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A NEW ALGORITHM FOR THE TUNING OF ANALOG FILTERS.(U)
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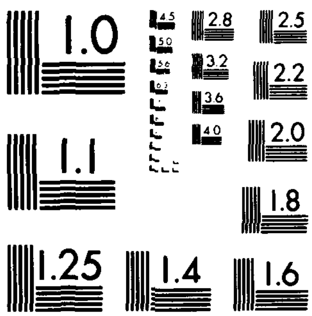

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A NEW ALGORITHM FOR THE [illegible]

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A084 958	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A NEW ALGORITHM FOR THE TUNING OF ANALOG FILTERS		5. TYPE OF REPORT & PERIOD COVERED (1) Technical Rep[redacted]t
		6. PERFORMING ORG. REPORT NUMBER (14) R-854; UILU-ENG-78-2247
7. AUTHOR(s) (10) Charles John Alajajian		8. CONTRACT OR GRANT NUMBER(s) (15) NSF ENG 78-11758; DAAB-07-72-C-0259; DAAG-29-78-C-0016
9. PERFORMING ORGANIZATION NAME AND ADDRESS Coordinated Science Laboratory University of Illinois Urbana, Illinois 61801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Joint Services Electronics Program		12. REPORT DATE (11) Aug[redacted]79
		13. NUMBER OF PAGES 104
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Filters
Tuning
Computer Algorithms
Active RC Filters

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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20. Abstract (Continued)

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UILU-ENG 78-2247

A NEW ALGORITHM FOR THE TUNING OF ANALOG FILTERS

by

Charles John Alajajian

This work was supported in part by the National Science Foundation under Grant NSF ENG 78-17758 and in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy and U.S. Air Force) under Contract DAAB-07-72-C-0259 and DAAG-29-78-C-0016.

A NEW ALGORITHM FOR THE TUNING OF ANALOG FILTERS

BY

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1979

Thesis Adviser: Professor T. N. Trick

Urbana, Illinois

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In this thesis a new algorithm is proposed for the tuning of analog filters. Tellegen's theorem and the adjoint network concept are used to relate large changes in a set of tuning elements to desired voltage changes in the manufactured filter. The component values in the manufactured circuit are measured and the deviation of the voltages in the manufactured circuit from the corresponding voltages in the nominal circuit design are computed at a set of critical frequencies. A set of tuning resistors is chosen and this information is entered into the large-change sensitivity expression. The changes in the tuning elements needed to reduce these voltages deviations to zero are computed by solving a set of linear algebraic equations whose rank is equal to the number of tuning elements

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ACKNOWLEDGEMENTS

I would like to thank Professor T. N. Trick, who was my teacher in computer-aided circuit design techniques and who introduced me to this fascinating area. I really appreciate the many helpful discussions with him and his encouragement. I would also like to thank Professor E. El-Masry for designing the filters in Chapters 4 and 6 and for his encouragement. I am also grateful to Professor M. E. Van Valkenburg who in addition to Professors Trick and El-Masry, offered suggestions which greatly improved the quality of the final revision. Also, thanks are due Mrs. Dee Wrather for her superb job of editing and typing this thesis. Finally, the support of the National Science Foundation under Grant No. ENG-7811758 is very much appreciated.

I would like to thank my parents, John and Marion Alajajian, for their financial and prayerful support and encouragement throughout the years I've been in school for which I am most grateful.

It's exciting to see God's hand at work in the world and in the lives of His people. As the Psalmist wrote

> God we heard it ourselves,
> our fathers told us what Your hand accomplished
> in their days,
> the days of long ago. . .
> Not by their sword did they take the land,
> nor did their arms win the victory.
> No, Your right hand, Your arm,
> and the light of Your face did it,
> because You were kind to them.
>
> Psalms 44:1,3 (Beck)

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1. INTRODUCTION

In the manufacture of high performance filters, it is often necessary to adjust some or all of the components of the filter in order to meet certain specifications, which are normally given in terms of the variation in gain with frequency. The process of adjusting component values to correct for deviations from the specified gain response is known as tuning.

Deviations from the specified gain response are due to two basic sources. The first of these is that the model assumed for the filter does not account for parasitic effects, such as losses in the capacitors, or the finite gain-bandwidth of the actual operational amplifier (op amp) being used. The second is that the components themselves will be perturbed from their nominal values due to the initial production tolerances.

Tuning is an important problem in industry, and considerable effort has been expended within industry and in research to develop algorithms that can be automated, and which maximize the yield. Another basic reason for the importance of tuning is the trade-off between component costs and tuning costs. Because the accuracy of the component values is inversely related to cost, it is usually more cost-effective to employ an efficient tuning algorithm, thereby significantly reducing manufacturing costs. In general the problem of minimizing component and tuning costs is complex, requiring the use of sophisticated computer aids for its solution [1,2,3]. Usually, practical considerations limit the tolerances that can be specified for the filter components so that some amount of tuning is needed. In addition, tuning can be a very expensive process, so that it is important to have a tuning algorithm that is efficient.

At the time of this writing, active components, such as op amps, are best fabricated by the silicon technology, while passive components over a wide range of values and with high precision are best fabricated by the thin and thick film technologies. A popular method of designing active filters is to use a combination of the thin or thick film technology and the silicon technology. In this hybrid approach, the silicon integrated circuit op amp chips are bonded to thin or thick film passive networks. While there are many trade-offs between the use of thin and thick films, thick films usually require less expensive production equipment, and for small quantities are less expensive to produce, whereas thin films are economically justifiable only for high production runs. In general thin films occupy less space than thick films and can be manufactured to tighter initial tolerances. Presently, thin and thick film resistors can be manufactured with initial production tolerances of $\pm$ 5 percent, and $\pm$ 20 percent, respectively, and can be adjusted with trimmed tolerances of $\pm$ .1 percent, and $\pm$ 1 percent, respectively, where these figures represent conservative values [4]. The value of a resistor R is determined by the sheet resistance R_s, in ohms/square, and the geometry, where l is the length and w is the width of the resistor. Then the value of the resistor R is given by

$$R = R_s\left(\frac{\ell}{w}\right) .$$

In trimming, only the width w of the resistor is adjusted. This is done by removing material from the width of the resistor by anodization, etchants, or abrasives, or by a laser beam [4]. Reducing the width of the resistor can only increase its value, so that the sheet resistance R_s is initially produced below its desired value so that the resistors may be trimmed up to their tuned values at the time of manufacture [5].

Thin film capacitors can be manufactured with an initial production tolerance of $\pm$ 5 percent at the time of this writing [6]. While thick film capacitors can be made, it is usually more economical to use discrete capacitors unless a large number is required [4]. The tolerances of thin film capacitors are determined by the accuracy of the film properties, such as thickness and dielectric constant, and also by the geometry. Thin film capacitors usually consists of a substrate, a layer of metal (commonly called the base electrode), a dielectric layer, and a top layer of metal (commonly called the counterelectrode). It is not feasible to adjust capacitors [6] perhaps due to the additional expenditure of effort required to cut through three layers of material as opposed to a single cut for resistor adjustment [7]. Also, automatic resistor apparatus is available which makes resistor trimming adjustments in a single operation thus making resistor adjustment more practical than capacitor adjustment on an assembly line [7].

A filter will be said to be tuned if it exhibits the same gain characteristic as the nominal (to within a constant) at its output. Thus, the purpose of the tuning resistors is solely to locate the poles and zeros of the transfer function of the manufactured filter in their nominal positions. Deviations in the dc level can usually be corrected by some other resistors in the circuit without affecting the pole-zero locations.

1.2 First-Order Sensitivity Methods

1.2.1 Transfer Function Sensitivity

First-order transfer function sensitivity methods use the differential [8] to relate small changes in the tuning elements to desired output voltage changes in the manufactured filter at a set of critical frequencies. The partial derivatives of the output voltage of the manufactured filter circuit with respect to each of the tuning elements can be efficiently and accurately obtained at each iteration via the adjoint circuit concept [9,10]. The solution of a set of linear algebraic equations gives the tuning element values at each iteration. The main difference between this method and the new method to be described in Section 1.2.8 is that the former method approximates the voltages in the tuned filter by the manufactured filter voltages (ΔV set equal to zero in equation (2.9)), whereas the latter method approximates the voltages in the tuned filter by the nominal filter voltages, which experience has shown, is a superior approximation. First-order transfer function sensitivity methods are useful if sufficiently small changes in the tuning elements will bring about the desired voltage changes in the manufactured filter. Otherwise, the method is slow to converge, or divergent.

1.2.2 Root Sensitivity Methods

First-order root sensitivity methods use the differential to relate small changes in the tuning elements to desired pole-zero (and possibly dc level) changes in the manufactured filter. The partial derivatives of the poles and zeros (and possibly dc level) of the manufactured filter circuit with respect to each tuning element are approximated by computing the changes in the pole-zero locations due to small variations in the tuning elements [11]. If the forward difference quotient [12] is used to approximate the derivatives and there are T tuning elements, then T+1 computations must be made. The central difference quotient [12] requires 2T computations.

In order to compute the desired pole-zero changes (and hence these partial derivatives) the pole-zero locations of both the nominal and manufactured filters must be found. To find the pole-zero locations of the manufactured filter, magnitude (or phase) measurements are made at a set of critical frequencies. This information is then used to compute the transfer function coefficients of the manufactured filter by solving a set of linear algebraic equations. Once these coefficients have been determined, a root solving subroutine is used to determine the pole-zero locations of the manufactured filter, and then of the nominal filter, whose transfer function coefficients are assumed to be known. Finally, the forward difference quotient or the central difference quotient is used to approximate the derivatives and this information is substituted into the differential expression resulting in another set of linear algebraic equations which is solved to give the changes in the tuning elements required to tune the filter. Of course, these steps would be iterated if the first pass calculations did not adequately tune the filter.

The usefulness of the technique is limited in practice to the tuning of simple, second or third-order filter stages, or such stages in cascade, as a result of computational and measurement inaccuracies. In particular, because the method requires calculation of the transfer function coefficients of the manufactured circuit (from which the roots of the manufactured circuit are computed) the roots are extremely sensitive to the accuracy with which the transfer coefficients are computed. The problem becomes acute when higher-order, narrow-band filter types are to be tuned [11,13]. There is little that can be done to overcome this problem except to use double precision in the computer calculations [13], or a frequency transformation which generates a new polynomial whose roots are further apart [13]. Also, some care should be taken in the selection of a root finding subroutine [11].

In addition to the large amount of computational effort required to find the approximate partial derivatives, the procedure is unsuccessful in tuning a simple second-order bandpass example when the capacitance values are 5 percent high and resistors are at their design values [11].

1.2.3 The ω_0 and Q Sensitivity Methods

First-order ω_0 and Q sensitivity methods use the differential to relate small changes in the tuning elements to desired changes in the parameters ω_0 and Q. Because these parameters are defined only for second-order systems, their use is limited to the tuning of simple second-order filters, or such filter stages in cascade.

The partial derivatives of the parameters ω_0 and Q of the manufactured circuit with respect to the tuning elements may be approximated by using either the forward or central difference quotient. The desired changes in the parameter values are calculated and the resulting information is substituted into the differential expression. The solution of a set of linear algebraic equations gives the values of the tuning elements required to tune the filter. Such parameter sensitivity methods lend some theoretical insight into the tuning problem but are not used in practice [14].

1.2.4 Functional Tuning

Functional tuning is a "seat of the pants" approach to filter tuning. At the time of manufacture the filter is powered up, or made functional, while a sinusoidal excitation is applied and measurements are made at a set of critical frequencies. The tuning elements are adjusted one at a time until required gain or phase specifications at these critical frequencies are met.

Functional tuning is straightforward enough in tuning second-order filter sections where there are simple relationships between the desired parameters and the circuit elements. However, because of interaction between the circuit elements it is often unclear which resistors to adjust and their order of adjustment. It is also possible to overshoot the tuned resistance values, since they are not known. Thus, an experienced technician is usually required to handle the tuning and testing. The method is a slow process; the obtainable accuracy is proportional to the number of iterations of the tuning steps [14].

It is not necessary to measure the component values or the parasitic elements of the filter when it is functionally tuned. As an added benefit, parasitics are automatically taken into account and effectively tuned out which makes it attractive to use in practice.

1.2.5 Deterministic Tuning (Coefficient Matching)

To be deterministically tuned, a thin or thick film filter is designed so that any passive element or parasitic element may be measured by connection to two appropriate pins on the side of the substrate. Thus, in the manufacturing process, the circuit elements are not all interconnected on the substrate. After tuning, certain pins are bridged and the filter is powered up and ready for service [6].

The solution of a set of nonlinear equations gives the required values of tuning resistance needed to tune the filter. To generate this set of equations, the symbolic transfer function including parasitic elements is found. The nonlinear equations are generated by matching the transfer function coefficients of the nominal filter with those of the manufactured filter. A set of tuning resistors is chosen and these resistors are solved for in terms of the measured capacitors, parasitic elements, remaining resistors, and the (known) nominal coefficients.

If the filter is to work properly when powered up, parasitic effects must be taken into account. This usually means including all first-order parasitic effects, and possibly second-order effects in the circuit model of the filter [14]. The degree and complexity of the resulting transfer function and the tuning resistor expressions increase markedly, making the tuning resistor expressions difficult to derive, and necessitating the use of powerful computer programs for their solution [6].

It is not known beforehand which set of resistors to choose for tuning the filter, or whether or not a solution to the deterministic tuning problem for the selected set of tuning resistors exists. There may be more than one set of tuning resistors which yields a solution, exactly one set which yields a unique solution, or there may not be a set which yields a solution, depending on the circuit topology and the component values. Even if a solution exists, there might be algebraic or numerical problems in finding it [6].

The main advantage of deterministic tuning over functional tuning is that tuning is accomplished by a non-interacting series of tuning steps.

1.2.6 Hybrid Tuning

In practice it is often most efficient to use a combination of functional and deterministic tuning. The filter is first determinisically tuned assuming ideal elements and then proceeded by a "functional touch-up" which is used to tune out undesirable parasitic effects [14].

1.2.7 Optimization

The general iterative circuit optimization process [12] consists of specifications, an initial approximation, and error evaluation. The specifications might be the nominal gain at a set of critical frequencies, for example. The allowable deviations from these specified values is also given. An initial circuit provided by the designer starts the process up. This circuit is the initial approximation and might be the gain of the first guess tuned circuit (measured at the same set of critical frequencies as the nominal) of one of the before mentioned methods. The initial approximation must be a good one or the optimization process may fail to converge, or converge to an inferior final tuned circuit.

The error is defined as the difference between the nominal response, and the tuned response, which changes, along with the tuning resistance, from iteration to iteration. Usually a single number called the performance index is used as a measure of the circuit performance at the ith iteration [12].

To determine whether or not the ith circuit is acceptable, it is compared to the nominal circuit and the (i-1)th circuit. If the deviation of the ith circuit from the nominal circuit is within acceptable limits the iteration stops. The iteration also stops if the tuning element values and/or the performance index does not change appreciably from one iteration to the next. In addition, the program should limit the maximum number of iterations by coming to a halt after a fixed number of iterations to avoid excessive computational costs, regardless of the circuit performance [12].

If the iteration continues, the tuning resistors are readjusted according to some rule which results in a decrease in the error at the next iteration [12].

Any of the before mentioned tuning methods (with the exception of functional tuning) may be improved upon using optimization. An example of this which casts the deterministic tuning problem as an optimization problem can be found in the literature [6]. To begin the process, the transfer function coefficient deviation vector $\underline{\Delta z}$ is found [6]. This is given by

$$\underline{\Delta z} = \underline{f}(\underline{G} + \underline{\Delta G}, \underline{C} + \underline{\Delta C}, \Delta G_p) - \underline{f}(\underline{G}, \underline{C}, \underline{0}) .$$

The first term on the right-hand side of the equation is the vector of coefficients of the manufactured transfer function; the second term is the vector of coefficients of the nominal transfer function. The transfer function coefficients are nonlinear functions of the resistors and capacitors in the circuit. In the manufactured circuit the jth resistor and capacitor are perturbed from their nominal conductance G_j and capacitance C_j by amounts ΔG_j and ΔC_j, respectively. In addition, there are parasitic elements ΔG_{pj} which are not present in the nominal circuit. Since it is not feasible to tune capacitors, the manufactured capacitors $\underline{C} + \underline{\Delta C}$ and the parasitic elements $\underline{\Delta G}_p$ are fixed, so that only the conductances $\underline{\Delta G}$ may be tuned. The objective of deterministic tuning is to solve for a subset of the conductances $\underline{\Delta G}$ (leaving the remaining conductances fixed) that drives the difference vector $\underline{\Delta z}$ to zero. By approximating $\underline{\Delta z}$ by the differential vector $\underline{dz}$, and then expressing it as a recursion relation and defining a performance index, the deterministic tuning problem is cast as an optimization problem [6,15].

The main advantage of optimization in this example is that problems of existence and uniqueness of a solution that occur in conventional deterministic tuning are overcome. It should be noted that all of the resistors are adjusted in this scheme.

There are several drawbacks to be considered here, however. Perhaps the most serious of these is the excessive computational cost of computing the partial derivatives of each numerator and denominator coefficient of the transfer function with respect to each passive and parasitic element for each filter stage. These costs are intensified when all first-order and some second-order parasitic effects are included in the circuit model of the filter, especially those of higher-order.

Multistage filters are tuned a stage at a time by this method. The algorithm has not been applied to the tuning of a multiple feedback structure [16,17], so that it is not known how successfully it tunes filters of this variety. As in deterministic tuning, the transfer function in symbolic form including parasitic elements is needed for each filter stage.

Moreover, individual tuning rules must be derived and programmed for each filter stage, which does not readily lend itself to tuning new filter products, and results in additional programming costs.

1.2.8 A New Tuning Algorithm for Analog Filters

This thesis presents a new algorithm for tuning analog filters. Tellegen's theorem [18] and the adjoint circuit concept [9] are used to develop a large-change sensitivity expression which relates large changes in the tuning elements to desired voltage changes in the manufactured filter at a set of critical frequencies. The partial derivatives are computed efficiently and accurately via the adjoint method [9,10]. The desired voltage changes are unknown, because the tuned circuit voltages are not known. However, the nominal voltages are known and are used to approximate the desired voltage changes. Experience has shown that this approximation yields an excellent first guess for the component values needed to tune the filter. The component values of the manufactured circuit are measured, and a set of tuning resistors are chosen. The voltages of the manufactured filter are either measured or simulated via the computer, along with the voltages of the (simulated) nominal filter at the critical frequencies, and this information is substituted into the large-change sensitivity expression. The solution of a set of linear algebraic equations of rank T+1 (where T is the number of tuning elements) gives a first guess for the tuned values of the filter. The process is iterated by simulating the first guess tuned circuit and substituting its voltages and the new approximation for the desired voltage changes into the large-change sensitivity expression. The resulting set of linear algebraic equations is solved to give the new tuned values. This process is continued until the specified error and stopping criteria are met. In most cases, the method converges very quickly, as will be seen in the forthcoming examples. In fact, one can obtain results that are almost identical to the nominal response in only two or three iterations.

2. DERIVATION AND IMPLEMENTATION OF THE ALGORITHM

2.1 Derivation of the Algorithm

In this section the tuning algorithm described in Section 1.2.8 will be derived and steps for its implementation will be given. The foundation of the derivation is the differential Tellegen's theorem [18], which is

$$\text{(2.1)} \qquad \sum_{k=1}^{b} (\Delta V_k \hat{I}_k - \Delta I_k \hat{V}_k) = - \sum_{j=1}^{m} (\Delta V_{pj} \hat{I}_{pj} - \Delta I_{pj} \hat{V}_{pj}) \,.$$

The terms on the right-hand side of equation (2.1) represent independent source branch voltages and currents including output port branch constraints, while the voltages and currents on the left-hand side of the equation represent the remaining branch constraints. Equation (2.1) was derived under the assumption that there are three topologically identical networks, N, $\hat{N}$, and N_Δ. The component values in the N_Δ network have been perturbed from those in N such that the voltages and currents in the N_Δ network are $V_k + \Delta V_k$, and $I_k + \Delta I_k$, respectively. Equation (2.1) relates changes in voltages and currents in the N network to the voltages and currents in the $\hat{N}$ network [19,20,21].

Since only resistors will be used as tuning elements, the conductance branch constraints in the manufactured network N are

$$\text{(2.2)} \qquad I_k = G_k V_k$$

and in the tuned network N_Δ

$$\text{(2.3)} \qquad I_k + \Delta I_k = (G_k + \Delta G_k)(V_k + \Delta V_k) \,.$$

Subtracting (2.2) from (2.3) gives

$$\Delta I_k = G_k \Delta V_k + G_k (V_k + \Delta V_k) \ . \tag{2.4}$$

If the branch constraints of $\hat{N}$ are chosen such that $\hat{N}$ is the adjoint circuit of the network N [9], for example,

$$\hat{I}_k = G_k \hat{V}_k \ , \tag{2.5}$$

then upon substitution of these adjoint network branch constraints and the differential branch constraints of equation (2.4) into equation (2.1) gives

$$\sum_{k=1}^{T} (V_k + \Delta V_k) \hat{V}_k \Delta G_k = \sum_{j=1}^{2} (\Delta V_{pj} \hat{I}_{pj} - \Delta I_{pj} \hat{V}_{pj}) \tag{2.6}$$

where T is the number of tuning elements and where a single input port and a single output port is assumed on the right-hand side of equation (2.6). Let port 1 denote the input port and port 2 denote the output port, and assume that the input port has a voltage source connected across its terminal pair. Since the purpose of tuning is to correct for deviations in gain at the output port (to within a constant), the quantities of interest are the output voltage V_0, or its change ΔV_0. On this basis, choose $\hat{V}_{p1} = 0V$ and $\hat{I}_{p2} = 1A$ so that equation (2.6) becomes

$$\sum_{k=1}^{T} (V_k + \Delta V_k) \hat{V}_k \Delta G_k = \Delta V_{p2} \ . \tag{2.7}$$

Requiring the output voltage of the tuned filter to be within a constant of the nominal value over the frequency spectrum results in

$$\Delta V_{p2}^{j} = c V_{0d}^{j} - V_{0m}^{j} \tag{2.8}$$

where V_{0m} and V_{0d} are the output voltages of the manufactured and nominal design circuits, respectively, the superscript j denotes the frequency at which the deviation is computed, and c is an unknown constant. Let n represent the number of critical frequencies at which the measurements are made. Then equation (2.7) becomes

$$(2.9)\quad \begin{bmatrix} (V_1+\Delta V_1)^1\hat{V}_1^1 & \cdots & (V_T+\Delta V_T)^1\hat{V}_T^1 & -V_{0d}^1 \\ \vdots & & \vdots & \vdots \\ (V_1+\Delta V_1)^n\hat{V}_1^n & \cdots & (V_T+\Delta V_T)^n\hat{V}_T^n & -V_{0d}^n \end{bmatrix} \cdot \begin{bmatrix} \Delta G_1 \\ \vdots \\ \Delta G_T \\ c \end{bmatrix} = \begin{bmatrix} -V_{0m}^1 \\ \vdots \\ -V_{0m}^n \end{bmatrix} .$$

The number of tuning elements and the number of frequencies must be chosen so that the rank of equation (2.9) is equal to the number of unknowns. Since the number of unknowns is T+1 and equation (2.9) is complex, this requires that $2n \geq T+1$.

The voltages V_k are the branch voltages in the manufactured circuit and the ΔV_k are the voltage changes which occur when the circuit is tuned. Usually, the term ΔV_k is neglected in equation (2.9) and the resulting expression is used to compute the first-order transfer function sensitivity

$$\left.\frac{\partial V_0}{\partial G_k}\right|_N = V_k\hat{V}_k$$

where the derivatives are evaluated about the manufactured circuit. However, below it is shown how to estimate the branch voltages ΔV_k which is the key to the new algorithm. Clearly the ΔV_k terms cannot be determined without a knowledge of the ΔG_k terms. However, since the tuned circuit will have essentially the same poles as the nominal circuit design, and since the zeros of the transfer functions for the internal branches will typically lie outside of the passband of the filter, then the branch voltages in the tuned circuit may be approximated as

$$(V_k + \Delta V_k)^j \approx V_{kd} \qquad (2.10)$$

where the V_{kd} are the branch voltages in the nominal circuit design. Thus, the tuning algorithm may be implemented as follows:

1) Select the number of tuning resistors such that T is less than or equal to the number of poles and zeros of the desired transfer function.
2) Select the number of frequencies n such that 2n = T+1, or if T+1 is odd, 2n = t+2. Select frequencies in the neighborhood of the band-edge of the filter, where the phase is changing most rapidly.
3) Analyze the circuit with its component values equal to their nominal design values at each of the critical frequencies in step 2, and substitute these branch and output voltages into equation (2.9) for $(V_k + \Delta V_k)$ and V_{0d}.
4) Measure the component values in the manufactured circuit.
5) Compute or measure V_{0m} at the critical frequencies in step 2.
6) Solve the adjoint circuit with its component values equal to those of the manufactured circuit at the critical frequencies in step 2 in order to obtain $\hat{V}_k$.
7. Solve equation (2.9) to obtain the ΔG_k terms, and the constant c.

8. Compute the error using the error criteria of Section 2.2. If the error is too large, proceed to step 9. Otherwise, stop the iteration.
9. If the maximum number of iterations specified has not been exceeded, generate a new manufactured circuit in the computer by replacing G_k by $G_k + \Delta G_k$, where the ΔG_k terms are obtained from step 7.

2.2 Error and Stopping Criteria

Of primary importance in the realization of the algorithm is a satisfactory means of halting the program when an acceptable tuned response is obtained. While a visual display or a hard copy graph could be made at each iteration, such an approach is time-consuming and uneconomical, although it does give an unmistakeably clear picture of whether or not the circuit at each iteration meets specifications. In lieu of this approach, the program examines up to three frequency bands specified by the user and halts execution when the program error criteria are met. In addition, the program limits the maximum number of iterations by coming to a halt after a fixed number of iterations (specified by the user) to avoid excessive computational costs, regardless of the tuned circuit performance. The error and stopping criteria for the various filter types will now be examined.

2.2.1 Criteria for a Lowpass Filter

In the design of a given filter type, the maximum gain deviation in the passband and the minimum attenuation in the stopband are parameters which are important to the circuit designer. Since these parameters are indicative of the gain response, they are a useful measure of the tuned circuit performance at each iteration. For the following discussion, refer to Figure 2.1. Let M_1 be the maximum gain in the passband (defined between ω_{p_1} and ω_{p_2}) and M_2 the minimum gain in the passband. Then define the maximum gain deviation in the passband D to be $D = M_1 - M_2$ dB. Let M_3 be the maximum gain in the stopband (defined between ω_{s_1} and ω_{s_2}). Then define $A = M_2 - M_3$ dB to be the minimum attenuation in the stopband as measured about the reference gain M_2.

The passband and stopband limits are specified by the user, along with the number of frequency points to be included in each band. The gain deviation that can be tolerated in the passband D_{max} is specified as well as the minimum attenuation acceptable in the stopband A_{min}. If these quantities are not specified, the program defaults to their (computed) nominal values. At each iteration the passband gain deviation D and the minimum attenuation A are calculated and compared to D_{max} and A_{min}, respectively. If these specifications are met, or if the change in tuning resistance from the previous iteration is less than one percent then the iteration is stopped.

It should be noced that for the special case of a Butterworth or Chebyshev filter, the maximum gain in the stopband occurs at the band-edge (at ω_{s_1} in Figure 2.1). This is a result of the monotonic property of Butterworth and Chebyshev polynomials in the stopband. Therefore in order to find the maximum gain in the stopband it is only necessary to compute the gain at the band-edge frequency for filters of this type.

2.2.2 Criteria for a Highpass Filter

The error and stopping criteria for a highpass filter is defined in the same manner as the lowpass filter criteria, and thus will not be repeated.

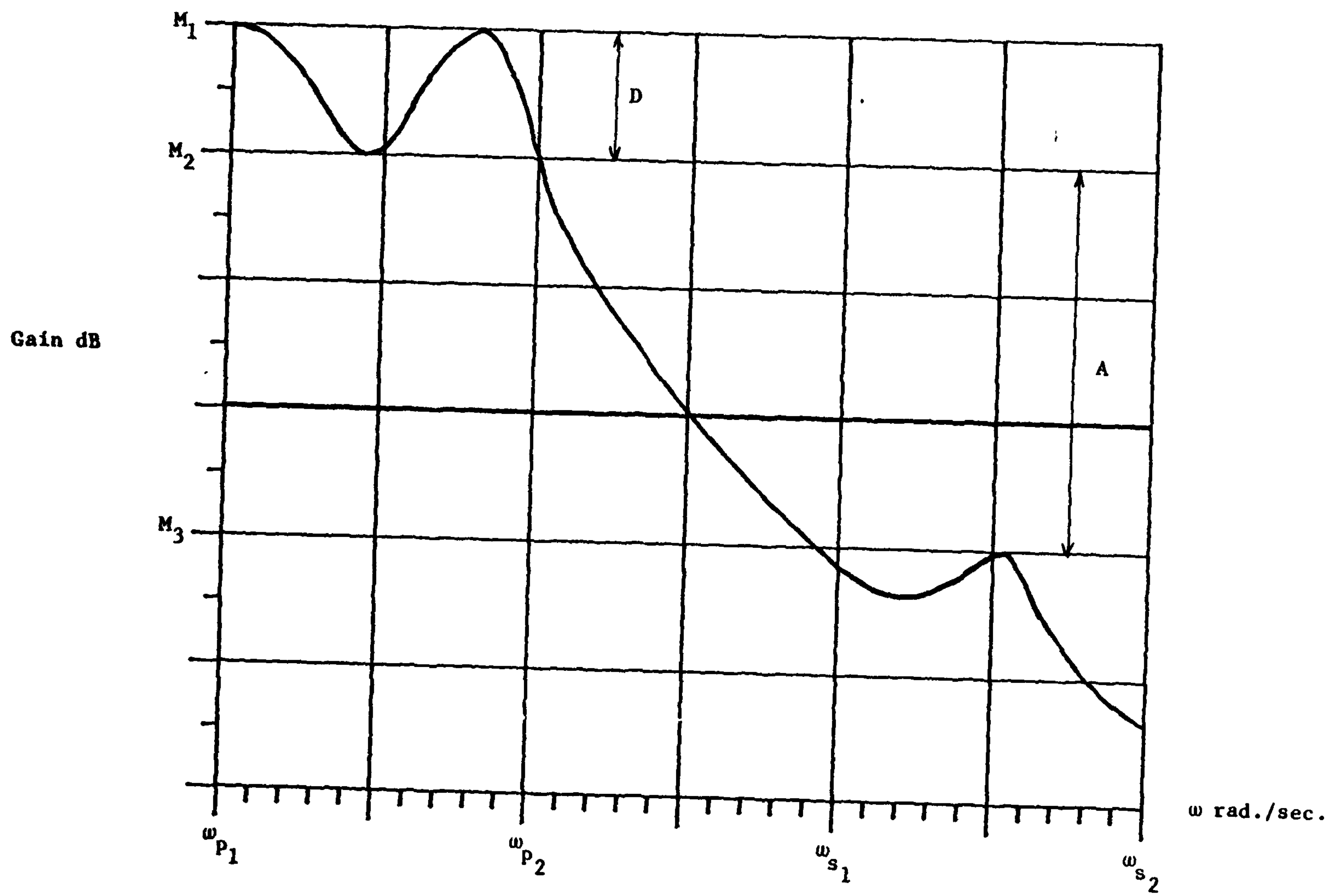

Figure 2.1 Typical Lowpass Gain Response

2.2.3 Criteria for a Bandpass Filter

The error and stopping criteria for a bandpass filter is similar to the lowpass filter criteria except for an additional stopband. For the following discussion, refer to Figure 2.2. Let M_1 and M_2 be the maximum and minimum gain in the passband, respectively, M_3 the maximum gain in the first stopband (defined between ω_{s_1} and ω_{s_2}) and M_4 the maximum gain in the second stopband (defined between ω_{s_3} and ω_{s_4}). Then define the maximum deviation in the passband D to be $D = M_1 - M_2$ dB. Another quantity of interest is the minimum attenuation in the first stopband, or the minimum attenuation in the second stopband, whichever is smaller (as measured about the reference gain M_2). Define the minimum attenuation A to be $A = M_2 - \max(M_3,M_4)$ dB where $\max(M_3,M_4)$ denotes whichever is greater, M_3 or M_4. The maximum gain deviation that can be tolerated in the passbank D_{max} and the minimum attenuation acceptable in the stopbands A_{min} are specified by the user and compared with D and A at each iteration. If these specifications are met, or if the change in tuning resistance from the previous iteration is less than one percent, then the iteration is stopped.

2.2.4 Criteria for a Band Reject Filter

In arriving at a suitable error and stopping criteria for a band reject filter, refer to Figure 2.3. Let M_1 be the minimum gain in the first passband (defined between ω_{p_1} and ω_{p_2}), M_2 the minimum gain in the second passband (defined between ω_{p_3} and ω_{p_4}) and M_3 the maximum gain in the stopband (defined between ω_{s_1} and ω_{s_2}). The quantity of interest is the minimum attenuation in the stopband as measured about the reference gain M_1 or M_2, whichever is smaller. Then define the minimum attenuation A to be $A = \min(M_1,M_2) - M_3$ dB where $\min(M_1,M_2)$ is the smaller of M_1

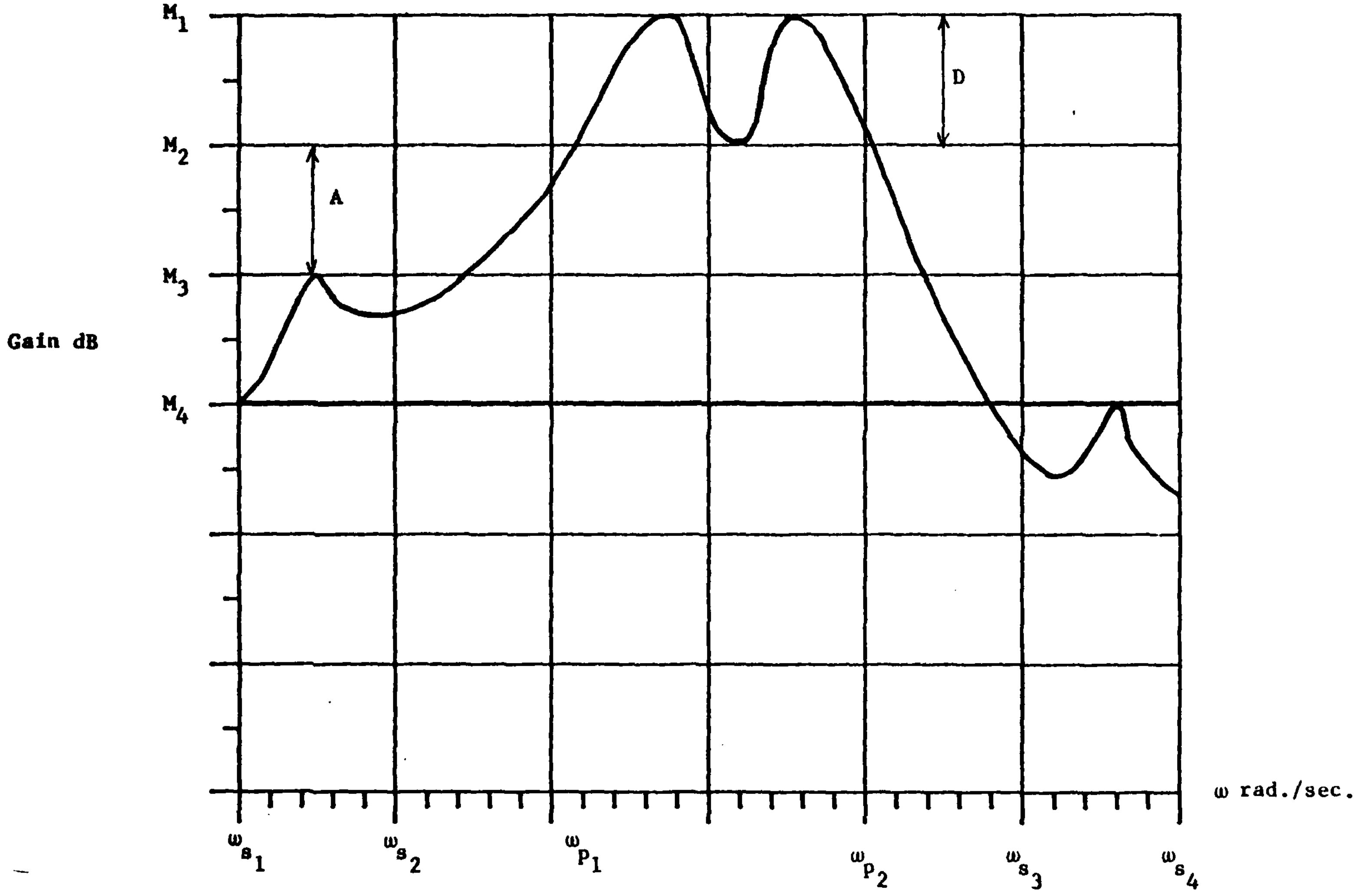

Figure 2.2 Typical Bandpass Gain Response

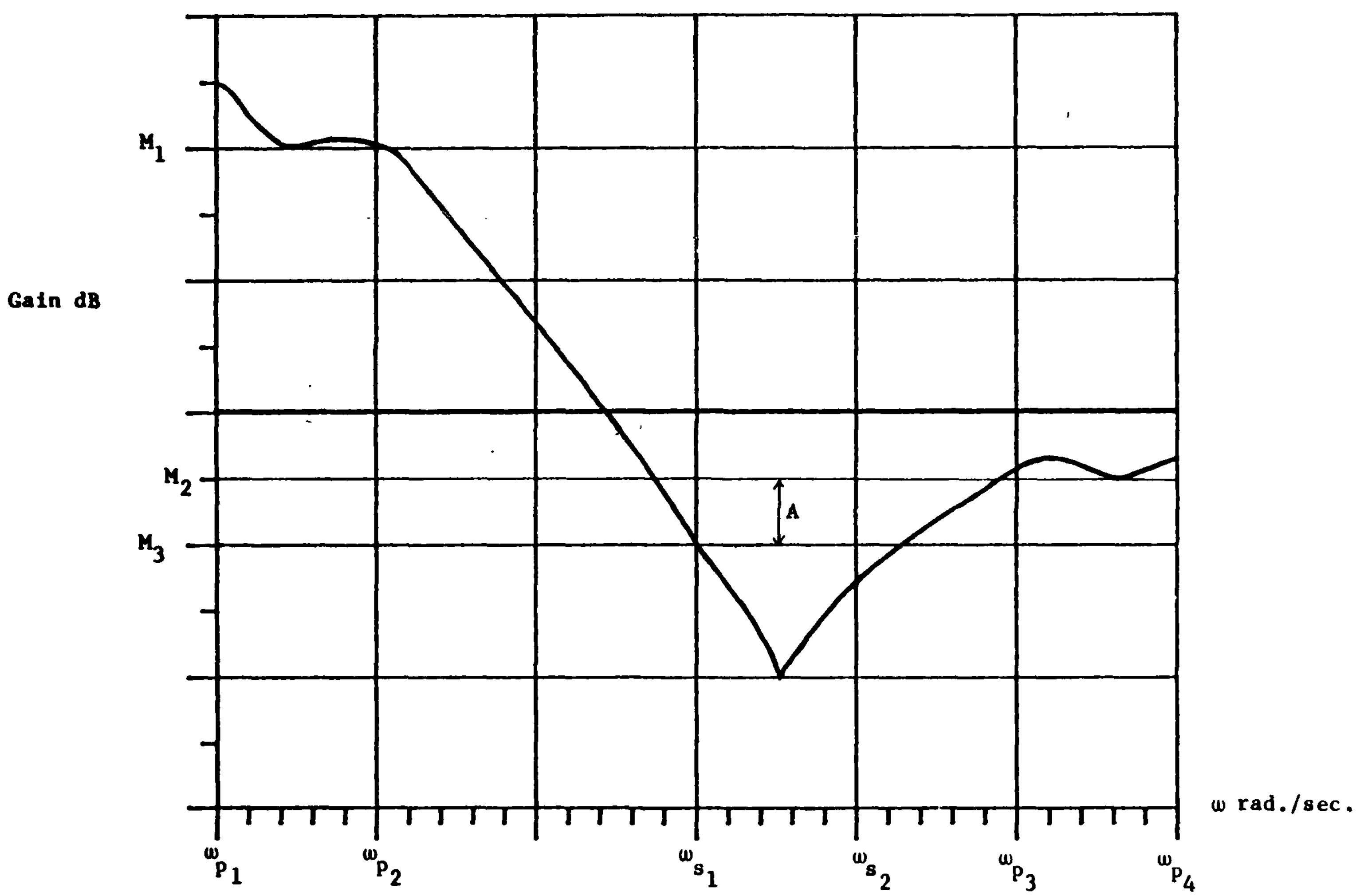

Figure 2.3 Typical Band Reject Gain Response

and M_2. The designer only specifies the minimum attenuation acceptable in the stopband A_{min}, which is compared to A at each iteration. If the specifications are met, or if the change in tuning resistance from the previous iteration is less than one percent, then the iteration is stopped.

2.2.5 Computer Program

A computer program written by the author which efficiently implements the tuning algorithm of Section 2.1 is available from the author. The ac steady state circuit analysis program (contained within and also written by the author) is used to simulate the manufactured, adjoint and nominal circuits. The circuit analysis program uses the modified nodal approach [23] to formulate the circuit equations, and the popular Decompose and Solve subroutines [24] to perform the Gaussian elimination. The adjoint branch voltages and currents are efficiently found from the LU factorization of the manufactured circuit [10].

3. TESTING THE ALGORITHM

In Section 1.2.8 a new algorithm was presented for the tuning of analog filters, which was subsequently derived in Chapter 2. It remains to test the algorithm on a number of filter samples which are representative of the manufacturing process. Before this can be done, the characteristics of the manufacturing process must be understood. As hybrid thin or thick film integrated circuits are a popular means of designing analog filters, in what follows a brief overview of some of the characteristics of the manufacturing process will be given.

3.1 Characteristics of the Thin Film Process

The initial production tolerance of a thin film resistor is $\pm$ 5 percent of its nominal value. This is primarily due to the sheet resistance of a thin film circuit, which can be controlled to $\pm$ 5 percent tolerances. The tolerances of the length or width of a resistor depend on the masking technique used, and can be made to within a range of $\pm$ .1 mil to $\pm$ .5 mil. The inaccuracy of the longer dimension is usually negligible with the length of the smaller one. Thus, if all the resistors on the substrate have the same smaller dimension (whichever is smaller, length or width) their values will all deviate from their nominal values by the same percentage. Furthermore, it is primarily the smaller dimension which determines the tolerance of the resistors [4].

All resistors can be routinely trimmed at the same time (by monitoring a single resistor) to within $\pm$ 1 percent if properly designed, and individually trimmed to an accuracy of $\pm$ .1 percent although at increased manufacturing cost. In the manufacturing process, the sheet resistance is initially produced below its desired value, the resistance values are measured and then the resistors are trimmed up to their desired values [4].

Parasitic effects of thin film resistors are quite small. These resistors usually perform better at high frequencies than their discrete counterparts. However, at frequencies in the megahertz range they can be modeled as shown in Figure 3.1. Typically the value of the shunt parasitic capacitance C_p is on the order of a few tenths of a picofarad [4].

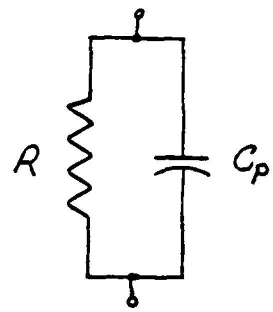

Figure 3.1 Model of a Thin Film Resistor Including Parasitic Effects

Normally, there is good tracking between resistors, especially those closest together on the substrate [4].

The initial production tolerance of a thin film capacitor is $\pm$ 5 percent of its nominal value [6]. The only parasitic effect to be considered in properly designed thin film capacitors is the series resistance R_{ps} which results from the finite conductivity of the two electrodes. This resistance usually varies from a few ohms in larger capacitors (on the order of .01 μF) to a few hundred ohms in very small capacitors. The shunt or leakage resistance R_{pp} is normally greater than 10^7 in which case its effects are negligible. A model for the thin film capacitor which includes parasitic effects is shown in Figure 3.2 [4].

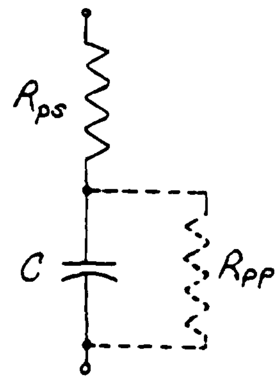

Figure 3.2 Model of a Thin Film Capacitor Including Parasitic Effects

Because conductors which interconnect the elements are usually very short, parasitic capacitance and parasitic inductance effects between conductors are usually less than their discrete circuit counterparts, and can be neglected. However, the resistance of the conductors can range from .1 to 1 ohm/square so that this effect must be included in the circuit model in cases where the circuit performance is critical [4].

3.2 Characteristics of the Thick Film Process

The initial production tolerance of a thick film resistor is ± 20 percent of its nominal value, and with trimming, resistor tolerances can be kept below 1 percent. As with thin films, it is the smaller dimension that primarily determines the tolerance of the resistors [4].

After screening and firing, the thick film resistance values are within ± 25 percent of the nominal values. Thus, in the manufacturing process, a good approach is to initially produce the sheet resistance 25 percent below the needed value, and then trim the resistors up to their desired values [4].

Although thick film capacitors can be made it is more economical to use discrete capacitors unless a large number is needed. It is more common to use thick films only for resistors and for conductors to interconnect the elements. Resistance of thick film conductors ranges from .1 to .01 ohms/square but this range can be decreased by a factor of ten less by solder coatings. In some instances it may be necessary to include this resistance in the circuit model [4].

3.3 Op Amp Characteristics

Discrete op amp chips are usually bonded to the thin or thick film circuits to form a hybrid thin or thick film filter. While an ideal op amp model facilitates the design of an active filter, a more accurate model might include parasitic elements which account for the finite input and output resistance and finite gain-bandwidth product of real op amps.

3.4 Generating a Sample Circuit

Armed with a knowledge of the hybrid thin or thick film manufacturing characteristics, it is now possible to generate some sample filters representative of the manufacturing process. In succeeding chapters the algorithm will be tested on a number of filter samples of various type, order and topology, and the yield of the simulated manufacturing process will be found.

Consider now the generation of a single sample circuit in which parasitic effects are omitted for simplicity. In order to simulate resistors in the sample filter circuit, it is assumed that in the manufacturing process the initial sheet resistivity is produced 25 percent below its desired value for example, that all resistors have been measured, and that tuning resistors have been individually trimmed 10 percent below their nominal value and the other resistors trimmed to their nominal value.

In light of the previous discussion the capacitors are modeled as follows. The first capacitor is assumed to be triangularly distributed in a five percent band about its nominal design value. Subsequent capacitors are assumed to be triangularly distributed in a two percent band about their nominal value plus the product of the nominal value and the percentage deviation of the first capacitor, and are not to exceed $\pm$ 5 percent of their nominal value (in which case they are truncated at + 5 percent or - 5 percent, whichever is closer).

To generate a single sample circuit, a random number generator is used to give uniformly distributed numbers in the interval [0,1]. There are a number of subroutines with this capability in the literature [25,26], and many computer systems include subroutines of this type in their libraries. A single sample circuit is generated as follows:

1) Set all tuning resistors 10 percent below their nominal value and all other resistors at their nominal value.

2) Assume a symmetric triangularly distributed probability density function $p(x)$ defined on some interval (x_l, x_h) with center $x_0 = (x_l + x_h)/2$. Find an expression for the probability distribution function $P(x)$ in terms of x, x_l, x_0 and x_h by integrating

$$P(x) = \int_{x_\ell}^{x} p(\xi)d\xi \qquad x_l \leq x \leq x_h$$

and using the property of probability density functions that

$$\int_{-\infty}^{\infty} p(\xi)d\xi = 1 \ ,$$

$P(x)$ is known to be a monotonically increasing function with maximum value of 1.

3) Find an expression for x in terms of x_l, x_0, x_h and P by solving for the inverse of P(x). That is

$$x = x(x_l, x_0, x_h, P) = P^{-1}(P(x)) .$$

4) Call the random number generator subroutine to give a real number α, $\alpha \in [0,1]$.

5) Since $P(x) \in [0,1]$, set $P = \alpha$.

6) If the first capacitance value has been generated go to step 7. Let C_{01} denote the nominal value of the first capacitor. Set $x_l = .95C_{01}$, $x_0 = C_{01}$, and $x_h = 1.05C_{01}$. Substitute these quantities and the value for α found in step 4 into the expression in step 3 and solve for x. Then x is the value of the first capacitor in the sample circuit. Define $k = (x - C_{01})/C_{01}$ to be the percentage deviation of the first capacitor.

7) Let C_{0j} denote the nominal value of the jth capacitor. To generate the jth capacitor in the sample circuit, repeat steps 4 and 5. Then set $x_0 = (1+k)C_{0j}$, $x_l = .98x_0$, and $x_h = 1.02x_0$. Substitute these quantities as well as the new value for α found in step 4 into the expression in step 3, and solve for x. Check to make certain that x lies in the interval $[.95C_{0j}, 1.05C_{0j}]$, and if not truncate x at $.95C_{0j}$ or at $1.05C_{0j}$, whichever is closer. Then x is the value of the jth capacitor in the sample circuit.

8) Repeat step 7 until all of the capacitors in the sample circuit have been generated.

In the actual manufacturing process, the actual trimmed resistance in step 1 may be measured and entered into the tuning algorithm, thus reducing the tolerance requirements on the resistor trimming equipment [6]. Also, any important parasitic effects can easily be included in the circuit model.

4. STATISTICAL ANALYSIS OF A SECOND-ORDER FRIEND HIGHPASS NOTCH FILTER

In order to test the tuning algorithm of Section 2.1, a second-order Friend circuit [27] was designed to realize the highpass notch transfer function given by

$$T(s) = 2.0 \frac{s^2 + 8\times10^6}{s^2 + 500s + 16\times10^6} \ .$$

The three frequency bands of the filter are defined as follows. The stopband ranges from 2814.97 to 2839.99 rad./sec., the first passband ranges from dc to 1600.0 rad./sec., and the second passband ranges from 4000.0 to 6283.19 rad./sec. The component values for the nominal design are given in Table 4.1 and the circuit diagram of the filter is shown in Figure 4.1. A minimum attenuation of 30 dB in the stopband is attainable with this design.

In order to simulate untuned filters of this type coming off the production line, five sample circuits were generated via the sequence of steps in Section 3.4. The percentage deviation in the component values of these sample circuits from the nominal are recorded in Table 4.1, along with the average value of the absolute percentage deviation in the capacitance values which will be referred to later. While the thin or thick film manufacturing process is not justified for such a small production volume as this, the number of sample circuits generated was restricted because of the rather high cost of simulating a large production volume.

Table 4.1 Component Values

Component	Nominal Value	% Deviation in the Component Values Sample Circuit 1	2	3	4	5
R_1	13.260 kΩ	0.00	0.00	0.00	0.00	0.00
R_2	93.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_3	214.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_4	2.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_5	2.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_6	12.467 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_7	10.00 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
C_1	.01 μf	-1.88	.58	-2.67	-3.24	.75
C_2	.01 μf	-1.35	.18	-3.42	-2.77	-.64
μ	10000.0	0.00	0.00	0.00	0.00	0.00
Average value of the Absolute % Dev. in the Capacitance Values		1.62	.38	3.05	3.01	.70

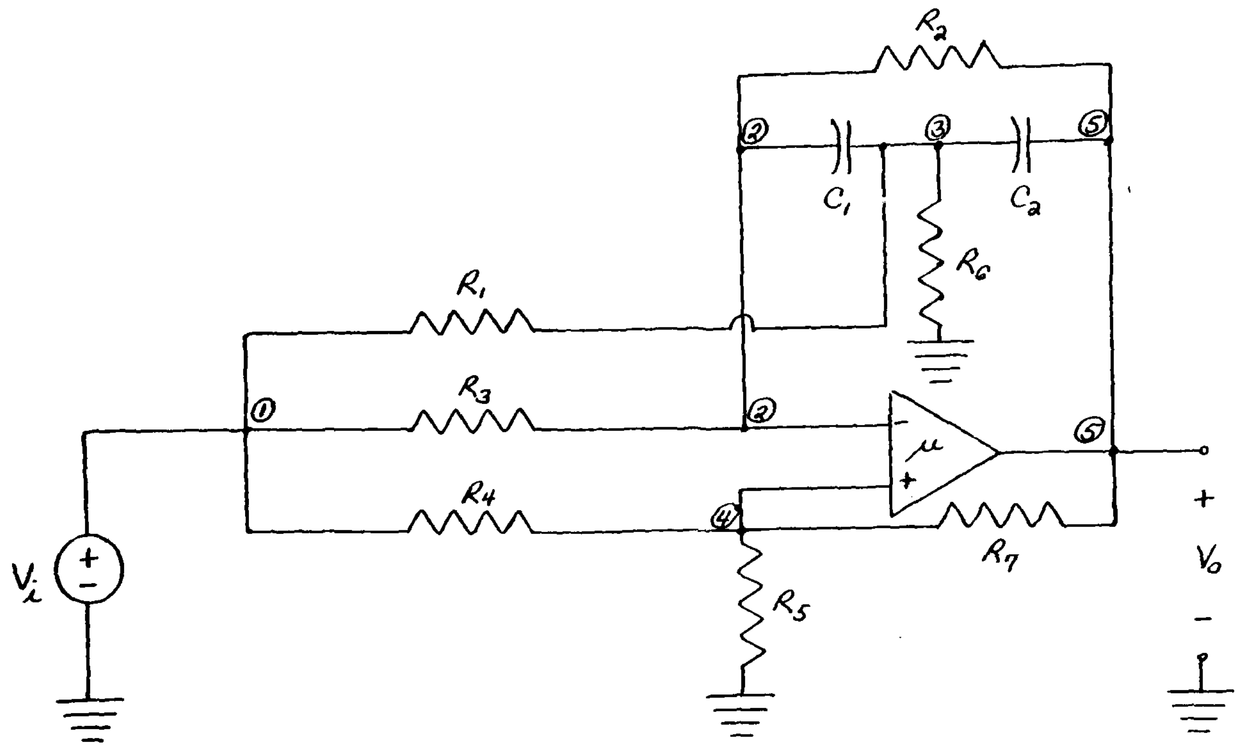

Figure 4.1 Circuit Diagram of a Second-Order Friend Highpass Notch Filter

The transfer function of an untuned filter is of the form

$$T(s) = K \frac{s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

where the transfer function coefficients K, b_1, b_0, a_1 and a_0 can be expressed in terms of the circuit components by using standard circuit analysis techniques. The resulting expressions (not required by the proposed large-change tuning algorithm) are complicated nonlinear functions of the resistors and capacitors in the circuit. Since it is not feasible to adjust capacitors, only resistors adjustments can be made to locate the poles and zeros of the manufactured circuit in their nominal locations. This can be done by driving the coefficients $b_1 \to 0$, $b_0 \to 8\times10^6$, $a_1 \to 500$ and $a_0 \to 16\times10^6$. To this end four resistors, R_3, R_5, R_6 and R_7 are selected to tune the filter, one for each coefficient. This requires that equation (2.9) be evaluated at three frequencies. These frequencies are chosen at 2828.4, 4000.0 and 4250.0 rad./sec. where the phase changes most rapidly. The coefficient K only effects the dc gain and has no effect on the pole-zero locations. However, deviations in the dc gain may easily be corrected within the filter if the symbolic transfer function is known. Otherwise it is a simple matter to append a single amplifier stage with gain (2/K) (or (1/C) in equation (2.9)) to correct for dc gain deviations.

Table 4.2 records the results of tuning the sample circuits. For each of the sample circuits the first iteration demonstrated a marked improvement in the minimum stopband attenuation, very nearly meeting the nominal specifications. All of the circuits met specifications in two iterations, at which time the algorithm converged to its final element

Table 4.2 Statistical Analysis of Friend HPN Filter Using Large-Change Transfer Function Sensitivity

Sample Circuit	Circuit Description	Min. Attenuation in Stopband (dB)
	Nominal	31.95
	Manufactured	2.48
1	Iteration 1	31.53
	Iteration 2	32.06
	Manufactured	.74
2	Iteration 1	31.48
	Iteration 2	31.96
	Manufactured	3.69
3	Iteration 1	27.50
	Iteration 2	32.41
	Manufactured	3.72
4	Iteration 1	27.96
	Iteration 2	32.41
	Manufactured	.98
5	Iteration 1	31.62
	Iteration 2	31.95

values. (In this chapter and in subsequent chapters, the last iteration recorded is the iteration where convergence occurs.) Finally, in each case the tuning resistors increased in value from their manufactured values.

The gain and phase response of the "worst case" sample circuit is depicted in Figures 4.2 and 4.3 respectively. The nominal response in this and in subsequent plots is always represented by a solid line, while a broken line may either represent the manufactured response or the tuned response depending on the context. By "worst case" circuit it is meant that circuit whose gain and phase response is the most distorted from the nominal. This is somewhat difficult to determine from just a knowledge of the minimum stopband attenuation so that the following measure will be used in this chapter and in the chapters to follow. Since the overall response (both gain and phase response) generally deteriorates as the circuit components are widely perturbed from the nominal (although some cancellations are possible), and because all of the sample circuits have the same resistance values, the percentage deviation in the capacitors gives some measure of how severely distorted the overall response will be. Therefore, the worst case circuit is found by averaging the absolute value of the percentage deviation in the capacitor values for each of the sample circuits and then designating the circuit with the highest average value as the worst case circuit. These averages (which were mentioned earlier) are recorded in Table 4.1, where the worst case circuit is seen to be circuit 3.

In just one iteration, the filter is very nearly tuned as the gain and phase plots of Figures 4.4 and 4.5 indicate. Here the broken lines represent the tuned response. (The tuned response plots in this chapter

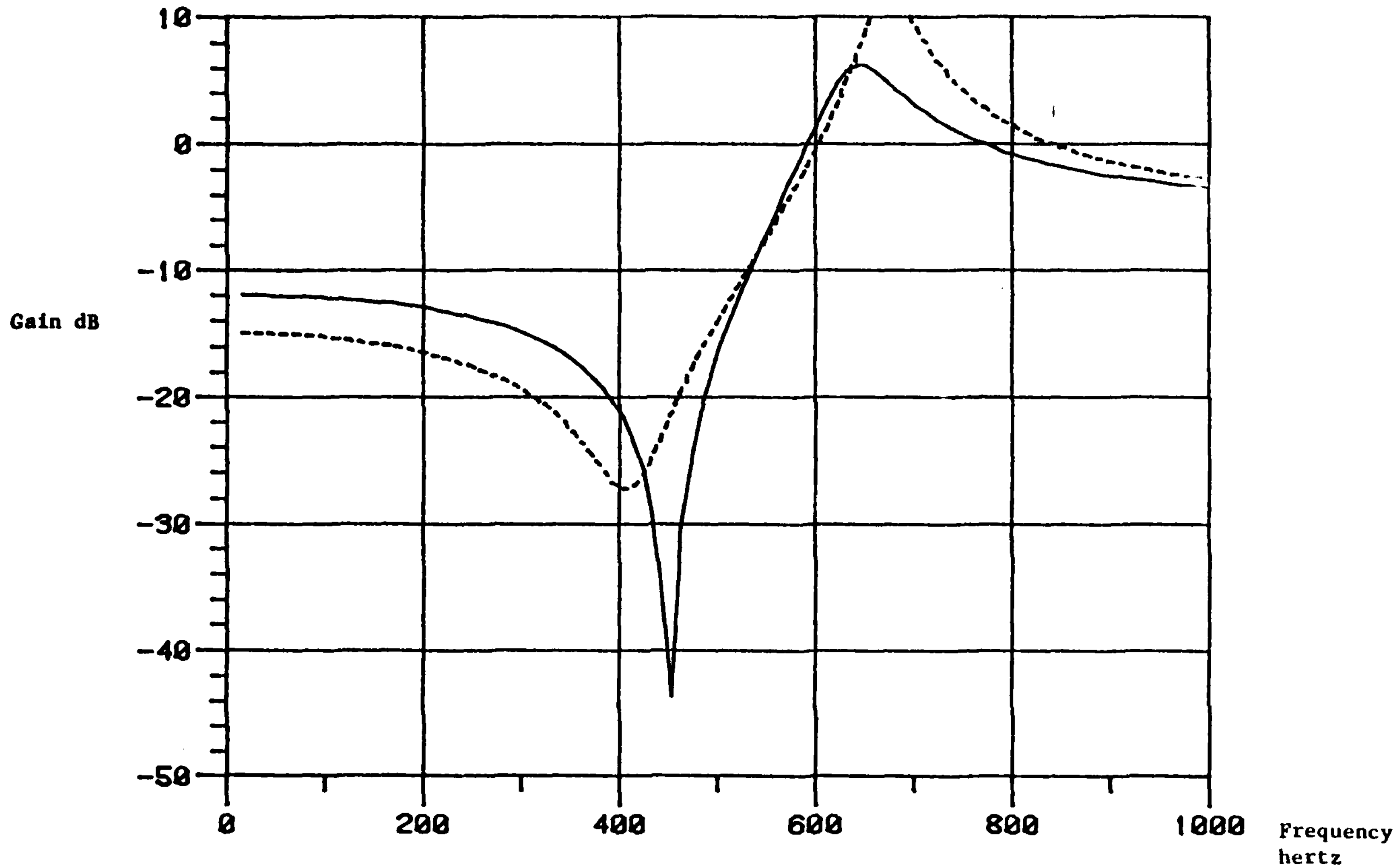

Figure 4.2 Gain Response of the Friend Highpass Notch Filer
—— Nominal Response
----- Manufactured Response

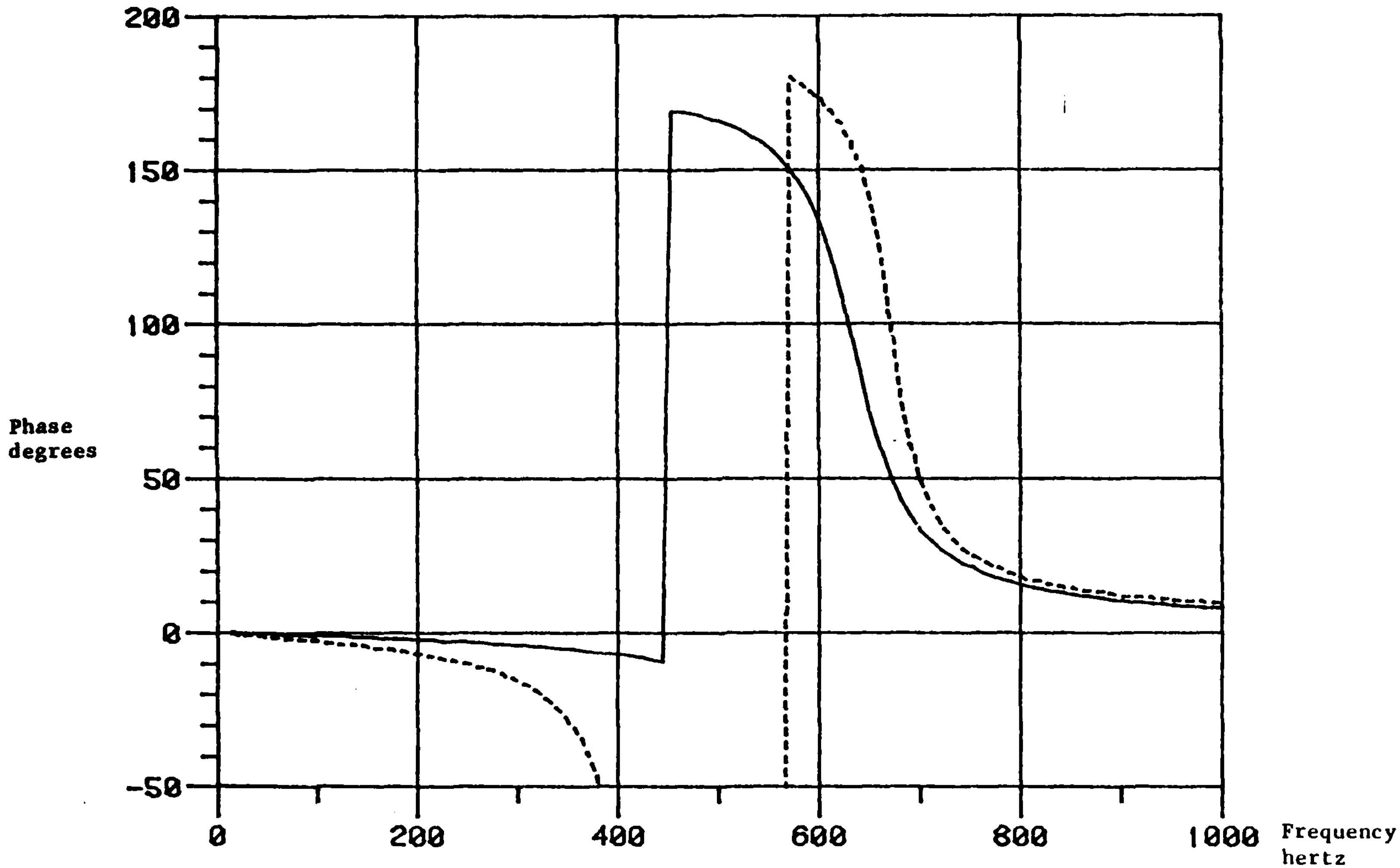

Figure 4.3 Phase Response of the Friend Highpass Notch Filter
——— Nominal Response
----- Manufactured Response

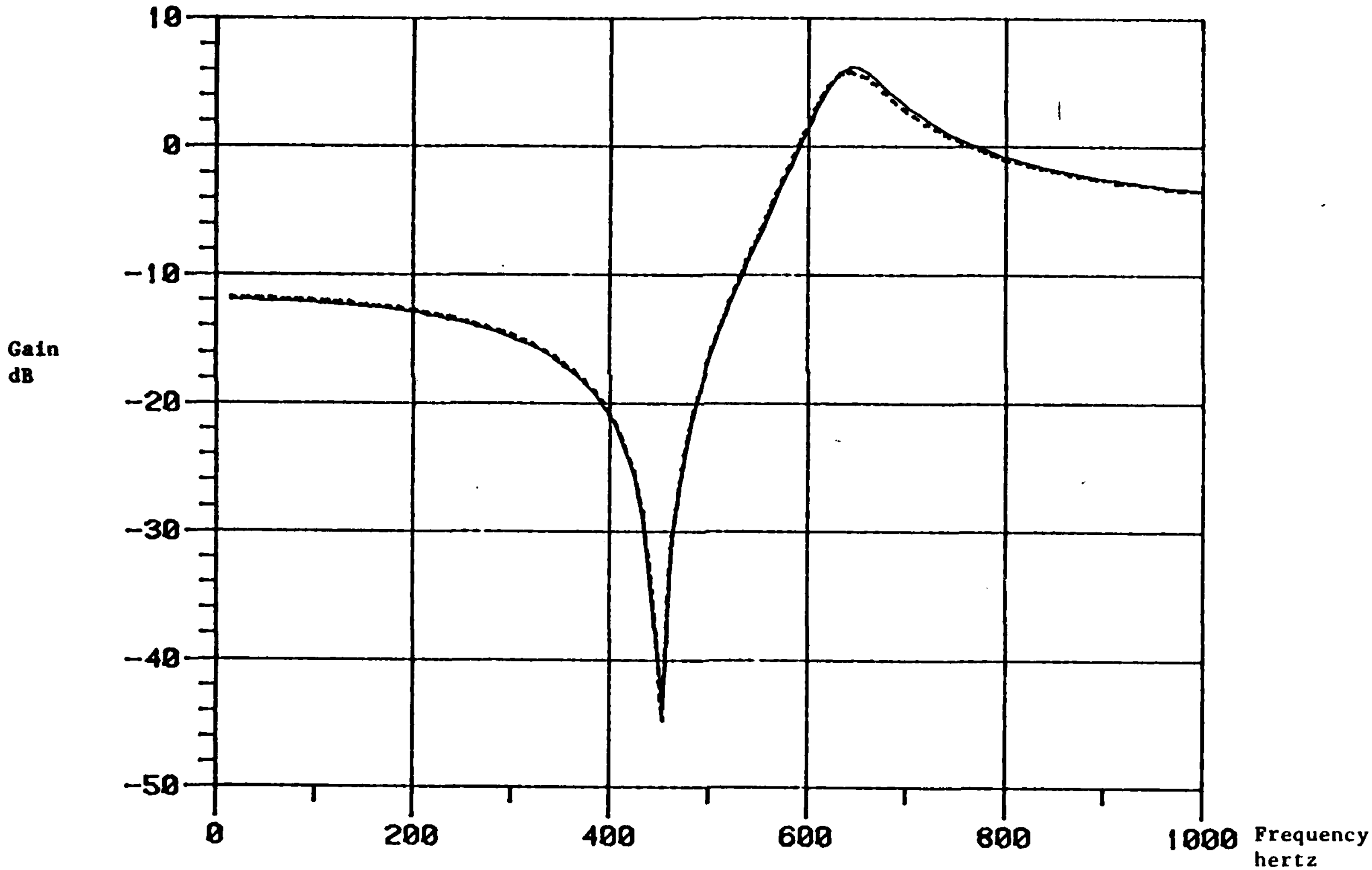

Figure 4.4 Gain Response of the Friend Highpass Notch Filter
—— Nominal Response
----- Tuned Response After One Iteration

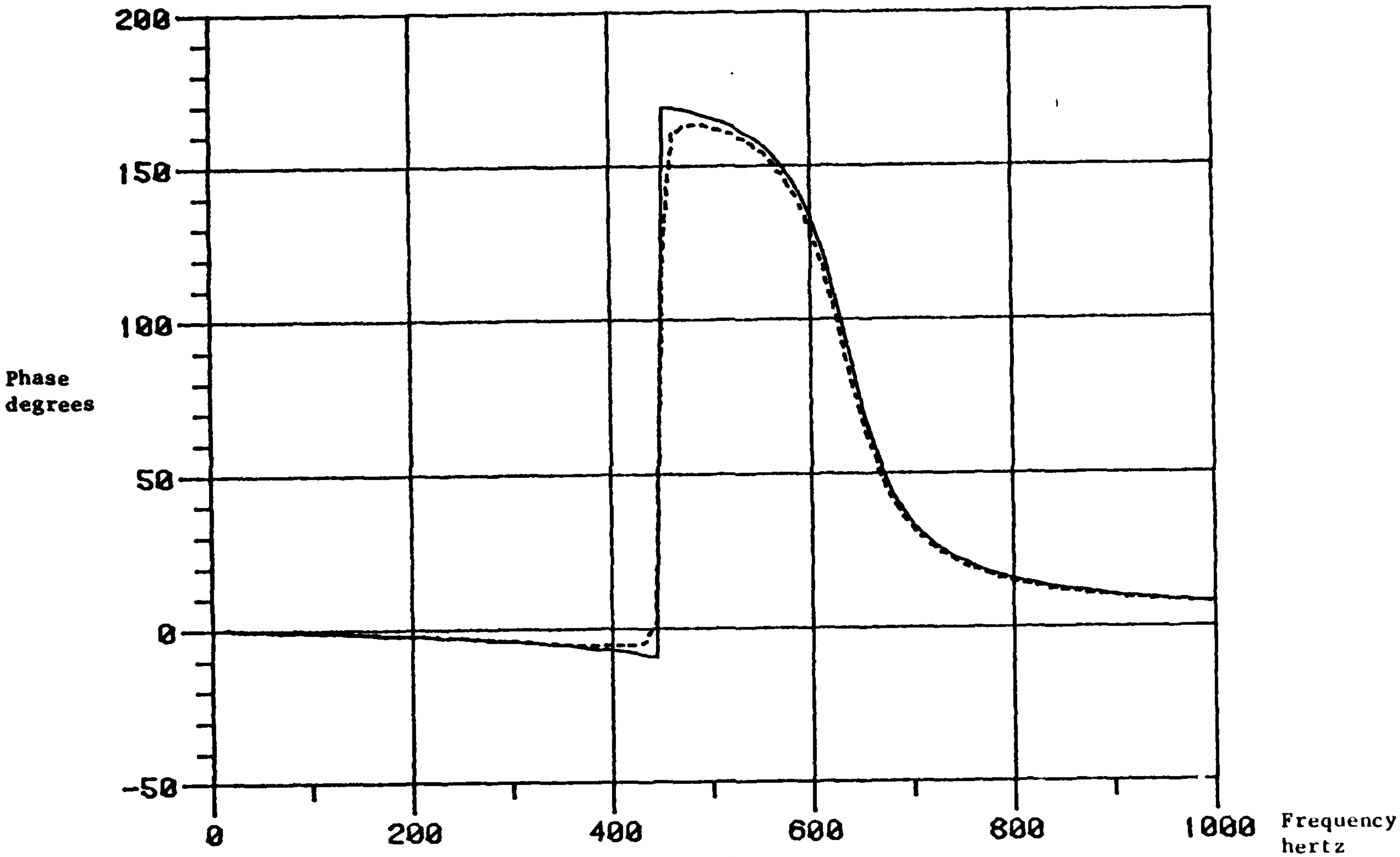

Figure 4.5 Phase Response of the Friend Highpass Notch Filter
——— Nominal Response
----- Tuned Response After One Iteration

and in the chapters to follow are shown with their corrected dc levels.) In one additional iteration the gain and phase response of the tuned circuit is virtually indistinguishable from the nominal gain and phase as shown in Figures 4.6 and 4.7. The final tuning element values at the iteration where convergence occurred are recorded in Table 4.3.

Finally, a first-order transfer function sensitivity method (ΔV terms set equal to zero in equation (2.9)) was tested using these same five sample circuits. The tuning results using this method are recorded in Table 4.4. It is interesting to note that the first-order method succeeded in tuning only those sample circuits whose average value of the absolute percentage deviation in the capacitance values was sufficiently small (sample circuits 2 and 5). Furthermore, a comparison of Tables 4.2 and 4.4 indicates that when the first-order sensitivity method worked it took twice as long to converge to the final element values as the large-change sensitivity method.

Although the circuits which could not be tuned by this first-order method exhibited a substantial improvement in the filter performance on the first iteration, in successive iterations the tuned circuit performance deteriorated substantially, eventually yielding one or more negative tuning element values, without converging to a final set of element values.

The first-order sensitivity method demonstrated 40 percent tuning reliability, although the circuits which it succeeded in tuning had only very minute changes in their capacitance values. This is to be compared to the 100 percent tuning reliability attained with the large-change sensitivity method which succeeded in tuning circuits with both small and large changes in their capacitance values. One might anticipate that in

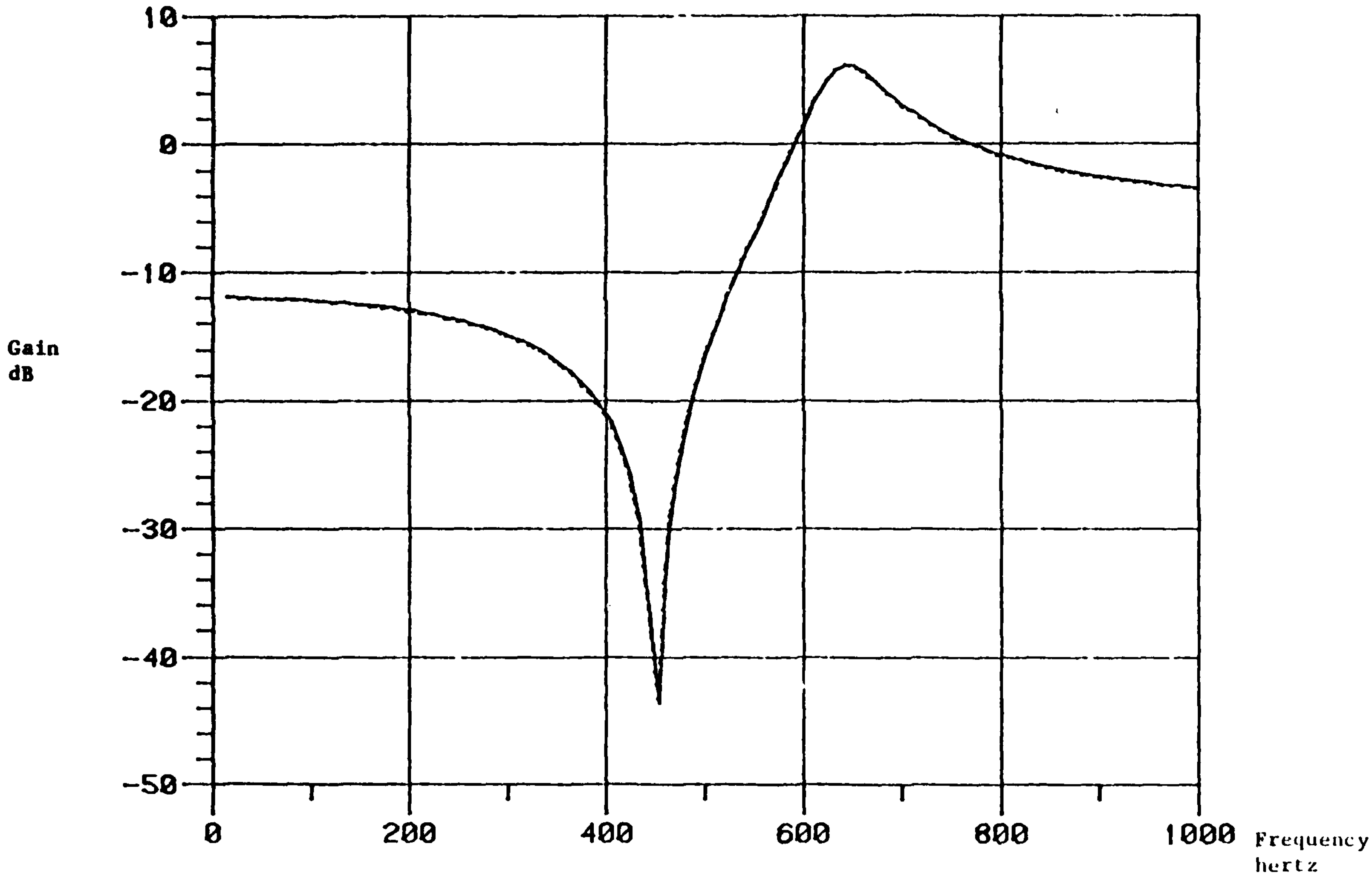

Figure 4.6 Gain Response of the Friend Highpass Notch Filter
——— Nominal Response
----- Tuned Response After Two Iterations

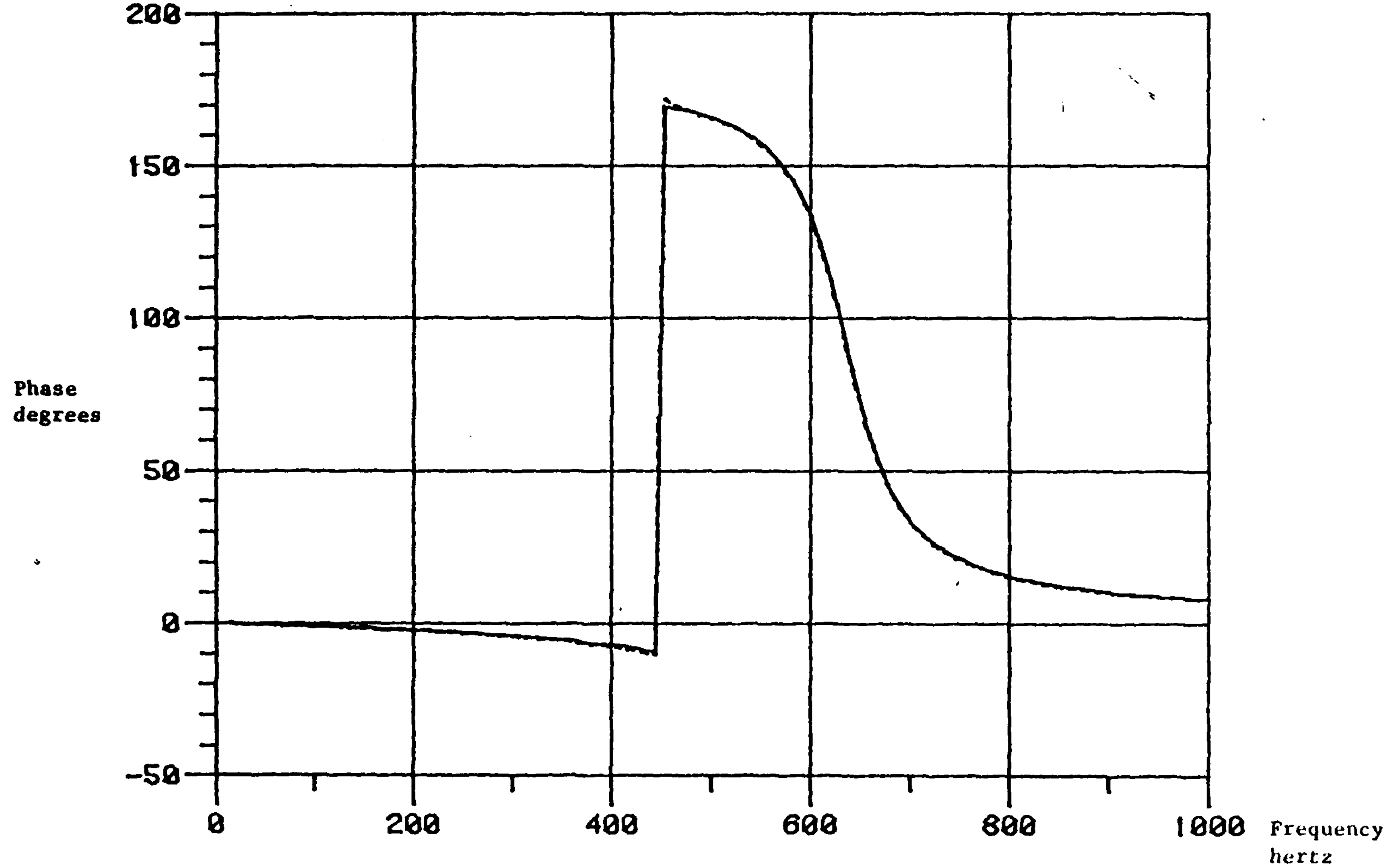

Figure 4.7 Phase Response of the Friend Highpass Notch Filter
——— Nominal Response
----- Tuned Response After Two Iterations

Table 4.3 Final Tuning Element Values

	Sample Circuit				
Element	1	2	3	4	5
R_3	202.8632 kΩ	216.6086 kΩ	192.9555 kΩ	193.4195 kΩ	214.1476 kΩ
R_5	2.123715 kΩ	1.972516 kΩ	2.242561 kΩ	2.238320 kΩ	1.996600 kΩ
R_6	13.18323 kΩ	12.30794 kΩ	13.87298 kΩ	13.84374 kΩ	12.45150 kΩ
R_7	9.985053 kΩ	10.01648 kΩ	10.05714 kΩ	9.999021 kΩ	10.06385 kΩ
c	1.030114	.9930847	1.057725	1.056805	.9991495

Table 4.4 Statistical Analysis of Friend HPN Filter Using First-Order Transfer Function Sensitivity

Sample Circuit	Circuit Description	Min. Attenuation in Stopband (dB)
	Nominal	31.95
1	Manufactured	2.48
	Iteration 1	25.64
	Iteration 2	29.57
	Iteration 3	16.52*
	Iteration 4	-18.74*
	Iteration 5	- .18*
2	Manufactured	.74
	Iteration 1	29.70
	Iteration 2	31.56
	Iteration 3	31.95
	Iteration 4	31.95
3	Manufactured	3.69
	Iteration 1	22.79
	Iteration 2	18.47
	Iteration 3	20.22*
	Iteration 4	13.16*
	Iteration 5	8.36*
4	Manufactured	3.72
	Iteration 1	22.92
	Iteration 2	17.74
	Iteration 3	16.91*
	Iteration 4	-11.16*
	Iteration 5	-16.08*
5	Manufactured	.98
	Iteration 1	29.01
	Iteration 2	31.40
	Iteration 3	31.93
	Iteration 4	31.95

*Iteration yielded negative element values.

higher-order filters, the ΔV terms in equation (2.9) become increasingly more important, so that the large-change estimate of the ΔV terms, given by equation (2.10) of the same section, becomes almost a necessity.

5. STATISTICAL ANALYSIS OF A FOURTH-ORDER BUTTERWORTH FREQUENCY-DEPENDENT NEGATIVE RESISTANCE (FDNR) FILTER

As another test of the proposed tuning algorithm a fourth-order Butterworth FDNR [28] lowpass filter was designed from a doubly-terminated passive prototype [29]. The admittances of the prototype filter are scaled by the complex frequency variable s to give a topologically equivalent network consisting of resistors, capacitors and frequency-dependent negative resistors [28]. The network is scaled to a new cutoff frequency of 6283.19 rad./sec. and the FDNR elements are realized by an active circuit due to Bruton [28] resulting in the circuit diagram of Figure 5.1.

The two frequency bands of the filter are defined as follows. The passband ranges from dc to 6283.19 rad./sec. and the stopband ranges from 12566.37 to 62831.85 rad./sec. The component values for the nominal design are given in Table 5.1. A maximum passband deviation of 2.98 dB and a minimum stopband attenuation of 21.04 dB is attainable with this design.

Five sample circuits were generated via the sequence of steps in Section 3.4 to simulate untuned filters of this type coming off the production line. The percentage deviation in the component values from the nominal, as well as the average value of the absolute percentage deviation in the capacitance values for each sample circuit is also given in Table 5.1.

The three resistors R_5, R_7 and R_8 are selected to tune this fourth-order filter, which requires the evaluation of equation (2.9) at two frequencies. These frequencies are chosen at 6283.0 and 5805.0 rad./sec.

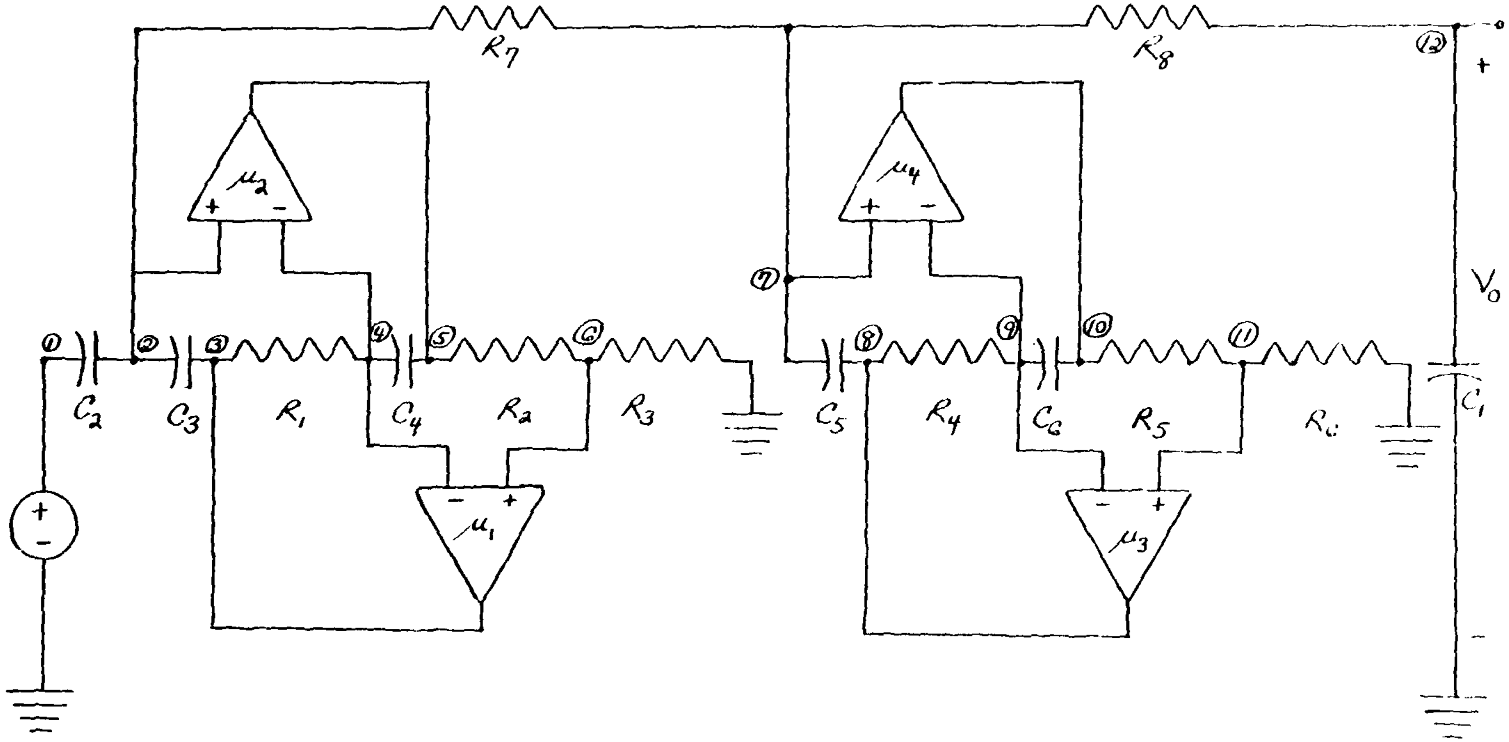

Figure 5.1 Circuit Diagram of a Butterworth FDNR Lowpass Filter

Table 5.1 Component Values

Component	Nominal Value	% Deviation in the Component Values — Sample Circuit 1	2	3	4	5
R_1	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_2	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_3	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_4	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_5	1.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_6	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_7	29.3 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_8	12.1 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
C_1	.01 μf	-1.88	-3.24	-.65	-4.01	-.72
C_2	.01 μf	-1.35	-2.77	-.57	-4.93	-1.68
C_3	.03493 μf	-1.65	-2.95	.38	-3.63	-.31
C_4	.03493 μf	-2.27	-4.57	-.83	-4.23	-1.16
C_5	.05422 μf	-2.93	-2.88	.08	-4.45	-.81
C_6	.05422 μf	-2.64	-3.46	-.78	-4.77	-1.00
μ_1	10000.0	0.00	0.00	0.00	0.00	0.00
μ_2	10000.0	0.00	0.00	0.00	0.00	0.00
μ_3	10000.0	0.00	0.00	0.00	0.00	0.00
μ_4	10000.0	0.00	0.00	0.00	0.00	0.00
Average Value of the Absolute % Dev. in the Capacitance Values		2.12	3.31	.55	4.34	.95

where the phase is changing most rapidly. Notice that the tuning resistors chosen consist of all of the resistors in the FDNR transformed lowpass prototype (corresponding to R_7 and R_8) and a single resistor in either grounded FDNR (corresponding to R_5).

The results of tuning the sample circuits are recorded in Table 5.2. In each case the filters met the specifications in a single iteration, at which time the iteration stopped. As an added plus, all of the tuning resistance values increased from their manufactured values.

Table 5.1 indicates that sample circuit 4 exhibits the worst overall response. The gain and phase response of this worst case circuit is shown for both the nominal and manufactured circuits in Figures 5.2 and 5.3, respectively. Figures 5.4 and 5.5 depict the respective gain and phase response of the tuned circuit after a single iteration. Some explanation of the phase plots is necessary. The actual phase of the nominal filter varies from 0 to -360 degrees, attaining a value of -180 degrees at f_c, -270 degrees at $2f_c$, and -360 degrees at about $10f_c$. However, because the built-in arctangent function on the computer is only capable of supplying angles between $\pm$ 180 degrees, the plots of Figures 5.3 and 5.5 result. The nominal phase is correct for frequencies up to and including the cutoff frequency. However, at a frequency lying just to the right of f_c, at f_c^+, the actual phase is $-(180 + \epsilon)$ degrees, where $\epsilon > 0$, which the built-in arctangent function interprets as $(180 - \epsilon)$ degrees, thus resulting in the sudden discontinuity in the phase at f_c. Similarly, as the frequency approaches $2f_c$, the actual phase approaches -270 degrees, which the built-in arctangent function interprets as 90 degrees. As the frequency approaches $10f_c$ (outside of the frequency limits of Figures 5.3 and 5.5) the actual phase approaches -360 degrees, which the built-in

Table 5.2 Statistical Analysis of Butterworth FDNR Lowpass Filter Using Large-Change Transfer Function Sensitivity

Sample Circuit	Circuit Description	Max. Dev. in Passband (dB)	Min. Atten. in Stopband (dB)
	Nominal	2.98	21.04
1	Manufactured	1.21	18.94
	Iteration 1	2.97	21.16
2	Manufactured	1.19	18.65
	Iteration 1	2.92	21.29
3	Manufactured	1.45	19.49
	Iteration 1	2.98	21.04
4	Manufactured	3.33	20.94
	Iteration 1	2.95	21.18
5	Manufactured	1.37	19.38
	Iteration 1	2.98	21.02

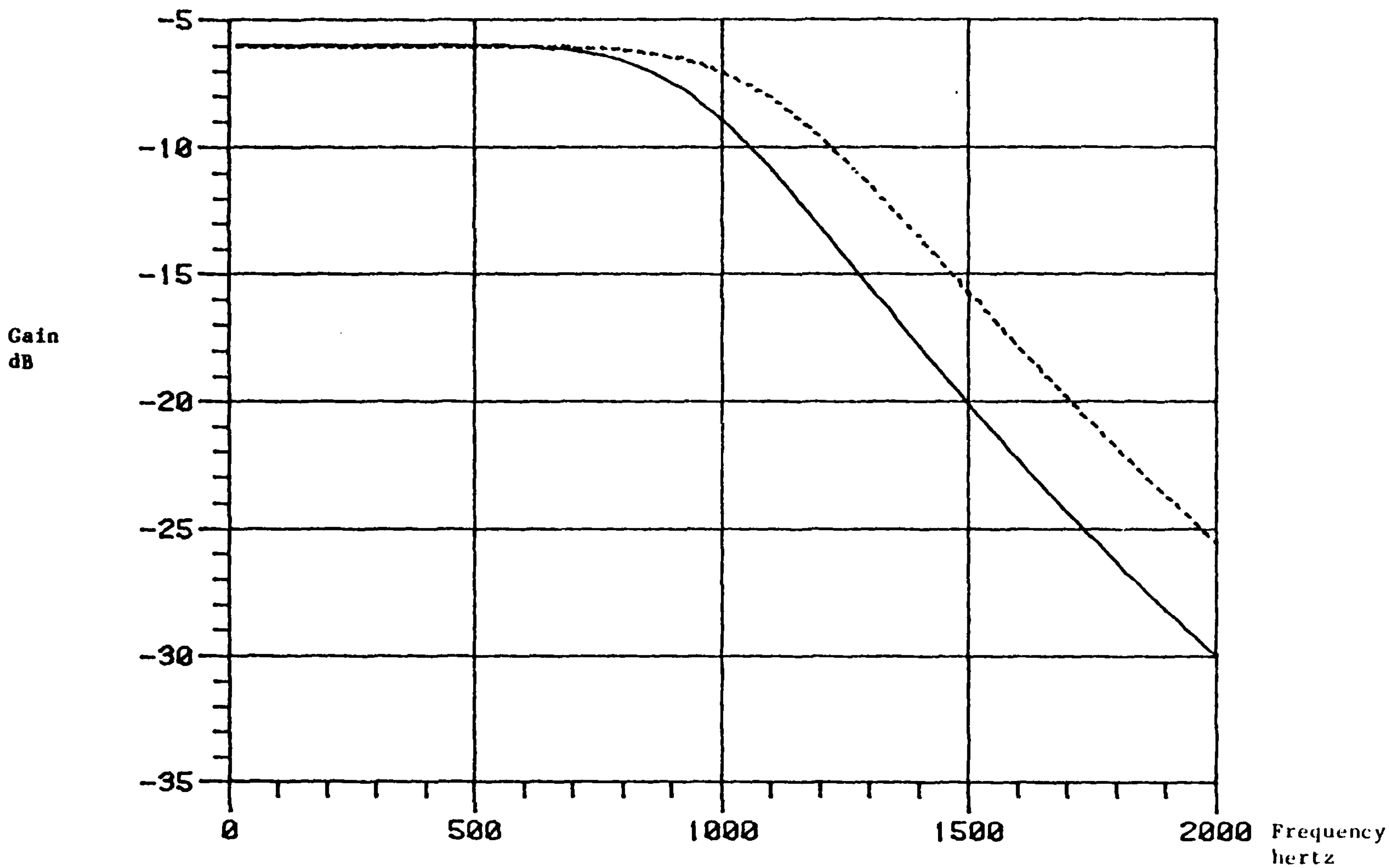

Figure 5.2 Gain Response of the Butterworth FDNR Lowpass Filter
—— Nominal Response
----- Manufactured Response

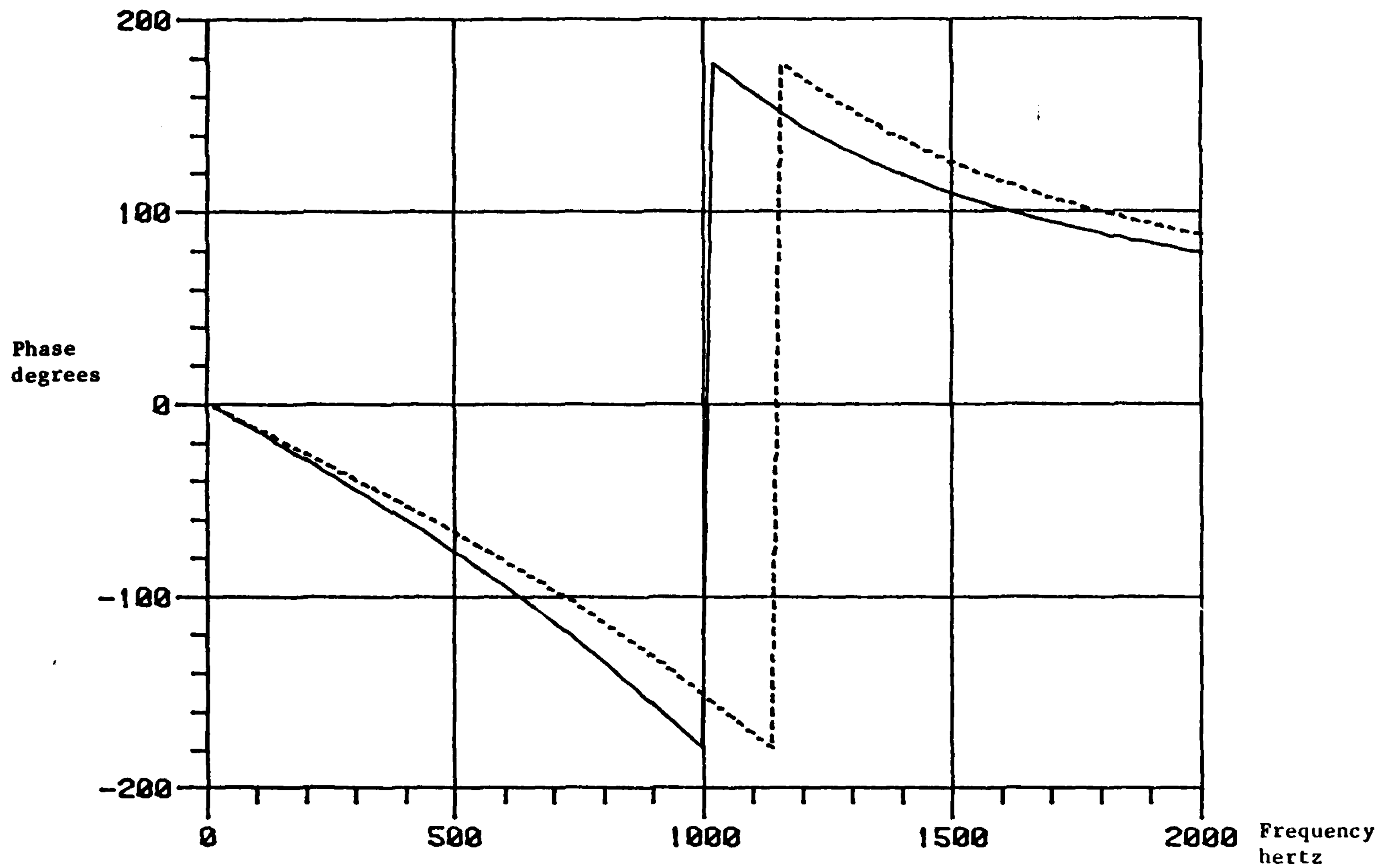

Figure 5.3 Phase Response of the Butterworth FDNR Lowpass Filter
—— Nominal Response
----- Manufactured Response

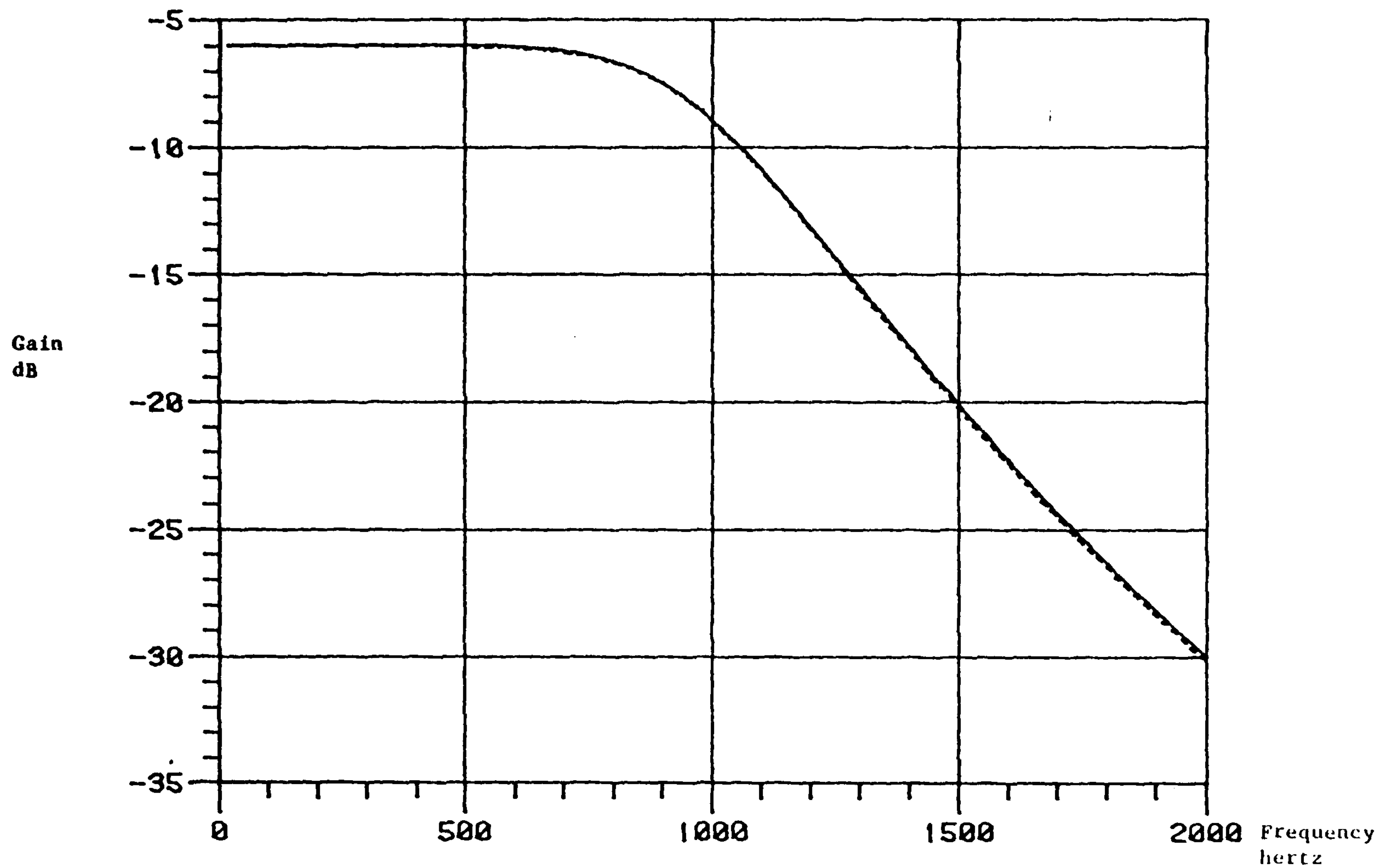

Figure 5.4 Gain Response of the Butterworth FDNR Lowpass Filter
—— Nominal Response
----- Tuned Response After One Iteration

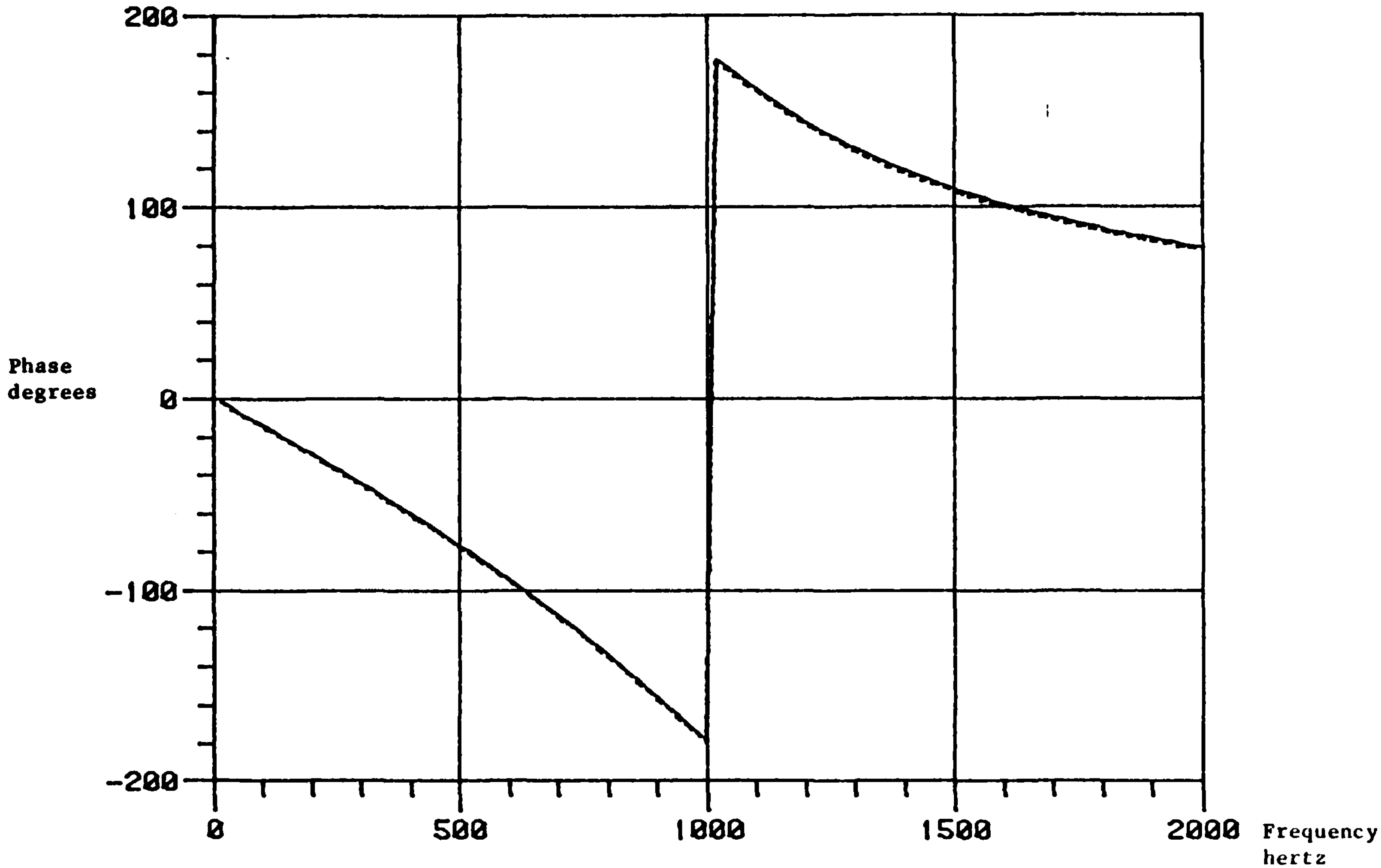

Figure 5.5 Phase Response of the Butterworth FDNR Lowpass Filter
—— Nominal Response
----- Tuned Response After One Iteration

arctangent function interprets as 0 degrees. Thus, Figures 5.3 and 5.5 do not give an accurate picture of the actual phase for frequencies in the vicinity of f_c (or for frequencies greater than f_c).

Table 5.3 records the final tuning resistance values at the iteration where convergence occurred for each of the sample circuits.

Finally, a first-order transfer function sensitivity method (ΔV terms set equal to zero in equation (2.9)) was tested using the same five sample circuits. Using this method, it was not possible to tune any of the circuits. For each of the sample circuits, the first iteration showed a deterioration in the circuit performance from the manufactured circuit performance, and yielded one or more negative elements. On subsequent iterations, the element values became increasingly smaller, eventually giving tuning resistance values on the order of 10^{-10}, or less. These small resistor values sometimes resulted in a singular matrix on the following iteration. In addition, the iterations never converged to a final set of element values. It is interesting to note that the first-order method failed even when the deviations in the capacitance values of a given circuit were small. Thus, in higher-order filters, the ΔV terms in equation (2.9) cannot evidently be neglected.

Table 5.3 Final Tuning Element Values

Element	Sample Circuit 1	2	3	4	5
R_5	1.007094 kΩ	.9711128 kΩ	1.004203 kΩ	.9946884 kΩ	.9972691 kΩ
R_7	30.68778 kΩ	31.90324 kΩ	29.38915 kΩ	32.34915 kΩ	30.00994 kΩ
R_8	12.86391 kΩ	13.61760 kΩ	12.15648 kΩ	13.31766 kΩ	12.16550 kΩ
c	1.003057	1.003108	1.000294	.9966631	.9956649

6. STATISTICAL ANALYSIS OF A SIXTH-ORDER CHEBYSHEV LEAPFROG BANDPASS FILTER

The circuit diagram of a sixth-order Chebyshev leapfrog bandpass filter which was used to test the proposed tuning algorithm is shown in Figure 6.1. The filter is designed from a doubly-terminated lowpass Chebyshev prototype with 1 dB passband ripple [29]. The lowpass to bandpass frequency transformation is used to give a bandpass filter with a center frequency of 6283.19 rad./sec. and a bandwidth of 628.32 rad./sec. Two negative feedback stages and a single state variable three amplifier biquadratic stage is used to realize an active filter via the leapfrog [16,17] concept. Multiple-feedback filters of this type are difficult to tune in practice, because of the large amount of interaction between the stages [30].

The frequency bands of the filter are defined as follows. The first stopband of the filter ranges from dc to 5400.0 rad./sec., the passband ranges from 5995.0 to 6603.0 rad./sec. and the second stopband of the filter ranges from 7289.0 to 12566.37 rad./sec. The component values for the nominal design are given in Table 6.1. A maximum passband deviation of .73 dB and a minimum stopband attenuation of 32.27 dB is attainable with this design.

Five sample circuits were generated via the sequence of steps in Section 3.4 to simulate untuned filters of this type coming off the production line. The percentage deviation in the component values of these sample circuits from the nominal, as well as the average value of the absolute percentage deviation in the capacitance values are also given in Table 6.1. The five tuning resistors R_2, R_7, R_9, R_{12} and R_{13}

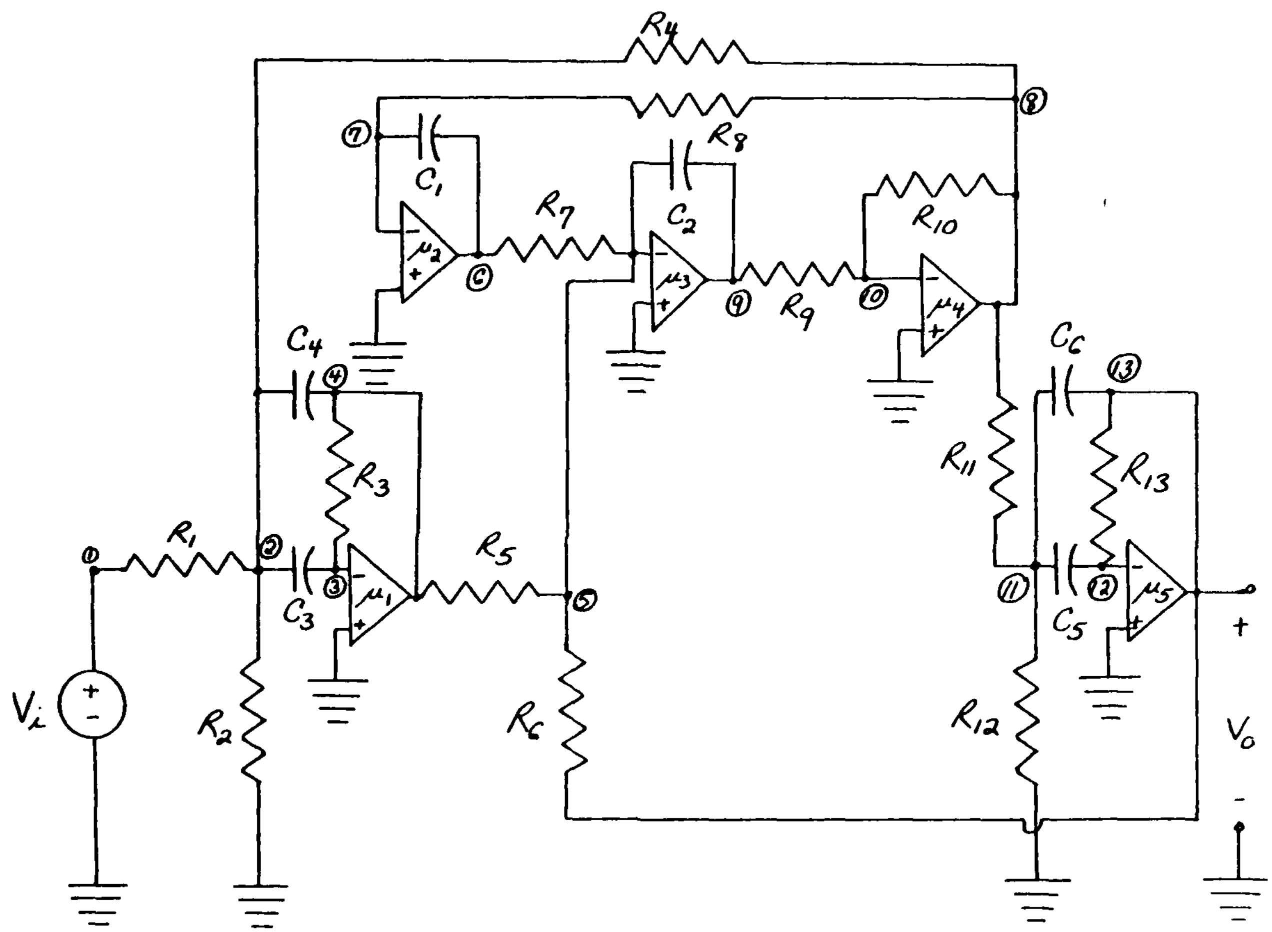

Figure 6.1 Circuit Diagram of a Chebyshev Leapfrog Bandpass Filter

Table 6.1 Component Values

Component	Nominal Value	% Deviation in the Component Values — Sample Circuit 1	2	3	4	5
R_1	81.9 kΩ	0.00	0.00	0.00	0.00	0.00
R_2	100.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_3	163.8 kΩ	0.00	0.00	0.00	0.00	0.00
R_4	81.9 kΩ	0.00	0.00	0.00	0.00	0.00
R_5	402.59 kΩ	0.00	0.00	0.00	0.00	0.00
R_6	402.59 kΩ	0.00	0.00	0.00	0.00	0.00
R_7	40.44 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_8	40.44 kΩ	0.00	0.00	0.00	0.00	0.00
R_9	10.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_{10}	10.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_{11}	81.9 kΩ	0.00	0.00	0.00	0.00	0.00
R_{12}	99.878 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_{13}	163.8 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
C_1	3930.0 pf	-1.88	-3.24	-.65	-4.01	-.72
C_2	3930.0 pf	-1.35	-2.77	-.57	-4.93	-1.68
C_3	.03932 μf	-1.65	-2.95	.38	-3.63	-.31
C_4	.03932 μf	-2.27	-4.57	-.83	-4.23	-1.16
C_5	.03932 μf	-2.93	-2.88	.08	-4.45	-.81
C_6	.03932 μf	-2.64	-3.46	-.78	-4.77	-1.00
μ_1	10000.0	0.00	0.00	0.00	0.00	0.00
μ_2	10000.0	0.00	0.00	0.00	0.00	0.00
μ_3	10000.0	0.00	0.00	0.00	0.00	0.00
μ_4	10000.0	0.00	0.00	0.00	0.00	0.00
μ_5	10000.0	0.00	0.00	0.00	0.00	0.00
Average Value of the Absolute % Dev. in the Capacitance Values		2.12	3.31	.55	4.34	.95

are selected to tune this sixth-order filter which requires the evaluation of equation (2.9) at three frequencies. Two of these frequencies are chosen close to the passband edge at 5975.88 and 6605.19 rad./sec. and the remaining frequency is chosen at the center frequency of the filter, at 6283.19 rad./sec.

The results of tuning the sample circuits are recorded in Table 6.2. Three of the five sample circuits converged to their final element values in just two iterations, while all of the circuits converged to their final element values in four iterations or less. In addition, in each case, all of the tuning resistors increased in value from their manufactured values. Table 6.1 indicates that sample circuit 4 exhibits the worst overall response. The gain response of the nominal and this worst case manufactured filter is shown in Figure 6.2. Figure 6.3 is a magnification of the gain curve in the passband. Because the gain of the manufactured circuit is so grossly distorted from the nominal it exceeds the grid limits of 1 dB and thus does not appear with the nominal curve in Figure 6.3. The phase response of the nominal and manufactured circuit is depicted in Figure 6.4.

After a single iteration the filter response markedly improves as Figures 6.5-6.7 indicate. In fact, the gain response improves so much that both the tuned and nominal gain curves appear in the magnified passband depicted in Figure 6.6. An additional iteration improves the response still further (Figures 6.8-6.10) until at the third iteration the tuned and nominal response curves are nearly identical, even when magnified (Figures 6.11-6.13)! In examining the magnified passband curves (Figure 6.12) it should be kept in mind that the entire vertical axis corresponds to 1 dB so that the gain deviation between the nominal and tuned circuit

Table 6.2 *Statistical Analysis of Chebyshev Leapfrog Bandpass Filter Using Large-Change Transfer Function Sensitivity*

Sample Circuit	Circuit Description	Max. Dev. in Passband (dB)	Min. Atten. in Stopband (dB)
	Nominal	.73	32.27
1	Manufactured	2.75	-31.84
	Iteration 1	1.53	31.87
	Iteration 2	.73	32.18
2	Manufactured	2.46	-34.55
	Iteration 1	2.86	30.03
	Iteration 2	.81	31.91
	Iteration 3	.80	32.20
	Iteration 4	.76	32.19
3	Manufactured	26.99	-29.76
	Iteration 1	.94	32.18
	Iteration 2	.74	32.25
4	Manufactured	22.55	-36.36
	Iteration 1	3.86	28.65
	Iteration 2	.85	31.81
	Iteration 3	.82	32.20
	Iteration 4	.75	32.18
5	Manufactured	27.52	-30.74
	Iteration 1	1.16	32.06
	Iteration 2	.75	32.21

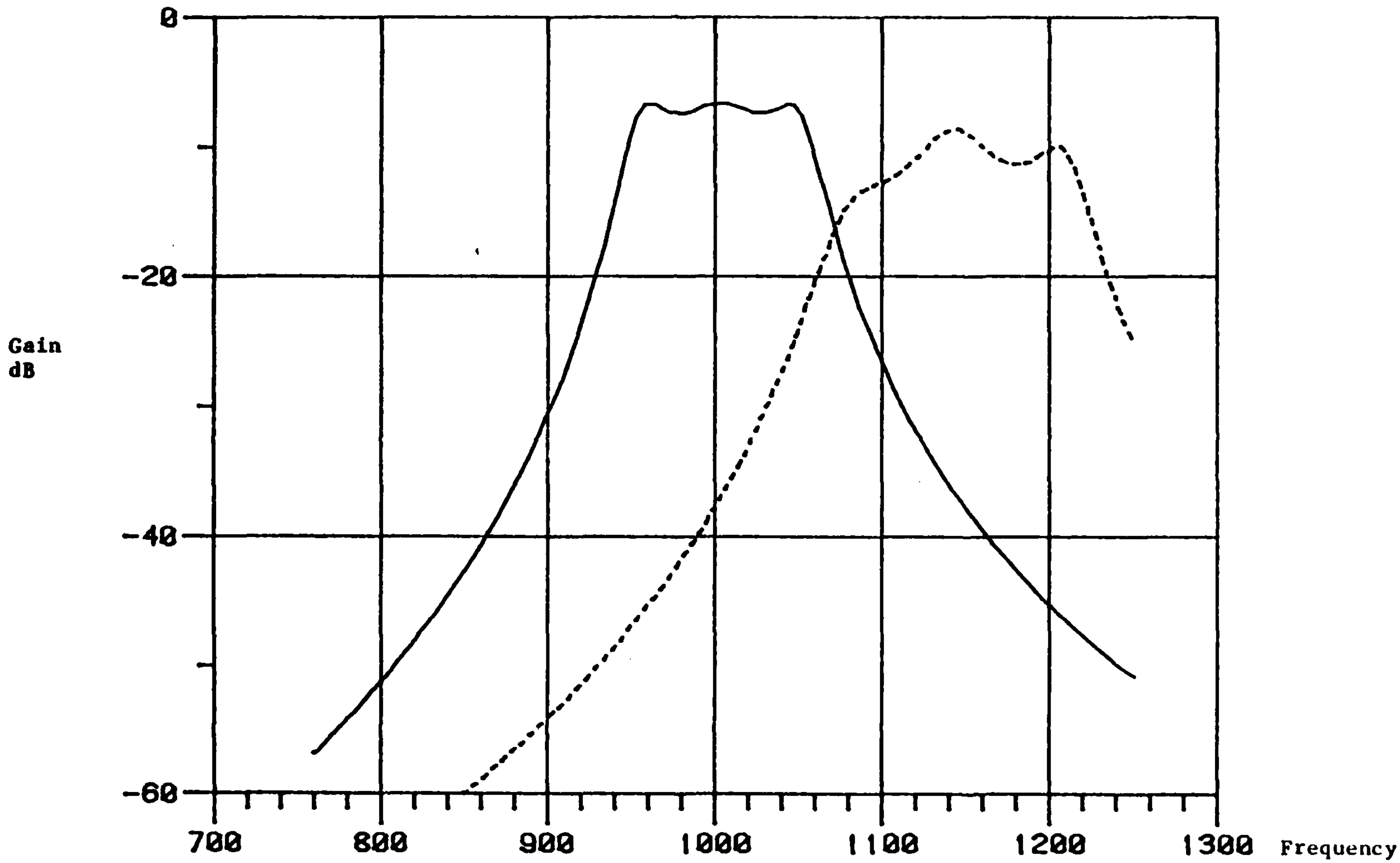

Figure 6.2 Gain Response of the Chebyshev Leapfrog Bandpass Filter
——— Nominal Response
- - - - - Manufactured Response

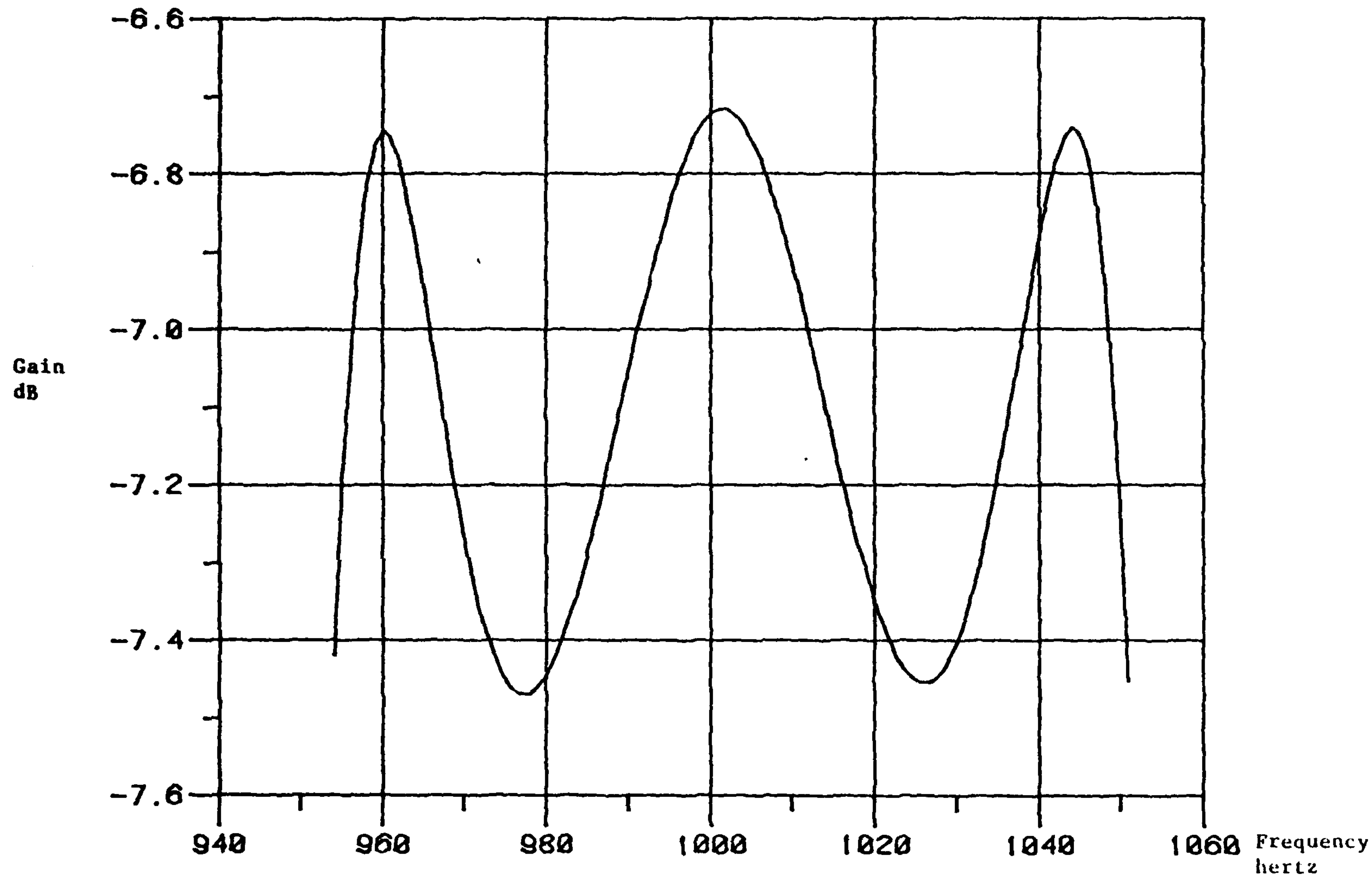

Figure 6.3 Magnified Gain Response of the Chebyshev Leapfrog Bandpass Filter in the Passband
—— Nominal Response
----- Manufactured Response

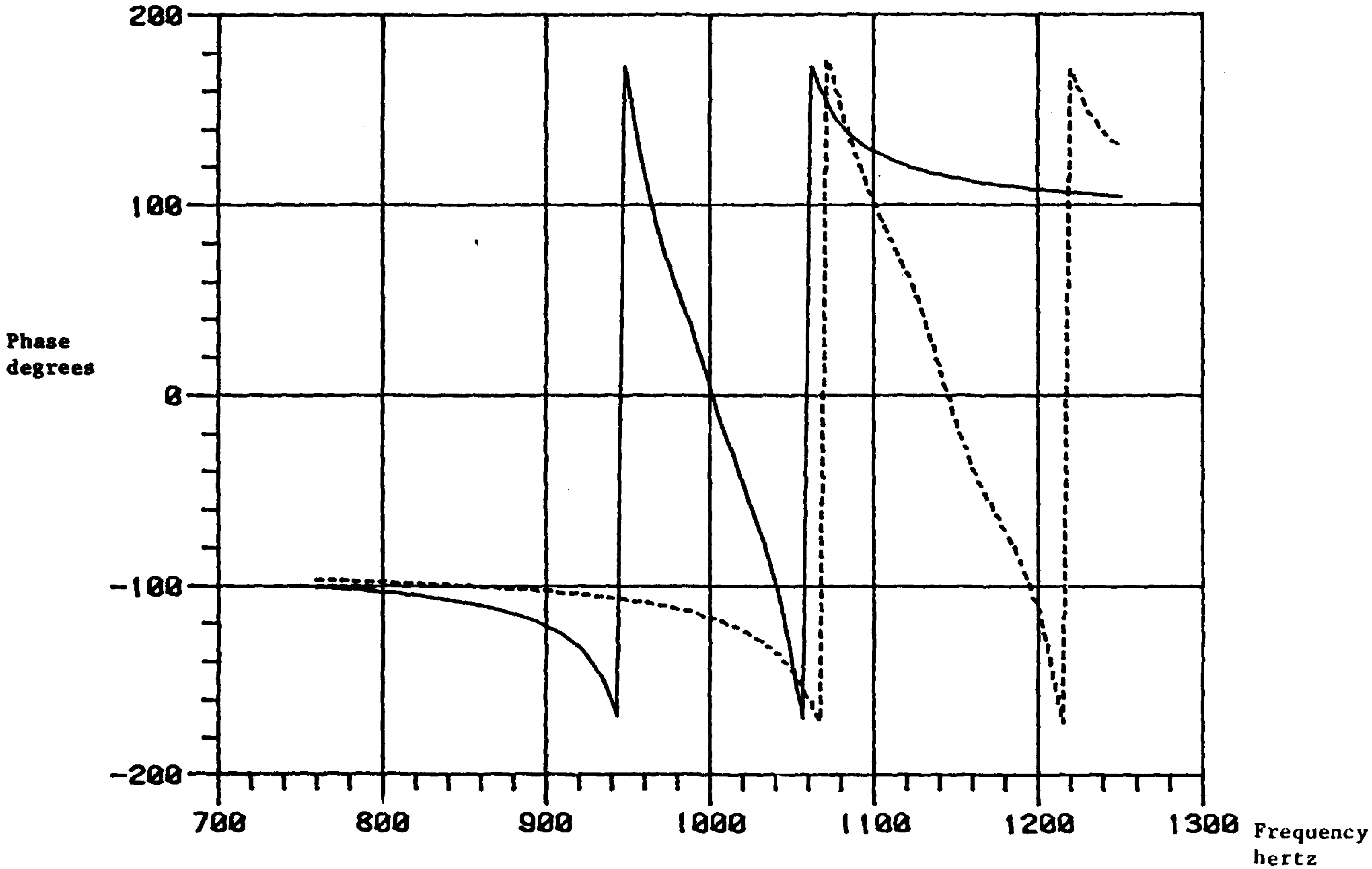

Figure 6.4 Phase Response of the Chebyshev Leapfrog Bandpass Filter
—— Nominal Response
----- Manufactured Response

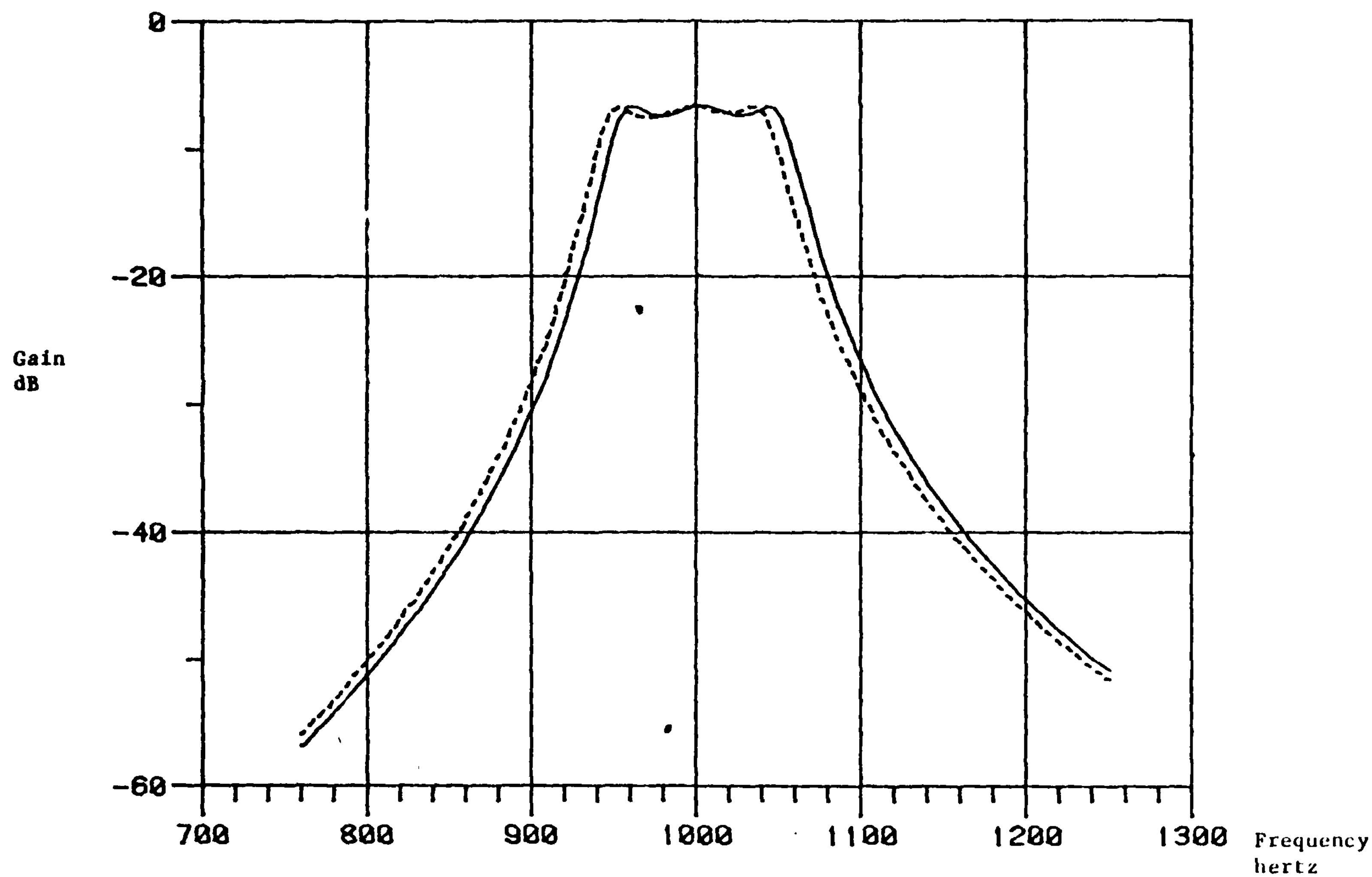

Figure 6.5 Gain Response of the Chebyshev Leapfrog Bandpass Filter
——— Nominal Response
- - - - Tuned Response After One Iteration

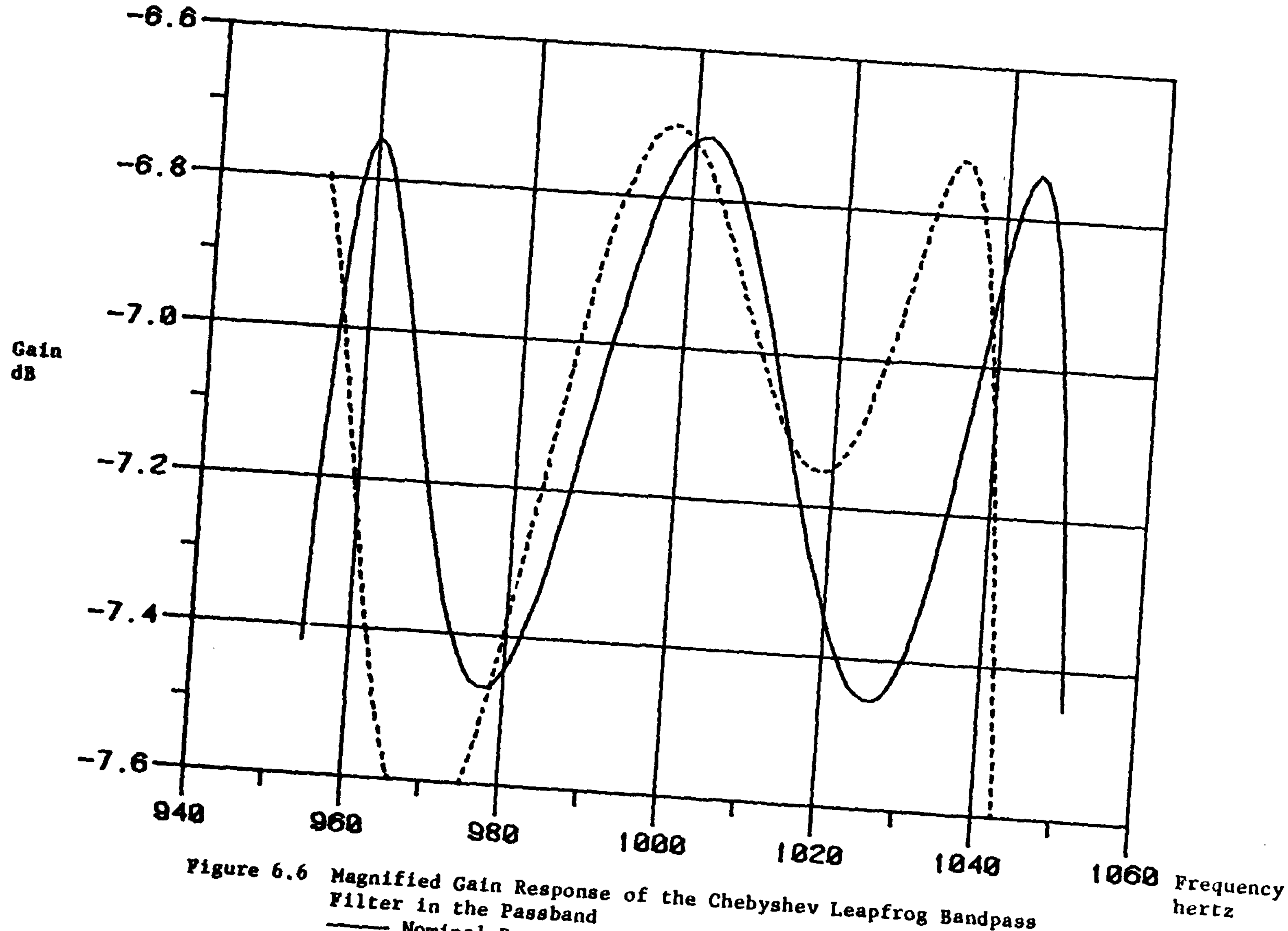

Figure 6.6 Magnified Gain Response of the Chebyshev Leapfrog Bandpass Filter in the Passband
——— Nominal Response
----- Tuned Response After One Iteration

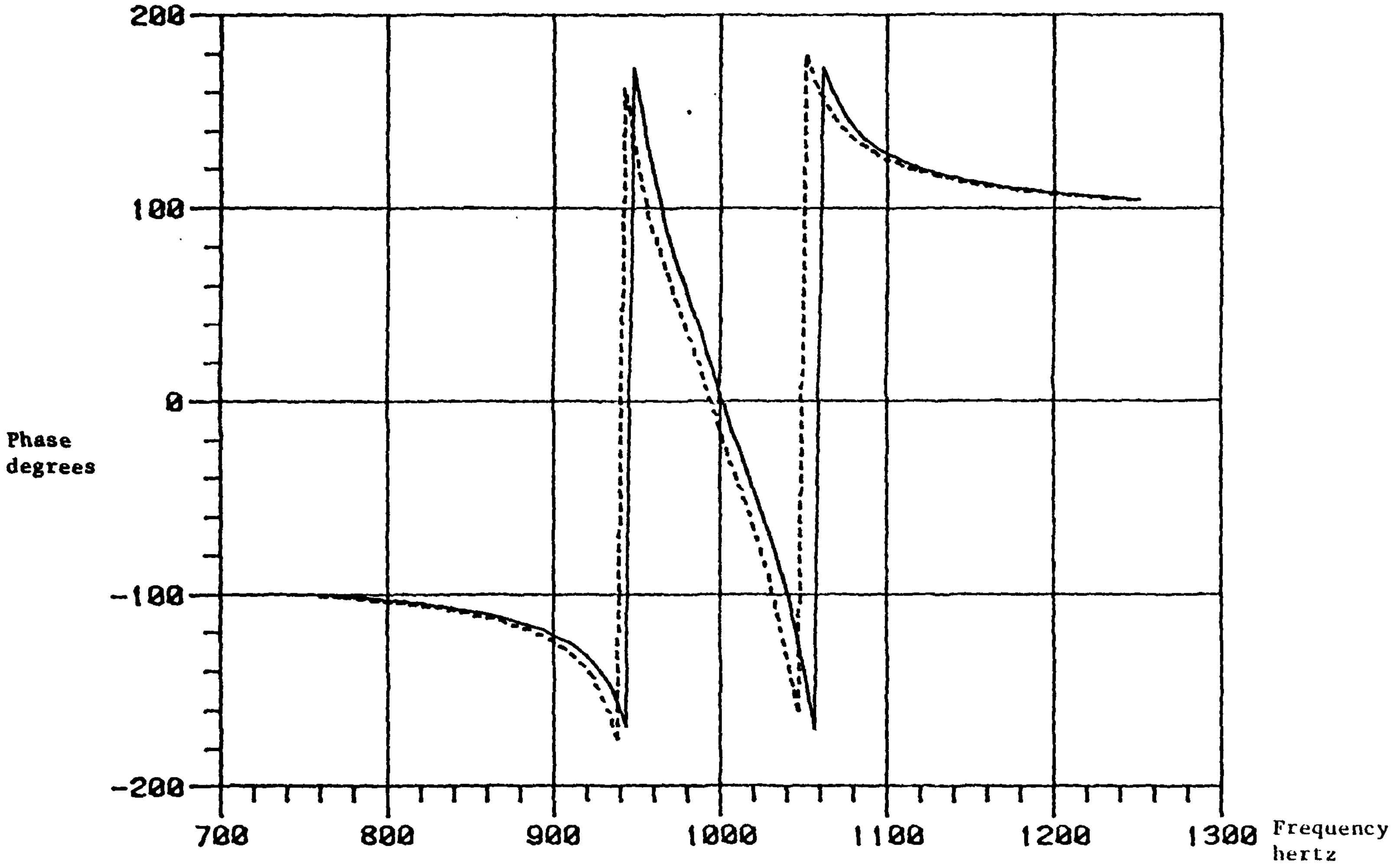

Figure 6.7 Phase Response of the Chebyshev Leapfrog Bandpass Filter
——— Nominal Response
----- Tuned Response After One Iteration

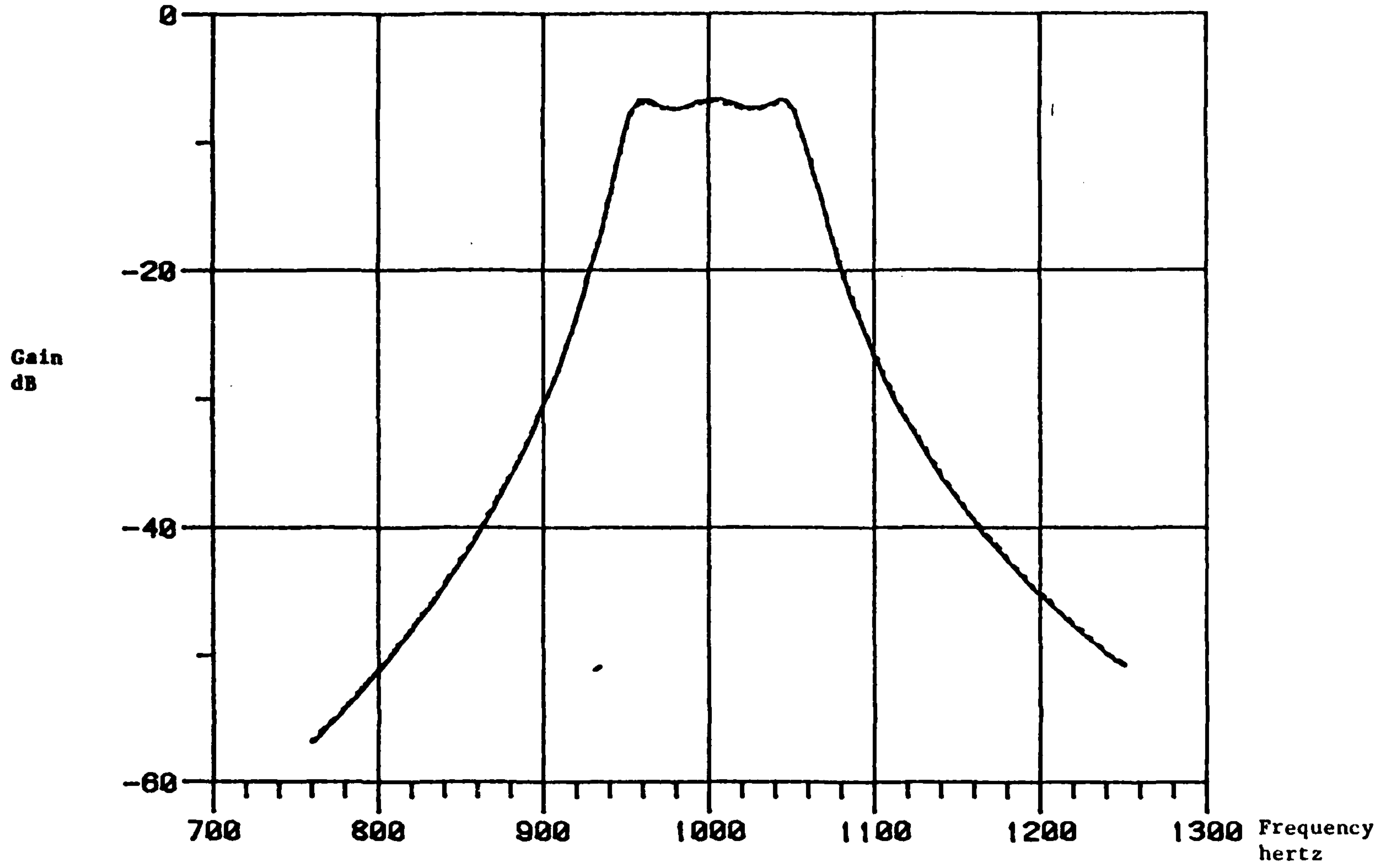

Figure 6.8 Gain Response of the Chebyshev Leapfrog Bandpass Filter
——— Nominal Response
----- Tuned Response After Two Iterations

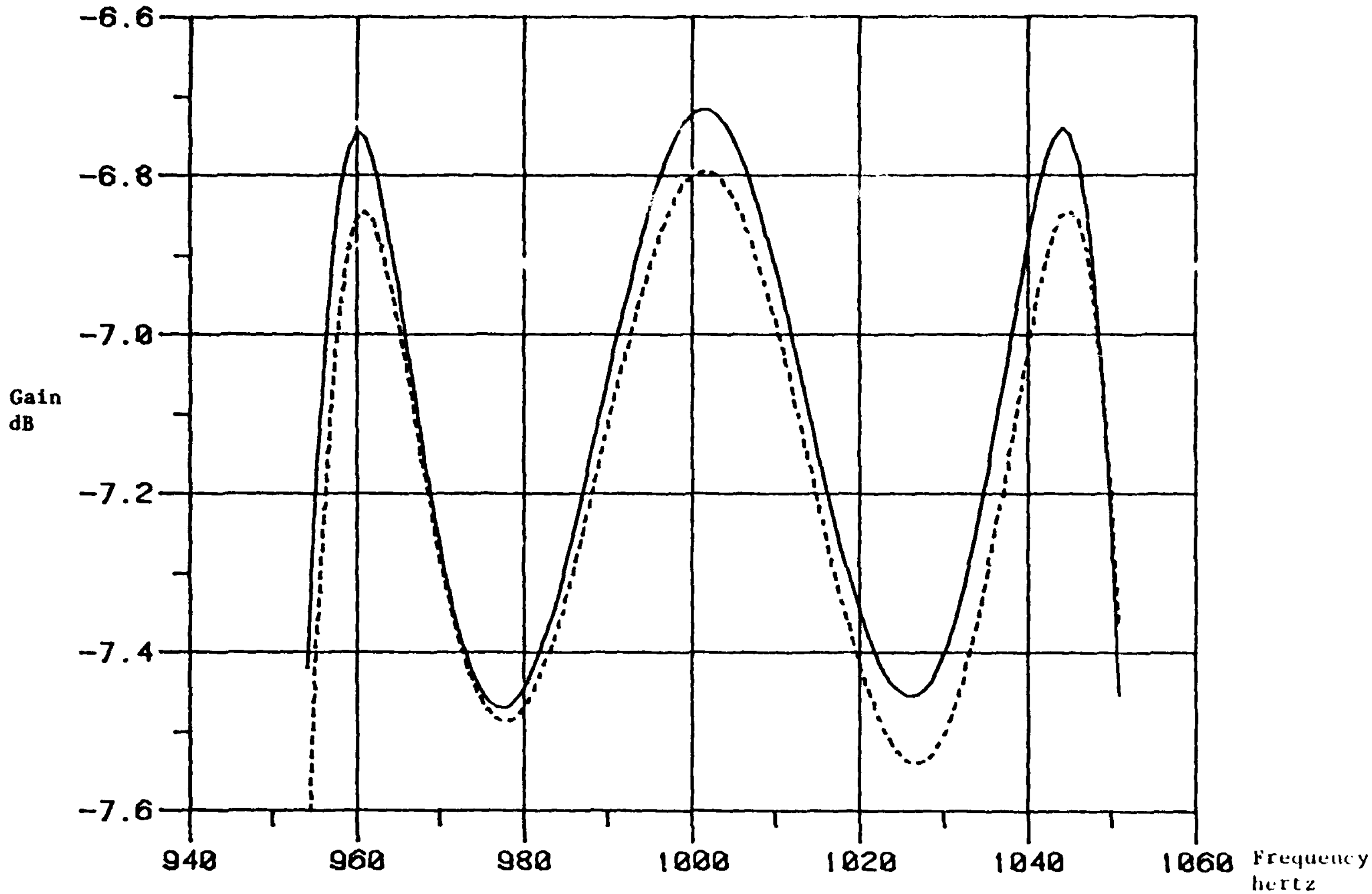

Figure 6.9 Magnified Gain Response of the Chebyshev Leapfrog Bandpass Filter in the Passband

——— Nominal Response

----- Tuned Response After Two Iterations

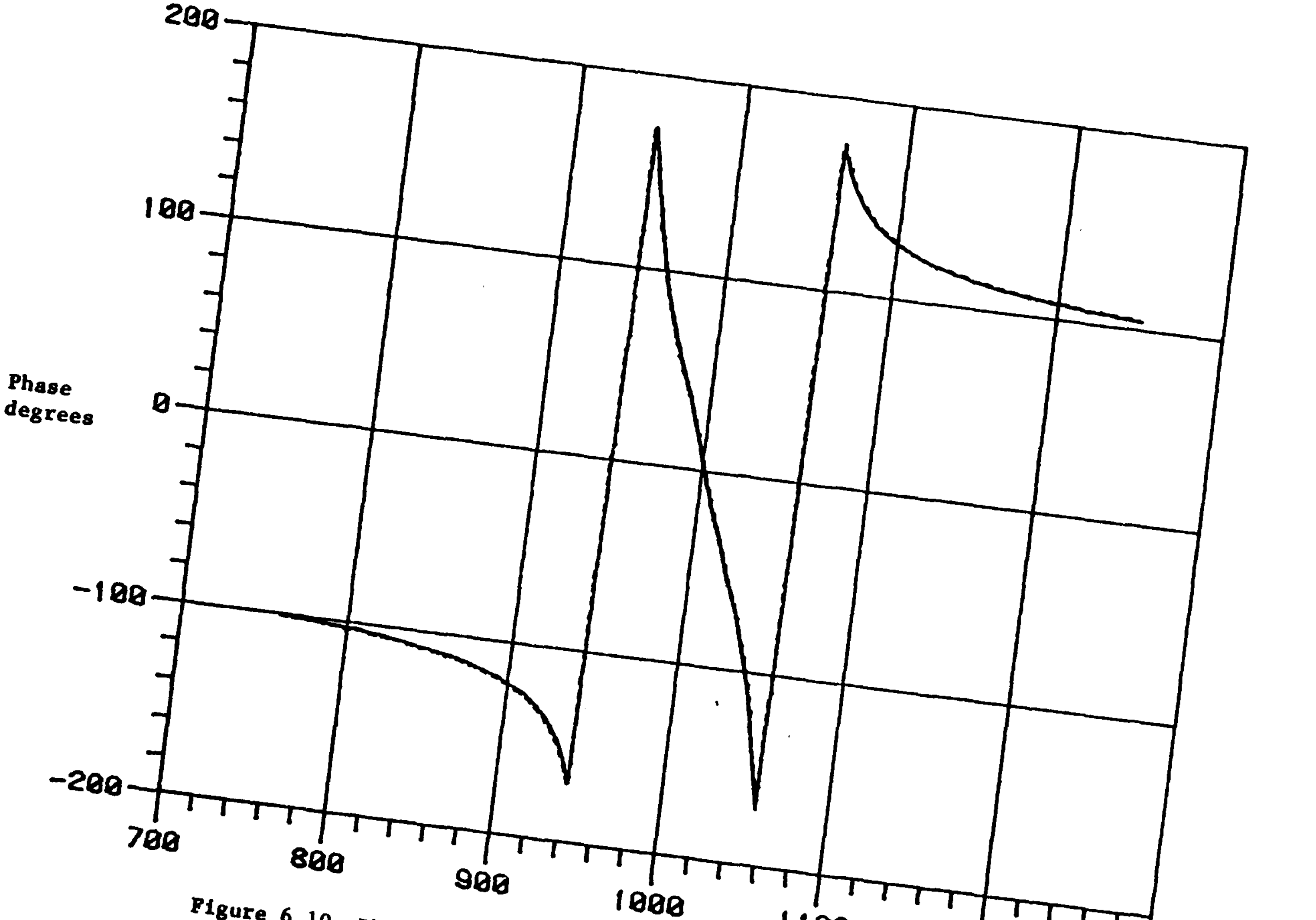

Figure 6.10 Phase Response of the Chebyshev Leapfrog Bandpass Filter

—— Nominal Response

----- Tuned Response After Two Iterations

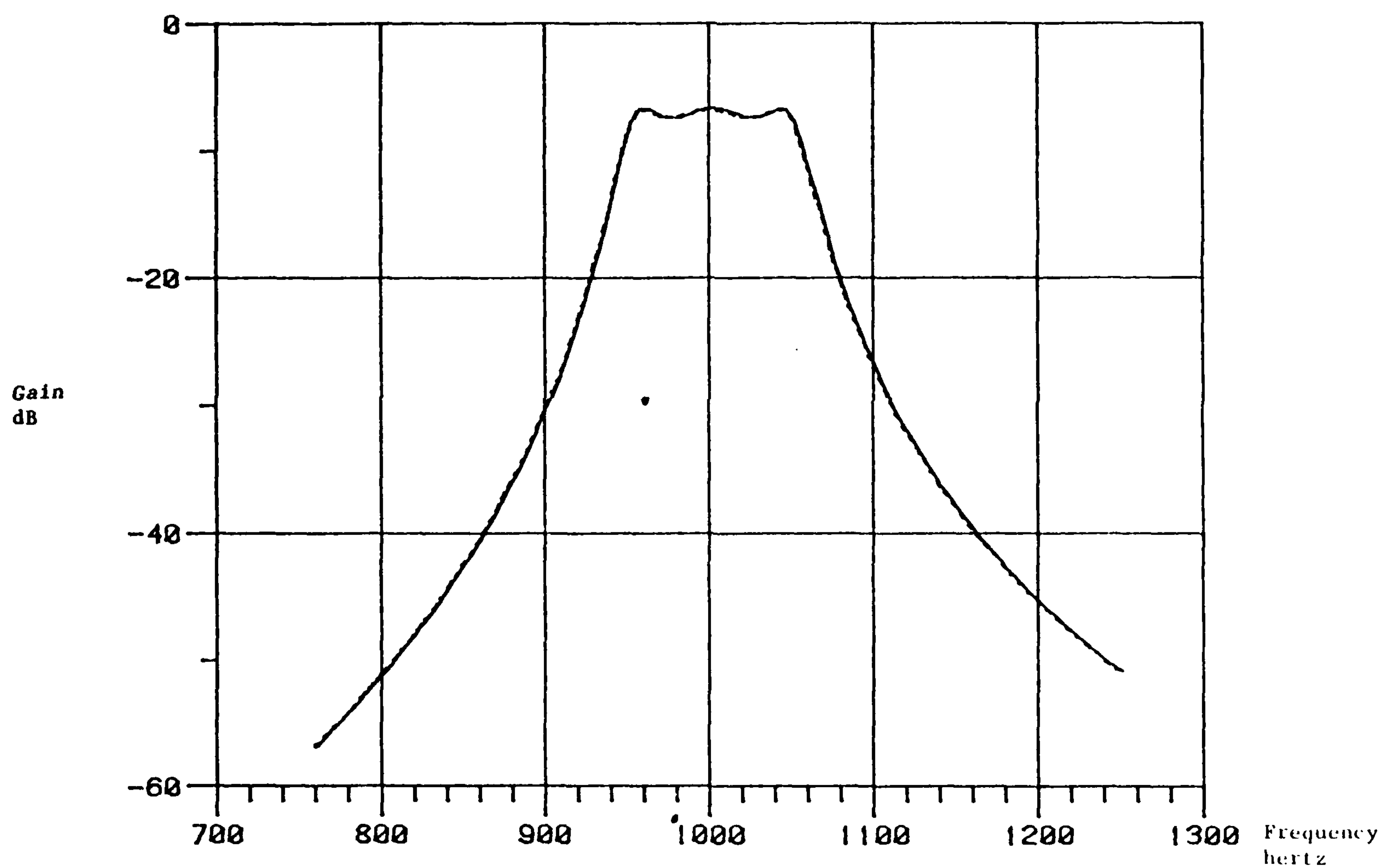

Figure 6.11 Gain Response of the Chebyshev Leapfrog Bandpass Filter
——— Nominal Response
----- Tuned Response After Three Iterations

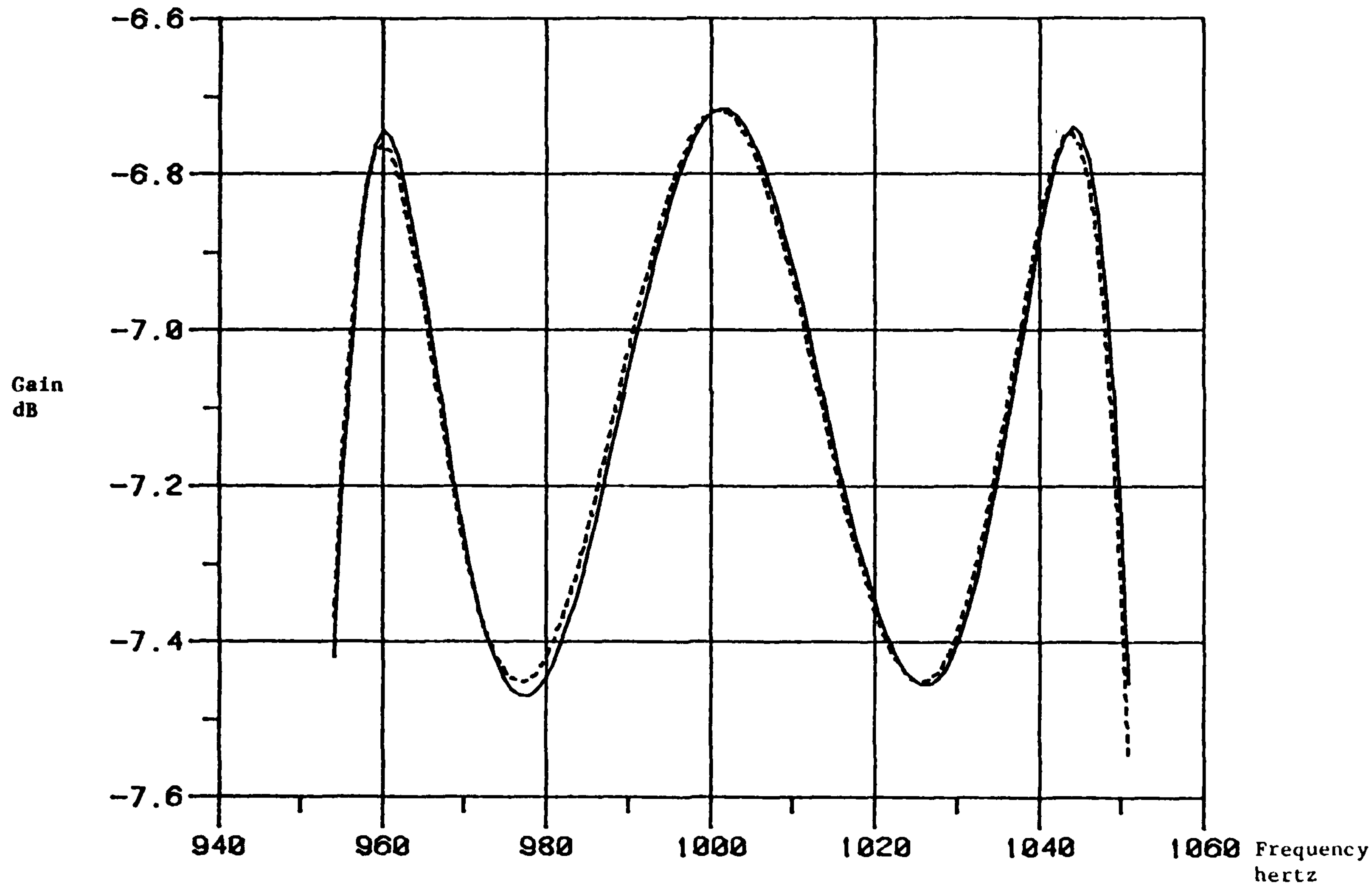

Figure 6.12 Magnified Gain Response of the Chebyshev Leapfrog Bandpass Filter in the Passband
—— Nominal Response
----- Tuned Response After Three Iterations

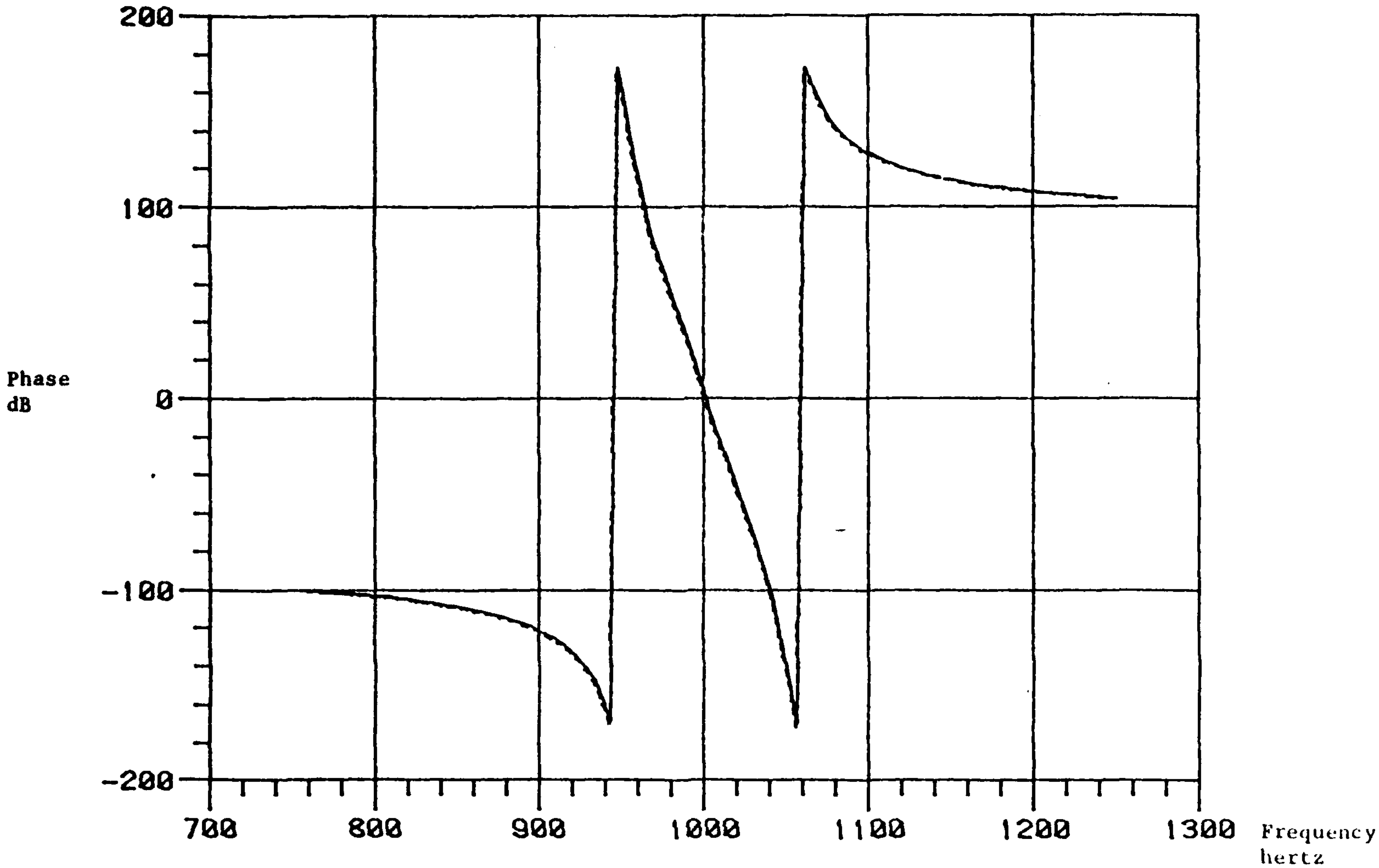

Figure 6.13 Phase Response of the Chebyshev Leapfrog Bandpass Filter
——— Nominal Response
----- Tuned Response After Three Iterations

at the third iteration is very minute. The algorithm finally converges at the fourth iteration resulting in a slight improvement in the passband (Figure 6.14).

The results of Table 6.2 must be interpreted with some care. Even though the table indicates that the maximum deviation in the passband is .82 dB at the third iteration, the magnified picture (Figure 6.12) indicates that there is very little difference between the results of the third and fourth iterations (Figure 6.14) although the fourth iteration indicates a gain deviation of .75 dB, some .07 dB less. Indeed, the element values change less than 1 percent from iteration 3 to iteration 4 so that a very small change in the overall tuned response is anticipated. The additional .07 dB gain deviation of iteration 3 occurs at the upper passband edge (at about 1051 hertz) and is attributed to minute deviations in the passband edge frequencies of the tuned filter as well as the digitized nature of the gain deviation computation. As another example of this consider the fact that the algorithm converged at the fourth iteration, attaining a maximum deviation in the passband of .75 dB, .02 dB greater than the nominal deviation of .73 dB. The additional .02 dB deviation occurs at the lower passband edge (about 954 hertz in Figure 6.14). Similar situations may occur in the stopband as well.

The final tuning resistance values at the iteration where convergence occurred are recorded in Table 6.3 for each of the sample circuits.

A word of caution needs to be said in regard to the selection of the tuning resistors. As a first choice one might logically select the feedback resistors R_4, R_6 and R_8 as tuning elements. However, with this choice of resistors, together with R_3 and R_{13}, the proposed large-change

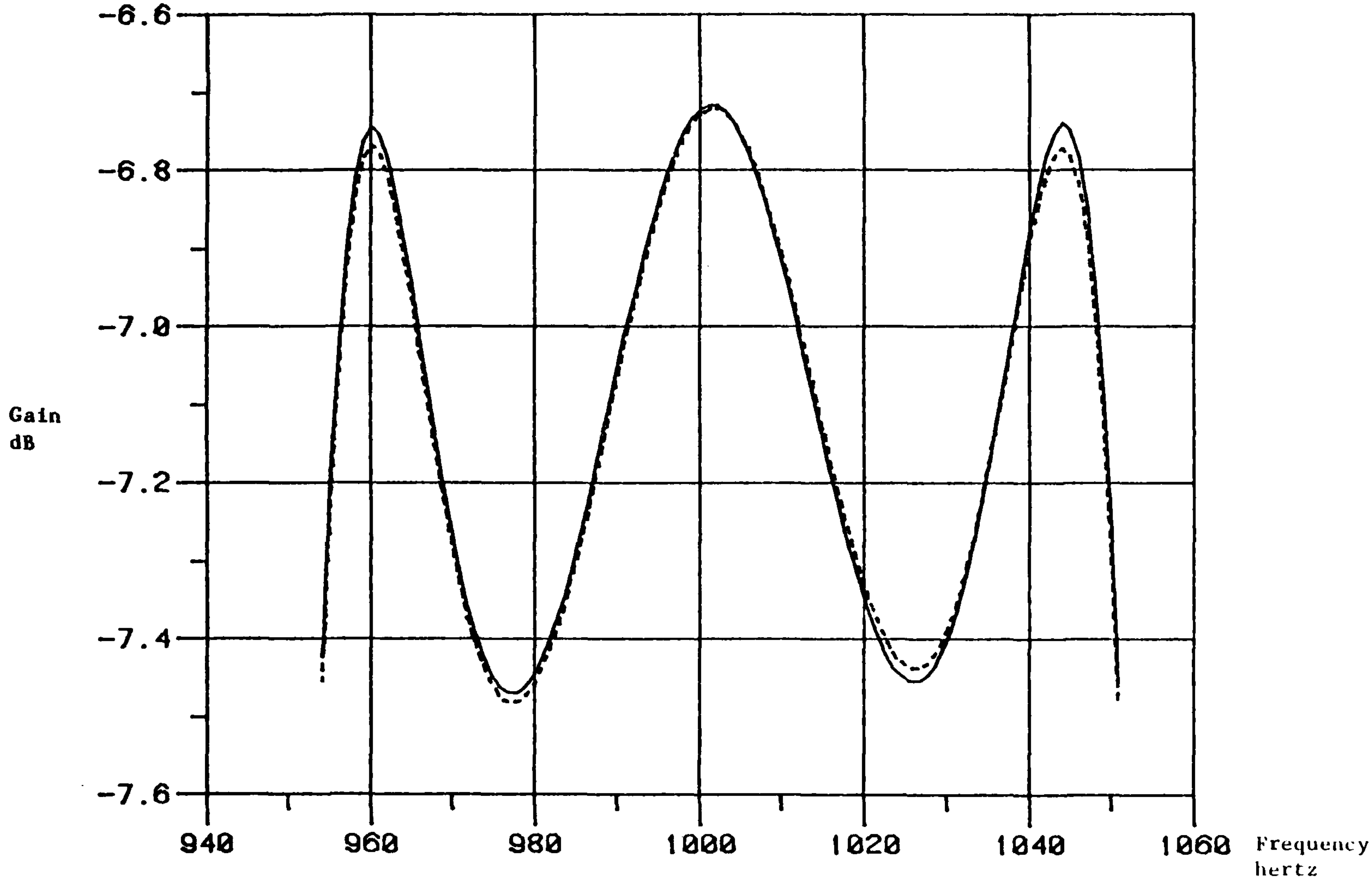

Figure 6.14 Magnified Gain Response of the Chebyshev Leapfrog Bandpass Filter in the Passband
——— Nominal Response
----- Tuned Response After Four Iterations

Table 6.3 Final Tuning Element Values

Element	Sample Circuit 1	2	3	4	5
R_2	103.5610 Ω	107.5127 Ω	100.0850 Ω	107.8176 Ω	100.9604 Ω
R_7	40.29796 kΩ	40.12631 kΩ	40.40745 kΩ	40.24049 kΩ	40.37877 kΩ
R_9	10.36586 kΩ	10.71301 kΩ	10.13031 kΩ	11.01189 kΩ	10.25855 kΩ
R_{12}	102.9451 Ω	100.7492 Ω	100.3952 Ω	102.4385 Ω	101.1761 Ω
R_{13}	168.8834 kΩ	173.8969 kΩ	164.7057 kΩ	176.2930 kΩ	165.5331 kΩ
c	1.019802	1.036307	1.008409	1.040429	1.010228

algorithm sometimes exhibited a thrashing behavior in which the first iteration would nearly tune the filter, but subsequent iterations would yield typically poorer results in both the passband and the stopband, sometimes culminating in one or more negative tuning elements. This strange behavior is perhaps an indication that no solution exists for the related deterministic tuning problem for the particular set of tuning resistors chosen. In the event that this behavior is encountered, another choice of tuning resistors must be made.

The five sample circuits were also used to test a first-order transfer function sensitivity method (obtained by setting the ΔV terms equal to zero in equation (2.9)). With this first-order method, it was not possible to tune any of the sample circuits. The first iteration resulted in a deterioration of the circuit performance for each of the five circuits, often yielding one or more negative tuning elements. Also, in subsequent iterations, the right-hand side of equation (2.9) was driven to zero (about 10^{-24}), yielding one or more negative tuning element values and/or extremely large tuning element values (about 10^{33}) probably indicative of a very ill-conditioned system of linear algebraic equations. In addition, the iterations never converged to a final set of element values. Ostensibly then, the ΔV terms in equation (2.9) are important in a higher-order filter such as this, and cannot be neglected.

7. STATISTICAL ANALYSIS OF A SIXTH-ORDER CHEBYSHEV FDNR BANDPASS FILTER

The circuit diagram of a sixth-order Chebyshev FDNR bandpass filter which was used to test the proposed large-change sensitivity tuning algorithm is shown in Figure 7.1. The circuit is designed from a doubly-terminated lowpass Chebyshev prototype with a .5 dB passband ripple [29]. The lowpass to bandpass transformation is used to give a bandpass filter with a center frequency of 6283.19 rad./sec. and a bandwidth of 628.32 rad./sec. The admittances of the resulting filter are scaled by the complex frequency variable s to give a topologically equivalent network consisting of resistors, capacitors and frequency-dependent negative resistors [28]. The grounded FDNR elements are realized by an active circuit due to Bruton [28], and the single floating FDNR is realized by two of these circuits connected back to back.

The three frequency bands of the filter are defined as follows. The first stopband of the filter ranges from dc to 5400.0 rad./sec., the passband ranges from 5995.0 to 6603.0 rad./sec. and the second stopband of the filter ranges from 7289.0 to 12566.37 rad./sec. The component values for the nominal design are given in Table 7.1. A maximum passband deviation of .52 dB and a minimum stopband attenuation of 30.09 dB is attainable with this design.

Five sample circuits were generated via the sequence of steps in Section 3.4 to simulate untuned filters coming off of the production line. The percentage deviation in the component values of these circuits from the nominal as well as the average value of the absolute percentage deviation in the capacitance values are also given in Table 7.1. The

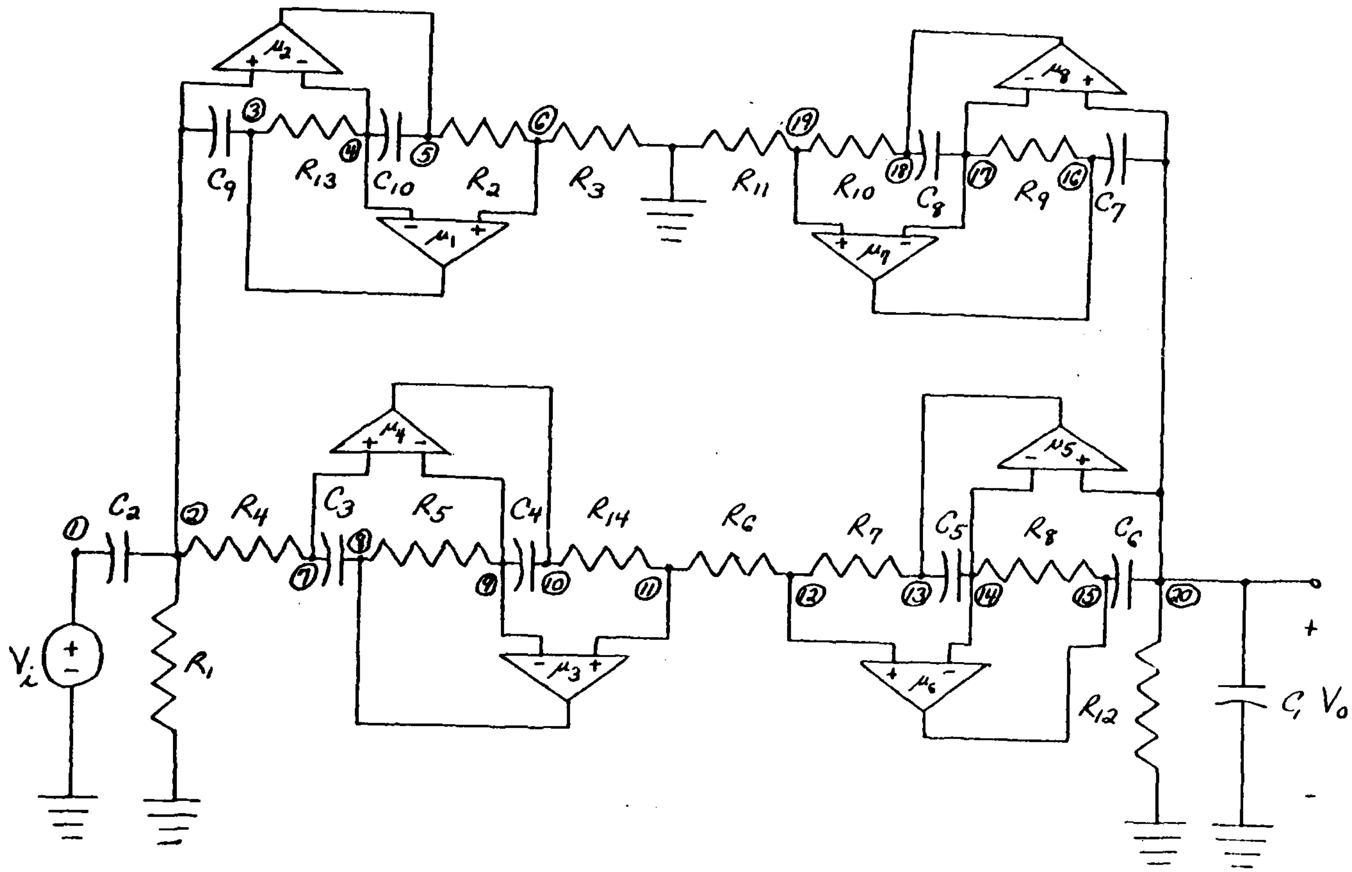

Figure 7.1 Circuit Diagram of a Chebyshev FDNR Bandpass Filter

Table 7.1 Component Values

Component	Nominal Value	% Deviation in the Component Values — Sample Circuit 1	2	3	4	5
R_1	997.02 Ω	-10.00	-10.00	-10.00	-10.00	-10.00
R_2	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_3	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_4	174.550 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_5	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_6	1.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_7	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_8	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_9	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_{10}	1.0 kΩ	-10.00	-10.00	-10.00	-10.00	-10.00
R_{11}	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_{12}	997.02 Ω	-10.00	-10.00	-10.00	-10.00	-10.00
R_{13}	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
R_{14}	1.0 kΩ	0.00	0.00	0.00	0.00	0.00
C_1	.01 μf	-1.88	.94	.99	3.51	-3.62
C_2	.01 μf	-1.35	.71	.76	2.44	-4.65
C_3	.01205 μf	-1.65	.68	.53	3.37	-3.47
C_4	.01205 μf	-2.27	1.02	.19	2.96	-2.66
C_5	.01205 μf	-2.93	1.99	.70	2.01	-2.62
C_6	.01205 μf	-2.64	.75	.03	3.45	-3.81
C_7	.1594 μf	-3.15	1.68	1.41	3.20	-3.27
C_8	.1594 μf	-1.40	.81	.55	3.38	-4.04
C_9	.1594 μf	-1.59	-.68	.90	3.63	-4.08
C_{10}	.1594 μf	-3.23	-.03	.71	4.11	-3.93
μ_1	100000.0	0.00	0.00	0.00	0.00	0.00
μ_2	100000.0	0.00	0.00	0.00	0.00	0.00
μ_3	100000.0	0.00	0.00	0.00	0.00	0.00

Table 7.1 (continued)

Component	Nominal Value	% Deviation in the Component Values Sample Circuit 1	2	3	4	5
μ_4	100000.0	0.00	0.00	0.00	0.00	0.00
μ_5	100000.0	0.00	0.00	0.00	0.00	0.00
μ_6	100000.0	0.00	0.00	0.00	0.00	0.00
μ_7	100000.0	0.00	0.00	0.00	0.00	0.00
μ_8	100000.0	0.00	0.00	0.00	0.00	0.00
Average Value of the Absolute % Dev. in the Capacitance Values		2.21	.93	.68	3.21	3.62

five tuning resistors R_1, R_4, R_6, R_{10} and R_{12} are selected to tune the sixth-order filter which requires the evaluation of equation (2.9) at three frequencies. The three frequencies are chosen at 5975.88, 6605.19 and 6283.19 rad./sec.

The results of tuning the sample circuits are recorded in Table 7.2. Table 7.1 indicates that sample circuit 5 exhibits the worst overall response. Three of the five circuits converged to their final element values in just three iterations. Only two of the circuits (one of which is the worst case circuit) required an additional iteration. In addition, in each case all of the tuning elements increased in value from their manufactured values. The gain response of the nominal and the worst case manufactured circuits is shown in Figure 7.2. Figure 7.3 shows the magnified curve in the passband. Because the gain of the manufactured circuit is so grossly distorted from the nominal it exceeds the grid limits of .6 dB and thus does not appear with the nominal curve in Figure 7.3. The phase response of the nominal and the manufactured circuits is shown in Figure 7.4. The response curves after one iteration are shown in Figures 7.5-7.7. Note that in the magnified passband (Figure 7.6) the filter response has vastly improved so that it now lies partially within the .6 dB grid limits. An additional iteration (Figures 7.8-7.10) improves the overall response still further so that the filter is nearly tuned in two iterations! The magnified passband (Figure 7.9) indicates that the tuned gain response does not fall exactly on top of the nominal gain response in the passband, but in light of the scale (.1 dB per division), this error is insignificant. Also note how the tuned gain response at the upper passband edge (about 1051 hertz) lies just inside the nominal curve at this frequency resulting in an additional

Table 7.2 Statistical Analysis of Chebyshev FDNR Bandpass Filter Using Large-Change Transfer Function Sensitivity

Sample Circuit	Circuit Description	Max. Dev. in Passband (dB)	Min. Dev. in Stopband (dB)
	Nominal	.52	30.09
1	Manufactured	19.80	-11.44
	Iteration 1	1.05	28.86
	Iteration 2	.55	30.04
	Iteration 3	.53	30.09
	Iteration 4	.54	30.05
2	Manufactured	9.59	7.86
	Iteration 1	.55	30.02
	Iteration 2	.52	30.09
	Iteration 3	.52	30.09
3	Manufactured	8.19	9.34
	Iteration 1	1.21	29.19
	Iteration 2	.53	30.13
	Iteration 3	.52	30.09
4	Manufactured	4.64	16.43
	Iteration 1	.69	29.95
	Iteration 2	.60	30.11
	Iteration 3	.54	30.07
5	Manufactured	24.13	-17.87
	Iteration 1	.89	29.15
	Iteration 2	.56	30.02
	Iteration 3	.53	30.08
	Iteration 4	.55	30.04

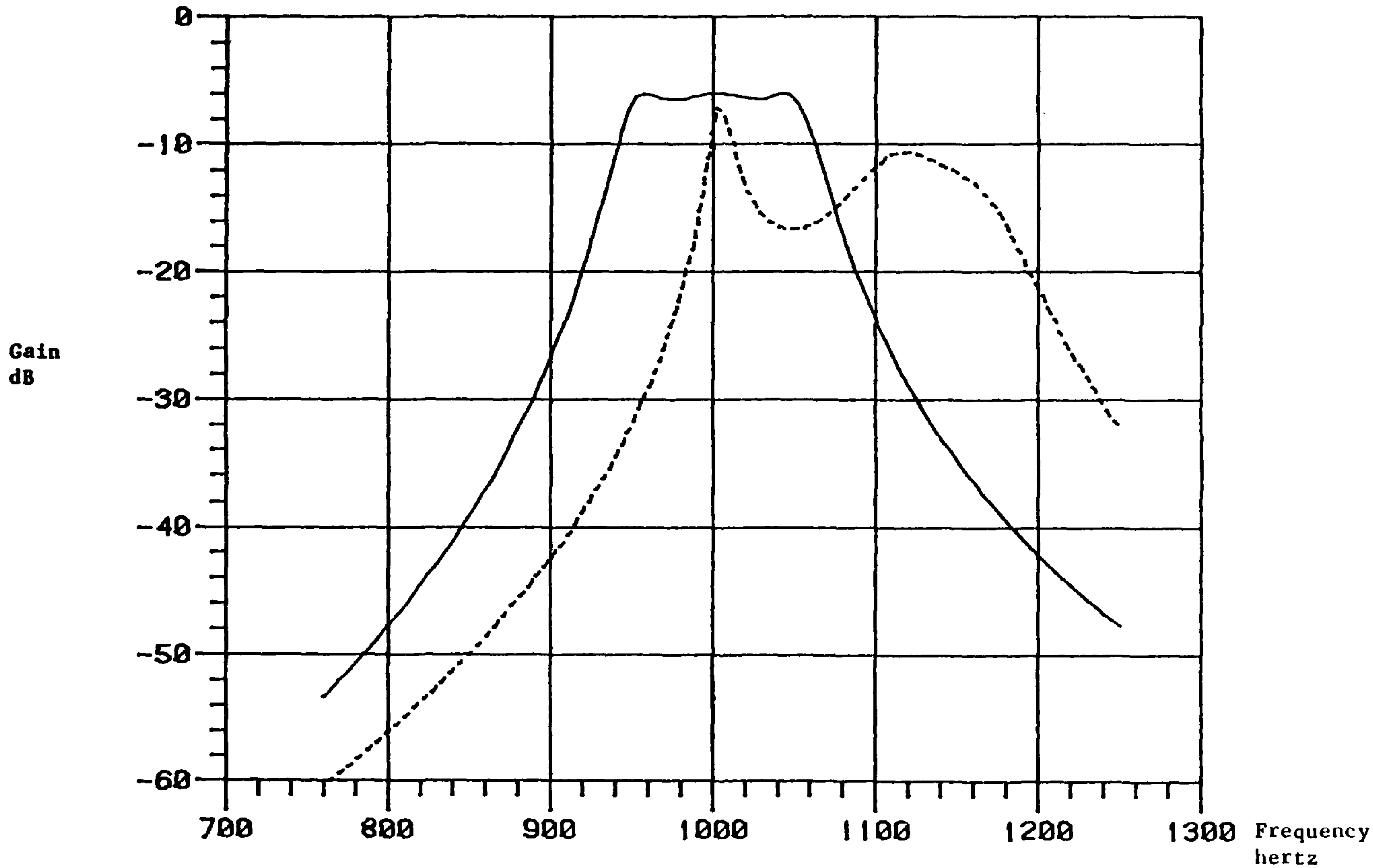

Figure 7.2 Gain Response of the Chebyshev FDNR Bandpass Filter
——— Nominal Response
----- Manufactured Response

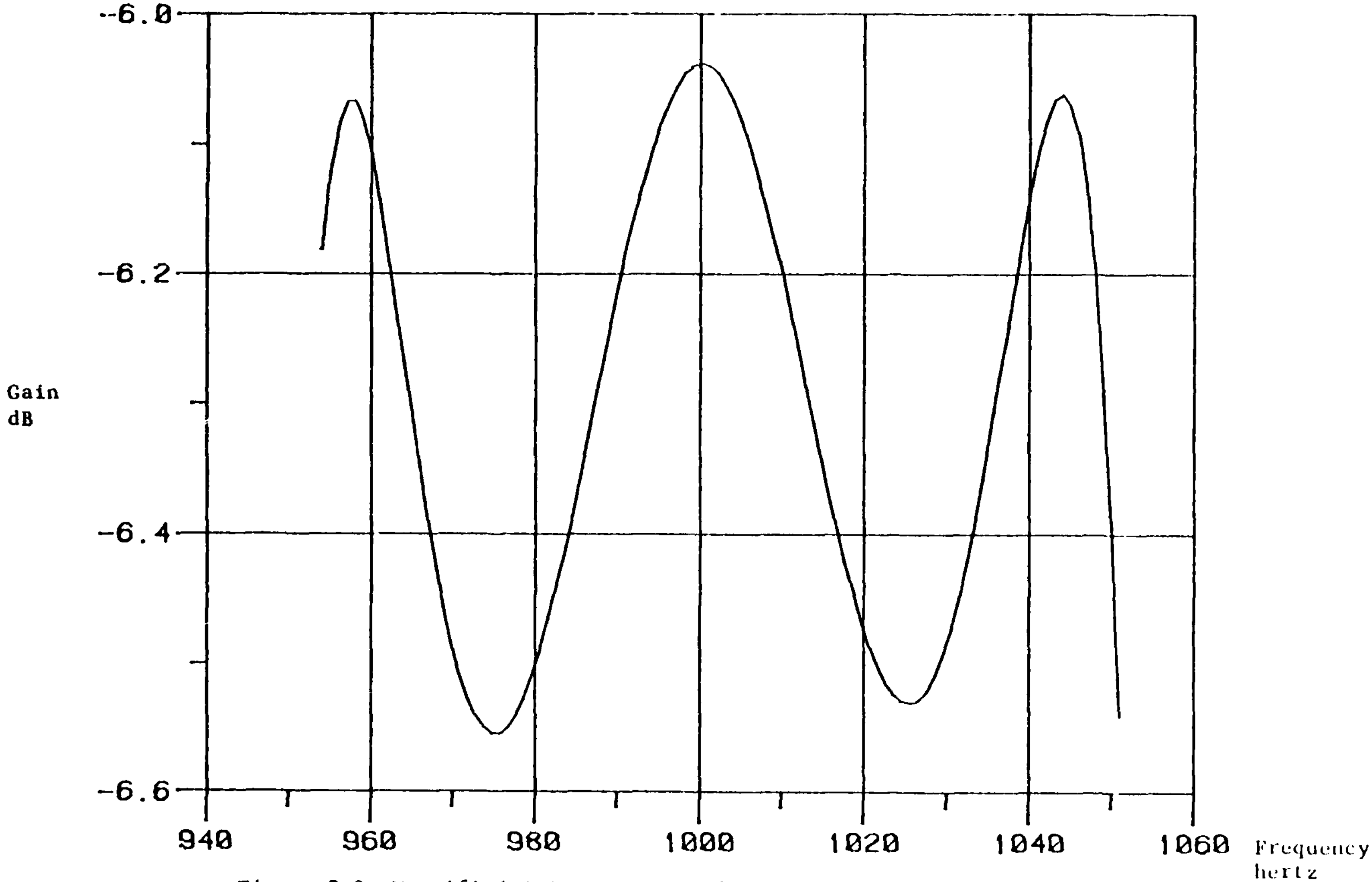

Figure 7.3 Magnified Gain Response of the Chebyshev FDNR Bandpass Filter in the Passband
—— Nominal Response
----- Manufactured Response

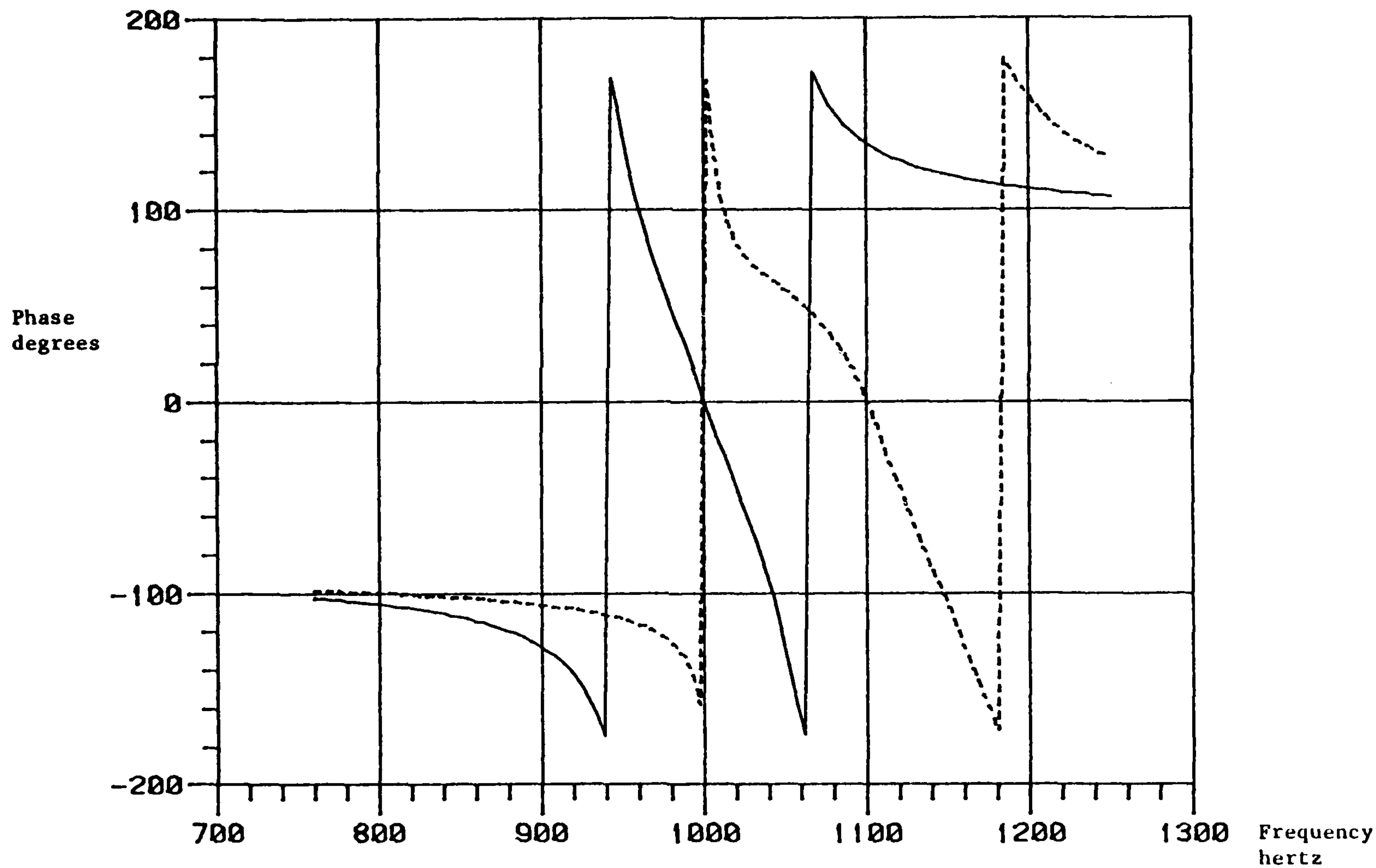

Figure 7.4 Phase Response of the Chebyshev FDNR Bandpass Filter
—— Nominal Response
----- Manufactured Response

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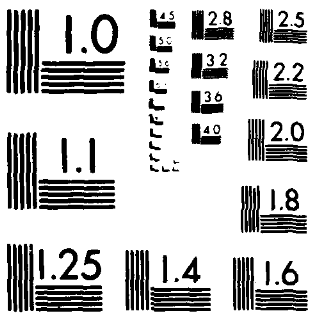
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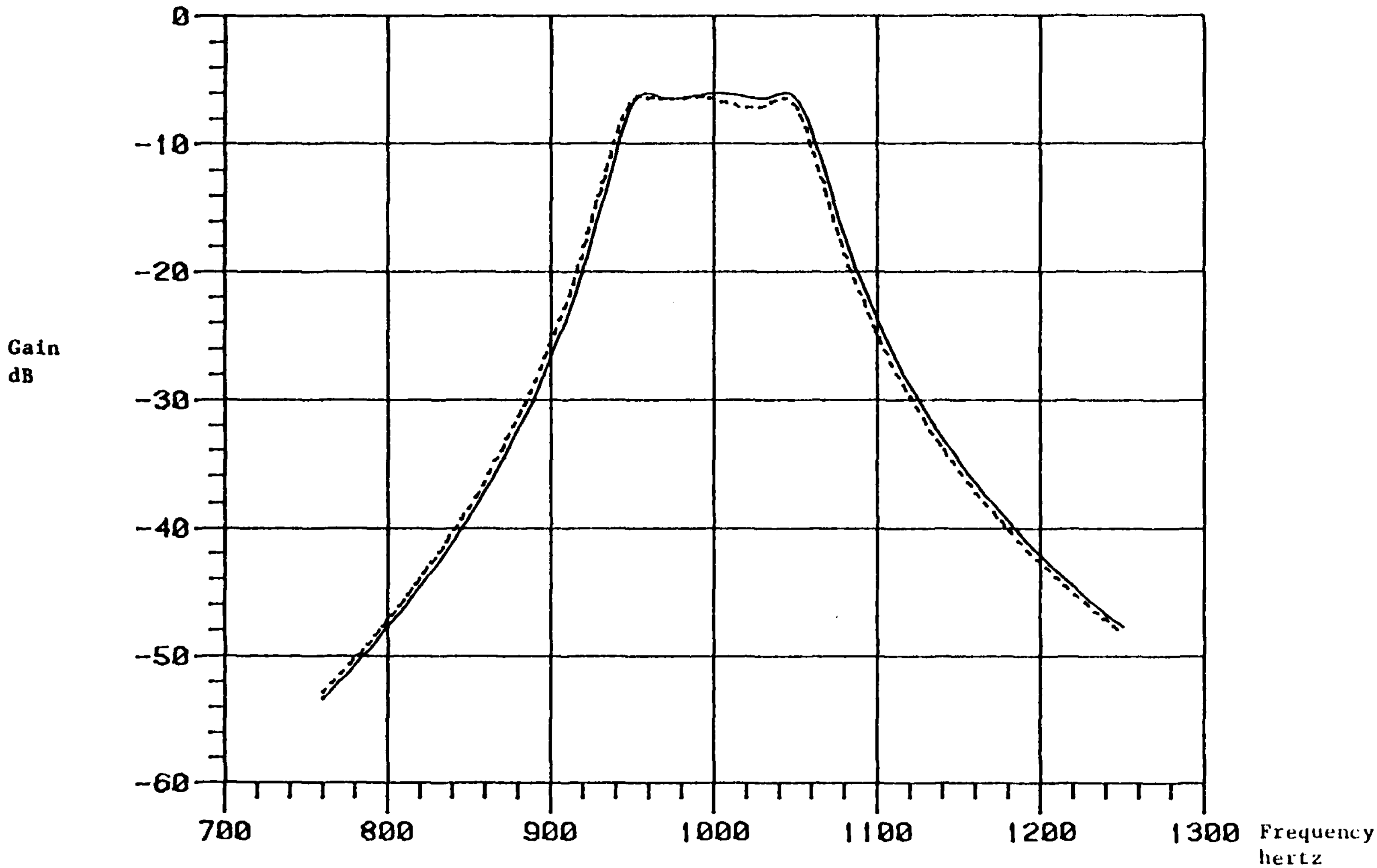

Figure 7.5 Gain Response of the Chebyshev FDNR Bandpass Filter
—— Nominal Response
----- Tuned Response After One Iteration

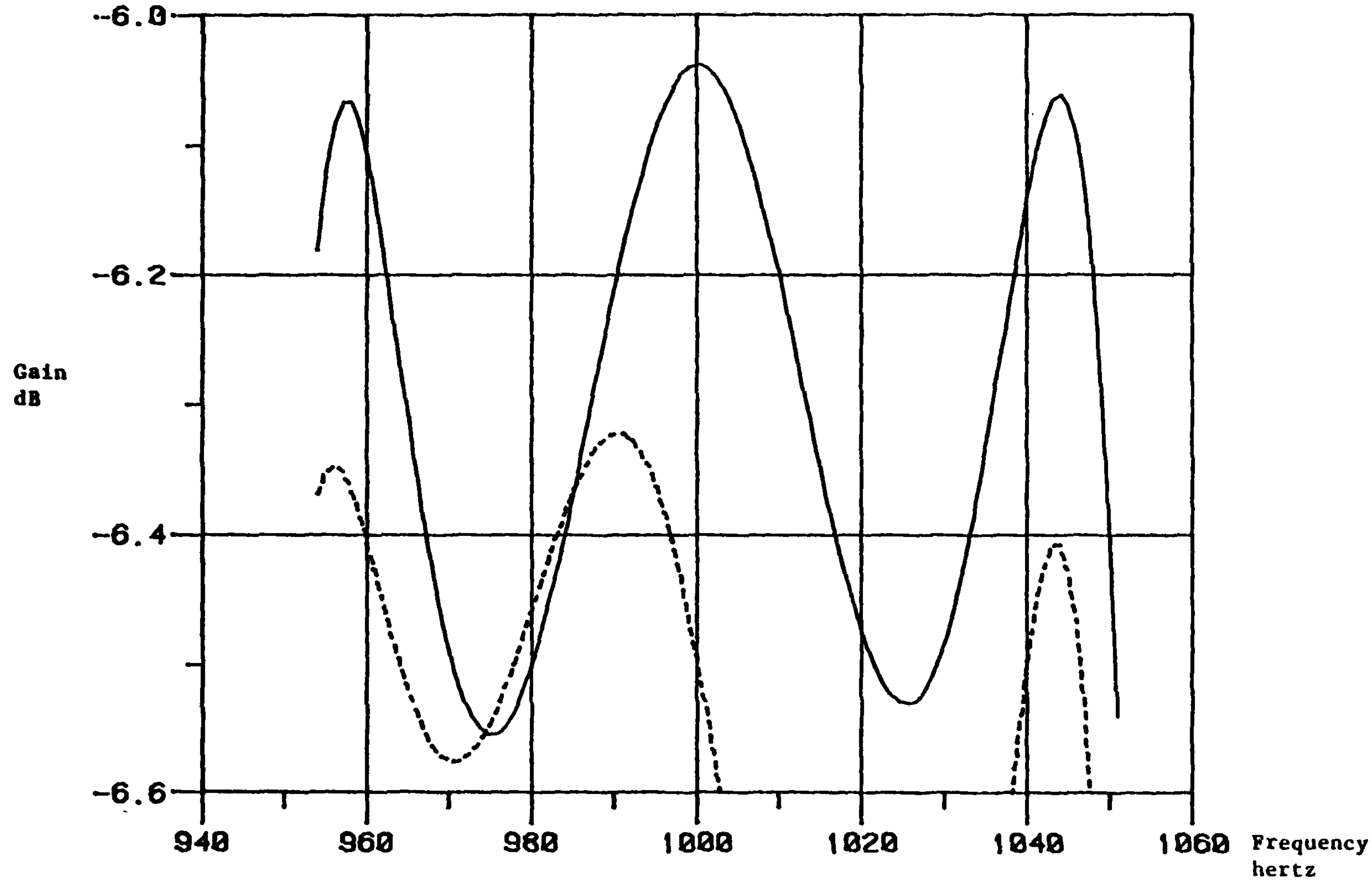

Figure 7.6 Magnified Gain Response of the Chebyshev FDNR Bandpass Filter in the Passband
—— Nominal Response
----- Tuned Response After One Iteration

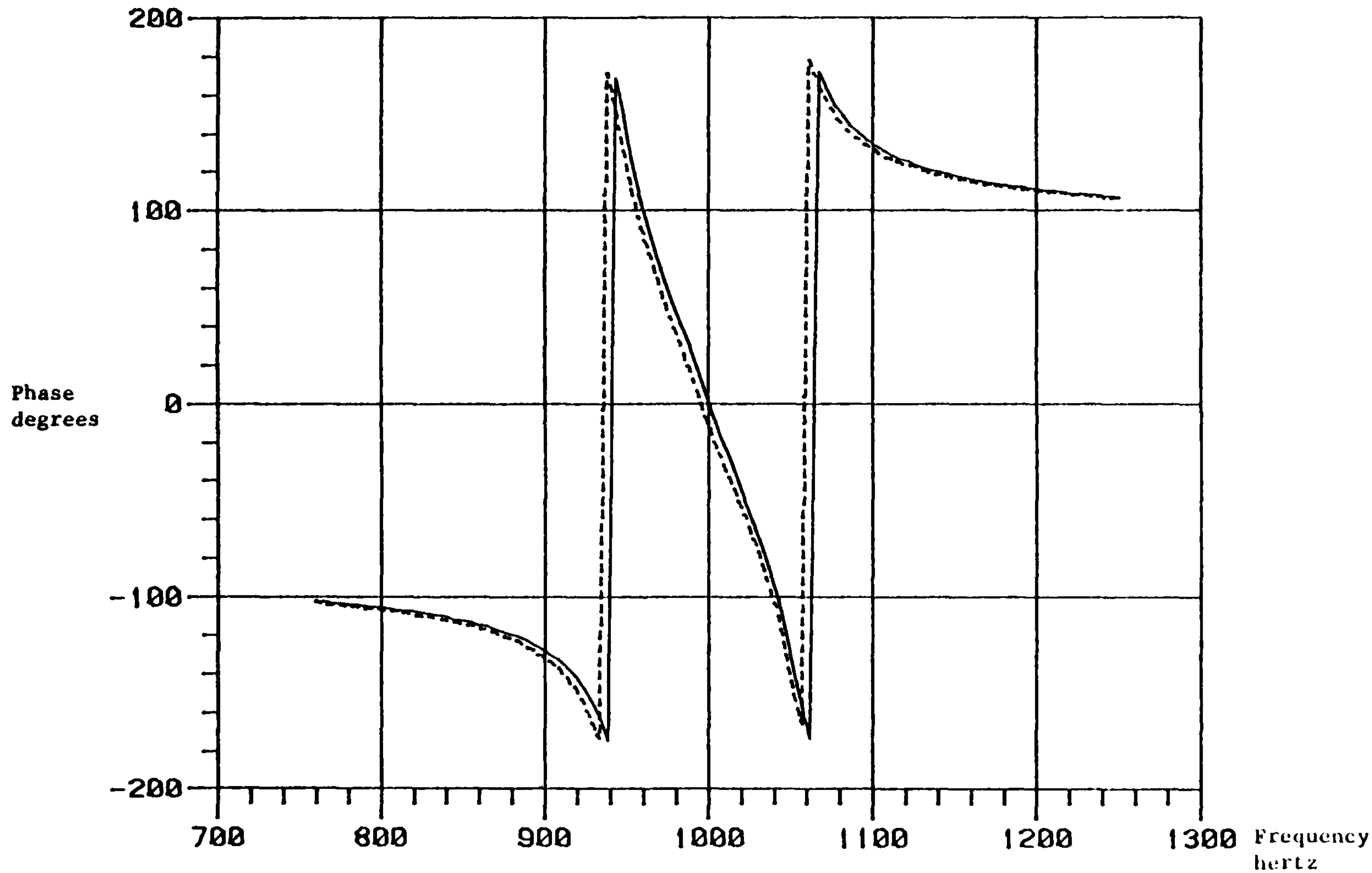

Figure 7.7 Phase Response of the Chebyshev FDNR Bandpass Filter
——— Nominal Response
\- - - - - Tuned Response After One Iteration

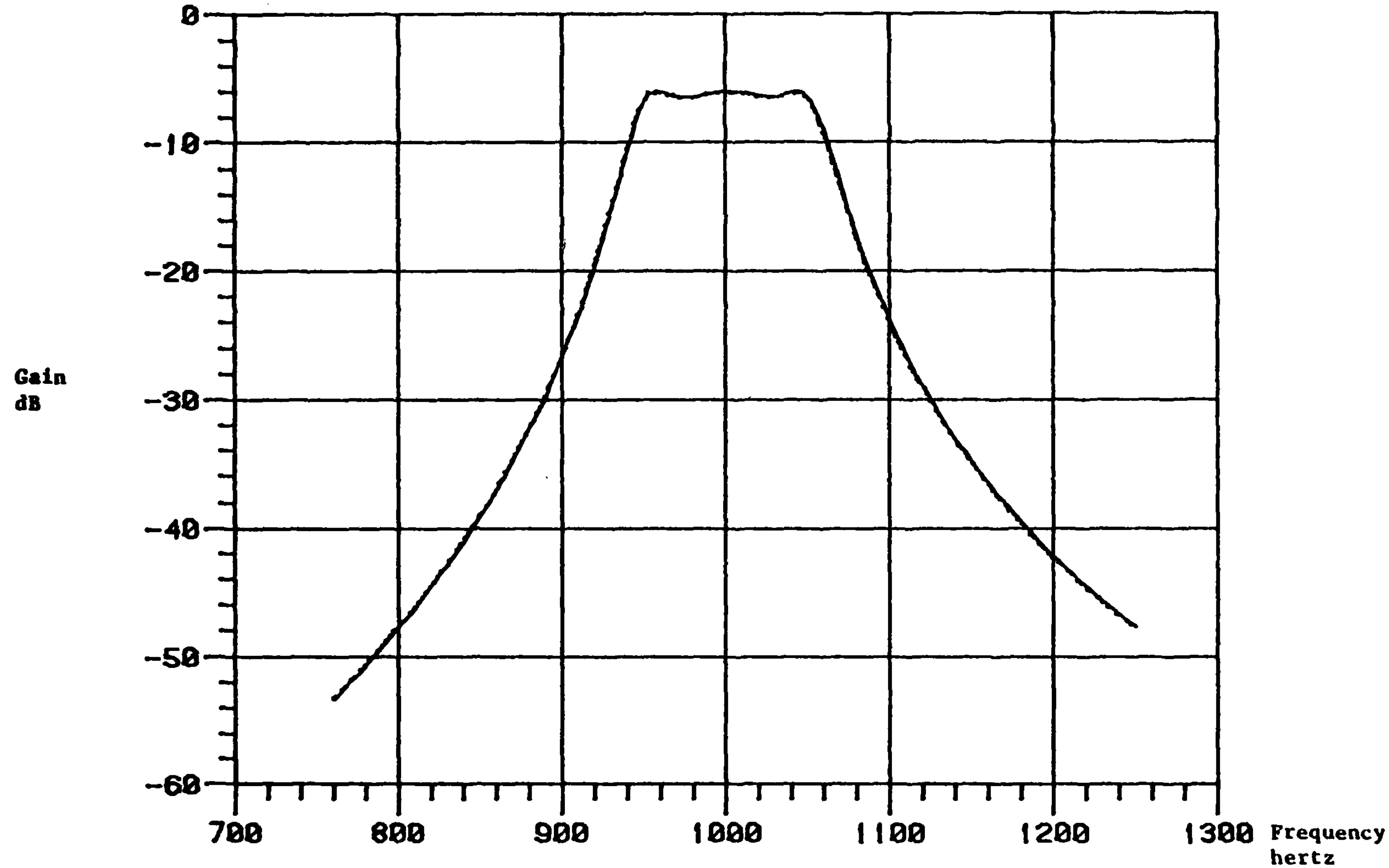

Figure 7.8 Gain Response of the Chebyshev FDNR Bandpass Filter
——— Nominal Response
----- Tuned Response After Two Iterations

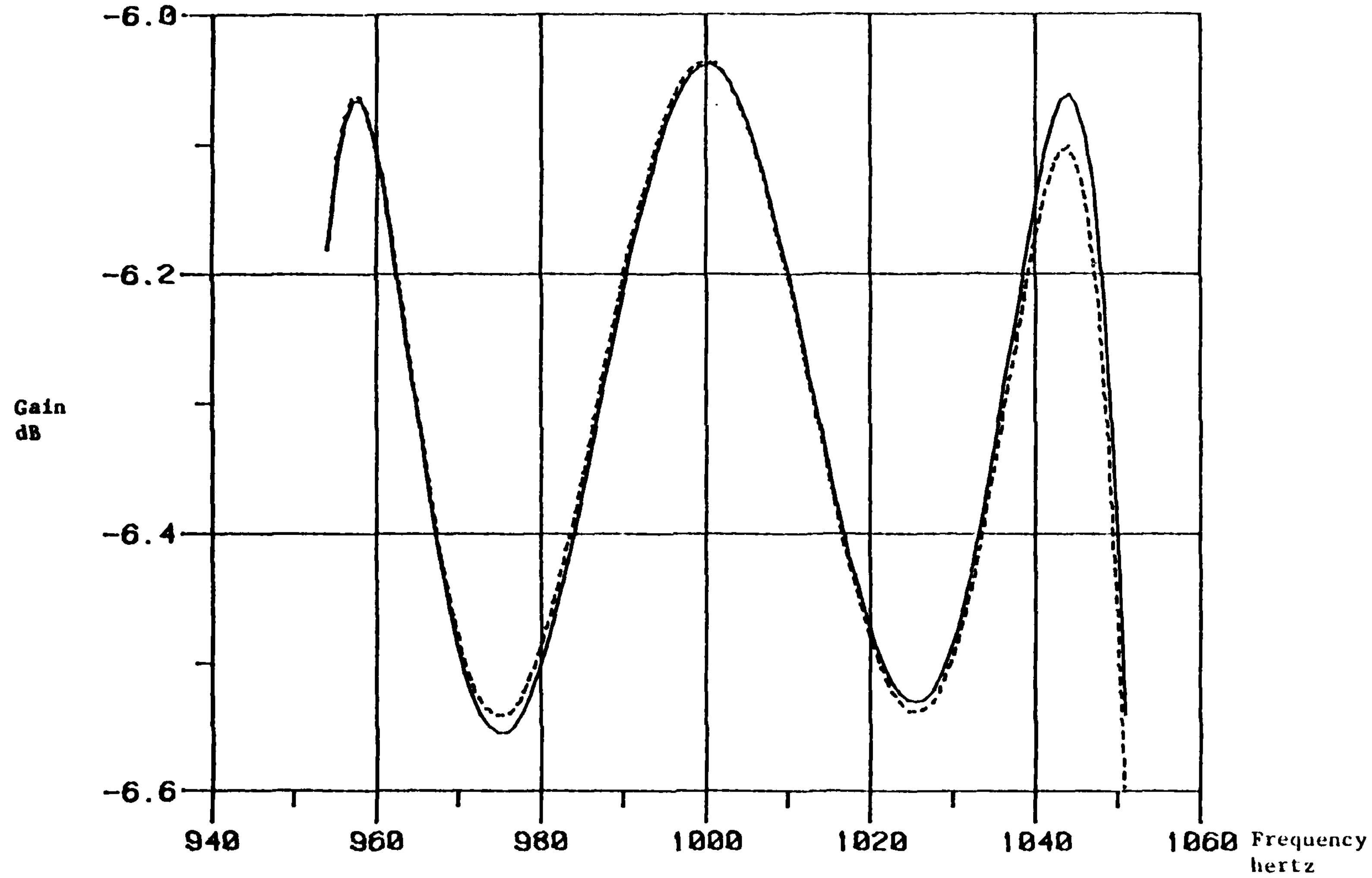

Figure 7.9 Magnified Gain Response of the Chebyshev FDNR Bandpass Filter in the Passband
——— Nominal Response
----- Tuned Response After Two Iterations

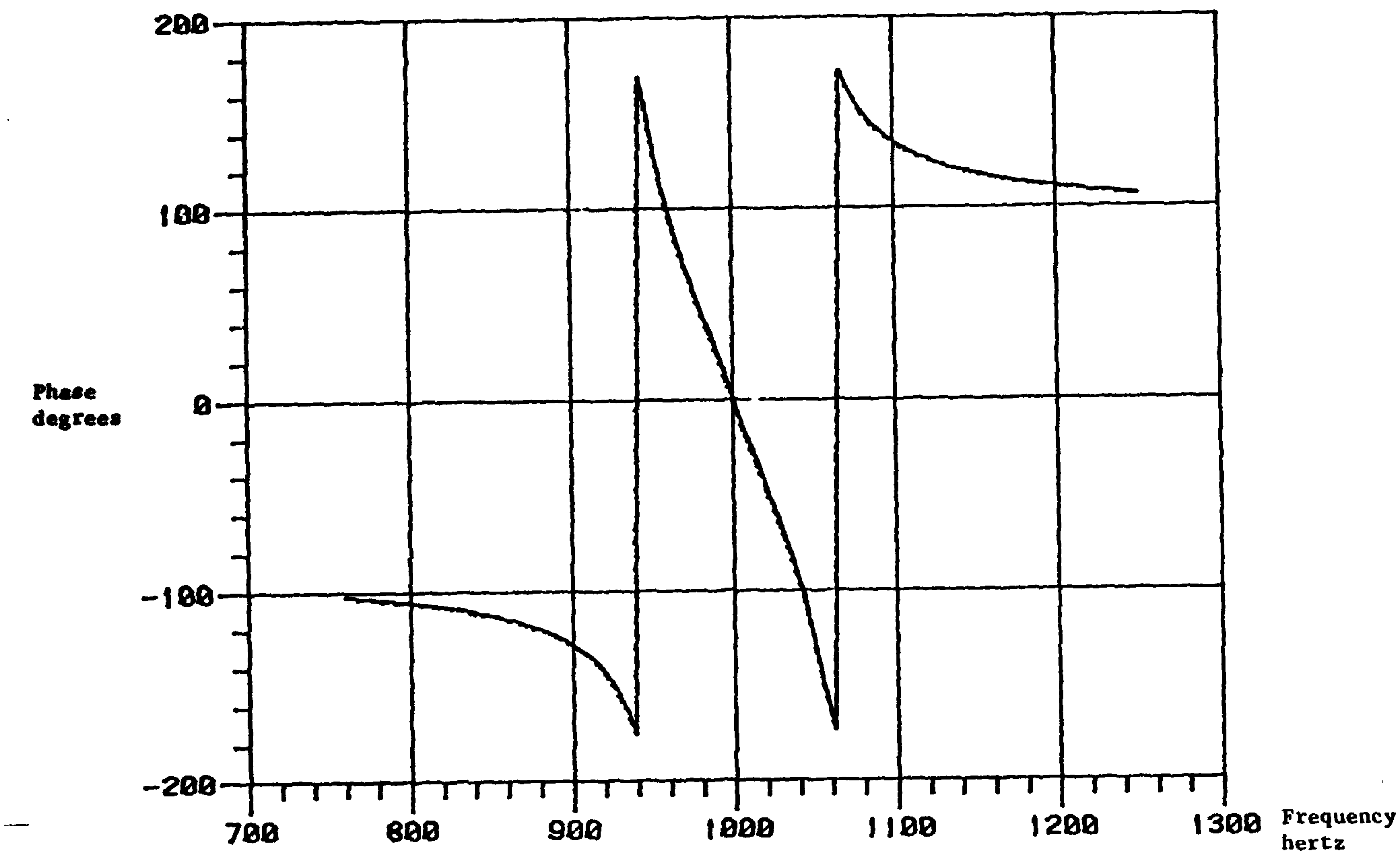

Figure 7.10 Phase Response of the Chebyshev FDNR Bandpass Filter
—— Nominal Response
----- Tuned Response After Two Iterations

gain deviation error as discussed in Chapter 6. The magnified passband at the final iteration is shown in Figure 7.11.

As with the sixth-order leapfrog filter of Chapter 6 some care must be exercised in the selection of the tuning resistors. For example, when R_1, R_4, R_7, R_{12} and R_{14} are chosen as tuning resistors the thrashing effects described in Chapter 6 result. However, when the tuning resistors mentioned earlier are used to tune the filter, no thrashing effects are evident. Notice that this set of tuning resistors consists of all of the resistors in the FDNR transformed bandpass filter (corresponding to R_1, R_4 and R_{12}) in Figure 7.1, a single resistor in the floating FDNR (corresponding to R_6) and a single resistor in either grounded FDNR (corresponding to R_{10}).

Table 7.3 records the final tuning resistance values at the iteration where convergence occurred for each of the sample circuits.

The same five sample circuits were used to test a first-order transfer function sensitivity method (ΔV terms set equal to zero in equation (2.9)). With this first-order sensitivity method it was not possible to tune any of the sample circuits even though the first iteration showed some improvement in the performance specifications of the filter, although by no means rivaling the improvement attainable with the large-change sensitivity method. In subsequent iterations the right-hand side of equation (2.9) was driven to zero (about 10^{-7}) yielding one or more negative tuning element values and/or abnormally large (sometimes abnormally small) tuning element values, probably indicative of an ill-conditioned system of linear algebraic equations. In addition, the iterations never converged to a final set of element values.

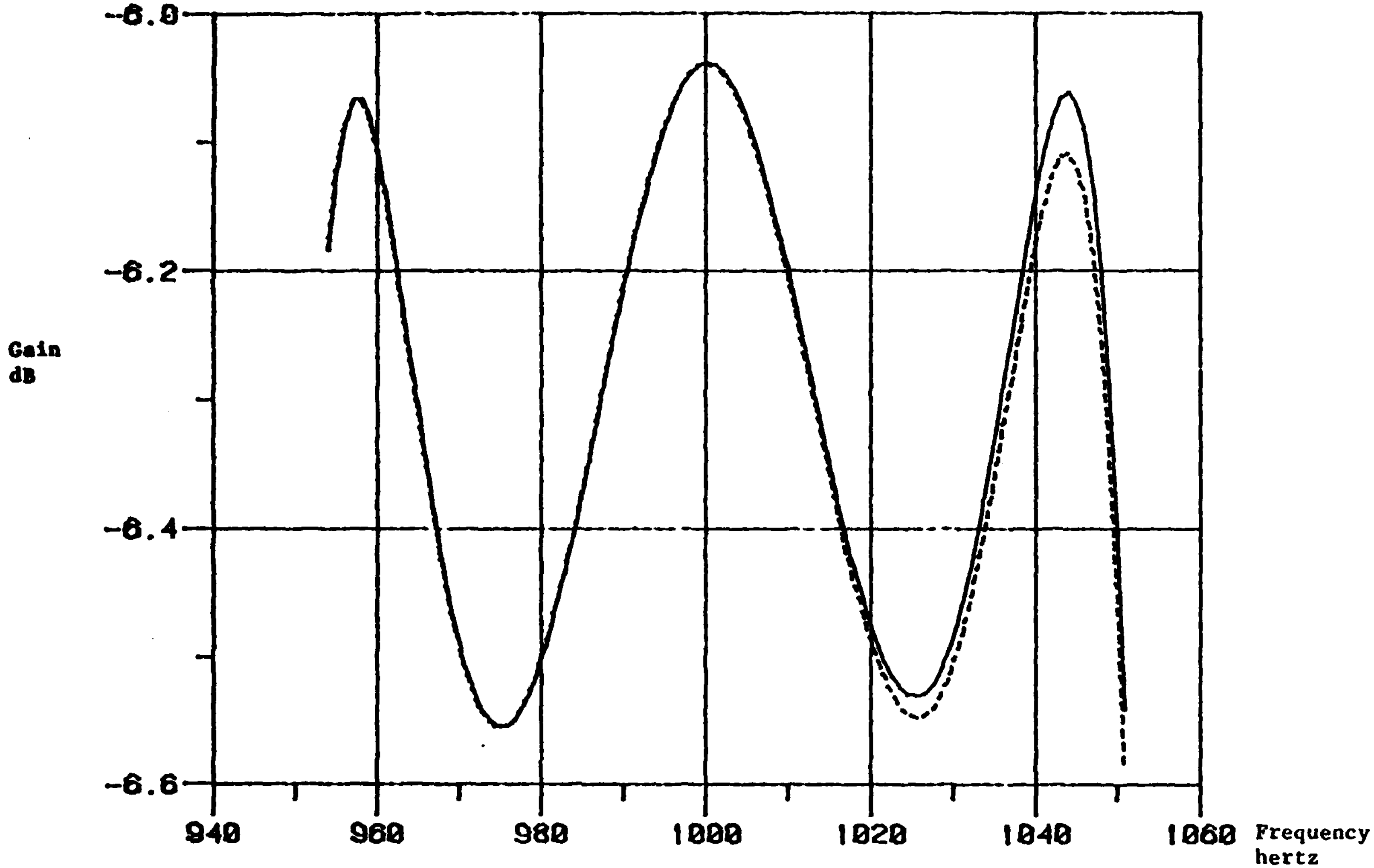

Figure 7.11 Magnified Gain Response of the Chebyshev FDNR Bandpass Filter in the Passband
—— Nominal Response
----- Tuned Response After Four Iterations

Table 7.3 Final Tuning Element Values

Element	Sample Circuit 1	2	3	4	5
R_1	1.043124 kΩ	1.005211 kΩ	.9806779 kΩ	.9211015 kΩ	1.077804 kΩ
R_4	175.7936 kΩ	173.9851 kΩ	173.0121 kΩ	168.2855 kΩ	181.9155 kΩ
R_6	.9680427 kΩ	1.013807 kΩ	.9983840 kΩ	1.026014 kΩ	.9792599 kΩ
R_{10}	1.064677 kΩ	.9988536 kΩ	.9821800 kΩ	.9244183 kΩ	1.071822 kΩ
R_{12}	.9839422 kΩ	.9727355 kΩ	.9959109 kΩ	1.014194 kΩ	1.005725 kΩ
c	.9942407	1.003968	.9988535	.9906004	.9930346

8. CONCLUSION

A large-charge transfer function sensitivity algorithm has been proposed for the tuning of analog filters. Tellegen's theorem and the adjoint network concept are used to relate large changes in a set of tuning elements to desired voltage changes in the manufactured filter at a set of critical frequencies [19,20,21]. Four filters of various type, order and topology were designed and the manufacturing process of each was simulated by generating a number of sample circuits. The sample circuits were used to test the proposed tuning algorithm which in every case yielded 100 percent tuning reliability. The proposed tuning algorithm is superior to the existing tuning methods in several aspects. While all of these methods require that the deviation in the element values at the time of manufacture not be too great, the proposed large-change algorithm can tolerate considerably larger element deviations and still tune the filter. This is aptly demonstrated by comparison of the proposed algorithm with the first-order transfer function sensitivity algorithm which yielded zero percent tuning reliability in the test filters of Chapters 5-7 and 40 percent tuning reliability in the test filters of Chapter 3. In this case only those sample circuits which had very small deviations in the capacitance values were successfully tuned. In addition, it took the first-order method twice as long to converge to the final element values for these circuits than when the proposed tuning algorithm was used, thus giving the latter an edge in speed of tuning and computational costs. As another example, Shockley and Morris [11] reported that the first-order root sensitivity method failed to tune a second-order, single amplifier bandpass filter in practice, when the capacitance values were five percent above the nominal and the resistors were at their design values. Contrast these results with the successful tuning of a sixth-order leapfrog band-pass filter with the large-change algorithm, when the capacitance values were, on the average, 4.34 percent below the nominal and the tuning resistors were 10 percent below the nominal with all other resistors at their design values. The results are even more staggering when one considers that the leapfrog filter and other multiple-feedback filters are difficult to tune in practice [30] as compared to the relative ease of tuning the test filter used by Shockley and Morris [11].

The proposed method is also superior to the previously cited methods in that it overcomes problems of accuracy, especially inherent in the root sensitivity method. Because the roots of the transfer function are calculated from the (calculated) transfer function coefficients in this method, there is a discrepancy between the accuracy of the coefficients and the roots, which becomes especially critical in the case of higher-order, narrow band filters [13]. The proposed tuning algorithm, on the other hand, has no difficulty in tuning filters of this kind, as is evident in the examples of Chapters 6 and 7. In addition the root sensitivity method (and the ω_0 and Q sensitivity methods) approximate derivatives by the forward difference quotient or the central difference quotient which require judicious choice of the change in the parameter in order to obtain a good approximation. In the large-change algorithm, however, the derivatives are computed accurately and efficiently via the adjoint circuit concept.

The proposed algorithm is also much more computationally efficient than the other algorithms, particularly than the deterministic method of approximating transfer function deviations using optimization [6] where the derivatives of each numerator and denominator coefficient of the transfer function with respect to each passive and parasitic element must be computed for each filter stage. Unlike the root sensitivity method, or deterministic tuning, the proposed tuning algorithm does not require powerful computer programs for the solution of equations of high degree.

The proposed tuning algorithm is also superior to the other methods in its flexibility. Because of problems with accuracy the root sensitivity method is limited to the tuning of second or third-order sections [11], which is not the case with the proposed tuning algorithm. Furthermore, the symbolic transfer function (including parasitic elements) is not required with the large-change tuning algorithm although it is required with the root sensitivity method and deterministic methods [6,11,14]. Unlike deterministic methods, it is not necessary to derive expressions for the tuning resistors in terms of the other components. Such expressions are difficult to derive in practice and may not even exist [6]. Also, individual tuning rules are not derived for each new filter product with the large-change algorithm, resulting in additional savings in programming costs.

Because there are usually more resistors than there are transfer function coefficients, it is not clear which resistors to select as tuning resistors when using the proposed tuning algorithm or deterministic tuning. The choice of a particular set of tuning resistors is important in both cases, as a given set may or may not yield a solution as was emphasized in Section 1.2.6. The poor selection of a set of tuning resistors sometimes results in the thrashing effects described in Chapter 6. Ostensibly, this is an indication that no solution exists to the related deterministic problem for the particular set of tuning resistors selected and the manufactured component values. Fixing the problem is a simple matter of selecting another set of tuning resistors. However, it is desirable to have some criteria for selecting a set of tuning resistors which alleviates this behavior, and the number of tuning resistors so required, thus warranting the need for further research.

Further research is also required in the area of more elaborate modeling of the thin or thick film components and the op amps for the filters of Chapters 4-7. New sample circuits which incorporate more sophisticated models (such as discussed in Chapter 3) would be used to test the proposed tuning algorithm. Since the new circuits containing these models would be more representative of actual thin or thick film hybrid filters coming off the production line, they would more accurately depict the reliability of the tuning algorithm in practice.

Finally, in some filter applications the variation in phase with frequency is of more importance than the gain response. Such applications typically occur in communications circuits where a linear phase or a constant group delay is often specified. If this is the case, the stopping and error criteria in Chapter 2 may be modified to stop the iteration when the desired phase specifications are met.

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VITA

Charles John Alajajian was born on May 26, 1952 in Cambridge, Massachusetts. He received the B.S. degree from Purdue University, West Lafayette, Indiana, in 1974 and the M.S. degree from the University of Illinois, Urbana-Champaign, Illinois, in 1976. In 1978 he received the Harold L. Olesen Award for excellence in undergraduate teaching by a graduate student in Electrical Engineering from the University of Illinois where he served as a teaching assistant from September 1974 to May 1975, and then as a graduate instructor from September 1975 to December 1978. In the summer of 1976 he worked as a research assistant at the Coordinated Science Laboratory in Urbana, Illinois on reduced-order modeling by error minimization, and in the following summer on a linear fault analysis algorithm. From January to August 1979 he worked as a research assistant at the Coordinated Science Laboratory on a new tuning algorithm for analog filters. His present publications are "Fault Analysis of Analog Circuits," co-authored with T. N. Trick, which appeared in the Proceedings of the 20th Midwest Symposium on Circuits and Systems in August of 1977, and "A New Tuning Algorithm for Analog Filters," co-authored with E. El-Masry and T. N. Trick which appeared in the Proceedings of the 22nd Midwest Symposium on Circuits and Systems in June of 1979. He was appointed Assistant Professor of Electrical and Computer Engineering at Clemson University, Clemson, South Carolina in 1979.