QUIET RADAR PROCESSOR ANALYSIS BY COVARIANCE MATRIX TRANSFORMATIONS

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The MICOM Quiet Radar program is a multi-year exploratory development effort to build and test a short-range air-defense system radar with Anti-Radiation Missile (ARM) immunity. By transmitting a low-power, bi-phase modulated, continuous-wave waveform in conjunction with ultra-low sidelobe antennas and a frequency-agile carrier frequency, it is possible to reduce ARM lock-on capabilities to ineffective ranges.
The objective of this effort was to determine probability of detection for a given false alarm rate for a candidate Quiet Radar processor by performing covariance matrix transformations. The analysis included range cell averaging CFAR.

Results were obtained for two possible configurations of the Quiet Radar processor. The results are given as plots where the probability of detection in each frequency cell is shown for various probabilities of false alarm and CFAR range cell window widths.

The analysis has verified previously determined CFAR losses and shown that processor performance is dependent on the low pass filter used for noise reduction.
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I. INTRODUCTION

The MICOM Quiet Radar program [1] is a multi-year, exploratory development effort to build and test a short-range air-defense system radar with Anti-Radiation Missile (ARM) immunity. By transmitting a low-power, bi-phase modulated, continuous-wave waveform in conjunction with an ultra-low sidelobes antenna and a frequency-agile carrier frequency, it is possible to reduce ARM lock-on capabilities to ineffective ranges.

Previous effort has determined the probability of false alarms and detections for the Quiet Radar Processor by using Monte Carlo simulations. [2] Further effort was directed toward determining the effect of Constant False Alarm Rate (CFAR) techniques on processor performance. [3]

The objective of this effort was to determine probability of detection for a given false alarm rate for the Quiet Radar Processor by performing covariance matrix transformations. The analysis included range cell averaging CFAR.

Section II contains the analytical development used in determining processor performance. Section III presents the matrices that describes the processor elements and manipulations required for a covariance analysis of the processor. Section IV presents the Quiet Radar system parameters used in the performance analysis. Section V presents performance results obtained from analysis.

II. PERFORMANCE ANALYSIS

The block diagram for the 2-D CFAR processor is shown in Figure 1. This is a linear system up to the point where \( \tilde{Z} \) is calculated. The system can be analyzed by a procedure contained in a Raytheon report. [4] A succinct presentation of the analysis follows. The input \( \tilde{X} \) is represented as a column matrix of the complex (i.e., I and Q channels) input sample values. It follows that:

\[
\tilde{a} = \tilde{A} \tilde{X} \quad \tilde{y} = \tilde{C} \tilde{a} \quad \tilde{g} = \tilde{W} \tilde{y} \quad \tilde{F} = \tilde{D} \tilde{g} . \quad (1-4)
\]
Due to the linearity, the output at $\bar{F}$ can be described by a Gaussian distribution when Gaussian signals are the input to the system. Also, the input can be separated into a sum of components, viz., ground clutter, noise, and target signal. Each component can be analyzed separately using superposition. The prime objective is to determine the variance at the FFT output. This is a mathematically tractable problem for the Gaussian signals. Let $\bar{M}$ represent the covariance matrix of $\bar{X}$, $\bar{M}_1$ the covariance matrix of $\bar{U}$, $\bar{M}_2$ the covariance matrix of $\bar{Y}$, and $\bar{M}_3$ the covariance matrix of $\bar{G}$. It follows that:

$$\bar{M}_1 = \bar{A} \bar{M} \bar{A}^T$$  \hspace{1cm} (5)

$$\bar{M}_2 = \bar{C} \bar{M}_1 \bar{C}^T$$  \hspace{1cm} (6)

$$\bar{M}_3 = \bar{W} \bar{M}_2 \bar{W}^T$$  \hspace{1cm} (7)

A similar transformation could be used to find the covariance matrix of $\bar{F}$. However, this is not necessary because the variance of each element of $\bar{F}$ is all that is required. Consequently, the analysis uses the L-point FFT algorithm, superposition of the I and Q signals, and separation of the real and imaginary parts of the $F_k$ element of $\bar{F}$ to obtain

$$\sigma_{R_k}^2 = \sum_{i,j=1}^{L} m_{ij} c_{ik} c_{jk}$$  \hspace{1cm} (8)

$$\sigma_{I_k}^2 = \sum_{i,j=1}^{L} m_{ij} d_{ik} d_{jk}$$  \hspace{1cm} (9)
where $m_{ij}$ terms are the elements of $M_J$,

$$c_{jk} = \cos\left[\frac{2\pi}{L} (j - 1)k\right]$$

(10)

$$d_{jk} = \sin\left[\frac{2\pi}{L} (j - 1)k\right]$$

(11)

and the $k$ subscript on the variance represents the $k^{th}$ frequency cell of the FFT output. Combining the I and Q channel results yields the real part of $F_k$, i.e., $R(F_k)$, and the imaginary part of $F_k$, i.e., $I(F_k)$ to each be normal with variance $\sigma_{Rk}^2 + \sigma_{Ik}^2$, i.e.,

$$R(F_k) \text{ and } I(F_k) \in N\left(0, \sigma_{Rk}^2 + \sigma_{Ik}^2\right).$$

(12)

This result holds for the $j^{th}$ range bin and the $k^{th}$ frequency cell for either ground clutter or noise. A change in notation is used to represent this feature, viz., for noise $N$

$$\sigma_{Njk}^2 = \sigma_{NRk}^2 + \sigma_{NIk}^2.$$  

(13)

Similar results hold for ground clutter, $g$. Thus,

$$\sigma_{jk}^2 = \sigma_{Njk}^2 + \sigma_{gjk}^2.$$  

(14)

The magnitude unit of Figure 1 will change the Gaussian distribution of $P_{jk}$ into an exponential distribution at $Z_{jk}$, i.e.,

$$P(Z_{jk}) = \frac{1}{2\sigma_{jk}^2} e^{-\frac{Z_{jk}^2}{2\sigma_{jk}^2}}.$$  

(15)
Calculation of the probability of false alarm for a fixed threshold \( V_{Tk} \), in the \( k \)th frequency cell yields

\[
P_{FA} = e^{-Y_{bk}}
\]

(16)

where

\[
Y_{bk} = \frac{V_{Tk}}{2\sigma_{jk}}^2
\]

(17)

When CFAR techniques are used, the threshold is not fixed but is a random variable. It is possible to calculate the expected value (i.e., average value) of the PFA as

\[
P_{FA} = \int_{0}^{\infty} P_{FA} p(Y_{bk}) \, dy_{bk}
\]

(18)

The density functions for the threshold are dependent on the CFAR techniques.

A 2-D CFAR which averages the \( k \)th frequency cell of an \( N \)-range-bin window will have

\[
P_{FA} = \frac{1}{1 + \frac{\tau_k K_2}{N}}^N
\]

(19)

where \( \tau_k \) relates the range bin of interest to the range bins used in the CFAR window and \( K_2 \) is a threshold constant used to specify a false alarm probability.
The development for the probability of detection follows a similar procedure, i.e.,

\[
\bar{P}_{Dk} = \int_{0}^{\infty} P_{Dk}(Y_{bk}) dY_{bk} 
\]

For a Swerling I target the results are

\[
P_{Dk} = \frac{-Y_{bk}}{1 + \bar{x}} \quad \text{for } 2-D \text{ CFAR}
\]

where \( \bar{x} \) is the signal-to-interference ratio for the range bin and frequency cell of interest.

At a given range, the ground clutter backscatter coefficients are assumed to be constant over the CFAR window. However, a Wiebull distribution \( p(\sigma^0) \) is assumed for the range dependency on these coefficients. The performance dependency on \( \sigma^0 \) is represented as \( P_{Dk}(\sigma^0) \). It follows that the expected value can be obtained from

\[
<\bar{P}_{Dk}> = \int_{0}^{\infty} \bar{P}_{Dk}(\sigma^0) p(\sigma^0) d\sigma^0
\]

These procedures were implemented by Raytheon in a computer program. The program is a hybrid of equation oriented calculations and Monte Carlo simulation. The Wiebull statistics of Equation (22) are evaluated by Monte Carlo procedures. The selection of a 2-D CFAR threshold, i.e., threshold from range bins below or above bin of interest, is not determined by Monte Carlo methods. Instead, the average values of the thresholds are determined and the largest average value is used to select the technique. The mathematical description of this is given below. The range
bins below the bin of interest yield a 2-D CFAR threshold of

\[ A_k = \tau_k \frac{K_2}{2N\sigma^2} \sum_{j=-1}^{-N} z_{jk} \]  

(24)

where \( z_{-1k} \) represents the "below" (-1) bins and the \( k^{th} \) frequency cell. The average value is

\[ \overline{A}_k = \frac{\tau_k K_2}{2} = \frac{K_2\sigma_{-1k}^2}{2\sigma_{ok}^2} \]  

(25)

Similar results hold for the 2-D CFAR threshold determined from the range bins above the bin of interest, i.e., \( B_k \) and \( \overline{B}_k \). The program determines

\[ \overline{Y_{bk}} = \text{Max}(\overline{A}_k, \overline{B}_k) \]  

(26)

The selection process is actually accomplished as

\[ 2\sigma_{ok}^2 \overline{Y_{bk}} = \text{Max}(K_2\sigma_{-1k}^2, K_2\sigma_{1k}^2) \]  

(27)

Once the threshold is selected, then the results of Equation (21) are used to calculate the probability of detection.

III. COVARIANCE TRANSFORMATION MATRICES

A digital processor which is a candidate for the Quiet Radar has been designated as D-8 [2]. A block diagram model used in the mathematical analysis is given in Figure 2 and corresponding input/output relationships are given in Table 1.

The D-8 processor will be analyzed by the covariance matrix performance analysis presented in the previous section. It is readily noticed that the systems, Figures 1
Figure 2. Model used for analysis of configuration D-8.

<table>
<thead>
<tr>
<th>MATRIX</th>
<th>DIMENSION</th>
<th>SYMBOL</th>
<th>RELATION TO INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>NIN X 1</td>
<td>( \bar{X} )</td>
<td></td>
</tr>
<tr>
<td>Coder Output</td>
<td>NIN X 1</td>
<td>( \bar{X}_C )</td>
<td>( \bar{B}_C \cdot \bar{X} )</td>
</tr>
<tr>
<td>MTI Output</td>
<td>M X 1</td>
<td>( \bar{a} )</td>
<td>( \bar{A} \cdot \bar{X}_C = \bar{A}\bar{B}_C\bar{X} )</td>
</tr>
<tr>
<td>Decoder Output</td>
<td>M X 1</td>
<td>( \bar{c} )</td>
<td>( \bar{B}<em>{SC} \cdot \bar{a} = \bar{B}</em>{SC}\bar{A}\bar{B}_C\bar{X} )</td>
</tr>
<tr>
<td>LPP Output</td>
<td>NFPT X 1</td>
<td>( \bar{v} )</td>
<td>( \bar{C} \cdot \bar{a} = \bar{C}\bar{B}_{SC}\bar{A}\bar{B}_C\bar{X} )</td>
</tr>
<tr>
<td>Window Output</td>
<td>NFPT X 1</td>
<td>( \bar{g} )</td>
<td>( \bar{W} \cdot \bar{v} = \bar{W}\bar{C}\bar{B}_{SC}\bar{A}\bar{B}_C\bar{X} )</td>
</tr>
</tbody>
</table>

Where NIN = Total No. of Inputs, M = No. of MTI Outputs, and NFPT = No. of FFT Points.
and 2, are the same with the exception that D-8 contains three additional elements. They are (1) coder, (2) analog-to-digital (A/D) converter, and (3) decoder.

The A/D converter is a non-linear device and this type of analysis is for linear systems only. Hence, the A/D converter is ignored. For many cases, the receiver noise will dominate over quantization noise. But if quantization noise is sufficient to effect processor performance, it could be applied to input of analysis as white or colored Gaussian noise.

A discussion on binary phase coding and its performance effect in the Quiet Radar is given in Reference 5. Therefore, only a few comments concerning the coder and decoder are presented. The coder is simply a multiplier used for coding the video signal (normally performed at RF prior to transmission) and the decoder is also a multiplier used to decode the video signal. The decoder provides range cell discrimination when coder and decoder have matched codes, i.e., the in-range channel. When coder and decoder have codes that are not matched, the decoder output will be a video signal modulated by a shifted version of the code, i.e., the out-of-range channel. $B_C$ and $B_{SC}$ are the coder and decoder matrices which are used in the covariance analysis.

It can easily be shown that

$$B_{SC} \cdot \overline{A} \cdot B_C = \begin{cases} \overline{A} & \text{for In-Range Channel} \\ \overline{A} \cdot B_{SC} & \text{for Out-of-Range Channels.} \end{cases}$$

Therefore, by neglecting the out-of-range channels, the output covariance matrix equation

$$\bar{M}_3 = (\bar{W} \cdot \bar{C} \cdot B_{SC} \cdot \overline{A} \cdot B_C) \cdot \bar{M} \cdot (\bar{W} \cdot \bar{C} \cdot B_{SC} \cdot \overline{A} \cdot B_C)^T$$

reduces to

$$\bar{M}_3 = (\bar{W} \cdot \bar{C} \cdot \overline{A}) \cdot \bar{M} \cdot (\bar{W} \cdot \bar{C} \cdot \overline{A}).$$
The following is a brief description of the matrices given in Table 2 for the covariance analysis. The input covariance matrix $\bar{M}$ is given as

$$
\bar{M} = \begin{bmatrix}
m_{1,1} & m_{1,2} & \cdots & m_{1,NIN} \\
m_{2,1} & m_{2,2} & \cdots & m_{2,NIN} \\
m_{NIN,1} & \cdots & \cdots & m_{NIN,NIN}
\end{bmatrix}
$$

This is a symmetric matrix and the element $m_{ij}$ can be determined from the interference, i.e., noise or clutter correlation function, by

$$
m_{ij} = R(\tau)\big|_{\tau=(i-j)T}
$$

Thus, only $NIN$ elements need to be calculated.

The MTI matrix $A$ for a two-pulse canceller is given as

$$
\bar{A} = \begin{bmatrix}
[10000 \ldots -1]_{1\times NDEL} & 0 \\
[0] & [10000 \ldots -1] \\
[00] & [10000 \ldots -1] \\
\vdots & \vdots \\
[0] & [10000 \ldots -1]
\end{bmatrix}
$$

where $NDEL$ is the number of input samples before an MTI output. $NDEL$ will be an integer multiple of the code length.
TABLE 2. COVARIANCE MATRICES RELATIONSHIPS FOR CONFIGURATION D-8

<table>
<thead>
<tr>
<th>MATRIX</th>
<th>DIMENSION</th>
<th>SYMBOL</th>
<th>TRANSFORMATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Covariance</td>
<td>NIN X NIN</td>
<td>$\bar{M}$</td>
<td>$\bar{M}<em>1 = (\bar{B}</em>{sc} \cdot \bar{A} \cdot \bar{B}_c) \cdot \bar{M}$</td>
</tr>
<tr>
<td>Coder</td>
<td>NIN X NIN</td>
<td>$\bar{B}_c$</td>
<td>$(\bar{B}_{sc} \cdot \bar{A} \cdot \bar{B}_c)^T$</td>
</tr>
<tr>
<td>MTI</td>
<td>M X NIN</td>
<td>$\bar{A}$</td>
<td></td>
</tr>
<tr>
<td>Decoder</td>
<td>NIN X NIN</td>
<td>$\bar{B}_{sc}$</td>
<td></td>
</tr>
<tr>
<td>LPF</td>
<td>NFFT X M</td>
<td>$\bar{C}$</td>
<td>$\bar{M}_3 = (\bar{W} \cdot \bar{C}) \cdot \bar{M}_1 \cdot (\bar{W} \cdot \bar{C})^T$</td>
</tr>
<tr>
<td>Window Function</td>
<td>NFFT X NFFT</td>
<td>$\bar{W}$</td>
<td></td>
</tr>
<tr>
<td>Window Output Covariance</td>
<td>NFFT X NFFT</td>
<td>$\bar{M}_3$</td>
<td></td>
</tr>
</tbody>
</table>

Where NIN = Total No. of Inputs, M = No. of MTI Outputs and NFFT = No. of FFT Points.

The LPF matrix $\bar{C}$ is given as

$$
\bar{C} = \begin{bmatrix}
    [C_1 C_2 \ldots C_{NFILT}] & [0] \\
    [0] & [C_1 C_2 \ldots C_{NFILT}] \\
    [0] & [C_1 C_2 \ldots C_{NFILT}] \\
    \vdots & \vdots \\
    [0] & \ldots \ldots \ldots \\
    [0] & [C_1 C_2 \ldots C_{NFILT}] \\
    \vdots & \vdots \\
\end{bmatrix}
$$
where the finite impulse response filter coefficients are $C_1, C_2, ... C_{N\text{FILT}}$ and each row initially contains $(N-1) \times NS\text{NEW}$ zeros where $N$ is the row number and $NS\text{NEW}$ is the sample rate reduction factor.

The window function matrix $\bar{W}$ is given as

$$\bar{W} = \begin{bmatrix} w_1 & 0 & \ldots & 0 \\ 0 & w_2 & \ldots & 0 \\ 0 & 0 & w_3 & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & w_{N\text{FFT}} \end{bmatrix}$$

where the diagonal elements are the window function coefficients.

Since the input covariance matrix $\bar{M}$ is an NIN x NIN matrix where NIN is typically 4000 to 6000, this prohibits use of simple matrix multiples to perform transformations. It would require an excessive amount of memory to store covariance matrix $\bar{M}$, i.e., $(4000)^2$ to $(6000)^2$ or 16 to 36 million words of memory. Therefore, it is necessary and possible to calculate one row of $\bar{W} \cdot C \cdot A$ and then one row of $(\bar{W} \cdot C \cdot A) \cdot \bar{M}$ and finally calculate one row of $\bar{M}_3 = (\bar{W} \cdot C \cdot A) \cdot \bar{M} \cdot (\bar{W} \cdot C \cdot A)$. Then continue this process until all NFFT rows of $\bar{M}_3$ are determined. The computer analysis takes advantage of this phenomenon. Appendix A contains the program listings and input list.

IV. QUIET RADAR SYSTEM PARAMETERS

Two different configurations of the Quiet Radar D-8 Processor will be studied. The appropriate system parameters for each processor are given in Table 3.
<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>PROCESSOR #1</th>
<th>PROCESSOR #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency f_c</td>
<td>10 GHz</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Code Length (PN Code) NC</td>
<td>31 Bits</td>
<td>63 Bits</td>
</tr>
<tr>
<td>Sample Rate f_s</td>
<td>4 MHz</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Samples/Code Bit NSC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>MTI Delay NDEL</td>
<td>62</td>
<td>126</td>
</tr>
<tr>
<td>LPF Length NFILT</td>
<td>124 Taps</td>
<td>166 Taps</td>
</tr>
<tr>
<td>*LPF Wait NWAIT</td>
<td>124</td>
<td>166</td>
</tr>
<tr>
<td>Sample Rate Reduction NSNEW</td>
<td>62</td>
<td>83</td>
</tr>
<tr>
<td>FFT Length NFFT</td>
<td>64 Points</td>
<td>64 Points</td>
</tr>
<tr>
<td>Look Time</td>
<td>1.023 msec</td>
<td>1.1042 msec</td>
</tr>
<tr>
<td>Weighting</td>
<td>Hamming</td>
<td>Hamming</td>
</tr>
</tbody>
</table>

*LPF wait represents number of input samples required before LPF output is used, i.e., NWAIT > NFILT.
V. PERFORMANCE RESULTS

Since the analysis contains several variable parameters, e.g., signal-to-noise ratio, clutter-to-noise ratio, clutter spread, CFAR window width, etc., all possible performance results are too numerous to perform. Therefore, the analyses were performed for the parameters that have been used in previous work [2], [3].

The performance study parameter set used for both processors is given in Table 4.

The results obtained are plotted in Figure 3-14. For both processor configurations using the parameter set in Table 4, the probability of detection for several probabilities of false alarm in each of the frequency cells are shown.

For both processors, several observations can be made about the performance. The performance degrades as the number of range bins in the CFAR window decreases. This is easily explained since a better estimate of the noise is obtained by use of more range cells.

The loss of processor performance in the 0th frequency cell is due to attenuation of both clutter and target at 0Hz by the MTI. For frequency cells close to cell zero, the clutter residue out of the MTI raises the threshold, thus lowering the probability of detection. The LPF reduces the performance in the upper frequency cells due to attenuation beyond the target velocities of interest. One other obvious observation is that as the input signal-to-noise ratio level is reduced, the processor performance decreases.

It is readily observed that processor #2 performs better than processor #1. The larger dwell time for processor #2 allows for more input samples and, therefore, the low-pass filter response of processor #2 can be improved over processor #1. One advantage for processor #1 is the hardware reduction possible by combining the decoder and LPF coefficient. This combination is made possible since the number of LPF coefficients is an integer multiple of the number of code bits, i.e., 124/31 = 4.
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/N (Input Signal-to-Noise)</td>
<td>-21, -24 dB</td>
</tr>
<tr>
<td>C/N (Input Clutter-to-Noise)</td>
<td>46 dB</td>
</tr>
<tr>
<td>σf (Clutter Spectral Width)</td>
<td>8 Hz</td>
</tr>
<tr>
<td>CFAR Window Width</td>
<td>4, 8, 16 Range Bins</td>
</tr>
<tr>
<td>PfA</td>
<td>$10^{-3}$, $10^{-4}$, $10^{-5}$, $10^{-6}$</td>
</tr>
</tbody>
</table>
Figure 3. Detection performance for 124-tap LPF D-8 processor with S/N = -21 dB and a 4 range bin CFAR.
Figure 4. Detection performance for 124-tap LPF D-8 processor with S/N -21 dB and an 8 range bin CPAR.
Figure 5. Detection performance of a 124-tap LPF D-8 processor with S/N = -21 dB and a 16 range bin CFAR.
Figure 6. Detection performance of a 166-tap LDF D-8 processor with $S/N = -21$ dB and a 4 range bin CFAR.
Figure 7. Detection performance of a 166-tap LPF D-8 processor with S/N = -21 dB and an 8 range bin CFAR.
Figure 9. Detection performance of a 166-tap LPF D-8 processor with $S/N = -24$ dB and 4 range bin CFAR.
Figure 10. Detection performance of a 166-tap LPF D-8 processor with $S/N = -24$ dB and an 8 range bin CFAR.
S/N = -24 dB, C/N = 46 dB, 8Hz GROUND CLUTTER, HAMMING WEIGHTS, 16 RANGE BINS IN CFAR WINDOW, 2-PULSE CANCELLER, 166 TAP FIR FILTER, 64 POINT FFT

Figure 11. Detection performance of a 166-tap LPF D-8 processor with S/N = -24 dB and a 16 range bin CFAR.
Figure 12. Detection performance of a 124-tap LPF D-8 processor with S/N = -24 dB and a 4 range bin CFAR.
Figure 13. Detection performance of a 124-tap LPF D-8 processor with $S/N = -24\,\text{dB}$ and an 8 range bin CFAR.
Figure 14. Detection performance of a 124-tap LPF D-8 processor with S/N = -24 dB and a 16 range bin CFAR.
APPENDIX A

PROGRAM LISTING
The development of the Quiet Radar Processor covariance analysis was performed on the AP-120B array processor and a PDP-11/10 host computer contained in the Data Acquisition and Analysis System [6].

The simulation was written in Fortran utilizing the high-speed processing capability of the AP-120B. The program contained calls to AP math library subroutines and to AP assembly language programs which simulate processor elements [2]. The input to the program is the same as the Monte Carlo simulation input in Reference 2. Hence, providing capability of ready comparison between Monte Carlo and covariance analysis results.
<table>
<thead>
<tr>
<th>CARD NO.</th>
<th>VARIABLES</th>
<th>FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>= Length of pseudo-random code</td>
<td></td>
</tr>
<tr>
<td>NSCLK</td>
<td>= Number of samples per code clock period</td>
<td></td>
</tr>
<tr>
<td>NFILT</td>
<td>= Number of FIR filter samples</td>
<td></td>
</tr>
<tr>
<td>NSNEW</td>
<td>= Sampling rate reduction factor</td>
<td></td>
</tr>
<tr>
<td>NWAIT</td>
<td>= Delay in FIR filter output</td>
<td></td>
</tr>
<tr>
<td>NP</td>
<td>= Number of two pulse canceller stages</td>
<td></td>
</tr>
<tr>
<td>NDEL</td>
<td>= Number of samples in MTI delay</td>
<td></td>
</tr>
<tr>
<td>NPFT</td>
<td>= Number of FFT samples</td>
<td></td>
</tr>
<tr>
<td>NCINT</td>
<td>= Number of non-coherent integration samples</td>
<td></td>
</tr>
<tr>
<td>NFA</td>
<td>= Number of different thresholds simulated</td>
<td></td>
</tr>
<tr>
<td>IC(I), I=1,..., NC</td>
<td>= One period of the code (consists only of ones and zeros)</td>
<td></td>
</tr>
<tr>
<td>FSAMP</td>
<td>= Sampling rate</td>
<td></td>
</tr>
<tr>
<td>VMAX</td>
<td>= A/D converter saturation voltage</td>
<td></td>
</tr>
<tr>
<td>NBIT</td>
<td>= Number of bits in A/D converter</td>
<td></td>
</tr>
<tr>
<td>NOISDB</td>
<td>= Noise power in dB</td>
<td></td>
</tr>
<tr>
<td>SIGDB</td>
<td>= Target return power in dB</td>
<td></td>
</tr>
<tr>
<td>FDOP</td>
<td>= Target doppler frequency in Hz</td>
<td></td>
</tr>
<tr>
<td>KT</td>
<td>= Target delay in number of samples</td>
<td></td>
</tr>
<tr>
<td>DISCDB</td>
<td>= Distributed clutter power in dB</td>
<td></td>
</tr>
<tr>
<td>SIGMAF</td>
<td>= Clutter spectral spread in Hz</td>
<td></td>
</tr>
<tr>
<td>XM</td>
<td>= Clutter DC-to-AC power ratio,m²</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>= WEIBULL parameter</td>
<td></td>
</tr>
<tr>
<td>FIXCDB</td>
<td>= Fixed clutter power in dB</td>
<td></td>
</tr>
<tr>
<td>KP</td>
<td>= Fixed clutter delay in number of samples</td>
<td></td>
</tr>
</tbody>
</table>
List of Inputs (Concluded)

<table>
<thead>
<tr>
<th>CARD NO.</th>
<th>VARIABLES</th>
<th>FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-8</td>
<td>Gaussian Curve fit parameters $[x(j)], [y(j)]$ and $[z(j)]$ for $j=1,\ldots,6$</td>
<td>6E12.5</td>
</tr>
<tr>
<td>9-33</td>
<td>FIR impulse response samples. NFILT numbers, five per card.</td>
<td>5E16.8</td>
</tr>
<tr>
<td>34-36</td>
<td>Pre-FFT weighting sequence. NFFFT numbers, five per card.</td>
<td>5E16.8</td>
</tr>
<tr>
<td>47</td>
<td>ALP(I) $I=1,\ldots,NFA$ = Multiplication factor to control the threshold levels</td>
<td>5E16.8</td>
</tr>
<tr>
<td>48</td>
<td>NRUN(I), $I=1,\ldots,NFA$ = Number of Monte-Carlo trials for each threshold setting.</td>
<td>16I5</td>
</tr>
<tr>
<td>49</td>
<td>CPARK(I), $I=1,\ldots,NFA$ = Multiplication Factors to Control Range Cell Averaging CFAR Threshold Level</td>
<td>5E16.8</td>
</tr>
<tr>
<td>50</td>
<td>CFAFK(I), $I=1,\ldots,NFA$ = Multiplication Factors to Control Frequency Cell Averaging CFAR Threshold Level</td>
<td>5E16.8</td>
</tr>
</tbody>
</table>

*Denotes inputs used by covariance analysis.
FORTRAN PROGRAM FOR PERFORMANCE ANALYSIS
OF QUIET RADAR SIGNAL PROCESSOR H Y
COVARIANCE MATRIX TECHNIQUES, THE
MATRIX SIZES REQUIRED USE OF AN AP-120
AND A PDP-11, THE MATRIX TRANSFORMATION
IS OF THE FORM (FwCA) M (FwCA)**T
WHERE
M IS THE INPUT COVARIANCE MATRIX
A IS CANCELER MATRIX
C IS LOW PASS FILTER MATRIX
w IS FFT WEIGHTS MATRIX
F IS FFT MATRIX
WHERE NFFT = NO. OF FFT POINTS
M = (NFFT-1)NSEN+NFILT = INPUTS TO LPF
NIN = NSEN=NFILT = M+NUEL=TL, NO, OF INPUTS
NFILT = NO. OF FEQ FILTER COEFFICIENTS
NSEN = SAMPLING RATE REDUCTION FACTOR
NUEL = NO. OF INPUTS BEFORE MARTI OUI OUI
NIN = INPUT SAMPLES PER CODE
NC = CODE LENGTH
N = NO. OF TRANSMISSIONS
TYPICAL NUMBERS
NFFT = 64
M = 4030
NIN = 4092
NFILT = 124
NSEN = 62
NUEL = 62
NIN = 2
NC = 31
N = 66
ASSUMES DECODING COMPLETED
COMMON/ARRAY/AN(64),CH(600),PM(9),FIL(200),S11(64),
* SIGC(64),SIGS(64),SCALE(64)
COMMON/COND/NC,NFILT,NSEN,MP,NUEL,NFFT,NIN,NFA,T,NIW,
* NOID,SIGD,S15CL1,NCFF1,A,SIGMA,INT,N
REAL NOID
CALL UOPK
CALL AFRUN
CALL SIGNO
CALL CFAM
STOP
END

COMMON/ARRAY/AN(64),CH(600),PM(9),FIL(200),S11(64),
* SIGC(64),SIGS(64),SCALE(64)
COMMON/COND/NC,NFILT,NSEN,MP,NUEL,NFFT,NIN,NFA,T,NIW,
* NOID,SIGD,S15CL1,NCFF1,A,SIGMA,INT,N
DIMENSION W2HTH(64)
DIMENSION AIN(100),ALP(9),APUN(9),CFAK(9),IC(100)
REAL NUIS,NUISUB

EQUIVALENCE (AIN,ALP,APUN,CFAK,IC,(*,FLG1))

INPUT
READ(5,1) NC,NSCLF,PFIL1,NSCHM,NSM11,VP,NUEL,
1 NPL,NSCN1,NSF
READ(5,2) (IC(I),I=1,NC)
1 FORMAT(10I5)
2 FORMAT(55L11)
3 READ(5,3) TSAMF,NSAM,NSF
READ(5,4) NUISUB,SIG1,SIG2,FLG1
4 FORMAT(3E15.3,15)
5 READ(5,5) DISCL1,SIG1,SIG2,NSCHM,A,A,FLG1,NSF
6 FORMAT(5E15.3,15)
7 READ(5,6) (XIN(I),I=13,10)
8 FORMAT(5E15.3)
9 READ(5,7) (PFIL1(I),I=1,NFIL1)
10 READ(5,8) (ALP(I),I=1,NSF)
READ(5,9) (APUN(I),I=1,NSF)
11 READ(5,6) (SIGS(I),I=0)
12 CONTINUE
DU 12 I=1,NSF+1
SIGS(I)=0
SIGS(1)=0,
13 CONTINUE
DU 17 I=1,NV0
CK(1)=0
14 CONTINUE
DU 10 I=1,NFIL1
CFIL1(I)=PFIL1(I)
15 CONTINUE
I=1/TSAMF
FAC1=0.23025K
UFL=1.5/TSAMF
NUIS=EXP(FAC1*NUISUB)
SIG=EXP(FAC1*SIG1)
DISCL1=EXP(FAC1*DISCL1)
NIN=NUIS
NIN=NIN*IC(NM)+NM1+1+NPM+1+NPM+1
NM1=M1(NI)+NM10
M1=M1+1
NI=NI+1
M1=NI+1
UFL=UFL+FLU(1,NI)
C

OUTPUT
PRINT 40,DWELL
40 FORMAT(10X,'PROCESSOR U=8'/
1 10X,'DWELL TIME = ',E12.5)
PRINT 11,NC,NSCLK,MP,NUID,FFT,N,FILT
11 FORMAT(10X,'CUTE PERIOD = ',15/
1 10X,'NU. OF SAMPLES PER CUTE PERIOD = ',15/

2 10X,'NU. OF PULSES CANCELLED IN M1 = ',15/
2 10X,'NU. OF SAMPLES IN M1 DELAY = ',15/
3 10X,'NU. OF FFT SAMPLES = ',15/
4 10X,'NU. OF RANGE CELLS IN CAM WINNOW = ',15/
5 10X,'SAMPLING RATE AT INPUT = ',E12.5,'Hz')
PRINT 13,NSNEW,NNAII,NFILT
13 FORMAT(10X,'SAMPLING RATE REDUCTION FACTOR = ',15/
1 10X,'NUMBER OF TRANSIENT SAMPLES DELETED = ',15/
2 10X,'NU. OF FIR IMPULSES RESPONSE SAMPLES = ',15)
PRINT 23
23 FORMAT(10X,'FILTER COEFFICIENTS'/)
PRINT 24,1F11.1,1=1,NFIL1
24 FORMAT(10X,'FILTER COEFFICIENTS'/)
PRINT 14,NSUB,NSIGM
14 FORMAT(10X,'TARGET RETURN POWER = ',E12.5,'db'/
1 10X,'NOISE POWER = ',E12.5,'db')
PRINT 15,DISCUB,NSIGM
15 FORMAT(10X,'DISCRIMINATED CLUTTER POWER = ',E12.5,'db'/
1 10X,'CLUTTER SPECTRAL SPREAD = ',E12.5,'Hz'/
2 10X,'WEIBULL PARAMETER = ',E12.5)
PRINT 26
26 FORMAT(10X,'LIGHTING COEFFICIENTS'/)
PRINT 16,(1F11.1,1=1,NFFT)
16 FORMAT(10X,'LIGHTING COEFFICIENTS'/)
C
AP CLEAN
CALL APCLK
CALL VCLK(4,1,32/67)
CALL APWK
RETURN
END

SUBROUTINE APRUN
COMMON/ARKAY/M(64),CR(64),CFARK(9),FILT(200),STI(64),
* SIGCS(64),SIGNS(64),SCALEI(64)
COMMON/CMSL/NC,NFILT,NSNEW,NP,NDEL,NSAMA,WAIT,
* NOIS,M,SIG,NVSCSK,MCFFT,A.SIGMAF,INT,N
DIMENSION M(6000)
REAL M,NOIS
DATA M/1*1.,5999*0.,/
AP INITIALZATION
P1=3, 141592654

GENERATING ONE ROW OF CA MAIIX

NO. OF CULS, EUAL NP*NUDL*NPFL1
NUDLZ=NUDL*2
NUDL1=NUDL+1
DU 6U J=1, NP
DU 5U I=NDL1, NUDL2
CK(I*NUDLZ)=CK(I*NUDL)
CK(I*NUDL+1)=CK(I*NUDL)
CK(I)=CK(I)*CK(1-NUDL)

CONTINUE

CONTINUE

AP DATA ENTRY
NCM=NUDL+1*NPFL1
NCU=2000+1*IN=1
NCUZ=NL0+1*IN=1
N1M=IN=1
NLZ=Z*IN=1

FOR GENERATING COUNTER AND NOISE ARRAYS
DU 100. IN=1, 2

GENERATING ONE ROW OF THE MAIIX

1(1414,1,1,1) GU 10 70
ANGZI*,1*1.SIGMA*1
DU 7 Y=1, W1
M(1)=((-ANGZ(I+1))*2)/2

CONTINUE

CONTINUE

CALL AUXRL
CALL VCLK(0,1,32/B1)
CALL APFI
CALL APFL1(I,1,32/1.2)
CALL APFL1(I1,100,1.2)
CALL APFL1(*,1,1.2)
CALL APFL
CALL MVX(CLK2,1,2000,0, -1)
1(1414,1) CALL VUSER(2000,1, 2000, 1, 1, 2)
CALL APFL
CALL APFL
CALL APFL
CALL APFL

CONTINUE

RETURN

END

SUBROUTINE APFL
COMM/AXLNI/4(234), CK(400), (FARR+, Y), P1LI(400), 11(4),
+ SIGMA(49), NLCN(100), SCAFL(1)
COMM/CURSL/4(1,231), 111(2), 1, 1, 1, 1, A, H, 1, H, 1, H, 1, 1, 1, 1, 1, 1, 1, 1.
I14b

C

M3 IS SIZED R0W BY ROW FROM STARTING ADDRESS 24000
LE=24000
ML=2000
DU 20 IN=1,IN

C

M3 IS AN NFP x NFP MATRIX

C

M3 IS SIZED ROW BY ROW FROM STARTING ADDRESS 24000
LE=24000
ML=2000
DU 20 IN=1,IN

C

M3 IS AN NFP x NFP MATRIX

C

LE=14000
MN=1
ND=1
CALL VMUL(100,1,1,LE,1,0,1,ML)
CALL SVRED(100,1,1,LE,ML)
CALL VMUL(ML,1,1,LE,1,1)
CALL AF

30 CONTINUE
ML=1+ML
LE=14000
DU 20 IN=1,IN

C

M3 IS SIZED ROW BY ROW FROM STARTING ADDRESS 24000
LE=24000
ML=2000
DU 20 IN=1,IN

C

M3 IS AN NFP x NFP MATRIX

C

LE=14000
MN=1
ND=1
CALL VMUL(100,1,1,LE,1,0,1,ML)
CALL SVRED(100,1,1,LE,ML)
CALL VMUL(ML,1,1,LE,1,1)
CALL AF

70 CONTINUE
LE=24000+1

20 CONTINUE
EXIT
END

SUBLTUTL:

C

M3 IS SIZED ROW BY ROW FROM STARTING ADDRESS 24000
LE=24000
ML=2000
DU 20 IN=1,IN

C

M3 IS AN NFP x NFP MATRIX

C

LE=14000
MN=1
ND=1
CALL VMUL(100,1,1,LE,1,0,1,ML)
CALL SVRED(100,1,1,LE,ML)
CALL VMUL(ML,1,1,LE,1,1)
CALL AF

40
C F IS SEPARATED INTO SIN AND COS MATRIXES
NCUS=0
NSIN=6000
ZERU=0.
CALL APPU(6,4000,1,1,1)
CALL APⅠU
FIN = (2,401)/4035
OU XU 1=1,4035
J=1-1
FINER IN *J
CALL APPU(4,1000,1,1,1)
CALL APⅠU
CALL Vkmp(32768,32/1,12000,1,401-1)
CALL VCU(l,2000,1,NCUS,401)
CALL VSIN(l,2000,1,NSIN,401,4035)
CALL ApⅠK
NCUS=NCUS+1
NSIN=NSIN+1
600 CONTINUE
C GENERATING P.M.
C PERFORMING P.M.
CALL Nmul(U,1,24000,1,12000,1,401,401)
CALL Nmul(U,1,24000,1,12000,1,401,401)
CALL MTrans(U,1,401,401)
CALL MTrans(U,1,401,401)
CALL ApⅠK
C DETERMINING DIAGONAL ELEMENTS OF P.M.P.
C STORE ELEMENTS AT ADDRESS 24000
MC=0
MS=6000
10=24000
NFTC=12000
NFTS=16000
OU 100 1=1,1013
CALL Vmul(NFTC,NFTS,MC,1,MC,1,NPIS)
CALL VAA(NPIS,NPIS,MS,1,MC,1,MS,1,NPIS)
CALL SVE(MS,1,10,1,1)
CALL ApⅠK
MC=MC+NPIS
MS=MS+NPIS
I0=10+1
NFTC=NFTC+1
NFTS=NFTS+1
100 CONTINUE
IF(IN,10,1) CALL APGET(SIGNS,24000,401,2)
IF(IN,10,2) CALL APGET(SIGCS,24000,401,2)
CALL ApⅠK
RETURN
EOD

THIS PAGE IS BEST QUALITY IMAGE FROM COPY FURNISHED TO DOG.
CALL ASSIGN(3,'UK1:RF MKR',0,'MEM')
DEFINT FI$ 6(200,128,0,0,0)
CALL *PUT(1,USCL,1,*CL)
IC=1
AA=1000
9 CONTINUE
DU 10 IC=1,9
AA=CFAPR(IC)*AA
PP$=1/A+IC*(AA/IC)**40
DU 501 K=1,AA+1
SUMP(K)=0.0
5010 CONTINUE
DU 5030 NA=1,NA
DU 5040 K=1,NA+1
SUMF(K)=SUMF(K+1)*NA
50400 CONTINUE
DU 6150 IC=1,6
SUMP(I)=SUMP(I)/NA
61500 CONTINUE
&IF(I<6,100)
100 FUNCTION(1,1,54,4,
,QUIET RADAR L=I RESULTS')
&IF(I<6,100) N,RAD,FRA
101 FUNCTION(100,9,0,0,10,15,6,
,CELLS IN CIRK WINDOW = I,15,6,
,LOW,INPUT THRESHOLD SCALING CONSTANT = I,E12.5,
,LOW,PROBABILITY OF FALSE ALARM = I,E12.5)
&IF(I<6,100)
102 FUNCTION(100,9,0,0,10,15,5,
,PROBABILITY OF DETECTION')
&IF(I<6,100) (I,SUMF(I),I=1,NA+1)
103 FUNCTION(50,0,15,12,5)
&IF(I<6,100) (CURE(H),I=1,NA+1)
&IF(I<6,100)
10 CUNTINUE
READ(3)CTAFA(1),I#1,NFA
READ(5,1)A
IF(N,N.E.0) GO TO 9
7 FUNCTION(5,10,6)
1  **F**UnMAT(1615)
   RETURN
   END
SUBROUTINE **B**ULL(D1SCLT,A,CL)
DIMENSION CL(1000)
DO 10 J=1,1000
   K=KAN(J,0)
   CL(J)=D1SCLT*K**2
10  CONTINUE
SUM=0.
DO 20 J=1,1000
   SUM=SUM+CL(J)
20  CONTINUE
SUM=SUM/1000.
DO 30 J=1,1000
   CL(J)=D1SCLT*CL(J)/SUM
30  CONTINUE
WRITE(6,50) SUM
50 FORMAT(/6X'**MEAN OF UNSCALED CLUTTER CROSS SECTION SEG IS '20.
   RETURN
END
SUBROUTINE **S**IGNU
COMMON/AKRAY=(64),CH(600),CFRAK(8),F11(200),S1L(64),
   * SIGCS(64), SIGS(64), SCALEI(64)
COMMON/CONS1/NC,NF11,NSNE,NP,NUEL,NFFT,NNI,NA,T,NAIT,
   * NUIS,SIG,DISCLT,CHIT,ASIGMA,INIT,A
REAL NUIS
DIMENSION T1(64)
NAIT=NAIT-1
PL=3.14159265
C COMPUTE FREQUENCY INCREMENTS
   FAC1=1./(NFFT*NSNE)
   ZEN=0.
C INITIALIZATION
   CALL APPUU(ZENU,32768,1,2)
   CALL APPUU(FILT,30/50,NF11,2)
   CALL APPUU(*,30600,NFFT,2)
   CALL APMD
   NUM=32000
   AK=2.*PI*FAC1
C LOOP FOR SIGNAL POWER CALCULATION
   DO 15 K=1,NFFT
      ANG=AK*(K-1)
      CALL APPUU(ANG,32768,1,2)
      CALL APMD
      CALL VRAMP(32768,32/6,12000,1,NNI)
CALL VSIN(12000,1,0,1,M1N)
CALL VCSU(12000,1,6000,1,m1n)
CALL M11(0,NUEL,NP,N1N)
CALL M11(6000,NUEL,NP,N1N)
CALL F1K1(0,30750,28400,NFIL1,T,NSNEW,NNWAIT,NFF1)
CALL F1K2(6000,30750,28401,NFIL1,T,NSNEW,NNWAIT,NFF1)
CALL W1G(28400,30600,NFF1)
CALL CFFT2(28400,29000,NFF1,1)
CALL CVMAGS(29000,2,29600,1,NFF1)
CALL MAXV(29600,1,NU,NFF1)
CALL APWR
15 CONTINUE
CALL APGE(1(SCALET,32000,NFF1,2)
CALL APWD
C SIGNAL, NOISE AND CLUTTER AVERAGE OUTPUTS
DO 10 I=1,NFF1
  NA11=ABS(SIGCS(I))/ABS(SIGCS(I))
  IF(NA11,L11,3) SIGCS(I)=0.
10 CONTINUE
  NA11=0,101)
101 FORMAT(1H1,//18X,6H SIGNAL, JAX, 5H NOISE, 37X, 7H CLUTTER/)
  NA11(0,100) (SCALE1(I),SIGNS(I),SIGCS(I),I=1,NFF1)
100 FORMAT(13X,E10.8,2/X,E16.8,2/X, E16.8)
DO 185 K=1,NFF1
  IL(k)=UNSLCT*SIGCS(K)+NOIS*SIGAS(K)
  TEST=11(K)
  IF(TESt.40,.3) TEST=1, L=10
  811(K)=(SIG*SCALE1(K))/TEST
185 CONTINUE
RETURN
END
APPENDIX B

RANGE CELL AVERAGING CFAR THRESHOLD
The range cell averaging CFAR threshold is calculated

\[ V_{Tk} = K \frac{1}{N} \sum_{j=1}^{N} Z_{jk} \]  

(B-1)

where \( V_{Tk} \) is estimated threshold

- \( Z_{jk} \) is FFT square law output
- \( N \) is CFAR window length
- \( K \) is scale factor for a specified average probability of false alarm
- \( j \) is range bin index
- \( k \) is frequency cell index.

For a system as shown in Figure B-1, if I and Q are Gaussian, then \( Z_{jk} \) is an exponential distribution. Thus,

\[ P(Z_{ik}) = \frac{1}{2\sigma_j} e^{-Z_{jk}/2\sigma_j^2} \]  

(B-2)

![Figure B-1. Detection System.](image)

It can be shown that, by knowing Equation (2), a probability density function can be derived for the threshold, obtained in Equation (1), and an average PFA can be found, i.e.,

\[ \overline{PFA}_k = \frac{1}{1 + \frac{\tau_k K}{N}} \]

where

\[ \tau_k = \frac{\sigma_{jk}^2}{\sigma_{ok}^2} \]

\( \sigma_{jk}^2 \) = variance of range cells of estimate

\( \sigma_{ok}^2 \) = variance of range cell of interest
Assume all range cells have identical noise, i.e., $\tau_k = 1$
or $\sigma_{jk}^2 = \sigma_{ok}^2$, then

$$K = N(\sqrt{\text{PFA}} - 1).$$

For example, let $\text{PFA} = 10^{-6}$ and $N = 4$, then

$$K = 4 \left( -4 \sqrt{10^{-6}} - 1 \right) = 122.5$$

CFAR threshold constants used in the analysis are given in Table B-1.
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