Numerical Experiments on the Stratospheric-Tropospheric Dynamical Interaction

Winston C. Chao
Science Applications, Inc.
McLean, Virginia 22102

AND

Mark R. Schoebel and Darrell F. Strobel
Geophysical and Plasma Dynamics Branch
Plasma Physics Division

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NAVAL RESEARCH LABORATORY
Washington, D.C.

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**AUTHOR**
Winston C. Chao*, Mark R. Schoebert and Darrell F. Strobel

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**ABSTRACT**
Numerical experiments with the UCLA-NEPRF tropospheric weather forecast model were performed to test the influence of stratospheric conditions which are representative of sudden warmings on tropospheric weather. Stratospheric conditions were represented by imposed vertical velocities at the upper boundary. An externally forced stratospheric disturbance as large as a sudden warming was found to generate an observable response in sea level pressure with a time constant of the order of a day. The numerical results are interpreted with the aid of an analytical model based on the linearized shallow water equation.


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I. INTRODUCTION

Most tropospheric weather prediction models impose an upper boundary condition of $\omega = \frac{dp}{dt} = 0$ at some finite height in the lower stratosphere. Lindzen et al. (1968) pointed out that this "rigid lid" boundary condition produces spurious reflections and introduces oscillations and standing waves whose amplitudes can mask the true solutions. In a more recent study Kirkwood and Derome (1977) investigated the effect of this boundary condition on forced stationary planetary waves. They used realistic mean zonal winds in their quasi-geostrophic study rather than a quiescent, isothermal atmosphere as Lindzen et al. (1968) adopted. The reflection and transmission of these ultralong waves are strongly dependent on the structure of the polar night jet centered near the stratopause (Charney and Drazin, 1961; Schoeberl and Geller, 1977). Kirkwood and Derome found that the stationary planetary wave structure could only be accurately modeled when the vertical resolution in the stratosphere adequately represented the mean zonal winds and Newtonian cooling ($\geq 21$ levels or $\Delta p < 10$ mb). Insufficient stratospheric resolution resulted in a poorly defined polar night jet and Newtonian cooling profile and spurious reflection at the upper boundary. They also suggested that misrepresentation of the structure of stationary planetary waves can lead to the generation of spurious external, free waves of which the 5 day wave (Geisler and Dickinson, 1976) has been found to be dominant in some forecast models (Lambert and Merilees, 1978).

The movement of tropospheric weather systems at middle and high latitudes is greatly influenced by these stationary, ultralong waves.
waves, particularly wave numbers 1 and 2. Therefore a good forecast of the former requires a good forecast of the latter. These planetary scale eddies are formed in the troposphere through baroclinic, diabatic, and orographic processes. During winter these waves propagate vertically, perturb the stratospheric circulation, transport heat and momentum from mid-latitudes into the polar regions, and generate sudden stratospheric warmings. Thus it is imperative to incorporate a sufficiently resolved stratosphere for tropospheric weather prediction.

In this report we document an effort to determine the influence of stratospheric conditions which are representative of a sudden warming on tropospheric weather. Quiroz (1977), McGuirk (1978), and O'Neill and Taylor (1979) showed that the 1976/77 sudden warming definitely affected tropospheric weather patterns. In addition Rosmond (private communications, 1978) has noted that Navy forecasts have lower skill during sudden warmings. The goals of this study were to ascertain the importance of stratospheric-tropospheric dynamical coupling and the time scale for stratospheric processes to affect tropospheric weather.
II. NUMERICAL EXPERIMENTS

The UCLA atmospheric general circulation model (Arakawa and Lamb, 1977) has been developed as a forecast model at the Naval Environmental Prediction Research Facility (NEPRF) (Payne, 1979) and was used for the numerical experiments discussed below. Two forecasts of 5-days duration each were performed with the same initialized data for January 19, 1977, generated from NEPRF's global model initialization program. During that week, record cold temperatures were experienced over most of the continental United States and an intense sudden warming had developed in the stratosphere. The reference forecast had the usual \( \omega = 0 \) upper boundary condition at 50 mb. The other forecast was performed with an imposed vertical velocity, \( \omega_1 \), at 50 mb based on the observed geopotential heights of wave number 1 and 2 during that intense sudden stratospheric warming.

The vertical velocity boundary condition has two wave components, \( \omega_1 = \omega_1^1 + \omega_1^2 \), where the superscripts represent wavenumber, and are defined as:

\[
\omega_1^1 = Ag(\Theta) f_1(\phi)
\]

\[
\omega_1^2 = Ag(\Theta) f_2(\phi),
\]

Here \( \Theta \) is the latitude and \( \phi \) is the longitude, \( g(\Theta) \) is defined to be zero between equator and \( 30^\circ N \) and to be a sine curve between \( 30^\circ N \) and \( 90^\circ N \) and is symmetric with respect to the equator. Also
where $\Theta_1$ corresponds to 120°W and $\phi_2$ to 0°. $A$ is the observed amplitude and equal to $-4.1 \times 10^{-5}$ mb/sec, or $-3.54$ mb/day. These values are crude estimates of the kind of vertical velocities which may be associated with sudden warming events. The values were obtained from the mechanistic model of Schoeberl and Strobel (1980). $\phi_1$ and $\phi_2$ are chosen such that $w_1$ and $w_2$ are 180° out of phase with their respective components of the observed geopotential height to insure that the vertical propagation of wave energy is upward. Momentum and heat are also transported through the model top by the imposed vertical velocity.
III. NUMERICAL RESULTS

From the detailed results of these two integrations, we have selected three grid points to illustrate the salient features. The general weather pattern is shown in Fig. 1 which is day 5 of the integration with imposed upper boundary vertical velocity. Point A is the location of the maximum negative value of $w_I (=6 \text{ mb day}^{-1})$ at the upper boundary, whereas point C is the maximum positive value of $w_I (=7.7 \text{ mb day}^{-1})$. Point B, which coincides with a deep low in the Gulf of Alaska, has $w_I \sim 0$ (actually $-0.5 \text{ mb day}^{-1}$). To emphasize stratospheric influence deep in the troposphere, sea level pressure differences from 1000 mb for the two experiments are shown in Fig. 2 to 4 for a 5 day period. Perceptible differences are evident after only 1 day at points A and C. At point A, after 4 days, the pressure difference between the two runs reaches 3 mb. At point B, a large change between day 3 and 4 reflects the movements and deepening of the low the pressure difference. A maximum difference of $\sim 6 \text{ mb}$ is reached after 3.5 days at point C.
IV. ANALYTIC STUDY OF THE ATMOSPHERIC RESPONSE TO THE IMPOSED VERTICAL VELOCITY.

Before interpreting the meaning of these results, it is instructive to investigate with an analytic model the expected planetary scale response of the atmosphere to the imposed vertical velocity. We hypothesize that the essential features of the response can be modeled with the linearized shallow water equations with a mean depth, \( H \), equal to the equivalent depth of an isothermal atmosphere (~10 km):

\[
\frac{\partial \phi'}{\partial t} + \mathbf{f} \times \mathbf{V} + \nabla \phi' = 0, \quad (1)
\]

\[
\frac{\partial \mathbf{V}}{\partial t} + g H \mathbf{V} \cdot \mathbf{V} = 0, \quad (2)
\]

where \( \phi' = gh' = g(h-H) \), \( h \) is the height of the water surface and is analogous to the sea level pressure, \( \mathbf{V} \) is the horizontal velocity and is constant with height, and \( Q/g \) is the mass source and represents the effect of imposed vertical velocity, \( \omega_I \).

From (1) we obtain the divergence and vorticity equations:

\[
\frac{\partial}{\partial t} \nabla \cdot \mathbf{V} - \mathbf{f} \cdot \nabla \mathbf{V} + \nabla^2 \phi' = 0, \quad (3)
\]

\[
\frac{\partial}{\partial t} k \cdot \nabla \mathbf{V} + f \mathbf{V} \cdot \nabla \mathbf{V} = 0. \quad (4)
\]
Substitution of (4) and (2) into (3) gives

$$\frac{\partial^2}{\partial t^2} \mathbf{v} \cdot \mathbf{v} + \mathbf{f} \cdot \mathbf{v} \cdot \mathbf{v} - gH \mathbf{v} \cdot (\mathbf{v} \cdot \mathbf{v}) + \nu^2 \mathbf{Q} = 0. \quad (5)$$

If \( \mathbf{Q} \) has wave numbers \( k \) and \( \ell \) in \( x \) and \( y \) directions, respectively, i.e.,

$$Q = \hat{Q} \exp \left[ i(kx + \ell y) \right],$$

the Laplacian can be written as

$$\nu^2 = -(k^2 + \ell^2) = -k^2.$$

If we use the notation

$$D = \mathbf{v} \cdot \mathbf{v} = D e^{i(kx + \ell y)}, \quad \text{Eq. (5) becomes} \quad (6)$$

$$\frac{\partial^2}{\partial t^2} \hat{D} + (gH \mathbf{k}^2 + \mathbf{f}^2) \hat{D} = K^2 \hat{Q}.$$

For the initial conditions we choose \( \hat{D} = \phi' = 0 \) at \( t = 0 \), which are consistent with the data initialization for the numerical experiments; thus,

$$\hat{D} = 0 \quad (\text{since } \mathbf{v} = 0 \text{ at } t = 0),$$

$$\frac{\partial \hat{D}}{\partial t} = 0 \quad (\text{from (3)}). \quad (7)$$

The solution is

$$\hat{D} = A \cos(\nu t) - A, \quad (8)$$

where

$$A = \frac{-K \hat{Q}}{\nu^2},$$

and

$$\nu = (gH \mathbf{k}^2 + \mathbf{f}^2)^{1/2}.$$
Eq. (2) can now be written

\[ \frac{\partial \phi'}{\partial t} + gh[A \cos(vt) - A] = \hat{\phi}, \]

\[ \hat{\phi}' = 0 \text{ at } t = 0. \]

The solution for \( \hat{\phi}' \) is

\[ \hat{\phi}' = \left[ \frac{-gHA}{v} \sin(vt) + (gHA + \hat{Q})t \right] e^{i(kx + ly)} \]

\[ \begin{align*}
&= \left[ \frac{gH^2Q}{v^3} \sin(vt) + \frac{f^2}{v^2} \hat{Q}t \right] e^{i(kx + ly)} \\
&= \left[ \frac{gH^2Q}{v^3} \sin(vt) + \frac{f^2}{v^2} \hat{Q}t \right] e^{i(kx + ly)} \tag{9}
\end{align*} \]

Since the surface height variation is analogous to the sea level pressure variation in the atmospheric model and \( Q \) represents the imposed vertical velocity at model top, Eq. (9) suggests the sea level pressure has a standing oscillation with frequency, \( v \), and a linear variation with time, which vanishes as \( f = 0 \). The standing oscillation is a direct consequence of the \( D = 0 \) initial condition and the imposed vertical velocity \( (Q \neq 0) \). If we choose \( D = -A \) instead for the initial condition along with \( \frac{\partial D}{\partial t} = 0 \), then Eq. (8) is just \( D = -A \) and the standing oscillation term in Eq. (9) vanishes. Thus the mismatch of initial and upper boundary conditions generates the oscillatory term which corresponds to the creation of two free modes traveling in opposite directions.

A numerical estimate for \( \hat{\phi}' \) is given below:
\[ g = 9.8 \text{ m/s}^2, \ H = 10^4 \text{ m}, \ f = 2\Omega \sin \theta = 1.3 \times 10^{-4} \text{s}^{-1}, \]
\[ \Omega = 7.3 \times 10^{-5} \text{s}^{-1}, \ \theta = \frac{\pi}{3}, \ a = 6.4 \times 10^6 \text{ m}, \text{ earth's radius}, \]
\[ k = \frac{2\pi}{L_x}, \ \lambda = \frac{2\pi}{L_y}, \ L_x = 2\pi \cos \theta = \pi a, \ L_y = \frac{2na}{3}, \]

Thus,
\[ k = \frac{2}{a}, \ \lambda = \frac{3}{a}, \]
\[ K^2 = k^2 + \lambda^2 = \frac{13}{a^2} = 3.2 \times 10^{-13} \text{ m}^{-2}, \]
\[ \nu = (gHK^2 + \lambda^2)^{1/2} = 2.2 \times 10^{-4} \text{s}^{-1}, \]
\[ \frac{gHK^2}{\nu^3} = 3.1 \times 10^3 \text{s}, \quad \frac{f^2}{\nu} = 0.32, \ T = \frac{2\pi}{\nu} = 2.9 \times 10^4 \text{ s}, \]
\[ \hat{\omega} = \max |\omega_1| = 4.1 \times 10^{-5} \text{ mb s}^{-1} \text{ for each wave number.} \]

Thus the amplitude of the standing oscillation is 0.17 mb and its period is 8 hr. The growth rate of the second term in Eq. (9) is 1.2 mb day \(^{-1}\).

Fig. 5 shows the sea level pressure difference at three locations in the first one and half days between the two runs, \( Pu - Pr \), where \( Pu \) is the sea level pressure of the run with imposed upper boundary vertical velocity, and \( Pr \), that of the reference run. Since \( Pu - Pr \) is analogous to the \( \phi' \) in the shallow water equation, the time variation of \( Pu - Pr \), shown in the figure as an oscillation superimposed on a linear change, corresponds well with that predicted by the shallow water model.
V. DISCUSSION

The study shows that the difference between the two simulations can be qualitatively explained by Eq. (9). In the shallow water equation case, the oscillatory part of the solution in Eq. (9) can be eliminated by using a different initial condition, but not the linear term.

However in a model with topographic forcing and differential heating due to sea-land contrast the imposed vertical velocity can be initialized as part of the stationary waves and then both terms in Eq. (9) are eliminated. In future proposed experiments the imposed vertical velocity upper boundary condition will be initialized as part of the stationary waves because it is kept constant in time. It is anticipated that the difference in sea level pressure between such a proposed experiment and the reference experiment will not be dominated by the direct effect of the imposed vertical velocity and will probably be smaller in amplitude than the results presented in Fig. 2-4. However the present experiment does demonstrate that an externally generated stratospheric disturbance as large as a sudden warming can generate an observable response in sea level pressure with a time constant of the order of a day.
Acknowledgments

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REFERENCES


Fig. 1 - General weather pattern on day 5
Fig. 2 - Sea level pressure - 1000 mb at point A

A 58° N \( \omega_l \sim -0.25 \text{ mb/hr} \)

170° W or 6 mb/day
Fig. 3 - Sea level pressure - 1000 mb at point B
Fig. 4 - Sea level pressure - 1000 mb at point C
Fig. 5 - Detailed sea level pressure difference between the two runs at point A, B, and C for the first 1.5 days.