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SYMMETRIC MISSILE DYNAMIC INSTABILITIES--A REVIEW

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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Dynamic instabilities observed for symmetric missiles and projectiles arise from a large variety of causes. These include unstable linear damping moments and differing nonlinear in-plane and out-of-plane damping moments for nonspinning re-entry vehicles, nonlinear Magnus moments for spinning missiles, and internal resonances with moving payload components. If aerodynamic trim is present, linear spin-yaw resonance can occur as well as nonlinear subharmonic motions and a number of other limit motions. This report gives a complete survey of these possibilities with a number of actual case histories.

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I. INTRODUCTION

Probably the first contribution of missile stability analysis was the placement of feathers on the arrow. This created a restoring moment and made the arrow a statically stable missile whose angular motion is approximated by a sine wave. Early bullet designers soon realized that feathers or fins are most inconvenient for gun launch and found it necessary to impart a high spin rate to the bullet or shell by means of rifled tubes. If sufficient spin is given the projectile, it is gyroscopically stable and its angular motion is approximated by the sum of two sine waves. From a dynamics point of view, the motion of a symmetric statically stable missile is a special case of the motion of a gyroscopically stable missile.

Until World War I, the primary concern of the designer was static or gyroscopic stability and the most important moments were the static moments, which were assumed to be linear in the angle of attack or sideslip:

$$C_m = C_{m_\alpha} \alpha + C_{m_0} \tag{1}$$

$$C_n = C_{n_\beta} \beta + C_{n_0} \tag{2}$$

The two transverse static moment terms can be combined into a convenient complex variables form:

$$C_m + i\, C_n = -i\, C_{M_\alpha} (\xi - \xi_a) + i\, \hat{C}_{M_\alpha} \bar{\xi} \tag{3}$$

where $\xi = \beta + i\alpha$

$$C_{M_\alpha} = (C_{m_\alpha} - C_{n_\beta})/2$$

$$\hat{C}_{M_\alpha} = (C_{m_\alpha} + C_{n_\beta})/2$$

$$\xi_a = (C_{m_0} + i\, C_{n_0})/(i\, C_{M_\alpha})$$

If $\hat{C}_{M_\alpha}$ is not zero, the aerodynamic moment of Eq. (3) is essentially asymmetric and the missile's angular motion has the complexity of aircraft motion. If both $\hat{C}_{M_\alpha}$ and ξ_a are zero, the aerodynamic moment is symmetric and the resulting angular motion is that of a body of revolution. For the intermediate case of zero $\hat{C}_{M_\alpha}$ and nonzero ξ_a, the missile has a slight aerodynamic asymmetry and its moment is that of a body of revolution with respect to the complex aerodynamic trim angle, ξ_a.

Since World War II, missile designers have encountered a number of surprises with regard to the dynamic stability of their designs. The dynamic stability is influenced by additional moment terms and determines the growth or decay of the oscillatory angular motion. In this paper, we will give a survey of dynamic instabilities that have been observed as well as some that are possible but not yet observed. For most of this paper, only symmetric missiles or missiles with slight asymmetries will be considered. A brief discussion of an almost symmetric missile ($|\hat{C}_{M_\alpha}| << |C_{M_\alpha}|$) as well as the effect of moving payloads will, however, also be given. Equal transverse moments of inertia will be assumed throughout the report.

II. NONSPINNING SYMMETRIC MISSILES

In addition to the static moment coefficients, the linear motion of a nonspinning missile is affected by four damping moment coefficients:

$$C_{m_q}, \; C_{n_r}, \; C_{m_{\dot\alpha}}, \; C_{n_{\dot\beta}}$$

For the oscillatory motion, the angular derivatives are related.

$$q \doteq \dot\alpha \quad , \qquad r \doteq -\dot\beta \tag{4}$$

The symmetry assumption, then, allows the following simple expression for the complete linear transverse moment:

$$C_m + i\, C_n = -\, i\, [C_{M_\alpha}\, \xi + (C_{M_q} + C_{M_{\dot\alpha}})\, \xi'\,] \tag{5}$$

where $\quad C_{M_q} = C_{m_q} = C_{n_r}$

$$C_{M_{\dot\alpha}} = C_{m_{\dot\alpha}} = -C_{n_{\dot\beta}}$$

The angular motion for a statically stable missile has the very simple form

$$\xi = K_1 e^{i\phi_1} + K_2 e^{i\phi_2} \tag{6}$$

$$K'_j = \lambda_j K_j \tag{7}$$

where

$$\lambda_j = - H/2$$

$$\phi'_j = \pm \sqrt{-M}$$

The motion is a damped ellipse with semi-major axis $K_1 + K_2$ and semi-minor axis $|K_1 - K_2|$. It is important to note that $\lambda_1 = \lambda_2$ implies that the eccentricity of the elliptical motion is maintained. Thus, initially planar motion remains planar and initially circular motion remains circular.

According to the definition of λ_j, the motion will be unstable if H is negative. The primary component of H is $C_{M_q} + C_{M_{\dot{\alpha}}}$ and, thus, this instability occurs when $C_{M_q} + C_{M_{\dot{\alpha}}}$ is positive. In other words, the aerodynamic moments act to increase the angular rates. This unexpected behavior has been observed at hypersonic speeds for slowly spinning re-entry shapes. This dynamic instability could be caused by the entropy gradient induced by the bow shock and reinforced by ablation[1]. At transonic speeds, unstable damping has been observed by MacAllister[2] and this has been explained by nose-induced separation[3].

An important feature of MacAllister's ballistic range measurements was that the initial almost-planar motion quickly became an oval almost-circular limit motion. The theoretical explanation of this limit motion requires the introduction of a rather strange damping moment. If we rewrite the linear damping moment terms of Eq. (5) using the polar form of the angle of attack, $\xi = \delta e^{i\theta}$, we have

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1. *L.E. Ericsson, "Unsteady Aerodynamics of an Ablating Flared Body of Revolution Including Effect of Entropy Gradient,"* *AIAA Journal 6*, *December 1968, pp. 2395-2401.*

2. *L.C. MacAllister, "Some Instability Problems with Re-entry Shapes," Ballistic Research Laboratories Memorandum Report No. 1224, August 1959, AD 377344.*

3. *L.E. Ericsson and J.P. Reding, "Dynamic Stability Problems Associated with Flare Stabilizers and Flap Controls,"* *Journal of Spacecraft and Rockets 7*, *February 1970, pp. 132-137.*

$$C_{M_d} = - i \left(C_{M_{\dot{\alpha}}} + C_{M_q}\right) (\delta' + i\theta' \delta)e^{i\theta} \tag{8}$$

Thus for linear theory, the damping moment in the plane of the total angle of attack is proportional to δ', the radial rate of change of this angle, the damping moment perpendicular to this plane is proportional to $\theta' \delta$, the circumferential rate of change of this angle, and the proportionality factors are equal.

The simplest nonlinear extension of this damping moment expression is

$$C_{M_d} = - i \left(d_0 + d_2\delta^2\right) (\delta' + i\theta' \delta)e^{i\theta} \tag{9}$$

A more general expression that retains the in-plane* and out-of-plane damping equality is obtained by making d_2 a function of δ^2. Even this assumption is not sufficient to generate the circular limit motion observed by MacAllister.

A successful approach[4] is to drop the equality of in-plane and out-of-plane damping:

$$\begin{aligned} C_{M_d} &= - i \left\{d_0 (\delta' + i\theta' \delta) + d_2\delta^2 \left[(1 + a)\delta' + i\theta' \delta\right] \right\} e^{i\theta} \\ &= -i \left[(d_0 + d_2\delta^2) \xi' + d_2 a\delta\delta' \xi\right] \end{aligned} \tag{10}$$

For constant d_2 and a, Eq. (10) introduces two cubic damping terms[5]. This nonlinear moment expression can be used in the usual quasilinear

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******An "in-plane moment" means a moment producing a rotation in the plane of the total angle of attack; the in-plane moment vector is thus normal to this plane. Similar remarks apply to the out-of-plane moment.*

4. *C.H. Murphy, "Circular Pitching and Yawing Motion of Nose Cone Configurations," Ballistic Missiles and Space Technology II, Pergamon Press, New York, 1961, pp. 328-336. (See also Ballistic Research Laboratories Report No. 1071, March 1959, AD 216341.)*

5. *C.H. Murphy, "Slender Body Estimates for Two Cubic Aerodynamic Damping Moments," AIAA Journal 4, March 1966, pp. 536-537.*

analysis[6-9]. According to this analysis, the nonlinear solution can be approximated by a solution of the form of Eqs. (6-7) in which the λ_j's become functions of the K_j's.

$$\lambda_1 = - [H_0 + H_2 (K_1^2 + a K_2^2)]/2 \tag{11}$$

$$\lambda_2 = - [H_0 + H_2 (K_2^2 + a K_1^2)]/2 \tag{12}$$

The behavior of a nonlinear solution can be described by trajectories in a K_1^2 vs K_2^2 amplitude plane. Since Eq. (6) for $\lambda_j = 0$ generates ellipses, each point in the amplitude plane identifies an elliptical motion and the trajectory through that point describes how this elliptical motion changes under the influence of nonlinear damping. Points on the amplitude plane axes represent circular motions and the line $K_1^2 = K_2^2$ is the locus of planar motions.

If H_0 and H_2 are opposite in sign, a circular singularity exists with amplitude $\delta_c = [-H_0/H_2]^{\frac{1}{2}}$. The amplitude plane for this case and for equal in-plane and out-of-plane damping (a = 0) is given in Fig. 1. The circular limit motions are unstable but there is a stable planar limit motion with amplitude $2\delta_c$. It can be shown that for $a < 1$ the circular motions are unstable but for $a > 1$ they are stable.

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6. *C.H. Murphy, "The Prediction of Nonlinear Pitching and Yawing Motion of Symmetric Missiles," Journal of the Aeronautical Sciences 24, July 1957, pp. 473-479. (See also Ballistic Research Laboratories Report No. 995, October 1956, AD 122221.)*

7. *C.H. Murphy, "Quasi-linear Analysis of the Nonlinear Motion of a Nonspinning Symmetric Missile," Journal of Applied Mathematics and Physics (ZAMP) 14, No. 5, September 25, 1963, pp. 630-643.*

8. *W.R. Haseltine, "Existence Theorems for Nonlinear Ballistics," J. Soc. Indust. Appl. Math. 11, September 1963, pp. 553-563.*

9. *C.H. Murphy, "Angular Motion of a Re-Entering Symmetric Missile," AIAA Journal 3, July 1965, pp. 1275-1282. (See also Ballistic Research Laboratories Report No. 1114, August 1960, AD 247271, and Ballistic Research Laboratories Memorandum Report No. 1358, June 1961, AD 266513.)*

Fig. 2 gives the amplitude plane for a = 2. Numerical integrations of the complete equations of motion verify that stable circular limit motions do exist for $a > 1$. Recently, several authors[10-13] have made a variety of wind tunnel measurements of out-of-plane damping and have shown that it can be quite different from in-plane damping for cones at supersonic and hypersonic speeds. Since $1 + a$ is the ratio of the planar damping to the circular damping, we see that this ratio must exceed two before stable circular motion can exist.

One common feature of Figs. 1-2 is that large-amplitude planar motions decay and small-amplitude motions grow. Although these motions go to different limit motions, the final motions are bounded. Thus, we could expect that for all cases when planar damping-in-pitch wind tunnel measurements show similar behavior, bounded flight motions would occur. Fig. 3, for H_0 and H_2 both negative and $a < -1$, shows that all motions go to large spiral motions although planar motions tend to the planar singular motion with amplitude $2[(1 + a)H_2/H_0]^{-\frac{1}{2}}$. This shows that intuitive arguments should be applied with care to nonlinear systems[14].

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10. *M. Tobak, L.B. Schiff and V.L. Peterson, "Aerodynamics of Bodies of Revolution in Coning Motion," AIAA Journal 7, January 1969, pp. 95-99.*

11. *L.B. Schiff and M. Tobak, "Results from a New Wind-Tunnel Apparatus for Studying Coning and Spinning Motions of Bodies of Revolution," AIAA Journal 8, November 1970, pp. 1953-1958.*

12. *G.W. Stone, E.L. Clark, Jr., and G.E. Burt, "An Investigation of Nonsymmetric Aerodynamic Damping Moments," AIAA Paper 72-29, San Diego, California, 1972.*

13. *O. Walchner and F.M. Sawyer, "'In-Plane' and 'Out-of-Plane' Stability Derivatives of Slender Cones at Mach 14," Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, Report ARL 73-0090, July 1973.*

14. *C.H. Murphy, "An Erroneous Concept Concerning Nonlinear Aerodynamic Damping," AIAA Journal 1, June 1963, pp. 1418-1419.*

III. LINEAR MAGNUS MOMENT

For a statically unstable missile such as a shell or bullet, high rates of spin are required for stability and a Magnus side moment must be added to the total aerodynamic moment. Since statically stable missiles are usually spun to reduce the effect of manufacturing asymmetries, this Magnus moment should be considered for finned missiles as well as bodies of revolution.

Before doing so, we must make a decision on the appropriate coordinate system. In order to avoid the algebraic complexities of a spinning missile-fixed XYZ axis system, we will use nonspinning aeroballistic coordinates $X\tilde{Y}\tilde{Z}$. The X-axis pitches and yaws with the missile. The $\tilde{Y}$-axis is selected to lie in the horizontal plane initially but it is very important to note that it does not remain there. If this axis were required to remain in the horizontal plane, the spin of the system would be nonzero and these axes would be called fixed-plane axes. Fixed-plane axes are useful when horizontal or vertical forces such as gravity are present[15]. Indeed, it has been shown that an ascending or descending missile with a constant horizontal control moment could have an instability due to the terms involving fixed-plane coordinates spin[16-17].

In aeroballistic coordinates, the complex angle of attack is written as $\tilde{\xi}$ and the transverse aerodynamic moment coefficients become

$$C_{\tilde{m}} + i\, C_{\tilde{n}} = \left[\phi' C_{M_{p_\alpha}} - i\, C_{M_\alpha} \right] \tilde{\xi} - i \left[C_{M_q} + C_{M_{\dot{\alpha}}} \right] \tilde{\xi}' \tag{13}$$

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15. C.H. Murphy, "Gravity-Induced Angular Motion of a Spinning Missile," Journal of Spacecraft and Rockets 8, August 1971, pp. 824-828. (See also Ballistic Research Laboratories Report No. 1546, July 1971, AD 730641.)

16. K.H. Lloyd and D.P. Brown, "Instability of Spinning Projectiles During Terminal Guidance," Journal of Guidance and Control 2, January-February 1979, pp. 65-70.

17. C.H. Murphy, "Instability of Controlled Projectiles in Ascending or Descending Flight," AIAA Paper 79-1669, August 1979. (See also USA ARRADCOM Ballistic Research Laboratory Memorandum Report No. 02915, April 1979, AD A072808.)

The first term on the right side of Eq. (13) represents the Magnus moment, which is proportional to the spin and the angle of attack. A positive Magnus moment coefficient represents a moment that acts to rotate the nose of the missile around the velocity vector in the direction of spin.

The presence of spin and Magnus moment does not change the form of the epicyclic solution of Eq. (6) but does give more complicated damping rates and frequencies.

$$\lambda_j = - \frac{H \phi_j' - P T + \phi_j''}{2 \phi_j' - P} \tag{14}$$

$$\phi_j' = \left[P \pm \sqrt{P^2 - 4M}\right] \Big/ 2 \tag{15}$$

Dynamic stability requires positive λ_j's and can be stated simply in terms of two stability factors, s_g and s_d, where

$$s_g = P^2/4M \tag{16}$$

$$s_d = 2T/H \tag{17}$$

The gyroscopic stability factor, s_g , is essentially the ratio of squared gyroscopic spin to static moment coefficient. Periodic motion occurs when this stability factor is greater than unity. The dynamic stability factor, s_d, is essentially the ratio of the Magnus moment coefficient to the sum of the damping moment coefficients. For dynamic stability[18]

$$1/s_g = \frac{4M}{P^2} < (2 - s_d)s_d \tag{18}$$

18. *C.H. Murphy, "Criteria for the Generalized Dynamic Stability of a Rolling Symmetric Missile," Journal of the Aeronautical Sciences 24, October 1957, pp. 773-774.*

Fig. 4 summarizes all the implications of this relation. Note that if s_d is outside the interval (0, 2), a statically unstable missile cannot be dynamically stabilized by spin and a statically stable missile can be made dynamically unstable by sufficiently high rates of spin. Very simple cone cylinders and finned cone cylinders have been shown to have s_d values outside this region[19]. The linear Magnus moment is largest for long projectiles with boattails at transonic speeds. Indeed, one 155mm developmental shell when fired at Mach numbers between 0.92 and 0.96 experienced a number of 3 km shorts due to Magnus instability.

IV. NONLINEAR MAGNUS MOMENT

In 1951, the Navy was faced with a very strange problem[20-21] during the development of the 12.75" antisubmarine ship-launched spinning finned rocket, the Weapon A. When fired to the port side of a high-speed destroyer, it performed well. When fired to the starboard side, however, its angular motion grew to a very large amplitude coning motion and its performance was completely unsatisfactory.

This dependence of missile stability on launch conditions is a characteristic of nonlinear differential equations and the cause, in the case of Weapon A, was a strongly nonlinear Magnus moment. The behavior can be easily predicted by the quasilinear theory for a simple cubic Magnus moment, i.e. a quadradic Magnus moment coefficient:

$$C_{M_{p_\alpha}} = \hat{c}_0 + \hat{c}_2 \delta^2 \qquad (19)$$

For this moment, the quasilinear exponential damping functions which determine trajectories in the amplitude plane become

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19. *C.H. Murphy, "Effect of Roll on Dynamic Instability of Symmetric Missiles," Journal of the Aeronautical Sciences 21, September 1954, pp. 643-644.*

20. *I.E. Highberg, "Suggested Mechanism for the Instability of Weapon A," NOTS TN-5036-94, 19 July 1951.*

21. *W.R. Haseltine, "Instability of Weapon A in Cross Winds," NAVORD Report 2057, September 1953, AD 023492.*

$$\lambda_1 = \lambda_{10} + \phi' \, b \, \hat{c}_2 \, (K_1^2 + 2 \, K_2^2) \tag{20}$$

$$\lambda_2 = \lambda_{20} - \phi' \, b \, \hat{c}_2 \, (K_2^2 + 2 \, K_1^2) \tag{21}$$

A possible amplitude plane for these λ_j's is given in Fig. 5. According to Eq. (15), the frequencies for a finned projectile ($M < 0$) are opposite in sign and thus its complex angular motion is described by the sum of oppositely rotating two-dimensional vectors. Gravity tip-off plus the crosswind produced by launch to the port of ships moving at thirty knots produces 10° amplitude clockwise angular motion. Launch to starboard produces 10° amplitude counterclockwise motion. If $\delta_c = 5°$, the motion associated with port launch lies to the left of the dashed curve (the separatrix) and will damp to a small-amplitude coning motion. Starboard launch is to the right of the separatrix and large-amplitude motion is successfully predicted by the theory.

V. AERODYNAMIC TRIM ($\xi_a \neq 0$)

The aerodynamic moment of a symmetric missile will not be zero at zero angle of attack if its body or fins are slightly deformed or its center of mass is not on its axis of symmetry. This trim moment causes the ξ_a in Eq. (3). In aeroballistic coordinates, the resulting trim angle rotates with the missile and has an amplitude that depends on the spin rate[22].

$$\tilde{\xi} = K_1 \, e^{i\phi_1} + K_2 \, e^{i\phi_2} + \xi_a \, k_3 \, e^{i\phi} \tag{22}$$

$$k_3 \doteq \phi_1' \cdot \phi_2' \left(\phi' - \phi_2'\right)^{-1} \left(\phi' - \phi_1' + i \, \lambda_1\right)^{-1} \tag{23}$$

For most spins, λ_1 can be neglected. Near resonance, $\phi' \doteq \phi_1'$, it is important and determines the maximum value of $|k_3|$. A plot of $|k_3|$

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22. *J.D. Nicolaides, "On the Free Flight Motion of Missiles Having Slight Configurational Asymmetries," Ballistic Research Laboratories Report No. 858, June 1953, AD 26405. (See also IAS Preprint 395, January 1953.)*

versus ϕ' is given in Fig. 6 and shows the need for missile designers to avoid resonance.

VI. INDUCED ROLL MOMENT ($\xi_a \neq 0$)

In general, roll motions are such that a missile's roll rate will vary through resonant spin rate and the missile will attain only a fraction of its maximum trim value[23-24]. If the roll moment at angle of attack is a function of θ, the roll orientation of the plane of the angle of attack[25], the roll moment coefficient can be written in the form

$$C_\ell = C_{\ell_0} + \phi' C_{\ell_p} + \delta C_{\ell_\delta}(\theta, \delta) \tag{24}$$

θ and δ can be computed from Eq. (22).

$$\delta e^{i\theta} = K_1 e^{i(\phi_1 - \phi)} + K_2 e^{i(\phi_2 - \phi)} + \xi_a k_3 \tag{25}$$

For most motions, θ varies rapidly and the rolling motion is unaffected by C_{ℓ_δ}. For pure trim motion ($K_1 = K_2$) or two-mode motion near resonance ($K_2 \doteq 0$, $\phi_1' \doteq \phi'$), θ is constant and C_{ℓ_δ} can have an important effect. Near resonance, δ can be quite large and the resulting induced roll moment (i.e., the third term in Eq. (24)) can force the roll rate to stay near resonance. This phenomenon of "roll lock-in" has been observed in flight as well as in a number of computer simulations.

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23. *R.J. Tolosko, "Amplification Due to Body Trim Plane Rotation," AIAA Paper 71-48, January 1971.*

24. *C.H. Murphy, "Response of an Asymmetric Missile to Spin Varying through Resonance," AIAA Journal 9, November 1971, pp. 2197-2201. (See also Ballistic Research Laboratories Report No. 1545, July 1971, AD 729772.)*

25. *J.D. Nicolaides, "Two Non-linear Problems in the Flight Dynamics of Modern Ballistic Missiles," IAS Report 59-17, January 1959.*

The induced roll moment can be caused by aerodynamic asymmetry present in a "symmetric" four-finned missile or by mass asymmetries in an aerodynamically symmetric missile. For example, the lift force on a symmetric missile at angle of attack acts on the center of mass of the missile. If the center of mass is not on the missile's axis of symmetry, it will produce a roll moment of the form of Eq. (24). It should be emphasized that the occurrence of "roll lock-in" depends on the details of the pitching motion and its coupling to the rolling motion through Eq. (24) and can only be determined by numerical integrations[26-28].

VII. INDUCED SIDE MOMENT ($\xi_a \neq 0$)

Although flight failures have been explained by the occurrence of resonance through "roll lock-in," in some cases angles of attack have been observed much larger than those predicted by Eq. (23). In 1959, Nicolaides[25] developed his "catastrophic yaw" theory by the introduction of induced side moments. The existence and the effect of these moments have been discussed by other authors[29-31]. Nicolaides' induced side moment term can be included in the aerodynamic moment expression of Eq. (13) by adding

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26. *L. Glover, "Effects on Roll Rate of Mass and Aerodynamic Asymmetries for Ballistic Re-entry Bodies," Journal of Spacecraft and Rockets 2, March-April 1965, pp. 220-225.*

27. *D.A. Price, Jr., "Sources, Mechanisms, and Control of Roll Resonance Phenomena for Sounding Rockets," Journal of Spacecraft and Rockets 4, November 1967, pp. 1516-1525.*

28. *D.A. Price, Jr., and L.E. Ericsson, "A New Treatment of Roll-Pitch Coupling for Ballistic Re-Entry Vehicles," AIAA Journal 8, September 1970, pp. 1608-1615.*

29. *F.J. Regan, V.L. Shermerhorn and M.E. Falusi, "Roll-Induced Force and Moments Measurements of the M823 Research Store," NOLTR 68-195, November 1968.*

30. *T.A. Clare, "Resonance Instability for Finned Configurations Having Nonlinear Aerodynamic Properties," Journal of Spacecraft and Rockets 8, March 1971, pp. 278-283.*

31. *T.R. Pepitone and I.D. Jacobson, "Resonant Behavior of a Symmetric Missile Having Roll Orientation-Dependent Aerodynamics," Journal of Guidance and Control 1, September-October 1978, pp. 335-339.*

$$\left[C_{SM_\alpha} (\theta, \delta) \right] \tilde{\xi}$$

to the right side of Eq. (13).

For flight conditions for which θ is constant, the primary effect of this term is to change λ_1 to

$$\lambda_1 = - \frac{H \phi_1' - P T + \phi_1''}{2 \phi_1' - P} - b C_{SM_\alpha} \tag{26}$$

The presence of the induced side moment coefficient in Eq. (26) introduces the possibility of a very small λ_1, which can cause a very large resonance value of k_3 in Eq. (23). An even worse possibility is a large positive value of λ_1, which would cause an exponential growth of k_3. This possibility is the "catastrophic yaw" of Nicolaides and may be the cause of some spectacular flight failures.

VIII. NONLINEAR AERODYNAMIC MOMENT ($\xi_a \neq 0$)

The combination of nonlinear aerodynamic moments with a trim moment can give rise to a rich variety of limit motions. Many of these motions have been produced by computer simulations but as yet none have been observed in flight. In this section, we will briefly consider the effect of (a) a cubic static moment, and (b) two cubic damping moments.

First, the static moment coefficient is assumed to have the form:

$$C_m + i C_n = - i \left[(c_0 + c_2 \delta^2) \xi - c_0 \xi_a \right] \tag{27}$$

For pure trim motion, this assumption replaces the linear Eq. (23) by a cubic equation for k_3. Near resonance, three values of k_3 can be computed but only two correspond to stable motion. Much more interesting limit motions have been found which are generalized subharmonic motions. For these motions, certain constant values of K_1, K_2, k_3 have been predicted and produced by computer integrations for spins far from resonance as well as quite near to resonance[32-33].

A second set of limit motions can be constructed for the damping moment expansion of Eq. (10):

$$C_m + i\, C_n = - i\, C_{M_\alpha} (\xi - \xi_a) + C_{M_d} \qquad (28)$$

A number of one-, two-, and three-mode limit motions have been predicted by the quasilinear theory and computed by numerical integration of the equations of motion[34].

IX. MOVING INTERNAL PARTS

In 1955, an eight-inch shell showed a strange spin decay coupled with a significant range loss[35] (Fig. 7). The range loss was due to a growth in the high-frequency component of the pitching motion ($\lambda_1 > 0$). Recent ballistic range tests of a developmental 20mm projectile with the M505 fuse have shown a growth of the high-frequency mode and unexplained spin decay[36]. In both cases, the projectiles carried a component that could move a small amount during flight.

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32. *C.H. Murphy, "Generalized Subharmonic Response of a Missile with Slight Configurational Asymmetries," AIAA Paper 72-972, September 1972. (See also Ballistic Research Laboratories Report No. 1591, June 1972, AD 749787.)*

33. *C.H. Murphy, B.A. Hodes and J.W. Bradley, "Stability of Subharmonic Limit Motions of a Slightly Asymmetric Missile," Ballistic Research Laboratories Memorandum Report No. 2494, June 1975, AD B005079.*

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These phenomena have been explained[37-38] by assuming either (1) a forced circular motion of an internal part about the axis of the projectile (hula-hoop motion) or (2) a forced precession of the spin axis of the internal part about the spin axis of the projectile. In both cases, only the Fourier component of the motion at the higher coning frequency of the projectile was considered. Thus, a resonance was considered for which the amplitude of the internal motion was constant.

The theory predicted a relationship between the growth of the higher frequency mode and the spin decay and in both cases excellent quantitative agreement was obtained. Therefore, the derived expressions can be used to set tolerances for manufacture of these projectiles.

X. ALMOST SYMMETRIC MISSILES

Throughout this report we have assumed that $C_{M_{\tilde{\alpha}}}$ in Eq. (3) was zero and neglected the essential asymmetry present when the pitch and yaw frequencies are not equal for a nonspinning missile. The influence of this airplane-like asymmetry on stability of a spinning vehicle was studied by Phillips[39] in 1948 and developed further in two recent very elegant papers by Hodapp[40-41]. The concept of an almost symmetric missile[42] was introduced in 1978 to show how a "little bit"

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37. *W.G. Soper, "Projectile Instability Produced by Internal Friction," AIAA Journal 16, January 1978, pp. 8-11.*

38. *C.H. Murphy, "Influence of Moving Internal Parts on Angular Motion of Spinning Projectiles," Journal of Guidance and Control 1, March-April 1978, pp. 117-122. (See also Ballistic Research Laboratory Memorandum Report No. 2731, February 1977, AD A037338.)*

39. *W.H. Phillips, "Effect of Steady Rolling on Longitudinal and Directional Stability," NACA TN 1627, June 1948.*

40. *A.E. Hodapp, Jr., "Effects of Unsymmetrical Stability Derivative Characteristics on Re-Entry Vehicle Trim Angle Behavior," Journal of Spacecraft and Rockets 11, May 1974, pp. 300-307.*

41. *A.E. Hodapp, Jr., "Effects of Unsymmetrical Stability Derivative Characteristics on Re-Entry Vehicle Transient Angular Motion," Journal of Spacecraft and Rockets 13, February 1976, pp. 82-90.*

42. *C.H. Murphy, "Angular Motion of Spinning Almost-Symmetric Missiles," Journal of Guidance and Control 2, November-December 1979, pp. 504-510. (See also USA ARRADCOM Ballistic Research Laboratory Technical Report No. 02121, November 1978, AD A063538.)*

of this essential asymmetry would affect the classical tricyclic theory of a symmetric missile with aerodynamic trim.

If $\hat{C}_{M_\alpha}$ is not zero, the pitch frequency of a nonspinning missile, ϕ'_α, is not equal to its yaw frequency, ϕ'_β . If the spin rate is outside the region between these two frequencies (the resonance region), the motion can be described by a pentacycle:

$$\tilde{\xi} = k_1 e^{i\phi_1} + k_2 e^{i\phi_2} + \xi_a k_3 e^{i\phi} + k_4 e^{i\phi_4} + k_5 e^{i\phi_5} \tag{29}$$

where $\phi'_4 = 2\phi' - \phi'_1$; $\phi'_5 = 2\phi' - \phi'_2$

$$\left.\begin{array}{l} k_4/k_1 \to 0 \\ k_5/k_2 \to 0 \end{array}\right\} \quad |\hat{C}_{M_\alpha}| \big/ |C_{M_\alpha}| \to 0$$

When the spin is in the resonance region,

$$\tilde{\xi} = \left(k_{1R}\, e^{\lambda s} + k_{4R}\, e^{-\lambda s} + \xi_a\, k_3\right) e^{i\phi} + k_2 e^{i\phi_2} + k_5 e^{i\phi_5} \tag{30}$$

According to Eq. (30), the motion grows exponentially when the spin is in the resonance region. Another unpleasant characteristic is the existence of peak values of $|k_3|$ at the endpoints of the region $\left(\phi' = \phi'_\alpha, \quad \phi' = \phi'_\beta\right)$.

Four specific characteristics of the motion of almost symmetric missiles $\left[\, |\hat{C}_{M_\alpha}| << |C_{M_\alpha}| \,\right]$ are:

(1) The general motion is well approximated by a symmetric missile with average coefficients.

(2) Far from zero spin or resonance spin rates, the first observable modification of the usual tricyclic motion for an almost-symmetric missile is the appearance of a $2\phi' - \phi_1'$ frequency, followed by the appearance of a $2\phi' - \phi_2'$ frequency as the asymmetry becomes greater.

(3) Near zero spin, both of these additional frequencies have substantial amplitudes, and near resonance, the $2\phi' - \phi_1'$ frequency has a substantial amplitude.

(4) For spin in the resonance region, large trims and exponential undamping are possible.

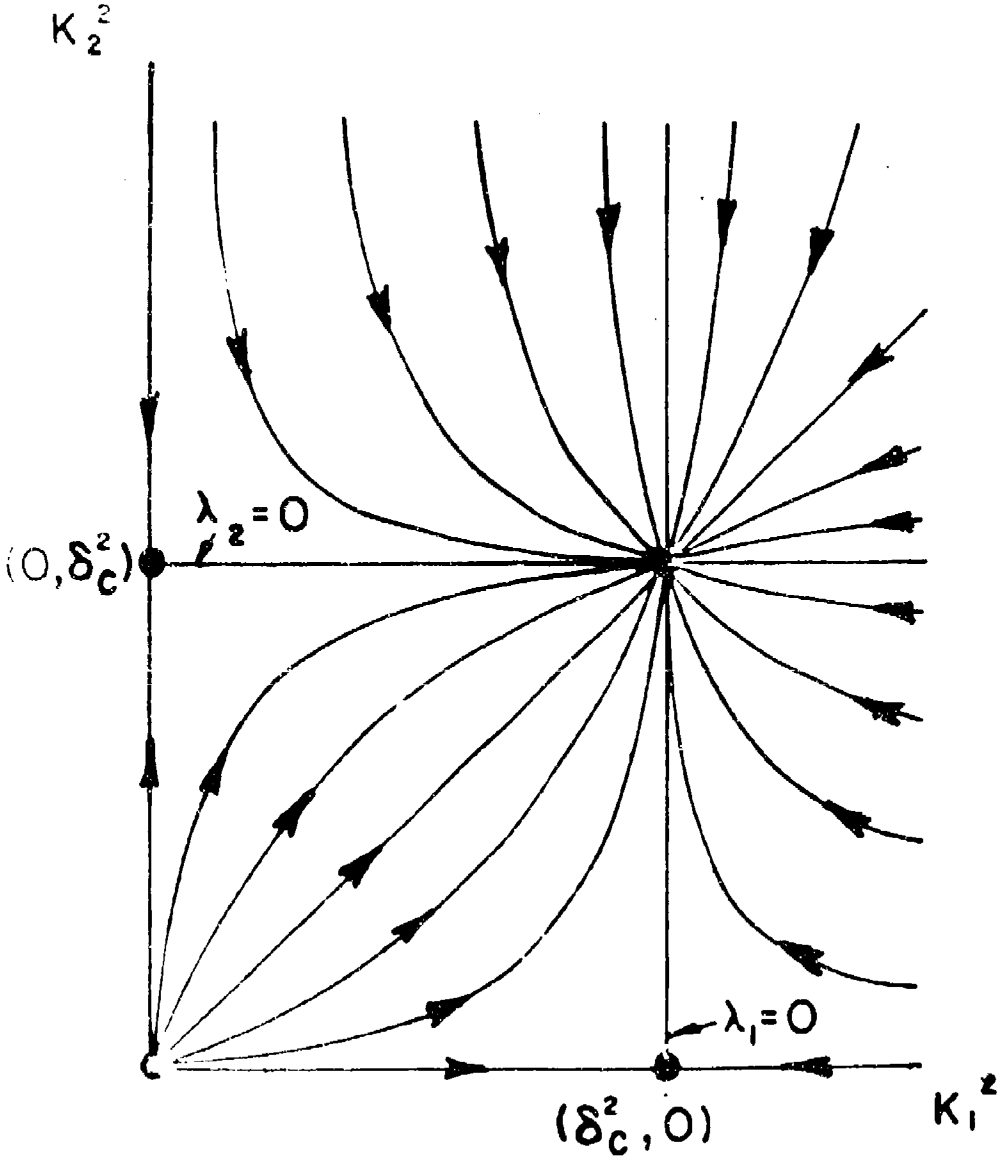

Figure 1. Amplitude plane for equal in-plane and out-of-plane damping moments ($a = 0$, $H_0 H_2 < 0$ in Eqs. (10-12)). The circular limit motions are unstable, but the planar limit motion is stable.

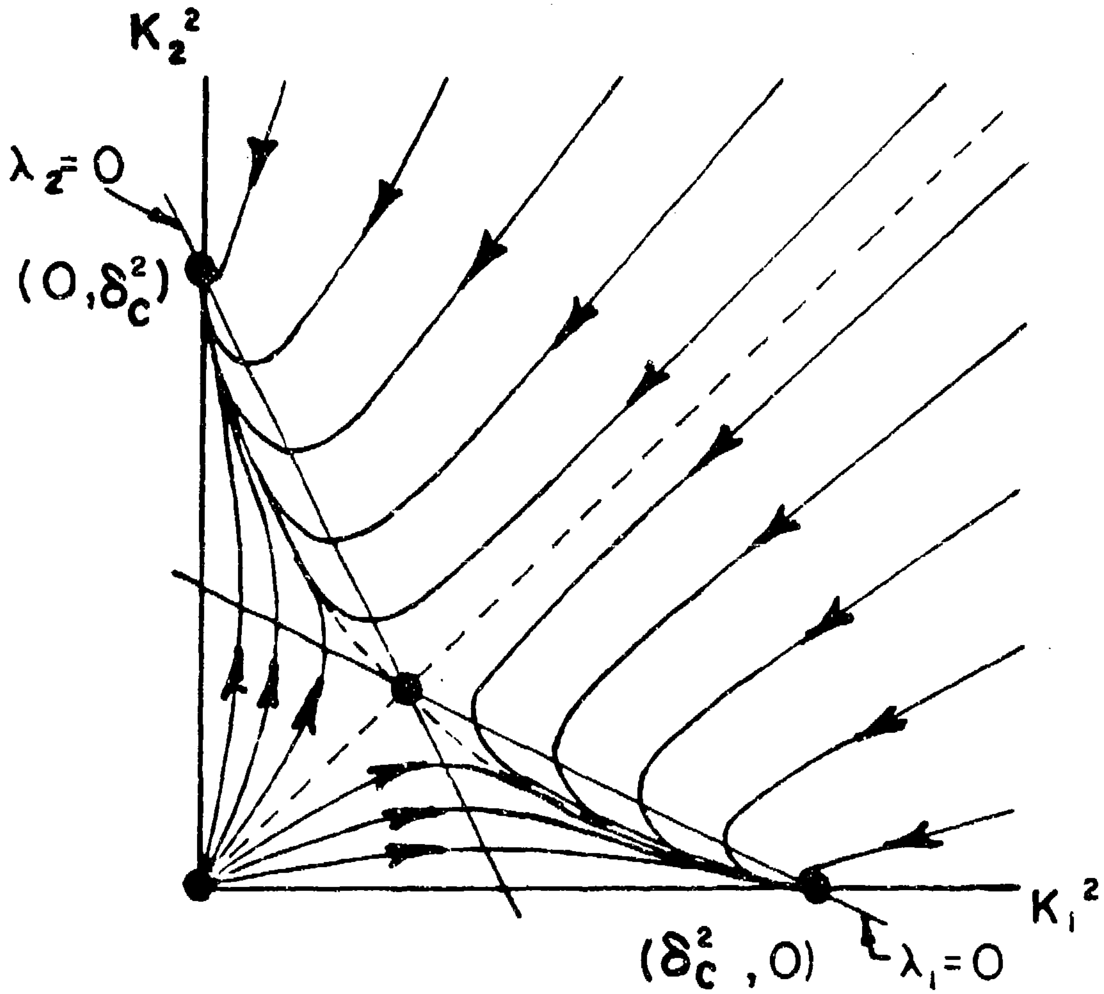

Figure 2. Amplitude plane for an in-plane cubic damping moment coefficient three times larger than the out-of-plane cubic damping moment coefficient ($a = ?$, $H_0 H_2 < 0$ in Eqs. (10-12)). The circular limit motions are stable, but the planar limit motion is unstable.

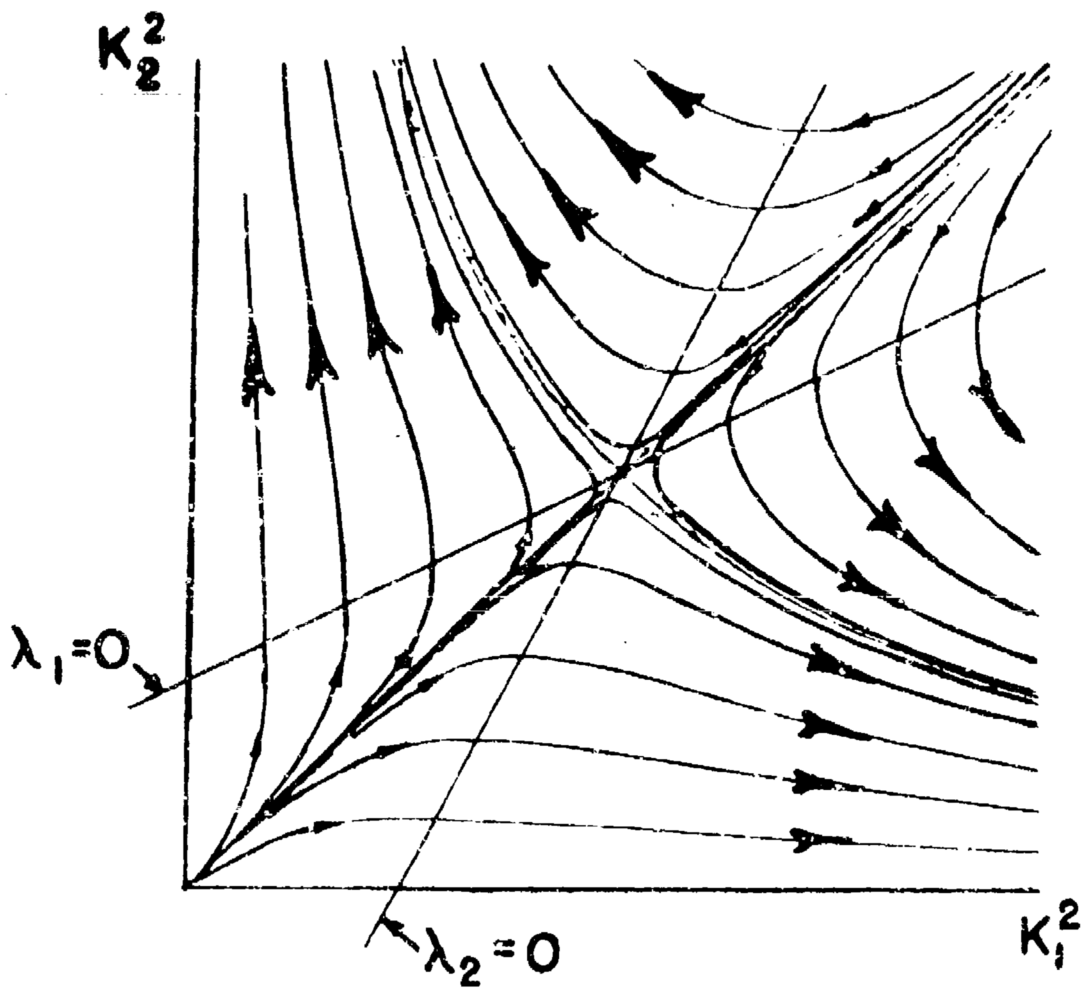

Figure 3. Amplitude plane for in-plane and out-of-plane cubic damping moment coefficients of opposite sign ($a = -2$, $H_0 < 0$, $H_2 < 0$ in Eqs. (10-12)). No circular limit motions exist; the planar limit motion is unstable.

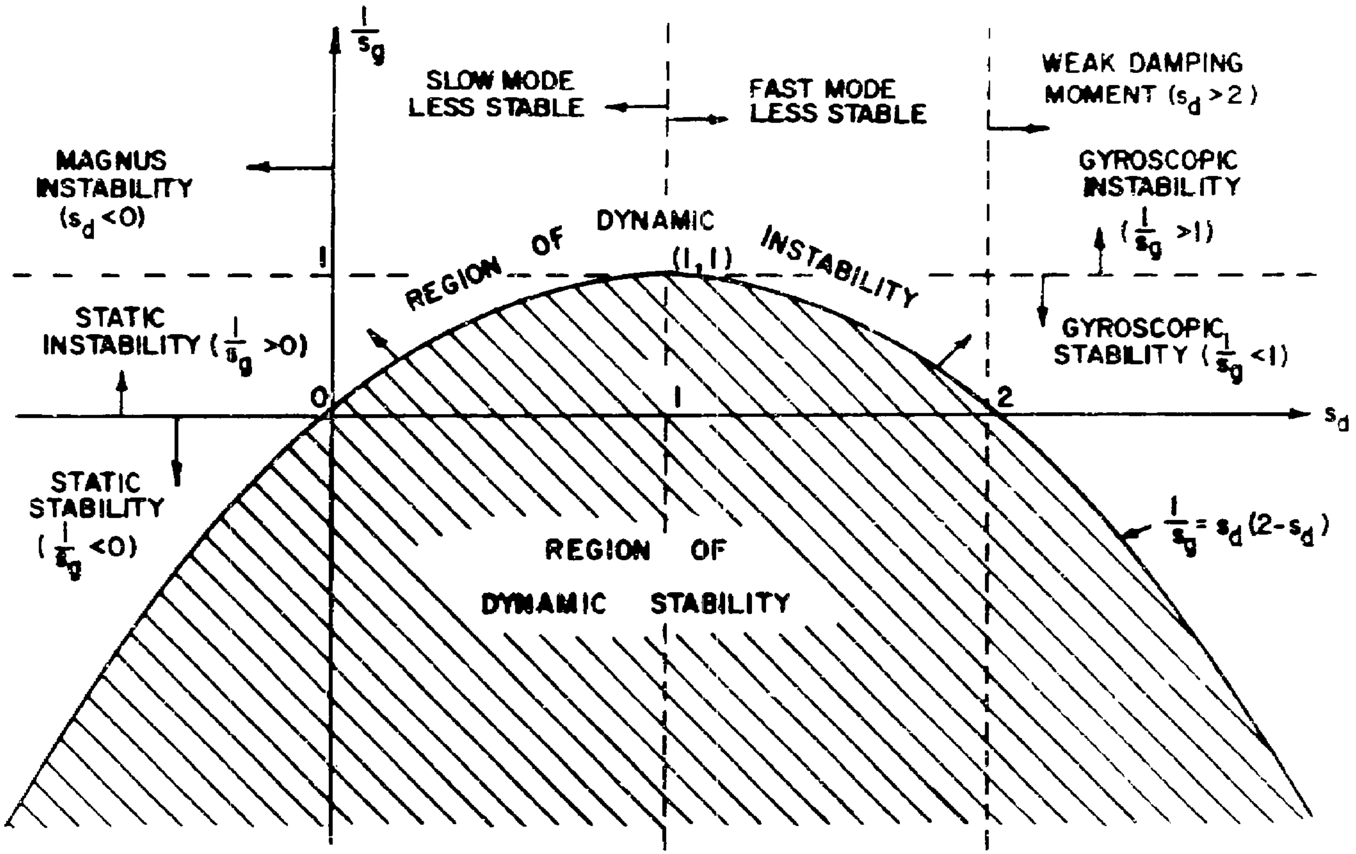

Figure 4. Stability conditions visualized by a plot of $1/s_g$ (the reciprocal of the gyroscopic stability factor) versus s_d (the dynamic stability factor).

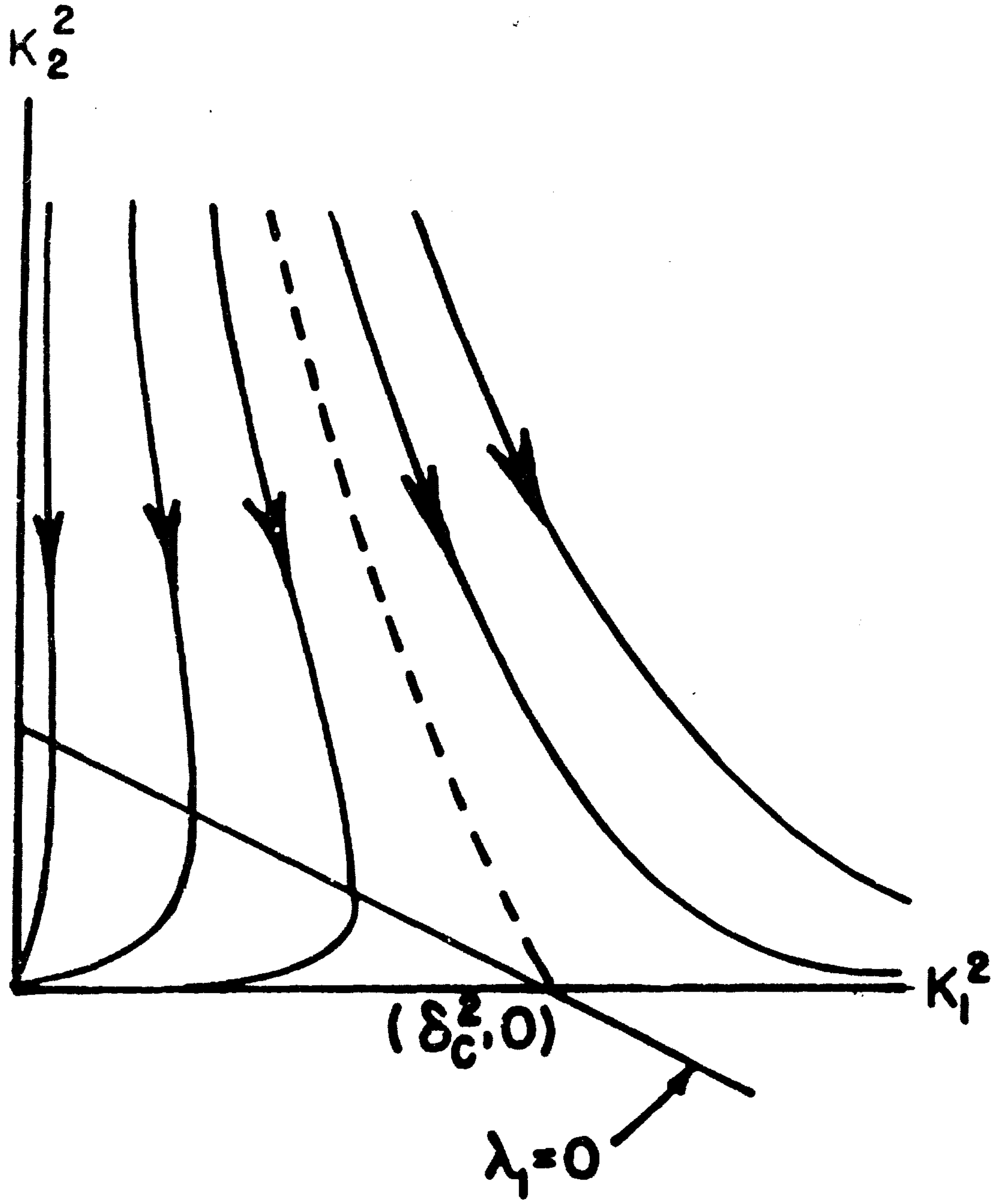

Figure 5. Amplitude plane for a cubic Magnus moment ($\lambda_{10}\ \lambda_{20} > 0$ in Eqs. (20-21)). The separatrix (dashed curve) divides all initial conditions into two families: those that yield a motion tending to zero amplitude and those that yield a motion that grows without bound.

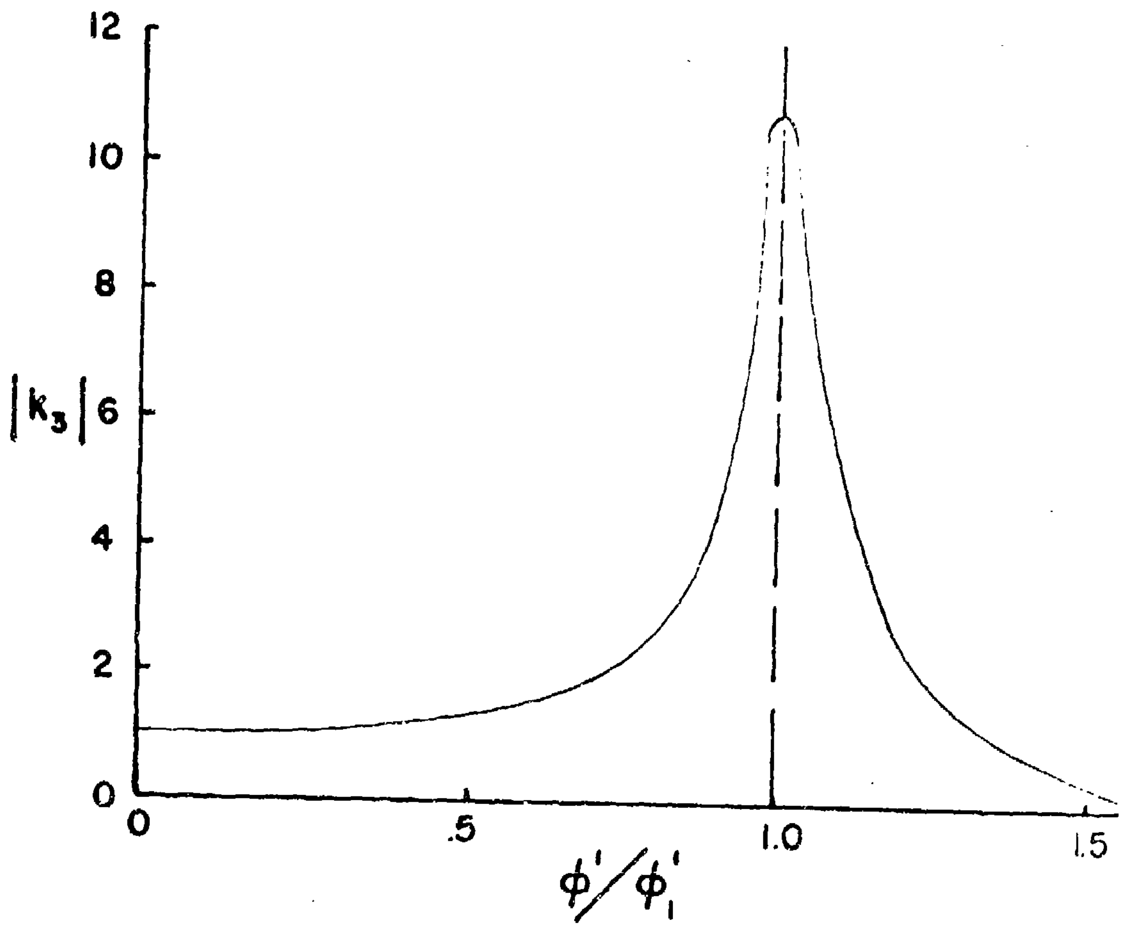

Figure 6. Trim amplitude (for constant spin rate) versus spin rate.

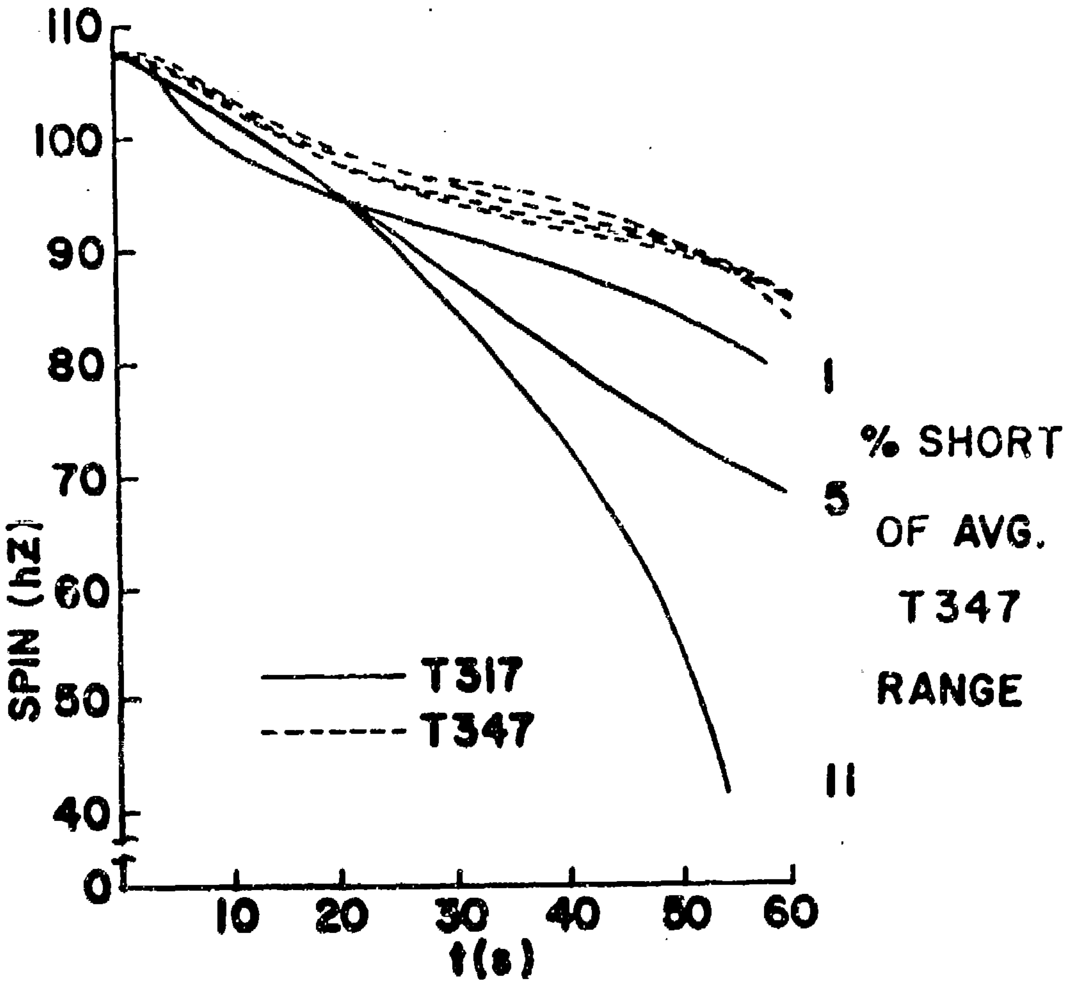

Figure 7. Measured spin histories for the T317 and T347 shell (the latter having the same shape, mass and moments of inertia as the T317, but no movable internal parts).

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LIST OF SYMBOLS

a	$\left(\dfrac{\text{cubic planar damping moment}}{\text{cubic circular damping moment}}\right) - 1$
b	$\dfrac{\rho\ S\ \ell^3}{2\ I_y \sqrt{P^2 - 4M}}$
C_D	$\dfrac{\text{drag force}}{(1/2)\ \rho\ S\ V^2}$
C_ℓ	$\dfrac{\text{roll moment}}{(1/2)\ \rho\ S\ \ell\ V^2}$
C_{L_α}	$\dfrac{\text{lift force}}{(1/2)\ \rho\ S\ V^2\ \delta}$
C_m, C_n	Y, Z components of $\dfrac{\text{aerodynamic moment}}{(1/2)\ \rho\ S\ \ell\ V^2}$
$C_{\tilde{m}}$, $C_{\tilde{n}}$	$\tilde{Y}$, $\tilde{Z}$ components of $\dfrac{\text{aerodynamic moment}}{(1/2)\ \rho\ S\ \ell\ V^2}$
C_{m_0}, C_{m_α}, C_{m_q}, $C_{m_{\dot\alpha}}$	coefficients in the expansion: $C_m = C_{m_0} + C_{m_\alpha}\ \alpha + C_{m_q}\ (q\ell/V) + C_{m_{\dot\alpha}}\ (\alpha' + \phi'\beta)$
C_{n_0}, C_{n_β}, C_{n_r}, $C_{n_{\dot\beta}}$	coefficients in the expansion: $C_n = C_{n_0} + C_{n_\beta}\ \beta + C_{n_r}\ (r\ell/V) + C_{n_{\dot\beta}}\ (\beta' - \phi'\alpha)$
C_{M_d}	damping moment part of $C_m + i\ C_n$
$C_{M_{p_\alpha}}$	$\dfrac{\text{Magnus moment}}{(1/2)\rho\ S\ \ell\ V^2\ \phi'\ \delta}$
C_{M_q}	$\left(C_{m_q} + C_{n_r}\right)/2$
C_{M_α}	$\left(C_{m_\alpha} - C_{n_\beta}\right)/2$

LIST OF SYMBOLS
(CONTINUED)

$\hat{C}_{M_\alpha}$	$\left(C_{m_\alpha} + C_{n_\beta}\right)/2$
$C_{M_{\dot\alpha}}$	$\left(C_{m_{\dot\alpha}} - C_{n_{\dot\beta}}\right)/2$
C_{SM_α}	$\dfrac{\text{induced side moment}}{(1/2)\rho\ S\ \ell\ V^2\ \delta}$
d_0, d_2	coefficient in Equations (9-10)
H	$\rho\ \frac{S\ell}{2m}\left[C_{L_\alpha} - C_D - k_t^{-2}\ (C_{m_q} + C_{M_{\dot\alpha}})\right]$
H_0	$\frac{\rho S\ell}{2m}\left[C_{L_\alpha} - C_D - k_t^{-2}\ d_0\right]$
H_2	$-\ \frac{\rho S\ell}{2m}\ k_t^{-2}\ d_2$
I_x, I_y	axial and transverse moments of inertia
k_a	$(I_x/m\ell^2)^{1/2}$
k_t	$(I_y/m\ell^2)^{1/2}$
K_1, K_2	magnitude of the 1- and 2-mode vectors describing the yawing motion
k_3	$M\ [(\phi')^2 - P\phi' + M + i\ (PT - H\phi')]^{-1}$
ℓ	reference length
m	mass
M	$\rho\ \frac{S\ell}{2m}\ k_t^{-2}\ C_{M_\alpha}$
P	$(I_x/I_y)\phi'$

LIST OF SYMBOLS (CONTINUED)

p, q, r	missile spin, pitch and yaw rates in the missile-fixed system
s_d	2T/H, the dynamic stability factor
s_g	$P^2/4M$, the gyroscopic stability factor
S	reference area
T	$\rho \frac{S\ell}{2m} \left[C_{L_\alpha} + k_a^{-2} C_{M_{p\alpha}} \right]$
V	magnitude of the velocity
XYZ	missile-fixed axes. The X-axis is along the longitudinal axis of the missile, positive forward
$X\tilde{Y}\tilde{Z}$	aeroballistic axes, where $\tilde{Y} + i\ \tilde{Z} = (Y + i\ Z)e^{i\phi}$
α	angle of attack in the missile-fixed system
β	angle of sideslip in the missile-fixed system
δ	$\lvert\xi\rvert$
δ_c	circular limit motion radius
θ	polar angle of ξ
λ_j	K'_j/K_j , j = 1, 2
ξ	complex angle of attack in the missile-fixed system, $\beta + i\ \alpha$
$\tilde{\xi}$	complex angle of attack in the aeroballistic system
ξ_a	$\left(C_{m_0} + i\ C_{n_0}\right) / i\ C_{M_\alpha}$
ρ	air density
ϕ	roll angle

LIST OF SYMBOLS
(CONTINUED)

ϕ'	$\frac{p\ell}{V}$, the roll rate (rad/cal)
ϕ'_α	zero-spin pitch frequency
ϕ'_β	zero-spin yaw frequency
ϕ_1, ϕ_2	orientation angles of the two epicyclic yaw arms
$(\bar{\ })$	complex conjugate
$(\ )^{\cdot}$	derivative with respect to time
$(\ )'$	derivative with respect to dimensionless arc length along the trajectory
$(\tilde{\ })$	component in the aeroballistic system

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Please take a few minutes to answer the questions below; tear out this sheet and return it to Director, US Army Ballistic Research Laboratory, ARRADCOM, ATTN: DRDAR-TSB, Aberdeen Proving Ground, Maryland 21005. Your comments will provide us with information for improving future reports.

1. BRL Report Number ______________________

2. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which report will be used.)

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3. How, specifically, is the report being used? (Information source, design data or procedure, management procedure, source of ideas, etc.) ______________________

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4. Has the information in this report led to any quantitative savings as far as man-hours/contract dollars saved, operating costs avoided, efficiencies achieved, etc.? If so, please elaborate.

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5. General Comments (Indicate what you think should be changed to make this report and future reports of this type more responsive to your needs, more usable, improve readability, etc.) ______________________

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6. If you would like to be contacted by the personnel who prepared this report to raise specific questions or discuss the topic, please fill in the following information.

Name: ______________________

Telephone Number: ______________________

Organization Address: ______________________

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