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QUIET RADAR PROCESSOR ANALYSIS BY COVARIANCE MATRIX TRANSFORMAT--ETC(U)
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QUIET RADAR PROCESSOR ANALYSIS BY COVARIANCE MATRIX TRANSFORMATIONS

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October 1979

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35809

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The MICOM Quiet Radar program is a multi-year exploratory development effort to build and test a short-range air-defense system radar with Anti-Radiation Missile (ARM) immunity. By transmitting a low-power, bi-phase modulated, continuous-wave waveform in conjunction with ultra-low sidelobes antenna and a frequency-agile carrier frequency, it is possible to reduce ARM lock-on capabilities to ineffective ranges.

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20. The objective of this effort was to determine probability of detection for a given false alarm rate for a candidate Quiet Radar processor by performing covariance matrix transformations. The analysis included range cell averaging CFAR.

Results were obtained for two possible configurations of the Quiet Radar processor. The results are given as plots where the probability of detection in each frequency cell is shown for various probabilities of false alarm and CFAR range cell window widths.

The analysis has verified previously determined CFAR losses and shown that processor performance is dependent on the low pass filter used for noise reduction.

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ILLUSTRATIONS (CONCLUDED)

I. INTRODUCTION

The MICOM Quiet Radar program [1] is a multi-year, exploratory development effort to build and test a short-range air-defense system radar with Anti-Radiation Missile (ARM) immunity. By transmitting a low-power, bi-phase modulated, continuous-wave waveform in conjunction with an ultra-low sidelobes antenna and a frequency-agile carrier frequency, it is possible to reduce ARM lockon capabilities to ineffective ranges.

Previous effort has determined the probability of false alarms and detections for the Quiet Radar Processor by using Monte Carlo simulations. [2] Further effort was directed toward determining the effect of Constant False Alarm Rate (CFAR) techniques on processor performance. [3]

The objective of this effort was to determine probability of detection for a given false alarm rate for the Quiet Radar Processor by performing covariance matrix transformations. The analysis included range cell averaging CFAR.

Section II contains the analytical development used in determining processor performance. Section III presents the matrices that describes the processor elements and manipulations required for a covariance analysis of the processor. Section IV presents the Quiet Radar system parameters used in the performance analysis. Section V presents performance results obtained from analysis.

II. PERFORMANCE ANALYSIS

The block diagram for the 2-D CFAR processor is shown in Figure 1. This is a linear system up to the point where $\bar{Z}$ is calculated. The system can be analyzed by a procedure contained in a Raytheon report. [4] A succinct presentation of the analysis follows. The input $\bar{X}$ is represented as a column matrix of the complex (i.e., I and Q channels) input sample values. It follows that:

$$\bar{\alpha} = \bar{A}\,\bar{X} \qquad \bar{\gamma} = \bar{C}\,\bar{\alpha} \qquad \bar{G} = \bar{W}\,\bar{\gamma} \qquad \bar{F} = \bar{D}\,\bar{G} \quad . \tag{1-4}$$

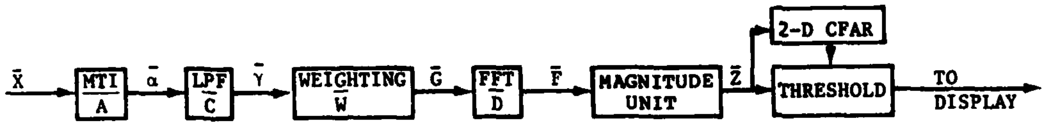

Figure 1. 2-D CFAR processor structure.

Due to the linearity, the output at $\bar{F}$ can be described by a Gaussian distribution when Gaussian signals are the input to the system. Also, the input can be separated into a sum of components, viz., ground clutter, noise, and target signal. Each component can be analyzed separately using superposition. The prime objective is to determine the variance at the FFT output. This is a mathematically tractable problem for the Gaussian signals. Let $\bar{M}$ represent the covariance matrix of $\bar{X}$, $\bar{M}_1$ the covariance matrix of $\bar{\alpha}$, $\bar{M}_2$ the covariance matrix of $\bar{Y}$, and $\bar{M}_3$ the covariance matrix of $\bar{G}$. It follows that:

$$\bar{M}_1 = \bar{A}\,\bar{M}\,\bar{A}^T \tag{5}$$

$$\bar{M}_2 = \bar{C}\,\bar{M}_1\,\bar{C}^T \tag{6}$$

$$\bar{M}_3 = \bar{W}\,\bar{M}_2\,\bar{W}^T \tag{7}$$

A similar transformation could be used to find the covariance matrix of $\bar{F}$. However, this is not necessary because the variance of each element of $\bar{F}$ is all that is required. Consequently, the analysis uses the L-point FFT algorithm, superposition of the I and Q signals, and separation of the real and imaginary parts of the F_k element of $\bar{F}$ to obtain

$$\sigma_{Rk}^{\;2} = \sum_{i,j=1}^{L} m_{ij}\, c_{ik}\, c_{jk} \tag{8}$$

$$\sigma_{Ik}^{\;2} = \sum_{i,j=1}^{L} m_{ij}\, d_{ik}\, d_{jk} \tag{9}$$

where m_{ij} terms are the elements of $\overline{M}_3$,

$$c_{jk} = \cos\left[\frac{2\pi}{L}(j-1)k\right] \tag{10}$$

$$d_{jk} = \sin\left[\frac{2\pi}{L}(j-1)k\right] \tag{11}$$

and the k subscript on the variance represents the k^{th} frequency cell of the FFT output. Combining the I and Q channel results yields the real part of F_k, i.e., $R(F_k)$, and the imaginary part of F_k, i.e., $I(F_k)$ to each be normal with variance $\sigma_{Rk}^2 + \sigma_{Ik}^2$, i.e.,

$$R(F_k) \text{ and } I(F_k) \; \varepsilon \; N\left(0, \sigma_{Rk}^2 + \sigma_{Ik}^2\right) . \tag{12}$$

This result holds for the j^{th} range bin and the k^{th} frequency cell for either ground clutter or noise. A change in notation is used to represent this feature, viz., for noise N

$$\sigma_{Njk}^2 = \sigma_{NRk}^2 + \sigma_{NIk}^2 . \tag{13}$$

Similar results hold for ground clutter, g. Thus,

$$\sigma_{jk}^2 = \sigma_{Njk}^2 + \sigma_{gjk}^2 . \tag{14}$$

The magnitude unit of Figure 1 will change the Gaussian distribution of F_{jk} into an exponential distribution at Z_{jk}, i.e.,

$$P(Z_{jk}) = \frac{1}{2\sigma_{jk}^2} e^{-Z_{jk}/2\sigma_{jk}^2} . \tag{15}$$

Calculation of the probability of false alarm for a fixed threshold V_{Tk}, in the k^{th} frequency cell yields

$$PFA = e^{-Y_{bk}} \tag{16}$$

where

$$Y_{bk} = V_{Tk}/2\sigma_{jk}^{2} \ . \tag{17}$$

When CFAR techniques are used, the threshold is not fixed but is a random variable. It is possible to calculate the expected value (i.e., average value) of the PFA as

$$\overline{PFA} \int_{0}^{\infty} PFA \ p(Y_{bk})dY_{bk} \ . \tag{18}$$

The density functions for the threshold are dependent on the CFAR techniques.

A 2-D CFAR which averages the k^{th} frequency cell of an N-range-bin window will have

$$\overline{PFA} = \frac{1}{\left(1 + \frac{\tau_k K_2}{N}\right)^{N}} \tag{19}$$

where τ_k relates the range bin of interest to the range bins used in the CFAR window and K_2 is a threshold constant used to specify a false alarm probability.

The development for the probability of detection follows a similar procedure, i.e.,

$$\bar{P}_{Dk} = \int_0^\infty P_{Dk}\, p(Y_{bk})\, dY_{bk} \quad . \tag{20}$$

For a Swerling I target the results are

$$P_{Dk} = e^{-Y_{bk}/(1 + \bar{x})} \tag{21}$$

$$\bar{P}_{Dk} = \frac{1}{\left(1 + \frac{\tau_k K_2}{N(1 + \bar{x})}\right)^N} \text{ for 2-D CFAR} \tag{22}$$

where $\bar{x}$ is the signal-to-interference ratio for the range bin and frequency cell of interest.

At a given range, the ground clutter backscatter coefficients are assumed to be constant over the CFAR window. However, a Wiebull distribution $p(\sigma^o)$ is assumed for the range dependency on these coefficients. The performance dependency on σ^o is represented as $P_{Dk}(\sigma^o)$. It follows that the expected value can be obtained from

$$\langle P_{Dk} \rangle = \int_0^\infty \bar{P}_{Dk}(\sigma^o)\, p(\sigma^o)\, d\sigma^o \quad . \tag{23}$$

These procedures were implemented by Raytheon in a computer program. The program is a hybrid of equation oriented calculations and Monte Carlo simulation. The Wiebull statistics of Equation (22) are evaluated by Monte Carlo procedures. The selection of a 2-D CFAR threshold, i.e., threshold from range bins below or above bin of interest, is not determined by Monte Carlo methods. Instead, the average values of the thresholds are determined and the largest average value is used to select the technique. The mathematical description of this is given below. The range bins below the bin of interest yield a 2-D CFAR threshold of

$$A_k = \tau_k \frac{K_2}{2N\sigma^2_{-1k}} \sum_{j=-1}^{-N} Z_{jk} \tag{24}$$

where $^{2}_{-1k}$ represents the "below" (-1) bins and the k^{th} frequency cell. The average value is

$$\bar{A}_k = \frac{\tau_k K_2}{2} = \frac{K_2 \sigma_{-1k}^2}{2\sigma_{ok}^2} . \tag{25}$$

Similar results hold for the 2-D CFAR threshold determined from the range bins above the bin of interest, i.e., B_k and $\bar{B}_k$. The program determines

$$\bar{Y}_{bk} = \text{Max}\left(\bar{A}_k,\ \bar{B}_k\right) . \tag{26}$$

The selection process is actually accomplished as

$$2\sigma_{ok}^2\ \bar{Y}_{bk} = \text{Max}\left(K_2\sigma_{-1k}^2,\ K_2\sigma_{1k}^2\right) . \tag{27}$$

Once the threshold is selected, then the results of Equation (21) are used to calculate the probability of detection.

III. COVARIANCE TRANSFORMATION MATRICES

A digital processor which is a candidate for the Quiet Radar has been designated as D-8 [2]. A block diagram model used in the mathematical analysis is given in Figure 2 and corresponding input/output relationships are given in Table 1.

The D-8 processor will be analyzed by the covariance matrix performance analysis presented in the previous section. It is readily noticed that the systems, Figures 1

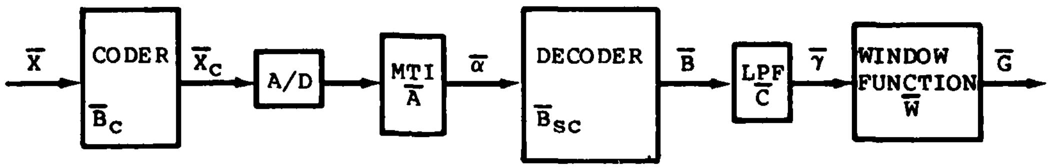

Figure 2. Model used for analysis of configuration D-8.

TABLE 1. INPUT/OUTPUT RELATIONSHIPS FOR CONFIGURATION D-8.

MATRIX	DIMENSION	SYMBOL	RELATION TO INPUT
Input	NIN X 1	$\overline{X}$	
Coder Output	NIN X 1	$\overline{X}_C$	$\overline{B}_C \cdot \overline{X}$
MTI Output	M X 1	$\overline{\alpha}$	$\overline{A} \cdot \overline{X}_C = \overline{A}\overline{B}_C\overline{X}$
Decoder Output	M X 1	$\overline{\beta}$	$\overline{B}_{SC} \cdot \overline{\alpha} = \overline{B}_{SC}\overline{A}\overline{B}_C\overline{X}$
LPF Output	NFFT X 1	$\overline{\gamma}$	$\overline{C} \cdot \overline{\beta} = \overline{C}\overline{B}_{SC}\overline{A}\overline{B}_C\overline{X}$
Window Output	NFFT X 1	$\overline{G}$	$\overline{W} \cdot \overline{\gamma} = \overline{W}\overline{C}\overline{B}_{SC}\overline{A}\overline{B}_C\overline{X}$

Where NIN = Total No. of Inputs, M = No. of MTI Outputs, and NFFT = No. of FFT Points.

and $\underline{2}$, are the same with the exception that D-8 contains three additional elements. They are (1) coder, (2) analog-to-digital (A/D) converter, and (3) decoder.

The A/D coverter is a non-linear device and this type of analysis is for linear systems only. Hence, the A/D converter is ignored. For many cases, the receiver noise will dominate over quantization noise. But if quantization noise is sufficient to effect processor performance, it could be applied to input of analysis as white or colored Gaussian noise.

A discussion on binary phase coding and its performance effect in the Quiet Radar is given in Reference 5. Therefore, only a few comments concerning the coder and decoder are presented. The coder is simply a multiplier used for coding the video signal (normally performed at RF prior to transmission) and the decoder is also a multiplier used to decode the video signal. The decoder provides range cell discrimination when coder and decoder have matched codes, i.e., the in-range channel. When coder and decoder have codes that are not matched, the decoder output will be a video signal modulated by a shifted version of the code, i.e., the out-of-range channel. B_C and $\overline{B}_{SC}$ are the coder and decoder matrices which are used in the covariance analysis.

It can easily be shown that

$$\overline{B}_{SC} \cdot \overline{A} \cdot \overline{B}_C = \begin{cases} \overline{A} \text{ for In-Range Channel} \\ \overline{A} \cdot \overline{B}_{SC} \text{ for Out-of-Range Channels.} \end{cases}$$

Therefore, by neglecting the out-of-range channels, the output covariance matrix equation

$$\overline{M}_3 = (\overline{W} \cdot \overline{C} \cdot \overline{B}_{SC} \cdot \overline{A} \cdot \overline{B}_C) \cdot \overline{M} \cdot (\overline{W} \cdot \overline{C} \cdot \overline{B}_{SC} \cdot \overline{A} \cdot \overline{B}_C)^T$$

reduces to

$$\overline{M}_3 = (\overline{W} \cdot \overline{C} \cdot \overline{A}) \cdot \overline{M} \cdot (\overline{W} \cdot \overline{C} \cdot \overline{A}).$$

The following is a brief description of the matrices given in Table 2 for the covariance analysis. The input covariance matrix $\bar{M}$ is given as

$$\bar{M} = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,NIN} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,NIN} \\ m_{NIN,1} & & \cdots & m_{NIN,NIN} \end{bmatrix}$$

This is a symmetric matrix and the element m_{ij} can be determined from the interference, i.e., noise or clutter correlation function, by

$$m_{ij} = R(\tau)\Big|_{\tau = \left|(i-j)\right| \cdot T}$$

Thus, only NIN elements need to be calculated.

The MTI matrix A for a two-pulse canceller is given as

$$\bar{A} = \begin{bmatrix} \underbrace{[10000 \ldots -1]}_{1XNDEL} & [\quad 0 \quad] \\ [0][10000 \ldots -1] & [\quad 0 \quad] \\ [00][10000 \ldots -1] & [\quad 0 \quad] \\ \vdots & \vdots \\ \cdots\cdots & \cdots\cdots \\ \vdots & \vdots \\ [\quad 0 \quad] & [10000 \ldots -1] \end{bmatrix}$$

where NDEL is the number of input samples before an MTI output. NDEL will be an integer multiple of the code length.

TABLE 2. COVARIANCE MATRICES RELATIONSHIPS FOR CONFIGURATION D-8

MATRIX	DIMENSION	SYMBOL	TRANSFORMATIONS
Input Covariance	NIN X NIN	$\overline{M}$	$\overline{M}_1 = (\overline{B}_{sc} \cdot \overline{A} \cdot \overline{B}_c) \cdot \overline{M} \cdot (\overline{B}_{sc} \cdot \overline{A} \cdot \overline{B}_c)^T$
Coder	NIN X NIN	$\overline{B}_c$	
MTI	M X NIN	$\overline{A}$	
Decoder	NIN X NIN	$\overline{B}_{sc}$	
LPF	NFFT X M	$\overline{C}$	$\overline{M}_3 = (\overline{W} \cdot \overline{C}) \cdot \overline{M}_1 \cdot (\overline{W} \cdot \overline{C})^T$
Window Function	NFFT X NFFT	$\overline{W}$	
Window Output Covariance	NFFT X NFFT	$\overline{M}_3$	

Where NIN = Total No. of Inputs, M = No. of MTI Outputs and NFFT = No. of FFT Points.

The LPF matrix $\overline{C}$ is given as

$$\overline{C} = \begin{bmatrix} [C_1\ C_2 \ldots C_{NFILT}] & [\quad & & 0 &] \\ [0]_{1XNSNEW} & [C_1\ C_2\ \ldots C_{NFILT}] & [& 0 &] \\ [0] & [0] & [C_1\ C_2\ \ldots C_{NFILT}] & [\ 0\ ] \\ \vdots & & & \\ [0]\ \ldots\ldots\ldots\ldots & & [0] & [C_1\ C_2\ \ldots\ C_{NFILT}] \\ & \ldots & & \end{bmatrix}$$

where the finite impulse response filter coefficients are $C_1, C_2, \ldots C_{NFILT}$ and each row initially contains (N-1) * NSNEW zeros where N is the row number and NSNEW is the sample rate reduction factor.

The window function matrix $\overline{W}$ is given as

$$\overline{W} = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & & 0 \\ 0 & & w_3 & \cdot \\ & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \\ \cdot & & & \\ 0 & & \cdots & w_{NFFT} \end{bmatrix}$$

where the diagonal elements are the window function coefficients.

Since the input covariance matrix $\overline{M}$ is an NIN x NIN matrix where NIN is typically 4000 to 6000, this prohibits use of simple matrix multiples to perform transformations. It would require an excessive amount of memory to store covariance matrix $\overline{M}$, i.e., $(4000)^2$ to $(6000)^2$ or 16 to 36 million words of memory. Therefore, it is necessary and possible to calculate one row of $\overline{W}\bullet\overline{C}\bullet\overline{A}$ and then one row of $(\overline{W}\bullet\overline{C}\bullet\overline{A})\bullet\overline{M}$ and finally calculate one row of $\overline{M}_3 = (\overline{W}\bullet\overline{C}\bullet\overline{A})\bullet \overline{M}\bullet(\overline{W}\bullet\overline{C}\bullet\overline{A})$. Then continue this process until all NFFT rows of $\overline{M}_3$ are determined. The computer analysis takes advantage of this phenomenon. Appendix A contains the program listings and input list.

IV. QUIET RADAR SYSTEM PARAMETERS

Two different configurations of the Quiet Radar D-8 Processor will be studied. The appropriate system parameters for each processor are given in Table 3.

TABLE 3. COMPARISON OF SYSTEM PARAMETERS FOR PROCESSOR #1 AND #2

PARAMETERS		PROCESSOR #1	PROCESSOR #2
Carrier Frequency	f_c	10 GHz	10 GHz
Code Length (PN Code)	NC	31 Bits	63 Bits
Sample Rate	f_s	4 MHz	5 MHz
Samples/Code Bit	NSC	2	1
MTI Delay	NDEL	62	126
LPF Length	NFILT	124 Taps	166 Taps
*LPF Wait	NWAIT	124	166
Sample Rate Reduction	NSNEW	62	83
FFT Length	NFFT	64 Points	64 Points
Look Time		1.023 msec	1.1042 msec
Weighting		Hamming	Hamming

*LPF wait represents number of input samples required before LPF output is used, i.e., NWAIT $\geq$ NFILT.

V. PERFORMANCE RESULTS

Since the analysis contains several variable parameters, e.g., signal-to-noise ratio, clutter-to-noise ratio, clutter spread, CFAR window width, etc., all possible performance results are too numerous to perform. Therefore, the analyses were performed for the parameters that have been used in previous work [2], [3].

The performance study parameter set used for both processors is given in Table 4.

The results obtained are plotted in Figure 3-14. For both processor configurations using the parameter set in Table 4, the probability of detection for several probabilities of false alarm in each of the frequency cells are shown.

For both processors, several observations can be made about the performance. The performance degrades as the number of range bins in the CFAR window decreases. This is easily explained since a better estimate of the noise is obtained by use of more range cells.

The loss of processor performance in the o^{th} frequency cell is due to attenuation of both clutter and target at oHz by the MTI. For frequency cells close to cell zero, the clutter residue out of the MTI raises the threshold, thus lowering the probability of detection. The LPF reduces the performance in the upper frequency cells due to attenuation beyond the target velocities of interest. One other obvious observation is that as the input signal-to-noise ratio level is reduced, the processor performance decreases.

It is readily observed that processor #2 performs better than processor #1. The larger dwell time for processor #2 allows for more input samples and, therefore, the low pass filter response of processor #2 can be improved over processor #1. One advantage for processor #1 is the hardware reduction possible by combining the decoder and LPF coefficient. This combination is made possible since the number of LPF coefficients is an integer multiple of the number of code bits, i.e., 124/31 = 4.

TABLE 4. LIST OF PERFORMANCE STUDY PARAMETERS

PARAMETER	VALUE
S/N (Input Signal-to-Noise)	-21, -24 dB
C/N (Input Clutter-to-Noise)	46 dB
σ_f (Clutter Spectral Width)	8 Hz
CFAR Window Width	4, 8, 16 Range Bins
P_{fA}	10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}

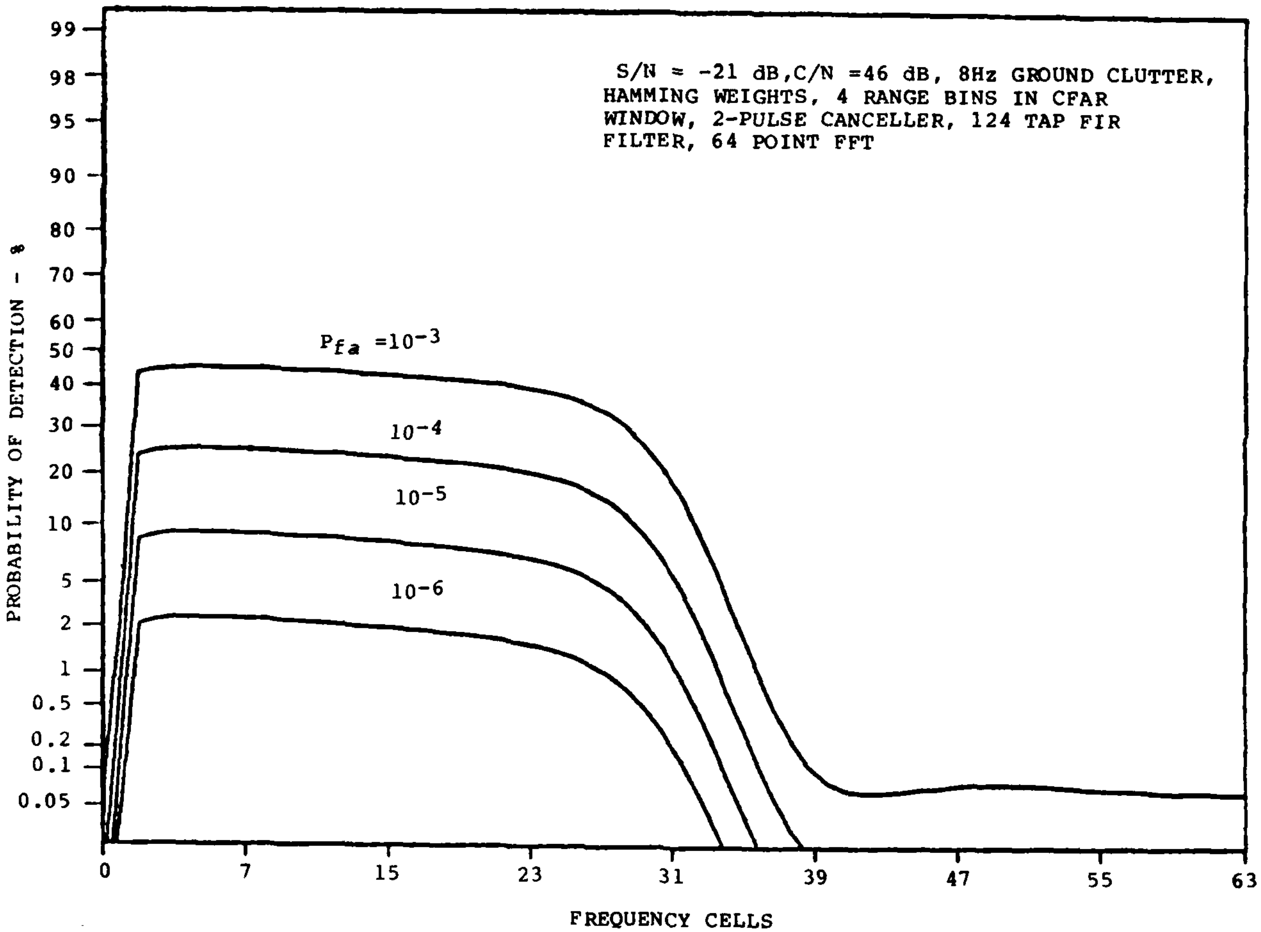

Figure 3. Detection performance for 124-tap LPF D-8 processor with S/N = -21 dB and a 4 range bin CFAR.

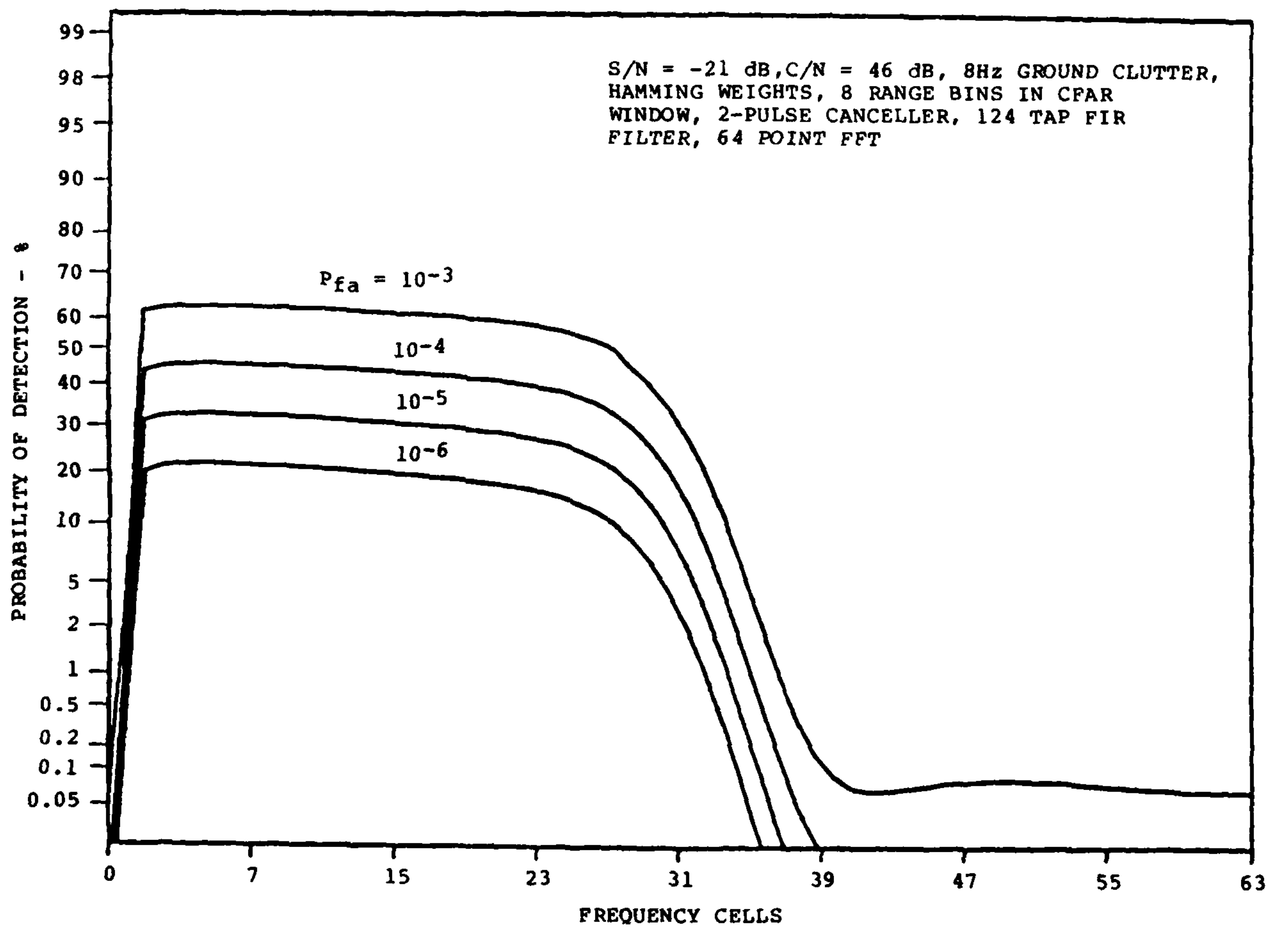

Figure 4. Detection performance for 124-tap LPF D-8 processor with S/N -21 dB and an 8 range bin CFAR.

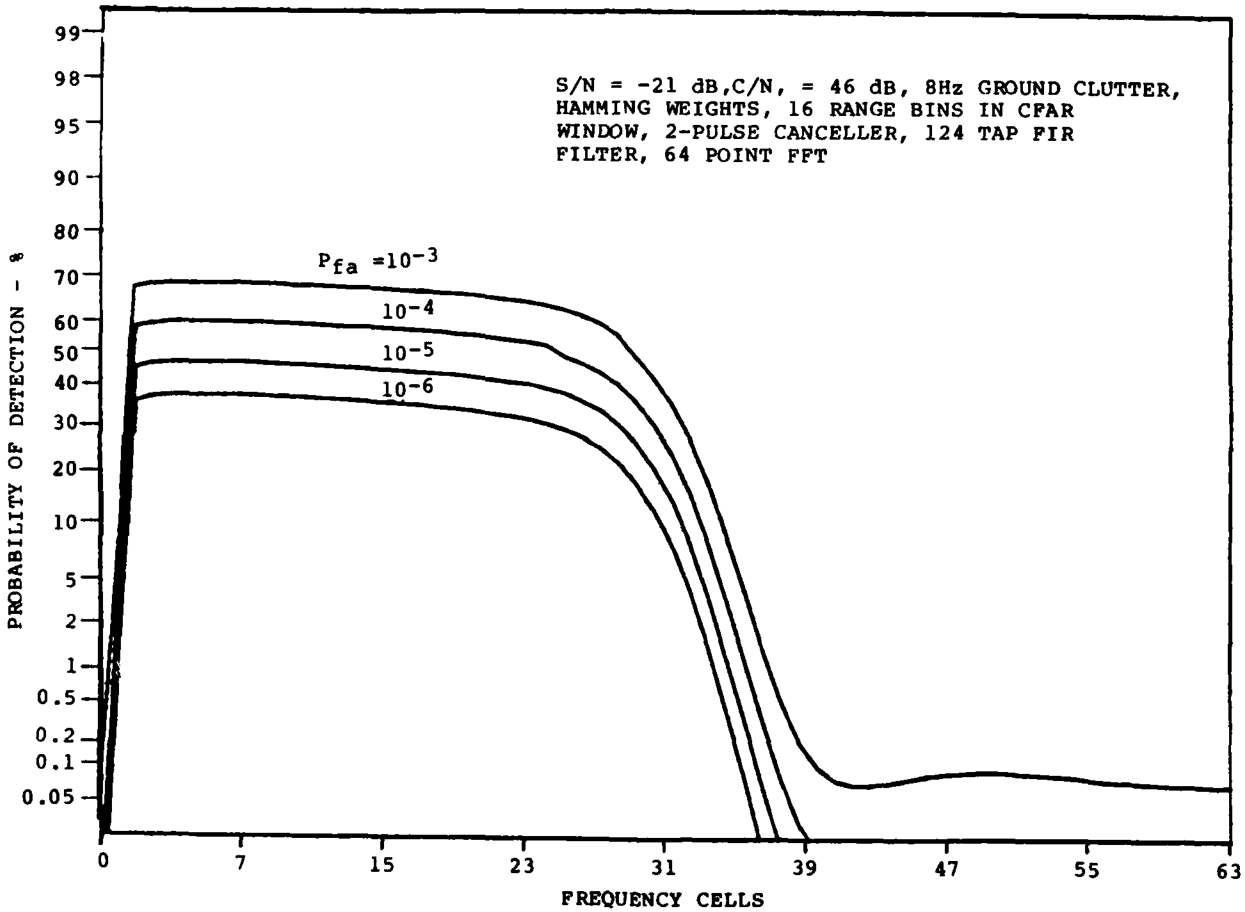

Figure 5. Detection performance of a 124-tap LPF D-8 processor with S/N = -21 dB and a 16 range bin CFAR.

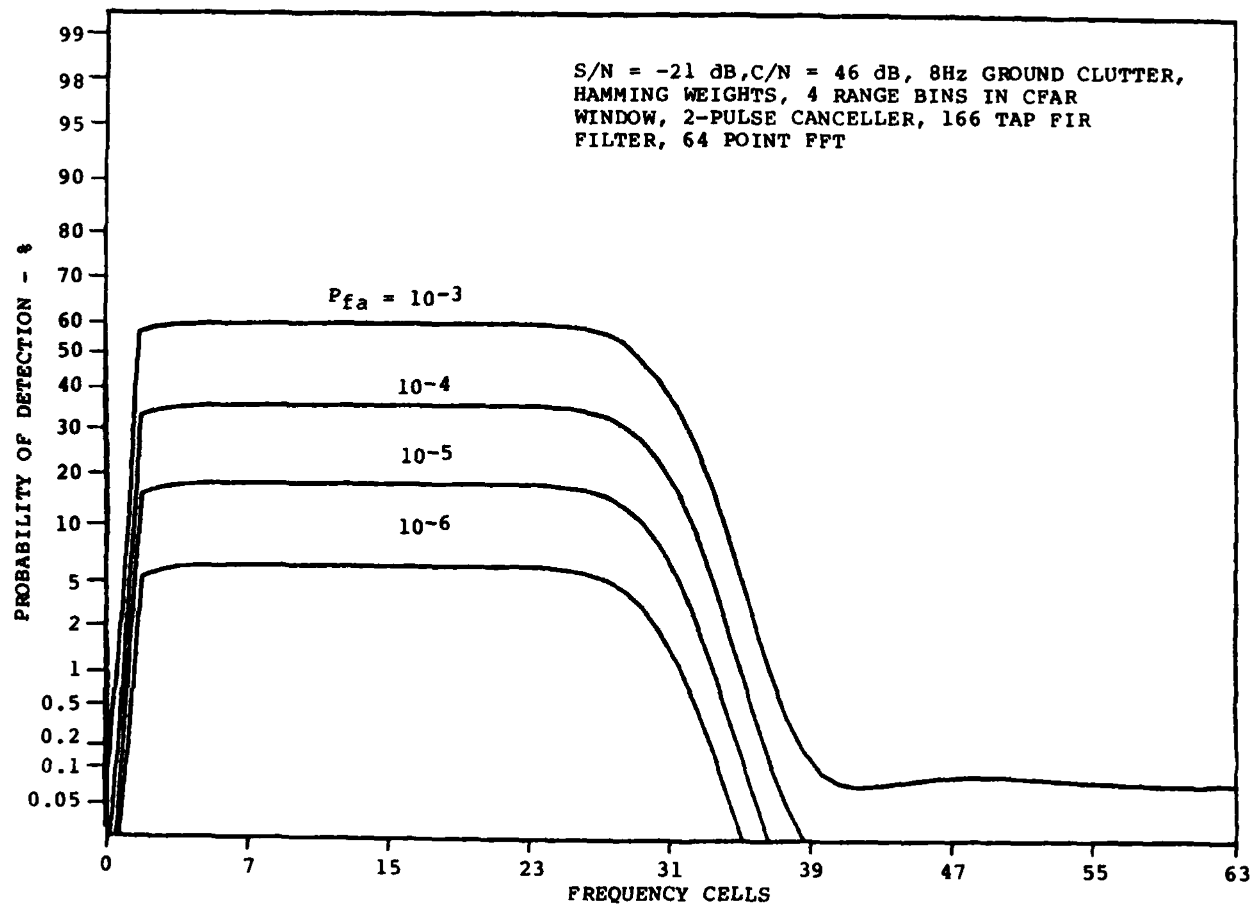

Figure 6. Detection performance of a 166-tap LPF D-8 processor with S/N = -21 dB and a 4 range bin CFAR.

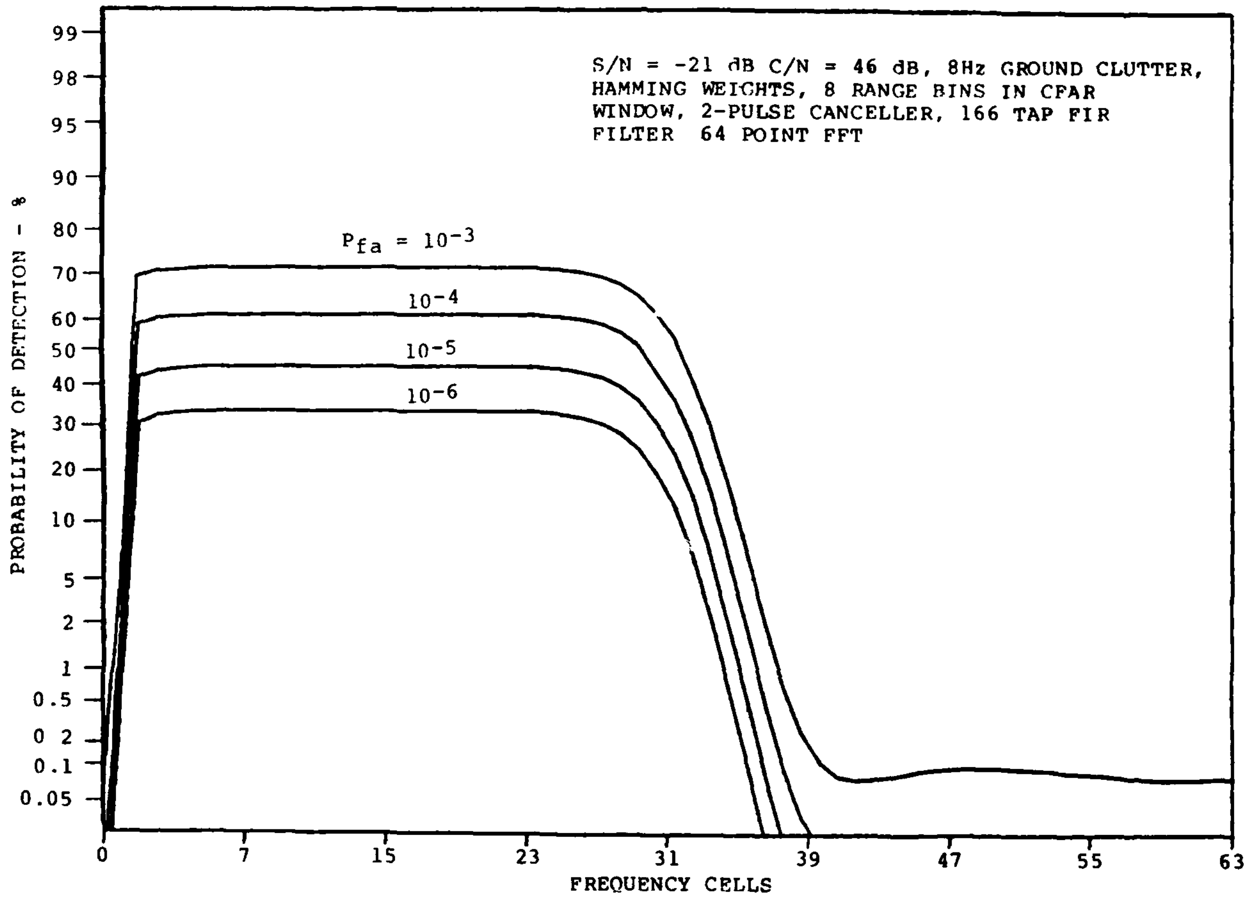

Figure 7. Detection performance of a 166-tap LPF D-8 processor with S/N = -21 dB and an 8 range bin CFAR.

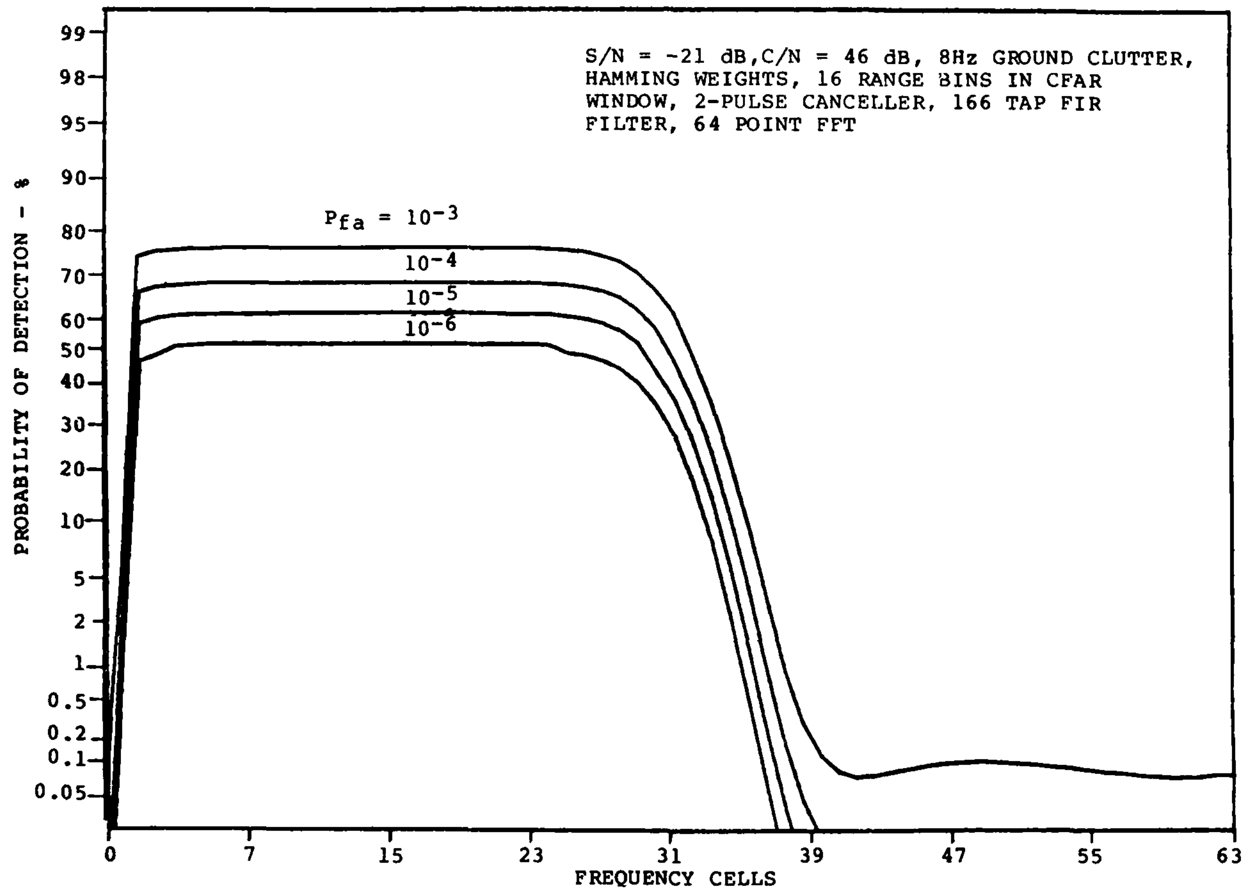

Figure 8. Detection performance of a 166-tap LPF D-8 processor with S/N = -21 dB and a 16 range bin CFAR.

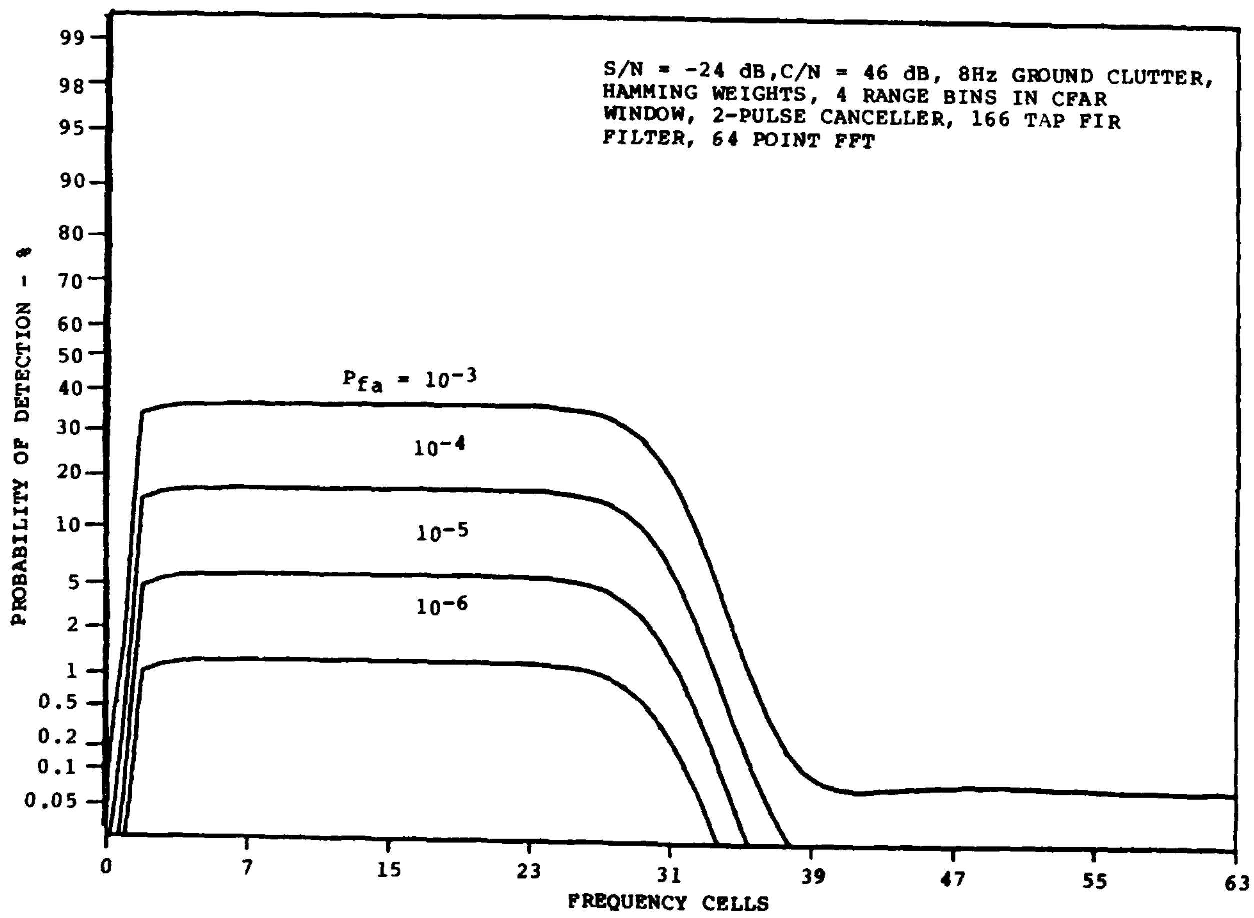

Figure 9. Detection performance of a 166-tap LPF D-8 processor with S/N = -24 dB and a 4 range bin CFAR.

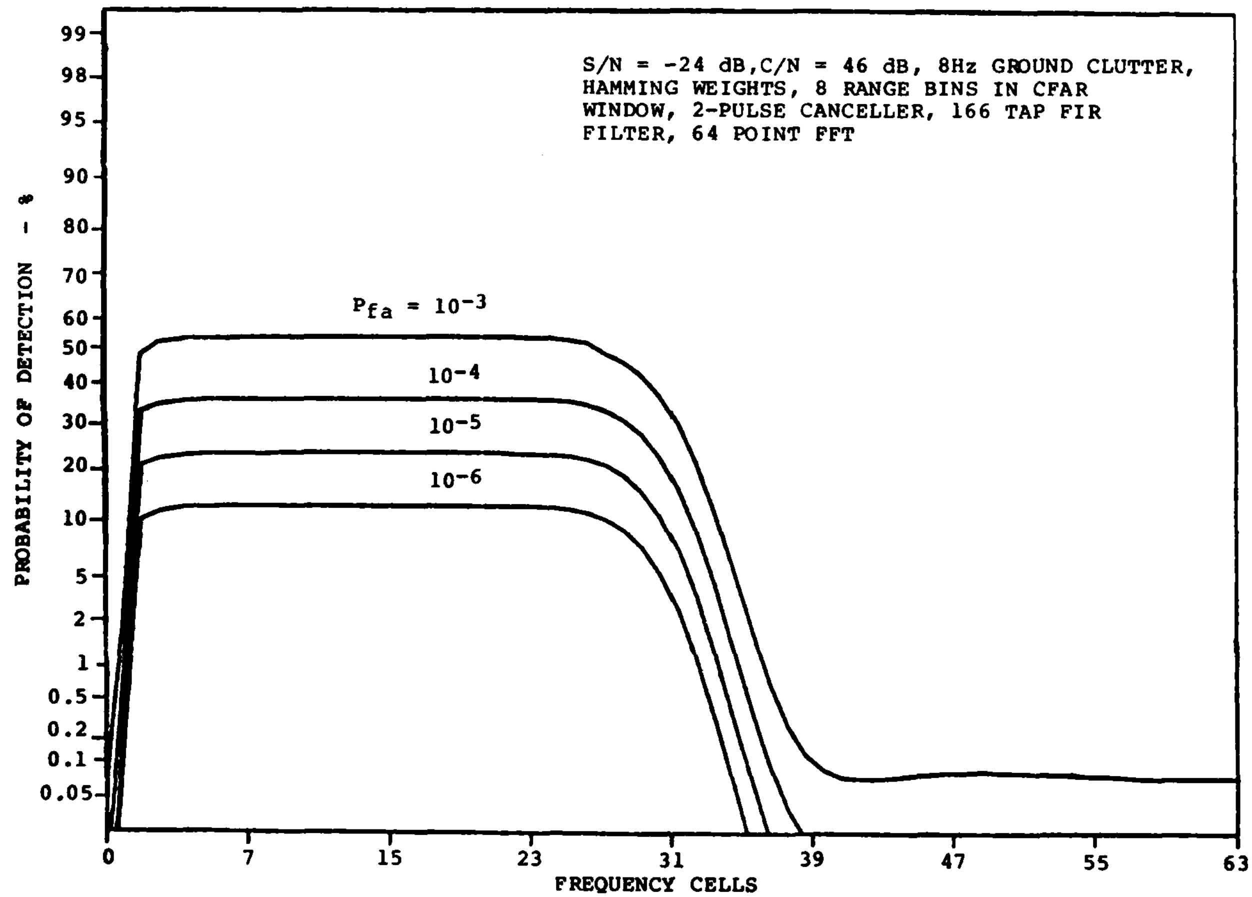

Figure 10. Detection performance of a 166-tap LPF D-8 processor with S/N = -24 dB and an 8 range bin CFAR.

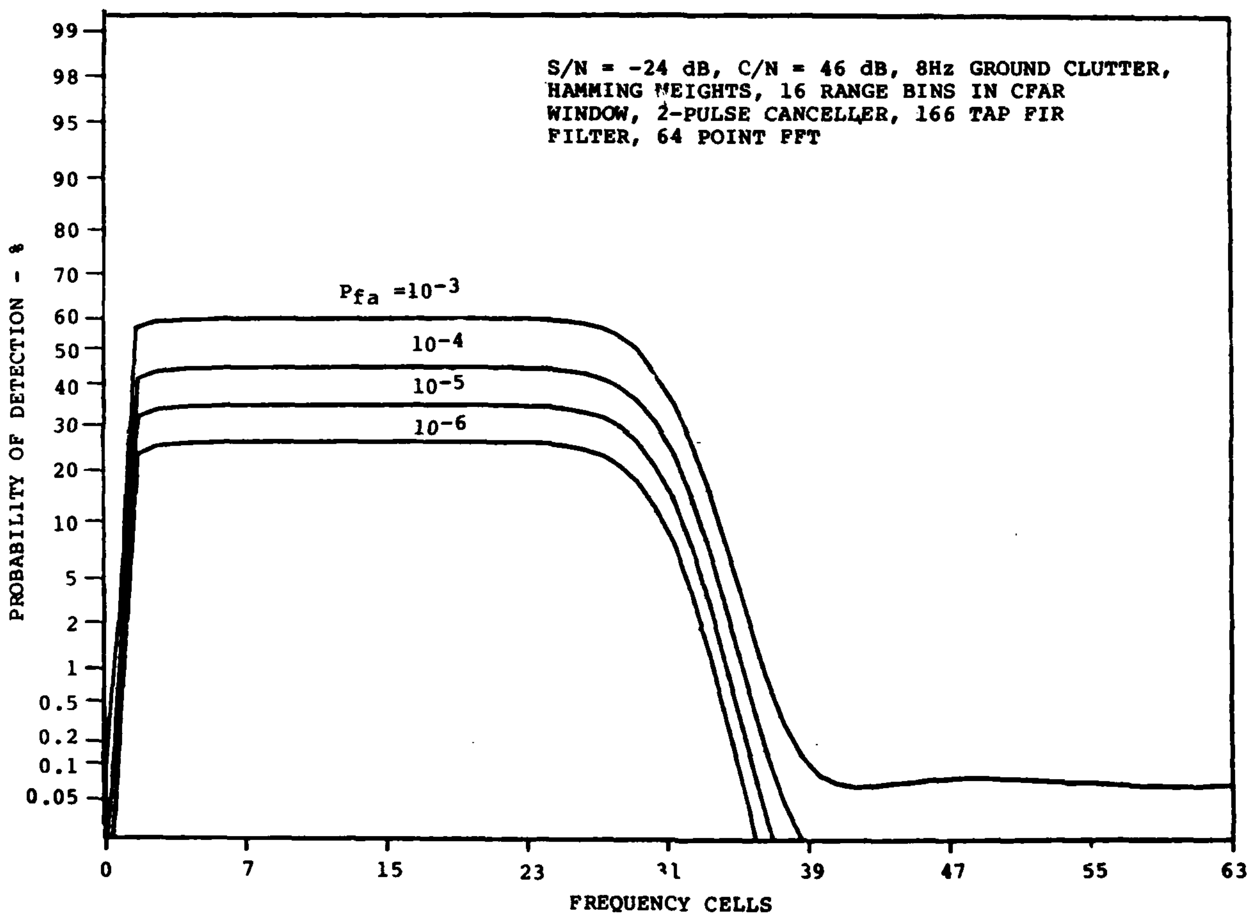

Figure 11. Detection performance of a 166-tap LPF D-8 processor with S/N = -24 dB and a 16 range bin CFAR.

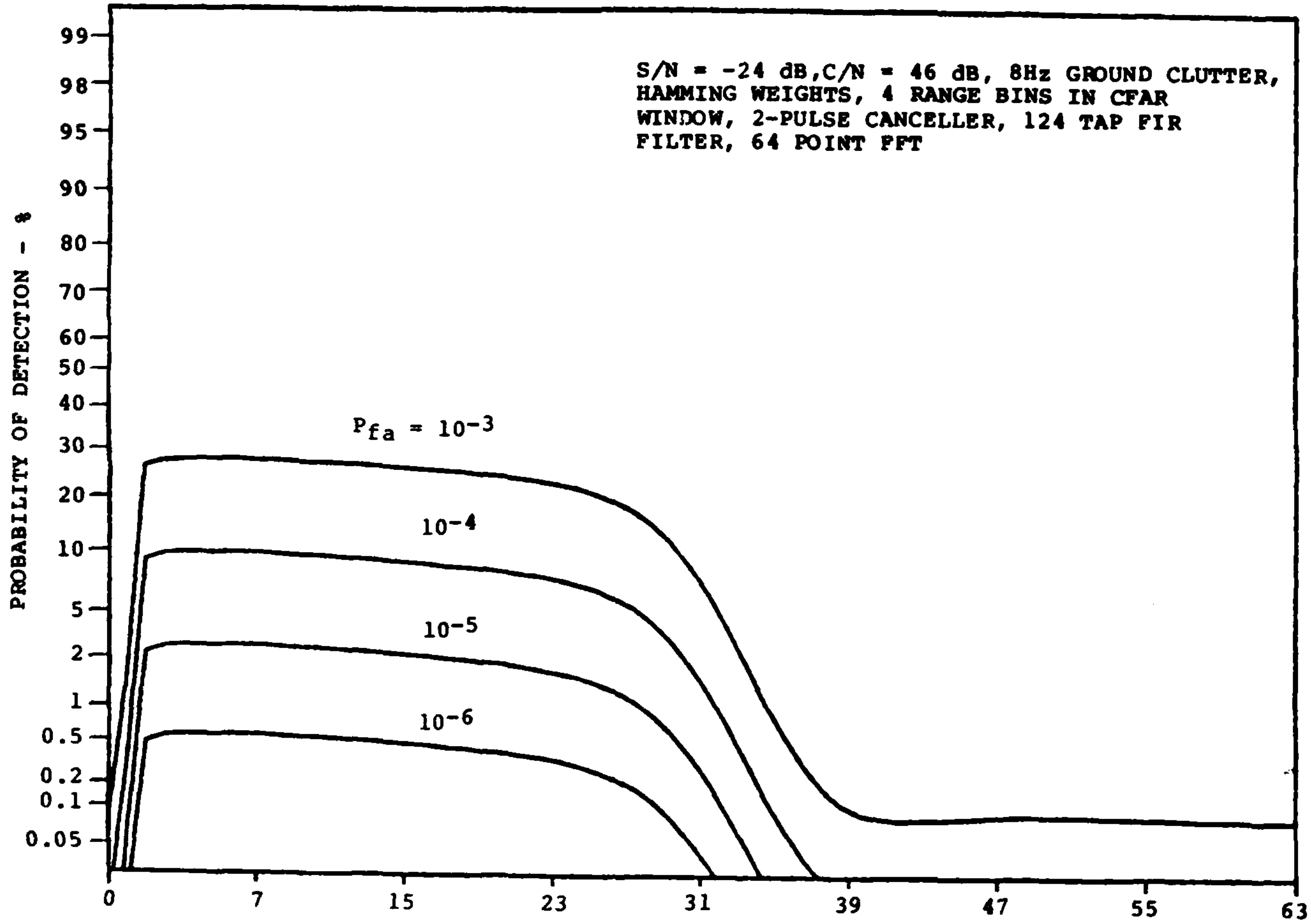

Figure 12. Detection performance of a 124-tap LPF D-8 processor with S/N = -24 dB and a 4 range bin CFAR.

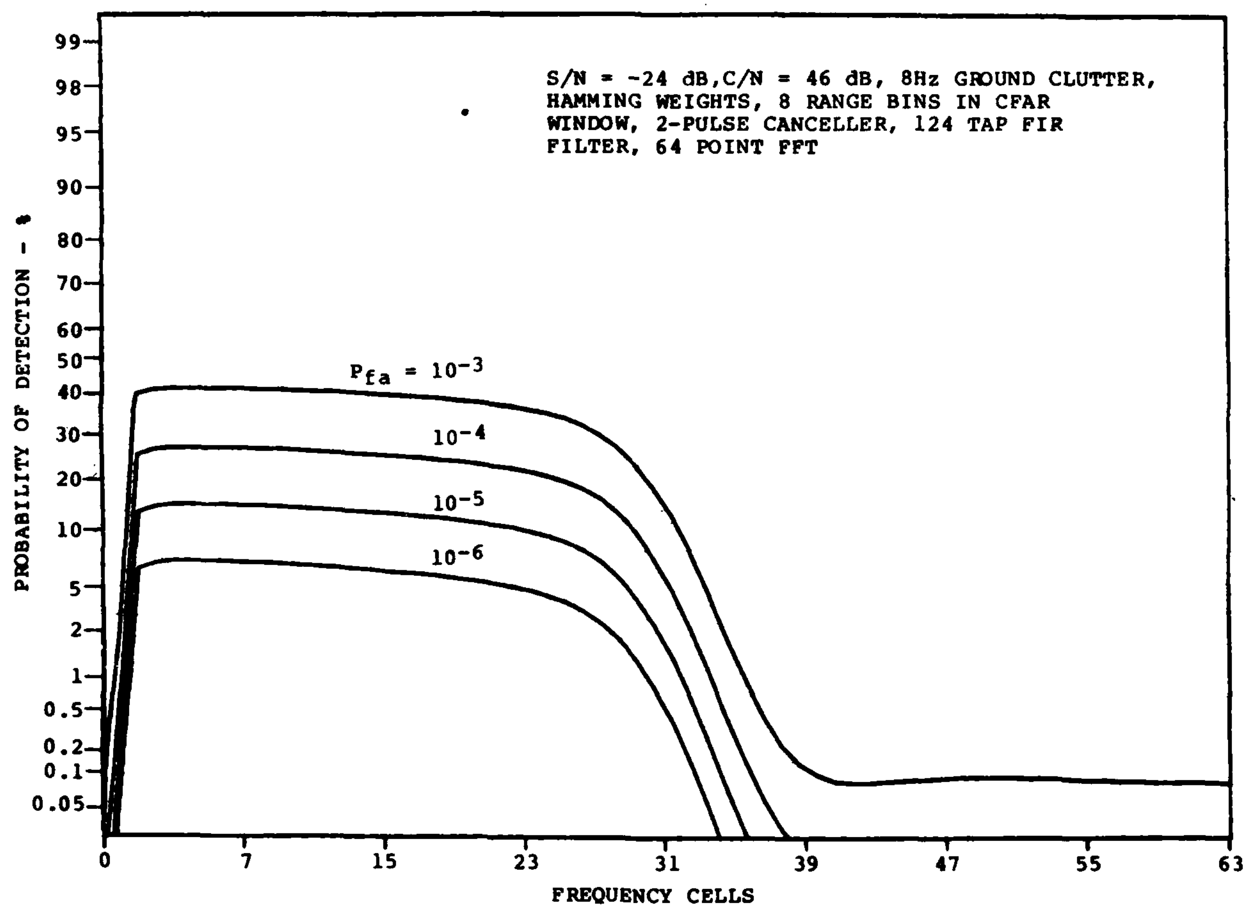

Figure 13. Detection performance of a 124-tap LPF D-8 processor with S/N = -24 dB and an 8 range bin CFAR.

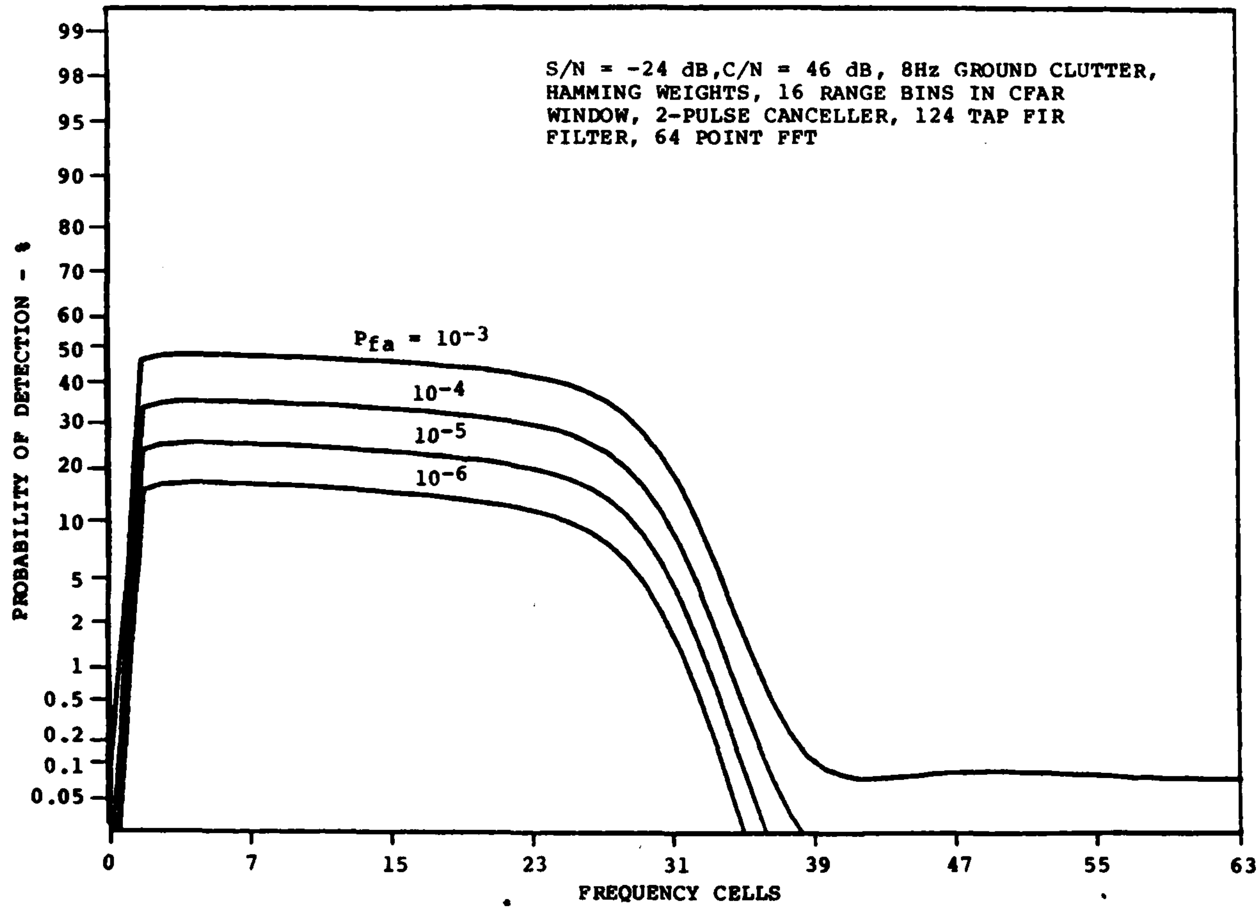

Figure 14. Detection performance of a 124-tap LPF D-8 processor with S/N = -24 dB and a 16 range bin CFAR.

APPENDIX A

PROGRAM LISTING

The development of the Quiet Radar Processor covariance analysis was performed on the AP-120B array processor and a PDP-11/10 host computer contained in the Data Acquisition and Analysis System [6].

The simulation was written in Fortran utilizing the high-speed processing capability of the AP-120B. The program contained calls to AP math library subroutines and to AP assembly language programs which simulate processor elements [2]. The input to the program is the same as the Monte Carlo simulation input in Reference 2. Hence, providing capability of ready comparison between Monte Carlo and covariance analysis results.

List of Inputs

CARD NO.	VARIABLES	FORMAT
1	NC = Length of pseudo-random code NSCLK = Number of samples per code clock period NFILT = Number of FIR filter samples NSNEW = Sampling rate reduction factor * NWAIT = Delay in FIR filter output NP = Number of two pulse canceller stages NDEL = Number of samples in MTI delay NFFT = Number of FFT samples NCINT = Number of non-coherent integration samples NFA = Number of different thresholds simulated	16I5
2	IC(I), I=1,..., NC = One period of the code (consists only of ones and zeros)	80I1
3 *	FSAMP = Sampling rate VMAX = A/D converter saturation voltage NBIT = Number of bits in A/D converter	2E15.4 I5
4 *	NOISDB = Noise power in dB SIGDB = Target return power in dB FDOP = Target doppler frequency in Hz KT = Target delay in number of samples	3E15.3 I5
5 *	DISCDB = Distributed clutter power in dB SIGMAF = Clutter spectral spread in Hz XM = Clutter DC-to-AC power ratio, m^2 A = WEIBULL parameter FIXCDB = Fixed clutter power in dB KF = Fixed clutter delay in number of samples	5E15.3 I5

List of Inputs (Concluded)

CARD NO.	VARIABLES	FORMAT
6-8	Gaussian Curve fit parameters [x(j)], [y(j)] and [z(j)] for j=1, ...,6	6E12.5
9-33 *	FIR impulse response samples. NFILT numbers, five per card.	5E16.8
34-36*	Pre-FFT weighting sequence. NFFT numbers, five per card.	5E16.8
47	ALP(I) I=1,...,NFA = Multiplication factor to control the threshold levels	5E16.8
48	NRUN(I), I=1,...,NFA = Number of Monte-Carlo trials for each threshold setting.	16I5
49 *	CFARK(I),I=1,...,NFA = Multiplication Factors to Control Range Cell Averaging CFAR Threshold Level	5E16.8
50	CFAFK(I),I=1,...,NFA = Multiplication Factors to Control Frequency Cell Averaging CFAR Threshold Level	5E16.8

*Denotes inputs used by covariance analysis.

```
C      FORTRAN PROGRAM FOR PERFORMANCE ANALYSIS
C      OF QUIET RADAR SIGNAL PROCESSOR BY
C      COVARIANCE MATRIX TECHNIQUES. THE
C      MATRIX SIZES REQUIRED USE OF AN AP-120
C      AND A PDP-11. THE MATRIX TRANSFORMATION
C      IS  OF THE FORM (FWCA).M.(FWCA)**T WHERE
C         M IS THE INPUT COVARIANCE MATRIX          NIN X NIN
C         A IS CANCELER MATRIX                       M  X NIN
C         C IS LOW PASS FILTER MATRIX              NFFT X M
C         W IS FFT WEIGHTS MATRIX                  NFFT X NFFT
C         F IS FFT MATRIX                          NFFT X NFFT
C     WHERE NFFT = NO. OF FFT POINTS
C              M = (NFFT-1)NSNEW+NFILT = INPUTS TO LPF
C            NIN = NSC*NC*K = M+NDEL=TOT. NO. OF INPUTS
C          NFILT = NO. OF FIR FILTER COEFFICIENTS
C          NSNEW = SAMPLING RATE REDUCTION FACTOR
C           NDEL = NO. OF INPUTS BEFORE MTI OUTPUT
C            NSC = INPUT SAMPLES PER CODE
C             NC = CODE LENGTH
C              K = NO. OF TRANSMISSIONS
C      TYPICAL NUMBERS
C           NFFT = 64
C              M = 4030
C            NIN = 4092
C          NFILT = 124
C          NSNEW = 62
C           NDEL = 62
C            NSC = 2
C             NC = 31
C              K = 66
C      ASSUMES DECODING COMPLETED
       COMMON/ARRAY/W(64),CR(600),CFARR(9),FILT(200),STT(64),
      *  SIGCS(64),SIGNS(64),SCALET(64)
       COMMON/CONST/NC,NFILT,NSNEW,NP,NDEL,NFFT,NIN,NFA,T,NWAIT,
      *  NOIS,SIG,DISCLT,NCFFT,A,SIGMAF,INT,N
       REAL NOIS
       CALL IOPR
       CALL AFRUN
       CALL SIGND
       CALL CFAR
       STOP
       END

       SUBROUTINE IOPR
       COMMON/ARRAY/W(64),CR(600),CFARR(9),FILT(200),STT(64),
      *  SIGCS(64),SIGNS(64),SCALET(64)
       COMMON/CONST/NC,NFILT,NSNEW,NP,NDEL,NFFT,NIN,NFA,T,NWAIT,
      *  NOIS,SIG,DISCLT,NCFFT,A,SIGMAF,INT,N
       DIMENSION WEIGHT(64)
```

```
      DIMENSION XIN(100),ALP(9),NRUN(9),CFAFK(9),IC(100)
      REAL NOIS,NOISDB
      EQUIVALENCE (XIN,ALP,NRUN,CFAFK,IC),(W,WEIGHT)
C     INPUT
      READ(5,1) NC,NSCLK,NFILT,NSNEW,NWAIT,NP,NDEL,
     1 NFFT,NCINT,NFA
      READ(5,2)(IC(I),I=1,NC)
    1 FORMAT(16I5)
    2 FORMAT(80I1)
    3 FORMAT(2E15.4,I5)
      READ(5,3) FSAMP,VMAX,NBIT
      READ(5,4)NOISDB,SIGDB,FDOP,KT
    4 FORMAT(3E15.3,I5)
      READ(5,5) DISCDB,SIGMAF,AM,A,FIXCDB,KF
    5 FORMAT(5E15.3,I5)
      READ(5,6) (XIN(I),I=13,30)
    6 FORMAT(6E12.5)
      READ(5,7) (FILT(I),I=1,NFILT)
    7 FORMAT(5E16.8)
      READ(5,7) (WEIGHT(I),I=1,NFFT)
      READ(5,7)(ALP(I),I=1,NFA)
      READ(5,8)(NRUN(I),I=1,NFA)
    8 FORMAT(16I5)
      READ(5,7)(CFARK(I),I=1,NFA)
      READ(5,1) N
      READ(5,7)(CFAFK(I),I=1,NFA)
      READ(5,1) NF
      DO 12 I=1,NFFT
      SIGCS(I)=0.
      SIGNS(I)=0.
   12 CONTINUE
      DO 17 I=1,600
      CR(I)=0.
   17 CONTINUE
      DO 10 I=1,NFILT
      CR(I)=FILT(I)
   10 CONTINUE
      T=1./FSAMP
      FACT=0.230258
      DEL=1./FSAMP
      NCFFT=NC*NFFT
      NOIS=EXP(FACT*NOISDB)
      SIG=EXP(FACT*SIGDB)
      DISCLT=EXP(FACT*DISCDB)
      NINT=NFFT
      NIN=(NINT-1)*NSNEW+NWAIT+NP*NDEL
      NMOD=MOD(NIN,NC)
      IF(NMOD.NE.0) NIN=NIN+NC-NMOD
      NCODE=NIN/NC
      DWELL=DEL*FLOAT(NIN)
```

```
C     OUTPUT
      PRINT 40,DWELL
 40   FORMAT(/10X,'PROCESSOR D-8'/
     1 10X,'DWELL TIME = ',E12.5)
      PRINT 11 ,NC,NSCLK,NP,NDEL,NFFT,N,FSAMP
 11   FORMAT(/10X,'CLOCK PERIOD = ',I5/
     1  10X,'NO. OF SAMPLES PER CLOCK PERIOD = ',I5/


     2  10X,'NO. OF PULSES CANCELLED IN MTI = ',I5/
     2  10X,'NO. OF SAMPLES IN MTI DELAY = ',I5/
     3  10X, 'NO. OF FFT SAMPLES = ',I5/
     4  10X,'NO. OF RANGE CELLS IN CFAR WINDOW = ',I5/
     5  10X,'SAMPLING RATE AT INPUT = ',E12.5,'  HZ')
      PRINT 13,NSNEW,NWAIT,NFILT
 13   FORMAT(/10X,'SAMPLING RATE REDUCTION FACTOR = ',I5/
     1  10X,'NUMBER OF TRANSIENT SAMPLES DELETED = ',I5/
     2  10X,'NO. OF FIR IMPULES RESPONSE SAMPLES = ',I5)
      PRINT 23
 23   FORMAT(/10X,'FILTER COEFFICIENTS'/)
      PRINT  24,(FILT(I),I=1,NFILT)
 24   FORMAT(5X,5E16.8)
      PRINT 14,SIGDB,NOISDB
 14   FORMAT(/10X,'TARGET RETURN POWER = ',E12.5,' DB'/
     1  10X,'NOISE POWER = ',E12.5,' DB')
      PRINT 15,DISCDB,SIGMAF,A
 15   FORMAT(/10X,'DISTRIBUTED CLUTTER POWER = ',E12.5,'  DB'/
     1  10X,'CLUTTER SPECTRAL SPREAD = ',E12.5,'  HZ'/
     2     10X,'WEIBULL PARAMETER = ',E12.5)
      PRINT 26
 26   FORMAT(/10X,'WEIGHTING COEFFICIENTS'/)
      PRINT 16,(WEIGHT(I),I=1,NFFT)
 16   FORMAT(5X,8E15.5)
C     AP CLEAR
      CALL APCLR
      CALL VCLR(0,1,32767)
      CALL APWR
      RETURN
      END

      SUBROUTINE APRUN
      COMMON/ARRAY/W(64),CR(600),CFARR(9),FILT(200),STI(64),
     *  SIGCS(64),SIGNS(64),SCALET(64)
      COMMON/CONST/NC,NFILT,NSNEW,NP,NDEL,NFFT,NIN,NFA,I,NWAIT,
     *  NOIS,SIG,DISCLT,NCFFT,A,SIGMAF,INT,N
      DIMENSION M(6000)
      REAL M,NOIS
      DATA M/1*1.,5999*0./
```

```
C       AP INITIALIZATION
        PI=3.141592654
C         GENERATING ONE ROW OF CA MATRIX
C       NP.LE.2
C       NO. OF COLS. EQUAL NP*NDEL+NFILT
        NDEL2=NDEL*2
        NDEL1=NDEL+1
        DO 60 J=1,NP
        DO 50 I=NDEL1,NDEL2
        CR(I+NDEL2)=-CR(I+NDEL)
        CR(I+NDEL)=-CR(I)+CR(I+NDEL)
        CR(I)=CR(I)-CR(I-NDEL)
   50   CONTINUE
   60   CONTINUE
C       AP DATA ENTRY
        NCM=NDEL*NP+NFILT
        NCOR=2000+NIN-1
        NCOR2=NCOR+NIN-1
        NINM=NIN-1
        NIN2=2*NIN-1
C       LOOP FOR GENERATING CLUTTER AND NOISE ARRAYS
        DO 100 INT=1,2
C       GENERATING ONE ROW OF THE M MATRIX
        IF(INT.EQ.1) GO TO 70
        ARG=2.*PI*SIGMAF*T
        DO 9 I=1,NIN
        M(I)=((-(ARG*(I-1))**2)/2.)
  9     CONTINUE
 70     CONTINUE
        CALL APCLR
        CALL VCLR(0,1,32767)
        CALL APWR
        CALL APPUT(W,1,NFFT,2)
        CALL APPUT(CR,100,NCM,2)
        CALL APPUT(M,NCOR,NIN,2)
        CALL APWD
        CALL VMOV(NCOR2,-1,2000,1,NINM)
        IF(INT.EQ.2) CALL VEXP(2000,1,2000,1,NIN2)
        CALL APWR
        CALL APEXD
        CALL APEFT
        CALL APWR
 100    CONTINUE
        RETURN
        END
        SUBROUTINE APEXL
        COMMON/ARRAY/W(64),CR(600),CFAPP(9),FILT(200),SII(64),
       *  SIGCS(64),SIGNS(64),SCALFT(64)
        COMMON/CONST/NC,NFILT,NSHEW,NF,NDEL,NFFT,NIN,NPA,T,NWAIT,
```

```
     1  NOIS,SIG,DISCLT,NCFFT,A,SIGMAF,INT,N
        REAL NOIS
        NCM=NDEL*NP+NFILT
C       AP EXECUTE
C        LOOP 20 GENERATES M3=(WCA).M.(WCA)**T
C       M3 IS AN NFFT X NFFT MATRIX

C       M3 IS STORED ROW BY ROW FROM STARTING ADDRESS 24000
        IEP=24000
        IM1=2000
        DO 20 IW=1,NFFT
C       LOOP 30 GENERATES ONE ROW OF M1 WHERE M1=(WCA).M
C       ONE ROW HAS NIN ELEMENTS
        DO 30 IML=1,NIN
        IM=NIN-IML+IM1
        IE=14000+IML-1
        CALL VMUL(100,1,IM,1,600,1,NCM)
        CALL SVE(600,1,1100,NCM)
        CALL VSMUL(IW,1,1100,IE,1,1)
        CALL APWR
   30   CONTINUE
        IM1=IM1+NSPEW
        IE=14000
C       LOOP 70 GENERATES ONE ROW OF M3 WHERE M3=(WCA).M.(WCA)**T
C       ONE ROW HAS NFFT ELEMENTS
        DO 70 IW2=1,NFFT
        CALL VMUL(100,1,IE,1,600,1,NCM)
        CALL SVE(600,1,1100,NCM)
        CALL VSMUL(IW2,1,1100,IEP,1,1)
        CALL APWR
        IE=IE+NSNEW
        IEP=IEP+NFFT
   70   CONTINUE
        IEP=24000+IW
   20   CONTINUE
        RETURN
        END
        SUBROUTINE APFFT
        COMMON/ARRAY/W(64),CH(600),CPARR(9),FILT(200),ST1(64),
       * SIGCS(64),SIGOS(64),SCALE1(64)
        COMMON/CONST/NC,NFILT,NSNEW,NP,NDEL,NFFT,NIN,NFA,1,NWAIT,
     1  NOIS,SIG,DISCLT,NCFFT,A,SIGMAF,INT,N
        REAL NOIS
        PI=3.141592654
C       PERFORMING F.M3.F**T
C       COMPUTE INTERFERENCE AFTER FFT
        NPTS=NFFT
C       GENERATING F MATRIX
C       F IS A  NFFT X NFFT MATRIX
```

```
C      F IS SEPARATED INTO SIN AND COS MATRICES
       NCOS=0
       NSIN=6000
       ZERO=0.
       CALL APPUT(ZERO,32766,1,2)
       CALL APWD
       FIN =(2.*PI)/NPTS
       DO 80 I=1,NPTS
       J=I-1
       FINC=FIN *J
       CALL APPUT(FINC,32767,1,2)
       CALL APWD
       CALL VRAMP(32766,32767,12000,1,NPTS)
       CALL VCOS(12000,1,NCOS,NFFT,NPTS)
       CALL VSIN(12000,1,NSIN,NFFT,NPTS)
       CALL APWR
       NCOS=NCOS+1
       NSIN=NSIN+1
 80    CONTINUE
C      GENERATING F**T
C      PERFORMING F.M3

       CALL MMUL(0,1,24000,1,12000,1,NFFT,NFFT,NFFT)
       CALL MMUL(6000,1,24000,1,18000,1,NFFT,NFFT,NFFT)
       CALL MTRANS(0,1,0,1,NFFT,NFFT)
       CALL MTRANS(6000,1,6000,1,NFFT,NFFT)
       CALL APWR
C      DETERMINING DIAGONAL ELEMENTS OF F.M3.F**T
C      STORE ELEMENTS AT ADDRESS 24000
       MC=0
       MS=6000
       IO=24000
       NFTC=12000
       NFTS=18000
       DO 100 I=1,NPTS
       CALL VMUL(NFTC,NPTS,MC,1,MC,1,NPTS)
       CALL VMA(NFTS,NPTS,MS,1,MC,1,MS,1,NPTS)
       CALL SVE(MS,1,IO,NPTS)
       CALL APWR
       MC=MC+NPTS
       MS=MS+NPTS
       IO=IO+1
       NFTC=NFTC+1
       NFTS=NFTS+1
 100   CONTINUE
       IF(INT.EQ.1) CALL APGET(SIGNS,24000,NFFT,2)
       IF(INT.EQ.2) CALL APGET(SIGCS,24000,NFFT,2)
       CALL APWD
       RETURN
       END
```

```
      SUBROUTINE CFAR
      COMMON/ARRAY/W(64),CR(600),CFARK(9),FIDI(200),STI(64),
     *  SIGCS(64),SIGNS(64),SCALET(64)
      COMMON/CONST/NC,NFIDI,NSHEW,DF,NDEL,NFFT,NIN,NFA,I,NWAIT,
     *  NOIS,SIG,DISCLT,NCFFT,A,SIGMAF,INI,N
      DIMENSION WCL(1000),TTT(64),STTT(64)
      DIMENSION PD(64),SUMPD(64)
      REAL NOIS
      CALL ASSIGN(3,'DK1:CFAR.FDT',0,'NEW')
      DEFINE FILE 3(200,128,U,JJ)
      CALL WBULL(DISCLT,A,WCL)
      ICI=1
      NWW=1000
9     CONTINUE
      DO 10 IC=1,NFA
      XKP=CFARK(IC)*N
      PFA=1./(1.+(XKP/N))**N
      DO 5061 K=1,NFFT
      SUMPD(K)=0.0
5061  CONTINUE
      DO 5030 NW=1,NWW
      DO 5040 K=1,NFFT
      TTT(K)=WCL(NW)*SIGCS(K)+NOIS*SIGNS(K)
      TEST=TTT(K)
      IF(TEST.EQ.0.)TEST=1.0E-10
      STTT(K)=(SIG*SCALET(K))/TEST
      PD(K)=1./(1.+XKP/(N*(1.+STTT(K))))**N
      SUMPD(K)=SUMPD(K)+PD(K)
5040  CONTINUE
5030  CONTINUE
      DO 6150 I=1,NFFT
      SUMPD(I)=SUMPD(I)/NWW
6150  CONTINUE
      WRITE(6,100)
100   FORMAT(1H1,54X,'QUIET RADAR L=N RESULTS')
      WRITE(6,101) N,XKP,PFA
101   FORMAT(//10X,'NO. OF CELLS IN CFAR WINDOW = ',I5,/
     1   10X,'CFAR THRESHOLD SCALING CONSTANT = ',E12.5/
     2   10X,'PROBABILITY OF FALSE ALARM = ',E12.5)
      WRITE(6,102)
102   FORMAT(////54X,'PROBABILITY OF DETECTION'//)
      WRITE(6,103) (I,SUMPD(I),I=1,NFFT)
103   FORMAT(54X,I5,5X,E12.5)
      WRITE(3'ICI) (SUMPD(II),II=1,NFFT)
      ICI=ICI+1
10    CONTINUE
      READ(5,7)(CFARK(I),I=1,NFA)
      READ(5,1)N
      IF(N.NE.0) GO TO 9
 7    FORMAT(5E16.8)
```

```
 1    FORMAT(16I5)
      RETURN
      END
      SUBROUTINE WBULL(DISCLT,A,WCL)
      DIMENSION WCL(1000)
      DO 10 I=1,1000
      R=RAN(0,0)
      WCL(I)=(ALOG(1./(1.-R)))**A
      WCL(I)=DISCLT*WCL(I)/2.
 10   CONTINUE
      SUM=0.
      DO 20 I=1,1000
      SUM=SUM+WCL(I)

 20   CONTINUE
      SUM=SUM/1000.
      DO 30 I=1,1000
      WCL(I)=DISCLT*WCL(I)/SUM
 30   CONTINUE
      WRITE(6,50) SUM
 50   FORMAT(/6X'MEAN OF UNSCALED CLUTTER CROSS SECTION SEQ IS 'E20.
      RETURN
      END

      SUBROUTINE SIGNO
      COMMON/ARRAY/W(64),CR(600),CFARR(9),FILT(200),STI(64),
     *  SIGCS(64),SIGNS(64),SCALET(64)
      COMMON/CONST/NC,NFILT,NSNEW,NP,NDEL,NFFT,NIN,NFA,T,NWAIT,
     *  NOIS,SIG,DISCLT,NCFFT,A,SIGMAP,INT,K
      REAL NOIS
      DIMENSION TI(64)
      NWAIT=NWAIT-1
      PI=3.14159265
C     COMPUTE FREQUENCY INCREMENTS
      FACT=1./(NFFT*NSNEW)
      ZERO=0.
C     INITIALIZATION
      CALL APPUT(ZERO,32766,1,2)
      CALL APPUT(FILT,30750,NFILT,2)
      CALL APPUT(W,30600,NFFT,2)
      CALL APWD
      NO=32000
      AR=2.*PI*FACT
C     LOOP FOR SIGNAL POWER CALCULATION
      DO 15 K=1,NFFT
      ARG=AR*(K-1)
      CALL APPUT(ARG,32767,1,2)
      CALL APWD
      CALL VRAMP(32766,32767,12000,1,NIN)
```

```
      CALL VSIN(12000,1,0,1,NIN)
      CALL VCOS(12000,1,6000,1,NIN)
      CALL MTI(0,NDEL,NP,NIN)
      CALL MTI(6000,NDEL,NP,NIN)
      CALL FIRT(0,30750,28400,NFILT,NSNEW,NWAIT,NFFT)
      CALL FIRT(6000,30750,28401,NFILT,NSNEW,NWAIT,NFFT)
      CALL WEIGHT(28400,30600,NFFT)
      CALL CFFTB(28400,29000,NFFT,1)
      CALL CVMAGS(29000,2,29600,1,NFFT)
      CALL MAXV(29600,1,NO,NFFT)
      CALL APWR
      NO=NO+1
   15 CONTINUE
      CALL APGET(SCALET,32000,NFFT,2)
      CALL APWD
C     SIGNAL,NOISE AND CLUTTER ARRAY OUTPUTS
      DO 10 I=1,NFFT
      RATI=ABS(SIGCS(I))/ABS(SIGCS(1))
      IF(RATI.LT.1.E-3) SIGCS(I)=0.
   10 CONTINUE
      WRITE(6,101)
  101 FORMAT(1H1///18X,6HSIGNAL,38X,5HNOISE,37X,7HCLUTTER/)
      WRITE(6,100) (SCALET(I),SIGNS(I),SIGCS(I),I=1,NFFT)
  100 FORMAT(13X,E16.8,27X,E16.8,27X,E16.8)
      DO 185 K=1,NFFT
      TI(K)=DISCLT*SIGCS(K)+NOIS*SIGNS(K)
      TEST=TI(K)
      IF(TEST.EQ.0.) TEST=1.E-10
      STI(K)=(SIG*SCALET(K))/TEST
  185 CONTINUE
      RETURN
      END
```

APPENDIX B

RANGE CELL AVERAGING CFAR THRESHOLD

The range cell averaging CFAR threshold is calculated

$$V_{Tk} = K \frac{1}{N} \sum_{j=1}^{N} Z_{jk} \qquad \text{(B-1)}$$

where V_{Tk} is estimated threshold

Z_{jk} is FFT square law output
N is CFAR window length
K is scale factor for a specified average probability of false alarm
j is range bin index
k is frequency cell index.

For a system as shown in Figure B-1, if I and Q are Gaussian, then Z_{jk} is an exponential distribution. Thus,

$$P(Z_{jk}) = \frac{1}{2\sigma_{jk}^2} e^{-Z_{jk}/2\sigma_{jk}^2} . \qquad \text{(B-2)}$$

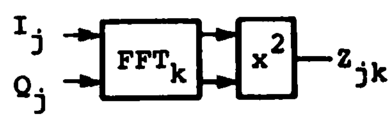

Figure B-1. Detection System.

It can be shown that, by knowing Equation (2), a probability density function can be derived for the threshold, obtained in Equation (1), and an average PFA can be found, i.e.,

$$\overline{PFA}_k = \frac{1}{\left(1 + \frac{\tau_k K}{N}\right)^N}$$

where

$$\tau_k = \frac{\sigma_{jk}^2}{\sigma_{ok}^2} = \frac{\text{variance of range cells of estimate}}{\text{variance of range cell of interest}}$$

Assume all range cells have identical noise, i,e, $\tau_k = 1$ or $\sigma_{jk}^2 = \sigma_{ok}^2$, then

$$K = N\left(\sqrt[-N]{\overline{PFA}} - 1\right).$$

For example, let $\overline{PFA} = 10^{-6}$ and $N = 4$, then

$$K = 4\left(\sqrt[-4]{10^{-6}} - 1\right) = 122.5$$

CFAR threshold constants used in the analysis are given in <u>Table B-1</u>.

TABLE B-1. CFAR THRESHOLD CONSTANTS

$\overline{PFA}$	N	K/N	K
10^{-3}	4	4.62	18.49
	8	1.37	10.97
	16	0.54	8.64
10^{-4}	4	9.00	36.00
	8	2.162	17.30
	16	0.778	12.45
10^{-5}	4	16.78	67.13
	8	3.22	25.74
	16	1.05	16.86
10^{-6}	4	30.62	122.05
	8	4.62	36.99
	16	1.37	21.94

REFERENCES

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