intensity continues, the company should realize that in another hour and 15 minutes the company will be almost completely annihilated. At this point (65% of original strength) the disciplined unit reaches a force level below which a certain anxiety sets in. If the intensity of the battle decreases and only one tank is lost over the next 80 minutes (attrition rate = 1 tank/.5 hr = 2 tanks/hr) the company can retain its composure and continue to fight to its original hypothesized breakpoint, 30% of its initial strength. However, if the intensity of the battle continues or becomes more severe, then the company may lose four or five tanks in the next 30 minutes and abandon its position at a force level of about 40% of its initial strength. Figure 27 illustrates these two situations.

In this case the tank company will break from the battle at a higher force level than the hypothesized breakpoint due to the rate of attrition exceeding some tolerable level.

The chart shows:

$-\frac{dx}{dt} = 5$ tanks/hr

65%

$-\frac{dx}{dt} = 2$ tanks/hr

$-\frac{dx}{dt} = 8$ tanks/hr

X_H 40%

battle
termination

X_{BP} 30%

$.5$ 1 1.5 7.0

time (hours)
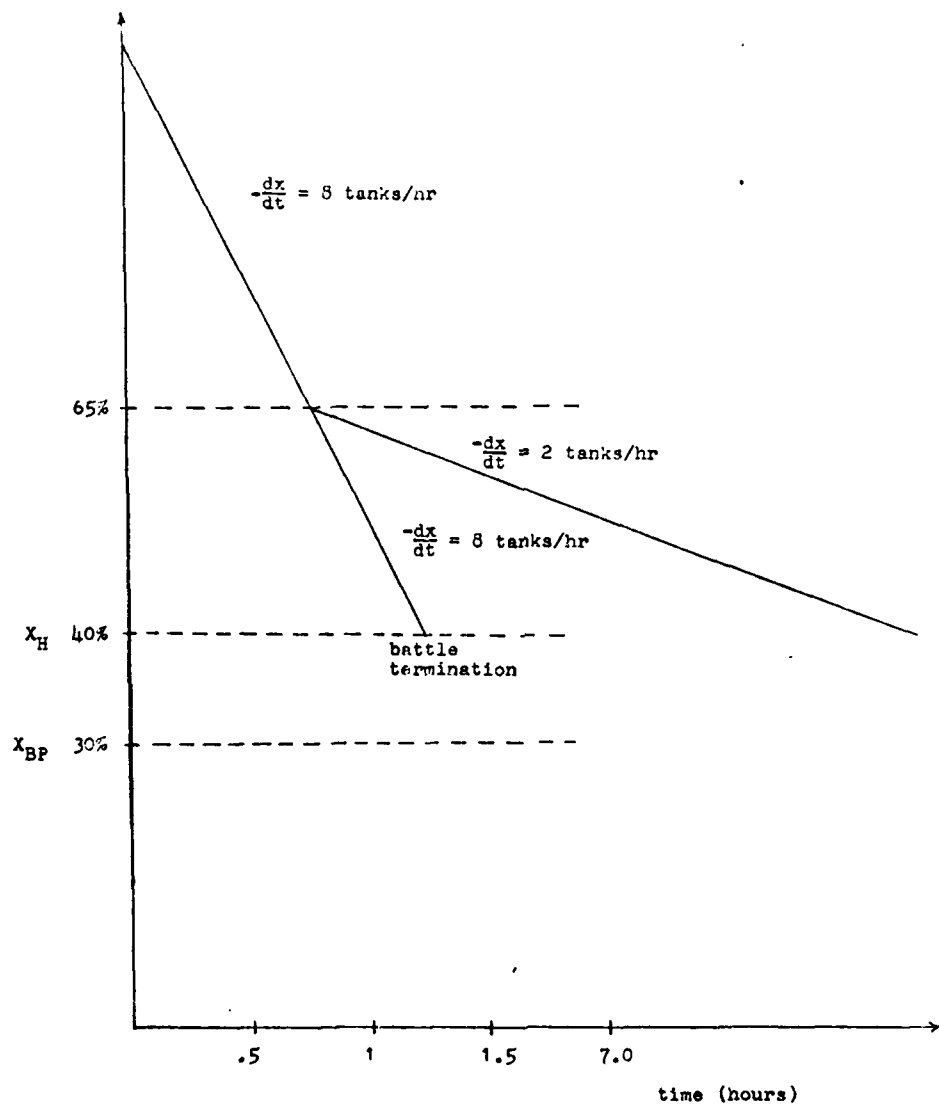
FIGURE 27

An illustration of a tank company breaking from an
engagement prior to its hypothesized breakpoint force
level due to a high attrition rate after passing its
anxiety level.

AN EXAMPLE OF A VARIABLE BREAKPOINT

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APPENDIX C

THE COMPUTER PROGRAM

The variables used in this program are listed in the order of their occurrence.

DT: the time increment

G: the transition factor that determines the speed of transition between area and aimed fire over the course of the CFA battle.

B: the initial Blue force level.

RR1: the initial Red first echelon force size.

R2: the initial Red second echelon force size.

BRMAX: the maximum number of available Blue reinforcements to the threatened MBA sector.

B1: the Blue covering force attrition coefficient.

R: the Red attrition coefficient, same for first and second echelon.

B2MAX: the maximum Blue attrition coefficient that can be attained by MBA forces.

B21: the attrition coefficient of the Blue MBA forces is they are surprised.

FR: the hypothesized breakpoint fraction of the Red forces (same for first and second echelon).

FCF: the hypothesized breakpoint fraction of the Blue covering force.

FMB: the hypothesized breakpoint fraction of the all Blue forces in the MBA.

TM: the time lapse between the end of the CFA battle and the start of the MBA battle when Blue wins the CFA battle.

TT: the time lapse between the end of the CFA battle and the start of the MBA battle when Red wins the CFA battle.

NU: the exponent of the functional increase in combat effectiveness as determined in subroutine NB2 (for a linear increase, $NU = 1$).

P: the percentage of Blue forces committed to the CFA ($0 < p < 1.0$).

BCFA(J):   the Blue covering force level at any instant of time.

R1(J):   the Red first echelon force level at any instant of time.

T1(J):   the time of the CFA battle at any instant.

TF1:   the duration of the CFA battle.

BCFAF: the Blue covering force level at the end of the CFA battle (covering force survivors).

TS: the time of the start of the MBA battle.

BMBA(M):   the force level of the original Blue MBA forces at any instant of time.

BR: the number of Blue reinforcing elements determined from Subroutine Thick.

RMBA(M):   the force level of Red forces attacking the MBA at any instant of time.

FMBAF(M): the total Blue force level in the MBA at any instant of time.

T2(M): the time of the MBA battle at any instant.

BMBAF: the Blue force level at the end of the MBA battle (Blue survivors).

RMBAF: the Red force level at the end of the MBA battle (Red survivors).

TF2: the duration of the entire defensive battle.

XLR: Red losses.

XLB: Blue losses.

XBR: Loss ratio.

XLD: Loss difference.

```
C     A PROGRAM THAT USES FINITE DIFFERENCE EQUATIONS TO APPROXIMATE A
C     LANCHESTER-TYPE ATTRITION MODEL THAT CAN BE USED AS A MEANS OF
C     ANALYSIS OF THE CONTRIBUTION OF THE COVERING FORCE TO THE DEFENSIVE
C     BATTLE.

      DIMENSION PCFA(2000),R1(2000),BMBA(2000),RMPACF(2000),TI(2000),
     1IT2(2000),RMPA(2000),XLR(81),XLC(81),XRR(81),XRR(81),XP(81),
     1IFMPAF(2000),RANGE(4)

C     INITIALIZE VARIABLES

      DT=.001
      MCOCUR=0
      X=0
      G=4.6
      B=1500.0
      RRFL=2000.0
      PF2=1000.0
      RFMAX=600.0
      R1=1.2
      R=0.2
      B2MAX=3.0
      R21=0.8
      FR=0.4
      FCCF=0.2
      FAB=0.5
      TM=.1
      TT=.25
      RL=1
      B2=3./(0.1*R)
      RANGE(1)=2.0
      RANGE(2)=0.0
      RANGE(3)=RR1+R2
      RANGE(4)=0.2
      WRITE(6,825) R,R1,RPL,R,R2,B2MAX,FR,FCF,FMF
      WRITE(6,350)
  350 FORMAT(1X,'NOTE: A ZERO INDICATES RED FORCES HAVE WON THE BATTLE'/
     1AND A ONE INDICATES BLUE FORCES HAVE WON THE BATTLE.')

C     INCREASE THE PERCENTAGE COMMITTED TO THE CFA FROM 10% TO 90% OF
C     TOTAL BLUE FORCE IN INCREMENTS OF 1%
```

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```
DO 600 I=1,900
RL=L
K2=I
K=K+1
XI=I
P=XI*.01
DO 600 J=1,900
```

CALCULATE THE INITIAL BLUE COVERING FORCE LEVEL : BCFA

```
BCFA(I)=B*P
```

DETERMINE INITIAL FORCE LEVEL OF RED 1ST ECHELON

```
R1(I)=RR1
```

INITIALIZE START OF THE CFA BATTLE AS TIME = C

```
T1(I)=0.0
```

CALCULATE THE TRANSITION FACTOR BETWEEN AREA AND AIMED FIRE AS A
FUNCTION OF TIME

```
E1=EXP(-G*T1(J))
```

FOR EACH TIME STEP OF SIZE DT DETERMINE THE NEW FORCE LEVEL OF
BLUE AND RED FORCES

```
BCFA(J+1)=BCFA(J)-(E1*RA*RCFA(J)+(1.0-E1)*R )*R1(J)*DT
R1(J+1)=R1(J)-R1*RCFA(J)*DT
```

INCREMENT TIME

```
T1(J+1)=T1(J)+DT
TF1=T1(J+1)
```

```
      BCFAF=BCFA(J+1)
      RIF=R1(J+1)
      N1=N1+1

C     DETERMINE BATTLE TERMINATION BY COMPARING INCREMENTAL FORCE
C     LEVELS VERSUS ESTABLISHED BREAKPOINTS

      IF(BCFA(J+1).LE.B*P+FCF) GO TO 610
      IF(R1(J+1).LE.RR1*FR) GO TO 620
600   CONTINUE
610   J1=0

C     IF RED WINS THE CFA BATTLE DETERMINE START TIME ,TS , OF MBA
C     BATTLE

      TS=TF1+TM
      GO TO 630
620   J1=1

C     IF BLUE WINS THE CFA BATTLE DETERMINE START TIME , TS , OF MBA
C     BATTLE

      TS=TF1+TT

C     DETERMINE THE INCREASED COMBAT EFFECTIVENESS OF MBA FORCES AS A
C     FUNCTION OF TS

630   CALL MB2 (TS,B2,R21,R2MAX,IND)

C     DETERMINE THE NUMBER OF REINFORCEMENTS THAT HAVE MOVED FROM ADJACENT
C     POSITIONS INTO THE THREATENED MBA SECTOR AS A FUNCTION OF TS

      CALL THICK (B,B2,R2MAX,TS)
      DO 700 N=1,500

C     CALCULATE INITIAL FORCE LEVEL OF BLUE MBA FORCES AND DIVIDE INTO
C     TWO HETEROGENEOUS FORCES WITH DIFFERENT COMBAT CAPABILITIES
```

```
RMRA(1)=R*(1.0-P)
RMRACF(1)=RCFAF+BR
QMRA(1)=R2+RIF
T2(1)=TS
FMRAF(1)=BMRA(1)+EMRACF(1)
BE=BMRA(N)+BMRACF(M)
```

FOR EACH TIME STEP OF DT DETERMINE THE NEW FORCE LEVELS OF BLUE
AND RED FORCES

```
RMRA(M+1)=RMRA(M)-(RMRA(M)/ED)*R*RMRA(M)*DT
BMRACF(M+1)=BMRACF(M)-(BMRACF(M)/EC)*R*RMPA(M)*DT
RMRACF(M+1)=RMRACF(M)-(B1*RMRACF(M)+B2*BMRA(M))*DT
FMRAF(M+1)=EMBA(M+1)+BMBACF(M+1)
```

INCREMENT TIME

```
T2(M+1)=T2(M)+DT
BMBAF=FMRAF(M+1)
FMRAF=RMBA(M+1)
TF2=T2(N+1)
N2=N2+1
```

DETERMINE BATTLE TERMINATION BY COMPARING INCREMENTAL FORCE LEVELS
VERSUS ESTABLISHED BREAKPOINTS

```
    IF (FMBAF(M+1).LE.FMRAF(1)*FMR) GO TO 710
    IF(RMRAF.LE.FRA(R2+RIF)) GO TO 720
700 CONTINUE
710 J2=0
    GO TO 730
720 J2=1
```

PLOT FORCE LEVEL ATTRITION CURVES FOR BOTH CFA AND MBA BATTLES
VERSUS TIME

```
730 CALL UTPLOT(TI,R1,N1,RANGE,1,1)
    CALL UTPLOT (TI,RCFA,N1,RANGE,1,2)
```

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```
      CALL UTPLOT (T2,FMBAF,N2,RANGE,1,2)
      CALL UTPLOT (T2,RMBA,N2,RANGE,1,3)
      WRITE(8,740)
740   FORMAT(3X,'GRAPH OF RED & BLUE ATTRITION VS TIME OVER CFA&MBA ')
      XF(K)=P

C     CALCULATE THE MEASURES OF EFFECTIVENESS FOR THIS ANALYSIS

C     RED LOSSES

      XLR(K)=FR1+R2-RMBAF

C     BLUE LOSSES

      XLE(K)=P-RMBAF

C     LOSS RATIO = BLUE LOSSES / RED LOSSES

      XFR(K)=XLB(K)/XLR(K)

C     LOSS DIFFERENCE = BLUE LOSSES - RED LOSSES

      XLC(K)=XLB(K)-XLR(K)

      WRITE (6,830)
830   FORMAT(2X,'CF',5X,'CF',2X,'WINNER (FA',2X,'TF',1X,'CF SURVIVORS',
     12X,'R1 SURVIVORS',1X,'TS',2X,'R2',3X,'BLUE MBA',2X,
     1'RED ATTACK',1X,'WINNER MBA',1X,'BLUE SURVIVORS',1X,
     1'FED SURVIVORS')
      WRITE (6,300) P,RCF1(1),J1,TF1,HCFAF,R1F,TS,B2,FMFAF(1),RMBA(1),
     1RMBAF,RMBAF
300   FORMAT(1X,F12.2,1X,F10.2,2X,F10.2,2X,I3,6X,F4.2,1X,F10.2,1X,F10.2,2X,F4.2,
     11X,F5.2,1X,F10.2,1X,F10.2,5X,I1,6X,F12.2,3X,F12.2)
      WRITE (6,50C)
800   CONTINUE
820   FORMAT(1X,'INITIAL BLUE FORCE=',F10.2,1X,'ATTRIT COEF=',F4.2,/,
     11X,'RED 1ST ECHELON,R1=',F10.2,1X,'ATTRIT COEF=',F4.2,/,
     11X,'RED 2D ECHELON,R2=',F10.2,1X,'WITH SAME ATTRIT COEF',/,
     11X,'MAX ATTRIT COEF POSSIBLE FOR BLUE MBA FORCE,B2MAX=',F5.2,/,
     11X,'RED BRKPT,FR=',F3.2,2X,'CONSTANT FOR R1 AND R2',/,
     11X,'BLUE CF BRKPT,FCF=',F3.2,3X,'BLUE MBA BRKPT,FMB=',F3.2,/,/,/)

C     PLOT MEASURES OF EFFECTIVENESS VERSUS P
```

```
      CALL PLOTP (XP,XLB,K,MODCUR)
      WRITE (6,440)
  440 FORMAT (30X,'GRAPH OF BLUE LOSSES VS P')
      WRITE (6,500)
      CALL PLOTP (XP,XBR,K,MODCUR)
      WRITE (6,450)
  450 FORMAT (30X,'GRAPH OF LOSS RATIO VS P')
      WRITE (6,500)
      CALL PLOTP (XP,XLD,K,MODCUR)
      WRITE (6,460)
  460 FORMAT (30X,'GRAPH OF LOSS DIFFERENCE VS P')
      WRITE (6,500)
      CALL PLOTP (XP,XLR,K,MODCUR)
      WRITE (6,470)
  470 FORMAT (30X,'GRAPH OF RED LOSSES VS P')
  500 FORMAT (1)
      STOP
      END
      SUBROUTINE NR2 (TS,B2,B21,B2MAX,NU)
C  SUBROUTINE THAT DETERMINES THE INCREASE IN PLUE COMBAT EFFECTIVENESS

      IF (TS.GE.0.0.AND.TS.LE..25) B2=B21
      IF (TS.GT.1.0) B2=B2MAX
      IF (TS.GT..25.AND.TS.LE.1.0) B2=(B2MAX-B21)*TS**NU+B21
      RETURN
      END
      SUBROUTINE THICK (E,BR,BRMAX,TS)
C  SUBROUTINE THAT DETERMINES THE # OF BLUE REINFORCEMENTS

      IF (TS.LE..25) BR=0.0
      IF (TS.GT.2.0) BR=BRMAX
      IF (TS.GT..25.AND.TS.LE.2.0) BR=(BRMAX/1.75)*TS-BRMAX/7.0
   50 RETURN
      END
```

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